Modeling and combining information within belief functions theory in Search And Rescue applications

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IMPORTANT INFORMATIVE STATEMENTS

Abstract

In this report, we study the expressiveness of the theory of belief functions in the particular case of Search And Rescue (SAR) operations. The theory of belief functions is a mathematical framework, which is used, as the probabilities, to represent the imperfections of a source of information. These imperfections are of different natures: reliability, vagueness, uncertainty, to name only a few. Representing and dealing with these imperfections in a SAR context is a major issue especially when one needs to combine and update information coming from heterogeneous and subjective sources.

After having reminded some background results of the theory of optimal search, we have suggested some approaches of the SAR problem founded on the theory of belief functions.

Résumé

Dans ce rapport, nous étudions l'expressivité offerte par la théorie des fonctions de croyance pour le problème de la planification d'une mission de recherche et sauvetage. La théorie des fonctions de croyance est un formalisme mathématique qui, au même titre que les probabilités, est utilisé pour représenter les imperfections d'une source d'information. Ces imperfections peuvent être liées à la non-fiabilité de la source, l'imprécision ou l'incertitude de l'information issue de la source. Modéliser toutes ces nuances pour un problème de recherche et sauvetage peut s'avérer fort utile pour combiner et mettre à jour les informations, surtout lorsque les informations dont on dispose pour définir les localisations possibles de l'objet disparu et la capacité à le retrouver sont de nature subjective.

Après avoir rappelé des résultats classiques issus de la théorie de la recherche op- timale, nous avons suggéré plusieurs approches de modélisation du problème de re- cherche et sauvetage dans le cadre la théorie des fonctions de croyance.

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Executive summary

Modeling and combining information within belief functions theory in Search And Rescue applications

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Introduction or background: Every year, there are hundreds of aeronautical Search and Rescue (SAR) incidents in Canada. In 2005, for example, there were 858 incidents. Al- though most of these incidents are of minor nature, some do evolve into major search operations requiring the use of numerous SAR resources over a period of many weeks. As a matter of fact, in 2005, there were 51 lost lives during aeronautical incidents and Canadian Forces aircraft have been tasked for 1091 missions[1]). Due to the dra- matic nature of these operations, it is crucial to ensure that the best available tools and technology are used to help plan and conduct search operations. In Canada, search planning is conducted by search mission coordinators who are highly trained individuals.

The use of search theory [2] and computer tools can be very helpful for planning an optimal search and maximizing the chances of finding survivors. The geographic decision support tool SARPlan was developed with this purpose in mind. It is based on search theory and uses a probabilistic framework to quantify the uncertain whereabouts of the searched object as well as the uncertainty surrounding detection capabilities.

One of the challenges in developing an appropriate optimization model for search planning is related to the adequate representation of the uncertainty surrounding a SAR case. The most common way is to use probability theory. There is, however, an alternative to probability theory, namely the use of belief functions from evidence theory. This powerful tool for representing uncertainty is also a practical formalism that allows one to merge several sources of information. This motivated us to explore the applicability of belief function theory to optimal search planning.

Results: We first review the framework of optimal search. Two kinds of probabilities are considered: The probability of containment which sums up the in- formation on the plane location and the probability of detection which describes the probability to detect a plane taking into account the interaction between the environment, the lost object and the observer. Using this approach, some strate- gies exist to split the effort available to search the lost object. In particular, we implemented the De Guenin algorithm. The main result of this work relies on the definition of several novel ways to represent the uncertainty surrounding the location of a missing aircraft in SAR operations. As a matter of fact, using belief functions has allowed us to take into account uncertainty in a finer, richer and more subtle way than the traditional probabilistic approach. The technique proposed relies on a discretization of the continuous space into a set of cells. One limitation of the proposed approach is directly linked to the granularity of the discretization as the computational complexity increases exponentially with the number of cells.

Significance: We showed that belief function has undeniable practical interests in SAR applications. This work can easily be extended to include reliability models, the ignorance of a source of information or the dependence between two sources of information by using more relevant combination rules than the conjunctive one. The results of this work can be adapted to situation monitoring problems with sensors or drones.

Future plans: Possible future works include the use of belief assignments obtained by applying the least commitment principle (choosing among a set of belief functions the least informative one). Also, new approaches will address the problem of finer granularity of the discretization of the continuous space: Continuous belief functions would be a good candidate.

Sommaire

Modeling and combining information within belief functions theory in Search And Rescue applications

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Introduction ou contexte : Des milliers d'incidents aéronautiques surviennent chaque année au Ca- nada. En 2005, il a fallu mettre en œuvre des opérations de secours lors de 858 acci- dents aériens [1]. Bien qu'ils soient dans la majorité des cas sans grande importance, des dizaines de personnes meurent ou disparaissent dans des incidents nécessitant la mise en œuvre de recherches de grande ampleur. En 2005, 51 personnes ont perdu la vie lors de catastrophes aériennes qui se sont produites au Canada [1]. Chaque année, il arrive que des opérations de recherches s'étalent sur plusieurs semaines. Dans ce cas, une bonne utilisation des moyens de recherche devient un élément crucial. De nos jours, cette planification est assurée par un coordonnateur de mission.

Tenir compte de manière formelle des informations physiques portant sur la nature du terrain, ses interactions avec l'objet disparu et les observateurs serait un pas en avant important, car celles-ci influencent directement la capacité des équipes de recherche à détecter l'objet porté disparu. C'est pour répondre à ce besoin que le système d'aide à la décision géographique SARPlan a été développé. Le but à terme est d'aider à la planification des efforts mis en œuvre pour retrouver un aéronef porté disparu sur le sol canadien.

Un cas de recherche et sauvetage reste une situation complexe à appréhender. Des modélisations des phénomènes mis en jeu existent, telle la théorie de la recherche optimale de Koopman qui repose sur une représentation probabiliste de l'information. La théorie des fonctions de croyance est une option de rechange à la théorie des probabilités et permet de représenter avec plus de richesse l'ignorance et l'incertitude inhérentes aux sources d'information. De plus, elle offre une panoplie de méthodes de combinaison d'information permettant de gérer le conflit entre les sources. Ainsi, la théorie des fonctions de croyance constitue un outil attrayant pour modéliser le problème de recherche et sauvetage. Dans ce rapport, nous proposons plusieurs modélisations du problème de recherche à l'aide des fonctions de croyance.

Résultats : Nous commençons par introduire le formalisme propre au problème de recherche et sauvetage, dans lequel on tient compte de deux types de probabilité : La probabilité de localisation nous renseigne sur l'information dont nous disposons *a priori* sur l'emplacement de l'avion porté disparu, et la probabilité de détection décrit la probabilité de détecter un objet tenant compte de l'interaction entre l'environnement, l'objet recherché et l'observateur. On peut ainsi proposer des stratégies pour répartir l'effort de recherché de façn optimale. Notamment, nous avons implémenté l'algorithme de De Guenin. Le principal résultat de ce travail est constitué par la définition de nouveaux modèles du problème de recherche et sauvetage à l'aide des fonctions de croyance, grâce à qui nous sommes parvenus à représenter les incertitudes sur la présence de l'avion dans la zone de recherche ainsi que l'évènement "détection" de manière plus fine et plus riche qu'avec les probabilités. Cela a des conséquences sur la façon dont les informations sur la localisation probable de l'avion sont mises à jour. Ayant testé plusieurs méthodes pour générer des fonctions de croyance, il est apparu que certains types de transformations utilisées pour les créer sont à éviter si l'on veut être en mesure de travailler avec des outils d'une complexité raisonable. En effet, l'approche proposée repose sur une discrétisation de l'espace continu en un ensemble de celulles, ce qui engendre une limitation directe- ment liée à la granularité de la discrétisation, puisque la complexité de calcul croît de façon exponentielle avec le nombre de cellules.

Importance : Nous avons proposé de nouveaux outils de fusion d'information et de nouveaux critères de décision fondés sur les fonctions de croyance pour le problème de recherche et sauvetage. Ayant posé les jalons d'une représentation du problème de recherche et sauvetage avec des fonctions de croyance, il est fa- cile d'imaginer des extensions de notre travail exploitant pleinement le potentiel de ce formalisme, de la modélisation de la fiabilité d'une source à la modélisation de l'ignorance ou encore celle de la dépendance de sources en utilisant des règles de combinaison de l'information dépassant le cadre de la simple règle conjonctive. Si cette étude a été faite à l'origine pour des problèmes de recherche et sauvetage d'avions, on peut facilement adapter ces résultats et représenter des problèmes de surveillance effectués par des capteurs ou des drônes.

Perspectives : De nombreux points sont encore à developer. En particulier, générer des fonctions de croyance en appliquant le principe de moindre engagement (choisir parmi un ensemble de fonctions de croyance compatibles la moins informative possible). Ensuite, l'utilisation des fonctions de croyance continues pourrait éviter une discrétisation de l'espace et ainsi résoudre le problème d'explosion combinatoire.

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1 Introduction

The theory of optimal search was developed during the World War II by B. Koopman [3, 4, 5]. One of the aims of his work was to detect German U-boats in the Atlantic Ocean. Current research efforts apply this theory in a Search And Rescue (SAR) context in order to develop optimal search plans [6]. The goal of an optimal search plan is to make the best use of available search resources in order to locate and rescue people in distress. There are two main components for modeling this problem: i) information on the possible location of the missing search object, ii) a sensor's capability to detect the search object. This capability depends on the search object type, on the environment where search missions are conducted, and on the sensor and amount of effort spent on searching. In the search and rescue literature, location and detection information are usually modeled using probabilities: the Probability Of Containment (POC) which describes the location distribution of the search object and the Probability Of Detection (POD) which is the conditional probability of detecting a search object with a given amount of effort.

A search plan defines the way effort is distributed over a search area. Optimal search planning consists of allocating the available resources in a way to maximize a given performance criterion. Today various sensors are available and are used in the context of searches such as drones for example [7]. In this report, we propose a new way to deal with sensors information based on the theory of belief functions. This theory is based on the work of Dempster and Shafer [8, 9], a mathematical framework useful to represent uncertainty and imprecision. Moreover, it is a powerful tool to merge several sources of information and take into account conflict and ambiguity. In Chapters 2 and 3, we present basic notions of search theory and belief functions theory, respectively. We then develop a belief function approach for the classical optimal search problem in Chapter 4 and extend it to the false alarm cases. Finally, in Chapter 5, we present simulations results to compare the different models and conclude on future works in Chapter 6.

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2 Theory of optimal search

The theory of optimal search was developed during the World War II. The earliest publications are by Bernard O. Koopman [3, 4, 5]. The application at the time was the detection of German U-boats in the Atlantic Ocean. There are two main components to a search theory problem: representing uncertainty and allocating the available effort. Uncertainty is normally represented using probability theory. Optimal effort allocation is obtained using optimization methods.

