STUDIES OF SYSTEMS-RELATED AND INHERENT DEFECTS IN COMMUNICATIONS NETWORKS

PHASE II

ANALYSIS OF QAM RECEIVER PERFORMANCE IN THE PRESENCE OF AN FM INTERFERENCE SIGNAL

by

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CHAPTER I

INTRODUCTION

The world of telecommunications is developing at a breathtaking rate. In the space of a century, telecommunications networks have become one of our most complex creations, with ramifications that extend worldwide. Up until the 1960s, over 90 per cent of telecommunications systems operated on the old principles of analogue communication; in particular, microwave systems are often of the FDM/FM type. Since then, of course, the new digital technology has become increasingly prevalent.

The predominance of digital systems is the result of a combination of factors, the most important among them doubtless being low manufacturing costs and good performance against interference. By the 1990s, according to BNR projections, 90 per cent of telecommunications networks will be digital systems.

With this dazzling development, the amount of data traffic on these systems is increasing by leaps and bounds. In order to meet the demand, very high output systems must be built; since the majority of frequency bands are relatively congested, digital systems having a high spectral efficiency will have to be introduced. The necessity for this has led companies in the communications industry to propose systems that are able to operate almost in the same frequency bands as the old analogue systems. The newest in this field is the RD-4A system from Northern Telecom Limited, which has been available on the international market since 1984. This system utilizes a 64-level quadrature modulation, operating at a rate of 20 Mbauds per second.

Given the very high speed of these new systems, their performance will become relatively sensitive to systems-related and inherent defects. To ensure proper operation, precautions will have to be taken in several respects: care in the design of electronic components, making transmitting and receiving antennas highly directional, and suitable separation of carrier frequencies. The latter aspect comes under the authority of the Department of Communications. At the request of the Systems Technology task force, headed at the time by Mr. G. De Couvreur and subsequently by Mr. M. Gaudreau, the Department has begun to examine the influence of FM systems on the performance of Quadrature Amplitude Modulation (QAM) systems. This influence depends naturally on the separation between the two carriers, their relative power, and their bandwidth.

In this report, the fundamental points of our problem are presented in Chapter II: modeling of QAM systems, passage of very broad bandwidth FM signals through a QAM receiver, method of calculating probability of error by moments, analysis of accuracy; numerical results are presented in Chapter III; and the report concludes with Chapter IV, in which a discussion of the work carried out is presented.

CHAPTER II

PASSAGE OF AN FM SIGNAL THROUGH A QAM SYSTEM

II.1 Introduction

In this chapter is presented the methodology used to solve the problem of interference caused by passage of an FM signal through a QAM receiver. In this methodology, we require an accurate model to describe the operation of a receiver in which behaviour toward FM interference will be examined. As this interfering signal is of very broad bandwidth, we will be able to justify use of the wide-sense stationary assumption to simplify analysis.

II.2 Quadrature Amplitude Modulation

Quadrature amplitude modulation is achieved by combining two carriers in quadrature, each amplitude-modulated. Let m and n be the number of levels on each carrier; we then obtain $M = m \times n$ possible states for the resulting two-dimensional signal. We will limit this report to the case where $m=n=4$, which is the case of 16-state modulation. Levels on each carrier are equidistant, giving Fig. 2.1.

The sixteen points of the constellation correspond to 16 four-bit words. These are formed by means of 4 binary trains grouped in twos. Let these trains be A_1, B_1, A_2 and B_2. Initially we have the formation of two-bit words A_1B_1 and A_2B_2, which will serve to modulate the two carriers by matching each value of the word with a signal level, with the following correspondence:

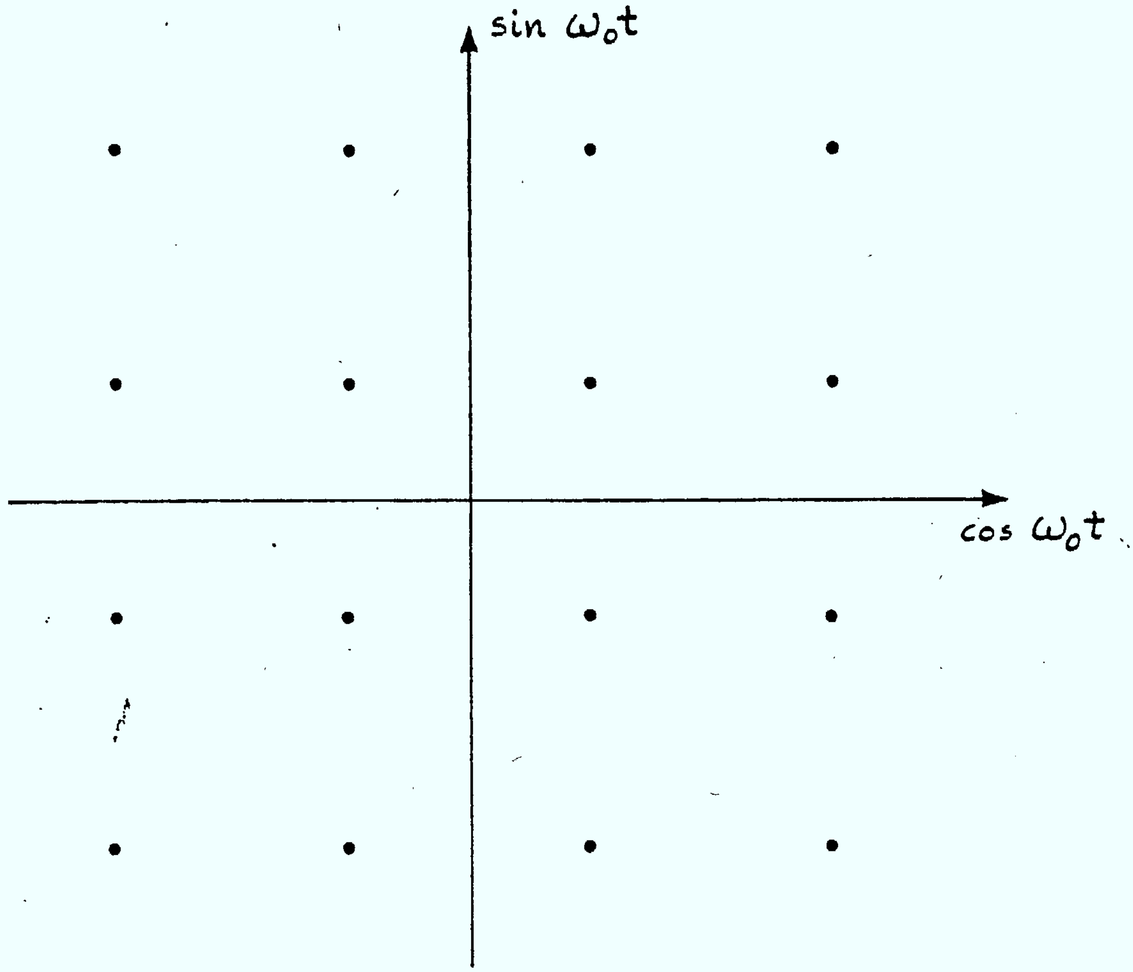


Fig. 2.1 - QAM-16 Constellation

A_i	B_i	Signal Level
1	1	3
0	1	1
0	0	-1
1	0	-3

Matching is done in such a way that words A_iB_i follow a Gray code, i.e., a code such that two adjacent words differ by only one bit. Addition of the two carriers thus determines 16 states for the words A_1B_1, A_2B_2. In practice, this type of modulation was selected for two reasons: first, performance is quite acceptable compared to the optimum (loss of 0.2 dB for a constraint on mean power); second, its implementation is simplified at the outset because it does not require any special technology.

II.3 Structure of QAM Modulator

The first component of the modulator is a coder that transforms the four binary trains into two quaternary trains and associates with each a signal with four amplitude levels. These signals attack the IF modulator following broadband filtering that serves to limit the spectrum. The two modulated carriers are then added to yield a 16-state signal. It will be noted that the two carriers in quadrature are obtained from a single oscillator, so as to retain phase coherence between the two channels. These operations are shown in Fig. 2.2

At the coder output, we find signals on channels x and y with four amplitude states determined by combinations A_iB_i (i = 1, 2). Matching is effected following a Gray code shown in Fig. 2.3. Decision areas are limited by the abscissas and ordinates -2,0 and 0,2. Because of coding, crossing a threshold results in only a single error.

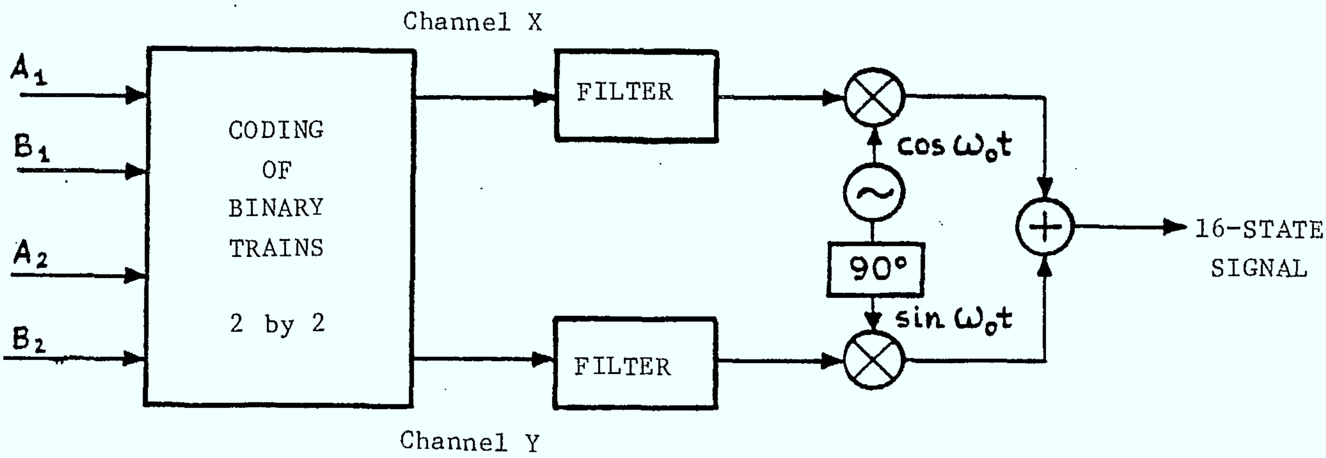


Fig 2.2 - Structure of QAM Modulator

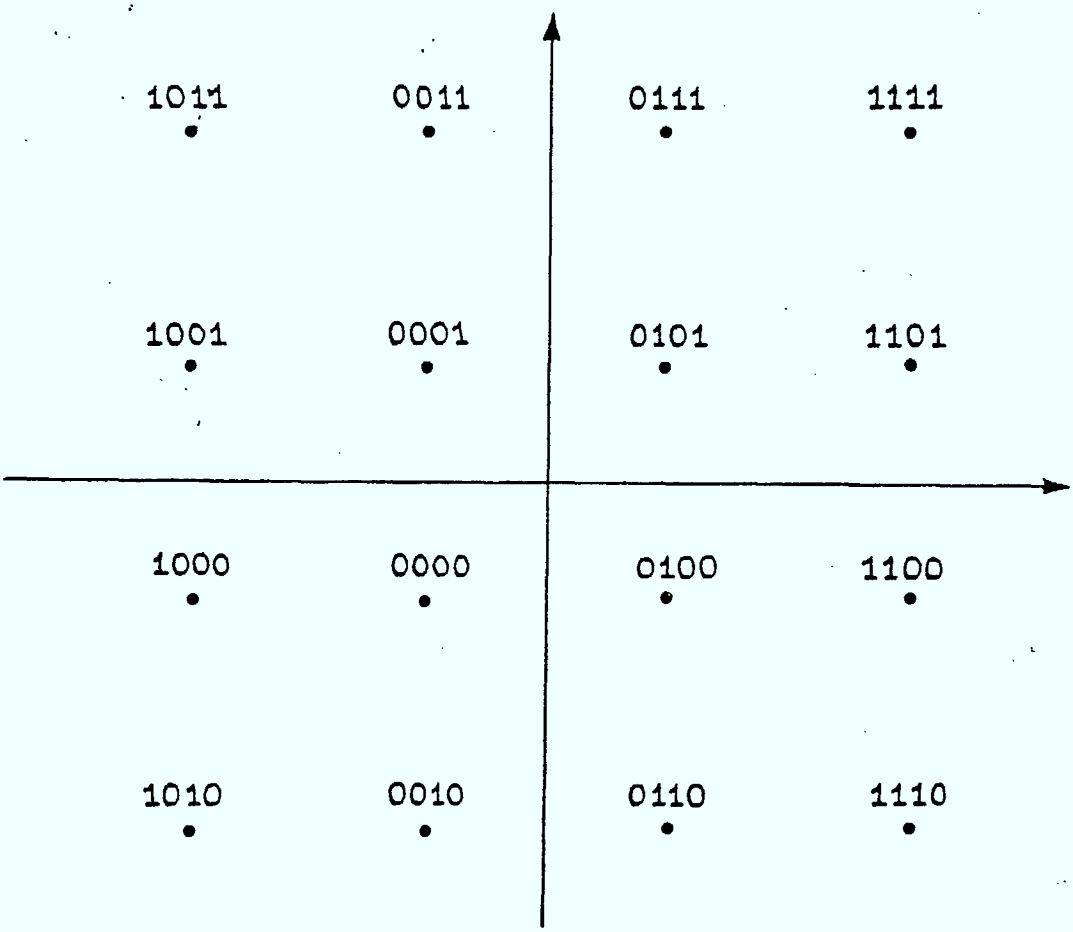


Fig. 2.3 - Gray Code in QAM

II.4 Statement of Problem

II.4.1 Diagram of Transmission Chain

Fig. 2.4 shows the overall appearance of component elements in the transmission system and the various defects encountered. The 16-state signal at the modulator output attacks a microwave oscillator that will transpose the signal into VHF. The need to place several channels side by side leads to filtering in order to limit their individual spectrums and fix their position in the transmission frequency band. This operation is achieved by FDM, Frequency Division Multiplex. Following passage of the VHF signal through the propagation medium, in this case free space, it is filtered to separate the channels, then transposed to IF for demodulation. The demodulator followed by the decoder yields an estimate of the original binary trains and serves to obtain phase references of the carrier and clock necessary for coherent demodulation and for operation of the decision circuits. Ideally, all of these operations will enable us to reconstitute the transmitted information signal correctly. Unfortunately, this is not the case in practice, as the IF signal undergoes a number of destructive effects on reception:

a) thermal noise (hence Gaussian white noise);
b) interference from adjacent channel signals in the multiplexer (hence, interfering FM signal);
c) intersymbol interference caused by filtering;
d) propagation defects (slow or rapid fading).

