

THE ROLE OF INFORMATION ACTIVITIES IN  
TOTAL CANADIAN MANUFACTURING

Separability and Substitutability

by

G. Warskett  
School of Public Administration  
Carleton University

Final Report to the  
Department of Communications  
CONTRACT 12ST. 36100-8-0968

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FOREWORD

This research was completed under contract (12ST. 36100-8-0968) for the Telecommunications Economics Branch, Department of Communications, Ottawa. The work forms part of the analysis being prepared by the DOC for the Working Party on Information, Computer and Communications Policy, Directorate for Science, Technology and Industry, Organization for Economic Cooperation and Development.

I have had the good fortune to obtain incalculable support and advice from Shirley Serafini and Michael Andrieu of the DOC. The important computer support work was excellently fulfilled by Mr. Aly Sherif of Queen's University. Ms. Sheila Nijhowne of Statistics Canada kindly helped with the input-output data preparation. Mr. Al Shackleton, also of DOC, kindly provided his estimates of expenditures on computer use in Canadian Manufacturing. I owe them all a debt of gratitude.

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## EXECUTIVE SUMMARY

In this study we examine the usefulness that information accounts have for economic analysis and policy. Limiting our attention to the Canadian manufacturing sector, we first discuss the technical necessity for reclassifying production inputs into information and noninformation services. Second, we review the concept of factor productivity and identify the problems attached to measures of information input productivity. Finally, using statistical estimates for coefficients which describe the technological relationship which holds between factor inputs and outputs, we obtain a rough measure of employment responses to factor price changes. In particular, we wish to estimate the impact a change in the price of information capital equipment has on information workers and production workers separately.

We choose the manufacturing sector because it possessed the most detailed data and similar studies are available to which our results can be compared. Our approach consists of statistically estimating, using econometric techniques, a "production function" for manufacturing. In fact, two production functions were estimated, one for manufacturing as a whole on annual observations for the period 1948-1973, and one for a "representative" major industry division on observations made up by pooling 5 years of data with 19 industries comprising manufacturing.

The logic of information accounts can be made more compelling if it is shown that the structure of manufacturing technology is such that information inputs are technically distinct from non-information inputs. For example, business applies the myriad of goods and services that are needed to produce the final output in different ways, and the main utility of national accounts is to reduce the large numbers of these and other commodities to some manageable size. This is achieved by a process of aggregation over "similar" commodities to

produce broad categories such as food, clothing, machinery and equipment, telecommunication services, etc. However, an examination of the commodities identified as information goods shows a great heterogeneity and one immediately suspects that information goods do not at all act as close substitutes, either in production or in consumption, an essential property necessary to make them eligible for aggration into a single commodity category.

Our production function analysis permits us to examine this problem of suitable information aggregates. Mathematical conditions can be specified which express the circumstance of aggregation and be embodied in the production function. A production function subject to these conditions (the maintained hypothesis) is fitted to the observations and so is another which, however, is free of these mathematical conditions for 'separation'. By seeing which specification, the maintained hypothesis or the alternative, fits the observations best we are using the data to help us judge the validity of information aggregates, at least as inputs into manufacturing. For example, we took the trouble to split capital stocks according to information capital and non-information capital. If we had found that the production function which fits the data best obeys the conditions suggesting that the split was needless, then the point of this capital subdivision is lost. This would imply that the accounts are thus at odds with the data, and arguments for national information accounts are thereby weakened. Our findings on this question are as follows. According to the fit over the time series data, in which only labour inputs are differentiated between information and non-information, it is statistically better to make this labour distinction rather than not. This agrees with similar studies unrelated to information accounts. With the pooled data we found inconclusive evidence on this issue. The evidence is not at variance with information accounts in that the division of capital and of labour in conformity with such accounts was deemed suitable by the statistical test. However, a



supplementary test showed no cause for belief that the technology distinguished between all information activity on the one hand and all non-information on the other.

The conclusion we arrive at regarding productivity of information work is that it cannot in any meaningful way be measured or examined independently of the other inputs. Measured productivity change for one input may well be caused by changing levels of other inputs and unless these additional causal factors are accounted for, no inference is possible from the observed changes.

Finally, we found from the estimates over the time series observations that information and non-information workers' employment are affected in different directions by changing capital prices. The implication of this result is that not all the work force has been affected adversely by increasing investment in capital. Greater investment in total machinery and equipment is accompanied by a lower share of revenue for production workers but a higher share of revenue taken by information workers. However, higher investment in construction capital goods has had the opposite impact on the revenue share for the two employee categories.

The production function estimated with the use of pooled data produced a pattern of employment impacts different from the foregoing. However, the aspect of the finding which is most striking is that a falling price for information capital does not seem to present a significant threat to manufacturing employment, even were there not to be any expansion in production. Of course, any presence of a falling capital price would lead to a fall in production costs and in all likelihood to increased production, and hence further employment. This result is important in policy terms since it suggests that fears for manufacturing employment based on the further incursion of information machinery and equipment are ill-founded. But before such a hard and fast conclusion is drawn, we remind the reader of the nature of the data used and their limitations. Some mention of this is made below but here the fact that the data period covers 1967-1971 is emphasized. Given that new capital is adopted piecemeal and only slowly to save

scrappage of productive capital in place, the results obtained over this estimation period should be good for the next five years or so from the end data, 1971. Thereafter, we cannot apply the estimates of substitution elasticities, through which we reckon the employment effects, with the same degree of confidence into 1980, say. Obviously, the more up-to-date data now being made available should be employed to further strengthen and test the results obtained so far.

The results seem to indicate some grounds for information accounting and the impact of capital prices on employment, but for several reasons these conclusions remain tentative pending further investigation and data improvements. Four of the most pressing developments that are needed are as follows. Firstly, the value added data used in both estimation approaches exclude observations on computer rental, probably representing a loss of a major source of computer use in manufacturing. Thus, one of the information items that would assume great importance in any information accounts is partly excluded from the study. This omission can be repaired, as it would be in the accounts, by subtracting from manufacturing business current expenditure the sum corresponding to computer charges and adding this sum to both the output and to the capital accounts expenditure. For the years prior to 1971 Statistics Canada possesses only rough industry estimates of this expenditure.

Secondly, as mentioned earlier, the pooled observations study can be extended to include the data becoming available for the years 1972-1976, years for which information investment became more important. Thirdly, the work can extend to other industries, such as 'finance, insurance and real estate' and 'trade, wholesale and retail' where computer services have had a significant impact. Finally, because the quality of information services has changed dramatically through the vast improvements made in computing and telecommunication services, the quantity and price indices for these services should be modified to allow for these improvements in quality. Procedures exist for this type of analysis.

Once these technical improvements have been satisfactorily settled we would be better able to study the vital issue brought by the arrival of the 'information revolution'; namely, the impact on manufacturing output prices of falling information capital prices, with its implications for Canada's balance of payments; and the overall medium run impact to be expected for employment in various industries resulting from these price reductions.

PART I

1. INTRODUCTION

One of the most significant developments in advanced industrial economies has been the growing importance of information activities. More and more employment is designated to produce, process and distribute symbols as opposed to things. In 1971, 40 per cent of the Canadian work force was thus occupied. The past two decades have moreover witnessed a dramatic growth in the stock of electronic information processing and distributing machinery and equipment, which now is poised threateningly over the jobs of information workers.

Many factors have combined to create an insatiable thirst for information in all sectors of the economy, including the private business sector. For the latter, not least of these factors are the size and complexity of the economic environment in which business operates and the increasing penetration of foreign markets, the opportunities offered by new technologies and new product lines, and the need to plan increasingly larger capital expenditures over a long time horizon. We see here the competitive drive to seek new markets, to hold on to old ones, and finally to reduce production costs.

These demands require information and data on conditions of current and prospective markets, improved techniques of production, optimal organization of production, as well as many other areas. The production and organization of this information is the task of the intellectual labour of market research, management, and research and development teams. In addition, information flows have to be maintained within each plant and between the various plants, outposts and the head office, which, in the past, called for a small army of clerical workers.

When seen in the wider context of economic and social history, our society is entering a new age of economic organization and development, with potentially great social implications. The considerable resources today which are devoted to public sector information activities, including social and economic planning, management and forecasting, education, and research, together with the resources applied in the private sector to the various information related functions mentioned earlier, cause a structural change in the economy whereby much less of total activities are directly applied to the production of final goods and services. Recent technological advances in micro-electronics are foreseen to hold both promises and threats. These developments leading to lower costs of processing, computing, storage, and transmission of data promise lower costs for both production and information work. The diffusion of the new technology in industry will mean further automation and control of production processes, with the use of less staff. Fabrication as well as assembly work can be turned over to robots as these evolve out of the stage of digital control power tools. Significantly, these developments are becoming economical for shorter production runs leading to wider application. With the phenomenal rise in the price of energy and materials, information activities and development will save on these expensive and scarce resources. The telecommunications infrastructure will substitute for physical transportation and allow for distributed data and word processing work, perhaps slowing or even reversing centralizing tendencies in economic activities. The labour-intensive administrative and information processing activities are becoming increasingly

subject to capital-using technological change that gives rise to such recent developments as electronics funds transfers, electronic mail, word processing machines and the use of smart machines in the retail trades.

For employment, these promises are rapidly becoming threats. The advent of these developments cries out for a scientific study of the possible social and economic impacts to come in its train. As with any science, a conceptual framework and whatever can be done in the way of qualitative and quantitative measurement have to be developed. Thus a call is out for revisions in national accounting to better reflect information activities. As we will see, a start at the macroeconomic level has already been taken, while in contrast the research here specifically studies the problems of the interrelationship between concepts and measurement at the microeconomic level, although at a very aggregative level.

The outline of this two part study is as follows. We review some of the issues in more detail and elaborate our line of attack for empirically testing the economic relevance of information accounts. In the first part an intermediate step is made through an econometric estimation of a production function for Canadian manufacturing. Much effort went into data preparation and this is reported together with an account of certain notable problems to be expected from the imperfect quality of the data. An incomplete study of information work productivity is also made, with a formulation for further work. Finally, with the help of estimated values for substitution elasticities, we conjecture the employment impacts for production and information workers

of changes in capital prices.

The second part of this study uses the same methodological approach to which the production function concept is central, but an improved data base allows many more of the relevant technical questions to be asked and tested.



## 2. THE ISSUES AND METHOD OF INVESTIGATION

### 1. The Policy Concerns

Following the work of Porat<sup>1</sup>, and earlier of Machlup<sup>2</sup>, in which a system of U.S. national accounts designed to highlight information activities was developed, widespread interest has been shown by the O.E.C.D. nations in the possibility of establishing such accounts for themselves. The project would entail considerable reworking of historical data and much redesigning of national accounts in order that the, largely non-market, information activities can be identified and receive due economic imputation of value. This would undoubtedly prove to be a large task, but the proponents of making national information accounts point to its utility, firstly, through the better grasp we will have on the pattern of growth in information activities and, secondly, through new time series relevant to policy matters of pressing importance, such as the over-all productivity of information work.

While the concept of national information accounts can be appealing, there remain some questions about the precise economic content of Porat's methodology.

One can take inputs and outputs, as does Porat, identify them as information goods and services or activities, and announce that a certain fraction of GDP comprises economic activities which are related to the production of information. As can easily be imagined, the size of this fraction is highly sensitive to how large or small is the catalogue of commodities and activities titled information. Such a catalogue, it has to be admitted, would be exceedingly arbitrary and

and must reflect to an uncomfortable degree the judgement of the compilers. The paramount question would naturally be, does any such catalogue make economic sense?

Let me illustrate the point. A motley collection of goods and services is endowed with the collective appellation "information" and is shown to comprise a larger and larger proportion of GDP through time. Likewise, more employment is directed to "information" production. Such secular trends are no doubt revealing in terms of the structural changes that are taking place in the economy, but quite what it is that is experiencing growth pains is hard to tell. The part of the 20th century we have survived so far has witnessed dramatic growth from zero in such things as automobiles, iron and steel production, aeroplanes, etc. and this phenomenon shows in a secular rise of manufacturing production. Manufacturing as an economic process -- the factory system -- is reasonably distinct from agriculture and the tertiary sector; and if not wholly by the methods of production, then possibly by its marketed products. Also we think, perhaps erroneously, we have a fair grip on the underlying demand and the supply schedule for manufactured goods. Can the same be said for the "information sector"?<sup>3</sup> What indeed is so special about this collection of goods and services that calls for a special treatment?

There is no quarrelling with the fact that the advanced industrial economies are undergoing some sort of decisive change and this change is linked with the processing and distribution of symbols, as opposed to concrete final goods and services. This is testified to by the rapid rise of computers and telecommunications, as well as the paper-shuffling bureaucracies both in

business and in government. Thus there is an outstanding phenomenon here which cries out to be researched and understood, otherwise much policy planning will go by the board. But to acknowledge this fact is not an automatic acceptance that the study of the "information sector" will be the key to a better understanding of future economic developments. Perhaps it is; but such a claim has to be substantiated before we embark on the production of national information accounts in a big way.

With this introduction we can motivate the present study. In economic analysis, when we group outputs or inputs we do so in the belief that the commodities comprising the aggregate exhibit some relationship to each other, either technologically or economically, which sets them apart from the remainder. We aggregate commodities in order to simplify our economic analysis but at the same time we endeavour to retain most that we see to be essential to the analysis.<sup>4</sup> This precaution requires that the components within an aggregate have a uniform response to prices of commodities outside the group. This similarity of response can be relatively weak or strong but it means in production that the degree of trade-off between goods and services within the group allowed by the technology is not, or at most only mildly, affected by the demand for those commodities not in the group. Under such conditions changes in the latter affect the demand for the aggregate components as if these formed a single entity; this is exactly the key property which allows the simplification by grouping in the first place. In view of these remarks the dichotomy between information and non-information activities from this economic

perspective has great merit if suitable conditions for this grouping are present. Otherwise, the benefit to be derived from such information accounts is not particularly evident to the economist.

To sum up, if the division between information and non-information is superfluous in the sense that no true aggregators as defined above exist, then economic studies with information at the focus may be empty, or worse, misleading. Without knowing that the structure of technology corresponds to this particular division of inputs, simulation studies and forecasts of information activities will be shrouded in unnecessary ambiguity since "information" would represent a mixed bag of goods and services rather than a relatively homogeneous commodity.

Productivity studies would suffer similarly, for the particular collective input factor called information whose productivity is under study would be, were the aggregation invalid, involved in the production of many types of output and these outputs would not be any less affected by other inputs. It follows again that a ratio of outputs to inputs, in spite of the common label of "information" for each, would be void of economic meaning.

The type of preliminary investigation the economist ought to carry out before setting up new accounts for use in economic studies should now be clear. The question to be answered is whether information goods and services constitute a true aggregate commodity. In technical terms, as we noted earlier, this question is one concerning the rate of substitution between the commodities constituting the aggregate, whether the aggregate in view is one consisting of factor inputs,

of outputs for final demand, or one of intermediate goods and services. Clearly the same question applies whether we are speaking of the macro-economy or of one specific sector. In this study we limit our analysis to one sector, examining the issue for labour and capital inputs into manufacturing, for which the production function approach to the specification of technology can be useful.

## 2. Production

The firms which comprise the private business sector of the economy are not exclusively involved with the process of production. A good portion of the work engaged in by many firms includes such non-production activities as marketing, research and development, management and administration. These activities, pertaining to the gathering, processing and distribution of information internal to the firm, influence output and consequently the firm's profits. The basic productivity question which arises is if the high cost of these information activities outweighs, at the margin, the benefit they bring to profit. This, and other questions we raised earlier, can be treated in terms of finding evidence to show that the production technology does indeed distinguish between information and non-information activities, and that goods within the aggregate are more closely substitutable among themselves than with those outside the aggregate. This issue can be posed in terms of the neo-classical production function (or the cost function) for the firm or the industry, and then subjected to various statistical tests.

In the present study we estimate a production function for the Canadian manufacturing industry using econometric techniques and apply certain statistical tests for "separability" of the function. This test is in terms of the hypothesis that the production function is best explained statistically by the separation of inputs as described above. The existence of such separability will support the view that the production technology inputs information activities separately from the inputs that go directly into the production process.

But it is not only as a test of the information sector concept that we undertake this research. More importantly, we wish to achieve a better understanding of the role of information activities in production. The results of this study will be an early step in this direction, if only a small one. This other objective is to examine the impact of price changes in information capital on the distribution of employment among information and non-information workers. Such an employment impact study may point to a methodology which can add to our understanding of the differential effects on employment produced by new technology.

The technology of a firm or industry can be described for econometric purposes by a production function, or equivalently, a cost function. The production function relates the maximum output obtainable under a fixed technology for given quantities of inputs;<sup>5</sup> the cost function relates the least cost bundle of inputs to the total cost of producing a given quantity of output. Here we are referring to a single output and by technology we denote the technical constraints which rule the quantity of inputs and the proportions

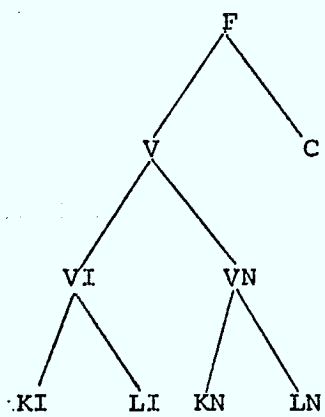
in which they can be used to produce a given output.

In fact we do not estimate the production function (cost function) directly but instead we estimate the factor demand equations. These requirements equations are derived from the production or cost function via the so-called first order efficiency conditions. The conditions themselves represent the efficient application of inputs to the production of a given output, at prices determined exogenously to the firm. The assumption here is that the firm is a price taker in all factor and goods markets and acts to maximize profits.<sup>6</sup>

As we hinted on earlier occasions, the hypothesis which is to be tested concerns the structure of manufacturing technology in Canada. The firm's value added<sup>7</sup> market output is seen to be the result of the co-operative effort of the primary inputs represented by a variety of activities carried out by labour and by capital services. The fact that there is no conceptual difficulty in accepting that productive activities stand in a different relation to the firm's output than do administrative activities makes it apparent there are distinctly different labour activities involved in manufacturing. The administrative work, or in our terminology, the information activities, have applications other than to the current production of physical output. By definition however, production is the exclusive concern of the production activities.

Because we are using the value added concept, the activities which enter the analysis exclude intermediate materials and energy, and involve only

the primary inputs, labour and capital services. Ideally the test for separability requires that the primary input be partitioned into information labour services (man-hours worked)  $HI$ , non-information labour services  $HN$ , information capital services  $KI$ , and non-information capital services  $KN$ . Our hypothesis of the process of production in manufacturing can be visualized through the simple tree diagram shown below:



In this diagram,  $F$  is the production function relating inputs to valued added output.  $V$  is a sub-production function nested within  $F$  along with  $C$ , and its appearance serves to express the supposed separation of total activities from construction. The functional arrangement in this fashion suggests that the rate of substitution between the two activities, information and non-information, are independent of the size of construction services.

The intermediate outputs denoted by  $VI$  and  $VN$  denote the contribution to final output of information activities and non-information activities respectively. This specification also alleges that the rate of substitution between capital and labour in the one activity is independent of the output



level of the other activity. This diagram illustrates the type of production specification we are testing. A rejection of the test (of the existence of VI , or both VI and VN) would suggest that the organization of the primary inputs into the information and non-information aggregates is spurious and does not mirror the real manufacturing process.

However, because of severe data limitations there are some problems in achieving this goal, one of which is the total lack of any statistical guide on how to assign the construction component of fixed capital formation to the two aggregate:

The parts of a building housing office staff and machines together with the labour and machines on the shop floor cannot in any obvious way be assigned to the titles information; and non-information; The existence of this fact in the data likely reflects a basic real phenomenon whereby the proportions in which the two types of activities take place inside the structure does not count for much, only the over-all size of the building. Consequently, in the following discussion KI , KN refer only to the machinery and equipment (M&E) component of capital services and not to construction (the inventory component is excluded throughout).

The second problem is the difficulty in making the correct assignment of machinery and equipment services to information and other activities. Not only is the data construction necessary to partition M&E capital stocks between the two activities a long and difficult task, but the practical assignment of capital services is full of ambiguities. One way to avoid

the difficulty is simply to leave the capital input undifferentiated, and in fact because of these concerns we make two distinct attacks on the separation problem. The first side-steps the issue of assigning capital services altogether by disaggregating labour services into two components but leaves capital in the aggregate form of machinery and equipment on the one hand and construction on the other. While of course this limited approach would not provide the full basis for a study of information inputs, since capital remains undifferentiated, it nonetheless allows us to discuss the important subject of information labour productivity, one of the major policy items under consideration. The capital attribution problem is encountered in the second attack, when both capital and labour are disaggregated appropriately. In both approaches we still have to confront the usual problem of finding a suitable statistical measure for capital utilization, by which services may be related to stock figures. We will discuss this subject further in the sections describing the estimation methods.

In order to carry out the tests described above it is necessary to specify a functional form which imposes no separability restrictions a priori. In this paper, we use the Transcendental Logarithmic Production Function (translog) to investigate the separability of information and non-information inputs into production in Canadian manufacturing.<sup>8</sup>

### 3. Substitution Elasticities and Production Functions

In the early 1970s a notable contribution was made in empirical production function analysis with the introduction of the "flexible form"

specification for production technology. The new functional forms, such as the translog<sup>9</sup> or the generalized Leontief<sup>10</sup>, prove to be particularly useful when considering more than two factor inputs to the production process.

