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Developing Integrated Multifactor Productivity Accounts in the Input-Output Accounting Framework: The Canadian Experience

By

René Durand

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Abstract

Statistics Canada's Multifactor Productivity Accounts are integrated into the Canadian System of National Accounts. Their originality rests, in part, on the application of the standard productivity formula to alternative but related sets of outputs and inputs in a bottom-up approach covering the whole business sector. The concept of vertical integration plays a central role in establishing relationships between alternative indices, including the relationships between static and dynamic indices. In the static framework, the stock of capital is exogenous. In the dynamic framework, capital goods become endogenous produced inputs. Establishments are seen as exchanging capital services across time periods. Time becomes a primary input of production, the productivity of which is associated with technical knowledge. A new measure of capital services together with an extended definition of economic efficiency are finally introduced that solve some paradoxical results that obtain with the conventional measure.

keywords: multifactor productivity, total factor productivity, national accounting, capital, waiting.

1 - Introduction

Statistics Canada's Multifactor Productivity Accounts are integrated into the Canadian System of National Accounts[1]. They are closely linked to the Input-Output Accounts[2]. The latter contain an incomparable set of integrated data for measuring productivity growth. Together with complementary labour hours worked and capital input data, they provide various measures of output with the corresponding inputs and associated prices that can be readily used to estimate alternative measures of productivity growth. These measures, in turn, can be related together using standard input-output modelling techniques.

Multifactor productivity growth, as usual in the literature, is defined as the difference between the weighted rates of growth of the outputs of a production process and the weighted rates of growth of the inputs used. In continuous time, it is often expressed by the Divisia index number formula. The weights, according to the Divisia formula, are the value shares of the various commodity outputs and inputs. We may write the productivity growth formula, D, as:

$$D = \sum c\dot{v} - \sum \omega \dot{x} \tag{1}$$

where the v's are the outputs of a production process in continuous time percentage rates of change (time derivatives of the logarithm noted as dotted symbols) weighted by their value shares c and the x's are the inputs also in rates of change and weighted by their cost shares ω. The Divisia indices are currently approximated by chained Törnqvist indices[3].

1. They are published annually in Statistics Canada's catalogue 15-204: *Aggregate Productivity Measures*.
2. They were initiated in 1987 following an earlier feasibility study carried out by Rymes and Cas that was recently published. See Cas and Rymes (1991).
3. Volume indices for Capital, Labour, Energy, Materials and Services and for the aggregate inputs and outputs are nevertheless computed using four alternative index number formulas: the Törnqvist, the Laspeyres, the Paasche and the Fisher Ideal. Implicit price indices as well as current price values are also available. See J. Johnson (1994).

The purpose of estimating multifactor productivity growth is basically to assess the role played by technical progress in economic growth and to separate its contribution from the contribution of inputs. In principle, productivity growth and technical progress are identical if: (1) the productivity calculations include all outputs and inputs, (2) there are no measurement errors, (3) there are constant returns to scale and (4) the markets are in competitive equilibrium. In practice, productivity estimates do not meet completely all four conditions and given their residual nature, they must be interpreted with some caution, particularly over the short-run. Technical progress is interpreted here, similarly to Denison, as advance in technical knowledge. It is assumed to be exogenous and, contrary to inputs, to be free.

All input and output data are adjusted so as to correspond to prices effectively received from the sale of output or the purchase of inputs. Commodity input prices, therefore, include commodity indirect taxes and are adjusted for subsidies where appropriate. Other indirect taxes comprise mostly property taxes and other taxes associated with the use of capital and they are added to the capital income. The latter income is therefore gross of direct income taxes and other indirect taxes. Similarly, labour income is gross of income taxes. Income taxes on capital and labour are included so as to make input prices equal to their effective cost to the establishments. Likewise industries' output prices are adjusted for subsidies. All final demand categories of the input-output tables are grouped into one commodity vector which is valued at primary factor prices to match the valuation of the primary inputs discussed above.

