Surveillance Planning in the International Littoral

A linear programming approach

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Abstract

A recognized problem in the field of surveillance is how to optimally schedule the deployment of a limited number of surveillance assets over a large operating area to provide the most information. Ensuring decision-makers achieve the maximum utility of their assets is essential for effective surveillance planning. Developing a core decision model that supports multiple objectives and a wide variety of operational constraints is required to construct a flexible decision aid. Ensuring a decision aid can support a wide variety of decision-maker considerations is critical to its extended use in an evolving operational environment. This paper provides a flexible linear programming model formulation that could form the core decision model of a surveillance planning decision aid. A detailed example is provided to illustrate its utility.

Résumé

La prévision de manière optimale du déploiement d'un nombre limité de biens de surveillance sur une importante aire d'opération, dans le but de fournir le plus d'information possible, représente l'un des problèmes connus du domaine de la surveillance. Il est essentiel pour la planification efficace de la surveillance, de s'assurer que les décideurs tirent profit au maximum de l'utilisation de leurs biens. Le développement d'un modèle de décision de fond qui appuie de multiples objectifs et une grande variété des contraintes opérationnelles est requis pour créer un outil d'aide à la décision souple. De plus, s'assurer que cet outil peut appuyer les nombreuses considérations des décideurs est essentiel à son utilisation étendue dans un environnement opérationnel changeant. Le présent rapport fournit la formulation d'un modèle de programmation linéaire souple qui pourrait constituer le modèle de décision de fond de l'outil d'aide à la décision lié à la planification de la surveillance. Un exemple détaillé démontre son utilité.

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Executive summary

Surveillance Planning in the International Littoral: A linear programming approach

Mark A. Stoddard; DRDC Atlantic TM 2010-294; Defence R&D Canada – Atlantic; December 2010.

Background: When operating in foreign littoral waters, one may be subjected to constraints on the duration of surveillance activities, frequency of surveillance activities, or simply restricted access to specific locations. This paper proposes the use of a general multi-period linear optimization model that utilizes the temporal, spatial, and spatiotemporal attributes of the operating area to construct surveillance plans in a constrained environment. The model formulation presented in this paper allows for the consideration of multiple decision maker objectives, and the constraints are flexible enough to consider changing resource limitations, political and legal restrictions, and geospatial priorities.

Results: The results collected in this paper illustrate the decision utility of the described model formulation to support decision making in the international littoral. Emphasis has been placed on the presentation of results to enable easy interpretation by decision makers. The optimal surveillance schedule produced in this report reflects the decision maker's objective to maximize a hypothetical cumulative Probability of Location (POL) of a Vessel of Interest (VOI) while operating within specifically defined operational constraints. In addition to the surveillance schedule, a minimal amount of analysis was performed on the model results to illustrate how additional questions can be answered using the developed model. The additional questions asked in this paper address the effects of additional resources on the cumulative POL, and changes to the level of operating constraint (political and geospatial).

Significance: The model presented in this paper provides a flexible model formulation to support multi-period surveillance planning in the international littoral. This model allows a decision maker to plan over longer planning horizons, providing a greater amount of continuity to surveillance plans. In addition to greater planning continuity, the model allows for consideration of a wide range of operational constraints. Operating in the international littoral brings its own unique set of considerations and constraints. In order to assure decision makers are getting maximum utility out of their available surveillance assets, it is essential to develop decision aids that support the construction of optimal surveillance plans that comply with these operational constraints.

Future plans: In order to incorporate the model presented in this paper into an integrated decision application, streamlining the process of constructing attribute data sets, running the model, and displaying the results are essential. Integrating the model output with an existing planning environment that can handle geo-referenced data such as C2PC would allow the results to be displayed graphically, providing a more user-friendly model output. The next step is to identify potential operational use-cases for this surveillance planning approach using linear programming. Once a use-case has been identified, the question of how to best integrate the model into an existing planning and decision making environment can be answered.

Sommaire

Surveillance Planning in the International Littoral: A linear programming approach

Mark A. Stoddard; DRDC Atlantic TM 2010-294; R & D pour la défense Canada – Atlantique; décembre 2010.

Introduction ou contexte : Lors des opérations dans les eaux littorales étrangères, le personnel peut être assujetti à des contraintes liées à la durée des activités de surveillance, à la fréquence des activités de surveillance ou simplement à l'accès limité de certains endroits. Le présent rapport propose l'utilisation d'un modèle général d'optimisation linéaire multipériode qui se sert d'attributs temporels, spatiaux et spatio-temporels d'une zone d'opération pour élaborer des plans de surveillance dans un environnement restreint. La formulation du modèle qui est présenté dans ce rapport permet de tenir compte de la considération de nombreux objectifs des décideurs. Par ailleurs, les contraintes sont assez souples pour examiner les restrictions changeantes de ressources, les restrictions légales et politiques et les priorités géospatiales.

Résultats : Les résultats recueillis dans le rapport montrent l'utilité de la décision de la formulation du modèle décrit pour appuyer la prise de décision sur le littoral international. L'accent a été mis sur la présentation des résultats afin que les décideurs aient le plus de facilité à les interpréter. Le plan de surveillance optimal produit dans ce rapport reflète l'objectif des décideurs visant à maximiser la probabilité hypothétique cumulative de l'endroit d'un navire d'intérêt, tout en effectuant des opérations en tenant compte de restrictions opérationnelles précises. En plus du plan de surveillance, une courte analyse a été effectuée sur les résultats du modèle afin de montrer comment il est possible de répondre à des questions supplémentaires à l'aide du modèle développé. Ainsi, les questions supplémentaires posées dans le présent rapport traitent des effets des ressources additionnelles sur la probabilité hypothétique cumulative et sur les changements apportés au niveau des contraintes opérationnelles (politiques et géospatiales).

Importance : Le modèle présenté dans le présent rapport fournit une formulation de modèle souple afin d'appuyer la planification de la surveillance multipériode sur le littoral international. Ce modèle permet à un décideur de planifier pendant de plus longues périodes de temps, tout en offrant une continuité plus importante pour les plans de surveillance. En plus d'offrir une meilleure continuité de planification, le modèle permet de considérer un vaste éventail de contraintes opérationnelles. Les opérations sur le littoral international présentent un ensemble unique de considérations et de contraintes. Pour s'assurer que les décideurs tirent profit au maximum de l'utilisation de leurs biens de surveillance, il est essentiel de développer des outils d'aide à la décision qui appuient l'élaboration de plans de surveillance optimale en fonction de ces contraintes opérationnelles.

