Implementation and Empirical Testing
of Inter-Temporal Cross-Subsidy Principles
Using the NPPS Model

Final Report

Prepared for and in collaboration with the
National Telecommunications Branch
Department of Communications

by

Laboratoire d'économétrie
Université Laval

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Preface

One of the main issues in the regulation of telecommunications is the determination of a pricing policy which reflects the social and economic priorities of the government. So, the central theme of this Report is how pricing policy related to the various objectives of the government. For reaching these goals, the Report is divided in two parts. In the first part, the policy objectives in the telecommunications industry are formalized in a tableau with a view to showing their hierarchy and their interrelationships; also the various means of regulation which the government possesses are matched with these policy objectives. In particular, the central role played by the tariffs structure is stressed, and in particular the four functions it performs. Finally these means are reviewed in the context of the NPPS model with a view to implementing empirically these policy objectives. In the second part, the various extensions of the cross-subsidy tests are reported as well as the results of some simulations performed with the NPPS model. In this sense, the present Report is the continuation of the works reported in the previous reports of the NPPS Project.

Although the present Report is the responsability of le Laboratoire d'économétrie de l'Université Laval, Part II of the Report represents the results of a combined effort by the following three groups with the names of the specialists involved

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Résumé

Le présent rapport est divisé en deux parties. Les objectifs principaux reliés à la première partie sont premièrement de chercher à relier les différents objectifs de politiques gouvernementales quant à l'industrie des télécommunications avec les différents moyens de réglementation que le gouvernement possède, deuxièmement de faire une évaluation de ces différents moyens par rapport à leur incorporation dans le modèle NPPS. Tout au long de cette partie, l'emphase est mise sur le rôle important joué par les tarifs afin de chercher à atteindre ces objectifs gouvernementaux. Finalement, un historique du projet NPPS est fait afin de situer ces préoccupations de politiques dans le contexte des modifications graduelles apportées au modèle NPPS.

La deuxième partie du rapport décrit les extensions qui ont été apportées au modèle NPPS afin d'accroître le champ d'application des tests d'interfinancement par rapport à: a) la dimension temporelle des services; b) l'introduction des élasticités-prix croisées parmi les services; c) la régionalisation des tests pour évaluer l'interfinancement entre exploitants. Les résultats de quelques simulations faites à l'aide du modèle NPPS et ayant trait aux sujets mentionnés précédemment sont également présentés. Finalement, une brève revue de la littérature concernant les différentes méthodes d'amortissement et une autre ayant trait à différentes règles de partage des coûts sont respectivement faites dans deux appendices.

Summary

The Report is divided in two parts. The main objectives of the first part are first to relate the various policy objectives assigned by the government to the industry of communications to the various means it possesses, and second to make an assesment of the various means already introduced in the NPPS model. Throughout all of this part, the crucial role played by the tariffs for reaching the policy objectives is stressed. Finally, the NPPS Project is reviewed with a view to relating these policy considerations in the context of the gradual modifications made in the NPPS model.

The second part of this Report describes the extensions that have been made to the NPPS model with a view to increasing the applicability of the cross-subsidy tests with respect to; a) the temporal aspect of the services; b) the introduction of some price cross-elasticity among the services; c) the regionalization of the tests for evaluating the inter-carriers cross-subsidization. The results of some simulations done with the NPPS model and in relation with the previous subjects are also presented. Finally, in two appendices, a brief review of the recent developments is presented concerning the various depreciation methods and certain cost separation rules.

PART I

FROM POLICY OBJECTIVES TO REGULATORY PROCESS IN TELECOMMUNICATIONS

1. Introduction

Any society through its representatives assigns to its members a set of objectives, sometimes called priorities, objectives defined most of the time in qualitative terms. Once these priorities are enumerated, the second task is to assign some desired values, or targets, to each of them and to establish an hierarchy among them. The problems related to the construction of a social welfare (or utility) function are well-known. Even if it is not the purpose of this Report to review all of them, at some places in it some difficulties pertaining to this construction will be mentioned.

In our free entreprise system, the achievement of the proposed objectives is mostly vested in the individual economic agents (consumers and producers). However, since at least the beginning of the 1930's, the governments, through their expenditures (think of the keynesian revolution) on one hand, through their regulatory means on the other, have gradually increased their importance in the social and economic domains in such a way that today one can say that they have enough power to contribute to the realization of the stated objectives.

The industry of telecommunications is a member of a set of industries referred to as public utilities. It is now generally agreed that a public utility entreprise is any entreprise which is subject to governmental regulation, including price regulation, barriers to entry, etc..., of a type designed primarily to protect the consumers in the long run. It is also agreed that this public interest aspect of the regulation is so because of some special features of the public utility entreprises, namely the necessity of them and their particular technical and economic characteristics. In effect, it is evident that a well-being of the transportation system, of the electric power and of the telephone system (three "classic" industries referred to as public utilities) are of life-and-death importance to every society. In other words, these industries can be seen as <u>instruments</u> for reaching some of the priorities already established by the governments.

The telecommunications industry is mainly regulated by two means: first by imposing an upper bound on the rate of return the carriers can earn, second by setting up some barriers to entry for the new carriers. The first means permits the determination, for a carrier as a whole, of a total revenue requirement which will be the starting point for the construction of a tariff structure. The second means reflects the presumption of the government that if full competition was allowed, the impact on the society would eventually become negative.

The big challenge for governments is how to utilize their instruments (tariffs structure, barriers to entry, quality of services, etc...) in such a way that the desired values of the stated objectives be approached as near as possible or to put it in more technical terms, that the social welfare function be as high as possible, given of course the existing constraints on the availability of resources.

It is in this context that the NPPS model as well as the HERMES model have been developed: for the NPPS model, the objective was to provide instruments in the hands of the Department of Communications for evaluating the impacts of the modifications of some control variables on the financial statements of the carriers; in the case of the HERMES model, the objection was to determine an optimal way of expanding the physical and the switching networks once the demands for the various services have been increased.

The objectives of the first part of this Report now become clear. It has already been established that every society must determine an hierarchy of objectives, some of them being very abstract in their nature, some being more easily measurable (think of the social indicators), and finally some being more operational. However, it is evident that the latter can contribute to the achievement of the former, and so on. For example, the industry of telecommunications can be viewed as an instrument for reaching governmental goals like the decentralization of the population in the territory or for reducing the mortality in the far northern regions. But, for trying to reach these objectives, one must assign some goals to the industry of telecommunication, like "accessibility and variability of the services", like "quality and reliability of communications", like "efficient and economical system of telecommunications", etc... And once again, these goals or objectives can be reached only by starting from more concrete or operational instruments in the hands of the government (it has to be recalled that the telecommunication industry is a regulated one). In the context of this hierarchy of objectives, the main objectives of this part of the Report can now be stated

a) to formalize the various objectives which are stated mostly in the "Proposals for Communications Policy for Canada", March 1973, Department of Communications, Ottawa;

b) to review the various means that the government posses-ses for regulating this industry, and expecially the crucial role played by the tariffs;

c) to relate, as closely as possible, the various objectives assigned to this industry and the various means already mentioned, stressing the compromise that most of the time must be made among the objectives;

d) to make an assesment of the various means already introduced in the NPPS model and the needed modifications for introducing more explicitly the tariffs in the model;

e) to try to determine some new policy options (or implications) and to study their possible implementation in the NPPS model.

However, before we go to the discussion of these items, a brief review of the characteristics of this industry will be made as well as a review of some theories of regulation and of the various definitions of the services supplied by it. The discussion of the last three items will be made in relation to their policy implications.

2. From Policy to NPPS

2.1 Economics and technics of telecommunications industry

The industry of telecommunications, being a member of the class of public utilities, is a regulated industry. It will be shown later that one of the important means of regulation among others are the tariffs. But before we go to these considerations, it is appropriate to review the characteristics of this industry, in order to appreciate how complex it is, and more importantly to review the difficulties of applying the marginal cost pricing principle to the industry. Also, this sub-section represents an introduction to the other sections of this Report as most of the discussion will be conducted in relation to these characteristics.

Among the main economic and technical characteristics of the industry of telecommunications, the following seven characteristics seem important. A brief comment will be made on each of them in relation to the problems they raise.

1. This industry supplies many services, switched and non-switched ones, the customers of which have very different income, culture, interest, etc... In other words, the characteristics of the demand can vary a great deal among the various customers of these services. More technically, the price elasticity of demand is very different from one market to another. Of course, this raises the question of trying to cross-subsidize the market with a larger price elasticity by the one having a small price elasticity.

The different price elasticities among the market is only one of the facet on the demand side. The second facet is the differences among the elasticities of substitution. This means that since the industry of telecommunications supplies many services, the existence or the non-existence of some substitutes varies enormously among these markets. For example, the private network can be seen as a substitute to the toll service (for the firms or the governments, of course). This raises the question of subsidizing the customers of the product where there exist close substitutes by the customers of products having no substitute. It can be expected that this problem will become more crucial in the future as the introduction of new services through competition will accelerate.

This is only one of several possible kinds of substitution in the industry. But there exists at least two other kinds and consideration of which can be fruitful for the determination of an optimal tariff structure: these are the "time" substitute and the "product" substitute.

For example, if one considers the "temporal unit" of observation as a week, it is evident that the displacement of the demand from the peak hours to the off-peak ones (for example the demand during the week-ends) is "more easy" for the private customer than for the entreprises. From the product substitution viewpoint, one can think about the industry of telecommunications as one among others which try to satisfy the "need" for communication, the other two being transportation and mail. Among them, the trade-off between transportation and communication will become more and more important in the future, as the energy cost will go up. And, it is evident that in the development of a policy for the telecommunications industry, and consequently for the tariffs strategy, that this facet of the problem must be taken into account. In particular, the development of an economic and efficient system of telecommunications can be an instrument for reaching the governmental objective of energy conservation.

In fact what is behind the above discussion is the definition of the services of telecommunications and the policy strategy that these definitions imply. A discussion of these subjects will be made in the sub-section 2.3.

2. The telecommunications services are for most of them not stockable for long periods of time. In consequence, there exists some very important variations in the demand during a typical period of time, i.e. there will exist some peak and some off-peak periods. It is evident that it is the demand at the peak period which creates the pressure for investment in capital in the network. This raises of course the problem of financing the user of the peak period by those of the off-peak periods. Also, and taken the various time elasticities into account, one can ask the following question: what will be the impact on the rate base of a reduction of the peak period of a certain percentage, even if the total demand remains unchanged? This question is important taken the pressure the telecommunications industry creates on the capital market.

3. The telecommunications industry is often referred to as a natural monopoly. By definition, a firm supplying only one service is referred to as a natural monopoly if it has decreasing average cost in the long run. And it is well-known that a tariff based on the marginal cost of supplying this service will not recover the fixed costs in providing the service. But since this industry is regulated by the constraint that it must be self-financed, it follows that some services have to be priced higher than their marginal costs. It can be noted that some authors (see, in particular, Waverman, L. "The Regulation of Intercity Telecommunications" in Phillips, A., ed., Promoting Competition in Regulated Markets, The Brookings Institution, Washington, D.C. 1975) have questioned the presence of economies of scale particularly concerning both the switching costs as well as the costs at the local level. In relation to the idea of this paragraph, several comments can be made:

a) the monopoly aspect of the industry can be questioned on the grounds that there exists some substitutes to the telecommunications services (see point 1 above);

b) The existence of a sole supplier may be economically justified even if some carrier produces at increasing average cost. In effect, the real question is the following: can a given quantity of service be provided more cheaply by a set of producers than by a sole one? In other words and assuming that the government eliminates for a moment every barrier to entry, is it evident that there will be incentive, i.e. positive economic profit, for some producers to go into the industry? Technically, the question refers to the existence of tariffs which would permit the sustainability of the monopolist. Under some reasonable assumptions, it can be shown that no viable competition can exist in this kind of industry (i.e. assuming decreasing average cost). Of course, this problem is strongly related, but no identical, to the cross-subsidization one. Some of these various questions in relation to the instrument of regulation will be taken again in the sub-section 2.5.

c) The definition of the natural monopoly concept is straightforward in the case of the monopolist offering one service. But, the carriers in this industry, in fact like most of the entreprises, supply many services. And it is now apparent in the literature that the definition of a natural monopoly is not so easy, the reason being that if outputs do not expand proportionately we do not know how to define an index of aggregate output by which to divide total cost, nor do we have any way of apportioning the joint and common costs so as to calculate an average cost, item by item. (See Baumol's article in the A.E.R., December 1977).

d) The definition of a natural monopoly even for a firm supplying many products is an easy task relative to testing for the presence of economies of scale. In effect, it seems (See Baumol, idem) that "because a claim of natural monopoly asserts that production by a single firm is cheaper than it would be in the hands of any and every possible combination of smaller firms, one must know the behaviour of the cost curve throughout its length in the interval between the origin and the particular output level considered" (pages 815 and 816). And one can imagine very easily that this is a rather difficult task.

e) The previous comments stressed the costing aspect as an element of definition for the products in the industry of telecommunications. But one may also question about the quantity supplied of the various services. In other words, it can be that the promotion of the output, even under the conditions of economies of scale, can lead to some results which are non-optimal, socially speaking. It can be noted that this problem of restricting the demand, in some way or another, is among the various goals behind the tariffs (see sub-section 2.5).

4. The technology of this industry is characterized by a relatively high importance of the capital. In consequence, the common and joint costs necessitate a separation rule in view of being assignable to some specific services or to particular units of a service. The existence of an "optimal" sharing rule and the knowledge of its implications are important for relating the cost of each service with its tariff and consequently for evaluating the presence or the absence of cross-subsidization among the services, among the customers and among the carriers. It seems that one of the main interests of the game theoretic approach recently developed in the literature, and in particular the Shapley value, is precisely to suggest, once one accepts some minor axioms, a precise cost separation formula. Moreover, the reader will find in the Appendix B of this Report a brief survey of the literature concerning the problem of the cost separation. Finally, it can be noted that in the NPPS model no such thing is needed in view of computing the supplementary cost of a service.

5. The capital input in this industry, and in fact in the other public utilities entreprises, is subject to some important indivisibilities. In relation to the marginal cost pricing approach this introduces at least two problems: first, the marginal cost cannot be uniquely defined, second, it is most probable that there will always be excess capacity in the network, even during the peak periods. From the interfinancing point of view, this raises the question of which user will pay for this excess capacity.

6. Most of the time, the investment projects of the carriers are made some periods in advance (see the preceding point 5), i.e. investing now for satisfying the forecasted demand with an horizon of three to four years ahead. Moreover, any new project necessitates many years of gestation and the rate of introduction of the new technologies is also planned for many years. The introduction of the dimension of the evolution of the demand as well as the replacement of the existing facilities has as a consequence that the industry of telecommunications is always in a disequilibrium situation and consequently that the decisions based on a short term marginal cost pricing are not the same as those based on a long term marginal cost one. It then follows that the proper time perspective is crucial for a policy based in part on the tariffs. Also, as it will be shown later on, the rate of depreciation is an important means in the hand of the state for regulating the rate of replacement of the existing facilities.

7. Many of the carriers in the industry are vertically integrated and own, totally or partially, some subsidiaries. In other words, many of the carriers supply services from point-to-point traffic. What it does mean is the fact that the carriers possess all the equipments from station to station. Consequently, if competition were allowed, it will be imaginable that various parts of the network can be owned by a particular entreprise, and some other parts by another firm. The vertical integration situation raises a lot of very difficult policy questions like the following:

a) should the manufacturing subsidiaries be financially separated from the carriers? Or, in other words, is there any possibility of financing the competitive services by the monopolistic ones?

b) what is the proper rate of return to allow for the manufacturers?

c) is the introduction of cost-reducing innovation made as fast as it is economically justified?

d) what are the probable impacts of allowing competition in the interconnect market?

e) what kinds of technical specification for the equipment are needed in view of reaching a certain degree of quality of service?

f) which objectives should the government like to reach by allowing competition in this industry?

2.2 Brief review of some theories of regulation

2.2.1 Introduction

Previously, it was said that the public utilities entreprises, and in particular the carriers in the industry of telecommunications were regulated by various means, including price regulation. Of course, one can try to suggest some reasons with a view to explaining the governmental intervention into the markets. This subject can by itself be the matter of a complete report. Here, our objectives are more limited.

From the point of view of regulation, the central tasks are the following three:

a) to determine who will receive the benefits or the burden of regulation;

b) what forms regulation will take;

c) to evaluate the effects of regulation upon the allocation of resources and upon the various governmental objectives.

In the present sub-section, point a) will be briefly discussed, keeping points b) and c) for the sub-section 2.5. Essentially, what we intend to do is to review critically the following two opposite theories of economic regulation:

a) <u>the "public interest" theory</u>: this theory holds that regulation is supplied in response to the demand from the public for the correction of inefficient or inequitable market practices.

b) the "capture" theory: this theory holds that regulation is supplied in response to the demands from interest groups struggling among themselves to maximize the incomes of their members. I can immediatly say that the critical review of these theories will be based essentially on the Posner's article "Theories of Economic Regulation" which has appeared in The Bell Journal of Economics, autumn 1974.

In the sub-section 2.4 where the various objectives in the industry of telecommunications are discussed, I will try to classify these objectives under one or the other of these theories. Here again, the reader should not expect too much because, to the knowledge of this writer, this is the first time that someone has tried to attack this problem in this context. But the reader will also easily recognized that the utilization of the various means of regulation will be very different if the governmental objectives pertaining to the industry of communications were interpreted in terms of one theory of regulation or the other one. Also, a discussion about vertical integration and about an eventual introduction of competition in this industry may be done in relation with these theories.

2.2.2 The "public interest" theory

There are at least three versions or formulations of this theory.

a) The original theory. Two hypotheses seem to be behind the economic thought concerning economic policy in the period starting from the year 1887 and up to 1958.

1. The first assumption was that economic markets are extremely fragile and apt to operate inefficiently (or inequitably) if left alone.

2. The other hypothesis was that government regulation is virtually costless.

In other words, it was assumed that behind each scheme of regulation, it is possible to discern a market imperfection, like the existence of such phenomena as increasing returns to scale or the presence of externalities, the existence of which supplied a complete justification for some regulation assumed to operate effectively and without cost.

This formulation is inacceptable for at least three reasons:

1. Most of the empirical works have shown that regulation is not positively correlated with the presence of external economies or diseconomies, or with monopolistic markets structure.

2. The conception of government as a costless and defendably effective instrument for altering market behaviour has also gone over the boards.

3. Finally, theoretical as well as empirical works have demonstrated that particular schemes of government regulation cannot be explained on the ground that they increase the wealth or, by any widely accepted standard of equity or fairness. (See, for example, Coase R.H. "The Federal Communications Commission". Journal of Law and Economics, Vol.2, no.2 Oct. 1959, pp.1-40).

b) A reformulation. The first reformulation of the "public interest" theory of regulation holds that regulatory agencies are created for bona fine public purposes, but are then mismanaged, with the result that those purposes are not always achieved.

This reformulation seems inacceptable for at least two classes of reasons:

1. First, it fails to recognize the facts that the socially undesirable results of regulation are frequently desired by groups influential in the inactment of the legislation setting up the regulatory scheme. For example, it seems that AT and T pressed for state regulation for ending competition among telephone companies.

2. Second, no sound theory or evidence have yet been proposed to explain why the agencies should be expected to be less efficient that other organizations.

c) A further reformulation. This reformulation now incorporates two new factors which were previously ignored:

1. The first factor is the non-operational character of many of the tasks that have been assigned to the regulatory agencies: of course, this does not explain why legislatures assign such tasks to agencies.

2. The second factor is the cost of effective legislative supervision of the agencies' performance.

Once one introduces these two factors in the "public interest" theory of regulation, one can consider more plausible the idea that regulation is an honest but frequently an unsuccessful attempt to promote the public interest.

2.2.3 The capture theory: the economic theory of regulation

Like the "public interest" theory which has many formulations, so is the case for the capture theory. However, in the following, solely the economic version will be reviewed. Essentially, this theory is based on the two following simple but important facts:

1. The first is that since the coercive power of government can be used to give valuable benefits to particular individuals or groups, the economic regulation can be viewed as a product whose allocation is governed by laws of supply and demand. This hypothesis has as a consequence to direct attention to the factors bearing on the value of regulation to those who value it the most on one hand, to focus attention to the factors bearing on the cost of obtaining regulation, on the other hand.

2. The second is that the theory of cartels (or cooperative game) may help to locate the supply and demand curves.

However there are at least two reasons why the pattern of regulation and the pattern of private cartelization are different.

1. First, the demand for regulation is greater among industries for which private cartelization is an unfeasible or very costly alternative.

2. Second, favorable regulation requires, in addition to the cooperative action of the firms, the intervention of the political process. And this political dimension of regulation requires two modifications of the theory of cartels when applied to regulation.

   a) The degree of participation in a coalition seeking protective regulation is greater, the greater is the asymmetry among the positions of the industry members.

   b) The determinants of political influence must be worked into the supply side of the market in regulation.

For concluding this review of this theory, two remarks will be made:

1. The economic theory of regulation is still not very well defined in the sense that it is at best a list of criteria, most of them coming from the theory of cartelization, relevant to predicting wether or not an industry will obtain favorable legislation.

2. This theory, when pushed to its logical extreme, excludes the possibility that a society might establish institutions that enable genuine public interest considerations to influence the formation of policy.

2.3 Various definitions of a good and their policy implications

It has been noted previously that the industry of telecommunications is itself an instrument in the hands of the governments for reaching some objectives. It then follows that this industry, by the various services it supplies, can be studied both from an efficiency point of view as well as from an equity one. For example, the government must take all the means to ensure that the services supplied by the carriers in this industry be such that they are provided at the lowest possible costs, the costs including of course a certain rate of return to the shareholders. This is the efficiency point of view. On the other hand, the government would like, for example, to reallocate the resources in the economy in such a way that this reallocation be more socially acceptable, for example guarantee a certain minimal revenue to all the citizens. This is the equity point of view. Taking into account this dual perspective is important first because the government can manipulate the tariff structure for reaching some objectives, second because the latter contains a mixture of both efficiency as well as equity aspects (see sub-section 2.6). Moreover, these two points of view can be antagonistic: in other words, sometimes one has to make a compromise between the efficiency point of view and the equity one. Of course, the resulting compromise depends on a lot of factors, but I think, it depends crucially on how one looks at the nature of the outputs or services supplied by the industry. I now would like to expand on this point; in the next sub-section I will try to match the various objectives with the various services provided by the telecommunications industry.

In the economic literature, three concepts of goods have been discussed: the concepts of private good, of a public good and finally of the merit one. Loosely speaking, a private good is a good where its consumption or utilization by an economic agent excludes its consumption (or utilization) by another one. Such is the case in the network when "all the lines are busy". By a (pure) public good, one means such a good that its consumption by an individual does not preclude its consumption by another agent. Such is the case with a television program: the fact that I am watching a movie does not preclude my neighbour to look the same movie. So is also the case with the military expenses. One can note, however, that some goods have a dimension of a public good but only under some limits of its utilization; these goods are referred to as "public goods subject to congestion". Such is the case with most of the transmission facilities: network in the industry of telecommunications, highways in the transportation industry. Finally, there is the so-called merit goods which by definition are those commodities or services which by their nature are private but the merits (or acquisition) of which are judged as powerful means by the society for reaching some redistributive objectives. Such is the case of the school system. In the following it will be shown that each of these classes of goods introduces different pricing policies and consequently have completely different policy implications.

In order to stress the idea that a pricing policy is strongly dependent on the nature of the product, suppose for instance that our economy contains solely private goods. In such economy, the following results, pertaining to the context, are worth to mention:

a) every firm will sell its product at the marginal cost in order to maximize its profit;

b) at the equilibrium, the tariffs will be such that an optimum allocation of resources will result. In other words, if all economic agents act in such a way as taking these "signals" as given, then it will not be possible in this economy to reallocate the various goods in such a way that nobody will be penalized (less satisfactory).

c) The optimal decision of the agents will be obtained in a decentralized manner, i.e. without having any knowledge of the decisions of the other agents. Of course, the price system is the instrument of decentralization.

d) Except to what has been said in b), no formal equity criteria is taken into account. In consequence, the resulting allocation can be judged inacceptable once one introduces some equity considerations in the analysis.

Now assume the introduction of a public good in such an economy, good which is supplied under the conditions of increasing return to scale. In this new "economy", the above results are modified as follows:

a) From an efficiency viewpoint, a tariff structure must now satisfy the next three criteria:

1. it must enable the total costs of the firm to be recovered;

2. it must be so designed that no customer willing to pay at least the marginal cost to serving him is turned away;

3. there should be no sales below marginal cost, and in fact, some products must be sold at a price higher that their marginal cost in order to pay for the fixed cost;

b) Ideally, the cost of the public good must be payed by those users or consumers of that good. But, as by their very nature this kind of goods can be consumed simultaneously by many persons, there is some incentive for somebody to become a "free rider", i.e. paying nothing for this good but still consuming it. In other words, the price system alone is insufficient for allocating optimally the resources; a coercive agent has to be introduced in this economy.

c) An immediate consequence of the previous point is that no decentralization in this economy is still possible: some knowledge of the actions of the other players becomes necessary for allocating optimally the resources. The presence of large economic externalities emphasizes this point.

Finally, assume that a merit good is introduced in our private economy. In relation to such an economy, the following problems arise:

a) As by its nature a merit good incorporates some redistributive aspect, it then follows that in our economy equity criteria must be defined from which a social welfare function can be derived. But the problems behind each of these steps are enormous.

b) As by definition a merit good is given free or partly free to certain groups in the society, a second question is by which means such an industry will be financed, be self-financed with its consequent cross-subsidy problem, or by some forms of taxation? However, in both cases one has to know the impact of the financing on the allocation of resources.

c) A consequence of the previous two points is the fact that an authority must exist in this kind of economy for coordinating the allocation of resources. It then follows that a price system alone is unable to permit to reach an optimum.

Concerning the services supplied by the industry of telecommunications, it is almost evident that they have certain features of each of these goods. For example, if one splits the various services in the following three components: access to the network, utilization of the facilities content of the service, one can say that it has correspondingly a public good perspective, a private good one and finally a merit good one. And consequently, one can imagine that there is no cost (or a fixed one) for having access to the network, that the utilization of the network be priced at its marginal cost and that the content of the television programs be subsidized by some means or another. Of course, there is a complementary aspect behind these components, but even there, dependent on which component the society would like to promote, a different tariff structure should result.

2.4 Policy objectives in the industry of telecommunications

The Government's objectives regarding canadian telecommunications have been identified in the Green Paper on communications policy for Canada. These objectives can be summarized by quoting from the last paragraph on page 35 of the Green Paper:

> "Economic, efficient and adequate communications, making the best use of all available modes, are essential to the sovereignty, social well-being, cultural development, economic property and safety of all Canadians".

It is evident from this quotation that the government sees the canadian telecommunications industry as a means or instrument among others, for reaching some more general, and broadly defined, objectives. But, having an economically efficient and adequate communications system in Canada represent specific objectives in their own right. So the following question can be asked: what are the means the canadian government possesses for the achievement of these more specific objectives for the telecommunications industry? This question will be taken up again in the next sub-section. For the time-being, this hierarchy of objectives will be formalized and some relations will be established among these objectives and the two afore-mentioned theories of regulation on one hand, and among these objectives and the various definitions of a good, on the other hand. As the reasons for establishing these relations have been motivated in the previous sub-sections, no further comment will be made here.

The hierarchy of the stated objectives and their relationships are formalized on the attached chart (this chart is a revised version of the one which appeared in the Report of March 31, 1977), where an arrow between two goals means that the initial objective contributes to the achievement of the final one. It can be noted first that some goals interact, second that there are many arrows going into the block "efficient and economical system" and "definition of collective objectives". This formal presentation reflects the above quotation about the necessity of an efficient system of telecommunications for reaching the stated objectives.

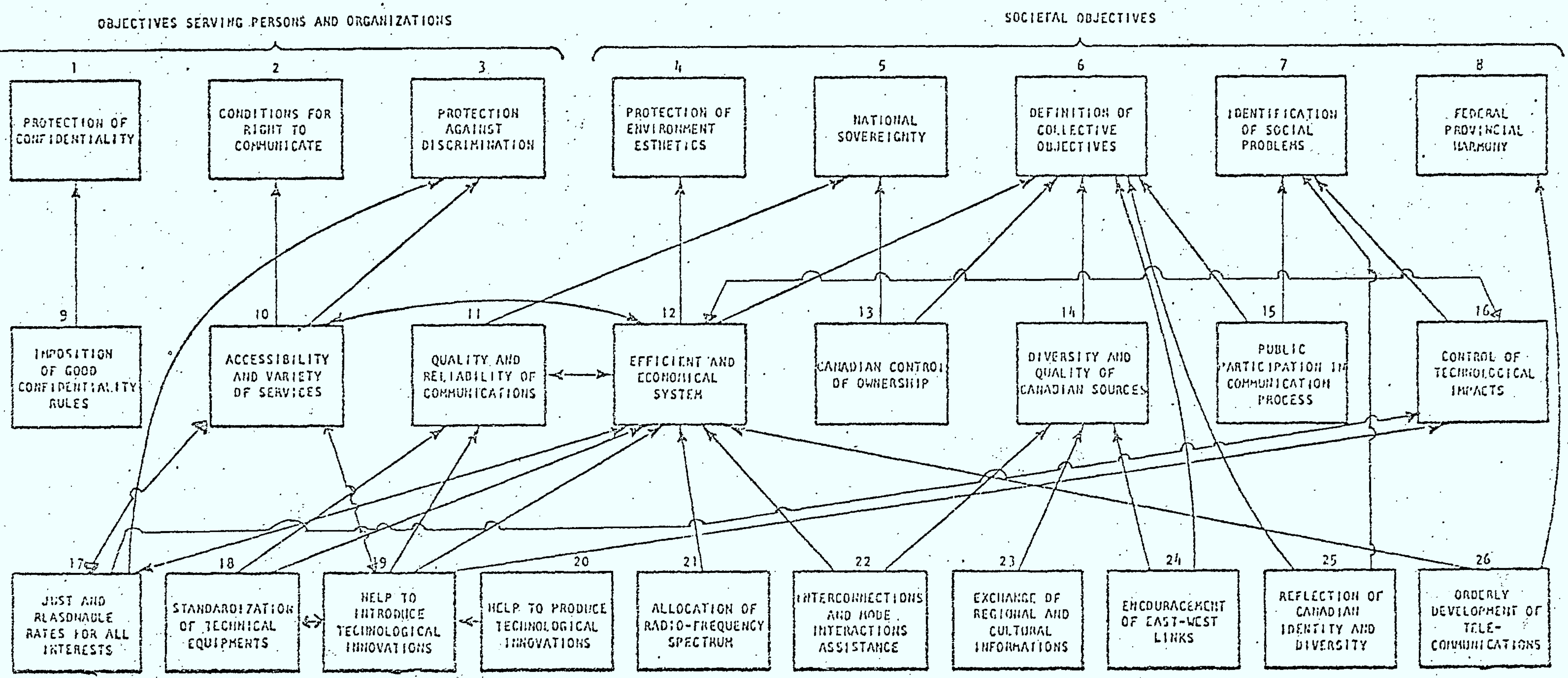
This chart can be studied from a great number of points of view, and in particular from the following ones:

a) <u>levels of aggregation of the objectives</u>: it can be said that most of the stated objectives are aggregation (or summary) of many components. For example, how does one define the environment esthetics, or the national sovereignty, etc...? It can be said that the first row of the tableau refers to some more general or abstract level of objectives, and, in this context, the industry of telecommunications is seen as an instrument, among many others, for reaching these objectives. The last two rows of the tableau refer to less aggregated levels of objectives, and it is obvious that they are more immediately peculiar to the industry of telecommunications.

b) <u>nature of the objectives</u>: more precisely are the objectives stated in qualitative terms or in quantitative ones, or can they be translated into quantitative ones? Of course, this question is strongly dependent on the level of aggregation mentionned in the previous paragraph. It can immediately be said that most of the objectives are stated in qualitative terms, but many of them can be translated into quantitative ones. For example, the objective "accessibility to the services" can be restated in quantitative terms by looking at the number of persons having a telephone set. This is the case for "quality and reliability of communications"; its quantitative equivalent being the probability of losing a call on a final route, or the number of calls which never reach their destination. This is also the case for the "canadian control of ownership"; it is possible to specify the number of shares that must be possessed by citizens.