The standard neo-classical theory of production posits certain well known properties for mathematical functions which are designed to describe the process of production. The flow of services from factors such as labour, materials, energy and capital, can be combined in various proportions to produce the flows of manufactured outputs, however only a restricted set of possible combinations are economically viable in the sense of not being wasteful of resources. This fact plus assumptions concerning divisibility of inputs and outputs gave rise to the characteristic properties of convex isoquants. Further assumptions yielded the homothetic property and the neo-classical production function can therefore be characterized as a "strictly quasi-concave homothetic function". While that description gave considerable scope, the major examples of production functions used in empirical analysis, the well-known Cobb-Douglas<sup>11</sup> and the CES\*<sup>12</sup> forms, are nevertheless unduly restrictive for purposes of hypothesis testing, especially when more than two factors of production are involved.

Both of these types of functional forms have built into them the assumption that the technical elasticity of substitution between factors is constant and, in the particular case of the Cobb-Douglas form the constant is constrained to equal unity. When only two factors are present, the CES form, unlike the Cobb-Douglas, does allow one to estimate the one elasticity of

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\*constant elasticity of substitution

substitution present but with three or more factors there are several partial elasticities to consider, and to assume them all equal is to miss some important economic phenomena. For example, there is evidence to show that non-production workers are complementary to capital while production workers are substitutable with both capital and other workers<sup>13</sup>. This observation has important employment implications, since a change in price of capital services can lead to further employment of one type of labour but, concurrently, to lay-offs for the other. The CES form lacks flexibility to allow the investigator to perceive the sign or size differences between the three partial elasticities that were examined in the above study, and other more suitable functional forms were introduced for the task, such as the flexible forms mentioned above.

Because of our concern with the price responsiveness of employment and the issue of input aggregation, this will determine our choice of available functional forms. In fact, for reasons to become apparent later, we use the translog function. This function, containing the Cobb-Douglas form as a special case, has the general form where the exponential logarithm of output is equal to a second degree polynomial in the logarithm of the inputs. This form gives a (second order) approximation to an arbitrary production function with the well-behaved properties mentioned earlier.

### 3. THE DATA AND ASSUMPTIONS

#### 1. Employment and Output in Canadian Manufacturing

Manufacturing employment in Canada over the past 3 decades underwent many developments. Aside from periodic movements attributable to business cycles, this evolution in work emerged mostly as a response to changes that took place in techniques of production, in products and in industry composition. Very briefly, we review the main events reflected in those employment time series which enter our empirical analysis, and start with trends in employment and real output for the major industry groups over the period 1961-1974.

In this period the Food and Beverage industry experienced a 50% increase in output while production employment increased only 10%. Meanwhile, the employment of information workers declined over the 1970s following a slight growth in the 1960s to produce for the full period a 5% fall in employment.

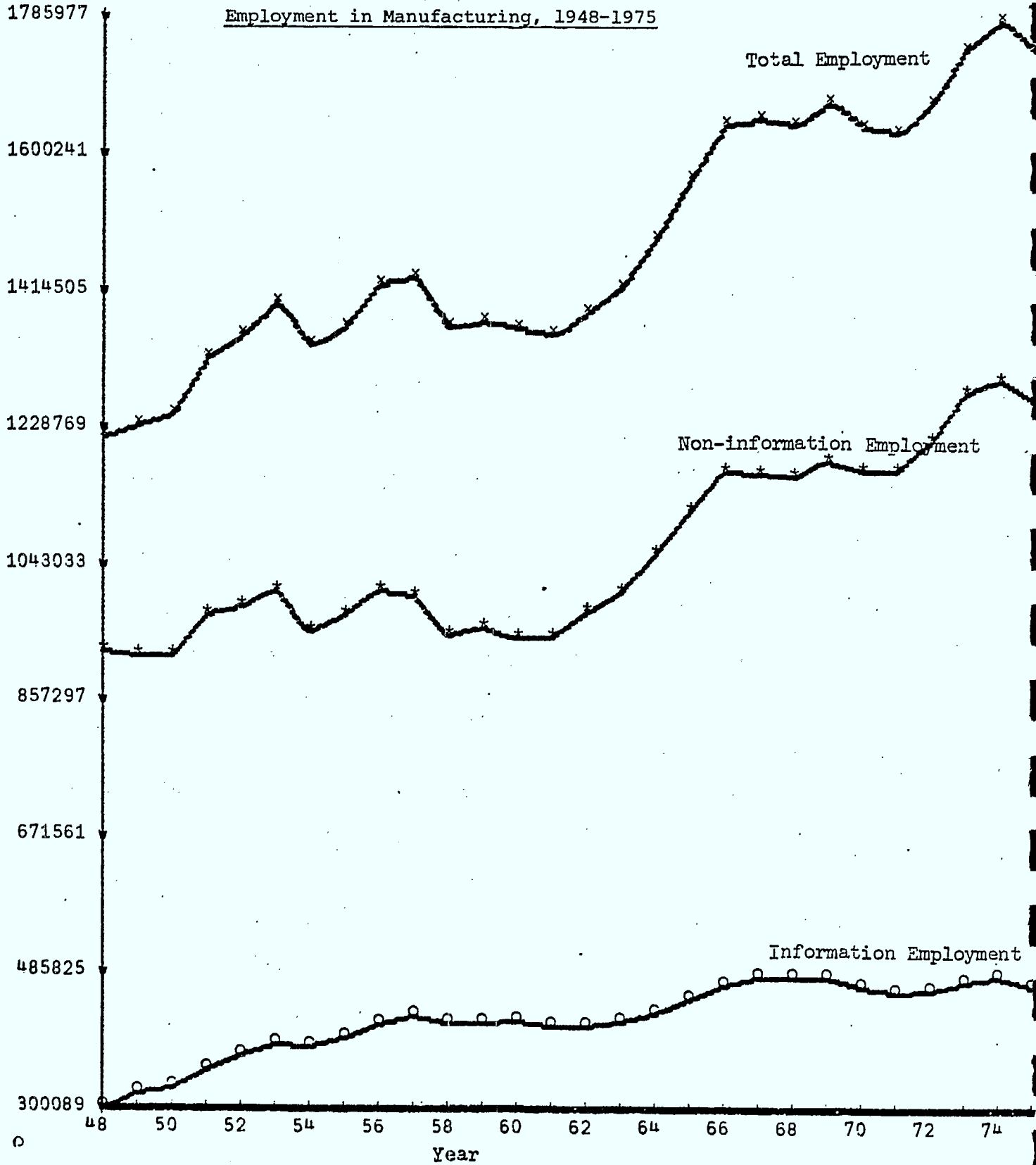
Paper and Allied Industries have had good growth over the 15 years, producing more than a doubling of output. Employment of production workers prospered with increasing production, but grew at a slower pace so that it rose only 30%. By contrast, information employment gained over 40% with growth levelling off in 1974 to produce in that year a 9% increase.

Many of the other industries followed a similar pattern of production employment, showing sensitivity to the business cycle in varying degrees. Information employment for all industries showed a great deal of cyclical insensitivity. But the 1970s proved a watershed in information employment

2,000,000

FIGURE 1

Employment in Manufacturing, 1948-1975



-Smaller...

for some industries when employment growth either stopped or turned down. A curious exception to the general pattern is provided by the Petroleum and Coal Products industry, in which production accelerated at an unusual rate to yield nearly a 240% increase. In 1974 production worker employment increased 12%, only to return to the 1961 level, and information employment by 4% for a 12% increase over the 1961 level.

The over-all employment picture for total Canadian manufacturing can be viewed from Figure 1. Over the period of the 1950s information employment grew significantly while production employment fluctuated widely about a barely perceptible trend line. This was an epoch of strong growth in manufacturing and rapid technological change. These improvements in techniques meant that the increase in output was accomplished with much less than a proportionate increase in the production work force. At the same time, the greater complexity of co-ordinating and controlling the larger production and market operations, the imperative of skillful planning behind the allocation of enormous investment funds marshalled for future production, the drive for research and development, all gave impetus to the growth in information employment.

The beginning of an end to the "long boom" of the 1950s and the 1960s can be perceived very clearly in the employment trends. The growth in both types of labour was severely hurt by the onset of the world-wide turndown, and information employment never recovered its momentum, even during the 1973-74 improvement in the domestic economy which was responsible for the latest surge in production employment.

More than one credible hypothesis can be advanced for the singular behaviour of information employment through these years. To begin with, it seems that the notable lack of cyclical sensitivity can be attributed to the relative inflexibility of the information factor input. In the initial stages of the "long boom" the manufacturing sector expanded its information activities capacity to produce an infrastructure possibly characterized by important indivisibilities. Once installed, this information infrastructure could perhaps handle the related administrative, R & D and management requirements over a substantial range of production levels. However, with the continued growth in manufacturing production, further additions to the information work force were called for in the middle 1960s. By contrast, subsequent years are marked by no growth or even contraction in information work. In the absence of a more detailed examination of the data the following explanation for this event is tentatively offered.

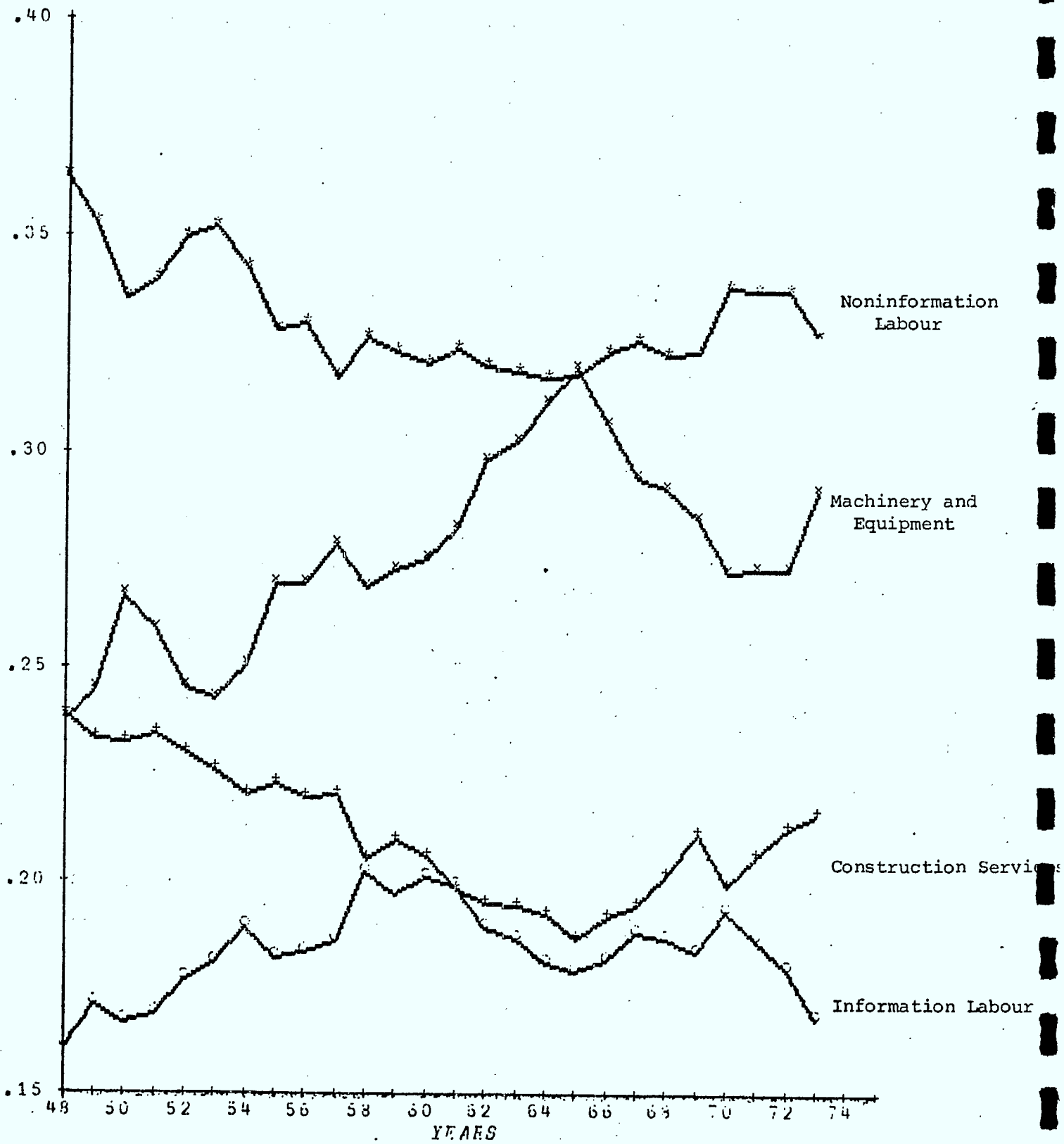
The introduction of computers into information processing activities was initially accompanied by net additions to the work force because of the new demand it brought for technicians, programmers and computer operators. Only later, once the computer penetrated every aspect of the labour-intensive information work, did the employment generating effect of computers reverse itself and become potentially a job displacement effect. This turnaround was possibly accelerated by successful drives to organize the white collar work force, in both the private and public sectors and throughout all industries



across the country, giving rise to strong wage demands in manufacturing. We see in the graphs of factors shares (Figure 2) that wage gains for information workers in this later period were, however, more illusory than real because of the effects of inflation, as shown indirectly by the falling share of costs spent on information work (others costs have risen faster), or because of job losses. It seems certain that both job displacement and inflation have held information labour costs severely in check in the first half of the present decade. At least for the manufacturing sector there does not appear to be any tendency for run-away information work costs. But before any conclusions can be drawn it would be important to determine three additional factors; the costs attributable to information capital; how much new information cost are borne by the public; and what costs are disguised by the value-added output data.

The situation in Canada is made particularly unusual by the deep penetration of foreign multinationals into the economy. The sway parent companies have over their Canadian branch-plants means employment and output decisions are formed by influences not exclusively associated with developments in the country's commercial markets. The importance of global corporate concerns to the operations of foreign controlled firms in Canada is hard to quantify, but case studies<sup>14</sup> undertaken by the Science Council of Canada are illuminating in their suggestion that forces outside the domestic economy play a powerful role. The clearest indication of this is given by the auto production rationalization agreement between Canada and the United States. According to the Science Council many

Figure 2: Factor Shares 1948-1973



managerial functions were eliminated following the auto pact as were research and development activities, despite increases in output and production work. The implication of this development for our empirical study based on statistics embracing this period is that a segment of Canadian manufacturing enacted an employment policy at variance with considerations of relative factor prices and the end product price.

The opposite effect on employment took place in the telecommunications industry, again for reasons exogenous to the Canadian economy. As a consequence of the 1956 United States anti-trust action taken against Western Electric, Canada rapidly acquired a mature telecommunication equipment manufacturing industry in the corporate form of Norther Electric. In terms of design capacity, 90 per cent of designs originated from a foreign source in 1960. By 1970 this preponderance of foreign source was reduced to 1 per cent. Over the period professional and non-professional staff grew 130 per cent while the growth in the former's quality was even more impressive.

These examples illustrate the importance foreign multi-national corporations have for the size of Canadian information employment. Work normally carried out in head offices and research laboratories are often conducted outside the country's border; table 1 below corroborates this observation. To the extent that management and research and development services are transferred across the border within the corporation the statistics on information work is biased downward. Likewise, any changes in employment over our sample period that are attributable to

Table 1

Table 1 - A Comparison of R & D per \$1 000 Sales in Canada and the United States: Manufacturing Industries by Sector, 1967

Sector	Canada			United States		
	Value <sup>a</sup> of Shipments (Sales)	Current <sup>b</sup> Intramural R & D Expenditures	R & D Expenditures per \$1 000 Sales	Value <sup>c</sup> of Shipments (Sales)	R & D <sup>d</sup> Expenditures	R & D Expenditures per \$1 000 Sales
	\$ million	\$ thousands	dollars	\$ million	\$ million	dollars
1. Food & kindred products	7 429.27	7 807	1.051	82 935	122	1.471
2. Tobacco products	493.26	m		4 957	m	
3. Textile mill products	1 404.939	3 700	2.634			
4. Knitting mills	325.543	m		19 767		
5. Apparel & related products	1 176.755	m		20 750	39	.963
6. Lumber & wood products	1 675.642	856	.511	10 875		
7. Furniture & fixtures	640.196	157	.245	7 634	11	.594
8. Paper & allied products	3 231.176	18 519	5.731	20 927	74	3.536
9. Printing, publishing & allied industries	1 297.275	m		21 677	m	
10. Chemicals & allied products	2 268.769	41 095	18.113	42 188	1 113	26.382
11. Petroleum & coal products	1 558.207	16 629	10.672	21 967	314	14.294
12. Rubber & plastics products	584.357	3 543	6.063	12 362	140	11.325
13. Leather & leather products	369.115	m		5 146	m	
14. Non-metallic mineral products	1 082.213	2 711	2.505	14 569	112	7.688
15. Primary metal industries	3 052.537	20 000	6.552	47 023	181	3.849
16. Metal fabricating industries	2 732.066	4 488	1.643	33 191	124	3.736
17. Machinery industries	1 516.875	13 062	8.611	49 077	1 033	21.049
18. Electrical products industry	2 312.519	83 261	36.004	43 606	2 755	63.179
19. Transportation equipment industries	4 720.876	43 161	9.143	70 539	4 421	62.675
20. Miscellaneous manufacturing (includes instruments & rel. prod.)	1 083.797	11 591*	2.442	26 673	407*	6.963
Totals	38 955.389	270 580	6.946	555 863	10 846	19.512

\*In addition to R & D expenditures in the miscellaneous manufacturing sector, these totals include R & D expenditures in all the above sectors which are denoted with an "m". Hence the ratios "R & D expenditures per \$1 000 sales" have as their base, the sales of these sectors as well as the sales of the miscellaneous sector.

<sup>a</sup>DUS, 1967 Annual Census of Manufacturers, Preliminary Bulletin, No. 31-208P, Table 2, pages 3-4. Value of shipments of goods of own manufacturers has been used proxy for sales.

<sup>b</sup>DBS, Industrial Research & Development Expenditures in Canada, 1967, No. 13-532, Table 4, page 31.

<sup>c</sup>U.S. Department of Commerce, Bureau of the Census, Statistical Abstract of the United States, 1969, Table 1109, pages 716-721. Value of shipments has been used as a proxy for sales.

<sup>d</sup>NSF, Research & Development in Industry, 1967, No. 69-28, Table 22, page 44.

Notes: U.S. figures for R & D include essentially the same costs as are included in the Canadian figures for current intramural expenditures. The only difference in the sets of figures is that the U.S. figures include depreciation and overhead, whereas the Canadian figures do not.

So that the Canadian and U.S. figures would be comparable, depreciation and overhead were abstracted from the U.S. figures. In Table 22 of the NSF publication, R & D costs are broken down into wages, materials and supplies, and other costs (see sample questionnaire on page 98 and explanation of questionnaire on pages 103 to 105). Therefore, the figures for the U.S. were arrived at by subtracting other costs from total R & D costs.

Source: Science Council, Special Study No. 22

these transfers or to trans-country corporate rationalization of production increases the amount of interfering 'noise' in our data and reduces the precision of our statistical tests.

2. A Review of the Assumptions

Computer service bureaus have grown rapidly in Canada since 1965, their output being services provided to other industries including the manufacturing industry. It is not known what proportion of manufacturing information costs is contracted out to service bureaus but whatever its importance, the cost does not appear in the value-added figures. As a result, the statistical data will tend to underestimate the amount of information input to manufacturing and this could account for a portion of the fall in the information work share shown by the data. In terms of the production function estimations this bias could possibly exaggerate the size of the other factors. Depending on the importance of this "value-added bias", the separability results can be affected by an unknown degree. Since the impact of this bias is likely to be pronounced, if at all, for only the latest few years, we assume the impact does not harm our conclusions of Chapter IV.

In the regression analysis we make three important assumptions regarding

the manufacturing sector needed to facilitate our research; namely, constant returns to scale, Hicks neutral technological change, and price taking behaviour.

Returns to scale is a concept which applies to the static production function independently of assumptions about technological progress. Under constant returns to scale and price taking behaviour (and free entry) no above-normal profits in the industry are possible in the long run. Clearly, then, the assumptions are very restrictive and have been subjected to much attack. The validity of the assumption of constant returns depends very much on the demand for manufactured goods being far in excess of the capacity of a few individual plants. For Canada, in steel and the automobile industry at the very least, this is far from the actual situation. Admittedly the foreign markets for Canadian manufacturing industry goods broadens the base of demand, but only for those specialized goods which are not imported in equal measure.

The obvious course in this exercise is to test if the production function exhibits any scale phenomenon, and if so, whether increasing or constant. At this stage, being short both of time and degrees of freedom for such a test, we take the modest view that constant returns to scale is the least harmful assumption pending further information. Yet clearly a more careful investigation of pricing behaviour and scale economies, and the impact that a misspecification of these may have on the regression results, is warranted.

Over the sample period much technological change has taken place, at

different pace in different industries. The assumption adopted for total Canadian manufacturing is that technical improvement is factor augmenting\* and at an equal rate for all factors. This again is a severe restriction, this time on the direction in which technical improvements take place, for we are saying that the productivity of all factors improved at the same rate. The analytical implication of having both the technical progress assumption and the constant returns to scale assumption is that all observations over inputs relate to a single isoquant. That is, according to these assumptions the observations in our data set, on quantity and prices of factors, reflect movements along a representative isoquant.