2.1 Search object modeling

The objective of developing an optimal search plan is to allocate the available resources in such a way that the survivors will be located as quickly as possible. To this end, we must firstly use the available information on the whereabouts of the searched object as well as the detection capabilities of the sensor in its surrounding environment. The Probability Of Containment (POC) describes the location distribution of the searched object, while the Probability Of Detection (POD) is a conditional probability of detecting the searched object with a given amount of effort. These two functions must be defined prior to developing a search plan.

2.1.1 Probability of containment

Let $\mathcal{R}$ be the search area. If $\mathcal{R}$ is continuous, the location of the searched object can be represented using a probability density function f_l. Hence following [2] we have:

$$\int_{x \in \mathcal{R}} f_l(x)\, dx = \beta \tag{1}$$

where $\beta \in [0, 1]$. A value of β lower than 1 allows us to consider that the object may be outside the search area.

If the search area $\mathcal{R}$ is discrete, we have:

$$\sum_{c \in \mathcal{R}} POC(c) = \beta \tag{2}$$

with $POC(c)$, the probability the searched object is in Cell c.

There are various ways to define the POC in the literature. The most traditional one consists in defining a bivariate Gaussian function centered on the Last Known Point

(LKP) and with standard deviation $\sigma^2 = \sigma_x^2 + \sigma_y^2$ and $\sigma_x = \sigma_y$, such that the radius of the circle in which there are 50% of chances of containing the searched object is equal to 1.1774σ. This method is appropriate when the only available information is the LKP. However, when the original destination of the searched object is also known, a line *datum* may be used. The POC is then required to be uniform on the paths of the line linking the LKP to the destination *datum* and to follow a Gaussian law on the orthogonal paths [6]. Other methods have been proposed to generate more complex probability of containment distributions. They use several *scenarii* to define possibility areas [10].

2.1.2 Probability of detection

The probability of detecting a searched object given that it is in a cell c ($POD(c)$) depends on several parameters such as the environment, the amount of search effort applied, the type of searched object and the sensor's performances.

2.1.2.1 Sweep width

To characterize the detection ability of a sensor, we use the lateral range function α [2] illustrated in Figure 1). $\alpha(r)$ provides the probability that an object, located at a
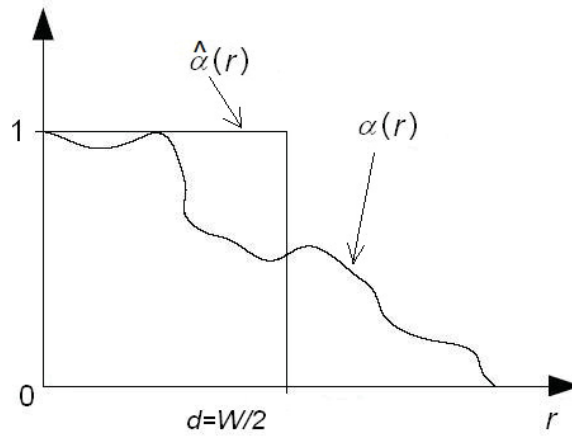
Figure 1: *Lateral range functions*

distance r perpendicular to the trajectory of the sensor, can be detected by a sensor (*cf.* Figure 2).
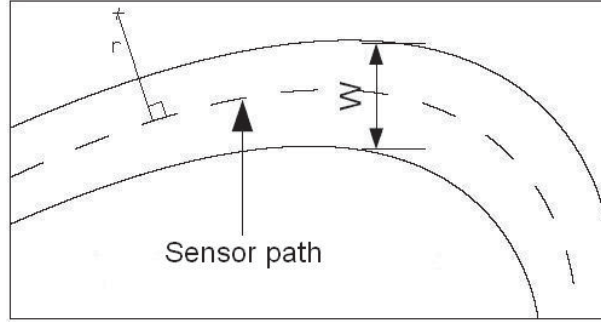
Figure 2: *Area swept by the sensor*

The integration of the lateral range function α gives the *sweep width W*:

$$W = 2 \int_0^\infty \alpha\left(r\right) dr \tag{3}$$

In the discrete case, we consider that the sweep width is uniform in a given cell and equal to $W\left(c\right)$.

A classical lateral range function is $\hat{\alpha}$ defined by [2]:

$$\hat{\alpha}\left(r\right) = \left\{ \begin{array}{ll} 1 & \text{for} \quad 0 \leq r \leq d \\ 0 & \text{for} \quad r > d \end{array} \right. \tag{4}$$

with $W = 2d$ (*cf.* Figure 2). A sensor described by this lateral range function $\hat{\alpha}$ is called a *definite-range law* sensor.

The concept of sweep width must be used carefully. As shown in Figure 3, for a same W, the number of detected targets (if we assume that they are uniformly distributed) is the same, but their location in space is different and directly connected to the function α [11].

2.1.2.2 Search effort

There are several ways to define the effort: as a length, as a time spent in an area, as the cost of a mission, etc. [2]. We chose here to define it as the length of the trajectory followed by the sensor. Let z be this length, V be the speed of the sensor and T the time spent in an area, then $z = V \cdot T$. The product of z by W provides an idea of the area swept by the sensor. In the literature [11, 6], C is defined as the area coverage. It is equal to $W \cdot z/A$ where A is the area to which the effort is applied.

(a) Sensor with any lateral range function α (b) A definite-range law sensor ($\hat{\alpha}$)
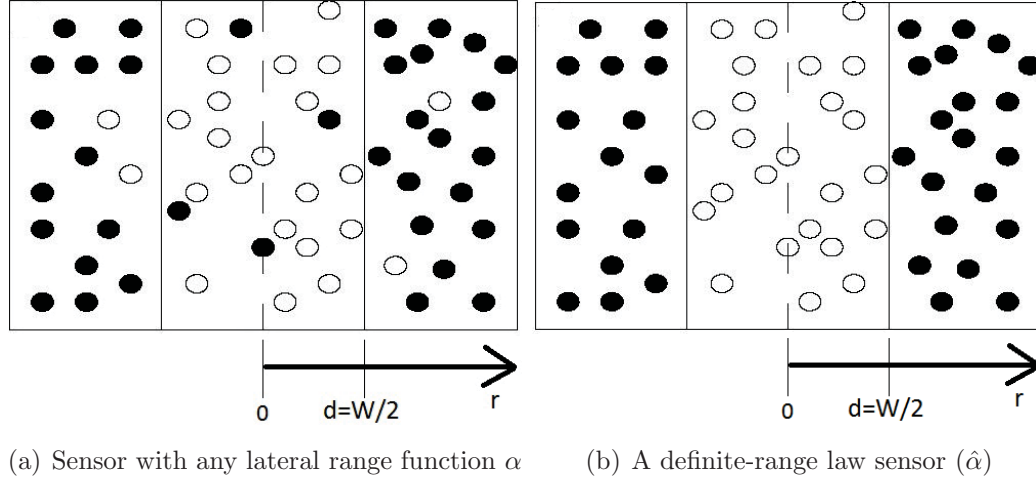
Figure 3: *Sensor detections: detected targets in white, non detected targets in black*

The smaller this *ratio*, the bigger the probability of detection. However, increasing C indefinitely will not contribute much to the POD.

2.1.2.3 Probability of detection

The probability of detection depends on both the lateral range function and the search effort. Let the sensor follow a definite range law of Equation (4), we can use W instead of α. If the sensor scans the search area with parallel paths spaced of W (*cf.* Figure 4), the probability of detection will be equal to [2]:

$$b\left(z\right) = \left\{ \begin{array}{ll} z \cdot W/A & \text{if} \quad z \leq A/W \\ 1 & \text{if} \quad z > A/W \end{array} \right. \tag{5}$$

This is the case where, for a given amount of effort and this type of sensor, the covered surface is as high as possible (*cf.* Figure 5). However, if the same type of sensor follows a random walk, the probability of detection will be [2]:

$$b\left(z\right) = 1 - \exp\left(-zW/A\right) \tag{6}$$

Hence the detection function given by (6) is a lower bound on the probability of detection obtained with $\hat{\alpha}$.

There are other detection models. For example, in [4] Koopman introduces the inverse cube detection model for visual searches where the sensor follows a specific α function and scans an area following parallel paths.
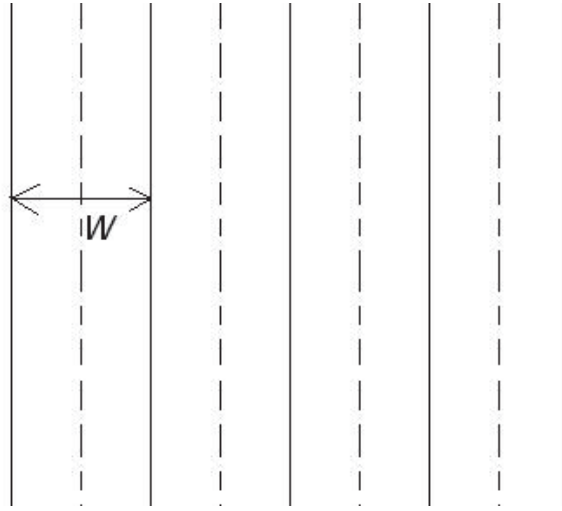
Figure 4: *Area swept with parallel paths*
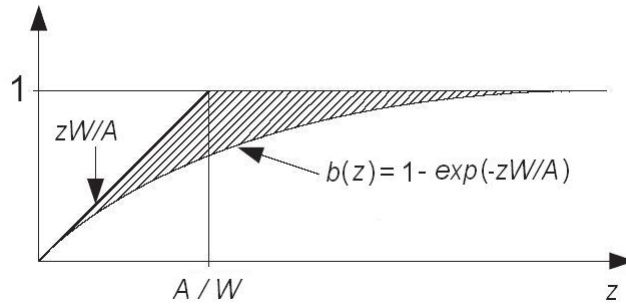
Figure 5: *Several kinds of POD*

2.1.3   Update of POC after a search

Between two search missions (search steps), the POC is updated by using the Bayes' rule accounting thus for unsuccessful searches. In discrete time, we have:

$$POC_n(c) = \frac{POC_{n-1}(c) \cdot (1 - POD_n(c))}{1 - POS_n} \tag{7}$$

with X_n, the amount X known at the n step. POS is the Probability Of Success (to be defined in Equation (14)) and is used as a normalization factor. A non-normalized POC can also be used:

$$POC_n(c) = POC_{n-1}(c) \cdot (1 - POD_n(c)) \tag{8}$$

Instead of redistributing the POC on all the search area, we assume that the searched object is somewhere else. This corresponds to lowering β.

2.1.4 Search with false alarms

In a real case, false alarms need to be considered [2]. To model the detection problem we have to consider two cases:

- We take the decision $\mathcal{D}_0$, *ie.* we decide there is nothing (hypothesis $\mathcal{H}_0$),

- We take the decision $\mathcal{D}_1$, *ie.* we decide there is an object (hypothesis $\mathcal{H}_1$).

