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Multi-Satellite Intelligence Collection Scheduling

Decision Model

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IMPORTANT INFORMATIVE STATEMENTS

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Abstract

Under the DRDC Joint Intelligence Collection and Analysis Capability (JICAC) project, this report proposes a formal multi-satellite intelligence collection scheduling deterministic decision model. A detailed description of the optimization model highlighting novelties is given. The underlying mathematical formulation is a mixed-integer quadratic programming model mapping intelligence requests to collection asset imaging opportunities in order to optimize collection value (value of information), subject to a variety of novel task, opportunity, capacity, temporal and cost constraints. Departing from known approaches, the new decision model is based on a new coverage approximation scheme to handle possible imaged area overlap while demonstrating desirable objective function convexity property making local optimal solution a global problem solution. The latter condition enables the utilization of powerful exact problem-solving techniques and the fast computation of a bound on the optimal solution, allowing sound comparative performance assessment over various problem-solving methods. The report also discusses the connection with ongoing decision models, potential problem-solving approaches expected to successfully solve the collection scheduling problem, and natural problem extensions.

Significance to Defence and Security

This report characterizes a new multi-satellite intelligence collection scheduling problem decision model, applicable to space-based intelligence, surveillance and reconnaissance. This work is suitably aligned with the RADARSAT Constellation Mission (RCM) project follow-on initiatives and some Canadian Armed Forces (CAF) priority on persistent intelligence, surveillance and reconnaissance in the Arctic and the North as well as all domain situation awareness to timely propose enhanced intelligence collection tasking solution. Part of an ongoing effort, it constitutes a first building block in the development of new science and technology approaches to provide near optimal intelligence collection for low density, high demand deployable collection assets, anticipated to be beneficial for the CAF.

Résumé

Sous la gouverne du projet de Capacité de Cueillette et d'Analyse de Renseignement Inter–armées ou « Joint Intelligence Collection and Analysis Capability » (JICAC) de RDDC, ce rapport propose un modèle déterministe de décision formel d'ordonnancement multi-satellites de cueillette du renseignement. Une description détaillée du modèle ainsi que les innovations proposées sont présentées. La formulation mathématique sous-jacente est un modèle de programmation quadratique mixte mettant en correspondance des requêtes de renseignements à des opportunités d'imagerie de ressources de cueillette afin d'optimiser la valeur de l'information cueillie, sujet à une nouvelle variété de contraintes de tâches, d'opportunités, de capacité, temporelles et de coûts. Se distinguant des approches connues, le modèle de décision proposé mise sur un nouveau schéma de calcul d'approximation de couverture prenant compte du chevauchement possible de régions imagées, permettant ainsi d'exploiter la propriété de convexité du problème d'optimisation résultant. Cette propriété permet l'accès à des méthodes exactes de résolution très efficaces ainsi que le calcul rapide d'une borne à la solution optimale supportant une évaluation comparative plus objective de différentes techniques de résolution. Le rapport discute également du lien avec les modèles de décision existants, des approches et algorithmes applicables pouvant potentiellement résoudre le problème d'ordonnancement, ainsi que les extensions naturelles du problème.

Importance pour la défense et la sécurité

Ce rapport caractérise un nouveau modèle de décision d'un problème d'ordonnancement multi-satellites de cueillette du renseignement applicable au renseignement, la surveillance et reconnaissance effectués à partir de l'espace. Ce travail est parfaitement aligné avec les initiatives de suivi du projet « RADARSAT Constellation Mission » (RCM) et des priorités des Forces Armées Canadiennes (FAC) telles la persistance du renseignement, la surveillance et reconnaissance dans l'Arctique, et le nord ainsi que l`éveil situationnel de multiples domaines, visant à proposer une solution améliorée de planification de cueillette du renseignement. Il constitue un premier jalon dans le développement de nouvelles approches de science et technologie aspirant à une cueillette du renseignement quasi optimale pour des ressources déployables de faible densité en forte demande, conduisant à des bénéfices anticipés pour les FAC.

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1 Introduction

Cost-effective space-based intelligence collection tasking targeted to suitable CAF Intelligence, Surveillance and Reconnaissance (ISR) application domains is key to maintain persistent multi-domain situational awareness over the Canadian territory, including the North region. Ultimately aimed at efficiently bridging the gap between information need and information-gathering, near-optimal resource management is primarily motivated by limited reasoning intelligence collection capability, cost-effectiveness, resource limitations, a low density high demand collection asset (satellite) and time-constrained uncertain environment.

DRDC is developing new intelligence collection technology concepts and automated decision support/planning capabilities applicable to satellite constellation resource management to support collection management awareness, space-based intelligence collection tasking enhancement and near optimal resource allocation. Based on a problem statement recently proposed in [1] to contextualize Multi-Satellite Intelligence Collection Scheduling, this document presents a formal mixed-integer quadratic programming problem formulation. The proposed deterministic problem model translates a satellite collection scheduling decision process leading to map a set of intelligence requests to a set of collection asset imaging opportunities to maximize collection value (value of information), subject to a variety of side constraints. Targeted to space-based ISR application domains, the new convex optimization problem model demonstrates computational complexity reduction with respect to commonly available state-of-the-art models, making possible the utilization of exact problem-solving methods in some cases, while providing a bound on the optimal solution. On top of traditional feature constraints largely reported in open literature, the promoted model introduces additional specifications, such as suitable task coverage thresholds, optional task mutual exclusion, task precedence, joint value task composition, imaging/service time windows, and, thermal constraints over individual and average orbits. The connection with ongoing decision models, potential problem-solving techniques and future model refinement are then briefly discussed. This work has been conducted under the DRDC JFD 4 Joint Intelligence Collection and Analysis Capability (JICAC) project between December 2015 and June 2016.

The report is outlined as follows. Chapter 2 briefly summarizes the targeted Multi-Satellite Collection Scheduling problem at hand, depicting its main novelties. A mixed-integer quadratic programming model formulation is then introduced in Chapter 3. It presents key data structure representations and the corresponding mathematical model. Chapter 4 discusses prospective methods and algorithms to solve the problem as well as future problem complexity refinements naturally envisioned. Finally, a conclusion is given in Chapter 5, briefly summarizing the main findings and the proposed way ahead.

2 Multi-Satellite Collection Scheduling

2.1 Background

A brief introduction to the satellite scheduling problem and its premise is described in [2]–[5]. Related work includes contributions pioneered by Lemaître et al. on the satellite planning/scheduling problem. An early survey was published in [6]–[7]. Based upon a low density high-demand collection asset premise, the general problem is known to be computationally hard, requiring exponential resources or run-time. For convenience purposes, early work mainly focused either on a single sensor satellite or small and homogeneous constellation problem settings. In most reported cases, it predominantly deals with simple point (spot) target observation tasks, and relies on new task clustering and preprocessing strategies to mitigate computational complexity. A more contemporary comprehensive technical survey on satellite image acquisition scheduling may be found in [8]–[11] and [4], respectively. Alternate work relates to downlink scheduling [12]–[14] and, joint observation and downlink scheduling optimization using combined [15]–[16] and decomposition [17] schemes coupled to approximate methods. Satellite scheduling contributions for some dynamic settings are also reported in [18]–[24]. Accordingly, task demand, priority, dominance and density, and/or mixed ground/on-board scheduling strategies are exploited to manage uncertainty and resolve conflicts in adjusting a flexible precomputed solution.

Open-loop satellite planning/scheduling has traditionally been connected to knapsack, time-constrained longest path, coverage path planning, constraint satisfaction, graph-based and mathematical programming problem formulations respectively, as reported in [4], [11]. Coverage path planning [25] is naturally well-suited for satellite surveying tasks. But most research contributions largely assume perfect coverage and sensing capability (no false alarms), as well as unbounded resources. A recent survey on coverage literature is given in [25]. Vasquez et al. [26]–[27] and Wolfe et al. [28] formulated satellite scheduling as the 0-1 knapsack problem under limited constraints. Original constraint satisfaction problem formulations and variants have been alternatively proposed by Lemaître et al. [29] and Verfaillie et al. [30]. Graph-based approaches using directed acyclic graph model [31]–[32], graph coloring [33]–[34], multi-objective frameworks [8], [21], [16], [35] have also been exploited to handle satellite scheduling. Finally, many approaches relying on a mathematical programming framework have been developed for satellite scheduling, first ignoring key memory and energy capacity constraints, as stated by Benoist et al. [36], Habet et al. [37]–[39] and Lemaître et al. [7]. Building from the original work, Liao et al. [40]–[41], Lin et al. [42]–[45] and Marinelli et al. [14] extended problem modelling introducing time-indexed integer programming formulations. Alternate integer programming models exploiting network flow [46]–[49], and longest path problem with time windows [50] formulations have also been suggested. A variety of related problem-solving approaches and variants ranging from operations research, artificial and computational intelligence techniques to heuristic methods have been developed and or adapted. Most popular metaheuristic methods include tabu search, genetic algorithms and ant colony, local search, hill-climbing and simulated annealing. Greedy algorithms, iterative and constructive solution methods and other multi-stage priority and conflict-avoidance -based heuristics have concurrently been developed. In counterpart, advocated exact methods include mathematical programming techniques, and branch and bound algorithms. However, most reported research contributions mainly focus on homogeneous satellites and single constellation settings, deal predominantly with point target tasks, and design new task clustering and preprocessing strategies to mitigate computational complexity. By and large, they mostly ignore large area coverage, complex task structure, joint value task composition, minimal task coverage thresholds, optional task mutual exclusion, task precedence, imaging/service time windows and satellite thermal and duty cycle constraints. Thus far, the proposed attempts to capture a multiple-visit imaging task (e.g., large areas of interest) have mainly introduced nonlinear modelling intricacies, leading to serious computational and solution quality limitations.

