

# ANALYSIS AND DESIGN OF MOBILE RADIO CELLULAR SYSTEMS WITH FIXED <br> CHANNEL ASSIGNMENTS 

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The report develops a needed comprehensive procedure for analysis and design of cellular systems with fixed channel assignments for the land mobile radio service. System cost, channel access delay $T$ or blocking probability P , output signal-to-noise ratio SNR and message traffic throughput are identified as primary performance criteria. These criteria are expressed in terms of the various system parameters such as cell size and location, number of cells, vehicle spatial density, vehicle call-attempt rate, average message length, total system bandwidth, channel spacing in Hz , number of channels per cell, assignment of channels to cells, cell carrier-to-noise ratio and cost indices. A procedure for optimization of system cost subject to constrains on $T$ or $P$ and SNR by selection of the system variables is proposed. Although the analysis procedure includes adjacentchannel interference effects, the optimization procedure assumes that such effects are negligible. Co-channel interference effects are included in the optimization procedure which is relatively easy to implement for systems with limited co-channel interference or cellular symmetries. Although the present study focuses specifically on analog FM systems, the general analysis and optimization procedures apply equally well to digital systems.

Additional work is needed to obtain reliable estimates of system costs, to improve the accuracy of output SNR expressions when good diversity is not available or when adjacent-channel interference is considerable, and to consider the relative merits of other types of systems including dynamic channel assignment systems, SSMA blanket broadcast systems, and contention access systems of the ALOAH type.

A more detailed executive summary is available in Section 1-4 of the report. Problems which warrant further studied are discussed in some detail in Section 1-5.

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## I INTRODUCTION

## I-1 Cellular Systems for Mobile Communications

The continually increasing demand for communication between landbased installations and mobile units is placing an ever increasing load on communication bandwidth. Bandwidth conservation proposals include schemes whereby the radio coverage area is divided into cells each of which contains a base station operating on one or more frequency channels. When there are a sufficient number of cells (typically 25 or more) channels may be reused, i.e. a specific channel may be reused in two or more cells which are sufficiently far apart geographically. At the present time, a satisfactory comprehensive procedure for analysis, design and evaluation of cellular systems seems unavailable. The purpose of this report is to develop such a procedure.

Varying estimates exist for the potential growth in mobile communications [M1,M2,R1,R2,S1,T1]; a projected growth rate of $15 \%$ is typical. Numerous authors advocate the development of a portable personal communications terminal [M1, M2, R1,T1].

Potential users of land mobile radio services include the repair trades, law enforcement agencies, transportation and delivery organizations, professionals, realtors, insurance agents, and other salesman. Reasons for increased demand by these users include increasing energy and other mileage costs, improved service and man-hour savings. It has been estimated that three vehicles equipped with communication facilities can do the work of four with no such facilities. It has also been suggested that developments of data base technology is needed to ensure full advantage of mobile communications.

The systems considered in this report use analog frequency modulation (FM). However, the approach developed here is also applicable to digital systems, with appropriate changes for expressions involving output sig-nal-to-noise ratio.

## I-2 Cellular Systems with Fixed Channel Assignments

In cellular systems with fixed channel assignments each frequency channel is permanently assigned to one or more specific radio cells[J1, S 2 , H1,M2,N1,F1]. In systems with dynamic channel assignment strategies, some or all channels are available for assignment to any cell, provided adequate geographic separation between each channel is maintained [J1, C1, C2, A1, E1].

In comparison with fixed channel systems dynamic assignment systems offer improved flexibility to changes in cellular communications traffic and reduce call terminations resulting from cell boundary crossings. Because of the flexibility in assigning channels to cells, mobiles need not be equipped to use as many channels as in the fixed channel case [S2]. However dynamic systems require a rather complex centralized control system, and are difficult to analyze. In fact, computer simulation seems to be required for analysis as well as for design and optimization. At low traffic levels dynamic assignment allows for more efficient use of bandwidth; however fixed assignment systems are more efficient at high traffic levels [J1].

A planar area can be completely covered by identically shaped cells which do not overlap only if the cells are triangular, square or hexagonal. The latter shape has the additional advantage of being reasonably close to circular, and has the smallest number (six) of adjacent neighbours. A hex-agonal-cell system with two rings of cells for channel reuse buffering requires seven sets of channels as shown in Fig. 1-1.


Fig. 1-1 Cellular System with Seven Sets of Channels.

The system must contain at least 20 cells before reuse occurs. A square-cell system requires 25 cells, and a triangular system requires even more [S2].

What cell size should be used? Small cells are desirable for two reasons; the resulting large number of cells would permit considerable bandwidth conservation via channel reuse, and the power levels required of transmitters would be low. Reduced power levels reduce costs of mobile and base transmitters and lessen electromagnetic interferences to other systems. However small cells require improved vehicle location accuracy and increase the probability that a call-in-progress will have to be transferred to another channel because of cell boundary crossings.

We do not consider specifically the important problem of vehicle location and monitoring in this report. Various publications on this separate topic are available [01, $\mathrm{R} 3, \mathrm{R} 4, \mathrm{~J} 1]$.

## I-3 Need for Analysis, Design and Assessment Procedures

As stated in Section 1-1, a satisfactory comprehensive procedure for analysis, design and evaluation of cellular systems, including fixed assignment systems seems unavailable. Some partial procedures are available. Considerable literature dealing with output signal-to-noise ratio calculations in mobile communications are available; Chapters 1-6, inclusive, in reference [J1] provide one of the best collections of results in this area. Queueing theory results are available to calculate channel access waiting times or channel blocking probability in terms of the number of channels available and comunication traffic [J1-Ch. $7, \mathrm{~K} 1, \mathrm{~K} 2$ ]. These queueing results have been used by Schiff [S2] and Cox and Reudink [C1, C2, J1-Ch.7] to calculate blocking probability in terms of number of channels and communication traffic. There is an apparent dearth of available published information on
system costs expressed in terms of relevant system parameters.
What is lacking is a comprehensive integration of performance criteria such as system cost, output signal-to-noise ratio, blocking probability (or channel access delay) and communication traffic expressed in terms of system parameters such as number of cells, cell area and location, assignments of channels to cells, number of channels per cell, total system bandwidth, base-station and mobile carrier-to-noise ratio, vehicle callattempt rate, average message length, vehicle spatial density and cost parameters. Consequently, analysis and design of existing and proposed systems tends to be heuristic, semi-quantitative or based on one performance criterion such as blocking probability to the exclusion of other equally important criteria.

There are many reasons for developing comprehensive analysis and design procedures for fixed assignment systems at this time. Actual cellular systems, at least in their initial years of operation, would likely employ fixed channel assignments for purposes of implementation simplicity. A comprehensive analysis and design procedure would facilitate an improved understanding of fixed channel systems, which understanding would seem to be a prerequisite for the analysis and design of dynamic assignment systems. A comprehensive procedure would provide a framework for comparison of fixed assignment systems with other systems including SSMA systems which do not use cellular concepts. Final assessments and comparisons would, of course, be based on field tests.

## I-4 Summary of the Present Report (Executive Summary)

The following rather detailed summary replaces the one normally found at the conclusion of each chapter.

Chapter 1 notes that an ever increasing demand for mobile communications motivates the use of cellular systems which conserve scarce bandwidth by providing for channel reuse. Currently lacking is a satisfactory comprehensive procedure for analysis, design and evaluation of such systems.

Chapter 1 concludes with topics which have been identified as requiring further study.

Chapter 2 deals with the development of primary performance criteria, which include system cost $C$, either channel access queueing delay $T$ or probability of channel blocking $P$, output signal-to-noise ratio SNR and message-traffic throughput $Q$. Quantities $T$ and $P$ for a given cell depend on the number of channels $M_{i}$ in cell $i$ and $\rho_{i}=L \lambda_{V} \int_{A_{i}} n(x, y) d x d y$ where $\rho_{i}$ is the cell traffic, $L$ is the average message length, $\lambda_{v}$ the vehicle callattempt rate, $A_{i}$ the cell area and $n(x, y)$ the spatial vehicle density. Because of the one-to-one relation between $P$ and $T$, only one of the two measures is used.

Reliable data on system cost behaviour seems unavailable. We suggest that system cost, which excludes costs of equipment located on mobiles, depends on number of base stations $N,\left\{M_{i}\right\}$ and carrier-to-noise ratios $\left\{\mathrm{CNR}_{\mathrm{i}}\right\}$ for each cell. Total system bandwidth is easily included as an independent cost variable if desired. We propose a simple functional form for cost in terms of the above variables. However other reasonable functional forms exist; the one most appropriate would be based on actual cost data as it becomes available. If desired, the cost of mobile equipment can be added to the function without difficulty.

Calculation of $\operatorname{SNR}$ in a mobile environment is more difficult than for conventional point-to-point links because of fast Rayleigh fading, slow log-normal fading, co-channel interference, adjacent-channel interference, frequency selective fading and random FM . Under conditions of high CNR or good diversity and no adjacent-channel intereference, SNR can be calculated in terms of the modulation index, CNR, base station power ratios, radio signal attenuation factor, cell areas and reuse distances. Extension of the SNR formula to include adjacent-channel interference is considered in Chapter 5.

Chapter 2 also discusses secondary performance measures including system adaptability, sensitivity and reliability and indicates how these are related to the primary performance measures.

Chapter 3 lists the system variables and indicates how these determine system performance. The basic variables are cell area and geometry, number of cells, average message length, vehicle density, vehicle callattempt rate, overall system bandwidth relative to voice bandwidth, channel separation in Hz , radio signal attenuation factor, total number of radio channels, assignments of radio channels to cells, and cost parameters.

The way in which these variables determine the performance measures is discussed. The important but often ignored role of the modulation index in performance determination is noted and discussed further in Chapters 4 and 5. Finally, typical system design and assessment scenarios are presented. In particular, the desirability of knowing the optimum performance attainable as well as the structure of the optimum system is emphasized.

Chapter 4 has a two-fold purpose: (1) to illustrate the application of the performance criteria to the analysis of systems and to system design and optimization (2) to demonstrate the optimization of simple cellular systems, defined as systems with limited channel reuse and negligible adjacent-
channel interference. Since actual systens in early years of operation would ve simple cellular systems this chapter is of immediate interest.

Chapter 5 extends the work in Chapter 4 in two directions: (1) algorithms are developed for the performance analysis of any cellular system and for the optimization of cellular systems with regular cell patterns (2) adjacent-channel interference and residual fading-related effects are included, and the previously used results for co-channel interference are shown to be a special case of the adjacent-channel case. The need for further development of relatively simple but reasonably accurate formula for adjacentchannel and fading-related interference is demonstrated. The role of FM spectral calculations is indicated.

Chapter 5 concludes with a discussion of alternatives for programming our analysis and design procedure. When all the variables are specified, even a small hand-held programmable calculator could be used to calculate access delays, SNR, total system cost and throughput. For simple systems, the same calculator could be used for system design and optimization. However when systems with uneven cell traffic levels or different size cells are used or when adjacent-channel interference is present, design and optimization would require a more powerful computational facility: A facility with graphics capabilities would be useful and would facilitate the analysis and design of simple systems as well as more complex ones.

Chapter 6 deals briefly with alternative system structures for the land mobile radio service. The alternatives considered include digital transmission systems, cellular systems in which radio channels are dynamically assigned, SSMA systems and packet switched systems. Digital transmission is a viable alternative to analog FM transmission, and may be used in either cellular or blanket broadcast systems. Unlike analog. FM, digital transmission is also suited and in fact necessary for use in packet-switched systems. The analysis and design procedure developed in this report is applicable for
digital transmission provided expressions for output SNR are appropriately modified.

As noted in Section 1-2, dynamic assignment of channels to cells seems to reduce the required system bandwidth, at low traffic levels, below that needed for fixed assignments and may lower the number of different channels required per mobile. Whether or not these advantages outweigh the increased system control costs can be decided only after a careful analysis based on the procedures developed in this report. Difficulties in obtaining closed-form expressions for channel access waiting time indicate that extensive computer simulations would be needed for such assessments.

With SSMA each mobile is assigned a unique binary code. Communication between the mobile and existing land service is via the base station closest (in terms of CNR) to the mobile in question. The need to locate the mobile is obviated, since the base station with the strongest received signal would be the one used for communication. Further study is needed to assess the SNR performance, bandwidth required and implementation costs of SSMA.

Packet switching is ideally suited to bursty data traffic and is currently used to transmit data, particularly computer data, over voice-: grade land lines and satellite radio links. Its applicability to the mobile radio environment has yet to be fully explored. Some transmitter and receiver storage would be required to smooth fluctuations in packet access and transmission variability. If mobiles are eventually designed for both data and voice capability the use of a common packet technology could be attractive.

In summary the objective of the present work is to present a methodology for analysis and design of land mobile radio services. The methodology can be used now and can be adapted to incorporate inevitably
better understanding of cost and SNR behaviour. The methodology elucidates and integrates the inherent overall relationships between the performance criteria and the numerous system variables.

## I-5 Recommendations for Further Study

The following topics have been identified as requiring further study.

1. Reliable estimates of the dependence of system costs on parameters such as the number of base stations, number of channels per cell and cell carrier-to-noise ratio are needed. Ultimately, cost data would be approximated by functions such as those suggest in Section 2-2.
2. Additional work is needed to develop relatively simple but reasonably accurate SNR expressions for the various diversity alternatives. Inclusion of co-channel interference requires approximations for FM spectra in terms of modulation index. Any pre-emphasis effects should also be included in SNR. In those situations where the effects of frequency selective fading and random FM are significant, these effects must also be included.
3. Application of the methodology developed in this report to a detailed comparative study of simple systems would be useful. Such systems are of immediate interest. The study, which would include optimization would provide insights for the eventual design and optimization of larger systems.
4. Further study and development direct toward the implementation of programed interactive systems with graphics capabilities for the design and optimization of large systems is needed. However the ultimate usefulness of such implementations will depend on the availability of cost data and expressions for SNR.
5. The relative advantages of using digital rather than analog FM transmission should be explored. The methodology developed in this report would be appropriate with SNR expressed in terms of source encoder bit rate, CNR, data rate and overall system bandwidth.
6. Design procedures based on the concept that channel reuse applies to adjacent-channel as well as co-channel interference should be developed and used for system design and evaluation.
7. Quantitative assessments of the relative merits and shortcomings of dynamic channel assignment schemes, SSMA multiplexing and packet switching for mobile services is needed. Such a study should not be unduly delayed, otherwise the opportunity to realize these alternatives, particularly SSMA or packet switching may be lost.

II-1 Desired Attributes of System Performance Criteria

Analysis, assessment and design of systems is u1timately based on how well a system accomplishes its tasks for a given cost. Objective evaluations are greatly facilitated by quantitative performance measures, whose specification is a crucial but often slighted aspect of systems engineering.

Desirable attributes of performance criteria include relevence, simplicity, and versatility. Performance criteria should be accurate enough to adequately describe system behaviour and yet simple enough to facilitate analysis, design, assessment and optimization. Compromises between accuracy and simplicity are nearly always necessary. It is often useful to describe system behaviour using progressively more accurate, albeit more complex characterizations.

Appropriate quantative performance criteria for mobile radio voice communication systems are message throughput, monetary cost, output signal-to-noise ratio and either waiting time for channel access or channel blocking probability. As shown in Section 2-3, knowledge of one of these latter two criteria specifies the other. These four are not independent but are related as described in Chapters III and IV.

Other performance criteria which are of interest include system sensitivity to changes in various parameters, system adaptability and system versatility. These (related) secondary criteria are considered briefly in Section 2-7.

## II-2 System Cost

System cost includes costs of all base stations and associated land links as well as supporting facilities such as license fees, design costs, equipment maintenance costs, initial capital outlays, capital additions and continuing system monitoring and evaluation. Facilities for locating and monitoring vehicles as well as equipment for channel assignments also contribute to system cost. We exclude costs of communication equipment located in mobiles, since such equipment would probably be purchased and maintained by the subscriber. Mobile costs can be included if desired as explained in Section 2-8.

Exclusion of mobile equipment costs suggests at least one potential economic ineffeciency; namely the remainder of the system might not be designed to minimize total system plus subscriber equipment costs. However there are at least two offsetting trends, namely potential insistence by regulatory authorities for a cost-effective distribution of total costs, and a realization by suppliers of the mobile service that a minimal total cost to the subscriber would normally maximize subscriber demand. Offsetting this latter factor is the further realization that maximization of subscriber demand does not necessarily maximize revenue to the suppliers of the mobile service [S3].

The problem of equitable distribution of costs between base stations and mobiles is not necessarily resolved favourably by allowing or requiring that mobile facilities be provided by base station suppliers, since monopolistic or oligopolistic inequities and ineffeciencies could result,

System costs are normally classified either as capital or operating costs [H2,S3,W1]; there is normally some latitude as to what fraction of total costs should be capitalized.

Total system costs can also be represented as fixed plus variable costs [S3,W1]. Fixed costs are those incurred in the establishment of a minimal facility. Variable costs are those additional costs required to provide an improved system which results from improved signal-to-noise ratio, increasing numbers of base stations and increasing numbers of channels.

The general behaviour of variable costs is normally governed initially by, increasing returns to scale and ultimately by diminishing returns [S3,M3,M4]. The actual functional dependence of variable costs on system variables depends on the specific variable involved. Fixed and variable costs change with technology and are often unavailable, particularly in situations involving new products and services [D1, $\mathrm{H} 2, \mathrm{M} 3, \mathrm{~V} 1, \mathrm{Y} 1, \mathrm{~L} 2]$.

In those situations where precise cost data is unavailable, there is considerable merit in using simple but reasonable cost dependencies in order to establish trends and to determine those cost measurements required for further studies. Accordingly, we propose to express total system cost in terms of the number of base stations $N$, the carrier-to-noise ratio $\mathrm{CNR}_{i}$ of each cell's base stations and the number of channels $M_{i}$ in each cell. In determining the carrier-to-noise ratio we exclude from the noise co-channel and adjacent-channel interference as well as frequency selective fading and random $F M$ which arises from vehicle motion. In defining $C N R_{i}$ a nominal base station and mobile separation is assumed, and signal level fluctuations due to fading which can be reduced by use of diversity are included; thus $C N R_{i}$ is based on average signal and noise levels.

