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By-Catches in Five Grand Bank Groundfish Fisheries
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## ABSTRACT

A linear programming technique was used to analyse the ratios of by-catch species to main species in five directed groundfish fisheries in NAFO Div. 3L, $3 N$, and $3 \emptyset$. Operating within allowable catch levels for 1981, the analysis provided the directed catch as well as the total catch for each species, based on yearly fishing patterns over the period 1976-79. The results showed that although the predicted directed catches for each species in 1981 varied considerably with each of the four fishing patterns, the total catch permitted for each species would be taken regardless of the fishing pattern used.

## RESUME

La programmation linéaire a servi à analyser les rapports entre les prises fortuites et celles de l'espèce visée dans cinq pêcheries dirigées des divisions 3L, 3 N et $3 \emptyset$ de 1'OPANO. Cette analyse, menée dans les limites de prises fixées pour 1981, a donné les quantités capturēes de l'espèce cible ainsi que le total de chaque espèce, fondées sur le rēgime annuel de pèche pendant la période 1976-79. Les résultats démontrent que, en dépit des fortes variations des prises dirigées prédites pour chaque espèce en 1981 à chacun des quatre régimes de pèche, les prises totales admissibles de chaque espèce auraient été atteintes, quel que soit le régime de pêche adopté.

## INTRODUCTION

In considering problems of by-catch, 5 directed fisheries by Canadian trawlers in NAFO Div. $3 \mathrm{~L}, 3 \mathrm{~N}$ and 30 combined were examined: cod, redfish, American plaice, witch and yellowtail. Only offshore fisheries i.e., those conducted using trawl nets, were considered. By-catch ratios were calculated for each of the 4 years 1976-1979. The linear programming model used by Brown et. al., $(1973,1975)$ is used for each of the 4 years' by-catch ratios to maximize total catch of all 5 species for 1981, the constraints being that the "allocation"1 for each species not be exceeded.

## MATERIALS AND METHODS

Catches for 1976, '77, '78 and '79 separately, for areas Div. 3LNO, for all trawl gear, by main species, were determined from ICNAF statistics (Tables 1, 2, 3, 4). From these, by-catch ratios were derived (Tables 5, 6, 7, 8). These were calculated, for each directed fishery, by dividing the catch of each species in the fishery by the catch of the directed species. The column headed "R" gives the sum of the ratios within each fishery i.e., the sum of the catch of the 5 species in the fishery divided by the catch of the main species.

Two methods were used to calculate the "allocations" for the Div. 3LNO trawl fisheries. The first method used the ratio of the Div. 3LNO catch by trawl in the 5 fisheries (Tables 1-4) to the Div. 3LNO catch by all gears for each year. These ratios were multiplied by the Canadian allocation for 1981 for each species to give an "allocation" for trawl gears for each species in the model for each year. For cod and witch, the portion of the Div. 2J3KL allocation which would theoretically be caught in Div. 3L was determined from catch ratios in Div. 2J3KL and Div. 3L in each year and this was added to the Div. 3NO allocation (Table 10) before the ratios were applied. The "allocations" calculated by this method appear in Table 11 . The second method involved the use of the actual 1981 Canadian allocations (Table 9). For redfish, American plaice, and yellowtail, the 1981 Div. 3LNO allocations were used directly as the Div. 3LNO "allocations" for the model (Table 17). For cod and witch, the Div. 3L portion of the Div. 2J3KL allocation was determined, and this was added to the Div. 3NO allocation. This was done for each species by taking the ratio of the Div. 3L catch to the Div. 2J3KL catch in a year and multiplying this by the Div. 2J3KL Canadian offshore allocation to produce a Div. 3L offshore "allocation". This was then divided by the Div. 2J3KL total Canadian allocation for that year and the resulting figure was multiplied by the 1981 Canadian allocation for Div. 2J3KL to produce the 1981 Div. 3L offshore "allocation". Results of these computations appear in Table 16.

[^0]The by-catch ratios and $R$ values were used in the linear programming model (Glicksman 1963; Llewellyn 1964). A value, $S$, which is the total catch of all species, was maximized so that none of the individual species "allocations" was exceeded (Appendix I). The solution to the set of equations gave the directed catches for each species, from which the total catches for each species were calculated. Program ZX3LP from IMSL was used in solving the linear programming setup. Results from the linear programming simulations appear in Tables 12-15 and 18-21.