The capital income shares are computed residually so that, in principle, the productivity indices are also corrected for the short run underutilization of capital goods (see Bernt and Fuss (1986)). Intuitively, this is because the capital income shares fall in periods of economic slack, thereby reducing the importance of the capital input growth in the productivity formula concurrent with the fall in output growth. This offsetting change in capital input growth, in theory, maintains the productivity growth estimates on a long term equilibrium path. In practice, this correction seems to be insufficient to remove all the pro-cyclical behaviour of the productivity indices when capital services are assumed to be proportional to the capital stock. However, using the generalized measure of capital services described in Section 4 resulted in slightly counter-cyclical productivity indices.

Categories of inputs and outputs such as "labour" are composed of highly heterogeneous elements differing by qualitative characteristics that cannot be directly quantified. Some authors (see, for instance, Kendrick, 1991) prefer not to adjust inputs for quality changes that they associate with technical progress on the ground that this incorrectly reduces the residual measure of technical progress. We believe, on the contrary, that quality changes must generally be dissociated from technical progress and that inputs and outputs be adjusted accordingly.

For instance, more investment in human capital cannot itself, as it is sometimes argued, be seen as a diffusion process of knowledge or as a form of technical progress since it is time consuming and time is not free. Skilled labour hours may be seen as more than current hours worked. They also include past leisure time invested in acquiring the better skills. On the other hand, technical progress is that process which changes the quality of labour input through time at no cost, that is, for a fixed investment in human capital (e.g. that yields better skills for the same number of schooling years). Technical progress increases the marginal product of both skilled and unskilled labour without necessarily changing their relative wages.

Quality changes are well taken into account in the productivity formula presented above provided that inputs classes are as homogeneous as possible. The rate of growth of each input is indeed weighted by its income share, which depends on its price. But this also means that input costs vary together with their quality. This suggests immediately an interpretation of the adjustment for quality. Increases in the quality of inputs have to be dissociated from technical progress as they imply an increase in input costs. Technical progress, on the other hand, involves no increases in factor costs. A quality index of any input can be computed by dividing the quality adjusted measure of input by its raw unadjusted measure.

A major characteristic of Statistics Canada's multifactor productivity program is its bottom-up approach to productivity measurement. Productivity indices are estimated with the most disaggregated data available that presently cover 122 industries. The industries cover the whole business sector to the exclusion of *owner-occupied dwellings,* and *government royalties on natural resources*[4]. Productivity indices are aggregated by steps up to the total business sector.

The bottom-up approach used in the productivity accounts serves two major purposes. First, as stated by many authors in the literature (see for instance Jorgenson (1990)), the assumptions that are necessary to admit the existence of an aggregate production function are heroic. Its existence requires that such a function be the same in all industries and that producers face identical factor prices. As shown by Jorgenson, estimates of productivity made at the aggregate level under these assumptions may differ significantly from the estimates obtained by aggregating detailed industry productivity results under less restrictive assumptions. However, the major reason standing behind the estimation of productivity at the disaggregated level is that aggregate productivity growth could not likely be understood without the analysis of the detailed results. Detailed results make it easier to distinguish between real and measurement factors associated with productivity growth.

Another important concept which is used throughout is that of vertical integration. Vertical integration occurs either when, in the real world, establishments buy their upstream suppliers or sometimes when, in a more artificial fashion, the statistician aggregates the data. To exemplify the latter case, when aggregation is carried from the industry level to the total business sector, the definition of output is generally modified from a gross output to a value added measure. Similarly, intermediate inputs are excluded from the input set which reduces to capital and labour alone. The aggregation process is thereby accompanied by a vertical integration process: sales and purchases of intermediate inputs are eliminated as if all establishments were vertically integrated together into a single large establishment covering the whole business sector. That single establishment buys only capital and labour services and sells all of its output on final markets. As can be seen, vertical integration is a process by which establishments decide, or are seen as deciding, to produce their own inputs instead of purchasing them from their suppliers. Vertical integration, therefore, pertains only to inputs that are themselves produced.

