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THE DERIVATION OF PLUME DISPERSION PARAMETERS FROM MEASURED THREE-DIMENSIONAL DATA

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THE DERIVATION OF PLUME DISPERSION PARAMETERS FROM MEASURED THREE-DIMENSIONAL DATA

by

H. Whaley*

ABSTRACT

It is well known that atmospheric diffusion models, however sophisticated and rigorous they may be mathematically, can only be as accurate as the input data used for model calibration and verification. In this paper, the method of finite differences is used to determine both the plume rise and the standard deviations of plume spread from three-dimensional data acquired by aerial probing. Comparisons are also made with corresponding values estimated by accepted empirical methods.

The derivation of reliable values for critical plume dispersion parameters, that are applicable to a given geographic region, significantly improves the precision of computations for specifying stack heights, selecting plant sites, and predicting ground-level pollutant concentrations.

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Direction des mines

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LA DÉRIVATION DES PARAMÈTRES DE DISPERSION DE PANACHE DES DONNÉES MESURÉES EN TROIS DIMENSIONS

par

H. Whaley*

RÉSUMÉ

C'est bien connu que les modèles de diffusion atmosphérique malgré leur sophistication mathématique peuvent seulement être aussi précis que les données en entrée utilisées pour l'étalonnage et la vérification du modèle. Dans ce rapport, l'auteur utilise la méthode de différences finies pour déterminer la montée de panache et la dérivation normale de la dispersion de panache des données en trois dimensions acquises par la méthode de sonde aérienne. Il a aussi fait des comparaisons avec les valeurs correspondantes estimées par les méthodes empiriques acceptées.

La dérivation des valeurs fiables pour les paramètres critiques de dispersion de panache qui sont applicables à une région géographique spécifique, améliore significativement la précision des calculs pour la spécification de l'hauteur des cheminées, pour la sélection des sites d'usines et pour la prédiction des concentrations de la pollution au niveau du sol.

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Nomenclature

A	- cross sectional area of plume (m^2)
C	- concentration of tracer (ppm SO_2)
q, Q	- pollutant mass flows as defined in equations (1) and (6) respectively (ppm - m^2)
h_s	- height of stack above ground at $X = 0$ (m)
S_y	- half-width of source as shown in FIG. 1 (m)
S_z	- height of plume fringe above centre of area source as shown in FIG. 1 (m)
X, Y and Z	- three-dimensional cartesian co-ordinate system denoting downwind distance, crosswind distance and height respectively (m)
Z_g	- height of ground elevation above $Z = 0$ at measuring location (m)
$\bar{y}$, $\bar{Y}$ and $\bar{z}$, $\bar{Z}$	- co-ordinates of the centre of pollutant mass flow in the crosswind and vertical dimensions respectively (m)
ΔC, ΔY and ΔZ	- finite difference forms of above defined variables (ppm or m)
$\overline{\Delta Z}$	- plume rise, $Z - h_s - Z_g$ (m)
$\overline{\Delta Z}_{C_{max}}$	- as for $\overline{\Delta Z}$, but based on the location of maximum concentration C_{max} (m)
$\Delta T/\Delta Z$	- vertical temperature gradient (°C/100m)
θ	- angle between actual traverse plane and normalized traverse plane (rad)
φ	- angle between plane of in-line stacks and plane normal to the plume axis (rad)
σ_y	- standard deviation of plume spread in the crosswind direction (m)
σ_z	- standard deviation of plume spread in the vertical direction (m)

1. INTRODUCTION

Empirical studies reported by WHALEY (1969) emphasized the need for reliable dispersion parameters which could be applied with confidence to plant siting and environmental management problems in Canada. In consequence, a comprehensive research program was initiated by the Canadian Combustion Research Laboratory (CCRL) to study the atmospheric dispersion of buoyant plumes emitted from tall stacks located in regions of Canada characterized by:

(i) land bordering large bodies of water
(ii) mountain and river valleys
(iii) flat terrain
(iv) foothill country
(v) arctic and sub-arctic.

Using an aerial probing methodology, a number of field research studies were conducted to obtain factual information on the dispersion of plumes emitted from both single and multiple sources under a variety of meteorological, topographical, and seasonal conditions. In each study, the plume was monitored in three dimensions for relevant pollution parameters using co-ordinated helicopter and vehicle-mounted instruments. Continuous tracking of the aerial probe by a navigational system employing radar principles was used to ensure that the spatial position of the plume was accurately known. Prior to and during each study, vertical profiles of temperature, wind speed, and wind direction were measured near the source using radiosonde and pilot balloons. In some instances, additional information was obtained from tower-mounted meteorological instruments.

The plume dispersion data, after reduction, were plotted as two-dimensional isopleth contour maps either in the crosswind (Y, Z) or downwind directions (X, Z) depending on the type of traversing procedure employed. These maps, which incorporate some degree of data interpretation, are used to construct isopleth drawings of the plan, side, and sectional views of the plume. The meteorological data obtained, together with synoptic weather maps, provide the background information necessary for further data interpretation.

Although data have been obtained from sources located in four of the five aforementioned geographic regions, this paper utilizes only data referred to by WHALEY et el (1971) and HIRT et al (1971) to illustrate a numerical method for deriving both the location of the plume axis and the standard deviations of plume spread from three dimensional measurements of SO_2 within plumes. The numerical method provides reliable input data for computations of plume rise and dispersion in a specific region in which only a few detailed studies of plume behaviour have been undertaken.

