# C.L.' Sheng/and C.M. Lam <br> School of Computer Science <br> University of Windsor Windsor, Ontario N9B 3P4 

for

Department of Communications Ottawa, Ontario
under
Department of Supply and Services
Contract Serial No. OSU81-00097

$$
\text { June 6, } 1981 \text { - June 10, } 1982
$$



## ACKNOWLEDGEMENTS

This research project was directed by the
Government Telecommunications Agency, Division of Development and Engineering under the University Research Program of the Department of Communications.

This research was performed to develop a generalized cost model for local communications network based on economic factors. The model was used to compare three types of services for a wide range of parameter values. The optimal combinations of schemes for the Ottawa Region were determined for the set of parameter values.

## CONTENTS

ACKNOWLEDGEMENTS ..... 1
S UMMARY ..... ii
LIST OF TABLES ..... iv
LIST OF FIGURES ..... v

1. INTRODUCTION ..... 1
2. A GENERALIZED COST MODEL ..... 4
2.1 Cost Components ..... 5
2.2 A Generalized Model ..... 6
3. SELECTION OF CERTAIN PARAMETERS ..... 9
3.1 Amortization Period and Interest Rate ..... 9
3.2 Station-to-Trunk Ratio ..... 10
4. SELECTION OF OPTIMAL SCHEME ..... 14
4.1 Case Study ..... 14
5. SUMMARY AND DISCUSSION ..... 21

## LIST OF TABLES

Table 1 - Cost Components: Initial Costs ..... 23
Table 2 - Cost Components - Recurring Costs ..... 24
Table 3 - Independent Variables for Initial Cost ..... 25
Table 4 - Independent Variables for Recurring Costs ... ..... 26
Table: 5 - Average Cost Per Station Per Month: $K=600$ ..... 27
Table 6 - Average Cost Per Station Per Month: $K=7.00$ ..... 30
Table 7 - Average Cost Per Station Per Month: $K=800$ ..... 33
Table 8 - Average Cost Per Station Per Month: $K=900$ ..... 36
Table 9 - Average Cost Per Station Per Month: $K=1000$ ..... 39
Table 10 - Average Cost Per Station Per Month: $\mathrm{K}=1100$ ..... 42
Table 11 - Optimum Choices for the Case Study ..... 45
Table 12 - Optimal Combinations of Schemes ..... 47

## LIST OF FIGURES

Figure 1 - Station-to-Trunk Ratio
for PBX Schemes $(B=0.05)$
(a) Number of Stations $\leq 3000 \ldots . . . . . . . .48$
Figure 1 - Station-to-Trunk Ratio
for PBX Schemes ( $B=0.05$ )
(b) Number of Stations $\leq 500$............... 49
Figure 2 - Optimal Costs for $K=700, ~ R=17.75 \%$........ 50
Figure 3 - Optimal Costs for $K=700, N=7 \ldots . . . . . . . . .$.
Figure 4 - Optimal Costs for $R=17.75 \%, N=7 \ldots . . . . .$.
Figure 5 - Selection Between EWD PBX and Leased PBX ... 53

Liberalization of terminal attachment regulation provides the opportunity for government users with alternate means to meet their telecommunication requirements. In addition to having a choice of sources for terminal equipment, the government has the opportunity to optimize communication cost by developing a shared local communication network in regions where there is a cluster of government departments. For a given set of user profiles, such as geographical distribution, user requirements, etc., there are a number of ways to intercomect the users to satisfy their communication needs. The users can subscribe to the Centrex Service. Each department can purchase or lease a private branch exchange switching system. The problem is to design the aetwork which has the lowest cost. The development of the methodology for optimal network design will emable the government to determine the most cost-effective network topology which satisfies the communcation need for a given situation. The objective of this research project is to achieve this goal.

The research consists of the following parts: to study telecommunication requirements in the government so as to identify typical user profiles; to develop cost model for telecommunication arrangements in the goverament; to develop a methodology for the design of optimal local
commuication network topology; to develop computer programs which will select. the most cost-effective network topology for a given user profile.

Existing telecommunication arrangements in government users fall into two categories: Centrex Service and PBX Service, Centrex Service will be defined as a government exchange wide service which meets the TBS guldelines for local telephone service. PBX Service will be defined as a peripheral (satellite) $P B X$ behind the centrex. The liberalization of terminal attachments means that the user can own certain equipment, such as a number of local private branch exchanges (PBX), lease a number of trunks, pay the appropriate installation and service charges, and connect the customer-owned equipment to the carrier's network. Therefore, both leased PBX's and customer-owned PBX's are possible. In summary, there are basically three schemes that a user can select: Centrex, Leased PBX, Purchased PBX. However, it is possible that new schemes may develop in the future.

In anticipation of changing needs in the future, we develop a generalized cost model which is adaptive to changes. The model is discussed in the next section. In the $P B X$ schemes, the user has a choice of station-to-trunk ratio. This ratio could have an important impact on the overall cost of the schemes as well as quality of service. We discuss how to select the proper ratio and the
significance of amortization period and interest rate in Section 3. In Section 4, we perform a case study to show how the model can be used and how to determine the optimal cost and scheme for a given department size. By using the user profile available to us, we show the minimum cost selections for a wide variety of parameter values as well. Finally, we summarize the achievements of this project in the last section.

## A GENERALIZED COST MODEL

Costs associated with each scheme can be separated into two categories according to the frequency of payment. In general, some inftial costs are required to acquire the necessary equipment and accessories, to install the equipment in the proper place, to train personnel for the job, etc. Once the system is in use, recurring costs, usually monthly, have to be paid to maintain the equipment, to gain access to the public telephone network, to employ personnel on the job, etc. Specific types of cost components incurred vary according to the scheme. They can be changed from time to time to reflect new tariff structures and charging methods. Tables 1 and 2 show the inftial and recurring cost components, not necessarily exhaustive, of each scheme, respectively. Before further discussion, some definitions of terms for the purpose of this project are in order. A main station is defined as the first telephone set provided with a telephone line, and an extension station is any subsequent set on the same line. A station is either a main or an extension station. A trunk is a telephone line from a central office to customer premises switch. A Ifne is a telephone line from a main telephone set to the nearest switching equipment.

Let $L, T, S, M$ and $E$ be the number of lines, trunks, stations, main stations and extension stations, respectively. For a centrex system, the number of main
stations usually is equal to the number of extension stations and the number of lines. Therefore,

$$
\begin{equation*}
M=E=L=\frac{S}{2} \tag{1}
\end{equation*}
$$

In the PBX schemes, usually the number of stations is equal to the number of lines, that is,

$$
\begin{equation*}
S=L \tag{2}
\end{equation*}
$$

Since there is a fixed relationship between $S$ and $L$ in either the centrex or the PBX system, one of the two can be eliminated in the discussion of the characteristics of cost component functions. We choose to eliminate $L$ in the following discussions.
2.1 Cost Components

Generally speaking, the cost components are functions of $T$ and $S$. More specifically the functions fall into one or more of the following categories:
i) constant
ii) Iinear
iii) step
iv) piecewise linear

Since constant, linear and step functions are special cases of piecewise linear functions, it can be said that the cost components are linear combinations of piecewise linear functions of $T$ and $S$. Tables 3 and 4 show the possible variables for each cost component.

In addition to the known cost components, new components
may be added from time to time. Also, additional independent
variables may be fntroduced. An example is the distance from the customer's premises to the central office. In summary, the cost components are not fixed all the time. The form of individual component functions may be different, and new independent variables may be introduced in the future. To avoid developing a cost model which will be out-dated quickly, a generalized model which is adaptive to changes is introduced.
2.2 A Generalized Model

Since the cost of each service scheme consists of two categories of cost components, each of which is the sum of a linear combination of piecewise linear functions of some independent variables, it can be represented by the following formula,

$$
\begin{equation*}
c=a f+g \tag{3}
\end{equation*}
$$

where
f is the total of all initial cost components, $g$ is the total of all recuring cost components, and
a $1 s$ a conversion factor from initial cost to equivalent recurring cost.