Integration may be restricted to the use of intermediate inputs only. Since intermediate inputs are inputs that are both produced and used within the same production periods, vertical integration over intermediate inputs may be analysed within a static accounting framework. Section 2 describes the many indices produced in the static input-output framework, their interrelationships and their aggregation rules. The economy is first assumed to be closed. The closed economy model is extended to the open economy model in which issues concerning imported inputs and the adjustment for changes in the terms of trade are successively discussed.

Vertical integration may also be carried over capital goods when the latter are considered as endogenous produced inputs. That extends the traditional static productivity accounting framework, in which capital goods are considered to be exogenous or non-produced inputs, so as to take explicitly into account the process of capital accumulation and the incidence that technical progress has on that accumulation. Section 3 introduces the dynamic framework and extends the usual measure of productivity found in the literature. Section 4 introduces an alternative measure of capital services together with a complementary definition of economic efficiency that go beyond the traditional framework of productivity and solve some paradoxical results that obtain with the conventional measure. One additional result of that section is that depreciation should be considered as an intermediate input and, consequently, that output should be defined as net of depreciation. Section 5 concludes.

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4. The first industry was excluded as it involves incomplete input imputations, in particular for the home owners' supply of labour. The second industry was excluded because government royalties could best be assimilated to taxes on the natural resource rental income.

2 - Measuring Productivity Within the Static Input-Output Framework

In the static framework, capital is considered as a *primary* input together with hours worked, that is as a *non-produced* input in contrast to the *intermediate* or produced inputs. Implicit is the assumption that capital services are proportional to the capital stock. The economy is first assumed to be closed. Results are then extended to the open economy case. The productivity measures are associated successively with industries' gross output, their gross output net of own use of inputs, their final demand commodity sales and their value added.

The input-output accounting framework breaks down the business sector output of the economy by industry and by commodity. The first logical building block of the productivity accounts would seem, therefore, to specify productivity indices for commodities produced by the industries. However, commodities are grouped into commodity bundles produced by industries in the productivity accounts rather than treated separately. This is because, in the Canadian rectangular input-output accounting framework, inputs are classified by industry only so that the information on the production process of individual commodities remains unknown.

The gross output industry indices

The *gross output industry* indices result from the application of the chained Törnqvist productivity index number formula to industry commodity gross output and their intermediate and primary inputs. Denoting the industry by commodity *real* output or make matrix by $\boldsymbol{V}$, the intermediate commodity by industry real input matrix by $\boldsymbol{U}$, and the matrices of primary inputs of capital and labour by type and by industry respectively by $\boldsymbol{K}$ and $\boldsymbol{L}$, the matrix equation of industry productivity indices on gross output is given by[5]:

$$\tau_g = \boldsymbol{C} \varnothing \dot{\boldsymbol{V}} - \boldsymbol{B}^{\mathrm{T}} \varnothing \dot{\boldsymbol{U}}^{\mathrm{T}} - \boldsymbol{H}_{\mathrm{L}}^{\mathrm{T}} \varnothing \dot{\boldsymbol{L}}^{\mathrm{T}} - \boldsymbol{H}_{\mathrm{K}}^{\mathrm{T}} \varnothing \dot{\boldsymbol{K}}^{\mathrm{T}} \tag{2}$$

where $\boldsymbol{C}$ is the matrix of output value shares by industry (rows) and commodities[6] (columns), $\boldsymbol{B}$, $\boldsymbol{H}_{\mathrm{L}}$ and $\boldsymbol{H}_{\mathrm{K}}$ are similarly the matrices of intermediate input value shares, labour input value shares by type and capital input value shares by type. The operator $\varnothing$ is used to indicate that the matrix product is limited to its diagonal elements only. For instance, the vector of industries' weighted rate of growth of outputs[7] is represented by:

$$\boldsymbol{C} \varnothing \dot{\boldsymbol{V}} = [\boldsymbol{c}_j^{\mathrm{T}} \dot{\boldsymbol{v}}_j] \quad \text{for all } j \tag{3}$$

The industry gross output productivity indices are thus given by subtracting the weighted rates of growth of the intermediate and primary inputs of industries from the weighted rates of growth of their gross commodity outputs.

