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FUNCTIONAL FORMS IN THE MODELLING
OF REGULATORY CONSTRAINT⁺

J. Breslaw^{*}
V. Corbo
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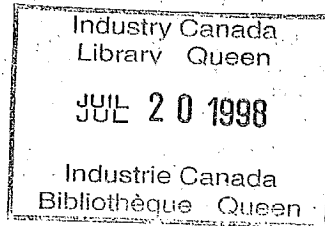
* Department of Economics, and Institute of Applied
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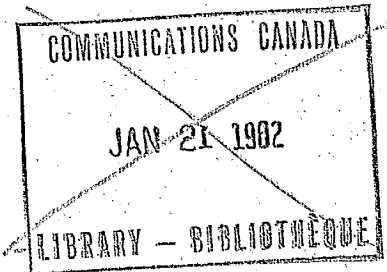
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INTRODUCTION

The Averch - Johnson (A-J) model of regulatory constraint has provided many theoretical insights into the behavior of regulated firms. Bailey (1973), in extending the A - J results, has pointed out a number of empirically verifiable implications of rate of return regulation. Spann (1974), Courville (1974), Petersen (1974) and Cowing (1978) have subsequently tested these implications of the A - J model. Although the specified models differ, the conclusions drawn for a cross-section of electric utilities generally support the existence of a binding regulatory constraint.

The model presented by Cowing represents a significant increase in generality over previous studies. Cowing hypothesized profit maximizing behavior¹ subject to rate of return constraint and used an extended version of Sheppard's Lemma to derive additional behavioral restrictions. For estimation, the regulated profit function was approximated by a (quadratic) flexible functional form. As such Cowing's model; (a) did not require that the regulatory multiplier be constant across firms (as in the Spann model), (b) allowed all inputs to be simultaneously determined (in a fashion superior to the Courville model) and finally (c) did not introduce restrictive variable definitions nor ignore cross-equation parameter constraints as in the Petersen model.

¹ Many public utilities are regulated by output price as well as rate of return. It is not impossible and, in fact, extremely likely that the demand for the aggregate of service outputs is inelastic. If this is the case then profit-maximizing is not in general an appropriate behavioral hypothesis. For example, the elasticity of demand for an aggregate of service outputs of Bell Canada is only -.38.

Notwithstanding Cowing's results, it is the purpose of this paper to demonstrate that additional restrictions are implied by rate of return regulation and when these conditions are considered it becomes evident that the A - J model cannot be properly tested using flexible functional forms of degree two in price.

In the first section, the A - J model is outlined and the additional conditions are derived. In the second section, the argument presented above is illustrated for a regulated firm for which the cost function is approximated alternatively by a translog quadratic, Diewert and generalized Cobb-Douglas function. Finally a third order approximation of the cost function is considered.²

² The restrictiveness problems demonstrated in this paper hold in addition to those demonstrated by Blackorby, Primont and Russel (1977) for flexible approximations to unregulated models of cost and production.

SECTION I

An A-J Model with Cost Minimization

Consider a firm that is simultaneously regulated with respect to price (and hence output from satisfaction of demand) and rate of return on economic capital. The firm behaves so as to minimize cost,³

$$C = wL + rK + vM \quad (1)$$

subject to the production function⁴

$$F(Q, K, L, M) = 0 \quad (2)$$

and a regulatory rate of return constraint

$$p \cdot Q - wL - vM - sK = 0 \quad (4)$$

where Q = output with price p ;

K = capital with rental rate r

L = labour with wage rate w

M = materials with price rate v

s = permitted rate of return on economic capital

The Lagrangian corresponding to the cost minimization problem is:

$$\mathcal{L} = wL + rK + vM + \lambda_1 F(Q, L, K, M) + \lambda_2 (p \cdot Q - wL - vM - sK) \quad (5)$$

where λ_1 is the Lagrange multiplier for the production function and λ_2 is the Lagrange multiplier for the regulatory constraint.

The first order (interior) conditions are:

$$\frac{\partial \mathcal{L}}{\partial L} = w(1 - \lambda_2) + \lambda_1 \frac{\partial F}{\partial L} = 0 \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda_2 s + \lambda_1 \frac{\partial F}{\partial K} = 0 \quad (7)$$

³ Treating just the three input case leads to no loss in generality.

