



QUANTIFYING THE PENALTY OF ADOPTING A
WORST PHILOSOPHY IN THE DEVELOPMENT
AND ANALYSIS OF SATELLITE ORBIT SCENARIOS

by

Dr. Robert W. Donaldson

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DEPARTMENT OF ELECTRICAL ENGINEERING
FACULTY OF APPLIED SCIENCE
THE UNIVERSITY OF BRITISH COLUMBIA

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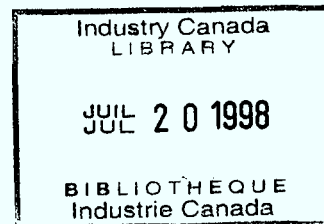
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FINAL REPORT

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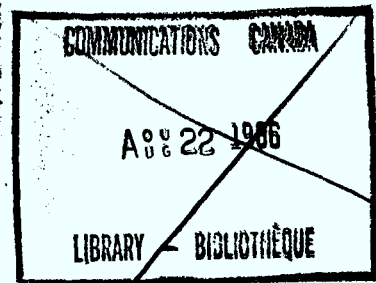
by



Dr. Robert W. Donaldson, Professor
Department of Electrical Engineering
Faculty of Applied Science
The University of British Columbia

for

Department of Communications
Ottawa, Ontario, Canada



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ABSTRACT

This report details a workable method to obtain carrier-to-interference (c/i) statistical distributions for geostationary satellite networks. The approach involves: (1) the convolution of the probability density functions of the link variables in dB to determine the probability density for each up-link and each down-link interferer; (2) transformation of the resulting density to power (non-dB) format; (3) convolution of these interference power densities for the different interfering links and; (4) transformation to dB format to obtain the aggregate i/c density and distribution function.

Small variations in link parameters are assumed to have a Gaussian probability density. Variation in off-boresight antenna gains and unmitigated rain fades cause large interference variations. The antenna gain variation density is obtained from an integral involving two probability densities; one for the actual antenna patterns relative to a standard reference pattern and another for the angular variations from nominal due to antenna mispointing and satellite station-keeping errors. Rain fade attenuation densities are also described and utilized in the analysis.

The method is used to obtain actual i/c distributions for various satellite networks, using realistic tolerances and orbit spacings. These distributions are then used to determine probabilities that i/c fails to exceed any given value. The results are then compared with those obtained using a conventional worst case analysis approach. A difference of several dB in calculated i/c levels can result, based on the two different approaches.

This work quantifies the penalty in using a worst case design approach for different nominal satellite separations when up to four interfering signals are present.

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I INTRODUCTION

An earlier report [1] developed a detailed model for small variations in interference and wanted signal levels in satellite networks. Variations in signal level at a particular earth or satellite location result from variations in transmissibility on the up-link and down-link, from pointing errors associated with transmit and receive earth station antennas and satellite antennas, from satellite station-keeping errors, and from transmit RF power level variations.

Up-link interference results from side-lobe emissions of earth stations transmitting to a satellite other than the one carrying the wanted signal. The up-link interference is combined with the wanted up-link signal and transmitted on the down-link path to the intended receiver. Also transmitted to this intended receiving earth station is down-link interference from other satellites intending to transmit to other earth stations.

Traditional approaches to analysis and design have involved worst-case scenarios, when in fact the variations in signal and interference levels are random. Our earlier report [1] compared the results of using a worst-case and a statistical approach, based on the assumption of "small" variations in signal and interference levels. Differences of several dB were seen to be possible.

The purpose of this present report is to present an analytical approach which will enable a statistical analysis, without assuming variations to be small and to present results based on this analysis. We show how the various probability density functions of the random variables causing signal and interference fluctuations combine to produce distributions for

carrier-to-interference levels for representative satellite network scenarios. Such distributions would enable link availability to be determined in terms of specified c/i confidence levels, and are used by us to compare our c/i results with those based on a conventional worst case analysis.

II CARRIER-TO-INTERFERENCE RATIO EQUATION

A. Basic Link Equation

Consider first the case where a wanted signal from a transmitting earth station passes through a satellite transponder to a receiving earth station. Using terms defined in Appendix I, the basic down-link equation for the carrier power at the receiving earth station is:

$$C = P_d - L_f - L_{ca} - L_m + G_s(\phi) - D + G_e(\theta) \quad (1)$$

Similarly, the wanted carrier power at the receiving satellite antenna is

$$C_s = P_u - L_f - L_{ca} + H_s(\alpha) - U + H_e(\beta) \quad (2)$$

These equations are in dB.

B. Interference

Added to the wanted up-link signal is up-link interference i_u from other earth stations. The dB power level of each up-link interference signal is given by an equation identical to (2).

The wanted signal plus up-link interference is transmitted on the down-link and is received together with down-link interference i_d from other satellites. Each down-link interfering signal from a satellite which does not carry the wanted signal has a link equation identical to (1).

C. Carrier-to-Interference Ratio

To determine c/i in power (non-dB) at the receiving earth station, we proceed as explained below.

For the k th up-link interfering signal in terms of power quantities

$$i_{uk}/c = \left(\frac{P_{uk}}{P_{uw} H_{ew} H_{sw}} \right) \left[\frac{H_{ek} H_{sk} U_w}{U_k} \right] \quad (3)$$

In (3) subscripts "k" and "w" denote the k th up-link interferer and wanted signal, respectively. Thus,

$$i_u/c = \sum_k i_{uk}/c \quad (4)$$

In writing (3), the terms involving small variations about nominal values are enclosed by "()", while those whose variations are potentially large are indicated by "[]". We assume that L_{ca} and L_f are identical for all up-link interferers and the wanted signal.

On the down-link, we cannot find i_d/c directly. Instead, we first find

$$B_j = i_{dj}/(c + i_u) \quad (5)$$

where i_{dj} denotes the j th down-link interfering signal. Assuming L_{ca} , L_f and L_m to be the same for all down-link signals gives

$$B_j = \left(\frac{P_{dj}}{P_{dw} G_{ew} G_{sw}} \right) \left[\frac{G_{ej} G_{sj} D_w}{D_j} \right] \quad (6)$$

In many cases all down-link fades D_j and D_w are equal. Power summing the total down-link interference yields

$$B = \sum_j B_j \quad (7)$$

To obtain i_d/c , we proceed as follows:

$$B = [i_d/c]/(1 + [i_u/c])$$

$$(i_d/c) = B(1 + [i_u/c])$$

$$i/c = (i_u/c)(1 + B) + B \quad (8)$$

Note that when $i_u/c \ll 1$, $B \approx i_d/c$. If in addition $i_d/c \ll 1$ then

$$i/c = (i_u/c) + (i_d/c) \quad (9)$$

III DETERMINATION OF C/I DISTRIBUTIONS

The approach for determining the distribution of i/c is as follows, assuming for now that c is constant.

1. Determine the probability densities for i_{uk}/c in (3) and B_j in (6).
2. Determine the probability density of i_u/c in (4) and B in (7).
3. Determine the density and then the probability distribution for i/c in (8).

Each of these steps requires operations involving probability density functions of the link variables P_u , $G_e(\theta)$, U , etc. Below we discuss the required operations in detail.

A Determination of Densities for i_{uk}/c and B_j

The quantities i_{uk}/c and B_j each result from multiplication of several random variables. There are two ways to obtain probability density functions for i_{uk}/c and B_j . One approach is to use the result in Appendix II for the probability density of a product of random variables. This approach requires that the densities of all link terms be expressed in power (non-dB) format.

An alternative approach is to write (3) and (6) in dB format. A sum of terms then results, and the density of i_{uk}/c and B_j in dB is the convolution of the respective dB densities of the link quantities, in accordance with Appendix II. Which approach is more convenient will depend, in part, on whether the link densities are based on dB or power levels. The density for

U or D, for example, is available with U and D expressed in either form [2]. The convolution operation on dB densities is probably easier to use numerically than is the product-density integral for power products, since the $|\beta|^{-1}$ factor in the latter integral may lead to numerical difficulties. However if the dB link quantities are used, i_u/c in dB will have to be transformed in accordance with the method in Appendix II, to a power (non-dB) quantity in order to obtain $\Sigma i_{uk}/c$ and ΣB_j .

We note here that it is always possible to transform from dB density to one for a power quantity as explained in Appendix II.

In determining the density for i_{uk}/c and B_j in (3) and (6), the terms in "()" would normally be small; and could reasonably be assumed to be Gaussian, either in dB or power format. The fact that either format implies Gaussian behaviour follows from the fact that for small variations, $10\log_{10}(1+x) \approx (10/\ln 10)x$. Here in the case of i_{uk}/c and B_j , respectively

$$x = (P_{uk}/P_{uw} H_{ew} H_{sw}) \quad (10)$$

and

$$x = (P_{dj}/P_{dw} G_{ew} G_{sw}) \quad (11)$$

The nominal (mean) value for x is $x = 1$ ($x = 0$ dB) and the variance of x is the sum of the variances of the individual normalized link term variances. This latter statement follows from the fact that for $\{a_i\}$ small

$$(1 + a_1)/(1 + a_2)(1 + a_3)(1 + a_4) \approx 1 + a_1 - a_2 - a_3 - a_4 \quad (12)$$

Here, $\{a_i\}$ corresponds to the normalized link variables P_{uk} , H_{ew} , etc.

The terms in "[]" in (3) and (6) are not necessarily small. In the absence of a rain fade mitigation philosophy, for example, U and D can vary up to 10 dB. Similarly, off-boresight values for H_e , H_s , G_e and G_s vary about their nominal values based on standard patterns, and the nominal angles α , β , θ and ϕ also vary due to antenna mispointing errors and satellite station-keeping errors. Determination of these antenna gain densities and rain fade densities is described in Sections IV and V.

B Determination of Densities for i_u/c and B

Having determined densities for i_{uk}/c and B_j , it is easy (in principle) to obtain densities for i_u/c and B by convolving the densities of i_{uk}/c and of B_j . In performing these convolutions we must use power (non-dB) quantities.

C Determination of the Density and Distribution for i/c

One sees from (8) that the probability density for i/c is the convolution of the density for B with that of the density for $(i_u/c)(1 + B)$. This latter density is obtained by either of the methods in Appendix I to determine the density for the products. The density for $(1 + B)$ is $f_B(\alpha - 1)$ where $f_B(\alpha)$ is the density for B .

In those cases where $i_u/c \ll 1$ and $i_d/c \ll 1$ the density for i/c is obtained by convolving densities for i_u/c and i_d/c .

Once the density for i/c is known, integration gives the distribution

$F_{i/c}(\alpha)$:

$$F_{i/c}(\alpha) = \int_{-\infty}^{\alpha} f_{i/c}(\beta) d\beta \quad (13)$$

A transformation to dB format gives the dB density for i/c and (13) then yields the distribution of i/c . From this distribution one obtains the probability that i/c exceeds a given value, since

$$\text{Prob}[(i/c) < \alpha] = F_{i/c}(\alpha) \quad (14)$$

and

$$\text{Prob}[(i/c) > \alpha] = 1 - F_{i/c}(\alpha) \quad (15)$$

The distribution function will depend on the relative up-link and down-link power levels P_u and P_d and their variances, the antenna gain functions and the statistics associated with these, and the link fades U and D .

The above discussion assumes that c is constant. This assumption is reasonable if transmitted power level variations of the wanted signal are negligible, and if rain fading of the wanted signal is perfectly mitigated or if on the down-link the wanted and interfering signals all pass through the same rain cell. If c is not constant, then the total interference is first determined, transformed to dB format and convolved with $-C$ (in dB) to yield the probability density of i/c .

IV RAIN FADE PROBABILITY DENSITY FUNCTIONS

The proposed model for rain attenuation in dB is the log-normal distribution [2-4]. Thus, the probability density $f_U(\alpha)$ which gives the dB reduction from the unfaded level is as follows:

$$f_U(\alpha) = \frac{10P_o}{(\ln 10)\sqrt{2\pi} \sigma \alpha} \exp[-(10\log \alpha - m)^2/2\sigma^2] + (1-P_o)\delta(\alpha) \quad (\text{dB}; \alpha > 0) \quad (16)$$

where P_o is the probability of rain. Note that m and σ represent the mean

and variance, respectively of $10\log U$, and that U itself is in dB. A similar density function applies to the down-link fade D . The parameters of the distribution are m , σ and P_o and are available from various sources [2-4].

In many cases we actually add $-U$ (or $-D$) to the link variables, as in (1) and (2). In this case the dB density which is convolved with the other link variables is given by (17) below. The derivation of (17) follows in accordance with the transformation $y = -x$ (see Appendix II). Thus

$$f_{-U}(\alpha) = \frac{10P_o}{(\ln 10)\sqrt{2\pi}\sigma(-\alpha)} \exp\left[-\frac{(10\log(-\alpha) - m)^2}{2\sigma^2}\right] + (1 - P_o)\delta(\alpha) \quad (\text{dB}; \alpha < 0) \quad (17)$$

To express the attenuation U in terms of power, we use the transformation $y = 10^{x/10}$ to obtain the log-log-normal density:

$$f_U(\beta) = \left[\frac{10P_o}{(\ln 10)^2 \sqrt{2\pi}\sigma\beta \log \beta} \right] \exp\left(-\frac{[10\log(10\log\beta) - m]^2}{2\sigma^2}\right) + (1 - P_o)\delta(\beta - 1) \quad (\text{power}, \beta > 1) \quad (18)$$

It is also convenient to obtain the density in power for $V = U^{-1}$. The transformation process described in Appendix II yields:

$$f_V(\beta) = \left[\frac{10P_o}{(\ln 10)^2 \sqrt{2\pi}\sigma\beta (-\log\beta)} \right] \exp\left[-\frac{(10\log(-10\log\beta) - m)^2}{2\sigma^2}\right] + (1 - P_o)\delta(\beta - 1) \quad (\text{power}; 0 < \beta < 1) \quad (19)$$

It is convenient to note, as a practical matter in comparing with others' equations that [2,3]

$$\ln 10 \log y = \ln y \quad (20)$$

Thus, an alternative way to write (16)-(19) follows with

$$\sigma_o = (\ln 10/10)\sigma \quad (21a)$$

$$\alpha_o = (\ln 10/10)m \quad (21b)$$

$$f_U(\alpha) = P_o (\sqrt{2\pi}\sigma_o \alpha)^{-1} \exp(-\frac{1}{2} [\ln(\alpha/\alpha_o)/\sigma_o]^2) + (1 - P_o)\delta(\alpha) \quad (\text{dB}; \alpha > 0) \quad (22)$$

$$f_{-U}(\alpha) = P_o (\sqrt{2\pi}\sigma_o [-\alpha])^{-1} \exp(-\frac{1}{2} [\ln(-\alpha/\alpha_o)/\sigma_o]^2) + (1 - P_o)\delta(\alpha) \quad (\text{dB}; \alpha < 0) \quad (23)$$

$$f_U(\beta) = P_o (\sqrt{2\pi}\sigma_o \beta \ln \beta)^{-1} \exp[-\frac{1}{2} \{\ln((\log_{10} \beta)/\alpha_o)/\sigma_o\}^2] + (1 - P_o)\delta(\beta - 1) \quad (\text{power}; \beta > 1) \quad (24)$$

$$f_{-U}(\beta) = P_o (\sqrt{2\pi}\sigma_o \beta [-\ln \beta])^{-1} \exp[-\frac{1}{2} \{\ln((-10 \log_{10} \beta)/\alpha_o)/\sigma_o\}^2] + (1 - P_o)\delta(\beta - 1) \quad (\text{power}; 0 < \beta < 1) \quad (25)$$

Typical values are $\alpha_o = 0.253\text{dB}$, $\sigma_o = 1.174$ and $P_o = 0.08$ during the worst month. These parameters correspond to 3.4 dB fading for 10^{-3} of the time and 8.7 dB fading for 10^{-4} of the time. These values have been used for NATO phase III System studies [2,3,5] and are representative for Eastern US Seaboard fading during the worst month of the year at X-band with a 17° antenna elevation angle.

V ANTENNA GAIN PROBABILITY DENSITIES

Actual antenna patterns vary from standard reference patterns used in interference calculations. Typical reference patterns appear in Appendix III. As well earth station mispointing, satellite mispointing and satellite station-keeping errors cause the actual interference angles seen by an antenna to vary from nominal values. These statements apply for both up-link and down-link interference.

The distribution of actual antenna gains over a sample of similar antennas and over the interference angles of interest are random. This

distribution of side-lobe levels has been modelled as Rayleigh for existing earth station antennas [6]. For offset parabolic-fed or Gregorian-type asymmetric antennas the side-lobe peaks follow a log-normal distribution [6,7].