Hence, four different probabilities need to be considered:

	$\mathcal{H}_1$	$\mathcal{H}_0$
$\mathcal{D}_1$	True detection $P\left(\mathcal{D}_1\|\mathcal{H}_1\right)$	False detection $P\left(\mathcal{D}_1\|\mathcal{H}_0\right)$
$\mathcal{D}_0$	False rejection $P\left(\mathcal{D}_0\|\mathcal{H}_1\right)$	True rejection $P\left(\mathcal{D}_0\|\mathcal{H}_0\right)$

In Figure 6, we assume that detector is created using the output of a sensor. If this value is higher than a threshold δ, the decision $\mathcal{D}_1$ is taken, otherwise it is the decision $\mathcal{D}_0$. The output of the sensor is modelized with a random variable. If $\mathcal{H}_0$ is true, it follows the distribution $\mathcal{N}\left(\mu_0, \sigma\right)$. If $\mathcal{H}_1$ is true, it follows distribution $\mathcal{N}\left(\mu_1, \sigma\right)$. Hence, for a given threshold, we obtain a probability of false detection and a probability of true detection.

For characterizing the sensor's performances, the ROC (Receiver Operating Characteristics) curve is commonly used. This curve gives the probability of a true detection as a function of the probability of a false detection and depends on the signal to noise ratio. In [12], the author suggests to consider the average number of false alarms n_f during a given interval of time and to associate it with a mean cost in terms of effort τ wasted in order to check the origin of the alarm. We have $T = T_s + n_f T_s \tau$, where T is the searching time and T_s is the effective searching time. To take into account the false alarm, the easiest way is to use W' equal to $W/(1 + n_f \tau)$ instead of the real W in order to compensate for the lack of efficiency of the sensor.
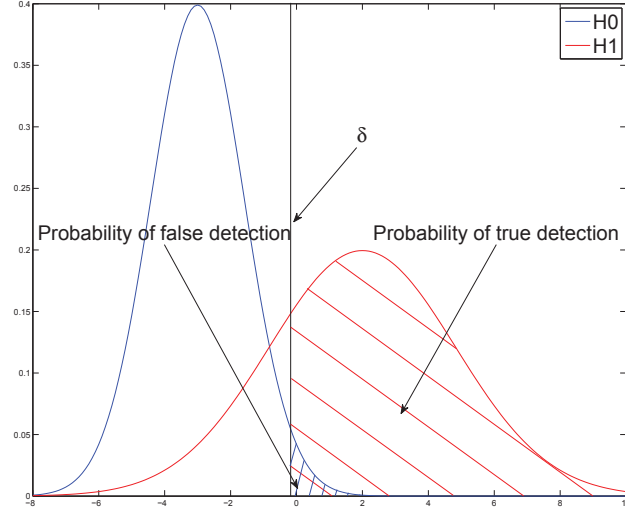
Figure 6: *Area under the curve gives the probability of true and false detection*

2.1.5 W **uncertain**

In some cases, W is unknown on the search area [13] and we thus need to represent the uncertainty surrounding it. W is then modeled as a random variable, with a distribution equal, to the probability that W is bigger than w for a given location x and a given w [2]:

$$G\left(w,x\right) \;\;=\;\; Pr\{W \leq w|\text{Target in } x\} \tag{9}$$

The POD becomes then [2]:

$$b\left(z,x\right) \;\;=\;\; \int_0^\infty B_w\left(z,x\right)G\left(dw,x\right) \tag{10}$$

with B_w, the probability of detection for a w.

In the literature [13, 2], W is assumed to be independent of the sensor's location. Therefore, G depends only on w. A classical example of a distribution proposed for W is the gamma distribution:

$$\frac{\partial}{\partial w}G\left(w\right) = w^{\nu-1}\gamma^\nu e^{\gamma w}/\Gamma\left(\nu\right) \tag{11}$$

where Γ is Euler's function, while γ and ν are the parameters that determine the shape of the distribution. The average of W is then equal to ν/γ and its standard deviation is equal to ν/γ^2 [2]. Under an exponential law assumption for the POD (*cf.* Equation (6)), we have:

$$B_w(z) = 1 - \exp(-zw) \tag{12}$$

Based on Equations (10), (11) and (12), we obtain:

$$b(z) = \int_0^\infty B_w(z) \frac{\partial}{\partial w} G(w)\, dw = 1 - (1 + z/\gamma)^{-\nu} \tag{13}$$

Considering the uncertainty on the sweep width modifies the values of the probability of detection and the way the effort is splitted.

2.2 Search planning

The goal of optimal search planning is to make the search as efficient as possible based on a given criterion.

2.2.1 Optimization criterion

To optimally plan a mission, the effort is distributed so that to optimize a relevant measure of performance. The criterion to maximize is often chosen as the probability of success (POS) defined as the likelihood of finding the searched object with an available effort Ξ [6]. In continuous space, the following quantity is to be maximized:

$$\begin{aligned} POS &= \int_{x \in \mathcal{R}} f_l(x)\, b(\xi(x))\, dx \\ \text{with } \Xi &= \int_{x \in \mathcal{R}} \xi(x)\, dx \end{aligned} \tag{14}$$

where $\xi(x)$ is the amount of effort applied on x.

2.2.2 Effort allocation - De Guenin's algorithm

If the effort is continuous and infinitely divisible, we can use the de Guenin's algorithm [14] to distribute it over the search area in an optimal way, as described by
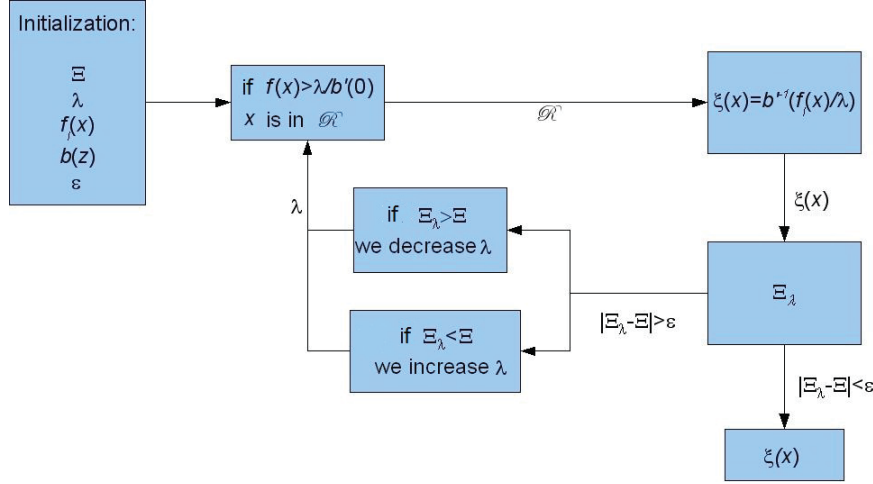
Figure 7: *Diagram describing the de Guenin's algorithm*

(14). The principle of this algorithm is simple. De Guenin proved that for a given amount of effort the POS is maximized if for all x of $\mathcal{R}$ we have:

$$f_l\left(x\right) b'\left(\xi\left(x\right)\right) = \lambda \tag{15}$$

with b' being the derivative in ξ of b and λ being a constant. So for a fixed λ, by inverting b' we can find the allocation of ξ_λ maximizing the POS on the search area for a global amount of effort. We then have $\Xi_\lambda = \int_{x \in \mathcal{R}} \xi_\lambda\left(x\right) dx$. The optimization problem is thus transformed. Our aim is now to find the λ that satisfies $\Xi_\lambda = \Xi$. The flowchart of De Guenin's algorithm is provided in Figure 7 and the MATLAB code in Appendix A.

If the effort is not infinitely divisible (for example, a finite number of indivisible observers) and if we suppose several stages of search (time is discrete), an alternative algorithm to de Guenin's one, called branch and bound algorithm, may be used to obtain a distribution of the effort [15].

For stationary objects, forward planing is done one step at a time. If the searched object is moving, we need to consider several planning steps and update the POC. In [16] and [15], some algorithms are proposed to solve this problem.

2.3   Limits

We have briefly presented the basic elements of search theory. The main observation regarding the theory is that, after a large amount of effort has been unsuccessfully applied in an area, it is assumed that the searched object is not within the area. This representation of uncertainty is unsatisfactory since a search area containing a searched object may be well covered without detecting the object. Belief functions theory allows us to take into account ignorance regarding the location of the searched object, and we will explore this representation in the upcoming chapter.

3 Belief functions theory

The theory of belief functions has been introduced by Arthur Dempster [8] and Glenn Shafer [9]. It is a mathematical framework useful to represent uncertainty and imprecision. It provides us powerful tools to combine pieces of information and to manage conflict.

Given a problem, a *frame of discernment* is a finite set of disjointed possible answers denoted Ω. The set of all subsets of Ω is called the power set of Ω and denoted by 2^Ω. In the case of an SAR application, we can consider that our search area is split in a grid. Hence the "Search Area", the frame of discernment, is composed by a set of disjoint elements.

A *basic belief assignment* (bba) is a mapping m^Ω from 2^Ω to $[0,1]$ such that $\sum_{A \subseteq \Omega} m^\Omega(A) = 1$. The value of $m^\Omega(A)$ for all A included in Ω is called the basic belief on A. This mapping allows us to assign a weight on a set of hypothesis without making any assumption on the hypothesis of the set. Hence, we can model ignorance or imprecision. For example, if we do not know where the lost object is, we can assign a mass of 1 on the set "Search Area". We do not have to assign mass on his subsets. A subset of Ω, A, such that $m^\Omega(A)$ is strictly positive is called a *focal element* of m^Ω.

In their original theory, Dempster and Shafer required the bba (that they called basic probability assignment) of the empty set to be 0, *i. e.* $m^\Omega(\varnothing) = 0$. In other words, that the frame of discernment is an exhaustive set of hypothesis. This restriction has been further dropped by Smets and Kennes in their Transferable Belief Model [17], allowing the empty set to be a focal element, an hypothesis they called the *open-world* assumption. There are several ways to explain the presence of a basic belief on the empty set. The most common one is to assume that the frame of discernment is not exhaustive and that the truth could be modelized by an hypothesis which is not included in Ω.