## RESULTS AND DISCUSSION

The linear programming model showed, for both sets of simulations, that except for the total "allocations" derived from the 1977 data, the total "allocations" would be caught using the corresponding fishing patterns. For the 1977 data, the total predicted 1981 catch from the "catch method" simulations represented 99.94\% of the total "allocation" and the total predicted 1981 catch from the "allocation method" simulations represented $99.97 \%$ of the total "allocation".

The increases in the 1981 total "allocations" using the "catch method" over the 4 years' data were due primarily to the increases in the predicted catch of cod. This was because the percentage of the total cod catch taken by trawl fisheries increased in each year. The "allocations" for the other species remained fairly stable over the 4 year range, the greatest difference being a $17.3 \%$ decrease in 1977-based witch "allocation" over the 1976-based one.

The differences in the 1981 total "allocations" using the "allocation method" were due primarily to the differences in the ratios of the Div. 3L offshore "allocation" to the Div. 2J3KL total allocation for cod (Table 16). For the 1976 data, which gave a 1981 Div. 3LNO "allocation" of 42,749 MT for cod, this ratio is highest, while for the 1978 data, which gave an "allocation" of 29,464 MT, the ratio was lowest. Witch did not have a great effect on the total 1981 "allocation" as its "allocations" range from 3,385 MT for the 1977 data to 4,456 MT for the 1976 data. "Allocations" for the other species were the same for each year's data, this being due to the fact that all the catch for these species was allocated to offshore fisheries. This was alright for redfish and yellowtail where there are no inshore fisheries, but was not the case for American plaice, where the inshore fishery takes a portion of the total allocated catch.

The increases in the directed catches of cod in the 4 "catch method" simulations were consistent with the increases in the "allocations" for cod in Div. 3L. Because of the low by-catch of redfish in any of the other fisheries (with the exception of the 1979 witch fishery, where total catches were negligible anyway), the directed catches of redfish showed little variation. The predicted directed catches of American plaice showed almost no difference in the 4 simulations. The total amounts of American plaice caught in the other fisheries were approximately equal in each of the 4 years, although the amounts caught in the individual fisheries showed some variation between years. The low directed catch for witch predicted by the 1976 data was due to the higher by-catch ratio of witch in the American plaice fishery that year ( 0.0429 ) compared to the other 3 years ( $0.0175,0.0170,0.0252$ ). The reasons
for the changes in the predicted directed catches of yellowtail are the differences in the yellowtail by-catch ratios in the cod and American plaice fisheries in the 4 years. For the 1977 data, which predicted the lowest directed catch of yellowtail, the by-catch ratios of yellowtail in both the cod and American plaice fisheries were higher than in the other 3 years.

For the "allocation method" simulations, much of the above reasoning is also true, as both sets of simulations used the same by-catch ratios. The lower ratios of directed catch to total catch for cod in the 1978 and 1979 based simulations were due to the higher by-catch ratios of cod in the other fisheries in those years, particularly in the American plaice fishery. Conversely, the higher directed-to-total ratio for cod in the 1976 simulation was due to the much lower by-catches of cod in the other fisheries. Other trends which were noticeable in the "catch method" simulations are also present in the "allocation method" simulations because of the identical by-catch ratios.

The model used shows how a maximum amount ( $\leq$ total "allocation") of several species can be caught using a known fishing pattern, in such a way that none of the individual "allocations" is exceeded. It predicts what the directed catch of each species in the model should be, based on the known fishing pattern. Some considerations to be made when applying the model:
A. It may not be correct to classify certain fisheries as being directed at one species. Some fisheries result in consistently high by-catches of other species. Thus, to say that a particular amount of effort is directed at a species may be misleading. The model classifies catches in terms of species caught in a directed fishery and therefore the directed catches predicted by the model may not be the result of a true directed fishery. An accurate assessment of by-catch ratios is therefore essential when using a model such as this.
B. The model predicts directed catch and total catch for each species based on the fishing pattern of a previous year. It is doubtful if this concept would be of any use in trying to regulate by-catches within fisheries from a practical viewpoint. Perhaps some sort of ongoing method, on a vessel by vessel basis, could be used in trying to control by-catches. Another possibility would be to break the yearly by-catch ratios into monthly ones. This would also involve the determining of "monthly allocations", based on either a monthly catch pattern or a division of the total allocation into month by month portions.