The intra-industry integrated productivity indices

The industry indices defined on gross output of industries are the most popular ones in the current economic literature. A variant of these indices used by the U.S. Bureau of Labor Statistics is based on the gross output net of intra-industry sales concept. Sales of establishments to other establishments belonging to the same

5. In the standard input-output notation, the dimensions of $\boldsymbol{V}$ are industries (rows) by commodities (columns) while the converse applies to $\boldsymbol{U}$, $\boldsymbol{K}$ and $\boldsymbol{L}$. Transposed matrices and vectors will bear the superscript T while diagonal matrices will be indicated with the ^ "hat" symbol. The dot over a symbol may be interpreted as the discrete time logarithmic changes with accompanying two-year moving averages of shares in which case the formula gives the Törnqvist index.
6. In the context of input-output modelling, $\boldsymbol{C}$ is often called the *product mix* matrix of industries.
7. The expression $\boldsymbol{C} \varnothing \boldsymbol{V}$ is a shortcut for the equivalent expression $(\boldsymbol{C} \bullet \boldsymbol{V})\boldsymbol{i}$ where the dot product indicates an element by element (Schurr) matrix product and where $\boldsymbol{i}$ is the summation (unit) vector of appropriate dimension.

industry are netted out both on the output and the input side of the productivity equation. It is as if the industries' establishments were merged into single large establishments or integrated together. These integrated establishments buy all their inputs outside their industry and equally sell all their output outside their industry. By contrast, the gross output industry productivity indices can be considered as measures of productivity at the establishment level of integration (within establishment flows of goods and services are netted out). Only the inputs coming into the establishments and the outputs going out of the establishments are considered in the computation of the indices. The formula for the intra-industry productivity indices is given by

$$\tau_{gn} = \boldsymbol{C}_n \varnothing \dot{\boldsymbol{V}}_n - \boldsymbol{B}_n^T \varnothing \dot{\boldsymbol{U}}_n^T - \boldsymbol{H}_L^T \varnothing \dot{\boldsymbol{L}}^T - \boldsymbol{H}_K^T \varnothing \dot{\boldsymbol{K}}^T \tag{4}$$

where the subscript n denotes that inputs and outputs are net of intra-industry sales[8]. The computation of these indices involves some subtleties associated with differences in the valuation of outputs and inputs and with non-business supply of some inputs such as imported inputs. This is dealt with in Appendix 1.

A more general vertical integration rule which applies to all industry indices is given in Appendix 2. According to that rule, the intra-industry integrated indices are related to the traditional gross output indices through the relation:

$$\tau_{gn} = \frac{\boldsymbol{g}}{\boldsymbol{g}_n} \bullet \tau_g \tag{5}$$

where the division (bar) and multiplication $\bullet$ of two vectors are carried element by element[9] and where $\boldsymbol{g}$ and $\boldsymbol{g}_n$ are respectively the vector of industry nominal gross output and nominal gross output net of intra-industry sales.

The interindustry integrated or commodity productivity indices

The idea of integration may be pushed one step further to cover not only intra-industry sales but also interindustry sales. The establishments of an industry may be integrated with their upstream suppliers which may themselves be integrated upstream with their own suppliers and so on. Under full vertical integration, the output of an industry becomes expressed as a function of the *direct* use of its own primary inputs and the *indirect* use of the primary inputs of all its upstream suppliers. In other words, all *direct* intermediate input uses of industries are replaced by their *indirect* primary input uses. Indeed, it makes sense to think of an industry purchasing commodities from another industry as being, in fact, indirectly using the inputs of that industry. The associated productivity measures are called the *interindustry* productivity indices as they are obtained by taking into account interindustry transactions. These indices were first introduced by Rymes (1972) and applied to Canada by Rymes and Cas (1985)[10]. Their formula, is given by:

$$\tau_{gi} = \boldsymbol{C} \varnothing \dot{\boldsymbol{V}} - [\boldsymbol{B}^T \varnothing (\dot{\boldsymbol{U}}^T - (\boldsymbol{i} \otimes \tau_{gi}^T \ \boldsymbol{D}))] - \boldsymbol{H}_L^T \varnothing \dot{\boldsymbol{L}}^T - \boldsymbol{H}_K^T \varnothing \dot{\boldsymbol{K}}^T \tag{6}$$

where $\boldsymbol{D}$ is the market share matrix of industries (their shares of each commodity output or row shares associated with $\boldsymbol{V}$) and $\otimes$ is the Kronecker matrix product. To interpret the last equation, note that this equation is obtained by replacing the rate of growth of each intermediate input, in the gross output industry

8. In general, τ is be used to denote productivity growth and its subscript to denote the output over which it is specified. Integrated measures are further subscripted by $\boldsymbol{i}$ when necessary.