The study presented in this report considers only the simultaneous presence of thermal noise, FM interference and filters.

II.4.2 Modeling

Taking into account the preceding assumptions, the transmission chain, defined as the succession of various components encountered by the signal, may be represented by Fig. 2.4. All modeling is carried out by means of the equivalent low-pass filter concept [10].

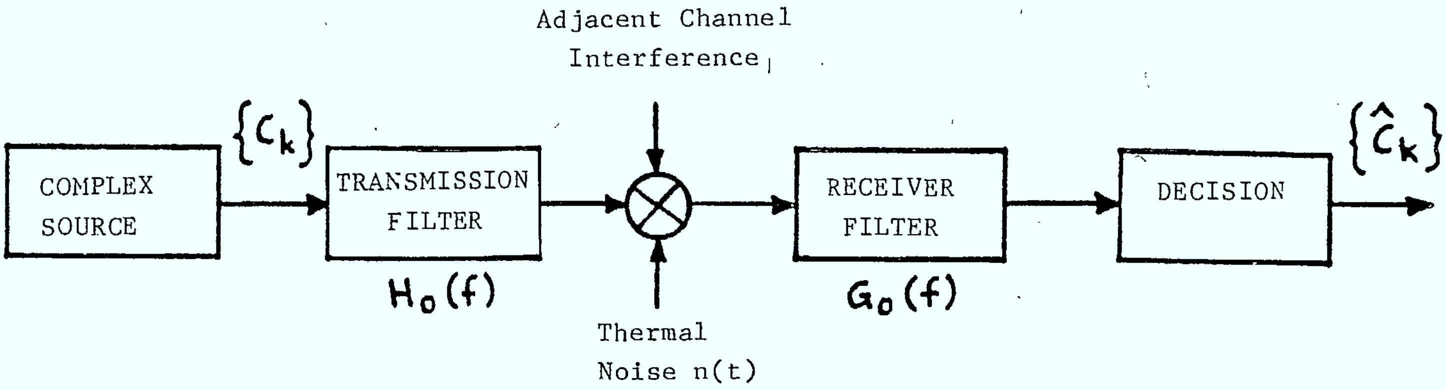


Fig. 2.4 - Equivalent Low-Pass System

Complex Source:

Let $\{a_k\}$ and $\{b_k\}$ be the quaternary messages at the modulator input. These are binary trains at the coder output. Let $\chi(t)$ and $s(t)$ be respectively the low-pass envelope and the modulator output signal. Generally speaking, we have:

$$\chi(t) = \begin{cases} 1 \text{ for } 0 < t < T \\ 0 \text{ otherwise} \end{cases} \qquad (1)$$

where T is the duration of a baud.

$$s(t) = A \sum_{k=-\infty}^{\infty} \{a_k \chi(t-kT)\cos\omega_o t + b_k \chi(t-kT)\sin\omega_o t\} \qquad (2)$$

Equation (2) can be put into a much more compact complex form:

$$s(t) = \mathrm{Re}\{s_e(t)e^{j\omega_c t}\} \qquad (3)$$

where

$$s_e(t) = \sum_{k=-\infty}^{\infty} (a_k + jb_k)\chi(t-kT) \qquad (4)$$

$s_e(t)$ is called the complex envelope of $s(t)$. We are then led to define an equivalent source of complex 16-state messages:

$$C_k = a_k + jb_k$$

Filters:

The transmission filter $H_o(f)$ brings together the VHF filters, video filters, and transmission medium. The video filter, of very broad bandwidth compared to the others, can be ignored, as its presence is unnecessary from a theoretical point of view. The receiving filter, placed at the demodulator input, has a transfer function $G_o(f)$. Its essential function is to limit reception interference and noise. It acts in the IF range, after a receiving preamplifier.

The signal thus sees an overall complex gain filter $H_o(f)G_o(f)$. Let $P(t) + jQ(t)$ be the response of this filter to the $\chi(t)$ signal. In the absence of any interference or additive noise, the demodulator input signal may be written in the following complex form:

$$x(t) = A \sum_{k=-\infty}^{\infty} (a_k + jb_k)\left[P(t-kT) + jQ(t-kT)\right] \tag{5}$$

To simplify writing the formula, let:

$$P_k(t) = P(t-kT) \quad \text{and} \quad Q_k(t) = Q(t-kT)$$

It follows that:

$$x(t) = \sum_{k=-\infty}^{\infty} (a_k + jb_k)\left[P_k(t) + jQ_k(t)\right]$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left[a_k P_k(t) - b_k Q_k(t)\right] + j\left[a_k Q_k(t) + b_k P_k(t)\right] \tag{6}$$

If the overall filter H_oG_o possesses parity of fading and group propagation time, i.e., its transfer function $H_o(f)G_o(f)$ represents a Hermitian symmetry around 0, let:

$$H_o(f)G_o(f) = \overset{*}{H_o}(-f)\overset{*}{G_o}(-f)$$

The term $Q_k(t)$ will then be identically zero. In this case, crosstalk becomes zero, and the demodulator input signal is simplified as:

$$x(t) = \sum_{k=-\infty}^{\infty} (a_k + jb_k)P_k(t) \quad (7)$$

Thermal Noise:

Thermal noise appears in the receiving preamplifier. It is additive, centred and flat-spectrum in a broad frequency band before the receiver band.

We will call the DSB power spectral density of the noise $N_o/2$. The noise power σ^2 affecting the probability of error is given by:

$$\sigma^2 = N_o T \int_{-\infty}^{\infty} |G_o(f)|^2 df \quad (8)$$

where $G_o(f)$ is the transfer function of the receiving IF filter. It is thus clear that an appropriate choice of $G_o(f)$ can limit additive noise power in the receiver.

Carrier-to-Noise Ratio:

This ratio, also referred to as signal-to-noise ratio, is the parameter that most affects the probability of error in reception, and is defined as the ratio of energy of a baud over power spectral density of the noise. Given the hypotheses of the problem stated by scientific delegates G. De Couvreur and M. Gaudreau, we may immediately adopt:

$H_o(f) = 1$

The modulated signal is then:

$$s(t) = A \sum_{k=-\infty}^{\infty} (a_k \cos\omega_o t + b_k \sin\omega_o t)\chi(t-kT)$$

where A is the separation between two adjacent amplitudes on the same channel. The mean energy per baud (or per transmitted word) is then:

$$E_m = \frac{A^2}{16} \left(\sum_{k=1}^{4} \frac{a_k^2 + b_k^2}{2} \right)$$

with

$a_k = \pm 1, \pm 3$ and $b_k = \pm 1, \pm 3$.

where

$E_m = 5\ A^2$

If we are interested in peak energy, we may adopt the definition:

$$E_c = A^2 \operatorname*{Max}_{k=1,4} \left\{ \frac{a_k^2 + b_k^2}{2} \right\} = 9\ A^2$$

Based on these values, we may define the ratios:

$$\frac{E_m}{N_o} \quad \text{and} \quad \frac{E_c}{N_o}$$

which lead back to terms to power with the help of duration of a baud T:

$$\frac{E_m}{N_o} = \frac{E_m/T}{N_o/T} \quad \text{and} \quad \frac{E_c}{N_o} = \frac{E_c/T}{N_o/T}$$

These are then ratios of signal power (average or peak) over noise power in the Nyquist band. Further in the report, results will be expressed as a function of the signal-to-noise ratio given by $\frac{E_m/T}{N_o/T}$, which locates S/N on the abscissas of the probability of error curves.

Automatic Gain Control

The purpose of automatic gain control (AGC) is to compensate for signal strength losses due to random fading the signal may suffer as it passes along the transmission chain. Practically speaking, this amounts to adjusting the gain to a constant level at the demodulator input. In analytical calculations, this operation enables the channel response to be normalized with respect to the maximum level sample. This normalization is aimed at obtaining results in a universal form.

II.4.3 Passage of FM Signal Through QAM Receiver

In the model shown in Fig. 2.4, interference results from the presence of one or more FM signals of very broad bandwidth. This interfering signal passes through the receiving filter $G_o(f)$ before being sampled at the same instant t_o as the digital QAM signal. It is clear that its effect on the decision device input is of the additive type, as is the effect of thermal noise. For the case where the interfering signal has a high modulation coefficient, the wide-sense stationary assumption may be used [6] to study the effect of filtering. It is known that in such conditions [5, 7], over a very short time interval, the signal behaves as a sinusoidal signal with a frequency that matches the instantaneous frequency of the FM signal. If the baseband signal is a Gaussian process, the spectrum of the very broad bandwidth FM signal takes Gaussian forms. This result is known as the Woodward Theorem [4]. According to these arguments, the distribution $p_B(f)$ of the instantaneous frequency of the FM signal admits the statistical form of the modulation signal. Thus:

$$p_B(f) = \frac{1}{\sqrt{2\pi}\,\Delta f_B} \exp\left[-\frac{(f-f_B)^2}{2\Delta f_B^2}\right] \qquad (9)$$

where is the effective deviation (i.e., RMS value) of the instantaneous frequency from the FM carrier. Under the wide-sense stationary assumption, the filter output is thus a sinusoidal signal the amplitude of which is amplified by the gain $|G_o(f)|$ and the signal may be any phase, without reference to the instant of sampling. In this way, the component due to the FM interference source at the decision device input is finally of the form:

$$I = B \cos \theta \qquad (10)$$

where B is the FM signal amplitude, amplified by the gain G(f), and θ a random variable uniformly distributed between 0 and 2π. As has just been observed, the wide-sense stationary assumption enables the problem to be simplified considerably.

In addition to taking account of this simplification, we also consider another, slightly more complex assumption: for a very broad bandwidth interference, its influence is equivalent to another interference of the same power spectral density. At first glance, this assumption would seem to have a noticeable weakness: the correlation effect between spectrum component phases is overlooked. However, it will be realized that, physically speaking, these phases could never be very coherent, as their total power is imposed by overall power spectral density. Taking into account this constraint on total power, the equivalence thus represents a noticeable avenue: an ensemble of harmonic signals could be constructed, situated at any frequencies whatsoever, as shown in Fig. 2.5.

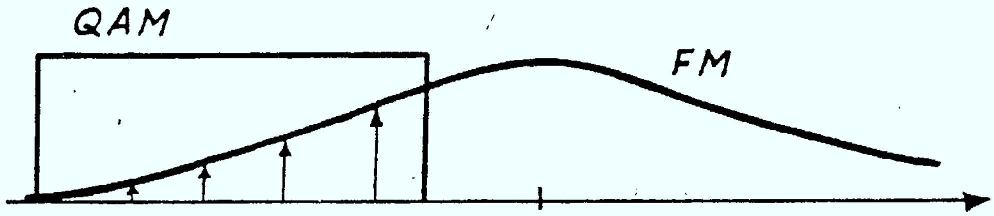


Fig. 2.5 - Spectrum Considerations of FM Signal
With Respect to QAM System

The ensemble of sinusoidal signals has amplitudes determined by the power spectral density of the FM signal, with the restriction that total power is equal to the partial power of the interference signal passing through the filter $G_0(f)$. The results obtained by the two approaches, "wide-sense stationary" and "power equivalence", apparently represent the two possible limits to overall performance of the probability of error. Because of the central limit theorem, the power equivalence assumption will sometimes be referred to as the Gaussian assumption.