When many factors enter the analysis, the supposition of equal factor augmentation rapidly loses its appeal. We can justifiably feel uneasy about supposing the productivity of construction services to improve at the same rate as information work, for example.

There is no pretending that our estimates of production function parameters give a clear and accurate image of production. The pollution and gaps in the data as well as the restrictions we impose on the functional specification will ensure that the image will be very imperfect. However we are interested in the broader aspect of production techniques, separability, and of what can be learned from information accounts. Our polluted data is our approximation to such accounts for manufacturing. Our production specification with its confining assumptions will likely not be too dissimilar to what would be applied on "ideal" information accounts. Thus our final results may prove

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\*Factor augmentation refers to the change in efficiency with which inputs are able to produce a given output. Such improvements in efficiency can follow from improved machines, better education or skill levels, or improved management techniques and practices.

to resemble those obtained with the help of ideal accounts, since the analytical limitations will be common to both. Hence reservations about the applicability of, or the confidence to be had in, the results, must also apply to what can be expected from a like analysis from information accounts.

### 3. The Two Data Sets and Estimated Production Functions

We mentioned earlier of independent statistical estimations made on two production functions, one where machinery and equipment in undifferentiated as to information or noninformation use, and the other where this differentiation is made. The first stage, being easier in terms of data preparation, acted as a trial study which was undertaken prior to the difficult second estimation task. Time series observations over the period 1948-1973 on total Canadian manufacturing provided the data base for the first production function. The estimated parameters of this function gives us a rough average indication of the flexibility there existed in the manufacturing sector as a whole for shifting the mix of information and noninformation labour, M&E services and construction services in response to factor price changes. As we will see in the following chapter the flexibility is indeed there, but by virtue of the use of time series there is no telling if the substitution between inputs which actually took place did so partial at the same machines and within the same structures or wholly because of the introduction of new capital goods and new labour skills. The parameters are also silent on how long a time it takes to produce the adjustment of input employments planned by



<b>Major Group 1 - Food and Beverage Industries</b>		<b>Major Group 12 - Primary Metal Industries</b>	
101	Meat and Poultry Products Industries	291	Iron and Steel Mills
102	Fish Products Industry	292	Steel Pipe and Tube Mills
103	Fruit and Vegetable Processing Industries	294	Iron Foundries
104	Dairy Products Industry	295	Smelting and Refining
105	Flour and Breakfast Cereal Products Industry	296	Aluminum Rolling, Casting and Extruding
106	Feed Industry	297	Copper and Copper Alloy Rolling, Casting and Extruding
107	Bakery Products Industries	298	Metal Rolling, Casting and Extruding, n.e.s.
108	Miscellaneous Food Industries		
109	Beverage Industries	<b>Major Group 13 - Metal Fabricating Industries (except Machinery and Transportation Equipment Industries)</b>	
<b>Major Group 2 - Tobacco Products Industries</b>		301	Boiler and Plate Works
151	Leaf Tobacco Processors	302	Fabricated Structural Metal Industry
153	Tobacco Products Manufacturers	303	Ornamental and Architectural Metal Industry
<b>Major Group 3 - Rubber and Plastics Products Industries</b>		304	Metal Stamping, Pressing and Coating Industry
162	Rubber Products Industries	305	Wire and Wire Products Manufacturers
165	Plastics Fabricating Industry, n.e.s.	306	Hardware, Tool and Cutlery Manufacturers
<b>Major Group 4 - Leather Industries</b>		307	Heating Equipment Manufacturers
172	Leather Tanneries	308	Machine Shops
174	Shoe Factories	309	Miscellaneous Metal Fabricating Industries
175	Leather Glove Factories	<b>Major Group 14 - Machinery Industries (except Electrical Machinery)</b>	
179	Luggage, Handbag and Small Leather Goods Manufacturers	311	Agricultural Implement Industry
<b>Major Group 5 - Textile Industries</b>		315	Miscellaneous Machinery and Equipment Manufacturers
181	Cotton Yarn and Cloth Mills	316	Commercial Refrigeration and Air Conditioning Equipment Manufacturers
182	Wool Yarn and Cloth Mills	318	Office and Store Machinery Manufacturers
183	Man-made Fibre, Yarn and Cloth Mills	<b>Major Group 15 - Transportation Equipment Industries</b>	
184	Cordage and Twine Industry	321	Aircraft and Aircraft Parts Manufacturers
185	Felt and Fibre Processing Mills	323	Motor Vehicle Manufacturers
186	Carpet, Mat and Rug Industry	324	Truck Body and Trailer Manufacturers
187	Canvas Products, and Cotton and Jute Bags Industries	325	Motor Vehicle Parts and Accessories Manufacturers
188	Automobile Fabric Accessories Industry	326	Railroad Rolling Stock Industry
189	Miscellaneous Textile Industries	327	Shipbuilding and Repair
<b>Major Group 6 - Knitting Mills</b>		328	Boatbuilding and Repair
231	Hosiery Mills	329	Miscellaneous Vehicle Manufacturers
239	Knitting Mills (except Hosiery Mills)	<b>Major Group 16 - Electrical Products Industries</b>	
<b>Major Group 7 - Clothing Industries</b>		331	Manufacturers of Small Electrical Appliances
243	Men's Clothing Industries	332	Manufacturers of Major Appliances (Electric and Non-Electric)
244	Women's Clothing Industries	333	Manufacturers of Lighting Fixtures
245	Children's Clothing Industry	334	Manufacturers of Household Radio and Television Receivers
246	Fur Goods Industry	335	Communications Equipment Manufacturers
248	Foundation Garment Industry	336	Manufacturers of Electrical Industrial Equipment
249	Miscellaneous Clothing Industries	338	Manufacturers of Electric Wire and Cable
<b>Major Group 8 - Wood Industries</b>		339	Manufacturers of Miscellaneous Electrical Products
251	Sawmills, Planing Mills and Shingle Mills	<b>Major Group 17 - Non-Metallic Mineral Products Industries</b>	
252	Veneer and Plywood Mills	351	Clay Products Manufacturers
254	Sash, Door and Other Millwork Plants	352	Cement Manufacturers
256	Wooden Box Factories	353	Stone Products Manufacturers
258	Coffin and Casket Industry	354	Concrete Products Manufacturers
259	Miscellaneous Wood Industries	355	Ready-Mix Concrete Manufacturers
<b>Major Group 9 - Furniture and Fixture Industries</b>		356	Glass and Glass Products Manufacturers
261	Household Furniture Manufacturers	357	Abrasives Manufacturers
264	Office Furniture Manufacturers	358	Lime Manufacturers
266	Miscellaneous Furniture and Fixtures Manufacturers	359	Miscellaneous Non-Metallic Mineral Products Industries
268	Electric Lamp and Shade Manufacturers	<b>Major Group 18 - Petroleum and Coal Products Industries</b>	
<b>Major Group 10 - Paper and Allied Industries</b>		365	Petroleum Refineries
271	Pulp and Paper Mills	369	Miscellaneous Petroleum and Coal Products Industries
272	Asphalt Roofing Manufacturers	<b>Major Group 19 - Chemical and Chemical Products Industries</b>	
273	Paper Box and Bag Manufacturers	372	Manufacturers of Mixed Fertilizers
274	Miscellaneous Paper Converters	373	Manufacturers of Plastics and Synthetic Resins
<b>Major Group 11 - Printing, Publishing and Allied Industries</b>		374	Manufacturers of Pharmaceuticals and Medicines
286	Commercial Printing	375	Paint and Varnish Manufacturers
287	Platemaking, Typesetting and Trade Bindery Industry	376	Manufacturers of Soap and Cleaning Compounds
288	Publishing Only	377	Manufacturers of Tissue Preparations
289	Publishing and Printing	378	Manufacturers of Industrial Chemicals
		379	Miscellaneous Chemical Industries
		<b>Major Group 20 - Miscellaneous Manufacturing Industries</b>	
		391	Scientific and Professional Equipment Industries
		392	Jewellery and Silverware Industry
		393	Sporting Goods and Toy Industries

\* major groups 6 and 7 are combined

manufacturing firms following changes in wage rates, capital prices and rentals. A further result, on which we will report fully later, shows a separation between information and non-information labour as well as a different response in the employment of each to a change in the amount of M&E laid down and in place. However, we are unable to preceive the impact of changes in information capital using this approach, a task which required a suitable disaggregation of M&E. Encouraged by the results obtained using the time series data, we proceeded to the difficult preparation of information and noninformation capital data for the second part of this study.

Whereas the first stage used 26 annual observations to estimate a production function for total manufacturing in Canada, in the second stage we have a short time series for a cross-section of major manufacturing industry divisions. The 19 divisions are shown in figure 3, and for each we use annual observations covering the years 1967-1971. This data thus gives us  $19 \times 5 = 95$  observations with which to estimate a 'representative' major division in manufacturing.

This second approach offers us greater opportunities to study information activities in manufacturing but also brings additional problems. For example, by pooling cross-section and time series data we gain a wealth of observation points relative to before, but these points belong to very different industries rather than to some unobserved representative industry. A partial correction for the difference between industries has been made but some loss in statistical 'degrees of freedom' is incurred thereby. This discussion is presented in more

precise fashion in section 4. The reader less interested in the technical aspects of regression analysis can skip that section without loss of continuity.

#### 4. Theoretical Model and Stochastic Specification

Two production functions are estimated, one for total manufacturing and the other for a representative industry within manufacturing, but we use the same specification of technology in both instances. The functional form used to represent technical conditions of production has a single output expressed as an exponential function whose exponent is a second order polynomial in the logarithm of the input quantities. Under the assumption that technical change takes place at a constant rate which is the same for all production factors<sup>15</sup> we write

$$Y = AF(HI, HN, KI, KN, C)$$

where

Y = output

A = rate of factor augmentation

HI = information labour hours of employment

HN = noninformation labour hours of employment

KI = information machinery and equipment services

KN = noninformation machinery and equipment services

C = construction services

The translog function in both instances is taken to be an approximation to the true structure of production technology fitted at the mid-point of the observations.

The M&E components are merged into a single aggregate, denoted as E, in the total manufacturing production function where the time series data does not permit a separation into components. Write

$$\ln Y - \ln A = \ln F$$

where  $\ln$  refers to the exponential logarithm, and we specify

$$\ln F = \sum_i \alpha_i \ln x_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln x_i \ln x_j$$

The indices  $i$  and  $j$  run over the relevant set of inputs.

$$i, j = 1, 2, \dots, 4 \quad \text{for total manufacturing}$$

$$i, j = 1, 2, \dots, 5 \quad \text{for representative industry}$$

By virtue of the assumption of constant returns we have the following imposed conditions on the parameters:

$$\sum_i \alpha_i = 1 \quad \sum_i \gamma_{ij} = 0 \quad \text{for all } j.$$

To these are added the symmetry conditions

$$\gamma_{ij} = \gamma_{ji} \quad , \quad \text{all } i, j,$$

corresponding to the property

$$\frac{\partial}{\partial x_j} \left( \frac{\partial F}{\partial x_i} \right) = \frac{\partial}{\partial x_i} \left( \frac{\partial F}{\partial x_j} \right).$$

The first order efficiency conditions for profit maximizations under price taking behaviour are

$$\frac{\partial AF}{\partial x_i} = \frac{P_i}{P} \quad \text{all } i$$

where  $p_i$  are the factor service prices and  $p$  is the output price. Since

$$\frac{\partial \ln AF}{\partial \ln x_i} = \frac{\partial \ln F}{\partial \ln x_i}$$

we can write

$$\frac{p_i x_i}{PAF} = \frac{\partial \ln F}{\partial \ln x_i}, \quad \text{all } i.$$

The left hand side is the share of production costs taken by the factor  $x_i$ , denoted by  $mx_i$ . Thus we have the factor share equations complete with random disturbance terms,  $u_{i0}$

$$mx_{i0} = \alpha_i + \sum_j \gamma_{ij} \ln x_{j0} + u_{i0}, \quad \text{all } i; \quad 0 \text{ run over observations.}$$

Since  $\sum_i mx_i = 1$ ,  $\sum_i \alpha_i = 1$ ,  $\sum_i \gamma_{ij} = 0$ , each factor share equation can be expressed in terms of the others. Because of this, one equation is deleted from the estimation system in order to avoid the presence of a singular variance-covariance matrix. The random disturbances are assumed to be both contemporaneously and temporally uncorrelated.

We regard the autocorrelation problem in the time series estimation as the

most important and accordingly allow for first order autocorrelation by assuming the disturbances  $u_{it}$  are generated by the autoregressive scheme

$$u_{it} = \rho_i u_{it-1} + e_{it} \quad t = 2, \dots, T$$

where for each  $i$  the random variables  $e_{it}$  are assumed to be normally and independently distributed with zero mean and constant variance. In this specification there will be a single autocorrelation coefficient with each regression equation.

Because the estimation involving pooled data is based on observations of a cross-section of industries over a five year period we should make allowance for the fact that not all the 19 industries have identical technological structures. We assumed that these differences between the industries would principally show up in their factor shares and this feature is embodied in the factor share equations by allowing the intercept term to vary from industry to industry. In conformity with the 'covariance model' for pooled data the same dummy variables were added to each equation to represent the deviation of industry value shares from the 0.1 level. So as to maintain degrees of freedom industries were grouped for this purpose into 5 sections. For each of the 4 requirements equations we have

$$mx_{i, \ell t} = \alpha_i + \sum_{j=1}^4 \gamma_{ij} \ln x_{i; \ell t} + \sum_{\ell=1}^4 \delta_{\ell} D_{\ell} + u_{i, \ell t}$$

$$i = 1, 2, 3, 4$$

$x_i$  = quantity of factor  $i$

$t = 1967, \dots, 1971$

$mx_i$  = value share of  $x_i$

$i = 1, 2, 3, 4$

$t = 1967, \dots, 1971$

$D_{\ell}$  = dummy variable ( =1 for section  $\ell$ ; 0 otherwise).

The method of sectioning the industries consisted of examining the tracking of all four value shares by the equation system estimated without dummies. In this fashion out-lying industries were identified by the unusual gaps existing between the actual value share and the plotted one. Certain industries were conspicuous for the poor tracking performance over all value shares in all five years of observations. Industries were grouped according to the similarity of their deviation from the fitted shares obtained from the regressions. The actual and fitted plots of value shares appear in the appendices.

The parameter estimates were obtained in two ways. One method consisted of maximizing the concentrated likelihood function with respect to the coefficients  $\alpha_i$ ,  $\gamma_{ij}$  and  $\delta_{\ell}$  taking into account the symmetry restrictions, and, when included, other constraints imposed by separability conditions. Computations were carried out using the full information maximum likelihood program in the Time Series Processing package. The other method involved the 'seemingly unrelated equations' approach available in the LSQ program of the T.S.P. package. Both methods give almost identical results, and hence only the latter are reported.

One final subject that remains to be raised concerns the rate at which factor inputs are utilized. The flow of services taken from a given stock of any input can vary from year to year and from one industry to the next. The work force, for example, due to strikes and lockouts, equipment breakdown, etc., can be employed but idle, and not yielding any services. At the other extreme, in the upswing of a production cycle, a given work force may be utilized relatively more intensively through overtime work or piece work. Capital obviously experiences fluctuating intensities of use owing to the fixed nature of plant and machinery and equipment. Again during periods of strike or lockouts, capital lies idle and no services are drawn from the stock although it continues to be an expense and suffer from deterioration. At other times machinery and plant can be worked at full intensity, dangerously near the point of breakdown. We see therefore there exists in practice a wide range of rates over which services are drawn from stock, rates which are capable of large fluctuations. The more fixed factor, capital in particular, will experience greater changes in the rate of utilization while the variable factor, most particularly ununionized workers without special or scarce skills, will be utilized at a more constant rate as hiring and layoffs may occur according to the movements of business demand. Salaried workers and those protected by union contracts most likely show characteristics of a 'quasi-fixed' factor, which is far from being perfectly variable but does not take as long as does capital to



adjust to the optimum level following changes in demand.

However we have no empirical measure for any under-utilization of labour. We can only assume that manhours worked is a reasonable direct measure of labour services, and thus the question of a measure for the utilization rate does not arise. On the other hand, we do not have a direct measure of capital services and we consequently made the assumption that services are proportional to the level of stocks. Clearly, in light of our discussion on fluctuating utilization rates, this is a pretty rough assumption to maintain. Fortunately we do have measures (quarterly by industry) of capacity utilization,  $u$ , and this is taken as proportional to the rate at which each unit of capital stock is utilized. The rate of utilization is assumed to apply to all capital commodities equally so that capital services are proportional to  $u_{jt} K_{ijt}$ , where  $u_{jt}$  is the capacity utilization rate for industry  $j$  in year  $t$  and  $K_{ijt}$  is the  $i$ th capital commodity type (construction, information M & E, noninformation M & E) employed in industry  $j$  in year  $t$ .

Finally, if we supposed it were appropriate to adjust all factors services in each industry by the same capacity utilization measure then in practice no adjustment need be made. This is so by virtue of the constant returns to scale assumption. Thus regression estimates based on data untreated for utilization rates correspond to the above supposition and does not preclude variable rates of utilization of inputs.

In the time series regression for total manufacturing the latter assumption is maintained (owing to insufficient number of

years of observation on utilization rates). In the representative industry approach capital services are adjusted by utilization rates as this gave a better fit than did the regression without such adjustment.

#### 4. INDICATED RESULTS AND CONCLUSIONS

##### 1. Total Manufacturing Estimates

The aggregate production structure for total manufacturing in Canada in terms of four inputs - information labour, non-information labour, machinery and equipment and construction - is satisfactorily explained by a translog specification fitted over the 26 data observations ( Table 2 ). All parameters show numerical values significantly different from zero as is indicated in Table 3. This result is conditional on two significant assumptions; constant return to scale and technical change with equal factor augmentation, in which all four factors improve their respective productivity equally over the years.

The production structure does not simplify in the sense that some factors of production do not affect the share of some other input. All factors play a role in determining the share of revenue spent on each factor. This conclusion that no  $\gamma_{ij}$  coefficients vanish is statistically tested through the maximum likelihood technique. Also the graphs of the actual and estimated cost shares under the several hypotheses that some of all such cross terms are zero suggest that all the hypotheses should indeed be rejected. Moreover, the point estimates of the substitution elasticities are not inconsistent with this conclusion.

The separation of various combinations of inputs from other combinations has also been tested, the one of leading interest being, of course, the separation of information labour from other inputs. No such separation was found to exist, nor any other possible separation. In particular there is no support for the

Table 2 Nonseparable Translog Function Total Manufacturing

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Information Labour Share  $MHI = \alpha_I + \gamma_{II} \ln\left(\frac{HN}{C}\right) + \gamma_{IN} \ln\left(\frac{LN}{C}\right) + \gamma_{IE} \ln\left(\frac{E}{C}\right)$

R - Squared = .7699

Durbin - Watson Statistic = 1.0423

Sum of Squared Residuals = .000631

---

Noninformation Labour Share  $MHN = \alpha_N + \gamma_{IN} \ln\left(\frac{HI}{C}\right) + \gamma_{NN} \ln\left(\frac{HN}{C}\right) + \gamma_{NE} \ln\left(\frac{E}{C}\right)$

R - Squared = .6581

Durbin - Watson Statistic = 1.2655

Sum of Square Residuals = .001408

---

M&E Share  $ME = \alpha_E + \gamma_{IE} \ln\left(\frac{HI}{C}\right) + \gamma_{NE} \ln\left(\frac{HN}{C}\right) + \gamma_{EE} \ln\left(\frac{E}{C}\right)$

R - Squared = .5597

Durbin - Watson Statistic = .4756

Sum of Squared Residuals = .005252

---

Log of Likelihood function = 318.5

The above equations are associated with the following production function:

$$2\ln F = \alpha_I \ln HI + \alpha_N \ln HN + \alpha_E \ln E + \alpha_C \ln C + \gamma_{II} (\ln HI)^2 + \gamma_{NN} (\ln HN)^2 + \gamma_{EE} (\ln E)^2 + \gamma_{CC} (\ln C)^2 + 2\ln HI (\gamma_{IN} \ln HN + \gamma_{IE} \ln E + \gamma_{IC} \ln C) + 2\ln HN (\gamma_{NE} \ln E + \gamma_{NC} \ln C) + 2\gamma_{EC} \ln E \ln C$$

Table 3

Table 3 Parameter Estimates for Nonseparable Translog Function:  
Total Manufacturing

Parameters	Estimates	Standard Error $\times 10^{-2}$	t-statistic
$\alpha_I$	.1936	.087	222.2
$\alpha_N$	.3183	.150	212.5
$\alpha_E$	.2746	.295	93.2
$\alpha_C$	.2135	.202	105.5
$\gamma_{II}$	.1432	.340	42.2
$\gamma_{IN}$	-.1532	.382	- 40.1
$\gamma_{IE}$	-.0392	.645	- 6.1
$\gamma_{IC}$	.0492	.952	5.2
$\gamma_{NN}$	.1752	.512	34.2
$\gamma_{NE}$	.0429	.715	6.0
$\gamma_{NC}$	-.0649	.883	- 7.4
$\gamma_{EE}$	.1726	1.94	8.9
$\gamma_{EC}$	-.1763	.622	- 6.7
$\gamma_{CC}$	.1920	4.05	4.7

usual practice of grouping labour into a single aggregate and grouping capital into a second aggregate. The parameter constraints associated with each test and the log of the likelihood function can be found in table 4. The level of disaggregation adopted in this work is likely the least necessary for studies of manufacturing in which the structural form of the production function is important as in, for example, simulation studies. In the appendix it is argued that even this degree of disaggregation is too meager for a conclusive study and, as suggested in the introduction, machinery and equipment should be subdivided into information and non-information components.