Some other considerations which apply to the model as it is used here i.e., for 5 Canadian Div. 3LNO groundfish fisheries conducted by trawl gears, are as follows:
A. Stocks of certain species overlap within the area, e.g. Div. 2J3KL cod and Div. 3NO cod. This presented some problems in determining "allocations".
B. Offshore fisheries only were considered. Of the 5 species considered, only cod and witch were caught in any significant (>10\%of total catch) quantities by inshore fisheries. Therefore, only by-catches occurring in the Div. 3LNO trawl fisheries were examined.
C. "Allocations" used in this model are based on catches or actual allocations in the 5 fisheries examined. By-catches of the species concerned occur in other fisheries in the area, but the total catches are not significant. Changes in other fisheries resulting in higher catches of the 5 species concerned could have an effect on the results, in which case the model would have to be expanded to include those fisheries.

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Table 1. Catches (MT) by Canadian vessels, 1976, Div. 3LNO, trawl gears.

| Directed fishery | Cod | Redfish | $\frac{\text { Species caught }}{\text { American plaice }}$ | Witch | Yellowtail |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cod | 1,059 | 25 | 367 | 41 | 111 |
| Redfish | 199 | 9,773 | 227 | 25 | 32 |
| American plaice | 2,649 | 57 | 40,852 | 1,752 | 2,763 |
| Witch | 37 | 0 | 563 | 1,291 | 55 |
| Yellowtail | 236 | 0 | 1,928 | 191 | 4,886 |

Table 2. Catches (MT) by Canadian vessels, 1977, Div. 3LNO, traw1 gears.

|  |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Directed fishery | Cod | Redfish | $\frac{\text { Species caught }}{\text { American plaice }}$ | Witch | Yellowtail |
| Cod | 2,949 | 51 | 1,032 | 23 | 511 |
| Redfish | 174 | 11,765 | 81 | 43 | 54 |
| American plaice | 4,029 | 65 | 34,924 | 611 | 5,517 |
| Witch | 57 | 5 | 316 | 2,559 | 16 |
| Yellowtail | 646 | 2 | 2,194 | 66 | 5,191 |

Table 3. Catches (MT) by Canadian vesse1s, 1978, Div. 3LNO, traw1 gears.

| Directed fishery | Cod | Redfish | Species caught <br> American plaice | Witch | Yellowtail |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Cod | 8,116 | 80 | 1,846 | 97 | 427 |
| Redfish | 80 | 4,737 | 13 | 24 | 7 |
| American plaice | 6,150 | 42 | 39,175 | 665 | 5,213 |
| Witch | 28 | 0 | 180 | 474 | 9 |
| Yellowtail | 898 | 8 | 3,050 | 95 | 9,428 |

Table 4. Catches (MT) by Canadian vessels, 1979, Div. 3LNO, trawl gears.

|  |  |  |  |  |  |
| :--- | ---: | :---: | :---: | ---: | ---: |
| Directed fishery | Cod | Redfish | $\frac{\text { Species caught }}{\text { American plaice }}$ | Witch | Yellowtail |
| Cod | 12,724 | 374 | 3,095 | 240 | 499 |
| Redfish | 741 | 9,661 | 228 | 89 | 45 |
| American plaice | 4,865 | 67 | 34,535 | 872 | 3,699 |
| Witch | 5 | 19 | 0 | 33 | 0 |
| Yellowtail | 1,769 | 21 | 3,987 | 102 | 13,238 |

Table 5. By-catch ratios, 1976. Derived from catches in Table 1. Div. 3LNO.

| Directed fishery | Species caught |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cod | 1.0000 | 0.0236 | 0.3466 | 0.0387 | 0.1048 | 1.5137 |
| Redfish | 0.0204 | 1.0000 | 0.0232 | 0.0026 | 0.0033 | 1.0495 |
| American plaice | 0.0648 | 0.0014 | 1.0000 | 0.0429 | 0.0676 | 1. 1767 |
| Witch | 0.0287 | 0.0000 | 0.4361 | 1.0000 | 0.0426 | 1.5074 |
| Yellowtail | 0.0483 | 0.0000 | 0.3946 | 0.0391 | 1.0000 | 1.4820 |

Table 6. By-catch ratios, 1977. Derived from catches in Table 2. Div. 3LNO.

| Directed fishery | Cod | Redfish | American plaice |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Table 7. By-catch ratios, 1978. Derived from catches in Table 3. Div. 3LNO.