2. ESTIMATION OF PLUME DISPERSION PARAMETERS

2.1 Diffusion Modelling

Most diffusion models, whether derived from statistical or physical principles, are Gaussian in nature. In such a model, the gases emitted from a stack become distributed across the plume according to the Gaussian or normal distribution function. Obviously, when considered in three dimensions and neglecting axial diffusion, this concept represents a bivariate normal distribution in the plane normal to the plume axis. One of the main virtues of the normal distribution is that it can be completely defined by its standard deviation if represented in a dimensionless form. Therefore, the standard deviations of plume spread, σy and σ_z, have become an accepted method of reporting diffusion parameters in the literature. Pioneers of this concept were PASQUILL (1961 and 1962) who estimated angular standard deviations and GIFFORD (1961) who converted these to linear dimensions.

2.2 Estimation Methods

Very little information on the estimation of the standard deviations of plume spread is available in the literature. Thus, it must be assumed that estimation procedures, in general, have been very imprecise. This point has been made recently by EIMUTIS and KONICEK (1972) where an attempt to improve the estimation of standard deviations was made employing regression techniques on available dispersion data.

When using photographic or other overall means of plume definition, such as may be obtained by lidar or other ground-based scanning techniques, it is very difficult to make precise measurements of the standard deviations of plume spread, particularly at downwind distances greater than 10 stack heights from the source. For instance, photographic information regarding vertical spread, depends on the degree of visibility of the plume boundaries. In such cases, a normal distribution within the plume is assumed and the vertical spread parameter σ_z is derived by simple calculation. However, this procedure is limited in accuracy because a normal distribution may not exist, especially either on a short-term basis or close to the source.

Even when isolated point measurements within a plume are available, it is often assumed that the distribution is bivariate normal with the values of σ_y and σ_z being estimated accordingly. This method yields an estimation of the plume spread at some measurable fraction of the peak concentration. For example with an assumed normal distribution, 10% of the peak concentration will be equivalent to a plume width or thickness of 4.3 σ.

3. DERIVATION OF PLUME DISPERSION PARAMETERS FROM MEASURED DATA

3.1 Theoretical Considerations

In the CCRL program, it has been found that the voluminous data obtained by aerial probing techniques can be evaluated best numerically by a three step procedure that employs the method of finite differences. This method, which is mathematically rigorous, eliminates any discrepancies introduced by the subjective approach used previously and minimizes errors due to acquiring data by instruments that have short response times.

The first step in the method involves the reconstruction of at least three crosswind sections of the plume to show spatial concentration isopleths that are plotted from SO_2 values measured at accurately determined points in space. The second step consists of digitizing the spatial co-ordinates (Y, Z) of each SO_2 contour of the plume section to establish the co-ordinates of the centre of pollutant mass flow, $\overline{Y}$ and $\overline{Z}$, and the standard deviations, σ_y and σ_z, according to the equations given below.

The mass of pollutant flowing across the area A occupied by the plume cross section is

$$q = \iint_A C \, dy.dz, \qquad (1)$$

the centre of pollutant mass flow is the first moment:

$$\overline{y} = \frac{1}{q} \iint_A Cy \, dy.dz, \qquad (2)$$

and the variance is the second moment about the axis

$$\sigma_y^2 = \frac{1}{q} \iint_A C \, (y - \overline{y})^2 \, dy.dz. \qquad (3)$$

Similarly for the vertical dimensions,

$$\overline{z} = \frac{1}{q} \iint_A Cz \, dz.dy, \qquad (4)$$

and

$$\sigma_z^2 = \frac{1}{q} \iint_A C \, (z - \overline{z})^2 \, dz.dy. \qquad (5)$$

If these integral equations are translated into finite difference form then (1) becomes

$$Q = \sum \Delta C \sum Y \Delta Z \qquad (6)$$

and co-ordinates of the centre of mass flow (2) and (4) respectively become

$$\bar{Y} = \frac{1}{2Q} \sum \Delta C \sum Y^2 \Delta Z \qquad (7)$$

$$\bar{Z} = \frac{1}{2Q} \sum \Delta C \sum Z^2 \Delta Y \qquad (8)$$

Likewise the variances (4) and (5) respectively become

$$\sigma_y^2 = \frac{1}{3Q} \sum \Delta C \sum (Y - \bar{Y})^3 \Delta Z \qquad (9)$$

and

$$\sigma_z^2 = \frac{1}{3Q} \sum \Delta C \sum (Z - \bar{Z})^3 \Delta Y \qquad (10)$$

In the third step, Equations (6), (7), (8), (9) and (10) are further simplified, computer programmed, and used in conjunction with digital input from the crosswind contour maps to derive values for critical parameters used in plume rise and dispersion computations.

3.2 Corrections for Traversing and Source Effects

3.2.1 Inclined Traverse Data

It is impossible without prior knowledge of the plume configuration to traverse perpendicular to the plume axis. Hence, it is usually necessary to correct measured plume parameters in the crosswind (Y) direction for the inclination of the traversing plane to the ideal plane perpendicular to the axis of the plume. Geometric considerations indicate a cosine correction as shown in FIG. 1, providing the planes do not differ by more than 60°. In cases where the difference is more than 60°, experience has shown that the section measured is more axial than crosswind and the diffusion parameters measured are too distorted for correction.