Interference for earth station antennas would normally be received at an angle beyond the first side lobe. The same comment would apply for narrow-beam satellite antennas. However for antennas with moderate beamwidths, interference could lie close to the main lobe. In this latter case, reliable statistics for probabilistic variations from standard templates do not appear to be available.

Typical mispointing tolerances upon installation of earth stations are 0.1° or 1dB, whichever is less [8]. Stations with auto-track capability could have tighter tolerances. For satellites, typical tolerances are 0.1° translation and 1.0° rotation [8]. Station keeping tolerances are also 0.1° from nominal. In the absence of details regarding the distribution of these errors, it is reasonable to assume a Gaussian distribution with a standard deviation equal to one-half or one-third the quoted tolerance.

A. Large Earth-Station Gain Probability Density

For earth station antennas, the reference pattern well beyond the side-lobe interference region is [6]:

$$G_e(\theta) = k \left[1 + \left(\frac{\theta}{\theta_r} \right)^n \right]^{-1} \quad (26)$$

Typically, $n = 2.5$ and

$$\theta_r = 15.85(D/\lambda)^{-0.6} \quad (27)$$

where D/λ is the antenna diameter to wavelength ratio. For $D = 30\text{m}$,

$\theta_r \approx 0.38^\circ$ at 5 GHz.

The following is a good approximation for (26) for $\theta \gg \theta_r$:

$$G_e(\theta) = k(\theta_r/\theta)^n \quad (28a)$$

$$10\log_{10} G_e(\theta) = 10\log k + 10n\log\theta_r - 10n\log\theta \quad (28b)$$

For large earth-station antennas, the choice $n = 2.5$ and $K = 10\log k + 10n\log\theta_r = 32$ provides a curve which lies above 90% of the side-lobe peaks, although for more recent antennas $K = 29$ dB may be more appropriate [6,9].

The median level of the side-lobe amplitude appears to be 7 dB below that of the 90% curve [6]. Thus, for large earth station antennas

$$m(\theta) = A - B\log\theta \quad (29)$$

where $A = B = 25$.

The probability density for the gain G can now be found by integrating over the density for θ :

$$f_G(y) = \int_{-\infty}^{\infty} f_G(y/x) f_\theta(x) dx \quad (30)$$

With σ_θ as the variance of the nominal interference angle due to mispointing, station keeping and other small errors, our Gaussian assumption yields:

$$f_\theta(x) = (\sqrt{2\pi}\sigma_\theta)^{-1} \exp(-x^2/2\sigma_\theta^2) \quad (31)$$

Data on which to base $f_G(y/x)$ are scarce; on the basis of [7] we use a Gaussian probability density:

$$f_G(y/x) = (\sqrt{2\pi}\sigma_G)^{-1} \exp[-(y - m(x))^2/2\sigma_G^2] \quad (32)$$

In (32), σ_G is the standard deviation in dB of the antenna gain variation

about the mean $m(x)$. This variance appears to be relatively independent of x [6]; however the mean depends on the angle off boresight in accordance with (29), beyond the first side lobe.

B. Small Earth Station Antennas and Satellite Antennas

For smaller earth station antennas an equation similar to (29) provides a nominal gain template $m(\theta)$ beyond the first side-lobe region [6,10] for use in (30). Values A and B vary with antenna type and often depend on D/λ (see Appendix III).

For narrow-beam satellite antennas [11] suggests that the above results for earth stations remain applicable. For wide-beam antennas, interference lies closer to the main lobe. The reference pattern in Appendix III for satellite antennas covers all angles and $m(\theta)$ is therefore available for use in (30). However, as noted earlier a reliable probability density for the distribution of gain levels near the main lobe seems unavailable.

Given the appropriate statistics and reference pattern, the method used to determine the gain probability density for large earth station antennas remains applicable in the cases considered here.

VI IMPLEMENTATION OF THE PROPOSED STATISTICAL ANALYSIS APPROACH

The approach presented here enables the aggregate c/i distribution to be determined. The dB density for i_u/c and i_d/c is obtained by convolution of the link dB densities and then transformed to a power (non-dB) link density. These link power densities are then convolved to obtain the density for i/c . This density is then transformed to dB, and the aggregate i/c distribution is then calculated. From this aggregate distribution one can obtain the probability that i/c exceeds (or fails to exceed) a given level a given

percentage of the time, in accordance with the parameters specified and constitutive probability densities employed.

The parameter values chosen for the link variables will affect the i/c distribution. Important parameters include the number of interfering signals N , their nominal off-boresight angles α , β , θ , and ϕ , the variance of the station-keeping, mispointing and other errors about these nominal values, the mean interference levels $m(\theta)$, $m(\phi)$, $m(\alpha)$ and $m(\beta)$ in (29) as given by A and B in (30), or by other constants in an alternative equation in Appendix III, the relative RF power levels, and the rain fade parameters P_0 , m and σ . Finally, the satellite beamwidth is important in determining $m(\theta)$ in (29), in accordance with the standard reference pattern. The combined variance of the small variation terms in (3) and (6) would probably have a relatively small effect on the aggregate i/c density.

The large number of system variables requires that specific scenarios be selected for detailed study. Some representative satellite networks and network scenarios are examined in the following sections.

To compare statistical design results with worst case results, it is necessary to relate worst case values to the parameters in the various link density functions used. The difficulty is to define "worst case". A reasonable approach is to use as worst case those values which are exceeded not more than a specified percentage of the time as calculated from a link parameter probability density.

Much of the statistical analysis involves convolutions and transformations which would be done numerically. Thus, the usual cautions are exercised to control potential errors associated with numerical techniques [14].

VII SATELLITE NETWORK CONFIGURATIONS

Figs. 1, 2 and 3 show three satellite network configurations which form the basis for subsequent analysis and results. Each case involves two interfering signals separated from the wanted signal at a nominal angle $\bar{\theta}$. In each case, one of the antennas is a broad-beam antenna, and the other antenna has a narrow beam. We later indicate how our results would be modified for the case where both the transmitting and receiving antennas have narrow beams. Our goal here is to use relatively simple but realistic networks, to enable clear focus on comparisons between statistical and worst case i/c calculations.

Fig. 1 shows two down-link interfering signals I_1 and I_2 together with the wanted signal W . The satellite antennas all have wide beams and the earth station antenna has a narrow beam. The actual positions of the satellites are indicated by solid lines. The nominal satellite positions in the absence of station-keeping errors are indicated by dotted lines. Angles θ_2 , θ_3 and θ_4 represent station-keeping angular errors, and θ_1 represents the pointing error of the earth station antenna. The actual separation angles θ_L and θ_R between the wanted and respective interfering signals I_1 and I_2 are as follows:

$$\theta_L = \bar{\theta} - [\theta_1 + \theta_2] + \theta_3 \quad (33)$$

$$\theta_R = \bar{\theta} + \theta_1 + \theta_2 + \theta_4 \quad (34)$$

Angles θ_1 , θ_2 , θ_3 and θ_4 are assumed to be statistically independent.

Thus the angular errors $\theta_R - \bar{\theta}$ and $\theta_L - \bar{\theta}$ are each random variables whose

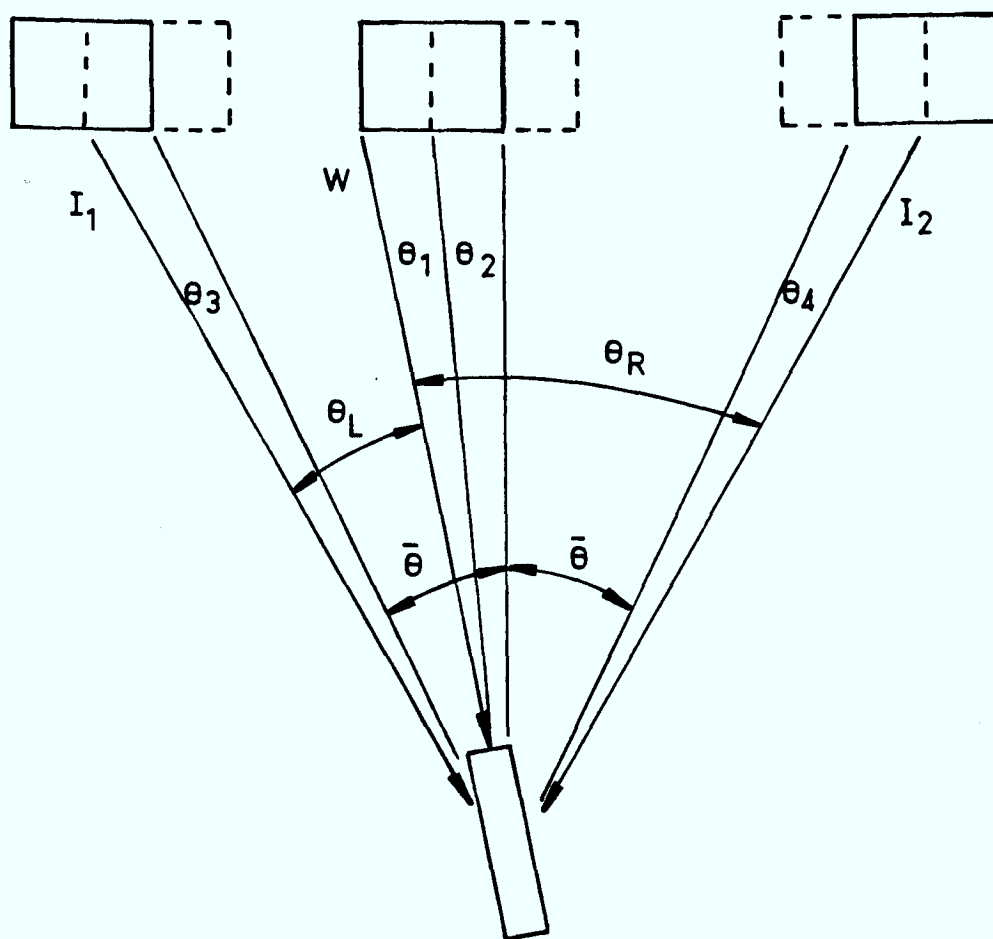


Fig. 1. Satellite network with down-link interference; wide-beam satellite antennas

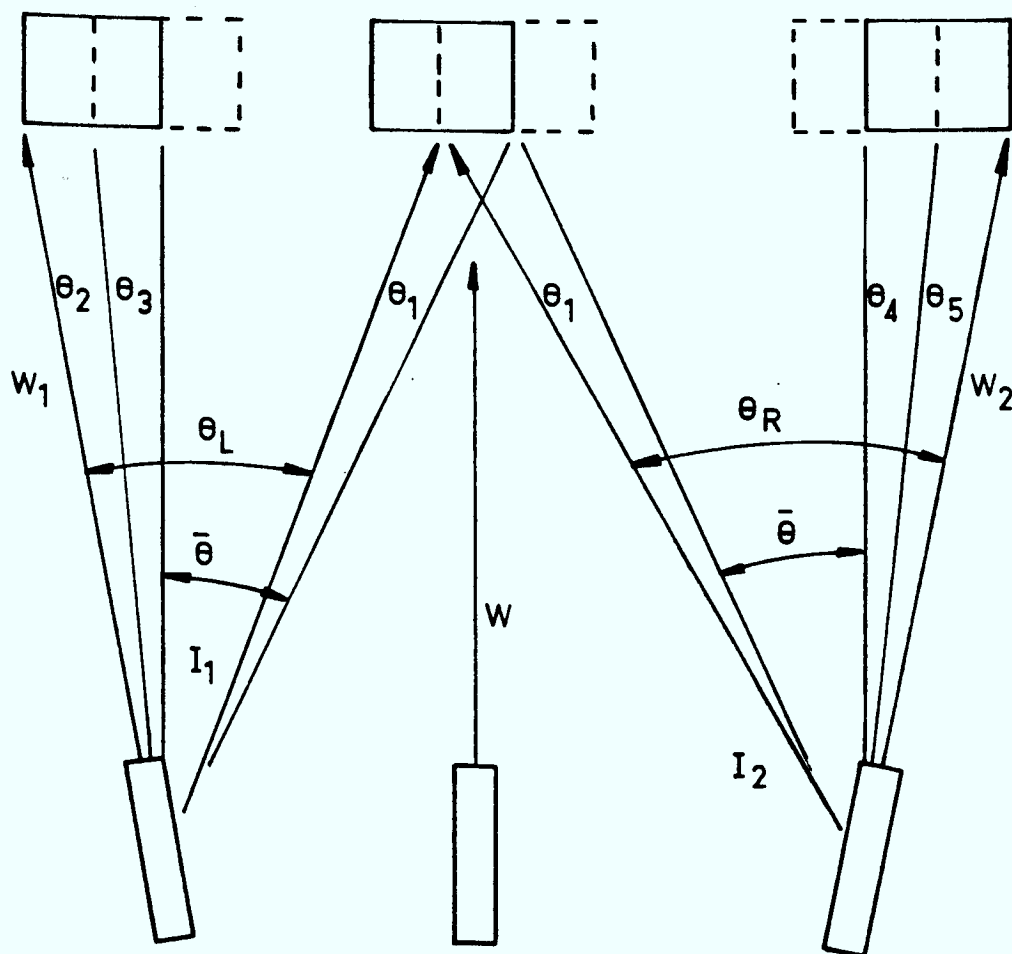


Fig. 2. Satellite network with up-link interference; wide-beam satellite antennas

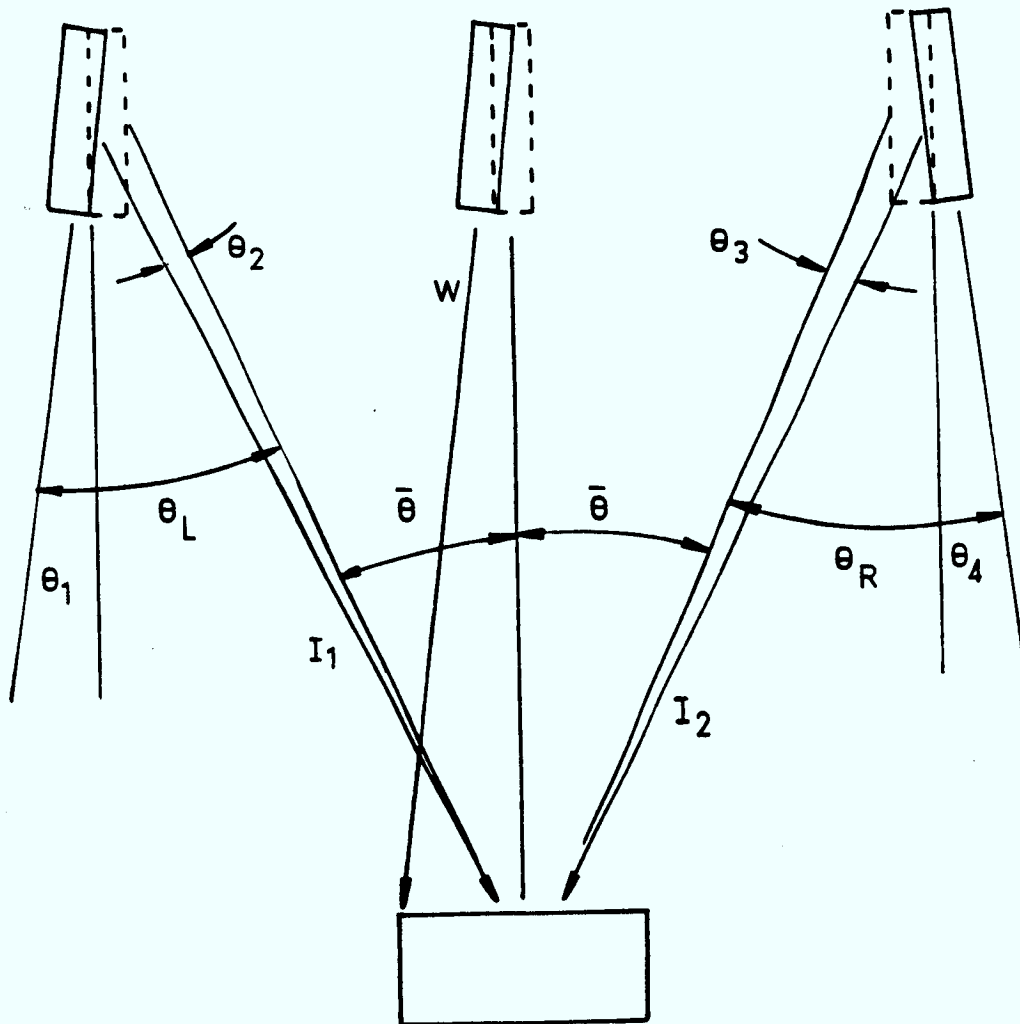


Fig. 3. Satellite network with down-link interference; narrow-beam satellite antennas

means and variances equal the sum of the means and variances, respectively of the corresponding error angles in (33) and (34).