It is to be noted that the quantitative equivalent of some objectives are not so easy to obtain. For example, think about the "control of technological impacts". The immediate question will be "on what":

INTERRELATIONSHIPS BETWEEN OBJECTIVES IN COMMUNICATIONS POLICY



on the tariffs, on the depreciation expenses, on the level of employment in the economy, etc...? In other words, the determination of a quantitative equivalent cannot be done without the help of an operational model, like the input-output table, like the NPPS model, like the HERMES model, etc... This was one of the motivating factors for building the NPPS and HERMES models.

c) <u>operationality versus the non-operationality of the objectives</u>: in the hierarchical context of the objectives, it is immediate that an objective which is both very disaggregated and which is stated in quantitative terms (or for which there exists an immediate quantitative equivalent) is an objective or instrument which is more operational than the one which does not have these characteristics. It will be shown in the next sub-section that it is possible to define a list of instruments which are more operational than most of the instruments which appear in the last two rows of this figure.

d) <u>hardware versus content of the telecommunications</u>: roughly speaking the last three "columns" or the right hand side of the tableau pertains more immediatly to the content of the telecommunications services, and the left hand side to the economic and technical characteristics of telecommunications networks. But, it is evident that some interrelationships must exist at a certain level: in other words, the state of the technology can impose certain constraints on the possible content or, in equivalent terms, on the variety of the services supplied. It can be noted that the control of content is an instrument of regulation for reaching some objectives like "exchange or regional, natural and cultural informations" as well as "reflection on canadian identity and diversity", both being seen as instruments for reaching the more general objectives of the "identification of social problems" and "definition of collective objectives".

e) <u>equity versus efficiency</u>: it is evident, taking for granted the aggregate character of most of the objectives, that each of them incorporates a mixture of both equity and efficiency aspects.

But more importantly, there is the fact that some objectives can be in conflict: for example, the fact that one would like to have an economical system of telecommunications may be somewhat incompatible with both the objectives that the rates be reasonable for <u>all</u> interested parties and that there must exist some control of the technological impact of innovations (which can postpone some cost-reducing innovation). In the same sense, the government can impose, on equity grounds, that all citizens have access to the telecommunications services, but this objective may conflict with the objective that the tariffs be just and reasonable for all interests. It will be shown in the next sub-section that even at the most operational levels, like the tariffs, some trade-offs must be made between equity and efficiency.

f) <u>depending on the interests served</u>: it can be seen that some of the objectives are more easily associable with the interests of the individual customers and/or entreprises, and some are stated more immediately for the benefits of all the citizens living in society (the so-called societal objectives).

Now, looking at the objectives in canadian telecommunications in this perspective, is it possible to conclude that these objectives reflect the "public interest" theory or the "capture" theory; can these two theories explain the intervetion of the government in the market, and especially in the telecommunications market? Recall that the "public interest" theory assumes that there is governmental intervention for protecting the public, in their role as customers, such protection is required for preventing the monopolist to charge unreasonable rates. Recall also that the "capture" theory justifies this intervention in view of improving the economic welfare of some groups in the society, and in particular the carriers. First, note that the telecommunications industry is most often viewed as a natural monopoly, thus using the "public interest" argument, one is justified in regulating this industry. This is the view of Bonbright, the most representative defender of this theory. But one can see some objectives as more peculiar to the protection of the carriers while others for the protection of the society as a whole, i.e. of the government itself. For example, some objectives like the "standardization of technical equipments", the "quality and reliability of communications" and "canadian control or ownership" can be seen as objectives for reducing competition in the industry, and consequently as an economic benefit to the carriers. For the society as a whole, the objectives like the "national sovereignty", "exchange of regional and cultural informations" and "encouragement of east-west links" can be seen as objectives reflecting societal objectives. Concerning these two ideas, one can say that in the telecommunications industry, the "capture theory" is not supported by evidence, if it has ever been, as competition is increasingly introduced in this industry (at least in U.S.A.). Also, it is evident that the societal objectives carry with them an element of cross-subsidization, so it may be more fruitful in the future to look at regulation as a (invisible) means of taxation. In general, taxation is an instrument used by the government in the pursue of equity objectives. To the extent that cross-subsidization is also an instrument for obtaining equity in the regulatory context, it can also be regarded as a form of taxation (Posner's article: Taxation by Regulation, Bell Journal, spring 1971, develops this idea).

g) nature of the services supplied: it was previously mentioned that the contract with the carriers for telecommunications services includes an access to a network, a possible utilization of it, and finally the content of the services. It is interesting to see that we can associate some objectives with each of these functions. For example, with the access to a network, one can associate the objectives of the "interconnections and mode interactions assistance", the "standardization of technical equipments" and the "accessibility and variety of services"; with the possible utilization of the facilities, one can associate the objectives of "imposition of good confidentiality rules" and the "quality and reliability of communications";

finally, with the content of the services, one can associate the objectives of "exchange of regional and cultural informations", and "reflection on canadian identity and diversity".

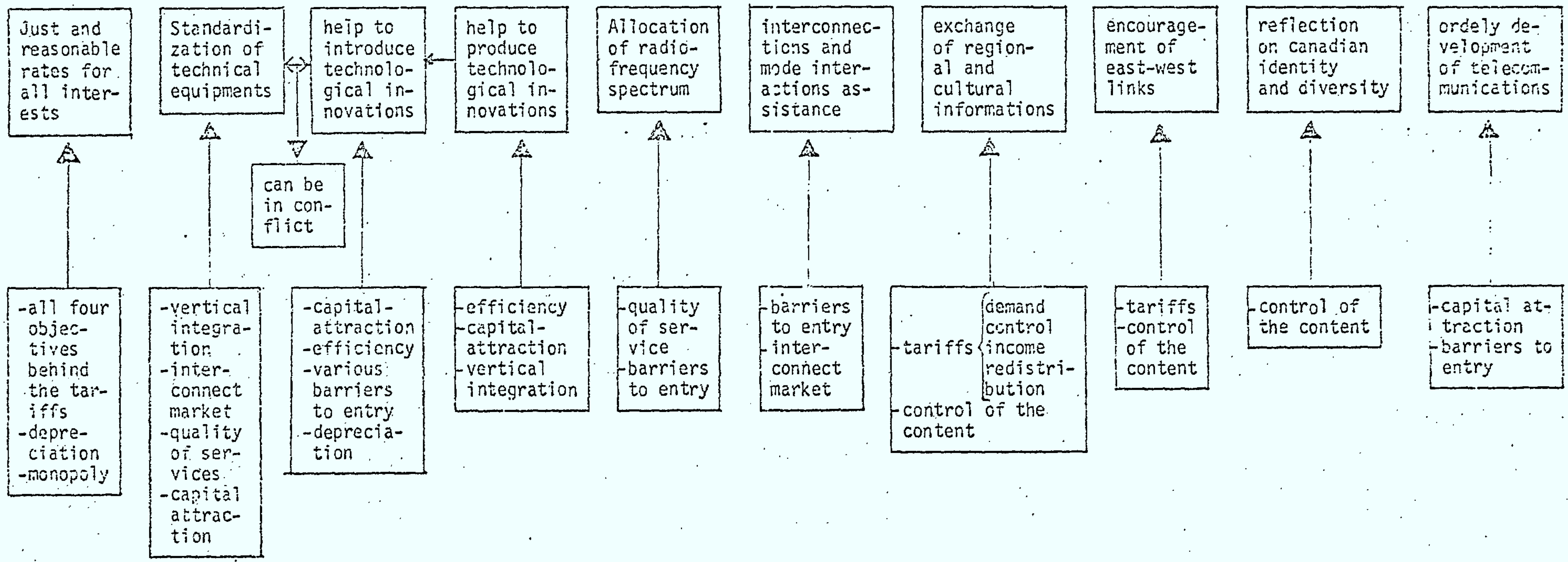
2.5 Means of regulation and their relation to the policy objectives

In the previous sub-section the various objectives which are behind the canadian communications policy were discussed as well as the various perspectives of studying them. In particular, the aggregate character of many objectives, the difficulties of quantifying and ponderating them were stressed. Finally, it was also noted that the industry of telecommunications is by itself an instrument for reaching some more general goals.

The industry of telecommunications is a regulated one, the means of regulation being diverse and numerous. Among them, one can mention the determination of the tariffs, the various barriers to entry and the depreciation methods (or the parameters defining each of them). One important question can be asked: hour these various means relate to the various objectives set out in the previous sub-section? In view of answering this question, one can start from the last row of the previous figure, i.e. from the most concrete objectives, and see how they can be reached by using the aforementioned means of control (and some others). In fact, one can construct a fourth row to this figure with some arrows going to some objectives, and evaluate the direct and indirect impacts on these objectives of the modifications of the value of the means. It will then be possible to see the complementaries among some of these goals for reaching a particular objective and also the antogonistic problems which are behind some means of regulation. These various points are the subject of the present sub-section where, in particular, the multiple objectives which are behind the tariffs will be stressed. The discussion will be based on the figure which appears in the next page.

The first means that the government possesses is the power to impose on the carriers the obligation to serve all the demands at reasonable cost. In the process of regulation, this obligation can be seen as an exchange with the carriers, the other side of that exchange being that the carriers will be the sole supplier of the services in a particular region, and be allowed to earn a certain rate of return on their

Interrelationship between objectives and means in the telecommunications industry



investments. Of course, this means of regulation can be more appropriately defined in terms of various barriers to entry and in terms of the process (or variables) which permits to earn this rate of return. But, more importantly, is the fact that this means carriers with it an idea of merit good aspect and consequently an idea of cross-subsidization. In effect, by imposing that all the demands be satisfied means that the government sees the telecommunications services as so important for reaching some other objectives like the attainment of some collective ones that nobody can be prevented to the supply of these services solely for costing reasons. And, even if this means incorporates some costing consideration, it seems that the pricing principle is more closely related to the "value of service" approach (or what the traffic can bear) then to the "cost of service" approach. Finally, it is expected that this means of regulation will be less important in the future as the technical innovations in several other markets will permit the creation of new services which will compete with the ones supplied by the monopolists.

The second means of regulation is the imposition of some minimal standard of quality in the provision of the services, means which has a direct impact on the technical specification of the equipments and on the objective of "quality and reliability of communications". For example, among the specifications is that the probability of loss in a final route cannot exceed 3% of the calls. It is clear that this means of regulation must be related in some way or another to the objective of "standardization of technical equipments" and to the "introduction of technological innovations". It is also clear that all of these objectives and means introduce some pressure on the rate base of the carriers, so they must be related to the capital attraction function of the tariffs (see below). An interesting policy question can be asked: what are the financial consequences of these technical rules? Or, in other words, is it possible to determine some trade-offs between the quality of service and, say, the rate base in such terms that a decrease of 1% in the quality of service can reduce the rate base by x%? The NPPS model described below has precisely for objective to give some quantitative answers to this kind of questions.

The third means of regulation can be applied to some extent only to the supply of some services: it concerns the control of the contents, for example of the television programs, and their minimal canadian content. This last objective can be seen as a means for reaching more general objectives like the creation of canadian industries in the domain of advertising and in the movies sector. And, as instrument for reaching the objectives like "reflections of canadian identity" and "exchange of regional and cultural informations".

The last three classes of means of regulation, and probably the most important ones, are respectively the tariff levels and tariff structure, the various barriers to entry, and the various depreciation methods. We will examine in sequence each of them.

The process of tariff regulation for the telecommunications industry can be divided into two steps: the first one consists of the determination of a total revenue requirement for the carrier, the second one being the determination of a particular tariff structure. Of course, behind the first step is the classification of the expenses between those which can be expensed and those which can be capitalized. The reader is referred to the Reports on the Cost Inquiry for a discussion on this subject. Here our intent is to reflect on the various functions or objectives served by the tariffs and to relate them to the objectives set out before. Among the objectives served by the tariffs, we shall concentrate on the following four:

a) a capital attraction function, i.e. the tariffs must be sufficiently high for generating sufficient revenues for the carriers so that they obtain a "fair" rate of return on their rate bases and to allow innovations. It should be noted that this industry must compete with many others for satisfying their financing needs. Also, the allowed rate of return is strongly related to the embedded cost of capital, contradicting with the rational utilization of the network which must be based on the prospective incremental cost (see Kahn, A.E., Between Theory and Practice; Reflections of a Neophyte Public Utility Regulator, Public Utilities Fornightly, jan. 2, 1975). Also, it can be noted that this function is also dependent of the structure of financing of the carriers, the different methods of depreciation and finally the income distribution function (customer versus shareholder). For example, it can be envisaged that the regulating agency imposes a relatively high debt-capital ratio in such a way that this gives some advantages to the customers as the interests paid on the bonds are deducted as an operating expenses. In the other hand, it can also impose a certain depreciation method which is such that it increases the burden of financing on the customers. This objective can also be in conflict with the desire of the government to make a proper developments of telecommunications: by allowing a certain rate of return, this can incitate the carriers to eliminate existing facilities for introducing new ones and/or to expand the existing ones. On the other hand, this objective can help for increasing the quality and the variety of the services. So even at this level of disaggregation, the tariffs levels reflect some objectives of the government.

b) a demand-control function, i.e. the tariffs must be such that the customer makes a rational use of the telecommunications services. This function is an essential means for preventing the customer for making an abuse use of the system and also for shifting the peak demand to the off-peak ones. Of course, the success of the controlling of the demand, and consequently its pressure on the rate base, is strongly dependant of the elasticity of the demand for the various services. Moreover, this objective can be in contradiction with the capital attraction one as the monopolist may try to incite the customer to use the various services. Finally, it can be noted that this objective can be discussed in relation with the various pricing schemes, like the two-part tariff, the first part representing a fixed cost for having access to the network, the second part costing the utilization of it.

c) an efficiency-incentive function, i.e. that the cost of supplying the services be as low as possible, or, in other words, that the regulating process does not create any distortion in the allocation of resources, like over-investment in capital. This objective can be in conflict with the various barriers to entry which exist, and also with the objective "control of technological impacts". Of course, this function can be in accordance with the capital-attraction function as it has a direct impact on the rate base. But from the carrier point of view, it can be a "non-interesting" objective as it is remunerated by a certain rate of return on this rate base, so it has interest that the rate base be as large as possible. Of course, one must distinguish between technological innovations which are interesting from a technical viewpoint but are economically unprofitable.

d) an income redistribution function, i.e. essentially the relationship between the customers of the services and the shareholders of the carriers, and also among the customers. It is immediate that this function is related to the determination of the tariff levels and consequently with the problems therein, i.e. the structure of corporate financing, the capital attraction function and the depreciation methods. Also, this function is pertaining to the tariff structure and its consequent problems of cross-subsidization. Note also that the tariffs seen in this context can give some worth to the Posner's argument about taxation by regulation.

The second most important classes of means of regulation are the various barriers to entry. In this context, one can distinguish the following four:

a) the non-possibility of supplying existing services by another carrier: this means of regulation is the counterpart of the first means of regulation previously mentioned. In this context a fundamental problem is the following: does there exist a tariff structure which, given the possibility of the introduction of competitor in this market, no firm will find profitable to go into the market? More technically, this question relates to the sustainability of natural monopoly? In other words, even if the government allows competition, it is possible to imagine a structure (i.e. a tariff structure) where no competition will take place. The existence of barriers to entry in the existing market can also be in contradiction with the efficiency-incentives function of the tariff.

b) the non-possibility of providing new services (cable, pay television, etc). This point is closely related to the regulation problem caused by the introduction of technological innovations and consequently the control of the technological impacts of these innovations, to the objectives of "accessibility and variety of services" and of "interconnections and mode interactions assistance". And moreover this means of regulation is intimately related to the four functions of the tariffs. In consequence, taking the large number of "externalities" this means creates, one must then pay attention to the various consequences that the suppression of these barriers can introduce.

c) the vertical integration between the supplier of the hardware and the carriers. Among the policy questions one can mention the following two (see Kahn vol. 2, pp. 291): 1) should the manufacturer be financially separated from the carrier? 2) what are the proper rate of return to allow for the manufacturer? This barrier to entry must also be related to the efficiency-incentive objective of the tariffs and also the rate of introducing cost - reducing innovations.

d) the interconnect market or the non-competition at the terminal level. This point is closely related with the previous one. Of course, this point must be related to the "technical specifications of the equipments" and consequently to the "quality and reliability of communications".

Finally, the last means of regulation is the depreciation methods with their corresponding parameters. This means is also very important, taken the relative importance of the capital structure in this industry; in fact, the depreciation expenses is the most important annual expenses. It is evident that this means of regulation is related to the capital-attraction and income redistribution of the tariffs. Moreover it is also crucial to every facet concerning the innovations in this industry. And, consequently, it carries with it an idea of intertemporal cross-subsidization.

2.6 The NPPS model

2.6.1 Introduction

In various places in the preceding pages, it was mentioned that the problems in the telecommunications industry can be evaluated from both a descriptive perspective as well as a normative one. For example, it is one thing to test if there exists cross-subsidization in the canadian network and it is another thing to try to evaluate the impacts on some stated goals of modification of, say, the tariff structure. It is evident that arguments presented so far are bond primarily to normative grounds, and second that the policy implications inherent to the industry of telecommunications in Canada are described mainly in qualitative terms. But it is clear that at some stage of the analysis some quantitative answer must be given, and these kinds of answers cannot be obtained without the construction of some operational model capable of simulating and optimizing the objectives. Such a model must be sufficiently disaggregated in order to approximate as closely as possible the activities of the carriers because it is at this level that the means of regulation mainly apply. It is in this spirit that the NPPS model was constructed: essentially it is an operational simulation model which is capable first of testing for the existence of cross-subsidies and if so, to evaluate their quantitative magnitudes, and second of supplying quantitative results, mostly in financial terms, to questions relating to certain policy instruments available to the regulator. As will be shown below, the treatment of this second class of problems necessitates some extensions in the NPPS model as it is presently constituted.

Now, what are the pricing implications of this perspective? It seems evident that if the government wishes to pursue objectives of providing universal access to the services and at the same time to promote some societal objectives, then the tariff structure must be such that there will be some cross-subsidies on one hand, and also tariff structure will be employed to limit the utilization of some facilities of a rational usage, on the other hand. In this context, one can recognize the concepts of public, merit and of private goods with their corresponding tariff strategies.

We will proceed as follows. In the next sub-section, the historic of the project will be briefly reviewed in order to show the temporal evolution of the objectives. In the sub-section 2.6.3 a brief description of the model will be made in order that the present section be self-contained. (The interested reader is referred to previous reports in view of having a better understanding of the complexities of the model). In the sub-section 2.6.4, the most important of this section, the various means of regulation already discussed in section 2.5 will be evaluated in relation to what has been done so far in the NPPS model, then it will be shown how extensions to the model can be made for the purpose of incorporating these regulatory instruments or means in the model. Finally, in the sub-section 2.6.5, some policy questions will be posed as well as their possible implementation in the NPPS model. It will be noted that in the section 4.4 of the second part of this Report some policy questions are posed and answers or solutions previously obtained from the model will be explained.

2.6.2 The historic of the NPPS Project

Since the beginning of the NPPS Project, initially called the IRA Project, in july 1973, one can distinguish three periods with their corresponding objectives:

a) the first year (July 1973 - March 1975). The objectives of this period were mainly to evaluate the impact on the financial statements of the carriers of some various settlement schemes. It was during this phase that it was decided to construct the Model by blocks, (see the next sub-section) each one being able to work alone or in an integrated manner with the others. It should be noted that this approach has proved useful as the Accounting Block has been developed and used independently of the other blocks for several applications since that time.

The second main objective was the incorporation in the model of a Policy Simulation Block. Since that time, development of this block has been delayed due to the difficulties encountered in quantifying policy objectives. However, the motivation and idea behind this remain, i.e. to try to answer some policy questions relevant to the canadian context of the industry of telecommunications. More technically, what the participants had in mind was the determination of some trade-offs between some financial, policy and technical variables. The interested reader can look to the "Final Report of the Second Phase of the IRA Project" for examples of the kinds of policy questions we have in mind. It should be noted that these policy questions are now the central theme of the present Report. Also, it seems that in order to give interesting answers to some policy questions, some feed-back loops have to be introduced in the model. These problems will be discussed more fully below.

It is worthwhile mentioning that one of the main technical achievements during this phase was the computation of some marginal costs of service (whichever defined) without having to introduce (or make any hypothesis about) any cost separation procedure.

b) The second period (March 1975 - March 1976). The studies conducted during this period consisted mainly of measuring, evaluating and empirically testing the cross-subsidization problem in the industry of communications. With this goal in mind, some cross-subsidy tests were developed and implemented in the NPPS model. All of cross-subsidy tests were derived from a game theoretic approach (in fact, this approach has many analogies with the economic theory of cartels already mentioned in the sub-section 2.2, once one reinterprets this theory in the context of a firm supplying many products). Essentially, these tests consisted of a so-called <u>stand-alone cost test</u> and a so-called <u>incremental cost test</u>. Behind the game theoretic approach, the core and the Shapley value are two very important concepts. The core can be defined as the set of allocations which cannot be blocked by any coalition (of services, of consumers) where it is said that a coalition can block an allocation when it can reallocate the goods, the costs or the tariffs among its members in such a way that at least one is better off in the new situation. Only the tariffs which are in the core are the subsidy-free tariffs. The Shapley value of a cooperative game is defined as a weighted average of incremental costs of say a service, where the weights refer to the probability of order of occurrence of that service. In fact, it is supposed that all orders of arrival in a coalition are equally likely (or that users with the same demands are to be charged equally). The importance of the Shapley value (or the fair allocation formula) in the context of the industry under study are twofold: first, due to the importance of fungible costs, the Shapley value can be seen as a way of allocating these costs among the services; second, as this industry is characterized by economies of scale, i.e. by decreasing long term average cost, pricing at marginal cost would introduce some deficits to the various carriers. In consequence, there must be some departure from marginal cost pricing; the Shapley value permits tariffing in such a way that the total revenue will be precisely equal to the total cost i.e. the revenue requirement is satisfied. One question can be asked: is the Shapley value always in the core of the price game? It happens the answer is no. However, it is in the core for the games having the property of being convex, i.e. having the property that the incremental cost of a coalition does not increase when the coalition grows. It is a mathematical way of discribing the phenomenon of economies of scale. In fact, the principles we incorporated in the Operating Block ensure that our "game" is convex and consequently that the Shapley value is in the core of the game. As a result, one can attempt to synthesize a tariff structure according to the Shapley value and consequently obtain a subsidy-free tariff structure. The problem is however not so easy as it seems since one has to take the reaction of the demand into account. In other words, one has to know the elasticities and the cross-elasticities of the various demands.

Regarding the implementation of the various cross-subsidy tests in the NPPS model, this objective has been reached by essentially increasing the interrelationship between the Operating and the Costing Blocks.

The second objective of the present phase was to formalize the fact that the tariffs are the most important instruments available to the government for reaching its various objectives, (the second most important instrument being various barriers to entry). Tarification is however a very complex problem. It has been shown in the previous sub-sections of this Report that the complexities of the problem arrive from many sources. In particular, the objectives offer conflict and are very ill-defined (being mostly in qualitative terms); some institutional arrangements must be taken into account; the various nature of the services supplied by this industry must be considered; finally, the fact that the tariffs serve at least four functions.

Generally speaking, the NPPS model is cost oriented. For this reason, it can determine relatively well if there exists some cross-subsidy in the canadian network, since it compares a posteriori the revenues generated by the demands versus the costs of the various services. But, if we are interested in modifying the tariff structure (for example to reflect the Shapley value), the model necessitates some extensions. This is the subject of the present phase of the project.

c) The present phase (March 76 - March 77): the objectives of this phase are essentially to generalize the cross-subsidy test for taking the non-zero cross-elasticities into account. Second, to introduce a time dimension in the model with a view to computing some prospective incremental cost, and finally to regionalize the tests for evaluating possible inter-carrier cross-subsidization. These extensions are described in the second Part of this Report. However, the central theme of these objectives is the fact that if the government wishes to modify the tariff structure on achieve a particular objective can the model be designed to a plausible answer? In other words, we are now going from a positive perspective (measurement of the existing cross-subsidy) to a normative perspective (modification of a particular tariff structure for achieving certain objectives). As it was previously said, the model in its present state is not very well equiped for considering these perspectives.

Historically, one can say that in the context of regulated industries the determination of the rates of the various services was based on two principles: the "value of service" (or what the traffic can bear) and the "cost of the service". The first one, long time privileged, was essentially related to the elasticities of the demands for the various services and equally to the cross-elasticities among the demands for them. In other words, this tariffing was demand oriented. However, the NPPS Model reflects the "cost of service" principle. In other words, it is supply oriented; one of the reasons being that the cost of each service is the basis for the tariff for that service. What we are now trying to do in the NPPS context is to simultaneously take both sides of the problem into account, without having either very much knowledge of the various cross-elasticities or demand functions. As far as we know, this has never been done previously. There is no doubt in our mind that if the problem is to be resolved, it will be done by simulation.

2.6.3 The functioning of the model

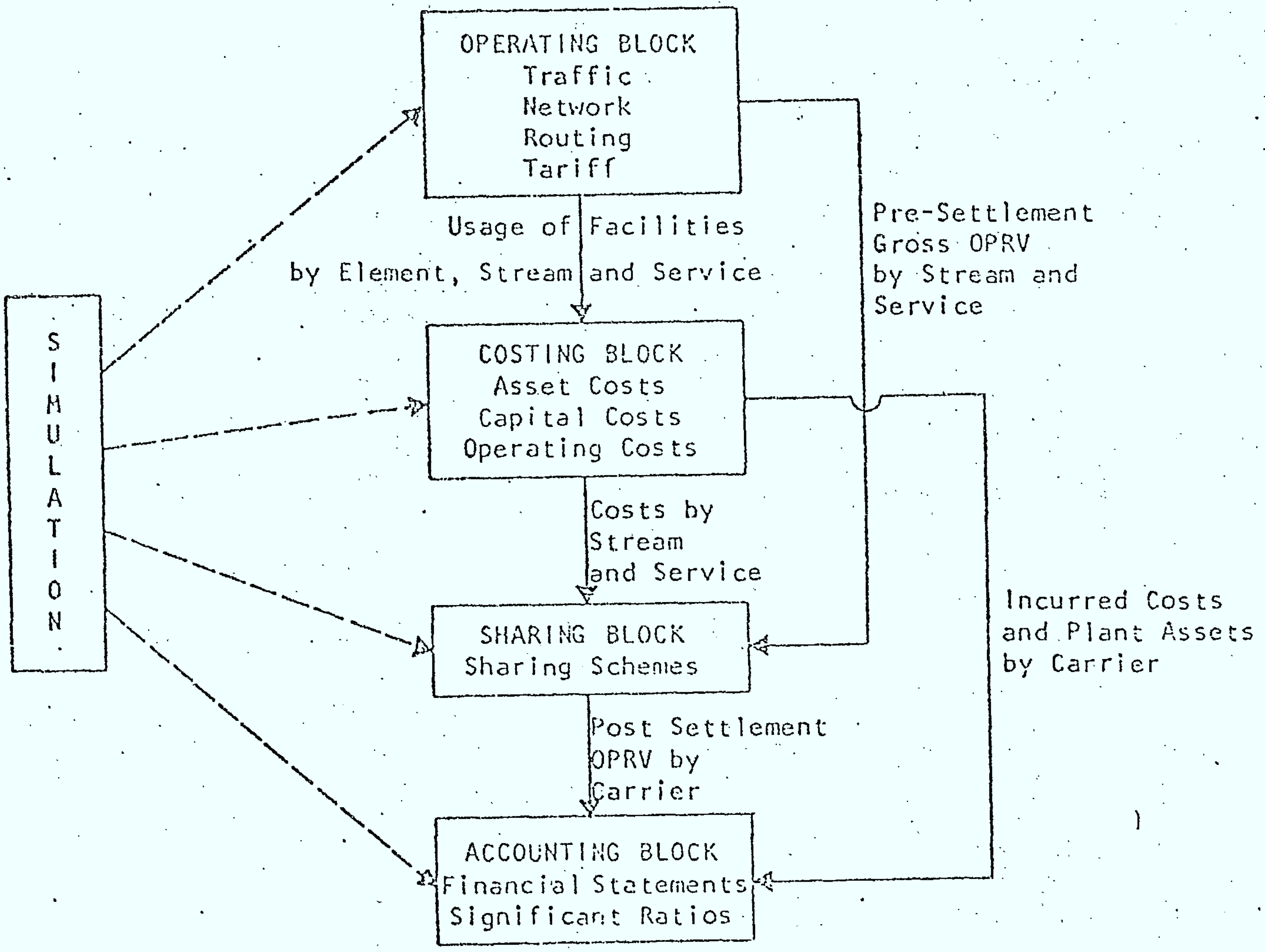
This sub-section presents a summary of the NPPS model. Apart form the logical structure of the model, the emphasis is put on the various inputs needed in each block and the outputs which result from each block once some algorithms are supplied. It is evident that the needed inputs can most frequently be seen as some instrument variables which the government can manipulate. Also, as it will become clear later, some of the outputs of a particular block are inputs for another blocks. As no comment will be made about the various algorithms needed for running the model, the reader is referred to the published reports for obtaining this knowledge.

The general structuring of the model, the various blocks and their interrelations are indicated by the flowchart, figure 1. It can be noted that these blocks refer respectively to the various operations of the carriers, to the costing of the services they provide, to the different settlement schemes for splitting the revenues and/or the costs of their interregional activities, and finally to their methods of accounting.