In general, $f$ can be written as

$$
\begin{equation*}
f=\sum_{i=1}^{I} d_{i} f_{i} \tag{4}
\end{equation*}
$$

$d_{i}=\left\{\begin{array}{l}1 \text { if } f_{i} \text { is applicable to the scheme } \\ 0 \text { otherwise }\end{array}\right.$
I is the total number of different cost components over all schemes,
and

$$
\begin{align*}
& f_{1}=C_{1}+\sum_{j=1}^{J} e_{i j} f_{i j}  \tag{5}\\
& e_{i j}=\left\{\begin{array}{l}
1 \text { if } f_{i} \text { is a function of } X_{j} \\
0 \text { otherwise }
\end{array}\right.
\end{align*}
$$

where
$C_{i}$ is the constant term in the fth cost component $J$ is the coral number of independent variables $X_{j}$,
and

$$
\begin{equation*}
f_{i j}=\sum_{k=1}^{\mathbb{K}_{i j}} f_{i j k}\left(X_{i j k 1}, X_{i j k 2}, Y_{i j k 1}, Y_{i j k 2}\right) \tag{6}
\end{equation*}
$$

where

$$
\begin{aligned}
& X_{i j} \text { is the number of partitions of independent variable } \\
& X_{j} \text { in the fth cost component, } \\
& X_{i j k l} \text { and } X_{i j k 2} \text { are the boundaries of region } k, \\
& Y_{1 j k l} \text { and } Y_{1 j k 2} \text { are the costs at the boundary points } \\
& \text { of region } k .
\end{aligned}
$$

In general, many of the $d_{r} ' s$ and $e_{i j}{ }^{\prime} s$ are 0 . At this time, we can assume that there are only two independent variables, namely $S$ and $T$. Obviously, there is no difficulty in extending it to include other variables in the future. A similar formula can be used for $g$.

For chose schemes in which certain equipment is bought at the start, heavy initial costs are incurred. At the end of an amortization period, the equipment may have
a salvage value. In general, the salvage value $h_{i}$ of cost component $f_{1}$ can be expressed as

$$
\begin{equation*}
h_{i}=m_{i} f_{i} \tag{7}
\end{equation*}
$$

where $w_{i}$ is the salvage value factor for initial cost componenti.

In summary, the equivalent per station cost of
a scheme is
$C^{\prime}=(C-b h) / S$
where
$h=\sum_{i=1}^{I} h_{1}$
and
b is a conversion factor from terminal cost to equivalent recurring cost.
3.

## SELECTION OF CERTAIN PARAMETERS

In this section, we shall discuss the choice of the three parameters: amortization period, interest rate, and station to trunk ratio.
3.1 Amortization Period and Interest Rate

In Eq. (3), the initial cost is converted to an equivalent recurring cost. This is essentially to amortize a capital cost over a certain period of time. It is observed that the cost distribution characteristics of the service schemes are different. In particular, the Purchased PBX Scheme has a very high initial cost and a small recurring cost. In contrast, the Centrex Scheme has a very small initial cost and a large recurring cost. The conversion factor, as a function of amortization period and interest, plays a very important role in the selection of the leastcost scheme. Generally speaking, long amortization period favours capital investment because the cost is spread over a longer period of time. Similarly low interest rate encourages capital investment because the installment is smaller.

Interest rate probably is dictated by management policy. Amortization period depends on a number of factors. Some of them are listed below:
i) equipment life expectancy
ii) technology
iii) office lease
iv) government policy

It is obvious that the amortization period cannot be longer than the life expectancy of the equipment. Consideration should also be given to aging of equipment. If the equipment needs frequent repairs involving high repair costs after a certain period, it may be preferable to have an early replacement before the end of the life expectancy. Technology changes may make existing equipment obsolete because maybe a new piece of equipment is much cheaper, has much more capability, and is more convenient to use. This is quite typical in electronic equipment. If the government department space is on lease basis, it may not be a good idea to make the amortization period past the lease expiration date. Government policy certainly is another factor that cannot be overlooked. If there is an indication that in the near future, the government is going to move a department to a different location or to restructure the department, then amortization period should not be too long.
3.2 Station-to-Trunk Ratio

The ratio of number of stations to number of trunks is an important factor which affects quality of service as well as cost. A low station-to-trunk ratio means that a
user will have a small probability of getting a busy tone because there are more trunks to be shared. It also means that the cost per station is higher than a larger station-to-trunk ratio because fewer stations share the cost of a trunk. For all practical purposes, we can assume that the ratio is 2 for the Centrex Scheme. For the PBX Schemes, a common practice is to select a ratio such that the probability of all trunks are busy is 0.05 . Exact probability of all trunks being busy is difficult to obtain because of uncertainties in call occurrence and call duration. However, if they can be assumed to follow certain distributions, then the problem reduces to a conventional queueing problem which may have an analytic solution. A model for telephone stations competing shared trunk lines follows.

Let $T$ be the number of trunk lines shared by $S$ stations. Here $S$ > otherwise there is no need to compete for a trunk line. If both the arrival time between calls and call duration time are exponentially distributed, and the fraction of time a station uses the telephone is p, then the probability that all $T$ trunks are used at a time is

$$
\begin{equation*}
B=\frac{\frac{(S p)^{T}}{T!} \frac{1}{1-S p / T}}{\sum_{k=0}^{T-1} \frac{(S p)^{k}}{k!}+\frac{(S p)^{T}}{T!} \frac{1}{1-\frac{S p}{T}}} \tag{9}
\end{equation*}
$$

Eq. (9) is based on the assumptions that
(i) The population is infinite
(ii) When all trunks are busy, the incoming call is put on hold, and wait for the first trunk available to complete the connection.

Strictly speaking, trunk sharing in telecommuncation does not satisfy these assumptions. However, they are reasonable assumptions. When the number of stations is large, say a hundred or more, the user population is large, and can be considered to be infinite for the purpose of the trunks. The justification for the second point is that when a person tries to make a call and cannot get through, in all likelihood he/she will try at least one more time. If there is a trunk available the call can be established. Therefore, in most instances it behaves like the person is waiting for the first available trunk. After comparing results obtained from other formulas, we find that this formula is the best approximation of the situation.

It can be shown that, for a given station-totrunk ratio, $B$ is reduced as $S$ increases. For example, 100 stations sharing 50 trunks has a smaller probability of all trunks being busy than 20 stations sharing 10 trunks. An intuitive explanation is that the chance of at least 50 out of 100 stations that want the use of a trunk simultaneously is smaller than at least 10 out of 20 stations that want to use the telephone.

The following steps can be used to determine $S / T$ for the $P B X$ schemes such that $B$ is 0.05 : For each $p$, and a given $S$, we estimate the equivalent $T$-value by
iteration for $T$ starting from 1 . Initially $B$ is greater than 0.05. As $T$ is increased, the $B$-value will be reduced. The smallest $T$-value which yeilds a B-value $\leq$ 0.05 is selected, hence the station-to-trunk ratio obtained. Figure 1 shows the $S / T$ values for $S \leq 3000$ for $0.08 \leq p$ $\leq 0.12$. It shows the following points of observation:
(i) As the number of stations increases the station-to-trunk ratio can be increased for the same probability that all trunks are busy.

As the utilization of individual station decreases (smaller p), for a given number of stations, a higher station-to-trunk ratio is possible to get the same quality of service.
(iii) The station-to-trunk ratio is asymptotically approaching $1 / p$ as the number of stations increases.

The first point of observation is expected, as discussed before. The explanation of the second point is that when the utilization is small, there are more idle times, so the chance of contention of use is small, therefore the number of trunks can be reduced. It can be seen that the overall system utilization is $S p / T$, and it cannot be larger than unity, that is,

$$
\frac{S p}{T}<1
$$

or

$$
S / T<1 / p
$$

This explains the third point.

## SELECTION OF OPTIMAL SCHEME

For a department with a given number of stations, the optimal scheme, that is, the one that costs the least per station, can be determined by computing the average monthly cost per station under each scheme and then choosing the one that costs least. A case study is discussed below.

### 4.1 Case Study

Consider an example of three schemes: Exchange Wide Dial PBX (EWD), Leased PBX and Purchased PBX. There are two independent variables, $S$ and $T$, seven different initial cost components and nine monthly cost components. Let $X_{1}=S$ and $X_{2}=T$. The seven initial cost components are:

Administrative Charge, $f_{1}=C_{1}=0 A C$
Premise Visit Charge, $f_{2}=C_{2}=P V C$
EWD Service Charge, $f_{3}=f_{31}=E S C * X_{1}$
Leased PBX Service Charge,

$$
\begin{align*}
& \mathrm{f}_{4}=\mathrm{f}_{41}=\mathrm{f}_{411}(80,150,5790.4,15949.5)+  \tag{4}\\
& \mathrm{f}_{412}(150,300,16650,39900)+\mathrm{f}_{413}(300,500, \\
& 39900,63250)+\mathrm{f}_{414}(500,1200,392890,940464) \\
& +\mathrm{f}_{415}(1200,3000,940464,2349706.2) \tag{5}
\end{align*}
$$

Trunk Service Charge,

$$
\begin{equation*}
\mathrm{f}_{5}=\mathrm{f}_{52}=\mathrm{ST} * \mathrm{X}_{2} \tag{6}
\end{equation*}
$$

PBX Equipment Purchase and Installation,

$$
f_{6}=f_{61}=K * X_{1}
$$

(7) Jack Service Charge,
$\mathrm{f}_{7}=\mathrm{f}_{72}=\mathrm{SJ} * \mathrm{X}_{2}$
There is no salvage value for all cost
components except item 6. Therefore,
for $i \neq 6, m_{i}=0$, and $m_{6}=m$.
The nine monthly cost components are
(1)