Fron the above discussion it follows that system cost is functionally expressed as follows:

$$
\begin{equation*}
\operatorname{cosT}=\mathrm{f}\left(\mathrm{~N} ; \mathrm{M}_{1}, \cdot \cdots, \mathrm{M}_{\mathrm{N}} ; \mathrm{CNR}_{1}, \cdot \cdots, \mathrm{CNR}_{\mathrm{N}}\right) \tag{2-1}
\end{equation*}
$$

It remains to specify function $f$. In doing so, we are guided by the following facts. One would expect $\operatorname{COST}$ to increase with $N$ and $M_{i}$ ( $i=1,2, . . ., N$ ) but at a decreasing rate due to economies of scale. One would also expect $\operatorname{COST}$ to increase with $\mathrm{CNR}_{\mathrm{i}}$ initially at a decreasing rate due to economies of scale but ultimatley at an increasing rate. In effect we are suggesting that the point of diminishing returns would not be reached for realistic values of $N$ and $M_{i}$ but would be reached for realistic values of $\mathrm{CNR}_{\mathrm{i}}$. If one assumes that effects of diminishing returns predominate the $\operatorname{COST}$ vs CNR behaviour then the following equation is consistent with the above assumptions, where $K_{o}$ is the fixed cost, $K_{1}$ the variable cost for $N=M_{i}=C N R_{i}=1$ and indices $a, b$ and $c$ determine the rate of change in variable costs:

$$
\left.\operatorname{CosT}=K_{o}+K_{1} N^{b} \begin{array}{c}
N  \tag{2-2}\\
i=1
\end{array} M_{i}^{a} \quad \operatorname{CNR}_{i}^{c}\right] / N
$$

The term in square brackets in (2-2) is the average variable cost per base station. If $C=\operatorname{cost} / K_{0}$ and $K=K_{I} / K_{0}$ then

$$
\mathrm{C}=1+\mathrm{KN}^{\mathrm{b}-1} \begin{align*}
& \sum_{i=1}^{N}  \tag{2-3}\\
& \mathrm{M}_{\mathrm{i}}^{\mathrm{a}} \mathrm{CNR}_{\mathrm{i}}^{\mathrm{c}}
\end{align*}
$$

where $C$ is the system cost relative to the fixed cost and $K$ is the variable cost at $N=M_{i}=C N R_{i}=I$ relative to the fixed cost.

It follows immediately that if all base stations have identical values for $\operatorname{CNR}_{i}$ and $M_{i}$ then with $C N R_{i}=\operatorname{CNR}$ and $M_{i}=m$

$$
\begin{equation*}
\mathrm{C}=1+\mathrm{KN}^{\mathrm{b}}{ }^{\mathrm{a}}(\mathrm{CNR})^{\mathrm{c}} \tag{2-4}
\end{equation*}
$$

From (2-4) and our earlier discussion we would expect $\mathrm{a}<1$, $\mathrm{b}<1$ and $c>1$.

It is also of interest to examine (2-3) when all base stations have identical values for $C N R$ and each channel is assigned to one and only one base station, all of which have an identical number of channels $M_{i}=$ $\mathrm{m}=\mathrm{M} / \mathrm{N}$. In this latter case:

$$
\begin{equation*}
\mathrm{C}=1+K N^{\mathrm{b}-\mathrm{a}} \mathrm{M}^{\mathrm{a}}(\mathrm{CNR})^{\mathrm{c}} \tag{2-5}
\end{equation*}
$$

One would normally expect $\mathrm{b}-\mathrm{a}>0$ since a cost increase accompanying an increase in the number of base stations would likely more than offset the corresponding decrease in the number of channels per base station.

In specifying the dependence of $\operatorname{COST}$ on CNR we note that various alternatives are available to obtain improved CNR , including use of diversity, higher transmitter power, or low-noise receivers. It is not unreasonable to assume that at any CNR value, the lowest-cost technique would be employed; thus the corresponding cost corresponds to the minimum system cost obtainable.

Other functions $f$ are available which conform to the general cost behaviour described above. More precise specification of cost dependence would seem to require data currently unavailable. Until such data becomes available, the above equations are proposed to ascertain cost trends, and to examine the interaction of costs with other system performance measures.

## II-3 Waiting Time and Blocking Probability

The average time $T_{o}$ that a subscriber waits before gaining access to a communication channel as well as the probability that his attempt to gain immediate access is unsuccessful are useful measures of mobile radio system performance [J1,S2].

In those situations (Erlang C[J1]) where requests for channe1 access are placed in a queue and served on a first-come-first-served (FCFS) basis by one of M channels, the normalized waiting time T for independent Poisson arrivals of messages of mean arrival rate $\lambda$ (message/sec) and exponentially distributed length with mean length $L(s e c)$ is as follows $[J 1, \mathrm{Kl}]$; where $T=T_{o} / L$ :

$$
\begin{align*}
& T=\left\{M[1-(\rho / M)]\left[1+[1-(\rho / M)] M!\sum_{k=0}^{M-1} \rho^{k-M} / k!\right]\right\}^{-1}  \tag{2-6}\\
& \rho=\lambda L \tag{2-7}
\end{align*}
$$

In those situations where calls arriving when all channels are busy are cleared (blocked) and not queued (Erlang B [J1]), the blocking probability $P$ is as follows for Poission arrivals and exponential message lengths:

$$
\begin{equation*}
P=\frac{\rho^{M}}{M!} / \sum_{k=0}^{M} \frac{\rho^{k}}{k!} \tag{2-8}
\end{equation*}
$$

It is clear from (2-6) and (2-8) that knowledge of $P$ fixes $T$ and conversely, since both $P$ and $T$ are monotonically related to both $\rho$ and $M$ provided $\rho<M$. If $\rho>M$ then the message traffic rate exceeds the system capacity and both $P$ and $T$ soon increase without bound. It must be emphasized that (2-6) and (2-8) apply only to steady state situations. Variations in $\rho$ are essumed to occur sufficiently slowly that (2-6) and (2-8) are accurate over any given time interval. Effects on $T$ and $P$ of rapid variations in $\rho$ are difficult to ascertain [K1].

It is necessary, for practical purposes to assume Poisson arrival rates and exponential message lengths in calculating $P$ and T. Fortunately, the Poisson assumption seems valid in most situations. The exponential length assumption is often not valid; however this distribution on a single
channel system gives waiting times which are longer than those for other typical distributions. Thus the Pollaczek-Khinchin formula for $T$ when $M=1$ (one channel) yields [K1]:

$$
\begin{equation*}
I=[\rho /(1-\rho)](\phi / 2) \tag{2-9}
\end{equation*}
$$

where $\phi=E\left(L^{2}\right) / L^{2}$ is the ratio of the expected value of the squared message length relative to the average length squared. Fig. 2-1 shows $\phi$ for various message length distributions $p_{\mathcal{L}}(\alpha)$ and demonstrates that $\phi$ is much larger for exponential distributions than for the others shown. Whether or not $T$ is largest for exponential message lengths on multi-channel systems is not known, since analytic results for $T$ are not available for $M>2$ and arbitrary length distributions [K1, p.19].

It is not difficult to determine the distribution $\mathrm{P}_{\mathrm{L}}(\alpha)$ in terms of the length distributions of various types of messages such as normal conversation, dispatch, and brief inquiries. Thus, if the Poisson arrival rate and length distribution for message type $i$ is $\lambda_{i}$ and $p_{L_{i}}(\alpha)$, respectively then:

$$
\begin{equation*}
P_{L}(\alpha)=\sum_{i}\left(\lambda_{i} / \lambda\right) p_{L_{i}}(\alpha) \tag{2-10}
\end{equation*}
$$

where

$$
\lambda=\sum_{i} \lambda_{i}
$$

The above equation (2-10) shows that if all messages types have exponential length distributions with different mean values then the length of the composite distribution is not exponentially distributed.

Between Erlang B and Erlang C service are an infinitude of possibilities where the user seeking access waits up to a certain time before giving up. In this situation both waiting time and blocking probability are non-zero;


Fig. 2-1: L, $E\left(L^{2}\right)$ and $\phi$ for various message length distributions.
the longer $T$ the smaller $P$ and conversely for a given value of $\rho$ and $M$. Thus, $T$ and $P$ in (2-6) and (2-8) represent upper bounds for any FCFS service discipline. Lower bounds are zero waiting time for Erlang $B$ and zero blocking probability for Erlang C.

In any call service discipline other than Erlang C, some calls will be blocked. Since some of these calls would normally be placed at a later time, the overall call-attempt rate should actually be modified to include any increase resulting from calls originally blocked.

Let $q$ be the probability that a blocked call will be attempted at a later time. Then the call-attempt rate $\lambda q$ due to both new calls and those originally blocked is:

$$
\begin{align*}
\lambda q & =\lambda\left(1+q P+(q P)^{2}+(q P)^{3}+\ldots\right) \\
& =\lambda /(1-q P) \tag{2-12}
\end{align*}
$$

Clearly $\lambda q$ increases with the product $q$ P. For $q=1$ and $P=0.1,(10 \%$ blocking with all blocked calls repeated) $\lambda q=1.11 \lambda$.

We have not considered effects of traffic level on $\rho$ except for the effects in (2-12) which arise from blocked calls. Increase in callattempt rates with the accompanying increase in $T$ and $P$ would likely have some effect on $\rho$. Thus, difficulty in obtaining a channel would probably encourage users to place only necessary calls. Offsetting this tendency to reduce $\rho$ by reducing $\lambda$ would be the increase in $\rho$ resulting from an increase in average message length $L$ caused by users holding channels unnecessarily for fear of not easily securing a channel at some later time.

The crossing of cell boundaries by vehicles also affects traffic statistics. Such boundary crossings have three effects on $\rho$, as follows:

1. Some calls terminate prematurely, decreasing L.
2. Prematurely terminated calls must be re-initiated, increasing $\lambda$.
3. When re-initiated calls are blocked, $L$ is also decreased.

The first two effects leave $\rho$ unchanged, while the third reduces $\rho$. For small values of $P$, the reduction is small, and in proportion to the vehicle boundary crossing probability which in turn is related to the vehicle velocity relative to the cell area. When calls queue, the third effect leaves $\rho$ unchanged.

Fig. 2-2 shows $T$ vs. $\rho / \mathrm{M}$ for several values of $M$. Of particular interest is the fact that an increase in both $\rho$ and $M$ which leaves $\rho / \mathrm{M}$ unchanged causes $T$ to decrease.

## II-4 Noise in a Mobile Radio Environment

In a conventional point-to-point analog AM communication system disturbed by additive white Gaussian noise of power spectral density $\mathrm{N}_{\mathrm{o}} / 2$, the output signal-to-noise ratio $S N R$ is given by the following expression where $P$ is the received information signal power and $W$ is the audio bandwidth [I1,T1]:

$$
\begin{equation*}
\mathrm{SNR}=\mathrm{P} / \mathrm{N}_{\mathrm{o}} \mathrm{~W} \tag{2-13}
\end{equation*}
$$

The bandwidth of the transmitted signal is either $W$ or $2 W$ depending on whether single or double sideband modulation is employed. If the total received signal power and carrier power is $P_{T}$ and $P_{C}$ respectively, then $P=P_{T}-P_{C}$ in (2-13) which applies to both DSB and SSB systems.

For conventional point-to-point above-threshold FM transmission of a Gaussian message in white Gaussian noise [L1, A2]

$$
\begin{equation*}
\operatorname{SNR}=(3 / 2) B^{2} \mathrm{P} / \mathrm{N}_{\mathrm{o}} \mathrm{~N} \tag{2-14}
\end{equation*}
$$



Fig. 2-2: Waiting Time $T$ vs. Per-Channel Traffic $\rho / M$
where

$$
\begin{equation*}
\beta^{2}=\overline{W_{c}^{2} / W^{2}} \tag{2-15}
\end{equation*}
$$

and $\overline{W_{c}^{2}}$ is the mean-square bandwidth of the transmitted FM signal [L1].
Thus:

$$
\begin{equation*}
\overline{W_{c}^{2}}=2 \int^{\infty}\left(f-f_{c}\right)^{2} S(f) d f / \int^{\infty} S(f) d f . \tag{2-16}
\end{equation*}
$$

where $S(f)$ is the power spectral density of the transmitted $F M$ signal. The notion of mean-square bandwidth is a useful description of FM bandwidth. The fact that an unfiltered FM signal has energy at frequencies arbitrarly distant from the carrier frequency $f_{c}$ makes any definition of FM bandwidth somewhat arbitrary.

It is seen that so long as the received power in an FM signal is above threshold, the SNR for FM systems transmitting Gaussian messages is better than that for AM systems by the factor $1.5 \beta^{2}$ where $\beta^{2}$ is the modulation index, which is alternatively referred to as the deviation ratio or bandwidth expansion factor. For non-Gaussian messages, the factor 1.5 is a good approximation to the true value which is virtually impossible to calculate precisely [LI]. The assumption that the received power in the signal from the transmitter remains above threshold is crucial, otherwise the noise, rather than the signal captures the FM receiver whose output SNR decreases very rapidly below threshold [T1, J1, H3, A3, P1]

In a mobile radio environment, SNR a given by (2-14) is often not applicable for one or more of the following reasons:

1. The radio transmission path characteristics between the mobile and base station change continuously as the mobile moves. In particular, multiple reflections of waves transmitted from a single source may interfere destructively to varying degrees. This interference is called fading and causes the received signal power $r$ to vary in intensity as the vehicle moves a few signal wavelengths in accordance with
the Rayleigh distribution $\mathrm{p}_{\mathrm{r}}(\alpha)$ as follows $[\mathrm{A} 3, \mathrm{~J} 1,02]$ :
$p_{r}(\alpha)= \begin{cases}0 & \alpha<0 \\ \left(\alpha / r_{0}\right) e^{-\alpha / 2 r_{0}} & \alpha>0\end{cases}$
The constant $r_{0}$ specifies mean of the distribution; in particular $\bar{r}=r_{0} \sqrt{\pi / 2}$ where $\bar{r}$ denotes the expected value of $r$.
2. The actual value of the mean received signal power $s=r_{0} \sqrt{\pi / 2}$ in (2-17) varies slowly with respect to wavelength as the distance between the base station and mobile changer. This slow variation is well approximated by a log-normal distribution. Thus, $\mathbf{r}_{\mathrm{o}}$ varies as $\mathrm{R}^{-\mathrm{n}}$ where $R$ is the distance between the base station and the mobile; the attenuation factor n lies between 2 and 4 [J1,02]. More specifically, the probability density $p_{s}(\alpha)$ of the mean value $s=r_{0} \sqrt{\pi / 2}$ of the received signal is [J1]

$$
\begin{equation*}
p_{s}(\alpha)=\frac{1}{\sqrt{2 \pi} \sigma} \exp \left[-\left(\alpha-\alpha_{o}\right)^{2} / 2 \sigma^{2}\right] \tag{2-18}
\end{equation*}
$$

where the expected value $\alpha_{0}$ of $s$ is

$$
\begin{equation*}
\alpha_{0}=A-10 n \log _{10}\left(R / R_{0}\right) \tag{2-19}
\end{equation*}
$$

Constant A in (2-19) depends on terrain and environment, antenna heights and gains, and carrier frequency; $R_{o}$ is a normalizing factor. Terrain and environment determines $n$ which increases with $R[A 3, B 1, H 3, J 1, K 3,02$, R5, S4].
3. Reuse of an FM channel by mobiles and base stations in geographically separated radio cells causes co-channel interference which is not normally present in conventional point-to-point communcation links. The actual level of interference depends on the geographic reuse separation, the modulation index $\beta$, the propagation factor $n$, message statistics, and the receiver structure [J1, P1, P2, M1].
4. Information transmitted on adjacent frequency channels provides another source of interference which may not occur in conventional point-to-point systems. The level of interference depends on frequency separation of the local and adjacent channe1, modulation index, geographic proximity of the adjacent channels, propagation factor and the receiver structure [J1, P1, P2,M1].
5. Because there are of ten several signal transmission paths between the mobile and base station, the interference between these paths may, at any given moment, be either destructive, constructive or neither depending on the frequency of the transmitted signal. Since the instantaneous frequency of a modultated FM signal depends on the amplitude of the message (or modulating) signal, fluctuations in received signal strength may occur even if the mobile is stationary. This effect referred to as frequency selective fading [J1, A3].
6. As a vehicle moves about the instantaneous phase of the received signal changes abruptly due to sudden changes in preferred (in terms of received energy) signal paths. The result is output output noise which is referred to as random FM [J1] which is proportional to $f_{m}^{-2}=(v / \lambda)^{-2}$ where $v$ and $\lambda$ denote, respectively, vehicle velocity and the wavelength of the transmitted radio signal. If random $F M$ is the only source of noise then in the presence of Rayleigh fading [J1, A3]

$$
\begin{equation*}
\operatorname{SNR} \simeq \mu \beta^{2} / f_{m}^{2} \tag{2-20}
\end{equation*}
$$

where $\mu$ is a constant which depends on the actual definition of system bandwidth and SNR is averaged over the received signal level. One sees that for sufficiently large vehicle velocity random FM can limit the obtainable output signal-to-noise ratio for a given value of $\beta$.

It has been shown that during vehicle motion random FM and frequency selective fading can interact to generate noise which is larger by an order of magnitude than each of the individual sources in the absence of the other [ JI ].

## II-5 Output Signal to Noise Ratio - Approaches to Analysis

The discussion in the previous section suggests that determination of the output SNR for a mobile radio channel subjected to fast Rayleigh fading, slower log-normal signal level variation, co-channel or adjacentchannel interference, frequency selective fading and random FM is no simple matter. The problem is further complicated by the following facts:

1. Calculation of co-channel or adjacent-channel interference effects leads to complex expressions for SNR even in the absence of Rayleigh fading. When there are a large number of interferers the total interference can be modelled as Gaussian noise which however is not white. The signal-to-interference ratio for this case in obtained in Section 5-4 assuming above-threshold behaviour. When there are a few interferers or a single interferer, the calculation is more difficu1t [P1, P2, C3, J1, R6, S5]. In either case however, the signal-to-interference ratio is proportional to $\beta^{3} P R / P R_{i}$ for above threshold behaviour where $P R / P R_{i}$ is the ratio of the average power in the received signal to that of the interference.
2. Calculation of co-channel and adjacent-channel interference requires knowledge of the spectral density of the transmitted FM signal. Spectral calculations for FM are tedious and nearly always involve approximations. Further, FM spectra depend on message statistics as well as $\beta$. Thus, for narrowband FM ( $\beta \approx 1$ )
the spectrum consists of an impulse at the carrier frequency plus the message spectrum [L1]. For wideband FM ( $\beta \gg 1$ ) a Gaussian message signal produces a Gaussian spectrum [L1,A2]. Actually the spectrum of any FM signal has components at virtually all frequencies.
3. Variation in received power due to both fast Rayleigh fading and slower log-normal variations of the mean signal level cause a conventional FM receiver to operate below threshold some of the time, in which case a more exact expression for SNR than (2-14) which assumes above-threshold behaviour, is needed to calculate output SNR even in the absence of co-channel and adjacent-channel interference, frequency selective fading and random FM. Analysis procedures are available but these yield complex expressions $[\mathrm{J} 1, \mathrm{P} 1, \mathrm{P} 2, \mathrm{~A} 4, \mathrm{~S} 5, \mathrm{R} 6, \mathrm{C} 3, \mathrm{D} 2]$.
4. To reduce both the primary and secondary effects of fading, diversity reception is used. Spatial diversity is the most commonly used scheme although feedback diversity or modifications thereof is also employed $[J 1, S 6, V 2, D 2, R 7]$. The design and optimization of diversity systems is itself a specialized subject area which will. not be discussed in detail here. Basically, the idea is to improve SNR on a Rayleigh fading channel by always observing the received signal at a point in space where it is not subjected to destructive self-interference; thus, two or more antennae may be placed a few wavelengths apart. Spatial diversity against shadowing of the average signal intensity may also be employed by placing directional antennas at the corners of a radio cell. Spatial transmission diversity may be used to provide a path free
of destructive multipath and shadowing.
In one sense the availability of diversity receivers further complicates the problem of calculating SNR which now depends not only on the various types of noise, but also on the detailed structure and amount of diversity employed. In another sense, however, diversity simplifies the problem of SNR calculations, as explained in the following paragraph.