A NATIONAL TELECOMMUNICATIONS PLANNING AND POLICY SIMULATION MODEL

CONCEPTUAL STRUCTURE

FIGURE 1



A) The Operating Block (see figure 2) takes the structure and the operations of the canadian network into account and the types of traffic it carries. Its main purposes or outputs are to transform the traffic patterns at peak demands into usage on the particular links contained in the physical network and to compute the presettlement gross operating revenues. The usages of the facilities provide an input to the Costing Block and the presettlement gross operating revenues, a component input for the revenue division process dealt with in the Sharing Block. The data bases (or inputs) for this block are as follows:

a) Traffic data base

i) For the non-switched traffic, point to point circuit requirements for a base period are given. Television and private lines are the only non-switched services considered so far.

ii) For switched traffic, point to point offered traffic profiles are provided in Erlangs or C.C.S. for typical days. Although the profiles can be modulated along 24 hours, oftentimes only the load for the peak hour is retained with factors of proportionality to convert the loads for other slices of the day. This type of traffic can be split into U.S. traffic, adjacent province traffic, and non-adjacent province traffic.

b) Switching network (S.N.) data base

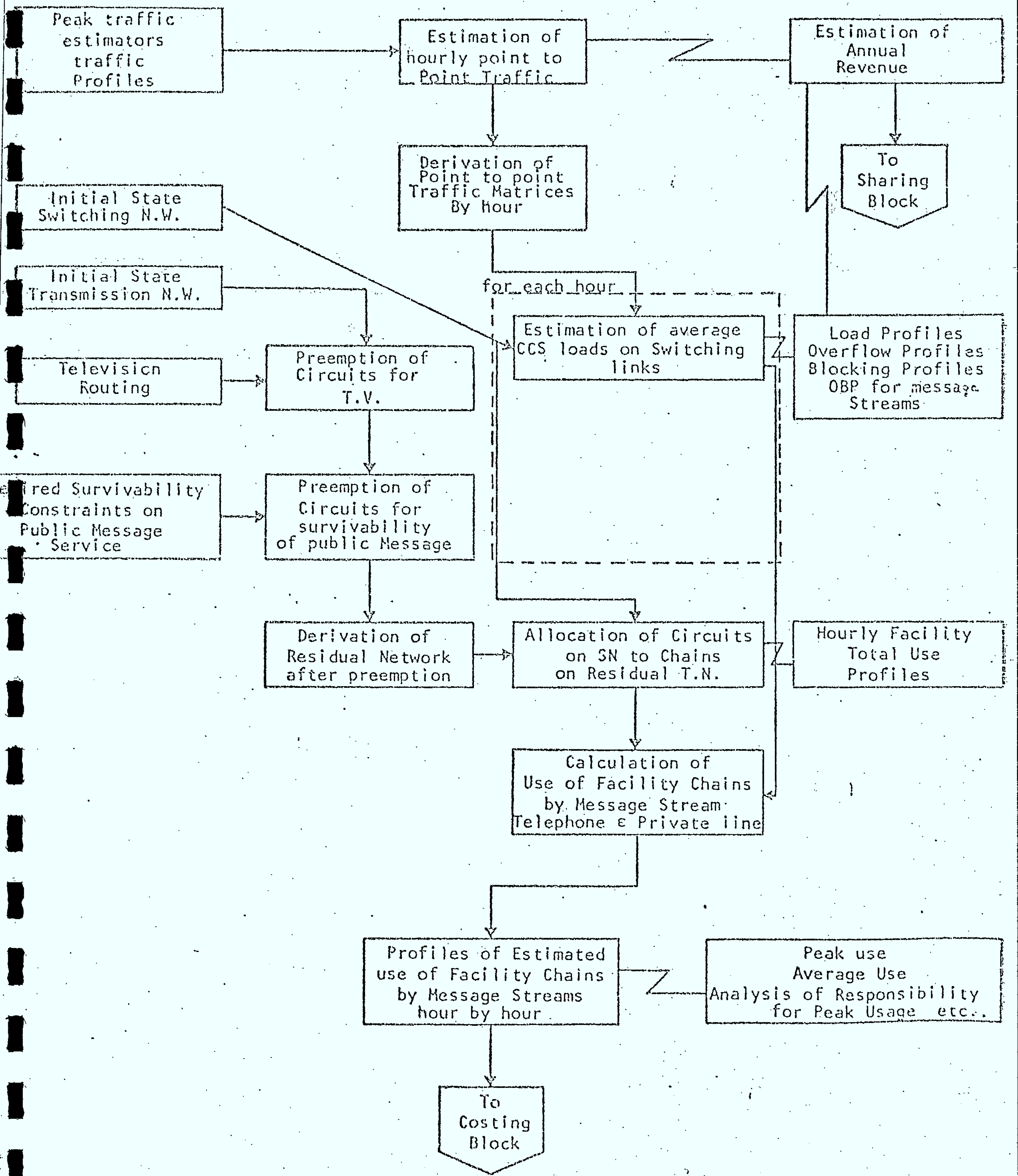
The S.N. is given with its configuration, its hierarchical structure and the rules of overflowing, its quality of service parameters (probability of loss on the ultimate trunk), the number of circuits on each link, the location of the switching machines, and an ownership tag for each facility.

c) Transmission network (T.N.) data base

The T.N. is given with its configuration, the link capacities (actual and ultimate), the ownership tags.

OPERATING BLOCK FLOWCHART

FIGURE 2



d) Tariffs data base

For our interests here, the rates are those of the Trans Canada Telecommunication System.

B) The Costing Block, (see figure 3) performs the functions (or outputs) of associating costs, whichever defined, to the physical facilities of the network, and to allocate costs to various services and streams of messages, using different principles (for example, under the present utilization of the network or assuming its full utilization). The cost by stream and service is fed into the Sharing Block.

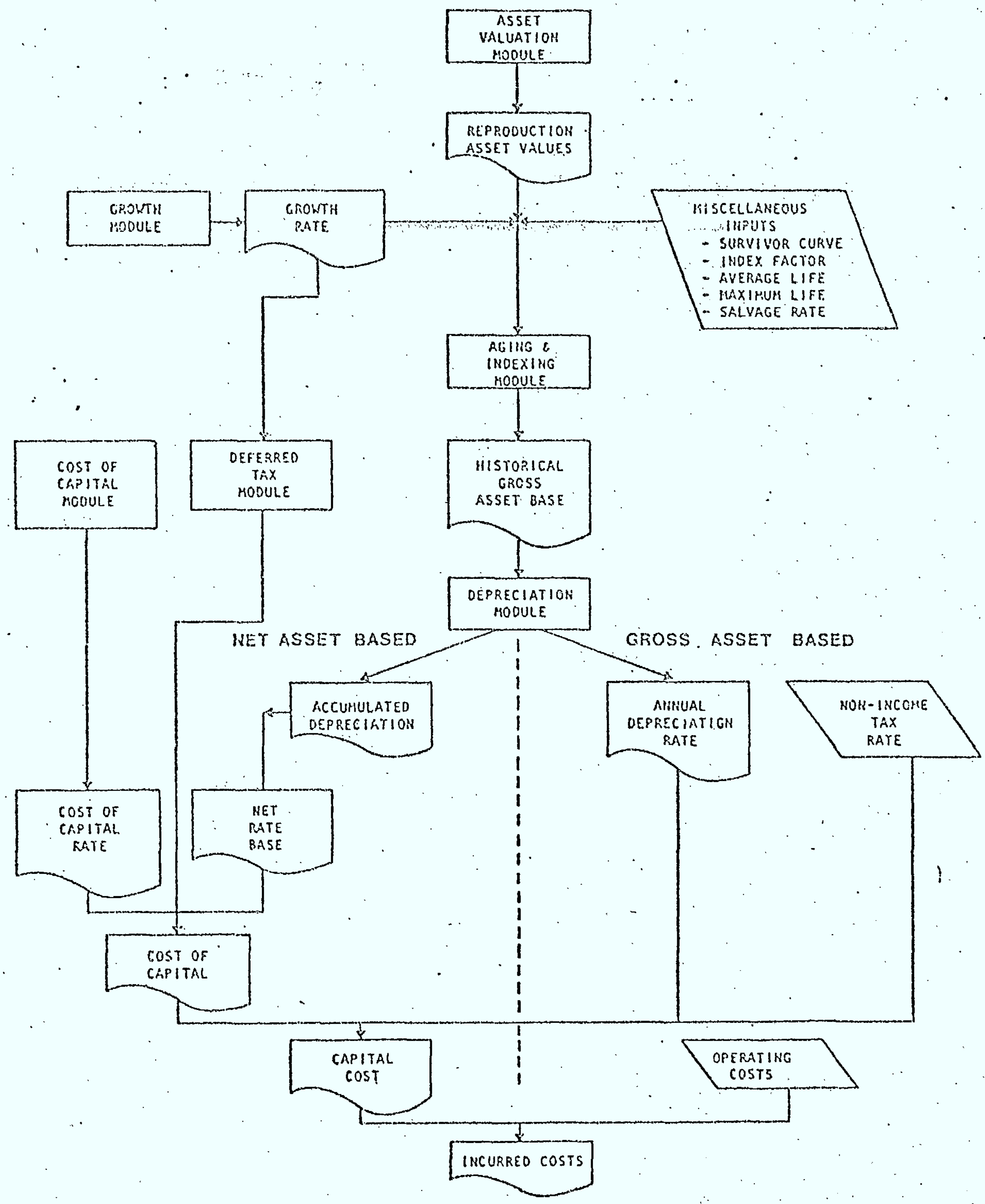
The Costing Block calculate the plant assets by categories of plant and by carrier in terms of both the gross investment and net investment, i.e. gross investment minus accumulated depreciation. This output is fed into the Accounting Block.

The incurred cost of each facility is the summation of the following components: depreciation, cost of capital, operating costs and finally the non-income tax. The incurred costs have been calculated both on a gross asset basis and net asset one, for each carrier and for the following accounting types of plant: switching, transmission, general equipment, building and land. The general logic for calculating these costs is shown by the Costing Flowchart, (figure 3). We will now review the important steps appearing in this diagram.

As can be seen in the figure 3, the starting point for calculating the incurred costs is the assets valuation of the assets, some at their reproduction costs and some at their historical ones. The Asset Valuation module provides, beginning-of-a-(particular)-year values for the aforementioned types of plant. The values for general equipment, building and land are taken directly from the financial statements of the carriers. However, such is not the case for switching and transmission equipments. For these elements, the assets valuation operation is performed by using some assets valuation functions: these functions are step ones in order to capture the economies of scale in the industry of

COSTING BLOCK FLOWCHART

FIGURE 3



telecommunications. Since in the NPPS model we are primarily concerned with the TCTS transmission network which consists essentially of microwave repeaters, multiplexing equipment and towers, these assets only will be evaluated for that network. For that purpose, all repeaters in the TCTS system have been classified, according to their function, into three categories: regular repeaters, branching or junction repeaters, terminal repeaters. For each of them, a cost function was established, the costs varying with the installed capacity (in RF channels).

In view of calculating the first component of the incurred costs, i.e. the depreciation, once the reproduction costs functions are available, the following steps must be performed. First, the growth rate of gross additions to the plants is required both in the Aging and Indexing Module and also in the Deferred Tax Module (the result of which being an input for the computation of the cost of capital). It is obtained as follows:

$$GA(y) = GA(0) \times R^y,$$

where GA(y) is the gross addition in year y; GA(0) is the gross addition in the initial base year and R^y is the compounded growth for y years. R is obtained by first transforming this equation in logarithm, second, by making a regression.

Up to now, the Asset Valuation Module which provides reproduction asset values, applies current cost functions and current technology to assets evaluated in the Operating Block. However these reproduction asset values must be transformed into <u>historical</u> asset values which are the ones which appear in company's books and also which are needed for regulatory purposes. In order to do this, we have to calculate the dollars surviving from the vintages, which we call the aging procedure, and to construct price indices relative to such vintages, which we call the indexing procedure. In consequence, the Aging Module applies various methods of depreciation, survival characteristics and growth rates. In the Aging and Indexing Module, the indexing factors are

a) pricing which reflects dollar inflation over time;

b) changes in technology which reflects differing real costs for the same capacity due to varying states of technology.

Two methods of depreciation are programmed in Depreciation Module. The first is the Average Service Life (SL), which depreciates on a straight line basis the whole of the vintage group. The second is Equal Life Groups (ELG) which depreciates the smaller equal life groups within the vintage group. However, for simulation purposes, it is possible to modify various parameters like the survival curve, the average life of the equipments, the maximum life, the inflation rate, the growth rate and finally the reproduction value.

The cost of capital is the second component for evaluating the incurred costs. It is calculated by using the following equation:

$$\text{Cost of capital} = \frac{1}{(1 - t)} [\text{RORE} \times (1 - \text{DCR})] + (i \times \text{DCR})$$

where t = tax rate, RORE = rate of return on equity: DCR = debt/capitalization; i = average interest rate on debt.

Finally, the last component is the operating costs which include among others the following elements:

- Maintenance: repairs to plant, station equipment, transmission power, buildings and grounds, etc.
- Marketing and commercial: advertising, sale expenses, salaries and wages, directory expenses;
- Traffic: mainly operators wages in the handling of messages;
- Provision for pensions and other employees benefits;
- Accounting: salaries and wages of Accounting and Statistical Departments;

- Engineering: principally expenses incurred in connection with planning for plant additions and changes and for equipment design for customer requirements and special projects;

- Other expenses: general office salaries and expenses, operating rental and miscellaneous expenses.

Up to now, these operating costs are estimated by applying particular ratios to the asset costs.

C) The Sharing Block

The Sharing Block contains a set of sharing schemes between carriers which aim at remunerating the owners for usages of their system by service streams passing through more than one system. In consequence, the output of this block will be, once a particular scheme is used, the determination of the postsettlement operating revenues by carrier. This provides an input for the Accounting Block.

The basic inputs consist of the pre-settlement gross operating revenues by stream of traffic, the facility usages and the facility costs. The output is the post-settlement revenues which are fed into the Accounting Block.

In Canada, revenue settlements occur in five cases:

1) Trans Canada Telecommunication System (TCTS) settlement. It applies to traffic originated and/or terminated in Canada and involving three or more carriers;

2) Adjacent members settlements;

3) Canada-United States settlement (TCTS versus ATT and CN-CP versus Western Union);

4) Teleglobe - CTO (Commonwealth Telecommunication Organization) settlement;

5) Teleglobe - Domestic carriers (TCTS and CN-CP) settlements.

For our experimental model, only three settlement schemes are retained:

1) The "Full Division Plan of Settlement" (TCTS): it starts by pooling all common system revenues; then each member receives from the pool an amount equal to the expenses it assigned to the provision of the revenue generating services. The balance of the pool revenues is shared according to the member's contribution in the provision of the service, this contribution being measured in terms of its share of the assigned plant value. Most separations during the cost assignment are made from relative use measures, and even the excess capacities are allocated.

2) The Old Commonwealth Scheme: it distributes the pooled revenues in the same proportion as the incurred expenses (operating expenses, depreciation and cost of capital)

3) The New Commonwealth Scheme: it equally divides the revenues of each stream between the terminal partners as well as the costs associated with the particular stream. Nodes and links unit costs are computed including capital and operating costs, then route unit costs are derived and stream costs are obtained by multiplying those route unit costs by the respective stream usages and summing for all routes of a stream. The carriers are terminal partners for certain streams and transit partners for others so that a kind of equilibrium is reached between the partners in sharing the revenues.

D) Accounting Block

The financial statements for each carrier, and possibly for all the carriers if desired, are the major outputs of the accounting block and also of the NPPS model. These financial statements basically consist of the following:

a) the Balance Sheet which consits of two main parts: assets and liabilities. The format specifically shows, in great details, the changes that take place during the current year and, in consequence, the situation of the carriers at the beginning of the year, the changes during the year and finally the situation at the end of the year of each carrier are printed.

b) the Income Statement which illustrates the operating revenues, operating expenses, other expenses, income taxes, debt service charges, extraordinary items and finally net income available for dividends and retained earnings.

c) the Sources and Uses of Funds Statement which shows how various funds are collected and from what sources. It also shows how these funds have been deployed, such as in gross construction expenditures investments, miscellaneous and increase in working capital. It should be noted that the sources and uses of funds algebraically equal the changes in the Balance Sheet.

Moreover some financial ratios are also automatically computed. Among them, one can mention the debt/capital ratio, the common dividend pay-out ratio, and the ratio of return for equity components.

Essentially these financial statements and ratios are obtained by utilizing a simultaneous equation system approach, system which contains as relations the main accounting definitions.

Finally a goal programming model is incorporated in the Accounting Block. This model permits to measure the impacts on some key variables once some constraints (for example, an upper bound on the rate of return) are imposed by a regulatory agency.

From the foregoing, the reader has surely noticed that the model has been shaped to utilize and maintain information, both exogenous and endogeneous, at a fine level of disaggregation. Also, by virtue of the "building block" concept and the structuring of the model in four main blocks, a very great flexibility in simulation capability has been provided. As indicated by the dotted line in the flowchart (figure 1), simulation scenarios may be constructed through accessing and changing some control variables or parameters in any one, in more than one or in all of the four main blocks. It follows that the various scenarios and their impact can be evaluated by running the model end-to-end or for intermediate output from any one of the blocks.

2.6.4 Means of regulation and their implementation in the NPPS model: an assesment

In the sub-section 2.5, seven means of regulation were reviewed as well as their importance for helping in reaching the policy objectives of the telecommunications industry. In the present subsection an operational means, i.e. the NPPS model, was described in view of implementing empirically the stated objectives. Then, it can be of interest to look more closely to what extent the various means are already incorporated in the NPPS model, and if not, to see the needed modifications in view of taking them into account in the model.

First consider the obligation to satisfy the demand at reasonable costs. This means is already included in the NPPS model, mostly in the Operating Block and the Costing one, the reasons being that in the former block one of the main inputs are the origin-destination calls at peak hours, and it is precisely one of the main functions in the latter block to compute the cost associated with the various services. Of course, there does not exist any objective measure of the reasonability of the costs.

The minimal quality of the services as well as the technical specifications of the equipments are also included as inputs in the Operating Block.

The means of "control of the contents" does not appear in the NPPS model, and it does not seem to exist any way of formalizing this means of regulation (except may be throughout many quantitative equivalents).

Concerning the depreciation methods, it can be said that for the time being two methods, with their parameters, are already included in the model and also these methods are now interrelated with an Aging and Indexing Algorithm. Of course, some simulations may be done by playing on the various parameters which define the methods.

Finally consider the last two means of regulation, tariffs and barriers to entry. Formally speaking, both of them are not already included in the NPPS model, but each one creates completely different problems. Take for example, the tariffs: it is evident that in some way or another one must incorporate in the model the reaction of the demand once the tariffs are modified, otherwise working with the revenues, as it is done presently, is insufficient. But one we are looking for modifying the existing tariffs structure, one must take the price-elasticities and cross-elasticities into account. The way of introducing them in the model are discussed in various places in the Report.

Now, look at the barriers to entry: here, the question is in the domain of trying to evaluate the possible consequences on some variables if there were no barriers. In other words, this kind of means of regulation has to be looked at in a simulation context. For example, if one would like to know the investment cost on each link in the transmission network, then the allocation model in the Operating Block has to be modified in such a way that the algorithm will have to choose, at each node, which technology is the cheapest. Of course, all the barriers to entry cannot be treated in this fashion, principally because level 5 is not yet incorporated in the model, and some of these barriers are mostly relevant to this level.

2.6.5 Policy implementation in the NPPS model

The NPPS model is a simulation model and it has be shown that it already incorporates some of the most important means of regulation. So, two classes of question remain to be answered. First, to what extent the NPPS model can supply some quantitative answers to policy questions like the tarifying of the excess capacities and its consequent inter-temporal cross-subsidy, the measurement of the cross-subsidy among the services, the determination of an "optimal" tariff structures, etc... Second, to what extent the NPPS model can be used for relating these instruments with the goals or objectives set out previously? We will comment on each class of questions.

As it was mentioned at several places in this Report, the NPPS model is a disaggregate model trying to be as close as possible on the various activities of carriers. Also, generally speaking, the NPPS model is cost oriented. For both reasons, the model is relatively well equiped for evaluating the importance of excess capacities in the networks and for measuring the presence or the absence of cross-subsidy among services and among carriers. But, for the time being, the model cannot determine such thing as an optimal tariff structure, the main reason being that the demand side of the telecommunications services must be taken into account. The reader is referred to the second part of the Report in view of appreciate the needed extensions that have to be made in the model.

The second class of questions is a very complex one at the technical level. By its construction, the NPPS model is shaped by blocks where in some optimization is allowed. But now what we have in mind is the superposition on the model as a whole of a global objective function which can take the various objectives into account. If this kind of approach were possible, one would have an immediate evaluation of the modifications of the values of the various means of regulation on the index which summarizes the objectives. It is the intent of Laval to study this technical problems during the next phase.

PART II

IMPLEMENTATION OF THE POLICY OBJECTIVES IN THE NPPS MODEL

3. Introduction

The purport of this second part of the report is twofold: first, to put some emphasis on the usefulness and the flexibility of the NPPS model in relating several applications of the model executed during the present phase of the contract; second, to give a deeper insight in some possible extensions of the NPPS model in view to render it a more compliant and more reliable tool when dealing with the complex situations arising in the industry of telecommunications.

All the subjects discussed in this part of the report are linked together by the ultimate quest of a satisfying answer to the intertemporal cross-subsidization problem. This objective, which directly aims at an adequate and fair tariff structure, would be reachable if the excess capacities, present in large amounts in both networks, can be precisely defined, appropriately treated and fairly apportioned amongst the services. A good treatment of the excess capacities is a necessary step to obtain a satisfying evaluation of the long-run incremental costs; so, any allocation of these capacities in excess signifies the introduction into the model of the temporal dimension, i.e. we need a redefinition of the services inside a model incorporating many periods. This multi-period framework directly refers to two things: the consideration of an expansion model with which demands become variable parameters and the generalisation of the cost-allocation formula which will take this temporal dimension into account in view to get a fairer distribution of the cost between services and generations. The transition from a static to a multi-period model makes more pressing the task of introducing the various elasticities of demand. With the passage of time, we have to consider the changes of the demands resulting from a modification of the rate structure; moreover, if we want to look at tariffs as an instrumental variable for the regulation authorities, we must envisage the existence of a demand block which will give the necessary information on the changing demands. So, all the discussed topics are closely related and they tend together to obtain a better determination of the tariff structure.

As previously said, the first section of the second part is concerned with the applications of the NPPS model realized during the past year. The first two sub-sections describe simulations translating a similar purpose: in a first phase, the model was run in view to allocate the whole excess capacity according to different schemes; in a second step, it was considered that only the pure excess capacity has to be apportioned, i.e. only the excess capacity not due to a potentially growing demand. Despite the fact that these two kinds of runs distribute differently the burden of excess capacities, they both lead to the conclusion that the amount of excess capacity to be shared is quite large relative to the used capacity.

The next sub-section applies the analysis of goal programming to the NPPS model, particularly to the accounting block. Many simulations were carried out with the 1976 data of Bell Canada, exploiting the advantage permitted by the goal programming method to reach several goals simultaneously by weighing them according to some ordering of priorities. The introduction of this multiple-objective approach in the NPPS model is completed and needs no further implementation before utilization. A last sub-section sums up three simulations made by Sorès this year: the first concerns the testing for the presence of intercarrier cross-subsidization with the respective pre-settlement or post-settlement revenues arrangements; the second run is the considering of the entry of Quebec Telephone in the TCTS club and to the consequences of this admittance upon its revenues according to different sharing schemes; the last simulation deals with the potential division of Bell Canada into two independent carriers. The applications of the NPPS model show its great adaptability to tackle different problems arising in the regulation of the industry of telecommunications.

The second section of part II of the Report analyses three main possible extensions in the NPPS model. The first sub-section studies the generalization of the standard cross-subsidy tests in taking into consideration the presence of cross-elasticities. After discussing the importance of these non-zero cross-elasticities for the regulation issues particularly, we summarize the tentative work done to introduce them and the resulting difficulties. The principal conclusion of this approach, is that one faces two alternatives: either we introduce the cross-elasticities directly into the cross-subsidy tests, which ones might become quite complex, or we consider the grafting on the NPPS model of a demand block. No matter which alternative is choosen, one has to consider the price-demand relationships.

The following sub-section deals with the introduction of an expansion model in the operating block in order to have a multi-period framework and to render possible the treatment of prospective incremental costs. The last sub-section proposes a generalization of the cost-allocation formula along a temporal dimension. This approach distinguishes services offered in distinct periods of time and take into consideration the ordering of arrival of the many services. Such a point of view seems to bring the cost-allocation formula much closer to the way decisions are taken concerning the services of telecommunications; moreover, the procedure looks operational. The last topic of this sub-section presents a simplification of the cost-allocation formula showing more explicitely the way incremental costs appear in this full-allocation cost scheme. There is a simplification of the formula from the fact that the ranking of the services according to their size makes possible the partial avoidance of the combinatorial calculations.

4. Applications of the NPPS model

4.1 Various treatments of the excess capacities

The presence of excess capacities is quite foreseeable in an industry like telecommunications exhibiting fast growing demands and enormous fixed costs. The existence of the capacities in excess can be explained from several points of view: simple error of planification, redundance to reach a survival objective, decreasing demand along a cycle or trend, indivisibility of optimal facilities associated with relatively small demands, growth reserve accumulated to protect against any brutal positive demand variation, growth reserve built to take advantage of economies of scale when the enterprise faces a sustained growing demand, etc... The excedents of capacity raise many interesting problems like who must bear the costs involved by these excess capacities, what are the planning rules adopted by managers to install these extra capacities, what are their extents in the actual network of telecommunications.

The NPPS model can be a helpful tool to answer some of the questions related to the concept of excess capacities and, in fact, last year simulations shed light on some of these issues. In the following paragraphs, we would like to sum up the information brought up by the model relating to its various treatments of the capacities in excess.

The first point to come up was the huge size of the capacity installed in excess of the used capacity. This finding was the result of a first series of simulations with the NPPS model trying to implement the cross-subsidization tests. These tests were performed on groups of services where cross-subsidy was suspected, i.e.:

- public messages versus private lines,
- short distance versus long distance toll traffic,

- peak traffic versus off-peak traffic,
- regional versus adjacent versus non-adjacent and U.S. traffic.

All these simulations gave the same conclusion: the incremental-cost test was always satisfied, with a ratio of revenues over incremental costs so large that it could hardly be believed that the cross-subsidy tests were passed only because approximations or defects in the model. For example, even with a better costing for transmission facilities and with an approximation for the multiplexing plan, the incremental costs of private lines were \$10.1 millions compared with generated revenues of \$41.6 millions, while the corresponding cost of excess capacity was \$98.7 millions.

Given the importance of the common costs and other non directly allocable costs, and since total costs must eventually be recovered, it seemed warranted to devise methods to take account of excess capacities when computing the incremental costs. We then proposed to use an "exhaustive" incremental cost which was the sum of the usual incremental cost and a new term representing a certain portion of the excess capacity imputed to a particular service. The idea behind this approach was to apportion the whole excess capacity between the services since one admitted that the cost of the extra capacity must be supported by the present customers. Two scenarios were envisaged: to share all the unused capacity among services, first according to the fair formula approach (the Shapley value), second proportionately to utilization.

Simulations were run to implement these two full allocations when considering only the scenario of public messages versus private lines. The results were similar for both cases: the full allocated costs of private lines were greater than the corresponding generated revenues. Moreover, the exhaustive incremental cost of private lines, evaluated according to the Shapley value, exceeded the estimated revenues; this situation stems probably from the fact that the Shapley value, in separating evenly the costs among all participating services, disfavors the private lines (and favors public messages) relatively to an allocation rule based on usage.

This double treatment of the excess capacity was not considered quite satisfying since it puts the burden of the entire excess capacity only on the shoulders of the present consumers. Moreover, this allocation was determined by the present relative utilization of the network, and this may be completely different from the future usages. In telecommunications network, protection facilities and indivisibilities leading to economies of scale are frequently associated with fast growing demands; in such a dynamic perspective, a portion of the excess capacity may be seen as a growth reserve which will benefit future as well as present generations. It is therefore justified to impute a part of the unused capacities to actual services; however, this does not imply or necessitate that the entire extra capacity be charged only to present generation.

One way to reach a fairer distribution of the burden may be to devide the excess capacity in two parts: a growth reserve which tends to meet an expanding demand as accurately as possible, and what is called pure excess capacity which is the surplus capacity over the sum of the used capacity and the growth reserve. The philosophy of this distinction lies on the hypothesis that only the growth reserve must be imputed to the customers and then allocated between the actual services; the pure excess capacity has to be borne by the carrier. This methodology represents an improvement since the charge imposed on present consumers corresponds only to their probable growing demand.

The implementation of the apportionment of the growth reserve necessitates the recourse to an expansion model and will be the subject of the next section.

4.2 Variations of the demands (expansion model)

The concept of a growth reserve implies the use of moving horizon (we will take three years) and the choice of growth rates for the demands translating the prospective usage of the equipment. The procedure consists to run the model successively for three years, increasing the demand for every service according to a growth rate particular to each service and determined exogenously. Even if the purpose of such a simulation is yet the allocation of some part of the excess capacity, we range it under the title of variation of demands since this approach may necessitate some expansion features in the model; in fact, after each year of growing demand some links could be saturated and block any future growth, though ample excess capacities may yet exist on most of the other links. No actual expansion scheme exists in the model right now, but all the links that became saturated during the process were handled by hand, pushing the limiting constraint so as to permit the expansion of the growing demand.

Two kinds of simulations were executed with the NPPS model in view to share the growth reserve among public messages and private lines: we allocated this spare capacity either according to utilization, or according to the fair allocation formula, taking a 12% uniform annual growth rate for public messages and a similar growth rate of 18% for private lines. In each kind of simulation we allocate the growth reserve either on present usage, or on future usage, but the destination does not make great difference. The results of the two simulations were quite similar, either when the sharing of the growth reserve was based on usage or on the Shapley value: the revenues of the private lines were greater than the fully allocated costs, in both cases, no matter what was the choosen planning horizon. The private lines fully allocated costs varied from \$25.5 to \$28.2 millions when based on usage, and from \$31.7 to \$35.1 millions when using the Shapley value, compared to generated revenues estimated at \$41.6 million. We can note the bias displayed by the Shapley value against the private lines.

These tests based on prospective expanding demands over a three-year period indicate that the exhaustive incremental cost of private lines is nearly proportional to the potential volume of service, given the growth rate of the demand. This result implies it is quite unlikely that the portion of the excess capacity imputed to a service might enlarge the exhaustive incremental cost so much as to exceed the generated revenues (even a doubling of the planning horizon and a relatively larger growth rate would not do that). We may infer from this that the growth reserve is a small portion of the unused capacities, and that the largest part of excess capacity is pure excess, even though some links can become saturated in the expansion of the demands over a three-year growth reserve.

A fuller treatment of the excess capacities, with a prospective causal responsability principle in view, would require, for example, a redefinition of the services in a temporal framework, the possibility of creating new services and demands, the utilization of growth rates sensitive to different kinds of links, the consideration of technological diffusion.

4.3 Goal Programming

4.3.1 The model

Dans le working paper no. 1 de septembre 1977, nous avions appliqué la programmation par objectifs à un modèle de 11 variables, 4 ratios et 25 équations dont 9 identités. Plusieurs variables endogènes devaient alors être fixées, a priori, à leur valeur historique.

Nous avons maintenant désagrégé le modèle pour le rendre plus pertinent. Nous obtenons ainsi 36 variables, 9 ratios et 14 identités. Lorsque nous utilisons toutes les contraintes du modèle, à savoir une borne inférieure et supérieure pour chaque variable et ratio, on obtient 104 équations.

Nous utilisions pour le modèle agrégé, un programme APL de programmation linéaire. En transformant la formulation du problème, nous obtenions les mêmes résultats qu'avec la programmation par objectifs. Les résultats des simulations étaient d'ailleurs donnés dans le working paper no. 1. Etant donné les dimensions que prend maintenant le problème, nous avons dû chercher un véritable programme de programmation par objectifs. Puisqu'il n'en existait aucun dans les programmes commerciaux couramment utilisés, nous avons utilisé celui présenté dans "Goal Programming for Decision Analysis"[1], et nous l'avons légèrement modifié. Nous le présentons en annexe avec un diagramme logique de son fonctionnement.

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(1) Lee, Sang M., Goal Programming for Decision Analysis, Auerbach, Philadelphie, 1972, 387 pages.

4.3.2 The software

Les 36 variables considérées sont les suivantes:

NETINC	NIA	TRANGV	REPL	DPRTVE	LAND
OPRV	EQ	DELCTI	DEPN	OPXP	GETV
DIVI	RE	DELDCR	DEPDIF	OTHINC	GTP
NEWDEB	PR	DELCL	CURDTX	IDC	UCC
DELEQ	L	DELINV	PRDTX	AD	OTHEXP
DELPR	GCE	DELDCH	DPRTVC	RET	CCA

Les 9 ratios dont on tient compte sont: DCR, PCR, DPR, ROREC, RORC, ITCAT, ITCBT, RORBI, RORBE. Notons enfin que les différents coefficients tels que T, IO, IN, etc... sont fixes et égaux à leur valeur historique.

Le lecteur trouvera dans l'appendix C le lexique des symboles utilisés dans le bloc comptable.