2.2 Problem

Based on the problem statement introduced in [1], a deterministic decision model is proposed for an open-loop, centralized, deterministic, multi-satellite Earth Observation intelligence collection tasking (image acquisition) problem. In that problem setting, a predetermined mission goal/task decomposition structure along with related tasks is assumed to be known in advance. Specified by the collection manager, the structure reflects information needs translating intelligence request breakdown derived from commander's critical information requirements. Hierarchical value apportionment over the goal/task structure from the root to the leaf nodes are assumed to be known as well. Accordingly, as displayed in Figure 1, it proposes a weighted task decomposition structure combining composite or abstract requests capturing new spatial and temporal dependencies, reflecting more realistic mission complexity. In Figure 1, v_k is the value associated to goal/task node *k*. The notion of value is made more explicit in Chapter 3.

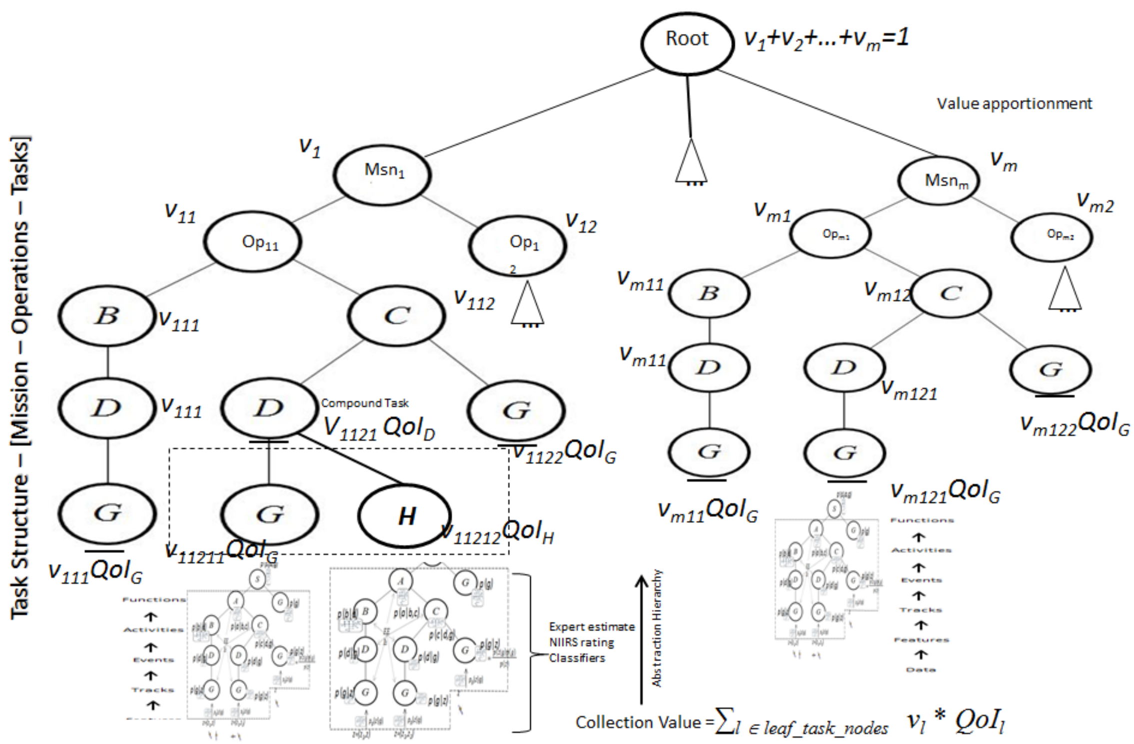

Figure 1: *Mission/task decomposition tree structure. Goals are assigned values hierarchically from the root to the leaves, distributing parent node values over child nodes (value apportionment). The problem objective is to allocate collection assets to maximize collection value translating serviced intelligence requests over all leaf nodes. Collection value of information is expressed in terms of nominal task value and expected quality of information/collection to be collected.*

The open-loop collection problem focuses on low-density high demand assets and involves a single episode. Focusing on image acquisition scheduling, multi-satellite collection tasking can be stated as follows: given a set of information requests (areas of interest to be observed) properly translated in weighted tasks, a set of heterogeneous collection assets, supporting resources such as ground stations, a collection tasking objective, a set of constraints defined over mission, task, operational, collector, supporting resource, communication, capacity, temporal and cost, the problem consists in allocating collection assets owned by single/multiple stakeholders to imaging observation task opportunities, over a predetermined time horizon to optimize single or multiple objectives. A typical objective consists of maximizing the value of information (collection value).

Figure 2 pictures possible areas of interest to be observed over the course of a satellite pass for different beam resolution modes. Imaging task opportunities refer to any possible observation actions of an area of interest within the field of view of an orbiting satellite sensor, along with various attributes (e.g., task identifier, imaging time interval: start/end time, duration, resolution, satellite identifier and related orbits). Opportunity construction for a given task imaging resolution requirement or beam width, is illustrated by the generation of parallel 'strips' as shown in Figure 3. Accordingly and based on sensor's field of view, basic strips with a corresponding beam width are first sequentially generated along the satellite ground track trajectory to meet geographical area of interest (AOI) boundaries. Basic strips are then subdivided to naturally synchronize intersecting time intervals, creating additional feasible imaging opportunities enriching scheduling flexibility. Figure 4 shows possible strip patterns generated for three satellite passes over different AOIs.

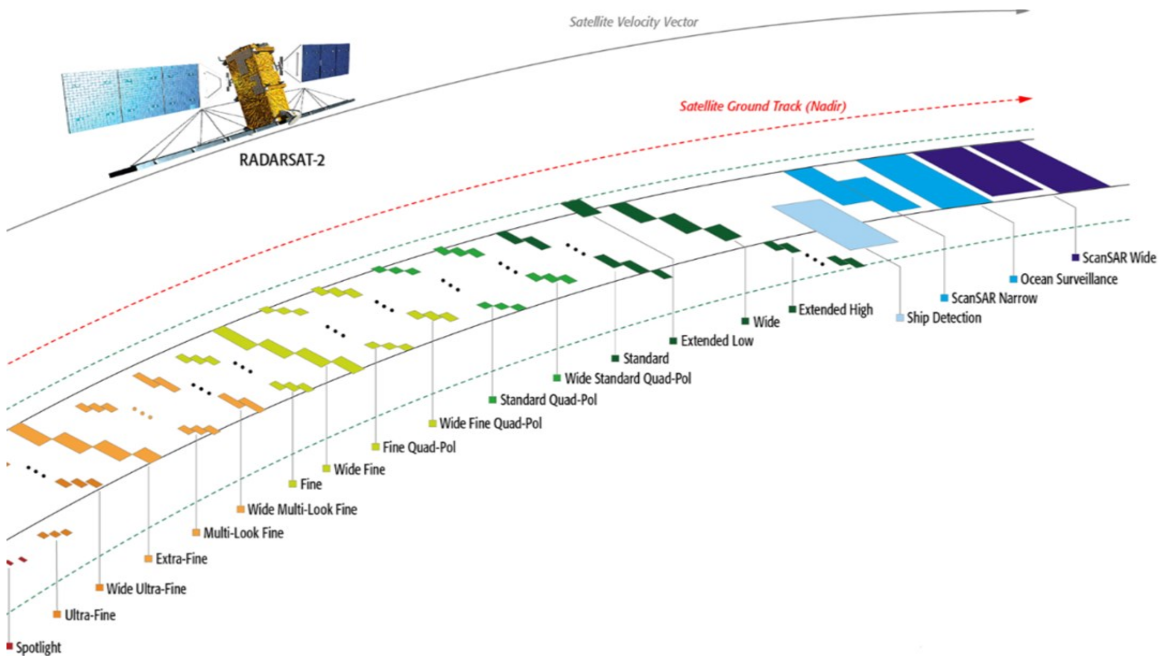

Figure 2: *Source: RADARSAT-2—Courtesy of the Canadian Space Agency [51].*

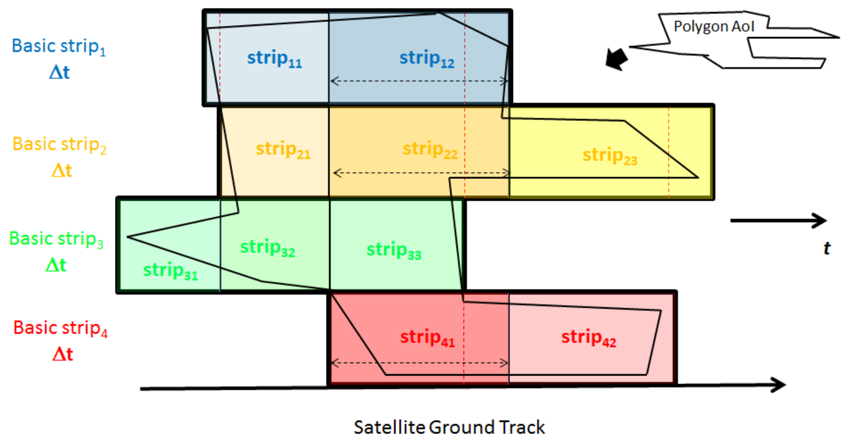

***Figure 3**: Large area (polygon defining AOI) coverage decomposition in strips, naturally defined along a satellite ground track trajectory.*