Using a bba m^Ω, we define the following functions:

Belief function:
$$bel^\Omega(X) = \sum_{A \subseteq X, A \neq \emptyset} m^\Omega(A), \forall X \in 2^\Omega \tag{16}$$

Plausibility function:

$$pl^{\Omega}(X) = \sum_{A \subseteq \Omega, A \cap X \neq \emptyset} m^{\Omega}(A), \forall X \in 2^{\Omega} \tag{17}$$

Pignistic probability [18]:

$$betP^{\Omega}(X) = \sum_{A \subseteq \Omega, X \in A} \frac{m^{\Omega}(A)}{|A|(1 - m^{\Omega}(\emptyset))}, \forall X \in 2^{\Omega} \tag{18}$$

The pignistic transformation is used to make a probability from a bba. It represents the bet an evidential source of information could have made in order to take a decision. The principle is to share the basic belief on a set to its constituent elements. The belief function is a kind of lower bound of probability. Indeed, this measure only considers pieces of evidence included in the set that is tested. On the contrary, the plausibility function is a kind of upper bound of probability because it includes all the pieces of evidence that are not in contradiction with the tested hypothesis.

In fusion applications, we assume we dispose of several sources of information. A major issue is to combine these sources of information. Let m_1^{Ω} and m_2^{Ω} be two bbas. The *conjunctive rule of combination [19]* of m_1^{Ω} by m_2^{Ω} denoted by $m_1^{\Omega} \otimes m_2^{\Omega}$ or by $m_{1 \otimes 2}^{\Omega}$ is given by:

$$m_{1 \otimes 2}^{\Omega}(A) = \sum_{X \cap Y = A} m_1^{\Omega}(X) m_2^{\Omega}(Y), \forall A \subseteq \Omega \tag{19}$$

This rule can be generalized to N bbas m_i^{Ω}:

$$\underset{i \in [1,N]}{\otimes} m_i^{\Omega}(A) = \sum_{C_1 \cap ... \cap C_N = A} \prod_{i \in [1,N]} m_i^{\Omega}(C_i), \forall A \subseteq \Omega \tag{20}$$

Such a rule is used under the assumption that the sources of information are independent. After the combination process, a part of the mass can be transfered to the empty set. In this context, we can assume that this phenomenon occurs because the different sources of information are in conflict. To deal with non independent and conflicting sources of information, we can use other operators of combination [20].

In several cases, it happens that the information transmitted by a source is compatible with a set of belief functions. When we have no clue about the one to choose,

it is usual to decide that the least committing one is the best suited. It is called the least commitment principle. Hence, we have to define an ordering between the belief functions to determine if a belief function is more or less committed than an other. One partial ordering for the belief functions is the relation of specialization. m_2^Ω is a specialization of m_1^Ω if and only if there is an operator s from 2^Ω to $|\,[0,1]$ such as for all A included in Ω :

$$m_2^\Omega (A) = \sum_{X \subseteq \Omega} s^\Omega (A, X)\, m_1^\Omega (X) \tag{21}$$

An operator of specialization splits the mass assigned into a set onto its subsets.

Let m^Ω be a bba representing the information given by an evidential source. We obtain an information that the truth is included in B, a subset of Ω. Hence we must update our bba using a conditioning process. Let $m^\Omega[B]$ be the bba obtained after a conditioning of m^Ω on B. According the conditioning rule of Dempster:

$$m^\Omega [B] (A) = \begin{cases} \displaystyle\sum_{X : X \subseteq \overline{B}} m (A \cup X) & \text{if } A \subseteq B, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

This bba fulfils the following condition. It is the least committed bba such as $bel^\Omega [B] (B) = 1$ and $m^\Omega [B]$ is a specialization of m^Ω.

When we handle different frames of discernment, we cannot always apply the conditioning rule of Dempster to update information. Let $\mathcal{T}$ be a frame of discernment linked to *a posteriori* data (measures) and Ω be the frame of decision. Let assume that we can define for each hypothesis $\omega \in \Omega$ the evidential knowledge on the measures $m^\mathcal{T} [\omega]$. According the general Bayesian theorem [17], if an information or measure reveals that the truth is in $t^* \subseteq \mathcal{T}$, we can deduce information on the frame of discernment Ω and build the bba $m^\Omega [t^*]$[1] :

$$m^\Omega [t^*] (A) = \prod_{\omega \in A} pl^\mathcal{T} [\omega] (t^*) \cdot \prod_{\omega \in \overline{A}} \left(1 - pl^\mathcal{T} [\omega] (t^*)\right) \tag{23}$$

In the following parts, we use belief functions to model problems associated to search and rescue applications.

---

[1]By using Equation (17), we have $pl^\mathcal{T} [\omega] (t^*) = \displaystyle\sum_{A \subseteq \Omega, A \cap t^* \neq \emptyset} m^\mathcal{T} [\omega] (A)$.

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4 Models

We use belief functions theory to represent the uncertainty on the information. Our objective is to show the richer expressiveness of this theory in modeling uncertainty and knowledge during a SAR operation. We begin by proposing models not accounting for false alarms in Section 4.1, and subsequently include the possibility of false alarms in Section 4.2.

4.1 Without false alarm

The classical approach in SAR is to consider that the detector (a human being), cannot transmit a false alarm. In this context, we will study how to represent the problem with belief functions.

4.1.1 Hypotheses

We use Koopman's theory to represent the SAR problem [3, 4, 5]. There are two sources of information: the sensor and the information on the system.

4.1.1.1 The sensor

We assume that the sensor does not produce any false detection. It is characterized by a detection function and a sweep width $W(i)$. Hence we know the conditional probability $POD(i)$ of detecting the searched object given it is in the cell i as a result of applying an effort $z(i)$.

4.1.1.2 Information on the system

We consider the information we have on the searched object at time t before applying any effort and initialize it with $POC(i)$, the *a priori* probability of containment. This probability can be viewed as the memory of the search planning tool.

4.1.2 Model 1: Working with a cell

We represent the information at the cell scale.

4.1.2.1 The frames of discernment

We distinguish two frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, which refers to the presence (p_i) or the absence $(\overline{p_i})$ of the searched object in Cell i.

- $D_i = \{d_i; \overline{d_i}\}$, which refers to the detection (d_i) or the non-detection $(\overline{d_i})$ in Cell i.

4.1.2.2 Basic belief assignment (bba)

We consider two bbas:

- $m_{ci,t}$ which is linked to the information the sensor in Cell i provides at time $t > 0$.

- $m_{i,t-1}$ which is linked to the information at time $t-1$ about the presence of the searched object in Cell i at time t. $m_{i,t-1}$ is a kind of memory for the system and can be updated by combining it with the information given by the sensor at time t.

Using the classical model in search theory [2], we can define $m_{ci,t}^{D_i}[p_i]$, and we have:

$$
\begin{aligned}
m_{ci,t}{}^{D_i}[p_i](d_i) &= POD(i) \\
m_{ci,t}{}^{D_i}[p_i](\overline{d_i}) &= 1 - POD(i)
\end{aligned}
\tag{24}
$$

This is the belief we have on the event detection in Cell i provided that the searched object is in Cell i.

The bba $m_{i,t-1}$ can be initialized based on the *a priori* information. Usually, a POC is assumed and we therefore use it to define the bba $m_{i,0}{}^{\Pi_i}$ which is null everywhere, except for:

$$
\begin{aligned}
m_{i,0}{}^{\Pi_i}(p_i) &= POC(i) \\
m_{i,0}{}^{\Pi_i}(p_i \cup \overline{p_i}) &= 1 - POC(i)
\end{aligned}
\tag{25}
$$

Indeed, since we only have information on the presence in a cell i, so the rest of the mass must be assigned to the ignorance.

At the end of a search, if the searched object has not been located, $m_{i,t-1}$ is updated. We must know $m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right]$, the belief function on the presence of the searched object

in Cell i given that there is no detection. We can set:

$$m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right](\overline{p_i}) = POD(i)$$
$$m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i}) = 1 - POD(i) \tag{26}$$

Indeed, there were $POD(i)$ chances to detect the searched object if it were present. We assume that there are $POD(i)$ chances not to find the searched object. The rest of the mass can be transferred to the ignorance since the absence of a detection does not provide any information about the presence of the object. Combining $m_{i,t-1}^{\Pi_i}$ with $m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right]$ allows to update the information about the location of the searched object and we obtain $m_{i,t}^{\Pi_i}$.

4.1.2.3 Combination of information

We chose the conjunctive rule of combination:

$$m_{i,t}{}^{\Pi_i} = m_{i,t}{}^{\Pi_i}\left[\overline{d_i}\right] = m_{i,t-1}{}^{\Pi_i} \otimes m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right] \tag{27}$$

Hence we obtain:

	p_i	$\overline{p_i}$	$p_i \cup \overline{p_i}$	$\emptyset$
$m_{ci,t}{}^{\Pi_i}\left[\overline{d_i}\right]$	0	$POD(i)$	$1 - POD(i)$	0
$m_{i,t-1}{}^{\Pi_i}$	$m_{i,t-1}{}^{\Pi_i}(p_i)$	$m_{i,t-1}{}^{\Pi_i}(\overline{p_i})$	$m_{i,t-1}{}^{\Pi_i}(p_i \cup \overline{p_i})$	$m_{i,t-1}{}^{\Pi_i}(\emptyset)$
$m_{i,t}{}^{\Pi_i}\left[\overline{d_i}\right]$	$m_{i,t-1}{}^{\Pi_i}(p_i)$ $(1 - POD(i))$	$m_{i,t-1}{}^{\Pi_i}(p_i \cup \overline{p_i})$ $POD(i)$ $+ m_{i,t-1}{}^{\Pi_i}(\overline{p_i})$	$m_{i,t-1}{}^{\Pi_i}(p_i \cup \overline{p_i})$ $(1 - POD(i))$	$m_{i,t-1}{}^{\Pi_i}(\emptyset)$ $+ m_{i,t-1}{}^{\Pi_i}(p_i)$ $POD(i)$

The belief assignment to the presence of the searched object in a cell decreases each time we update it as it does in the probabilistic case (Equation (8)). The mass transferred to the ignorance Ω decreases also following the application of effort. In probability theory, the probability of the presence of the searched object outside of the search area $(1 - \beta)$ increases. In belief function theory, the mass on $\emptyset$ increases because of the conflict between the two sources of information considered. At the end of a search, even if a great amount of effort has been used, the use of the proposed belief function approach does not allow to state that the searched object is not in the cell that was already searched. This is quite reasonable since it is very possible to look in an area containing the searched object without finding it.

4.1.3 Model 2: Extension of Model 1 to a grid

Now, we work at the grid scale.

4.1.3.1 The frames of discernment

We consider again two frames of discernment:

- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the searched object in the different N cells.

- $D = \{d_1; \overline{d_1}; \ldots; d_N; \overline{d_N}\}$, the detection of the searched object in the N possible cells.