3.2.2 Line Sources

The derived values of the diffusion parameters σ_y and σ_z after correction for inclination of the traverse plane may also be applied to diffusion from line sources. However, for comparison with published data on single source emissions some corrections must be applied. In the case of a source such as multiple in-line stacks the correction is quite simple and based on angular similarity. If the stacks are symmetrically spaced they may be considered to be a line source of length 2Sy. If the line of the stacks is inclined at an angle φ to the perpendicular to the plume axis and the traverse plane at an angle θ as shown in FIG. 1, then the corrected value of σ_y is:

$$\sigma_y = \sigma_y' \ \mathrm{Cos}\,\theta \left(1 - \frac{S_y \ \mathrm{Cos}\,\varphi}{\sqrt{3}\ \sigma_y' \ \mathrm{Cos}\,\theta} \right) \qquad (11)$$

where σ_y' is the uncorrected value of the horizontal standard deviation and the term $S_y/\sqrt{3}$ is the equivalent horizontal standard deviation of a line emission. It is clear from Equation (11) that source effects tend to become negligible at large distances from the source, because the term

$$\frac{S_y \ \mathrm{Cos}\,\varphi}{\sqrt{3}\ \sigma_y' \ \mathrm{Cos}\,\theta} \longrightarrow 0.$$

as the downwind distance increases.

3.2.3 Small Area Sources

In the case of small area sources a similar correction can be made to σ_y, but consideration must also be given to the correction of the measured value of σ_z. Obviously each source configuration will be different but, as an example consider the simplified case of a uniform area source, the emission profile of which represents uniform flow from an arbitrarily defined centre plane as shown in FIG. 1. If the height of the emission profile is S_z, the equivalent standard deviation of uniform profile is $S_z/2\sqrt{3}$. Hence, the correction to the vertical standard deviation is:

$$\sigma_z = \sigma_z' \left(1 - \frac{S_z}{2\sqrt{3}\ \sigma_z'}\right) \qquad (12)$$

where σ_z' is the uncorrected vertical standard deviation. Obviously, it may sometimes be necessary to apply other simplifying assumptions to particular source configurations.

Both Equations (11) and (12) maintain angular similarity with equivalent single source plumes after PASQUILL (1961) and the corrections applied are implicit in the Gaussian model. However, it must be noted that other than the traverse angle correction to σ_y', the derived values must be used in diffusion computations for a particular emission source.

4. COMPARISON OF DERIVED AND ESTIMATED PLUME DISPERSION PARAMETERS

4.1 Plume Axis

The plume axis is usually a vague term that has different interpretations. In a simple model, the plume axis in the horizontal plane (X, Y) is defined as a line in the mean wind direction and in the vertical plane (X, Z) as a line at the effective height of the plume centreline. At the other extreme, fairly detailed wind and temperature data may be used to show complex variations in the location of the axis of a model plume.

TABLE 1

COMPARISON OF DERIVED AND CALCULATED PLUME RISE VALUES

Stability Class	Neutral[1]			Stable[2]		
Mean Heat Flux/Stack (mcal/s)	16.9			7.4		
Mean Wind Speed (m/s)	11.85			11.80		
Axial Distance (km)	0.66	3.50	6.12	1.3	4.1	12.5
Plume Rise (m)						
$\overline{\Delta Z}$	66	259	213	87	66	40
$\overline{\Delta Z}_{C_{max}}$	70	188	88	128	68	103
BRIGGS (1969)	87.5	158	158	84.9	-	-
LUCAS (1963)	112	-	-	91.6	-	-
CONCAWE (after BRUMMAGE 1966)	9.9	-	-	8.1	-	-
CCRL-2 (after WHALEY 1969)	63.9	-	-	52.2	-	-
MOSES (1967)	58.4	-	-	38.8	-	-

1/After WHALEY et al (1971) see FIG. 2 and 3.
2/ After HIRT et al (1971) see FIG. 4, 5, and 6.

Referring to Section 3, it can be seen that for three-dimensional plume data, the plume axis is defined by the derived coordinates of the centre of pollutant mass flow $\bar{Y}$ and $\bar{Z}$. Examples of how the plume axis, as defined by these co-ordinates, compares with the axis defined by the peak concentration location can be seen in FIG. 2, 3, 4, 5, and 6. FIG. 2, 3, and 4 are typical crosswind sections of a plume showing SO_2 isopleths and FIG. 2, 3, and 4 are a side view and a plan view respectively derived from the relevant crosswind sections. FIG. 4, in particular, is an excellent example of the displacement of the peak concentration from the centre of pollutant mass flow.

4.2 Plume Rise

It can be seen that the derived value of $\bar{Z}$, the vertical location of the centre of pollutant mass flow, represents the height of the plume axis above the zero plane Z = 0, and consequently the plume rise $\overline{\Delta Z}$ can be defined by the equation

$$\overline{\Delta Z} = \bar{Z} - (h_s + Z_g) \qquad (13)$$

where h_s is the height of the stack above the ground at X = 0, and Z_g is the height of the ground above a reference level, Z = 0, at the location of $\bar{Z}$. This parameter, unlike the location of the peak concentration in the plume, is not prone to significant fluctuations on a short-term observation basis. Some examples of plume rise derived from measured crosswind plume sections are compared with other methods of estimation in TABLE 1.

It is interesting to note in FIG. 5 that in the case of an isothermal condition in which an elevated inversion was observed above the plume and where the terrain rose more rapidly than the inversion base, $\overline{\Delta Z}$ decreased with distance downwind as might be expected. However, the value of $\overline{\Delta Z}$ based on the peak concentration height ($\overline{\Delta Z}_{C_{max}}$) does not show this effect, nor do any of the plume rise equations in the literature.