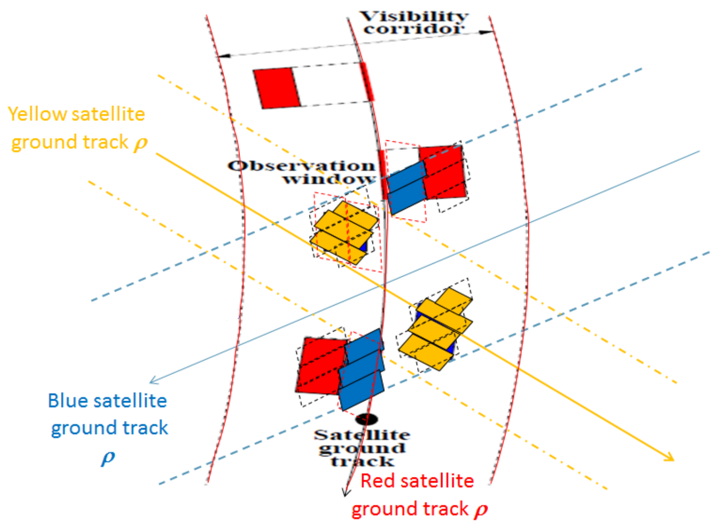

***Figure 4:** Task area decomposition into disjoint parallel strips for a given sensor resolution requirement is generally constellation/satellite or even possibly orbit-dependent, defining possible overlapping areas for multiple satellite constellations. Heterogeneous satellite composition further exacerbates the variability of task area decomposition patterns.*

Task area decomposition is opportunity-driven which is satellite and orbit-dependent. This contrasts with the limited single "fixed-strip" pattern generated for an arbitrary reference satellite trajectory, traditionally advocated in open literature. The latter usually turns out particularly convenient in the case of a unique constellation composed of homogeneous satellites since satellite members follow similar trajectories, observing near similar areas when sufficiently close to one another. But, imposing a "fixed-strip" pattern approach may fall apart, unnecessarily restricting or overlooking real alternate imaging opportunities. Such a static area decomposition imposed to multiple and heterogeneous or virtual constellations involving various satellite geometries, as illustrated in Figure 4, would make on-board maneuver costly or unrealistic, and opportunity coverage calculations very complex. But, these considerations still remain largely ignored in past reported work. In contrast, the current setting promotes generalization, making opportunity generation satellite-driven rather than constellation-driven.

Problem description may be briefly summarized as follows:

Input: Requests/orders, goal/task structure, satellite collection assets, time horizon (number of orbits/passes/revolutions).

Goal: Allocate resources to maximize collection value utility function.

Task: Preprocessing: Observation request area subdivision in strips.

Specifications: Target (spot) /area (polygon); area definition along azimuthal and elevation coordinates.

Requirements including National Image Interpretability Rating Scale (NIIRS) rate or resolution (waveform/beam—predefines tasks and satellite combinations/parameters); mono/stereo.

Collection assets: Heterogeneous collection platforms and supporting resource (e.g., ground station communication asset).

Imaging opportunity calculation based on satellite propagation model.

Constraints: Task visibility/precedence/priority, mandatory visits, minimal coverage, mission single and cumulative maximum imaging cost, temporal and resource capacity. Temporal constraints include set-up/service and imaging duty cycle, time windows, conflicting opportunity transition whereas resource capacity constraints comprise energy budget, on-board storage and communication. An imaging observation cannot be interrupted.

Output: collection schedule of image acquisitions/observations.

Despite prior goal/task decomposition knowledge, satellite-task allocation is not necessarily determined in advance. This paves the way to a hybrid/mixed partial planning and scheduling problem approach depending whether prior task assignment is known or not. The above collection tasking problem presents similarity with well-known operations research “longest path” and capacitated “vehicle routing with time windows” (VRPTW) problem variants. In the latter case, the satellite scheduling problem is reduced to a constrained VRPTW. Satellites represent vehicles having limited visibility on tasks associated with customers, imaging observations are mapped with servicing customers, and orbits correspond to periodic vehicle tours or routes, subject to a variety of task and resource capacity constraints (e.g., energy budget, on-board memory storage, communication bandwidth).

2.3 Novelty

The proposed hybrid decision model combines a classical coarse-grained model component generally designed for complex tasks such as large area coverage, in which a limited number of task plans must be precomputed, to a fine-grained model component relaxing the need for any predetermined task plans, focusing on non-complex tasks and further expanding search space exploration for better solutions. The fine-grained component explicitly embeds task area coverage modelling, rather than relying on complex area precomputations. The hybrid approach generalizes point target tasks, introduces composite request abstracting primitive request definition and combine multiple related tasks whenever required. It further extends the basic concept of homogeneous trailing satellite constellation to any heterogeneous satellite constellation configuration to properly and efficiently handle area task coverage, coupling disjoint and overlapping strip task decomposition schemes in a single model.

Largely limited to single constellation and homogeneous satellite collection scheduling problems for convenience purposes, currently known model formulations from the open literature fail to explicitly capture coverage contributions involving multiple intersecting areas, while being computationally prohibitive to generate provably optimal solutions. As a result, problem-solving has primarily been restricted to the use of heuristic methods and metaheuristic techniques. In contrast to exclusively develop approximate problem-solving techniques to an exact hard problem model, the development of an approximate model using exact techniques can be alternately contemplated as well. Featuring heterogeneous satellites and constellations, the proposed integer programming optimization model includes a composite coverage approximation function combining arbitrarily overlapping imaging opportunity/strip areas emerging from various sensor geometry. Accordingly, the proposed reformulation permits to capture any overlapping imaging opportunity strip area configurations defining a target/task as pictured in Figure 5–6 for multiple concurring satellite opportunities, rather than exclusively relying on disjoint contiguous imaging opportunity areas target/task decomposition requirements as shown in Figure 3 which negatively impact solution quality by deliberately overlooking alternate potential overlapping opportunities. In its simplest form, area coverage second-order approximation consists in subtracting the sum of opportunity areas covered from the sum of mutually intersecting areas over all covered opportunity area pair combinations. This approach tends to naturally penalize solutions having significant area intersections among selected opportunities, alternately encouraging opportunity composition presenting minimal overall intersecting areas in covering the targeted area of interest.

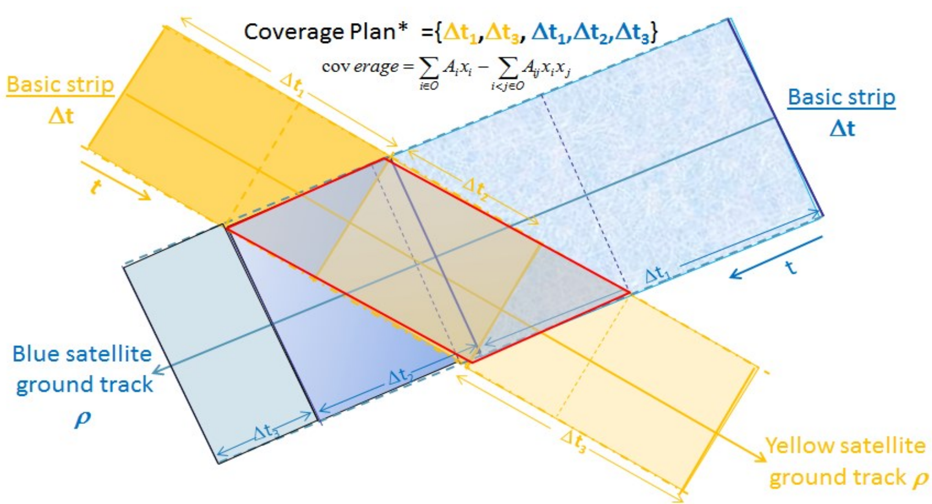

Figure 5: *Overlapping imaging opportunities (strips) for a given resolution for two mutually crossing satellite ground tracks moving downward. A satellite pass is assumed to show three opportunities. A 5-opportunity coverage plan {Δt_1, Δt_3, Δt_1, Δt_2, Δt_3} is illustrated, minimizing intersecting areas.*

A third-order approximation would further add area contributions from all intersecting opportunity triplet combinations to improve overall coverage precision. Based on a reasonable assumption over typical solution quality expectations, which consists in focusing on low order coverage contributions to minimize desirable area intersections as much as possible, the advocated coverage approximation is computationally convenient while significantly decreasing the number of decision variables otherwise required. The approach could further be adapted to restrict consecutive collection asset observations to the most likely feasible opportunity transitions whenever required resulting in prospective computational gains. Unsuitable transitions involving distant task opportunities or incurring prohibitive delays are simply disregarded in advance.