II.5 Calculation of Probability of Error

II.5.1 General Form

We will now place ourselves in the last part of the transmission chain, at the output of filter $G_0(f)$; the observed signal is written as:

$$x(t) = \sum_{k=-\infty}^{\infty} \{a_k P_k(t) - b_k Q_k(t)\} + j\{b_k P_k(t) + a_k Q_k(t)\}$$

$$+ B_x(t) + jB_y(t) + U(t) + jV(t) \qquad (11)$$

The demodulator followed by the sampler examines the real and imaginary portions of x(t) at instants $t_k = t + kT$. At instant τ, we have:

$$\mathrm{Re}\{x(t)\} = \sum a_k P_k(\tau) - b_k Q_k(\tau) + I_x(\tau) + U(\tau)$$

$$\mathrm{Im}\{x(\tau)\} = \sum\{b_k P_k(\tau) + a_k Q_k(\tau)\} + I_y(\tau) + V(\tau)$$

We may also separate $x(\tau)$ into five terms:

$$x(\tau) = (a_0 + jb_0)P_0(\tau) - (b_0 - ja_0)Q_0(\tau)$$

$$+ \sum_{k \neq 0}\{a_k P_k(\tau) - b_k Q_k(\tau)\} + j\{a_k Q_k(\tau) + b_k P_k(\tau)\}$$

$$+ B_x(\tau) + jB_y(\tau) + U(\tau) + jV(\tau) \qquad (12)$$

These different terms represent respectively:

- usable signal
- crosstalk signal, which translates coupling between the two channels x and y

- intersymbol interference $II(\tau)$
- interference $B(\tau)$
- additive noise.

The sum of $II(\tau)$ and $B(\tau)$ may be put into the form:

$$I(\tau) = II(\tau) + B(\tau) = X(\tau) + jY(\tau)$$

X and Y are random variables centred by the same law of symmetrical distribution F(I) around 0. $P_k(\tau)$ and $Q_k(\tau)$ are determinist terms, and a_k and b_k independent random variables taking four equally probable states.

To each point (i), representing one of 16 states, is associated a decision area D_i, in the sense of a maximum likelihood criterion. The probability of error on the states is written:

$$P_e = \sum_{i=1}^{16} P_i P\{r \epsilon D_i | i \text{ be transmitted}\}$$

where P_i is the a priori probability of state (i), and r the observation (signal at receiver output). As the 16 states are assumed to be equally probable, we have $P_i = 1/16$, and P_e is simplified as:

$$P_e = \frac{1}{16} \sum P\{r \epsilon D_i | i \text{ be transmitted}\}$$

In the presence of interference I, P_e becomes:

$$P_e = \int_{II} \sum P_i P\{r \epsilon D_i | i \text{ be transmitted and } I = b\} dF(b)$$

where II is the ensemble of all possible values $I(\tau)$ may take. The problem thus consists of evaluating the conditional probability of error on transmission of (i) and for a given value of I = X + jY. We will in fact directly evaluate the probability of error for each binary train and not for the overall signal. These are measurements of estimated trains, which are realized in practice. We will see, nonetheless, how to link the error probabilities on elementary trains to overall probability. Let us first normalize the received signal with respect to $P_o(\tau)$:

$$x_N(\tau) = (a_o + jb_o) - (b_o - ja_o)\frac{Q_o(\tau)}{P_o(\tau)} + \frac{X+jY}{P_o(\tau)} + \frac{U+jY}{P_o(\tau)}$$

In order not to burden the notation, we will reutilize $x(\tau)$ in place of $x_N(\tau)$, which amounts to setting $P_o(\tau) = 1$. Let:

$$x(\tau) = (a_o + jb_o) - (b_o - ja_o)Q_o(\tau) + (X + jY) + (U + jV)$$

Let us set $\alpha = -b_oQ_o(\tau) + X$ and $\beta = a_oQ_o(\tau) + Y$.

Let us call P_e^A1 (respectively P_e^B1) the probability of error on Train A1 (respectively B_1). We may demonstrate (see Appendix 1) that:

$$P_e^A1 = \frac{1}{4}\sum_{b_o=\pm 1,\pm 3}\frac{1}{2}\int_{-\infty}^{\infty}\operatorname{erfc}\left(\frac{A+\alpha}{\sigma\sqrt{2}}\right)dF(x)$$

$$P_e^B1 = \frac{1}{2}P_e^A1$$

As has been discussed previously, intersymbol interference and crosstalk are not important components in our work; we may then choose the optimum case for our analysis, i.e., the case of Hermitian symmetry filters. It follows that:

$$Q(t) \equiv 0$$

and then:

$$P_e^A1 = \frac{1}{2}\int_{-\infty}^{\infty}\operatorname{erfc}\left(\frac{A+X}{\sigma\sqrt{2}}\right)dF(x) \tag{13}$$

$$P_e^B1 = \frac{1}{2}P_e^A1$$

In the absence of any form of interference, the results are simplified to yield:

$$P_e^{A_1} = \frac{1}{2}\,\mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right)$$

$$P_e^{B_1} = \frac{1}{2}\,P_e^{A_1}$$

This amounts to assuming that the channel is an ideal (Nyquist) channel. The latter expressions of probability then serve to calculate performance in the presence of additive Gaussian noise only. For this ideal channel, however, which completely eliminates intersymbol interference, the problem of calculating the interfering signal clearly becomes more complex; for this reason we will henceforth take into account intersymbol interference caused by a Butterworth filter with linearly equalized phase.

II.5.2 Calculation of Integral by Ho and Yeh Method

In expression (12), we have to calculate the integral:

$$J = \int_{-\infty}^{\infty} \mathrm{erfc}\left(\frac{A+X}{\sigma\sqrt{2}}\right) dF(X) \qquad (14)$$

The Ho and Yeh method consists of developing the erfc function in Taylor series in the vicinity of point $A/\sigma\sqrt{2}$:

$$\mathrm{erfc}\left(\frac{A+x}{\sigma\sqrt{2}}\right) = \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) + \sum_{k=1}^{\infty} \left(\frac{x}{\sigma\sqrt{2}}\right)^k \frac{1}{k!} D_k$$

with

$$D_k = \left.\frac{d^k \mathrm{erfc}(x)}{dx^k}\right|_{x = \frac{A}{\sigma\sqrt{2}}} \qquad (15)$$

Hence:

$$J = \int_{-\infty}^{\infty} \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) dF(x) + \sum_{k=1}^{\infty} \frac{1}{k!} D_k \int_{-\infty}^{\infty} \left(\frac{x}{\sigma\sqrt{2}}\right)^k dF(x)$$

This expression may be simplified by noting that $\int_{-\infty}^{\infty} dF(x) = 1$ and setting $M_k = \int_{-\infty}^{\infty} x^k dF(x)$:

$$J = \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) + \sum_{k=1}^{\infty} \frac{1}{k!} D_k M_k \frac{1}{(\sigma\sqrt{2})^k}$$

Note immediately that D_k is linked to the Hermite Polynomials $H_n(x)$ [8]:

$$\frac{d^k}{dx^k} \mathrm{erfc}(x) = (-1)^k \frac{2}{\sqrt{\pi}} H_{k-1}(x)\exp(-x^2)$$

Finally:

$$J = \mathrm{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) + \frac{2}{\pi} \exp\left(-\frac{A^2}{2\sigma^2}\right) \sum_{k=1}^{\infty} \frac{1}{k!(\sigma\sqrt{2})^k} H_{k-1}\left(\frac{A}{\sigma\sqrt{2}}\right) M_k \qquad (16)$$

Parameters M_k in (16) are called the k order moments of the random variable X.

Calculation of moments M_k

Let X and F(x) be respectively a random variable and its probability distribution function. By definition, its k order moment is given by:

$$M_k = \int_{-\infty}^{\infty} x^k dF(x)$$

These moments may be obtained from its characteristic function $\phi_X(\omega)$ which is the Fourier transform of its probability density:

$$\phi_X(\omega) = E\{e^{j\omega X}\} = \int_{-\infty}^{\infty} e^{j\omega x} dF(x)$$

Deriving $\phi_X(\omega)$ successively, we obtain:

$$\phi_X(\omega) = 1 + j\omega M_1 + \ldots + (j\omega)^k \frac{M_k}{k!} + \ldots$$

Thus, if we arrive at calculation of the characteristic function $\phi_X(\omega)$ of the random variable X, the sum of all interference present on channel X, i.e.:

$$X = \sum_{k \neq 0} a_k P(\tau - kT) + B_X(\tau) \tag{17}$$

Since $B_X(\tau)$ and all variables a_k are statistically independent, the characteristic function is quite simply the product of all characteristic functions corresponding to each term of equation (17). In their research, Ho and Yeh proposed a recursive method to calculate successive moments. Unfortunately, this recursivity is very sensitive to accumulated rounding errors; thus it very often leads to even-order negative moments, which contradicts the mathematical definition [9]. This phenomenon requires that moments be calculated by the direct method; as this is an exhaustive method, it clearly takes up more computer time. Before presenting this calculation method, let us observe that the random variables present in (17) are all statistically independent; it is then known that the moments of their sum are combinations of their moments. We need only consider X as the sum of two statistically independent random variables X_1 and X_2 having even probability densities: it follows immediately that:

$$X = X_1 + X_2$$

$$E\{X^k\} = E\{(X_1 + X_2)^k\}$$

$$= \sum_{\ell=0}^{k} C_\ell^k \, E\{X_1^\ell\} E\{X_2^{k-\ell}\}$$

As X_1 and X_2 are distributed symmetrically, we have:

$$E\{X_1^{2\ell+1}\} = E\{X_2^{2\ell+1}\} = 0$$

which immediately implies that uneven-order moments $E\{X^{2\ell+1}\}$ are identically zero. Only the even-order moments $M_{2k} = E\{X^{2k}\}$ exist and are given by:

$$M_{2k} = \sum_{p=0}^{2k} C_{2p}^{2k} M_{2p}^{1} M_{2k-2p}^{2} \qquad (18)$$

where M_{2p}^{1} and M_{2k-2p}^{2} are respectively 2p order moments and 2k-2p order moments of random variables X_1 and X_2. Equation (18) is thus the key to the direct method of calculating moments of a sum of independent random variables.

Thus, in order to apply the direct method to study of the influence of a very broad bandwidth FM signal, we need only calculate moments of intersymbol interference and moments of the FM signal separately. The latter are calculated very simply by means of the model proposed by equation (10). Moments of intersymbol interference are dealt with in the following section.

II.5.3 Direct Method of Calculating Moments [2]:

Let U be the number of samples of channel response at an interval $\chi(t)$, the low-pass envelope of a baud, as described by (1). There are thus U - 1 terms of intersymbol interference; as the transmitted signal may take one of the four values (±1, ±3), the number N of possible configurations is:

$$N = 4^{U-1}$$

The intersymbol interference random variable X is written:

$$X = \sum_{k \neq 0} a_k P_k$$

where:

$$P_k = P(\tau+kT)$$

Moments $M2p$ of X become:

$$M_{2p} = \sum_{a_k} P[X_k = X]X^{2p}$$

$$M_{2p} = \frac{1}{N}\sum_{a_k} (\sum_{k\neq 0} a_k P_k)^{2p}$$

As the calculation of M_{2p} in this case is independent for each component, accuracy may be chosen arbitrarily.

It now remains to prove convergence of the method and to deduce the criteria enabling this to be verified. Through the use of computers, we are led to make truncations at various levels in calculating numerical series. Taking into account parity of dF(x), the integral J is written:

$$J = \operatorname{erfc}\left(\frac{A}{\sigma\sqrt{2}}\right) + \frac{2}{\sqrt{\pi}} \exp\left(-\frac{A^2}{2\sigma^2}\right) \sum_{k=1}^{\infty} \frac{M_{2k}}{(2k)!} \times \frac{H_{2k-1}\left(\frac{A}{\sigma\sqrt{2}}\right)}{(\sigma\sqrt{e})^{2k}}$$

Let U_k be the general term for the series. It follows that:

$$\frac{U_{k+1}}{U_k} = \frac{M_{2k+2}}{M_{2k}} \times \frac{H_{2k+1}\left(\frac{A}{\sigma\sqrt{2}}\right)}{H_{2k-1}\left(\frac{A}{\sigma\sqrt{2}}\right)} \times \frac{1}{(\sigma\sqrt{2})^2} \times \frac{1}{(2k+1)(2k+2)} \qquad (19)$$

For very large values of k, we have [8]:

$$\left|H_{2k-1}(x)\right| < \frac{2^k}{\sqrt{2}} (2k-3)!! \sqrt{2k-1} \exp\left(\frac{x^2}{2}\right)$$

where:

$$(2k-3)!! = (2k-3)(2k-1)\ldots 3.1$$

Thus equation (19) is simplified to yield:

$$\left|\frac{U_{k+1}}{U_k}\right| \simeq \frac{M_{2k+2}}{M_{2k}} \frac{(2k-1)!!}{(2k-3)!!} \frac{1}{(\sigma\sqrt{2})^2} \frac{1}{(2k+1)(2k+2)} \sqrt{\frac{2K+1}{2K-1}} \qquad (20)$$