⁴ The production is assumed positive monotonistic in (K, L, M) and quasi-concave.

$$\frac{\partial \mathcal{L}}{\partial M} = v(1-\lambda_2) + \lambda_1 \frac{\partial F}{\partial M} = 0 \quad (8)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = F(Q, L, K, M) = 0 \quad (9)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = p \cdot Q - wL - vM - sK = 0 \quad (10)$$

In principle, the optimal level of each input - L^* , M^* and K^* , as well as the Lagrange multipliers (λ_1^* , λ_2^*) can be solved in terms of (w, r, v, s, p, Q). Thus the minimum cost can be written:

$$\begin{aligned} C^* &= wL^* + vM^* + rK^* \\ &= C(w, r, v, s, p, Q) \end{aligned} \quad (11)$$

However, several restrictions are implied by optimization.

1 The cost function must be homogeneous of degree one in (w, r, v, s, p) instead of the standard case of homogeneous of degree one in (w, r, v). This restriction should normally be tested in empirical work.

2 The first order conditions imply restrictions on optimally chosen factors. In particular,

$$\frac{\partial L^*}{\partial r} = \frac{\partial K^*}{\partial r} = \frac{\partial M^*}{\partial r} = 0 \quad (12)$$

This follows immediately from the system of equations (6) - (10). Define:

$$\theta = \lambda_1 / (1 - \lambda_2) \quad (13)$$

Equations (6), (8), (9) and (10) can then be used to solve for (θ^* , L^* , M^* , K^*) in terms of (s, w, v, p, Q). Hence, at the optimum, θ^* , L^* , K^* and M^* are independent of r . From equations (7) and (13), it follows that λ_1 and λ_2 can be solved directly as:

$$\lambda_1^* = (s-r) A(s, w, v, p, Q) \quad (14)$$

$$\lambda_2^* = 1 - (s-r) B(s, w, v, p, Q) \quad (15)$$

Bailey (1973) has shown that restriction (12) must hold for profit maximization. It is important to recognize that these conditions hold also in the less restrictive case of cost minimization. In fact it is these restrictions which ultimately undermine the usefulness of employing price functions to model regulated firms. This argument is demonstrated in the section which follows.

SECTION 2

Restrictiveness of Flexible Functional Forms

In line with the recent econometric analysis of regulated firms, the cost function⁵ described by (11) can be approximated to second order by a number of flexible functional forms in (s, w, r, v, Q, p) . As well, additional information can be introduced into the estimated model by considering the optimization process which defined the regulated cost function. An extended version of Sheppard's Lemma can be used to show that:

$$\frac{\partial C}{\partial w} = (1 - \lambda_2) L^* \quad (16)$$

$$\frac{\partial C}{\partial r} = K^* \quad (17)$$

$$\frac{\partial C}{\partial s} = -\lambda_2 K^* \quad (18)$$

$$\frac{\partial C}{\partial v} = (1 - \lambda_2) M^* \quad (19)$$

$$\frac{\partial C}{\partial p} = \lambda_2 Q \quad (20)$$

In principle, equations (17) and (18) can be used to eliminate the unobservable variable λ_2 from equations (16), (18), (19) and (20) and these new equations can be used along with the cost function to estimate characteristics of the regulated production technology as well as the effectiveness of rate of return regulation.

From a statistical point of view, the model can be estimated in a straightforward fashion. As well, the independence restrictions given in (12) can be used to constrain the cost model of

⁵ The reader may wish to verify that even when technology is of the Cobb-Douglas variety, closed form solutions for L , K and M and consequently C do not exist. Thus, under regulatory constraint even Cobb-Douglas cost and production functions are not self-dual.

the regulated firm. However, condition (13)² implies restrictions which, dependent on the functional form of the cost function, will, in general, not be met by the data.

In the general case; using conditions (12) and (16)-(19)

$$\frac{\partial C}{\partial w} = (1-\lambda_2) L^* \therefore \frac{\partial^2 C}{\partial w \partial r} = -\frac{\partial \lambda_2}{\partial r} L^* \quad (21)$$

$$\frac{\partial C}{\partial r} = K^* \therefore \frac{\partial^2 C}{\partial r^2} = 0 \quad (22)$$

$$\frac{\partial C}{\partial s} = -\lambda_2 K^* \therefore \frac{\partial^2 C}{\partial s \partial r} = -\frac{\partial \lambda_2}{\partial r} K^* \quad (23)$$

$$\frac{\partial C}{\partial v} = (1-\lambda_2) M^* \therefore \frac{\partial^2 C}{\partial v \partial r} = -\frac{\partial \lambda_2}{\partial r} M^* \quad (24)$$