TABLE 2

COMPARISON OF DERIVED AND CALCULATED σ VALUES

Stability Class	Neutral[1] $\Delta T/\Delta Z$ = - 0.88°C/100m			Isothermal with Elevated Inversion[2] $\Delta T/\Delta Z$ = 1.5°C/100m		
Axial Distance (km)	0.66	3.5	6.1	1.3	4.1	12.5
σ_y (finite difference)	37.4	189	517	91.7	314	1147
σ_y (10% of peak value)	59	185	285	80.7	297	954
σ_y (Pasquill-Gifford)	48[3]	210	350	63[4]	180	580
σ_z (finite difference)	18	55	184	45.4	38.4	65.2
σ_z (10% of peak value)	36	63	118	13.2	14.6	52.3
σ_z (Pasquill-Gifford)	23[3]	72	101	25[4]	51	97

1/WHALEY et al (1971)
2/HIRT et al (1971)
3/Class D Stability
4/Class E Stability

4.3 Plume Standard Deviations (σ values)

The derived values of σ_y and σ_z are shown in TABLE 2 for two atmospheric conditions, near neutral and stable with a capping inversion. Comparison of the values with those attributed to PASQUILL (1961 and 1962) and GIFFORD (1961) are made in TABLE 2.

It can be seen that under neutral conditions reasonable agreement with Pasquill-Gifford values for Class D stability are obtained by both the finite difference and the 10% of peak value methods for determining values of σ_y and σ_z. However, for stable conditions, the latter two methods give values of σ_y that are higher and values of σ_z that are lower than the corresponding Pasquill-Gifford values, particularly at distances greater than 4 km from the source. This is attributed to the fact that the isothermal layer was capped by an inversion at about 150 m above ground level which restricted the vertical diffusion and increased the lateral diffusion as illustrated in FIG. 5 and 6. As noted earlier, the restriction of vertical diffusion caused by the stable layer resulted in a decrease of plume rise with increasing distance downwind.

5. CONCLUSIONS

Plume dispersion parameters can be accurately determined from SO_2 distributions measured within the plume by using the method of finite differences. As shown in FIG. 2, 3, and 4, these numerically derived values of $\overline{Y}$, $\overline{Z}$, σ_y and σ_z effectively eliminate errors caused by subjective and arbitrary interpretation of measured data.

The analysis of detailed data by this method also minimizes errors caused by transient phenomena such as plume fragments of high concentration which produce asymmetrical concentration gradients across the plume.

The derivation of reliable values for critical plume dispersion parameters, particularly $\overline{Z}$ and σ_z, significantly improves the precision of computations for selecting stack heights and for predicting ground-level pollutant concentrations.

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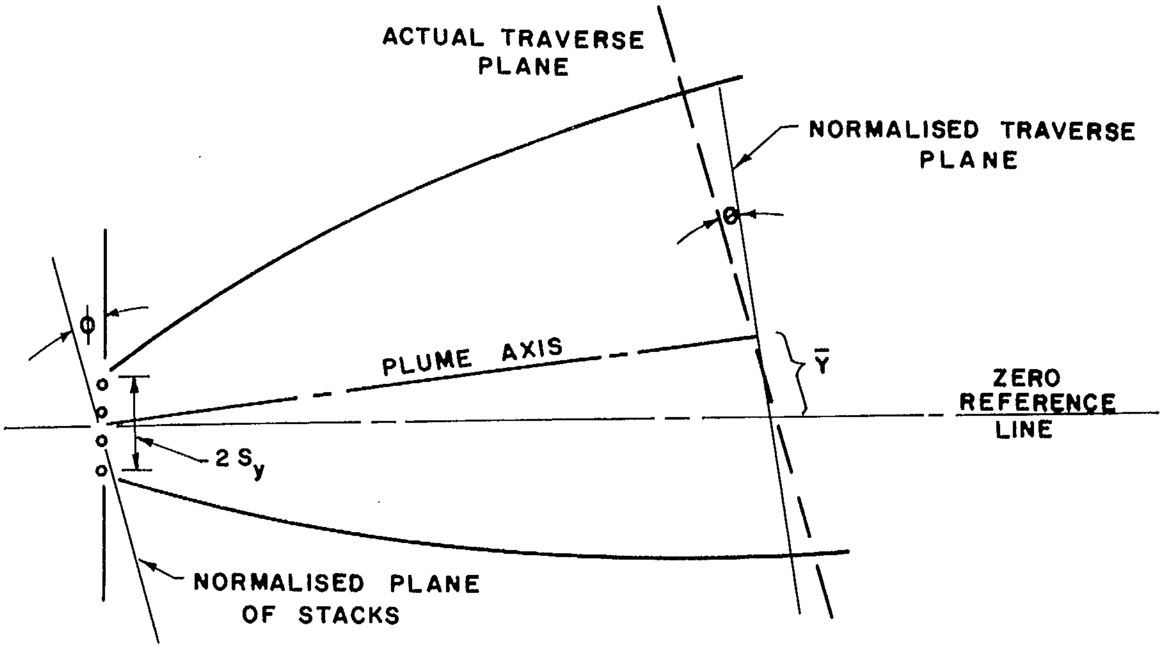
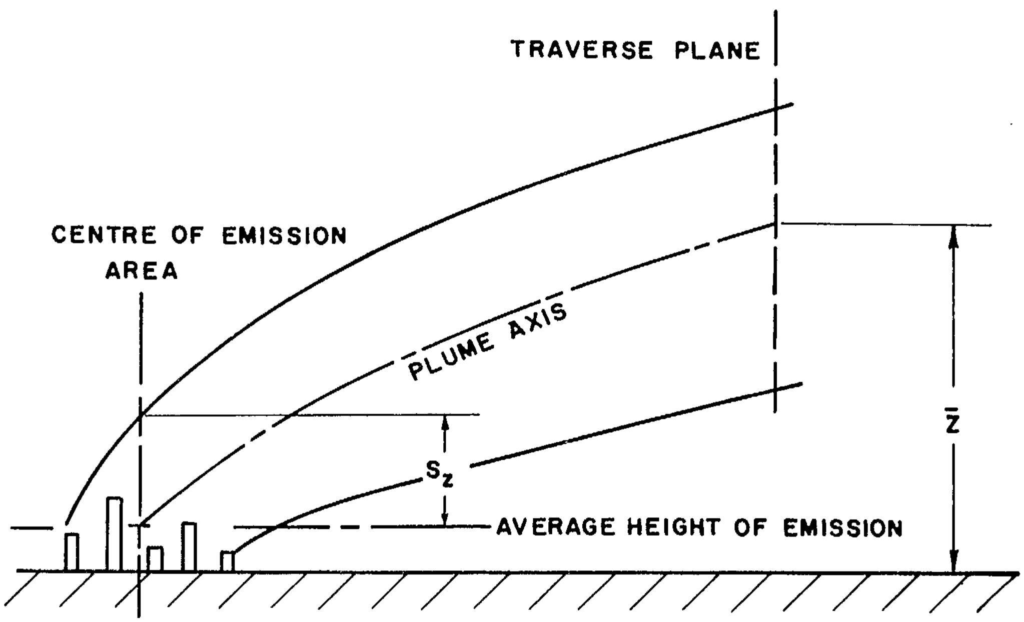