Also, we have:

$$M_{2k} = \int_{-\infty}^{\infty} X^{2k} dF(x) < \int_{-\infty}^{\infty} [Sup(x)]^{2k} dF(x)$$

where:

$$X = \sum_{\ell \neq 0} a_\ell P(\tau-kT)$$

hence:

$$Sup(x) = 3 \sum_{\ell \neq 0} \left|P(\tau-\ell T)\right|$$

and then:

$$\frac{M_{2k+2}}{M_{2k}} < \left(3 \sum_{\ell \neq 0} \left|P(\tau-\ell T\right|\right)^2$$

Equation (20) becomes:

$$\left|\frac{U_{k+1}}{U_k}\right| < \left(\frac{3 \sum_{\ell \neq 0} \left|P(\tau-\ell T)\right|}{\sigma\sqrt{2}}\right)^2 \sqrt{\frac{2k-1}{2k+1}} \frac{1}{k+1}$$

The Alembert criterion ensures convergence for:

$$3 \sum_{\ell \neq 0} \left|P(\tau-\ell T)\right| < \sigma\sqrt{2}$$

This condition is realized only if the channel displays relatively low distortion. Practically speaking, this convergence is linked to accuracy of the method, which is represented by error R_N due to truncation of the series beginning at the Nth term:

$$R_N = \sum_{k=N}^{\infty} \frac{M_{2k}}{(2k)!} \frac{H_{2k-1}(\frac{A}{\sigma\sqrt{2}})}{(\sigma\sqrt{2})^{2k}} \frac{2}{\sqrt{\pi}} \exp(-\frac{A^2}{2\sigma^2})$$

To study the behaviour of R_N, we will utilize the increases of M_{2k} and $H_{2k-1}(x)$ as previously, replacing the factorial for the Sterling approximation:

$$k! \sim k^k e^{-k} \sqrt{2\pi K}$$

It follows that:

$$R_N < \sqrt{\frac{2}{\pi}} \exp(\frac{-A^2}{4\sigma^2}) \sum_{k=N}^{\infty} (\frac{3\sum_{\ell} |P(\tau-kT)|}{\sigma\sqrt{2}\ \sqrt{k}})^{2k} \times \frac{1}{\sqrt{2K-1}} \frac{1}{k^k e^{-k}\sqrt{2\pi k}} \qquad (21)$$

Since:

$$\sqrt{2\pi N} \leq \sqrt{2\pi k}$$

equation (21) may be rewritten as:

$$R_N < \sqrt{\frac{2}{\pi}} \exp(\frac{-A^2}{4\sigma^2}) \sum_{k=N}^{\infty} (\frac{3\sum_{\ell \neq 0} |P(\tau-\ell T)|}{\sigma\sqrt{2k}})^{2k} \times \frac{1}{e^{-k}} \frac{1}{\sqrt{2N-1}\ \sqrt{2\pi N}} \qquad (22)$$

By setting $k = p + N$, the series of (22) becomes:

$$\frac{1}{e^{-N}} (\frac{3\sum_{\ell \neq 0} |P(\tau-\ell T)|}{\sqrt{N}\ \sigma\sqrt{2}})^{2N} \sum_{p=0}^{\infty} [(\frac{3\sum_{\ell \neq 0} |P(\tau-\ell T)|}{\sqrt{N}\ \sigma\sqrt{2}})^2 \frac{1}{e^{-1}}]^p \qquad (22')$$

The form of (22') is a geometric series in which the result is well known: thus, the upper limit of R_N defined by (22) is obtained as:

$$R_N < \sqrt{\frac{2}{\pi}} \frac{\exp(\frac{-A^2}{4\sigma^2})}{N!\ \sqrt{2N-1}} \frac{\overline{M}_N}{(\sigma\sqrt{2})^{2N}}$$

where:

$$\overline{M_N} = \frac{\left(3 \sum_{\ell \neq 0} \left|P(\tau - \ell T)\right|\right)^{2N}}{1 - e\left(\frac{3 \sum_{\ell \neq 0} \left|P(\tau - \ell T)\right|}{\sqrt{N}\ \sigma\sqrt{2}}\right)^2} \qquad (23)$$

This increase in truncation error produced on the computer, at the same time as calculation of the truncated moments, will thus serve a practical purpose as a criterion for stopping in the calculation of intersymbol interference. Thus, the series is stopped when the relative error reaches 10^{-5} in our program.

II.6 Conclusion

We have presented quite fully in this chapter the methodology enabling performance of a QAM receiver in the presence of an FM interference signal to be analyzed. The method consists of considering this interference to be simply an additive component at the decision device input. Thus, the method proposed by Ho and Yeh to study BPSK systems may be adapted to resolving our problem. However, the recursivity of their method is very sensitive to truncation error, leading us to calculate moments of overall interference directly. This interference, the sum of a number of independent random variables, may be analyzed systematically. It is of interest to note that the accuracy of this direct method is controllable by use of a previously defined quantitative criterion.

In spite of a fairly high calculation time, the flexibility of the method allows interference of all kinds to be taken into account, a noticeable advantage over other known methods in the literature. Operation of this method and programming are dealt with in the following chapter.

CHAPTER III

PRESENTATION OF RESULTS AND DISCUSSION

III.1 Method of Moments

III.1.1 Introduction

The beginnings of our research were directed to development of a PASCAL program to calculate the probability of error in a QAM receiver by the Ho and Yeh method [1].

This method, which will be called the method of moments, was developed by its authors to calculate the probability of error in the presence of ISI. It has been emphasized, however, [2] that the method is sufficiently flexible to permit calculation of P_e in the presence of one or even more than one kind of interference. In the latter case, on the assumption that the different kinds of interference are independent of each other, we need only calculate moments for each kind of interference separately, then combine them using an appropriate formula.

To check proper operation of the programs developed, we calculated P_e in the presence of ISI, then with sinusoidal interference, and compared our results with published results [3]. Once this stage was completed, we next undertook to calculate P_e in the presence of FM interference, as we had proposed to do in the preliminary phase.

III.1.2 Calculation of Moments of ISI

III.1.2.1 Method of Calculation

Intersymbol interference (ISI) appears in reception with transmission of several consecutive symbols, and results from the fact that channel response is not completely zero at multiples of the sampling period. It is a random variable in the form:

$$X(t_o) = \sum_{i \neq 0}^{\infty} a_i P(t_o - iT) \qquad \text{(path in phase)}$$

where p(t) is the real portion of the low-pass channel response at an interval (or any other form of pulse), and a_i is the symbol transmitted at instant $t_o - iT$, for which possible values depend on the number of QAM new and are equally probable. We wish to calculate the 2k order moments of X. Hence:

$$E\{X^{2k}\} = E\left[\sum_{i \neq 0}^{\infty} a_i P(t_o - iT)\right]^{2k}$$

We did this calculation by the direct method, which consists of averaging all possible values of X, even though there are other methods available. In practice, infinite summation is truncated at U terms, so as to minimize the number of calculations. However, we must be certain that convergence of moments is reached. The calculation is made without too much difficulty if the different channel response samples decrease rapidly. If this is not the case, the huge number of calculations may render the task impossible. If we take a fifth-order Butterworth filter and a rectangular pulse, 17 samples are sufficient to permit convergence of moments. In spite of this, the number of calculations is still considerable. For example, if we take the QAM-64, we have $8^{16} = 2.8 \times 10^{14}$ values of X to average. This number is still too high for results to be produced in a reasonable length of time. Fortunately, it is possible to reduce calculation time by taking into account the fact that the different symbols transmitted are independent. If we separate the variable X into two independent variables X_1 and X_2, each having

U-1/2 terms, each of which can be separated in turn into two other variables, we have for the calculation of moments:

$$E\{X^{2k}\} = E\{(X_1 + X_2)^{2k}\} = \sum_{j=0}^{k} C_{2k}^{2j} \, E\{X_1^{2j} X_2^{2k-2j}\}$$

and

$$E\{X_1^{2j} X_2^{2k-2j}\} = E\{X_1^{2j}\} \, E\{X_2^{2k-2j}\}$$

as X_1 and X_2 are independent. We may then use this formula to combine the moments of X_1 and X_2 or to combine the moments of any kinds of interference whatsoever, provided they are independent. For example, using this formula, we may without difficulty combine ISI with a sinusoidal interference, as we shall see in the following section.

III.1.2.2 Description of QAM

Calculation of ISI by the direct method and of P_e by the method of moments has been achieved by the QAM program. A simplified flow chart for the program is shown in Fig. 3.1. Results of these initial trials were compared with those given in [3] for the QAM-4, 16, 36 and 64, with a fifth-order Butterworth filter and a rectangular pulse. Convergence of moments was checked for the specific case discussed. However, if one wished to change the filter rate or the pulse, it would have to be checked each time that convergence had been reached. If it is not, the number of channel response samples will have to be increased, and consequently calculation time would increase proportionally. However, procedures for calculating ISI moments will have to be modified each time. The QAM program program prints out two values of P_e. P_{E1} is the probability of error in the presence of Gaussian noise only, and P_{E2} the probability of error with Gaussian noise and ISI. These results depend on three parameters: BT, N and SNR. BT is the filter bandwidth normalized to 1/T. N is the parameter that selects the type of QAM utilized, where $M = (2N)^2$ and M is the number of states. Finally, SNR is the ratio of mean power of the QAM signal to the noise power in dB.

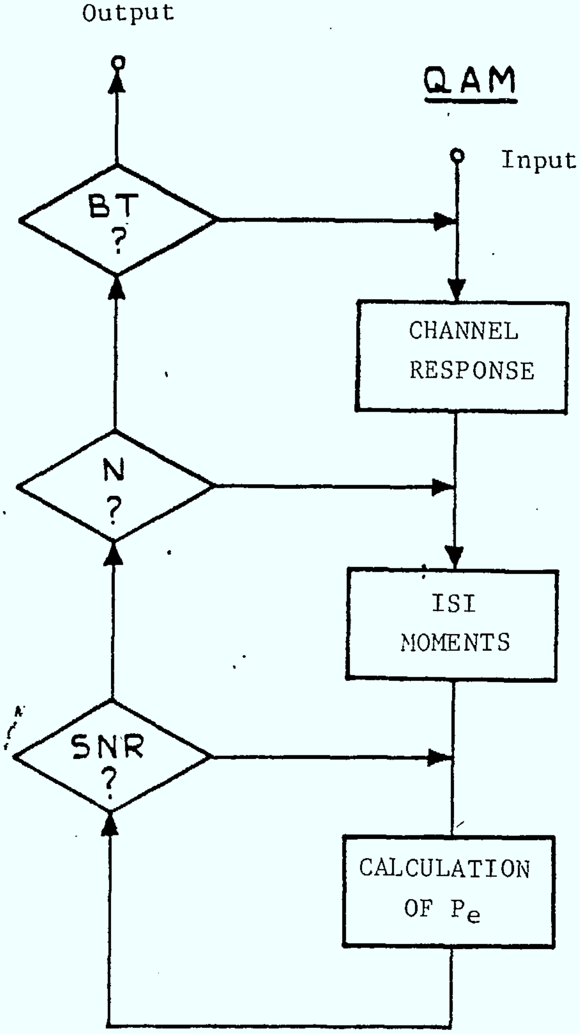


Fig. 3.1 - QAM Flow Chart

$$SNR = 10 \log_{10} \frac{p_m}{N_o/2T}, \quad \text{where} \quad p_m = \frac{A^2}{T} \frac{((2N)^2-1)}{3}$$

Calculating moments of ISI for modulation types QAM-4, 16, 36 and 64 is done by procedures MOM4, MOM16, MOM36 and MOM64, using the direct method as described. Calculation of channel response is done numerically by the Romberg integration method, by functions REP, ROM and FREQ. These functions result in the following integral:

$$s(kT) = 2 \int_0^L \left(\frac{\sin\pi\Omega}{\pi\Omega}\right)\left(\frac{1}{(1+(2\Omega/BT)^{10})^{\frac{1}{2}}}\right) \cos 2\pi k\Omega \, d\Omega$$

where L is chosen so as to limit errors and calculation time. Accuracy of the results is fixed in ROM→ at 10^{-12} or by the number of iterations (15). This acuracy may be modified if a saving in calculation time is desired. In addition, the channel response rate may be modified by altering the function to be integrated, which is in FREQ. In this function, the integration variable is normalized to 1/T and $B = 2f_c$. As concerns calculation of P_e with ISI (calculation of series), this is realized in HERMITE. This function requires that moments of ISI up to a certain 2K order be calculated beforehand. Thus, the FIN (END) parameter must be chosen, determining a sufficiently large number of moments to be calculated so as to permit convergence of the series, for which accuracy has been fixed at 10^{-5}. The HERMITE function yields the following calculation:

$$y = \frac{e^{-x^2}}{\sqrt{\pi}} \sum_{k=1}^{N} E_k M_{2k}$$

where

$$E_k = \frac{x^{2k}}{(2k)!} \left[\left(1 - \frac{4k-5}{2x^2}\right)E_{k-1} - \frac{(k-2)}{(k-1)} E_{k-2}\right]$$

with

$$E_1 = x^3 \quad \text{and} \quad E_2 = \frac{x^4}{24}(8x^3 - 12x)$$

The argument x is given by:

$$x = \frac{Ap_o}{\sqrt{2}\ \sigma}$$

A is calculated from SNR, and σ^2 is the noise power at the decision device input. P_o is the channel response value at the instant of sampling. Thus, by combining the following equations:

$$SNR = 10 \log_{10} \frac{P_m}{N_o/2T}, \quad \text{where} \quad P_m = \frac{A^2}{T} \frac{((2N)^2-1)}{3}$$

and

$$\sigma^2 = \frac{N_o}{2} (BT)(kT)$$

we obtain:

$$x = P_o \left[\frac{3 \times 10^{SNR/10}}{2((2N)^2-1)BT \cdot kT}\right]^{\frac{1}{2}}$$

BT is the normalized bandwidth, and kT is the channel filter noise band. For a Butterworth filter, we have for kT:

$$kT = \frac{\pi}{2n\sin(\pi/2n)}$$

if n = 5, kT = 1.0166.