EWD monthly charge, $g_{1}=g_{11}=1.75 * E M R * X_{1}$
EWD Local Service Charge,
$g_{2}=g_{21}=L S S / 2 * X_{1}$
Leased PBX Monthly Charge,
$g_{3}=g_{31}=g_{311}(80,150,1327.2,2289)+$
$g_{312}(150,300,2296.5,3828)+g_{313}(300,500$,
$3828,6130)+\mathrm{g}_{314}(500,1200,2585,5292)+$
$g_{315}(1200,3000,5292,12295.38)$
(4)

Trunk Charge, $g_{4}=g_{42}=M T * X_{2}$
(5) Trunk Equivalency Charge, $g_{5}=g_{52}=M T E * X_{2}$
(6) GTA Trunk Charge, $g_{6}=g_{62}=M T G * X_{2}$
(7)
(8)

Floor Space Charge, $g_{7}=g_{71}=M F S * X_{1}$
PBX Maintenance Charge, $g_{8}=g_{81}=M M * X_{1}$
Jacks Charge, $g_{9}=g_{92}=M J * X_{2}$
The initial and monthly cost components for
the three schemes are as follows:

## EWD PBX

$$
\begin{aligned}
& f_{E W D}=f_{1}+f_{2}+f_{3} \\
& g_{E W D}=g_{1}+g_{2}
\end{aligned}
$$

## Leased PBX

$$
\begin{aligned}
& f_{L X}=f_{1}+f_{2}+f_{4}+f_{5} \\
& g_{L X}=g_{3}+g_{4}+g_{5}+g_{6}+g_{7}
\end{aligned}
$$

## Purchased PBX

$$
\begin{aligned}
& f_{P X}=f_{5}+f_{6}+f_{7} \\
& g_{P X}=g_{4}+g_{5}+g_{6}+g_{7}+g_{8}+g_{9}
\end{aligned}
$$

The equivalent per station monthly cost for each scheme is:

END PBX: $\quad C_{E W D}^{\prime}=\left(a f_{E W D}+g_{E W D}\right) / X_{1}$
Leased PBX: $\quad C_{L X}^{\prime} \dot{=}\left(a f_{L X}+g_{L X}\right) / X_{1}$
Purchased PBX: $\quad C_{P X}^{\prime}=\left(a f_{P X}+g_{P X}-b m f_{6}\right) / X_{1}$
where
$a=\frac{(R / 12)(1+R / 12)^{12 N}}{(1+R / 12)^{12 N}-1}$
$b=\frac{R / 12}{(1+R / 12)^{12 N}-1}$
$N$ is the amortization period in years,
$R$ is the interest rate.
For this study, the parameter values can be
taken as follows:
$O A C=23$
$P V C=9$
$E S C=86.42$
$\mathrm{ST}=48$
$S J=40.5$
$E M R=13.78$
LS $=1.75$
$\mathrm{MT}=48.9$
$\mathrm{MTE}=1.5$
MTG $=17.5$
$\mathrm{MFS}=0.5$
$M M=3$
$M J=0.85$
The remaining parameters $K, N$ and $R$ vary depending on the situation and management decision. For this study, we assume their values are

$$
600 \leq K \leq 1100
$$

$5 \leq N \leq 10$
$0.15 \leq R \leq 0.2$
The following combinations of $K, N$ and $R$ are
included in the study:
$K=600,700,800,900,1000$ and 1100
$\mathrm{N}=5,7$ and 10
$R=0.15,0.1775$ and 0.2
Taking into consideration of removal cost, rapid changes in technology, and the fairly large value of $N$, the salvage value of the purchased $P B X$ is assumed to be 0 , i.e., $m=0 ; p$ is set to 0.1 . The study covers the number of stations up to 3000 .

Tables 5 to 10 show the average cost per station per month of each scheme for selected number of stations between 20 and 3000. Table ll summarizes the results. The following inference can be made:
(1) In general, for small number of stations (S $\leq 200$ ), except for low cost $P B X$ and long amortization period, EWD $P B X$ is the cheapest. This is expected because for both Leased PBX and Purchased PBX, there are high initial cost components.
(ii)
(iii)

For large number of stations ( $S \geq 600$ ), Purchased PBX is the cheapest if its cost per station is low ( $K \leq 700$ ) and the amortization period is at least seven years. If the period is five years or less, Purchased $P B X$ is not the best choice except if the price of the $P B X$ is very low ( $K \leq 600$ ) and the number of stations is very high (S > 2000).

For medium number of stations (200<S 400),
Leased PBX appears to be the best option,
except when the price of the $P B X$ is very cheap and the amortization period is long.

To find the sensitivity of the optimal cost
to the three important parameters $K, R$ and $N$, the optimal cost curves are plotted in Figures 2 to 4. In each figure, one of the parameters is varied while the others are held constant. The following observations can be made:
(i)

Sensitivity to $N$ - Purchased $P B X$ is the most sensitive to $N$. Large $N$ makes the scheme more favourable. Note that when $N=10$, Purchased $P B X$ is the cheapest even for $S=200$. Leased $P B X$ is the best choice for $200<S \leq 400$, regardless of amortization period. For $S \leq 100$, EWD PBX is the choice.

Sensitivity to R - Both EWD PBX and Leased PBX are not sensitive to R. Again, Purchased $P B X$ is the most sensitive to $R$. However, in this particular case, the choice is almost independent of $R$ for $\mathrm{S} \leq 400$, namely, for $\mathrm{S} \leq 100$, EWD PBX is the cheapest, for $200<\mathrm{S} \leq 400$, Leased PBX is the choice. For large number of stations ( $S \geq 600$ ), at relatively low interest rates, Purchased $P B X$ is the choice; otherwise EWD PBX is the cheapest. Sensitivity to $K$ - Both EWD PBX and Leased PBX are not functions of $K$, therefore the variation in the choice is a result of wide variations in the cost of Purchased PBX as a function of $K$. For very low cost $(K=600)$, Purchased PBX is the cheapest, even when $S$ is small. On the other hand, when $K \geq 800$, it is not a choice regardless of the number of stations.

Since both EWD PBX and Leased PBX are not functions of $K$, the two schemes can be compared for a range of $N$ and $R$ values. The results are plotted in Figure 5. It is observed that

For $S$ 200, EWD PBX is cheaper.
For $200<S \leq 400$, Leased PBX is cheaper.
(iii)

For $600 \leq S, E W D$ PBX is cheaper except when $N$ is large and $R$ is small.

It is interesting to find out how significant
it is to choose the optimal scheme for a given set of user profiles. As an example, the Ottawa region is used. In the region, there are close to 60,000 stations in over 500 buildings. The number of stations per building varies from 1 to close to 3000. The optimal cost is compared with the all-EWD PBX selection. The results for the 54 parameter combinations are shown in Table 12. It can be seen that the total cost is about 1.5 million dollars per month, and the savings vary from about $1.4 \%$ to $14.7 \%$, or from about $\$ 22,600$ to $\$ 229,000$ per month.

Telecommunication is a highly volatile field. New products are designed and developed from time to time. New types of arrangements may be made with vendors and public carriers to secure telecommunication needs as a result of competition, new technological development and law changes. Therefore, new cost schemes may be evolved. Moreover, for a given scheme, the cost components and cost functions may be changed too because a different charging method may be adopted by the vendors or carriers. Hence, it is difficult to develop a detailed and specific model which is valid for a long time. In view of this, we have developed a generalized cost model, which assumes that
(i) costs under each scheme can be separated into two categories, namely initial and recurring costs; each cost component can be represented as a Iinear combination of piecewise linear functions of independent variables.

The generalized approach has the advantage that the model can be used as long as the assumptions are valid.

In the $P B X$ schemes, the customers have to
select the number of trunks to satisfy their need. Therefore, there is a choice of station-to-trunk ratio. The tradeoff is between cost and convenience. High
station-to-trunk ratio means a high probability of getting a busy signal, but a lower cost. The opposite is true for low station-to-trunk ratio. We have developed a method to determine the proper station-totrunk ratio which gives a specified probability that all trunks are busy.

After the basic model and formulas have been developed, we have shown how they can be used in a case study. The example determines the optimal cost and scheme for a range of number of stations. The following observations $c a n$ be made from the study: Average monthly cost of the Purchased PBX scheme is very sensitive to the initial cost of the equipment. As a result, the choice of optimal scheme depends very much on the initial cost. In general, for small number of stations (S $\leq 200$ ), EWD PBX, a Centrex Service, is the cheapest; for medium number of stations, ( $200<\mathrm{S} \leq 400$ ), Leased PBX is the choice;
for large number of stations, ( $\mathrm{S} \geq 2000$ ), Purchased PBX is the best.

Finally, it is found that by choosing the appropriate scheme for each building in the Ottawa region, a substantial saving can be achieved.