The objective of diversity is to provide one path for transmission of a radio signal between mobile and base station. In the limit, diversity can virtually eliminate the effects of both Rayleigh fading of the desired signal and its secondary effects, namely random $F M$ and frequency selective fading, with the result that SNR is again given by (2-14) in the absence of co-channel and adjacent-channel interference. Of course, the received information signal power varies as $R^{-n}$ where $R$ is the mobile-base station separation. By letting $R$ equal the cell radius, the minimum value of $S N R$ is obtained for a radio cell. It follows that if co-channel interference effects are present then for good diversity and/or CNR sufficiently high to keep the signal above threshold, the minimum value of $\operatorname{SNR}$ for a cell is as follows:

$$
\begin{equation*}
\operatorname{SNR}=\left\{\left[\Gamma \sum_{i=1}^{I}\left(\frac{R}{D_{i}}\right)^{n} \frac{P_{i}}{P}\right] \beta^{-3}+\left[\frac{3}{2} \frac{\beta^{2}}{R^{n}} C N R\right]^{-1}\right\}^{-1} \tag{2-22}
\end{equation*}
$$

In (2-22) $P / P_{i}$ is the ratio of the radiated power of the signal of interest to that of co-channel interferer $i ; D_{i}$ is the geographic distance between the distant transmitter and local receiver; I is the member of co-channel interferers; $\Gamma$ is a constant which depend on $I$, the amount of fading suffered by both the signal and the interferers, and on the message signal characteristics; and CNR equals $P / N_{o} W$ as defined in (2-13) and (2-14) at a unit distance from the transmitter as in Section 2-2.

As noted in Chapter $5,(2-22)$ can be modified slightly to include ad-jacent-channel interference from both the local and distant cells.

We have not yet specifically stated whether SNR in (2-22) applies to the base-station or mobile receiver. In fact, we assume that the SNR value is the same for both, on the basis that one normally seeks to design voice communications systems with acceptable SNR performance for all users. In an actual system, the base station would probably radiate more power and use a better receiver than a mobile. The number of mobiles per base station clearly exceeds unity; consequently it would be cost effective to make mobile receivers and transmitters relatively inexpensive. Similarly CNR would be the same for both.

Another important question is the following: is the base-station or mobile more vulnerable to co-channel interference? Which is the most troublesome source of interference, mobiles or base stations?

Since base stations would normally radiate more power than mobiles, the former would be the more troublesome unless mobiles were sufficiently close to the object of their interference. Because of receiver sensitivity differences, base stations would be the more vulnerable to co-channel interference, unless the interferer were much closer to a mobile than to its associated base station.

A conservative approach based on the above arguments is to assign to $D_{i}$ in (2-22) a value equal to the distance between base stations less the radius $R$ of the local cell, and to regard $P / P_{i}$ as base-station power ratios.

The SNR as given by (2-22) is an average over the cellat radius $R$. Local variations are caused by variations in terrain, buildings, antenna heights and other environmental conditions [J1, 02, B1, K3, M5, R5, S4].

Recently computer models [ $\mathrm{B} 3, \mathrm{~L} 3, \mathrm{~S} 4$ ] have been used to predict local variations of SNR from the average.

## II-6 Message Throughput

Message throughput $Q$ for a mobile radio system is simply the product of the system call-completion rate and the average holding time or message length $L$. The call-completion rate is the product of the observed call-attempt rate $\lambda$ and $1-\mathrm{P}$ where P is the blocking probability in (2-8). Thus

$$
\begin{align*}
Q & =\lambda L(1-P) \\
& =\rho(1-P) \tag{2-23}
\end{align*}
$$

In an actual system, the observed call-attempt rate is less than the actual rate $\lambda_{a}$ by the factor $F$ which denotes the probability that the mobile is in a radio "hole", i.e. that the signal-to-noise ratio is inadequate for communication at any particular spatial location. Thus,

$$
\begin{equation*}
\lambda=F \lambda_{a} \tag{2-23}
\end{equation*}
$$

Factor $F$ can be calculated using the log-normal signal level probability. Typically, $F \simeq 0.85[J 1]$ and $\lambda L \simeq 0.03$ erlangs/mobile for public calls and $\simeq 0.004$ erlangs/mobile for dispatch messages. Since public and dispatch calls typically occur with probabilities of 0.7 and 0.3 , respectively, a value of $\lambda L \simeq 0.022$ erlang/mobile is typical. (One erlang is a load that engages one channel completely for one hour).

In systems which use an Erlang C service discipline, calls not served immediately are queued. In this case $P=0$ and $Q=\rho$. Finally, when each of $N$ cells has a total call attempt rate $\lambda_{i}$ then

$$
\begin{equation*}
Q=\sum_{i=1}^{N} \rho_{i}\left(1-P_{i}\right) \tag{2-25}
\end{equation*}
$$

where $\rho_{i}=\lambda_{i} L$ and $P_{i}$ is the blocking probability for cell $i$.

II-7 Other Performance Criteria
Performance criteria other than cost, delay (or blocking probability), output signal-to-noise ratio and information throughput are of interest. Additional relevant criteria include system sensitivity, reliability and versatility.

Versatility implies relatively easy accommodation to new situations, including incorporation of new technologies, system expansions, and applications not necessarily envisioned during the design of the initial system.

Sensitivity relates to amounts by which performance measures change in response to changes in system parameters, and is of particular interest when information such as traffic statistics and costs used as a basis of system design change rapidly and by large amounts. Such changes are characteristic of a growth industry such as mobile radio.

Reliability is perhaps the most difficult of all performance measures to quantify. Basically reliability is a measure of the likelihood that a system will operate as desired.

To some extent, these and other additional performance criteria can be evaluated by intelligent use of our basic performance criteria. Sensitivity to system parameter changes is obtainable by varying the parameters and calculating the resulting changes in performance. Effects of small parameter changes can be estimated using the linear terms in a series expansion about nominal parameter values.

Versatility can be assessed in part by considering how an existing design might be modified to meet changes in cost and traffic data. In designing a mobile system, some consideration should be given to expansion of the number of base stations with minimal disruption to those already in
place. Again, our basic performance criteria can be calculated under a variety of assumptions for comparison purposes.

Reliability is, to some extent, inherent in our definitions of SNR and $T$ (or P). However, equipment failures and channel outages are not reflected in these measures. A suitable and all-encompassing definition of reliability is not likely to be soon forthcoming.

## II-8 ConcIuding Remarks Regarding the Performance Criteria

Perhaps the most important comment is that both the equations and the variables which define our performance criteria are inexact. Unfortunately we don't really know the extent to which the equations may be in error, particularly the cost equation. Nor is it easy to ascertain the reliability of estimates for message traffic parameters, attenuation factor, or cost indices and constants.

There is clearly a need to obtain reasonably accurate descriptions of cost behaviour. Also needed is additional study regarding the cumulative effects of the various kinds of FM disturbances. This problem is of particular relevance when there are a few co-channel or adjacent-channel interferers present, in which case application of the central limit theorem is inappropriate, or when diversity available is well short of the achievable limit. We note here that system cost was assumed to be independent of system bandwidth. If regulatory agencies charge a substantial fee based on bandwidth used, bandwidth would have to be included as a cost variable. Thus, if regulatory authorities set license fees in proportion to total system bandwidth $B, K$ in (2-2) through (2-5) would be replaced by $K B$, and B would appear as an independent variable in (2-1).

Notwithstanding the limitations inherent in the primary performance criteria, they are useable as design tools, as will be demonstrated
in Chapter 4. As our knowledge of mobile radio sytems increases, so will the precise dependence on the system variables proposed for the performance Criteria, as well as the accuracy of the variables themselves. The general approach to the uses of the criteria should not change substantially.

If desired, a subset of the proposed criteria can be used. For example, one may wish to exclude cost considerations, and minimize queueing delay $T$ subject to a constraint on $S N R$, although it is difficult to conceive of designing or assessing a system without regard to its cost.

Some would argue that both cost and throughput should be normalized with respect to coverage area $A$ and total system bandwidth B. Such normalization is easily accomplished by dividing $C$ and $Q$ by BA.

To include in (2-5) the cost of mobile communication equipment, a term similar to the variable cost term in (2-2) is added where $\rho / \lambda_{v} L$ is the number of mobiles, $\widetilde{M}_{i}$ the average number of channels per mobile in cell $i$ and $C N R_{i}$ as previously defined. The value of $K, a, b$, and $c$ might, of course, be different for mobiles than for base stations. Some care is required when all channels in a given cell are not available to all mobiles in the ce11 [S2].

## III-1 System Variables

Listed below are those system variables on which mobile radio system design and assessment is based. Most of the symbols were introduced in Chapter 2. All symbols are listed here for convenience.

The symbols are listed either as: (1) specified at the outset as known quantities, (2) variables subject to selection during the design phase, or (3) variables derived from those in category (1) and (2). There is some ambiguity as to whether or not some variables should be in category (1) or (2). For example, bandwidth expansion factor $\beta$, normalized separation d between frequency channels, and reuse interval $D_{i j}$ are listed below as design variables, even though some would insist that some or all of these variables should be specified at the outset. Our contention is that specification of these variables at the outset unnecessarily constrains the final system, and that these variables should, therefore, be subject to optimization.

We have included for completeness $\mathrm{v}(\mathrm{x}, \mathrm{y})$ the vehicle velocity.
In this report we assume that its effect in altering traffic $\rho$, as explained in Section 2-3, is negligible.

1. System Variables Specified at the Outset

A - system coverage area
B - total system bandwidth
$\lambda_{v}$ - mean call-attempt rate per vehicle
L - nean call duration
$n(x, y)$ - vehicle spatial density
$\mathrm{v}(\mathrm{x}, \mathrm{y})$ - vehicle spatial velocity

W - voice bandwidth

K - normalized ratio of variable to fixed cost
a - cost exponent for number of channels
$b$ - cost exponent for number of cells in the system.
c - cost exponent for normalized carrier-to-noise ratio
$n$ - radio signal attenuation factor
2. System Variables Selected During System Design

M - total number of channels
$M_{i}$ - total number of channels in cell i
$\beta$ - ratio of root mean-square channel bandwidth to voice bandwidth (bandwidth expansion factor)
$N$ - number of cells
$A_{i}-$ area of cell $i$
d - normalized frequency separation between adjacent channels
$D_{i j}$ - geographic distance between local cell $\mathbf{i}$ and distance cell j
$\mathrm{CNR}_{i}$ - output signal to voice-band noise ratio at unit distance from the base station transmitter for cell i
$P_{i}$ - base station transmitter power in cell i
G(f) - IF filter transfer characteristic
$\Gamma(x)$ - function which defines the level of co-channel and adjacentchannel interference. In the absence of adjacent-channel interference $x=0$ and $\Gamma(0)=\Gamma$
3. Derived Variables
$\lambda_{i}-$ cell call rate

$$
\begin{equation*}
\lambda_{i}=\lambda_{v} \int_{A_{i}} n(x, y) d x d y \tag{3-1}
\end{equation*}
$$

$\rho_{i}$ - ce1l utilization factor or throughput

$$
\begin{equation*}
\rho_{i}=\lambda_{i} L \tag{3-3}
\end{equation*}
$$

$p$ - system utilization factor or throughput

$$
\begin{equation*}
\rho=\lambda L \tag{3-4}
\end{equation*}
$$

$R_{i}$ - radius of cell $i$ (distance from the centre to the farthest point on the cell)

The relationship between the cell radius and cell area depends on the shape of the cell as follows (all cells are symmetric and identical):

$$
\begin{array}{ll}
R_{i}=\sqrt{A_{i} / 3} . & \text { (hexagonal ce11s) } \\
R_{i}=\sqrt{A_{i} / 2} & \text { (square cells) } \tag{3-5b}
\end{array}
$$

The relationship between the minimum number of sets $S$ of channels, the reuse distance and the cell radius depends on cell geometry, as follows:

$$
\begin{array}{ll}
S=D^{2} / 3 R^{2}=D^{2} / A & \text { (hexagonal cells) } \\
S=D^{2} / 2 R^{2}=D^{2} / A & \text { (square cells) } \tag{3-6b}
\end{array}
$$

In the following section we establish and discuss the following relationship:

$$
\begin{equation*}
B M=B / d W \tag{3-7}
\end{equation*}
$$

## III-2 Relationship Between Bandwidth Expansion, Audio Bandwidth, Channel

 Spacing, Number of Channels and System BandwidthOf considerable interest is the relationship between the variables
listed in the heading of this section. These relationships are characterized as follows, where $\overline{W_{c}^{2}}$ is the mean square bandwidth of the transmitted signal as defined by $(2-16)$ and $\Delta$ is the actual distance in Hertz between adjacent frequency channels.

$$
\begin{align*}
& B^{2}=\overline{W_{C}^{2}} / W^{2}  \tag{3-8}\\
& \Delta M=B  \tag{3-9}\\
& \Delta=d \sqrt{W_{C}^{2}} \tag{3-10}
\end{align*}
$$

Typically, the channel separtaion is two or three times the mean square bandwidth which makes $2<d<3$.

From the above equations one obtains (3-7) which shows that given $B / W, M$ varies inversely with $\beta$. In effect, then, $\beta$ (or M) affects $C, T$ (or P) and SNR. There is an additional indirect effect when each channel is assigned to a unique cell or to a selected number of cells; namely the total number of cells $N$ is somewhat dependent on $\beta$ (and hence $M$ ) as well as on the reuse distance $\left\{D_{i j}\right\}$.

The nature and implications of these relationships are further explored in the remainder of this report.

## III-3 System Design and Assessment Scenarios

Typical system design and assessment scenarios are as follows:
Scenario 1: A system has been completely designed, and merely requires assessment.

Since all variables are specified at the outset one can immediately calculate $C, T(o r P), S N R$ and $Q$ using the equations in Chapter 2. Scenario 2: A system has been completely designed, and most variables are known. However, there is some uncertainty about some of these, in particular, there is uncertainty concerning future values of traffic level $\rho_{i}$ and cost indicies $a, b$ and $c$.

In this case one can readily determine $C, T$ (or $P$ ), $S N R$ and $Q$ and in paritcular can observe how these performance criteria depend on the possible range of values of the variables in question.

Scenario 3: Many variables are specified, but some major decisions such as the size, shape and location of the cells, as well as the assignment of Channels to cells and the number of cells is not finalized.

In this case one could calculate $C, T$ (orP), $S N R$ and $Q$ for various Competing designs in order to select the best.

In this situation two difficulties may arise. First, there may be no single scheme which is best in terms of all performance criteria. Second, undiscovered designs may be better than any of those proposed. Scenario 4: Virtually nothing is specified other than anticipated traffic levels $^{\text {and }}$ cost parameters and even these are subject to some uncertainty. The objective is to design the best system.

This last scenario is the ultimate in both challenge and importance. If one can calculate the performance of the best system, a benchmark is available against which to compare all other systems. If the
structure of the optimum system can be deduced, insight regarding implementation of the actual system is often obtained.

The usual approach to optimization with four performance criteria is to either minimize or maximize [K1] one criterion while constraining the other three. In our situation, it would be meaningful to minimize cost for a given throughput, with constraints on SNR and $T$ (or P) . The free system variables such as number, location and size of cells, and assignment of channe1s to cells would be selected to achieve optimization.

Thus, the appropriate equations which describe performance in terms of the system variables are (2-2), (2-6) for Erlang C service or (2-8) for Erlang B service, (2-22) assuming good diversity and no adjacent-channel interference, and (2-23). Equation (2-2) assumes a specific cost relationship which is often replaced in discussions by the more general relationship (2-1). In section 5-3 and 5-4 (2-22) is modified to include adjacent-channel interference.

Examination of these equations shows that a particular system Variable affects more than one performance function. Thus, $\left\{\mathrm{CNR}_{\mathbf{i}}\right\}$ affects both cost and SNR; $\left\{M_{i}\right\}$ affects both cost and $T$ (or $P$ ); $\left\{\rho_{i}\right\}$ affects both $Q$ and $T$ (or $P$ ) and, from (3-7) $\beta$ affects $S N R$ and $T$ (or $P$ ). The effects of these relationships are explored in Chapters 4 or 5 , and are seen to complicate system design and optimization.

## IV-1 Introductory Comments

In this chapter we apply previously developed ideas to the analysis and design of mobile systems having relatively simple cellular structures. Our purpose is to develop further insight into the specific ways in which the proposed performance measures interact with system variables and with each other, and to lead up to Chapter 5 which develops analysis and design procedures applicable to arbitrary cellular systems.

We regard as simple those cellular systems in which a specific radio channel is assigned to, at most, one cell (no channel reuse), or which have relatively few cells (say fewer than 10 ) and very limited reuse of channels. In addition, adjacent channel interference is absent, and diversity Is assumed sufficiently good that the received signal is always above threshold, with the result that primary and secondary effects of fading do not degrade SNR. In many cases it would be advantgeous to remove adjacent-channel interference.

## Iv-2 Single-Cell System

In a single cell system $N=1$. The performance criteria, when expressed in terms of the system variables are as follows:

$$
\begin{align*}
\mathrm{C} & =1+\mathrm{KM} \mathrm{M}^{\mathrm{a}} \mathrm{CNR}^{\mathrm{C}}  \tag{4-1a}\\
& =\mathrm{C}(1, \mathrm{M}, \mathrm{CNR})  \tag{4-1b}\\
\mathrm{SNR} & =1.5 \beta^{2} \mathrm{CNR} / \mathrm{R}^{\mathrm{n}}  \tag{4-2a}\\
& =\operatorname{SNR}(3, \mathrm{CNR})  \tag{4-2b}\\
\mathrm{T} & =\mathrm{T}(\rho, \mathrm{M})  \tag{4-3}\\
\mathrm{Q} & =\rho \tag{4-4}
\end{align*}
$$

General functional representation has been used to emphasize that much of the ensuing discussion is valid even if different specific functional relationships for cost and SNR are used. Function $T$ is specified by (2-6).