If a single interfering signal is present, then the interference density due to these angular variations is given by (30) and (31), with

$\sigma_{\theta_L} = \sigma_{\theta_1} + \sigma_{\theta_2} + \sigma_{\theta_3}$. If we further assume that any transmitter power level variations due to ageing, manufacturing tolerances and power level variations due to boresight errors for the wanted signal are negligible, that all down-link rain fades among the wanted and interfering signals are perfectly correlated and that all satellites transmit at equal power, then from (6) and (9) (upper-case denotes dB quantities):

$$I_1/C = I_d/C = G_e(\theta) - G_e(0) \quad (35)$$

When two down-link interferers are present, we must use i_1/c and i_2/c to obtain i/c . However, now i_1/c and i_2/c are dependent, through the angle $\Delta = \theta_1 + \theta_2$. A positive Δ value reduces by Δ the angular separation between I_1 and W but increases by Δ the separation between I_2 and W . For small Δ values, under the same assumptions regarding power levels and rain fading as above, these two effects tend to cancel, as we show below:

$$\begin{aligned} I_1/C &= -G_e(0) + G_e(\theta_L) \\ &= -G_e(0) + 25(1 - \log \theta_L) \\ i_1/cG_e(0) &= (10/\theta_L)^{2.5} \\ &= [10/(\theta + \theta_3 + \Delta)]^{2.5} \\ &\approx [10/(\theta + \theta_3)]^{2.5}(1 - 2.5\Delta) \end{aligned} \quad (36)$$

$$i_2/cG_e(0) \approx [10/(\theta + \theta_4)]^{2.5}(1 + 2.5\Delta) \quad (37)$$

If $\theta_3 \ll \bar{\theta}$ and $\theta_4 \ll \bar{\theta}$ then adding (36) to (37) yields

$$1/c G_e(0) \approx 2[10/(\bar{\theta} + \theta_4)]^{2.5} \quad (38)$$

Thus, it is as if $\theta_1 = \theta_2 = 0$, and the only variation in θ arises from station-keeping errors of the satellites which generate the interference. It is this kind of careful analysis that is ignored in worst case design situations.

Fig. 2 shows a network where two narrow-beam earth station antennas generate up-link interference. The separations in this case are

$$\theta_L = \bar{\theta} - \theta_1 + \theta_2 + \theta_3 \quad (39)$$

$$\theta_R = \bar{\theta} + \theta_1 + \theta_4 + \theta_5 \quad (40)$$

In the case of a single interfering signal (i.e. $I_2 = 0$) the $1/c$ distribution is identical to that for Fig. 1, assuming that any rain fading is perfectly mitigated, that transmitted power level variations are negligible, and that all earth station antennas transmit at equal power levels.

When two interfering signals are present in Fig. 2, then variations in θ_1 effectively cancel, and it is as if two independent random variables, i_1/c and i_2/c with random angular variations $\theta_2 + \theta_3$ and $\theta_4 + \theta_5$ were present. Convolution of i_1/c and i_2/c yields i/c .

Fig. 3 shows the case where all three satellites have narrow-beam antennas and the earth station antenna has a wide beam. In this case, the angular variations of the interfering signals are due solely to the station-keeping and pointing errors of the satellite antennas. Under the assumptions noted earlier i/c is obtained by convolving the densities of i_1/c and i_2/c , with

$$\theta_L = \bar{\theta} + \theta_1 + \theta_2 \quad (41)$$

$$\theta_R = \bar{\theta} + \theta_3 + \theta_4 \quad (42)$$

In those cases where rain fading or transmitter power level variations are present, the densities of i_1 and i_2 must first be obtained by convolution of dB quantities prior to convolution of i_1 and i_2 (power) densities, followed by dB convolution of I and $-C$ as explained earlier.

VIII NETWORK SCENARIO ONE: SINGLE INTERFERING SIGNAL

We begin with the simplest possible network configuration, depicted in Fig. 1, which operates in accordance with the following assumptions:

- 1a. All interference is from a single down-link interfering satellite transmission.
- 1b. Both the wanted (W) and interfering (I) signal are from wide-beam satellites nominally separated by $\bar{\theta}^\circ$. Both satellites transmit at equal power levels to a narrow-beam earth station antenna.
- 1c. Any rain fading experienced by W and I is perfectly correlated.

For this case i/c is obtained from (35). The sole cause of variations in i/c is variation in θ . These variations result from satellite station-keeping errors and earth station and satellite pointing errors. All errors have a nominal 0.1° tolerance. We have translated this 0.1° tolerance into a standard deviation of $0.1/\sqrt{6}$. The $\sqrt{6}$ factor is chosen because it is the variance of a triangular probability density with a 0.1° peak deviation.

It follows that $\sigma_\theta = 0.1\sqrt{3}/\sqrt{6}$ in (31) since the three tolerance errors are reasonably assumed to be independent.

The value for σ_G in (32) is $\sigma_G = 3.91$ dB, a value which is less than the

5 or 6 dB reported in [7], but which is consistent with the 5 dB measured spread between the median and 90% peak sidelobe levels reported in [6].

Figs. 4 and 5 show $F_{I/C}$ distributions for $\bar{\theta} = 2^\circ$ and $\bar{\theta} = 4^\circ$, respectively where $\bar{\theta}$ is the nominal satellite spacing. The nominal median side-lobe attenuation by the earth station against the interfering down-link signal in $m(\bar{\theta}) = 25(1 - 10 \log \bar{\theta})$ from (29).

Using the printout corresponding to Figs. 4 and 5 (see Appendix IV), one easily obtains $\alpha_{0.9}$ such that $F_{I/C}(\alpha_{0.9}) = 0.90$:

$$\alpha_{0.9} + G_e(0) = \begin{cases} 22.5 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 15.0 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (43)$$

We can compare these values with those based on a worst case design, assuming that the interference 90% side-lobe levels lie 5 dB above the median [6,7]:

$$\alpha_{0.9}]_{WC} + G_e(0) = \begin{cases} 30 - 25 \log_{10}(1.7^\circ) = 24.24 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 30 - 25 \log_{10}(3.7^\circ) = 15.79 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (44)$$

The difference between a worst case and statistical design is:

$$\Delta_{WC-STAT} = \begin{cases} 1.7 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 0.79 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (45)$$

We also show in Figs. 4 and 5 the effects of tolerance angles $B > 0.1^\circ$. One sees that the distributions are not strongly dependent on tolerances below 0.5° for $\bar{\theta} = 2^\circ$ and below 1.0° for $\bar{\theta} = 4^\circ$. Again, all angular errors are Gaussian with equal standard deviations of $B/\sqrt{5}$.

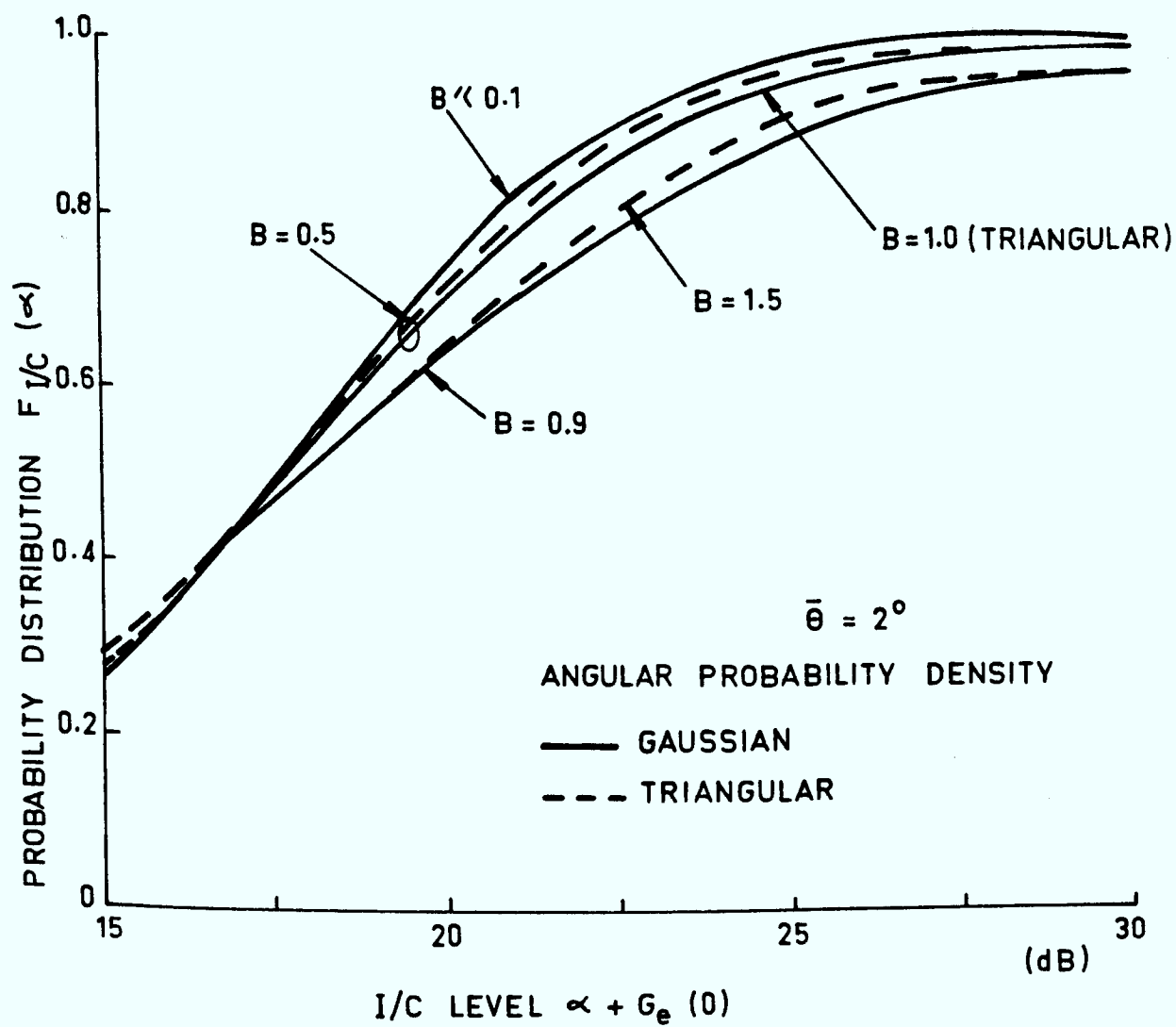


Fig. 4. Probability distributions $F_{I/C}(\alpha)$ for a single interfering signal; $\bar{\theta} = 2^\circ$

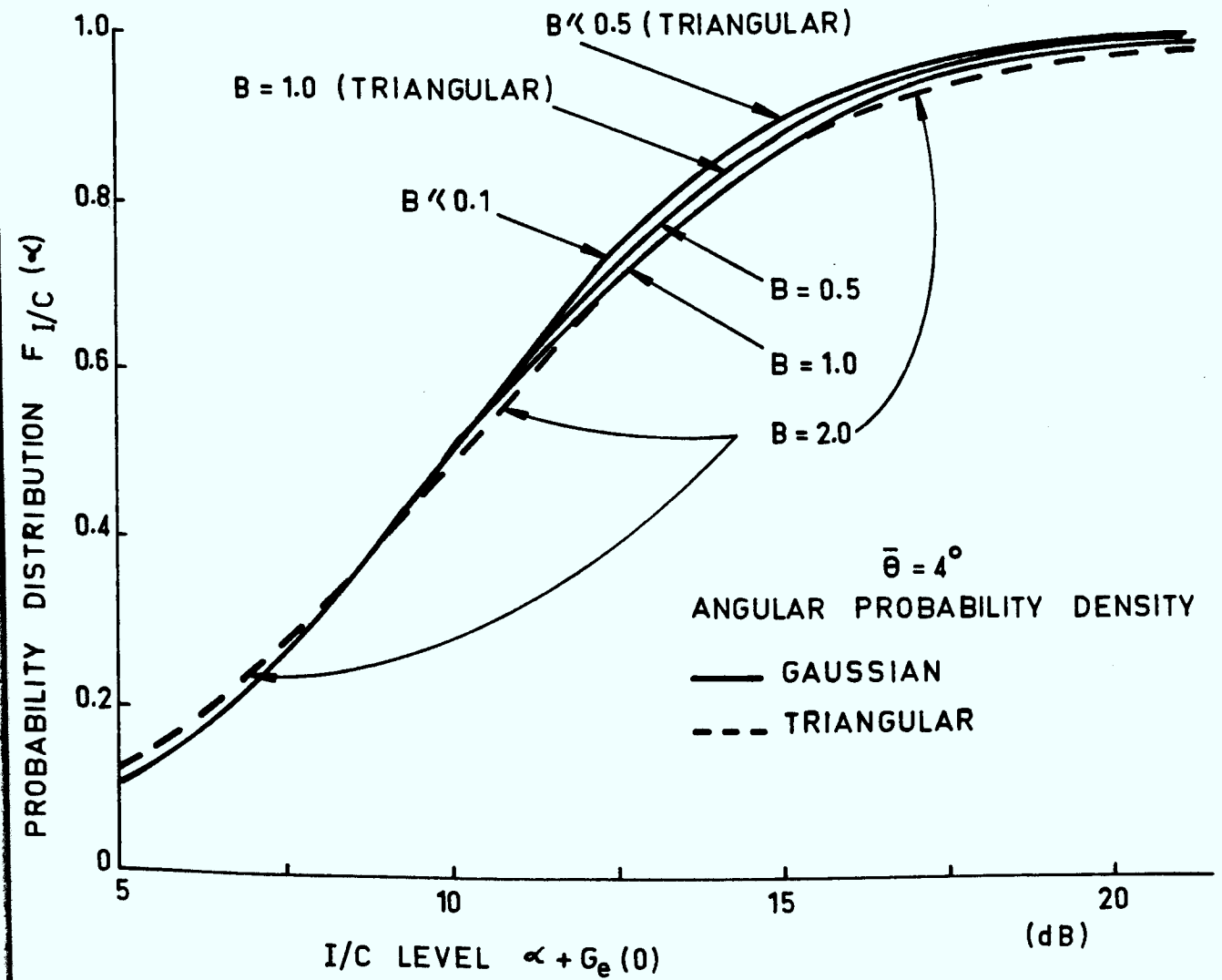


Fig. 5. Probability distributions $F_{I/C}(\alpha)$ for a single interfering signal; $\bar{\theta} = 4^\circ$

Figs. 4 and 5 also indicate the effects on the distributions of using a triangular distribution rather than a Gaussian distribution for the cumulative effects of angular pointing and station-keeping errors. One sees that for a given overall standard deviation B , the variation in I/C is largest in the Gaussian case, although the difference is negligible for 0.1° tolerances. The difference is small for tolerances below 0.5° for $\bar{\theta} = 2^\circ$ and below 1.0° for $\bar{\theta} = 4^\circ$.

From the above paragraphs, which in essence state that small changes in B do not much affect the I/C distributions, it follows that the results in Figs. 4 and 5 are applicable to the network configurations in Figs. 2 and 3, as well as the one in Fig. 1. In particular, the results apply to the following two additional cases, except that for the network in Fig. 3 the worst case results would be based on an angular error of 0.2° rather than 0.3° .

2. All interference is from a single down-link interfering satellite transmission. Both the wanted (W) and the interfering (I) signals are from narrow-beam satellites nominally separated by $\bar{\theta}^\circ$. Both satellites transmit at equal power levels to a wide-beam earth station antenna.

Any rain fading experienced by W and I is perfectly correlated.

3. All interference is from a single up-link interfering signal. Both the wanted (W) and interfering (I) signals are from narrow-beam earth station antennas pointing at wide-beam satellites nominally separated by $\bar{\theta}^\circ$. Both earth stations transmit at equal power levels.

Any rain fading experienced by W and I is perfectly mitigated.

The distributions in Figs. 4 and 5 closely approximate the normal curve

for $B \gtrsim 0.5^\circ$ and $B \gtrsim 1.0^\circ$, respectively. It follows that dB link variations of other variables which are Gaussian can be easily combined with I/C variations depicted in Figs. 4 and 5, since the convolution of two Gaussian densities is a Gaussian density [12].