The production function is well behaved for all observation years for it possesses the important properties of monotonicity, that is, positive marginal productivity for all inputs, and convexity. Strong necessary conditions for the latter property are negative values for all own substitution elasticities and it is found that these conditions do in fact hold over all years, for all own elasticities. Thus, fittingly, the demand schedule for each factor of production is downward sloping.

## 2. Elasticities and Responses to Price Changes

The Allen elasticity of substitution (AES) offers a measure of the technical trade-off or substitution between input services for a fixed level of output. These estimates provide an indication of the direction in which factor demand will be affected by relative factor price changes under fixed output. Unfortunately we do not possess direct AES estimates but the elasticities for the midpoint of the series can be calculated using the

Table 4. Testing for Separability: Total Manufacturing

The hypothesized aggregation of inputs is denoted by parentheses which group the symbols denoting those inputs belonging to the aggregate. Thus [(HI, HN), E, C] signifies the separation of labour from the capital inputs, i.e. the aggregation of labour.

Type of separation (log of likelihood)	Description	Nonlinear constraints on parameters
1. [(HI,HN,E), C] ( 279.3 )	Construction services separate from other inputs	$\gamma_{II} = \frac{\alpha_I}{\alpha_N} (\gamma_{IN} + \gamma_{NN} + \gamma_{NE}) - \gamma_{IN} - \gamma_{IE}$ $\gamma_{EE} = \frac{\alpha_E}{\alpha_N} (\gamma_{LN} + \gamma_{NN} + \gamma_{NE}) - \gamma_{IE} - \gamma_{NE}$
2. [(HN,E),HI,C] ( 269.5 )	Production activities separate from others	$\gamma_{IE} = \frac{\alpha_E}{\alpha_N} \gamma_{IN}$ $\gamma_{EE} = \frac{\alpha_E}{\alpha_N} (\gamma_{NN} + \gamma_{NE}) - \gamma_{NE}$
3. [(HI,HN), (E,C)] ( 299.0 )	aggregate labour-aggregate capital	$\gamma_{NE} = \frac{\alpha_N}{\alpha_I} \gamma_{IE}$ $-\gamma_{II} = \gamma_{IN} + \frac{\gamma_{IE}}{\alpha_E} (1 - \alpha_I - \alpha_N)$ $-\gamma_{NN} = \gamma_{IN} + \frac{\gamma_{IE} \alpha_N}{\alpha_I \alpha_E} (1 - \alpha_I - \alpha_N)$
4. [HI, (HN,E,C)] ( 260.1 )	Information labour separated	$\gamma_{IE} = \frac{\alpha_E}{\alpha_N} \gamma_{IN}$ $-\gamma_{II} = \frac{\gamma_{IN}}{\alpha_N} (1 - \alpha_I)$
5. [(HI,C),HN,E] ( 268.9 )	Information labour and construction services separate from other inputs	$-\gamma_{NN} = \gamma_{NE} + \frac{\gamma_{IN}}{\alpha_I} (1 - \alpha_N - \alpha_E)$ $-\gamma_{EE} = \gamma_{NE} + \frac{\gamma_{IE}}{\alpha_I} (1 - \alpha_N - \alpha_E)$

Table 4. (continued)

6. [(HI,C), (HN,E)]  
( 260.0 )

aggregate information work  
with construction v. production  
activity

$$\gamma_{IN} = \frac{\alpha_N}{\alpha_E} \gamma_{IE}$$

$$-\gamma_{NN} = \gamma_{NE} - \frac{\alpha_N}{\alpha_E} (\gamma_{NE} + \gamma_{EE})$$

$$-\alpha_I = \frac{\gamma_{IE}(1 - \alpha_N - \alpha_E)}{\gamma_{NE} + \gamma_{EE}}$$

The test is formulated as follows: The null hypothesis corresponds to one of the separability situations above, while the alternative hypothesis in every case corresponds to the unconstrained equation system. It is a well known statistical fact that for large samples, minus twice the log of the ratio of likelihood functions (associated with the null and alternative hypotheses) is distributed as  $\chi^2$  with degrees of freedom equal to the number of constraints. Thus our test consists of finding if the calculated  $\chi^2$  lie in the critical region outside the .95 probability interval. If so, the null hypothesis is rejected; otherwise it is accepted.

The separability constraints apply to a production structure for which the translog function is an approximation located about the 1961 observation.



parameter estimates and values of relative shares in costs; the details of the derivation are given in part II.

The values for the Allen elasticities of substitution are given in Table 5. In fact, they all appear rather large in light of other studies. One point of agreement with similar studies is the substantial degree of substitutability in Canadian and U.S. manufacturing between production and non-production workers, which are close to our two types of labour, non-information and information workers respectively. While it is true our data base is somewhat different, the uniformity in the studies indicate an important structural characteristic in manufacturing processes. However, with respect to the sign of the elasticities between the labour types and capital, our time series results do not agree with the U.S. studies, although the pooled data estimates do have this agreement with the U.S. results.

The time series results tell us that a rise in the price of one type of labour will induce greater employment in the other type, with output constant. Much more interesting, however, is the impact on employment of a change in capital services price. With no change in output, a fall in the price of M&E services will have an adverse effect on non-information employment and at the same time will strengthen information employment. This differential effect of M&E on the two labour types forcefully indicates the inconsistency of aggregating labour, since the two labour components react diametrically opposite to changes in the M&E stock or prices. As the signs in the elasticities signify, the reaction of labour

TABLE 5

Allen Elasticities of Substitution

Total Canadian Manufacturing

1961

$\sigma_{II}$	-50.0
$\sigma_{IN}$	30.5
$\sigma_{IE}$	-28.8
$\sigma_{IC}$	37.0
$\sigma_{NN}$	-21.2
$\sigma_{NE}$	20.4
$\sigma_{NC}$	-22.3
$\sigma_{EE}$	-16.6
$\sigma_{EC}$	17.0
$\sigma_{CC}$	-22.2

I = information labour

N = non-information labour

E = machinery and equipment

C = construction

to construction services is the reverse to that for M&E. A fall in the price of construction services will affect the employment of the two labour types in the opposite direction to the effect produced by a fall in the service price of M&E.

Regarding the two components of capital we find them to be as highly substitutable as was found for the two types of labour. But because of their differential relationship with the two labour types a consistent capital aggregate may not exist, and hence their separation from the labour inputs cannot be held to be true, as we discovered through statistical tests.

The regression results reveal a particular technical relationship to hold between the various inputs, thus reflecting a specific structure of technology. This structure is in agreement with the following intuitive picture of the manufacturing process. Firstly, production is not in accordance with the fixed coefficient technology whereby invariably all factors are employed in a fixed ratio. According to our results, for each level of output machines can displace production workers as a consequence of factor price changes which makes machines relatively cheaper to use. On the other hand, it is true that with output increasing and prices in fixed ratio, employment will increase in a fixed ratio by virtue of our constant returns to scale assumption. However, the historical fact is that the factor price ratios have undergone significant changes over the observation period, and these developments must be taken into account in modelling production processes.

Secondly, more machines per unit of output appears to be accompanied by increased information employment. The reason for this phenomenon may lie in

the greater demand for professionals required to service and manage the machinery and equipment. The fact that the displacement of production workers implies less payroll administration does not seem to affect administrative work to the extent of offsetting the prevailing trend. We might guess from these remarks that personnel administration is relatively insensitive to fluctuations in the industry's work force but the employment patterns exhibited by the sub-aggregate industries within manufacturing vary to the extent it makes it hard to have confidence in any interpretations of direct causality underlying the aggregate complementarily - substitution effects.

The role of construction services appear rather surprising in this context. Perhaps it cost less to shelter and provide ancillary facilities for machines than for production worker, at least this is what we can gather from construction services' substitubility with M&E and its complementarily with noninformation or production workers. Also construction services substitutes for information labour, a fact which is hard to rationalize purely in causative terms. Maybe this is a phenomenon characteristic of the aggregate function and one that fails to emerge with the same consistency at the sub-aggregate level of the major industry groups. This is a point of comparison to which we shall return when discussing the results for the representative industry production function. It may also be due to the fact we were unable to assign construction to information and noninformation functions.

The outcome for the regression analysis is a well-behaved, constant returns to scale production function which shows no sign of a technology which separates inputs. The equivalent conclusion that every factor affects every other factor's cost share means, for one thing, that the productivity of information labour is responsive to the level of employment of other factors.

### 3. Information Worker's Productivity

In his work on the information economy, Porat presented the interesting conclusion that U.S. information work has experienced a steady secular decline in productivity since the 1930s. Using input-output tables, Porat made his analysis by aggregating all information outputs and all information inputs for each of the census years that the table was available. The ratio of the aggregate "real" output to the value of the secondary information input he called information productivity, and this was calculated for the years 1929-1974.

This measure is not entirely in keeping with the statistical tradition of measuring productivity. In Canada, labour productivity for each major industry group is published as the ratio of real value-added output to employment or to manhours worked, the second being recognized as a better alternative.

This readily understood and appealing measure has severe conceptual shortcomings, not least of which is its imprecision of meaning. The chief problem is that changes in the productivity measure could well be due to causes other than changes in the level of employment of the factor in question. Developments in capital, in the employment of a second category of labour, and changes in the overall production function will each affect the evolution of information labour productivity. Thus, the fact that more of one type of labour is being employed per unit of output does not necessarily draw one to the conclusion that each worker is contributing less to production. It can well

be that the greater employment of information work is the result of saving one the use of other factor inputs, perhaps because the productivity of the latter has fallen into relative decline! In terms of policy action, very little weight can be placed on an output to input ratio measure of productivity.

However, Porat's productivity measure has even less validity than that possessed by the real output to real input ratio. Despite the author's use of the term "real" output it is applied in a sense different from the usual economic meaning of a value deflated by an appropriate price index. As is evident from Porat's figure 9.3 and the related text, his productivity measure is in terms of dollar output to dollar input. The "real" aspect of GNP consists in removing the element of measured output attributed to "nonproductive" information work. In the appendix we present a procedure for extending the present research to a productivity study, but for the present let us translate the method used by Porat to our analysis.

Thus, in keeping with Porat's framework, "real output" is here defined as total value-added of manufacturing production in Canada, in our notation  $py$ , where  $p$  and  $y$  are respectively price and quantity indices of output, less expenditure on information labour  $p_I x_I$ , where  $p_I$  and  $x_I$  are respectively price and quantity indices of information labour.

Then in our notation,

$$\text{Productivity of } I = \frac{PY - P_I x_I}{P_I x_I}$$

or  $1 + \text{productivity of } I = \frac{1}{\text{MHI}}$ , where MHI is the share of information labour in cost.

By this definition, the information labour share in costs provides all information on the productivity of information I . We can therefore immediately conclude that the state of employment of the other factors will have a direct impact on the productivity level, as defined by Porat, since the cost share equations do not exclude the presence of any factor.

The graph in Figure 3 also does not bear out the same trend obtained by Porat. For manufacturing we find a general tendency for an improvement in information work productivity since the late 1950s and early 1960s. A bold interpretation of this trend could be to attribute this event to the introduction of computers. However, a more suitable measure of productivity is needed before we can draw any reasoned conclusions.

#### 4. The 'Representative' Sub-industry Production Function Estimates

In contrast to the time series study we now consider the effect of distinguishing between information capital and non-information capital. In this study of the 'representative' industry within total manufacturing we are enabled to examine if the structure of technology agrees with information accounting. With this object in view, we apply our regression estimates to the set of four hypotheses, namely, aggregation of information activities, aggregation of non-information activities, aggregation of labour and finally, aggregation of machinery and equipment. The first aggregation hypothesis deals with the information accounts question, as in a way does the second since it is complementary to the first test. The third and fourth tests, if affirmative would provide counter evidence to the view that manufacturing technology sharply separates information and non-information activities.

With only five years of observations available we increased the number of observations by taking a cross-section of 19 major industry groups within manufacturing. The resulting 95 observations are conceived as being observations on five years of production by a fictitious representative or average industry. Consequently the estimated parameter values do not describe the technology of any one of the 19 industries, but instead they offer a description of a fictitious average technology. The response to any policy or price change that may be simulated by way of such parameters again represent some average for the entire industry.

The 19 industries feature a wide variance in capital-labour ratios and production techniques. These differences in structure show in the variation in the factor shares from industry to industry. Ideally, these differences should not be too wide for regression purposes and an attempt has



been made to identify the worst cases of 'out-liers' and to compensate for them with dummies variables for the offending industry groups. Details of the procedure is given in the previous section, but it should be noted here that all regressions were repeated with the dummies absent as a check on the robustness of the separability tests. It was deemed important to undertake this check since the inclusion of dummies and their effect on the statistical results do not exactly correspond to the theoretical concerns which lead to their use. The dummies undoubtedly lead to better fits, as is testified by the larger values for the maximum likelihood function, but the procedure we use for their inclusion is unavoidably somewhat arbitrary by virtue of our choice of industry groupings. Several configurations of groups were tried and the one producing the best log likelihood figure was selected for reporting herein along with the regressions without dummy variables.

The estimations obtained for the 'representative' industry within manufacturing are given in the 'Value Share Equation Estimates' below where HI, HN, KI, KN, C refer to information labour, noninformation labour, information capital, noninformation capital and construction, respectively. The letter M refers to the marginal value share of revenue. Two sets of estimates have been obtained, to reflect different hypotheses regarding the rate of retirement of KI stocks, but only one is shown, corresponding to a uniform 10 year life. From the results we can see that any one factor of production experiences a fall in value share with an increase in employment of any other factor. All but two parameters have a numerical value significantly different from zero as can be inferred from the t-statistics in table 6. This indicates that the production structure belonging to the representative

industry for the manufacturing sector is satisfactorily explained by a translog specification fitted over 95 observations . The 19 industries comprising the cross-sectional observations are listed by title in figure 3 (page 29) together with the titles of their sub-components. Table 7 shows the factor share equation system, estimated by the method of Full Information Maximum Likelihood, as well as the equation statistics.

Figure 4

Value Share Equation Estimations (1967-1971)

Parameter values are given to 3 decimal places. The symmetric matrix below contains the estimates for the  $\gamma_{ij}$  parameters,  $i, j = 1, 2, 3, 4$ . The  $\gamma_{i5}$   $\gamma_{55}$  parameters can be derived using the conditions given in section 3.3. The vector of parameters contain the estimates for  $\alpha_i$ ,  $i = 1, \dots, 4$ .

Estimated KI lifetime equal to 10 years for all industries. Life times for KN are taken from S.C. estimates.

a. With 4 Dummies

$$\begin{bmatrix} \text{MHI} \\ \text{MHN} \\ \text{MKI} \\ \text{MKN} \end{bmatrix} = \begin{bmatrix} .300 \\ .279 \\ .128 \\ .156 \end{bmatrix} + \begin{bmatrix} +.115 & & & \\ & -.093 & & \\ & & +.119 & \\ & & & +.038 \\ & & & & -.036 \\ & & & & & +.138 \end{bmatrix} \begin{bmatrix} \ln \frac{\text{HI}}{\text{C}} \\ \ln \frac{\text{HN}}{\text{C}} \\ \ln \frac{\text{KI}}{\text{C}} \\ \ln \frac{\text{KN}}{\text{C}} \end{bmatrix}$$

All parameters are significantly different from zero at the 95% level, excepting  $\gamma_{23} + \gamma_{24}$ . The matrix is symmetric by virtue of the imposed symmetry assumption ( $\gamma_{ij} = \gamma_{ji}$ ).

b. Without Dummies

$$\begin{bmatrix} \text{MHI} \\ \text{MHN} \\ \text{MKI} \\ \text{MKN} \end{bmatrix} = \begin{bmatrix} .307 \\ .288 \\ .120 \\ .156 \end{bmatrix} + \begin{bmatrix} +.116 & & & \\ & -.094 & & \\ & & +.127 & \\ & & & +.033 \\ & & & & -.033 \\ & & & & & +.143 \end{bmatrix} \begin{bmatrix} \ln \frac{\text{HI}}{\text{C}} \\ \ln \frac{\text{HN}}{\text{C}} \\ \ln \frac{\text{KI}}{\text{C}} \\ \ln \frac{\text{KN}}{\text{C}} \end{bmatrix}$$

All parameters are significantly different from zero at the 95% level, excepting  $\gamma_{14} + \gamma_{23}$ .

Table 6 - Parameter Estimates for Nonseparable Translog Function:  
Representative Industry

a. With 4 Dummies

Parameters	Estimates	Standard Error x 10 <sup>-2</sup>	t-statistic
$\alpha_1$	.30014	.38797	77.4
$\alpha_2$	.27941	.69130	40.4
$\alpha_3$	.12795	.25129	50.9
$\alpha_4$	.15550	.77086	20.2
$\gamma_{11}$	.11467	.25035	45.8
$\gamma_{12}$	-.09304	.19901	46.7
$\gamma_{13}$	.00672	.12638	-5.32
$\gamma_{14}$	.00360	.23959	-1.50
$\gamma_{22}$	.11867	.33829	35.1
$\gamma_{23}$	-.00170	.09819	-1.73
$\gamma_{24}$	-.00266	.35398	-.751
$\gamma_{33}$	.03751	.10993	34.1
$\gamma_{34}$	-.03614	.16914	-21.4
$\gamma_{44}$	.13822	.49392	28.0

b. Without Dummies

$\alpha_1$	.30714	.64625	47.5
$\alpha_2$	.28820	1.0089	28.6
$\alpha_3$	.12005	.3625	33.1
$\alpha_4$	.15619	1.3524	11.5
$\gamma_{11}$	.11617	.37168	31.3
$\gamma_{12}$	-.093973	.30094	-31.1
$\gamma_{13}$	-.00664	.17454	-3.8

$\gamma_{14}$	-.00020	.37431	-.053477
$\gamma_{22}$	..12641	.43250	29.2
$\gamma_{23}$	-.00030	.13916	-.218
$\gamma_{24}$	-.01230	.50566	-2.43
$\gamma_{33}$	.03354	.14244	23.5
$\gamma_{34}$	-.03272	.24085	-13.6
$\gamma_{44}$	.14320	.82257	17.4

Table 7. Nonseparable Translog function: Representative Industry Factor

Share Equations

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Information Labour Share  $MHI = \alpha_1 + \gamma_{11} \ln \frac{HI}{C} + \gamma_{12} \ln \frac{HN}{C} + \gamma_{13} \ln \frac{KI}{C} + \gamma_{14} \ln \frac{KN}{C}$

WITH DUMMIES                      WITHOUT DUMMIES

Durbin = Watson Statistic =            .8658                      .5076

Sum of Squared Residuals =            .01482                      .04401

---

Noninformation Labour Share  $MHN = \alpha_2 + \gamma_{12} \ln \frac{HI}{C} + \gamma_{22} \ln \frac{HN}{C} + \gamma_{23} \ln \frac{KI}{C} + \gamma_{24} \ln \frac{KN}{C}$

Durbin - Watson Statistic =            .6423                      .4173

Sum of Squared Residuals =            .10713                      .19030

---

Information M&E Share  $MKI = \alpha_3 + \gamma_{13} \ln \frac{HI}{C} + \gamma_{23} \ln \frac{HN}{C} + \gamma_{33} \ln \frac{KI}{C} + \gamma_{34} \ln \frac{KN}{C}$

Durbin - Watson Statistic =            .9080                      .5269

Sum of Squared Residuals =            .00272                      .00671

---

Noninformation M&E Share  $MKN = \alpha_4 + \gamma_{14} \ln \frac{HI}{C} + \gamma_{24} \ln \frac{HN}{C} + \gamma_{34} \ln \frac{KI}{C} + \gamma_{44} \ln \frac{KN}{C}$

Durbin-Watson Statistic =            .5354                      .3760

Sum of Squared Residuals =            .11405                      .28809

---

Log of Likelihood Function = 1161.66                      1006.01

The above equations are associated with the following production function:

$$F = \sum_{i=1}^5 \alpha_i \ln x_i + \sum_{i=1}^5 \sum_{j=1}^5 \gamma_{ij} \ln x_i \ln x_j$$

where the  $x_i$ ,  $i = 1, \dots, 5$  refer to quantities of the 5 inputs.

The two regression results shown in figure 4 yield results that exhibit a high degree of consistency. The dominant term in each value share equation, the  $\alpha$ 's, turn out to have similar values in both regressions. The greatest difference is that shown in the MKI equation and this by only 6%. In both regressions all diagonal terms of the matrix are positive while the off-diagonal ones are all negative in sign. The absolute values of the  $\gamma_{ij}$  parameters show greater deviation than do the  $\alpha_i$ , although the large diagonal terms are much closer, with the exception being the MKI equation again, where the two estimates for  $\gamma_{33}$  differ by 13%. However, perhaps the most significant difference between the two regression appear in those off-diagonal terms which are insignificantly different from zero. The regression employing dummies indicates that while the information labour share and both the capital shares are all influenced by the level of employment in every input, the same is not the case for non-information labour. With dummies, the regression results seem to suggest that the non-information labour value share is uninfluenced by the level of employment of machinery and equipment. It should be noted that this result is not tantemont to a reduction of the elasticity of substitution between HN and capital to unity as it might first appear.