Table 1 - Cost Components: Initial Costs

| Component | Centrex | Leased PBX | Purchased PBX |
| :---: | :---: | :---: | :---: |
| 1. G.o.C. Premises Equipment |  |  |  |
| 1.1 common equipment | N/A | N/A | x |
| 1.2 line equipment | " | " | x |
| 1.3 trunk equipment | " | " | x |
| 1.4 basic station sets | " | " | x |
| 1.5 feature sets | " | " | x |
| 1.6 consoles | " | " | x |
| 1.7 installation materials | " | " | x |
| 1.8 users manuals | " | " | x |
| 1.9 engineering | " | " | x |
| 1.10 installation | " | " | x |
| 1.11 service charges | x | x | N/A |
| 1.12 initial training | x | x | x |
| 1.13 in-house wiring | N/A | x | x |
| 2. Local Tandem Switch | N/A | x | x |
| 3. PBX-CO Truck Charges | N/A | x | x |
| 4. Existing Equipment |  |  |  |
| 4.1 removal | x | x | x |
| 4.2 salvage value (G.O.C. owned equipment on 1y) | x | x | x |

Table 2 - Cost Components - Recurring Costs

| $\underbrace{\text { Component }}$ | Centrex | Leased PBX | Purchased PBX |
| :---: | :---: | :---: | :---: |
| \. G.O.C. Premises Equipment |  |  |  |
| 1.l maintenance contract | N/A | N/A | x |
| - 1,2 training | x | x | x |
| 1.3 equipment rearrangements | x | x | x |
| 1.4 space rental | N/A | x | X |
| 1.5 power | N/A | x | x |
| 1.6 operator loaded salaries | x | x | x |
| ```1.7 attendant/receptionist loaded salaries``` | x | x | x |
| 1.8 Gov"t supplied maintenance personnel-loaded salaries | N/A | x | x |
| 1.2 directory ciarges | x | $x$ | x |
| 1.1Q USOC charges | x | x | N/A |
| ?. Local Tandem Switch Costs | N/A | x | x |
| 3. PBX-CO Trunk Charges | N/A | x | $\mathbf{x}$ |
| 4. Other Costs |  |  |  |
| 4.1 local shared services charges | x | x | x |
| 4.2 IX Network Charges | x | $x$ | $x$ |
| 4.3 debt servicing charges | x | $\pm$ | x |

N/A - not applicable

Table 3 - Independent Variables for Initial Cost


Legend:

```
- not applicable
x function of T or S as indicated
```

Table 4 - Independent Variables for Recurring Costs


Legend:

```
- not applicable
x function of }T\mathrm{ or }S\mathrm{ as indicated
```

rable 5 - Averaje Cost Per Station Per Month: $K=600$

$a)$
7
6
9
11
14
17
29
52
75
97
119
173
226
279
331
is)
T
6
9
11
14
17
29
52
73
67
119
173
236
279
331

$N=$| $5, \quad P=0$ |
| ---: |
| $E W D$ |
| 27.21 |
| 27.19 |
| 27.19 |
| 27.18 |
| 27.18 |
| 27.18 |
| 27.17 |
| 27.17 |
| 27.17 |
| 27.17 |
| 27.17 |
| 27.17 |
| 27.17 |
| 27.17 |

c)
$\mathrm{N}=$
EWD
27.32
27.30
27.29
27.29
27.29
27.28
27.28
27.28
27.28
27.28
27.28
27.28
27.28
27.28

Leased

31.12
30.59
27.80
25.36
34.59
34.06
33.74
33.28
33.01
32.85
32.72
Purchased
40.73 35.39 32.43
31.34 31.34
31.48 29.71 28.64 28.28 28.02
27.86 27.86
27.60 27.43
27.33 27.33
27.24
d)

| 5 | $T$ |
| ---: | ---: |
| 20 | 0 |
| 40 | 6 |
| 60 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2070 | 226 |
| 2500 | 279 |
| 3000 | 331 |

e)

| 5 | $T$ |
| ---: | ---: |
| 20 | 6 |
| 10 | 9 |
| 60 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

$N=7, P=0.1$,
Leased

30.54
29.90
26.87
24.38
28.98
28.45
28.13
27.67
27.40
27.25
27.12
) $\mathrm{N}=7, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.15$
) $N=7, P=0.1, m=0.0, \quad R=0.2$

| 5 | 1 |
| ---: | ---: |
| 20 | 6 |
| 40 | 9 |
| 60 | 11 |
| 80 | 14 |
| 190 | 17 |
| 200 | 29 |
| 400 | 52 |
| 000 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

EMD
26.94
26.93
26.92
26.92
26.92
26.91
26.91
26.91
26.91
26.91
26.91
26.91
26.91
26.91
Leased

30.78
30.18
27.25
24.78
31.25
30.72
30.40
29.94
29.67
29.51
29.38
Purchased
$38 \cdot 04$
32.73
29.73
29.79
29.20
28.85
28.85
27.08
26.02
25.66
25.40
25.24
25.24
25.94
24.98
24.81
$24 \cdot 72$
$24 \cdot 6.3$
6)
$N=10, \quad P=0.1, \quad n=0.0$
$\mathrm{R}=0.15$


|  | $\mathrm{h})$ | $\mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{~K}=0.1775$ |  |  |
| ---: | :---: | :---: | :---: | :---: |
| S | r | EWD | Leased | Purchased |
| 20 | 0 | 26.56 |  | 35.31 |
| 40 | 9 | 26.55 |  | 30.04 |
| 50 | 11 | 26.54 | 27.11 |  |
| 30 | 14 | 26.54 | 30.42 | 26.52 |
| 100 | 17 | 26.54 | 29.77 | 26.17 |
| 200 | 29 | 26.54 | 26.69 | 24.41 |
| 400 | 52 | 26.53 | 24.19 | 23.36 |
| 600 | 75 | 26.53 | 27.86 | 23.01 |
| 800 | 97 | 26.53 | 27.33 | 22.74 |
| 1000 | 193 | 26.53 | 27.01 | 22.58 |
| 1500 | 173 | 26.53 | 26.55 | 22.33 |
| 2000 | 226 | 26.53 | 26.28 | 22.16 |
| 2500 | 273 | 26.53 | 26.13 | 22.06 |
| 3000 | 331 | 26.53 | 26.00 | 21.97 |


|  | i) | $\mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$ |  |  |
| ---: | ---: | :---: | :---: | :---: |
| S | r | EWD | Leased | Purchased |
| 20 | 6 | 26.69 |  | 36.23 |
| 40 | 9 | 26.68 |  | 30.95 |
| 60 | 11 | 26.67 |  | 28.01 |
| 90 | 14 | 26.67 | 30.54 | 27.43 |
| 100 | 17 | 26.67 | 29.91 | 27.07 |
| 200 | 29 | 26.66 | 26.88 | 25.31 |
| 400 | 52 | 26.66 | 24.39 | 24.26 |
| 600 | 75 | 26.66 | 29.00 | 23.90 |
| 300 | 97 | 26.66 | 28.47 | 23.64 |
| 1000 | 119 | 26.66 | 28.16 | 23.48 |
| 1500 | 173 | 26.66 | 27.69 | 23.22 |
| 2000 | 226 | 26.66 | 27.43 | 23.06 |
| 2500 | 279 | 26.66 | 27.27 | 22.96 |
| 3000 | 331 | 26.66 | 27.14 | 22.87 |

Table 6 - Average Cost Per Station Per Month: $K=700$
a) $N=5, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{~K}=0.15$

| $S$ | $T$ |
| ---: | ---: |
| 20 | 6 |
| 40 | 9 |
| 60 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 53 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |


|  | $\mathrm{B})$ | $\mathrm{N}=\mathrm{S}, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.1775$ |  |  |
| ---: | ---: | :---: | :---: | :---: |
| S | T | EWD | Leased | Purchased |
| 20 | 6 | 27.21 |  | 42.48 |
| 40 | 9 | 27.19 |  | 37.15 |
| 60 | 11 | 27.19 |  | 34.19 |
| 80 | 14 | 27.18 | 31.02 | 33.60 |
| 100 | 17 | 27.18 | 30.47 | 33.25 |
| 200 | 29 | 27.18 | 27.64 | 31.47 |
| 400 | 52 | 27.17 | 25.19 | 30.41 |
| 600 | 75 | 27.17 | 33.63 | 30.05 |
| 800 | 97 | 277.17 | 33.10 | 29.79 |
| 1000 | 119 | 27.17 | 32.78 | 29.63 |
| 1500 | 173 | 277.17 | 32.31 | 29.37 |
| 2000 | 226 | 277.17 | 32.05 | 29.20 |
| 2500 | 279 | 27.17 | 31.89 | 29.10 |
| 3000 | 331 | 27.17 | 31.76 | 29.01 |