Assume for the moment that $\rho$ and $B / d W$ are specified as is cell radius $R$. If $B$ is also specified, then so is $M$, since $B M=B / d W$. In accordance with (4-4) throughput $Q$ is fixed as is waiting time $T$ in accordance with (4-3). It follows that in a single-cell system, specifying $\beta$ at the Outset defines queueing delay T , which may or may not satisfy existing constraints. With $\beta$ and $\rho$ specifled at the outset, the only way to be sure of meeting constraints on $T$ is to allow $B / d W$ to become a design variable, in which case the constraint on $T$ specifies $B / d W$ and therefore $M$. The SNR constraint fixes CNR in (4-2) which together with M determines systems cost in (4-1).

We now examine the way in which the above statements are modified if $\beta$ is not fixed at the outset but is a design variable. In this case $\beta$ and hence $M$ would be selected to meet the constraint on delay T. From (4-2) CNR would be chosen to satisfy the SNR constraint, and this choice Would determine cost $C$. By leaving $B$ to be specified as a design variable, Constraints on $T$ and SNR can be met, since CNR can normally be made suffiCiently large to keep the FM signal above threshold provided cell size is not so large that unreasonable values of $C N R$ are required. The above Statement is also based on the assumption $\beta>1$. If the number of channels required is so large that $B<1$, then either $B$ must be increased or voice bandwidth $W$ or channel spacing $d$ decreased as explained in Section 3-3.

In an actual design situation, various types of transmitters, receivers and supporting hardware and software would be considered with
different cost functions. The system which yielded acceptable values of SNR and $T$ and minimum cost would normally be selected.

## IV-3 N-Cell System Without Channel Reuse-Identical Cells

We consider next an N -cell system, some examples of which appear in Fig. 4-1. We do not specify the detailed shape of the cell, but agree that each cell has identical base station parameters CNR, R, m and $\beta$, and identical traffic throughput $\rho / \mathrm{N}$. We assume that each channel is assigned to one and only one cell, in which case $m=M / N$. The performance criteria are as follows:

$$
\begin{align*}
\mathrm{C} & =1+\mathrm{K} \mathrm{~N}^{\mathrm{b}-\mathrm{a}} \mathrm{M}_{\mathrm{CNR}}{ }^{\mathrm{c}}  \tag{4-5a}\\
& =\mathrm{C}(\mathrm{~N}, \mathrm{~m}, \mathrm{CNR})  \tag{4-5b}\\
\mathrm{SNR} & =1.5 \beta^{2} \mathrm{CNR} / \mathrm{R}^{\mathrm{n}}  \tag{4-6a}\\
& =\operatorname{SNR}(\beta, \mathrm{CNR}, \mathrm{R})  \tag{4-6b}\\
\mathrm{T} & =\mathrm{T}(\rho / \mathrm{N}, \mathrm{~m}) \tag{4-7}
\end{align*}
$$

where function $T$ is specified by (2-6) and

$$
\begin{equation*}
Q=\rho \tag{4-8}
\end{equation*}
$$

For a given value of N and R , the comments made regarding a single-cell system also apply to the present case. In particular, freedom to select $\beta$ enables constraints on $T$ and SNR to be satisfied and cost to be uniquely determined. If $\beta$ is specified at the outset, however, it may be necessary to increase $B / d W$ beyond its initially specified value to ensure that constraints on $T$ can be met.

Cell areas decrease as $N$ increases, as does traffic per
Cell $\rho / \mathrm{N}$. The actual shape of the cell would determine the nominal value of $R$ as well as the need for antennas with some directivity. As $N$ increases, $R$ decreases and so does the required value of CNR required to meet SNR constraints as indicated by the explicit dependence of SNR on $R$ in (4-6b).

This CNR decrease tends to decrease the per-base-station cost, as does the decrease in $m$ which occurs because of the per-cell decrease in traffic which accompanies an increase in $N$. Offsetting these two effects is the increase in total cost due to increasing $N$. Knowledge of the functional dependence of cost on $N, m$ and CNR permits selection of the value of $N$ which minimizes cost.

## IV-4 N-Ce11System Without Channel Reuse-Arbitrary Cells

The situation here is similar to the previous one, except that the geographic location, area $A_{i}$ and shape of the cells are arbitrary. Thus, cell traffic $\rho_{i}$, nominal cell radius $R_{i}$ and number of channels $M_{i}$ will not normally be identical for all cells.

The performance criteria are as follows:

$$
\begin{align*}
C & =1+K N^{b-1} \sum_{i=1}^{N} M_{i}^{a} \mathrm{CNR}_{i}^{c}  \tag{4-9a}\\
& =C\left(N, M_{1}, M_{2}, \ldots, M_{N}, \mathrm{CNR}_{1}, \ldots, \mathrm{CNR}_{N}\right) \tag{4-9b}
\end{align*}
$$

$\operatorname{SNR}=1.5 \beta^{2} C N R_{i} / R_{i}^{n}$
$=\operatorname{SNR}\left(\beta, \operatorname{CNR}_{i}, A_{i}\right)$
(i $=1,2, \ldots, N$ )
$T=T\left(\rho_{i}, M_{i}\right) \quad(i=1,2, \ldots, N)$
$Q=\sum_{i=1}^{N} \rho_{i}$
We assume that $B / d W$, and $\lambda_{v} \operatorname{Ln}(x, y)$ are specified at the outset. Thus $N, B$ and $\left\{A_{i}\right\}$ are design variables subject to the constraint

$$
\begin{equation*}
A=\sum_{i=1}^{N} A_{i} \tag{4-13}
\end{equation*}
$$

The dependence of nominal radius $R_{i}$ on $A_{i}$ is indicated in the transition from (4-10a) to (4-10b).

Design proceeds as follows:

1. Select N
2. Specify $\left\{A_{i}\right\}$ subject to (4-13). Calculate $\left\{\rho_{i}\right\}$ from $\rho_{i}=\left[\lambda_{v} \int_{A_{i}} n(x, y) d x d y\right] L$.
3. Determine $\left\{M_{i}\right\}$ required to meet the constraint on $T$ for each cell. The fact that ( $4-11$ ) consists of $N$ uncoupled equations, where the $i^{\text {th }}$ equation includes on $1 y \rho_{i}$ and $M_{i}$ as variables make determination of $\left\{M_{i}\right\}$ relatively easy.
4. Determine $M=\sum_{i=1}^{N} M_{i}$ and $\beta=B / M d W$
5. Determine $\left\{\operatorname{CNR}_{1}\right\}$ from the SNR constraints (4-10). Again, separability of these $N$ equations makes solution easy.
6. Determine cost from (4-9). If an analytical expression for cost is unavailable, cost can still be determined using catalogue estimates since all system variables are known.
7. To optimize the design with respect to $\left\{A_{i}\right\}$ repeat the above algorithm using different sets of $\left\{A_{i}\right\}$.
8. To optimize the design with respect to $N$, repeat the above algorithm for various values of N .

If $\beta$ is specified at the outset, then the total number of available channels $M$ is fixed, with the result that there may be an insufficient number of channels available to meet the delay constraints. In this case, the best that can be done for a given $\left\{A_{i}\right\}$ and $N$ is to allocate channels to cells in such a way that $T$ is equal for all cells. The procedure for meeting SNR constraints and determining cost is unchanged. The importance of B as a design variable is again evident.

Finally, we note that the above optimization algorithm can be applied for various numerical constraint values on SNR and $T$ as well as for
various values of system parameters such as $\rho, B / d W$ and cost indices. In this way system cost can be obtained vs $T$, SNR, throughput and selected system parameters.

## IV-5 5-Cell System with Channel Reuse - Symmetrical Cells

The cellular pattern labelled potential cellular reuse in Fig. 4-1 is among the simplest cellular reuse schemes suitable for traffic distributed over two spatial dimensions.

We assume spatial symmetry such that the area and traffic ( $A_{2}$ and $\rho_{2}$ respectively) of each of the outer cells is identical. The area $A_{1}$ of the inner cell is then obtained from

$$
\begin{equation*}
A=4 A_{2}+A_{1} \tag{4-14}
\end{equation*}
$$

It follows that

$$
\begin{equation*}
Q=4 \rho_{2}+\rho_{1} \tag{4-15}
\end{equation*}
$$

Consider a reuse scheme such that the north and south cells use the same $M_{2}$ frequency channels and the east and west cells also reuse a different set of $M_{2}$ channels. If the centre cell uses $M_{1}$ channels then

$$
\begin{equation*}
M=M_{1}+2 M_{2} \tag{4-16}
\end{equation*}
$$

The performance criteria for this case are as follows:

$$
\begin{align*}
& \mathrm{C}=1+\mathrm{KN}^{\mathrm{b}-1}\left[\mathrm{M}_{1}{ }^{\mathrm{a}} \mathrm{CNR}_{1}{ }^{\mathrm{c}}+4 \mathrm{M}_{2}{ }^{\mathrm{a}} \mathrm{CNR}_{2}{ }^{\mathrm{C}}\right]  \tag{4-17a}\\
& =\mathrm{f}\left(\mathrm{~N}, \mathrm{M}_{1}, \mathrm{M}_{2}, \mathrm{CNR}_{1}, \mathrm{CNR}_{2}\right)  \tag{4-17b}\\
& \operatorname{SNR}=1.5 \beta^{2} \mathrm{CNR}_{1} / \mathrm{R}_{1}{ }^{\mathrm{n}} \\
& =\operatorname{SNR}_{1}\left(B, \mathrm{CNR}_{1}, \mathrm{~A}_{1}\right) \text { (for all inner cell channels) }  \tag{4-18a}\\
& \left.\mathrm{SNR}=\{\mathrm{T}\}^{-3}\left[\mathrm{R}_{2} /\left(2 \mathrm{R}_{1}+\mathrm{R}_{2}\right)\right]^{\mathrm{n}}+\left[1 \cdot 5 \beta^{2} \mathrm{CNR}_{2} / \mathrm{R}_{2}{ }^{\mathrm{n}}\right]^{-1}\right\}^{-1} \\
& =\operatorname{SNR}_{2}\left(B, \mathrm{CNR}_{2}, \mathrm{~A}_{1}, \mathrm{~A}_{2}\right) \text { (for all outer cell channels) }  \tag{4-18b}\\
& T=T\left(\rho_{i}, M_{i}\right) \quad(i=1,2)  \tag{4-19}\\
& Q=o_{1}+4 \rho_{2} \tag{4-20}
\end{align*}
$$



Fig. $\mathrm{A}_{-1}$ Simple Cellular Systems (a) Single-Cell System (b) 3-Cell System (c) 5-Cell System Without Channel Reuse (d) 5-Cell System with Reuse Potential.

The dependence of $R_{i}$ on $A_{i}$ has been recognized in (4-18) where $D_{i j}=2 R_{1}+R_{2}$. Although SNR for the four outer cells now depends on the nominal radius $R_{1}$ of the inner cell, the solution procedure described in the previous section is directly applicable with $N=5$ and with obvious simplifications resulting from the present outer cell symmetries. Thus, one selects $A_{1}$ and $A_{2}=A_{3}=A_{4}=A_{5}$, which fixes $\rho_{1}$ and $\rho_{2}=\rho_{3}=\rho_{4}=\rho_{5}$. The constraint on $T$ then determines the number of channels per cell, as well as $\beta$. If $\beta$ is specified at the outset, then meeting the queueing delay constraints may require expansion of the system bandwidth; otherwise one may have to be content with selecting $M_{1}$ and $M_{2}$ to equalize delays in each cell. Once $\beta$ is known, $\mathrm{CNR}_{1}$ and $\mathrm{CNR}_{2}$ are obtained from (4-18), and cost is then obtained from (4-17).

## IV-6 5-Ce11 System with Channel Reuse-Arbitrary Ce11s

We now relax the symmetry considerations in the previous section, and number the cells as in Fig. 4-1. Again we assume that cells 2 and 4 may use identical frequency channels, that cells 3 and 5 may use a different set of identical channels, and that the number of channels in each of the five cells may be different. For example, cell 2 may use 10 channels and cell 4, 7 channels all of which are reused by cell 2 .

The appropriate equations are as follows $(\mathrm{N}=5)$ :

$$
\begin{align*}
A & =\sum_{i=1}^{5} A_{i}  \tag{4-21}\\
Q & =\sum_{i=1}^{5} \rho_{i}  \tag{4-22}\\
C & =1+\mathrm{KN}^{b-1} \sum_{i=1}^{5} M_{i}^{a} \mathrm{CNR}_{i}{ }^{c}  \tag{4-23a}\\
& =f\left(5 ; M_{1}, \ldots, M_{5} ; \mathrm{CNR}_{1}, \ldots, \mathrm{CNR}_{5}\right)  \tag{4-23b}\\
S N R & =1.5 B^{2} \mathrm{CNR}_{i} / R_{i}{ }^{n} \quad(i=1,2, \ldots, 5)
\end{align*}
$$

$$
\begin{align*}
&=\operatorname{SNR}_{1}\left(\beta, \mathrm{CNR}_{1}, A_{1}\right) \quad \text { (inner cell channels) } \\
& \operatorname{SNR}=\left\{\left[\Gamma \beta^{-3}\left(P_{j} / P_{i}\right)\left[R_{i} /\left(2 R_{1}+R_{j}\right)\right]^{\mathrm{n}}+\left[1 \cdot 5 \beta^{2} C N R_{i} / R_{i} n^{n}\right]^{-1}\right\}^{-1}\right. \\
&=\operatorname{SNR}_{2}\left(\beta_{i} P_{i} / P_{j}, A_{j}, A_{1}, C N R_{i}, A_{i}\right)\left\{\begin{array}{c}
(i=2,3,4,5) \\
\text { outer cell channels }
\end{array}\right\}  \tag{4-24b}\\
& T=T\left(\rho_{i}, M_{i}\right) \quad i=1,2, \ldots, 5
\end{align*}
$$

The value of $j$ in ( $4-24 b$ ) depends on the value of $i$ in an obvious way; $j=i+2$ if $i=2$ or 3 and $j=i-2$ if $i=4$ or 5.

In examining the above equations, particularly (4-24b) a new problem becomes evident, namely SNR for a reused channel in any outer cell depends on the ratio of its base station's power to that of the opposite cell. Another potentially independent system parameter has emerged to frustrate our efforts to bring to a manageable level the number of such system parameters. In evaluating the performance of an existing or proposed system there is, of course, no difficulty since the power ratios $P_{i} / P_{j}$ would be specified at the outset. However, in design and optimization studies another free parameter may be available.

In many situations this power ratio would be fixed (often at Unity) or specified as a function of either $A_{i} / A_{j}$ or $C N R_{i} / C N R_{j}$ or both. (For example, in many cases $P_{i} / P_{j}=C N R_{i} / C N R_{j}$ which implies that receivers $i$ and $j$ have the same noise characteristics). Minimization of cost subject to constraints on $S N R$ and $T$ is then in accordance with the algorithm in Section $4-4$ since $P_{i} / P_{j}$ is not an independent design variable.

If $\left\{P_{i} / P_{j}\right\}$ is an independent variable set then optimization begins with the first four steps of the algorithm in Section $4-4$ with $N=5$, following which $\beta$ and $\left\{M_{i}\right\}$ have been determined in terms of $\left\{A_{i}\right\}$ and $N$. $\mathrm{CNR}_{1}$ is then determined from (4-24a). At this point $\left\{\mathrm{P}_{i} / \mathrm{P}_{j}\right\}$ is specified and $C N R_{i}$ is obtained from (4-24b) for $i=2,3,4$ and 5. Cost $C$ is then
determined from $(4-23),\left\{P_{i} / P_{j}\right\}$ is then varied (within its allowable limits) and $\mathrm{CNR}_{\mathbf{i}}(i=2,3,4,5)$ and costs are again calculated. In this way $\left\{\mathrm{P}_{\mathrm{i}} / \mathrm{P}_{\mathrm{j}}\right\}$ is optimized. The entire procedure is then repeated for different initial choices of $\left\{A_{i}\right\}$.

The above discussion assumes $\beta$ to be a design variable. If $\beta$ is specified at the outset, then either $B / d W$ must be a design variable or the designer must be prepared to relax constraints on $T$, if necessary.

Finally, we note one further complication in this section relative to that in the last section, namely that some channels in outer cells may not be reused. Since we tacitly assume that all channels in a cell have Identical CNR values, the $S N R$ constraint will be met by any non-reused channel if it is met by all reused channels in the cell.

## IV-7 Design Example

Concepts developed to this point will be illustrated using an example based on Fig. 4-1, and on the following assumptions:

1. The communication traffic $\lambda_{V} \operatorname{Ln}(x, y)$ is uniform over the coverage area.
2. Cost constant $K$ is sufficiently large that in the parameter range of interest, fixed costs can be ignored, with the result that

$$
\begin{equation*}
\mathrm{C}=\mathrm{KN}^{\mathrm{b}} \sum_{\mathrm{i}} \mathrm{M}_{\mathbf{i}}{ }^{\mathrm{a}} \mathrm{CNR}_{i}{ }^{\mathrm{c}} \tag{4-25}
\end{equation*}
$$

3. A11 transmitters and receivers have identical noise performance; thus $P_{i} / P_{j}=C_{i N R}^{i} / \operatorname{CNR}_{j}$
4. The constraint on the normalized access delay $T$ is such that in the absence of channe 1 reuse, a system having four or more cells of equal area violates the delav constraint.
5. For each cell we require $\operatorname{SNR} \ll \mathrm{X}$.

We first compare the costs of the single-cell and 3 -cell systems in Fig. 4-1. If $R$ is the radius of the circle covering the entire area then a three-cell system requires three circles of radius $(\sqrt{3} / 2) R=0.87 R$ for total coverage. With such a circle some coverage overlap will occur. In an actual system directional antennas would reduce the effective radius below $0.87 R$.

Since the delay constraint is satisfied for the three-cell system the constraint would also be satisfied for the single-cell system as explained in Section 2-3.

The SNR constraint requires:

$$
\begin{array}{ll}
C N R>X^{n} / 1 \cdot 5 \beta^{2} & \text { (single-cell system) } \\
\text { CNR }>\left[\mathrm{XR}^{\mathrm{n}} / 1 \cdot 5 \beta^{2}\right] 0.87^{\mathrm{n}} & \text { (three-ce } 11 \text { system) } \tag{4-27}
\end{array}
$$

The costs are as follows:'

$$
C / K=C N R M^{2}
$$

$$
\begin{equation*}
>\left(\mathrm{X} / 1 \cdot 5 \beta^{2}\right)^{c} \mathrm{R}^{\mathrm{nc}} \mathrm{M}^{\mathrm{a}} \text { (single-ce } 11 \text { system) } \tag{4-28}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{C} / \mathrm{K}>\left(\mathrm{X} / 1.5 \beta^{2}\right)^{c} \mathrm{R}^{\mathrm{nc}} \quad 0.87^{\mathrm{nc}} \mathrm{M}^{\mathrm{a}} \mathrm{~N}^{\mathrm{b}-\mathrm{a}} \underset{\substack{(\mathrm{~N}=3 ; \\ \text { system })}}{ } \text { three-cell } \tag{4-29}
\end{equation*}
$$

It is seen that the three-cell system is preferred to a singlecell system if and only if:
$(0.87)^{\mathrm{nc}} \mathrm{N}^{\mathrm{b}-\mathrm{a}}<1 \quad(\mathrm{~N}=3)$
which implies

$$
\begin{equation*}
(b-a) / n c<-\log 0 \cdot 87 / \log 3 \tag{4-30}
\end{equation*}
$$

Clearly, knowledge of system cost trends is needed in order to determine the preferred system.