(a) Schematic plan of plume from in-line stacks.



(b) Schematic side view of plume from area source.

FIG. 1. Determination of correction factors for different traversing and source configurations.

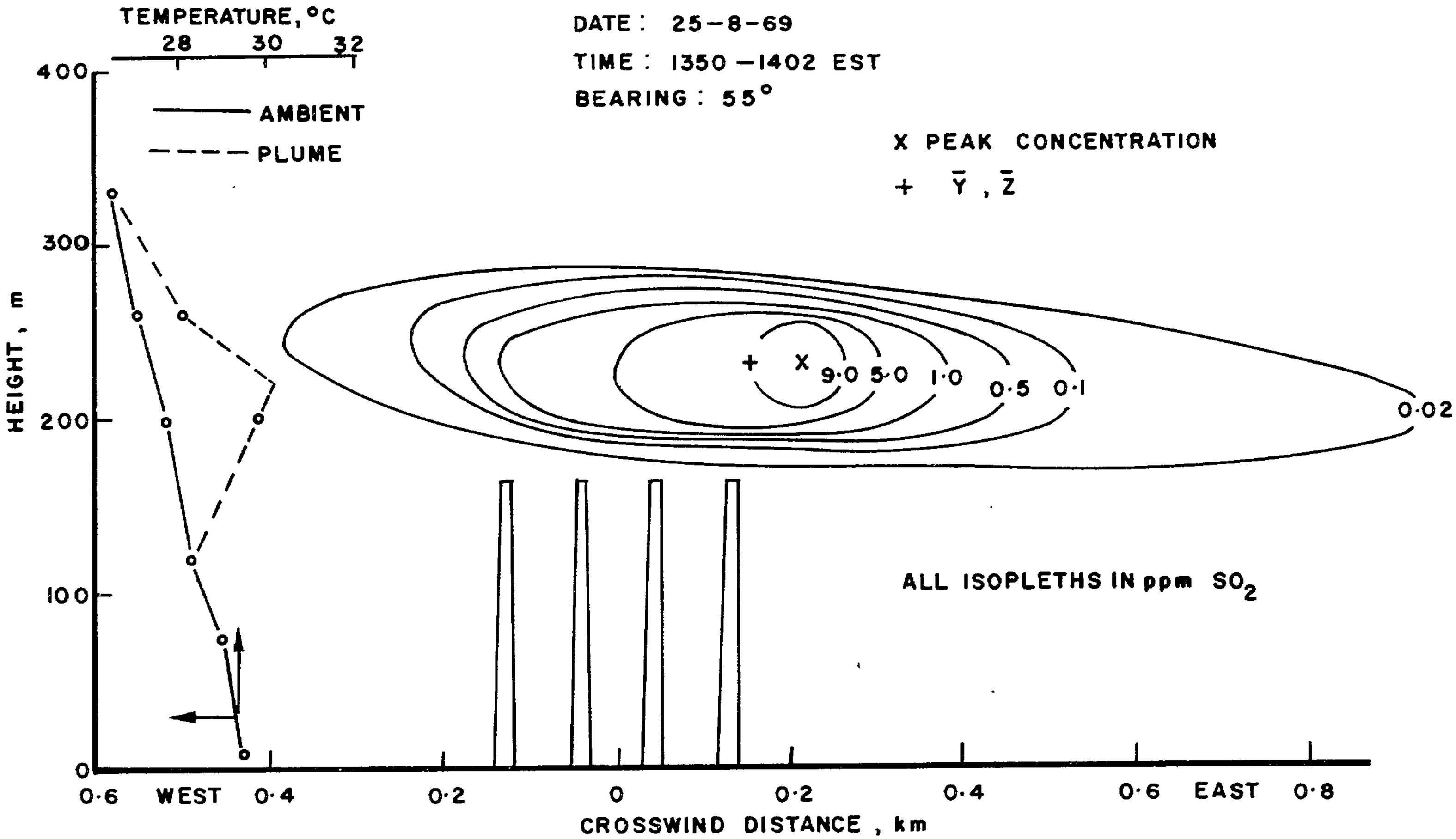


FIG. 2. Vertical crosswind section of the plume, at an axial distance of 0.66 km from the source, 25-8-69.