4.1.3.2 Basic belief assignment

We use the bba described in Model 1 in Equation (24). Instead of using $m_{i,t}{}^{\Pi_i}$ we rather use $m_t{}^\Pi$. Hence the information is memorized at the grid scale and not at the cell scale:

$$
\begin{aligned}
m_0{}^\Pi (p_i) &= POC(i) \\
m_0{}^\Pi \left(\bigcup_{j \in [1;N]} p_j \right) &= 1 - \sum_{j \in [1;N]} POC(j)
\end{aligned}
\tag{28}
$$

We apply the same idea put forward in Model 1 but we consider $m_{ci,t}{}^\Pi$ instead of $m_{ci,t}{}^{\Pi_i}$. If the searches are not successful, we update the information set. Using the same results as in the previous model (Equation (26)), we represent the information returned by the sensor following an unsuccessful search in Cell ci:

$$
\begin{aligned}
m_{ci,t}{}^\Pi \left[\overline{d_i} \right] \left(\bigcup_{j \in [1;N]-\{i\}} p_j \right) &= POD(i) \\
m_{ci,t}{}^\Pi \left[\overline{d_i} \right] \left(\bigcup_{j \in [1;N]} p_j \right) &= 1 - POD(i)
\end{aligned}
\tag{29}
$$

We can combine the several sources of information to obtain $m_t{}^\Pi$, the updated bba of $m_{t-1}{}^\Pi$.

4.1.3.3 Combination of information

Again, we use the conjunctive rule of combination:

$$m_t{}^\Pi = m_t{}^\Pi \left[\overline{d_i}\right]_{i\in[1;N]} = m_{t-1}{}^\Pi \otimes_{i\in[1;N]} \left(m_{ci,t}{}^\Pi \left[\overline{d_i}\right]\right) \tag{30}$$

This is a result of the computations we conducted[2]:

	p_i		$\Pi_{\setminus p_i}$	$\Pi_\cup$	$\emptyset$
$m_{ci,t}{}^\Pi \left[\overline{d_i}\right]$	0	$\ldots$	$POD(i)$	$1 - POD(i)$	0
			$\vdots$		
$m_{t-1}{}^\Pi$	$m_{t-1}{}^\Pi(p_i)$		$m_{t-1}{}^\Pi\left(\Pi_{\setminus p_i}\right)$	$m_{t-1}{}^\Pi\left(\Pi_\cup\right)$	$m_{t-1}{}^\Pi\left(\emptyset\right)$
$m_t{}^\Pi \left[\overline{d_j}\right]_{j\in[1;N]}$	$m_t{}^\Pi \left[\overline{d_j}\right](p_i)_{j\in[1;N]}$		$m_t{}^\Pi \left[\overline{d_j}\right]\left(\Pi_{\setminus p_i}\right)_{j\in[1;N]}$	$m_t{}^\Pi \left[\overline{d_j}\right]\left(\Pi_\cup\right)_{j\in[1;N]}$	$m_t{}^\Pi \left[\overline{d_j}\right]\left(\emptyset\right)_{j\in[1;N]}$

When we develop the computations:

$$
\begin{aligned}
m_t{}^\Pi \left[\overline{d_j}\right]_{j\in[1;N]}(p_i) &= m_{t-1}{}^\Pi(p_i)\left(1 - POD(i)\right) \\
&\quad + m_{t-1}{}^\Pi\left(\Pi\right)\left(1 - POD\left(i\right)\right) \prod_{j\in[1;N]-\{i\}} POD\left(j\right) + \ldots \\
m_t{}^\Pi \left[\overline{d_j}\right]_{j\in[1;N]}\left(\Pi_{\setminus p_i}\right) &= POD(i) \prod_{j\in[1;N]-\{i\}} POD\left(j\right) m_{t-1}{}^\Pi\left(\Pi_\cup\right) \\
m_t{}^\Pi \left[\overline{d_j}\right]_{j\in[1;N]}\left(\Pi_\cup\right) &= \prod_{j\in[1;N]} m_{t-1}{}^\Pi\left(\Pi_\cup\right) \\
m_t{}^\Pi \left[\overline{d_j}\right]_{j\in[1;N]}\left(\emptyset\right) &= m_{t-1}{}^\Pi\left(\emptyset\right) + m_{t-1}{}^\Pi\left(p_i\right) POD(i) \ldots + \ldots
\end{aligned}
\tag{31}
$$

We did not developed all the expressions because this kind of model may not be applicable due to computational complexity issues (*i.e.* computation made on the powerset). A part of the mass is transferred to the $\emptyset$ due to the conflict between the

---

[2]We have $\Pi_{\setminus p_i} = \bigcup_{j\in[1;N]-\{i\}} p_j$ and $\Pi_\cup = \bigcup_{j\in[1;N]} p_j$.

sources and to a missing information: the searched object could be outside the grid. The assignment to $\emptyset$ can be compared with the probability that the searched object is outside the grid $1 - \beta$. Although this representation is more appropriate than the previous one, it is not usable if the number of cells is too large.

4.1.4 Model 3: Another model on the grid

In order to avoid having a large powerset, we propose a new way to represent the information with a bba.

4.1.4.1 The frames of discernment

Two elements are important: the presence of the searched object in a cell and the detection event. We consider three frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, the presence of the searched object in cell i.

- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the searched object in any of the cells.

- $D = \{d_1; \overline{d_1}; \ldots; d_N; \overline{d_N}\}$, the detection in a cell.

4.1.4.2 Basic belief assignments

We consider the following two bbas:

- $m_{i,t}$, the information given by the sensor in Cell i at time $t > 0$ given the information on the system at time $t - 1$.

- m_t, the information on the system at time t.

We initialize m_t:

$$m_0^\Pi(p_i) = POC(i)$$
$$m_0^\Pi\left(\bigcup_{i \in [1;N]} p_i\right) = 1 - \sum_{i \in [1;N]} POC(i) \tag{32}$$

We then have:

$$m_t^{\Pi_i}(p_i) = m_t^\Pi(p_i)$$
$$m_t^{\Pi_i}(p_i \cup \overline{p_i}) = 1 - m_t^\Pi(p_i) \tag{33}$$

As in Model 1, we use:

$$m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i) = m_{t-1}^{\Pi_i}(p_i) \cdot (1 - POD(i)) \tag{34}$$

To avoid having a large powerset, we assign the rest of the mass to the ignorance:

$$m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i}) = 1 - m_{i,t}^{\Pi_i}(p_i) \tag{35}$$

Then we apply the following ballooning extension:

$$\begin{aligned} m_{i,t}^{\Pi}\left[\overline{d_i}\right](p_i) &= m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i) \\ m_{i,t}^{\Pi}\left[\overline{d_i}\right]\left(\bigcup_{j\in[1;N]} p_j\right) &= m_{i,t}^{\Pi_i}\left[\overline{d_i}\right](p_i \cup \overline{p_i})\,; \end{aligned} \tag{36}$$

To update m_t^{Π}, we combine all the $m_{i,t}{}^{\Pi}\left[\overline{d_i}\right]$.

4.1.4.3   Information combination

To update m_t, we use the conjunctive rule of combination. Then we have:

$$m_t^{\Pi} = m_t^{\Pi}\left[\overline{d_i}\right]_{i\in[1;N]} = \otimes_{i\in[1;N]}\left(m_{i,t}{}^{\Pi}\left[\overline{d_i}\right]\right) \tag{37}$$

We defined the bbas to avoid an explosion of the number of focal elements. The belief remains on the set of Pi and on the ignorance. When we update the information, a part of the belief is transferred to the $\emptyset$. This is the consequence of the conflict between the sources and an incomplete frame of discernment.

4.2   With false alarms

In this part, we consider false alarms. This is common particularly when drones are used [7]. To represent false alarms, we use the model described by Pollock [12].

4.2.0.4 The sensor

We assume that the sensor sometimes returns false alarms, i.e. detects an object while none is present. Assuming a detection function and sweep width $W(i)$, for a given effort $z(i)$, we know $POD(i)$. The effort $z(i)$ is assumed to be discrete, indivisible, and to correspond to one sensor scan. The probability of a false alarm (or false detection) is defined by $POD'(i)$.

4.2.0.5 Information on the system

This is the information available on the searched object at time t before any effort has been applied. It can be considered as the memory of the search planning tool. We initialize it with $POC(i)$, the prior probability of containment.

4.2.0.6 The frames of discernment

Two elements are important: the presence of the search object in a cell and the observation of a bip or no bip in a cell. In this model, a bip of the sensor is considered as an uncertain detection. We therefore have three frames of discernment:

- $\Pi_i = \{p_i; \overline{p_i}\}$, the presence of the search object in cell i.

- $\Pi = \{p_1; \ldots; p_N\}$, the presence of the search object in any cell of the grid.

- $B_i = \{b_i; \overline{b_i}\}$, a bip (b_i) or a no bip $(\overline{b_i})$ in cell i.

4.2.0.7 Basic belief assignments

We consider two bbas:

- $m_{ci,t}$, the information of the sensor in Cell i at time $t > 0$.

- m_{t-1} the *a priori* information we have on the presence of the search object in Cell i at time t. It can be updated by combination with the information on the sensor at time t. It represents a kind of memory of the system.

We chose[3]:

$$
\begin{aligned}
m_{ci,t}{}^{B_i}\,[p_i]\,(b_i) &= POD(i)\\
m_{ci,t}{}^{B_i}\,[p_i]\,(\overline{b_i}) &= 1 - POD(i)\\
m_{ci,t}{}^{B_i}\,[p_i]\,(b_i \cup \overline{b_i}) &= 0\\
m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i) &= POD'(i)\\
m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(\overline{b_i}) &= 1 - POD'(i)\\
m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i \cup \overline{b_i}) &= 0
\end{aligned}
\tag{38}
$$

According to the general Bayesian theorem (23), we have:

$$
\begin{aligned}
m_{ci,t}{}^{\Pi_i}\,[b_i]\,(p_i) &= pl_{ci,t}{}^{B_i}\,[p_i]\,(b_i) \cdot \left(1 - pl_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i)\right)\\
m_{ci,t}{}^{\Pi_i}\,[b_i]\,(\overline{p_i}) &= pl_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i) \cdot \left(1 - pl_{ci,t}{}^{B_i}\,[p_i]\,(b_i)\right)\\
m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(p_i) &= pl_{ci,t}{}^{B_i}\,[p_i]\,(\overline{b_i}) \cdot \left(1 - pl_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(\overline{b_i})\right)\\
m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(\overline{p_i}) &= pl_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(\overline{b_i}) \cdot \left(1 - pl_{ci,t}{}^{B_i}\,[p_i]\,(\overline{b_i})\right)
\end{aligned}
\tag{39}
$$