The above analysis assumes that $M$ (and $\beta$ ) are fixed at the outset. If the delay constraint could not be met for $N=3$, $M$ could be increased by reducing $\beta$, in which case the costs would be as follows, with M1 and M3 denoting the total number of channels in the one-cell and three-cell system, respectively:

$$
\begin{align*}
& \mathrm{C} / \mathrm{K}>(\mathrm{X} / 1.5)^{\mathrm{c}} \mathrm{R}^{\mathrm{nc}}(\mathrm{dW} / \mathrm{B})^{2 \mathrm{c}} \mathrm{Ml}^{\mathrm{a}+2 \mathrm{c}} \text { (single-cell system) }  \tag{4-31}\\
& \mathrm{C} / \mathrm{K}>(\mathrm{X} / 1.5)^{\mathrm{c}} \mathrm{R}^{\mathrm{nc}(\mathrm{dW} / \mathrm{B})^{2 c}(0.87)^{\mathrm{nc}} \mathrm{~N}^{\mathrm{b}-\mathrm{a}} \mathrm{M3}^{a+2 \mathrm{c}}} \\
& (\mathrm{~N}=3 ; \text { three-cell system) } \tag{4-32}
\end{align*}
$$

The three-cell system is preferred if and only if

$$
\begin{equation*}
(b-a) \log 3+(a+2 c) \log (M 3 / M 1)<n c(-\log 0.87) \tag{4-33}
\end{equation*}
$$

The ratio $M 3 / \mathrm{Ml}$ would be determined by the delay constraint.
The smaller is $M 3 / M 1$, the more likely is inequality ( $4-33$ ) to be satisfied.
An analysis similar to the above can be undertaken for other values of $N$ with no channel reuse. The effective radius will, of course, change from 0.87 R , depending on N and the cell layout.

Consider now the 5 -cell system. In accordance with the fourth assumption, channel reuse is necessary to meet the delay constraint, either because $\beta$ is irrevocably fixed at the outset or because a sufficiently large value of $M$ would make $\beta<1$. Following the algorithm in Section $4-4$ we set $N=5$ and select the radii $R_{1}$ and $R_{2}$ of the inner and outer (identical) cells, respectively. Assume for the moment that $\beta$ is fixed at the outset and that $\left\{M_{i}\right\}$ can be selected to meet the delay constraint.

The SNR constraints for the inner and outer cells are as follows:

$$
\begin{align*}
& \mathrm{CNR}_{1}>\left[1 \cdot 5 \beta^{2}\right]^{-1} \times \mathrm{R}_{1}^{\mathrm{n}}  \tag{4-34}\\
& \left\{\left[1.5 \beta^{2} \mathrm{CNR}_{2} / \mathrm{R}_{2}^{\mathrm{n}}\right]^{-1}+\Gamma \beta^{-3}\left(\frac{\mathrm{R}_{2}}{\mathrm{R}_{2}+2 \mathrm{R}_{1}}\right)^{n}\right\}^{-1}>\mathrm{x}  \tag{4-35a}\\
& \left\{\left[1.5 \mathrm{CNR}_{2}\right]^{-1}+\Gamma \beta^{-1}\left(\alpha \mathrm{R}+\mathrm{R}_{1}\right)^{-n}\right\}^{-1}>\mathrm{XR}_{2}^{n} / \beta^{2} \tag{4-35b}
\end{align*}
$$

In moving from (4-35a) to (4-35b) we have used $\alpha R=R_{1}+R_{2}$ where $\alpha>1$ is a correction factor to account for the fact that some overlap in coverage may occur.

Assuming that $\Gamma \beta^{-1}$ and $\beta^{2}$ are known, specification of $R_{1}$ yields $\mathrm{CNR}_{1}$ from (4-34) and $R_{2}$ which yields $\mathrm{CNR}_{2}$ from (4-35). Cost $C$ is then obtained from (4-25). In this way $\dot{C}$ is obtained vs. $R_{1}$ and the value of $R_{1}$ which minimizes $C$ subject the constraint on $T$ defines the optimum system.

As a free variable $\beta$ should be selected to provide just enough channels to meet the delay constraints. Cost $C / K$ is determined vs. $R_{1}$, which is selected to minimize total cost. Clearly $0<R_{1} \leqslant R$. If the delay constraints require $\beta<1$ then these constraints cannot be met.

The final step involves comparison of the minimum cost with that for the one-cell and three-cell systems. Further algebraic manipulations demonstrate the need to know the cost parameters to determine the minimum cost system.

## V. ALGORITHMS FOR THE ANALYSIS AND DESIGN OF MOBILE RADIO SYSTEMS

## V-1 Introduction

Following the developments in the previous chapter, we are now able to generalize the algorithm in Section $4-4$ to the design of any cellular system with arbitrary channel reuse provided adjacent-channel interference is absent. As in Chapter 4 we assume that diversity is of sufficiently high quality or CNR is sufficiently high that operation is maintained above threshold. When this assumption does not apply the general operation of the algoritnm would still be as in Section 4-4. However specific equations for SNR, which are available, [J1, Ch4] would become more complex. Similarly, further study may yield a cost function different from the specific one proposed below. However the general approach to analysis and design would not change. In particular inclusion of total system bandwidth $B$ as a cost variable is seen to be straightforward and presents no difficulties since Cost is normally determined as a final step.

Inclusion of adjacent-channel interference for purposes of analysis of fully specified systems is straightforward in principle, although actual calculation of the effect on SNR requires FM spectral calculations. Expliotation of the relative channel frequency separation $d$ as design variable seems very difficult in the absence of cellular symmetry.

In calculating the interference multiplier $\Gamma(x)$ we assume a
Sufficiently large number of interferers that Gaussian interference results. With few interferers, other approaches are available to calculate $\Gamma(x)$ [ $P 1$, P2, J1, 3 3,R6, S5] which could alternatively be obtained by measurement or Simulation.

## V-2 Arbitrary Channel Assignments with Co-channel Interference

We assume that adjacent-channel interference is negligible, and that relative available bandwidth $B / d W$ and traffic density $n(x, y)$ are specified at the outset. On the basis of the discussion in Section 4-6, the following equations apply:

$$
\begin{align*}
A & =\sum_{i=1}^{N} A_{i}  \tag{5-1}\\
Q & =\sum_{i=1}^{N} \rho_{i}  \tag{5-2}\\
C= & 1+K N^{b-1} \sum_{i=1}^{N} M_{i}^{a} C N R_{i}^{c}  \tag{5-3a}\\
= & f\left(N_{i} M_{1}, \ldots \ldots, M_{N} ; C N R_{1}, \ldots \ldots, C N R_{N}\right)  \tag{5-3b}\\
S N R_{i}= & \left\{\Gamma \beta_{j}^{-3} \sum_{j C F_{i}}\left(P_{j} / P_{i}\right)\left(R_{i} / D_{i j}\right)^{n}+\left[1.5 \beta^{2} C N R_{i} / R_{i}^{n}\right]^{-1}\right\}^{-1}  \tag{5-4}\\
T_{i}= & T\left(\rho_{i}, M_{i}\right) \tag{5-5}
\end{align*}
$$

Again we have recognized by ( $5-3 \mathrm{~b}$ ) that a cost function other than the one in (5-3a) may be most appropriate. Constant $\Gamma$ is calculated as explained in Section 5-5. Set $\mathrm{F}_{\mathbf{i}}$ includes all co-channel interferers observed in cell i. If all $M_{i}$ channels in cell $i$ have identical CNR values (as would normally be the case) then that channel whose co-channel interference is largest will determine $\mathrm{SNR}_{\mathrm{i}}$.

From the above discussion one sees that calculation of SNR and $T$ for each cell, system cost $C$ and throughput $Q$ is straightforward given all the system parameters. A formal analysis algorithm is depicted in Fig. 5-1.


Fig. 5-1 Algorithm for the Analysis of Cellular Mobile Radio Systems with Good Diversity and No Adjacent-Channel Intereference.

Extension of the design optimization algorithm articulated in Chapter 4 seems difficult for arbitrary cel. geometries and locations, although the first three steps of the algorithm are appropriate. Following step 3, the required number of channels $\left\{M_{i}\right\}$ for each cell is known. From this point trial and error is needed if the system consists of many cells of arbitrary shape. However, for a regular cell structure with equal-area cells with $R_{i}=R$, equal values of $P_{i}=P$ and $C N R_{i}=$ CNR, and equal reuse distance $D_{i j}=D$ for all $i, j$, the required number of distinct channels $M$ can be calculated, and so therefore can $B$. The co-channel interference term in (5-4) can then be calculated. If this term exceeds the $S N R$ constraint then $D$ must be increased, which increases $M$ and decreases $\beta$. If $\beta>1$ is eventually obtained then the required CNR value can be calculated from (5-4). Finally the cost is obtained using (5-3). The CNR and cost calculations can then be repeated using different reuse distances $D$ and correspondingly different values of $M, \beta$ and $C N R$ to yield the minimum cost system. There may be no value of $\beta>1$ such that constraints on both SNR and $T$ are satisfied for the given value of $B / d W$. However it may be possible to meet these constraints using an irregular cellular topology.

The above optimization procedure is formalized in Fig. 5-2 where NF and CNRF denote upper limits on $N$ and CNR, respectively, which limits would be present in a practical situation. The co-channel interference term in (5-4) is SIR. Constant $\alpha$ is the amount by which the reuse distance $D$ is incremented, and MIN and MAX denote minimum and maximum values. With negligible adjacent-channel interference the switch is in position 2, $\mathrm{dMIN}=\mathrm{dMAX}=\mathrm{d}$ and $\mathrm{dINC}=0$. The algorithm lists all system configurations which meet the constrains. The algorithm is Fig. 5-2 extends to those cases where $P_{i} / P_{j}$
is a function of $C N R_{i}, C N R_{j}, R_{i}$ and $R_{j}$ so long as cellular regularity enables knowledge of the size and location of one cell to determine that of all others. If $P_{i} / P_{j}$ depends $C N R_{i}$ and $C N R_{j}$ (for example $P_{i} / P_{j}=C N R_{i} / C N R_{j}$ implies that all base station receivers have identical noise figures) then (5-4) implies solution of $N$ coupled rather than uncoupled equations for $\left\{\mathrm{CNR}_{i}\right\}$. When non-uniform cell sized is involved, reuse distance $D$ would vary with cell size.

When interchannel interference is not negligible and $d$ is a design variable, the switch in Fig. $5-2$ is in position 1 as explained in Section V-5.

## V-3 Adjacent-Channe 1 Interference

For an understanding of the effects of adjacent channel interference, refer to Fig. 5-3, which shows spectra of the local channel and of adjacent channels which may fall within the transfer characteristic $G(f)$ of the $I F$ filter of the receiver for channel $i$.

We assume that there are a sufficient number of adjacentchannel interferers that the totality of noise resulting from adjacentchannel interference, co-channel interference and receiver noise is Gaussian. We also assume the various sources of interference to be uncorrelated in which case the interference spectrum $I_{i}(f)$ for a local channel in cell i is as follows where $S(f)$ is the power density of the broadcast FM signal:

$$
\begin{equation*}
I_{i}(f)=A \sum_{\substack{j, k \\ j \neq i}}\left[P_{j} / D_{i j}{ }^{n}\right]|G(f)|^{2} S(f-k \Delta) \tag{5-6}
\end{equation*}
$$

onstant $A<1$ accounts for the fact that because of fast Rayleigh fading the average received interference power is somewhat less than the attearated radiated power $P_{j} / D_{i j}{ }^{n}$. The other symbols are defined in Section $3-1$. The sumation is, strictly speaking, over all cells other than the local cell and over all channels $k$. In practice, only those cells and channels which contribute significantly to interference would be included.
6. MINIMUM, MAXIMUM d: dMIN, d MAX 7. COVERAGE AREA: A



Fig. 5-3 Illustrating Adjacent-Channel Interference.

The assumption regarding uncorrelated interference will be valid for interfering sources from different cells. Correlations between channels from the same cell would be weak and are ignored. The dependence on $\beta$ of the FM signal spectrum is specifically noted.

In calculating the output signal-to-interference ratio $\operatorname{SIR}_{\mathbf{i}}$ for channels in cell i we again assume good diversity which implies that the IF signal-to-noise ratio is above threshold. The standard FM analysis procedures can then be employed to yield $S I R_{i}$ as follows:

$$
\operatorname{SIR}_{i}=\frac{\beta^{3}}{\sum_{j, k}\left(\frac{P}{P_{i}}\right)\left(\frac{R_{i}}{D_{i j}}\right) \Gamma\left(k d^{\beta}\right)}
$$

i¥j and $k \neq 0$
$\Gamma(x)$ is calculated as explained in the following section.
Gaussian noise is normally present in addition to interference. Such noise is normally white, with the result that its signal-to-noise ratio is given by (2-14), assuming above-threshold behaviour. Addition of these two noise sources yields the signal-to-noise ratio $\operatorname{SNR}_{1}$ for a channel in cell i:

$$
\begin{equation*}
\operatorname{SNR}_{i}=\left(\operatorname{SIR}_{i}^{-1}+\left[(3 / 2) \beta^{2} \operatorname{CNR}_{i} / R_{i}^{n}\right]^{-1}\right)^{-1} \tag{5-8}
\end{equation*}
$$

Al1 previous expressions for $\operatorname{SNR}$ are special cases of (5-8).
It is seen that above threshold co-channel and adjacent-channel interference effects decrease as $\beta^{3}$ while Gaussian noise effects decrease as $z^{2}$.

## V-4 Calculation of Interference Mültiplier $\Gamma(x)$

The calculation of $\Gamma(x)$ in (5-7) begins with consideration of Fig. 5-4 where

$$
\begin{equation*}
r(t)=\sqrt{2 P} \quad \cos \left(2 \pi f_{0} t+\theta+\int_{-\infty}^{t} m(\tau) d \tau\right)+n(t) \tag{5-9}
\end{equation*}
$$

In (5-9), $f_{o}$ is the FM carrier frequency, $m(t)$ the message signal of interest, $\theta$ a constant but unknown phase angle and $n(t)$ is Gaussian noise which, contrary to the usual assumption is non-white with power spectral density $N(f)$.

Over any time interval which is small in comparison with $f_{o}^{-1}$, $m(t)$ is constant; thus at time $t=t_{0}$ the instantaneous frequency $f_{a}=f_{0}+$ $m(t) / 2 \Pi$.

We can now write

$$
\begin{align*}
& n(t)=n_{c}(t) \cos \left(2 \pi f_{a} t+\theta\right)+n_{s}(t) \sin \left(2 \pi f_{a} t+\theta\right)  \tag{5-10}\\
& r(t) \simeq \sqrt{2 P} \cos \left(2 \pi f_{a} t+\theta-n_{s}(t) / \sqrt{2 P}\right) \tag{5-11}
\end{align*}
$$

where we assume that the received carrier is well above threshold with the result that for most of the time

$$
\begin{align*}
& \left|n_{c}(t)\right| \ll \sqrt{2 P}  \tag{5-12a}\\
& \left|n_{s}(t)\right| \ll \sqrt{2 P} \tag{5-12b}
\end{align*}
$$

The instantaneous frequency is

$$
\begin{equation*}
2 \pi f_{a}=2 \pi f_{o}+m(t)-\dot{n}_{s}(t) / \sqrt{2 P} \tag{5-13}
\end{equation*}
$$

where $\dot{n}_{s}(t)$ is the time derivative of $n_{s}(t)$.
The power density spectrum of $\ddot{n}_{s}(t)$ is

$$
\begin{equation*}
N_{\dot{s}}(f)=\frac{(2 \pi f}{2 P}_{2}^{2}\left[N\left(f+f_{a}\right)+N\left(f-f_{a}\right)\right] \tag{5-14}
\end{equation*}
$$



Fig. 5-4 FM Receiver.
and the noise power from the audio filter of bandwidth $W$ and unit passband amplitude is

$$
\begin{equation*}
N_{o}=\left(4 \pi^{2} / P\right) \int_{0}^{W} f^{2}\left[N\left(f+f_{a}\right)+N\left(f-f_{a}\right)\right] d f \tag{5-15}
\end{equation*}
$$

Finally, $N_{o}$ must be averaged over instantaneous frequency $f_{a}$, which involves averaging with respect to the amplitude of message $m(t)$ in accordance with (5-13). Thus, with average denoted by an overhead bar

$$
\begin{equation*}
\left.\bar{N}_{0}=\left(4 \pi^{2} / P\right) \int_{0}^{W} f^{2}\left[\overline{N\left(f+f_{a}\right.}\right)+\overline{N\left(f-f_{a}\right)}\right] d f \tag{5-16}
\end{equation*}
$$

The signal-to-noise ratio $\operatorname{SNR}$ is

$$
\begin{equation*}
\mathrm{SNR}=\overline{\mathrm{m}}^{2} / \overline{\mathrm{N}}_{\mathrm{O}} \tag{5-17}
\end{equation*}
$$

Use of (2-16) and $2 \overline{\mathrm{~m}}^{2}=4 \pi^{2} \overrightarrow{\mathrm{~W}}_{\mathrm{c}} 2$ in (5-17) yields

$$
\begin{equation*}
S N R=\bar{W}_{c}^{2} P / 2 \int_{0}^{W} f^{2}\left[\overline{N\left(f+f_{a}\right)}+\overline{\left.N\left(f-f_{a}\right)\right] d f}\right. \tag{5-18}
\end{equation*}
$$

In the special case where $N(f)=N_{o} / 2$ (white Gaussian noise) averaging is unnecessary and the familiar expression (2-14) results.