Consider, for example, that the transmitted power levels of the wanted and interfering satellite signals each vary by ± 0.5 dB from nominal values. Such variations occur as a result of ageing and manufacturing tolerances and also account for mispointing of the main lobe. Each of these variations might typically be Gaussian with standard deviations each of $\sigma_A = 0.5$ dB about the nominal power level. Inclusion of these power level variations would not much alter the curves in Figs. 4 and 5, since the variances would increase from $\sigma_\theta = 3.91$ dB to

$$\begin{aligned}\sigma &= \sqrt{\sigma_\theta^2 + 2 \sigma_A^2} \\ &\approx 3.97 \text{ dB}\end{aligned}\tag{46}$$

However, a worst case design would assume that the wanted signal was faded by 0.5 dB while the interfering signal was 0.5 dB above its nominal level. The difference between the worst case design value and the statistical value would increase by almost 1 dB from that in (45). Thus

$$\Delta_{\text{WC-STAT}} = \begin{cases} 2.7 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 1.79 \text{ dB} & (\bar{\theta} = 1^\circ) \end{cases}\tag{47}$$

The curves Figs. 4 and 5 can be used to determine the effects of rain fading. Assume, for example that the wanted signal suffers a 5 dB fade while the interfering signal does not fade at all. Such a case could easily occur for up-link interference. The effect of fading is the same as if $G_e(0)$ in

Fig. 4 and 5 were reduced by 5 dB, which would move the curves 5 dB to the right. For 0.1° tolerances, a 90% confidence level with no rain fading would become a confidence level of 50% under a 5 dB wanted signal fade for both $\bar{\theta} = 2$ and 4 degrees. A 99% confidence level would reduce to 87% for both $\bar{\theta} = 2$ and 4 degrees.

Similarly, a 5 dB fade on the interfering signal would move the curves 5 dB leftward. A 90% confidence level with no rain fading becomes a confidence level in excess of 99% with a 5 dB rainfade on the interfering signal.

A similar analysis applies if known fades or enhancements occur because of changes in other link variables.

IX NETWORK SCENARIO TWO: TWO INTERFERING SIGNALS

We consider now the effects of two interfering signals for the systems depicted in Figs. 1-3, inclusive. The two interfering signals can both be on the down-link, both on the up-link, or one on the down-link and one on the up-link. We assume that both interfering signals are separated by the same nominal angle $\bar{\theta}$ from the wanted signal, that both the wanted and any interfering signals on the up-link are transmitted at equal power levels, that the same statement applies to any interfering signals and the wanted signal on the down-link. Rain fades on the wanted and interfering down-link signals are assumed perfectly correlated and any up-link rain fading is perfectly mitigated.

Figs. 6 and 7 show the distribution $F_{I/C}(\alpha)$ for 2 interfering signals for $B < 0.5^\circ$. The horizontal axis is $\alpha + G_e(0) - N_{dB}$ where N_{dB} is the number of interferers in dB; here $N_{dB} = 3$ dB. The distribution was obtained by

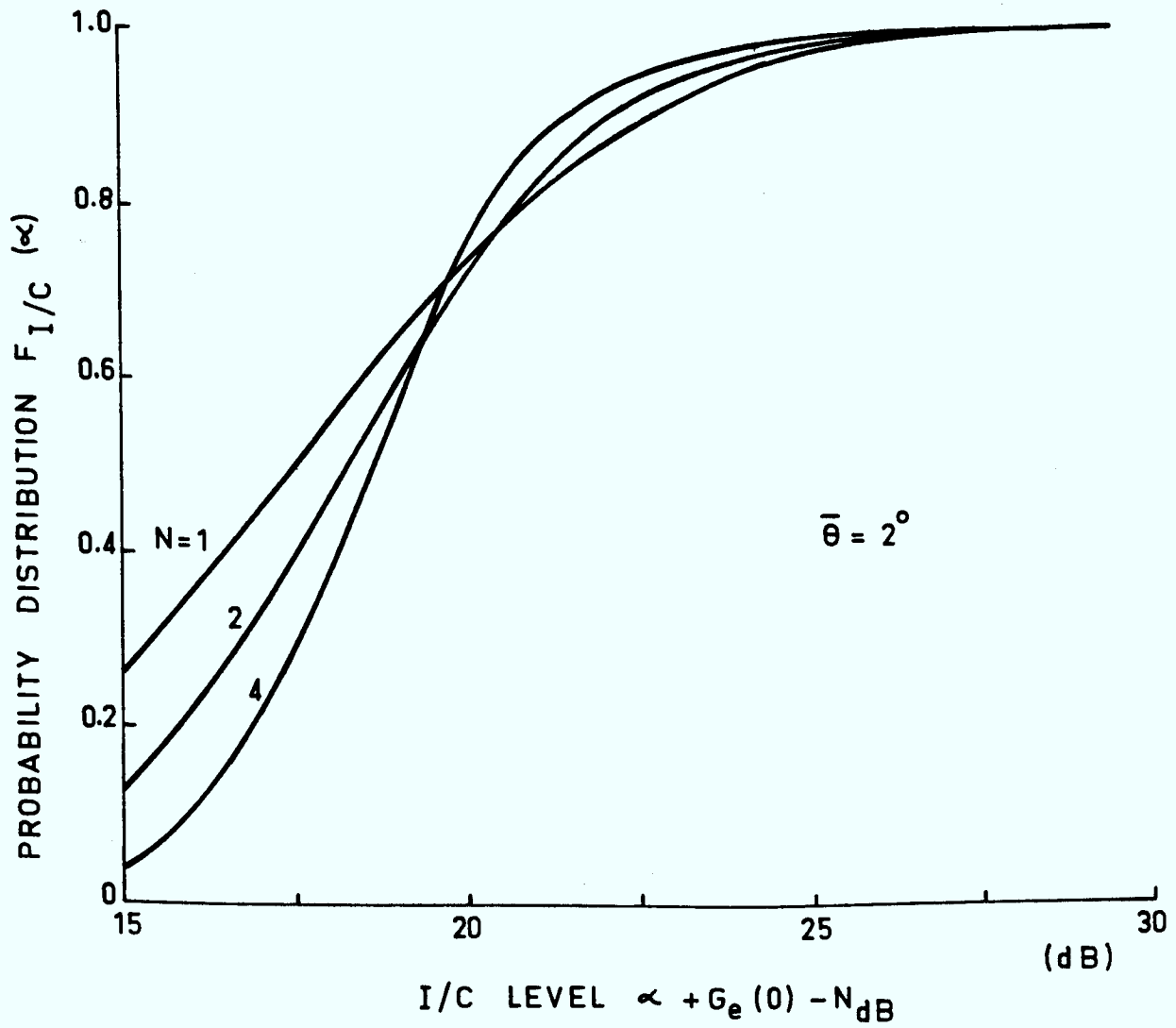


Fig. 6. Probability distributions $F_{I/C}(\alpha)$ for 1, 2 and 4 interfering signals; $\bar{\theta} = 2^\circ$

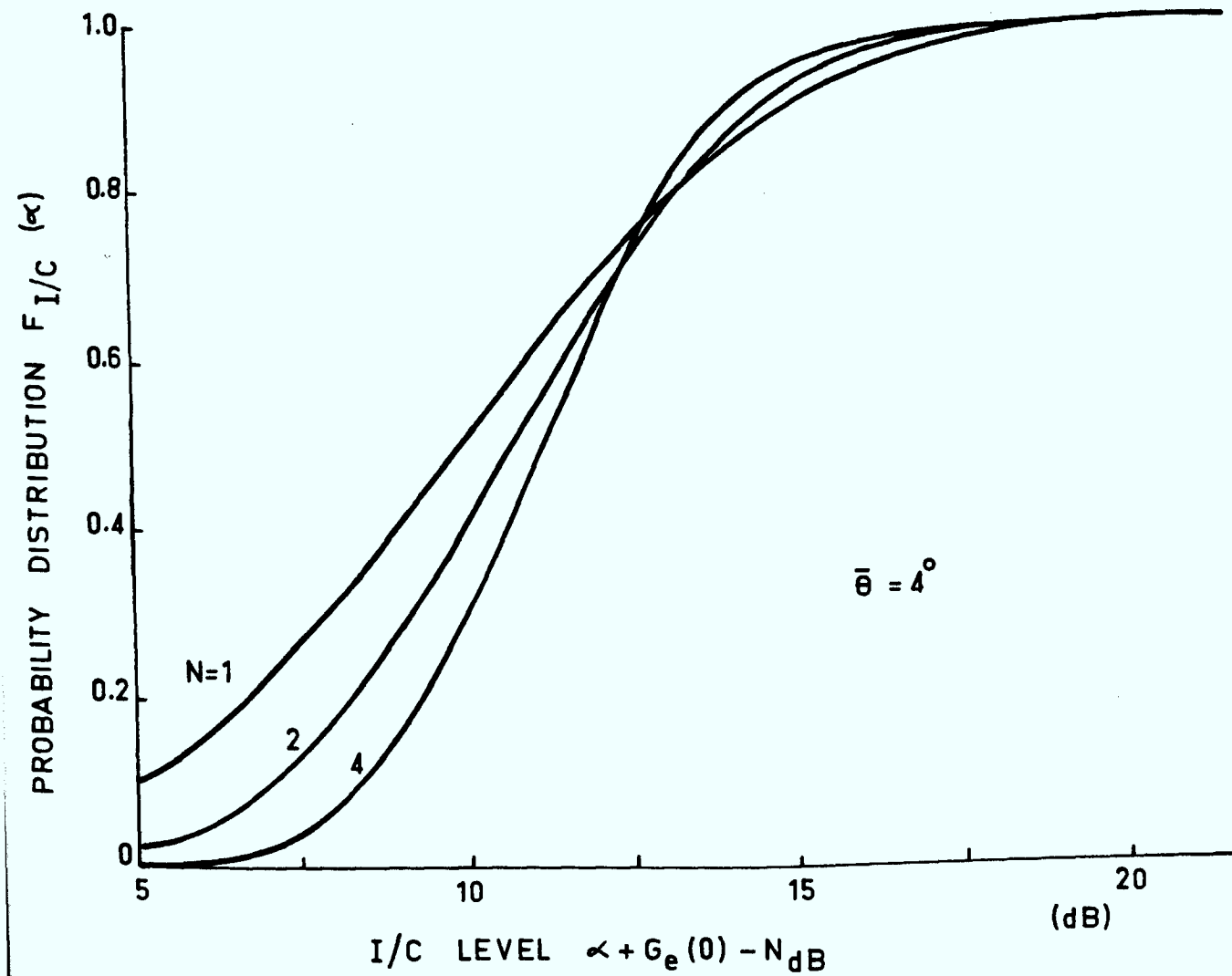


Fig. 7. Probability distributions $F_{I/C}(\alpha)$ for 1, 2 and 4 interfering signals; $\bar{\theta} = 4^\circ$

numerical self-convolution of the i/c density (power quantity) followed by conversion to dB format. The program listing appears in Appendix IV. From Figs. 6 and 7, the 90% points are easily obtained, as follows:

$$\alpha_{.9} + G_e(0) - N_{dB} = \begin{cases} 22.1 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 14.6 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (48)$$

A worst case design would assume both interfering signal beams to be separated from the wanted signal beam by the worst cast amount; namely $\Delta = 0.3^\circ$ for the networks in Figs. 1 and 2 with $B = 0.1^\circ$ and $\Delta = 0.2^\circ$ for the Fig. 3 case. Using $\Delta = 0.3^\circ$, worst case analysis yields, in the absence of power level variations:

$$\begin{aligned} I/C]_{WC} + G_e(0) - 3 &= 30 - 25 \log (\bar{\theta} - 0.3) \\ &= \begin{cases} 24.2 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 15.8 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \end{aligned} \quad (49)$$

Variations of ± 0.5 dB in all transmitted power levels would increase the values in (49) by 1.0 dB, since the wanted signal would be assumed to be reduced by 0.5 dB while all interfering signals would be increased by 0.5 dB above nominal values.

The difference in dB between worst case and statistical analysis is, with no power variation:

$$\Delta_{WC-STAT} = \begin{cases} 2.1 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 1.2 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (50)$$

This difference increases by 1.0 dB with power level variation.

The effects of rain fading are again easily seen from Figs. 6 and 7.

For a 5 dB fade on the signal, the 90% confidence level is reduced to 33% for

$\bar{\theta} = 2^\circ$ and to 36% for $\bar{\theta} = 4^\circ$. The reduction is larger than for $N = 1$ interfering signal, because the distribution is less flat for $N = 2$ than for $N = 1$.

A 5 dB fade on each interfering signal, with the wanted signal unfaded increases the confidence level above 98%.

The effect of a 5 dB fade from a 99% confidence level is to reduce the confidence level to approximately 90% for $\bar{\theta} = 2^\circ$ and for $\bar{\theta} = 4^\circ$. These values are approximate but conservative; the curves become very flat above the 95% confidence level and this causes some difficulty in determining graphically the confidence level reduction from 99%.

X NETWORK SCENARIO THREE: FOUR INTERFERING SIGNALS

We now consider the effects of four interfering signals, all separated by the same nominal value $\bar{\theta}$ from the wanted signal beam centre. Normally, two of the interfering signals would be up-link signals and two would be down-link signals.

Figs. 6 and 7 show I/C distributions for this case for $B < 0.5^\circ$. The 90% confidence points are as follows:

$$\alpha_{.9} + G_e(0) - N_{dB} = \begin{cases} 21.6 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 14.1 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (51)$$

Comparison with the worst case design results in (44) yields the following differences between worst case and statistical analysis results for 0.1° tolerance angles:

$$\Delta_{WC-STAT} = \begin{cases} 2.7 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 1.8 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \quad (52)$$

These results assume no power level variations. If a ± 0.5 dB variation occurs in transmitted power levels, then 1.0 dB is added to the values in (52). In the $\bar{\theta} = 2^\circ$ case, the difference then becomes 3.7 dB.

A 5 dB rain fade of the wanted signal results in a reduction from a 90% confidence level to 15% for both $\bar{\theta} = 2^\circ$ and $\bar{\theta} = 4^\circ$. The same 5 dB fade from a 99% confidence level reduces the confidence level to 92% for $\bar{\theta} = 2^\circ$ and 4° . These and earlier results appear in Table 1.

A 5 dB fade on all four interfering signals results in an increase in the 90% confidence level to a value above 97%.

XI NARROW-BEAM SATELLITE NETWORKS

The networks in Figs. 1-3, inclusive, include one wide-beam and one narrow-beam antenna on each up-link and down-link path. We now consider networks which consist solely of narrow-beam antennas. We assume that all side-lobe levels are as given in Appendix III for large diameter earth station antennas. In this case, for a single interfering signal (denoted by "j"):

$$(i/c) = \left[\frac{P_j}{P_w G_e(0) G_s(0)} \right] \left(\frac{G_e(\theta) G_s(\phi) D_w}{D_j} \right) \quad (53)$$

In the absence of power level variations and rain fading, the two random quantities are $G_e(\theta)$ and $G_s(\phi)$. To determine the distribution $F_{I/C}(\alpha)$ in this case we convolve the dB distributions for $G_e(\theta)$ and $G_s(\phi)$. Since these distributions are assumed identical and are essentially Gaussian for $B < 0.5^\circ$, the convolution yields another Gaussian density whose mean value is

TABLE 1 COMPARISON OF WORST CASE AND STATISTICAL ANALYSIS OF I/C AT 90 PER CENT CONFIDENCE LEVELS

NUMBER OF INTERFERING SIGNALS	θ	$\Delta_{WC-STAT}(dB)$	CONFIDENCE LEVEL FOR 5dB SIGNAL FADE	
			FROM 90%	FROM 99%
1	2°	1.7	50%	87%
1	4°	0.8	50%	87%
2	2°	2.1	33%	>90%
2	4°	1.2	36%	>90%
4	2°	2.7	15%	>92%
4	4°	1.8	16%	>92%

Note: For ± 0.5 dB variations in transmitted power levels, add 1.0 dB to $\Delta_{WC-STAT}$.

TABLE 2 COMPARISON OF WORST CASE AND STATISTICAL ANALYSIS OF I/C AT 90 PER CENT CONFIDENCE LEVELS, NARROW-BEAM NETWORKS

NUMBER OF INTERFERING SIGNALS	θ	$\Delta_{WC-STAT}(dB)$	CONFIDENCE LEVEL FOR 5dB SIGNAL FADE	
			FROM 90%	FROM 99%
1	2°	5.9	64%	94
1	4°	4.1	56%	93
2	2°	6.1	56%	>95
2	4°	4.5	56%	>95
4	2°	6.5	47%	>96
4	4°	4.9	43%	>96%

Note: For ± 0.5 dB variations in transmitted power levels, add 1.0 dB to $\Delta_{WC-STAT}$.

twice that of $G_e(\theta)$ and $G_s(\phi)$, i.e. $\bar{\theta} = \bar{\phi}$

$$m(\bar{\theta}) = 50(1 - \log \bar{\theta}) = m(\bar{\phi}) \quad (54)$$

The density standard deviation σ_{θ} is $\sqrt{2}$ times that of each individual density.