Plans futures : Afin d'ajouter le modèle présenté dans le présent rapport à l'application de décision intégrée, il faut simplifier le processus de création d'ensembles de données d'attribut, faire l'essai du modèle et afficher les résultats. L'intégration du modèle présenté à l'environnement de planification actuel, qui peut tenir compte des données géoféréfencées (p. ex., C2PC), permettrait d'afficher les résultats à l'aide de graphiques et fournirait un modèle plus convivial pour les utilisateurs. La prochaine étape vise à cerner des cas d'utilisation possible aux

fins d'opérations pour cette approche de planification de la surveillance en utilisant une programmation linéaire. Une fois que des cas auront été cernés, il sera possible de répondre à la question liée à la façon optimale d'intégrer le modèle à la planification et à l'environnement de prise de décision en place.

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Table of contents

List of figures

List of tables

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1 Introduction

With the development of the Canada First Defence Strategy, the Canadian Forces has identified the ability to lead and/or conduct a major international operation for an extended period as a required core capability as we plan for the future [1]. Canada continues to increase its participation in the global fight against piracy; therefore, the Canadian Forces (CF) finds itself operating in the international littoral. When conducting surveillance operations in foreign littoral waters, one may be subjected to constraints on the duration of surveillance activities, frequency of surveillance activities, or simply restricted access to specific coastal locations.

On 10 December 1982, the United Nations Convention of the Law of Sea (UNCLOS) [2] set a comprehensive regime of law and order in the world's oceans and seas, thus establishing rules governing all uses of the oceans and their resources. Section 1 - Article 2 of the UNCLOS discusses the legal status of the territorial sea (not exceeding 12 nautical miles), of the air space over the territorial sea and its bed and subsoil [2]. Most notably for the purpose of this paper is the extension of sovereignty to the air space over the territorial sea. The extension of sovereignty defined in the UNCLOS does not preclude the right for innocent passage through territorial waters, although, several activities with a direct bearing on surveillance planning have been identified as violations of innocent passage. These include:

1. Any act aimed at collecting information to the prejudice of the defence or security of the coastal state;

2. The launching, landing or taking on board of any aircraft;

3. The launching, landing or taking on board of any military; and

4. Any other activity not having a direct bearing on passage.

In addition to these steady state conditions, Section 3 – Article 25 of the UNCLOS allows any coastal state to temporarily suspend innocent passage through specified areas of its territorial waters. Furthermore, the United Nations Security Council (UNSC) may periodically pass resolutions that if adopted, may over-ride portions of the UNCLOS. A noted example is UNSC Resolution 1816 [3], applies to the territorial waters of Somalia. By the terms of resolution 1816 (2008), which was unanimously adopted, the Council decided that the States cooperating with Somalia's transitional Government would be allowed, for a period of six months, to enter the territorial waters off Somalia and use "all necessary means" to repress acts of piracy and armed robbery at sea, in a manner consistent with relevant provisions of international law. Article 25 and UNSC Resolution 1816 highlight how an operating area can continuously evolve as conditions change and planning boundaries expand and contract.

Given these planning challenges, there is a need to develop robust decision aids that support surveillance planning in the international littoral. Understanding the operational constraints, and how they influence our operations, is a critical enabler of effective command decision making in the international littoral. In areas where surveillance activities are heavily constrained due to political / legal restrictions, decision aids must exist that allow us to explore alternate futures. To demonstrate this requirement, we will refer back to UNSC Resolution 1816, and to better understand the impact of UNSC Resolution 1816, a decision maker may want to answer the following questions:

1. What is the impact of UNSC Resolution 1816 on our current activities?

2. How should we modify our current surveillance policy to take into account Resolution 1816?

3. Do we require additional assets or a different type of asset?

4. Do we have to modify the critical information requirements for decision makers to support the changes to our operating area?

Having the decision aids to address high level questions is critical to sustaining *in-situ* operations in the international littoral where you are exposed to many external factors and an evolving operating environment. Further complicating the problem is the decision maker's goal to construct multi-period surveillance plans that provide a greater amount of planning continuity, while exploiting the unique attributes of the operating area, to produce the most information [4].

2 Operating Area Attributes

Operating area attribute data can come from a variety of sources, and in a variety of formats. The attributes can be temporal or spatial in nature, or for the purpose of this paper, they can exist as spatiotemporal attributes. Constructing spatiotemporal data is a complex task involving intricate issues, such as the representation of an object's position in time, and spatial attributes that change values depending on specific locations in the time domain [5].

For example, in response to the increased levels of piracy off the coast of Somalia, The United Nation Institute for Training and Research (UNITAR) has begun to extensively analyze Somali pirate activity. The approach taken by the UNITAR is to conduct studies on the number of reported incidents, their spatial distribution, density, and temporal variation throughout the year. Figure 1 depicts a UNITAR pirate attack density plot for the Gulf of Aden, observed during 2008. The data used to generate Figure 1, and the other data sets being produced by the UNITAR, are perfect examples of operating area attribute data which can be exploited for surveillance planning. Understanding the spatial distribution and the density of reported acts of piracy is invaluable information to a decision maker tasked with planning surveillance activities. Combining this information, and turning it into a spatially defined attribute of the operating area can enable optimal surveillance planning.