| ${ }^{5}$ | T | EWD | Leased | Purchased |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 6 | 27.32 |  | 43.37 |
| 40 | 9 | 27-30 |  | 38.04 |
| 60 | 11 | 27.29 |  | 35.08 |
| 80 | 14 | 27.29 | 31.12 | 34. 49 |
| 100 | 17 | 27.29 | 30.59 | 34.13 |
| 200 | 29 | 27.28 | 27.80 | 32.35 |
| 400 | 52 | 27.28 | 25.36 | 31.29 |
| 600 | 75 | 27.28 | 34.59 | 30.93 |
| 800 | 97 | 27.28 | 34.06 | 30.67 |
| 1090 | 113 | 27.28 | 33.74 | 30.51 |
| 1500 | 173 | 27.28 | 33.28 | 30.25 |
| 2000 | 226 | 27.28 | 33.01 | 30.08 |
| 2500 | 279 | 27.28 | 32.85 | 29.98 |
| 3000 | 331 | 27.28 | 32.72 | 29.89 |

d) $\quad v=7, p=0.1, m=0.0, \quad k=0.15$

| S | T | EWD | Leased | Furchased |
| :---: | :---: | :---: | :---: | :---: |
| 30 | 6 | 26.65 |  | 38.15 |
| 40 | 9 | 26.57 |  | 32.86 |
| ¢0 | 11 | 26.67 |  | 25.93 |
| 30 | 14 | 26.67 | 30.54 | 29.34 |
| 100 | 17 | 26.66 | 29.90 | 28.99 |
| 200 | 23 | 26.66 | 26.87 | 27.22 |
| 400 | 52 | 26.66 | 24.38 | 26.17 |
| 600 | 75 | 26.66 | 28.98 | 25.82 |
| 300 | 97 | 26.60 | 28.45 | 25.55 |
| 1000 | 119 | 26.66 | 28.13 | 25.39 |
| 1500 | 173 | 26.66 | $27 \cdot 67$ | 25.13 |
| 2000 | 220 | 26.66 | 27.40 | 24.97 |
| 2500 | 279 | 26.66 | 27.25 | 24.87 |
| 3000 | 331 | 26.66 | 27.12 | 24.78 |

e) $\mathrm{N}=7, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.1775$

| $S$ | $T$ | EWD | Leased | Purchased |
| ---: | :---: | :---: | :---: | :---: |
| 20 | 6 | 26.83 |  | 39.29 |
| 10 | 9 | 26.81 |  | 33.99 |
| 60 | 11 | 26.80 |  | 31.05 |
| 80 | 14 | 26.80 | 30.67 | 30.46 |
| 100 | 17 | 26.80 | 30.05 | 30.11 |
| 200 | 29 | 26.80 | 27.08 | 28.35 |
| 400 | 52 | 25.80 | 24.60 | 27.29 |
| 600 | 75 | 26.79 | 30.21 | 26.93 |
| 800 | 97 | 26.79 | 29.63 | 26.67 |
| 1000 | 119 | 26.79 | 29.36 | 26.51 |
| 1500 | 173 | 26.79 | 28.90 | 26.25 |
| 2000 | 226 | 26.79 | 28.63 | 26.09 |
| 2500 | 279 | 26.79 | 28.47 | 25.99 |
| 3000 | 331 | 26.79 | 28.34 | 25.90 |


|  | $f)$ | $N=$ | $7, P=0.1$, | $m=0.0$, | $R=0.2$ |
| ---: | ---: | ---: | ---: | ---: | ---: |
| $S$ | $T$ | $E W D$ | Leased | Purchased |  |
| 20 | 5 | 26.94 |  | 40.26 |  |
| 40 | 9 | 26.93 |  | 34.96 |  |
| 60 | 11 | 26.92 |  | 32.01 |  |
| 80 | 14 | 26.92 | 30.78 | 31.42 |  |
| 100 | 17 | 26.92 | 30.18 | 31.07 |  |
| 200 | 29 | 26.91 | 27.25 | 29.30 |  |
| 400 | 52 | 26.91 | 24.78 | 28.24 |  |
| 600 | 75 | 26.91 | 31.25 | 27.88 |  |
| 800 | 97 | 26.91 | 30.72 | 27.62 |  |
| 1090 | 119 | 26.91 | 30.40 | 27.46 |  |
| 1500 | 173 | 26.91 | 29.94 | 27.20 |  |
| 2000 | 226 | 26.91 | 29.67 | 27.04 |  |
| 2500 | 279 | 26.91 | 29.51 | 20.94 |  |
| 3000 | 331 | 26.91 | 29.38 | 26.85 |  |


i) $\mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$

| S | T | EWD | Leased | Purchased |
| ---: | ---: | ---: | ---: | ---: |
| 20 | 6 | 26.69 |  | 38.17 |
| 40 | 9 | 26.68 |  | 32.88 |
| 00 | 11 | 26.67 |  | 29.95 |
| 80 | 14 | 26.67 | 30.54 | 29.36 |
| 100 | 17 | 26.67 | 29.91 | 29.01 |
| 200 | 29 | 26.66 | 26.88 | 27.24 |
| 400 | 52 | 26.66 | 24.39 | 26.19 |
| 600 | 75 | 26.66 | 29.00 | 25.84 |
| 800 | 97 | 26.66 | 28.47 | 25.57 |
| 1000 | 119 | 26.66 | 28.16 | 25.41 |
| 1500 | 173 | 26.66 | 27.69 | 25.15 |
| 2000 | 226 | 26.66 | 27.43 | 24.99 |
| 2500 | 279 | 26.66 | 27.27 | 24.89 |

Table 7 - Average Cost Per Station Per sonth: $\mathrm{A}=\mathrm{SCO}$


|  | b) | $\mathrm{N}=5, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{~K}=0.1775$ |  |  |
| ---: | ---: | ---: | ---: | ---: |
| S | T | EWD | Leased | Purchased |
| 20 | 6 | 27.21 |  | 45.00 |
| 40 | 9 | 27.19 |  | 39.68 |
| 60 | 11 | 27.19 |  | 36.72 |
| 80 | 14 | 27.18 | 31.02 | 36.13 |
| 100 | 17 | 27.18 | 30.47 | 35.77 |
| 200 | 29 | 27.18 | 27.64 | 34.00 |
| 400 | 52 | 27.17 | 25.19 | 32.93 |
| 600 | 75 | 27.17 | 33.63 | 32.58 |
| 800 | 97 | 27.17 | 33.10 | 32.31 |
| 1000 | 119 | 27.17 | 32.78 | 32.15 |
| 1500 | 173 | 27.17 | 32.31 | 31.89 |
| 2000 | 226 | 27.17 | 32.05 | 31.73 |
| 2500 | 279 | 27.17 | 31.89 | 31.63 |
| 3000 | 331 | 27.17 | 31.76 | 31.54 |


|  | c) | $\mathrm{N}=5, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$ |  |  |
| ---: | ---: | ---: | ---: | ---: |
| S | T | EWD | Leased | Purchased |
| 20 | 6 | 27.32 |  | 46.02 |
| 40 | 9 | 27.30 |  | 40.69 |
| 60 | 11 | 27.29 | 31.12 | 37.73 |
| 80 | 14 | 27.29 | 37.14 |  |
| 100 | 17 | 27.29 | 30.59 | 36.78 |
| 200 | 29 | 27.28 | 27.80 | 35.00 |
| 400 | 52 | 27.28 | 25.36 | 33.94 |
| 600 | 75 | 27.28 | 34.59 | 33.53 |
| 800 | 97 | 27.28 | 34.06 | 33.32 |
| 1000 | 119 | 27.28 | 33.74 | 33.16 |
| 1500 | 173 | 27.28 | 33.28 | 32.90 |
| 2000 | 226 | 27.28 | 33.01 | 32.73 |
| 2500 | 279 | 27.28 | 32.85 | 32.63 |
| 3000 | 331 | 27.28 | 32.72 | 32.54 |


~) $N=10, \quad \mathrm{i}=0.1, \quad \mathrm{~m}=0.0, \quad \mathrm{R}=0.15$

n)

| $S$ | $T$ |
| ---: | ---: |
| 20 | 0 |
| 10 | 9 |
| 60 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

i)

| $S$ | $T$ |
| ---: | ---: |
| 20 | 6 |
| 10 | 9 |
| 00 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

E日D
26.41
20.40
26.39
26.39
26.39
26.39
26.39
26.39
26.38
26.38
26.38
26.38
26.38
26.38
Leased

30.28
29.60
26.46
23.36
26.52
25.99
25.671
24.94
24.78
24.65
Purchased
37.46
32.20
29.27
28.69
28.34
26.58
25.53
25.18
24.92
24.76
24.50
24.34
24.24
24.15
$N=10, P=0.1, m=0.0, \quad R=0.1775$
$E W D$
26.56
26.55
26.54
26.54
26.54
26.54
26.53
26.53
26.53
26.53
26.53
26.53
26.53
26.53
Leased