The same structure for MN does not emerge from the regression without dummies, at least not in detail. In this case non-information capital affects the HN share but not the HI share. It is in these particulars that the two regressions yield differences deserving of attention.

It is important to determine if the estimated value share equations correspond to a well-behaved production function. Such a function have the fundamental properties of (i) positive output for non-zero inputs and (ii) non-decreasing, concave form, the latter signifying diminishing marginal returns to individual factors. These two conditions are not automatically met in the translog specification since the quadratic terms will violate both over a certain range of positive inputs. It is appropriate then to regard the translog specification as an approximation to the 'true' production function, valid within a neighbourhood of the region of observations, provided the two conditions are met there, and having no assured validity outside that neighbourhood. Even so, some of the observations may fail to satisfy these conditions if the fit about the regression hyper-planes is not tremendously good, as in the case discussed here. Thus the region of validity may be more narrow than first supposed, located only within a neighbourhood about a point at which the approximation is deemed exact. For our purposes, this point about which the approximation is taken is associated with the mean point of the input quantities.

We have two regressions to examine for well-behavedness, namely with and without the addition of 4 dummy variables. In both cases the first condition is met except for a few observations in the KI fitted value shares. The observations for which the fitted shares are negative correspond in the main to industries with the lowest observed KI shares in the data. In the regression without dummies,



the condition is violated in all five years in the Petroleum and Coal Products industry group, and in the first two years in the Food and Beverage industry group. The former industry group has such low share values for KI that we can attribute the negative fitted values to statistical errors. The two observations in Food and Beverage are unlikely to be due to random errors and probably count as violation of the first condition for a well-behaved production function.

The second regression, with the 4 dummies included, gives a better performance. The only industry for which the fitted KI share show negative values is Leather, and this for only 3 years. All other fitted shares are positive. Again the Leather group has very low values for the KI shares and we attribute the negative fitted values to random errors and not to a violation of the condition.

In spite of the appearance of negative fitted shares at a few observations, all apart from two in the regression without dummies can be accounted to random error and we can allow that the regression past the first test. Moreover, all value shares are positive at the mean value of input quantities.

The concavity test has proved a more exacting one for the regression to pass. The condition applies to the matrix of the second derivatives of the production function, bordered by the value shares. For concavity this matrix must be negative definite

at the point of approximation. Necessary conditions for negative definiteness are negative values for the own elasticity of all 5 inputs. This weaker condition corresponds to downward sloping demand schedules for inputs. A check on these elasticities over the observations has revealed that the second well-behavedness condition is violated over most of the data points. In the regression without dummies, only four industries - Food and Beverage, Furniture, Electrical Products and Miscellaneous Manufacturing - show negative own elasticity for all inputs in all 5 years. Metal Fabricating pass in 1971 only. The remaining industries show at least one own elasticity with positive sign. Clearly this regression cannot serve as an approximation to a well-behaved production function (with decreasing marginal returns) over every observation.

A similar story emerges for the estimation with dummies included, but here Food and Beverage fail while the other three pass in terms of own elasticities. The industry which is satisfactory for 1971 now turns out to be Non-Metallic Mineral Products. All others suffer in having at least one positive own elasticity.

However, at the point about which the approximation is taken, namely the mean of input quantities, we have the correct sign for all own elasticities in the case of the regression without the dummy variables (see Table 8). This is not true however for the regression with dummies added.

From these observations we conclude that the production function (estimated without dummies) is well-behaved and possesses all the appropriate properties within a neighbourhood of the point of approximation.

Table 8

Allen Elasticities of Substitution (AES)  
Representative Industry

AES	Without Dummies	With Dummies
HI, HI	-41.8	-66.3
HI, HN	22.0	34.0
HI, KI	- 3.75	20.9
HI, KN	8.29	4.66
HI, C	-12.3	-12.2
HN, HN	-14.5	-20.1
HN, KI	- .5	-12.1
HN, KN	- 2.6	.132
HN, C	9.5	7.02
KI, KI	-59.	-59.8
KI, KN	-21.4	- 7.6
KN, C	57.	40.8
KN, KN	- 2.1	.98
KN, C	3.9	- 6.3
C, C	-24.2	.89

Four types of separation were tested statistically using the maximum likelihood method as described under table 4. As before, the hypothesized separation was treated as the null hypothesis while the unconstrained system of equations corresponded to the alternative hypothesis. The tests are to apply to the mean point of the quantity inputs. The separability constraints can be found in table 9, and the values of the log of the likelihood function shown there, when compared to that for the unconstrained system, indicates the rejection of all four hypotheses. From these results we learn that there is no evidence for (a) labour aggregation, (b) capital aggregation, (c) information activity aggregation and (d) noninformation activity aggregation. By inference we cannot expect a concurrent aggregation of information activity on the one hand and of noninformation activity on the other to be accepted under statistical testing.

Table 9 Testing for Separability: Representative Industry

For parameter labels we use the following notation: HI = 1, HN = 2, KI = 3, KN = 4. The type of aggregation under test is indicated by parentheses.

Type of separation (log of likelihood)	Description	Nonlinear constraints on parameters
1. {(HI,KI), HN, KN, C} With Dummies (1038.68) Without Dummies (923.96)	aggregation of information activities	$\gamma_{12} = \frac{\alpha_1}{\alpha_3} \gamma_{23}$ $\gamma_{14} = \frac{\alpha_1}{\alpha_3} \gamma_{34}$ $\gamma_{33} = \frac{\alpha_3}{\alpha_1} (\gamma_{11} + \gamma_{13}) - \gamma_{13}$
2. {(HN,KN), HI, KI, C} (1066.95) (939.8)	aggregation of non-information of activities	$\gamma_{12} = \frac{\alpha_2}{\alpha_4} \gamma_{14}$ $\gamma_{23} = \frac{\alpha_2}{\alpha_4} \gamma_{34}$ $\gamma_{44} = \frac{\alpha_4}{\alpha_2} (\gamma_{22} + \gamma_{24}) - \gamma_{24}$
3. {(HI,HN), KI, KN, C} (1146.45) (993.42)	aggregation of labour	$\gamma_{13} = \frac{\alpha_1}{\alpha_2} \gamma_{23}$ $\gamma_{14} = \frac{\alpha_1}{\alpha_2} \gamma_{24}$ $\gamma_{22} = \frac{\alpha_2}{\alpha_1} (\gamma_{11} + \gamma_{12}) - \gamma_{12}$
4. {(KI,KN), HI, HN, C} (1070.6) (956.13)	aggregation of capital	$\gamma_{13} = \frac{\alpha_3}{\alpha_4} \gamma_{14}$ $\gamma_{23} = \frac{\alpha_3}{\alpha_4} \gamma_{24}$ $\gamma_{44} = \frac{\alpha_4}{\alpha_3} (\gamma_{33} - \gamma_{34}) - \gamma_{34}$

## 5. Conclusion

The technological structure in Canadian manufacturing industries, according to our data, does not appear to place special emphasis on information inputs. There is no evidence for sub-aggregation of primary inputs having an information function on the one hand and a sub-aggregate of production related inputs on the other. Each type of capital and labour services are used in conjunction with all others, and there is no sign that sub-groups of inputs are employed in proportions independently of the employment of others. This means that there is no total information activity which has an existence absolutely distinct from production activities. For manufacturing, the one is closely intertwined with the other, producing the important result that information labour productivity is determined not only by the level of its own employment and effort, but also by the amount of employment and effort associated with every other input. Any movement up or down in productivity has to be corrected for changes occurring in companion inputs before any inference can be made about changes in the contribution given factor makes to output.

The implications for information accounting are not clearly directed in any one way. In the first place, it is better to disaggregate inputs into information and noninformation components rather than not, since manufacturing production is thus better "explained" statistically. A valuable result of this fact is that we obtain a more detailed account of how employment can be affected by changes in the price of different capital services, as we noted earlier. Such

detail is genuine and cannot be inferred from the more aggregative structure of inputs. In the second place, however, the negative evidence for any separation of information from noninformation activity makes one suspicious that an organization of the data according to schemes other than the information one could have equal validity. Until that suspicion is removed, the particular inference we can indeed make in favour of information accounts continues somewhat under a cloud, for it remains that it has to be shown that information accounting is superior to all others and is not simply arbitrary.

Yet we do see value in the detail the information/noninformation breakdown offers. This position is engendered by the differential and distinct effect capital price changes have on employment ratios, effects which were described earlier, and which can only be appreciated once the data is in the disaggregate form. Relative factor price changes cause adjustments to take place in the industry's demand for factor inputs even as output is maintained fixed. This phenomenon show up in the values of the substitution elasticities given in table 8. In contrast, following an increase in the level of production no substitution takes place provided meanwhile factor prices remain constant or do not undergo a relative change with respect to each other. Thus by virtue of the constant returns to scale assumption any increase or cutback in production is accompanied by a proportionate increase or fall in all factor inputs. But a relative factor price change will induce substitution to take place between inputs so that the mix of factor employment shifts (when referring to a change in a

specific price or prices it is implicitly assumed that other prices are constant).

The results obtained for the representative industry in table 8 can only be regarded as rough estimates of the true AES values. We have no independent statistical test of their accuracy or of the confidence to be attached to the values. In many respects the signs of the cross-elasticities do not agree with that obtained in our time series (total manufacturing) estimates although they agree, in as far a comparison is possible, with a U.S. study.<sup>15</sup> We take the results to be indicative but not absolutely accurate.

The effect of a rise (or fall) in the price of a factor on its own employment can be more or less severe. Information labour and information capital seem to have a strong response to a change in own price. For instance, a fall in the price of information capital will induce a rapid and strong demand for the accumulation of further information capital goods as well as their more intensive use when in place ready for use. In comparison the non-information factors respond rather sluggishly to a change in their price. Construction has somewhat an intermediate effect on its demand.

Other effects of a fall in the service price of information capital is a small enhancement in demand for information labour and no effect to speak of on non-information labour. The two information factors are complementary services. The increase in demand produced by the same price change on non-information capital will be much more significant, while the effect on construction will be in the



opposite direction, curtailing demand. Information capital substitutes for construction while it has non-information capital as a complementary good.

The two types of labour are strong substitutes whereas non-information labour is weakly complementary to machinery and equipment and information labour substitutes with non-information capital.

To sum up, the impact on labour of a small change in price of manufacturing information capital presents little danger to manufacturing employment. It should be recalled that the type of information capital we have in the study is somewhat different from the modern mix whereby computers are a much more predominant feature. An adjustment to the value added output figures to include rental services and an extension of the time series nearer to the present would offer us an excellent opportunity to re-evaluate the elasticities with computers adequately accounted for. It would be of considerable interest to determine in this manner if the figures in table 8 are corroborated or denied.

The elasticities in the time series case paint a different picture regarding complements and substitutes as we already reported in the text following table 5. We might expect the time series figures to be generally lower since given time a much greater degree of adjustment may take place. But we do have 5 years within the pooled data and this fact rather weakens the argument. We are talking of course of two different production functions, total manufacturing

in the time series instance and the fictitious 'representative' industry in the other instance, but it was expected that at the level of technology aggregated at the level of 5 inputs a sufficient similarity between the sub-industries within manufacturing and total manufacturing would exist to cause at least some consistency between the factors that are complementary and those that are substitutes. One possible explanation for the difference can be found in the fact that the two studies give approximations to a production function in neighbourhoods about different input mixes. This is because the relative factor prices in the two sets of data are sufficiently diverse to suggest that widely different sections of the isoquant are involved in the estimations. It is theoretically possible for substitution to occur at one section of the isoquant while complementarity holds at another. Thus, there is no inherent inconsistency in the two results given by the time series and the pooled data estimates.

In terms of the issue of job displacement, the relevance of the studies might be questioned on the grounds that the emerging technology bears no resemblance to current technology, constituting a structural change in production methods. A structural change of such a magnitude it can be argued, will result in technical relations between inputs outside the province of the study, based as it is on historical time series. The argument is not so conclusive as it seems, since new production techniques are introduced rather slowly. At the level of the individual firm, investment in new methods of production and quality changes in the final product can of course constitute a radical transformation. However, not all firms introduce new techniques at once and many make good use of their existing capital for a long time to come. The point is that the price of current machinery and

equipment does not suddenly plunge to zero with the emergence of an alternative, more productive capital. Replacement investment naturally will be dominated by new technology, but this constitutes only a fraction of total capital. Capital of old vintage works along side the new and investment over a five year period hardly brings about the dramatic change which is tantamount to a new structure for manufacturing production. It follows that the conclusions about the job displacement effects of information capital given in the representative industry study have a bearing on how the factor input mix needed to produce each unit of output will change in the years following the estimation period, under the scenario of falling information capital price. Of course, further away from the estimation period the validity of the conclusions becomes less firm.

By this reasoning, we see that the employment impact reported for the representative industry may be good for the five years following 1971, but will unlikely present an accurate picture of the sort of capital-labour trade-offs that will take place in 1981. To obtain a better grasp of modern events we need the up-dated data which is now becoming available, as we stressed on several occasions earlier in the report. We are encouraged by the results obtained thus far and it would be greatly satisfying if more recent data bears out the empirical findings reported herein.

PART II

5. SEPARABILITY

Consider a production function with  $n$  inputs  $y = F(x_1, \dots, x_n)$ . Partition the set of integers  $N = \{1, \dots, n\}$  into  $p$  mutually exclusive and exhaustive subsets  $[N_1, \dots, N_p]$  to be called the partition  $P$ . The production function  $F(x)$  is said to be weakly separable<sup>16</sup> with respect to the partition  $P$  if the marginal rate of substitution (MRS) between any two inputs  $i$  and  $j$  from any subset  $N_s$ ,  $s = 1, \dots, P$ , is independent of the quantities of inputs outside of  $N_s$ , i.e.

$$\frac{\partial}{\partial x_k} \frac{F_i}{F_j} = 0 \text{ for all } i, j \in N_s \text{ and } k \notin N_s$$

where  $F_i$  denotes the first order partial derivative  $\partial F(x)/\partial x_i$ . Weak separability with respect to the partition  $P$  is equivalent to the production function  $F(x)$  being of the form  $F(v_1, \dots, v_p)$  where  $v_s$  is a function of the elements of  $N_s$  only. For example suppose  $x_1$  stands for input of information services,  $x_2$  for non-information labour services,  $x_3$  for machinery and equipment, and  $x_4$  for structures. Then the weak separability of the two types of labour on the one hand and machinery and equipment and structures on the other can be expressed functionally as  $F(v(x_1, x_2), x_3, x_4)$ . Note that the sub-function  $v$  is independent of the amount of services issuing from structures or machinery and equipment, and that the shape of the isoquant curve in the  $(x_1, x_2)$  plane is unaffected by the levels of  $x_3$  or  $x_4$ .

Clearly a stronger form of functional separability is available to us and this also has economic meaning. Define strong separability with respect to the partition  $P$  if the MRS between any two inputs from different subsets  $N_s$  and  $N_t$  does not depend on the quantities of inputs outside of  $N_s$  and  $N_t$ , i.e.

$$\frac{\partial}{\partial x_k} \frac{F_i}{F_j} = 0 \text{ for all } i \in N_s, j \in N_t, s \neq t, \text{ and } k \notin N_s \cup N_t.$$

The corresponding functional form is additive, i.e.  $F(v_1 + \dots + v_p)$ .

By carrying out the operation of differentiation both types of separability conditions can be written as

$$F_{ik} F_{jk} - F_{jk} F_{ik} = 0,$$

where  $F_{ij}$  is the second partial derivative of  $F$ . Of course, the  $i, j$  indexes belong to one subset in the case of weak, to different subsets in the case of strong, separability. This equation can be re-written in terms of non-linear constraints on the estimated values of the parameters of the production function.

The Allen partial elasticity of substitution (AES) between inputs  $x_i$  and  $x_j$  can be expressed in terms of the price elasticities of derived demand as follows:

$$\sigma_{ij} = E_{ij}/w_j, \text{ all } i$$

where

$$E_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i}, \text{ } p_i \text{ is the service price of the } i^{\text{th}} \text{ input and } w_j$$

is the share of the  $j^{\text{th}}$  input in total cost, equal to  $p_j x_j / \sum p_i x_i$ . The AES correspond to conventional comparative statistics analysis; they measure the response of derived demand to an input price change, holding output and all other input prices fixed. Note moreover that

$$\sigma_{ij} = \frac{p_j}{x_i} \frac{\partial x_i}{\partial p_j} \frac{1}{w_j} = \frac{p_j}{x_i} \frac{\partial x_i}{\partial p_j} \frac{c}{p_j x_j} = \frac{\partial x_i}{\partial p_j} \frac{c}{x_i x_j}$$

Here  $c = \sum p_i x_i$ . By duality (see next section) and Shephard's lemma

$$x_i = \frac{\partial c}{\partial p_i} \quad (\text{to be denoted by } c_i) \quad \text{all } i.$$

$$\text{Also } \frac{\partial x_i}{\partial p_j} = \frac{\partial^2 c}{\partial p_i \partial p_j} \quad (\equiv c_{ij})$$

Thus we have the useful formula for calculating the AES from an estimated cost function:

$$\sigma_{ij} = \frac{c_{ij}}{c_i c_j}$$

The separation of inputs 3 and 4 from inputs 1 and 2 can equivalently be expressed as:<sup>15</sup>

- (i) functional separability:  $F(x_1, x_2, x_3, x_4) = H[J(x_1, x_2), x_3, x_4]$
- (ii) equality of the AES:  $\sigma_{13} = \sigma_{23}$  ;  $\sigma_{14} = \sigma_{24}$ .

The equivalence between function separability and the expression (i) can be proved, but only with some difficulty. To gain an intuitive grasp for this equivalence consider the following less general argument. Firstly, the sufficiency is readily demonstrated. For,

$$F_i = \frac{\partial H}{\partial x_i} = J_i H_J, \quad i = 1, 2.$$

Thus  $\frac{F_1}{F_2} = \frac{J_1}{J_2}$  and is independent of  $x_3$ .

Demonstrating necessity is more difficult. Assume, for simplicity,  $F$  is linear homogenous. Then

$$\begin{aligned} F &= F_1 x_1 + F_2 x_2 + F_3 x_3 \\ &= F_2 \left( \frac{F_1}{F_2} x_1 + x_2 \right) + F_3 x_3. \end{aligned}$$

Now  $F_1/F_2 = f(x_1, x_2)$ , a function independent of  $x_3$ . Assume  $f(x_1, x_2)$  is a rational function so that we may write

$$f(x_1, x_2) = \frac{J_1(x_1, x_2)}{J_2(x_1, x_2)}$$

Hence

$$F = \frac{F_2}{J_2} (J_1 x_1 + J_2 x_2) + F_3 x_3.$$

Define the function

$$J(x_1, x_2) = J_1(x_1, x_2)x_1 + J_2(x_1, x_2)x_2$$

where

$$\frac{\partial J}{\partial x_1} = J_1 \quad \text{and} \quad \frac{\partial J}{\partial x_2} = J_2.$$

This is possible, but will not be proved here. Then  $J$  is linear homogenous as is necessary if  $F$  is to be so, and moreover

$$F = \frac{F_2}{J_2} J + F_3 x_3$$

and  $F_J = \frac{F_2}{J_2}$  by linear homogeneity.



But this latter is consistent with the definitions made earlier, since

$$\frac{\partial}{\partial x_2} F(J(x_1, x_2), x_3) \equiv F_2 = F_{J_2}$$

To show the relation (ii) consider from the first order conditions for maximization, for goods  $x_1$  and  $x_2$ , say;

$$\frac{p_1}{p_2} = \frac{F_1}{F_2}$$

We can use this equalization condition between the relative prices and the slope of the isoquant in the (1,2)-plane as follows:

$$\frac{\partial}{\partial x_3} \left( \frac{F_1}{F_2} \right) = \frac{\partial}{\partial x_3} \left( \frac{p_1}{p_2} \right) = p_2 \frac{\partial p_1}{\partial x_3} - p_1 \frac{\partial p_2}{\partial x_3} = 0$$

The last equality obtains from the separability condition. Thus

$$\frac{1}{p_1} \frac{\partial p_1}{\partial x_3} = \frac{1}{p_2} \frac{\partial p_2}{\partial x_3}$$

which, on multiplying by  $x_3/w_3$  yield the required result:

$$\sigma_{13} = \sigma_{23}$$

Similarly,  $\sigma_{14} = \sigma_{24}$ .

The factor share equations corresponding to the translog production function are written as

$$m_i = \alpha_i + \sum_j \gamma_{ij} \ln x_j \quad i = 1, \dots, n$$

where, under constant returns,  $m_i$  are the cost share of the  $i^{\text{th}}$  input in total cost. Note that the condition  $\sum_i m_i = 1$  is satisfied by the set of conditions  $\sum_i \alpha_i = 1$  and  $\sum_j \gamma_{ij} = 0$  which as we saw are implied by constant returns to scale.