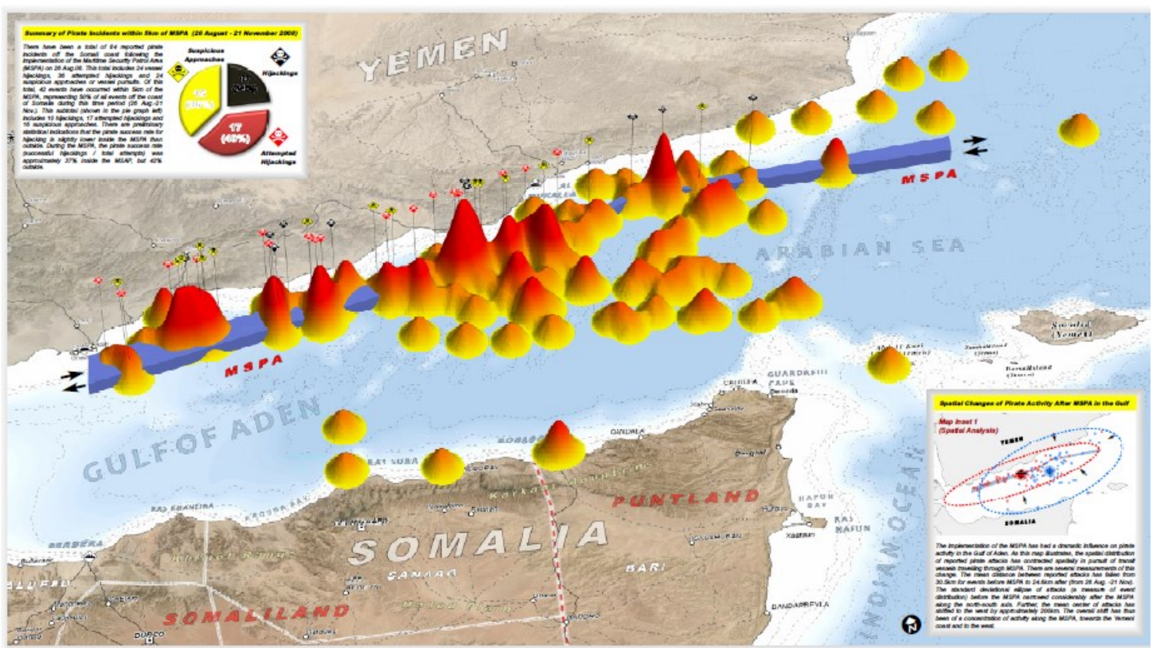

Figure 1: Pirate Attack density in the Gulf of Aden (2008), UNITAR

Attributes are easily assigned when an operating area is broken up into a collection of smaller, unique operating area management cells. A simple method of breaking up an operating area is to superimpose a grid over the operating area, such as the latitude / longitude gridding shown in Figure 2. Each map cell will have its own unique spatial attributes, and could be managed independently. If this was the case, the assignment of a management prescription to each map cell would be considered a tactical-level decision, the reason for which is that the decision made only concern the optimal management of a single map cell, while ignoring the rest of the operating area. Operational-level decisions exist when the operating area is managed as a collection of individual management cells. In this case, the objective is not the optimal management of an individual map cell, but the optimal management of the entire operating area.

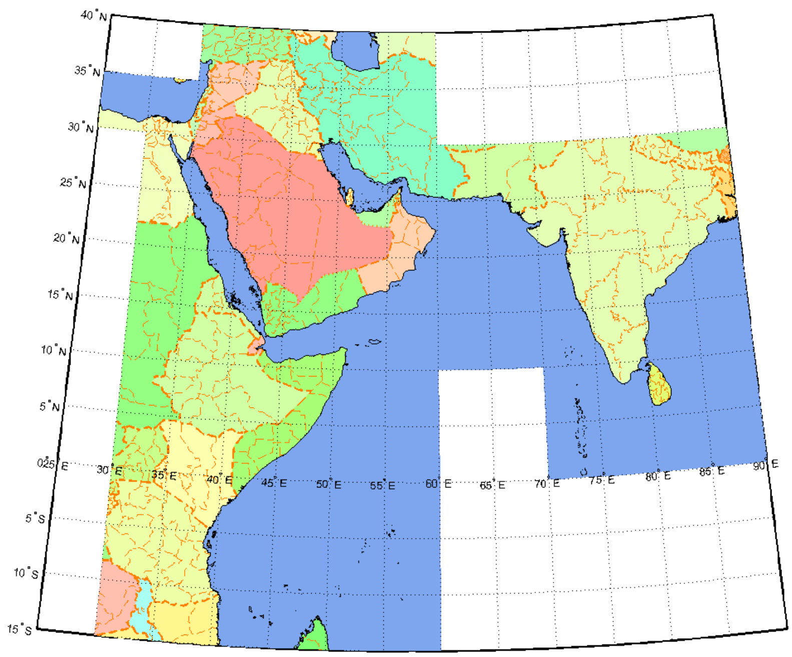

Figure 2: Base map of operating area with superimposed grid, VOIR User Manual [6]

The problem of multi-period surveillance scheduling is often complicated by constraints on the number of operating area map cells that can be observed at any point in time, due to resource constraints. This problem is not important if we are able to observe the entire operating area at any point in time. What remains of interest to the decision maker is how to construct an optimal surveillance pattern when an observation constraint is enforced. Several additional sources of decision complexity also exist:

- The size of the operating area;
- The number of available surveillance assets and downtime;
- Changing political / legal constraints;
- Asset dependant surveillance characteristics (swath size, speed, and flight time); and
- Time-of-day (TOD), or time-of-year (TOY) variation in attribute values.

This paper will demonstrate, through the use of linear programming, how a limited number of surveillance assets can be optimally assigned to an operating area based on attributes maps. The model will consider a multi-period planning horizon incorporating spatiotemporal attributes, thereby constructing optimal multi-period surveillance patterns.

3 Linear Programming

Linear Programming (LP) is a very general optimization technique. LP was designed to be used primarily to solve managerial problems. LP is applicable to many problems concerned with the allocation of scarce resources between competing activities in order to achieve the best value of a stated objective [7]. This definition describes the situation often faced by military decision makers. The resources with which they work, be they people, aircraft, time, or money, are always limited. Many of the activities that decision makers control compete for these resources. For example, one decision maker may want to increase the number of hours spent patrolling in a specific geographic area, but then less hours would be available in another area. Another decision maker might want to dedicate one aircraft for training purposes, but then fewer aircraft would be available for patrolling. Another aircraft could be purchased, but then less money would be available to support operations. And so on. No matter what course of action they chose, decision makers always face constraints that limit the range of their options.

In practice, such problems can involve hundreds or even thousands of variables and constraints, as well as additional complications like integer variables. Even large and complex LP problems that arise in practice can, once formulated, be solved rapidly using standard computer software. Such software not only gives an optimum solution to the stated mathematical problem, but routinely provides sensitivity analysis to help inform the decision making process.

3.1 Decision Variables

Decision variables are the variables within a model that one can control and their values determine the solution of the model. In any LP model, the decision variables should completely describe the decisions to be made. This could be the number of hours spent patrolling a geographic area, or the number of aircraft dedicated to patrolling. With this in mind, one could define

$$x_1 = \text{number of hours spent patrolling}$$

$$x_2 = \text{number of aircraft dedicated to patrolling}$$

Decision variables are most often non-negative and either have a real or integer value. Non-negativity implies that the decision variable cannot take a negative value. For example, the variable x_1 would have to be non-negative because the number of hours spent patrolling cannot be less than zero. In addition, the variable x_1 could be either real or integer, depending on how we define units of time. If we had specified that time spent patrolling must be allocated in discrete one hour blocks then the variable x_1 would have to be integer to reflect this.

3.2 Objective Functions

The objective function in LP models is a linear function that expresses the relationship between the decision variables. A function $f(x_1, x_2, \ldots, x_n)$ of $x_1, x_2, \ldots, x_n$ is a linear function if and only if for some set of constants $c_1, c_2, \ldots, c_n$, $f(x_1, x_2, \ldots, x_n) = c_1x_1 + c_2x_2 + , \ldots, + c_nx_n$. This linear function must either be minimized or maximized to be called an objective function. The fact that an objective function for a LP must be a linear function of the decision variables has two implications [8].

1. The contribution of the objective function from each decision variable is proportional to the value of the decision variable. For example, the contribution to the objective function from allocating three hours to patrolling is exactly three times the contribution to the objective function from allocating one hour.

2. The contribution to the objective function for any variable is independent of the values of the other decision variables. For example, referring to the previously defined decision variables, no matter what the value of x_2, the allocation of x_1 patrolling hours will always contribute x_1 hours to the objective function.