Using the definition (17), we obtain:

$$
\begin{aligned}
m_{ci,t}{}^{\Pi_i}\,[b_i]\,(p_i) &= m_{ci,t}{}^{B_i}\,[p_i]\,(b_i) \cdot \left(1 - m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i)\right) = POD(i)\,(1 - POD'(i))\\
m_{ci,t}{}^{\Pi_i}\,[b_i]\,(\overline{p_i}) &= m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(b_i) \cdot \left(1 - m_{ci,t}{}^{B_i}\,[p_i]\,(b_i)\right) = POD'(i)\,(1 - POD(i))\\
m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(p_i) &= m_{ci,t}{}^{B_i}\,[p_i]\,(\overline{b_i}) \cdot \left(1 - m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(\overline{b_i})\right) = POD'(i)\,(1 - POD(i))\\
m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(\overline{p_i}) &= m_{ci,t}{}^{B_i}\,[\overline{p_i}]\,(\overline{b_i}) \cdot \left(1 - m_{ci,t}{}^{B_i}\,[p_i]\,(\overline{b_i})\right) = POD(i)\,(1 - POD'(i))
\end{aligned}
\tag{40}
$$

So:

$$
\begin{aligned}
m_{ci,t}{}^{\Pi_i}\,[b_i]\,(p_i \cup \overline{p_i}) &= 1 - \left(m_{ci,t}{}^{\Pi_i}\,[b_i]\,(p_i) + m_{ci,t}{}^{\Pi_i}\,[b_i]\,(\overline{p_i})\right)\\
m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(p_i \cup \overline{p_i}) &= 1 - \left(m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(p_i) + m_{ci,t}{}^{\Pi_i}\,\left[\overline{b_i}\right]\,(\overline{p_i})\right)
\end{aligned}
\tag{41}
$$

If the probability of a false alarm is null we obtain Model 2. Taking into account false alarms justifies the representation of ignorance. Indeed, a bip can indicate the presence as well as the absence of the search object (false detection). Assuming multiple drones, we can combine the information of several cells [7] with the following ballooning:

---

[3]ex: $m_{ci,t}{}^{B_i}\,[p_i]\,(b_i)$ as the value of the bba for b_i given p_i at time t, on the frame of discernment B_i.

$$m_{ci,t}^{\Pi} [\beta_i] (p_i) = m_{ci,t}^{\Pi_i} [\beta_i] (p_i)$$

$$m_{ci,t}^{\Pi} [\beta_i] \left(\bigcup_{j \in [1;N]-\{i\}} p_j \right) = m_{ci,t}^{\Pi_i} [\beta_i] (\overline{p_i})$$

$$m_{ci,t}^{\Pi} [\beta_i] \left(\bigcup_{j \in [1;N]} p_j \right) = m_{ci,t}^{\Pi_i} [\beta_i] (p_i \cup \overline{p_i})$$

(42)

with $\beta_i \in B_i$. We initialize m_t^{Π} with the POC:

$$m_0^{\Pi} (p_i) \quad = \quad POC(i)$$

$$m_0^{\Pi} \left(\bigcup_{i \in [1;N]} p_i \right) \quad = \quad 1 - \sum_{i \in [1;N]} POC(i)$$

(43)

Hence we can combine these bbas to update m_{t-1}^{Π}.

4.2.0.8 Combination of information

We use the conjunctive rule of combination to update the information:

$$m_t^{\Pi} = m_t^{\Pi} [\beta_i]_{i \in [1;N]} = m_{t-1}^{\Pi} \underset{i \in [1;N]}{\otimes} m_{cj,t}^{\Pi} [\beta_j]$$

(44)

with $\beta_i \in B_i$.

Within belief functions theory, we can easily combine pieces of information coming from several sources. However, the number of focal elements that can ultimately reach the size of the powerset is an important problem since it leads to computational issues.

This mathematical framework is also useful for expressing uncertainty, ignorance and conflict between several sources of information.

5 Illustrations

In this part, we illustrate the comparative behaviors of the four models proposed in the representation of uncertainty and the combination of several sources of information.

5.1 Comparison of the models without false alarms

A way to compare belief function theory with probability theory is to transform belief functions into probability ones through the pignistic probability, as defined by Equation (18). Hence, instead of maximizing the POS like in the probabilistic case, we can maximize the average product of the pignistic probability of the event p_i by the conditional pignistic probability of d_i given p_i. Therefore, in Model 1 we maximize [4]:

$$
\begin{aligned}
& \sum_{i\in[1;N]} betP^{\Pi_i}_{i,t-1}(p_i)\, betP^{D_i}_{ci,t}[p_i](d_i) \\
&= \sum_{i\in[1;N]} \left(\sum_{A\in 2^{\Pi_i},p_i\in A} \frac{m^{\Pi_i}_{i,t-1}(A)}{|A|\left(1-m^{\Pi_i}_{i,t-1}(\emptyset)\right)} \sum_{A\in 2^{D_i},d_i\in A} \frac{m^{D_i}_{ci,t}[p_i](A)}{|A|\left(1-m^{D_i}_{ci,t-1}[p_i](\emptyset)\right)} \right) \\
&= \sum_{i\in[1;N]} \left(\frac{m^{\Pi_i}_{i,t-1}(p_i)}{1-m^{\Pi_i}_{i,t-1}(\emptyset)} + \frac{m^{\Pi_i}_{i,t-1}(p_i\cup\overline{p_i})}{2\left(1-m^{\Pi_i}_{i,t-1}(\emptyset)\right)} \right) POD(i)
\end{aligned}
\tag{45}
$$

For Models 2 and 3 we maximize:

$$
\begin{aligned}
& \sum_{i\in[1;N]} betP^{\Pi}_{t-1}(p_i)\, betP^{D_i}_{ci,t}[p_i](d_i) \\
&= \sum_{i\in[1;N]} \left(\sum_{A\in 2^{\Pi},p_i\in A} \frac{m^{\Pi}_{t-1}(A)}{|A|\left(1-m^{\Pi}_{t-1}(\emptyset)\right)} \sum_{A\in 2^{D_i},d_i\in A} \frac{m^{D_i}_{ci,t}[p_i](A)}{|A|\left(1-m^{D_i}_{ci,t-1}[p_i](\emptyset)\right)} \right) \\
&= \sum_{i\in[1;N]} \left(\sum_{A\in 2^{\Pi},p_i\in A} \frac{m^{\Pi}_{t-1}(A)}{|A|\left(1-m^{\Pi}_{t-1}(\emptyset)\right)} POD(i) \right)
\end{aligned}
\tag{46}
$$

For simulation purposes, we assume that the sensor follows the exponential detection function (6). To initialize the search, we use a grid of POC, of W, of area and a global amount of available effort Z (determined at 100), as shown in Figure 8.
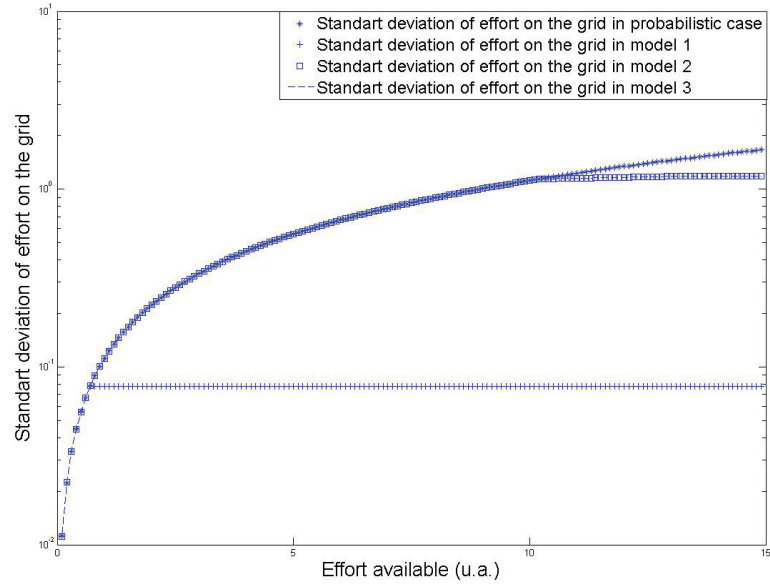
---

[4]See Model 1 for notations.

Figure 8: *Parameters of the problem on a grid*

If the effort is infinitely divisible, we may use the de Guenin's algorithm [15] (*cf.* Appendix A).

Figure 9: *Standard deviations of the effort as a function of the total effort allocated*

When we use probabilities to represent uncertainty, we obtain the results in Figure 10(a).

The results obtained with Model 1 are shown in Figure 10(b). Models 2 and 3 use the same prior information therefore the distribution of effort is the same in both cases.

(a) Original *POC*.

(b) Model 1

(c) Model 2

(d) Model 3

Figure 10: *Repartition of effort with the de Guenin algorithm and updated probability with the original probabilistic model and evidential models 1, 2 3*

However, since they are not updated in the same fashion the resulting information is different (See Figures 10(c) and 10(d)).

To evaluate the models, it can be interesting to study the distribution of information on the grid. We can use the standard deviation to verify if the distribution of a parameter is spatially homogeneous. We fix W to a uniform value in all the grid cells. In order to decide in which model the information on the searched object's location is degraded the fastest, we only use the information on the location of the searched object to allocate effort.

We used a 4×4 grid for the test with equal cell areas of 1 and equal sweep widths W of 1. Furthermore, the grid of the POC is defined with a bivariate Gaussian distribution, as defined in Section 2.1.1 (Appendix B).
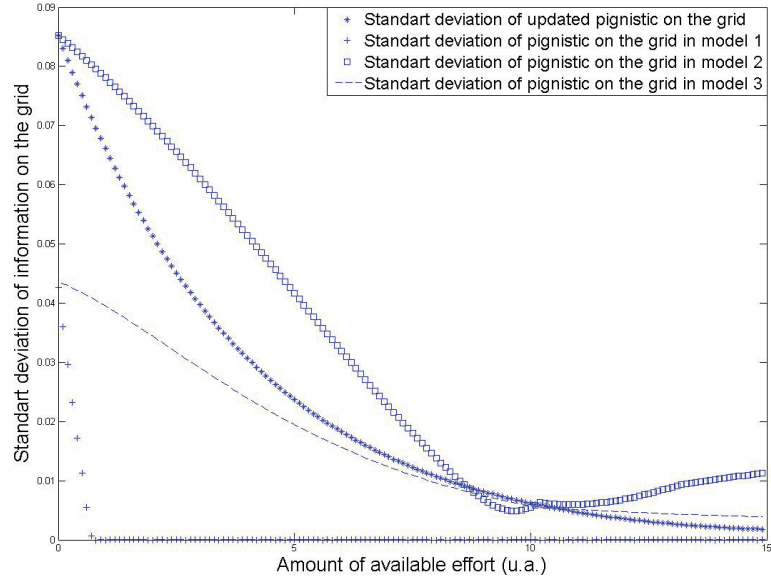
Figure 11: *Regarding the information on the presence of the searched object as a function of the total effort available*

In Figure 9, we observe that the standard deviation of the effort is the same for Models 2 and 3 since the same prior information is used. In Model 1, the standard deviation is less important, so the effort is more homogeneously distributed on the grid. For each model, there is a threshold above which an additional effort will not

change the standard deviation of effort. This indicates that the information used to distribute the effort is not relevant anymore. As a matter of fact, we observe in Figure 11) that this occurs when the spatial standard deviation of the information on the missing object after an update is near zero. In Model 2, the increase of the curve above the threshold is due to a computational approximations required when working with a too large number of focal elements.