EQUATIONS SIMULTANEES

1) -NETINC + DIVI - NEWDEB - DELEQ - 0.955 DELPR + GCE + TRANGV + DELCTI -DELDCR - DELCL + DELINV + DELDCH + REPL - DEPN -DEPDIF - CURDTX -PRDTX - DPRTVC - DPRTVE = 9699.055

2) -NETINC + .50364 OPRV - .02203 NEWDEB + .01977REPL - .50364 DEPN -.50364 DEPDIF - .50364 OPXP + .85109 OTHINC + IDC = 83057.13

3) NETINC - .045 DELPR - NIA = 27326.63

4) -DELEQ + EQ = 1,435,305

5) DIVI - NIA + RE + TRANGV = 594,095

6) -DELPR + PR = 343,211

7) -NEWDEB + L + REPL = 2,100,392

8) AD - DEPN - DEPDIF - DPRTVC - DPRTVE + RET = 1,732,457

9) -.01292GCE + DEPN + .02991 RET + .02991 LAND + .02991 GETV = 372,947.69

10) -GCE + RET + GTP = 6,433,396

11) IDC - .00895 GCE = 6672.80

12) -GCE + DPRTVC + IDC + UCC + OTHEXP = 3,921,368

13) .10043 UCC - CCA = 6.77

14) .49636 DEPN + .49636 DEPDIF + CURDTX - .49636 DPRTVC + .49636 DPRTVE -.49636 IDC - .49636 OTHEXP - .49636 CCA = 0

CONTRAINTES SUR LES RATIOS

$$-L_B(EQ + RE + PR) + (1 - L_B)L - d^+ = 4474552L_B - 2,100,392$$

(DCR)

$$-U_B(EQ + RE + PR) + (1 - U_B)L + d^- = 4474552U_B - 2,100,392$$

$$-L_B(EQ + RE + L) + (1 - L_B)PR - d^+ = 4474552L_B - 343,211$$

(PCR)

$$-U_B(EQ + RE + L) + (1 - U_B)PR + d^- = 4474552U_B - 343,211$$

$$DIVI - L_B NIA - d^+ = 0$$

(DPR)

$$DIVI - U_B NIA + d^- = 0$$

$$NETINC + .04375NEWDEB - \tfrac{1}{2}L_B(EQ + RE + PR + L) - .03926REPL - d^+ = 2237276L_B - 164,992.78$$

(RORC)

$$NETINC + .04375NEWDEB - \tfrac{1}{2}U_B(EQ + RE + PR + L) - .03926REPL + d^- = 2237276U_B - 164,922.78$$

$$NIA - \tfrac{1}{2}L_B(EQ + RE) - d^+ = 1,015,474.5L_B$$

(ROREC)

$$NIA - \tfrac{1}{2}U_B(EQ + RE) + d^- = 1,015,474.5U_B$$

$$NETINC - (L_B - 1)(.04375NEWDEB - .03926REPL) - d^+ = 164,922.78(L_B - 1)$$

(ITCAT)

$$NETINC - (U_B - 1)(.04375NEWDEB - .03926REPL) + d^- = 164,922.78(U_B - 1)$$

$$NETINC + .49636(OPRV - DEPN - OPXP) + .04375(1 - T - U_B)NEWDEB + .03926(L_B + T - 1)REPL + .14891\,OTHINC - d^+ = 164{,}922.78(L_B + T - 1) \quad \text{(ITCBT)}$$

$$NETINC + .49636(OPRV - DEPN - OPXP) + .04375(1 - T - U_B)NEWDEB + .03926(U_B + T - 1)REPL + .14891\,OTHINC + d^- = 164{,}922.78(U_B + T - 1)$$

$$-NETINC - .04375NEWDEB + .03926REPL - \tfrac{1}{2}L_B(CURDTX + PRDTX + AD - GTP) + OTHINC = 164922.78 - 1769903.5L_B \quad \text{(RORBI)}$$

$$-NETINC - .04375NEWDEB + .03926REPL - \tfrac{1}{2}U_B(CURDTX + PRDTX + AD - GTP) + OTHINC = 164{,}922.78 - 1{,}769{,}903.5U_B$$

$$-NETINC - .04375NEWDEB - .13355L_BGCE + .03926REPL + OTHINC + IDC - \tfrac{1}{2}L_B(CURDTX + PRDTX + AD - GTP) + d^- = 164{,}922.78 - 1670316L_B \quad \text{(RORBE)}$$

$$-NETINC - .04375NEWDEB - .13355U_BGCE + .03926REPL + OTHINC + IDC - \tfrac{1}{2}U_B(CURDTX + PRDTX + AD - GTP) - d^+ = 16{,}4922.78 - 1{,}670{,}316U_B$$

où L_B = borne inférieure
U_B = borne supérieure

Identification des membres de droite des équations comptables

EQUATION 1: $-PRO \cdot RHO + OTHADJ - ADJP + ADJR + ADJA + NSV - DELOCA$

EQUATION 2: $(1-T)\ IO \cdot LO$

EQUATION 3: $PRO \cdot RHO$

EQUATION 4: EQO

EQUATION 5: $REO + OTHADJ$

EQUATION 6: PRO

EQUATION 7: LO

EQUATION 8: $ADO + ADJR + NSV$

EQUATION 9: $DEPRAT \cdot [\, GTPO + \frac{1}{2}\ (- PUCO - LANDO - GETVO + ADJP)]$

EQUATION 10: $GTPO + ADJP$

EQUATION 11: $\frac{1}{2} \cdot IC \cdot PUCO$

EQUATION 12: $UCCO - ADJU$

EQUATION 13: — (6.77 représente une erreur d'approximation)

EQUATION 14: —

EQUATION 15: L_B (CONST 15) $- LO$

EQUATION 16: U_B (CONST 15) $- LO$

EQUATION 17: L_B (CONST 15) $- PRO$

EQUATION 18: U_B (CONST 15) $- PRO$

EQUATION 19: 0

EQUATION 20: 0

EQUATION 21: $\frac{1}{2} \cdot L_B\ (EQO + REO)$

EQUATION 22: $\frac{1}{2} \cdot U_B\ (EQO + REO)$

EQUATION 23: $\frac{1}{2} \cdot L_B$ (CONST 15) $- IO \cdot LO$

EQUATION 24: $\frac{1}{2} \cdot U_B$ (CONST 15) $- IO \cdot LO$

EQUATION 25: $(L_B - 1) \cdot IO \cdot LO$

EQUATION 26: $(U_B - 1) \cdot IO \cdot LO$

EQUATION 27: $(L_B + T - 1) \cdot IO \cdot LO$

EQUATION 28: $(U_B + T - 1) \cdot IO \cdot LO$

EQUATION 29: $L_B\,[-\frac{1}{2}(GTPO - ADO) + DEFTXO + ADJU + ADJB] + IO \cdot LO$

EQUATION 30: $U_B\,[-\frac{1}{2}(GTPO - ADO) + DEFTXO + ADJU + ADJB] + IO \cdot LO$

EQUATION 31: $L_B\,[\ \frac{1}{2}(PUCO - GTPO + ADO) + DEFTAXO] + IO \cdot LO$

EQUATION 32: $U_B\,[\ \frac{1}{2}(PUCO - GTPO + ADO) + DEFTAXO] + IO \cdot LO$

L_B = borne inférieure

U_B = borne supérieure

CONST 15 = EQO + REO + LO + PRO

Pour chacune des simulations on utilisera les 14 équations simultanées mais le choix des contraintes sera différent. On devra imposer des contraintes sur chacune des variables pour éviter des résultats aberrants tels que des revenus d'opération nuls. Ces contraintes n'ont pas été présentées car elles sont simplement de la forme

$$X_i - d^+ = L_B$$

$$X_i + d^- = U_B \qquad i = 1 \ldots 36.$$

Les contraintes sur un ratio en particulier ne seront utilisées que lorsque ce même ratio sera dans la fonction objectif.

Dans un premier temps, nous avons testé le modèle en fixant les bornes inférieures égales aux bornes supérieures et égales aux valeurs historiques. Les résultats furent très concluants car toutes les valeurs historiques furent retrouvées avec une marge d'erreur ne dépassant pas 0.01%.

A titre d'exemple, nous présentons dans ce rapport deux simulations dont les résultats sont donnés aux tableaux 1 et 2. La première simulation visait un DCR = 0.45, tout en respectant les bornes décrites au tableau 1. Le deuxième exemple simule une augmentation des dépenses de construction (GCE) de 10%, financées au tiers par un accroissement de la dette. On veut encore une fois un DCR = 0.45 mais on désire aussi garder les revenus d'opération (OPRV) et DPR à leur valeur historique. Malgré ces contraintes, on obtient toujours une solution réalisable et tous les objectifs sont atteints.

Tableau 1

Résultats des simulations

NOM DE LA VARIABLE	VALEUR HISTORIQUE	SIMULATION #1			SIMULATION #2		
		BORNE INFERIEURE	SOLUTION	BORNE SUPERIEURE	BORNE INFERIEURE	SOLUTION	BORNE SUPERIEURE
NETINC	238,633	100,000	218,866	500,000	100,000	371,211	500,000
OPRV	1,903,924	1,000,000	1,592,945	3,000,000	1,000,000	1,903,915	3,000,000
DIVI	143,969	–	–	300,000	–	235,997	300,000
NEWDEB	255,180	–	–	500,000	330,000	330,000	500,000
DELEQ	41,082	–	100,000	100,000	–	538,518	800,000
DELPR	33,782	–	100,000	100,000	–	0	100,000
NIA	209,786	50,000	187,039	500,000	50,000	343,884	500,000
EQ	1,476,387	1,435,000	1,535,301	1,600,000	1,435,000	1,973,816	2,300,000
RE	659,912	300,000	781,134	1,000,000	300,000	601,981	1,000,000
PR	376,997	343,000	443,211	600,000	343,000	343,211	600,000
L	2,266,172	2,100,000	2,099,997	2,700,000	2,100,000	2,230,387	2,700,000
GCE	900,692	500,000	500,000	1,500,000	990,000	990,000	1,500,000
TRANCV	0	–	0	100,000	–	100,000	100,000
DELCTI	0	–	0	100,000	–	100,000	100,000
DELDCR	1,141	–	0	100,000	–	0	100,000
DELCL	77,922	25,000	25,000	200,000	25,000	25,000	200,000
DELINV	523	–	0	100,000	–	100,000	100,000
DELDCH	1,348	–	39,498	100,000	–	80,724	100,000
REPL	89,400	–	391	200,000	–	200,000	200,000
DEPN	381,878	200,000	380,336	500,000	200,000	379,693	500,000
DEPDIF	0	–	0	100,000	–	0	100,000
CURDTX	87,638	50,000	200,000	200,000	50,000	152,599	200,000
PRDTX	0	–	0	100,000	–	0	100,000
DPRTVC	5,247	–	98,024	100,000	–	0	100,000
DPRTVE	5,247	–	0	100,000	–	0	100,000
OPXP	990,245	900,000	900,000	2,000,000	900,000	900,000	2,000,000
OTHINC	50,493	–	150,000	150,000	–	150,000	150,000

Tableau 1 (suite)

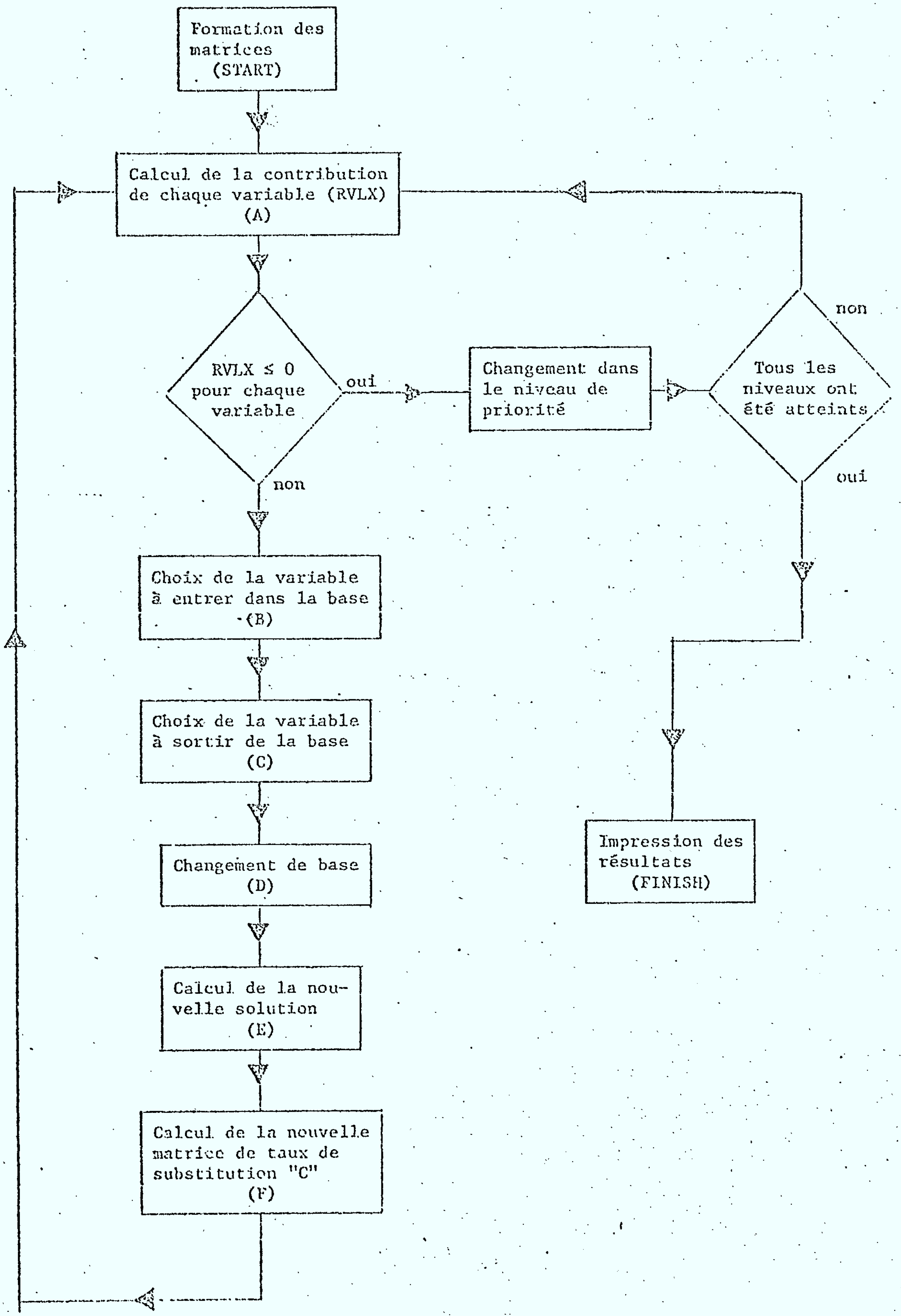
NOM DE LA VARIABLE	VALEUR HISTORIQUE	BORNE INFERIEURE	SOLUTION	BORNE SUPERIEURE	BORNE INFERIEURE	SOLUTION	BORNE SUPERIEURE
IDC	14,734	–	14,335	100,000	–	15,533	100,000
AD	1,999,212	1,500,000	1,710,808	3,000,000	1,500,000	1,612,144	3,000,000
RET	125,168	–	500,000	500,000	–	500,000	500,000
LAND	152,110	–	0	500,000	–	0	500,000
GETV	83,787	–	0	200,000	–	0	200,000
GTP	7,208,470	6,000,000	6,433,393	9,000,000	6,000,000	6,923,388	9,000,000
UCC	4,733,779	3,000,000	4,106,549	6,000,000	3,000,000	4,695,826	6,000,000
OTHEXP	68,300	–	200,000	200,000	–	200,000	200,000
CCA	475,407	200,000	436,384	800,000	200,000	471,595	800,000

Tableau 2

Valeurs prises par les ratios

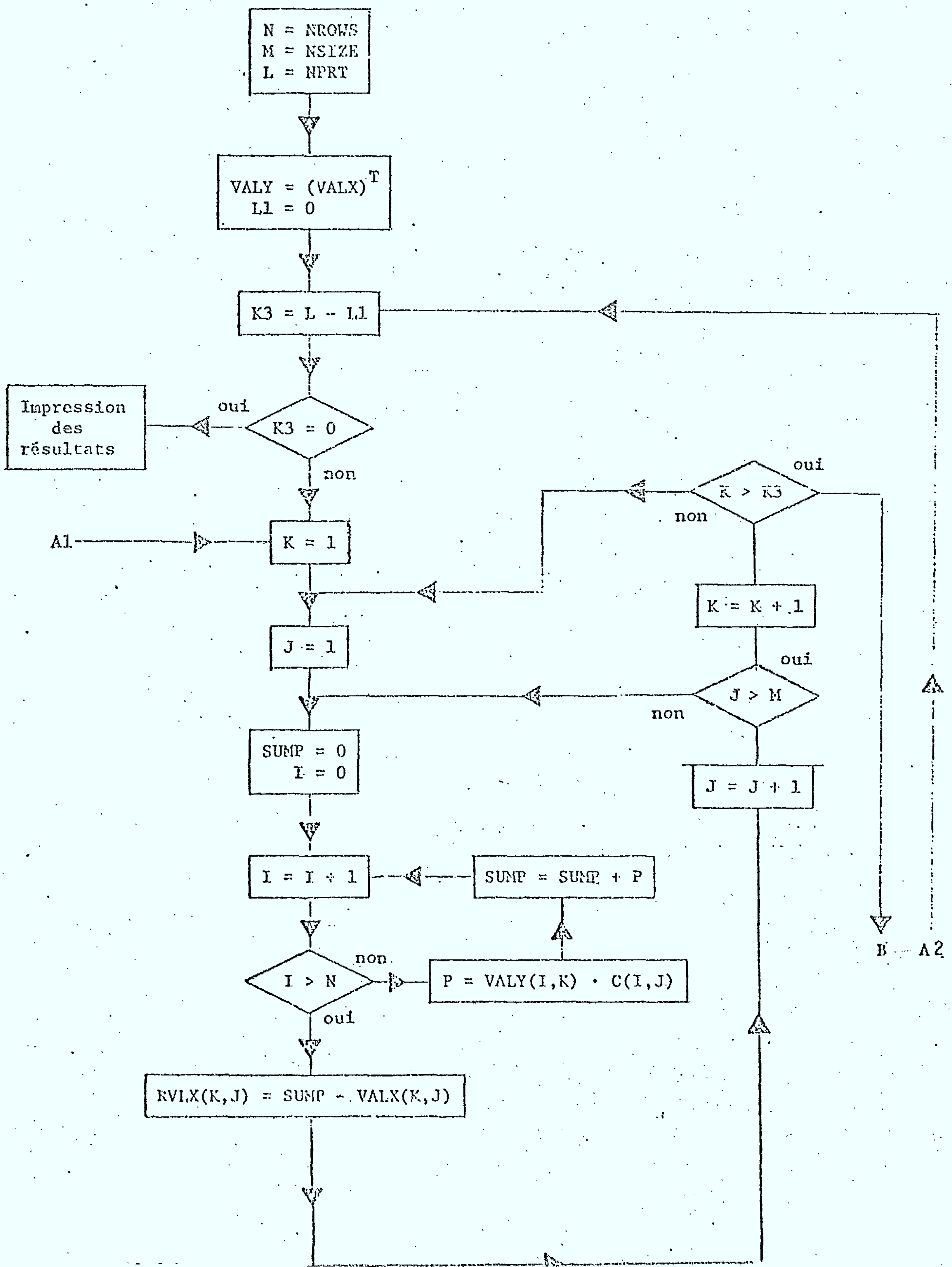
RATIO	VALEUR HISTORIQUE	SIMULATION #1	SIMULATION #2
DCR	0.47186	0.45000	0.45000
PCR	0.07783	0.08425	0.07132
DPR	0.68627	0.00000	0.68627
ROREC	0.10068	0.08605	0.14930
RORC	0.08887	0.08223	0.11278
ITCAT	2.38282	2.32721	3.16440
ITCBT	3.45963	2.90723	4.60485
RORBI	0.08329	0.05799	0.09030
RORBE	0.08416	0.05678	0.09161

SCHEMA GENERAL

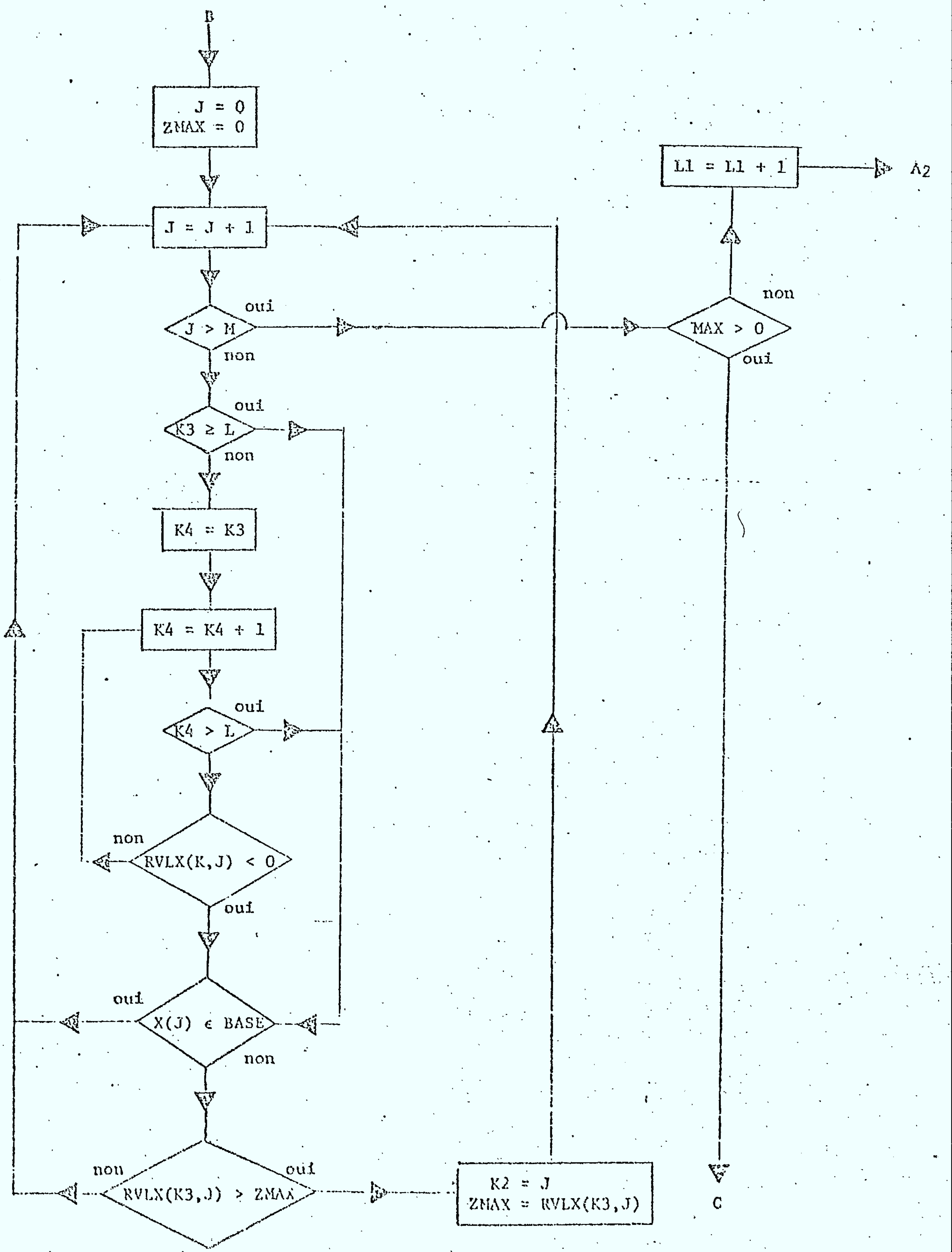


PROGRAMME PRINCIPAL

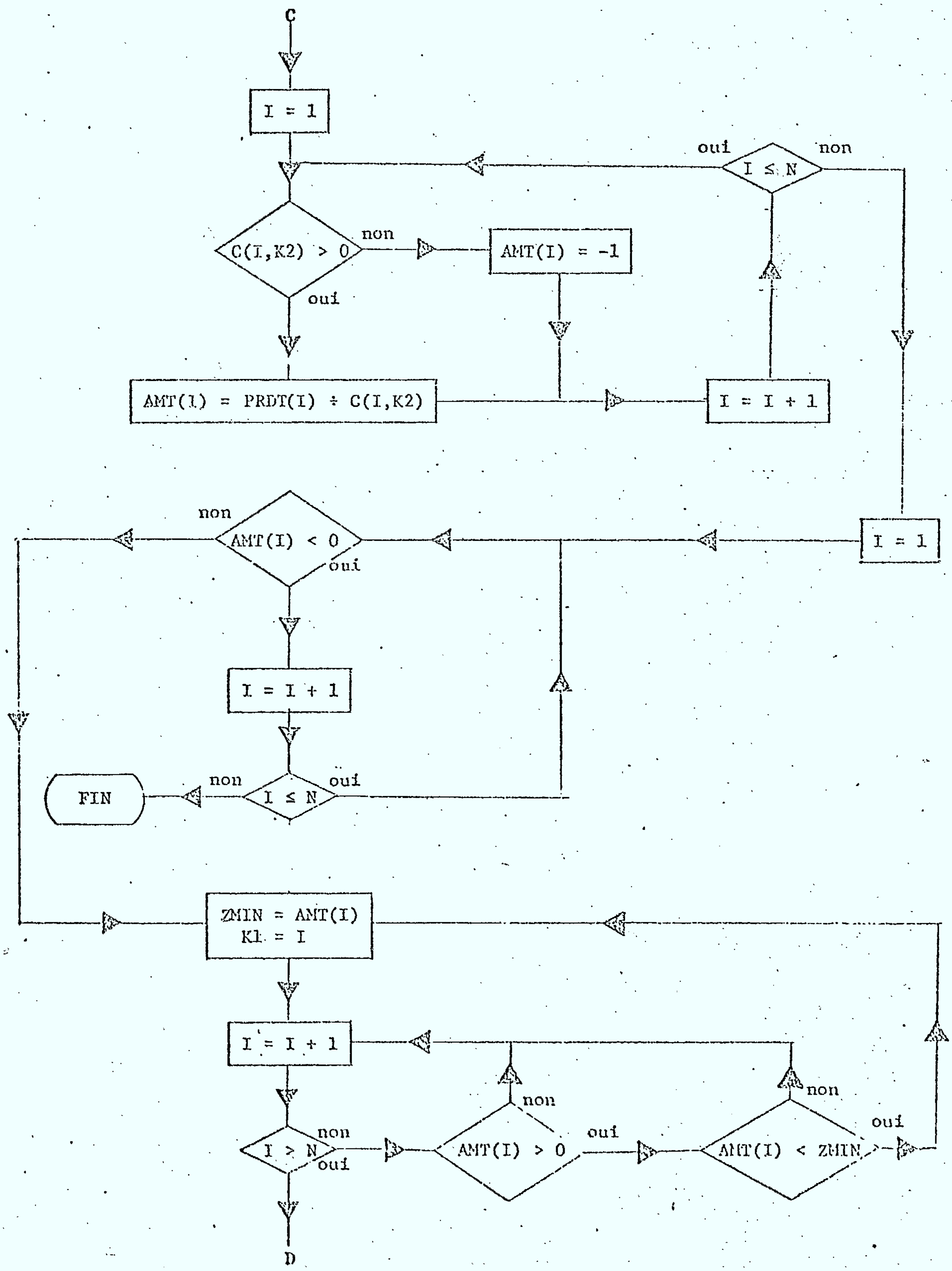
PARTIE "A"



PARTIE "B"


B
J = 0
ZMAX = 0
J = J + 1
oui
J > M
non
oui
K3 ≥ L
non
K4 = K3
K4 = K4 + 1
oui
K4 > L
non
RVLX(K,J) < 0
oui
oui
X(J) ∈ BASE
non
non
RVLX(K3,J) > ZMAX
oui
K2 = J
ZMAX = RVLX(K3,J)
L1 = L1 + 1
A2
non
MAX > 0
oui
C

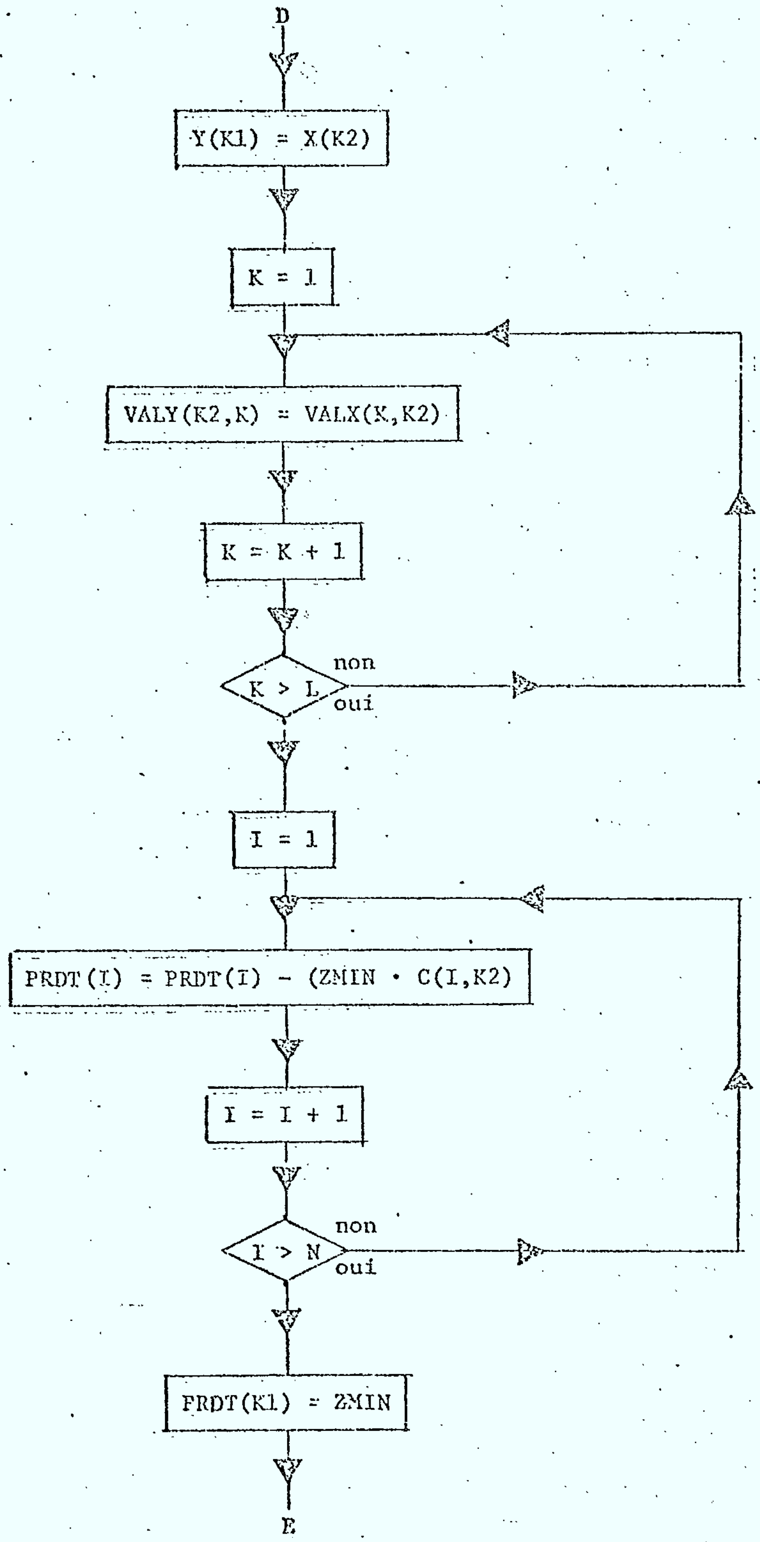
PARTIE "C"


C
I = 1
oui
I ≤ N
non
C(I,K2) > 0
non
AMT(I) = -1
oui
AMT(I) = PRDT(I) ÷ C(I,K2)
I = I + 1
non
AMT(I) < 0
oui
I = 1
I = I + 1
FIN
non
I ≤ N
oui
ZMIN = AMT(I)
K1 = I
I = I + 1
non
non
I > N
non
oui
AMT(I) > 0
oui
AMT(I) < ZMIN
oui
D