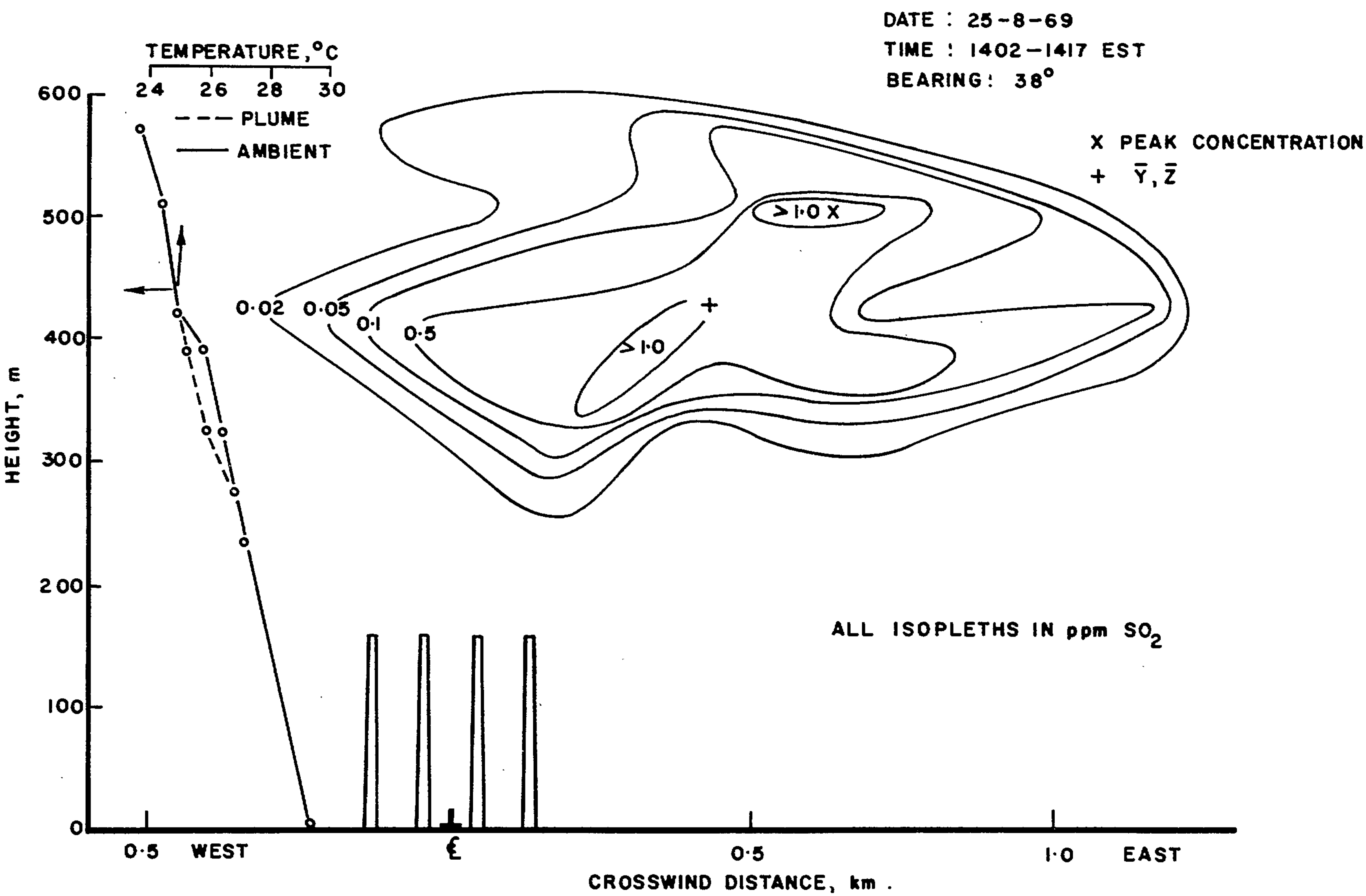


FIG. 3. Vertical crosswind section of the plume, at an axial distance of 3.5 km from the source, 25-8-69.