In the case when $n(t)$ consists solely of adjacent-channel interference, of which co-channel interference is a special case, $N(f)=I_{i}(f)$ where $I_{i}(f)$ is given by (5-6). It follows that with $P=P_{i} / R_{i}^{n}$ in (5-18) the signal-to-interference ratio $S I R$ is as follows, where $i$ is the local cell, $j$ the cell from which the interference eminates on a channel separated in frequency by $k \Delta H z$ from the local channel:
where

$$
\begin{equation*}
F(f, k \Delta)=\left[\overline{G\left(f+f_{a}\right) S\left(f+f_{a}-k \Delta\right)}+\overline{G\left(f-f_{a}\right) S\left(i-f_{a}-k \dot{L}\right)}\right] \tag{5-20}
\end{equation*}
$$

The change of variable $(f / W)=y$ enables $(5-19)$ to be rewritten as follows:

$$
\begin{equation*}
\left.\operatorname{SIR}=\left[\frac{\sqrt{W_{S}^{2}}}{W}\right]_{\substack{j, k \\ j \neq i}}^{3} \sum_{\substack{P_{i} \\ P_{i} \\ P_{j}}}^{\left(\frac{R_{i}}{D_{i j}}\right)} \int_{0}^{n} y^{2} H\left(y, \frac{k \Delta}{W}\right) d y\right]^{-1} \tag{5-21}
\end{equation*}
$$

where

$$
\begin{equation*}
H(y, x)=2 A F(y, x) \sqrt{\frac{W_{c}^{2}}{2}} \tag{5-22}
\end{equation*}
$$

If we now define

$$
\Gamma(x)=\int_{0}^{1} y^{2} H(y, x) d y
$$

and recall the definition of $\beta$ in (3-8) and that $\Delta=d \beta W$, (5-7) follows imnediately. The amplitude of $S(f)$ would tend to be propor-
tional to $\left\{\sqrt{W_{c}^{2}}\right\}^{-1}$ since the total power in an interfering signal is constant. Function $G(f)$ would be frequency-scaled in proportion to $\sqrt{W_{c}^{2}}$. Thus, $r(x)$ would depend on $\beta$ through variations in the spectral shape of $S(f)$; over a narrow range of $\beta, \Gamma(x)$ would be virtually independent of $\beta$, since $H(y, x)$ would be relatively independent of $\beta$.

With co-channel interference only, $x=0$ in (5-23) and $\Gamma(0)=\Gamma$.
Calculation of $\Gamma(x)$ requires knowledge of the following:

1. Constant $A$ introduced in (5-6); A depends on the amount of Rayleigh fading of the interferers relative to that of the signal. Thus, A depends on the effectiveness of the diversity employed. An upper bound on $\Gamma(x)$ and hence on $S T R^{-1}$ is obtained with $A=1$.
2. the IF filter characteristic $G(f)$.
3. the amplitude probability density of the voice signal which may be approximated by either a Laplacian [C4, S7] or a Gamma [C4,R8] distribution.
4. the normalized $F M$ signal spectrum $S$ (f) $\sqrt{W_{c}{ }^{2}}$. For a narrow range of $\beta$, this dependence on $\beta$ should be relatively weak. Various approaches exist for calculating. FM spectra [A1, P1, P2, R9]
5. the product $d \beta$ when adjacent-channel interference is present. The Appendix details calculation of $S(f)$ in (5-20)

## V-5 Programming the Analysis and Design Algorithms

We now consider programming the analysis and design procedures articulated in Sections 5-2 and implied by the discussion in Sections 5-3 and 5-4.

Some flexibility exists as to the precise organization of data entry. If items 10-14 inclusive are to be calculated all of the data items 1-9 are required. Regarding item 3, the location, shape and area of of the cells could be entered in one of several ways, for example by representing the coverage area by a matrix and assigning each point in the matrix to a cell, or by entering cell boundary co-ordinates using a light pen or stylus-tablet. Regarding item 6, each channel would have following its identity number the numbers of all those cells to which it was assigned. If items 10,11 and 12 are supplied as input . data then entry of items 1 and 3 becomes unnecessary. Constant $\Gamma=\Gamma(0)$ could either be calculated or measured prior to entry in which case $\Gamma$ would be specified numerically, or calculated.
using (5-23) with $\mathrm{x}=0$.
If all of the independent items 1 through 14 were entered as data then a hand-held programmable calculator such as the HP-97 could be used to calculate $\mathbf{T}$ (or P) and SNR for each cell, C and Q. For symmetrical systems with relatively few cells, the same calculator could be used for optimization using the algorithm in Section 4-4, or its extension for cochannel interference in Fig. 5-2, since selection of $\left\{A_{1}\right\}$ in steps 2 and 7 would be determined by the current value of N and symmetry requirements.

For analysis situations involving many different non-symmetrical cellular layouts the algorithm in Fig. 5-1 would be programed on a larger computational facility which would calculate items 10-14, inclusive. If the analysis was actually being conducted during trial-and-error design of a large cellular system then an interactive graphics facility would be of the greatest assistance.

The above comments regarding analysis extend in a general way when adjacent-channel interference is included in SNR calculations. However the need to calculate $\Gamma(x)$ vs $x$ makes the actual computation more difficult. It would be useful to obtain an approximate closed-form expression for $\Gamma(x)$ either by analysis or measurement; if this could be done a hand-held programmable calculator might again be used for analysis of a fully specified system.

Concerning design, two additional degrees of freedom have been introduced with adjacent-channel interference, namely relative frequency separation $d$ and IF filter characteristic $G(f)$. In all our analysis we assumed that $G(f)$ did not significantly distort the output message. In fact, it might be useful to permit some distortion of the output message in exchange for removal of noise and interference from adjacent channels.

Even with $G(f)$ fixed, the additional freedom available in selection of $d$ makes the number of possible system designs very large. One can now choose between relatively large values of d which implies co-channel interference only, and smaller values of $d$ thereby increasing adjacent-channel interference to the point where it can no longer be ignored. However the accompanying increase in $M$ reduces the co-channel interference. Where is the best trade-off? At present we don't know. Trail-and-error design seems to be the only available design approach unless cellular regularity is present, in which case the algorithm in Fig. 5-2 applies with the switch in position 1. The algorithm could be further modified to accommodate optimization of $G(f)$ with respect to some parameters; $G(f)$ would normally be frequency scaled in proportion to $\sqrt{\mathrm{W}_{\mathrm{c}}{ }^{2}}$.

Clearly, desion which includes adjacent-channel interference requires a rather powerful computational facility prefereably with graphics capabilities. However, some study to improve our understanding of alternative design possibilities should probably preceed development of such a facility.

Intentional acceptance of some adjacent-channel interference as a zeans of increasing the total number of channels has also been proposed by Mikulski [M1] who did not, however, present a method for its calculation.

# VI. ALTERNATIVE SYSTEM STRUCTURES <br> FOR THE LAND MOBILE RADIO SERYICE 

## VI-1 Digital Transmission Systems

Digital rather than analog FM technology may be employed to transmit a voice message over individual frequency channels. Digital transmission is attractive for the following reasons:

1. The bandwidth required for each voice channel may be less than for FM which normally occupies either 25 or 30 KHz .
2. Eventually, mobile services will include data (both text and video) as well as voice messages. Digital transmission of voice permits use of a single technique for transmission and storage of all message types. Potential cost reductions are therefore possible.
3. Digital transmission facilitates encryption for purposes of privacy. 4. Digital transmission provides the opportunity for trade-offs between speech quality, channel bandwidth and implementation costs.
4. Channel encoding of digital transmissions may be used to combat multipath fading.
5. Digital transmission may permit a SNR which is higher than for FM which uses the same channel bandwidth.
6. The growing interest in other mobile radio system configurations including spread-opectrum multiple access (SSMA) and packet switched radio. is more compatible with and in some cases requires digital transmission.

Disadvantages of digital technology include:

1. Possibly larger channel bandwidth than for FM
2. Higher implementation costs, at least initially until digital voice approaches the technological maturity of analog FM.
3. Synchronization complexities and difficulties.

Fig. 6-1 shows a differential encoder for digital voice transmission;
$m(t)$ and $\hat{m}(t)$ denote the input and reconstructed voice signal, respectively. If the predictor and associated feedback connections are absent in the encoder and decoder and if sampling rate $f_{s}=2 W$ where $W$ is the bandwidth of the pre- and post-filters, then a standard PCM system results. Toll quality PCM speech requires a seven-bit quantizer which implies a data transmission rate $r=42 \mathrm{~K} \mathrm{bits} / \mathrm{sec}$ for $\mathrm{W}=3 \mathrm{KHz}$. The bandwidth $W_{C}$ of the channel normally lies between $r / 2$ which requires single-sideband and 2 r ; typically $\mathrm{W}_{\mathrm{c}} \simeq 1.5 \mathrm{r}$ [Jl] which implies a 63 KHz bandwidth, which is more than twice that of a. channel normally used for analog FM transmission.

Inclusion of the feedback 100p in Fig. 6-1 reduces to six the number of quantization bits required for toll quality speech, which number can be further reduced to 5 or 4 bits if an adaptive quantizer and entropy coder used. In this latter case the required bandwidth is 30 or 24 KHz . If some degredation in speech quality is acceptable, further bit-rate and corresponding bandwidth reduction is possible. For example, Cohn and Melsa [C5] have demonstrated that a 5 -level adaptive quantizer with entropy coding of the quantizer output levels results in $9.6 \mathrm{Kbits} / \mathrm{sec}$ speech of quality comparable to that between 4- and 5-bit PCM, while a 7-level quantizer with $16 \mathrm{Kbits} / \mathrm{sec}$ yields quality between that of 5- and 6-bit PCM. In both cases 3.2 KHz speech was sampled at 6.4 KHz . Hanson and Donaldson [H4] conducted extensive


Fig. 6-1. Differential encoder/decoder for digital voice transmission
subjective tests on a system like Cohn's and Melsa's and found further flexibility to be available by removing the Cohn-Melsa restriction of Nyquistrate sampling. Both studies assumed noise-free digital channels, although Cohn and Melsa reported little degradation in reconstructed speech quality for random bit-error rates as high as $10^{-3}$.

When the quantizer in Fig. 6-1 contains two levels a delta modulation ( $\Delta$-mod) system results. The great advantage of $\Delta$-mod systems, which typically operate at sampling rates in excess of three times the Nyquist rate, is their simplicity. The quantizer is a simple comparator, and word synchronization is not required. At low bit rates well-designed $\Delta$-mod systems give a higher SNR than conventional PCM systems; at high bit rates the converse is true [J2, J3, C6, G1, N2].

The SNR of a digital voice transmission systems depends on the bit-rate of the source encoder and on the error statistics of the digital channel.

For a memoryless digital channel the error statistics are summarized by the per-bit error probability $p$ which depends on the bit rate $r$ as well as the bandwidth and carrier-to-noise ratio of the actual physical Channel. Error-rate $p$ also depends on the digital data modulator and demodulator, and enormous effort has been devoted to the design of these to maximize the data rate while keeping the error rate to an acceptable minimum; typically $p<10^{-5}$ [L4, S8, B2, F2].

Contributions to SNR from digital channel errors increase with $r$, while those from the source encoder decrease with $r$. It follows that an ${ }^{\circ} \mathrm{p}$ timum value of r exists, and such existence has been demonstrated [ $C$, Y2]. The fact that $r$ can be varied for performance optimization in accordance
with the bandwidth and noise characteristics of the physical channel gives mobile system designers, operators and users a degree of operational flexibility not available in the case of $F M$ systems. Such flexibility could be used to make available two or more transmission rates $r$; users with infrequent and non-critical voice requirements might choose a channel having a lower bit rate with a correspondingly lower cost. In SSMA systems (Section 6-3) This flexibility in rate selection could be employed to combat excessive noise caused by too many active users. In packet radio systems (Section 6-4) r could be adjusted to reduce the overall traffic level thereby reducing access delays.

Source encoders other than the type depicted in Fig. 6-1 have been proposed for voice. Tree coders [A5, A6] make efficient use of dependencies between samples of the voice signal to operate at approximately $12 \mathrm{Kbits} / \mathrm{sec}$. Vocoder-based encoders which make use of the speech production model of the human vocal system [F3, F4] have demonstrated $1.5 \mathrm{Kbits} / \mathrm{sec}$ [S9]. These and'other types of encoders, although efficient in terms of bit rate seem uneconomic at present.

Calculation of error-rate $p$ in a mobile radio environment containing all of the degredations listed in Section 3-4 as well as intersymbol interference is a formidable task. Some results are available [B3, G2, M6, P3, T3, Z1]. Arguments similar to those used to obtain (5-8) indicate that $p$ depends on all of those variables affecting SNR in (5-8). The dependense on bit-rate $r$ would be reflected in a relationship bteween $r$ and $\beta$. Delay T or blocking probability $P$ remain unchanged. Thus, the optimization algorithms developed in Chapters 4 and 5 apply to digital as well as FM transmission over dedicated channels. However SNR must be expressed in terms
of the system variables for a given modulation/demodulation scheme. The availability of numerous modulation/demodulation and diversity alternatives requires preselection of the most promising of these prior to system design and optimization.

Our discussion on analog FM systems implied use of diversity to combat fading, and our SNR equations are based on the best obtainable diversity performance. In using digital transmission diversity may be replaced to some extent by channel encoding/decoding (see Fig. 6-1) to combat bursts of errors resulting from signal fades. Design of the coder/decoder requires knowledge of the burst statistics which may be in the form of $P(m, n)$, the probability of $m$ errors in block of $n>m$ bits [C8, L4] or by a Markov 2-state (or multi-state) channel model which describes burst statistics [L4, G3]. One way to combat bursts' is to use interleaving of bits and random error-correcting codes [L5]. Alternatively or as well special burst error correcting codes may be used [L4, C9, L5]. Powerful codes exist and can be implemented using relatively inexpensive digital logic [C10 L5]. The cost of implementing channel coding/decoding must, of course, be weighed against potential benefits. Also the effects on SNR of random and burst errors must be compared [J4, A3] in order to assess the need for interleaving.

In those applications involving transmission of data, channel encoding would normally be employed for error detection [L4, L5], which is more easily implemented than error correction. Upon detection of one or more errors in a block of bits, the receiver requests retransmission of the block in question [B4]. Although the real-time nature of speech transmission on conventional channels as well as the non-critical role of channel errors


#### Abstract

discourages a retransmission strategy, the error detection circuitry used for data could be augmented for correction of errors in the voice transmission.


Digital transmission of video information for display on small video screens will eventually follow transmission of voice and data. Much work has been done on efficient source encoding of video [H5, H6, W2, L6]; some of these schemes will eventually be implemented.

VI-2 Cellular Systems with Dynamic Channel Assignments

In mobile systems with fixed cellular assignments, the channels in some cells will be more heavily loaded than those in other cells. An obvious strategy to improve overall system performance is for idle channels from lightly loaded cells to be available to cells encountering unusually heavy traffic.

Three questions now present themselves: (1) What strategy should be used to assign unused channels to cells? (2) What is the resulting improvement in performance? (3) What additional system cost and complexity results?

Question (1) can be phrased in rather general terms as follows: Given a total number of channels per cell, what fraction of these should be permanently assigned to each cell, and what fraction should be available for general use? Recently Rich and Schwartz [R10] examined this question in the context of buffer assignment for purposes of queueing messages. Their analysis assumed Poisson message arrivals and exponentially distributed service times and showed that approximately two-thirds of the available
buffer space should be permenantly assigned to individual message queues with the remaining one-third available for sharing as needed. Blocking probability was the performance criterṭon which was optimized. Kamoun [K4] considered the same problem and obtained more general results which, however are not as easily interpreted.

Direct extension of the results of Rich and Schwartz to mobile radio channel assignment is complicated by the fact that all of the unused channels may not be available to a particular cell because of reuse constraints. It is not immediately evident how to modify the unconstrained buffer allocation analysis to accomodate channel reuse constraints. However simulation results corroborate those of Rich and Schwartz and indicate that approximately $70 \%$ of the total number of channels should be permenantly assigned to cells, with the remaining $30 \%$ assigned to a common pool [J1]. The resulting improvement in performance is significant, particularly for low traffic levels; for a 160 channel system with square cells of equal size, equal traffic and a reuse distance of 4 (10 channels per ce11), dynamic assignment of $30 \%$ of the 160 channe1s results in $5 \%$ blocking probability. A fixed assignment system would require 172 channels to achieve a similar performance which represents a saving of $7 \%$ in the number of channels required [J1]. This saving decreases as $\rho$ increases. Chapter 7 of Jakes [J1] book provides an excellent summary of the various dynamic assignment schemes proposed by others [A1, C1, C2].

There is some question as to whether or not the increase in system control costs needed to implement dynamic channel assignments is warranted, particularly in view of the fact that dynamic assignment strategies seem
to offer limited improvement when such improvement is needed most, namely in heavy traffic. The performance results quoted above are based on switching calls in progress from a dynamic channel to a fixed channel when one becomes free. In assigning channels dynamically reuse constraints must always be checked.

Use of the analysis and design methodology developed in previous chapters requires simulation to determine delay $T$ in terms of cell traffic $\rho_{i}$ and $M_{i}$ which would now denote an average number of channels per cell. Alternatively, results obtained from simulation could be fitted by functions or listed in tabular form.

VI-3 SSMA Systems

Spread spectrum multiplexing [E3,C11,S10,D4] has been proposed for the land mobile radio service. Spread spectrum involves sharing of the entire system bandwidth among all users, each of whom has a unique waveform which is used both as a carrier signal and as an identifier.

Spread spectrum signals have been categorized as direct sequence, frequency hopping, pu1se-FM (chirp) and time hopping [D4], In a direct sequence system the message signal, which is normally digital (but could be analog) is multiplied by a pseudo-random binary waveform and the resulting product signal is then transmitted. The basic pulse frequency of the pseudorandom signal is many times larger than that of the modulation bit rate and determines the bandwidth of the transmitted signal. In a combined frequency and time hopping system, the frequency-time plane is divided into rectangular
regions of equal size. Each carrier wayeform consists of a unique timesequence of frequency pulses.

In either system the receiver obtains the original message by correlating a stored replica of its spread-spectrum carrier against the received signal. The noise is, for the most part, generated by other users. Care is needed in selecting spread spectrum carrier waveforms with desirable correlation properties; otherwise rejection of other users' carrier signals will not be possible.

A spread spectrum system has several advantages, as follows:

1. The number of users which can be handled by a single base station or by the system as a whole has no hard limit. As the number of users increases the bit-error probability decreases; the SNR decreases and the synchronization delay $T$ increases.
2. From item 1 above it follows that no user is denied access to the channel; blocking probability $\mathrm{P}=0$.
3. Because each user has his own identity signal, channel switching as mobiles cross cell boundaries is unnecessary. The forced call termination problem characteristic of FM cellular systems does not occur. 4. User privacy against casual eavsdroppers is provided by virtue of personalized address waveforms which could be keyed on the basis of social insurance numbers, vehicle license numbers or vehicle serial numbers. Privacy against professional eavsdroppers with appropriate equipment is not assured, however.
4. The wide bandwidth of the carrier signal provides considerable protection against Rayleigh fading; not all of the spectrum would be
subject to fading at any one time.
5. Priority for a fee or, in the case of emergency services for need is possible by regulation of permissible transmission power levels. The higher the power level the lower $T$ and higher SNR.
6. A spread spectrum system may co-exist with conventional systems which occupy a portion of the band occupied by the spread spectrum system. Thus the possibility exists for phasing spread spectrum systems into operation without unduly disrupting the operation of existing systems.
7. The fact that all user hardware would be standardized and digital in nature provides for potential economies of scale. Each user's identity would be keyed in prior to initial operation.
8. In systems with many small cells, the SSMA bandwidth may be less than that used by conventional FM.