3.3 Constraints

In a LP model, constraints limit the values that decision variables can take. For a constraint to be reasonable, all terms in the constraint must have the same units. Constraints are often described as a set of linear inequalities. For any linear function $f(x_1, x_2,..., x_n)$ and any number b, the inequalities $f(x_1, x_2,..., x_n) <= b$ and $f(x_1, x_2,..., x_n) >= b$ are linear inequalities. The fact that each LP constraint must be a linear inequality or linear equation has two implications.

1. The contribution of each variable to the left-hand side of each constraint is proportional to the value of the variable.

2. The contribution of a variable to the left-hand side of each constraint is independent of the values of the variable. For example, no matter how many x_1 hours we decide to spend patrolling, the number of aircraft dedicated to patrolling will use x_2 number of aircraft.

Constraints can be classified as binding or non-binding. A constraint is binding if the left-hand side and the right-hand side (*b*) of the constraint are equal when the optimal values of the decision variables are substituted into the constraint. A constraint is non-binding if the left-hand side and the right-hand side (*b*) of the constraint are unequal when the optimal values of the decision variables are substituted into the constraint.

3.4 Optimal Solution

A set of constraints bound a feasible region in which the optimal solution is contained. The optimal solution represents the set of decision variable values that maximize or minimize the objective function. This equates to searching the feasible region for the combination of decision variable values that maximize or minimize the objective function. For LP problems, the optimal solution always occurs at an extreme point in the feasible region. This depends on the fact that both the objective function and the constraints are linear functions. Therefore, finding the optimal solution equates to evaluating all the extreme points in the feasible region to determine the extreme point that minimizes or maximizes the objective function. For a description of how this is achieved, please refer to [8]. Even the large and complex LP problems that arise in practice can, once formulated, be solved rapidly using standard computer software. Such software not only gives an optimum solution to the stated mathematical problem, but routinely provides sensitivity analysis to help inform the decision making process.

4 Multi-Period Surveillance Planning Model (MPSP)

4.1 Overview

At the operational level, the goal of the decision maker is to optimally manage an entire operating area. This differs from the tactical-level goal because one is no longer concerned with the optimal management of an individual map cell. At the operational level, one must accept certain sub-optimal tactical-level decisions to realize operational-level gains [7, 9]. Often, many additional constraints exist at the operational-level that are not present in tactical-level models. Examples of these may include required observations at specific map locations regardless of the spatiotemporal attribute value; this could relate to a high value map area containing critical infrastructure. In the following paragraph we present a simple illustrative example of the MPSP using the attribute of POL.

For simplicity, assume a 5 x 3 map grid representing the operating area. Each cell in the grid is represented by a variable X_i, and each variable X_i has an associated POL (Table 1). Cells in the grid with a high POL indicate they provide greater target localization potential than cells with a low POL. Since our goal is to maximize total detection over the entire planning horizon, our objective function will attempt to select cells with a higher POL when possible. This very small problem serves to illustrate how the model works. It is too small to illustrate the complexity associated with large-scale operating areas (> 100000 map cells), and with more operational constraints. In the next section we will introduce the sets, data, decision variables, constraints, and the objective function we wish to maximize.

Table 1: Area map with associated POL values for each map cell in the operating area

X_1	X_2	X_3	X_4	X_5		0.07	0.3	0.9	0.09	0.01
X_6	X_7	X_8	X_9	X_{10}	→	0.1	0.4	0.07	0.03	0.01
X_{11}	X_{12}	X_{13}	X_{14}	X_{15}		0.9	0.05	0.01	0.01	0.01

Area Map **POL Map (period 1)**

4.2 Sets, Data, Decision Variables, and Observation Prescriptions

The following three sets are used in the MPSP:

- I = The set of all map cells *i,*
- J = The set of all observation prescriptions *j,* and
- T = The set of all time periods *t*

The data sets used in this model describe the assignment of a surveillance prescription to each map cell. In addition to the assignment of a prescription, a data set also exists that contains the attribute data for each map cell. The data contained in the attribute data set is crucial to the decision maker, as it is primarily used in the objective function. Other data includes the area of each map cell and the right hand side values for each constraint in the linear program:

- $O_{i,j,t}$ = Observation made in map cell *i*, using observation prescription *j*, in period *t;*
- $A_{i,t}$ = Attribute data for map cell *i*, in period *t;*
- $Area_i$ = Area of map cell *i;*
- NumObs = Maximum number of observations; and
- ReqObs = Required number of observations for a given map cell.

The only decision variable included in this simple model formulation represents the prescription assigned to each map cell. It is this variable that describes the timing of all surveillance actions that will occur at a particular map cell in the operation area:

- $x_{i,j}$ = Integer variable representing the assignment of prescription *j*, to map cell *i.*

A prescription can be described as the set of time periods when surveillance activity will occur at an individual map cell. For the purpose of this model formulation, an observation prescription only provides the time-periods when observation activities will occur on an individual map cell. An observation prescription does not provide the details of how the map cell is to be searched. It is the decision maker's responsibility to deploy assets and to assign the search pattern to be used. It is assumed that these decisions will achieve the associated POL for that map cell.

4.3 Objective Function

The goal of the MPSP is to construct a surveillance pattern that maximizes a given spatiotemporal attribute over multiple time periods. Assuming we have historical or simulated attribute data on an operating area, we can use this information to provide variable weightings in the following objective function:

$$\text{Maximize} \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} O_{i,j,t} \times A_{i,t} \times x_{i,j} . \tag{1}$$

In Equation 1 we sum over all time periods, map cells, and observation prescriptions, the cumulative attribute value (in this case, POL) by selecting decision variables $x_{i,j}$.

4.4 Constraints

In this simple model formulation, only the following three constraints are considered:

A constraint that ensures we conduct surveillance on the entire map cell, (2)

$$\sum_{j \in J} x_{i,j} = A_i \quad \forall i \in I,$$

a constraint that limits the number of observations we can make,

$$\sum_{i \in I} \sum_{j \in J} O_{i,j,t} \times x_{i,j} \leq NumObs \quad \forall t \in T, \tag{3}$$

and, a constraint that ensure we meet our surveillance goals,

$$\sum_{t \in T} \sum_{j \in J} O_{i,j,t} \times x_{i,j} \geq \mathrm{Re}\,qObs \quad \forall i \in I. \tag{4}$$

Equation 2 ensures that all map cells receive an observation prescription, assuring that none of the map cells in the operating area are ignored. A valid observation prescription includes a "do nothing" prescription that allows us to ignore a map cell location if desired. Equation 3 limits the total number of observations that can be made in any given period. Lastly, Equation 4 allows us to enforce a minimum number of observations made at each map cell location. Equation 4 would be used if one wanted to examine each map cell location regardless of its associated attribute value.

5 MPSP Illustrative Example

5.1 Preamble

Note: In this hypothetical example, the Canadian Forces are assumed to have control of all surveillance assets, organic and fixed wing. This example does not accurately reflect the way surveillance is conducted in the region. Instead, it is designed to simply highlight potential surveillance objectives and constraints, and illustrate the types of questions that can be answered using this modeling approach.