Note in passing that there is an optimal BT where P_e is minimal. In the case of a fifth-order Butterworth filter and a rectangular pulse, we find BT = 1.05. This value will recur often in our results. We would also add that this value of BT is optimal only for this type of filter and pulse. For example, with an ideal filter, we obtain BT = 1.0. It is possible as well to change the value of the order of the filter, which is set at 5 by the parameter z. Be sure in each case to remember to alter the value of kT accordingly. It should also be ensured that convergence of moments is reached in each case. The trials we ran revealed that a Butterworth filter higher than 50th order was necessary to be equivalent to an ideal filter as concerns ISI.

III.1.3 Calculation of Moments of Sinusoidal Interference

III.1.3.1 Method of Calculation

With the programs developed up to now, in order to calculate the probability of error in the presence of sinusoidal interference, we needed only to know the moments of this interference.

Let us assume this interference at the decision device input is in the form:

$I = B\cos\theta$, where θ is equally distributed.

The 2k order moments of I may be easily calculated by:

$$M_I^{2k} = E\left\{B^{2k}\cos^{2k}\theta\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} B^{2k}\cos^{2k}\theta \, d\theta$$

$$M_I^{2k} = \frac{B^{2k}(2k)!}{2^{2k}(K!)^2} \quad \text{and} \quad M_I^{2k+2} = \frac{B^2}{2}\,\frac{(2k+1)}{(k+1)}\,M_I^{2k}$$

This calculation is done in the MOMBS procedure, and calculation of P_e in the presence of ISI and this interference is done in the QAMBS program.

III.1.3.2 Description of QAMBS

The QAMBS program, as may be seen by examining the listings in the Appendix and the flow chart (Fig. 3.2), is essentially the QAM program to which the MOMBS procedure has been added. This procedure, as is known, calculates moments of sinusoidal interference. The calculation is followed by the combination of ISI moments and sinusoidal moments in the UNION procedure. After this point, calculation continues as in QAM.

QAMBS

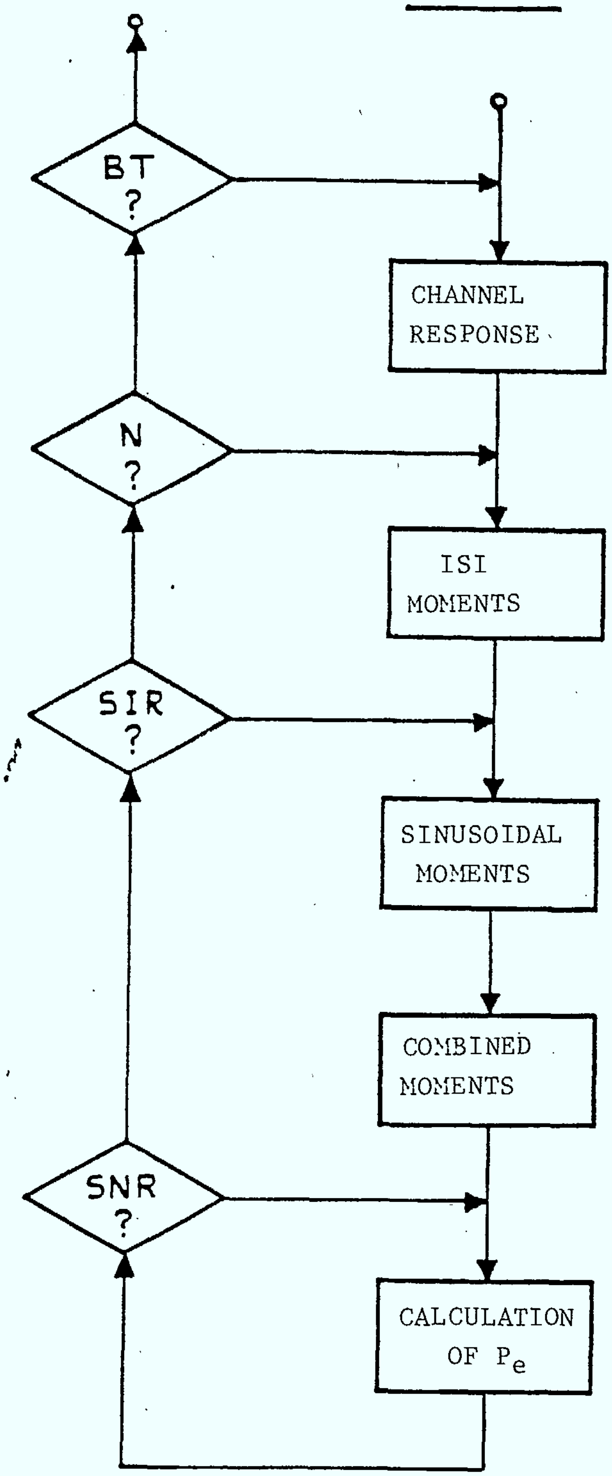


Fig. 3.2 - QAMBS Flow Chart

In the MOMBS procedure, calculation of moments necessitates knowing B^2. This value is determined from the SIR, signal to interference ratio in dB.

$$SIR = 10 \log_{10} \frac{P_m}{B^2/2T}, \quad \text{where} \quad P_m = \frac{A^2}{T} \frac{((2N)^2-1)}{3}$$

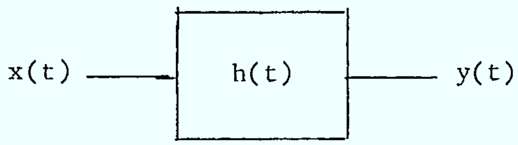
Hence:

$$B^2 = \frac{2((2N)^2-1)*10^{-SIR/10}}{3}$$

III.2 Moments of FM Interference

III.2.1 Introduction

With the programs developed up to the present time as a tool, we dwelled at some length on the problem of calculating moments of an FM interference. Before describing the first method we used in our research, we would like to emphasize the difficulty involved in calculating FM moments in an exact way. The difficulty arises from the fact that in order to calculate the response of a linear system to a non-Gaussian random process, a thorough knowledge of statistics of all orders is generally necessary. Let us assume the following linear system:



If we calculate the nth order statistic of output y(t), we obtain [4]:

$$E\{y(t_1)y(t_2)\ldots y(t_n)\} = E\{x(t_1)x(t_2)\ldots x(t_n)\}*h(t_1)\ldots *h(t_n)$$

where * is the convolution operator. This expression represents a considerable amount of calculating, putting any explicit solution out of reach of the majority of computers. When x(t) is Gaussian, knowing $E\{x(t_1)x(t_2)\}$ is sufficient for the calculation of statistics of any order of y(t). In addition, the properties of a Gaussian type function permits predicting the result of the calculation without doing any calculating. The convolution of a Gaussian function with any function whatsoever will yield another Gaussian function for which the only parameters to be determined are the mean and the variance. Of course the properties of an FM signal do not enable us to arrive at such a simplification, even if in theory it is possible to calculate moments of any order of an FM signal by non-linear transformation.

It would thus appear necessary to resort to simplifying assumptions that will enable asymptotically valid solutions to be obtained or which represent an error limit of some kind.

The first assumption we studied was the wide-sense stationary assumption. This assumption is valid asymptotically when the modulating FM signal varies very slowly compared to the receiver's sampling period. Thus, seen from the receiver, FM interference appears as a sinusoidal interference in which frequency and amplitude are constant over one or more sampling periods, but vary slowly with the statistics of the modulating signal.

The second assumption on which we worked was the Gaussian assumption. This assumption represents the limit case, or, as we shall see, the upper limit of a very large sum of different independent interferences. If the power spectral density of our FM signal is broken down into a very large number of narrow bands, the result is a sum of sinusoidal interferences whose amplitudes will be weighted by the value of the power spectral density at the frequency of each little band. Now, if the phase of each of these bands is independent of the others, the result at the limit is a Gaussian interference.

III.2.2 Wide-Sense Stationary Assumption

III.2.2.1 Method of Calculation

This assumption has been used by Morinaga and Namekawa [5] for the case of multiple FM interference in a PSK receiver. However, no discussion of the validity of such an assumption was presented.

Thus, if we start with the wide-sense stationary assumption, we will have a signal at the decision device input in the form:

$$s(kT) = KG(f_i)\cos\theta$$

where G(f) represents the frequency response of the receiver and θ is equally distributed. The random variable f_i represents the instantaneous low-frequency response of the FM signal, for which the statistics depend on the modulating signal. We chose a Gaussian distribution for f_i. This remains a realistic choice, and enables calculations to be simplified. In addition, G(f) may be chosen to simplify calculations even more. The simplest case is the one where G(f) is an ideal filter. In this case, the random variable $G(f_i)$ becomes a binary variable, taking the value 1 or 0 depending on whether f_i is inside the receiver's pass band or not. The procedure that follows is very simple:

1) calculate P_e with no interference (P_1);
2) calculate P_e with sinusoidal interference $I = K\cos\theta$: (P_2);
3) take the weighted average of the two values.

If $P_k = P_r(|f_i| < B/2)$, then we have:

$$P_e = p_k P_2 + (1-p_k)p_1 = P_1 + P_k(p_2-p_1) \qquad p_2 > P_1$$

P_k is calculated by:

$$P_k = \frac{1}{2}\left[\operatorname{erfc}\left(\frac{\tilde{f}+B/2}{\sqrt{2}\,\Delta f}\right) - \operatorname{erfc}\left(\frac{\tilde{f}-B/2}{\sqrt{2}\,\Delta f}\right)\right]$$

where $\tilde{f}$: frequency separation between FM and QAM carriers,
Δf : RMS frequency deviation of FM signal.

For purposes of calculation, $\tilde{f}$ and Δf are normalized to B/2, and

III.2.2.2 Numerical Difficulties

As can be seen, the resulting calculation is very simple. By contrast, the use of an ideal filter, while facilitating calculation of FM interference, complicates calculation of ISI. It will be necessary to check that convergence of moments of ISI has in fact been reached for this new type of filter. The decrease of channel response samples is very slow, and convergence is possible in practice, with a not too large number of samples, only for BT = 1.0. In addition, for this type of filter and a rectangular pulse, a relatively high level of ISI interference is obtained as compared to a real situation (see Fig. 3.3). We were able to verify, by calculating P_e for different orders of Butterworth filter, that as the order of the filter tends toward infinity, ISI increases in a constant manner and reaches its maximum value for an ideal filter (see Fig. 3.4).

Our subsequent efforts were directed toward reducing this level of ISI, while still covering our ideal filter. Our tests bore mainly on the type of pulse at transmission. We expected a priori that use of a cosine-squared pulse would diminish ISI as compared to a rectangular pulse. In fact, results demonstrated the contrary, as ISI was higher in the case of the cosine-squared pulse. The result could have been foreseen by examining the spectral rate of each of these pulses.

In both cases, we have a spectrum similar in form (see Fig. 3.5). We expect ISI to be lower if H(BT/2) is nearly unity. If $H(\Omega)$ is constant in the BT band, ISI will be zero. It can be verified that:

H(BT/2) = 0.64	rectangular pulse
H(BT/2) = 0.50	cosine-squared pulse with recovery over two intervals.

Thus, ISI will be higher for a cosine-squared pulse than for a rectangular pulse. This result obviously applies only for an ideal filter.