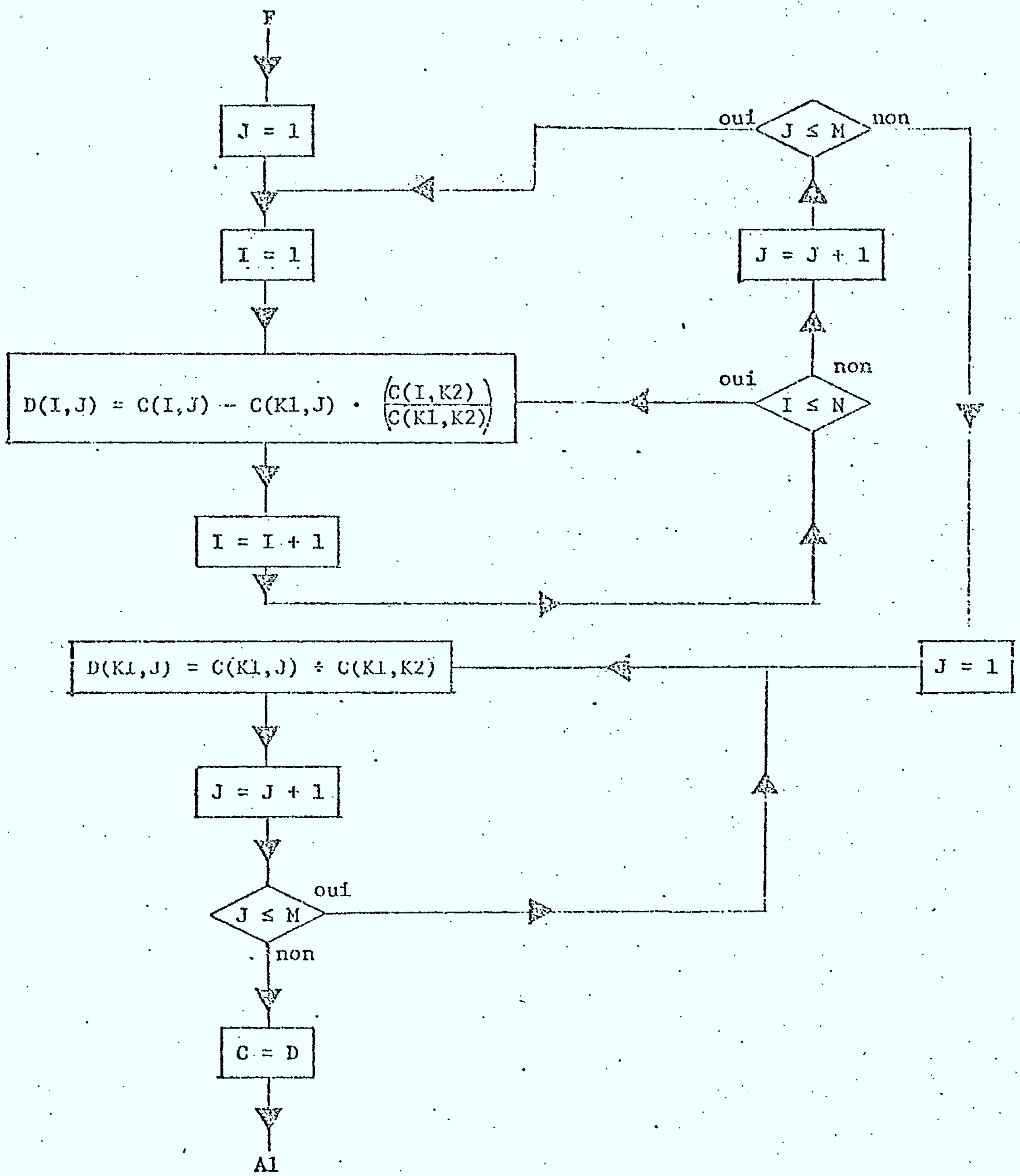
PARTIES "D" ET "E"

P A R T I E "D"

P A R T I E "E"



PARTIE "F"


F
J = 1
I = 1
D(I,J) = C(I,J) - C(K1,J) · (C(I,K2) / C(K1,K2))
I = I + 1
oui
I ≤ N
non
J = J + 1
oui
J ≤ M
non
J = 1
D(K1,J) = C(K1,J) ÷ C(K1,K2)
J = J + 1
J ≤ M
oui
non
C = D
A1

SOUS-ROUTINE START

On définit d'abord les variables suivantes

NROWS = nombre de rangées

NVAR = nombre de variables

NFLDS = nombre d'écarts positifs + nombre d'écarts dans les deux sens

NPRT = nombre de priorités

NSIZE = NROWS + NVAR + NFLDS

On forme ensuite

- 2 matrices nulles: RVLX = (NPRT + 1) $\begin{bmatrix} & & & \\ & & & \end{bmatrix}$
  NSIZE

  VALY = NROWS $\begin{bmatrix} \\ \\ \\ \end{bmatrix}$
  NPRT + 1

- La matrice des taux de substitution: C

$$\text{NROWS}\left[\begin{array}{cccc|cccccc|ccc} 1 & & & & -1 & & & & & & x_{11} & \cdots & x_{1m} \\ & 1 & & & & -1 & & & & & \vdots & & \vdots \\ & & 1 & & & & -1 & & & & & & \\ & & & \ddots & & & & -1 & & & & & \\ & & & & & & & & -1 & & & & \\ & & & 1 & & & & & & -1 & x_{n1} & \cdots & x_{nm} \end{array}\right]$$

NROWS NFLDS NVAR

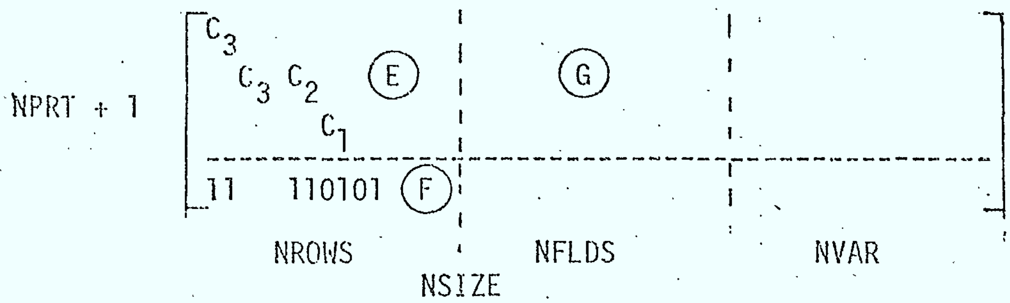
NSIZE

C est formé: - d'une matrice identité (NROWS x NROWS)
- d'une matrice (NROWS x NFLDS) où la valeur -1 à la ligne i

correspond à un écart positif ou dans les 2 sens, pour la contrainte i

- d'une matrice NROWS x NVAR où sont representées les contraintes des équations

- La matrice de la fonction objectif: VALX



La valeur 1 dans le sous-vecteur F correspond à une égalité ou à un écart positif.

La matrice E est composé des priorités à atteindre par des valeurs inférieures. La dernière ligne de E correspond à la première priorité. Si le coefficient est à la colonne J, le $J^{\text{ième}}$ ligne est dans la fonction objectif. Les priorités à atteindre par des valeurs positives sont dans la matrice G. Les autres sous-matrices sont nulles.

- Le vecteur des termes indépendants: RHS(1 x NROWS)

- Deux vecteurs X = [1,2,3,...,NSIZE]

  Y = [1,2,3,...,NROWS]

SOUS-ROUTINE FINISH

Que le programme soit exécuté en entier ou non, toutes les données sont imprimées telles que perforées sur les cartes, et ce dans le but de repérer plus facilement une erreur dans l'entrée des données. Puis le vecteur des termes indépendants et la matrice de la fonction objectif sont imprimés. Le programme original de Lee prévoyait aussi l'impression de la matrice des taux de substitution, mais étant donné son ordre de grandeur dans notre problème (100 x 150), nous n'avons pas jugé bon de la donner. Il sera toujours possible, pour des cas spécifiques, de faire sortir cette matrice.

Le programme mentionne par la suite le nombre de contraintes, de variables (réelles et d'écart), de priorités, de priorités supplémentaires et enfin le nombre d'itérations nécessaires à la résolution du problème.

Nous entrons maintenant vraiment dans la sous-routine "finish" qui imprime
- la base optimale
- la matrice finale de la fonction objectif (ZJ - CJ)
- une évaluation de la fonction objectif
- une analyse des écarts pour chaque contrainte
- la valeur des variables et des ratios
- la non-réalisation de chacun des objectifs.

Lors de l'impression de la base et de la matrice (ZJ - CJ), les 36 premières variables (colonnes) correspondent aux 36 variables du problème; les n suivantes aux variables d'écart positives et les autres aux variables d'écart négatives.

4.3.3 Result of simulations

VARIABLE ANALYSIS

NUMERO DE LA VARIABLE	NOM DE LA VARIABLE	BORNE INFERIEURE	VALEUR OPTIMALE	BORNE SUPERIEURE	VALEUR HISTORIQUE
1	NETINC	100000.	265465.	500000.	238633.
2	OPRV	1903924.	2094309.	2094316.	1903924.
3	DIVI	0.	300000.	300000.	143969.
4	NEWDEB	330000.	330000.	500000.	255180.
5	DELEQ	0.	0.	800000.	41082.
6	DELPR	0.	100000.	100000.	23786.
7	NIA	50000.	233639.	500000.	209786.
8	EQ	1435000.	1435305.	2300000.	1476387.
9	RE	300000.	427734.	1000000.	659912.
10	PR	343000.	443211.	600000.	376997.
11	L	2100000.	2270390.	2700000.	2266172.
12	GCE	990000.	1092518.	1500000.	900692.
13	TRANGV	0.	100000.	100000.	0.
14	DELCTI	0.	********** 0.0	100000.	0.
15	DELDCR	0.	100000.	100000.	1141.
16	DELCL	25000.	200000.	200000.	77922.
17	DELINV	0.	0.	100000.	523.
18	DELDCH	0.	100000.	100000.	1348.
19	REPL	0.	200000.	200000.	89400.
20	DEPN	200000.	396894.	500000.	381678.
21	DEPDIF	0.	100000.	100000.	0.
22	CURDTX	50000.	94960.	200000.	87638.
23	PRDTX	0.	********** 0.0	100000.	0.
24	DPRTVC	0.	100000.	100000.	5247.
25	DPRTVE	0.	100000.	100000.	5247.
26	OPXP	900000.	931490.	2000000.	990245.
27	OTHINC	0.	0.	150000.	50493.
28	IDC	0.	16451.	100000.	14734.
29	AD	1500000.	2429341.	3000000.	1999212.
30	RET	0.	0.	500000.	125618.
31	LAND	0.	0.	500000.	152110.
32	GETV	0.	0.	200000.	83787.
33	GTP	6000000.	7525909.	9000000.	7208470.
34	UCC	3000000.	4697426.	6000000.	4733779.
35	OTHEXP	0.	200000.	200000.	68300.
36	CCA	200000.	471756.	800000.	475407.

.PCR	PCR	DPR	ROREC	RORC	ITCAT	ITCBT	RORBI	RORBE	
.48560	0.08727	1.28403	0.12000	0.09698	2.54784	4.26815	-0.00000	-0.00000	V. historique
.47186	0.07783	0.68627	0.10068	0.08887	2.38282	3.45963	0.08329	0.08416	

Objectifs à atteindre

BELL CANADA 1976

VARIABLE ANALYSIS

NUMERO DE LA VARIABLE	NOM DE LA VARIABLE	BORNE INFERIEURE	VALEUR OPTIMALE	BORNE SUPERIEURE	VALEUR HISTORIQUE
1	NETINC	100000.	334508.	500000.	238633.
• 2	OPPV	1000000.	2059213.	3000000.	1903924.
3	DIVI	0.	0.	300000.	143965.
4	NEWDEB	0.	199608.	500000.	255180.
5	DELFO	0.	800000.	800000.	41087.
6	DELPR	0.	0.	100000.	33786.
7	NIA	50000.	307262.	500000.	209786.
8	EO	1435000.	2235297.	2300000.	1476387.
9	FE	300000.	854800.	1000000.	659912.
10	PR	343000.	343210.	600000.	376997.
11	L	2100000.	2099997.	2700000.	2266172.
12	GCE	500000.	1273151.	1500000.	900692.
13	TRANGV	0.	46556.	100000.	0.
14	DELCTI	0.	100000.	100000.	0.
15	DELOCR	0.	0.	100000.	1141.
16	DELCL	25000.	25000.	200000.	77922.
17	DELINV	0.	100000.	100000.	523.
18	DELOCH	0.	100000.	100000.	1348.
19	REPL	0.	200000.	200000.	89400.
20	DEPN	200000.	364962.	500000.	381878.
21	DEPDIF	0.	0.	100000.	0.
22	CURDTX	50000.	65853.	200000.	87638.
23	PRDTX	0.	0.	100000.	0.
24	DPRTVC	0.	0.	100000.	5247.
25	DPRTVE	0.	0.	100000.	5247.
26	OPXP	900000.	900000.	2000000.	990245.
27	OTHINC	0.	0.	150000.	50493.
28	IDC	0.	18068.	100000.	14734.
29	AD	1500000.	1597408.	3000000.	1999212.
30	RET	0.	500000.	500000.	125618.
31	LAND	0.	500000.	500000.	152110.
32	GETV	0.	200000.	200000.	83787.
33	GTP	6000000.	7206541.	9000000.	7208470.
34	UCC	3000000.	5176444.	6000000.	4733779.
35	OTHEXP	0.	0.	200000.	68300.
36	CCA	200000.	519863.	800000.	475407.

DCR	PCR	DPR	ROREC	RORC	ITCAT	ITCBT	RORBI	RORBE	
0.41971	0.06859	0.0	0.12000	0.10000	3.01800	4.89938	0.11042	0.11317	V. historique
0.47186	0.07783	0.68627	0.10068	0.08887	2.38282	3.45963	0.08329	0.08416	

• Objectifs à atteindre

4.4 Summary of Sorès' work

In this section, we want to present a short summary of three simulations made by Sorès during this phase of the project, showing the usefulness and the versatility of the NPPS model in the way it can tackle different problems that governmental authorities have to face concerning the industry of telecommunications. These three runs executed with the aid of NPPS are the testing for interregional cross-subsidization, some plausible consequences of the admittance of Quebec Telephone into the TCTS club, and the treatment of an eventual division of Bell Canada into two distinct companies, one for Ontario and one for Quebec.

Two kinds of interregional cross-subsidy tests were implemented, both of them were incremental-cost tests. The first scenario was designed to check whether the interregional rate structure was subsidy-free; to do that, it compared the incremental cost on the whole network of interregional traffic originating in one carrier's territory with pre-settlement revenues collected by the considered carrier. If this test is passed, this would imply that the total TCTS member costs incurred for interregional traffic stemming from one company is at least covered by the revenues gained by this company. The second scenario proposed to examine whether post-settlement revenues obtained by a carrier cover the costs incurred for all the TCTS interregional traffic using the considered carrier's facilities. The failure of this second test would signify that the sharing scheme utilized in the process discriminates against the given carrier.

These two interregional cross-subsidy test were carried on only for the Eastern companies, NBT and MTT, since we previously knew from NPPS that post-settlement revenues exceed fully allocated costs corresponding to TCTS adjacent and non-adjacent traffic for all but these two carriers; this means that all other carriers already satisfy the second test, for these fully allocated costs are larger than the needed incremental costs. On one hand, pre-settlement revenues for NBT and MTT were greater than the incremental costs to TCTS carriers of interregional traffic originating in these two companies by a factor of 17; on the other hand, NBT and MTT post-settlement revenues far exceeded the incremental costs of the whole interregional traffic using their facilities for all the three sharing schemes used (New Commonwealth, Old Commonwealth, TCTS). These results imply that the two interregional incremental-cost tests are easily satisfied, particularly by Eastern carriers, and that model qualifications such that multiplexing, servivability and excess capacities can hardly invalidate this conclusion.

The simulation of the admittance of Quebec Telephone in the TCTS club may be treated in the NPPS model assuming that Quebec Telephone belongs to TCTS and by using different sharing schemes in order to estimate its post-settlement revenues. Of course, the representation of Quebec Telephone in the NPPS model is incomplete since this carrier has so far never been explicitely taken into consideration. For example, two switching points with relatively few inhabitants are missing and some transmission facilities such as a RF link are also absent; nevertheless, the simulation is worthwhile though the results must be qualified accordingly.

The costing block of NPPS gives an amount of total assets for Quebec Telephone of $15.6 millions with corresponding incurred costs of $4 millions a year. The estimated peak-hour C.C.S. traffic originating in Quebec Telephone amounts to around 500 for adjacent traffic, a value which is more than ten times greater than that for non-adjacent traffic, while the regional traffic is itself not significant; so, most of the traffic stemming from Quebec Telephone's territory is towards adjacent companies, a situation quite different from Bell's position which is highly developed for regional traffic. Using this estimated traffic, it can be seen that the usage of the switching network is very low and, moreover, may be quite asymetric which can have a sensible effect on fully allocated costs. Quebec

Telephone transmission facilities are used only in proportion varying between 6% and 29%; moreover, the description of the transmission network included in NPPS seems incomplete since there are large discrepancies between the number of circuits getting to and coming from some nodes.

On the revenue side, NPPS indicated that total revenues collected by Quebec Telephone amount to $1 million, with nearly nine tenths generated by adjacent traffic. When these revenues are compared to fully allocated costs of about $4 millions, this gives a ratio of collected revenues / total incurved costs of 0.25 for Quebec Telephone; the corresponding ratio for Bell Canada is 2.18 ($360 millions/$165 millions). The difference between these two ratios is too large to be tied only to some limitations of the model, and must be viewed as a consequence of the presence of economies of scale and indivisibilities.

The sharing block of the NPPS model, and its up-grading for getting results at a selected origin-destination level, permitted the calculation fo Quebec Telephone post-settlement revenues under the three main sharing schemes (New Commonwealth, Old Commonwealth, TCTS), assuming Quebec Telephone belongs to TCTS. The striking result of this simulation is the unlikelihood of the implementation of a New Commonwealth sharing scheme with a unique TCTS tarrif structure. This conclusion stems from the fact that Quebec Telephone's traffic and most of other carriers' traffic originating or terminating in Quebec Telephone do not recover their costs; the reason lies in the underutilization of the facilities of Quebec Telephone, which renders their usage very expensive when evaluated at fully allocated costs. The situation is quite different under the other two schemes (TCTS and Old Commonwealth), i.e. Quebec Telephone can recoup its costs, since the high cost of underutilization is then apportionned among all origin-destination pairs; these two rules are therefore less costly for and more faborable to Quebec Telephone than the first scheme. These remarks must be qualified by the fact that fully allocated costs are probably too high since we miss regional traffic nodes for Quebec Telephone; nevertheless, the margin between revenues and costs under the New Commonwealth scheme is large enough to render unlikely the reversal of the conclusion.

The third application made with the aid of the NPPS model was the eventual division of Bell Canada into two distinct companies, Bell-Ontario and Bell-Quebec. The costing block separated Bell Canada facilities (nodes, links, switching machines) between Bell-Ontario and Bell-Quebec in the proportions of about 2/3 and 1/3 respectively; the corresponding split ratios for total assets and total costs are closer to 3/4 and 1/4 respectively. So, if Bell Canada were divided, Bell-Ontario would be at least two times greater than Bell-Quebec.

The treatment of the breaking up of Bell Canada by the sharing block is more complex since the traffic between Quebec and Ontario becomes adjacent traffic. The ratios of pre-settlement revenues /incurred costs are 1.95 and 3.27 respectively for Bell-Ontario and Bell-Quebec, while the same ratio for Bell Canada is 1.97. When we take account of the sharing schemes, post-settlement revenues are greater for Bell-Ontario and smaller for Bell-Quebec than their respective pre-settlement revenues, no matter which sharing scheme (New or Old Commonwealth, TCTS) is used. The New Commonwealth sharing scheme would discriminate against Bell-Ontario, while both TCTS and Old Commonwealth sharing schemes would unfavor Bell-Quebec. This conclusion may be explained by the great magnitude of interregional revenues relative to costs for Bell-Quebec (the revenues/costs ratio is 3.27). Since most of these collected revenues are generated by traffic with Bell-Ontario and since the greater part of the facilities for this traffic is inside Bell-Ontario's territory, any sharing scheme based either on incurred costs or on total assets will discriminate against Bell-Quebec.

In the preceding paragraphs, we have seen that the NPPS model, due particularly to its disaggregate structure, can be applied to current regulatory problems to give valuable informations.

5. Extension in the NPPS model

5.1 Generalization of the cross-subsidy tests

5.1.1 Review

The industry of telecommunications is noticeable for its long-term decreasing average costs, the jointness of its multi-service supply and the existence of a lot of indivisibilities. These economies of scale and those common costs make really hard the task of pricing in a fair way the services offered by this industry. In such a case, regulation policies are a straightforward recourse frequently used by governments for giving the public the benefits of minimal cost production while impeding monopoly abuses like undue profits or excessive rates of return. Nonetheless, regulation is far from being an absolute panacea. In the economics of regulation, cross-subsidization, in which a certain price structure unduly favors the consumers of one service at the expense of the consumers of another service, may be possible since there is some restriction for the entry into the industry or because the existence of some degree of vertical integration. Cross-subsidy thus reflects market imperfections.

Many definitions, hence many tests, have been proposed with the purpose of trying to precise and to quantify cross-subsidization. Two tests have mainly retained our attention when we want to evaluate the extent of cross-subsidization between the services if the tariff structure is already given. The generalized incremental-cost test (GICT) says that the firm's price vector $(p_1, \ldots, p_m)$ for the n services is subsidy-free if and only if:

(1) $R(S) \geq C(N) - C(N-S)$, for all subsets S of N.

This means that the revenues from providing the group S of services must at least equal the added costs necessary to offer this group of services. The generalized stand-alone test (GSAT), on the other hand, asserts that the tariff structure is subsidy-free if and only if:

(2) $R(S) \leq C(S)$, for all subsets S of N.

This test requires that the stand-alone costs of fulfilling only this set of services be not more than covered by the portion of the total revenues of the company generated by the provision of this coalition of services.

In the precedent phase of the NPPS project, we have looked at cross-subsidy tests in the framework of the theory of n-person cooperative games so as to get an easier recognizable structure for the "game" of cross-subsidization. This approach has permitted us to note that the core of this game was precisely the set of revenues passing GSAT, or equivalently, passing GICT (since these two tests were identical under the underlying hypothesis of fixed demands). Moreover the reference to the game theory has allowed the application of some results of that theory to the cross-subsidization problem. It is well known from the theory of n-person cooperative games that any game fulfilling the subadditivity condition has a non-empty set of imputations. This implies that, as long as we assume the existence of economies of joint production, there is at least one vector of revenues passing SAT. This implication is interesting because the hypothesis necessary is not really severe since it corresponds to the notion of a natural monopoly.

Another result stemming from this cost-sharing game is the following theorem (see G. Faulhaber, Am. Ec. Rev., vol. 65, no. 5, p. 966-977): if we assume that

a) cross-elasticities are zero, i.e. $\delta q_i/\delta p_j = 0$ for all $i \neq j$,

b) prices are not perverse, i.e. $\delta\pi(S)/\delta p_i > 0$ for all $i \in S$,

then the core of the cost-sharing game is identical to the set of subsidy-free prices. The theorem signifies that if the revenues are in the core of the game, i.e. pass GICT, and if conditions a and b are satisfied, then no customer coalition could obtain lower prices by splitting off from the grand coalition and the tariff structure presents no subsidy between services. Hence, the game theoretic approach applied to the treatment of cross-subsidization lies critically, so far, on two assumptions: the nullity of all the elasticities and the anti-perversity of the prices. The cross-subsidy tests (GICT, GSAT) are limited in the same way by these two hypotheses. Another constraint comes from the fact that the approach focused on revenues as the payoff variables though the prices would be more relevant as parameters in the determination of a tariff structure. It seemed then appropriate to criticize these two assumptions and to concentrate more on prices as key variables in view to come closer to a politic of tariff determination.

The hypothesis of anti-perversity of prices does not seem limitative for our purpose. If a service is being subsidized, and then failed (say) the incremental-cost test, hence the other services would be better off without it since the remaining revenues should exceed the remaining costs. Nevertheless, the consumers of the remaining services are only better off if they face lower prices. To obtain this, excess revenues must be returned to consumers by the way of lower prices, and in that manner we have to impose a new hypothesis, called anti-perversity: if profits are defined by $\pi(S) = R(S) - C(S)$, then i in S implies

$$\frac{\delta\pi(S)}{\delta p_i} > 0.$$

The reasonableness of this assumption lies on the following fact. Considering a profit function which is concave and negative if all prices are zero; if we assume there were a price p^* at which $\delta\pi(S)/\delta p^* < 0$, then we can find a price $p^{**} < p^*$ yielding the same profits to the firm, but with $\delta\pi(S)/ p^{**} > 0$. Hence both the firm (more production and more revenues with identical profits) and the consumers (lower prices) would be better off. One would have to be perverse to operate at p^* rather than at p^{**}. We note that this anti-perversity assumption is equivalent to the restriction of choosing pareto-optimal prices and is not at all restrictive for our work.

What happens if self-elasticities are no longer zero? In the analysis so far, the demands were not modified relative to changes in the coalition structure; tests then resumed to comparisons between costs and revenues for the different coalitions, all the calculations being based on the initial vectors of prices and demands. Cross-subsidy tests were therefore actual price tests. This result seems a priori no more valid if we introduce functions of demand which vary with their own prices, i.e. $q_i = q_i(p_i)$, $i = 1, \ldots, n$. Even if revenues are in the core given the initial price and quantity vectors, the secession of a coalition T from the grand coalition implies a modification in the supply structure; this may induce prices and quantities to vary in order to meet the additional zero-profit constraints appearing with the formulation of new opposing supplies. Is it possible that the new structure generates lower prices for consumers of T though satisfying the general zero-profit constraint? The theorem quoted at the end of the last paragraph answers negatively this question.

In fact, the whole analysis of the preceding paragraphs, and then all the cross-subsidy tests, is yet valid when self-elasticities are non-zero provided two assumptions are met. These two assumptions are conditions a and b in the above-mentioned theorem. Thus, if cross-elasticities are zero and if prices are not perverse, SAT, ICT, GSAT and GICT are adequate tests when demands vary only with their own prices; moreover, all the calculations may be accomplished with only considering the initial price and quantity vectors.

What about the assumption of zero cross-elasticities? Non zero cross-elasticities modify the type of demand functions we are refering to; these demands must now be expressed as follows:

$$q_i = q_i(p_1, p_2, \ldots, p_n), \; i = 1, \ldots n.$$

The variation of the price of a commodity influences not only the quantity demanded of this commodity, but also the quantity demanded of every other commodity whose cross-elasticity relative to the first commodity is not zero. The presence of cross-elasticities drastically entangles the situation: the cross-subsidy tests are no more valid and the core of the cost sharing game is modified. Before giving a closer look at the problems we have to face when considering non-zero cross-elasticities, we would like to stress attention on the very importance of these elasticities in the treatment of cross-subsidization.

5.1.2 Importance of the non-zero cross-elasticities

All the cross-subsidy tests (incremental-cost test, stand-alone test, scenario one test) that were previously defined in terms of a game theoretic approach are in fact conceived under the rigid hypothesis of perfectly inelastic demands. This was a valid point of view when the only objective was the reckoning of the cross-subsidization, given a certain tariff structure; the quoted cross-subsidy tests are then appropriate to measure the extent of a posteriori cross-subsidization.

The story is quite different, however, if we are coming at a rate-making policy, i.e. if we require the possibility to modify the rate structure in view of reaching certain governmental goals. Hence, to implement a politic of determining the tariffs in order to realize some predetermined objectives, we have to focus on the tariffs themselves and also to try to evaluate the effects of these modifications on the initial demands. The needed information on the reactions to tariff changes are precisely this one yielded by the cross-elasticities (including the self-elasticities).

The introduction of cross-elasticities in the NPPS model modifies, in a certain sense, the perspective of the model. In other terms, up to now the NPPS model was cost oriented, i.e. it was constructed in order to compute the "cost" of every service. Now, we are trying to take the value of the service into account or, in other words, we are considering "what the traffic can bear". This new dimension of the NPPS model is worthwhile since it provides a much more active tool to the regulatory body, enjoying in this case a greater flexibility on the road to the fulfillment of its policy objectives.

The cross-elasticities, being some kind of reaction coefficients to a variation in the prices, are intimately related to other economic variables and may have influence on some of them. In particular, cross-elasticities have some impacts on:

a) the realized rate of return;

b) the utilization of the various equipments;

c) the quality of services;

d) the technological innovation and consequently the introduction of new services;

e) the relationship between the competitive and the monopolistic services;

f) the various barriers to entry.

So, cross-elasticities are plenty of pertinent information when an agency of regulation wants to realize some chosen objectives having economic flavor.

The introduction of the cross-elasticities in the NPPS model may be worthwhile yet in another direction. We know, in some informal manner, that the private customers are sensible to the peak and off-peak tariffs, while they are much less influenced, if they are at all, by the alternative use of private lines versus toll messages. The converse seems plausible for the private enterprise: greater sensitivity to the rates of private versus public lines, while a much weaker response to time differentiated tariffs. If we can know more precisely these respective coefficients of reaction and can introduce them in some way into the NPPS model, then the simulations could pretend to be more reliable and closer to the reality and, moreover, it may be possible to evaluate the different implications of a tariff policy onto customers exhibiting distinct elasticities.

When we wanted to study a particular service, we had to confront the revenues it generates with the additional costs necessary to its provision. It was a simple comparison between added revenues and added costs. In presence of cross-elasticities, this procedure is no more adequate. The revenues which accrued from the service in question do not represent any more the incremental revenues due to that service, since revenues from other services may rise (if the service considered is a net complement) or diminish (if it is a net substitute). The signs of the coefficients of the cross-elasticities have then a crucial role, as we shall see in the following section, and we must then modify the cross-subsidy tests to take account of the presence of the cross-elasticities.

The final remark concerns the economy as a whole. The industry of telecommunications is one among a large number of industries competing for the provision of services which can be viewed as substitutes for the telecommunications services. Among them, one can mention the mail industry and the transport sector. This signifies that any modification in the tariff structure of the telecommunications industry may have impacts onto the market shares of several close industries. In fact, it would be valuable to know not only the cross-elasticities between the services of telecommunications, but also the cross-elasticities between services offered by related industries. There is no doubt that it will be impossible to consider this feature in the present NPPS model.

5.1.3 Difficulties of the generalization

All the cross-subsidy tests so far implemented in the NPPS project were in fact conceived under the hypothesis of perfectly inelastic demands. Given the importance of the cross-elasticities and the desire to render the NPPS model more flexible and more reliable, it was natural hence to wish to introduce the concepts of elasticities and cross-elasticities. The purpose was twofold: first, to have, with the NPPS model, a more realistic description of the industry of telecommunications and second, to put the emphasis on prices and tariffs rather than revenues. The objective of taking the various elasticities into account raised more serious difficulties than we had imagined at the beginning; in the following paragraphs, we shall try to recap the evolution of the work done and to have a closer look at the experience gained during the process.

The main result obtained in the case of perfectly inelastic demands asserts that the core of the cost-sharing game is identical to the set of subsidy-free prices if the cross-elasticities are zero and if the prices are not perverse. This means that, under these conditions, the generalized incremental-cost test (or equivalently the generalized stand-alone test) is the pertinent test for evaluating cross-subsidization between services. Moreover, this result is still valid when self-elasticities are non-zero provided that the two preceding conditions are yet satisfied. Thus, if the cross-elasticities are zero and if the prices are not perverse, then SAT, ICT, GSAT, and GICT are adequate tests when demands vary only with their own prices, and the calculations may be performed with only considering the initial price and quantity vectors.