| Directed fishery | Cod | Species caught |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Redfish | American plaice | Witch | Yellowtail | R |
| Cod | 1.0000 | 0.0099 | 0.2275 | 0.0120 | 0.0526 | 1.3020 |
| Redfish | 0.0169 | 1.0000 | 0.0027 | 0.0051 | 0.0015 | 1.0262 |
| American plaice | 0.1570 | 0.0011 | 1.0000 | 0.0170 | 0.1331 | 1.3082 |
| Witch | 0.0591 | 0.0000 | 0.3797 | 1.0000 | 0.0190 | 1.4578 |
| Yellowtail | 0.0952 | 0.0008 | 0.3235 | 0.0101 | 1.0000 | 1.4296 |

Table 8. By-catch ratios, 1979. Derived from catches in Table 4. Div. 3LNO.

| Directed fishery | Cod | Redfish | Species caught |  | Yellowtail | R |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | American plaice | Witch |  |  |
| Cod | 1.0000 | 0.0294 | 0.2432 | 0.0189 | 0.0392 | 1.3307 |
| Redfish | 0.0767 | 1.0000 | 0.0236 | 0.0092 | 0.0047 | 1.1142 |
| American plaice | 0.1409 | 0.0019 | 1.0000 | 0.0252 | 0.1071 | 1.2751 |
| Witch | 0.1515 | 0.5758 | 0.0000 | 1.0000 | 0.0000 | 1.7273 |
| Yellowtail | 0.1336 | 0.0016 | 0.3012 | 0.0077 | 1.0000 | 1.4441 |

Table 9. 1981 Canadian Allocations (MT)

| Species | Division | (MT) |
| :---: | :---: | :---: |
| Cod | 2 J 3 KL | 185,000 |
|  | 3Nø | 9,800 |
| Redfish | $\begin{aligned} & 3 L N \\ & 30 \end{aligned}$ | $\begin{aligned} & 8,000 \\ & 7,500 \end{aligned}$ |
| American plaice | 3LNØ | 54,200 |
| Witch | 2 J 3 KL | 3,000 |
|  | 3NØ | 3,000 |
| Yellowtail | 3 LN Ø | 20,500 |

Table 10. Amount of 1981 Div. 2J3KL Canadian Allocations (MT) allowed for Div. 3L, based on yearly catches.

|  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | 1976 | 1977 | Year | 1978 |
| Species | 1976 | 1979 |  |  |
| Cod | 109,829 | 106,134 | 98,316 | 96,244 |
| Witch | 2,089 | 534 | 635 | 1,516 |

Table 11. 1981 Canadian "allocations" (MT) for trawl fisheries in Div. 3LNØ, based on yearly catches.

|  |  |  | Year |  |  |
| :--- | ---: | ---: | ---: | ---: | :---: |
|  | 1976 | 1977 | 1978 | 1979 |  |
| Species | 12,672 | 18,903 | 27,223 | 30,188 |  |
| Cod | 15,039 | 14,731 | 15,025 | 15,385 |  |
| Redfish |  |  |  |  |  |
| American <br> plaice | 49,814 | 49,068 | 49,330 | 48,535 |  |
| Witch | 3,738 | 3,092 | 3,109 | 3,432 |  |
| Yellowtail | 20,337 | 20,489 | 20,490 | 20,500 |  |
| Totals | 101,600 | 106,283 | 115,177 | 118,040 |  |

Table 12. Linear programming simulation of 1981 catches, based on 1976 pattern. "Catch method" used. Catches in MT. Div. 3LNO.

| Species <br> sought | 1981 "allocation" for <br> trawl <br> fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | ---: | ---: |
| Cod | 12,672 | 8,988 | 12,672 |
| Redfish | 15,039 | 14,772 | 15,039 |
| American plaice | 49,814 | 39,347 | 49,814 |
| Witch | 3,738 | 1,013 | 3,738 |
| Yellowtail | 20,337 | 16,643 | 20,337 |
| Totals | 101,600 | 80,763 | 101,600 |

Table 13. Linear programming simulation of 1981 catches, based on 1977 pattern. "Catch method" used. Catches in MT. Div. 3LNO.