Figs. 8 and 9 show the distributions $F_{I/C}(\alpha)$ for $N = 1, 2$ and 4 interfering signals for narrow-beam networks with $\bar{\theta} = 2^\circ$ and 4° . The $N = 2$ and 4 cases are obtained by convolving the (power) densities i/c as was done for the curves in Figs. 6 and 7. From these distributions the 90% confidence levels are obtained.

To compare with confidence levels using a worst case approach we assume that the actual separation angle θ is reduced from the nominal angle by $2(0.1) = 0.2^\circ$ for a 0.1° tolerance on pointing and satellite station-keeping errors. Thus, the interference level under worst-case conditions is

$$\begin{aligned} I/C]_{WC} + G_e(0) + G_s(0) - N_{dB} &= 60 - 25[\log(\bar{\theta} - 0.3) + \log(\bar{\theta} - 0.2)] \\ &= \begin{cases} 47.9 \text{ dB} & (\bar{\theta} = 2^\circ) \\ 31.3 \text{ dB} & (\bar{\theta} = 4^\circ) \end{cases} \end{aligned} \quad (55)$$

The differences $\Delta_{WC-STAT}$ between a worst case and statistical design approach are listed in Table 2 and are seen to be much larger than the corresponding values in Table 1.

Also shown in Table 1 are the effects of a 5 dB rain fade on the wanted signal, from 90% and 99% confidence levels. The degradations in confidence level are less than the values in Table 1, because the curves are less steep in Figs. 8 and 9 than in Figs. 6 and 7.

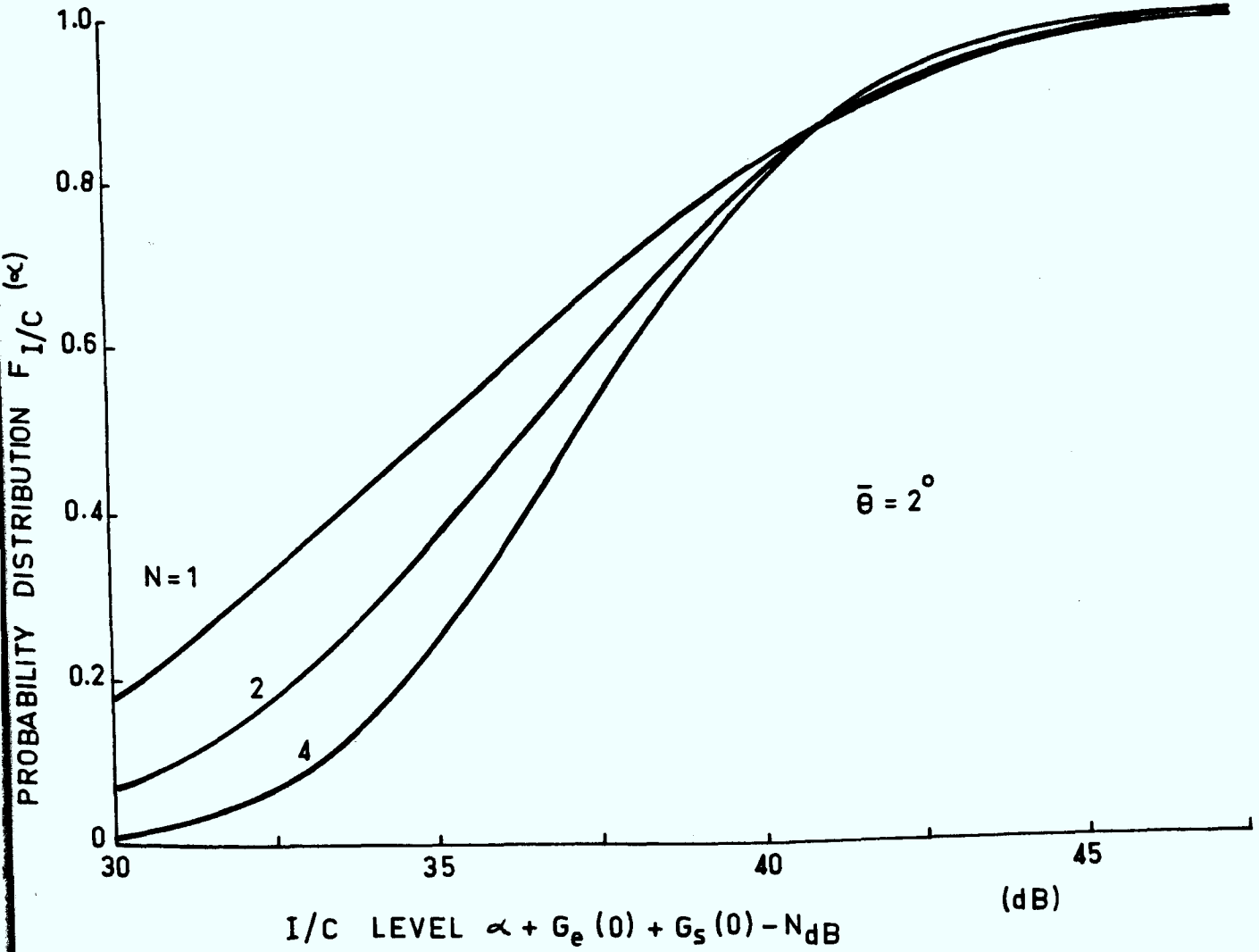


Fig. 8. Probability distributions $F_{I/C}(\alpha)$ for 1, 2 and 4 interfering signals; narrow-beam satellite network; $\bar{\theta} = 2^\circ$

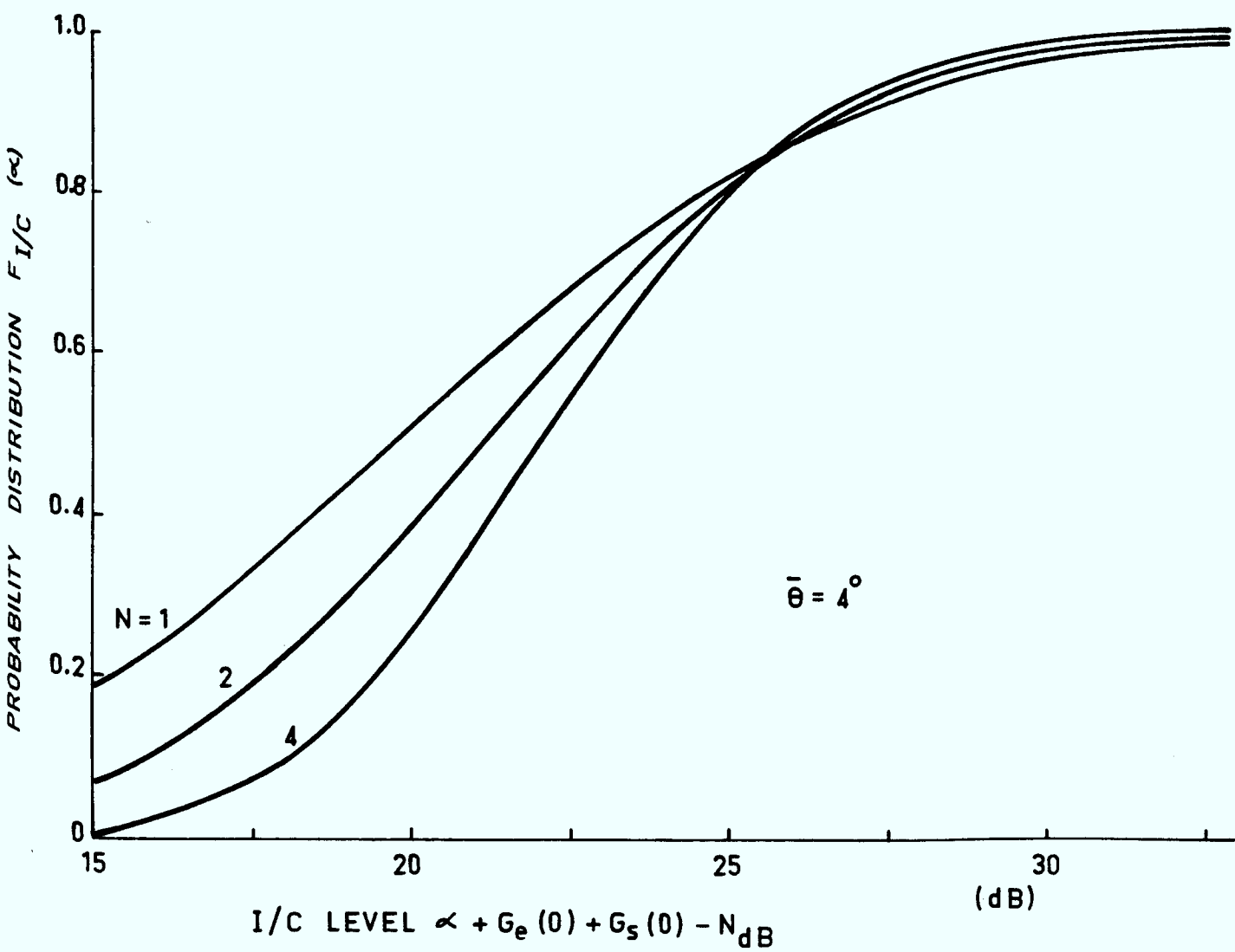


Fig. 9. Probability distributions $F_{I/C}(\alpha)$ for 1, 2 and 4 interfering signals; narrow-beam satellite network; $\bar{\theta} = 4^\circ$

The effects of a 5 dB fade simultaneously on all interfering signals yields an improvement in confidence level to approximately 98% or more for $N = 1, 2$ and 4 with $\bar{\theta} = 2^\circ$ and $\bar{\theta} = 4^\circ$.

To obtain results for other side-lobe level distributions, it is necessary only to convolve the satellite and earth station dB probability densities for these side-lobe level variations. As explained earlier probability density data is scarce, particularly for satellite antennas in the intermediate beamwidth range.

XII RAIN FADING EFFECTS

To see the effects of rain fading we first determine the probability distribution from the density in (22). The "wanted signal fade" curve in Fig. 10 is obtained by integration of (22) using (14) with $\alpha_0 = 0.253$ dB, $\sigma_0 = 1.174$ dB, and $P_0 = 1$. This curve shows the I/C distribution with one interfering signal present and with fading of the wanted signal only assuming that all other link parameters remain constant. This curve shows the signal fade to be less than 1.06 dB with 90% probability during rainfall times.

Fig. 10 also shows the I/C distribution when the sole interfering signal fades, with all other link variables held constant. Lying between this curve and the "wanted signal fade" curve is the I/C distribution which results when the wanted signal and the sole interfering signal fade independently. This latter curve is obtained by convolution of (22) and (23) followed by integration of the resulting probability density function. In this latter case the 90% probability is reduced from 1.06 dB when the wanted signal only fades, to 0.8 dB.

The curves in Fig. 10 were plotted assuming equal transmitted power

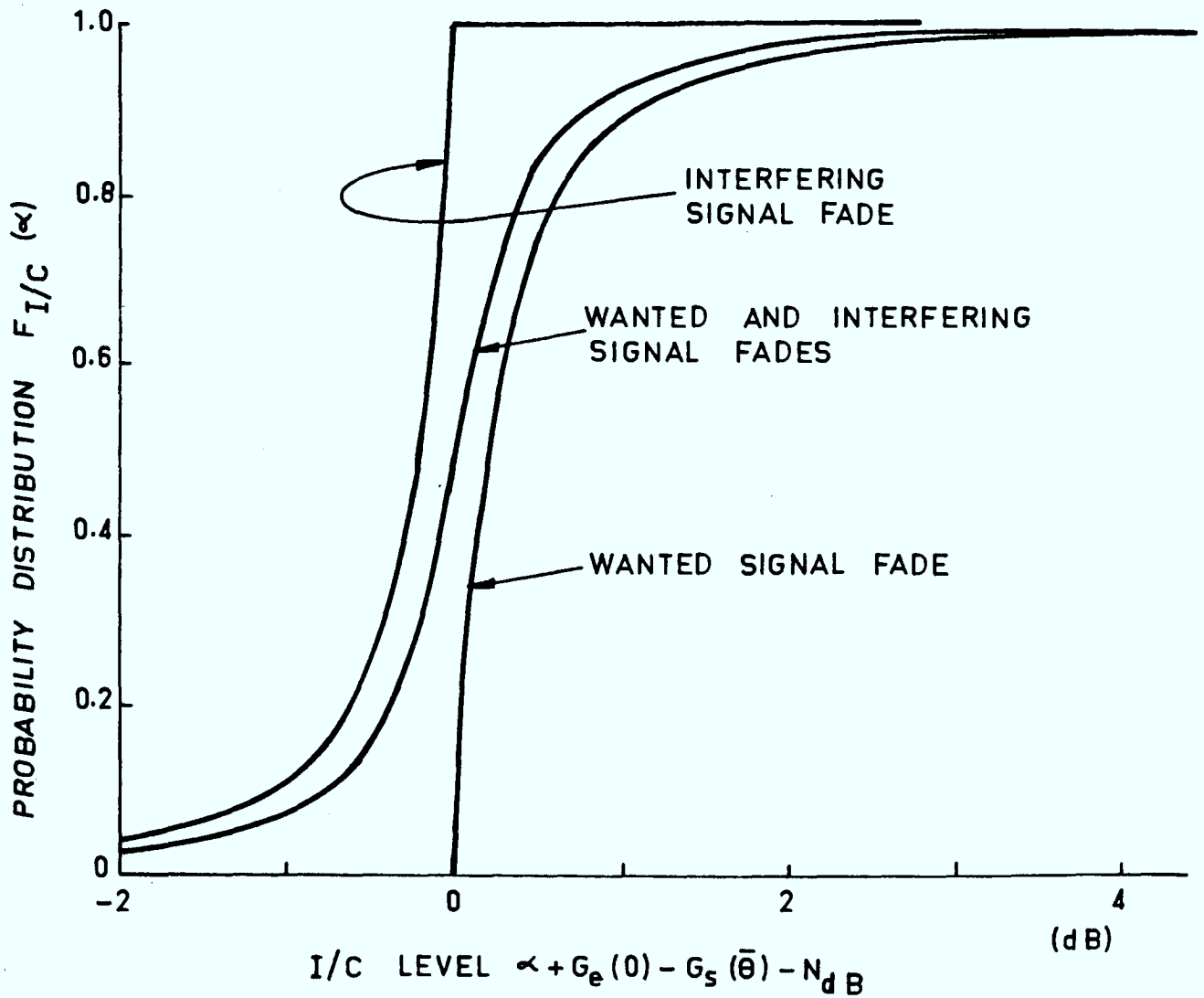


Fig. 10. Probability distribution $F_{I/C}(\alpha)$ with rain fading, for a single interfering signal.