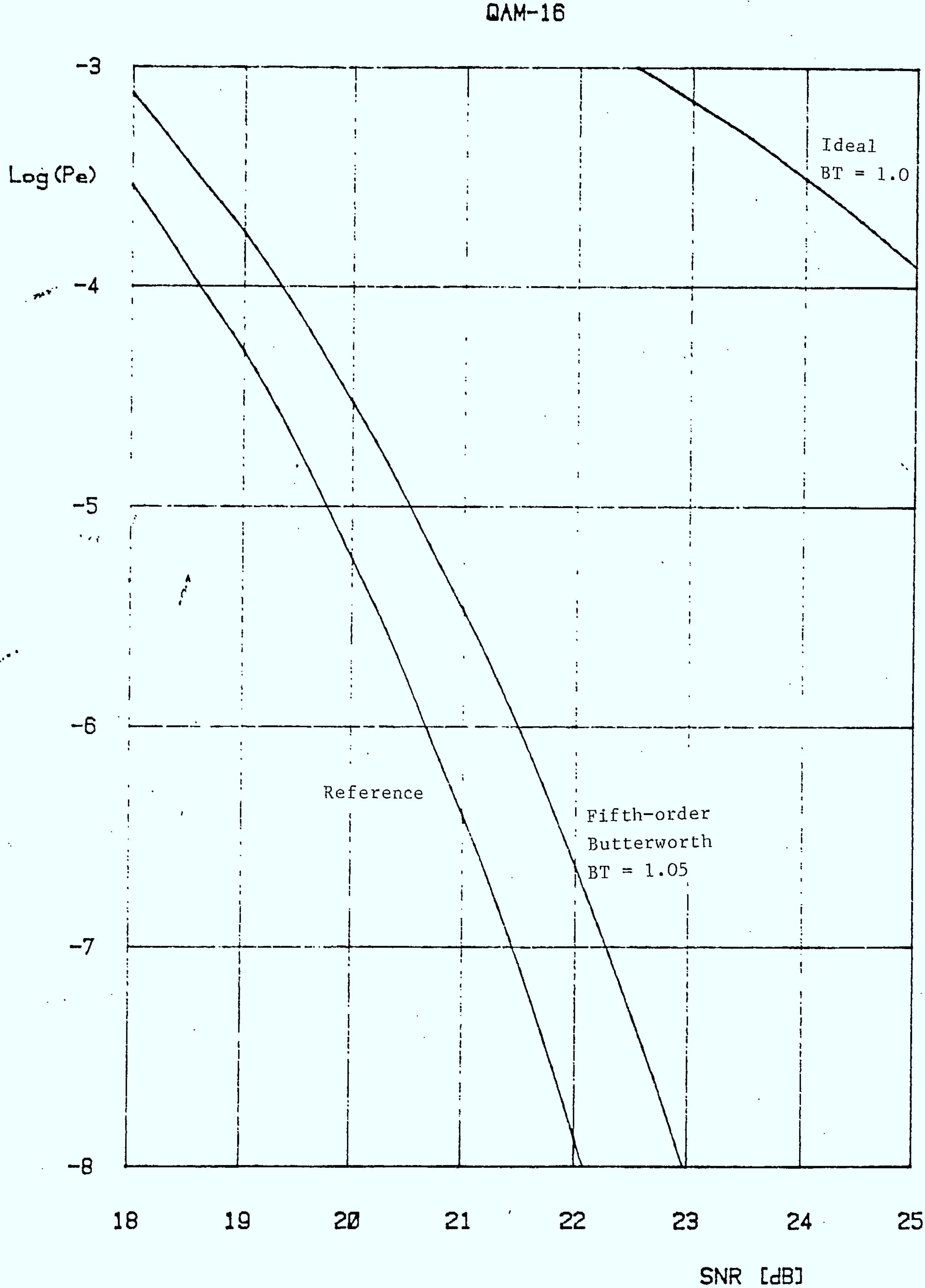

QAM-16
Log (Pe)
-3
-4
-5
-6
-7
-8
Ideal
BT = 1.0
Reference
Fifth-order
Butterworth
BT = 1.05
18
19
20
21
22
23
24
25
SNR [dB]

FIGURE 3.3

20

QAM-16 - NTH ORDER BUTTERWORTH FILTER - BT = 1.05

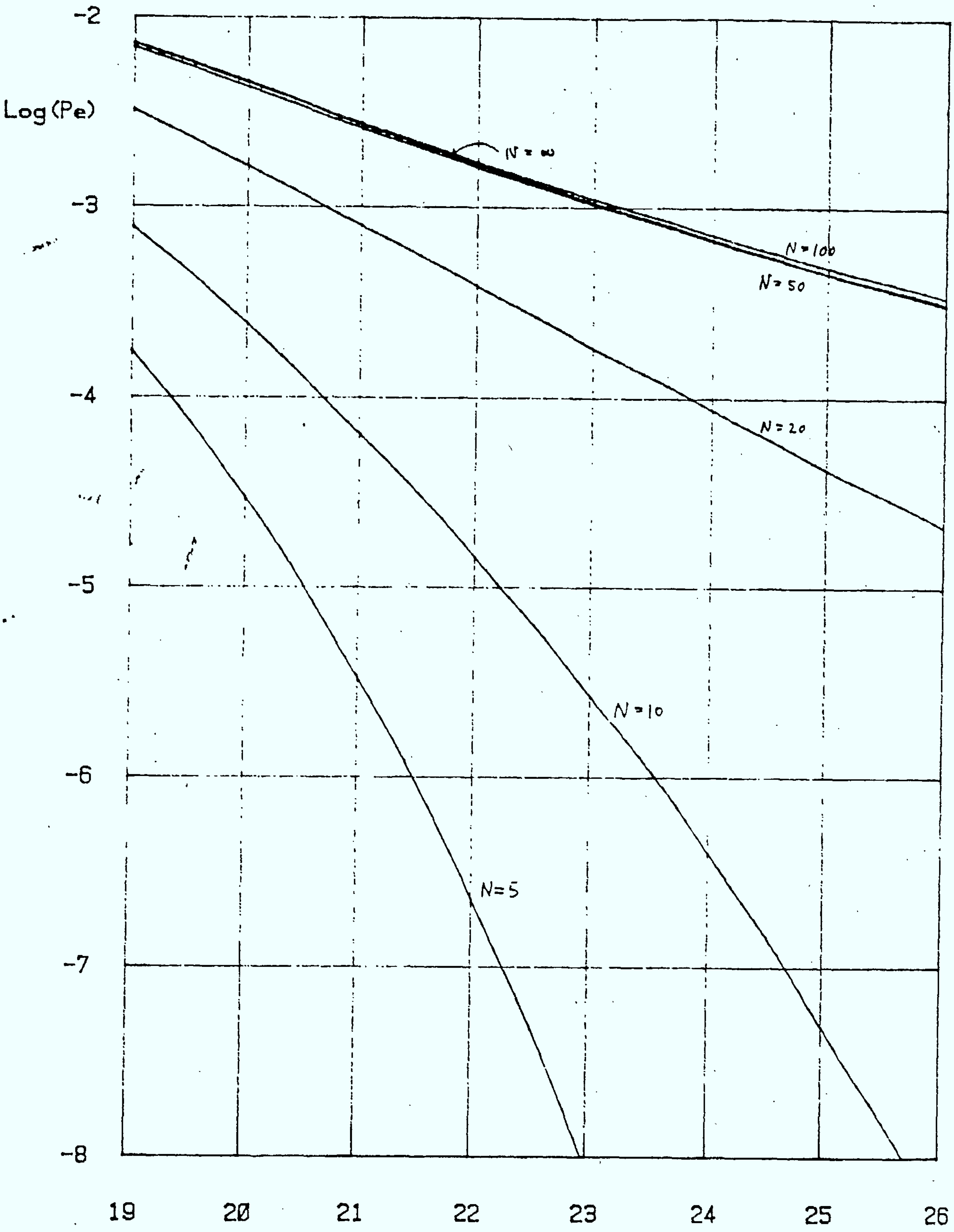


FIGURE 3.4

III.2.2.3 Theoretical Difficulties

The use of an ideal filter renders the wide-sense stationary assumption even more unlikely, as will be seen in studying the Weiner limit [6].

Let y(t) be an FM signal at the output of a real linear filter for which frequency response is rational.

$$y(t) = v(t) + v_c(t)$$

where v(t) is the wide-sense stationary term, and $v_c(t)$ an error term we wish to be as low as possible. Weiner, in his article, gives the following limit:

$$|v_c(t)| < \Delta\omega |f'(\tau)|_{max} \sum_{y=1}^{n} \frac{|K_y|}{(\alpha_y)^3} = \Delta\omega |f'(\tau)|_{max} K(n)$$

As H(p) is rational, then $H(p) = \sum_{y=1}^{n} \frac{K_y}{p-p_y} \quad p_y = \alpha_y + j\beta_y$

K(n) may be calculated for a Butterworth filter. The result is presented in Table 3.1. As can be observed, K(n) increases with n, and tends toward infinity for an ideal filter. It thus becomes impossible to find reasonable values for $\Delta\omega |f'(\tau)|_{max}$ that will render $|v_c(t)|$ as small as desired.

III.2.2.4 Results

To overcome the difficulties encountered, both numerical and theoretical, we preferred to present the results with a fifth-order Butterworth filter. With this type of filter, calculation of FM moments is no longer as simple, as it is necessary to calculate G(f) moments by a non-linear transformation. We feel, however, that these complications can be avoided by considering the problem in another way. Let us assume the band of our Butterworth filter is divided into three disconnected zones (see Fig. 3.6). Zones 1 and 3 represent the preceding case of ideal filtering, as the value of the response is 1 or 0; in the transition zone (zone 2), the response may take

n	K(n)
1	1
2	4
3	10.24
4	20.91
5	37.40
10	311.20
15	2,324.32
20	26,397.98
25	385,365.80
30	6,143,691.96

Table 3.1

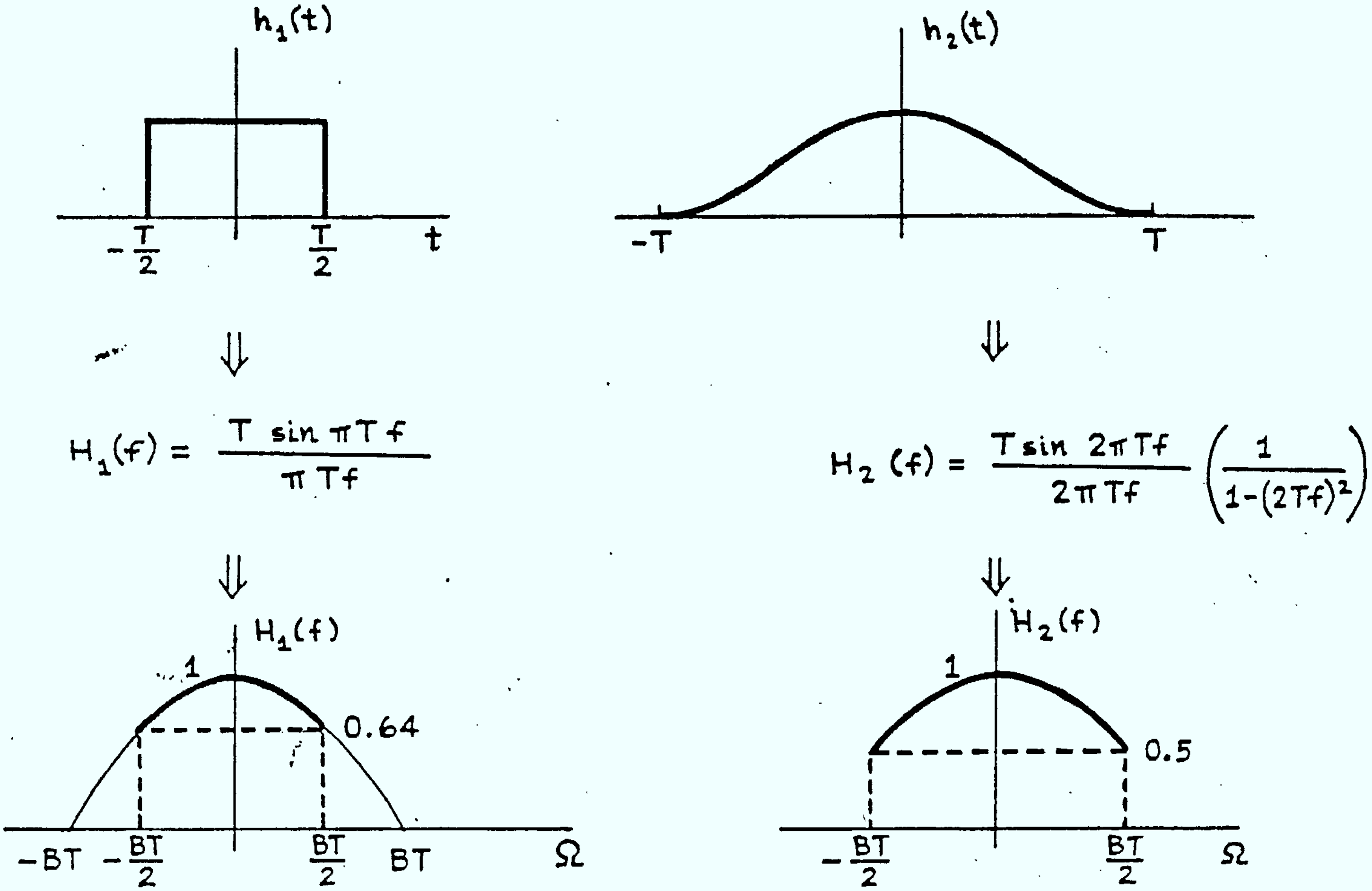


Fig. 3.5 - Comparison of two different pulse forms

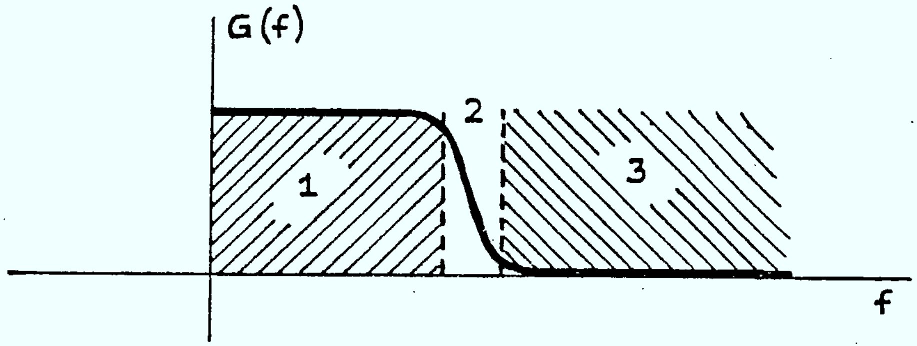


Fig. 3.6 - Filtering model for purposes of calculating FM interference

a continuous value between 0 and 1. We may assign the value 1/2 to this zone and take an average of the three corresponding values of P_e. However, since the probability of the instantaneous frequency being in this zone is low, as this zone is generally narrow, we can simply overlook this zone and do the calculation as in the ideal case, without committing any major errors.