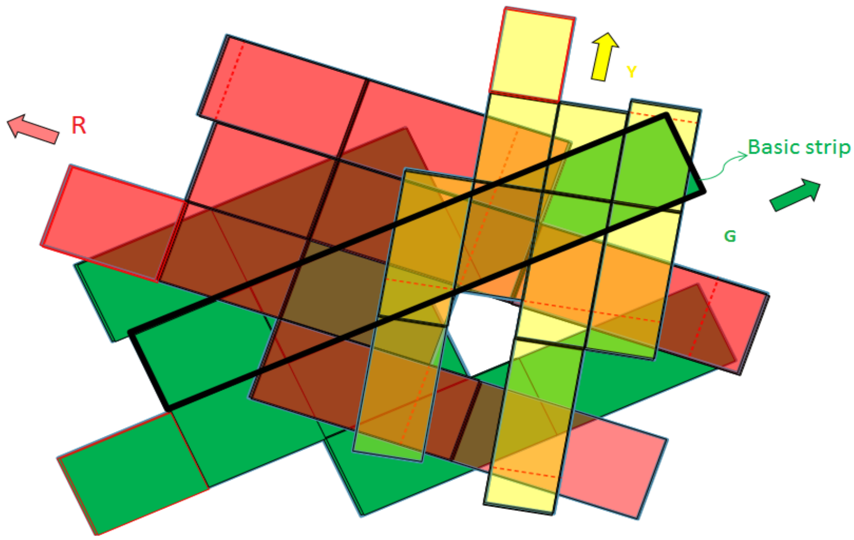

Figure 6: *Overlapping imaging opportunities/strips for a given sensor resolution resulting from three different concurring satellite passes (left/R, right/G, up/Y), reflected through different tones, over a large area of interest.*

The proposed mixed-integer quadratic collection scheduling (tasking) network flow problem reformulation extends classical models while reducing combinatorial complexity. The model presents suitable objective function and constraint convexity properties (see Chapter 3), making well-known powerful optimization technology at our disposal to optimally solve large problems or compute optimality gap. The approach assumes a predefined mission decomposition scheme reflecting a hierarchical goal structure. At each level, a nominal value reflecting priority on information need is attached to a goal node. Nominal value apportionment is achieved from the top level goal down to the task level. Lower level goals correspond to collection tasks. Multiple objectives are currently represented via a single main objective along with subordinate objectives alternatively described as constraints. The decision model introduces the notion of quality of information/service coupled to partial task completion/satisfaction, heterogeneous satellites from multiple constellations, overlapping imaging opportunities. It includes primitive and compound tasks while incorporating multiple task coverage requirement. The network flow formulation alternately makes use of a collection graph to conveniently capture feasible opportunity transition and facilitate constraint expressivity, implicitly encoding conflicting opportunity or certain time constraints. The proposed decision model presents a variety of new constraints, including task precedence or mutual exclusion; opportunity transition including conflicting opportunity over consecutive orbits which improperly remain largely ignored in literature; satellite utilization/imagery budget or cost, resource, as well as orbital and average satellite duty cycle. Moreover, in cases where opportunity transition time impacting energy budget constraints is transition-independent (e.g., satellite-dependent constant as for synthetic aperture radar satellites) computational gains are further expected due to a reduction in the number of decision variables required. The underlying approach also questions and revisits common task priority handling practice, often arbitrarily imposing task partial ordering to schedule orders, regardless of alternate factors such as task opportunity contention reflecting the number of opportunities that can possibly service a task. The basic principle of priority is meant to be a conflict resolution or a tie-breaking mechanism, and not a discrimination policy being improperly used as a precedence constraint to partially order task or to preempt task when unnecessary. Finally, contrarily to most state-of-the-art reported work, the new problem model formulation provides a solution quality /collection value upper bound which can be rapidly computed from the relaxed problem decision model (decision variable integrity relaxation), presenting an objective measure to compute optimality gap to properly compare relative performance from alternate techniques.

3 Mixed-Integer Quadratic Programming Model Formulation

A mixed-integer quadratically-constrained problem formulation for the deterministic open-loop decision problem described in Section 2 is proposed. It revisits standard nonlinear collection tasking problem modelling resulting to a mixed-integer quadratic program (MIQP). The MIQP model exploits an acceptable second or third-order approximation to estimate coverage calculations while reducing undesirable area opportunity intersections or overlap leaning towards unnecessarily increasing redundancies and cost. The formulation relies on a directed acyclic graph representation to depict possible imaging observation moves for each satellite revolution and, a mathematical model to explicitly capture decision problem objective and constraints. The graph representation proves particularly convenient to easily specify problem constraints in the decision model.

3.1 Collection Graph Representation

A satellite network representation pictured under the form of a directed acyclic graph is exploited to capture satellite observation moves/collection opportunities over a given orbit as illustrated in Figure 7. The collection opportunity network proves very convenient to significantly simplify constraint modelling during problem model construction. It implicitly reflects conflicting intra-orbit opportunity transition constraints. Such constraints comprise orbit duration or itinerary path length, legal moves, and single satellite orbit/path solution. Multiple disconnected subtours over a given orbit are prohibited. Let H be the time horizon, SAT be a set of heterogeneous earth observation satellites $sat = 1,2,\ldots,|SAT|$, and ΔT_{sat} be the satellite sat revolution period. Orbit $\rho(t, sat) \in P$ is satellite (sat) and cycle (t) -dependent where $sat \in SAT$ and $t \in \{1,2,..., H/\Delta T_{sat}\}$. Let R be the set of requests r with nominal value V_{r0} and, R_ρ the set of complete/partial feasible requests (tasks) in corridor visibility of orbit ρ ($R_\rho \subseteq R$) matching sensor orbit ρ capability, S_r the set of task s with nominal value V_{rs0} composing request r ($V_{r0} = \sum_s V_{rs0}$), O_ρ the set of image acquisition opportunities o over orbit ρ, and $O_{rs\rho}$ the set of opportunities associated to request r and task s during orbit ρ ($O_\rho = \cup_{rs} O_{rs\rho}$) respectively. Opportunities are generated using a special purpose software calculating feasible imaging options based on satellite kinematics, on-board sensor characteristics and geometry. The basic directed acyclic graph description can then be summarized as follows. Let $\mathcal{G}_\rho=(\mathcal{V}_\rho, \mathcal{A}_\rho)$ be a directed acyclic graph coupled to satellite orbit $\rho \in P=\{1,...,|P|\}$ where $|P| = |SAT| * H/\Delta T_{sat}$. $\mathcal{V}_\rho$ denotes a set of vertices reflecting satellite opportunities $\{o(r,s)\}$ in orbit ρ ($V_\rho=O_\rho$). Alternatively, $\mathcal{A}_\rho$ designates the set of arcs (o,o') connecting $o(r,s)$ to $o'(r',s')$, encoding feasible opportunity transition in orbit ρ where $o, o' \in \mathcal{V}_\rho$. An arc may alternately reflect a specific collector decision regarding the next imaging move to execute. A partial view of a satellite orbit network is exhibited in Figure 8.

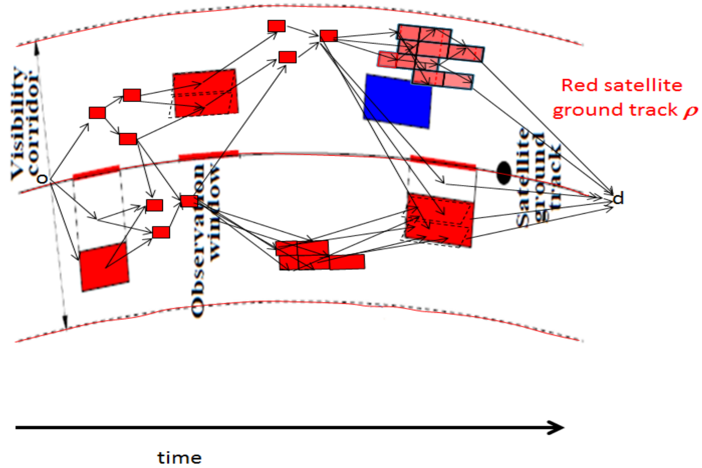

Figure 7: *Ground track projection of a satellite moving to the right over a given orbit. A network representation is used to capture possible observations or collection opportunities. A node reflects an imaging opportunity o whereas an arc connecting two nodes translates timely feasible, legal transition moves between two consecutive opportunities o and o'. For clarity purposes, only a subset of feasible transitions has been shown.*