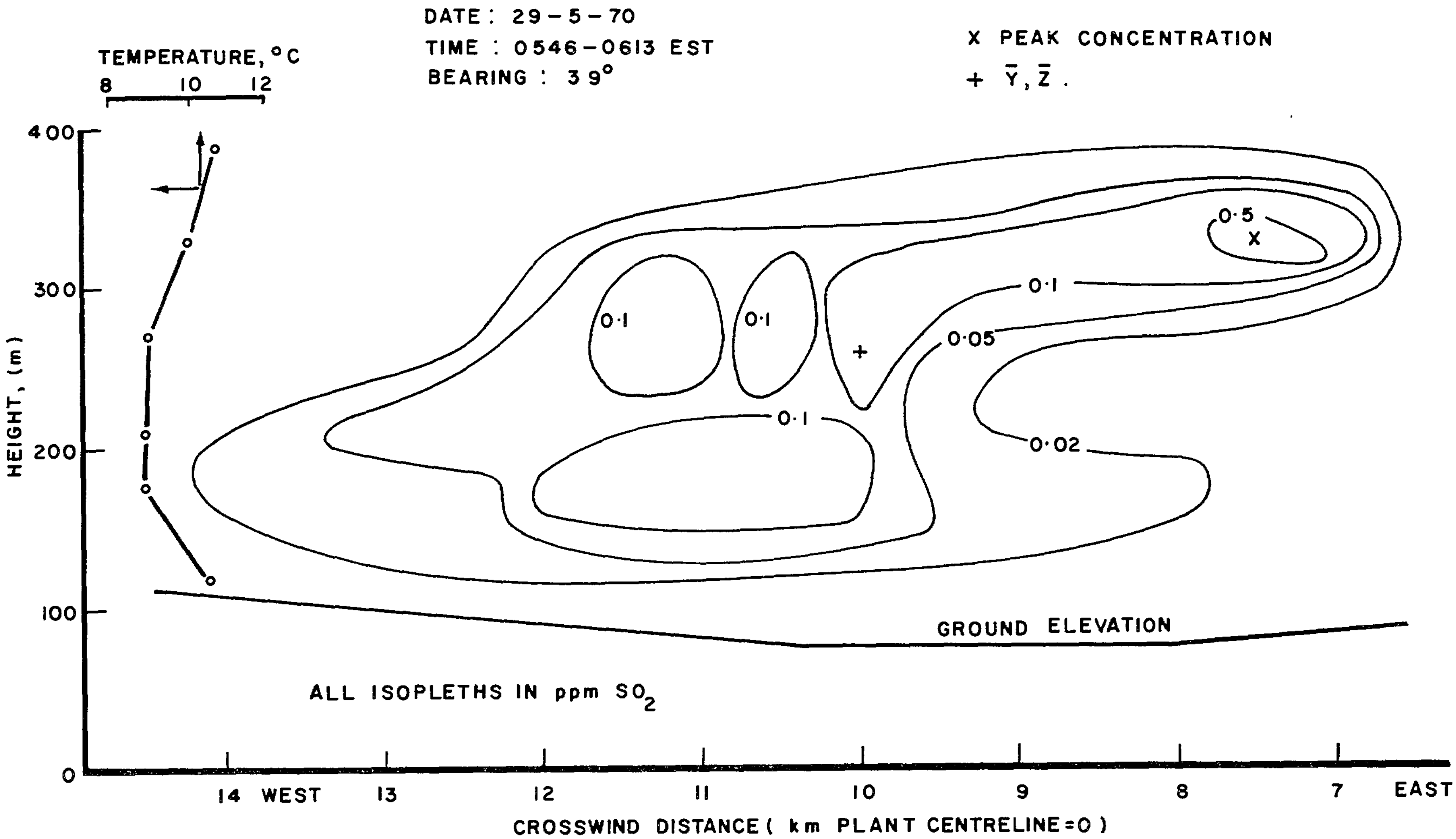


FIG. 4. Vertical crosswind section of the plume at an axial distance of 12.5 km from the source, 29-5-70.