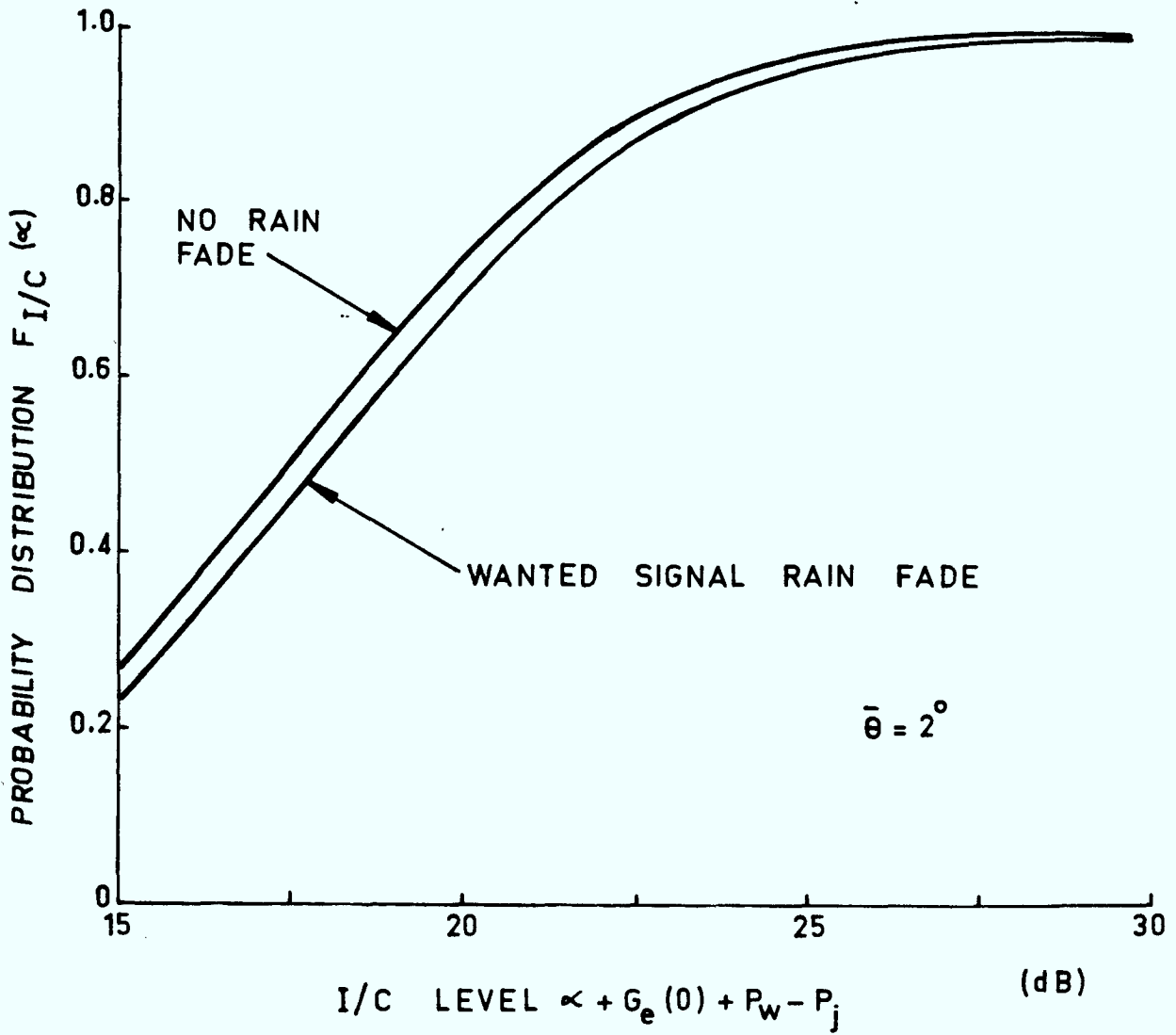


Fig. 11. Probability distributions $F_{I/C}(\alpha)$ with rain fading, for a single interfering link; $\bar{\theta} = 2^\circ$

levels for the wanted and interfering signal. If these levels are unequal, then the abscissa should be relabelled: $\alpha + G_e(0) - G_e(\bar{\theta}) + P_w - P_j$. If more than one interfering signal is present and if these and the wanted signal are all transmitted at equal power levels and are all subjected to rain fading then the resulting distribution lies between the curves labelled "wanted signal fade" and "wanted and interfering signal fade". The curves in Fig. 10 apply to satellite networks with one wide-beam and one narrow-beam antenna on each link. By adding the terms $G_s(0) - G_s(\bar{\phi})$ to the horizontal axis, they also apply to narrow-beam networks. The curves apply to up-links as well as down-links.

We next consider the effects of rain fading together with other random variations of the link variables. Fig. 11 shows the I/C distribution with and without rain fading of the wanted signal, assuming that the sole interfering signal is not subjected to rain fading. The horizontal distance between the two curves is approximately 0.5 dB. If both the wanted and interfering signal fade independently then the distribution lies very close to the "no rain fade" curve. From this last statement one concludes that with the rain fade parameters used here, the I/C distribution is not changed very much from the unfaded cases considered earlier. This statement is much strengthened by the fact that the probability of rain is typically much less than unity, which moves any rain fading curve much closer to the corresponding unfaded curve. These statements apply to narrow-beam networks as well as to those depicted in Figs. 1-3, for various $\bar{\theta}$ values.

A worst case analysis of interference effects would assume "maximum" fading of the wanted signal and no fading of the interfering signals. Therefore, in calculating $\Delta_{WC-STAT}$, rain fading adds approximately 1.0 dB to the otherwise determined values in Tables 1 and 2.

XIII SUMMARY AND CONCLUDING REMARKS

A viable procedure has been developed and used to obtain probability distributions for the interference-to-signal ratio for various representative satellite networks. Actual results are presented for nominal orbital separation angles of $\bar{\theta} = 2^\circ$ and 4° for up to four interfering signals. All interfering signals were assumed to be at the minimum orbital separation angle $\bar{\theta}$ from the wanted signal since these interferers have the most effect on the interference level.

The I/C probability distributions were determined on the basis of Gaussian variations in the angular separations between the beams of the interfering and wanted signals. For tolerances below 0.5° for satellite station keeping and antenna mispointing, the probability distribution in dB of the antenna side-lobe levels remains essentially unchanged from its assumed Gaussian distribution. This same statement applies if the cumulative variations in θ from nominal values follow a triangular probability density, except that in such case the tolerances can be even larger, up to 1.0° . There is no firm data on the actual density for variations in θ ; our choices seem reasonable.

The I/C distributions were used to compare I/C confidence levels with those based on a conventional worst case analysis. The difference $\Delta_{WC-STAT}$ can be substantial, up to 3.7 dB with ± 0.5 dB variation in transmitted power

levels for $\bar{\theta} = 2^\circ$, with 4 interfering signals on networks where each link has one wide-beam and one narrow-beam antenna. For networks where all antennas have narrow beamwidths, the difference is much larger, up to 7.5 dB with 4 interfering signals at $\bar{\theta} = 2^\circ$.

The above results apply with rain fading perfectly mitigated on all up-links and perfectly correlated among all down-link signals. If the wanted signal is subject to a known dB rain fade, then the probability that I/C exceeds a given level can again be determined from the distribution curves, and the same comment applies if all interfering signals simultaneously undergo the same rain fades. In the case of different rain fades among different interfering signals, the result on I/C confidence levels could be conservatively estimated by assuming that all interfering signals undergo the minimum rain fade.

An alternate way to account for rain fading effects is to convolve the rain fade probability density with the dB probability densities of the other link variables, as explained in the previous section. For typical rain fade distributions the dB variance of the rain fade probability density is typically 0.5 dB or less, while that of the probability distribution due to variations of antenna patterns is close to 4 dB. The variance of the sum probability density remains close to 4 dB and the distribution remains virtually unchanged from its value when rain fading effects are ignored.

The results presented here quantify what has been previously articulated in a qualitative way; namely that a worst case design is overly conservative in predicting actual I/C levels, averaged over space and time. The results

in this report could be used, together with other data including antenna gains and transmitted power levels, to predict the probability that a satellite link would operate at a given performance level. As stated earlier [15] "Being able to state results in probabilistic terms, and being able to state the cost of increasing those probabilities, is a new approach to orbit management." We would suggest that the probabilistic approach is more realistic than the conventional worst case approach and is comparable to means used to quantify the reliability behaviour of other complex systems.

The probabilistic approach could also be used to determine satellite network spectrum-orbit capacity. Use of a worst case analysis to determine the minimum nominal satellite separation $\bar{\theta}$ required to provide a wanted I/C ratio yields an unnecessarily large value for $\bar{\theta}$. Use of I/C distribution curves like those presented here would yield a lower value of $\bar{\theta}$ and a correspondingly larger capacity. The actual determination of spectrum orbit capacity is complex [16] but an approach based on link I/C probability distributions is a reasonable one worthy of careful examination.

Estimates of the difference between orbit capacities based on worst-case and statistical evaluations are possible. For example on networks with four interfering signals and a nominal 2° spacing, (51) indicates that with 90% Probability $I/C + G_e(0) = 27.6$ dB. A worst case analysis shows

$$I/C + G_e(0) = 6 + 30 - 25 \log(\bar{\theta}_{WC} - 0.3) \quad (56)$$

The value of $\bar{\theta}_{WC}$ needed for a 27.6 dB value for $I/C + G_e(0)$ is easily obtained from (56):

$$\begin{aligned}\bar{\theta}_{WC} &= 0.3 + 10^{(8.4/25)} \\ &= 2.47^\circ\end{aligned}\quad (57)$$

The spectrum-orbit capacity appears to have been reduced under worst case analysis to approximately 81% of what is obtainable with $\bar{\theta} = 2^\circ$ under probabilistic analysis. If we add the effects of power level variations of ± 0.5 dB and rain fading of 1.0 dB for the wanted signal then

$$\begin{aligned}\bar{\theta}_{WC} &= 0.3 + 10^{(10.4/25)} \\ &= 2.91^\circ\end{aligned}\quad (58)$$

In this case the orbit-capacity reduction is to 69% of the statistically determined value at $\bar{\theta} = 2^\circ$.

For narrowbeam networks with $\bar{\theta} = \bar{\phi} = 2^\circ$ and 4 interfering signals, Fig. 8 indicates that with 90% probability $I/C + G_e(0) + G_s(0) - N_{dB}$ is less than 41.8 dB. A worst case analysis which includes ± 0.5 dB transmitter power level variations and 1.0 dB rain fading of the wanted signal yields the following $\bar{\theta}_{WC}$ for 41.8 dB:

$$41.8 \approx 8 + 60 - 50 \log(\bar{\theta}_{WC} - 0.25) \quad (59)$$

$$\begin{aligned}\bar{\theta}_{WC} &= 0.25 + 10^{(26.2/50)} \\ &= 3.59^\circ\end{aligned}\quad (60)$$

The resulting reduction in orbit capacity is to 56% of the $\bar{\theta} = 2^\circ$ value.

The above analysis of orbit-capacity reductions is somewhat oversimplified but does indicate the conservatism of a worst case analysis. In practise, users would find at $\bar{\theta} \approx 3^\circ$, data rates or signal-to-interference ratios would be higher than what is expected from a worst case analysis based on $\bar{\theta} = 2^\circ$. This is in fact what is happening, and is one of the factors which motivated this study.

APPENDIX I: DEFINITION OF LINK PARAMETERS

Satellite link parameters are defined as follows:

Down-Link

- P_d : RF output power from the satellite transponder.
- $G_s(\phi)$: satellite antenna gain in the direction of the receiving earth station antenna at angle ϕ .
- L_m : multiplexing loss in the satellite after the power amplifier
(≈ 1 or 2 dB)
- L_f : free space loss between satellite and earth station (including a 20 log(f) component, ≈ 205 dB at 12 GHz)
- L_{ca} : clear air loss (fraction of a dB at SHF)
- D : rain-attenuation loss (highly variable, up to ≈ 10 dB at 12 GHz)
- $G_e(\theta)$: earth station receiving gain in the direction of the satellite at angle θ

Up-Link

- P_u : RF output power of the wanted signal from the earth station transmitter
- $H_s(\alpha)$: satellite receiving antenna gain in the direction of the transmitting earth station at angle α
- U : rain attenuation loss on up link
- $H_e(\beta)$: earth station transmitter gain in the direction of the satellite receiving antenna at angle β

When both the wanted signal and interference are under consideration, the subscript "w" is appended for the wanted signal, "u" for the up-link interference and "d" for the down-link interference.

APPENDIX II: SUMS, PRODUCTS AND TRANSFORMATIONS OF RANDOM VARIABLES

Important results involving sums, products and transformations of random variables are summarized below for easy reference. Further details are available elsewhere [12,13].

A. Sums of Random Variables

Let x and y be two random variables, and let $z = x + y$. Then the density $f_z(\alpha)$ in terms of the joint density $f_{xy}(\beta, \gamma)$ is

$$f_z(\alpha) = \int_{-\infty}^{\infty} f_{xy}(\alpha - \beta, \beta) d\beta \quad (\text{AII-1-a})$$

$$= \int_{-\infty}^{\infty} f_{xy}(\beta, \alpha - \beta) d\beta \quad (\text{AII-1-b})$$

If x and y are statistically independent then f_z is the convolution of x and y :

$$f_z(\alpha) = \int_{-\infty}^{\infty} f_x(\alpha - \beta) f_y(\beta) d\beta \quad (\text{AII-2-a})$$

$$= \int_{-\infty}^{\infty} f_x(\beta) f_y(\alpha - \beta) d\beta \quad (\text{AII-2-b})$$

B. Product of Random Variables

Let $z = xy$. Then

$$f_z(\alpha) = \int_{-\infty}^{\infty} |\beta|^{-1} f_{xy}\left(\frac{\alpha}{\beta}, \beta\right) d\beta \quad (\text{AII-3-a})$$

$$= \int_{-\infty}^{\infty} |\beta|^{-1} f_{xy}\left(\beta, \frac{\alpha}{\beta}\right) d\beta \quad (\text{AII-3-b})$$

If x and y are statistically independent

$$f_z(\alpha) = \int_{-\infty}^{\infty} |\beta|^{-1} f_x\left(\frac{\alpha}{\beta}\right) f_y(\beta) d\beta \quad (\text{AII-4-a})$$

$$= \int_{-\infty}^{\infty} |\beta|^{-1} f_x(\beta) f_y\left(\frac{\alpha}{\beta}\right) d\beta \quad (\text{AII-4-b})$$

C. Transformation of Random Variables

Let x and y be single-valued random variables, with $y = g(x)$. Then

[13]

$$f_y(\beta) = [f_x(\alpha)/|g'(\alpha)|] \quad \alpha = g^{-1}(\beta) \quad (\text{AII-5})$$

We have assumed a one-to-one relationship of x to y . If more than one x value yields the same y value (AII-5) is readily extended [13].

As an example, consider the case where x in dB is a Gaussian random variable; thus

$$f_x(\alpha) = (\sqrt{2\pi}\sigma)^{-1} \exp[-(\alpha-m)^2/2\sigma^2] \quad (\text{AII-6})$$

The power level y is related to x :

$$y = 10^{x/10} \quad (\text{AII-7})$$

Thus, $g(x) = 10^{(x/10)}$

$$= e^{(\ln 10/10)x}$$

$$g'(x) = (\ln 10/10) 10^{(x/10)}$$

$$= (\ln 10/10)y$$

$$f_y(\beta) = \frac{(10/\ln 10)}{\sqrt{2\pi} \sigma \beta} \exp\left[-\frac{(\log_{10} \beta - m)^2}{2\sigma^2}\right] \quad (\text{AII-8})$$

Thus, y is a log-normal random variable.

APPENDIX III - ANTENNA GAIN REFERENCE PATTERNS

We include on the following four pages the standard antenna reference Pattern for earth station antennas [6] and for satellite antennas [11], over the entire angular region. Pages 47 and 48 apply to earth station antennas, while pages 49 and 50 apply to satellite antennas. These standard patterns are under continuing review by various organizations.

ANNEX 1

REFERENCE PATTERN OF THE WARC-79

The reference pattern in Fig. 12, as agreed to by the WARC-79, is given by the following extract from Appendices 28 and 29 of the Radio Regulations:

Determination of the antenna gain

"The relationship $\varphi(\alpha)$ may be used to derive a function for the horizon antenna gain, $G(\text{dB})$ as a function of the azimuth α , by using the actual earth station antenna pattern, or a formula giving a good approximation. For example, in cases where the ratio between the antenna diameter and the wavelength is not less than 100, the following equation should be used:

$$G(\varphi) = G_{\max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda} \varphi \right)^2 \quad \text{for } 0 < \varphi < \varphi_m \quad (39a)$$

$$G(\varphi) = G_1 \quad \text{for } \varphi_m \leq \varphi < \varphi_r \quad (39b)$$

$$G(\varphi) = 32 - 25 \log \varphi \quad \text{for } \varphi_r \leq \varphi < 48^\circ \quad (39c)$$

$$G(\varphi) = -10 \quad \text{for } 48^\circ \leq \varphi \leq 180^\circ \quad (39d)$$

where:

D : antenna diameter }
 λ : wavelength } expressed in the same unit

G_1 : gain of the first sidelobe = $2 + 15 \log \frac{D}{\lambda}$

$$\varphi_m = \frac{20\lambda}{D} \sqrt{G_{\max} - G_1} \text{ (degrees)}$$

$$\varphi_r = 15.85 \left(\frac{D}{\lambda} \right)^{-0.6} \text{ (degrees)}$$

When it is not possible, for antennas with $\frac{D}{\lambda}$ of less than 100, to use the above reference antenna pattern and when neither measured data nor a relevant CCIR Recommendation accepted by the administrations concerned can be used instead, administrations may use the reference diagram as described below:

$$G(\varphi) = G_{\max} - 2.5 \times 10^{-3} \left(\frac{D}{\lambda} \varphi \right)^2 \quad \text{for } 0 < \varphi < \varphi_m \quad (40a)$$

$$G(\varphi) = G_1 \quad \text{for } \varphi_m \leq \varphi < 100 \frac{\lambda}{D} \quad (40b)$$

$$G(\varphi) = 52 - 10 \log \frac{D}{\lambda} - 25 \log \varphi \quad \text{for } 100 \frac{\lambda}{D} \leq \varphi < 48^\circ \quad (40c)$$

$$G(\varphi) = 10 - 10 \log \frac{D}{\lambda} \quad \text{for } 48^\circ \leq \varphi \leq 180^\circ \quad (40d)$$

where:

D : antenna diameter }
 λ : wavelength } expressed in the same unit

G_1 = gain of the first sidelobe = $2 + 15 \log \frac{D}{\lambda}$

$$\varphi_m = \frac{20\lambda}{D} \sqrt{G_{\max} - G_1} \text{ (degrees)}$$

The above patterns may be modified as appropriate to achieve a better representation of the actual antenna pattern.