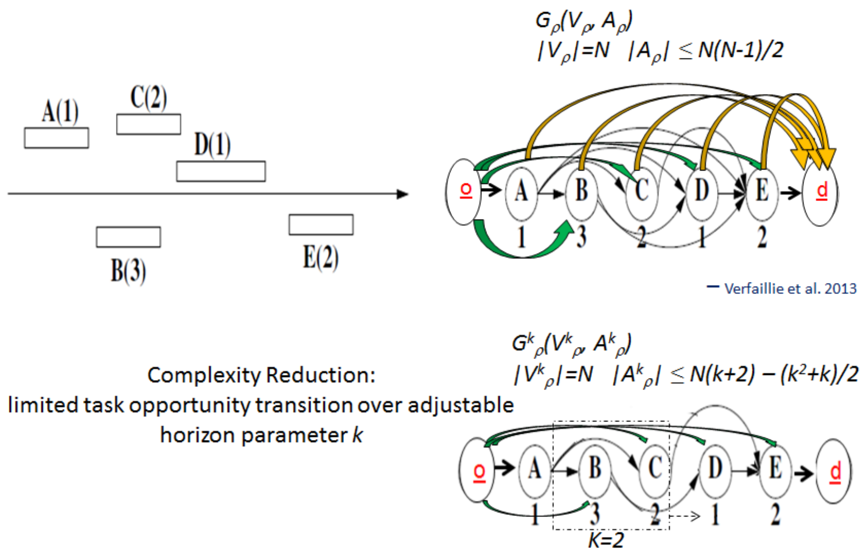

Figure 8: *Directed acyclic graph illustrating opportunities and feasible transitions for a given satellite orbit. The top network diagram shows opportunities A-E (N=5) and all possible legal transitions. Fictitious origin (o) and destination (d) location nodes are artificially introduced to conveniently define legitimate paths in the graph. The network shown at the bottom is a simplified subnetwork keeping most likely possible transition moves in order to reduce computational complexity.*

An integer binary decision variable $x_{rso\rho}$ related to node visit $o(r,s) \in \mathbb{V}_\rho$ defines a basic satellite path's construct. Accordingly, path solution for satellite orbit ρ includes vertex $o(r,s)$ when $x_{rso\rho} = 1$. These decision variables are coupled to continuous 'resource' decision variable $ts_{rso\rho}$ referring to the start time of imaging task (r,s) over opportunity o and orbit ρ. Flow decision variables $u_{rsor's'o'\rho}$ characterizing opportunity transition from $o(r,s)$ to $o'(r',s')$ are however required to represent variable resource consumption constraints (e.g., energy budget). The variable assignment $u_{rsor's'o'\rho}=1$ indicates a transition between task s from request r over imaging opportunity o and task s' from request r' over imaging opportunity o' during orbit ρ where $(o(r,s), o'(r',s')) \in \mathcal{A}_\rho$. Therefore, a feasible satellite path may be built traveling along arcs connecting o to d nodes, instantiating flow decision variables. Constraints on resource variables $ts_{rso\rho}$ prevent unsuitable solution generation composed of disjoint subtours to occur during path construction. The resulting network structure depicting a directed acyclic graph may then be used to construct a legal satellite path solution for a given orbit through a temporal sequence of binary integer flow decision variable instantiations. In order to reduce computational complexity, the original acyclic graph may be further revisited, restricting consecutive satellite observations to most likely opportunity transitions. Accordingly, opportunity transitions involving unacceptable delays are deliberately ignored. This is exemplified in the bottom diagram of Figure 8. Preliminary network/graph generation and exploitation allows simplifying decision model construction substantially.

3.2 Collection Tasking Mathematical Model

The proposed mathematical collection tasking decision model is a convenient quadratic approximation of the decision model and the nonlinear objective function commonly used in the literature [4]. It relaxes a non-convex problem model to a convex formulation, approximating collection value. The proposed convexity property emerges from the semi-definite positive matrix used to describe the quadratic formulation. That property guarantees a local optimal solution to be a global optimum. The basic idea exploits the fact that total coverage resulting from n swaths may be expressed as a sum of contributions involving all possible i^{th}–order ($i = 1..n$) overlap combinations, corresponding to i swaths intersection terms. Accordingly, a polynomial approximation implicitly constraining location coverage to a limited number of visits to minimize overlap contributions turns out to be sufficient and quite acceptable in practice to efficiently estimate quality of collection (area coverage) and guide solution space exploration for small area targets. As a result, a second order decomposition scheme may therefore be used to estimate coverage for any feasible candidate collection task plan, significantly reducing combinatorial complexity while relaxing the need for a parallel strip AOI decomposition structure, usually restricted and valid only for traditional trailing satellite constellations.

3.2.1 Parameters and Variables

The parameters and variables used to specify the basic problem model formulation are described as follows:

Parameters:

- H: time horizon
- SAT: set of heterogeneous earth observation satellites. A satellite may be agile or non-agile. The former refers to a platform having multiple degrees of freedom in motion and control whereas the latter denotes a satellite that can simply maneuver in a single rolling dimension perpendicular to the orbit direction.
- ΔT_{sat}: satellite sat revolution period, $sat \in SAT$
- P: collection of orbits/revolution/track/pass/path $\rho \in \{1,..,|P|\} = |SAT|*|P_{\text{sat}}|$ (number of orbits per satellite). Orbits are sorted in increasing order.

$|P_{\text{sat}}| = \lceil H/\Delta T_{sat} \rceil$
$\rho = \rho\,(t,\ sat)$, cycle $t \in \{1,2,\ldots,\ H/\Delta T_{sat}\}$, $sat \in SAT$
Sensors: electro-optical (EO), infrared (IR) or synthetic aperture radar (SAR)
P can be partitioned in ascending and descending orbits: $P = P_A \cup P_D$

P_{rs}: collection of orbits/revolution/track/pass ρ related to task (r,s). $P_{rs} \subseteq P$

P_{sat}: collection of orbits for satellite *sat*

T_ρ: satellite orbit ρ revolution period

R: set of candidate requests r. A request r defines either a point target (spot) or a polygon area (region) A_r to be covered with beam (waveform) B. Each request r is decomposed into a set S_r of tasks. A point target generally corresponds to a primitive request comprising a single task whereas a compound request includes a set of tasks s.

A_r: area of interest (AOI) of request r

A_{rs}: area of interest of strip task s under request r, associated with area of interest A_r

R_ρ: set of complete/partial requests in orbit ρ corridor (track) visibility ($R_\rho \subseteq R$) and matching sensor orbit ρ capability (feasible pairing—matching). Partially visible requests are included.

R_π: set of generally complex requests coupled with optional predefined (user-defined) collection plans to select from

R_{stereo}: set of imaging requests demanding two nearly concurrent observations from different perspectives (stereo). (Note that sophisticated stereo requests presenting complex observation conditions rather impose $r \in R_\pi$)

$\Delta T_{stereo,r}$: acceptable delay between observations to satisfy a stereo request r

S_r set of task s composing request r (e.g., subareas/regions/strips composing an AOI)

$[\,\underline{\omega}_r,\ \overline{\omega}_r\,]$ completion time window for servicing (imaging) request r and related tasks

$VIS_{rs\rho}$: binary visibility matrix indicating if task s under request r is within sensor footprint/field of view (projected sat track area) over orbit ρ. $vis_{rs\rho} = 1$ if $r \in R_\rho$ and task s is visible to orbit ρ, assuming on-board sensor successfully matches task requirements, otherwise $vis_{rs\rho} = 0$.

$PREC_{rsr's'}$: binary matrix mapping partially ordered task pairs $(r,s;\ r',s')$, defining a precedence relationship. $PREC_{rsr's'}=1$ imposes (r,s) to precede(r',s').

$ME_{\ rsr's'}$: binary matrix mapping mutually exclusive task pairs (r,s) and (r',s'), imposing to service at most one conflicting task

V_{r0}: nominal value of request r ranging over [0,1]

V_{rs0}: nominal value of request-task (r,s). $\sum_{s \in S_r} V_{rs0} = V_{r0}$. For a coverage request, $V_{rs0} = V_{r0}\frac{A_{rs}}{A_r}$

$prop_{rs\ THR}$: minimal relative task (r,s) area coverage threshold (proportion) requirement

$\theta_{rso\rho}$: pointing/look angle for imaging task (r,s) opportunity o on orbit ρ

$ts_{rso\rho}$: imaging task (r,s) opportunity o start time (s) on orbit ρ

$te_{rso\rho}$: imaging task (r,s) opportunity o end time (s) on orbit ρ

$o_{rs\rho}$: imaging/collection task (r,s) opportunity o on orbit ρ. It is defined as follows:
<identifier o.ID, request r, task s, beam mode B, look/incidence angle $\theta_{rso\rho}$,
time interval $[ts_{rso\rho},\ te_{rso\rho}]$, satellite direction (Ascending/Descending) *Dir*,
orbit ρ> = < o.ID, r, s, B, $\theta_{rso\rho}$, $[ts_{rso\rho},\ te_{rso\rho}]$, *Dir*, ρ>

O: set of all collection opportunities ($\cup_{\text{rs}} O_{rs}$ or $\cup_\rho O_\rho$)

O_{rs}: set of collection opportunities (over all possible orbits) for task (r,s)

O_ρ: set of collection opportunities (over all tasks) during orbit ρ.