9. Which may alternatively be written $\tau_{gn} = \hat{\boldsymbol{g}}\hat{\boldsymbol{g}}_n^{-1}\tau_g$.

10. Their study was later published as Cas A. and T.K. Rymes (1991). See also Wolfe (1991).

formula (2), by its rate of growth deflated by the productivity gains of its originating industry[11]. If productivity gains are positive, this means that the weighted rate of growth of the intermediate inputs is larger than the weighted rate of growth of the primary inputs used in their production. Therefore, in general, τ_{gi} is larger than τ_g.

More generally, integration usually increases the value of productivity measures as intermediate inputs are replaced by primary inputs (interindustry integrated indices) which increase at a lower rate or are simply eliminated on both sides of the productivity equation (intra-industry integrated indices). Conceptually, this comes by enlarging the set of productive processes involved in the computation of the productivity gains to include those of all upstream suppliers of the industry. Comparing equation (6) with (2), one can also express the interindustry indices as a function of the gross output industry indices as follows:

$$\tau_{gi} = [\boldsymbol{I} - \boldsymbol{B}^{T}\boldsymbol{D}^{T}]^{-1}\tau_{g} \tag{7}$$

Equation (7) shows that the industry and interindustry indices are related through the transpose of the nominal *impact* matrix of the standard square input-output model, that is through the Leontief inverse, relating gross output to the final demand deliveries of industries. The interindustry indices can therefore be seen as weighted averages of the industry indices. The weights are the industries' input shares. The weights of the Leontief inverse sum to more than one which formally accounts for the impact of integration when going from the industry to the interindustry indices. Integration proceeds over industries so that the resulting indices refer to the productivity growth associated with the production of the many bundles of goods and services by the economy as a whole. In that sense, the interindustry indices associate productivity growth with commodities rather than with industries.

One may note that, in the above formula, the rate of growth of intermediate inputs is deflated by $\boldsymbol{D}^{T}\tau_{gi}$ rather than by the interindustry indices alone. This is because the interindustry indices are defined in the industry space rather than the commodity space and refer to industry bundles of commodities rather than to individual commodities. The transformation of the indices with the market share matrix defines individual commodity indices. These are defined, in a rectangular input-output framework in which technology is available by industry only, as the average of the productivity growth rates of industries from which the commodities originate.

The final demand commodity productivity indices

From the discussion of the previous paragraph, it follows that if $\boldsymbol{e}$ is the vector of final demand by commodity then the vector τ_e of productivity gains associated with these commodities may be related to the interindustry indices by:

$$\tau_{e} = \boldsymbol{D}^{T}\tau_{gi} \tag{8}$$

These indices may equivalently be expressed by the following relationship:

$$\tau_{e} = \dot{\boldsymbol{e}} - \Omega_{L}\varnothing\dot{\boldsymbol{L}}_{e} - \Omega_{K}\varnothing\dot{\boldsymbol{K}}_{e} \tag{9}$$

where Ω_L and Ω_K refer to the labour and capital direct and indirect cost shares by commodity and where $\boldsymbol{L}_e$ and $\boldsymbol{K}_e$ refer to matrices of labour and capital inputs directly and indirectly used by commodity.

11. As discussed in Section 3 below, Cas and Rymes also extend identically the transformation of the traditional productivity equation to the capital stock.