It follows from (14) and (15) that if the production constraint is binding, $\partial \lambda_2 / \partial r \neq 0$ ⁶. Hence:

$$\frac{L^*}{K^*} = \frac{\partial^2 C / \partial w \partial r}{\partial^2 C / \partial s \partial r} \quad (25)$$

$$\frac{M^*}{K^*} = \frac{\partial^2 C / \partial v \partial r}{\partial^2 C / \partial s \partial r} \quad (26)$$

1. Quadratic Cost Function⁷

$$C = C_0 + \sum_i C_i X_i + \frac{1}{2} \sum_i \sum_j C_{ij} X_i X_j \quad (27)$$

$$X_i \in X : X = (w, v, r, s, p, Q)$$

$$\frac{\partial^2 C}{\partial r^2} = C_{rr} \quad \frac{\partial^2 C}{\partial w \partial r} = C_{wr} \quad \frac{\partial^2 C}{\partial v \partial r} = C_{vr} \quad \frac{\partial^2 C}{\partial s \partial r} = C_{rs}$$

$$\text{From (25) } L^*/K^* = C_{wr}/C_{rs} \quad (28)$$

$$\text{From (26) } M^*/K^* = C_{vr}/C_{rs} \quad (29)$$

or constant factor ratios must hold for each data point.

⁶ Suppose to the contrary that $\frac{\partial \lambda_2}{\partial r} = 0$. Then, from (15) $B(s, w, v, p, Q) = 0 \Rightarrow \lambda_2 = 1$. However, this implies either $\lambda_1 = 0$ or $\theta = \infty$. The former negates the production constraint being binding. The latter violates the monotonicity and quasi-concavity assumptions. Hence $\frac{\partial \lambda_2}{\partial r} \neq 0$.

2. Translog Cost Function

$$\text{Log } (C) = C_0 + \sum_i C_i \text{Log } (X_i) + \frac{1}{2} \sum_i \sum_j C_{ij} \text{Log } (X_i) \text{Log } (X_j) \quad (30)$$

$$\frac{\partial^2 C}{\partial r^2} = r^2 K^{*2} - C_r K^* + C^2 C_{rr} = 0 \quad (31)$$

$$\frac{\partial^2 C}{\partial w \partial r} = \frac{1}{C} (1 - \lambda_2) L^* K^* + \frac{C}{w r} C_{wr} = -\frac{\partial \lambda_2}{\partial r} L^* \quad (32)$$

$$\frac{\partial^2 C}{\partial v \partial r} = \frac{1}{C} (1 - \lambda_2) M^* K^* + \frac{C}{v r} C_{vr} = -\frac{\partial \lambda_2}{\partial r} M^* \quad (33)$$

Thus from (31)

$$\frac{K^* r}{C} = \frac{1}{2} [1 \pm (1 - 4 C_{rr})^{.5}] \quad (34)$$

and from (32) and (33)

$$\frac{L^* w}{M^* v} = \frac{C_{wr}}{C_{vr}} \quad (35)$$

Thus, the cost share of each factor is constant, for each data point.

3. Diewert Cost Function

$$C = \sum_i \sum_j C_{ij} X_i^{\frac{1}{2}} X_j^{\frac{1}{2}} \quad (36)$$

$$\frac{\partial^2 C}{\partial r^2} = \frac{-1}{2r} [K^* - C_{rr}]$$

$$\frac{\partial^2 C}{\partial w \partial r} = \frac{1}{2} \frac{C_{wr}}{\sqrt{w r}} \quad \frac{\partial^2 C}{\partial v \partial r} = \frac{1}{2} \frac{C_{vr}}{\sqrt{v r}} \quad \frac{\partial^2 C}{\partial s \partial r} = \frac{1}{2} \frac{C_{rs}}{\sqrt{r s}}$$

$$\text{From (22)} \quad K^* = C_{rr} \quad (37)$$

$$\text{From (25)} \quad \frac{L^*}{K^*} = \frac{C_{wr}/\sqrt{w r}}{C_{rs}/\sqrt{r s}} \Rightarrow \frac{L^* \sqrt{w}}{K^* \sqrt{s}} = \frac{C_{wr}}{C_{rs}} \quad (38)$$

$$\text{From (26)} \quad \frac{M^*}{K^*} = \frac{C_{vr}/\sqrt{v r}}{C_{rs}/\sqrt{r s}} \Rightarrow \frac{M^* \sqrt{v}}{K^* \sqrt{s}} = \frac{C_{vr}}{C_{rs}} \quad (39)$$

⁷Neither the quadratic cost function nor the Diewert cost function can exhibit homogeneity of degree one in all prices, since terms in Q enter additively. The exercise is valid however for profit functions which are not subject to homogeneity constraints.

ie. capital is constant over all data points, and the relationship shown in Equations (38) and (39) also holds across all data points.