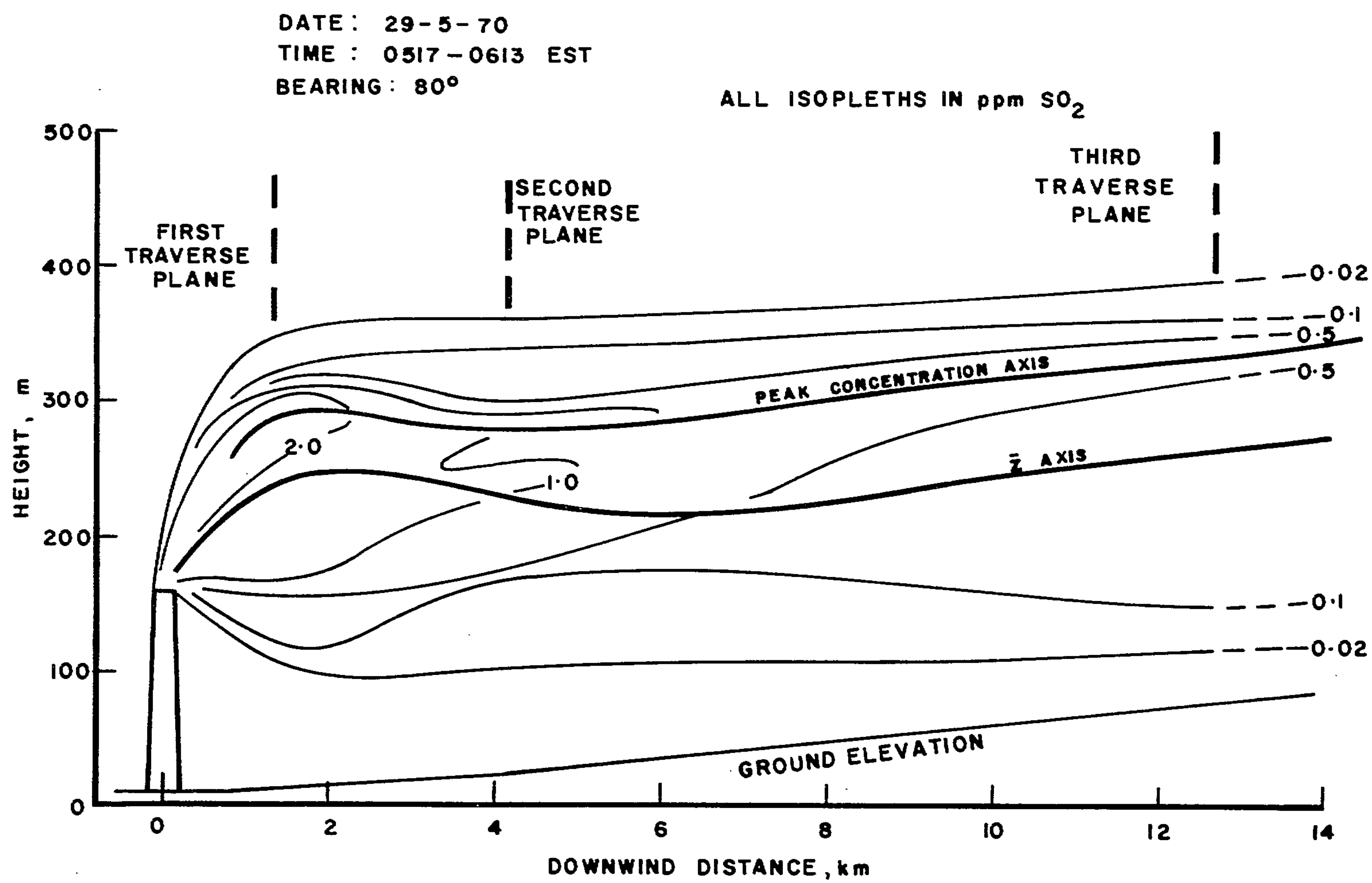


FIG. 5. Side view of a plume derived from crosswind traverses, 29-5-70.

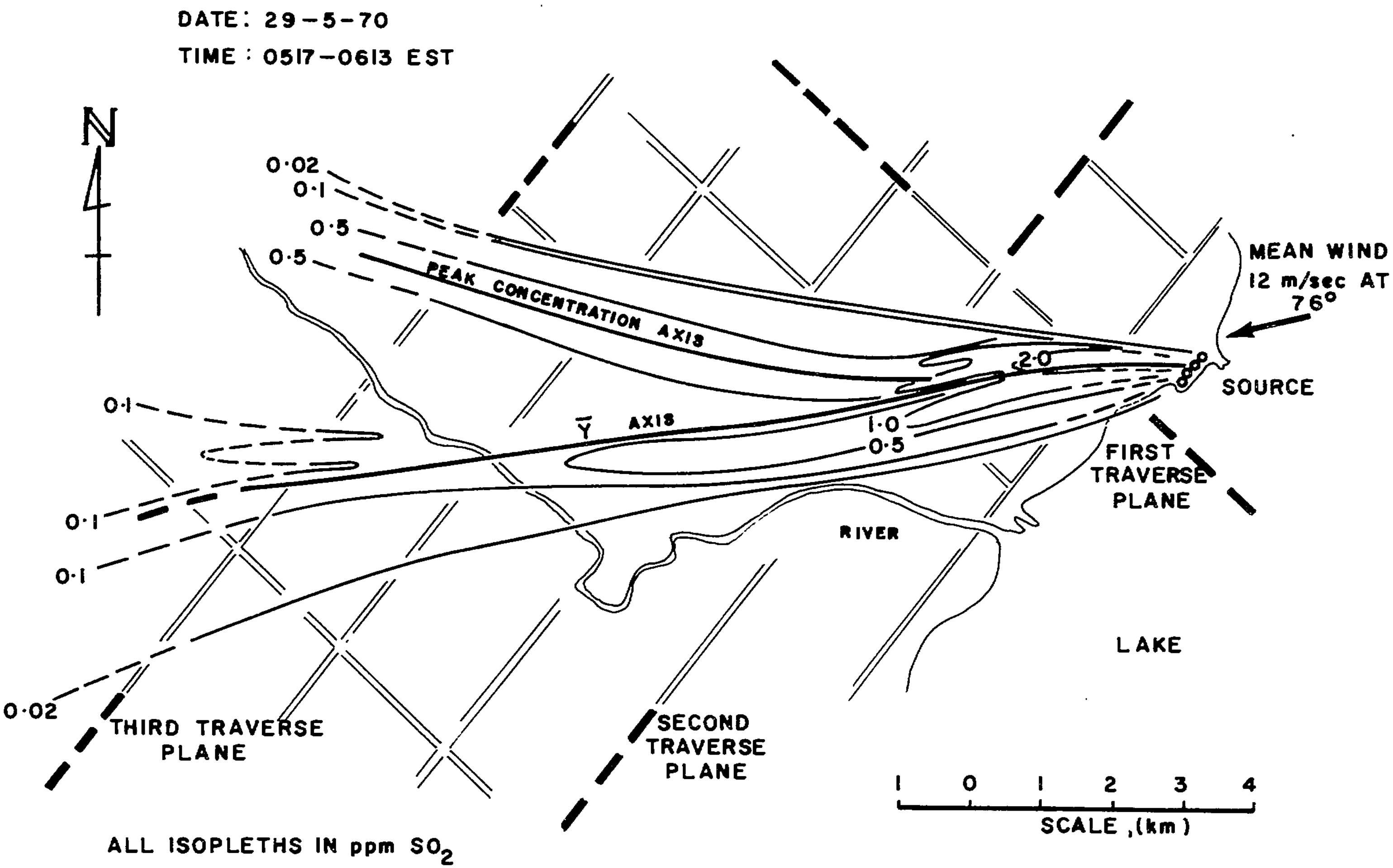


FIG. 6. Plan view of a plume derived from crosswind traverses, 29-5-70.