Formula (9) represents, by definition, the vector of Divisia indices of productivity growth associated with final demand commodities. Indeed, it equates productivity gains on each commodity to the difference between the rate of growth of that commodity and the weighted rate of growth of the primary inputs used in its production. Values of direct and indirect primary input requirements are obtained by the application of the usual current price impact matrix of the input-output model to the current price diagonal matrix formed with the vector of final demand. Real input requirements are obtained by deflating nominal values by input prices.

Equation (8) relates final demand individual commodity indices to the interindustry commodity bundle indices. They can be seen as averages of the productivity gains of the industry from which they originate. However, the latter productivity gains referred to the gross output of these commodities rather than to their final demand deliveries. That the indices do not change when shifting from a gross to a net output basis is shown in Appendix 3. Intuitively, this comes from the fact that productivity measures are uniquely defined by the transformation processes that are considered and these are the same in the case of the commodity indices, whichever measure of output is used.

In the case of industry indices, these transformation processes are not the same for all indices. The specification of the industry indices depends on the level of integration of the associated output measure. The industry production function may or may not be the same as the establishment production function depending on whether intra-industry inputs and outputs are or are not included in the specification of the production process of the industry. Hence, contrary to the current belief, productivity measures are not dependent as such on the definition of output, whether gross, net of intra-industry sales or otherwise. Their value depends only on the degree of integration of the production processes being considered. This integration level may and usually does change with the choice of the level of output but not necessarily so. Thus, it would be possible to associate the non-integrated productivity measure of an industry (gross output industry index) with its direct final demand deliveries.

Returning to the individual commodity indices, though they could, in principle, be computed, they are not regularly produced. These indices would merely refer to the productivity gains associated with industry technologies rather than to their proper commodity technology which remains unknown in the rectangular input-output framework. For this reason, only the interindustry indices which pertain to commodity bundles are calculated and published.

The commodity productivity index definition is useful, however, as it leads us one step ahead in the analytical derivations of the productivity indices. First, the assumption that technology is specific to commodities or, as an approximation, to industry commodity bundles, as opposed to the assumption that technology is specific to industries leads us to the definition of productivity indices associated with the value added of industries, which we turn to next. Secondly, commodity indices are necessary to derive dynamic index number formulas as discussed in Section 3.

The value-added productivity indices

Productivity indices may be associated with the value added of industries and their primary inputs of capital and labour, provided that real value added be properly defined. Using standard input-output relationships, the value added on each commodity delivered to final demand can be traced back to the contributing industries. Looking from the other side of the pipe, one may see that industry value added may be broken down by commodity. Any industry productivity gain may therefore be seen as a weighted average of the productivity gains made on the commodities delivered to final demand, the weights being the value share of each commodity in the total value added of industries. Letting the matrix of the distribution of the nominal value added by industry and by commodity be denoted by $\boldsymbol{Y}$, that matrix may be derived by applying the impact matrix to final (diagonal) demand expenditure:

$$\boldsymbol{Y} = \hat{\lambda}\left[\boldsymbol{I} - \boldsymbol{DB}\right]^{-1}\boldsymbol{D}\hat{\boldsymbol{p}}\hat{\boldsymbol{e}} \tag{10}$$

where $\hat{\lambda}$ is the diagonal matrix of the shares of nominal value added in the nominal value of gross output of industries. The vector of total value added by industry, $\boldsymbol{y}$, may be obtained from (11) by summing over commodities. Defining the value shares of commodities in the value added of industries by:

$$\boldsymbol{C}_y = \hat{\boldsymbol{y}}^{-1}\boldsymbol{Y} \tag{11}$$

Productivity on real value added by industry may be expressed as:

$$\tau_y = \boldsymbol{C}_y\tau_e \tag{12}$$

Since the productivity gains made on final output can be related to the productivity gains made by industries on their gross output, it follows that productivity measures on value added can be related to productivity measures on gross output. Appendix 3 shows that productivity gains on value added, τ_y, are related to productivity gains on gross output, τ_g, by

$$\tau_y = \hat{\lambda}^{-1}\tau_g \tag{13}$$

This relationship also follows more directly from the general integration rule of Appendix 2. The indirect route followed here conveys a more intuitive interpretation of these indices, which refer to production processes of industries that are integrated downward to final markets. They include the contribution of the primary inputs of an industry to the productivity gains of the whole chain of industries producing final goods. These measures, consequently, exhaust the productivity gains associated with industrial activity. It follows that their aggregation weights, provided by the nominal value added shares of industries in total value added, sum to one contrary to the gross output productivity indices. The latter indices consider intermediate inputs as exogenous and do not account for the productivity gains made in their production. Hence, they only account partially for the productivity gains associated with industrial activities.