4. Generalized Cobb-Douglas Cost Function

$$C = \pi \pi_{ij} \left(\frac{X_i + X_j}{2} \right)^{C_{ij}} \quad (40)$$

$$\frac{\partial^2 C}{\partial w \partial r} = \frac{-C C_{wr}}{(w+r)^2} + \frac{(1-\lambda_2) L^* K^*}{C} = -\frac{\partial \lambda_2}{\partial r} L^* \quad (41)$$

$$\frac{\partial^2 C}{\partial v \partial r} = \frac{-C C_{vr}}{(v+r)^2} + \frac{(1-\lambda_2) M^* K^*}{C} = \frac{\partial \lambda_2}{\partial r} M^* \quad (42)$$

$$\text{Hence } \frac{L^*}{M^*} \frac{(w+r)^2}{(v+r)^2} = \frac{C_{wr}}{C_{vr}} \quad (43)$$

Thus, in this case, the relationship shown in Equation (43) must hold across all data points.

3rd Order Cost Functions

The restrictions implied by the four functional forms discussed above are sufficient to invalidate the use of standard second degree functional approximations of the cost function for a regulated firm. As seen above, the second order approximation is not sufficiently general.

A third order translog approximation of the cost function is:

$$\begin{aligned} \text{Log } (C) = & C_0 + \sum_i C_i \text{Log } (X_i) + \frac{1}{2} \sum_{ij} C_{ij} \text{Log } (X_i) \text{Log } (X_j) \\ & + \frac{1}{6} \sum_{ijk} C_{ijk} \text{Log } (X_i) \text{Log } (X_j) \text{Log } (X_k) \end{aligned} \quad (44)$$

from which:

$$\frac{\partial^2 C}{\partial r^2} = r^2 K^{*2} - CrK^* + C^2 \left[C_{rr} + \sum_k C_{rrk} \text{Log}(X_k) \right] = 0 \quad (45)$$

$$X_k = [w, v, r, s, p, Q]$$

$$\frac{\partial^2 C}{\partial w \partial r} = (1-\lambda_2) \frac{L^* K^*}{C} + C \left[\frac{C_{wr}}{wr} + \frac{1}{wr} \sum_k C_{wrk} \text{Log}(X_k) \right] = \frac{\partial \lambda_2}{\partial r} L^* \quad (46)$$

and a similar term for $\frac{\partial^2 C}{\partial v \partial r}$

Thus solving (45)

$$\frac{K^*}{C} = \frac{1}{2} \left[1 \pm \left(1 - 4 \left[C_{rr} + \sum_k C_{rrk} \text{Log}(X_k) \right] \right)^{.5} \right] \quad (47)$$

and

$$\frac{L^* w}{M^* v} = \frac{C_{wr} + \sum_k C_{wrk} \text{Log}(X_k)}{C_{vr} + \sum_k C_{vrk} \text{Log}(X_k)} \quad (48)$$

There is no theoretical reason why the system of equations (11), (16), (17), (19), (20)⁸, (47) and (48) should not be used to estimate the parameters of the cost function. However, in the case of $C = C(w, v, r, s, p, Q)$ there will be 84 parameters, assuming symmetry. Imposing homogeneity of degree one in prices imposes 11 restrictions. Hence with 7 equations and (say) a sample period of 40 data point years, there will be 73 parameters to be estimated and 207 degrees of freedom. Although econometrically this is not unreasonable, computationally the system will be too large.

⁸ Using (18) to eliminate λ_2

CONCLUSIONS

In this paper it has been shown that the first order conditions for cost minimization of a firm subject to rate of return regulation imply a set of restrictions on the second derivatives of the cost function. Further, a second order approximation of a cost function has been shown to be, in general, too restrictive to satisfy most data when these conditions are imposed. If the cost function is made more general (for example, a third order approximation) the function is shown to be sufficiently general to satisfy the second derivative conditions, but the estimation problems arising from a model of this size may well be insurmountable, given present computational technology.

Thus, it would appear that the production function approach provides the most promising direction of future empirical research into regulation. The problem that arises using this methodology is that, unlike the cost function approach, there is no way of eliminating λ_2 from the estimation process.⁹ However, since λ_2 will in general take a different value each period, the number of λ_2 parameters equals the number of data points. The authors have designed a tentative approach to solve this problem (Breslaw et al, 1979); unfortunately it also promises to be computationally expensive.

⁹ If Equation (7) is excluded from the model, the λ problem is solved, but it is the experience of the authors that in this case many of the parameters of the model cannot be estimated with precision.

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