| Species <br> sought | 1981 "allocation" for trawl fisheries | Directed catch | Total catch |
| :---: | :---: | :---: | :---: |
| Cod | 18,903 | 12,628 | 18,903 |
| Redfish | 14,731 | 14,429 | 14,731 |
| American plaice | ce 49,068 | 39,142 | 49,007 |
| Witch | 3,092 | 2,102 | 3,092 |
| Yellowtail | 20,489 | 12,036 | 20,489 |
| Totals | 106,283 | 80,337 | 106,222 |

Table 14. Linear programming simulation of 1981 catches, based on 1978 pattern. "Catch method" used. Catches in MT. Div. 3LNO.

| Species <br> sought | 1981 "allocation" for <br> trawl <br> fisheries | Directed <br> catch | Total <br> catch |
| :--- | :---: | :---: | ---: |
| Cod | 27,223 | 19,297 | 27,223 |
| Redfish | 15,025 | 14,779 | 15,025 |
| American plaice | 49,330 | 39,569 | 49,330 |
| Witch | 3,109 | 1,987 | 3,109 |
| Yellowtail | 20,490 | 14,148 | 20,490 |
| Totals | 115,177 | 89,780 | 115,177 |

Table 15. Linear programming simulation of 1981 catches, based on 1979 pattern. "Catch method" used. Catches in MT. Div. 3LNO.

| Species <br> sought | 1981 "allocation" for <br> trawl <br> fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | :--- | ---: |
| Cod | 30,188 | 21,396 | 30,188 |
| Redfish | 15,385 | 13,612 | 15,385 |
| American plaice | 48,535 | 38,344 | 48,535 |
| Witch | 3,432 | 1,817 | 3,432 |
| Yellowtail | 20,500 | 15,491 | 20,500 |
| Totals | 118,040 | 90,660 | 118,040 |

Table 16. Calculation of "allocations" for 1981 D.ivisions 3LNO trawl fisheries for cod and witch

| Year | Species | Ratio of 3L total catch to 2J3KL total catch | ```2J3KL offshore allocation MT``` | ```3L offshore "allocation" MT``` | ```2J3KL total allocation MT``` | ```1981 3L offshore "allocation" MT``` | ```1981 3LNO offshore "allocation' MT``` |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1976 | Cod | 0.5937 | 24,000 | 14,248 | 80,000 | 32,949 | 42,749 |
|  | Witch | 0.6963 | 4,600 | 3,203 | 6,600 | 1,456 | 4,456 |
| 1977 | Cod | 0.5737 | 17,750 | 10,183 | 67,750 | 27,806 | 37,606 |
|  | Witch | 0.1779 | 6,500 ${ }^{1}$ | 1,156 | 9,000 | 385 | 3,385 |
| 1978 | Cod | 0.5314 | 20,000 | 10,629 | 100,000 | 19,664 | 29,464 |
|  | Witch | 0.2117 | 6,500 | 1,376 | 9,000 | 459 | 3,459 |
| 1979 | Cod | 0.5202 | 30,000 | 15,607 | 130,000 | 22,210 | 32,010 |
|  | Witch | 0.5056 | 6.,500 | 3,286 | 9,000 | 1,095 | 4,095 |

${ }^{1}$ Estimated. Based on 1976, 1978 and 1979 figures.

Table 17. 1981 Canadian "allocations" for trawl fisheries in Divisions 3LNO based on yearly allocations

| Species | Year |  |  |  |
| :--- | ---: | ---: | ---: | ---: |

Table 18. Linear programming simulation of 1981 catches, based on 1976 fishing pattern. "Allocation method" used to detemmine "allocations". Catches in MT.in Div. 3LNO.

| Species <br> sought | 1981 "allocation" <br> for trawl fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | ---: | ---: |
| Cod | 42,749 | 39,533 | 42,749 |
| Redfish | 15,500 | 14,519 | 15,500 |
| American Plaice | 54,200 | 34,274 | 54,200 |
| Witch | 4,456 | 872 | 4,456 |
| Yellowtail | 20,500 | 13,955 | 20,500 |
| Totals | 137,405 | 103,153 | 137,405 |

Table 19. Linear programming simulation of 1981 catches, based on 1977 fishing pattern. "Allocation method" used to determine "allocations". Catches in MT.in Div. 3LNO.

| Species <br> sought | 1981 "allocation" <br> for trawl fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | ---: | ---: |
| Cod | 37,606 | 31,749 | 37,606 |
| Redfish | 15,500 | 14,869 | 15,500 |
| Americancelaice | 54,200 | 38,959 | 54,156 |
| Witch | 3,385 | 2,289 | 3,385 |
| Yellowtail | 20,500 | 8,759 | 20,500 |
| Totals | 131,191 | 96,625 | 131,147 |