As part of hypothetical Canadian Forces anti-piracy commitment off the Coast of Somalia, it is assigned an operating area in which to conduct surveillance activities. The assigned operating area is broken down into 15 individual map cells (see Figure 3). Prior to arrival in the operating area, the operations commander received a detailed report on vessel of interest (VOI) activity in each of the 15 individual map cells. The detailed report provides a 24-hour description of the probability of locating a VOI in each map cell, if surveillance activities were to occur. Each 24-hour period is partitioned into three 8-hour periods, with the corresponding probability of locating a VOI during that given time of day (see Table 1). Due to resource constraints, the Canadian Forces are only able to observe 5 individual map cells during an 8-hour period, regardless of their proximity to each other.

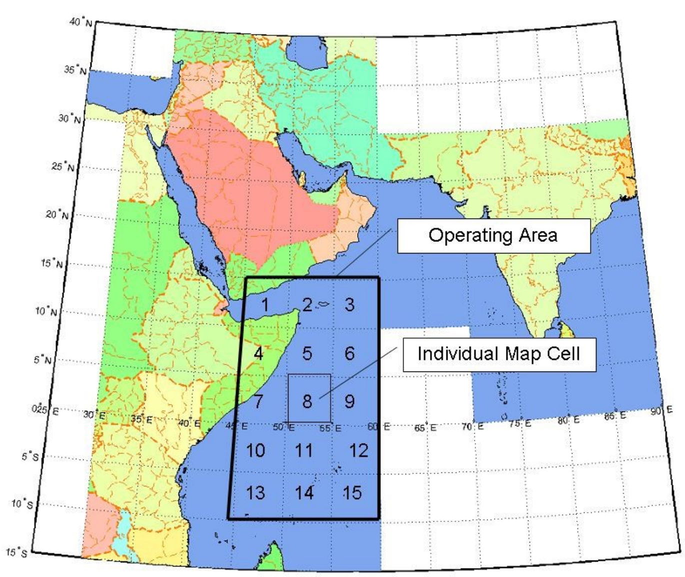

Figure 3: Example operating area divided up into individual map cells

Prior to arrival, in addition to the attribute table regarding VOI activity in the operating area, the Canadian Forces were informed of three operating area constraints (see Figure 4):

1. Coastal areas may only be searched for 8-hours in a 24-hour period (due to agreements made with local governments granting the CF access to territorial waters and air space);

2. Map cells 9, 11, and 13 lie directly on a shipping lane between Mumbai and Madagascar and have been identified as a high priority for surveillance activities. The Canadian Forces are required to search each of the three map cells for no less than two 8-hour periods over a 24-hour span; and

3. In support of the current NATO Afghan mission, it has been requested that each navy operating in the area have at least 1 surveillance asset that can be tasked for an 8-hour period to conduct surveillance of known drug smuggling routes, if requested.

Table 2: Attribute table containing the probability of Locating a VOI in each operating area map cell partitioned into 3, 8-hour periods

	TIME		
MAP CELL	00:00 - 08:00	08:00 - 16:00	16:00 - 24:00
1	.6	.1	.4
2	.5	.2	.3
3	.2	.3	.1
4	.1	.2	.1
5	.5	.5	.5
6	.5	.3	.5
7	.3	.4	.3
8	.3	.3	.2
9	.2	.4	.1
10	.3	.2	.5
11	.1	.2	.2
12	.1	.1	.1
13	.2	.5	.4
14	.2	.2	.2
15	.1	.1	0

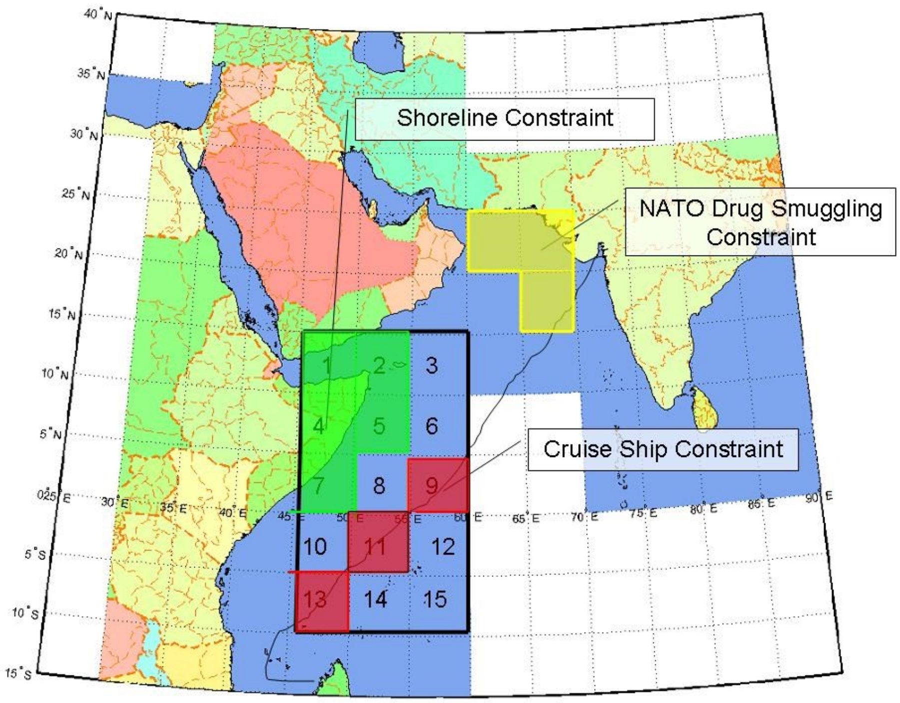

Figure 4: Updated operating area map containing strategic level objectives for the region.

Finally, the overall objective for the Canadian Forces operating in the area is to construct a 24-hour surveillance schedule that allows them to maximize the cumulative POL of a VOI, while satisfying the constraints applied to the operating area. The results of the model must provide the decision maker with a surveillance plan for each 8-hour period, and must be displayed graphically so that it can be easily interpreted. Lastly, in addition to the surveillance plan, the senior strategic analyst for the Canadian Forces would like to know the following information:

1. What is the impact of the coastline constraint on the cumulative probability of location?

2. If the surveillance constraint on the shipping lane was reduced to a single 8-hour period, what would the effect be on the cumulative probability of location?

3. If the Canadian Forces decided to dedicate one of their surveillance assets to support the drug smuggling effort, what is the effect on their existing mission?

4. Provide a basic analysis that could be used to support the Canadian Forces desire to purchase additional surveillance assets for use in the operating area.

5.2 Model Overview

In this section, the linear programming formulation of the objective function and the constraints described in the model preamble will be presented. Table 3 contains the sets, data, and decision variables for the model, and Table 4 contains the description of each of the 7 observation prescription that will be used in the model. The observation prescriptions form the basis of the surveillance plan. Each map cell will receive a prescription that details the timing of observations at that location. The model formulation in this section is similar to the model described earlier with the addition of a new set of decision variables. Due to the addition of the NATO drug constraint, a variable must be created that allows us to include the constraint in the optimization model. This variable controls when the Canadian Forces decide to assign a surveillance asset to the NATO drug mission.