The translog production, owing to its quadratic nature, is an ill-behaved production function globally. When at least one  $\gamma_{ij} \neq 0$  there exists configurations of inputs such that neither monotonicity nor convexity is satisfied. We have to check that the estimated production function is well-behaved for each data point. This involves checking that the estimated expression for  $m_i$  are all positive and that the bordered Hessian matrix of first and second partial derivatives of  $F$  is negative definite.<sup>17</sup>

For the production function  $F$  we noted that the separability conditions can be expressed in terms of the partial derivatives

$$F_{ik} F_j - F_{jk} F_i = 0,$$

where the  $i^{\text{th}}$  and  $j^{\text{th}}$  factors are separated from factors  $k \neq i, j$ . From this we see that the necessary and sufficient condition for the translog function to be separable is

$$m_j \gamma_{ik} - m_i \gamma_{jk} = 0,$$

since  $F_j = m_j$ . Monotonicity requires  $m_i > 0$ , all  $i$ , so that if separability holds and if  $\gamma_{jk} = 0$ , then  $\gamma_{ik} = 0$ . Suppose, however, that  $\gamma_{jk} \neq 0$ ,  $\gamma_{ik} \neq 0$ . Since  $m_i = \alpha_i + \sum_j \gamma_{ij} \ln x_j$  the above condition can be rewritten as

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} + \sum_m (\gamma_{im} \gamma_{jk} - \gamma_{jm} \gamma_{ik}) \ln x_m = 0$$

and the global conditions (holding for all values of the  $x_i$ ) for inputs  $i$

and  $j$  to be separated from  $k$  become

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$$

$$\gamma_{im} \gamma_{jk} - \gamma_{jm} \gamma_{ik} = 0 \quad m = 1, \dots, n$$

Alternatively, we can write (for  $\gamma_{ik}$ ,  $\gamma_{jm}$  nonzero)

$$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{ik}}{\gamma_{jk}} = \frac{\gamma_{im}}{\gamma_{jm}} \quad m = 1, \dots, n$$

Production functions specified to have a quadratic form (such as the translog function) do not exhibit the desired non-decreasing property (and hence isoquants convex to the origin) required of production functions over all non-negative input vectors, i.e. globally. However, we can regard the translog specification as a second order approximation to the true production function over data set, the first order part of which is the familiar Cobb-Douglas form. From this perspective the conditions to be satisfied for separability are greatly simplified. This is because we make a local test for the conditions within the range of the data set, and in fact apply the test to the mid-point in the time series (1961 for example). The data for 1961 is normalized to unity so that naturally,  $\ln x_i(1961) = 0$ , for all  $i$ . Consequently the local condition for input  $i$  and  $j$  to be separated from  $k$  becomes

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$$

i.e. only the first of the two set of conditions shown above. One form of separability can be associated with zero values for the cross terms,

$$\gamma_{ik} = \gamma_{jk} = 0.$$

## 6. CONSTRUCTION OF THE TIME SERIES DATA

In many crucial respects the raw data on labour services and value-added output are incomplete and flawed by various definitional changes made throughout the sample period. Considerable effort has been made to reconstruct missing observation points as well as to impose a modest degree of consistency in each time series and between the labour services and output series.

### 1. The Labour Series

Our strategy involved no more than satisfying the very basic requirement for econometric work. This consisted of producing consistent series over the time period on the factor inputs which at the same time properly corresponded to the figures for value-added production. In Canada this preliminary task is no easy matter because of the many changes in the classification, concept, and coverage which have had such profound impacts on each of the series. As a result we are confronted by two major discontinuities in all series and, worse still, by breaks in some of them.

By the assumption of linear homogeneity we are able to calculate the value of total capital services as the difference between value-added input and the value of total labour services. This present discussion the, relates to output and labour services only. The labour data series which are available are disaggregated into two groups: "non-information" workers, consisting of production workers in manufacturing operations, employees in new construction, outside piece workers, and other production and related workers; and "information" workers consisting of administrative and office employees, sales and distribution workers, and employees at other locations. These series for each group are

man-hours worked, number of employees, and wages and salaries. The man-hours worked series is different from man-hours paid by excluding time accounted for by holiday, vacation and sick leave. Unfortunately the disaggregated time worked series begin only in 1961 whereas they are required from 1948 onwards. Secondly, the series have a discontinuity in 1969. On the other hand, the number of employees series and the compensation series suffer from two discontinuities, one at 1961 and another at 1959. In 1961 there exists a one-year overlap, and at the other discontinuity the overlap is for the three years 1957-1959.

All series are spliced at the point of discontinuity by the simple and crude method of taking the ratio of the values in the overlap year and using this to rescale the numbers on one side of the splice in line with those on the other side, usually so that it is the shorter part of the series which is modified. Thus in this way the post-1969 man-hours paid series were modified to conform with the observations of the earlier years.

The backward extension of the man-hours series was achieved through a least squares fit of man-hours (H) to employment (W), following a logarithmic transformation of the data. A number of regressions were run for this backcasting.

The two following equations provided the best fit:

$$\ln IH = 7.01251 + 1.04101 \ln IW$$

(12.0136)      (23.2451)

t-values in brackets

$$R^2 = .9846$$

$\ln IH$  = the natural logarithm of the number of man-hours worked  
of information workers

$\ln IW$  = the natural logarithm of the number of information workers  
in Canadian manufacturing

$$\ln NH = 9.37796 + .872480 \ln NW$$

(20.8796)      (27.0775)

t-values in brackets

$$R^2 = .9868$$

$\ln NH$  = the natural logarithm of the number of man-hours worked  
of non-information workers (no adjustments required).

$\ln NW$  = the natural logarithm of the number of non-information  
workers in Canadian manufacturing.

These equations were then used as a basis for estimating information/non-information hours worked for the period 1947 - 1960.

A glance at the first equation will show that the average weekly hours (for a 50-week work-year) for information workers is around 40 hours, with a slight tendency to decline in later years when employment is higher. If not so transparent, a similar situation holds for the non-information workers in which on average about 40 hours per week is worked and this has a slowly falling secular trend. The mix between straight time and overtime, one would think, may help explain the appearance of the non-information equation but this is purely speculative.

The employment series had its own peculiar complications. The post-1961 series differs from the earlier years in that more non-production workers are included who formally were not enumerated by the survey. These consist mainly

of research and development employees, clearly information workers, and sales personnel. These extra employees we desire to see included as information workers, so the earlier two splices in the series are inflated appropriately in the manner discussed above, thereby simulating the count of those employees not covered. However, this action required a corresponding inflation of the value-added output figures. Fortunately we have an overlap in the output produced by the narrow coverage of workers and the output produced by the broad coverage of workers (inclusive of sales and R & D) for the period 1961-1975. The ratio of the means of these output series was used to inflate the pre-1961 value-added series to produce a series in output reconciled with those of the repaired input series.

## 2. The Capital Data

The real stock of construction and machinery and equipment for Canadian manufacturing are directly derivable from Statistics Canada sources containing series on the mid-year net capital stock of total Canadian manufacturing annually since 1926 in constant 1961 dollars. The data are broken down into four components: building construction, engineering construction, machinery and equipment, and capital items charged to operating expenses (smaller types of equipment normally charged by respondents to the Capital Expenditures Survey to current accounts and having a serviceable life greater than one year). These four components were aggregated into two: construction - the sum of the first two, and machinery and equipment - the sum of the second two.

To go from real stock to service flows we assume that the quantity of capital services rendered by construction and M&E is proportional to the corresponding real stocks. Denote the stocks by  $K_C$  and  $K_M$  respectively. The quantities of service flows at time  $t$ , denoted by  $Q_{C,t}$  and  $Q_{M,t}$ , are computed as

$$Q_{C,t} = uK_{C,t}$$

$$Q_{M,t} = uK_{M,t}$$

where  $u$  is a constant "quality of capital" index.

The replacement price of these two components is derived by dividing the current dollar gross fixed capital formation by the corresponding constant dollar gross fixed capital formation. The capital service price is then derived from the replacement price using a formula, described more fully in chapter 7, which accounts for depreciation, capital gains and returns. The series developed by Denny and Pinto were used.<sup>18</sup> The service prices in their series do not include a capital gains term, on the assumption that realized capital gains did not affect the service price because they were unanticipated at the time decisions were made.

We note that the procedure adopted in the estimation may be an oversimplification, as we have assumed not only that the service flow rendered by construction and M&E are proportional to the real stock, but that they are in the same proportion. By contrast, for U.S. manufacturing, Berndt has used a  $u$  of .19 for structures. Our  $u$  was derived implicitly by subtracting the value of labour inputs from the value of output, to arrive at the aggregate value of capital services. The value of capital services was then further



disaggregated between M&E and construction, by the ratio of the values of the two capital components.

### 3. The Data Base

The results of the data base reconstruction can be seen from the tables and graphs in this section. Table 10 shows very clearly the extent and locations of the discontinuities in the time series. The zero entries in the TVA and VA columns denote empty observations. The data sets A, B and C are the parts of the time series that required splicing in the manner described in the previous section.

Table 11 shows our reconstituted data set which was used as the base for all subsequent empirical work in the study. The manner of its construction was described earlier. Figure 5 reveals a reasonably smooth backward extension of the measure of annual output, total value-added. In Figure 6 we see the relative size and movements in the hours worked by labour type. As we noted earlier, non-information employment fairly consistently charts the boom and recession cycles in Canada over the period in view, but the same cannot be said for the information workers.

Figure 2 gives the value shares of the two types of labour over the data period and these graphs tend to indicate an independence of movement which is inconsistent with the two labour types being grouped together by the production technology separately from the other factors.

Figure 5 gives also a picture of how the TVA series was constructed over the period of empty observations so as to track the movements in the restructured

VA series. The gap between the two represents activities reported in later surveys but not included in the earlier years. Most of this additional activity relates to the "head office" of establishments.

Table 10 ORIGINAL DATA SETS

	YEAR	NH	TH	NW	TW	VA	TVA
Set A	1948	957491	1155721	1876773	2409368	4938787	0
	1949	949656	1171207	1963463	2591891	5330566	0
	1950	952244	1183297	2078634	2771267	5942058	0
	1951	1010588	1258375	2459566	3276281	6940947	0
	1952	1025355	1288382	2713715	3637620	7443533	0
	1953	1053226	1327451	2940339	3957018	7993069	0
	1954	989030	1267966	2821586	3896688	7902124	0
	1955	1010992	1298461	2995267	4142410	8753450	0
	1956	1051723	1353020	3298666	4570692	9605425	0
	1957	1045177	1359061	3416226	4819628	9822085	0
	1958	981735	1289602	3333172	4802496	9792506	0
	1959	997907	1303956	3543456	5073074	10320963	0
Set B	1957	1035333	1340948	3391803	4778040	0	0
	1958	972468	1272686	3305975	4758614	9454954	0
	1959	988991	1287809	3517599	5030128	10154277	0
	1960	971610	1275476	3565124	5150503	10371284	0
	1961	969276	1264946	3646113	5231447	10682138	0
Set C	1961	939413	1352605	3532943	5701651	10434832	10931561
	1962	974376	1389516	3834514	6096174	11429644	11986666
	1963	1003566	1425440	4095916	6495289	12272734	12875073
	1964	1057502	1491257	4513633	7080939	13535991	14247184
	1965	1115892	1570299	5012345	7822925	14927764	15785311
	1966	1172943	1646024	5575206	8695890	16351740	17260256
	1967	1168651	1652827	5869085	9254190	17005696	18049639
	1968	1160226	1642352	6278429	9905504	18332204	19483614
	1969	1189887	1675332	6921525	10848341	20133593	21456276
	1970	1167063	1637001	7232256	11363712	20047801	21417748
	1971	1167810	1628404	7819050	12129897	21737514	23187881
	1972	1213106	1676130	8763104	13414609	24264829	25981742
	1973	1275985	1751066	10060062	15220033	28716119	30766506
1974	1300792	1785977	11637073	17556982	35084752	37654465	
1975	1272051	1741545	12672237	19160724	36139301	38715600	

NH = noninformation hours worked

VA = value added by manufacture

TH = total hours worked

TVA = value added - total activity

NW = noninformation wage

TW = total wage

Table 11. DATA BASE (revised)

YEAR	NH	TH	NW	TW	VA	TVA
1948	919251	1219340	1805520	2603264	4658147	4958725
1949	911729	1235679	1888919	2800476	5027663	5352086
1950	914213	1248434	1999717	2994287	5604408	5966047
1951	970227	1327645	2366187	3539942	6546536	6968968
1952	984404	1359304	2610687	3930361	7020563	7473583
1953	1011162	1400524	2828707	4275462	7538873	8025338
1954	949530	1337764	2714462	4210277	7453096	7934025
1955	970615	1369938	2881549	4475774	8256046	8788788
1956	1009719	1427500	3173430	4938522	9059609	9644203
1957	1003435	1433874	3286526	5207492	9734487	10362630
1958	942507	1360881	3203362	5186320	9236059	9832039
1959	958521	1377052	3408418	5482237	9919192	10559253
1960	941675	1363865	3454468	5613432	10131175	10784914
1961	939413	1352605	3532943	5701651	10434832	10931561
1962	974376	1389516	3834514	6096174	11429644	11986666
1963	1003566	1425440	4095916	6495289	12272734	12875073
1964	1057502	1491257	4513633	7080939	13535991	14247184
1965	1115892	1570299	5012345	7822925	14927764	15785311
1966	1172943	1646024	5575206	8695890	16351740	17260256
1967	1168651	1652827	5869085	9254190	17005696	18049639
1968	1160226	1642352	6278429	9905504	18332204	19483614
1969	1189887	1675332	6921525	10848341	20133593	21456276
1970	1167063	1637001	7232256	11363712	20047801	21417748
1971	1167810	1628404	7819050	12129897	21737514	23187881
1972	1213106	1676130	8763104	13414609	24264829	25981742
1973	1275985	1751066	10060062	15220033	28716119	30766506
1974	1300792	1785977	11637073	17556982	35084752	37654465
1975	1272051	1741545	12672237	19160724	36139301	38715600

Figure 5 Value Added Output

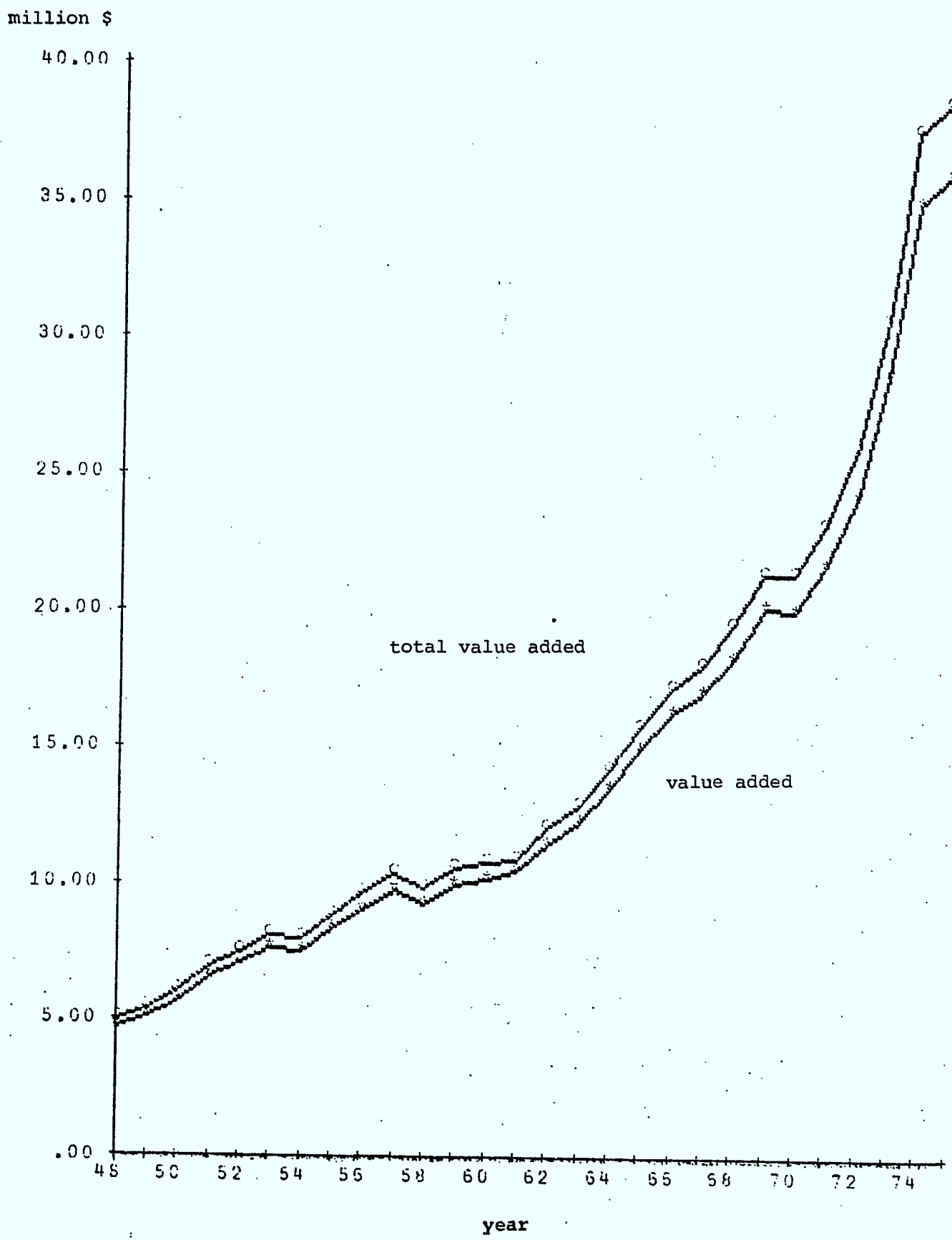
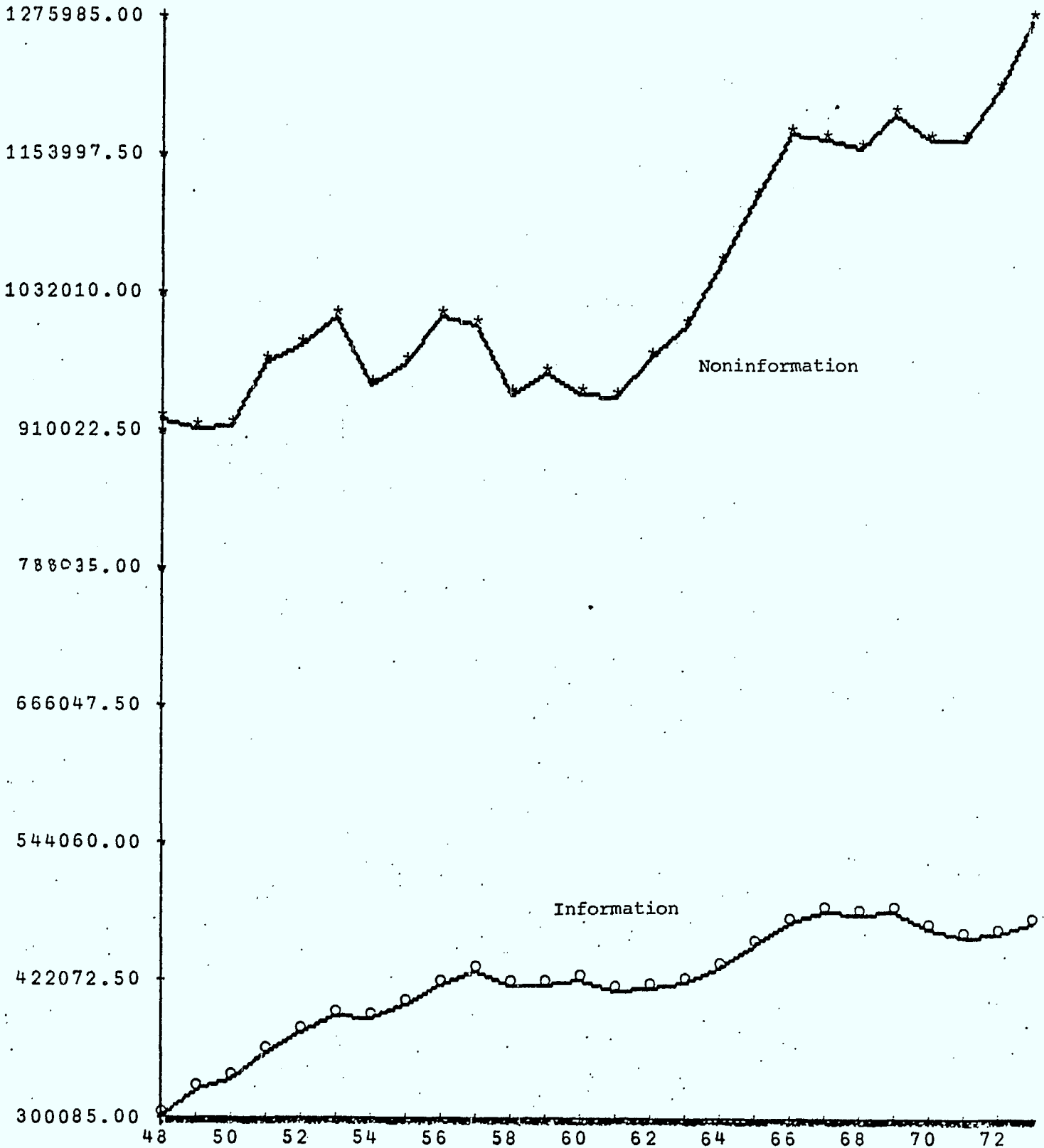


Figure 6 Hours of Work

hours worked per annum



7. CONSTRUCTION OF THE POOLED DATA

1. Manhours and Data Base

Statistics on manhours worked and value-added output for the period 1967-1971 were used as obtained from Statistics Canada. The statistical material is the same as that described in Chapter 6 for the 1961-1974 period except that here it is disaggregated by major industry groups. The value of hours worked, information and noninformation, by industry for each of 5 years, are given in Table 12.