The picture, however, is quite modified when we want to introduce the various cross-elasticities. The variation of the price of a commodity, influences not only the quantity demanded of this commodity, but also the quantity demanded of other commodity whose cross-elasticity with respect to the first commodity is not zero. The presence of cross-elasticities drastically complicates the situation since the cross-subsidy tests, as previously formulated, are no more valid; this signifies that these tests are no longer equivalent and that the core of the game is modified. A new task now confront us: we have to construct another game and another core, i.e. redefine what we mean by subsidy-free prices, if we wish to consider demand functions sensitive to the prices of many (perhaps all) commodities.

With more sophisticated demand functions which imply the presence of cross-elasticities, we must look at a more complex game whose value is no more a cost function, but a profit function, constrained to be non-negative, and whose payoffs are no more the revenues, but the prices. We shall then refer to this game as the profit-sharing game since profits have to be shared between players by choosing a price vector. The core of this new game can be described as follows. The price vector $p = (p_1, \ldots, p_n)$ belongs to the core of the profit-sharing game if and only if:

α) $\pi(N,p) = 0$

β) there does not exist a subset $S \subset N$, $S = \{i_1, \ldots, i_S\}$, and prices $p^* = \{p^*_{i_1}, \ldots, p^*_{i_S}\}$ such that

I: $\pi(S,p^*) \geq 0$ for any feasible choice of p^*_k, $k \notin S$,

II: $p^*_j < p_j$, $j \in S$.

The vector p* in this definition is in fact the minimax solution to the non-cooperative game S versus N - S, i.e. players of S look for the minimal prices that will keep their coalition solvent ($\pi(S,p^*) \geq 0$) no matter what prices will be charged to the services in N - S, provided N - S remains solvent too. If $p^* < p$, where p is in fact the solution to the cooperative game N, then the coalition S would be better off by refusing to cooperate. On the other hand, if a price vector lies in the core of the profit-sharing game, then there is no economic incentive for any customer group to quit the grand coalition N and this price structure will be subsidy-free.

The only clear result obtained from this approach is that the core of the profit-sharing game is smaller or larger than the core of the cost-sharing game (with fixed demands) depending on whether the service is a substitute or a complement for the other services. Thus, not only the magnitudes of the cross-elasticities are of matter but their signs also, which ones can enlarge or diminish the set of subsidy-free prices.

With the aid of a numerical example concerning two services, we had illustrated the point that the order of stringency of the three tests, SOT, SAT, and ST is completely determined by the sign of the cross-elasticities. If the two services are gross substitutes (positive cross-elasticities), then SOT is more stringent than ST which is more stringent than SAT; this means, for example, that prices which are subsidy-free with respect to SAT are not necessarily so relative to ST. On the other hand, if the two services are complements (negative cross-elasticities), then

SAT is more stringent than ST which is more stringent than SOT.

We can represent the two possible cases when considering only two services. Figure 1 and figure 2 represent the sets of subsidy-free prices relative to SAT, ST and SOT when the two services are gross substitutes or gross complements respectively.

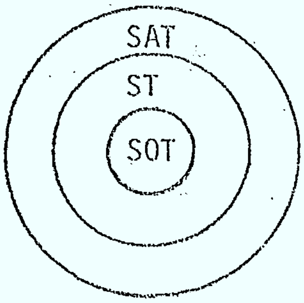


Figure 1

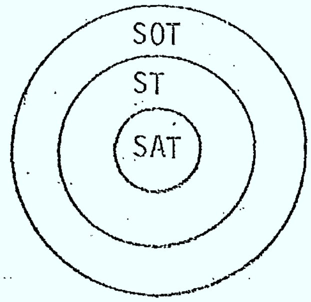


Figure 2

If for example, these two services are gross substitutes (positive cross-elasticities), then the set of subsidy-free prices according to the stable test (ST) is smaller than (and in fact included in) the subsidy-free set associated with the stand-alone test(SAT). Remembering that SAT is the core of the cost-sharing game (with fixed demands) and ST is the core of the profit-sharing game (with demands varying according to their cross-elasticities), the order of stringency obtained for these two tests has the following important consequence: when, for sake of simplicity, we omit to consider cross-elasticities and suppose they are all zero, then it is quite possible that every cross-subsidy test (equivalent to GSAT) is well passed without implying that no cross-subsidization exists. This deceptive conclusion results from the fact that there may be a whole bunch of prices which are subsidy-free with fixed demands (i.e. according to SAT) but which would be tarnished with cross-subsidization if we had taken the presence of cross-elasticities into consideration. Thus, cross-subsidy tests with fixed demands are too easy to be satisfied when we can presume the existence of cross-elasticities between two substitute services. The converse is true if the two services are complements: SAT is too much stringent if we can pretend that cross-elasticities are not zero. Since in the industry of telecommunications it seems that there is no services which are complements, then only the first case is pertinent, i.e. SAT is too loose as a test.

Thus, when considering only two services, the relationship between the cost-sharing game (SAT) and the profit-sharing game (ST) is quite precise relative to their order of inclusion; however, no easy relationship seems attainable between these two games with respect to their absolute size, i.e. it is very dubious that we can develop a tractable criterion, based on the magnitudes and the signs of the cross-elasticities, which can determine the prices to be added or subtracted to the core of the cost-sharing game for getting the new set of subsidy-free prices and take the influence of the cross-elasticities into consideration.

For the next step, we worked out several examples dealing with three services to look for a better generalization. Although the calculations are more laborious, the whole procedure is quite similar to the two-service case. To compute the different sets of subsidy-free prices, we have again to find solutions of quadratics but now with many more crossed terms, and we need to resolve simultaneous equations, with more variables in each equation and with a greater number of equations. Nevertheless, no direct and palatable result is reachable, even about the stringency of the several tests. We cannot obtain a single order of stringency for the tests considered since many combinations of services and of relations between services (concerning substitutability and complementarity) are now possible. In general, the previous correspondence between the order of inclusion of the subsidy-free sets and the signs of the cross-elasticities can only be checked when looking at two services at a time, i.e. only when concentrating in each plane defined by a pairwise selection of the services.

The computation of the several examples concerning two or three services brought to surface an entirely different kind of difficulty stemming from the very definition of subsidy-free prices. According to the formulation of the tests when taking the cross-elasticities into account, we said that the price of a service was subsidy-free if it were not greater than the price we get if this service were the only service offered. Two approaches were used to compute the limit prices: on one hand, we put the prices of all other services so high as to choke off completely their demands; on the other hand, we create as many subsidiaries as there are services to provide, each firm facing a one-dimensional demand function. These two alternatives permit us to calculate the limit prices by isolating each service at a time, but the procedure of treating every service independently of the others gives rise to a serious shortcoming. Indeed, it seems natural to take linear cost functions as a possible good approximation over a certain range of output; this was in fact the approach adopted in the several examples we worked out. This proxi, however, has a severe economic defect since each subsidiary firm may be economically unsound (i.e. facing negative profits) when we put attention on the real market situation of the industry represented by a multidimensional price-demand relationship. This result is a direct consequence of the assumption of decreasing average costs: the splitting of a unique firm enjoying economies of joint production in many subsidiary enterprises may generally imply reduced demands in the real market and hence greater average costs.

All the examples we have manipulated with three services lead in fact to the conclusion that the subsidiaries we imagined for computing the limit prices are economically unsound, i.e. their profits are negative. This is in a complete harmony with the theoretical results obtained by I.W. Sandberg (Two theorems on a justification of the multiservice regulated company, Bell J. of Econ., spring 1975; p. 346-356). Since the approximation of the cost functions over a certain range by linear functions seems quite natural, this outcome stresses the need to reexamine the definition of subsidy-free prices conveyed by the previous tests; as a matter of fact, without any additional hypothesis on the relevant functions, it seems too much severe to impose a subsidy-free price structure to a regulated multiservice enterprise when the comparison subsidiaries are themselves economically shaky. Thus, when the calculations lead to the absence of subsidy-free prices, as was the case in the examples worked out, this does not necessarily reflect a certain kind of inherent unfairness in the operation of a multiservice regulated firm, but it may just be a direct consequence of the rigid procedure we chose to compute the subsidy-free prices. One way to get out of the dilemma posed by this approach may be as follows: we continue to work with SOT and ST as adequate tests, i.e. we follow the same procedure as before for computing the different limit prices, but we redefine what we mean by subsidy-free prices. If each price of the regulated firm is inferior to its limit price, then we say that prices are subsidy-free; if some prices of the regulated firm are greater than their respective limit prices and if one or more subsidiary firm is economically unsound with respect to the real market, then we consider the tests as inconclusive; finally, if some prices are greater than their respective limit prices and if each subsidiary faces nonnegative profits, then we say that the prices indicate cross-subsidization.

5.1.4 Strategy for the future

This brief review of the main points of the work done up to now on the introduction of the cross-elasticities in the treatment of the cross-subsidization issue has stressed the very importance of the cross-elasticities. Any omission of these coefficients of reaction may invalidate the usual cross-subsidy tests if we want to use these tests as a tool to determine a subsidy-free tariff structure. On the contrary, if the cross-subsidy tests are viewed as means to evaluate the extent of cross-subsidization when we consider the rate structure as given, then the usual tests are quite appropriate since the impacts of the cross-elasticities are already incorporated in the revenues employed in computing the tests.

These two alternative interpretations of cross-subsidy tests have compelled attention on a new way to look at the problem of taking account of the various cross-elasticities. The first approach is the one we have adopted so far, i.e. trying to take account directly of the cross-elasticities in order to modify the cross-subsidy tests. Among the main difficulties of this view are that we require a certain knowledge of the demand functions, we need a refinement of the concept of subsidy-free prices and more investigation would be necessary to find out an adequate cross-subsidy test, perhaps like ST or SOT, which one would be reliable and operational.

The second approach considers the cross-subsidy tests developed up to now as quite meaningful criteria for evaluating cross-subsidization on an a posteriori basis. Once the rate structure is established, all the previous tests, GICT and GSAT in particular, are very pertinent instruments to verify the presence or the absence of cross-subsidization, even when cross-elasticities are non zero since we look at the rate structure and at demands when all the mechanisms of markets have already played their role. The crucial focus of study now becomes the construction of a demand block which can be grafted to the NPPS model and inside of which the cross-elasticities will enter into action to make the different demands sensible to the variations in the rate structure. The dynamics of this second approach is as follows: we start with a given tariff structure and we test if there is some cross-subsidization; if we find cross-subsidy that is judged undue, so we modify the initial tariffs in consequence; we let the demand block works so as to take account, via the included cross-elasticities, of this variation of the rates and we test again the resulting revenues to see if some improper cross-subsidization persists; we repeat the process until a satisfying rate structure is obtained.

Finally, the only point we want to mention before closing this section is that whatever alternative we choose to attacking the problem, the knowledge of the price-demand relationships will always be required if we want to have flexibility and adequacy in the NPPS model to determine a "proper" tariff structure. Either we introduce the cross-elasticities directly in the cross-subsidy tests, a way we have indicated the difficulties, or we let the cross-subsidy tests unchanged and permit the same work to be done by a certain kind of demand block. So, any further analysis of the issue of introducing the value of a service in the NPPS model has to encompass a closer study of demand relations.

5.1.5 Model incorporating some coefficients of reaction

Another avenue for introducing the cross-elasticities may be by the use of coefficients of reaction for the demands, making some parametrization on them.

We would like to present in this sub-section a formal way of considering these coefficients, model on which it might be usefull to bring more thought.

The model is an optimization one: it consists of minimizing the variations of the revenues which will cover the annualized investment cost variation. Formally, the model is the following:

$$\text{Min} \sum_i g_i(1-\eta_i)\, dp_i$$

subject to

$$\sum_i g_i(1-\eta_i)\, dp_i - c\Delta v \geqq 0$$

$$Ax \qquad\qquad -\Delta v \leqq 0$$

$$Bx + [\eta_i(g_i/p_i)]\ dp = g + rg$$

$$\underline{dp} \leq dp \leq \overline{dp}$$

$$dp \geq 0$$

where η_i is the price elasticity, and rg refers to some autonomous growth of the demand.

5.2 Expansion model in the Operating Block

5.2.1 Companies' practices

The planning horizon for new systems (like Telenet) is rather long, 5 to 10 years, but for routine growth it is shorter, 3 to 5 years. Usually the planning of facility installation is combined with old facility replacement and it is likely that the planning is made for separate parts of the network and co-ordinated at the vice-president level. We are not able to model the full diversity of capital deployment procedures (see for instance "Project Portfolio Approach" in Multiple Criteria Decision Making by James L. Cochrane and Milan Zeleny, p. 439. "Capital Rationing in the Face of Multiple Organizational Objectives" by J.D. Forsyth, Queen's University and D.J. Laughhunn, Duke University"). Rather, the expansion in the Operating Block is viewed as a global approximation for a short horizon.

5.2.2 The network expansion literature

The literature on network construction is already a huge one. The journal "Networks" has just issued a bibliography (vol. 7. no 2, 1977) that we are exploring for new titles. As soon as multi-period setting is envisaged, the computing effort becomes arduous and the researches are more and more involved in trying to decompose large-scale problems (see, for instance, for a general overview: D.M. Himmelblau, editor, Decomposition of Large-scale problems, North-Holland and American Elsevier, 1973). Some interesting lines of thought merit to be explored and evaluated for our own problems. Let mention: H.P.F. Nguyen and R.R. Vemuganti: Topological Properties of Multi-commodity Dynamic Networds, 1975, Working paper. T.E. Morton: Forward Algorithms and Planning Horizons for Dynamic and Linear Programming, 1975, Carnegie-Mellon University, Management Sciences Research Report, no 358.

The multicommodity feature of the telecommunication expansion problem is a nightmare even for linear costs. On the other hand, some results for a growing demand of one product, with a discrete time structure, a fixed planning horizon and concave cost functions can be found in: A.S. Manne and A.F. Veinott, jr., in Investment for Capacity Expansion, Size, Location and Time Phasing, A.S. Manne, Editor, G. Allen and Unwin Ltd, London, 1957. We suggest that these results could be used for the pending links in the network.

For a one period model, it is really encouraging to read the article signed by a researcher of the Bell Telephone Laboratories, specifically: CIJ. McCallum Jr., A Generalized Upper Bounding Approach to a Communications Network Planning Problem, Networks, 7: 1-23, 1977. His results show that our urge to develop the software along these lines in order to increase the size capacity of the model was well justified. The reader of this article will note the similarity with the allocation expansion model in the NPPS model except for the column generator which does not exist in McCallum's paper but for post-optimization purpose.

5.2.3 The Operating Block Status from the expansion point of view

Up to now, the NPPS model, to the exception of the Accounting Block, is a one period model. More precisely, it generally gives results for one current year but we used it also for some tests on prospective use of equipment, applying some growth rates to the requirements and pushing "manually" the capacity limits when needed. But, if we are ready to accept similar cost coefficients for an excess capacity and for expansion on a particular link, the actual software could be used for expansion with a minor effort.

5.2.4 An operational definition of the prospective incremental cost

As it has been said in 5.2.2 several avenues are explored, but to begin with, an earlier proposal not yet retained is again outlined as a comparative simple scheme. The definition of the prospective incremental cost is made in a framework of a fixed time horizon, discrete time structure and perfect forecasts; if the forecasts are shaky, the model can be re-run with different forecast values but for the moment, no stochastic programming features are explored.

The assumptions and data are as follows:

a) On the demand side:

- for the switched traffic, instead of a unique traffic matrix giving the peak demand in C.C.S. or Erlangs for all relevant pair of demand points, we need a sequence of matrices St_0, St_1, St_2, ... (St for switched traffic);

- for the non-switched traffic, instead of a unique circuit requirement matrix for one year for all relevant pairs of demand points, we need also a sequence of matrices NSR_0, NSR_1, NSR_2, ... (NSR for non-switched traffic);

- the subscripts are for the decision periods (one year long);

- the sequences can be given entirely from the outside or - better for space and computation saving - they can be built as functions of time, like $y = a + bt$ where a is the initial demand and b is the arithmetic rate of growth or $y = a(1 + r)^t$, where r is the geometric rate of growth. A particular combination of a and b, or a and r, can be chosen for each pair of demand points;

- for indivisible block of demand, like T.V., we can use the same type of functions since t takes only integer values;

- the proposed families of functions are for simplicity sake, but any nondecreasing function could do it. At that stage, it is not clear if the nondecreasing characteristic is an empirical observation and/or a computational convenience for getting a solution;

- at each period the values of the demands can be combined for the switched traffic first, and after dimensioning, the resulting circuit requirements between each pair of adjacent nodes of the switching network can be added to the non-switched requiremente;

- the incremental demand becomes, in the prospective view, a full sequence of requirements for a given subset of demand pairs, given a certain time horizon; but we could imagine an incremental demand for a sub-sequence only. In the cross-subsidy tests, that incremental demand is added (or subtracted) to the total demand in order to compute the corresponding incremental cost;

b) On the supply side:

- for the switching capacity, we must assign to each switching node a sequence of cost functions and the initial switching capacity. Some cost functions describing the cost of new technology can be available only from a given period in the sequence. There will be no choice between the new and the old technology; the new one will supersede the old one if the unit cost, at full capacity, of the former is lower than the corresponding cost of the latter;

- for the transmission capacity, we must assign to each transmission link a sequence of our standard cost functions; but, in that case, we will allow at most two technologies for each link and a link will then be duplicated using fictitious node and link assuming that no technology dominates the other in terms of costs. There will be no interconnection cost between technologies;

- as for the switching machines, some technologies can be available only after a given time;

- the cost functions could be indexed to show the effect of inflation;

- if a technology is reproducible on the same link, the ultimate link capacity for any period could be twice or three times the standard ultimate capacity since later on we propose a way of climbing steps in the allocation-expansion model;

- the initial capacity must be known for each transmission link;

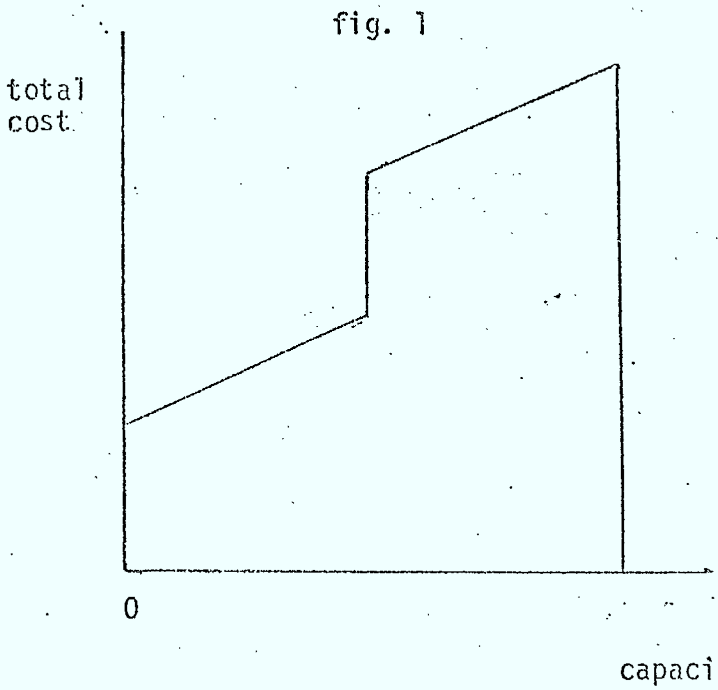
- the multiplexing costs, which are approximately linear, can be integrated in the cost functions;

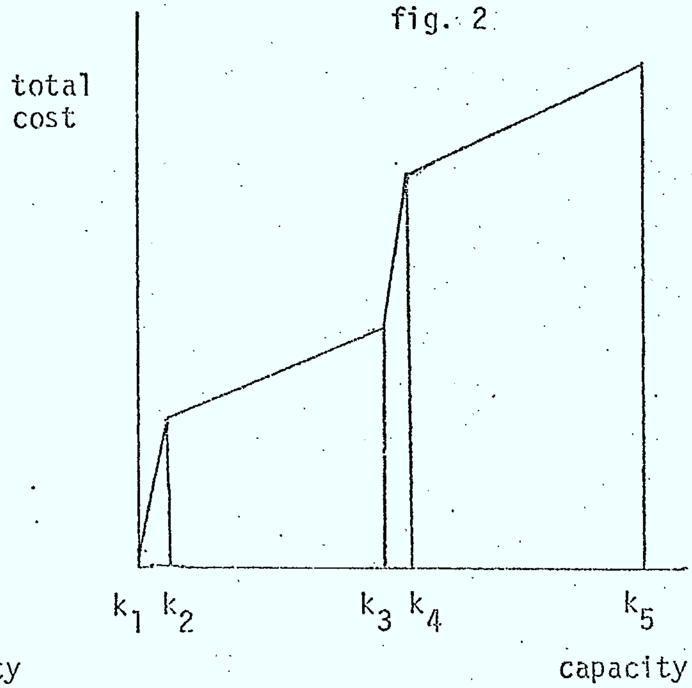
- it is not clear at that stage how the annual costs related to capacities (maintenance, for instance) will be introduced: at each period, or as an equivalent present value imputed at the vintage period of the facilities and based on some average useful life;

- the number of links can vary as time goes on when new links are envisaged, but the network must be kept connected;

- each cost value in the cost functions must be tranformed to a present value using the proper discounting coefficient; the reference time will be the beginning of period one;

- whenever we want to keep the possibility of climbing a fixed cost step in a cost function, an approximating piecewise linear function will be substituted to the step function as shown in figures 1 and 2;





- it can be shown (see: IJIRI, Management Goals and Accounting for Control, North-Holland, 1965, 0. 13-22) that the piecewise representation can be incorporated in a standard linear programming, the cost to be paid being one more constraint per linear piece plus one upper bound constraint.

From these assumptions and data the computing of prospective incremental cost of a set of services could be run as follows, if we stick to a one period model:

1. Fix the number of years of the planning horizon T.

2. For the switching network (S.N.):

2.1 Given the sequences of traffic matrices, dimensionning parameters (loss probabilities, diverse ratios) and network configuration, find the circuits requirements on the links.

2.2 From the sequence of circuits requirement incident to switching nodes, find the sequence of sizes for the switching machines.

2.3 Using the sequence of cost functions, the initial capacities and the initial values, find the sequence of additions and their values.

2.4 Convert in a sequence of annual costs (depreciation, plus cost of capital, plus diverse operating costs linked to value of facilities like maintenance and taxes).

3. For the transmission network (T.N.):

3.1 Add the sequence of circuits requirements for the S.N. to the sequence of circuits requirements for the non-switched traffic.

3.2 Run the one period allocation-expansion model along the lines of "Note technique, 28 avril 1975, Claude Autin, Guy St-Cyr, Laboratoire d'économétrie, Sujet: Expansion sur plusieurs périodes", and therefore find the sequence of new capital spending.

3.3 Convert the initial and new facility values in sequence of annual costs.

It must be clear that the strategy adopted for implementing such a procedure is heavily dependent on the existing computing capacity. If we choose to work with a multi-period model, decomposition techniques will have to be developed.

5.3 Cost allocation formula (Shapley value)

5.3.1 Review

It is well-know that one of the main technical characteristics of the industry of telecommunication is the relative importance of common (or fungible) costs. Consequently, to determine the cost of a service, one needs a procedure for allocating these costs among all the services supplied by a carrier. Several methods exist to separate common costs. Loehman and Whinston (The Bell Journal of Economics and Business Science, Vol. 2, no 2, Autumn 71) have deduced, from a set of axioms, a meaningful formula of social incremental cost. From these axioms and assuming that n users with fixed positive demands agree to use a collective facility, they have shown that individual charges for use of the facility must be related to the following formula:

$$F(i) = \sum_{\substack{G \subset N \\ i \in G}} \frac{(n-g)!\,(g-1)!}{n!} [C(G) - C(G-i)]$$

where G are subsets of size g of the whole group of users n, and C(G) is the minimum cost of fulfilling the demand D_G for the sub-group G. If the potential users accept the fairness of the axioms behind this formula they must then also accept the cost allocation formula F(i).

In the case of three services, the cost allocation formula is written as follows:

$$F(1) = \frac{1}{3}C(1) + \frac{1}{6}[C(1, 2) - C(2)] + \frac{1}{6}[C(1, 3) - C(3)] + \frac{1}{3}[C(1, 2, 3) - C(2, 3)]$$

and similarly for the other two services. Here C(1) is the social incremental cost due to the first service, if it is the only service considered; [C(1, 2) - C(2)] the social incremental cost once the first service comes right after the second service but before the third; etc...

Among the main properties of this formula, one can mention:

a) total costs of the service are covered by the charge scheme;

b) although price per unit according to this formula is not uniform, users with the same quantities demanded will pay the same charge;

c) the charge is a function only of the incremental costs due to one user;

d) if incremental costs are all multiplied by a constant, then the charge will also be multiplied by the same constant.

The computation of the cost allocation formula is not an easy thing when the number of services becomes important, due at least to the combinatorial character of the problem. In consequence if it were possible to find a way of simplifying this formula, this should be very welcomed. In sub-section 5.3.3 such a procedure will be discussed: of course, more reflexions will be needed to implement this simplification in the telecommunications industry context.

Another limitation of the present formulation of the Shapley value is its static character, i.e. time is completely ignored when allocating the common costs. We all know that the passage of time can modify the very nature of a service and so can influence the concept of cost associated with the provision of a service. In the next sub-section we shall introduce the temporal dimension in the Shapley value.

In the sub-section 5.1.1, the game theoretic approach to the subsidization problem was summarized and, in particular, the usefulness of the core concept has been stressed in order to deduce a subsidy-free price structure. There is a link between this approach and the cost allocation formula which is worth to mention. If the resulting values of this formula were in the core of the "cost sharing game", this would imply that a price structure based on this formula would be subsidy-free. Unfortunately such a result is not true in general and even more, the core of the "cost sharing game" can be empty i.e. there can exist no price structure which is subsidy-free. In the next paragraphs we will discuss the concept of a "convex game", a game which has the property that its core always exists and moreover, that the cost allocation formula (also called the Shapley value) is always in it. The adequacy of this concept to the domain of telecommunications will also be mentioned.

As it was previously mentioned, two of the problems with the game theoretic approach are that the core of the price game can be empty on one hand, and that the cost allocation formula needs not be in the core of the game on the other hand. However, it is shown by Shapley (Cores of Convex Games, International Journal of Game Theory, Vol. 1, no 1, 1971, pp. 11-26) that for the case of convex game, the core is not empty (in fact, it is quite large) and that the cost allocation formula (the Shapley value) is an element of the core. Intuitively, a convex game characterizes the property that the incentives for joining a coalition increase as the size of the coalition grows. More formally, a convex game is a game which possesses the following property

$$C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T)$$

for all $i \in N$ and all $S \subset T \subset N - \{i\}$. In other words, the supplementary cost of the service i does not increase if the number of elements in a subset increases. This concept is analogous to the increasing returns to scale associated with convex production function in economics. Of course one has to verify if this hypothesis is verified for our problem.

Essentially, it must be verified that the objective function in the allocation model, seen as a function of the right-hand side of the contraints, is a convex function. In fact, it can be shown that as far as there is some increase in the demands which does not necessitate any expansion of the capacity of the network (and a fortiori once it is needed) the aforementioned function is a convex one; this will be true if the ultimate capacity concept is retained. Finally, as the supplementary cost of a service (however defined) is the difference between the value of two convex objective functions, it will also be convex knowing that the difference between two convex functions is also convex.

We can then conclude that being given the approach taken in the NPPS model for determining the supplementary cost of a service, the core of the game is never empty and also that the cost allocation formula is always in it. Or, in other words, if tariffs are based on this formula, they will be necessarily subsidy-free. Beyond this result, one can ask whether using the convex game approach is possible with the view to formulate new subsidy tests more stringent than the ones so far used in the NPPS model?

5.3.2 The temporal aspect of the Shapley value

The fairness of the cost-allocation formula comes from the assumptions that all subgroups of a coalition of users of a common facility can occur, and that each ordering of arrival of consumers within a subgroup is equally likely. Another facet of the attractiveness of this separation scheme was that the individual charges it implies cover the sum of the total costs. So, the individual costs of the use of a common facility are given by F(i).

One characteristic of this formula is its static aspect; when applied to the services of telecommunications industry, the preceding cost-allocation scheme treats each service without any reference to a temporal dimension. We know, however, that time is a crucial variable in the very definition of the telecommunications services, and its pertinence to the concepts of excess capacity and in the characterization of the prospective incremental costs is evident. In the following paragraphs, we shall present a preliminary analysis of the introduction of time in considering the services of telecommunications and in the way this new generalization can affect the cost-allocation scheme.

Suppose, for example, that we consider two services and two periods of time. The easiest way to introduce the temporal dimension in the definition of the services is to mark each service with a time index; hence, we distinguish the present state from the future state of a service as two different services (of course, the future can be heard here as meaning a lapse of time of three years from now). We obviously could look at many periods of time and decompose each service in as many new services as there are time intervals, but it seems more advantageous here to concentrate on only two periods of time in order to make the illustration of the procedure more striking. Thus, we have the following set of services:

$$J = \{P_1, F_1, P_2, F_2\} = \{1, 2, 3, 4\},$$

where P_1 and F_1 represent the first service in its present and future state respectively, and similarly for P_2 and F_2 with respect to the second service. The relabeling of the four services in the same order with the numbers 1, 2, 3 and 4 is only to facilitate the notation when speaking of the possible coalitions.

The most important feature brought by the introduction of the time dimension seems that, contrary to the previous static approach, not all coalitions are now possible and the orders of arrival in a permissible coalition have not all the same probability of occurebce, These major modifications are obscured by a certain veil of arbitrariness, for they depend very much on the way we conceive the definition of a service in a temporal setting. We shall illustrate the methodology and the new kind of cost-allocation schemes that result with the aid of two simple examples. It is worth mentionning that no general cost-separation formula can be obtained since the probabilistic coefficients associated with each incremental cost are completely dependent on the way we eliminate certain coalitions and certain orders of arrival inside each would-be coalition. Although the process is the same for all situations, the properties of the resulting formulas rest heavily on the definition of a service in a temporal context.

The procedure can be grossly characterized in four phases. The first step is the definition of the services with reference to the time dimension. As said previously, this signifies that we add a temporal index to each service; thus, we multiply the number of services by the number of time periods considered. Though this is the unique explicit operation associated with this phase, the way we conceive the role of time in the definition of the services is a very crucial stage since it can determine the subsequent steps of the process. The second phase is the identification of the permissible coalitions. This step is fundamentally dependent on the way we have characterized the many services. The third phase is the enumeration of all the possible orders of arrival in each permissible coalition; this step also depends heavily on the definition of the services. The last phase, and the more laborious one relative to the necessary calculations, is the determination of the coefficients associated with each incremental cost in the final cost-allocation formula. Let us pass to the two examples to illustrate what we mean by all this.

With the first example we do not mind about the very content of a service but we are only interested in the constraints imposed by the passage of time on the possible formation of subgroups. For example, we only permit coalitions of different services in the same period of time and of the same service in different intervals of time. We will not try to justify this view of the coalitions formation, for we admit its complete arbitrariness; the purpose of this example is to find a symetric situation close enough to the Shapley value in order to serve as a first step towards a more realistic definition of the services of telecommunications. The possible pairwise coalitions are thus represented by the shaded areas in the first tableau; the permissible coalitions are thus 12, 13, 24, 34, and the forbidden ones are 14, 23 (note that the order is irrelevant in a coalition). The structure of the twin coalitions being symetric, any order of arrival in these coalitions in equally likely.

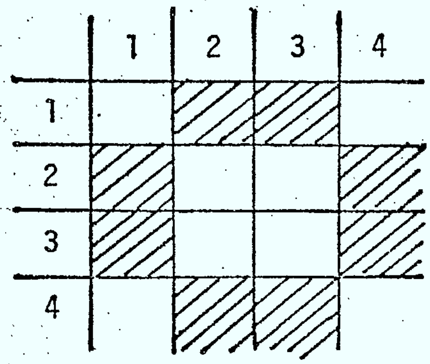


Tableau 1

Having at hand now the allowed twin coalitions, we must scrutinize the orders of arrival in each possible bigger coalition. The set of all permissible coalitions for the example considered is

P = {1, 2, 3, 4, 12, 13, 24, 34, 123, 124, 134, 234, 1234}.