The disadvantages of SSMA systems are as follows:

1. Control of transmitted power would be needed to prevent users near a base station from overpowering those far away (The near-far problem).
2. Some form of vehicle location would be needed to monitor vehicle movement from cell-to-cell. However accurate monitoring of the type needed for multichannel cellular systems is not required.
3. Signal encoding-decoding is more complex than for the mature and well-engineered conventional FM technology.
4. In systems with large cells, SSMA bandwidth may be larger than required for conventional FM.
5. Synchronization of SSMA signals is more difficult than for FM or conventional digital signals.

Provision of power control. would not seem to be an unduly
difficult problem. Initial transmission from a base station could be at maximum power. The received power levels could be used to adjust power levels for subsequent transmissions, and could also be used to estimate mobile to base station distances. Vehicle location could be implemented by listening with all base stations capable of hearing a given mobile. The continuing decrease in cost of digital hardware should narrow the gap between the cost of SSMA and FM hardware. Actually, the need for less precise vehicle location hardware in SSMA may favour SSMA over conventional FM systems.

All of the above arguments support those who favour a careful examination SSMA as an alternative to both fixed assignment and dynamic assignment cellular systems. Such an examination requires comparison of system costs for given SNR and delay constraints. As indicated earlier delay would be due to the time required to synchronize both the individual carrier waveform chips and the carrier frame (word). Although much relevant work has been done on synchronization [L7, L8, D4, Q1, S11, H7], explicit results which display synchronization delay $T$ (which is akin to queueing delay in dedicated-channel system) seem unavailable. Clearly $T$ would increase with the number of users per base station for two reasons; the noise level would increase and so would the number of different identifier waveforms. Delay $T$ would decrease as system bandwidth $B$ increases. One of the reasons that synchronization is difficult both to acquire and maintain is that (sinusoidal) carrier recovery if attempted at all would probably have to be performed after achieving frame synchronization of the pseudonoise or other SSMA waveform.

Calculation of bit-error probability on which SNR depends is not particularly easy especially if intersymbol effects of the SSMA chips are considered [H8], however bounds are available [P4, K5, D4]. It is of interest that no one seems to have seriously considered shaping the individual SSMA waveform chips or use of sophisticated receivers to reduce intersymbol interference.

System cost data seems unavailable. However it would be of interest to compare SSMA systems with conventional FM systems and frequency multiplexed digital systems solely on the basis of required bandwidth. A preliminary study [E3,C11] which compares an SSMA cellular system with an FM cellular system at equal values of $S N R=30 d B$ seems to favour the SSMA system on the basis of number of users per Hz per unit area. Time delays and system costs were not compared. !

## VI-4 Packet-Switched Systems

Standard telephone networks use circuit (or line) switching, whereby the conversing parties hold physical circuit connections until completion of their call. While circuit switching provides for a dedicated connection between two users, the time taken to establish the connection, the rather high probability of a busy signal on one or more of the links in the transmission circuit and the unavailability of the circuit to other users during periods of silence (which constitutes at least 30 percent of the call time [G1]) are all disadvantages of circuit switching.

Telegraph systems, many of which originated with the railroads, were and are used to transmit coded messages. In early systems messages arriving at a switching centre were received and stored on punched paper. tape. The tape holding the incoming message was placed in appropriate outgoing tape readers for transmission to the next node. The message was transmitted even if subsequent links in the total communication path between source and destination were temporarily unavailable. The price of this convenience includes message storage facilities at each node, as well as variable nodal delays.

Packet switching involves decomposition of messages into fixedlength packets and transmission of these individually, possibly over different routes, to the message destination where message reassembly occurs. Packet switching is particularly attractive when many intermediate nodes lie between the message's origin and destination. As soon as the first packet of a message is received by the first node in the chain, it can be forwarded while the next packet can be sent from the origin to the first node. If there are more intermediate nodes than packets in a given message the entire message can be moving towards its destination with each packet on a different link. This pipelining effect reduces the source-to-destination message delay [K1,S2,S13,A7]. Packetizing of messages can reduce the potentially large nodal storage required to handle very long messages and can provide for additional flexibility regrading message transmission priorities. Thus single message packets can be interleaved with a sequence of packets from a long message which may have no urgency regarding delivery time.

Either message or packet switching is indicated when traffic is bursty, in which case a high bandwidth low duty-cycle channel is needed.

Long file transfers favour circuit switching. When traffic consists of both interactiye conyersations and file transfers, packet switching is indicated, for reasons previously stated. Packet switching involves problems additional to those inherent in message switching, which problems include packets arriving at the destination out of sequence, in duplicate or not at all [K1, S12, S 13, S14].

The desision as to which form of switching to use is not an easy one. Some studies have been conducted in order to compare switching techniques but a comprehensive treatment seems unavailable. Because we do not yet fully understand all the issues involved in the design of packet switched networks, particularly the issues involving network management and protocols, a definitive comprehensive comparison of switching methods seems unlikely at present. However; enough is known concerning the advantages, design and operation of packet switched networks to warrant their continued use and study.

Selection of packet length is important. If $H$ is the packet overhead, including bits for packet synchronization, addressing and error detection, and if $L$ is the packet's maximum length, the following facts are relevant [R11]:

1. The ratio of useful information to overhead is (L-H)/H; hence the network throughput increases as $L$ increases.
2. The larger $L$, the larger the probability of a packet error and subsequent retransmission with an accompanying reduction in throughput if the error rate is too high.
3. Increasing $L$ increases network delay for short packets which must. queue behind one or more long packets, some or all of which may be
a part of a long message. This particular problem can be obyiated by giving short. packets priority over long packets. Howeyer this priority queuing discipline discriminates against long messages.
4. It is desirable to make $L$ sufficiently large that most message lengths will not exceed L. In this case, overheads and occurences of out-of-sequence packet arrivals at the destination are minimized.
5. The larger the number of nodes between source and destination, the shorter the desired value of $L$ in riew of delay reductions via the pipeline effect.

The few quantitative results available suggest that network performance does not change rapidly as the packet length varies about the apparent optimum value [R11, K1, S14].

Transmission of voice via packet switching presents both opportunities and problems [G1, F7]. A' long silent period can be coded by a number representing its length with a resulting reduction in traffic. Data rates of source encoders/decoders could be reduced or expanded in response to network loadings. Packet switched networks may eventually be equipped for multiple addressing of packets; this capability could be extended to voice. Novel multiplexing schemes particularly suited to packet radio systems would be available. Some of these multiplexing techniques are discussed briefly later.

Disadvantages of packetized voice include unpredictable and possibly excessive packet delays, the need to buffer messages prior to both transmission and reassembly, and the reduction in throughput caused by data packet overhead and network control packets. These overheads can constitute up to $90 \%$ of the traffic [K1]. Out-of-order and lost packets
also cause delays in reassembly as well as buffer overflows. Howeyer these two problems are not as severe for voice with its natural redundancy as for data; a voice message could be reassembled using packets available, and an intelligible message would result. The delays themselves could be reduced by not retransmitting voice packets. Delay variations could be reduced by using packet switching to establish a specific transmission route for all voice packets in a given conversation [G1, F7, F8]. Unfortunately these last two "remedies" require different nodal processing of voice and data packets.

A packet radio system would be similar to an SSMA system in that a cellular structure would not be needed. Consequently, virtually all of the SSMA advantages and disadvantages listed in the previous section would also apply to packet radio.

Various multiplexing schemes developed specifically for packet radio have been developed [A8, F8, G4, K1, K6, K7, L9, L10, M7, T4, T5, T6]. The pure ALOAH [A8, K1] technique is simplest; users transmit at any time and if their packets are obliterated by those of other users they retransmit. The advantage of this protocol is its simplicity; the disadvantages are the low throughput whose maximum value is $18.4 \%$ of the traffic offered and the potential collapse of the channel due to a traffic jam caused by more and more frequent collisions. By use of increasing complex protocols, the throughput can be increased and stability problems reduced. Alignment (slotting) of packet transmission times, power control, channel sensing, and reservation schemes are among the available techniques. It is not always easy to determine whether or not the increased cost justifies the increase in throughput.

Further studies on packet switched voice for mobile radio applications are needed and will undoubtedly be forthcoming. The analysis and design methodology developed in this report is applicable, although approximations based on simulations would be required to represent delay vs throughput for some multiplexing protocols. Lost packet probabilities would have to be estimated to calculate SNR ; these estimates would be based in part on the variability in end-to-end packet delay.

## APPENDIX - FM Spectral Calculations

In order to calculate the effects of adjacent-channel interference, $F(f, k \Delta)$ in (5-20) must be obtained. Equation (5-20) requires knowledge of the amplitude probability density of the instantaneous frequency $f_{a}$ which for FM with no pre-emphasis is proportional to the amplitude probability density of speech. Two functions which provide reasonable fits to speech densities $P(\alpha)$ are the Laplacian [C4, S7] and Gamma [C4,R8] functions, defined as follows:

$$
\begin{align*}
& p(x)=\frac{1}{2 b} \exp (-|x-a| / b) \quad \text { (Laplacian) }  \tag{A-1}\\
& p(x)=\frac{c}{2 \Gamma(d)}[c|x|]^{d-1} \exp (-c|x|) \quad \text { (Gamma) } \tag{A-2}
\end{align*}
$$

Non-negative constants $a, b, c$ and $d$ are selected to fit the actual data; $\Gamma(x)$ is the Gamma function defined by $\Gamma(x)=x \Gamma(x-1)$ for any $x>0$. If $x$ is an integer $\Gamma(x)=x!$

Since $p(x)$ in $(A-1)$ and (A-2) is even in $x, J(f)=\overline{G\left(f+f_{a}\right) S\left(f+f_{a}-k \Delta\right)}$
$=\overline{G\left(f-f_{a}\right) S\left(f-f_{a}-k \Delta\right)}$ where $J(f)$ is obtained by convolution as follows:

$$
\begin{equation*}
J(f)=\int_{-\infty}^{\infty} G\left(f-f_{a}\right) S\left(f-k \Delta-f_{a}\right) p\left(f_{a}\right) d f_{a} \tag{A-3}
\end{equation*}
$$

Standard computer routines are available for fast convolution.
It remains to calculate $S(f)$, the power spectral density of the $F M$
signal

$$
\begin{equation*}
s(t)=\sqrt{2 P} \cos \left(2 \pi f_{0} t+\theta+\int_{-\infty}^{t} m(\tau) d \tau\right) \tag{A-4}
\end{equation*}
$$

where $\theta$ is a random phase angle uniformly distributed between zero and $2 \pi$. Unless $m(t)$ is Gaussian calculation of $S(f)$ seems virtually impossible. If
$m(t)$ is Gaussian then [R9, P1, P5].

$$
\begin{align*}
& S(f)=\int_{-\infty}^{\infty} h(\tau) e^{-j 2 \pi f \tau} d \tau  \tag{A}\\
& h(\tau)=\exp \left[-2 \int_{-\infty}^{\infty} M(f)\left(\frac{\sin \pi f] \tau]}{f}\right)^{2} d f\right] \tag{A-6}
\end{align*}
$$

where $M(f)$ is the power spectral density of $m(t)$. The following equation has been obtained as a good approximation to $M(f)$ [J1,F5]:

$$
\begin{equation*}
M(f)=f^{4} /\left(f^{2}+70^{2}\right)\left(f^{2}+180^{2}\right)\left(f^{2}+400^{2}\right)\left(f^{2}+700^{2}\right) \tag{A-7}
\end{equation*}
$$

It is clear that speech is not Gaussian. Unfortunately we cannot calculate $S(f)$ unless we assume that speech is Gaussian. This we do, in view of the fact that the amplitude density of speech bears some resemblance to the Gaussian be11-shape, and in view of the many other approximations and assumptions inherent in developing our analysis and design methodology.

In order to determine $h(\tau)$ in (A-6) we first write $M(f) / f^{2}$ as follows, where $a_{1}=70^{2}, a_{2}=180^{2}, a_{3}=400^{2}$ and $a_{4}=700^{2}$.

$$
\begin{equation*}
M(f) / f^{2}=\sum_{i=1}^{4} A_{i} /\left(f^{2}+a_{i}^{2}\right) \tag{A-8}
\end{equation*}
$$

Constants $A_{i}$ would be selected in the usual ways.

$$
\begin{align*}
& \text { Determination of } h(\tau) \text { now requires evaluation of } \\
& \begin{aligned}
I(a, \tau) & =\int_{-\infty}^{\infty} \frac{(\sin \pi f|\tau|)^{2}}{\left(f^{2}+a^{2}\right)} d f \\
& =\frac{1}{2} \int_{-\infty}^{\infty} \frac{d f}{f^{2}+a^{2}}-\frac{1}{2} \int_{-\infty}^{\infty} \frac{\cos 2 \pi f|\tau|}{f^{2}+a^{2}} d f
\end{aligned} \tag{A-9}
\end{align*}
$$

Closed form expressions for each of the above integrals exist;
thus

$$
\begin{align*}
I(a, \tau) & =2 \pi / a-(2 \pi / a) \exp (-2 \pi a|\tau|) \\
& =(2 \pi / a)(1-\exp (-2 \pi a|\tau|)) \tag{A-11}
\end{align*}
$$

From (A-6) to (A-11), inclusive one obtains

$$
\begin{align*}
h(\tau) & =\exp \left[-2 \sum_{i=1}^{4} A_{i}\left(2 \pi / a_{i}\right)\left\{1-\exp \left(-2 \pi a_{i}|\tau|\right)\right\}\right] \quad(A-12)  \tag{A-12}\\
& =\exp \left[-2 \sum_{i=1}^{4} A_{i}\left(2 \pi / a_{i}\right)+2 \sum_{i=1}^{4} A_{i}\left(2 \pi / a_{i}\right) \exp \left(-2 \pi a_{i}|\tau|\right)\right] \quad(A-13)  \tag{A-13}\\
& =\prod_{i=1}^{4} \exp \left(-b_{i}\right) \prod_{j=1}^{4} \exp \left[b_{j} \exp \left(-2 \pi a_{j}|\tau|\right)\right] \quad(A-14) \tag{A-14}
\end{align*}
$$

where

$$
\begin{equation*}
b_{i}=4 \pi A_{i} / a_{i} \tag{A-15}
\end{equation*}
$$

Substitution of (A-14) into (A-5) yields $S(f)$.
Note that $S(f)$ can be expressed as follows:

$$
\begin{equation*}
S(f)=\left[\prod_{i=1}^{4} \exp \left(-b_{i}\right)\right] H_{1}(f) \otimes H_{2}(f) \otimes H_{3}(f) \otimes H_{4}(f) \tag{A-16}
\end{equation*}
$$

where $\otimes$ denotes convolution and $H_{i}(f)$ is the Fourier transform of $\exp \left[b_{i} \exp \left(-2 \pi a_{i}|\tau|\right)\right]$.

Representation of $\exp \left[b_{i} \exp \left(-2 \pi a_{i}|\tau|\right)\right]$ by an infinite series yields

$$
\begin{equation*}
\exp \left[b_{i} \exp \left(-2 \pi a_{i}|\tau|\right)\right]=\sum_{k=0}^{\infty} \frac{\left[b_{i} \exp \left(-2 \pi a_{i}|\tau|\right)\right]^{k}}{k!} \tag{A-17}
\end{equation*}
$$

It follows from ( $A-16$ ) that


Since each term in the series decreases rapidly as $k$ increases, the first few terms may be used to approximate $H_{i}(f)$ and the error can be calculated. Thus, either (A-18) together with (A-16) or (A-14) together with (A-5) can be used to calculation $S(f)$.

An approach similar to the one leading to (A-18) was used by Ferris [F6] to calculate the spectral characteristics of FDM-FM multiplexed signals.

Techniques are available to replace $S(f)$ by bounds [P1, R9, P5, A9] which, unfortunately are rather complex.

Examination of the above, equations confirms the validity of the comments in Section V-4 regarding the behaviour of $S(f)$ in relation to its mean square bandwidth. Thus, if $M(f)$ in ( $A-7$ ) is multiplied by $V$ so is $\overline{W_{c}{ }^{2}}$ and the coefficients $A_{i}$. From (A-6) and (A-5) one sees that $h(0)=\int_{-\infty}^{\infty} S(f) d f$ is independent of $V$, which means that the area under $S(f)$ is independent of $V$; $V$ increases the bandwidth of $H_{i}(f)$ and $S(f)$ increases in accordance with ( $A-18$ ) and ( $A-16$ ) and $S(0)$ must therefore $S$ decrease.

## REFERENCES

Al L.G. Anderson, "A simulation study of some dynamic channel assignment algorithms in a high capacity mobile telecomunications system", IEEE Trans. Veh. Technol., vol. VT-22-pp. 210-217, Nov. 1973.

A2 N. Abramson, "Bandwidth and spectra of phase - and frequency - modculated waves", IEEE Trans.Commun. Syst., vol. CS-11, pp. 401-414, Dec. 1963.
G.A. Arrendondo and J.I. Smith, "Voice and data transmission in a mobile radio channel at $850 \mathrm{MHz}^{\prime \prime}$, IEEE Trans. Veh. Techno1., vol. VT-26, pp. 88-93, Feb. 1977.

A4 W.L. Aranguren and R.E. Langseth, "Baseband performance of a pilot diversity system with simulated Rayleigh fading signals and cochannel interference", IEEE Trans. Commun., vol. COM-21,pp. 12481257, Nov. 1973.

A5 Anderson J.B. and C.-W.P. Ho, "Architecture and construction of a hardware sequential encoder for speech", IEEE Trans. Commun., vol. COM-25, pp. 703-707, June 1977.

A6 J.B. Anderson and J.B. Bodie, "Tree encoding of speech", IEEE Trans. Inform. Theory, vol. IT-21, pp. 379-387, July 1975.

A7 N. Abramson and W.W. Chu, Ed., Computer-Communication Networks. Englewood Cliffs, N.J.: Prentice-Hall, 1973.
N. Abramson, "The throughput of packet broadcast channels", IEEE Trans. Commun., vol. COM-25, pp. 117-128, Jan. 1977.

A9
V.R. Algazi, "Bounds on the spectra of angle-modulated waves", IEEE Trans, Commun. Technol., vol. COM-16, pp. 561-566, Aug. 1968.
K. Bullington, "Radio propagation for vehicular communications", IEEE Trans. Veh. Technol, vol. VT-26, pp. 295-308, Nov. 1977.
T. Berger and D.W. Tufts, "Optimum pulse amplitude modulation, part I: transmitter receiver design and bounds from information theory", IEEE Trans. Inform, Theory, vol. IT-13, pp. 196-208, Apr. 1967.

B3
D.F. Boyko and P.J. McLane, "Error rate bounds for digital PAM transmission with phase jitter and intersymbol or co-channel interference", IEEE Trans, Commun., vol. COM-25, pp. 536-541, May 1977.

B4
H.O. Burton and D.D. Suillivan,' "Errors and Error Control", Proc. IEEE, vol. 60, pp. 1293-1301, Nov. 1972.