Thus, the results presented in Appendix A are valid for a fifth-order Butterworth filter with BT = 1.05 and for the QAM-16. On these curves, the Y parameter corresponds to the RMS frequency deviation normalized to B/2, and the FM position corresponds to the X parameter, which is the separation between the FM and QAM carriers normalized to B/2. The SIR parameter has been previously defined.

The first type of curve presents Log(P_e) as a function of X for different values of SIR and Y. The second type of curve presents the DSNR (increase in SNR to keep P_e constant) as a function of SIR for $P_e = 10^{-5}$ with and without ISI for two values of Y. These curves are far from forming a full ensemble, but do give an adequate idea of the appearance of the results.

These results were realized with the QAMFM program. It is essentially similar to QAMBS, with the calculation of P_k added. The QAMFM flow chart is presented in Fig. 3.7. In addition to the value of different parameters, the program prints six different values of probability error. These correspond to the following situations:

P_{E1} : P_e with Gaussian noise only
P_{E2} : P_e with Gaussian noise and constant sinusoidal interference
P_{E3} : as in P_{E2}, with ISI added
P_{E4} : as in P_{E1}, with ISI
P_{E5} : P_e with FM interference, with ISI
P_{E6} : P_e with FM interference, without ISI

The values traced on the curves correspond to P_{E5}.

QAMFM

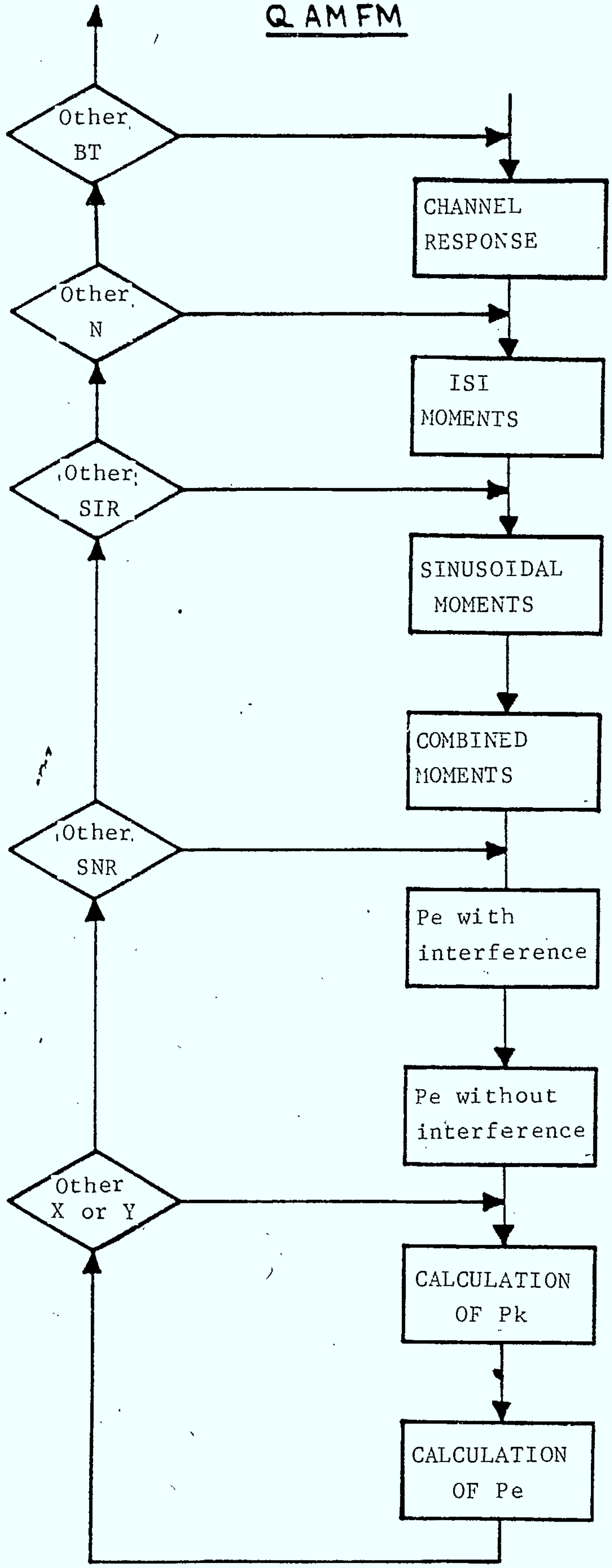


Fig. 3.7 - QAMFM Flow Chart

III.2.3 Gaussian Assumption

III.2.3.1 Introduction

Let G(f) be the power spectral density of our FM interference. We wish to divide the band of this power spectral density into n disconnected bands. At the limit, when n is large, each of these little bands could be seen as a sinusoidal interference in the form:

$$I_i = B_i \cos\theta_i$$

where the index i represents the band i situated at frequency f_i, $B_i = \sqrt{G(f_i)}$ and θ is equally distributed. Thus, our FM interference will be made up of the sum of n sinusoidal interferences which, if we assume the values of θ_i to be independent, will have a Gaussian distribution when n tends toward infinity. Thus, the Gaussian assumption represents the limit case of an infinite sum of different independent interferences.

III.2.3.2 Method of Calculation

We concerned ourselves initially with the case of a finite number of sinusoidal interferences for white G(f) and G(f) in Gaussian form. To do this, the method of moments proved very useful. However, calculation time becomes very high when the number of bands increases substantially. This is due to the fact that moments for each interference must be combined, requiring a fairly long calculation time. Nonetheless, it enabled us to observe the behaviour of P_e when n increases. Examining Fig. 3.8, we see that P_e increases in a monotonous manner with n. Thus, the Gaussian assumption, which is the limit case when n is infinite, represents an upper limit for a sum of independent interferences. The same behaviour is observed regardless of the form of G(f). Thus, in order to calculate the moments of our Gaussian interference, we need only calculate the variance, which in fact represents the power of our interference in the receiver band. Next, we may proceed to calculate P_e by the method of moments. Actually, calculation by the method of moments is no longer necessary, since the interference can be assimilated into the Gaussian noise. In the end, then, the calculation can be reduced to a few simple operations:

QAM-16 MULTIPLE SINUSOIDAL INTERFERENCE, WITHOUT ISI

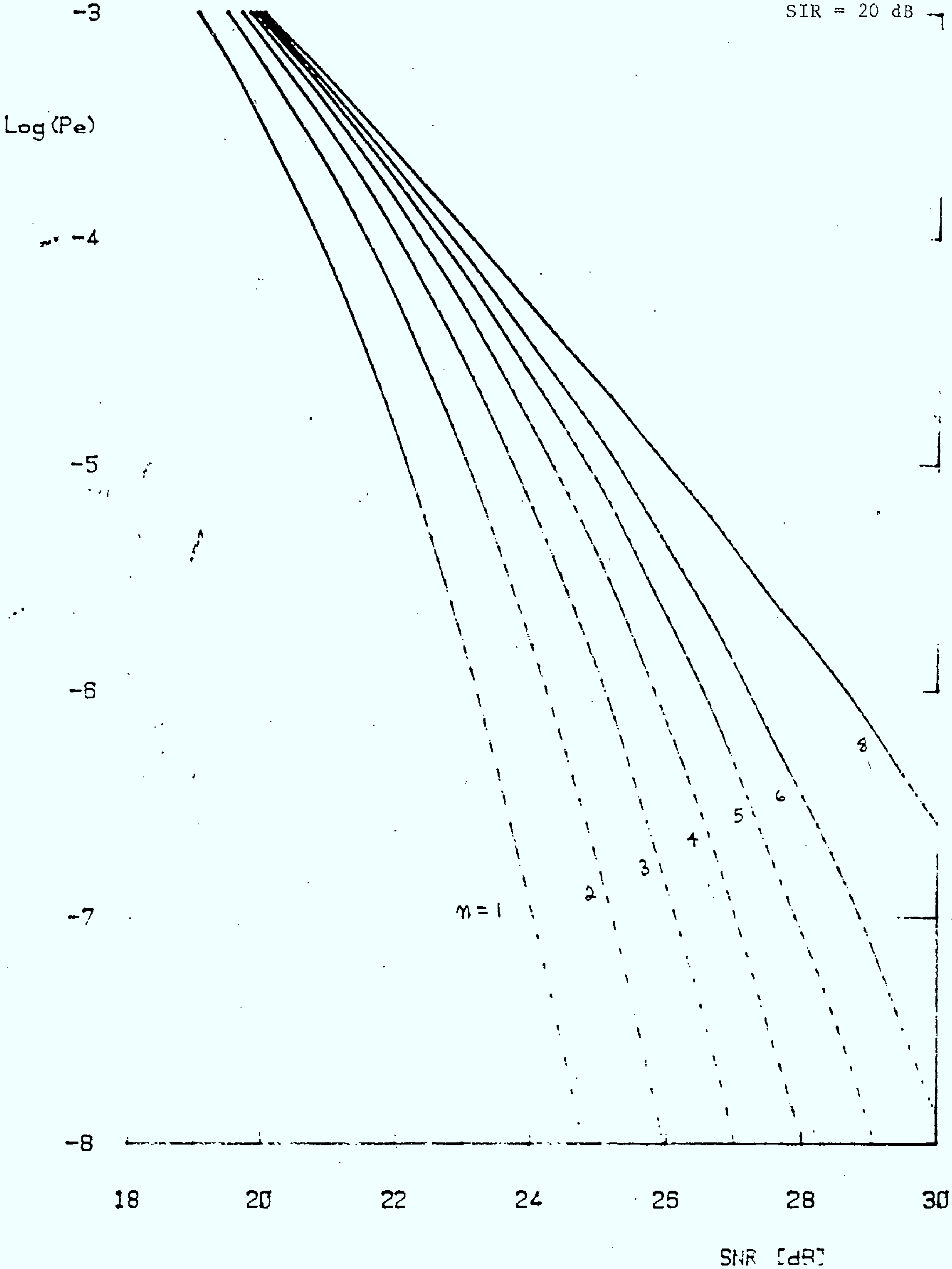


FIGURE 3.8

20

1) Calculate interference power at the decision device input, incorporating the power spectral density in the receiver band. In our results, this was realized with a power spectral density in Gaussian form, which simplifies calculation somewhat.

2) A new SIR is calculated, called SIRC:

$$SIR = 10 \log_{10} \frac{P_m}{P_I}$$

where P_m = mean power of QAM signal, and
P_I = mean power of interference at receiver input

If we have $P = \int_{-\infty}^{\infty} G(f)\left|H(f)\right|^2 df$ % of interference power entering receiver.

where H(f) is the receiver response [H(f) is ideal in our results] and G(f) is normalized, we will have:

$$SIRC = 10 \log_{10} \frac{P_m}{P.P_I} = SIR - 10 \log_{10} P$$

3) A new SNR is calculated, called SNIR, signal to noise plus interference ratio:

$$SNIR = 10 \log_{10} \frac{S}{N+I} = 10 \log_{10} \left[10^{-SNR/10} + 10^{-SIRC/10}\right]^{-1}$$

4) P_e is calculated by:

$$P_e = \tfrac{1}{2}\,(2-1/N)\,\mathrm{erfc}(S)$$

where

$$S = \left(\frac{3\times 10^{SNIR/10}}{2((2N)^2-1}\right)^{\frac{1}{2}}$$

The entire calculation can be done on a small calculator, the only numerical difficulty being calculation of a numerical integral for calculation of erfc(x) and P. This was realized in the OFMRG program (see Fig. 3.9). Clearly, all this is very simple, as ISI is overlooked. To include ISI, it would simply be necessary to return to the method of moments, which should not pose any problem.

III.2.3.3 Description of QFMRG

This program provides, for purposes of comparison, the two values of P_e corresponding to the situation with or without interference. The program allows any G(f) to be used. One need only modify the FT function in the BAND function. The two parameters relating to FM interference are DF and FC: these are normalized to B/2 as previously.

FC has the same significance as in the wide-sense stationary assumption: it is the deviation between the FM and QAM carriers, normalized to B/2. DF, on the other hand, has a different significance: here, it represents the standard deviation of G(f) normalized to B/2, whereas before, the Y parameter represented the RMS frequency deviation of the FM signal, which is in fact the standard deviation of the modulating signal normalized to B/2.