Table 20. Linear programming simulation of 1981 catches, based on 1978 fishing pattern. "Allocation method" used to determine "allocations". Catches in MT.in Div. 3LNO.

| Species <br> sought | 1981 "allocation" <br> for trawl fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | ---: | ---: |
| Cod | 29,464 | 20,852 | 29,464 |
| Redfish | 15,500 | 15,234 | 15,500 |
| American.Plaice | 54,200 | 44,211 | 54,200 |
| Witch | 3,759 | 2,244 | 3,459 |
| Yellowtail | 20,500 | 13,453 | 20,500 |
| Totals | 123,123 | 95,994 | 123,123 |

Table 21. Linear programming simulation of 1981 catches, based on 1979 fishing pattern. "Allocation method" used to determine "allocations". Catches in MT.in Div. 3LNO.

| Species <br> sought | 1981 "allocation" <br> for traw7 fisheries | Directed <br> catch | Total <br> catch |
| :--- | ---: | ---: | ---: |
| Cod | 32,010 | 22,454 | 32,010 |
| Redfish | 15,500 | 13,394 | 15,500 |
| American.Plaice | 54,200 | 43,950 | 54,200 |
| Witch | 4,095 | 2,326 | 4,095 |
| Yellowtail | 20,500 | 14,850 | 20,500 |
| Totals | 126,305 | 96,974 | 126,305 |

## APPENDIX I

Example of the linear programming model used by Brown et al., (1973)
3 species A, B, C.
Directed fisheries occur for each. By-catches of other 2 species occur in each of the 3 fisheries. Catches are broken down by main species as follows:

| Species | Species Caught |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Sought | $A$ | $B$ | $C$ |  |
| $A$ | $x_{1}$ | $x_{2}$ | $x_{3}$ |  |
| $B$ | $y_{1}$ | $y_{2}$ | $y_{3}$ |  |
| C | $z_{1}$ | $z_{2}$ | $z_{3}$ |  |

Where $x, y \& z$ represent the catches in MT. From this, a table of ratios is calculated. Within each fishery, each species is given as its proportion to the directed species. The table is as follows:

| Species | Species Caught |  |  |
| :---: | :---: | :---: | :---: |
| Sought | A | B | C |
| A | 1.00 | $\frac{x_{2}}{x_{1}}$ | $\frac{x_{3}}{x_{1}}$ |
| B | $\frac{y_{1}}{y_{2}}$ | 1.00 | $\frac{y_{3}}{y_{2}}$ |
| C | $\frac{z_{1}}{z_{3}}$ | $\frac{z_{2}}{z_{3}}$ | 1.00 |

Values for $R_{1}, R_{2}$, and $R_{3}$ are obtained by adding across the above table so that:

$$
\begin{aligned}
& R_{1}=1.00+\frac{x_{2}+x_{3}}{x_{1}} \\
& R_{2}=1.00+\frac{y_{1}+y_{3}}{y_{2}} \\
& R_{3}=1.00+\frac{z_{1}+z_{2}}{z_{3}}
\end{aligned}
$$

If we let the predicted directed catch for species $A, B$, and $C$, be equal to $A^{\prime}, B^{\prime}$, and $C^{\prime}$ respectively, then we can formulate the problem as follows:

Maximize a value $S$ equal to $R_{1} A^{\prime}+R_{2} B^{\prime}+R_{3} C^{\prime}$ so that
Total catch of Species $A=1.00 A^{\prime}+\frac{y_{1}}{y_{2}} B^{\prime}+\frac{z_{1}}{z_{3}} C^{\prime} \leq$ "allocation" for Species $A$
Total catch of Species $B=\frac{x_{2} A^{\prime}}{x_{1}}+1.00 B^{\prime}+\frac{z_{2}}{z_{3}} C^{\prime} \leq " a l l o c a t i o n "$ for Species $B$
Total catch of Species $C=\frac{x_{3}}{x_{1}} A^{\prime}+y_{y_{2}} B^{\prime}+1.00 C^{\prime} \leq " a l l o c a t i o n "$ for Species $C$.


[^0]:    "allocation" is used throughout this paper to mean an amount allowed to be caught. It is not a true allocation, although it is calculated from such.