30.42
27.77
26.69
24.19
27.86
27.33
27.01
26.55
26.28
26.13
26.00
Purchased
38.39
33.61
30.63
30.09
29.74
27.38
26.93
26.58
26.31
26.16
25.90
25.73
25.64
25.55
$N=10, \quad P=0.1, \quad \mathrm{~m}=0.0, \mathrm{R}=0.2$
EWD
26.69
26.68
26.67
26.67
26.67
26.66
26.60
26.66
26.66
26.66
26.66
26.66
26.66
26.66

Leased

30.54
29.91
26.38
24.39
29.00
28.47
28.16
27.69
27.43
27.27
27.14
Purchased
40.10
34.81
31.88
31.29
30.94
29.18
28.12
27.77
27.50
27.35
27.09
26.92
26.82
26.73

Table 8 - Averare Cost Per Station Per Month: $K=900$
a) $N=5, \mathrm{P}=0.1, \mathrm{a}=0.0, \mathrm{R}=0.15$

| $S$ | $\Gamma$ | EWD | Leased | Purchased |
| ---: | ---: | :---: | :---: | :---: |
| 20 | 0 | 27.08 |  | 46.17 |
| 40 | 3 | 27.07 |  | 40.85 |
| 60 | 11 | 27.06 |  | 37.90 |
| 80 | 14 | 27.06 | 30.90 | 37.31 |
| 100 | 17 | 27.05 | 30.33 | 36.36 |
| 200 | 29 | 27.05 | 27.45 | 35.10 |
| 400 | 52 | 27.05 | 24.93 | 34.12 |
| 600 | 75 | 27.05 | 32.48 | 33.77 |
| 800 | 97 | 27.05 | 31.95 | 33.50 |
| 1000 | 119 | 27.05 | 31.64 | 33.34 |
| 1500 | 173 | 27.05 | 31.17 | 33.08 |
| 2000 | 226 | 27.05 | 30.90 | 32.92 |
| 2500 | 273 | 27.05 | 30.75 | 32.82 |
| 3000 | 331 | 27.05 | 30.62 | 32.73 |

b) $\quad N=5, P=0.1, m=0.0, \quad R=0.1775$

| S | $\Gamma$ | EWD | Leased | Purchased |
| ---: | ---: | ---: | ---: | ---: |
| 20 | 6 | 27.21 |  | 47.53 |
| 40 | 9 | 27.19 |  | 42.20 |
| 60 | 11 | 27.19 |  | 39.25 |
| 80 | 14 | 27.18 | 31.02 | 38.65 |
| 160 | 17 | 27.18 | 30.47 | 38.30 |
| 200 | 29 | 27.18 | 27.64 | 36.53 |
| 400 | 52 | 27.17 | 25.19 | 35.46 |
| 000 | 75 | 27.17 | 33.63 | 35.11 |
| 800 | 97 | 27.17 | 33.10 | 34.84 |
| 1000 | 119 | 27.17 | 32.78 | 34.68 |
| 1500 | 173 | 27.17 | 32.31 | 34.42 |
| 2000 | 226 | 27.17 | 32.05 | 34.25 |
| 2500 | 279 | 27.17 | 31.89 | 34.15 |
| 3000 | 331 | 27.17 | 31.76 | 34.06 |

c) $N=5, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$

| S | T | EWD | Leased | Purchased |
| ---: | :---: | :---: | :---: | :---: |
| 30 | 6 | 27.32 |  | 48.67 |
| 40 | 9 | 27.30 |  | 43.34 |
| 50 | 11 | 27.29 |  | 40.38 |
| 20 | 14 | 27.29 | 31.12 | 39.79 |
| 100 | 17 | 27.29 | 30.59 | 39.43 |
| 200 | 29 | 27.28 | 27.80 | 37.65 |
| 400 | 52 | 27.28 | 25.36 | 36.39 |
| 600 | 75 | 27.28 | 34.59 | 36.23 |
| 300 | 97 | 27.28 | 34.06 | 35.97 |
| 1090 | 119 | 27.28 | 33.74 | 35.81 |
| 1500 | 173 | 27.23 | 33.28 | 35.54 |
| 2000 | 226 | 27.28 | 33.01 | 35.38 |
| 2500 | 279 | 27.28 | 32.85 | 35.28 |
| 3000 | 331 | 27.28 | 32.72 | 35.19 |


$\mathrm{g}) \quad \mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.15$

| $S$ | $T$ | EWD | Leased | Purchased |
| ---: | ---: | :---: | :---: | :---: |
| 20 | 6 | 26.41 |  | 39.07 |
| 40 | 9 | 26.40 |  | 33.81 |
| 60 | 11 | 26.39 |  | 30.89 |
| 00 | 14 | 26.39 | 30.28 | 30.30 |
| 100 | 17 | 26.35 | 29.60 | 29.95 |
| 200 | 29 | 26.39 | 26.46 | 28.20 |
| 400 | 52 | 26.39 | 23.96 | 27.14 |
| 600 | 75 | 26.39 | 26.52 | 26.79 |
| 800 | 97 | 26.38 | 25.99 | 26.53 |
| 1000 | 119 | 26.38 | 25.67 | 26.37 |
| 1500 | 173 | 26.38 | 25.21 | 26.11 |
| 2000 | 226 | 26.38 | 24.94 | 25.95 |
| 2500 | 279 | 26.38 | 24.78 | 25.85 |
| 3000 | 331 | 26.38 | 24.65 | 25.76 |

n)

| $S$ | $T$ |
| ---: | ---: |
| 20 | 6 |
| 40 | 9 |
| 60 | 11 |
| 80 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

$\mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0$,
$R=0.1775$
EWD
26.56
26.55
26.54
26.54
26.54
26.54
26.53
26.53
26.53
26.53
26.53
26.53
26.53
26.53

Leased
Purchased
40.67
35.40
32.47
31.8
31.53
29.77
29.77
28.72
28.36
$28 \cdot 10$
27.94
27.68
$27 \cdot 52$
27.42
$27 \cdot 33$
i) $N=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$


| - |
| :---: |
|  |  |

EWD
26.69
26.68
26.67
26.67
26.67
26.66
26.60
26.66
26.66
26.66
26.66
26.66
26.66
26.66
Leased
Purchased 42 . 03 36.75 33.81
33.22
30.54
29.91
26.88
24.39
29.00
28.47
28.16
27.69
27.43
27.27
27.14
33.22
32.87 31.11
30.05
29.70
29.74
29.44
29.28
29.02
29.02
28.86
28.76
28.67

Tasle - Averaze Cost Per station Per Month: $k=1000$

| 5 | T | EWD | Leased | Purchased |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 0 | 27.05 |  | 48.55 |
| $+0$ | 9 | 27.07 |  | 43.23 |
| 60 | 11 | 27.06 |  | 40.28 |
| 80 | 14 | 27.06 | 30.90 | 39.69 |
| 1.00 | 17 | 27.05 | 30.33 | 39.34 |
| 200 | 23 | 27.05 | 27.45 | 37.56 |
| 400 | 52 | 27.05 | 24.99 | 36.50 |
| 600 | 75 | 27.05 | 32.48 | 36.15 |
| 800 | 97 | 27.05 | 31.95 | 35.88 |
| 1000 | 119 | 27.05 | 31.64 | 35.72 |
| 1500 | 173 | 27.05 | 31.17 | 35.46 |
| 2000 | 226 | 27.05 | 30.90 | 35.30 |
| 2500 | 279 | 27.05 | 30.75 | 35.20 |
| 3000 | 331 | 27.05 | 30.62 | 35. 11 |

b) $N=5, P=0.1, m=0.0, R=0.1775$

| $S$ | $\tau$ | EWD | Leased | Purchased |
| ---: | ---: | ---: | ---: | ---: |
| 20 | 6 | 27.21 |  | 50.05 |
| 10 | 9 | 27.19 |  | 44.73 |
| 60 | 11 | 27.19 |  | 41.77 |
| 80 | 14 | 27.18 | 31.02 | 41.18 |
| 100 | 17 | 27.18 | 30.47 | 40.83 |
| 200 | 29 | 27.18 | 27.64 | 39.05 |
| 400 | 52 | 27.17 | 25.19 | 37.99 |
| 000 | 75 | 277.17 | 33.63 | 37.6 .3 |
| 300 | 97 | 27.17 | 33.10 | 37.37 |
| 1000 | 119 | 277.17 | 32.78 | 37.21 |
| 1500 | 173 | 27.17 | 32.31 | 36.95 |
| 2000 | 226 | 27.17 | 32.05 | 36.73 |
| 2500 | 273 | 27.17 | 31.39 | 36.68 |
| 3000 | 331 | 27.17 | 31.76 | 36.59 |