5.2.1 Sets, Data, and Decision Variables

Table 3 contains a detailed list of the sets, data, and decision variables as described in the model preamble.

Table 3: Description of the sets, data, and decision variables presented in the model preamble

Item	Description
$i = 1..15$	The set of management cells
$j = 1..7$	The set of prescriptions
$t = 1..3$	The set of time periods
$O_{i,j,t}$	Data on action made at management cell *i*, using prescription *j*, in period *t*
$POL_{i,t}$	Data of probability of location of VOI at management cell *i*, in period *t*
$x_{i,j}$	Decision variable for assignment of prescription *j*, to management cell *i*
d_t	Decision variable for assignment of asset to NATO drug mission, in period *t*

Table 4: Description of tactical-level observation prescriptions available to each map cell (0 – No Action, 1 – Observation Made)

Prescription	00:00 – 08:00	08:00 – 16:00	16:00 – 24:00
1	0	0	0
2	0	1	0
3	0	0	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

5.2.2 Linear Programming Model Representation

This section contains the complete description of MPSP model formulation for this problem. The problem has been formulated as a multi-period maximization problem.

$$\text{Maximize} \sum_{t=1}^{3}\sum_{i=1}^{15}\sum_{j=1}^{7} \mathrm{P}_{i,j,t} \times POL_{i,t} \times x_{i,j} \qquad \text{(Objective Function)} \tag{5}$$

SUBJECTED TO

$$\sum_{j=1}^{7} x_{i,j} = A_i \quad \forall i = 1..15 \qquad \text{(Area Constraint)} \tag{6}$$

$$\sum_{i=1}^{15}\sum_{j=1}^{7} P_{i,j,t} \times x_{i,j} + d_t \leq 5 \quad \forall t = 1..3 \qquad \text{(Total Observation Constraint)} \tag{7}$$

$$\sum_{t=1}^{3}\sum_{j=1}^{7} P_{i,j,t} \times x_{i,j} \geq 2 \quad \forall i = 9,11,13 \qquad \text{(Shipping lane Constraint)} \tag{8}$$

$$\sum_{t=1}^{3}\sum_{j=1}^{7} P_{i,j,t} \times x_{i,j} <= 1 \quad \forall i = 1,2,4,5,7 \qquad \text{(Coastline Constraint)} \tag{9}$$

$$\sum_{t=1}^{3} d_t >= 1 \quad \forall t = 1..3 \qquad \text{(NATO Drug Constraint)} \tag{10}$$

Equation (5) states that the objective of this linear program is to maximize the probability of locating a VOI over the entire planning horizon, for the entire operating area. Equations (6) to Equation (10) provide the constraints under which our objective function is to be maximized against. Equation (6) ensures that each map cell in the operating area receives a prescription. Equation (7) ensures that we never exceed our available surveillance resources in a given period. Equation (8) enforces our cruise ship constraint, as described in the list of strategic-level constraints for the operating area. Equation (9) enforces the shoreline constraint along coastal areas. Lastly, Equation (10) ensures that we assign 1 surveillance asset to support the NATO drug mission during a 24-hour period.

5.2.3 Model Implementation and Run Statistics

Mathematical Programming Language (MPL) 4.2 by Maximal Software was used for all aspects of model development and solving [10]. MPL 4.2 is an advanced modeling system that allows the model developer to formulate complicated optimization models in a clear, concise, and efficient way. In addition, MPL works with the world's fastest optimization engines, such as CPLEX and XPRESS, and many other commercially available solvers.

The following model statistics and solution were automatically produced by MPL 4.2:

Model Statistics

Parsing time:	0.06 sec
Solver name:	CPLEX (11.2.1)
Objective value:	0.353
Solution time:	0.09 sec
Constraints:	29
Variables:	108
Integers:	108
Nonzeros:	315
Density:	10 %

Model Solution

Optimal integer solution found

MAX Cumulative POL = 35.3%

5.3 Results

5.3.1 Optimal Prescription Assignments

Table 5 contains the prescription assignments for each map cell in our observation area. The observation sequence associated with each prescription details the periods that observation activity occurred at that map cell location. The observation sequences assigned to each map cell detail the optimal solution to the MPSP example.

Table 5: Prescription assignment for each map with the associated observation sequence

Map Cell	Prescription	Observation Sequence (Period)
1	4	(1 , 0 , 0)
2	3	(0 , 0 , 1)
3	1	(0 , 0 , 0)
4	1	(0 , 0 , 0)
5	2	(0 , 1 , 0)
6	5	(1 , 0 , 1)
7	2	(0 , 1 , 0)

8	1	(0 , 0 , 0)
9	6	(1 , 1 , 0)
10	3	(0 , 0 , 1)
11	5	(1 , 0 , 1)
12	1	(0 , 0 , 0)
13	7	(1 , 1 , 1)
14	1	(0 , 0 , 0)
15	1	(0 , 0 , 0)
NATO	1	(0 , 1 , 0)

5.3.2 Operational-Level Optimal Surveillance Plan

Figures 5, 6, and 7 provide a graphical display of the optimal surveillance plan for each 8-hour period. Map cells highlighted in red indicate that the area is searched in that period. These surveillance plans satisfy all constraints and serve to maximize the cumulative probability of locating a VOI over a 24-hour period. Figure 5 provides the surveillance plan from 00:00 to 08:00, Figure 6 provides the surveillance plan from 08:00 to 16:00, and Figure 7 provides the surveillance plan from 16:00 to 24:00.