For example, consider the coalition 123. The set of all the orders of arrival are:

1(23), 1(32), 2(13), 2(31), 3(12), 3(21)
(12)3, (21)3, (13)2, (31)2, (23)1, (32)1.

The notation 1(23) means that coalition 123 was formed by 1 joining first, and followed by the subcoalition 23 in which 2 joins first. Since coalition 23 is forbiden, we have to write off four orders of formation and we are left with the eight orders:

2(13), 2(31), 3(12), 3(21), (12)3, (21)3, (13)2, (31)2.

The case of coalition 1234 is a bit more complex. Introduce the following notation: $1(\overline{234})$ means that coalition 1234 was formed by 1 joining first followed by coalition 234; 234 was formed by 2 joining first followed by coalition 34 in which 3 was the first member. Similarly, (12) (34) means that coalition 12 joins first followed by coalition 34 and in each one 1 and 3 were respectively the first member. The complete listing of all the orders of arrival in coalition 1234 contains 120 members. Without writing all these orders, let us just assert that 80 ones are left after discarding the forbidden coalitions.

The enumeration of all the possible orders of arrival in each permissible coalition is the necessary step to calculate the probability that service i follows R-i in coalition R. In turn, these probabilities are used to compute the coefficients associated with the incremental costs in the desired cost-allocation formula. We will not explicit the manipulations involved in the computation of this formula for they are fastidious and mechanical; let us instead quote the resulting individual charges and compare them with the familiar cost-allocation scheme based on the Shapley value. The four individual charges are:

$$\begin{aligned} F(1) = 1/4\, C(1) &+ 1/8\, [C(12) - C(2)] + 1/8\, [C(13) - C(3)] \\ &+ 1/8\, [C(124) - C(24)] + 1/8\, [C(134) - C(34)] \\ &+ 1/4\, [C(1234) - C(234)] \end{aligned}$$

$$\begin{aligned} F(2) = 1/4\, C(2) &+ 1/8\, [C(12) - C(2)] + 1/8\, [C(24) - C(4)] \\ &+ 1/8\, [C(123) - C(13)] + 1/8\, [C(234) - C(34)] \\ &+ 1/4\, [C(1234) - C(134)] \end{aligned}$$

$$\begin{aligned} F(3) = 1/4\, C(3) &+ 1/8\, [C(13) - C(1)] + 1/8\, [C(34) - C(4)] \\ &+ 1/8\, [C(123) - C(12)] + 1/8\, [C(234) - C(24)] \\ &+ 1/4\, [C(1234) - C(124)] \end{aligned}$$

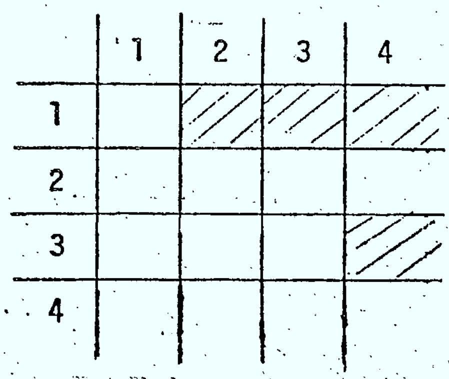
$$\begin{aligned} F(4) = 1/4\, C(4) &+ 1/8\, [C(24) - C(2)] + 1/8\, [C(34) - C(3)] \\ &+ 1/8\, [C(124) - C(12)] + 1/8\, [C(134) - C(13)] \\ &+ 1/4\, [C(1234) - C(123)]. \end{aligned}$$

Just for way of comparison, let us cite the individual charge for service 1 that would result from the familiar Shapley value:

$$\begin{aligned} F'(1) = 1/4\, C(1) &+ 1/12\, [C(12) - C(2)] + 1/12\, [C(13) - C(3)] \\ &+ 1/12\, [C(14) - C(4)] + 1/12\, [C(123) - C(23)] \\ &+ 1/12\, [C(124) - C(24)] + 1/12\, [C(134) - C(34)] \\ &+ 1/4\, [C(1234) - C(234)]. \end{aligned}$$

The two straightforward differences between the Shapley value and the modified cost-allocation formula are that some incremental costs vanished in the process and hence the coefficients have been modified. Since the introduction of time in the definition of the services renders some coalitions unrealizable, this implies the disappearance of the corresponding incremental cost in the final individual charges. As a consequence, this forces a readjustment of the coefficients for the sum of all the coefficients inside an individual charge must add to one. So, we observe a great similarity between the traditional Shapley value and treatment of this example. This likeness lies on the symetric aspect of the considered example; the coefficients of every C(i) and of C(N) are the same in both cases, and in each individual charge the rest of the cost is split evenly between all the permissible incremental costs.

Let us consider now a second example, closer to the services of telecommunications, and which has not the symetric property. The first service will be public messages, and the second service will be private lines; so, 1 and 2 represent present and future public messages respectively, and 3 and 4 represent present and future private lines. We assume that the network is principally built for present public messages, which implies a preponderant role for service 1. The set of permissible twin coalitions is represented by tableau 2. Services 2, 3 and 4 may join service 1 in a pairwise coalition, but they have to follow 1 in the coalition. Likewise, service 4 can join service 3, but has to follow it. All the other pairs of services are prohibited, either because the leading position of present public messages or because the ordering of services imposed by the passage of time.



As previously, the next step is the analysis of the orders of membership in each bigger group. The complete list of the permissible coalitions is for this example:

$$P = \{1, 2, 3, 4, 12, 13, 14, 34, 123, 124, 134, 234, 1234\}.$$

Because the preponderant role of service 1 and the very restricted number of possible twin coalitions, the number of arrivals in a coalition is drastically diminished. For example, coalition 123 can have only two orders of arrival: (12)3 and (13)2; similarly, the orders of membership in 124 are (12)4 and (14)2, while coalition 134 is formed with the arrivals 1(34) and (13)4. On the other hand, the coalition 234 is unrealizable since 2 cannot be the first member. Finally, for the global coalition 1234 the possibilities of formation are reduced to the following seven ones: $1(2\overline{34})$, $1(\overline{34}2)$, (12) (34), $(\overline{12}3)4$, $(\overline{13}2)4$, $(1\overline{34})2$ and $(\overline{13}4)2$. With the same mechanics as for the preceding example, we can utilize the information given by these orderings of formation and calculate the coefficients for each incremental cost. The final individual charges are then:

$$F(1) = C(1)$$

$$\begin{aligned} F(2) = {} & 3/7\, C(2) + 3/7\, [C(12) - C(1)] + 1/14\, [C(123) - C(13)] \\ & + 1/14\, [C(124) - C(14)] - 3/7\, [C(234) - C(34)] \\ & + 3/7\, [C(1234) - C(134)] \end{aligned}$$

$$F(3) = 1/2\, [C(13) - C(1)] + 1/2\, [C(123) - C(12)]$$

$$\begin{aligned} F(4) = {} & 1/14\, [C(14) - C(1)] + 4/7\, [C(34) - C(3)] \\ & - 1/14\, [C(124) - C(12)] \\ & + 3/7\, [C(134) - C(13)] \\ & - 4/7\, [C(234) - C(2) - C(3)] \\ & + 4/7\, [C(1234) - C(123)] \end{aligned}$$

This allocation of total costs between the different services is very distinct from that proposed by the Shapley value. The coefficients associated with the incremental costs can take many different values which reflect the great asymetry of the structure of the twin coalitions, and which seems much closer to the spirit of decision makers when they manage the separation of common costs, considering that the network is already in place and was built particularly for providing specific services. What seems much more astonishing is the negative sign attached to some of these coefficients; it is difficult at first glance to give a meaningful interpretation of this observation. However, it is interesting to note that the preponderant position of service 1 is translated by the equality between the individual charge of service one and its stand-alone cost.

The approach propounded in this section to generalize the cost-allocation scheme of a temporal setting appears really promising while more thought must be devoted to simplify the computational procedure and to find out a satisfying interpretation to the resulting individual charges. Finally, it is worthwile to mention that the approach proposed may be judge as unfair since users with identical demands do not necessarily support identical charges: these depend on the weight the users have in the different coalitions. However, we can say that the procedure is not so unrighteous because it takes account of the bargaining power of each service and does not treat them independently of their importance or their ordering.

5.3.3 Simplification of the Shapley formula

Suppose the following situation: assume that the cost of any subset of services, defined in a proper way, is equal to the cost of the "largest" service in that subset. For example, it is evident that if one considers switched and non-switched services in a same coalition, the cost of this coalition will be equal to the cost of providing the switched services. Another possibility is the following: if in a same coalition one considers peak and off-peak demands between certain O-D pairs, the cost of this coalition will be equal to the cost of providing the services at peak demands.

In such cases, it has been shown by Littlechild and Owen (A Simple Expression for the Shapley Value in a Special Case, Management Science, Vol. 20, No 3, Nov. 73, pp. 370-372) that the cost allocation formula (the Shapley value can be simplified to the following expression

$$F(i) = \sum_{k=1}^{j} (c_k - c_{k-1}) / r_k$$

for $i \in N_j$, $j = 1, \ldots, m$. Here, we have n services, denoted by $i = 1, \ldots, n$, which are partitioned in m classes or types. N_j is the set of services of type j, and n_j is the number of services in class N_j. Each class N_j is defined so that every service included in this type is characterized by a cost c_j; without loss of generality, we ordered these types such that

$$0 = c_o < c_1 < c_2 < \ldots < c_m.$$

Finally, $r_k = \sum_{j=k}^{m} n_j$, and $\sum_{j=1}^{m} n_j = n$

In fact, considering services, we put together those services which imply the same cost, and we rank these types of services according to an increasing sequence of cost. The above-mentioned cost-allocation formula results when the cost of providing any coalition of services is equal only to the cost of the more expensive service in it. Such a situation may be interesting in the industry of telecommunications since when we consider the provision of a coalition of switched and non-switched services. or of peak and off-peak demands, it is well plausible the manager figures that the cost of fulfilling such a coalition is completely determined by the most expensive service in the coalition.

A possible interpretation of the rule implied by this modified Shapley value is the following:

a) Divide the cost of supplying for the cheapest type of services equally among the total number of service;

b) Divide the incremental cost of supplying for the second cheapest type of services equally among the number of all services but those contained in the cheapest type. Continue thus the procedure until the incremental cost of the more expensive type is equally apportioned among the number of services included in this lattest type of services.

Apart from the fact that this interpretation of the modified Shapley value presents some interest, particularly in the way we can aggregate the kinds of services we considered, this new formulation of the cost-allocation scheme is even more important for its computational simplification. In fact, instead of computing the incremental costs for every possible coalition of services, we only have to calculate the differences of costs for each consecutive pair of costs in the increasing sequence previously mentioned. Although this modified Shapley value will give the same separation of costs as the initial cost allocation scheme, it has the advantage of putting emphasis on the structure of the incremental costs and on the services as viewed primarily from a cost causation perspective. It is worth noting that such an application relies heavily on the definition of the services and on the particular way we aggregate them, another characteristic of this new version of the cost-separation scheme reserves a short mention for it compels attention on the notion of avoidable costs. Since services are ranked according to their increasing costs, the passage from one type of services to another type gives us some information on the costs that would be avoided if the last, more expensive type of service is discontinued. So, this way of looking at the Shapley value indicates that the allocation of common costs proposed by this scheme is in fact grounded on some concept of avoidable costs.

APPENDIX A : Méthodes de dépréciation

Introduction

En observant l'évolution de la structure industrielle moderne, on constate que le capital physique et par conséquent les charges fixes, représentent une part grandissante dans les coûts d'exploitation. L'industrie des télécommunications ne fait pas exception à la règle. Bien au contraire, en 1976, les frais d'amortissement représentaient près de 30% des dépenses d'exploitation de Bell Canada. Malgré cela, on a parfois tendance à estimer l'amortissement sans se soucier des fondements économiques sous-jacents à ce concept.

Nous analyserons les problèmes posés par l'amortissement de même que les différentes méthodes servant à l'estimer. Nous ferons un rapide survol de son application dans le modèle NPPS et nous examinerons les problèmes posés par les impôts reportés qui découlent de la diffé-ce entre l'amortissement comptable et l'allocation du coût en capital. Enfin, nous regarderons les modifications qui pourraient être apportées au calcul de l'amortissement.

1. Considérations générales

Le problème de l'amortissement provient du fait que l'entreprise achète à un moment donné des équipements qui lui rendent des services pendant plusieurs années. "L'amortissement comptable a pour objet de répartir le coût ou la valeur d'un élément d'actif immobilisé corporel (ou d'un groupe de biens), moins sa valeur de récupération, sur sa durée d'une façon systématique et rationnelle. L'amortissement vise à répartir le coût d'un bien et non à l'évaluer. L'amortissement annuel est la perte du coût total attribué à un exercice en particulier". (cf.[9])

La première difficulté tient à l'évaluation de la vie utile de l'équipement. Celle-ci peut être écourtée par l'insuffisance ou la désuétude car il est parfois difficile de prévoir l'expansion de l'entreprise à moyen terme de même que l'innovation technologique.

La seconde difficulté a trait au rythme d'amortissement pendant la vie de l'équipement. Une formulation générale de l'amortissement au temps t serait:

$$a_t = V_{t-1} - V_t + i_t V_{t-1}$$

où a_t = amortissement au temps t

V_t = valeur de l'installation au temps t

i_t = taux d'intérêt au temps t

La première partie $(V_{t-1} - V_t)$ représente la dépréciation de l'installation pendant l'année alors que $(i_t V_{t-1})$ représente l'intérêt perdu sur la valeur de l'équipement. Ce manque à gagner est généralement absent du calcul comptable de l'amortissement. On le retrouve dans le calcul économique.

Mais cette méthode générale suppose qu'on connaît à tout moment la valeur de rachat des installations d'où la nécessité d'un marché secondaire bien développé. Si un tel marché existe pour certains produits (l'automobile par exemple), il n'en va pas ainsi pour la majorité des facteurs à amortir. Alors on devra estimer leurs valeurs tout au long de leur vie utile.

2. Méthodes d'amortissement

Au lieu d'estimer rigoureusement, à chaque année, la valeur des installations, on fixe habituellement un rythme d'amortissement qui sera appliqué jusqu'à ce que l'équipement soit remplacé. Or il existe plusieurs méthodes d'amortissement. Nous exposerons ici celles qui sont le plus couramment utilisées.

2.1 Méthode de l'amortissement linéaire (straight line method)

L'amortissement annuel est calculé en divisant le coût du bien à amortir moins sa valeur de récupération par sa vie utile estimée.

Soient V_0 = coût du bien

S = la valeur de récupération

N = la vie utile estimée

$$a_t = \frac{V_0 - S}{N}$$

Cette méthode donne un amortissement constant à chaque année. Son seul avantage est la simplicité de son calcul, car elle ne repose sur aucune justification économique. Ainsi, elle peut conduire à des résultats erronés. Malgré cela, elle est sans aucun doute la méthode la plus utilisée.

2.2 Méthode de l'amortissement à taux constant (decline balance method)

On calcule l'amortissement en multipliant la valeur comptable des installations par un taux constant d'amortissement. La valeur comptable est égale au coût du bien moins l'amortissement accumulé. Ce taux d'amortissement γ/N est généralement compris entre $1/N$ et $2/N$. On obtient donc

$$a_1 = \frac{V_0 \gamma}{N}$$

$$a_2 = \frac{(V_0 - a_1)\gamma}{N} = \frac{V_0 \gamma}{N}\left(1 - \frac{\gamma}{N}\right)$$

d'où
$$a_t = \frac{V_0 \gamma}{N}\left(1 - \frac{\gamma}{N}\right)^{t-1}$$

De plus l'amortissement accumulé est égal à $V_o\left(1 - \frac{\gamma}{N}\right)^t$.

Lorsque le taux d'amortissement est égal à deux fois celui utilisé dans la méthode linéaire, c'est-à-dire que $\gamma = 2$, on appelle en anglais cette méthode: "double declining balance method" (DDB).

Quelque soit la valeur de γ, l'amortissement est dégressif car pendant les premières années, il est plus élevé qu'à la fin de la vie utile de l'installation. Enfin, notons que la valeur de récupération sera fonction de la valeur de γ et N.

2.3 Méthode d'amortissement proportionnel à l'ordre numérique renversé des années (sum of the years' digits method) (SYD)

On calcule l'amortissement annuel en multipliant le coût de l'installation moins sa valeur de rachat par une fraction variable dans le temps. La fraction a comme dénominateur constant la somme des chiffres représentant les différentes années de la vie utile de l'installation soit

$$1 + 2 + 3 + \ldots + N + N(N+1)/2$$

Le numérateur, qui change à tous les ans, est égal au nombre d'année de la vie utile restant au début de l'année courante soit (N-t+1).
On a donc, comme amortissement annuel,

$$a_t = \frac{2(V_o - S)(N-t+1)}{N(N+1)}$$

L'amortissement accumulé est égal à $\frac{(V_o - S)[t(2N-t+1)]}{N(N+1)}$

2.4 Méthode d'amortissement généralisée

Récemment, Buck et Hill ont généralisé ces différentes méthodes d'amortissement. (cf. [2])

Soit $a(t) = V(t-1) - V(t)$

où $V(t)$ représente la valeur de l'installation au temps t.

On a alors 2 contraintes à respecter

- lorsque la Nième période est atteinte, la méthode d'amortissement doit faire arriver la valeur des équipements V(N) égale à S, déterminé à l'avance
- la charge sur les N périodes doit être égale à $V_o - S$.

Soit $a(t+1) = \alpha a(t) + \beta$

α, β sont des constantes et a(1) est fixé à l'avance.

Alors l'amortissement pour la période t est donné par

$$a(t) = a(1) + (t-1)\beta \qquad \alpha = 1$$

$$a(t) = \alpha^{t-1}a(1) + \beta(1-\alpha^{t-1})/(1-\alpha) \qquad \alpha \neq 1$$

On peut vérifier que les méthodes précédentes sont des cas particuliers de cette méthode générale en remplaçant les constantes par les valeurs suivantes:

méthode d'amortissement	a(1)	α	β
linéaire	$\frac{V(0) - S}{N}$	0	$\frac{V(0) - S}{N}$
SYD	$\frac{2[V(0) - S]}{N+1}$	1	$\frac{-2[V(0) - S]}{N(N+1)}$
DDB	$\frac{2V(0)}{N}$	1 - 2/N	0

L'avantage d'une telle méthode est de permettre plus de flexibilité dans l'évaluation de l'amortissement et de pouvoir arriver à une méthode optimale.

De façon générale, quelque soit la méthode utilisée, elle devrait représenter la perte d'utilité que nous procure l'installation. Or comme, avec le temps, les frais d'entretien augmentent et le rendement diminue, l'amortissement devrait être plus élevé au début, ce qui élimine la méthode d'amortissement linéaire. Mais nous reviendrons plus tard sur le choix de la méthode appropriée.

3. Amortissement d'un ensemble d'installations

Lorsqu'on veut amortir un ensemble d'installations, plusieurs problèmes supplémentaires se posent. Il est évident qu'on ne peut amortir séparément chacun des équipements. On doit donc les grouper d'une façon ou d'une autre. Cependant, ils n'ont pas tous été installés en même temps de même qu'ils n'ont pas tous la même vie utile.

Deux paramètres déterminent les caractéristiques de service d'une installation: la vie moyenne de survie et son schéma de dispersion autour de cette moyenne. Des courbes de survie ont été estimées dont les plus connues sont celles publiées par le Iowa State College Engeneering Experiments Station.

Indépendamment de ces différentes courbes estimées on peut en plus, distinguer deux groupes d'équipements particuliers par rapport à la dispersion de leur schéma de retrait. Certains, installés à une date donnée, peuvent être retirés graduellement et indépendamment les uns des autres. C'est le cas des véhicules, câbles, matériel de bureau, etc... Il s'agit alors d'équipements indépendants (mass-properties). L'autre groupe est constitué des équipements intégrés (integrated properties). Ceux-ci, formés d'unités complexes, sont de nature telle qu'ils devront être retirés simultanément. Le retrait de la majorité des composantes n'est donc pas indépendant du retrait de l'installation prise comme un tout. Certaines composantes pourront évidemment être remplacées indépendamment de l'ensemble.

Lorsqu'il s'agit d'amortir ces installations, on utilise généralement deux méthodes: "the Average Service Life (ASL) method et the Equal Life Group (ELG) method". La méthode ASL amortit l'ensemble des installations sur la base de leur vie moyenne alors que la méthode ELG sépare les installations selon leur vie utile et les amortie séparément à l'intérieur de chaque groupe. Le taux d'amortissement dans la méthode ASL est constant et égal à 1/L (où L = Vie utile moyenne). De plus ce taux est indépendant de la dispersion des retraits d'équipements autour de la vie moyenne. Dans le cas de la méthode ELG, le taux d'amortissement est plus élevé que le taux ASL dans les premières années et moins élevé à la fin. L'écart sera d'autant plus grand que le schéma de dispersion sera étendu.

On peut conclure que la méthode ELG sera avantageuse dans le cas d'une entreprise en croissance car le taux d'amortissement de cette méthode est fonction du taux de croissance de l'entreprise ce qui n'est pas le cas pour l'autre méthode.

Dans le modèle NPPS, ces deux méthodes ont été considérées.

Posons: L = Age moyen

T = Age maximum

X = Age

SRV(x) = % des installations restantes (installées X années auparavant et calculées à partir des courbes de survie)

TAR(x) = % des installations dédommagées

GTP = valeur au livre des installations au temps présent

GTPR = valeur de remplacement des installations au temps présent

NSV = taux de valeur de rachat

GA = valeur au livre des installations initiales

Y = 100 • X/L

R = taux de croissance des installations.

On obtient alors $GA = GTP \,/\, \sum_{x=0}^{T} [R^{T-X} \cdot SRV(Y)]$

$$\bar{X} = \frac{\sum_{X=0}^{T} [SRV(Y) \cdot R^{T-X} \cdot X]}{\sum_{X=0}^{T} [SRV(Y) \cdot R^{T-X}]}$$

$$E(x) = \frac{\sum_{X=\bar{X}}^{T} [(SRV(Y) + SRV(Y-1))/2]}{SRV(\bar{Y})} \quad \text{où } \bar{Y} = 100 \cdot \bar{X}/L$$

La méthode ASL nous donne un amortissement accumulé ADASL

$$ADASL = (1 - E(x)/L) \cdot GTP \cdot (1 - NSV)$$

La méthode ELG nous donne un amortissement accumulé ADELG

$$ADELG = \sum_{X=0}^{T} \{[TAR(Y) - (1 - SRV(Y))] \cdot [1 - NSV] \cdot R^{T-X} \cdot GA\}$$

et un taux d'amortissement DEPRAT

$$DEPRAT = \frac{\sum_{X=0}^{T} \{[TAR(Y) - TAR(Y-1)] \cdot R^{T-X} \cdot GA\}}{\sum_{X=0}^{T} \{[(SRV(Y) + SRV(Y-1))/2] \cdot R^{T-X} \cdot GA\}}$$

Dans le cas des propriétés intégrées, on a considéré que la méthode ELG.

Bien que ces équations ne représentent pas la version finale telle qu'elle apparaît dans le modèle, elles sont données ici pour représenter l'approche qui fut prise dans l'application des deux méthodes d'amortissement.

4. <u>Amortissement comptable et allocation du coût en capital</u>

Nous avons jusqu'à présent parlé de l'amortissement comptable des immobilisations. Or pour les fins de l'impôt, l'amortissement doit être calculé tout autrement.

Selon la loi de l'impôt, les biens amortissables sont répartis en différentes catégories auxquelles se rattache un taux d'amortissement. Ce taux constant est appliqué sur le solde non amorti des immobilisations. L'allocation du coût en capital ainsi obtenu diffère de l'amortissement comptable et lui est généralement supérieur. Il s'ensuit que le revenu imposable et par conséquent les impôts dus pendant l'année seront inférieurs à ce qu'ils auraient été en utilisant l'amortissement comptable. La différence entre les impôts calculés selon les deux méthodes constitue les impôts reportés.

Or, contrairement aux autres, les entreprises réglementées sont laissées libres dans l'allocation de ces impôts reportés. Ils peuvent soit les comptabiliser immédiatement dans leur profit (méthode du "flow through"), soit les accumuler dans une réserve pour impôts reportés (méthode de "normalisation").

L'effet de la première méthode peut se refléter par une baisse des tarifs à court terme car les impôts reportés sont comptabilisés directement dans les profits. Cependant, à plus long terme, il y a un risque de voir les tarifs augmenter si les impôts reportés deviennent négatifs. La normalisation réduit les tarifs d'un montant égal à la réduction de la base tarifaire. Cependant, cette réduction est définitive.

5. Allocation des impôts reportés

Nous établirons maintenant à l'aide du modèle développé par Lenhart, laquelle des deux politiques présentées plus haut est préférable pour le consommateur et pour l'investisseur. (cf. [5])

Définissons immédiatement toutes nos variables.

C = cash-flow

δ = rapport dette/capital

D_{bj} = amortissement comptable pour l'année j

D_{tj} = allocation du coût en capital pour l'année j

i = taux d'intérêt sur la dette

k = investissement initial

N = réserves pour fin d'impôts reportés (normalisation)

$PW_r(a)$ = valeur présente d'une suite $a_1, a_2, \ldots,$ estimé pour un taux d'intérêt r

R = revenus

RR = revenus requis

r = taux d'intérêt

ρ = taux de rendement

T = impôts

T_j = taux d'imposition

X = base tarifaire

Y base tarifaire (sous "normalisation")

Etablissons maintenant le modèle. L'amortissement sur toute la durée de la vie utile doit égaler la valeur de l'installation (valeur de rachat nulle)

$$\sum_{j=1}^{\infty} D_{bj} = \sum_{j=1}^{\infty} D_{tj} = k. \tag{1}$$

Les revenus requis (net des autres dépenses) doivent égaler les coûts suivants

$$RR_j = \rho X_j + T_j + D_{bj}. \tag{2}$$

Comme les paiements d'intérêt et l'amortissement est déductible d'impôt, on aura

$$T_j = T_o(RR_j - D_{tj} - i\delta X_j). \tag{3}$$

En résolvant simultanément (2) et (3), on obtient

$$RR = \frac{1}{1 - T_o}[(\rho - T_o i\delta)X + D_b - T_o D_t] \tag{4}$$

$$T = \frac{T_o}{1 - T_o} [(\rho - i\delta)X + D_b - D_t] \tag{5}$$

Si on a normalisation, la réserve pour impôts reportés sera

$$N_j = \sum_{k=1}^{j-1} T_o(D_{tk} - D_{bk}). \tag{6}$$

L'équation (2) devient

$$RR_j = \rho Y_j - T_j + D_{bj} + T_o(D_{tj} - D_{bj}) \tag{2b}$$

où $Y_j = X_j - N_j$.

En résolvant simultanément (2b) et (3) on obtient

$$RR = \frac{1}{1 - T_o} (\rho - T_o i\delta)Y + D_b \tag{4b}$$

$$T = \frac{T_o}{1 - T_o} (\rho - i\delta)Y + T_o(D_b - D_t). \tag{5b}$$

Examinons maintenant l'intérêt du consommateur. Nous posons comme hypothèse qu'il cherche à minimiser la valeur présente de ses déboursés donc des revenus requis par l'entreprise.

$$\text{Posons } \Delta RR_j \equiv RR_{FTj} - RR_{Nj}. \tag{7}$$

De (4) et (4b) on obtient

$$\Delta RR_j = \frac{1}{1 - T_o} [(\rho - T_o i\delta)N_j + T_o(D_{bj} - D_{tj})] \tag{8}$$

$$\text{Donc } PW_r(\Delta RR) = \frac{1}{1 - T_o} [(\rho - T_o i\delta)PW_r(N) + T_o PW_r(D_b - D_t)] \tag{9}$$

Or comme

$$PW_r(N) = \frac{T_o}{r} PW_r(D_b - D_t) \qquad (10)$$

Alors on obtient

$$PW_r(\Delta RR) = \frac{T_o}{1 - T_o} [1 - (1/r)(\rho - T_o i\delta)] PW_r(D_b - D_t) \qquad (11)$$

Comme $PW_r(D_b - D_t) > 0$ pour tout $r > 0$, la préférence du consommateur dépendra de $r_1 = \rho - T_o i\delta$. Il préfèrera la normalisation lorsque son propre taux d'intérêt sera plus petit que r_1 qui constitue le coût du capital net de l'entreprise.

Si nous estimons $\rho = .12$

$T_o = .50$

$i = .08$ (données approximatives pour Bell Canada)

$\delta = .50$

Alors on aura $r_1 = 0.10$.

L'investisseur pour sa part, cherchera à maximiser le taux de rendement moins les dépenses d'intérêt soit

$(\rho - i\delta)X$ lors du flowthrough

$(\rho - i\delta)Y$ lors d'une normalisation.

Or comme $X_j > Y_j$ pour tout j, l'investisseur préfèrera le flowthrough s'il ne tient pas compte du risque attaché à cette méthode. On ne peut cependant rien conclure au sujet du comportement de la firme avec un tel modèle. Notons enfin que les mêmes conclusions s'appliquent à une entreprise en croissance.

6. Application au modèle NPPS

Alors que précedemment, nous étions portés à favoriser un amortissement au livre accéléré, ces derniers développements nous font réfléchir.

Si nous posons que le taux d'actualisation du consommateur est plus petit que le coût du capital net de l'entreprise (ce qui est fort probable), alors on devra maximiser

$$PW_r(d_b - D_t) = PW_R(D_b) - PW_r(D_t).$$

Etant donné la méthode d'amortissement fiscale, on devra allouer à la compagnie le maximum d'amortissement permis en vertu de la loi de l'impôt. Cependant, on devrait tenter de minimiser l'amortissement au livre tout en respectant les principes comptables. Alors qu'on favorisait la méthode ELG, on devrait ici, dans l'intérêt du consommateur, employer la méthode ASL.

Certaines simulations, provoquant un changement important dans le rapport dette équité, pourraient justifier un changement de la méthode d'amortissement. Il en serait de même pour un changement exogène du taux d'intérêt ou du taux de rendement permis.

Alors le passage automatique d'une méthode à l'autre, pourrait être incorporé dans le modèle si nous fixions a priori le taux d'actualisation du consommateur; toutes les autres variables pouvant être (ou étant déjà) calculées par le modèle.

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APPENDIX B

Various treatments of the common costs

La présente section a pour but de faire une certaine revue de la littérature sur la séparation des coûts communs et de son application au domaine des télécommunications. Il va s'en dire que beaucoup de travail reste à faire d'abord pour approfondir notre connaissance du problème, et pour l'incorporation de certaines procédures dans le cadre du modèle NPPS. Le problème de l'allocation des coûts communs pour des produits est très complexe et quelques auteurs ont essayé de le résoudre par différentes méthodes. En particulier, Kaplan et Thompson(1) ont proposé une solution via des modèles de programmation mathématique, modèles qui seront brièvement exposés dans la présente section.

Considérons une firme qui fabrique n produits en utilisant m ressources et supposons que tous les coûts sont communs et fixes, c'est-à-dire qu'on ne peut les attribuer à une ressource en particulier. Soit le vecteur $x = (x_1, x_2, \ldots, x_n)^T$ où x_j représente les variables de décision pour le produit j. Alors cette firme cherchera la meilleure décision en ce qui concerne la production (la meilleure allocation des coûts des ressources pour chaque produit) pour maximiser ses profits. Elle devra:

$$\max px$$
$$\text{sujet à: } Ax \leq b$$
$$x \geq 0$$

où - a_{ij} est le montant de la ressource i qui entre dans la fabrication du produit j; A, une matrice m x n.
- b_i est le montant disponible pour la ressource i au cours d'une période; b, un vecteur m x 1.
- p_j est le profit associé au produit j; P, un vecteur 1 x n.