5.2   Model with false alarms

We study the consequences of a bip or a no bip in this model. We begin by settling a grid of POC, of area, of POD and of POD'. We use the data of the previous section to define the POC and the area and define the grid of POD and POD' (See Figure 12).


Figure 12: *The probability of a bip following a sensor scan of the cell*

Referring to Figure 8, we first consider that the sensor in the cell of area 81 returns a bip as illustrated in Figure 13. The pignistic probability that the searched object is in this cell therefore increases.

Figure 13: *Updated pignistic probability*

Next, we assume that the sensor in the cell of area 27 does not return a bip. The pignistic probability on the grid is less heterogeneous (See Figure 13). The behavior observed with these two examples must not be considered representative since the evolution of the pignistic probability depends on the *ratio POD/POD′*. To verify this, we conducted tests (*cf.* Appendix C) on a 4×4 grid on which the pignistic probability is uniform. We set two ROC curves that represent the behaviors of a sensor used by day or by night as represented in Figure 14.
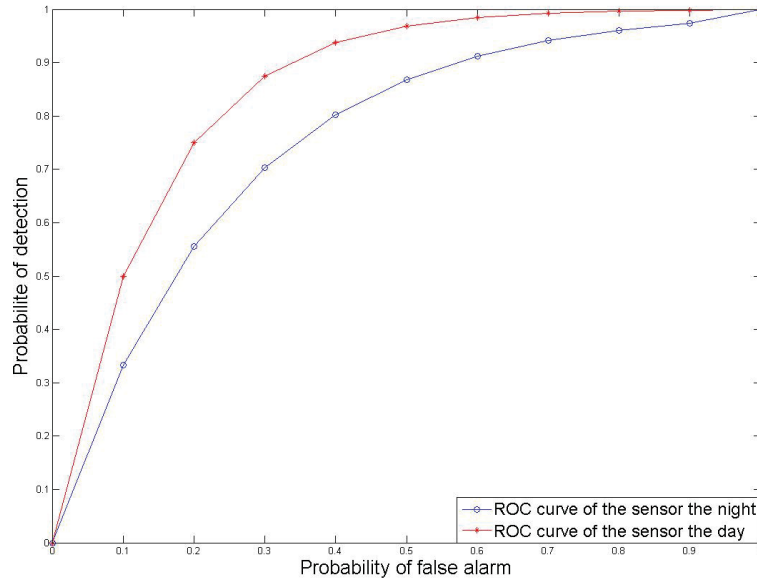
Figure 14: *ROC curves of the sensor*

We now assume that the sensor scans the search area for each point of the ROC curves and that random bips are generated. We obtain the number of bips for each point of the curves shown in Figure 15 and the spatial distribution of pignistic probability at the end of search shown in Figure 16. According to the point we use on the ROC curves, the information we obtain on the plane location is more or less relevant. If we do not have relevant prior information on plane location, the more informative point of a ROC curve is the one where the *ratio POD/POD′* is the biggest.

Among all the models we developed, the most interesting ones appear to be Models 2 and 4. However, the use of belief functions in these models require a particular
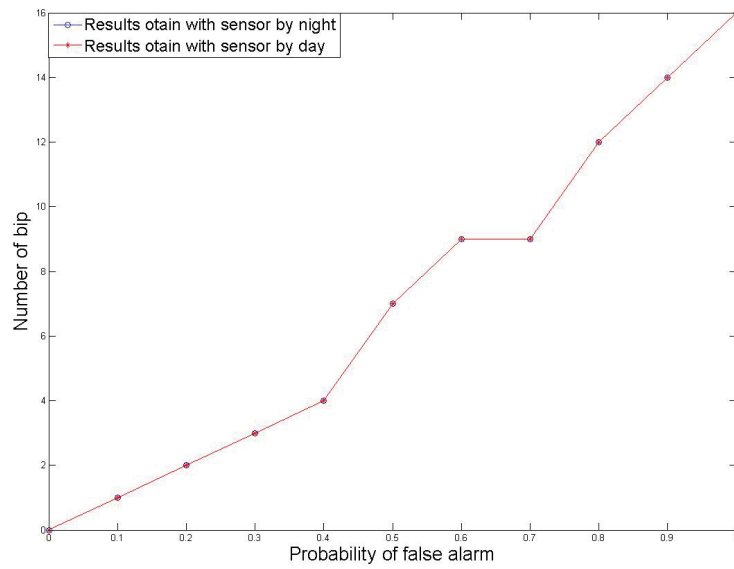
Figure 15: *Number of bips as a function of the sensor's configuration*

attention to avoid the "explosion" of the number of focal elements created during the combination process.
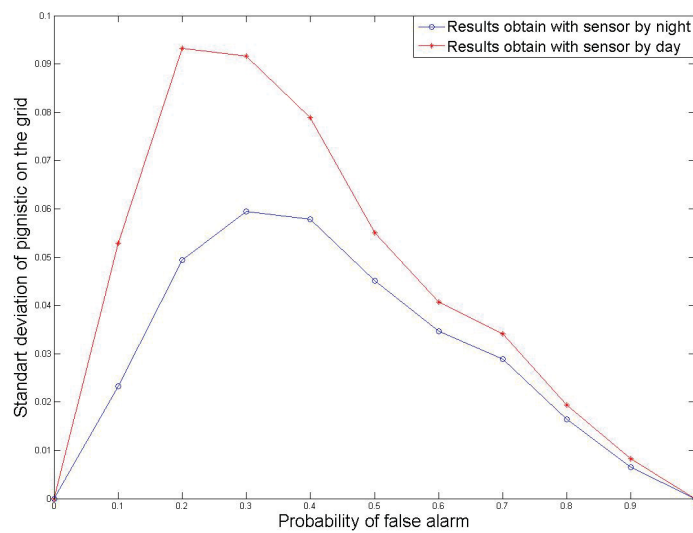
Figure 16: *Standard deviation in the spatial distribution of pignistic probability as a function of the sensor's configuration*

6 Conclusions

This report proposes an original way to model and manage the information in a Search And Rescue (SAR) problem. Instead of using the traditional probabilistic framework we propose a representation of information based on belief functions theory which allows to model and combine several sources of information in a finer and richer way than probabilities do.

As a background introduction to this work, we first summed up the state of the art in optimal search theory, introducing models and concepts commonly used in SAR applications, and presented the belief functions theory. We proposed several models for the SAR problem using this framework which account or not the false alarms, either at the cell or at the grid level. We compare the results obtained in information combination and update.

We observed that the allocation of effort on the search area highly depends on the model used, which highlight the impact of the theoretical framework used. The model also affects the way prior information on location is updated after mission planning. It appears that in some cases, using belief functions to update information and split the effort is more relevant than using probabilities. However, the framework of belief functions theory may involve computational complexity and thus the model must be selected with care.

As a conclusion, the results obtained are promising and encouraging. The uncertainty is better handled within the framework of belief functions theory, and the model has a clear impact on the effort allocation over the search area. The quality is planned to be quantified and analyzed in further works.

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Annex A: De Guenin's algorithm

This program was written for MATLAB 7.5.0 (R2007b). The detection law is exponential (*cf.* eq.(6)).

```
function [phi,POS] = algodeGuenin(POC,w,area,phitotal)



lambda = 1; % parameter to estimate
epsilon = 0.001;
ok = 0; % to know if we continu the algo.
delta =lambda/2; % Step of the dichotomy
phi = zeros(size(POC)); % Intitialization of effot on cells



while(ok == 0 )
    % if x is in the search area
    tmp = POC-lambda./(w./area);
    indice = find(tmp>0);
    % No effort in the cell
    phi = zeros(size(POC));
    % To be sure we have something in the search area
    test = size(indice);
    if( test(1) ~= 0)
        % Repartition of the effort
        phi(indice) = 1./(w(indice)./area(indice)) ...
            .*log((w(indice)./area(indice)) ...
            .*POC(indice)/lambda);
    end
    % To know if we must continue
    philambda = sum(phi(indice));
    if(phitotal+epsilon<philambda)
        % Increase lambda
        lambda = lambda + delta;
        delta = delta/2;
    elseif(philambda<phitotal-epsilon)
        % Decrease lambda
        lambda = lambda - delta;
```

```
            delta  =  delta /2;
        else
            ok  =  1;
        end
end

% Compute POS
POD  =  1−exp(−w.∗phi );
POS  =sum(sum(POD.∗POC) ) ;
```

Annex B: Distribution of effort according to the models (1,2,3).

This program was written for MATLAB 7.5.0 (R2007b). The detection law is exponential (*cf.* eq.(6)).

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
close all
clear all


% Init POC
mu = [0  0]; % Mean
Sigma = [1  0;  0  1]; % Standard deviation
% Set the grid
x1 = -3:2:3;  x2 = -3:2:3;
[X1,X2] = meshgrid(x1,x2);
POC = mvnpdf([X1(:)  X2(:)],mu,Sigma);
POC = reshape(POC,length(x2),length(x1));
% Beta
prob=0.8;
% Normalization of POC
POC = POC/(sum(sum(POC)))*prob;
% w uniform on the grid
w = ones(size(POC));
% area uniform on the grid
area = ones(size(POC));
% Set amount of effort
phito=0.1;

% For several amounts of available effort
for indice=1:150
    phitotal=phito*(indice-1);

    % Probabilistic model
    [phi,POS] = algodeGuenin(POC,w,area,phitotal);
    POD = 1-exp(-w.*phi);
```

```matlab
POC1 = POC.*(1-POD);

% Model 1
% Pignistic at the beginning
BETP = 1/2+POC/2;
% Distribution of effort
[phi1,POS] = algodeGuenin(BETP,w,area,phitotal);
% Pignistic update
POD = 1-exp(-w.*phi1);
BETP1 = BETP.*(1-POD);

% Model 2
% Pignistic at the beginning
BETP2 = POC+(1-prob)/(length(x2)*length(x1));
% Distribution of effort
[phi2,POS] = algodeGuenin(BETP2,w,area,phitotal);
% Initialize bba
POC2=reshape(POC,length(x2)*length(x1),1);
mm{1}=[POC2;1-sum(sum(POC2))];
FF{1}=zeros(length(POC2)+1,length(POC2));
FF{1}(1:length(POC2),1:length(POC2))=eye(length(POC2));
FF{1}(length(POC2)+1,1:length(POC2))=ones(1,length(POC2));
POD = 1-exp(-w.*phi2);
POD2=reshape(POD,length(x2)*length(x1),1);
% Update bba
% We use a T. Denoeux's and de Ph. Smets' Matlab library
for ind=2:length(POD2)+1
    mm{ind}=[POD2(ind-1);1-POD2(ind-1)];
    FF{ind}=zeros(2,length(POD2));
    FF{ind}=ones(2,length(POD2));
    FF{ind}(1,ind-1)=0;
end
% Combination of bba
K=2^length(POC2);
version='out';
[m,Fv,F,C]=comb_coars(mm,FF,K,version);
% Compute Pignistic
BETP3=zeros(size(POC2));
```