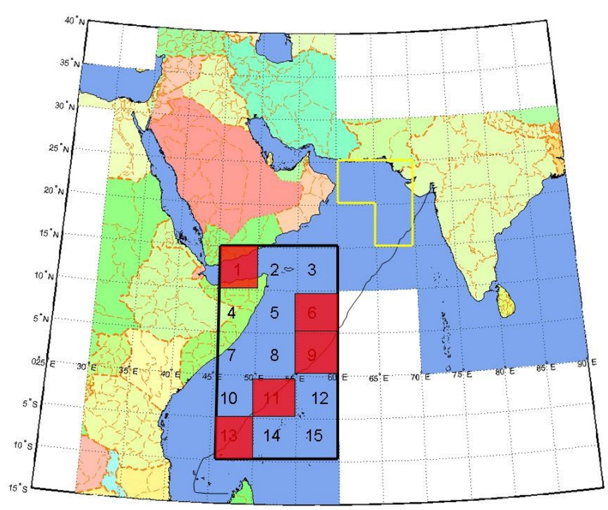

Figure 5: Graphical display of optimal surveillance plan from 00:00 to 08:00

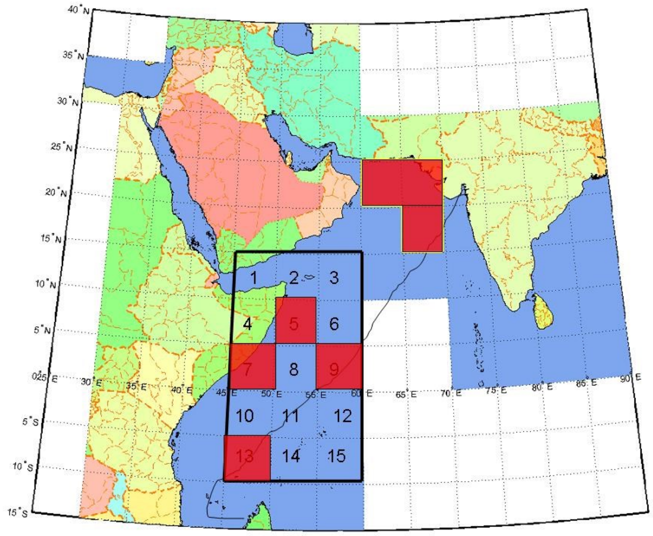

Figure 6: Graphical display of optimal surveillance plan from 08:00 to 16:00

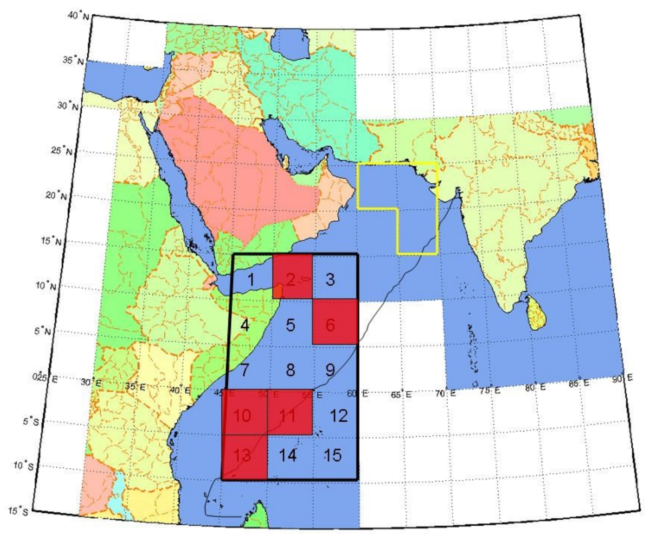

Figure 7: Graphical display of optimal surveillance plan from 16:00 to 24:00

5.4 Analysis for Senior Strategy Analyst

As described in the model preamble, additional analysis is required to satisfy a requirement of a senior strategy analyst for the Canadian Forces. Using the model developed in this section, these questions are easily answered using sensitivity analysis, which allows us to examine how variation in the model formulation or inputs quantitatively affects the output (solution) from the model. The analysis begins with question 1:

1. What is the impact of the coastline constraint on the cumulative probability of location?

a. *If there was no constraint on the number of visits we could make to coastline areas, the objective function would increase from 35.3% to 36.67%, making for an increase of only 1.4%. This indicates that, in terms of the cumulative POL, the coastline constraint has only a slight impact on the operation.*

2. If the surveillance constraint on the shipping lane was reduced to a single 8-hour period what would the effect be on the cumulative probability of location?

    a. *If our requirement to provide surveillance over the shipping lane was reduced to only a single 8-hour period in the three management cells, the objective function would increase from 35.3% to 38.67%. This accounts for an increase of 3.4%.*

3. If the Canadian Forces decided to dedicate one of their surveillance assets to support the drug smuggling effort, what would be the effect on their existing mission?

    a. *If the Canadian Forces decided to dedicate one of their surveillance assets to support the NATO mission, the cumulative probability of detection when all other original constraints were enforced would decrease to 34.67%. This accounts for a decrease of only 0.7%. This indicates that dedicating 1 of the 5 surveillance assets to support the NATO drug mission would not greatly affect the mission off the coast of Somalia.*

4. Provide a basic analysis that could be used to support the Canadian Forces desire to purchase additional surveillance assets for use in the operating area.

    a. *Based on the constraints applied to the operating area, there is no benefit to the Canadian Forces to exceed 12 surveillance assets. Figure 8 shows a plot of how the cumulative probability of location increases as the number of surveillance assets is increased. Additionally, if economics are considered, Figure 9 shows a plot of the incremental increase in the cumulative probability of location as the number of surveillance asset allocations is increased. Under the constraints enforced in our model, and with the objective of maximizing the cumulative POL, the optimal number of asset allocations would be 5, which represent the peak incremental increase in the POL as the number of asset allocations is increased.*

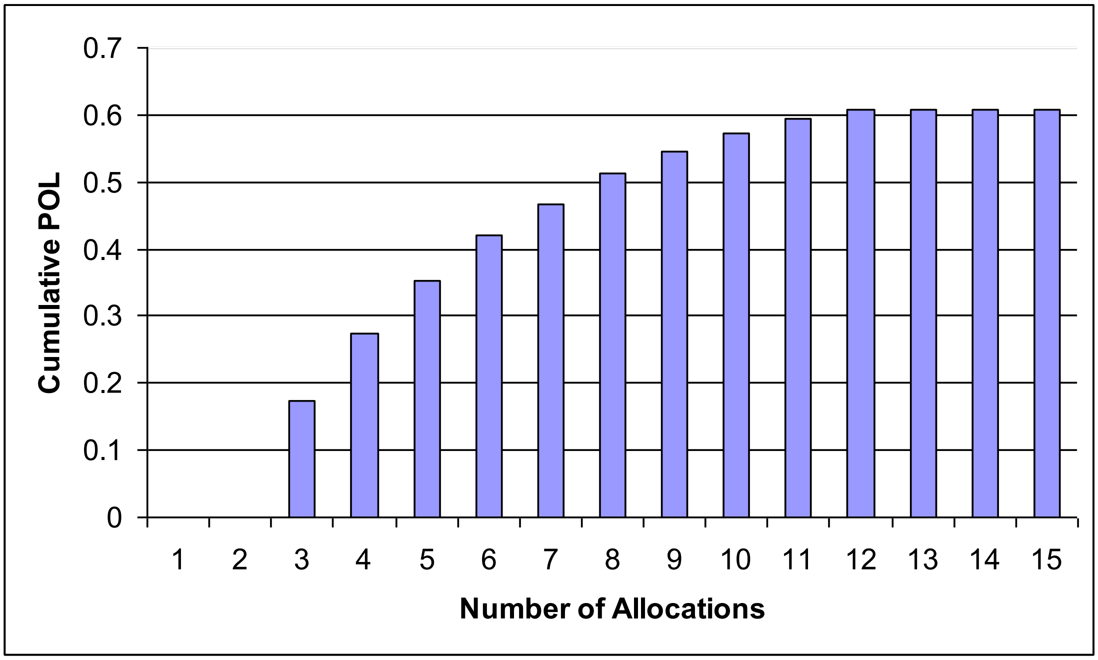

Figure 8: Plot of the Cumulative Probability of Location as the Number of Surveillance Assets Increases