| S | T | EWD | Leased | Purchased |
| :---: | :---: | :---: | :---: | :---: |
| 20 | 6 | 27.32 |  | 51.32 |
| 40 | 3 | 27.30 |  | 45.99 |
| 60 | 11 | 27.29 |  | 43.0 .3 |
| . 30 | 14 | 27.29 | 31.12 | 42.44 |
| 100 | 17 | 27.29 | 30.59 | 42.08 |
| 200 | 29 | 27.28 | 27.30 | 40.30 |
| 400 | 52 | 27.28 | 25.36 | 39.24 |
| 600 | 75 | 27.28 | 34.59 | 38.88 |
| 800 | 97 | 27.28 | 34.06 | 38.61 |
| 1000 | 110 | 27.28 | 33.74 | 38.45 |
| 1500 | 173 | 27.28 | 33.28 | 38.19 |
| 2000 | 225 | 27.28 | 33.01 | 38.0 .3 |
| 2500 | 279 | 27.28 | 32.85 | 37.93 |
| 3000 | 331 | 27.28 | 32.72 | 37.84 |


| c) |  | 7, $\quad 3=0.1, m=0.0, k=0.15$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | $\Gamma$ | EVD | Leased | Purchased |
| 20 | 6 | 26.69 |  | 43.93 |
| 40 | 9 | 26.07 |  | 38.65 |
| 50 | 11 | 20.67 |  | 35.71 |
| 50 | 11 | 26.67 | 30.54 | 35. 13 |
| $1)^{10}$ | 17 | 26.66 | 29.90 | 34.78 |
| 200 | 23 | 26.66 | 26.87 | 33.01 |
| 400 | 52 | 26.66 | 24.38 | 31.96 |
| 600 | 75 | 26.60 | $28 \cdot 98$ | 31.60 |
| 300 | 97 | 26.66 | 28.45 | 31.34 |
| 1000 | 119 | 26.66 | 28.13 | 31.18 |
| 1500 | 173 | 26.66 | 27.67 | 30.92 |
| 2000 | 226 | 26.66 | 27.40 | 30.76 |
| 2500 | 279 | 26.66 | 27.25 | 30.66 |
| 3000 | 331 | 26.66 | 27.12 | 30.57 |


$R=0.1775$

| EWD | Leased | Purchased <br> 26.83 |
| :---: | :---: | :---: |
| 26.51 |  | 40.55 |
| 26.80 |  | 37.31 |
| 26.80 | 30.67 | 36.73 |
| 26.30 | 30.05 | 36.37 |
| 26.80 | 27.08 | 34.61 |
| 26.80 | 24.60 | 33.55 |
| 26.79 | 30.21 | 33.20 |
| 26.79 | 29.68 | 32.93 |
| 26.79 | 29.36 | 32.77 |
| 26.79 | 28.90 | 32.51 |
| 26.79 | 28.63 | 32.35 |
| 26.79 | 28.47 | 32.25 |
| 26.79 | 28.34 | 32.16 |

f)
$\mathrm{N}=7, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$

| $S$ | T |
| ---: | ---: |
| 20 | 6 |
| 40 | 3 |
| 60 | 11 |
| 30 | 14 |
| 100 | 17 |
| 200 | 29 |
| 400 | 52 |
| 600 | 75 |
| 800 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

EWD
26.94
26.93
26.92
26.92
26.92
26.91
26.91
26.91
26.91
26.31
26.91
26.91
26.91
26.91
Leased

30.78
30.18
27.25
24.78
31.25
30.72
30.40
29.94
29.67
29.51
29.38

Purchased
46.92
41.62
38.67
38.08
37.73
37.73
35.96
34.90
34.55
34.28
34.28
34.12
34.86
33.70
33.60
33. 60
g) $N=10, \quad v=0.1, \quad m=0.0, \quad \mathrm{R}=0.15$

1
5
9
11
14
17
29
52
75
97
119
173
226
279
331
n)
5
20
40
60
80
100
200
400
600
800
1000
1500
2000
2500
3000

1
6
9
11
14
17
29
52
75
97
119
173
226
279
331
$N=10, P=0$
$E W D$
26.56
26.55
26.54
26.54
26.54
26.54
26.53
26.53
26.53
26.53
26.53
26.53
26.53
26.53
EVD
26.41
26.40
26.39
26.39
26.39
26.39
26.39
26.39
26.38
26.38
26.38
26.38
26.38
26.38
Leased

30.28
29.60
26.46
23.96
26.52
25.09
25.67
25.21
24.94
24.78
24.65
Purchased
40.65
$35 \cdot 42$
32.50
31.92
31.56
29.81
28.76
28.41
28.14
27.99
27.73
27.56
27.47
27.38
Leased

30.42
29.77
26.69
24.19
27.86
27.33
27.01
26.55
26.28
26.13
26.00
$\mathrm{R}=0.1775$
Purchased
42.46
37.18
34.25
33.67
33.31
31.56
30.50
30.15
29.89
29.73
29.47
29.31
29.21
29.12
$\mathrm{N}=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$

| EWD | Leased | Purchased |
| :---: | :---: | :---: |
| 26.69 |  | 43.96 |
| 26.68 |  | 38.68 |
| 26.67 | 30.54 | 35.74 |
| 26.67 | 29.91 | 34.80 |
| 26.67 | 26.88 | 33.04 |
| 26.66 | 24.39 | 31.99 |
| 26.66 | 29.00 | 31.63 |
| 26.66 | 28.47 | 31.37 |
| 26.66 | 27.16 | 31.21 |
| 26.66 | 27.69 | 30.95 |
| 26.66 | 27.43 | 30.79 |
| 26.66 | 27.67 | 30.69 |
| 26.66 | 27.60 |  |

Fable 10 - Average Cost Per Station Per sonth: K=110,

3
20
40
60
60
100
200
400
600
800
1000
1500
2000
2530
3000
d)
$N=7, \quad N=0 \cdot 1, \quad u=0.0, \quad k=0.15$
$T$
6
11
11
17
23
52
73
147
173
226
273
331
e)
5
20
40
50
20
100
200
400
600
800
1000
1500
2000
3500
3000
1
6
9
11
14
17
29
52
75
97
119
173
220
273
331

| $+$ | Hocmatnonvorombor <br>  - कracle |
| :---: | :---: |
|  |  |


| 7. $P=0.1, ~ m=1) .0$, |  |
| :---: | :---: |
| E日D | Leased |
| 26.94 |  |
| 26.93 |  |
| 26.92 |  |
| 26.92 | 30.78 |
| 26.92 | 30.18 |
| 26.91 | 27.25 |
| 26.91 | 24.78 |
| 26.91 | 31.25 |
| 26.91 | 30.72 |
| 26.91 | 30.40 |
| 26.91 | 29.94 |
| 26.91 | 29.67 |
| 26.91 | 29.51 |
| 26.91 | 29.38 |

$\mathrm{R}=0.2$
Purchased
49.14
43.84
40.89
$40 \cdot 30$
39.95
38.18
37.12
36.77
36.50
36.34
36.08
35.92
35.82
g) $\quad \mathrm{N}=10, \mathrm{P}=0.1, \quad \mathrm{~m}=0.0, \quad \mathrm{k}=0.15$

| $S$ | $T$ | $E W D$ | Leased | Purchased |
| ---: | ---: | :---: | :---: | :---: |
| 20 | 5 | 26.41 |  | 42.30 |
| 40 | 9 | 26.40 |  | 37.04 |
| 00 | 11 | 26.39 |  | 34.11 |
| 80 | 14 | 26.39 | 30.28 | 33.53 |
| 100 | 17 | 26.39 | 29.60 | 33.18 |
| 200 | 29 | 20.39 | 26.46 | 31.42 |
| 400 | 52 | 26.39 | 23.86 | 30.37 |
| 600 | 75 | 20.39 | 26.52 | 30.02 |
| 300 | 97 | 20.38 | 25.99 | 29.76 |
| 1090 | 119 | 26.38 | 25.67 | 29.60 |
| 1560 | 173 | 26.38 | 25.21 | 29.34 |
| 2000 | 226 | 26.38 | 24.94 | 29.18 |
| 2300 | 279 | 26.38 | 24.78 | 29.08 |
| 3000 | 331 | 26.38 | 24.65 | 28.99 |

in)

| 5 | 2 |
| ---: | ---: |
| 20 | 6 |
| 10 | 9 |
| 50 | 11 |
| 60 | 14 |
| 100 | 17 |
| 200 | 20 |
| 400 | 52 |
| 600 | 75 |
| 300 | 97 |
| 1000 | 119 |
| 1500 | 173 |
| 2000 | 226 |
| 2500 | 279 |
| 3000 | 331 |

$\mathrm{N}=10, \quad \mathrm{P}=0.1$,
EWD
26.56
26.55
26.54
26.54
26.54
26.54
26.53
26.53
26.53
26.53
26.53
26.33
26.53
26.53