The observations sequence in Table 12 is in chronological order of 5 years for each of the 19 industries in turn. Thus the first 5 observations refer to major group 1, the next 5 to major group 2, and so on.

TABLE 12

## INPUT QUANTITIES without CAPITAL UTILIZATION

	LI	LN	KI	KP	C1
1	154839.	289865.	30688.5	.170886E+07	.126100E+07
2	158570.	286204.	31028.6	.175225E+07	.130150E+07
3	150798.	282720.	31932.6	.179279E+07	.136630E+07
4	145276.	287008.	34730.1	.185386E+07	.142490E+07
5	139975.	282247.	36169.7	.189386E+07	.148000E+07
6	4581.52	14783.8	7128.75	52563.8	61200.0
7	4510.41	13718.9	7168.70	53198.8	64000.0
8	4416.01	13122.7	7301.66	53988.4	64800.0
9	4379.36	13181.7	7170.54	53154.2	64900.0
10	4380.18	12356.6	7232.66	53273.4	65699.9
11	23153.6	60934.2	4827.98	147669.	154600.
12	24503.2	63187.0	4988.66	152625.	162700.
13	23921.2	66136.1	5316.93	168256.	176100.
14	22979.6	64075.7	5673.22	174566.	195300.
15	23492.0	63729.9	7283.94	207457.	219000.
16	9977.73	51676.4	781.380	30585.4	54200.0
17	10471.8	52749.1	807.620	32130.0	54600.0
18	9861.96	51605.8	832.450	33429.7	55100.0
19	8614.91	47089.4	798.700	32835.7	85099.9
20	8184.21	47144.4	794.400	32855.4	54900.0
21	31291.5	118841.	29042.2	644097.	290900.
22	27928.8	114376.	27771.7	635416.	297000.
23	27976.2	117262.	27241.4	634582.	304300.
24	27714.4	109588.	27441.9	633050.	315400.
25	26993.1	109206.	27505.1	630874.	321100.
26	37461.9	203732.	27023.5	109483.	65300.0
27	36808.8	205199.	27558.7	109337.	64100.0
28	35237.4	210789.	30353.9	112689.	64800.0
29	32752.9	207257.	31684.7	113547.	64400.0
30	32741.6	204764.	33469.8	115445.	62300.0
31	27489.2	159917.	10644.9	509238.	272600.
32	27852.0	159458.	11043.7	526654.	275700.



(Table 12 Cont'd)

	LI	LN	KI	KN	CI
33	27638.6	162298.	12890.1	594232.	293100.
34	26192.5	151662.	14839.7	644560.	321200.
35	27773.9	159000.	16937.7	702583.	342900.
36	19609.6	72797.7	4911.02	72967.8	70000.0
37	19671.7	70796.6	5220.16	76464.5	77099.9
38	19522.6	72813.9	5312.96	78219.7	79500.0
39	18198.4	69313.9	5281.12	78834.1	80699.9
40	18103.1	70573.9	5363.23	80106.8	82199.9
41	48971.1	180152.	40499.2	.242773E+07	.141050E+07
42	49503.2	177321.	40020.6	.242068E+07	.147200E+07
43	51853.5	183851.	40666.2	.244514E+07	.152770E+07
44	51792.8	179253.	44159.5	.255695E+07	.161050E+07
45	51021.7	175036.	46760.8	.265416E+07	.169620E+07
46	66709.1	92390.1	46645.0	358350.	259100.
47	68333.8	92074.9	46865.5	362804.	262800.
48	70181.3	92469.7	47931.8	368562.	267600.
49	66272.8	95875.5	50176.9	376232.	273300.
50	67533.9	93529.6	52929.1	385807.	279600.
51	44968.0	173350.	48825.0	.204913E+07	.125870E+07
52	46971.6	170306.	48122.9	.202318E+07	.131170E+07
53	48813.1	164253.	50398.6	.204734E+07	.135490E+07
54	49260.3	174780.	55112.8	.212626E+07	.139200E+07
55	49824.2	168983.	59515.4	.219549E+07	.143970E+07
56	69390.4	211660.	27185.3	599982.	485000.
57	72553.4	205498.	29190.1	621568.	497500.
58	71465.1	213129.	30391.5	652858.	509100.
59	65984.6	215542.	33005.4	675271.	524400.
60	64375.2	208607.	33618.8	682212.	532800.
61	60393.9	97833.4	28805.5	241148.	233400.
62	62653.4	89776.9	29488.2	249040.	244100.

(Table 12 Cont'd)

	LI	LN	KI	KN	CI
63	63649.5	98147.2	30666.2	258359.	253000.
64	63714.2	94778.4	32115.7	268455.	268200.
65	45259.7	90676.7	32484.6	269405.	280000.
66	76078.1	218015.	85837.7	956206.	692300.
67	77901.4	221823.	84408.5	970365.	722100.
68	82207.7	232079.	91061.7	.100656E+07	742100.
69	81774.1	205265.	106526.	.108204E+07	765300.
70	77152.6	217839.	104118.	.108465E+07	778000.
71	89265.7	152551.	35329.4	428286.	292700.
72	84381.6	150079.	35675.9	433446.	307800.
73	82616.3	156677.	36497.6	448523.	319300.
74	83733.1	145730.	38461.3	460418.	334300.
75	88927.0	142668.	40705.9	475334.	350500.
76	24847.9	80784.9	43995.2	768044.	447500.
77	25400.7	80144.7	41786.4	746986.	463400.
78	25512.1	80681.3	41223.4	735074.	475300.
79	24868.0	74735.1	41156.5	730754.	492000.
80	24650.3	78613.6	37680.7	704546.	498600.
81	14489.3	14366.5	2265.07	224514.	.129660E+07
82	14571.3	14307.5	2270.86	235076.	.136120E+07
83	14094.9	13341.2	2021.28	229135.	.144350E+07
84	14149.7	13456.5	1889.62	226704.	.158250E+07
85	14585.3	13640.9	1814.99	226368.	.175180E+07
86	67954.5	75354.3	55256.3	.122801E+07	.101850E+07
87	70959.6	77098.2	58199.0	.125886E+07	.112040E+07
88	72783.2	79739.0	58001.0	.124411E+07	.124260E+07
89	73069.5	79654.0	57142.3	.123624E+07	.135020E+07
90	72989.4	75675.4	55705.8	.122096E+07	.143910E+07
91	33631.8	80475.2	11617.2	152592.	123200.
92	33428.8	80069.1	12480.0	163771.	131700.
93	36025.1	83420.2	14162.0	182847.	138400.
94	33204.7	79969.2	14352.2	185507.	146100.
95	33655.9	80614.0	14975.7	192838.	152700.
	1	2	3	4	5

## 2. The Formation of the Capital Stock Data

The definition of information and non-information machinery and equipment should exactly correspond to the same two categories of labour services. By this principle, information machinery and equipment is simply the machinery and equipment operated by information workers, and likewise for the non-information factors of production. Unfortunately, this separation of capital services is not entirely clearcut since both types of workers could conceivably be making use of the same type of capital goods if not the same physical machine, such as a computer. The nature of the data is such that we cannot directly and precisely attribute so much of capital services to information activity and the remainder to noninformation activities. The data identifies net capital formation by commodity type rather than by use, and consequently we are forced into some guesswork as to which labour type uses which capital commodity. Clearly this imperfect procedure of assigning capital services is open to many mistakes, but most of the commodities have descriptions which reduces the chance of serious mis-assignment of machinery and equipment between the two categories to accept levels. Unfortunately, there exists no opportunity for assessing the potential bias inherent in this method, so we have no check on the data errors thus introduced. We mentioned earlier the difficulty with construction (both building and engineering) as a category of capital, and because of this it remains in aggregate form.

We can identify commodities destined to become capital goods via the input-output structure of the Canadian economy, which is available for the years 1961-1971 (now extended to 1974). Thus we have an 11-year period of annual

observations on business fixed capital formation by commodity groups for 19 manufacturing industries. Of concern to us is the machinery and equipment component of fixed capital formation and the following commodity list shows the commodities that have been identified as comprising information M&E:

<u>Standard Commodity Classification</u>	<u>Commodity Title</u>
17000	Carpets, etc.
20500	Office Furniture
20600	Special purpose furniture
32900	Office Machinery and equipment
35700	TV, Radio, Record players
35800	Telecommunications, etc.
49800	Laboratory and scientific
50300	Photographic
51200	Advertising goods
52000	Phonograph records

The remaining commodities entering fixed capital formation are identified as comprising non-information M&E .

For each industry the detailed commodities were aggregated to form aggregates using the Divisia chain index method. Hence, (discarding the suffix denoting industry),

$$\ln q_{I,t} - \ln q_{I,t-1} = \sum_{i \in I} w_{i,t} (\ln q_{i,t} - \ln q_{i,t-1})$$

- where
- $q_{I,t}$  = information FCF in year t
  - $q_{i,t}$  = deflated component FCF with information M&E ,  $i \in I$  , using Statistics Canada commodity deflators.
  - $I$  = index set for information type commodities
  - $w_{i,t} = \frac{1}{2} (v_{i,t} + v_{i,t-1})$
  - $v_{i,t}$  = value share of commodity i in total information M&E

Similarly,

$$\ln q_{N,t} - \ln q_{N,t-1} = \sum_{i \in N} w_{i,t} (\ln q_{i,t} - \ln q_{i,t-1})$$

where  $q_{N,t}$  = noninformation FCF in year  $t$ .

$N$  = index set for noninformation type commodities.

A price index for each of the two aggregate commodities was obtained implicitly by dividing the Divisia quantity into the respective current dollar value.

The short investment series of 11 years is insufficient to produce a base year capital stock for each of 19 industries and yet reserve enough observations for the actual regressions. For the construction of a base year capital stock we require a time series on gross fixed capital formation reaching back for as long as the average life of the machinery and equipment. So for the regression to have at least minimal degrees of freedom and thus useful tests of significance to emerge, we need to reserve at least four years of observations for pooling with the 19 industry observations. The remaining seven years scarcely encompasses the lifetime of an "average machine". There is only one way in which to repair this shortcoming in the length of the series and that is to reconstruct the information and noninformation components of M&E for a sufficient number of years prior to 1961. This is done in the following way.

Annual data are available from Statistics Canada on gross fixed capital formation in current dollars of total M & E for each industry to the beginning of the century and earlier. We also have estimates of M & E lifetimes by industry from the same source, as well as M & E stock estimates. We note from the 11 years of observations on current dollar information and non-information

M & E large fluctuations in the two investment series, but also a fair constancy in the investment ratio between the two. This agreeable pattern in the observations suggested we would not be totally in error by assuming that the proportion of each year's investment destined for information M & E and for noninformation M & E held relatively stable and remained the same as the average proportion revealed in the observation period. Following this suggestion we produced a Divisia aggregate of M & E gross investment from our two M & E components for each of the 11 observation years, and compared these totals with the totals given by Statistics Canada. Owing to definitional differences and differences of source data, a moderate discrepancy emerged between our and Statistics Canada's series, which was removed by rescaling our series in line with their 'reference' series. For the majority of industries the movement of our series and the reference total M & E fixed capital formation series corresponded almost exactly while for the remaining few industries the totals tallied much closer but the movement in investment agreed significantly less closely, as revealed by the correlation coefficient. (see tables 7 and 8).

The investment reference series thus provided us with a total M & E series for as many years back as we need to go for the purpose of cumulating investment into the base year capital stock. The total was partitioned into information and noninformation M & E in the fixed proportions as indicated above, yielding the investment series for the two types of M & E that we require.

Table 13

Machinery and Equipment Fixed Capital Formation; Industry Groups 1-10

	Q1	Q2	Q3	Q4	Q5
1961	178916.	11627.3	25035.9	5526.89	43004.8
2	185974.	9813.08	26063.6	5612.69	57789.7
3	173457.	11572.0	23268.0	5479.82	71101.0
4	194588.	10265.7	32000.9	5574.15	120355.
5	213786.	16047.7	33036.5	5006.10	123649.
6	244756.	14593.6	54769.8	7450.44	119910.
7	270549.	16260.0	38646.5	6093.36	92334.9
8	244408.	14584.6	41354.8	7702.21	75957.2
9	245581.	15121.6	59907.4	7657.12	88825.0
1970	282668.	12300.5	48374.2	5303.77	88468.7
.1	256935.	13915.6	94482.9	6044.28	87034.7

	Z1	Z2	Z3	Z4	Z5
1961	143100.	8599.99	23700.0	4300.00	28000.0
2	143000.	7000.00	24500.0	4600.00	36200.0
3	134100.	8299.99	22900.0	4600.00	45000.0
4	141500.	7000.00	29200.0	4400.00	74599.9
5	151900.	10800.0	31700.0	4000.00	80299.9
6	172000.	9599.99	49100.0	5799.99	70699.9
7	199200.	11300.0	37500.0	5200.00	58100.0
8	180600.	10200.0	42800.0	6500.00	49700.0
9	181600.	10500.0	63300.0	6400.00	59600.0
1970	211900.	8700.00	49400.0	4600.00	61800.0
.1	192300.	9700.00	87299.9	5200.00	59100.0

	Q6	Q7	Q8	Q9	Q10
1961	17404.5	48785.1	7228.79	247793.	44474.0
2	21984.2	46840.0	8481.16	259409.	47156.6
3	23277.3	59140.8	10148.9	302319.	52374.0
4	25307.1	70652.6	13767.1	443894.	65616.6
5	23888.5	81090.1	14345.6	500415.	52664.3
6	29740.8	73508.4	15100.3	669037.	71435.0
7	21924.2	69767.2	15405.2	596590.	57447.6
8	25470.9	78705.5	16941.5	376707.	62381.4
9	34597.9	149196.	14279.6	431025.	66450.1
1970	29686.9	133906.	12458.8	588630.	72449.9
.1	32564.1	150023.	13781.9	580733.	77982.6

	Z6	Z7	Z8	Z9	Z10
1961	12700.0	41500.0	4700.00	162000.	32700.0
2	14500.0	36900.0	5899.99	172600.	32600.0
3	15500.0	46600.0	7000.00	206300.	37200.0
4	15700.0	53100.0	9299.99	294500.	44800.0
5	15200.0	60400.0	10000.0	340800.	35100.0
6	18900.0	54700.0	10400.0	450300.	45900.0
7	14900.0	50100.0	10500.0	399900.	38100.0
8	17400.0	58800.0	11900.0	251700.	42400.0
9	24600.0	118600.	10200.0	293800.	46000.0
1970	22000.0	108700.	9200.00	417800.	53600.0
.1	24000.0	116300.	9700.00	402400.	56700.0

Table 14 Machinery and Equipment Fixed Capital Formation: Industry Groups 11-19

	Q11	Q12	Q13	Q14	Q15
1	191018.	55977.7	30026.0	66106.6	42342.0
2	314911.	73690.2	32319.2	72257.6	55192.0
3	263504.	69046.6	38680.7	98248.2	59106.5
4	395117.	96928.6	57330.6	163261.	65912.1
5	346947.	123346.	53500.9	256959.	81597.3
6	503883.	157454.	58422.8	253150.	122577.
7	359847.	136044.	61982.9	223954.	112752.
8	282730.	141513.	57322.2	158785.	83562.9
9	367179.	160368.	61647.6	210331.	101439.
10	467086.	153048.	64988.7	297852.	100370.
11	463852.	128763.	51209.4	157913.	108118.

	Z11	Z12	Z13	Z14	Z15
1	125500.	40800.0	23300.0	46200.0	30000.0
2	205300.	51300.0	24800.0	48100.0	37900.0
3	169300.	47800.0	29400.0	65999.9	40300.0
4	249100.	64400.0	41700.0	106700.	43000.0
5	222000.	82899.9	37400.0	172800.	53500.0
6	319400.	103500.	40100.0	164800.	79899.9
7	222400.	93499.9	44600.0	150800.	76899.9
8	170800.	97699.9	41700.0	104800.	57500.0
9	236300.	108000.	43800.0	140500.	69899.9
10	316200.	106000.	47500.0	207000.	72099.9
11	310300.	90499.9	37900.0	108400.	77699.9

	Q16	Q17	Q18	Q19
1	165234.	7822.86	169629.	23375.8
2	118938.	15943.7	115459.	31638.1
3	115929.	15186.9	142788.	32865.1
4	175825.	7769.03	177084.	32528.6
5	208864.	17110.8	322786.	42390.1
6	282084.	15982.8	331451.	55609.7
7	203850.	34544.7	310857.	51632.3
8	175203.	45616.6	256839.	55366.2
9	209730.	19906.7	182278.	70885.2
10	239275.	24934.5	190360.	49463.0
11	137561.	28061.6	175804.	57585.5

	Z16	Z17	Z18	Z19
1	44000.0	5400.00	117300.	15400.0
2	49700.0	11400.0	75399.9	19000.0
3	48100.0	10600.0	95499.9	18900.0
4	71799.9	4900.00	114100.	17200.0
5	85699.9	11300.0	215600.	22200.0
6	115300.	10200.0	216000.	31900.0
7	86499.9	23500.0	201000.	29100.0
8	73199.9	31800.0	168800.	27600.0
9	89599.9	13800.0	120800.	31700.0
10	105400.	17900.0	130100.	23300.0
11	57800.0	20100.0	120300.	26500.0



The stock estimate for the two capital goods was constructed from the investment series using the double declining balance method. We use the standard approach of taking depreciation to be at a constant percentage rate for groups of assets having a given useful economic life,  $T$ . The depreciation rate,  $\delta$ , under the double declining balance approach, equals twice the inverse of the estimated life by which 2/3 of the value of an asset evaporates in the first half-life.\*

The stock of a capital asset at time  $t$  is thus the stock carried over from the preceeding period less depreciation plus gross investment in the current period:

$$K_t = K_{t-1}(1 - \delta) + I_t, \quad \delta = 2/T.$$

For the stock in period  $t$ , cumulation is began from period  $t - T$ . In all, we have  $2 \times 19 = 38$  such series for information and noninformation M & E by industry. The cumulated stock can be expressed in equivalent terms as a series involving present and past investment:

$$K_t = I_t + I_{t-1}(1 - \delta) + I_{t-2}(1 - \delta)^2 + \dots + I_{t-T}(1 - \delta)^T$$

From this it is evident that recent year investment make up most of today's stock of assets with depreciation progressively reducing the proportion of past investment which remains as machines get older. Implicit in the expression is also the assumption that machines of different vintage have the same characteristics, apart from losses through wear and tear or reduced efficiency with aging. The machines operated in any one industry are worked

\*The life-time of non-information machines are taken as equal to the estimated life given by Statistics Canada. Two alternative assumptions on life-time are applied to information machines; the same as for noninformation on the one hand, and a uniform 10 years on the other.

with the same ratio of labour services to capital services as measured in the aggregate.

The price of capital services, for each capital good, has to be calculated from the asset price, the interest rate and the rate of depreciation. These rental prices were computed under the assumption that rentals are chosen such that the rate of return on holding one dollar worth of each of the capital goods are equal. The service price can be expressed as the sum of the cost of capital and the current cost of replacement less the cost of capital loss on the value of the asset

$$P_t^S = P_{t-1}^A r_t + P_t^A \delta - (P_t^A - P_{t-1}^A)$$

$P_t^S$  = service price at period  $t$

$P_t^A$  = asset price at period  $t$

$r$  = rate of return (discount or interest rate' the rate of return on corporate bonds is used as proxy variable).

Capital services,  $K^S$  are assumed to be proportional to the amount of capital assets,  $K_{ij}^S = u_j K_{ij}$ , where  $i = 1, 2, 3$  runs over the capital goods and  $j = 1, \dots, 19$  refer to the industries. Capital goods within an industry are utilized at the same rate according to this assumption, a rate which can be measured by the estimates for capacity utilization by industry,  $u_j$ . The procedure we follow is less exact. Under the assumption that utilization has been constant over the estimation period (not a good assumption in view of the trough which occurred in 1970), the utilization rate variable becomes irrelevant in the translog specification of production activities, a fact we put to use.

Up to a constant, the factor share equations are linear in the logarithm of input quantities. Denote the  $i$ th equation by the form

$$m_i = \alpha_i - \sum_j \gamma_{ij} \ln(x_j/c)$$

where  $c$  denotes construction services and  $j$  sum over inputs save construction.