$O_{rs\rho}$: set of collection opportunities for request r task s during orbit ρ

$d_{rso\rho}$: imaging task (r,s) opportunity o duration (dwell time) ($te_{rso\rho}$ - $ts_{rso\rho}$) over orbit ρ

τ_ρ: orbital imaging duty cycle period—maximal (absolute) cumulative imaging time (s) deemed acceptable over orbit ρ imposed by thermal capacity constraints

$\bar{\tau}_{sat}$: maximal average imaging time (s) over orbit ρ imposed by thermal capacity constraints

W_ρ: memory storage capacity (memory units) in orbit ρ

w_ρ: memory consumption rate (memory units/s) by an observation in orbit ρ

E_ρ: energy capacity (Joule) in orbit ρ

eo_ρ: energy consumption rate (Joule/s) by an observation in orbit ρ

es_ρ: energy consumption rate (Joule/s) for task transition by a sensor in orbit ρ

$\Delta t_{0\rho}$: set-up time (s) for imaging opportunity transition

$\Delta t_{rsor's'o'\rho}$: transition time (s) from task (r,s) collection opportunity o to task (r',s') collection opportunity o', over orbit ρ posterior to the set-up phase.
(e.g., $\Delta t_{rsor's'o'\rho} = |\theta_{rso\rho} - \theta_{r's'o'\rho}|/sl_\rho$ for an optical satellite, where sl_ρ is the slewing rate related to orbit ρ)

$\Delta t^c_{rsor's'o'\rho_{sat}}$: transition time(s) over consecutive orbits ρ_{sat} and $(\rho+1)_{sat}$ for a same satellite platform sat (e.g., $\Delta t^c_{rsor's'o'\rho_{sat}} = \left|\theta_{rso\rho_{sat}} - \theta_{r's'o'(\rho+1)_{sat}}\right| \big/ sl_{\rho_{sat}}$ for an optical satellite)

$CONFL_{oo'\rho_{sat}}$: binary matrix mapping time-conflicting opportunity pairs over consecutive orbits ρ_{sat} and $(\rho+1)_{sat}$ for a satellite sat where $o \in O_{\rho sat}$ and $o' \in O_{(\rho+1)sat}$.
$CONFL_{o(r,s,\rho_{sat})\, o'(r',s',(\rho+1)_{sat})\rho_{sat}} = 1$ indicates mutually exclusive imaging opportunities

if $$ts_{r's'o'(\rho+1)_{sat}} < te_{rso\rho_{sat}} + \left(\Delta t_{0\rho_{sat}} + \Delta t^c_{rsor's'o'\rho_{sat}}\right) \quad \forall r,r' \in R, s \in S_r, s' \in S_{r'}, \rho_{sat}, (\rho+1)_{sat} \in \mathrm{P}_{sat}, sat \in SAT, (o,o') \in CONFL$$

Π_{rs}: set of user/pre -defined (sub) task plans $\{\pi_{rs\varphi}\}$ that can cover (sub) task (r,s) ($r \in R_\pi$, $\varphi \in \Phi_{rs} = \{0,1,2,\ldots,|\Pi_{rs}|\}$), $\Pi_{rs} = \{\varnothing, \pi_{rs1}, \pi_{rs2}, \ldots, \pi_{rs\varphi}, \ldots, \pi_{rs|\Pi rs|}\} = \{\pi_{rs\varphi}\}$, where $\varnothing$ represents the null plan.

$\pi_{rs\varphi}$: φ^{th} (sub) task plan defining a subset of collection imaging opportunities selected from a given set O_{rs} to achieve/cover task (r,s) ($r \in R_\pi$) - $\pi_{rs\varphi} = \{o^\pi_{rs\varphi 1}, o^\pi_{rs\varphi 2}, \ldots, o^\pi_{rs\varphi|\pi\_rs\varphi|}\}$

$A_{rso\rho}$: area coverage, overlapping area of interest A_{rs} defining task s under request r, associated with opportunity o on orbit ρ

$A^{strip}_{rso\rho}$: imaging strip area (swath) coverage associated with opportunity o on orbit ρ for task s from request r ($A^{strip}_{rso\rho\ \rho} \geq A_{rso\rho}$)

$A_{rso\rho o'\rho'}$: overlapping area between opportunity o, orbit ρ, and, opportunity o', orbit ρ', over the area of interest A_{rs} associated with task s under request r

$cost_{rso\rho}$: imaging opportunity o cost associated with task s, request r during orbit ρ

$cost_{rs\ max}$: imaging budget available for request r, task s

$cost_{max}$: overall imaging budget available

Decision variables:

v_r: continuous variable over [0,1] capturing the collection value of request r

$visit_{rs}$: binary variable indicating if task s under request r has been visited over the time horizon. $visit_{rs} \in \{0,1\}$

$w_{rs\varphi}$: binary variable referring to the selection of plan φ among possible plans in the set Π_{rs} to cover task (r,s).

$x_{rso\rho}$: binary variable indicating whether task s under request r is scheduled to be serviced by opportunity o on orbit ρ. $x_{rso\rho} \leq vis_{rs\rho}$

$z_{rso\rho o'\rho'}$: binary variable indicating whether task s under request r is scheduled to be serviced by opportunity o on orbit ρ and, opportunity o' on orbit ρ'

$u_{rsor's'o'\rho}$: orbit ρ network flow binary decision variable, indicating a transition between imaging task s under request r opportunity o and, imaging task s' under request r' opportunity o', in orbit ρ (i.e., (r,s) imaging precedes (r',s') image acquisition)

The decision model is given as follows:

$$\max \quad \sum_{r \in R} v_r \tag{1}$$

Subject to the constraint set:

Complex task (coarse-grained model):

$$v_r \leq \sum_{s \in S_r} \sum_{\varphi \in \Phi_{rs}} V_{rs0} w_{rs\varphi} \qquad r \in R_\pi \tag{2}$$

$$w_{rs\varphi} \leq \frac{1}{\|\pi_{rs\varphi}\|} \sum_{\rho \in \mathrm{P}_{rs}} \sum_{o \in O_{rs\rho} \cap \pi_{rs\varphi}} x_{rso\rho} \quad r \in R_\pi, s \in S_r, \varphi \in \Phi_{rs} = \{1,2,...,|\Pi_{rs}|\}, \pi_{rs\varphi} \in \Pi_{rs} \tag{3}$$

$$\sum_{\varphi \in \Phi_{rs}} w_{rs\varphi} \leq 1 \quad r \in R_\pi, s \in S_r \tag{4}$$

Non-complex task (fine-grained model):

$$v_r \le \sum_{s\in S_r} V_{rs0}\left(\sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{rs\rho}} C_{rso\rho}x_{rso\rho} - \sum_{\rho\in \mathrm{P}_{rs}}\sum_{\substack{o\in O_{rs\rho}\\(\rho,o)<(\rho',o')}}\sum_{\rho'\in \mathrm{P}_{rs}}\sum_{o'\in O_{rs\rho'}} D_{rso\rho o'\rho'}\left[\frac{\left(x_{rso\rho}+x_{rso'\rho'}\right)^2-\left(x_{rso\rho}+x_{rso'\rho'}\right)}{2}\right]\right)$$
$$r\in R \mid R_\pi \tag{5}$$

$$C_{rso\rho}=\frac{A_{rso\rho}}{A_{rs}},\qquad D_{rso\rho o'\rho'}=\frac{A_{rso\rho o'\rho'}}{A_{rs}}$$

Minimal coverage:

$$\frac{1}{\sum_{\rho\in \mathrm{P}_{rs}}\left|O_{rs\rho}\right|}\sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{rs\rho}} x_{rso\rho} \le visit_{rs} \quad r\in R \mid R_\pi, s\in S_r \tag{6}$$

$$\sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{rs\rho}}\frac{A_{rso\rho}}{A_{rs}}x_{rso\rho} - \sum_{\rho\in \mathrm{P}_{rs}}\sum_{\substack{o\in O_{rs\rho}\\(\rho,o)<(\rho',o')}}\sum_{\rho'\in \mathrm{P}_{rs}}\sum_{o'\in O_{rs\rho'}}\frac{A_{rso\rho o'\rho'}}{A_{rs}}\left[\frac{\left(x_{rso\rho}+x_{rso'\rho'}\right)^2-\left(x_{rso\rho}+x_{rso'\rho'}\right)}{2}\right]$$
$$\ge prop_{rs\_THR}visit_{rs} \quad r\in R \mid R_\pi, s\in S_r \tag{7}$$

Task Precedence:

$$PREC_{rsr's'}\left(\frac{ts_{rso\rho}+d_{rso\rho}}{ts_{r's'o'\rho'}}\right)\left(x_{rso\rho}+x_{r's'o'\rho'}-1\right)\le x_{r's'o'\rho'} \quad r,r'\in R,\rho,\rho'\in \mathrm{P},$$
$$o\in O_{rs\rho}, o'\in O_{r's'\rho'} \tag{8}$$

$$PREC_{rsr's'}\ x_{r's'o'\rho'} \le \sum_{\rho\in \mathrm{P},o\in O_{rs\rho}} x_{rso\rho} \qquad r,r'\in R,\rho'\in \mathrm{P}, s\in S_r, s'\in S_{r'}, o'\in O_{r's'\rho'} \tag{9}$$