Corresponding to (13), there exists a measure of real value added of industries whose rate of growth is given by adding the productivity gains on value added to the primary input growth. This measure differs from the standard measure of real value added obtained by the double-deflation method (see Durand, 1994a).

Before concluding the presentation of the alternative productivity indices in the static framework, it is important to note first, that the appropriate choice of a formula depends on the analytical purpose at hand. When interpreting and comparing the numerical results obtained with all these alternative indices, one must also note that the results are comparable only if the indices are at the same integration level. Hence, an establishment's productivity gains, usually measured on the basis of its gross output, are comparable to its industry's productivity gains measured on the same basis but they are not comparable to the intra-industry integrated productivity gains. Comparability, however, is maintained between different levels of aggregation as aggregated indices are regular weighted averages of component indices.

Finally, one must note that in most input-output accounting systems, the definition of the gross output of industries is not the same across industries. For instance, in manufacturing, gross output is usually defined as total sales adjusted for inventory changes, while in the wholesale and retail trade industries, gross output is defined as a gross margin (including value added plus operating costs but excluding the cost of the goods purchased for resale). Comparability of gross output based productivity gains is not maintained between industries in those cases. Comparability is always maintained, however, when productivity gains are established on the basis of industry value added. That conveys a definite advantage to the latter indices. Still, when establishing international comparisons, it would appear to be preferable to assess the joint productivity of all industries involved in the production of competing goods, that is to use the interindustry/commodity productivity indices rather than any industry indices. All industries are indeed involved in the competitiveness game, either directly or indirectly.

Aggregation rules

Aggregating productivity indices is a simple matter when it is realized that an aggregate productivity index is simply an index computed on concatenated data. Thus, let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m$ be the output vectors of m production processes and $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m$, be the corresponding input vectors. Consider the concatenated vectors $\boldsymbol{v}$ and $\boldsymbol{x}$, given by:

$$\boldsymbol{v}^T = [\boldsymbol{v}_1^T, \boldsymbol{v}_2^T, \ldots, \boldsymbol{v}_m^T] \qquad \boldsymbol{x}^T = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \ldots, \boldsymbol{x}_m^T] \tag{14}$$

Aggregate productivity growth is simply computed by applying the productivity formula on the vectors $\boldsymbol{v}$ and $\boldsymbol{x}$. It can be easily shown, by decomposing the output and input indices, that the aggregate index is also a weighted average of the component indices with weights equal to the value shares of the components into the aggregate. Hence, aggregation of productivity gains over industries or some other dimension can be seen as a simple process by which input and output vectors are concatenated together before calculating productivity growth or as a process whereby the aggregate productivity gains are obtained from disaggregated productivity gains by taking their weighted average.

Aggregation is done here with the weighting rule. Thus, the aggregate Divisia index of productivity over all commodities is simply the weighted average of the interindustry/commodity indices with weights equal to their value shares α in total value added:

$$\alpha^T = \frac{\boldsymbol{p}^T \hat{\boldsymbol{e}}}{\boldsymbol{p}^T \boldsymbol{e}} \tag{15}$$