In cases where $\frac{D}{\lambda}$ is not given, it may be estimated from the expression $20 \log \frac{D}{\lambda} \approx G_{\max} - 7.7$, where G_{\max} is the main lobe antenna gain in dB."

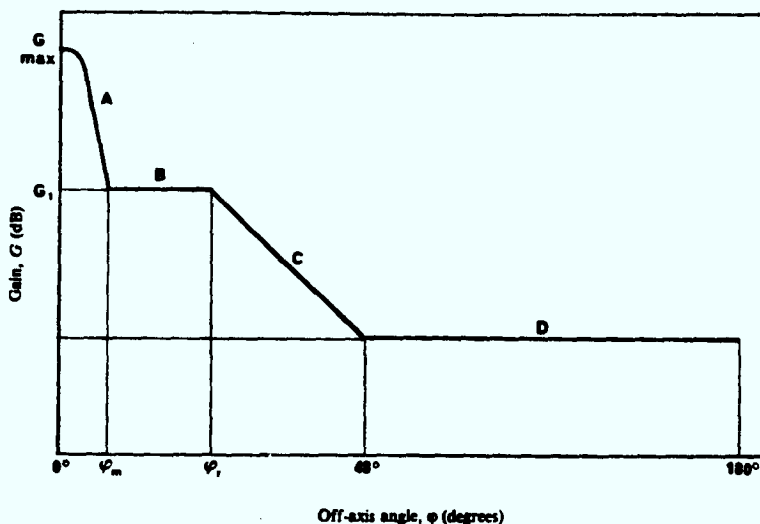


FIGURE 12 - Reference radiation pattern of an earth-station antenna (after the WARC-79)

- A: main lobe
- B: first side lobe
- C: other side lobe
- D: residual gain

The equations quoted above include the evaluation of antenna radiation pattern close to the axis of the main beam, which is not a part of the radiation pattern currently quoted in the CCIR and recent experimental data has indicated that it may be necessary to modify the equation quoted above for the gain of the first side lobe.

Measurements made on a number of symmetric Cassegrain antennas have shown that the relative first side-lobe levels (generally < -14 dB) do not exhibit a clear dependence on D/λ . Figure 13 shows the data which has been converted into absolute first side-lobe levels from the knowledge of the peak gain of each antenna. It can be seen that the above formula for G_1 under predicts the first side-lobe gain particularly for larger antennas. Based on these considerations, the following equation is considered to be a more appropriate representation of the first side-lobe gain:

$$G_1 = 20 \log (D/\lambda) - 7 \quad \text{dBi}$$

Whereas this equation represents an approximate mean of the measured data, it is evident that individual design features of an antenna, e.g. aperture illumination efficiency, would produce variations in the first side-lobe levels as is indicated by the spread of the data shown in Fig. 13.

For angles beyond 1° to 1.5° , however, the above equations simplify to those now used in the CCIR for antenna of $D/\lambda > 100$.

Furthermore, it is recommended that the compatibility of these formulas with the search for an efficient utilization of the geostationary orbit should be studied.

4. Satellite antenna reference radiation pattern

It appears desirable to postulate guidelines for a pattern as a basis for further consideration for satellite antennas which have relatively simple pattern envelopes, such as those having simple circular main lobes. It is also desirable to have an interim reference for these conditions for the co-ordination computations of Appendix 29 of the Radio Regulations (Final Acts of the World Administrative Radio Conference), if actual patterns are not available.

As noted previously, the radiation pattern of the satellite antenna is important in the region of the main lobe as well as the farther side lobes. Thus, the following postulated pattern commences at the -3 dB contour of the main lobe and is divided into four regions.

$$G(\theta) = G_m - 3 \left(\frac{\theta}{\theta_0} \right)^2 \quad \text{dB for } \theta_0 < \theta < 2.6 \theta_0 \quad (1)$$

$$G(\theta) = G_m - 20 \quad \text{dB for } 2.6 \theta_0 < \theta < 6.3 \theta_0 \quad (2)$$

$$G(\theta) = G_m - 25 \log \left(\frac{\theta}{\theta_0} \right) \quad \text{dB for } 6.3 \theta_0 < \theta < \theta_1 \quad (3)$$

$$G(\theta) = -10 \quad \text{dB for } \theta_1 < \theta \quad (4)$$

where:

$G(\theta)$: gain at the angle (θ) from the axis.

G_m : maximum gain in the main lobe.

θ_0 : one-half the 3 dB beamwidth in the plane of interest (3 dB below G_m).

θ_1 : value of (θ) when $G(\theta)$ in equation (3) is equal to -10 dB.

These functions are shown in Fig. 13.

Equation (1) is one of a number of functions which may be utilized to approximate the main lobe pattern of a simple (un-shaped) beam. In the region from -3 dB to -20 dB as postulated herein, this function provides gain values which are generally higher than those encountered with actual simple beam antennas. Equation (2) covers the region of the first, or the first few side lobes, and is based on typical values achieved when no attempt is made to reduce the first side-lobe levels. Equation (3) covers the region of the farther side lobes. A slope of -7.5 dB/octave is utilized as in the earth-station reference patterns. The fourth region, equation (4), is also derived from the earth-station reference pattern.

Difficulties arise, however, in attempting to apply the postulated pattern to an elliptical beam, as shown in [CCIR, 1974-78]. Administrations are therefore requested to submit measured radiation patterns for antennas with other than simple circular beams, including elliptical beams.

5. Conclusions

From the standpoint of satellite antenna design, it cannot be assumed that efficient orbit utilization will be obtained unless it is specifically sought. In general, apertures which are larger than those required to achieve the necessary e.i.r.p., will enhance orbit utilization over a coverage area. Therefore, satellite antenna radiation pattern objectives appear to be desirable. To enhance orbit utilization, the spacecraft antenna should have the following general characteristics:

- the main lobe patterns of the satellite antennas should conform to the coverage area as closely as possible (beam shaping in the plane normal to the axis of propagation is desirable);
- the side lobes should be controlled outside the coverage area. The utilization of techniques to reduce the first side-lobe level and to increase the far side-lobe envelope slope are to be encouraged;
- the position of a geostationary satellite should not be unduly restricted by steerability limits of narrow beam antennas.

There are many parameters involved with complex satellite antenna patterns which affect orbit utilization, and additional study is required before any general conclusions can be drawn.

It is not known at present whether a spacecraft antenna reference pattern can be developed which will be applicable to the large variety of complex patterns which may be utilized.

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APPENDIX IV: COMPUTER PROGRAM DOCUMENTATION

The computer programs used to obtain the results presented are listed below, together with some sample numerical output. Considerable care was taken in developing these programs, to enable easy readability, modification of key variables, and initial experimentation to obtain preliminary results.

In all programs, key variables are:

- B: tolerances for mispointing and satellite station keeping
- THETA: nominal satellite separation angle
- STHETA: standard deviation in degrees of the cumulative angular errors from mispointing and satellite station keeping
- SDB: standard deviation in dB of the interference side lobe level
- N: number of panels used in the numerical Simpson's rule integration of (30)
- M: number of panels in the numerical trapezoidal rule integration to perform numerical convolutions, and in the distribution function calculations using (13)
- F%: a flag to control the number of convolutions, in accordance with the number of interfering signals
- ALPHAM: median rain fade
- SIGMA: raid fade standard deviation

All programs were written and executed in IBM advanced microcomputer basic. The language was fully adequate for our purposes and is easily converted into FORTRAN 77, if desired. Multiline statements proved very useful in writing compact programs, and multicharacter variable names (up to 255 characters) facilitate program readability. The fact that subroutine

variables are globally defined did not prove inconvenient for our applications.

Program SCENAR1C.BAS was used to obtain the I/C distributions for Gaussian angular variations with a single interferer (Figs. 4-9, inclusive).

Program SCENARIT.BAS was a modification of 1C, and was used to obtain the curves in Fig. 4 and 5 for a triangular probability density for the cumulative angular variation due to mispointing and station keeping errors.

Program SCENAR2C.BAS was used to obtain the I/C distributions in Figs. 6-9, inclusive, for 2 and 4 interfering signals. The convolutions were obtained by using a trapezoidal integration rather than the conventional first-order hold approach, to increase the numerical accuracy [14]. Convolution involved interference power quantities with equal increments in power, which implied highly unequal steps in dB. Many points ($M = 1000$) were needed to obtain adequate numerical accuracy. It was found that for the cases considered here, convolutions involving only the first M output values gave sufficient accuracy, since the output probability density values from $M + 1$ to $2M$ were very small, relative to values in the range from 0 to M .

Program SCENAR1R.BAS was used to obtain the rain fading probability distributions in Fig. 10 labelled "wanted signal fade" and "interfering signal fade". Program SCENAR2R.BAS yielded the other Fig. 10 curve. Program SCENAR3R.BAS was used to obtain the rain fade distribution in Fig. 11.

```

10 LPRINT "PROG NAME: SCENARIC.BAS"
20 LPRINT
30 '
40 B=.1:      : THETA = 2:      N = 200:      M = 200:      PI = 3.141593
50 STHETA=(B*SQR(3))/SQR(6):      SDB=3.91
60 '
70      DEF FNA(X) = 25*(1-(LOG(X)/LOG(10)))
80      DEF FNGAUSS(X,S) = EXP(-(X/S)^2)/2)/(SQR(2*PI)*S)
90 '
100     RANGETH=5*STHETA:      DELTATH=(2*RANGETH)/N
110     LLIMTH=THETA-RANGETH:      ULIMTH=THETA+RANGETH
120     RANGEDB=3*SDB:      DELTADB = (2*RANGEDB)/M
130     LLIMDB=FNA(THETA)-RANGEDB:      ULIMDB=FNA(THETA)+RANGEDB
140 '
150 LPRINT "B=";B,      "THETA=";THETA,      "N=";N,      "M=";M
160 LPRINT "FNA(THETA)=";FNA(THETA),      "STHETA=";STHETA,      "SDB=";SDB
170 LPRINT
180 LPRINT "RANGETH=";RANGETH,      "DELTATH=";DELTATH
190 LPRINT "LLIMTH=";LLIMTH,      "ULIMTH=";ULIMTH
200 LPRINT"CHECK: LLIMTH+N*DELTATH="; LLIMTH+DELTATH*N
210 LPRINT
220 LPRINT "RANGEDB=";RANGEDB,      "DELTADB = ";DELTADB
230 LPRINT "LLIMDB=";LLIMDB,      "ULIMDB=";ULIMDB
240 LPRINT "CHECK: LLIMDB+M*DELTADB=";LLIMDB+M*DELTADB
250 '
260 DIM X(201),      DENSI(201),      DISTI(201)
270 FOR J = 0 TO M:      X = LLIMDB+J*DELTADB:      X(J)=X
280 SUMOD = 0
290 FOR I = 1 TO N-1 STEP 2
300 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
310 SUMOD = SUMOD+FNGAUSS((X-ADB),SDB)*FNGAUSS((Y-THETA),STHETA)
320 NEXT I
330 SUMEV = 0
340 FOR I = 2 TO N-2 STEP 2
350 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
360 SUMEV = SUMEV+FNGAUSS((X-ADB),SDB)*FNGAUSS((Y-THETA),STHETA)
370 NEXT I
380 AL = FNA(LLIMTH):      AU = FNA(ULIMTH)
390 SUML=FNGAUSS((X-AL),SDB)*FNGAUSS((LLIM-THETA),STHETA)
400 SUMU=FNGAUSS((X-AU),SDB)*FNGAUSS((ULIM-THETA),STHETA)
410 FSUM=SUML+SUMU+2*SUMEV+4*SUMOD
420 DENSI(J) = (DELTATH/3)*FSUM
430 '
440 IF J = 0 THEN DISTSUM = 0 ELSE DISTSUM = DISTSUM+DENSI(J-1)+DENSI(J)
450 DISTI(J)=(DISTSUM/2)*DELTADB
460 NEXT J
470 '
480 LPRINT
490 LPRINT"K" TAB(20) "dB" TAB(40) "DENSI(K)" TAB(60) "DISTI(K)": LPRINT
500 FOR K = 0 TO N STEP 2
510 LPRINT K TAB(20) X(K) TAB(40) DENSI(K) TAB(60) DISTI(K)
520 NEXT K
530 END

```

PROG NAME: SCENAR1C.BAS

B= .1 THETA= 2 N= 200 M= 200
FNA(THETA)= 17.47425 STHETA= 7.071069E-02 SDB= 3.91

RANGETH= .3535535 DELTATH= 3.535535E-03
LLIMTH= 1.646447 ULIMTH= 2.353554
CHECK: LLIMTH+N*DELTATH= 2.353554

RANGEDB 11.73 DELTADB = .1173
LLIMDB= 5.744252 ULIMDB= 29.20426
CHECK: LLIMDB+M*DELTADB= 29.20426

K	dB	DENSI(K)	DISTI(K)
0	5.744252	1.171285E-03	0
2	5.978853	1.39755E-03	3.008057E-04
4	6.213453	1.661588E-03	6.590528E-04
6	6.448053	1.968478E-03	1.084192E-03
8	6.682653	2.323746E-03	1.586918E-03
10	6.917253	2.733368E-03	2.179275E-03
12	7.151852	3.203746E-03	2.874759E-03
14	7.386452	3.741704E-03	3.688418E-03
16	7.621053	4.354439E-03	4.636945E-03
18	7.855653	5.049471E-03	5.73876E-03
20	8.090252	5.834597E-03	7.014083E-03
22	8.324852	6.717797E-03	8.484981E-03
24	8.559452	7.707154E-03	1.017541E-02
26	8.794052	8.81074E-03	1.211123E-02
28	9.028652	1.003649E-02	1.432016E-02
30	9.263252	1.139207E-02	1.683177E-02
32	9.497852	1.288471E-02	1.967737E-02
34	9.732452	1.452104E-02	2.288991E-02
36	9.967052	1.630693E-02	.0265038
38	10.20165	1.824726E-02	3.055471E-02
40	10.43625	2.034578E-02	3.507933E-02
42	10.67085	2.260489E-02	4.011508E-02
44	10.90545	2.502543E-02	4.569976E-02
46	11.14005	2.760654E-02	5.187115E-02
48	11.37465	3.034546E-02	5.866663E-02
50	11.60925	.0332374	.0661227
52	11.84385	3.627533E-02	7.427446E-02
54	12.07845	3.944999E-02	8.315512E-02
56	12.31305	4.274977E-02	.0927954
58	12.54765	4.616064E-02	.1032231
60	12.78225	4.966624E-02	.1144623
62	13.01685	5.324788E-02	.1265332
64	13.25145	5.688457E-02	.1394511
66	13.48605	6.055333E-02	.1532262
68	13.72065	6.422929E-02	.1678633
70	13.95525	6.788584E-02	.1833609
72	14.18985	7.149521E-02	.1997112
74	14.42445	7.502844E-02	.2168998
76	14.65905	7.845605E-02	.2349053
78	14.89365	.0817482	.2536994
80	15.12825	8.487531E-02	.273247
82	15.36285	8.780838E-02	.2935059
84	15.59745	9.051949E-02	.3144272
86	15.83205	9.298215E-02	.3359558
88	16.06665	9.517186E-02	.3580304
90	16.30125	9.706642E-02	.3805845