Mutual Task Exclusion:

$$ME_{rsr's'}x_{r's'o'\rho'} \leq 1 - \frac{1}{|O_{rs}|}\sum_{\rho\in \mathrm{P}}\sum_{o\in O_{rs\rho}} x_{rso\rho} \qquad r,r'\in R, s\in S_r, s'\in S_{r'}, \rho'\in \mathrm{P}, \quad o'\in O_{r's'\rho'} \tag{10}$$

Orbit task visibility constraints:

$$x_{rso\rho} \leq vis_{rs\rho} \qquad r\in R_\rho, s\in S_r, \rho\in \mathrm{P}, o\in O_{rs\rho} \tag{11}$$

Conflicting opportunities over consecutive orbits for a particular satellite platform:

$$CONFL_{oo'\rho_{sat}}\left(x_{rso\rho_{sat}} + x_{r's'o'(\rho+1)_{sat}}\right) \leq 1 \qquad r,r'\in R, s\in S_r, s'\in S_{r'}, sat\in SAT, \quad \rho_{sat}\in \mathrm{P}, o\in O_{rs\rho_{sat}}, o'\in O_{r's'(\rho+1)_{sat}} \tag{12}$$

Stereo imaging/observations:

$$z_{r\,s=1\,o\rho o'\rho'} \leq x_{r\,s=1\,o\rho} \quad r\in R_{stereo}, \rho, \rho'\in \mathrm{P}_{rs}, o\in O_{r\,s=1\,\rho}, o'\in O_{r\,s=1\,\rho'} \tag{13}$$

$$z_{r\,s=1\,o\rho o'\rho'} \leq x_{r\,s=1\,o'\rho'} \quad r\in R_{stereo}, \rho, \rho'\in \mathrm{P}_{rs}, o\in O_{r\,s=1\,\rho}, o'\in O_{r\,s=1\,\rho'} \tag{14}$$

$$\sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{r\,s=1\,\rho}} x_{r\,s=1\,o\rho} \leq 2 \quad r\in R_{stereo} \tag{15}$$

$$\sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{r\,s=1\,\rho}} x_{r\,s=1\,o\rho} \leq 2 \sum_{\rho\in \mathrm{P}_{rs}}\sum_{o\in O_{r\,s=1\,\rho}}\sum_{\substack{\rho'\in \mathrm{P}_{rs} \\ (\rho,o)<(\rho',o')}}\sum_{o'\in O_{r\,s=1\,\rho'}} z_{r\,s=1\,o\rho o'\rho'} \quad r\in R_{stereo} \tag{16}$$

$$\left|ts_{rs=1o\rho} - ts_{r\,s=1\,o'\rho'}\right|\left(2\left(x_{r\,s=1\,o\rho} + x_{r\,s=1\,o'\rho'}\right) - 3\right) \leq \Delta T_{stereo,r} \qquad r\in R_{stereo}, \rho, \rho'\in \mathrm{P}_{rs}, \quad o\in O_{r\,s=1\,\rho}, o'\in O_{r\,s=1\,\rho'} \tag{17}$$

Energy constraints over orbit ρ:

$$\sum_{r\in R_\rho}\sum_{s\in S_r}\sum_{o\in O_{rs\rho}} eo_\rho\left(te_{rso\rho}-ts_{rso\rho}\right)x_{rso\rho}+$$

$$\sum_{r\in R_\rho\cup\{\underline{o}\}}\ \sum_{\substack{s\in S_r\\ ((rso),(r's'o'))\in A_\rho\\ G_\rho(V_\rho,A_\rho)}}\ \sum_{o\in O_{rs\rho}}\ \sum_{r'\in R_\rho\cup\{\underline{d}\}}\ \sum_{s'\in S_{r'}}\ \sum_{o'\in O_{r's'\rho}} es_\rho\left(\Delta t_{0\rho}+\Delta t_{rsor's'o'\rho}\right)u_{rsor's'o'\rho}\le E_\rho \quad \rho\in \mathrm{P} \tag{18}$$

$$\sum_{r'\in R_\rho}\ \sum_{s'\in S_r}\ \sum_{o'\in O_{r's'\rho'}} u_{rsor's'o'\rho}=x_{rso\rho}\quad r\in R_\rho, s\in S_r, \rho\in \mathrm{P}, o\in O_{rs\rho} \tag{19}$$

Opportunity flow conservation:

$$\sum_{\substack{r\in R_\rho\\ ((rso),(r's'o'))\in \mathcal{A}_\rho,\mathcal{G}_\rho(\mathbb{V}_\rho,\mathcal{A}_\rho)}}\ \sum_{s\in S_r}\ \sum_{o\in O_{rs\rho}} u_{rsor's'o'\rho}-\sum_{\substack{r''\in R_\rho\\ ((r's'o'),(r''s''o''))\in \mathcal{A}_\rho,\mathcal{G}_\rho(\mathbb{V}_\rho,\mathcal{A}_\rho)}}\ \sum_{s''\in S_{r''}}\ \sum_{o''\in O_{r''s''\rho}} u_{r's'o'r''s''o''\rho}=0$$

$$r'\in R_\rho, s'\in S_{r'}, \rho\in \mathrm{P}, o'\in O_{r's'\rho} \tag{20}$$

$$x_{\underline{o}11\rho}=1\quad x_{\underline{d}11\rho}=1\quad \rho\in \mathrm{P} \tag{21}$$

Image acquisition cost:

$$\sum_{\rho\in \mathrm{P}}\ \sum_{r\in R_\rho}\ \sum_{s\in S_r}\ \sum_{o\in O_{rs\rho}} cost_{rso\rho}\ x_{rso\rho}\le cost_{\max} \tag{22}$$

$$\sum_{\rho\in \mathrm{P}}\ \sum_{o\in O_{rs\rho}} cost_{rso\rho}\ x_{rso\rho}\le cost_{rs\max}\quad r\in R, s\in S_r \tag{23}$$

On-board memory storage capacity:

$$\sum_{r\in R_\rho}\sum_{s\in S_r}\sum_{o\in O_{rs\rho}} w_\rho\left(te_{rso\rho}-ts_{rso\rho}\right)x_{rso\rho}\le W_\rho\quad \rho\in \mathrm{P} \tag{24}$$

Imaging duty cycle:

$$\sum_{r\in R_\rho}\sum_{s\in S_r}\sum_{o\in O_{rs\rho}} d_{rso\rho}x_{rso\rho} \le \tau_\rho \qquad \rho\in \mathrm{P} \tag{25}$$

Average imaging duty cycle over multiple consecutive orbits:

$$\begin{aligned}\sum_{t\in\{1,\ldots,t'\}}\sum_{r\in R_\rho}\sum_{s\in S_r}\sum_{o\in O_{rs\rho}} d_{rso\rho_{sat\ t'}}x_{rso\rho_{sat\ t'}} \le t'\bar{\tau}_{sat} \qquad & sat\in SAT, 2\le t'\le \frac{H}{\Delta T_{sat}},\\ & \rho_{sat\ t'}\in \mathrm{P}\end{aligned} \tag{26}$$

Decision variables:

$$\begin{aligned}&u_{rsor's'o'\rho}, x_{rso\rho}, visit_{rs}, \in\{0,1\} \quad \rho\in\mathrm{P}, r, r'\in R_\rho, s\in S_r, o\in O_{rs\rho}, s'\in S_{r'}, o'\in O_{r's'\rho}\\ &z_{rso\rho o'\rho'}\in\{0,1\} \quad r\in R, s\in S_r, \rho,\rho'\in \mathrm{P}_{rs}, o\in O_{rs\rho}, o'\in O_{rs\rho'}\\ &v_r\in[0,1] \quad r\in R\\ &w_{rs\varphi}\in\{0,1\} \quad r\in R_\pi\end{aligned} \tag{27}$$