```
gg=sum(F');
for i=1:length(POC2)
    for ind=1:length(m)
        if(F(ind,i)==1)
            BETP3(i)=BETP3(i)+m(ind)/(gg(ind)*(1-m(1)));
        end
    end
end
BETP3 = reshape(BETP3,length(x2),length(x1));

% Model 3
BETP4=BETP2;
phi3=phi2;
% Update POC
POC3 = POC2.*(1-POD2);
% bba after combinating
m5=POC3;
m5_ignorance=1; % Mass on ignorance
for i=1:length(POC3)
    m5_ignorance=m5_ignorance*(1-POC3(i));
    for j=1:length(POC3)
        if(i~=j)
            m5(i)=m5(i)*(1-POC3(j));
        end
    end
end
% Mass on the empty set
m5_empty=1-(sum(sum(m5))+m5_ignorance);
% Pignistic
BETP5=m5+m5_ignorance/((1-m5_empty)*length(m5));
BETP5 = reshape(BETP5,length(x2),length(x1));

% reshaping data
POC1_r=reshape(POC1,length(x2)*length(x1),1);
BETP1_r=reshape(BETP1,length(x2)*length(x1),1);
BETP3_r=reshape(BETP3,length(x2)*length(x1),1);
BETP5_r=reshape(BETP5,length(x2)*length(x1),1);
phi_r=reshape(phi,length(x2)*length(x1),1);
```

```matlab
        phi1_r=reshape(phi1,length(x2)*length(x1),1);
        phi2_r=reshape(phi2,length(x2)*length(x1),1);
        phi3_r=reshape(phi3,length(x2)*length(x1),1);
        % Compute the standard deviation
        POC1_r_sqrt(indice)=sqrt(var(POC1_r));
        BETP1_r_sqrt(indice)=sqrt(var(BETP1_r));
        BETP3_r_sqrt(indice)=sqrt(var(BETP3_r));
        BETP5_r_sqrt(indice)=sqrt(var(BETP5_r));
        phi_r_sqrt(indice)=sqrt(var(phi_r));
        phi1_r_sqrt(indice)=sqrt(var(phi1_r));
        phi2_r_sqrt(indice)=sqrt(var(phi2_r));
        phi3_r_sqrt(indice)=sqrt(var(phi3_r));
end

% Draw
figure;
semilogy(0:0.1:phitotal,phi_r_sqrt,'*');
hold on;
semilogy(0:0.1:phitotal,phi1_r_sqrt,'+');
semilogy(0:0.1:phitotal,phi2_r_sqrt,'s');
semilogy(0:0.1:phitotal,phi3_r_sqrt,'--');
xlabel('Effort_available_(u.a.)');
ylabel('Standard_deviation_of_effort_on_the_grid');
h = legend('Standard_deviation_of_effort_on_the_grid_in_...
_____probabilistic_case', ...
  'Standard_deviation_of_effort_on_the_grid_in_model_1', ...
  'Standard_deviation_of_effort_on_the_grid_in_model_2', ...
  'Standard_deviation_of_effort_on_the_grid_in_model_3',4);
set(h,'Interpreter','none');
figure;
plot(0:0.1:phitotal,POC1_r_sqrt,'*');
hold on;
plot(0:0.1:phitotal,BETP1_r_sqrt,'+');
plot(0:0.1:phitotal,BETP3_r_sqrt,'s');
plot(0:0.1:phitotal,BETP5_r_sqrt,'--');
xlabel('Effort_total_disponible_(u.a.)');
ylabel(''Standard deviation of information on the grid');
h = legend(''Standard deviation of updated pignistic  ...
```

```
                    on the grid'',␣...
'Standard deviation of pignistic on the grid in model 1',
'Standard␣deviation␣of␣pignistic␣on␣the␣grid␣in␣model␣2',
'Standard␣deviation␣of␣pignistic␣on␣the␣grid␣in␣model␣3',4);
set(h,'Interpreter','none');
```

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Annex C: Model with false alarms(4).

This program was written for MATLAB 7.5.0 (R2007b). The detection law is exponential (*cf.* eq.(6)).

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

close all
clear all

% Initalize POC
mu = [0 0]; % Mean
Sigma = [1 0; 0 1]; % Standard deviation
% Init grid
x1 = -3:2:3; x2 = -3:2:3;
POC = ones(length(x2),length(x1));
% Beta
prob=0.8;
% Normalization of POC
POC = POC/(sum(sum(POC)))*prob;
% Pignistic at the beginning
BETP = POC+(1-prob)/(length(x2)*length(x1));
% Init bba
POC1=reshape(POC,length(x2)*length(x1),1);
mm{1}=[POC1;1-sum(sum(POC1))];
FF{1}=zeros(length(POC1)+1,length(POC1));
FF{1}(1:length(POC1),1:length(POC1))=eye(length(POC1));
FF{1}(length(POC1)+1,1:length(POC1))=ones(1,length(POC1));
% Points on ROC curves
POD_f0 = 0:0.1:1; % Prob of false alarm
POD_f1 = [POD_f0,POD_f0];
POD1(1) = 0;
POD2(1) = 0;
for ind=1:9
    % Prob bip search object by day
    POD1(ind+1) = POD1(ind)+(1/3)*(1-POD1(ind));
    % Prob bip search object by night
```

```matlab
        POD2(ind+1) = POD2(ind)+1/2^ind;
end
POD1(11) = 1;
POD2(11) = 1;
POD = [POD1,POD2];
% Random bip
alea = rand(length(x2)*length(x1),1);
% Number of false alarms
nbbip =zeros(length(POD));
% FOR ALL THE POINTS
for ind=1:length(POD)
    % Probability of having a bip
    Pbit{ind} = POC1*POD(ind)+ ...
            (ones(size(POC1))-POC1)*POD_f1(ind);
    % bba linked to the sensor
    for index=2:length(POC1)+1
        % If bip
        if((alea(index-1))<(Pbit{ind}(index-1)))
            nbbip(ind) = nbbip(ind)+1;
            mm{index}=[POD(ind)*(1-POD_f1(ind)); ...
                    POD_f1(ind)*(1-POD(ind)); ...
                    1-(POD(ind)*(1-POD_f1(ind)) ...
                    +POD_f1(ind)*(1-POD(ind)))];
            FF{index}=zeros(3,length(POC1));
            FF{index}(1,index-1)=1;
            FF{index}(2:3,1:end)=ones(2,length(POC1));
            FF{index}(2,index-1)=0;
        else
            mm{index}=[POD_f1(ind)*(1-POD(ind)); ...
                    POD(ind)*(1-POD_f1(ind)); ...
                    1-(POD(ind)*(1-POD_f1(ind))+ ...
                    POD_f1(ind)*(1-POD(ind)))];
            FF{index}=zeros(3,length(POC1));
            FF{index}(1,index-1)=1;
            FF{index}(2:3,1:end)=ones(2,length(POC1));
            FF{index}(2,index-1)=0;
        end
    end
```

```matlab
        % Combination of bba
        K=2^length(POC1);
        version='out';
        [m, Fv, F, C]=comb_coars(mm, FF, K, version);
        % Compute updated pignistic
        BETP1=zeros(size(POC1));
        gg=sum(F');
        for i=1:length(POC1)
            for indice=1:length(m)
                if (F(indice,i)==1)
                    BETP1(i)=BETP1(i)+m(indice)/(gg(indice)*(1-m(1)));
                end
            end
        end
        % Standard deviation
        BETP1_r_sqrt(ind)=sqrt(var(BETP1));
end
% trace
figure;
plot(POD_f0, POD1, '-ob');
hold on;
plot(POD_f0, POD2, '-*r');
xlabel('Probability_of_false_alarm', 'fontsize', 20);
ylabel('Probability_of_detection', 'fontsize', 20);
h = legend('ROC_curve_of_the_sensor_by_night', ...
            'ROC_curve_of_the_sensor_by_day', 2);
set(h, 'Interpreter', 'none');
figure;
plot(POD_f0, BETP1_r_sqrt(1:11), '-ob');
hold on;
plot(POD_f0, BETP1_r_sqrt(12:22), '-*r');
xlabel('Probability_of_false_alarm', 'fontsize', 20);
ylabel('Standard_deviation_of_pignistic_on_the_grid', ...
        'fontsize', 20);
h = legend('Results_obtained_with_sensor_by_night', ...
            'Results_obtained_with_sensor_by_day', 2);
figure;
plot(POD_f0, nbbip(1:11), '-ob');
```

```
hold on ;
plot ( POD_f0 , nbbip ( 12:22 ) , '−∗r ' ) ;
xlabel ( ' Probability of false alarm ' , ' fontsize ' ,20);
ylabel ( 'Number of bips ' , ' fontsize ' ,20);
h = legend ( ' Results obtained with sensor by night ' ,    . . .
             ' Results obtained with sensor by day ' , 2 ) ;
```

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In this report, we study the expressiveness of the theory of belief functions in the particular case of Search And Rescue (SAR) operations. The theory of belief functions is a mathematical framework, which is used, as the probabilities, to represent the imperfections of a source of information. These imperfections are of different natures: reliability, vagueness, uncertainty, to name only a few. Representing and dealing with these imperfections in a SAR context is a major issue especially when one needs to combine and update information coming from heterogeneous and subjective sources.

After having reminded some background results of the theory of optimal search, we have suggested some approaches of the SAR problem founded on the theory of belief functions.

Dans ce rapport, nous étudions l'expressivité offerte par la théorie des fonctions de croyance pour le problème de la planification d'une mission de recherche et sauvetage. La théorie des fonctions de croyance est un formalisme mathématique qui, au même titre que les probabilités, est utilisé pour représenter les imperfections d'une source d'information. Ces imperfections peuvent être liées à la non-fiabilité de la source, l'imprécision ou l'incertitude de l'information issue de la source. Modéliser toutes ces nuances pour un problème de recherche et sauvetage peut s'avérer fort utile pour combiner et mettre à jour les informations, surtout lorsque les informations dont on dispose pour définir les localisations possibles de l'objet disparu et la capacité à le retrouver sont de nature subjective.

Après avoir rappelé des résultats classiques issus de la théorie de la recherche op- timale, nous avons suggéré plusieurs approches de modélisation du problème de recherche et sauvetage dans le cadre la théorie des fonctions de croyance.

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belief functions; fusion; SAR; search; rescue; Koopman

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