(1) Kaplan,R. et Thompson,G., "Overhead Allocation via Mathematical Programming Models", The Accounting Review, avril 1971, pp. 352-364.

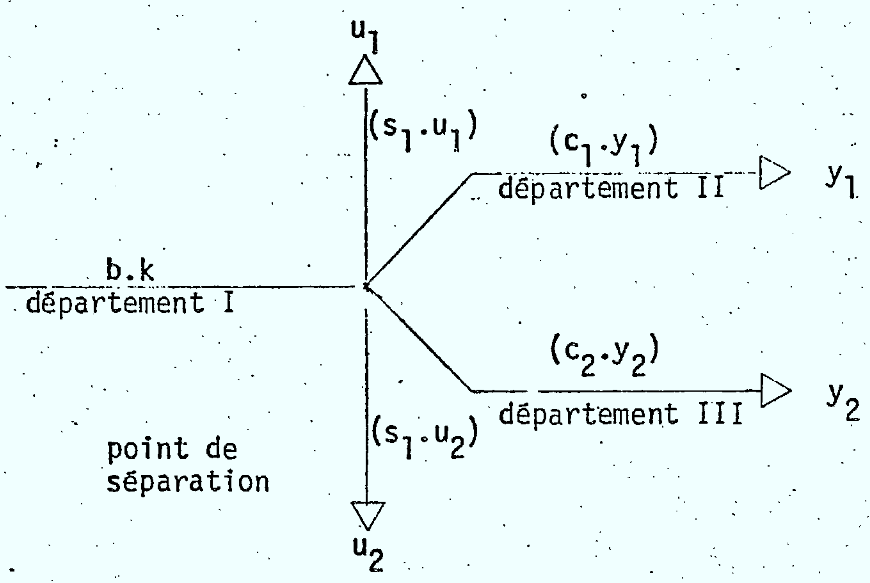
Notons que Kaplan et Thompson ont fait l'hypothèse, que ce modèle de programmation linéaire se situe dans un marché de concurrence parfaite; ce qui implique que les prix sont donnés. Si par contre, on suppose que la firme a des coûts attribuables à certaines ressources, alors le problème de décision pour la production afin de maximiser les profits se traduira comme suit:

$$\max\ (p - BA)\ x$$
$$\text{sujet à:}\quad Ax \leq b$$
$$x \geq 0$$

où B_i est le coût moyen par unité attribué à la ressource i.

D'autre part, plusieurs auteurs interprètent les coûts communs comme des coûts joints. A cet effet, ils ont suggéré plusieurs modèles ou méthodes pour l'allocation des coûts joints de produits. Regardons la signification de ces coûts joints pour différents auteurs et les modèles développés dans ce domaine.

Une situation de coût joint a lieu lorsqu'un input sert à fabriquer deux ou plusieurs produits qui peuvent être issus d'un processus de production soit en proportions fixes ou en proportions variables. Sous l'hypothèse de proportions fixes, trois auteurs ont montré que pour maximiser les profits, il n'est pas nécessaire d'allouer les coûts de l'input aux produits joints. Dans les pages qui vont suivre, ces différents modèles seront élaborés à l'aide de l'exemple suivant: soit une compagnie qui produit avec b unités d'input (k) dans le département I, a_1 unités de x_1 et a_2 unités de x_2. Toute la production ou une partie de x_1 peut être vendue au point de séparation à s_1 dollars par unité ou bien elle peut entrer dans le département II pour être transformée, puis vendue par la suite. Le produit x_2 ou une partie de x_2 peut aussi être vendu au point de séparation à s_2 dollars par unité ou bien, il peut entrer dans le département III pour être transformé et vendu par la suite. Graphiquement, cela peut être représenté comme ceci:



$$a_1k = x_1 = y_1 + u_1$$
$$a_2k = x_2 = y_2 + u_2$$

Figure 1: 2 produits joints

1) Le modèle de Manes et Smith[1]

Sous les hypothèses que les excès d'output d'un produit joint peuvent être sans coût et que toute la production doit être vendue seulement après le point de séparation, Manes et Smith ont montré qu'il est alors possible de faire une allocation sous certaines conditions pour connaître les fonctions de demande des produits joints. Par conséquent, cette allocation implique pour les produits joints, une investigation des possibilités de combinaison. Alors Manes et Smith, établissent le problème du coût joint pour deux fonctions de demande indépendantes et connues, de deux produits x_1 et x_2 comme suit:

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(1) Manes, R.P. et Smith, V.L., "Economic Joint Cost Theory and Accounting Practice", The Accounting Review, janvier 1965, pp. 31-35.

<u>modèle I</u>: max Profit = $R_1(x_1) + R_2(x_2)$ - Coût $(x_1 + x_2)$ (1)

sujet à la contrainte: $x_1 = \alpha x_2$

où α est la constante de proportionnalité entre x_1 et x_2.

Nous pouvons réécrire la fonction objectif en termes d'une variable comme ceci:

$$\text{max Profit: } R_1(\alpha x_2) + R_2(x_2) - \text{Coût } (\alpha x_2 + x_2) \quad (2)$$

et en considérant les conditions du 2e ordre, nous obtenons:

$$\frac{dP}{dx_2} = \frac{dR_1}{dx_2} + \frac{dR_2}{dx_2} - \frac{dc}{2x_2} = 0 \quad (3)$$

On résout cette équation et on obtient l'optimum recherché, i.e., le prix optimal des produits joints. Comme il n'est pas possible de dégager le coût marginal du revenu marginal individuel, alors une allocation significative de coûts pour les produits peut être faite. Cependant, si nous admettons la possibilité d'écarter un des produits joints, un résultat légèrement différent sera obtenu. Naturellement, dans ce cas nous déduirons que le produit écarté n'a pas de coût. Une meilleure solution est suggérée par le modèle suivant lorsque nous écartons un des produits joints:

<u>modèle II</u>: max Profit = $R_1(y_1) + R_2(y_2) - C(x_1, x_2)$

sujet à: 1) $x_1 = \alpha x_2$

2) $x_1 - y_1 = u_1 \geq 0$

3) $x_2 - y_2 = u_2 \geq 0$

où y_1 et y_2 = quantités de produits joints vendus

x_1 et x_2 = quantités de produits joints fabriqués

u_1 et u_2 = quantités invendues qui n'ont pas de coûts.

Par substitution des variables x_1 et x_2, nous obtenons la fonction de revenu net suivante qui sera la fonction à maximiser:

$$\phi = R_1(y_1) + R_2(y_2) - C(y_1 + u_1, y_2 + u_2) - \lambda[u_1 + y_1 - \alpha (u_2 + y_2)]$$

Après certaines transformations, on revient au modèle précédent et on peut trouver le prix optimal de chaque produit joint. Donc la répartition des unités d'un produit ne peut pas rapporter de projet maximum à moins que ce produit soit rendu au point où l'élasticité de la demande est unitaire, c'est-à-dire où le revenu marginal = 0 (Cm = Rm). Dans un tel cas, il n'y a pas de coûts d'input qui seront chargés à ce produit mais tous ces coûts seront transférés sur le deuxième produit joint. Dans le modèle de Manes et Smith, on pourrait parler de sous-produits et de produit principal.

L'approche traditionnelle de la comptabilité des coûts d'un sous-produit a été de ne pas allouer de coûts d'input aux sous-produits et d'indiquer le revenu net du sous-produit comme une réduction dans les coûts de vente du produit principal ou encore comme un revenu supplémentaire. Alors on peut constater que le modèle II exposé par Manes et Smith rejoint l'approche traditionnelle de la comptabilité des coûts.

2) Le modèle de Jensen(1)

L'analyse de Jensen est basée sur la détermination d'un modèle pour la prise d'une décision de production - prix à court terme et cela pour deux produits joints complémentaires fabriqués dans des proportions fixées (x_1, x_2). Par conséquent, les résultats de cette analyse peuvent s'étendre aux cas de proportions variables.

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(1) Jensen, D.L., "The Role of Cost in Pricing Joint Products: a Case of Production in Fixed Proportions", The Accounting Review, juillet 1974, pp. 465-476.

Le modèle que Jensen présente, diffère de celui de Manes et Smith de plusieurs façons:

1- La fonction de coût est donnée dans une forme explicite qui est consistante avec les caractéristiques de la comptabilité des coûts d'une production jointe.

2- L'exigence qu'une portion d'un des produits joints soit éliminée est relâchée i.e. que l'excès de production d'un produit joint n'est pas éliminé et par le fait même, il implique des coûts.

3- Le problème est formulé explicitement comme une décision de prix plutôt qu'une décision de production i.e. prix optimal au lieu de production optimale.

Notons aussi que les politiques de prix optimal et de production optimale sont équivalentes dans ce sens que l'un implique l'autre et vice-versa.

Conséquemment Jensen cherche la politique de prix optimal pour maximiser les profits. Dans ce modèle, Jensen fait l'hypothèse qu'au point de dissociation on vend une partie de la production sur un marché de concurrence parfaite (les prix sont connus) tandis qu'après le processus de séparation, le reste de la production est vendu sur un marché de concurrence imparfaite (les prix sont à déterminer). Le problème est d'obtenir les prix p_1 et p_2, les quantités de production jointe au point de dissociation (u_1 et u_2) et la quantité totale d'input qui maximisent les profits de deux produits joints fabriqués dans des proportions fixes où chaque produit peut être poussé au-delà du point de dissociation.
On veut

$$\max \Pi = p_1 D_1 + p_2 D_2 - c_1 D_1 - c_2 D_2 + s_1 u_1 + s_2 u_2 - bk$$

sujet à: 1) $D_1 + u_1 = a_1 k$

2) $D_2 + u_2 = a_2 k$

3) $p_1, p_2, u_1, u_2, k \geq 0$

où D_1 et D_2 sont les fonctions de demande des 2 produits

u_1 et u_2 = quantités vendues ou éliminées au point de dissociation

s_1 et s_2 = prix des quantités vendues au point de dissociation en concurrence parfaite

c_1 et c_2 = prix des quantités vendues après le point de dissociation (après avoir passé dans les départements II et III) en concurrence imparfaite.

Il y a une condition nécessaire pour solutionner ce problème: le profit (Π) doit être une fonction concave des prix, des quantités et des capacités. En maximisant le profit, Jensen a obtenu deux types de solutions optimales c'est-à-dire deux sortes de prix optimal.

<u>Solution I</u>: Lorsque la politique de production - prix, requiert la vente d'une partie de la production au point de dissociation et le reste, après le processus de séparation.

<u>Solution II</u>: Lorsque la politique de production - prix optimal exige que toute la production des deux produits soit vendue après la séparation.

Dans les deux cas, Jensen trouve un prix optimal. Par conséquent, il conclut que l'on peut accorder un coût joint aux produits fabriqués dans des proportions fixes quand 1^o le profit est maximisé par la vente de la production entière après le processus de séparation et 2^o quand la maximisation du profit requiert la vente d'une partie de la production au point de séparation et le reste, après ce point. Dans le 1er cas, la demande basée sur l'allocation doit être dérivée d'une politique de prix optimal tandis que dans le 2e cas, la demande basée sur l'allocation assigne la totalité des coûts joints à l'autre produit qui est vendu après le processus de séparation.

De plus, à partir de ce modèle, on pourrait incorporer des contraintes supplémentaires telles que le coût de l'équipement entrant dans la fabrication, les inventaires; c'est ce que R. Hartley[1] a fait.

3) Le modèle de Colberg[2]

Dans ce modèle, Colberg fait la même hypothèse que Manes et Smith, notamment l'excès de production d'un produit joint est sans coût mais par contre, il ne requiert pas que toute la production soit vendue seulement après le processus de séparation. Il essaie lui aussi de maximiser ses profits et de trouver le prix optimal pour chaque produit joint.

En résumé, la capacité de l'input qui entre dans la fabrication des deux produits joints dans des proportions fixes est la même. Ce qui est différent, ce sont les coûts marginaux entrant dans la détermination de chaque modèle. Mais chaque coût marginal inclut le coût joint marginal de séparation. Ce dernier démontre que l'implantation d'une politique optimale ne requiert pas une allocation de tel coût sous aucun des modèles considérés. Autrement dit, pour maximiser les profits, il n'est pas nécessaire d'allouer les coûts de l'input aux produits joints si ceux-ci sont fabriqués dans des proportions fixes.

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(1) Hartley, Ronald V., "Decision Making When Joint Products Are Involved", The Accounting Review, octobre 1971, pp. 746-755.

(2) Colberg, Marshall R., "Monopoly Prices Under Joint Cost: Fixed Proportions", Journal of Political Economy, 1941, pp. 103-110.

Regardons maintenant comment l'allocation des coûts joints peut se faire dans le domaine des télécommunications ou les entreprises sont réglementées. Il y a plusieurs bases d'allocation qui peuvent être suggérées pour les coûts joints dans ce domaine. W.R. Scott(1) a relaté les trois méthodes suivantes:

1) Méthode basée sur la valeur des ventes relatives.

Dans cette méthode, l'allocation est basée sur la capacité de payer. Alors les produits joints de haute valeur reçoivent des coûts élevés d'allocation. Cette approche tend à forcer le profit comptable à se rapprocher du profit marginal.

2) Méthode basée sur des mesures de capacités physiques.

Cette méthode est basée sur le principe d'emploi actuel c'est-à-dire sur une unité de mesure (par exemple: message - minute - mille pour un réseau de télécommunication). Elle est dans un sens, opposée à la première car une haute et une basse valeur de service employant le réseau pour un même message - minute - mille recevraient la même allocation. En effet, les coûts d'opportunité de services variés sont très différents, aussi le profit marginal s'éloigne du profit comptable.

Une autre difficulté dans l'application des mesures relatives à l'industrie des télécommunications serait que l'allocation devrait être affectée par une structure de taux. Si un service est marqué par un bas taux par période, alors le coût marginal du client employant ce service est nul.

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(1) Scott, W.R., Certain Accounting Aspects of Telecommunication Regulation, study 6, pp. 332-338.

Cette méthode est expliquée par Charles Phillips[1] à l'aide de l'exemple d'une compagnie de téléphone. Il se pose la question suivante: comment séparer les coûts entre les services téléphoniques à l'intérieur de l'état (local) et ceux entre les états (interurbain). Soit une entreprise fournissant des services de communications entre les états et à l'intérieur de l'état. Les méthodes de séparation des coûts entre les différents services sont construites autour du principe d'emploi actuel. Ce principe signifie que 1^o tous les coûts d'équipements employés uniquement pour un service soient assignés directement à ce service et que 2^o tous les coûts d'équipements employés conjointement pour deux ou plusieurs services soient alloués parmi les services sur une base d'emploi actuel laquelle considérera l'occupation et le temps relatif dans la mesure de l'emploi.

Les mesures du temps de l'emploi sont déterminées sur une unité de base (exemple: unité d'encombrement par appel) dans une étude d'encombrement sur 24 heures plutôt que seulement sur le volume des heures d'affaires. Alors, le circuit de téléphone est séparé sur la base des minutes d'emploi (minutes de temps pour une conversation). Cependant les taxes, les dépenses généralisées, etc..., sont séparées sur la même base que les coûts courants ou la valeur aux livres des coûts des équipements relatifs.

Pour le règlement des coûts, tous les revenus collectés sur les opérations entre états sont mis en commun chaque mois; les dépenses et les taxes allouées à ces mêmes opérations pour chaque compagnie sont enlevées de ces revenus. Les revenus nets résultant de cette opération, représentent le montant des profits. Ce montant sera distribué selon la base d'investissement net que chaque compagnie a apporté au total des investissements nets du service entre états

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(1) Phillips, Charles F., The Economics of Regulation, ed. Irwin, 1969, pp. 153-162.

(i.e. la part de contribution de chacun). Si par exemple, un département ou une compagnie fournit 35% du total des investissements nets, il(elle) recevra 35% des profits obtenus.

3) Approche par la programmation

Certains auteurs disent que si les proportions des produits joints sont variables, alors la décision concernant la production mixte est basée sur les coûts d'opportunité . Mais cependant Shillinglaw[1] montre que ce n'est pas garanti que les coûts d'opportunité de chaque produit joint s'additionnent et soient égaux aux coûts joints totaux. Alors dans ce cas, les coûts d'opportunité n'aideraient pas beaucoup dans l'allocation des coûts joints pour déterminer le profit relatif. Cependant, une nouvelle approche pour l'allocation des coûts joints, laquelle apparaît indépendante des proportions fixes ou variables, a été suggérée par Weil[2]. Selon cet auteur, pour maximiser les profits d'une firme il suffit de connaître les fonctions de coûts et de demande de la firme et l'application de cette méthode devient très facile. En effet, connaissant les fonctions de demande pour deux produits joints, on peut

$$\max \Pi = P_1Q_1 + P_2Q_2 - 10C$$

sujet à: 1) $Q_1 < C$

2) $Q_2 < C$

où C est la quantité d'input achetée au coût de $10.00 l'unité.

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(1) Shillinglaw, G., Cost Accounting: Analysis and Control, (Revised Edition, Homewood, Ill.: R.D. Irwin, Inc. 1967), p. 247.

(2) Weil, Roman, "Allocation Joint Cost", The American Economic Review, décembre 1968, pp. 1342-45.

Mais, pour une firme réglementée, la spécification de ces fonctions devient un problème difficile. Par contre, Lerner et Moag, ont trouvé un moyen de pallier à ce problème en passant par les méthodes de programmation. Ils suggèrent un modèle qui minimise le prix chargé, étant donné un niveau de production demandé.

W.R. Scott applique la méthode de Weil pour l'allocation des coûts joints sur un exemple pris dans le domaine des télécommunications. Soit les trois services suivants:

Service 1: Mesuré en unité de milles par mois. Par exemple, le nombre d'appareils téléphoniques domestiques en service.

Service 2: Les appels longue distance, mesurés en milliers d'appels par mois.

Service 3: Transmission spéciale des données offerte durant les heures de pointes du service 2, mesurée en milliers de message-minutes.

Les trois services sont produits par des machines communes qui peuvent produire k unités de service 1 ou s unités de capacités maximum pour le service 2. Il définit aussi:

v = un coefficient qui relate les capacités maximum au nombre total d'unités du service 2 demandé dans le mois.

y = Coût d'amortissement et coût d'opération par machine par mois. (Coût joint par machine). Ils sont constants.

i = Temps moyen que le service 2 occupe sur le système.

t = Milliers de minutes dans la période.

$$p_1 = a_1 - b_1 q_1$$
$$p_2 = a_2 - b_2 q_2$$
$$p_3 = a_3 - b_3 q_3$$

Ce sont les courbes de demande de chaque produit (service). a_i, $b_i > 0$ et $i = 1, 2, 3$.

Il pose comme hypothèse:

1 - Il n'y a pas d'interaction entre ces fonctions de demande.

2 - P_1 et P_2 sont les prix maximum que l'organisme de réglementation impose. Seul P_3 n'est pas imposé.

Etant donné les courbes de demande, chacun de ces prix maximum spécifie une quantité minimum de Q_1 et Q_2 respectivement. Soit p_1 et q_1 les prix et la quantité du service 1,..., etc., et m le nombre de machines dans le réseau, d'où les coûts joints totaux seront ym. Alors notre problème peut s'exprimer comme suit:

$$\max \Pi = p_1 q_1 + p_2 q_2 + p_3 q_3 - ym \qquad (1)$$

$$\text{sujet à:} \quad q_2 \leq mvs - \frac{vs}{k} q_1 \qquad (2)$$

$$q_3 \leq (ms - \frac{s}{k} q_1)(t - vi) \qquad (3)$$

$$q_1 \geq Q_1 > 0 \qquad (4)$$

$$q_2 \geq Q_2 > 0 \qquad (5)$$

$$q_3 \text{ et } m \geq 0 \qquad (6)$$

L'inégalité (2) représente les possibilités de production entre 1 et 2 et l'inégalité (3) donne le nombre maximum de minutes pour le service 3. Notons que le service 3 emploi le système quand le service 2 ne l'emploie pas. Si on veut maximiser les profits d'une firme non réglementée, il faut enlever les contraintes (4) et (5). Cependant, le but de Scott n'est pas seulement de maximiser les profits mais aussi de faire l'allocation des coûts joints. A cet effet, après avoir résolu l'équation (1) sujette à ses contraintes, on peut trouver le revenu marginal de chaque service au niveau optimum de production. D'où $Rm_i = \frac{\partial \Pi}{\partial q_i} = p_i - q_i$. Maintenant, en faisant l'hypothèse que le revenu marginal est égal au coût marginal pour chaque service, on peut trouver l'allocation de l'équipement joint de chaque service. Soit pour le service i: $Rm_i = \frac{Cm_i}{y}$(coût joint par machine) $= x_i$ machines pour le service i. Si on veut trouver le revenu net pour chaque service, on doit prendre le revenu de ce service ($q_i \times p_i$) et lui enlever le coût joint d'allocation de ce service qui est Cm_i. Une autre façon de déterminer le revenu net de chaque service est de prendre le nombre de machines affectées au service i (x_i) multiplié par la quantité produite de ce service (q_i).

Cette méthode donne une bonne allocation des coûts joints car l'allocation est basée sur les coûts d'opportunité et elle est objective.

Un autre problème qui serait bon d'exposer et qui se rapproche de l'allocation des coûts joints est celui de l'allocation des coûts des services réciproques. Le problème d'allocation des coûts des services réciproques est très simple. Il suffit de trouver une réponse au problème suivant: quel est le coût approprié qui peut être attribué à chacun des départements pour les services reçus et encourus? Supposons le cas suivant: dans une entreprise, il y a deux départements de production P_1 et P_2, et deux départements de services S_1 et S_2. Le département S_1 accorde 40% de ses services à P_1, 40% à P_2 et le reste à S_2. En ce qui concerne le département S_2, il accorde lui aussi 40% de ses services à P_1, 50% à P_2 et 10% à S_1. Etant donné que les coûts alloués directement à S_1, S_2, P_1, P_2 sont \$9,000., \$6,000., \$20,000., \$16,000., respectivement, quelle est l'allocation appropriée du coûts des services entre les départements?.

Pour résoudre ce problème, il y a plusieurs solutions qui ont été proposées mais D. Ashton(1) nous en explique trois. Il s'agit du modèle de Williams et Griffiths(2), celui de Manes(3) et celui de Minch et Petri(4). Ces modèles ont été solutionnés à l'aide de l'approche matricielle. Ashton nous les présente avec leur solution matricielle et par la suite, il refait les calculs en appliquant les principes comptables.

Soit la matrice $B = \begin{pmatrix} 0 & 0.1 \\ 0.2 & 0 \end{pmatrix}$ qui représente l'allocation entre les départements de services.

Soit le vecteur $S = (S_1 \quad S_2)^T$ $= (9{,}000 \quad 6{,}000)^T$ qui représente l'allocation directe des coûts des départements de services.

le vecteur $P = (P_1 \quad P_2)^T$ $(20{,}000 \quad 16{,}000)^T$ qui représente l'allocation directe des coûts des départements de production.

la matrice $A = \begin{pmatrix} 0.4 & 0.4 \\ 0.4 & 0.5 \end{pmatrix}$ qui représente l'allocation des services aux départements de production.

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(1) Ashton, Daniel, "Solutions to the Reciprocal Service Cost Allocation Problem", Centre for Industrial Economic and Business Research, University of Warwick, décembre 1973.

(2) Williams et Griffiths, "Matrix Theory and Cost Allocation", The Accounting Review, July 1964, pp.671-678.

(3) Manes, R.P., "Comment on Matrix Theory and Cost Allocation", The Accounting Review, July 1965, pp. 640-643.

(4) Minch et Petri, "Matrix Models of Reciprocal Service Cost Allocation", The Accounting Review, July 1972, pp. 576-580.

1) Le modèle de Williams et Griffiths

Dans ce modèle, les auteurs incluent les crédits pour les services accomplis après que les charges pour les services reçus ont été incluses. Pour résoudre le problème, il faut trouver les vecteurs S' et P' tel que $S' = (I - B)^{-1} S$ et $P' = A(I - B)^{-1} S$. Alors $S' = \begin{pmatrix} S_1' \\ S_2' \end{pmatrix} = \begin{pmatrix} 9796 \\ 7958 \end{pmatrix}$ et $P' = \begin{pmatrix} P_1' \\ P_2' \end{pmatrix} = \begin{pmatrix} 27{,}102 \\ 23{,}898 \end{pmatrix}$.

D'où le coût total des départements des services est $17,754 (9796 + 7958) tandis que le coût total pour l'allocation directe des départements des services égale $15,000. (9,000. + 6,000.). Ici, le modèle ne fait pas une bonne allocation des coûts entre les départements des services.

2) Le modèle de Manes

Il incorpore simultanément les charges pour le travail fait par les autres départements de services et il crédite le travail fait pour les autres. Par conséquent, il cherche les vecteurs S' et P' satisfaisant à:

$$S' = S + BS' - (I - Z) S' \qquad (1)$$

$$P' = P + AZ^{-1} S' \qquad (2)$$

où la matrice diagonale Z, représente la somme des pourcentages des départements de production $Z = \begin{pmatrix} 0.8 & 0 \\ 0 & 0.9 \end{pmatrix}$. Dans ce modèle,

$S' = \begin{pmatrix} S_1' \\ S_2' \end{pmatrix} = \begin{pmatrix} 8{,}077 \\ 6{,}923 \end{pmatrix}$ et $P' = \begin{pmatrix} P_1' \\ P_2' \end{pmatrix} = \begin{pmatrix} 27{,}115 \\ 23{,}885 \end{pmatrix}$; il y a une meilleure allocation des coûts entre les départements des services, comparativement au 1er modèle.

3) Le modèle de Minch et Petri

Pour Minch et Petri, les crédits pour les services accomplis sont considérés dans le coût d'allocation réciproque. Soit le vecteur $x = (x_1\ x_2)^T$ qui reflète les crédits pour les services accomplis. Alors $x' = \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = S' - Bx'$ et de plus, $S' = x' + Bx'$,

$P' = P' + AMS'$ où $M = \begin{pmatrix} 1/0.8 & 0 \\ 0 & 1/0.9 \end{pmatrix}$. Donc, $S' = \begin{pmatrix} S_1' \\ S_2' \end{pmatrix} = \begin{pmatrix} 8045 \\ 6955 \end{pmatrix}$ et

$P' = \begin{pmatrix} P_1' \\ P_2' \end{pmatrix} = \begin{pmatrix} 27{,}114 \\ 23{,}886 \end{pmatrix}$.

Ashton conclut que théoriquement, le deuxième modèle, celui de Manes, est le meilleur mais par contre, le troisième donne les mêmes résultats que le deuxième. De plus, en appliquant les méthodes comptables, Ashton arrive aux mêmes résultats que ceux obtenus par l'approche matricielle.

APPENDIX C

Lexique des symboles utilisés dans le bloc comptable

ACCREV - Account receivable
AD - Accumulated depreciation, end of years
ADO - " " , beginning of years
ADJA - Adjustments to calculate DPRTVE
ADJB - Regulatory adjustments to rate base (RORBE)
ADJD - Adjustments to deferred taxes (CURDTX)
ADJO - " " other expense
ADJP - " " gross telephone property (GTP)
ADJR - " " accumulated depreciation (AD)
ADJU - " " undepreciated capital cost (UCC)
ADVGV - Advances by government
ALPHA - Flowthrough coefficient of deferred taxes

BETA - Ratio of taxable other income/other income (excl. IDC)

CAM - Commercial and Marketing expenses
CCA - Capital cost allowance
CCARAT - " " " rate
CCL - Debt issue expenses as percentage of gross proceeds of issue
CE - Issue expense rate, common shares
CL - Current liabilities
CLO - " " , beginning of year
CNRF - Charges to construction not requiring funds
CP - Issue expense rate, preferred shares
CTI - Cash and temporary investments
CTIO - " " " " , beginning of year
CURDTX - Current deferred taxes

DCD - Deferred charges - debt
DCDO - " " - " , beginning of year
DCH - " " - total,
DCHO - " " - " , beginning of year
DCO - " " - other
DCOO - " " - " " " "
DCR - Debt/capitalization ratio
DELCL - Change in current liabilities
DELCTI - " " cash and temporary investments (CTI)
DELDCH - " " deferred charges
DELDCR - " " " credits
DELDTX - " " " taxes
DELEQ - " " equity
DELINV - " " investments
DELOCA - " " other current assets
DELODCR - " " " deferred credits
DELPR - " " preferred stock
DELWK - " " working capital
DEPDIF - Depreciation difference
DEPN - " expense
DEPRAT - " rate
DFTAX - Deferred taxes
DFTAXO - " " , beginning of year
DIVI - Dividends on common shares
DPNONC - Depreciation on other non-cash charges
DPR - Dividends payout ratio
DPRTVC - Depreciation - tools and vehicles - capitalized
DPRTVE - " " " - expensed
DSC - Debt services charges

EQ - Common stock
EQO - " " , beginning of year

GAMMA	-	Ratio of plant under construction (PUC)/GCE (end of year)
GCE	-	Gross construction expenditure
GEDRAT	-	Depreciation rate on general equipment (tool & vehicles)
GETV	-	General equipment tool & vehicles
GETVO	-	" " " " " , beginning of year
GTP	-	Gross telephone property
GTPAC		" " plant at cost
GTPACO	-	" " " " " , beginning of year
GTPO	-	" " property, beginning of year

IC	-	Interest on construction (PUC)
IDC	-	" during construction
IN	-	" rate on new debt
INCTAX	-	Income taxes accrued
INV	-	Investments
INVO	-	" , beginning of year
IO	-	Old (embedded) interest rate
IT	-	Income taxes
ITC	-	Interest time coverage
ITCAT	-	" " " incl. taxes
ITCBT	-	" " " excl. taxes

L	-	Debt
LAND	-	Land
LANDO	-	" , beginning of year
LO	-	Debt, " " "
LOCAL	-	Local revenue
LTD	-	Long term debt

MAINT	-	Maintenance expense
MISOPRV	-	Miscellaneous operating revenue

NETINC - Net Income
NEWDEB - New debt
NIA - Net income available to common shareholders
NOI - Non taxable other income
NOTES - Short term notes
NSV - Net salvage value

OCA - Other current assets
OCAO - " " " , beginning of year
ODCR - " deferred credits
ODCRO - " " " , beginning of year
OMEGA - Ratio of regulatory working capital to operating expense
OPRV - Operating revenue (gross)
OPXP - " expenses other than depreciation
OTHADJ - Adjustments to retained earning (RE)
OTHCA - Other current assets
OTHEXP - " expenses
OTHINC - " income - total
OTHOPXP - " operating expense
OTHTX - " taxes

PCR - Preferred capital ratio
PDIVI - Dividends on preferred stock
PLAMPS - Plant acquired - Plant sold
PR - Preferred Stock
PRO - " " , beginning of year
PRDTX - Prior year's deferred taxes
PUC - Plant under construction
PUCO " " " , beginning of year

RE - Retained earning
REO - " " , beginning of year

REPL	-	Repayment of long-term debt and note
RET	-	Retirements
RHO	-	Dividend rate on old preferred share
RHON	-	" " " new " "
RORBE	-	Rate of return - asset based excl. (IDC + PUC)
RORBI	-	" " " - " " incl. (" " ")
RORC	-	" " " - total capital
RORERC	-	" " " - " equity capital
SIE	-	Share issue expenses
SPLIT	-	Proportion of depreciation on tools and vehicles which is expensed
T	-	Taxe rate
TOOL	-	Tool revenue
TOTASS	-	Total assets
TOTLIA	-	" liabilities
TOTOTHINC	-	" other income
TRAF	-	Traffic expense
TRANGV	-	Transfers to government
UCC	-	Undepreciated capital cost
UCCO	-	" " " " , beginning of year
UNCOLL	-	Uncollectible accounts
Zeta	-	Ratio of common stock dividends to transfer to government owners

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