The quadratically constrained program consists of maximizing collection value of servicing a set of requests composed of tasks, as described in (1) subject to linear constraints (2)–(4), (6), (8)–(27) and quadratic constraints (5) and (7). Inequality (2) captures the collection value of complex tasks given predefined feasible task collection plans to choose from. Imaging opportunity assignments composing such a plan is reinforced by (3), while equation (4) ensures that at most a single plan is selected. In contrast, approximate collection value of a non-complex task is expressed by the quadratic constraint set (5). It accounts for individual imaging opportunity coverage and, all opportunity pair partial coverage overlap combinations. Note that equation (5) could be directly integrated in the objective (1) as well. Minimal approximate relative task coverage requirement is depicted through constraint sets (6)–(7). Note that (5) and (7) can be linearized, substituting the quadratic expression [.] reflecting the product $x_{rso\rho}x_{rso'\rho'}$, by the integer variable $z_{rso\rho o'\rho'}$ along with the constraint sets $z_{rso\rho o'\rho'} \le \min(x_{rso\rho}, x_{rso'\rho'})$ and $z_{rso\rho o'\rho'} \ge x_{rso\rho} + x_{rso'\rho'} - 1$ respectively. This is otherwise implicitly achieved by commercial solvers during a preprocessing phase. Constraint sets (8)–(9) capture task precedence relationships (e.g., detection (*r,s*) must precede identification (*r',s'*)) whereas constraint set (10) ensures mutual task exclusion on some limited, partially ordered tasks. Inequality (11) translates intrinsic request/task visibility over some satellite orbits limiting the number of decision variables. In contrast to acyclic graphs implicitly capturing feasible intra-orbit imaging opportunity transitions, legal opportunity transitions over two consecutive orbits for a given satellite is explicitly handled through constraint set (12). This may occur for a specific satellite connecting the last and the first observations from consecutive orbits respectively. Those constraints are easily generated using consecutive same satellite path collection networks to identify mutually exclusive (time-conflicting) inter-orbit imaging opportunity pairs. It should be noted that this condition remains largely ignored from published decision models, leading possibly to unfeasible solutions. Primitive stereo imaging constraints are governed by equations (13)–(17), in which both observations must take place almost simultaneously. Sophisticated stereo imaging requests are rather managed as complex tasks through predefined possible task collection plans, as depicted trough constraint sets (2)–(4). Constraint sets (18)–(19) govern energy budget/capacity over satellite orbit ρ. Energy consumption essentially originates from imaging and transition activities. Graph opportunity flow conservation for orbit ρ is alternatively ensured through constraints sets (20)–(21). The latter allows connecting fictitious origin ($\underline{o}$) and destination ($\underline{d}$) nodes to build feasible satellite orbit path (legal sequence of observations). In other respect, inequalities (22)–(23) refer to the overall mission and task-specific financial budget (capacity) for image acquisition. An image cost is typically assumed proportional to the required task resolution and coverage. The proposed modelling represents a convenient approach to alternately cope with the financial problem objective dimension. On-board memory storage limitation is governed by constraint set (24). As satellites pass in and out of the Earth's shadow, thermal conditions may fluctuate drastically, imposing constraints on sensor use. Average imaging duty cycle constraint restricting maximum cumulative observation time per orbit and over multiple consecutive orbits due to thermal conditions, are described by inequalities (25)–(26) respectively. Binary integer and continuous decision variables are finally specified in expressions (27).

4 Discussion

The proposed multi-satellite collection scheduling problem formulation can be naturally reduced to the single standard constellation problem commonly found in open literature for homogeneous satellite cases. Accordingly, such a constellation setting generally involves identical trailing satellite separated by a small time lag, therefore naturally duplicating imaging opportunity swaths (perfect overlap) for a given task. A task area can then be defined through a set of disjoint parallel strips similar for each satellite member. As a result, for problem instances in which single imagery visits are required, the index '*o*' standing for opportunity in the proposed decision model may be alternately substituted for the related 'fixed' strip occurrence, taking advantage of imaging opportunity area duplication for each constellation satellite. Consequently, under disjoint parallel strips opportunity area definition and single visit (e.g., due to perfect sensor observation) conditions, mapping notation '*o*' to a strip area makes no longer quadratic the objective function and related constraints, but linear, reducing the approach to a simpler and well-known mixed-integer linear programming formulation further subject to a single task visit constraint, and exploiting homogeneous constellation collection scheduling problem properties. It is worth noticing though, that even in the homogeneous case, partially intersecting feasible opportunity swaths resulting from dissimilar neighboring satellite orbits are often deliberately ignored for convenience purposes, artificially reducing problem complexity at the expense of optimality. Further exploring feasible task opportunity combinations is anticipated to directly impact problem complexity and solution quality. The proposed quadratic formulation turns out to be a generalization of the standard single constellation multiple homogeneous satellite collection scheduling problem, accounting for mixed satellites and/or many constellations. The value of the problem model is intended to be first demonstrated over a single large area task use case of interest for comparison purposes, and then, on multiple tasks.

Scheduling algorithm design will be inspired from operations research and artificial intelligence, namely, through the development and reutilization of heuristics, metaheuristics and soft computing procedures (e.g., genetic, tabu search, ant colony), including commercially available exact mixed-integer programming optimization methods or a combination of them. Problem-solving techniques reutilization from related problems such as the multiple knapsack, vehicle routing and constrained longest path and variants will also be exploited. Baseline comparison with ongoing user-defined or knowledge-based procedures and naïve scheduling heuristics (e.g., highest utility/duration imaging opportunity ratio) will be highlighted. Despite available modern optimization machinery/technology able to successfully handle large problems, algorithm scalability remains a real challenge and will be handled separately as dictated by targeted problem instances.

The proposed model is intended to pave the way to hierarchical complexity refinement, including uncertainty management, task multiplicity and diversity. Problem complexity will eventually expand to integrate downlink scheduling. It aims at combining simultaneously satellite imaging and image downloading, relying on proper communication asset utilization such as ground stations and communication satellites networks. The resulting problem naturally relates to time-constrained pick-up and delivery combinatorial optimization problem variants and extensions.

5 Conclusion

In the context of the development of new intelligence collection technology concepts and automated decision planning capabilities applicable to satellite constellation to support collection management awareness and near-optimal resource management for low density high demand collection asset (satellite), a formal multi-satellite intelligence collection scheduling decision model has been presented. The proposed mixed-integer quadratic programming model is a mathematical formulation mapping a set of intelligence requests to a set of collection asset imaging opportunities to maximize collection value, subject to a variety of novel task, opportunity, capacity, temporal, memory and cost constraints. It departs from traditionally known and intractable approaches, relying on a new coverage approximation to deal with partial area coverage superimposition conferring the objective function and constraints of the optimization problem with desirable convexity property. This enables exploitation of available and powerful exact problem-solving techniques as well as efficient bound computation on the quality of the optimal solution giving credibility and soundness to comparative assessment of computed solutions and techniques. The proposed convex optimization problem introduces additional constraints including task precedence, temporal and capacity constraints as well as multiple performance threshold requirements. Relationship to existing models and promising/potential and problem-solving candidate algorithms have also been briefly discussed.

Future work is envisioned to further extend the decision model to reflect hierarchical problem complexity refinements and new features such as virtual satellite constellation heterogeneity, expanded task type diversity, uncertainty management and generalized quality of observation. Alternate efforts will consist to develop and implement problem-solving concepts and algorithms, conduct performance comparison analysis, and investigate practical limitations for possible use cases. Further directions include the investigation of problem-solving approach scalability, problem modelling generalization to dynamic and distributed settings as well as the growing integration of sensemaking/analysis and resource allocation.

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List of Symbols/Abbreviations/Acronyms/Initialisms

AOI	Area of Interest
CAF	Canadian Armed Forces
DRDC	Defence Research and Development Canada
EO	Electro-Optical
IR	Infra-Red
ISR	Intelligence, Surveillance and Reconnaissance
JICAC	Joint Information Collection and Analysis Capability
MIQP	Mixed-integer Quadratic Program
MOP	Measure of Performance
NIIRS	National Image Interpretability Rating Scale
RCM	RADARSAT Constellation Mission
RDDC	Recherche et Développement Defense Canada
S&T	Science and Technology
SAR	Synthetic Aperture Radar
VRPTW	Vehicle Routing with Time Windows

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Under the DRDC Joint Intelligence Collection and Analysis Capability (JICAC) project, this report proposes a formal multi-satellite intelligence collection scheduling deterministic decision model. A detailed description of the optimization model highlighting novelties is given. The underlying mathematical formulation is a mixed-integer quadratic programming model mapping intelligence requests to collection asset imaging opportunities in order to optimize collection value (value of information), subject to a variety of novel task, opportunity, capacity, temporal and cost constraints. Departing from known approaches, the new decision model is based on a new coverage approximation scheme to handle possible imaged area overlap while demonstrating desirable objective function convexity property making local optimal solution a global problem solution. The latter condition enables the utilization of powerful exact problem-solving techniques and the fast computation of a bound on the optimal solution, allowing sound comparative performance assessment over various problem-solving methods. The report also discusses the connection with ongoing decision models, potential problem-solving approaches expected to successfully solve the collection scheduling problem, and natural problem extensions.

Sous la gouverne du projet de Capacité de Cueillette et d'Analyse de Renseignement Inter–armées ou « Joint Intelligence Collection and Analysis Capability » (JICAC) de RDDC, ce rapport propose un modèle déterministe de décision formel d'ordonnancement multi-satellites de cueillette du renseignement. Une description détaillée du modèle ainsi que les innovations proposées sont présentées. La formulation mathématique sous-jacente est un modèle de programmation quadratique mixte mettant en correspondance des requêtes de renseignements à des opportunités d'imagerie de ressources de cueillette afin d'optimiser la valeur de l'information cueillie, sujet à une nouvelle variété de contraintes de tâches, d'opportunités, de capacité, temporelles et de coûts. Se distinguant des approches connues, le modèle de décision proposé mise sur un nouveau schéma de calcul d'approximation de couverture prenant compte du chevauchement possible de régions imagées, permettant ainsi d'exploiter la propriété de convexité du problème d'optimisation résultant. Cette propriété permet l'accès à des méthodes exactes de résolution très efficaces ainsi que le calcul rapide d'une borne à la solution optimale supportant une évaluation comparative plus objective de différentes techniques de résolution. Le rapport discute également du lien avec les modèles de décision existants, des approches et algorithmes applicables pouvant potentiellement résoudre le problème d'ordonnancement, ainsi que les extensions naturelles du problème.