Leased

30.42
29.77
26.69
24.19
27.86
27.33
27.01
26.55
26.28
26.13
26.00
$\mathrm{q}=0.1775$
Purchased
44.24
38.37
36.04
35.45
35.10
33.34
32.29
31.94
31.67
31.51
31.26
31.09
30.93
30.90
i) $N=10, \mathrm{P}=0.1, \mathrm{~m}=0.0, \mathrm{R}=0.2$

EWD
26.69
26.68
26.67
26.67
26.67
26.66
26.66
26.66
26.66
26.66
26.66
26.66
26.60
26.66

Leased
Purchased 45. 90 40.61 37.68
37.09
36.74
34.98
$33 \cdot 92$
33. 57
33.30
33. 14
32.89
32.72
32.72
32.62
$32 \cdot 62$
32.53

Table 11 - Optimum Choices for the Case Study

| K | N | S Range | Choice |
| :---: | :---: | :---: | :---: |
| 600 | 5 | $\begin{aligned} & S \leq 200 \\ 200 & \leq S \leq 400 \\ 600 \leq S & \leq 2000 \\ 2000<S & \leq 3000 \end{aligned}$ | ```EWD LP EWD when R=20%, otherwise PP PP``` |
|  | 7 | $\begin{array}{r} s \leq 100 \\ 200 \leq s \leq 400 \\ 600 \leq s \leq 3000 \end{array}$ | EWD <br> Interest rate sensitive, no clear choice. <br> PP except when $S=400$ and $R=17.75 \%$. In that case LP is cheaper. |
|  | 10 | $\begin{aligned} \mathrm{S} & \leq 60 \\ 80 \leq \mathrm{S} & \leq 3000 \end{aligned}$ | EWD <br> PP except when $R=20 \%$, EWD is cheaper for $\mathrm{S} \leq 100$. |
| 700 | 5 | $S \leq 200$ $200<S \leq 400$ $600 \leq S \leq 3000$ | $\begin{aligned} & \text { EWD } \\ & \text { LP } \\ & \text { EWD } \end{aligned}$ |
|  | 7 | $\begin{aligned} S & \leq 200 \\ 200<S & \leq 400 \\ 600 \leq S & \leq 3000 \end{aligned}$ | ```EWD ``` |
|  | 10 | $\begin{aligned} & S \leq 100 \\ & 200 \leq S \\ & \leq 400 \\ & 600 \leq S \leq 3000 \end{aligned}$ | EWD <br> No clear choice PP |
| 800 | 5 | $\begin{aligned} S & \leq 200 \\ 200<S & \leq 400 \\ 600 \leq S & \leq 3000 \end{aligned}$ | EWD <br> LP <br> EWD |
|  | 7 | $\begin{aligned} & S \leq 200 \\ & 200<S \leq 400 \\ & 600 \leq S \leq 3000 \end{aligned}$ | EWD <br> LP <br> EWD |
|  | 10 | $S$ $\leq 200$ <br> $200<S$ $\leq 400$ <br> $600 \leq S$ $\leq 3000$ | ```EWD LP PP except when R=20%, EWD is slightly cheaper.``` |


| K | N | $S$ Range | Choice |
| :---: | :---: | :---: | :---: |
| 900 | 5 | $\begin{aligned} s & \leq 200 \\ 200<S & \leq 400 \\ 600 \leq S & \leq 3000 \end{aligned}$ | $\begin{aligned} & \text { EWD } \\ & \text { LP } \\ & \text { EWD } \end{aligned}$ |
|  | 7 | $\begin{aligned} s & \leq 200 \\ 200<S & \leq 400 \\ 600 \leq S & \leq 3000 \end{aligned}$ | EWD <br> LP <br> EWD |
|  | 10 | $\begin{aligned} S & \leq 200 \\ 200<S & \leq 400 \\ 600 \leq S & \leq 3000 \end{aligned}$ |  |
| $\begin{gathered} 1000 \\ \text { or } \\ 1100 \end{gathered}$ | 5,7 | $\begin{aligned} & S \leq 200 \\ & 200<S \leq 400 \\ & 600 \leq S \leq 3000 \end{aligned}$ | $\begin{aligned} & \text { EWD } \\ & \text { LP } \\ & \text { EWD } \end{aligned}$ |
|  | 10 | $\begin{array}{r} S \leq 200 \\ 200<S \leq 400 \\ 600 \leq S \leq 3000 \end{array}$ | ```EWD LP Low rate favours LP; high rates favour EWD``` |

```
EWD - EWD PBX
LP - Leased PBX
PP - Purchased PBX
```

Table 12-Optimal Combinations of Scheraes

ALL EWD
1,5051
$1,695,321$
$1,612,172$
$1,575,301$
$1,533,367$
$1,530,205$
$1,539,098$
$1,567,928$
$1,575,448$
$1,538,321$
$1,605,838$
$1,612,172$
$1,575,301$
$1,533,367$
$1,530,205$
$1,559,098$
$1,567,925$
$1,575,448$
$1,598,321$
$1,605,838$
$1,612,172$
$1,575,301$
$1,583,367$
$1,590,205$
$1,559,098$
$1,567,928$
$1,575,448$
$1,508,321$
$1,605,838$
$1,612,172$
$1,575,301$
$1,583,367$
$1,590,205$
$1,559,098$
$1,567,928$
$1,575,448$
$1,598,321$
$1,605,838$
$1,612,172$
$1,575,301$
$1,583,367$
$1,590,205$
$1,559,098$
$1,567,928$
$1,575,443$
$1,598,321$
$1,605,838$
$1,612,172$
$1,575,301$
$1,533,367$
$1,530,205$
$1,559,098$
$1,567,928$
$1,575,448$
$1,661,010$
1


| \%SAVE | $\cdots \mathrm{EWD}$ | $\%$ LP | \% pp |
| :---: | :---: | :---: | :---: |
| 3.5 | 22.0 | 24.5 | 53.3 |
| 2.0 | 33.5 | 24.8 | 41.7 |
| 1.4 | 73.2 | 24.8 | 0.1 |
| 9.0 | $1+2$ | 3.3 | 52.5 |
| 6.8 | 17.3 | 21.0 | 61.7 |
| 5.2 | 22.0 | 24.8 | 53.3 |
| 14.7 | 9.1 | 0.0 | 00.9 |
| 11.5 | 11.2 | 0.0 | 83.8 |
| 9.0 | 14.2 | 4.8 | 81.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.5 | 75.2 | 24.8 | 9. 0 |
| 1.4 | 75.2 | 24.8 | $0 \cdot 0$ |
| $4 \cdot 6$ | 21.6 | 25.1 | 33.3 |
| 2.6 | 29.6 | 24.8 | 45.7 |
| 1.6 | 70.3 | 24.8 | 4.7 |
| 9.3 | 13.7 | 7.0 | 79.3 |
| 6.5 | 17.3 | 24.0 | 53.2 |
| 4.5 | 21.6 | 25.1 | 53.3 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.5 | 75.2 | 24.8 | O. ${ }^{\text {( }}$ |
| 1.4 | 75.2 | 24.8 | 0.0 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.7 | 75.2 | 24.8 | 0.0 |
| 1.6 | 75.2 | 24.8 | 0.0 |
| $5 \cdot 5$ | 21.3 | 25.5 | 53.3 |
| 2.9 | 26.8 | 25.5 | 47.7 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.5 | $75 \cdot 2$ | 24.8 | 0.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.4 | 75.2 | 24.8 | 0.0 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.7 | 75.2 | 24.8 | 0.0 |
| 1.6 | 75.2 | 24.8 | $0 \cdot 0$ |
| 3.6 | 28.9 | 71.1 | 0.0 |
| 1.9 | 62.4 | 37.6 | 0.0 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.4 | 75.2 | 24.8 | 0.0 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.7 | 75.2 | 24.8 | 0.0 |
| 1.6 | 75.2 | 24.8 | $0 \cdot 0$ |
| 3.6 | 28.9 | 71.1 | 0.0 |
| 1.9 | 62.4 | 37.6 | 0.0 |
| 1.8 | 74.9 | 25.1 | 0.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.5 | 75.2 | 24.8 | 0.0 |
| 1.4 | 75.2 | 24.8 | 0.0 |
| 1.3 | 74.9 | 25.1 | 0.0 |
| 1.7 | 75.2 | 24.8 | 0.0 |
| 1.6 | 75.2 | $2+.8$ | 0.0 |
| 3.6 | $2 \mathrm{S.9}$ | $71 \cdot 1$ | 0.0 |
| 1.9 | 62.4 | 37.6 | 0.0 |
| 1.8 | 74.9 | 25.1 | $0 \cdot 0$ |
| 1.6 | 34.6 | 22.3 | 43.1 |

$E W D-E W D P 3 X$
$L P=$ LeasedPSX
$P P=$ PurctasedPbX

Figure 1 - Station-to-Trunk Ratio


Number of Stations




Number of Stations