The least square estimator of  $\gamma_{ij}$  will consist of terms containing the sum of products involving the deviation of  $\ln(x_j/c)$  from its sample mean

$$\ln x_{jt} - \frac{1}{T} \sum \ln(x_{jT}/c_T)$$

or

$$\ln \frac{x_{jt}/c}{(\prod_T (x_{jT}/c))}$$

Evidently, if the service at time  $t$ ,  $x_{jt}$  is related to a stock  $X_{jt}$  through a constant,  $x_{jt} = u_j X_{jt}$ , and likewise  $c_t = u C_t$ , the following equality holds:

$$\ln \frac{x_{ht}/c_t}{(\prod_T (x_{jT}/c_T))} = \ln \frac{x_{jt}/c_t}{(\prod_T (X_{jT}/C_T))}$$

Thus, to the degree that the assumption of a constant rate of utilization is reasonable, our estimation results will be relatively unaffected by the use of stock data rather than service figures.

In the representative industry case we do not assume a constant rate for all factors. Capital is assumed to have variable utilization, equal for all three types and equal to capacity utilization for the industry. The input quantities adjusted for capital utilization are shown in Table 15.

TABLE 15

## INPUT QUANTITIES with CAPITAL UTILIZATION

	LI	LN	KI	KN	C1
1	154839.	289865.	30381.7	.169177E+07	.124839E+07
2	158570.	286204.	30097.7	.169968E+07	.126245E+07
3	150798.	282720.	30655.3	.172107E+07	.131165E+07
4	145276.	287008.	32993.6	.176117E+07	.135365E+07
5	139975.	282247.	34361.2	.179917E+07	.140600E+07
6	4581.52	14783.8	6558.45	48358.7	56304.0
7	4510.41	13718.9	6093.39	45219.0	54400.0
8	4416.01	13122.7	6206.41	45890.1	55080.0
9	4379.36	13181.7	6166.75	45712.6	55814.0
10	4380.18	12356.6	6220.09	45815.1	56502.0
11	23153.6	60934.2	4345.18	132902.	139140.
12	24503.2	63187.0	3990.93	122100.	130160.
13	23921.2	66136.1	4253.54	134604.	140880.
14	22979.6	64075.7	4141.45	127433.	142569.
15	23492.0	63729.9	4807.40	136922.	144540.
16	9977.73	51676.4	726.683	28444.4	50406.0
17	10471.8	52749.1	775.315	30844.8	52416.0
18	9861.96	51605.8	774.178	31089.7	51243.0
19	8614.91	47089.4	686.882	28238.7	73185.9
20	8184.21	47144.4	691.128	28584.2	47763.0
21	31291.5	118841.	18587.0	412222.	186176.
22	27928.8	114376.	19162.5	438437.	204930.
23	27976.2	117262.	20975.9	488628.	234311.
24	27714.4	109588.	19483.7	449466.	223934.
25	26993.1	109206.	20903.9	479464.	244036.
26	37461.9	203732.	16484.3	66784.8	39833.0
27	36808.8	205199.	18188.7	72162.1	42306.0
28	35237.4	210789.	20944.2	77755.3	44712.0
29	32752.9	207257.	22179.3	79483.1	45080.0
30	32741.6	204764.	25437.1	87738.2	47348.0
31	27489.2	159917.	9580.43	458315.	245340.
32	27852.0	159458.	9939.36	473989.	248130.

(Table 15 Cont'd)

	LI	LN	KI	KN	CI
33	27638.6	162298.	11472.2	528866.	260859.
34	26192.5	151662.	11723.3	509202.	253748.
35	27773.9	159000.	13719.6	569093.	277749.
36	19609.6	72797.7	4518.14	67130.4	64400.0
37	19671.7	70796.6	4593.74	67288.7	67847.9
38	19522.6	72813.9	4675.41	68833.3	69960.0
39	18198.4	69313.9	4172.09	62278.9	63753.0
40	18103.1	70573.9	4236.95	63284.4	64938.0
41	48971.1	180152.	35234.3	.211213E+07	.122713E+07
42	49503.2	177321.	34417.7	.208178E+07	.126592E+07
43	51853.5	183851.	37412.9	.224953E+07	.140548E+07
44	51792.8	179253.	38860.3	.225012E+07	.141724E+07
45	51021.7	175036.	39279.0	.222949E+07	.142481E+07
46	66709.1	92390.1	44312.8	340433.	246145.
47	68333.8	92074.9	44053.5	341036.	247032.
48	70181.3	92469.7	45535.2	350134.	254220.
49	66272.8	95875.5	46664.5	349896.	254169.
50	67533.9	93529.6	49224.0	358800.	260028.
51	44968.0	173350.	41989.5	.176225E+07	.108248E+07
52	46971.6	170306.	43310.6	.182086E+07	.118053E+07
53	48813.1	164253.	44350.8	.180166E+07	.119231E+07
54	49260.3	174780.	49601.5	.191364E+07	.125280E+07
55	49824.2	168983.	50588.1	.186617E+07	.122374E+07
56	69390.4	211660.	24738.6	545984.	441350.
57	72553.4	205498.	25395.4	540764.	432825.
58	71465.1	213129.	30087.6	646329.	504009.
59	65984.6	215542.	28054.6	573980.	445740.
60	64375.2	208607.	27903.6	566236.	442224.
61	60393.9	97833.4	24484.7	204975.	198390.
62	62653.4	89776.9	23590.5	199232.	195280.

(Table 15 Cont'd)

	LI	LN	KI	KN	CI
63	63649.5	98147.2	26372.9	222189.	217580.
64	63714.2	94778.4	26334.9	220133.	219924.
65	45259.7	90676.7	25987.7	215524.	224000.
66	76078.1	218015.	66095.0	736278.	533071.
67	77901.4	221823.	70059.1	805403.	599343.
68	82207.7	232079.	81044.9	895843.	660469.
69	81774.1	205265.	77763.7	789892.	558669.
70	77152.6	217839.	85376.5	889416.	637960.
71	89265.7	152551.	32503.1	394023.	269284.
72	84381.6	150079.	31751.6	385767.	273942.
73	82616.3	156677.	33577.8	412641.	293756.
74	83733.1	145730.	31922.9	382147.	277469.
75	88927.0	142668.	34192.9	399280.	294420.
76	24847.9	80784.9	34756.2	606755.	353525.
77	25400.7	80144.7	33011.3	590119.	366086.
78	25512.1	80681.3	32566.5	580709.	375487.
79	24868.0	74735.1	29632.7	526143.	354240.
80	24650.3	78613.6	29767.7	556592.	393894.
81	14489.3	14366.5	2061.21	204308.	.117991E+07
82	14571.3	14307.5	2180.03	225673.	.130675E+07
83	14094.9	13341.2	1920.22	217678.	.137132E+07
84	14149.7	13456.5	1738.45	208567.	.145590E+07
85	14585.3	13640.9	1633.49	203731.	.157662E+07
86	67954.5	75354.3	47520.4	.105609E+07	875910.
87	70959.6	77098.2	50051.2	.108262E+07	963543.
88	72783.2	79739.0	51620.9	.110726E+07	.110591E+07
89	73069.5	79654.0	49142.4	.106316E+07	.116117E+07
90	72989.4	75675.4	48464.1	.106224E+07	.125202E+07
91	33631.8	80475.2	8480.54	111392.	89935.9
92	33428.8	80069.1	9609.57	126103.	101409.
93	36025.1	83420.2	11188.0	144449.	109336.
94	33204.7	79969.2	10764.1	139131.	109575.
95	33655.9	80614.0	11681.1	150414.	119106.
	1	2	3	4	5

## 8. POINTS OF ANALYSIS

In this chapter we gather together some technical points relevant to the analysis but whose interest is chiefly to the technically inclined economist.

### 1. Price Elasticity and Allen Elasticity of Substitution (AES)

Given the production function  $y = F(x)$ ,  $x = (x_1, x_2, \dots, x_n)$  and profit maximizing behaviour we obtain the first order conditions

$$pF_i(x) = p_i \quad i = 1, 2, \dots, n$$

where  $p$  is the Lagrangian multiplier and  $p_i$  are the input prices. Since our objective is to obtain a formula for price elasticities it is appropriate to transform the above equilibrium expressions into the equivalent nature logarithmic form;

$$f_i = m_i \quad i = 1, 2, \dots, n$$

subject to constant output,

$$\sum f_i d \ln x_i = 0,$$

where  $f_i$  denotes  $\partial \ln F / \ln x_i$  and  $m_i$  the marginal value share of the  $i$ th factor input, i.e.  $p_i x_i / pF$ . The elasticity parameters are a measure of the response of the equilibrium to a small change in prices. The manner in which the system responds is calculated by taking the total derivative of the foregoing equations. Denote the Hessian matrix of  $\ln F$  by  $f_{xx} = (f_{ij}) \equiv (\partial^2 \ln F / \partial \ln x_i \partial \ln x_j)$  and the  $n \times 1$  vector of

of first partial derivatives by  $f_x \equiv (f_i) = (m_i) = m$ . The  $1 \times n$  transpose is denoted by  $f_x' = m$ . Also note that

$$dm_i = m_i (d \ln p_i - d \ln x_i - d \ln F - d \ln p)$$

(The fact that  $d \ln F = 0$  (constant output) is incorporated in the side condition but not at this juncture.) The total derivative of the first order conditions above is

$$\sum_j (f_{ij} - m_i \delta_{ij}) d \ln x_j - m_i d \ln p = m_i d \ln p_i + m_i \sum_j m_j d \ln x_j$$

i.e.

$$\sum_j (f_{ij} - m_i \delta_{ij} - m_i m_j) d \ln x_j - m_i d \ln p = m_i d \ln p_i$$

again with the side condition

$$\sum_j m_j d \ln x_j = 0$$

In matrix form we have

$$\begin{bmatrix} f_{xx} - mm' - M & m \\ m' & 0 \end{bmatrix} \begin{bmatrix} \eta \\ \mu' \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} M$$

Where  $M = (m_i \delta_{ij})$  is the  $n \times n$  diagonal matrix whose diagonal elements are the marginal value shares ;  $\eta = (\eta_{ij}) = (d \ln x_i / d \ln p_j)$  is the  $n - 1$  matrix of price elasticities and the  $n \times 1$  vector

$\mu = (d \ln p / d \ln p_i)$  gives the response of the product price to change in inputs prices. Denote by  $G$  the  $n+1 \times n+1$  matrix on the far left of the equation. The matrix is symmetric and non-singular since  $f_{xx}$  has these properties. Thus  $G^{-1}$  exists, is symmetric and

$$\begin{bmatrix} \eta \\ \mu' \end{bmatrix} = G^{-1} \begin{bmatrix} M \\ 0 \end{bmatrix}$$



Because of the presence of the M matrix on the right hand side the matrix of price elasticities cannot be symmetric except in the special case where all value shares are equal. This asymmetry can be remedied by rewriting the above expression as

$$\begin{bmatrix} \eta \\ \mu' \end{bmatrix} M^{-1} = G^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Define the matrix  $\sigma = (\sigma_{ij}) = \left( \frac{1}{m_j} \frac{\partial \ln x_i}{\partial \ln p_j} \right)$  then the matrix  $\sigma$  is symmetric as

$$\begin{bmatrix} \eta M^{-1} \\ \mu' M^{-1} \end{bmatrix} = \begin{bmatrix} \sigma \\ \mu M^{-1} \end{bmatrix} = G^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix}$$

$\sigma$  is the matrix of Allen elasticities of substitution (AES) and has the symmetry property  $\sigma_{ij} = \sigma_{ji}$ . The inverse of the G matrix yields the AES plus the elasticity of the product price with respect to input prices adjusted by the appropriate value share.

To show that  $\sigma$  is a negative definite matrix, first of all observe that the inverse of a negative definite matrix is also negative definite. For, if  $\alpha$  is an arbitrary non-zero  $n \times 1$  vector and H is a negative definite matrix, then by Schwarz' inequality

$$(\alpha' H \alpha) (\beta' H \beta) \geq (\alpha' H \beta)^2 > 0$$

Set  $\beta = H^{-1} \alpha$ ,

$$(\alpha' H \alpha) (\alpha' H^{-1} \alpha) > 0$$

Thus  $\alpha' H^{-1} \alpha$  has the same sign as  $\alpha' H \alpha$ . Secondly  $[f_{xx} - mm' - M]$  is negative definite since  $f_{xx}$  has this property. Since

$$\alpha' (f_{xx} - mm' - M) \alpha = \alpha' f_{xx} \alpha - (\alpha' m)^2 - \alpha' M \alpha$$

$$\text{and } (\alpha' m)^2 - \alpha' M \alpha = (\sum m_i \alpha_i)^2 - \sum m_i \alpha_i^2$$

we need to show that

$$(\sum m_i \alpha_i)^2 \leq \sum m_i \alpha_i^2$$

that is, the convex combination ( $\sum m_i = 1$ ) of squares is no less than the square of the convex combination. This type of relationship is in fact the definitional relation for convex functions i.e.  $f(\alpha)$  is convex if and only if

$$f(\sum m_i \alpha_i) \leq \sum m_i f(\alpha_i)$$

Since the second derivative of the square function is positive,  $f(\alpha) = \alpha^2$  is convex. We have shown  $H = [f_{xx} - mm' - M]$  is negative definite and it follows that  $H^{-1}$  is negative definite also.

The inverse of the partitioned matrix

$$G = \begin{bmatrix} H & m \\ m' & 0 \end{bmatrix}$$

when pre-multiplied into  $\begin{bmatrix} I \\ 0 \end{bmatrix}$  can be expressed

$$\text{as } G^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} = \frac{1}{m' H^{-1} m} \begin{bmatrix} (m' H^{-1} m) H^{-1} - H^{-1} m (H^{-1} m)' \\ m' H^{-1} \end{bmatrix}$$

so that the AES matrix is

$$\sigma = H^{-1} - \frac{1}{m' H^{-1} m} (H^{-1} m) (H^{-1} m)'$$

Finally we now show that  $\sigma$  is also negative definite.

We have for any  $\alpha$ ,

$$\alpha' \sigma \alpha = \frac{1}{m H^{-1} m} \left[ (\alpha' H^{-1} \alpha) (m H^{-1} \alpha) - (\alpha' H^{-1} \beta)^2 \right]$$

By Schwarz' inequality the term in the bracket is positive excepting for the special case of  $\alpha$  being a scalar multiple of  $m$ , when the term vanishes. Thus  $\alpha' \sigma \alpha$  takes the sign  $m' H^{-1} m < 1$  for all nonzero vectors  $\alpha$ . It follows for example that factor demand is downward sloping as  $\sigma_{ii} < 0$  for all  $i$ .

Explicitly the  $(n - 1) \times (n - 1)$  symmetric matrix  $G$  is

$$G = \begin{bmatrix} \gamma_{11} - (mx_1)^2 - mx_1 & \gamma_{12} - mx_1 mx_2 & \dots & m_1 \\ \gamma_{12} - mx_1 mx_2 & \gamma_{22} - (mx_2)^2 - mx_2 & \dots & m_2 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ m_1 & m_2 & \dots & 0 \end{bmatrix}$$

and

$$\sigma_{ij} = \frac{|G_{ij}|}{|G|}, \text{ or the } i, j \text{ element of the inverse of the } G \text{ matrix.}$$

## 2. Separation: Comparison of True Separation with Test for Conjectured Separation

Consider a four factor cost function for which the true separation is as follows:

$$c = c(a(p_1, p_2), p_3, p_4)$$

This corresponds to the following constraints on the AES

$$\sigma_{13} = \sigma_{23}, \quad \sigma_{14} = \sigma_{24}$$

However, suppose data are available only in the aggregate form thus  $(x_1, g(x_2, x_4), x_3)$ , so that we have  $c = c(x_1, g(x_2, x_4), x_3)$  and the true situation is not known to us. Consider now the exercise of testing for the separation of  $g$  and  $x_3$  from  $x_1$ , i.e. the test for

$$\sigma_{1g} = \sigma_{13}.$$

Given the true situation will the test turn out affirmative? Not necessarily, as we can see from  $\sigma_{ij} = c_{ij} c / c_i c_j$  and the fact that the true state yields the conditions

$$\frac{c_{13}}{c_1} = \frac{c_{23}}{c_2} \quad \text{and} \quad \frac{c_{14}}{c_1} = \frac{c_{24}}{c_2}$$

or

$$\frac{c_1}{c_2} = \frac{c_{13}}{c_{23}} = \frac{c_{14}}{c_{24}}.$$

Note that

$$c_i = c_g \frac{\partial p_g}{\partial p_i}, \quad i = 2, 4; \quad c_{23} = c_{g3} \frac{\partial p_g}{\partial p_2}$$

$$\text{and } c_{14} = c_{1g} \frac{\partial p_g}{\partial p_4}$$

$$c_{24} = c_g \frac{\partial^2 p_g}{\partial p_2 \partial p_4} - c_{gg} \frac{\partial p_g}{\partial p_2} \frac{\partial p_g}{\partial p_4}$$

$$(c_g = \frac{\partial c}{\partial p_g}) .$$

Assume, for sake of simplicity, the aggregator,  $g$ , is linear, so that  $\partial^2 p_g / \partial p_2 \partial p_4 = 0$ .

The conditions met in the actual situation can be translated in terms of the aggregate variable  $g$  to read

$$\frac{\partial p_g}{\partial p_2} \frac{c_1}{c_3} = \frac{c_{13}}{c_{3g}} = \frac{c_{1g}}{c_{gg}}$$

However the condition we are seeking is

$$\frac{c_{1g}}{c_g} = \frac{c_{13}}{c_3} .$$

Thus the two coincide only if  $\frac{c_{3g}}{c_3} = \frac{c_{gg}}{c_g}$ .

But this implies  $\frac{\partial}{\partial g}(c_g/c_3) = 0$ , or  $c_g = kc_3$ , where  $k$  = constant of integration independent of  $p_g$ . By Shephard's lemma this corresponds to the very unlikely case that the aggregate factor  $g$  and the factor 3 are hired (fired) in fixed ratio wherever the price of  $g$  falls (rises). Thus using data in too aggregative a form can lead to misleading results except in very special and unlikely cases. For this reason both capital and labour inputs should be suitably disaggregated when studying the separation of information and non-information activities.

### 3. Productivity Growth

In the context of the production function estimations of this study the following approach to productivity is feasible. Let  $Q$  and  $g$  denote the level and the growth rate of real value added output respectively.

Write  $y \equiv (\ln x_1, \ln x_2, \dots, \ln x_n)'$ , the matrix  $\Gamma = (\gamma_{ij})$  and

$= (\alpha_1, \alpha_2, \dots, \alpha_n)'$ , where ' denotes transpose. Then the translog specification can be written as

$$\ln Q = \alpha'y - \frac{1}{2}y' \Gamma y$$

Let  $\rho = (\rho_1, \rho_2, \dots, \rho_n)'$  where

$$\rho_i = \frac{d \ln x_i}{dt}$$

be the vector of the  $i$ th input growth rate. Denote by  $g$  the observed rate of growth output, then

$$g = (\alpha - \Gamma y)$$

Let  $w = (w_1, w_2, \dots, w_n)'$

where

$$w_t = \alpha - \Gamma y_t$$

is the vector of relative share of inputs in total costs at year  $t$ .

Write  $W = (w_{it})$ , then  $\rho$  can be estimated by least squares from

$$g = W\rho$$

From this regression we can obtain an estimate,  $\rho_I$ , of the average growth rate for information labour over the estimation period. Now  $\rho_I$  can be interpreted as a composite of two growths; a growth in employment plus the growth in information labour "quality". An average for the employment growth,  $g_I$ , can be directly calculated from the data so that the difference  $(\rho_I - g_I)$  gives the average growth (positive or negative) in quality. The data set can be subdivided into two or more parts and the various growth rates calculated for each part. Then one can explore changes which took place in the quality of information labour.

FOOTNOTES

1. Porat (1977).
2. F. Machlup (1962).
3. Department of Communications (1978).
4. The issue of information accounts involves more than simply the niceties of consistent aggregation of inputs or of outputs. A much more serious question involves the appropriateness and rigor of the conceptual framework behind the exercise and its correspondence with economic behaviour. See my "Economic Theory and the Information Economy", Department of Communications (1978).
5. This description of representation of the technical structure of production through a functional relation between (value added) output and inputs refers to a given "state of the art" in production. The techniques of manufacturing can and do improve over time and the specification is later extended to allow for such technological progress to take place.
6. This and other assumptions are discussed in terms of their reality and impact on the results subsequently.
7. The value added concept involves an a priori assumption regarding the separability of intermediate inputs from the primary inputs. This assumption for Canadian manufacturing has been examined and attacked by M. Denny and D. May (1977).
8. A different specification, the Generalized Leontief, is becoming popular for the cost function.
9. L.R. Christensen, D.W. Jorgenson, and L.J. Lau, (1971).
10. E. Erwin Diewert (1971).
11. C. Cobb and P.H. Douglas (1928).
12. K. Arrow, H. Chenery, B. Minhas and R. Solow (1961).
13. E.R. Berndt and L.R. Christensen (1974).
14. Science Council, Special Study No. 22.
15. This is a time series (1929-1968) study by Berndt and Christensen, Oct. 1973.

16. If constant returns to scale holds, factor augmenting technical change with equal rates of augmentation is equivalent to Hicks-neutral technical change. This shows why the Hicks-neutral type of change is readily handled and also shows why it is not a particularly plausible case for a study of information and noninformation activities. Technical change seems to have been labour saving over the period in question but this issue requires a more careful attention and will be looked at in a further study.
17. This section closely follows the analysis of Berndt, E.R. and L.R. Christensen (1973a).
18. Theorem 4.5, R.T. Rockefeller (1970).
19. M. Denny and C. Pinto (1976).



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