92	16.53585	9.864631E-02	.4035463
94	16.77045	9.989509E-02	.4268401
96	17.00505	.1007996	.4503867
98	17.23965	.1013504	.4741041
100	17.47425	.1015414	.4979086
102	17.70885	.1013707	.5217156
104	17.94345	.1008401	.5454402
106	18.17805	9.995523E-02	.5689986
108	18.41265	9.872555E-02	.5923088
110	18.64725	9.716393E-02	.6152915
112	18.88185	9.528667E-02	.6378704
114	19.11645	9.311306E-02	.6599739
116	19.35105	9.066524E-02	.6815348
118	19.58565	8.796756E-02	.702492
120	19.82025	8.504639E-02	.7227896
122	20.05485	8.192959E-02	.7423786
124	20.28945	7.864613E-02	.7612163
126	20.52405	7.522557E-02	.7792672
128	20.75865	7.169771E-02	.7965026
130	20.99325	.0680921	.8129008
132	21.22785	6.443768E-02	.8284471
134	21.46245	6.076236E-02	.8431331
136	21.69705	5.709275E-02	.8569573
138	21.93165	5.345387E-02	.8699238
140	22.16625	4.986879E-02	.8820425
142	22.40085	.0463586	.8933289
144	22.63545	4.294212E-02	.9038023
146	22.87005	3.963586E-02	.9134869
148	23.10465	3.645398E-02	.9224104
150	23.33925	3.340822E-02	.9306031
152	23.57385	3.050797E-02	.9380983
154	23.80845	2.776037E-02	.944931
156	24.04305	2.517033E-02	.9511374
158	24.27765	2.274072E-02	.9567551
160	24.51225	2.047253E-02	.9618218
162	24.74685	1.836497E-02	.9663751
164	24.98145	1.641576E-02	.9704527
166	25.21605	1.462122E-02	.9740911
168	25.45065	1.297651E-02	.9773261
170	25.68525	1.147583E-02	.9801924
172	25.91985	1.011259E-02	.9827229
174	26.15445	8.879575E-03	.9849487
176	26.38905	7.769157E-03	.9868999
178	26.62365	6.773412E-03	.988604
180	26.85825	5.884277E-03	.9900871
182	27.09285	5.093665E-03	.9913736
184	27.32745	4.393592E-03	.9924852
186	27.56205	3.776251E-03	.9934426
188	27.79665	3.234104E-03	.9942639
190	28.03126	2.759936E-03	.9949661
192	28.26586	2.346907E-03	.9955642
194	28.50046	1.988588E-03	.996072
196	28.73506	1.678981E-03	.9965014
198	28.96966	1.412533E-03	.9968636
200	29.20426	1.184141E-03	.9971676

```

10 LPRINT "PROG NAME: SCENAR1T.BAS"
20 LPRINT
30 '
40 B=.1:      ; THETA = 2:      N = 100:      M = 100:      PI = 3.141593
50 STHETA=(B*SQR(3))/SQR(6):      SDB=3.91
60 '
70 DEF FNA(X) = 25*(1-(LOG(X)/LOG(10)))
80 DEF FNGAUSS(X,S) = EXP(-(X/S)^2)/2)/(SQR(2*PI)*S)
85 DEF FNTRI(X,A)=(.5/A)*(1-(ABS(X)/A))*(((X+A)/ABS(X+A))-((X-A)/ABS(X-A)))
90 '
100 RANGETH=B:      DELTATH=(2*RANGETH)/N
110 LLIMTH=THETA-RANGETH:      ULIMTH=THETA+RANGETH
120 RANGEDB=3*SDB:      DELTADB = (2*RANGEDB)/M
130 LLIMDB=FNA(THETA)-RANGEDB:      ULIMDB=FNA(THETA)+RANGEDB
140 '
150 LPRINT "B=";B,      "THETA=";THETA,      "N=";N,      "M=";M
160 LPRINT "FNA(THETA)=";FNA(THETA),      "STHETA=";STHETA,      "SDB=";SDB
170 LPRINT
180 LPRINT "RANGETH=";RANGETH,      "DELTATH=";DELTATH
190 LPRINT "LLIMTH=";LLIMTH,      "ULIMTH=";ULIMTH
200 LPRINT "CHECK: LLIMTH+N*DELTATH="; LLIMTH+DELTATH*N
210 LPRINT
220 LPRINT "RANGEDB=";RANGEDB,      "DELTADB = ";DELTADB
230 LPRINT "LLIMDB=";LLIMDB,      "ULIMDB=";ULIMDB
240 LPRINT "CHECK: LLIMDB+M*DELTADB=";LLIMDB+M*DELTADB
250 '
260 DIM X(201),      DENSI(201),      DISTI(201)
270 FOR J = 0 TO M:      X = LLIMDB+J*DELTADB:      X(J)=X
280 SUMOD = 0
290 FOR I = 1 TO N-1 STEP 2
300 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
310 SUMOD = SUMOD+FNGAUSS((X-ADB),SDB)*FNTRI((Y-THETA),B)
320 NEXT I
330 SUMEV = 0
340 FOR I = 2 TO N-2 STEP 2
350 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
360 SUMEV = SUMEV+FNGAUSS((X-ADB),SDB)*FNTRI((Y-THETA),B)
370 NEXT I
410 FSUM=2*SUMEV+4*SUMOD
420 DENSI(J) = (DELTATH/3)*FSUM
430 '
440 IF J = 0 THEN DISTSUM = 0 ELSE DISTSUM = DISTSUM+DENSI(J-1)+DENSI(J)
450 DISTI(J)=(DISTSUM/2)*DELTADB
460 NEXT J
470 '
480 LPRINT
490 LPRINT "K" TAB(20) "dB" TAB(40) "DENSI(K)" TAB(60) "DISTI(K)": LPRINT
500 FOR K = 0 TO N STEP 2
510 LPRINT K TAB(20) X(K) TAB(40) DENSI(K) TAB(60) DISTI(K)
520 NEXT K
530 END

```

```

10 LPRINT "PROG NAME: SCENAR2C.BAS"
20 LPRINT
30 '
40 B=.1:      : THETA = 2:      N =100:      M =1000:      PI = 3.141593
50 STHETA=(B*SQR(2))/SQR(6):      SDB=5.53:      C1=10/LOG(10):      FX=0
60 '
70      DEF FNA(X) = 50*(1-(LOG(X)/LOG(10)))
80      DEF FNGAUSS(X,S) = EXP(-((X/S)^2)/2)/(SQR(2*PI)*S)
90 '
100 RANGETH=5*STHETA:      DELTATH=(2*RANGETH)/N
110 LLIMTH=THETA-RANGETH:      ULIMTH=THETA+RANGETH
120 RANGEDB=3*SDB:      DELTADB=(2*RANGEDB)/M
130 LLIMDB=FNA(THETA)-RANGEDB:      ULIMDB=FNA(THETA)+RANGEDB
140 LLIMPWR=10^(LLIMDB/10):      ULIMPWR=10^(ULIMDB/10)
150 DELTAPWR=(ULIMPWR-LLIMPWR)/M
160 '
170 LPRINT "B=";B,      "THETA=";THETA,      "N=";N,      "M=";M
180 LPRINT "FNA(THETA)=";FNA(THETA),      "STHETA=";STHETA,      "SDB=";SDB
190 LPRINT
200 LPRINT "RANGETH=";RANGETH,      "DELTATH=";DELTATH
210 LPRINT "LLIMTH=";LLIMTH,      "ULIMTH=";ULIMTH
220 LPRINT"CHECK: LLIMTH+N*DELTATH="; LLIMTH+DELTATH*N
230 LPRINT
240 LPRINT "RANGEDB";RANGEDB,      "DELTADB = ";DELTADB
250 LPRINT "LLIMDB=";LLIMDB,      "ULIMDB=";ULIMDB
260 LPRINT "CHECK: LLIMDB+M*DELTADB=";LLIMDB+M*DELTADB
270 LPRINT
280 LPRINT "LLIMPWR=";LLIMPWR,      "ULIMPWR=";ULIMPWR,      "DELTAPWR=";DELTAPWR
290 LPRINT
300 '
310 DIM X(1001),      DENSI(1001),      DISTI(1001),      DENSIC(1001)
320 FOR J = 0 TO M:      XPWR = LLIMPWR+J*DELTAPWR
330 X = C1*LOG(XPWR):      X(J)=XPWR
340 SUMOD = 0
350 FOR I = 1 TO N-1 STEP 2
360 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
370 SUMOD = SUMOD+FNGAUSS((X-ADB),SDB)*FNGAUSS((Y-THETA),STHETA)
380 NEXT I
390 SUMEV = 0
400 FOR I = 2 TO N-2 STEP 2
410 Y = LLIMTH+I*DELTATH:      ADB = FNA(Y)
420 SUMEV = SUMEV+FNGAUSS((X-ADB),SDB)*FNGAUSS((Y-THETA),STHETA)
430 NEXT I
440 AL = FNA(LLIMTH):      AU = FNA(ULIMTH)
450 SUML=FNGAUSS((X-AL),SDB)*FNGAUSS((LLIM-THETA),STHETA)
460 SUMU=FNGAUSS((X-AU),SDB)*FNGAUSS((ULIM-THETA),STHETA)
470 FSUM=SUML+SUMU+2*SUMEV+4*SUMOD
480 DENSI(J) = (C1/XPWR)*(DELTATH/3)*FSUM
490 NEXT J
500 LPRINT "FINISHED CALCULATING DENSI(J)"
510 '
520 FOR J = 0 TO M
530 SUMC = 0
540 FOR K = 0 TO J-1
550 SUMC = SUMC + DENSI(K)*DENSI(J-K) + DENSI(K+1)*DENSI(J-(K+1))
560 NEXT K
570 GOSUB 790
580 NEXT J
585 '
670 LPRINT
680 LPRINT"K" TAB(20) "dB" TAB(40) "DENSIC(K)" TAB(60) "DISTI(K)": LPRINT
690 FOR K = 0 TO M STEP 20
695 XDBOUT = C1*LOG(X(K)+LLIMPWR*(1+2*FX))-3*(1+FX)
700 LPRINT K TAB(20) XDBOUT TAB(40) DENSIC(K) TAB(60) DISTI(K)
710 NEXT K
711 LPRINT
712 LPRINT
713 IF FX=0 THEN FOR J=0 TO M: DENSI(J)=DENSIC(J): NEXT J: FX=1: GOTO 520
715 END
720 '
730 'DISTRIBUTION CALC. SUBROUTINE
740 '
790 DENSI(J)=(DELTAPWR/2)*SUMC
800 '
810 IF J = 0 THEN DISTSUM = 0 ELSE DISTSUM = DISTSUM+DENSIC(J-1)+DENSIC(J)
820 DISTI(J)=(DISTSUM/2)*DELTAPWR
830 RETURN

```

```

10 LPRINT "PROG NAME: SCENAR1R.BAS"
20 LPRINT
30 '
40 ALPHAM=.235:          SIGMA=1.174:          M =1000:          PI = 3.141593
50 LPRINT "ALPHAM=";ALPHAM,          "SIGMA=";SIGMA,          "M=";M
60 LPRINT
70 '
80 DEF FNRAIN(X) = EXP(-.5*(LOG(X/ALPHAM)/SIGMA)^2)/(SQR(2*PI)*SIGMA*X)
90 '
100 DIM X(1001),          DENSIR(1001),          DISTI(1001)
110 DELTARAIN=(LOG(ALPHAM)+5*SIGMA)/M:          X(0)=0:          DENSIR(0)=0
120 DISTSUM=0
125 '
130 FOR J=1 TO M:          X=J*DELTARAIN:          X(J)=X
140 DENSIR(J) = FNRAIN(X)
150 '
160 DISTSUM = DISTSUM+DENSIR(J-1)+DENSIR(J)
170 DISTI(J)=(DISTSUM/2)*DELTARAIN
180 NEXT J
190 '
200 LPRINT
210 LPRINT "K" TAB(20) "dB" TAB(40) "DENSIR(K)" TAB(60) "DISTI(K)" :LPRINT
220 FOR K = 0 TO M STEP 20
230 LPRINT K TAB(20) X(K) TAB(40) DENSIR(K) TAB(60) DISTI(K)
240 NEXT K
250 END

```

```

10 LPRINT "PROG NAME: SCENAR2R.BAS"
20 LPRINT
30 '
40 ALPHAM=.235:          SIGMA=1.174:          M =1000:          PI = 3.141593
50 LPRINT "ALPHAM=";ALPHAM,          "SIGMA=";SIGMA,          "M=";M
60 LPRINT
70 '
80 DEF FNRAIN(X) = EXP(-.5*(LOG(X/ALPHAM)/SIGMA)^2)/(SQR(2*PI)*SIGMA*X)
90 '
100 DIM X(1001),          DENSIR(1001),          DENSIC(1001),          DISTI(1001)
110 DELTARAIN=(LOG(ALPHAM)+5*SIGMA)/M:          X(0)=0:          DENSIR(0)=0
120 DISTSUM=0
130 '
140 FOR J=1 TO M:          X=J*DELTARAIN:          X(J)=X
150 DENSIR(J) = FNRAIN(X)
160 NEXT J
170 '
180 DISTI(M)=1
190 FOR J=0 TO M-1
200 SUMC=0
210 FOR K=J TO M-1
220 SUMC = SUMC + DENSIR(K)*DENSIR(K-J) + DENSIR(K+1)*DENSIR(K+1-J)
230 NEXT K
240 DENSIC(J)=(SUMC/2)*DELTARAIN
250 '
260 IF J=0 THEN DISTSUM=0 ELSE DISTSUM = DISTSUM+DENSIC(J-1)+DENSIC(J)
270 DISTI(J)=(DISTSUM/2)*DELTARAIN
280 NEXT J
290 '
300 LPRINT
310 LPRINT "K" TAB(20) "dB" TAB(40) "DENSIR(K)" TAB(60) "DISTI(K)" :LPRINT
320 FOR K = 0 TO M STEP 20
330 LPRINT K TAB(20) X(K) TAB(40) DENSIR(K) TAB(60) .5+DISTI(K)
340 NEXT K
350 END

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```

10 LPRINT "PROG NAME: SCENAR3R.BAS"
20 LPRINT
30 '
40 ALPHAM=.253:      SIGMA=1.174:      M =1000:      PI = 3.141593
50 THETA=2:          SDB=3.91
60 LPRINT "ALPHAM=";ALPHAM,          "SIGMA=";SIGMA,          "SDB=";SDB,
    "THETA=";THETA,          "M=";M
70 LPRINT
80 '
90 DEF FNRAIN(X) = EXP(-.5*(LOG(X/ALPHAM)/SIGMA)^2)/(SQR(2*PI)*SIGMA*X)
100 DEF FNGAUSS(X,S) = EXP(-(X/S)^2)/2)/(SQR(2*PI)*S)
110 '
120 DIM X(2001), DENSIR(2001), DENSIG(2001), DENSIC(2001), DISTI(2001)
130 DELTADB=(6*SDB)/M:      LLIMDB=-3*SDB
140 LPRINT "DELTADB=";DELTADB,      "LLIMDB=";LLIMDB
150 DISTSUM=0
160 '
170 FOR J=0 TO M:      X=LLIMDB+J*DELTADB:      X(J)=X
180 DENSIG(J) = FNGAUSS(X,SDB)
190 IF J=(M/2) THEN DENSIR(J)=0 ELSE DENSIR(J) = FNRAIN(X)
200 NEXT J
210 FOR J=M+1 TO 2*M:      X(J)=LLIMDB+J*DELTADB
220 DENSIG(J)=0:      DENSIR(J)=0
230 NEXT J
240 '
250 DENSIC(0)=0
260 FOR J=1 TO 2*M
270 IF J <= (M/2) THEN DENSIC(J) =0: GOTO 350
280 IF J >= ((3*M)/2)+1 THEN DENSIC(J)=0: GOTO 350
290 SUMC=0
300 FOR K=0 TO J-1
310 SUMC = SUMC + DENSIG(K)*DENSIR(J-K) + DENSIG(K+1)*DENSIR(J-1-K)
320 NEXT K
330 DENSIC(J)=(SUMC/2)*DELTADB
340 '
350 DISTSUM = DISTSUM+DENSIC(J-1)+DENSIC(J)
360 DISTI(J)=(DISTSUM/2)*DELTADB
370 NEXT J
380 '
385 STOP
390 LPRINT "K" TAB(20) "dB" TAB(40) "DENSIC(K)" TAB(60) "DISTI(K)" :LPRINT
400 FOR K = 0 TO 2*M STEP 20
410 Y=X(K)+LLIMDB+25*(1-(LOG(THETA)/LOG(10)))
420 LPRINT K TAB(20) Y TAB(40) DENSIC(K) TAB(60) DISTI(K)
430 NEXT K
440 END

```

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