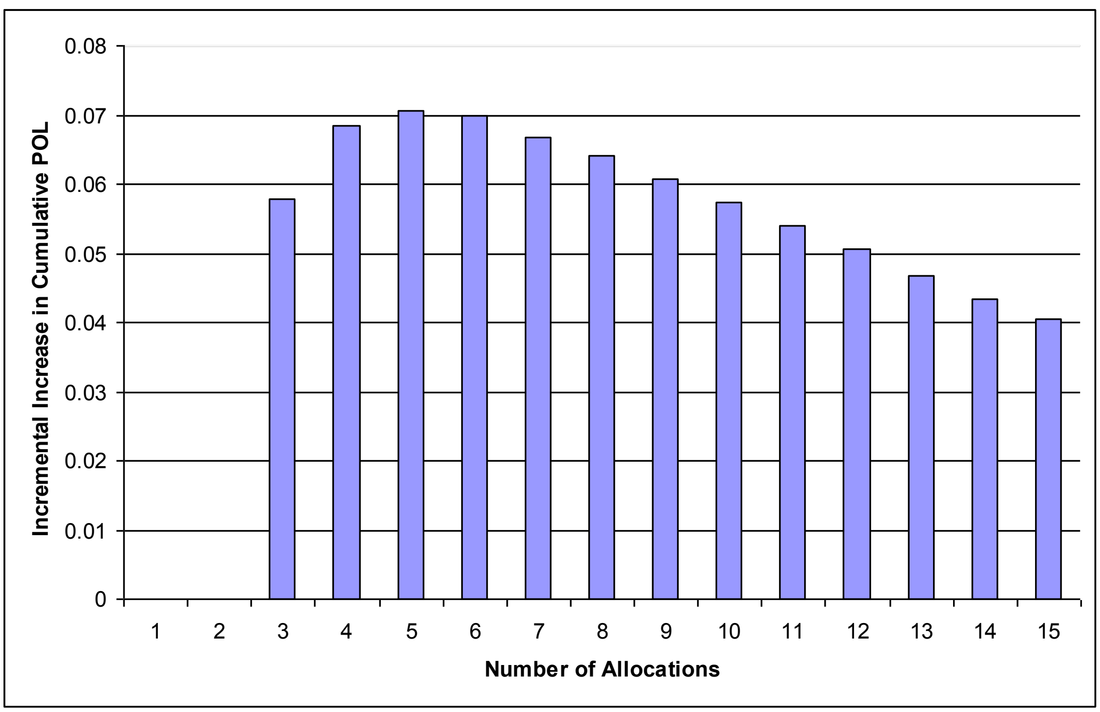

Figure 9: Plot of the incremental change in POL with respect to the number of surveillance assets allocations

6 Conclusion

Linear programming provides a powerful tool for the construction of multi-period surveillance planning models. The results of linear programming models are easily interpreted, and provide great decision utility. The use of temporal, spatial, and spatiotemporal attributes allows for the construction of robust surveillance plans based on the availability of quantitative data. By identifying and classifying attributes that can be used in optimization models, it is possible to begin to architect data requirements for decision models that support maritime domain awareness.

In addition to understanding information requirements, it is essential to construct decision aids that allow for the inclusion of a wide variety of operational constraints. As Canada increases its presence in the international littoral, it will be exposed to an evolving decision environment. As objectives change, or political instabilities increase, the Canadian Forces must ensure they have the ability to adapt. Decision makers must have the tools that allow them to continue to achieve mission objectives and to conduct and sustain operations in an uncertain environment.

The need for surveillance plans that optimize the use of limited resources is critical to the long-term sustainability, and success of surveillance operations in the international littoral. This paper provides a general model formulation that can be expanded to consider numerous objectives and a multitude of operational-level constraints.

In order to incorporate the model presented in this paper into an integrated decision application, streamlining the process of constructing attribute data sets, running the model, and displaying the results is essential. Integrating the model output with an existing planning environment that can handle geo-referenced data such as C2PC[1] would allow the results to be displayed graphically, providing a more user-friendly model output. The next step is to identify potential operational use-cases for this surveillance planning approach using linear programming. Once a use-case has been identified, the question of how to best integrate the model into an existing planning and decision making environment can be answered.

[1] Developed for the U.S. Marine Corps, C2PC displays the COP from a Global Command and Control System (GCCS)-based server or tactical data from other C2PC workstations. Users can view and edit the COP, apply overlays, display imagery, send and receive tactical messages and gain overall battlefield situational awareness.

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List of symbols/abbreviations/acronyms/initialisms

DND	Department of National Defence
DRDC	Defence Research & Development Canada
DRDKIM	Director Research and Development Knowledge and Information Management
LP	Linear Programming
MPL	Mathematical Programming Language
MPSP	Multi-Period Surveillance Planning Model
NATO	North Atlantic Treaty Organization
POL	Probability of Locating
TOD	Time of Day
TOY	Time of Year
UNCLOS	United Nations Convention on the Laws of the Sea
UNITAR	United Nations Institute for Training and Research
UNSC	United Nations Security Counsel
VOI	Vessel of Interest

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A recognized problem in the field of surveillance is how to optimally schedule the deployment of a limited number of surveillance assets over a large operating area to provide the most information. Ensuring decision-makers achieve the maximum utility of their assets is essential for effective surveillance planning. Developing a core decision model that supports multiple objectives and a wide variety of operational constraints is required to construct a flexible decision aid. Ensuring a decision aid can support a wide variety of decision-maker considerations is critical to its extended use in an evolving operational environment. This paper provides a flexible linear programming model formulation that could form the core decision model of a surveillance planning decision aid. A detailed example is provided to illustrate its utility.

La prévision de manière optimale du déploiement d'un nombre limité de biens de surveillance sur une importante aire d'opération, dans le but de fournir le plus d'information possible, représente l'un des problèmes connus du domaine de la surveillance. Il est essentiel pour la planification efficace de la surveillance, de s'assurer que les décideurs tirent profit au maximum de l'utilisation de leurs biens. Le développement d'un modèle de décision de fond qui appuie de multiples objectifs et une grande variété des contraintes opérationnelles est requis pour créer un outil d'aide à la décision souple. De plus, s'assurer que cet outil peut appuyer les nombreuses considérations des décideurs est essentiel à son utilisation étendue dans un environnement opérationnel changeant. Le présent rapport fournit la formulation d'un modèle de programmation linéaire souple qui pourrait constituer le modèle de décision de fond de l'outil d'aide à la décision lié à la planification de la surveillance. Un exemple détaillé démontre son utilité.

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Linear Programming, Surveillance, Decision Aid