C1 D.C. Cox and D.O. Reudink, "Dynamic Channel reassignment in twodimensional large-scale mobile radio systems", Bell Syst. Tech. J., vol. 51, pp. 1611-1630, Sept. 1972.

C2 D.C. Cox and D.o. Reudink, "Increasing channel occupancy in large scale mobile radio systems: dynamic channel reassignment", IEEE (Joint) Trans. Commun. Veh. Technol., pp. 1302-1306, Nov. 1973.

C3 C. Chayavadhanangkur and J.M. Park, Jr., "Analysis of FM systems with co-channel interference using a click model", IEEE Trans. Commun., vol. COM-24, pp. 903-910, Aug. 1976.
D. Chan and R.W. Donaldson, "Subjective evaluation of pre- and postfiltering in PAM, PCM, DPCM voice communication systems", IEEE Trans.

Commun. Technol., Vol. COM-19, pp. 601-612, Oct. 1971; also in Waveform Quantization and Coding. N.S. Jayant, Ed. New York: IEEE Press, 1976.

C5 D.L. Cohn and J.L. Melsa, "The residual encoder - an improved ADPCM system for speech digitization", IEEE Trans, Commun., vol. COM-23, pp. 935-941, Sept. 1975.

C6 P. Cumminski, "Single integration, adaptive delta modulation", Bell Syst. Tech. J., vol. 54, pp. 1463-1474, Oct. 1975.

C7 K.Y. Chang and R.W. Donaldson, "Analysis, optimization and sensitivity study of differential PCM systems operating on noisy communication channels", IEEE Trans. Commun., vol. COM-20, pp. 338-350, June 1972. R.T. Chien, A.H. Haddad and B. Goldberg, "An analytic error model for real channels", in Conf. Rec.,'IEEE Int. Conf. Commun. Philadelphia, PA, June 1972, pp. (15-7)-(15-12).
D. Chase and L.W. Weng, "Multiple-burst correction techniques for slowly fading channels", IEEE Trans. Inform. Theory, vol. IT-22; . pp. 505-514, Sept. 1976.
R.T. Chien, V.E. Clayton, P.E. Boudreau and R.R. Locke, "Error correction in a radio-based data communication system", IEEE Trans. Commun., vol. COM-23, pp. 458-462, Apr. 1975.
G.R. Cooper and R.W. Nettleton, "A spread spectrum technique for high capacity mobile communications", Proc. 27th Annual Veh. Technol. Conf., Orlando, FA, 1977.

DI B.E. Davis, G.J. Caccapolo and M.A. Chaudry, "An econometric planning model for the American Telephone and Telegraph Company", Bell J. Econ. and Mang. Sci., vol. 4, pp. 29-36, Spring 1973.

D2 B.R. Davis, "FM moise with fading channels and diversity", IEEE Trans. Commun. Tech., vol. COM-19, pp. 1189-1199, Dec. 1972.

D3
J. Durkin, "Computer prediction of service areas for VHF and VHF land mobile radio services", IEEE Trans. Veh. Technol., vol. VT-26, pp. 323-327, Nov. 1977.

D4 R.C. Dixon, Spread Spectrum Systems. New York: Wiley, 1976.
E1 J.S. Engel and M.M. Peritsky, "Statistically-optimum dynamic server assignment in systems with interfering servers", IEEE Trans. Veh. Technol., VT-22, pp. 203-209, Nov. 1973.

E2 R.E. Eaves and A.H. Levesque, "Probability of block error for very slow Rayleigh fading in Gaussian noise", IEEE Trans. Commun., vol. COM-25, pp. 368-374, March 1977.

E3 R.P. Eckert and P.M. Kelly, "Implementing spread spectrum technology in land mobile radio services", IEEE Trans. Commun., vol. COM-25, pp. 867-869, Aug. 1977.

F1. Z.C. Fluhr and E. Nussbaum, "Switching plan for a cellular mobile. telephone system", IEEE Trans. Veh. Technol., vol. VT-22, pp. 197202, Nov. 1973.
D.D. Falconer and F.R. Magee, Jr., "Evaluation of decision feedback equalization and Viterbi algorithm detection for voice-band data transmission - part I', IEEE Trans. Commun., vol. COM-24, pp. 11301138, Oct. 1976; part. II, pp. 1238-1245, Nov. 1976.
J.L. Flanagan, "Computers that talk and listen", Proc. IEEE, vol. 64, pp. 405-415, Apr. 1976.

F4
J.L. Flanagan, Speech Analysis, Synthesis and Reception New York: Springer Verlag, 1972.

Van Nostrand, 1953.
C.C. Ferris, "Spectral Characteristics of FDM-FM signals", IEEE Trans. Commun., vol. COM-16, pp. 233-238, Apr. 1968.
J.W. Forgie and A.G. Nemth, "An effecient packetized voice/data network using statistical flow control', in Conf. Rec., Int. Conf. Commun., Chigaco IL. June 1977.
M.J. Fischer and T.C. Harris, "A model for evaluating the performance of an integrated circuit - and packet-switched multiplex structure", IEEE Trans, Commun., vol. COM-24, pp. 195-203, Feb. 1976. B. Gold, "Digital speech networks", Proc. IEEE, vol. 65, pp. 16361658, Dec. 1977.
F.E. GLave and A.S. Rosenbaum', "An upper bound analysis for keying with co-channel, adjacent-channel and intersymbol interference", IEEE Trans. Commun., vol. COM-23, pp. 586-597, June 1975. E.N. Gilbert, "Capacity of a burst noise channel", Bell Syst. Tech. J., vol. 39, pp. 1253-1265, Sept. 1960.
I. Gitman, R.M. Van Slyke and H. Frank, "Routing in packet-switching broadcast radio networks", IEEE Trans. Commun., vol. COM-24, pp. 926-931, Aug. 1976.
V. Machenburg, B.D. Holm and J.I. Smith, "Data signalling functions for a cellular mobile telephone system", IEEE Trans. Veh. Technol., vol. VT-26, pp. 82-88, Feb. 1977.
A.D. Hall, III, "An overview of economics of scale in existing communications systems", IEEE Trans. Syst., Man and Cybern., vol. SMC-5, pp. 2-14, Jan. 1975.

H3 F. Hansen and F.I. Meno, "Mobile fading - Rayleigh and lognormal superimposed", IEEE Trans, on Veh. Technol., vol. VT-26, pp. 332335, Nov. 1977.

H4 B.A. Hanson and R.W. Donaldson, "Subjective evaluation of an adaptive differential voice encoder with oversampling and entropy coding", IEEE Trans, on Commun., vol. COM-26, Feb. 1978, in press.
A. Habibi, "Survey of adaptive image coding techniques", IEEE Trans. Commun., vol. COM-25 pp. 1275-1285, Nov. 1977.

H6 T.S. Huang, "Coding of two-tone images", IEEE Trans. Commun., vol. COM-25, pp. 1406-1425, Nov. 1977.

H7 P.M. Hopkins, "A unified analysis of pseudonoise synchronization by envelope correlation", IEEE Trans. Commun., vol. COM-25, pp. 770778, Aug. 1977.

H8 P.M. Hopkins and R.S. Simpson, "Probability of error in pseudonoise (PN) - modulated spread spectrum binary communication system", IEEE Trans. Commun. Vol. COM-23, pp. 467-472, Apr. 1975.

J1 W.C. Jakes, Jr. Ed., Microwave Mobile Communications. New York: Wiley, 1974.
N.S. Jayant, "Step size transmitting differential coders for mobile telephony", Bell Syst. Tech. J., vol. 54, pp. 1557-1582, Nov. 1975.

K1 L. Kleinrock, Queueing Systems, Volume 2: Computer Applications. New York: Wiley, 1976.

K2 L. Kleinrock, Queueing Systems, Volume 1: Theory. New York: Wiley, 1975.

K3 S. Kozono and K. Watanabe, "Influence of environmental buildings on UHF land mobile radio propagation", IEEE Trans. Commun., vol. COM-25, pp. 1133-1143, Oct. 1977.

K4 F.A. Kamoun, "Design Considerations for Large Computer Communication Networks", UCLA-ENG Rept. 7642, Los Angeles, CA, Apr. 1976:

K5 P.A. Kullstam, "Spread spectrum performance analysis in arbitray interference", $\mathbb{I E E E}$ Trans. Commun., vol. COM-25, pp. 848-853, Aug. 1977.
L. Kleinrock and F.A. Tobagi, "Packet switching in radio channels: part 1: carrier sense multiple access modes and their throughputdelay characteristics", IEEE Trans. Commun., vol. COM-23, pp. 14001416, Dec. 1975.

K7.
L. Kleinrock and S. Lam, "Packet switching in a multiaccess broadcast channel: performance evaluation", IEEE Trans. Commun., vol. COM-23, pp. 410-423, Apr. 1975.

L1 B.P. Lathi, An Introduction to Random Signals and Communication Theory. Scranton, PA: International Textbook Co., 1968.

L2 R.N. Lane, "Spectral and economic effeciencies of land mobile radio systems", IEEE Trans. Veh. Technol., vol. VT-22, pp. 93-104, Nov. 1973.

L3 S.A. Leslie, "Computer model for paging system coverage", IEEE Trans. Veh. Technol., vol. VT-26, pp. 172-180, May 1977.

14 R.W. Lucky, J. Salz and E.J. Weldon, Principles of Data Communication. New York, N.Y.: McGraw-Hill, 1968.
S. Lin, An Introduction to Error-Correcting Codes. Englewood Cliffs, N.J.: Prentice-Hall, 1973.

L6 T.L.R. Lei, N. Scheinberg and D.L. Schilling, "Adaptive delta modulation systems for video encoding", IEEE Trans. Commun., vol. COM-25, pp. 1302-1315, Nov. 1977.

L7 W.C. Lindsay and M.K. Simon, Telecommunications Systems Engineering, Englewood Cliffs, N.J.: Prentice-Hall, 1973.

L8 W.C. Lindsay, Synchronization Systems in Communication and Control. Englewood Cliffs, N.J.: Prentice-Hall, 1972.

L9 S. Lam and L. Kleinrock, "Packet switching in a multiaccess broadcast channel: dynamic control procedures", IEEE Trans. Commun., vol. COM-23, pp. 891-904, Sept. 1975.

L10 K.D. Levin, "The overlapping problem and performance degredation of mobile digital communication systems", IEEE Trans. Commun., vol. COM-23, pp. 1342-1347, Nov. 1975.

M1 J.J. Mikulski, "A system plan for a $900-\mathrm{MHz}$ portable radio telephone", IEEE Trans. Veh. Technol., vol. VT-26, pp. 76-81, Feb. 1977.
J. Martin, Future Developments in Telecommunications. Englewood Cliffs, N.J.: Prentice-Hall, 1977. of speech encoding systems", Bell Syst. Tech. J., vol. 54, pp. 16151636, Nov. 1974.

01
W.H. Melody, "Relations between public policy issues and economies of scale", IEEE Trans. Syst., Man and Cybern., vol. SMC-5 pp. 14-22, Jan. 1975.
L.H. Mantell, "Some estimates of returns to scale in the telephone industry", IEEE Trans. Syst., Man and Cybern, vol. SMC-5, pp. 23-29, Jan. 1975.
J.H. McMahon, "Interference and propagation formulas and tables used in the Federal Communications Commission Spectrum Management Task Force land mobile frequency assignment mode1", IEEE Trans. Veh. Techno1., vol. V「-23, pp. 129-134, Nov. 1974.
J.E. Mazo and J. Salz, "Theory of error rates in digital FM", Bell Syst. Tech. J., vol. 45, pp. 1511-1535, Nov. 1966.
J.J. Metzner, "On improving utilization in ALOAH networks", IEEE Trans, Commun., vo1. COM-24, pp. 447-448, Apr. 1976.

1 T. Nomura and N. Yoshikawa, "Multiple radio zone plans in mobile radio systems", IEEE Trans. Veh. Technol., vol. VT-25, pp. 68-74, May 1975.
P. Noll, "A comparative study of various quantizing schemes for speech encoding", Bell Syst. Tech. J., vol. 54, pp. 1597-1614, Nov. 1975.

3 P. Noll, "Effects of channel errors on the signal-to-noise performance

1 G.D. Ott, "Vehicle location in cellular mobile radio systems", IEEE Trans, Veh. Technol., vol. VT-26, pp. 43-46, Feb. 1977.
V.K. Prabhu, "Spectral density bounds on an FM wave" IEEE Trans. Commun., pp. 980-984. Oct. 1972.

Q1 S.U.H. Qureshi, "Fast start-up equalization with periodic training sequences", IEEE Trans. Inform. Theory, vol. IT-23, pp. 553-563, Sept. 1977.

R1
Y. Okumura, E. Ohmori, T. Kawano and K. Fukuda, "Field Strength and its variability in VHF and UHF land-mobile radio service", Rev. Elec. Comm. Lab., vol. 16, Sept. - Oct. 1968; also in: Communication Channels: Characterization and Behaviour, B. Goldberg, Ed. New York, N.Y.: IEEE Press, 1976.
V.K. Prabhu and H.E. Rowe, "Spectral density bounds of a PM wave", Be11 Syst. Tech. J., vol. 48, pp. 769-811, March 1969.
V.K. Prabhu and L.H. Enloe, "Interchannel interference consideration in angle modulated systems", Be11 Syst. Tech. J., vol. 48, pp. 2333-2357, Sept. 1969.
V.K. Prabhu, "PSK-type modulation with overlapping baseband pulses", IEEE Trans. Commun., vol. COM-25, pp. 980-990, Sept. 1977.
M.B. Pursley, "Performance evaluation for phase-coded spreadspectrum multiple-access communication - part I: system analysis", IEEE Trans. Commun, vol. COM-25, pp. 795-799, Aug. 1977.
L. Roberts, "Extensions of packet communication technology to a hand held personal terminal", in AFIPS Cong. Proceedings, SJCC, vo1. 40, Montvale, N.J.: AFIPS Press 1972, pp. 295-298, also in Advances in Computer Communications, W.W. Chu, Ed., Dedham Mass.: Artech House, 1976.
M.A. Rich and M. Schwartz, "Buffer sharing in computer-communication network nodes", IEEE Trans. Commun., vol. COM-25, pp. 958-971, Sept. 1977.

R11 R.D. Rosner, R.H. Bittell and D.E. Brown, "A high throughput packetswitched network technique without message reassembly", IEEE Trans. Commun., vol. COM-23 pp. 819-828, Aug. 1975.
A.C. Stocker, "Application of the small-zone concept to police communication in population centers: an exploratory study", IEEE Trans. Veh. Technol., vol. VT-25, pp. 50-54, May 1976.
L. Schiff, "Traffic capacity of three types of common-user mobile radio communication systems", IEEE Trans. Commun. Technol. vol. COM-18, pp. 12-21, Feb. 1970.
P.A. Sammuelson, Economics, 3rd Canadian Ed. New York: McGraw-Hill 1971.
H. Suzki, "A statistical model for urban radio propagation", IEEE Trans. Commun., vol. COM-25, pp. 673-679, May 1977.
0. Shimbo, "Threshold characteristics of FM signals demodulated by an FM discriminatur", IEEE Trans, Inform. Theory, vol. IT-15, pp. 540-549, Sept. 1969.
L. Schiff, "Statistical suppression of interference with diversity in a mobile-radio environment", IEEE Trans. Veh. Technol., vol. VT-21, pp. 121-128, Nov. 1972.
B. Smith, "Instantaneous companding of quantized signals", Bell Syst. Tech. J., vol. 36, pp. 653-709, May 1957.
J. Salz, "Optimum mean-square decision feedback equalization", Bell Syst. Tech. J., vol. 52, pp. 1341-1374, Oct. 1973.
M. Sambur, "An effecient linear prediction vocoder", Bell Syst. Tech. J., vol. 54, pp. 1693-1724, Dec. 1975. R.A. Scholtz, "The spread spectrum concept", IEEE Trans. Commun., vol. COM-25, pp. 748-756, Aug. 1977.

S11 M.K. Simon, "Noncoherent pseudonoise code tracking performance of spread spectrum receivers", IEEE Trans. Commun., vol COM-25, pp. 327-345, Mar. 1977.
M. Schwartz, R.R. Boorstyn and R.L. Pickholtz, "Terminal-oriented computer-communication networks", Proc. IEEE, vol. 60, pp. 14081423, Nov. 1972; also in Advances in Computer Communications, W.W. Chu, Ed. Dedham, MA. Artech House, 1976.

S13 M. Schwartz, Computer-Communication Network Design and Analysis. Englewood Cliffs, N.J.: Prentice-Hall, 1977.
C.A. Sunshine, "Interprocess Comunication Protocols for Computer Networks", Stanford Elect. Lab. Tech. Rept., Stanford Univ., Stanford, CA, Dec. 1975.

T1 K. Tsujimura and M. Kuwabara, "Cordless telephone system and its propagation characteristics", IEEE Trans. on Veh. Technol., vol. VT-26, pp. 367-371, Nov. 1977.

Taub and D.L. Schilling, Principles of Communication Systems. New York: McGraw-Hill, 1971.
R.M. Tobin and K. Yao, "Upper and lower bounds for coherent phase-shift-keyed (CPSK) systems with cochannel interference", IEEE Trans. Commun., vol. COM-25, pp. 281-287, Feb. 1977.
F.A. Tobagi and L. Kleinrock, "Packet switching in radio channels: part 4 - stability considerations and dynamic control in carrier sense multiple access", IEEE Trans. on Commun., vol. COM-25, pp. 1103-1119, Oct. 1977.

IEEE Trans. Commun., vol. COM-24, pp. 832-845, Aug. 1976.
F.A. Tobagi and L. Kleinrock, "Packet switching in radio channels: Part 2 - the hidden terminal problem and the carrier sense multiple access mode with a busy tone", IEEE Trans. Commun., vol. COM-23, pp. 1417-1433, Dec. 1975.

V1 M.D. Vinod, "Non-homogeneous production functions and applications to telecommunications", Bell J. of Econ. and Manag. Sci, vol. 3, pp. 531-543, Fall 1972.

W1 J.F. Weston and E.F. Brigham, Managerial Finance, 4th Edition. Holt, Rinehart and Winston, Hinsdale, Illinois, 1972.
L. Weiss, I Pas and D.L. Schilling, "Videi encoding using an adaptive digital delta modulator with overshoot suppression", IEEE Trans. Commun., vol. COM-23, pp. 905-920, Sept., 1975.
B. Yaged, Jr., "Economies of scale, networks, and network cost elasticity", IEEE Trans. Syst., Man and Cybern., vol. SMC-5, pp. 30-40, Jan. 1975.
J. Yan and R.W. Donaldson, "Subjective effects of channel transmission errors on PCM and DPCM voice communication systems", IEEE Trans. Commun., vol. COM-20, pp. 281-290, June 1972.

Z1 L.E. Zegers and C.B. Dekker, "A comparison of digital transmission techniques for standard FM mobile radio sets", IEEE Trans. Commun., vol. COM-25, pp. 364-368, March 1977.