Results are shown in Appendix B for the QAM-4, QAM-16 and QAM-64. In each case, a series of P_e curves is provided as a function of SNR for different values of DF, FC and SIR. An additional curve is presented for the case of a centered interference of narrow bandwidth (FC = 0 and DF = 0.1) for different values of SIR. All these results apply to an ideal filter, with no ISI.

III.3 Discussion

III.3.1 General Presentation

Let us begin by taking the results presented in Appendix 2 concerning the wide-sense stationary assumption. It will be noted first that these results are very partial and apply only to the QAM-16. This is in contrast to the results shown in Appendix B for the Gaussian assumption. In these, a fairly complete ensemble of curves is given for the QAM-4, QAM-16 and

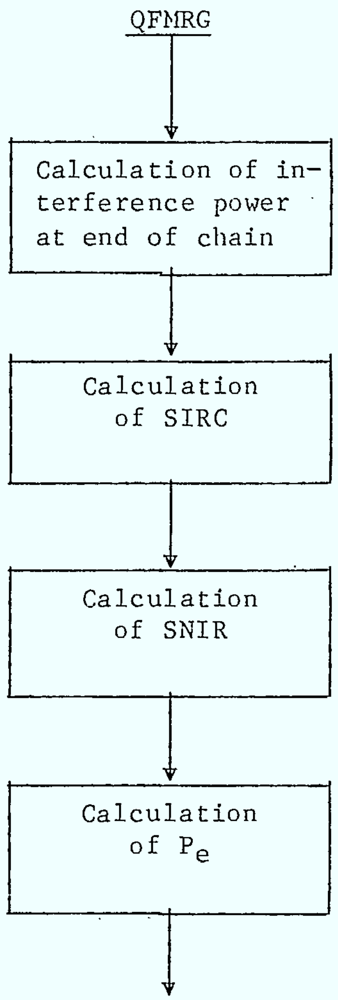


Fig. 3.9 - QFMRG Flow Chart

QAM-64. First, we feel the results in Appendix 3 are more realistic, and in addition are easier to obtain, as the calculations can be done on a small computer.

Other differences in presentation may be noted, for example the horizontal scale of the curves. In the first case, $\log(P_e)$ is presented as a function of the FM position (X parameter) for different values of SNR. In the other case, $\log(P_e)$ is presented as a function of SNR for different values of FC. We feel that the first presentation is preferable to the second, as it immediately permits the variation of P_e to be more easily seen as a function of a parameter of FM interference. In the second case, on the other hand, the SNR parameter has been kept on the horizontal scale in anticipation of calculating an upper limit. In fact, this upper limit enables us to determine the SNR range in which the Gaussian assumption would be an absolute upper limit.

There is another point to be noted in Appendix 2 with regard to the curves: these are presented taking ISI into account, while those in Appendix 3 do not take it into account. It is certain that if we wish to compare the curves with each other, we would have to know a priori the value of P_e to subtract due to ISI alone. Examining the curves, whichever assumption is used, similar behaviour is observed. P_e is inversely proportional to SNR and SIR, which is not surprising. It will also be noted that P_e decreases in a monotonous manner as the deviation between QAM and FM carriers increases, for a given FM spectral dispersion. This result is also self-evident. In addition, it is noted that for a fixed deviation between carriers, P_e may vary in one direction or the other as a function of FM spectral dispersion (broad or narrow spectrum). If the deviation is small, a narrow FM spectrum will cause more degradation in QAM than will a broad spectrum. By contrast, if the deviation is large, the reverse process is observed. For example, let us take the curves in Figs. A.2.1 and A.2.2 for SNR = 30 dB. For X between 0 and 1.5, the value of P_e obtained for Y = 1 is higher that the value obtained for Y = 2. On the other hand, for X = 1.5 to X = 5, the reverse occurs. This behaviour is general, and is found in the results in Appendix 3 as well.

This may be understood if we know that in both cases P_e is a function solely of total power of the interference entering the QAM receiver. This implies that for a centred interference, in which all the power passes into the receiver, P_e is independent of the power spectral density of the interference. The latter observation hints at a degree of inaccuracy in the two assumptions used. We feel that in reality broadband interference causes more degradation of P_e than narrowband interference for the same total power entering the receiver, but our two approaches do not permit this phenomenon to be taken into account.

III.3.2 Comparisons

In order to better compare the results obtained with the two assumptions, in Fig. 3.10 we traced P_e for the QAM-16 without ISI, with SIR = 20 dB, as a function of SNR for the case where all the interference power enters the receiver. Referring to this case, we see three curves numbered 1, 2 and 3. Curve #1 is simply P_e obtained with Gaussian channel noise only. Curve #2 represents P_e obtained with constant sinusoidal interference. All curves obtained with the wide-sense stationary assumption for different values of X and Y will be located somewhere between curves 1 and 2. Curve #3 presents P_e obtained with the Gaussian assumption, and all curves obtained in this way for different values of FC and DF will be located between curves 1 and 3.

We thus see clearly the considerable deviation that exists between curves 2 and 3, and hence between the two assumptions, the wide-sense stationary assumption clearly being more conservative. It is for this reason as well that, after obtaining our initial results with the wide-sense stationary assumption, we sought another method that would yield more realistic results. Under the Gaussian assumption, we felt we would be in an intermediate case, between the worst case and the wide-sense stationary assumption. The Gaussian assumption assumes independence of the phases between different spectral bands. This condition is certainly not the worst case, nor the best. The worst case would be one in which all the different interference components were always in phase, not a very realistic case. Thus, we feel the Gaussian assumption might yield valid results if we compare the results with those for the wide-sense stationary assumption, which are clearly overconservative, and for the worst case, which would be totally unrealistic.

There is another point to be noted with regard to Fig. 3.10. It will be observed that curve #2 always seems to overlap curve #1, whatever the SNR, while curve #3 seems to tend toward a limit value of P_e as SNR tends toward infinity. The latter behaviour is easily explained: when the SNR is small, it is channel noise that predominates, while when SNR is large, interference predominates. At the limit, for SIR = 20 dB, when SNR tends toward infinity, P_e should tend toward the value that would be obtained on curve #1 with SNR = 20 dB. As concerns curve #2, the same behaviour is not observed at all, probably because equivalence in terms of P_e between sinusoidal interference and Gaussian noise occurs at a very high SNR level or at a very low P_e for an SIR = 20 dB.

III.3.3 Upper Limit

We have noted previously that the Gaussian assumption was more realistic than the wide-sense stationary assumption, without being able to state the extent to which it is close to reality. To remedy this shortcoming, we thought of calculating an upper limit for P_e that would enable us to see whether the Gaussian assumption in a certain SNR range could be considered as an absolute upper limit for the case of any non-Gaussian interference. There are several methods described in the literature, and the calculation should theoretically pose no major problems. However, time constraints prevented us from carrying it out. We thus deferred this operation to the framework of future work.

QAM-16 - NO ISI - SIR = 20 dB

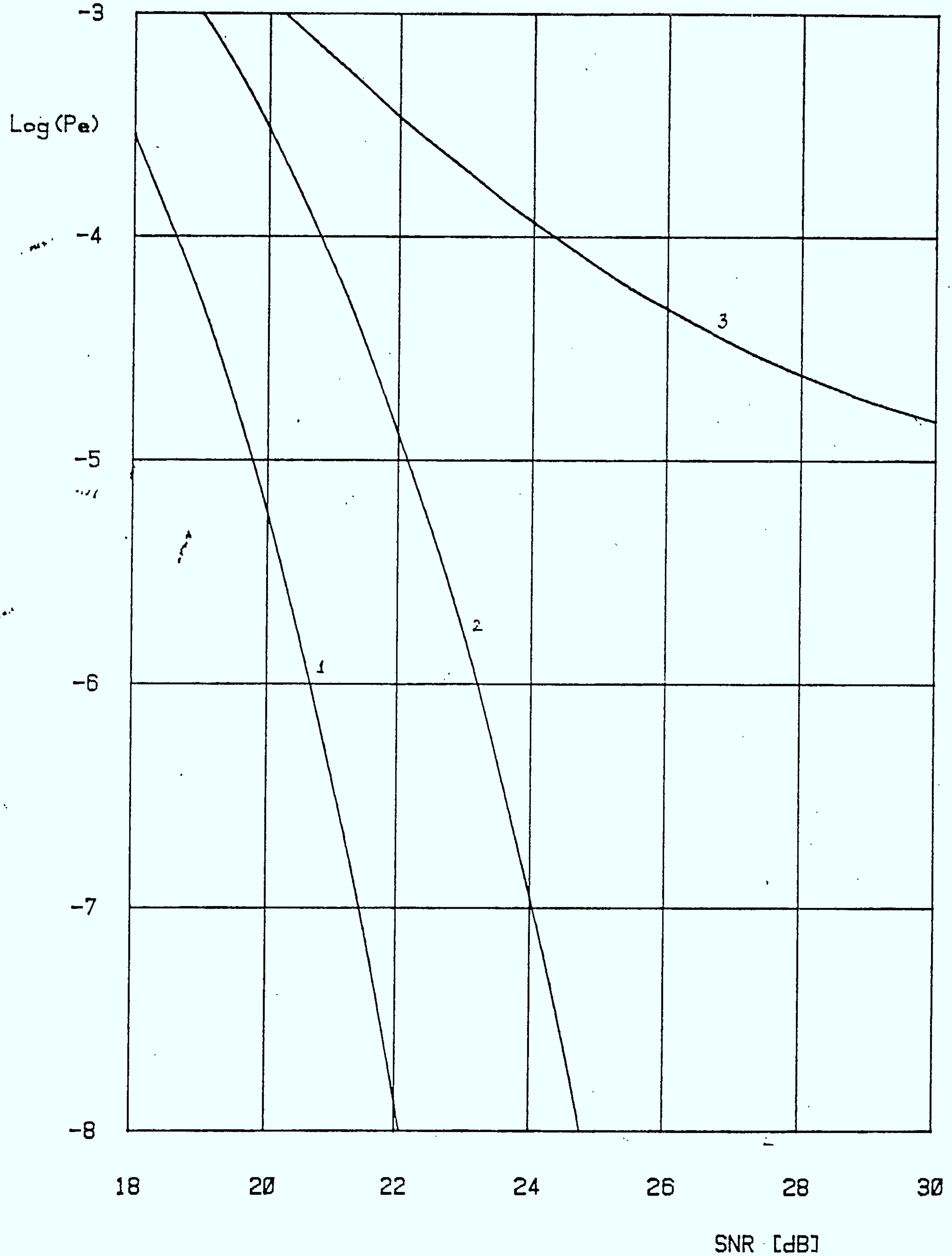


FIGURE 3.10

CHAPTER IV

CONCLUSION

We have presented in this paper a flexible methodology for studying the performance of QAM receivers in the presence of interference of any kind, including FM signals from adjacent channels.

This methodology was based on a simple concept proposed by Ho and Yeh. This was simply to express the probability of error as a function of moments of the variable representing the sum of all interference present at the receiver input. As these forms of interference are independent, we needed only calculate the moments of each separately, and subsequently combine them according to precise rules.

The interference component of greatest interest to us was a very broad bandwidth FM signal. Based on the wide-sense stationary assumption, its behaviour over short time intervals was equivalent to a sinusoidal signal whose frequency corresponded to the instantaneous frequency of the FM signal. This simplifying assumption enabled us to deal in a simple manner with interference from an FM signal.

We also proposed an assumption based on the concept of "power equivalence". This assumption was justified physically by the constraint imposed on total power in the QAM pass band. However, it was expected that this assumption would lead to more pessimistic results than those obtained by the wide-sense stationary assumption.

Because of our choice of mathematical formulation, a Butterworth filter was selected as a model of the transmission channel instead of a Nyquist channel. This choice automatically introduced intersymbol interference at the receiver output. This factor was minimized by a judicious choice of parameters involved, such as: instant of sampling, bandwidth and order of filter for which phase was linearly equalized.

Results obtained were finally presented in the form of a family of normalized curves. They are thus usable for any situation. It is interesting to note that these results appear incomplete in our view. We obtained results for interference caused by an FM signal. To obtain them numerically, however, we were required to utilize both assumptions: one of them very optimistic, the other very pessimistic. It remains to be seen whether these results fall within general optimistic and pessimistic limits. Calculation of these limits will be carried out under the next contract, in which we propose to analyze narrow band FM interference.

Last but not least is the complexity of the method used. On the one hand, the method consumes a great deal of calculation time, although this no longer represents a real drawback, with the advent of mini- and microcomputers. On the other hand, this complexity has necessitated a very serious effort (one engineer-year) to successfully implement the method. The program, written in PASCAL, is modular and flexible, enabling users to vary all the parameters they wish, while ensuring relatively reasonable operation. This development cost would seem at first glance to be fairly high. However, the program is now available on a semi-public basis. It should not be overlooked that before the contract giving rise to this report was awarded, programs of this nature were the jealously guarded private property of companies like BNR, AT&T, Ericsson, etc. Thus a giant step forward has been taken; we can only hope for similar fruitful cooperation between academic researchers and public agencies.

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