Aggregate business sector multifactor productivity growth τ is therefore given by

$$\tau = \alpha^T \tau_e = \alpha^T \boldsymbol{D}^T \tau_{gi} \tag{16}$$

This aggregate index can also be expressed in terms of the disaggregated industry indices on the basis of (7):

$$\begin{aligned} \tau &= \alpha^T \boldsymbol{D}^T [\boldsymbol{I} - \boldsymbol{B}^T \boldsymbol{D}^T]^{-1} \tau_g \\ &= \frac{\boldsymbol{g}^T}{\boldsymbol{p}^T \boldsymbol{e}} \tau_g = \beta^T \tau_g \end{aligned} \tag{17}$$

where the weights β are defined by the third equality. These weights are the aggregation-integration weights of Domar (1961)[12]. The weights of (17) are composed of the industry integration weights $\hat{\lambda}^{T}$ and the value-added aggregation shares ε used to aggregate the value-added productivity indices to the total business sector:

$$\beta = \frac{\hat{\boldsymbol{y}}}{\boldsymbol{p}^{T}\boldsymbol{e}}[\ \hat{\boldsymbol{g}}\ \hat{\boldsymbol{y}}^{-1}] = \hat{\varepsilon}\hat{\lambda}^{-1} \tag{18}$$

The interpretation of Domar's rule, therefore, becomes quite clear: to aggregate industry multifactor productivity indices defined on gross output to the business sector index defined on value added, one has first to integrate the indices, that is, to express industries' productivity gains in terms of their use of the economy's primary inputs. That gives value-added productivity indices by equation (13). The latter gains are thereafter aggregated with the nominal value-added shares of industries which, contrary to Domar's weights, sum to one.

The aggregation of the intra-industry integrated indices is, by definition of these indices, always carried out with some vertical integration as, when industries are aggregated together, their former interindustry sales become intra-industry sales that must be netted out. Their aggregation-integration weights are the value shares of the gross output net of intra-industry sales of industries in the aggregate value of gross output net of intra-industry sales. These shares sum to more than one.

Aggregation and integration

As just seen, vertical integration changes the measure of productivity gains and, therefore, constitutes a fundamental characteristic of the productivity indices. There are various ways of integrating production activities vertically leading to different measures of productivity gains. Each one, however, pertains to a uniquely defined set of production activities. It can thereby be seen that productivity gains related to specific productive activities have a unique measure. Different choices of output and input sets at the industry level lead to different measures of productivity gains only because they refer to different sets of production activities. In that perspective, the debates about which measures are right and which are wrong is a false debate. The question must always be: To which set of production activities do we want to apply a productivity measure? Once that set is chosen, its productivity measure is unique. Note also that aggregation and integration of production processes are independent operations that have often been confused and this has lead to unproductive debates in the literature.

Opening the economy

Opening the economy raises two sets of issues. The first one pertains to the treatment of imported inputs. These are produced inputs but, contrary to other intermediate inputs, the productivity gains made in their production must be allocated to the trading partner economies. Hence, it would appear that, from the domestic economy's point of view, these inputs would best be considered as primary non-produced inputs. Such a treatment of primary inputs was suggested by Gollop (1982).

The second issue concerns the terms of trade effect. Some authors, like Diewert and Morrison (1986), have suggested to correct the productivity indices by a terms of trade effect as if an improvement in the terms of trade of the economy were equivalent to an outward movement of its production possibility frontier. Traditional developments in that area rather consider the terms of trade effect as a pure consumption effect and it is dealt with by a movement in the consumption possibility frontier. Gains from terms of trade do not change aggregate output and, therefore, productivity gains. The trade balance remains deflated by a double deflation procedure similar to the double deflation of the industry value added.

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12. See also Hulten (1978).

With respect to the first issue, part of the answer comes from looking at the productivity of the integrated set of economies trading together. Treating their imported inputs as primary inputs would yield productivity estimates for each of them that would not aggregate to the productivity gains of all economies taken together. Indeed, for the individual economies, the computation of aggregate productivity gains would exclude the productivity gains made in the production of imported inputs. However, for the integrated group, all imported inputs reduced to standard intermediate inputs. The productivity gains associated with their production are therefore taken into account by using the integrated productivity indices on value added discussed above, either by commodity or by industry. Hence, individual country's productivity gains would have to be weighted by inflated shares summing to more than one to give the aggregate productivity gains of the set of countries in a manner similar to the application of Domar's rule to the gross industry indices.

In the latter case, however, there is no issue as to which economy these productivity gains must be allocated. In the present situation, inflating the weights or, what amounts to the same thing, treating imported inputs as intermediate inputs raises the issue of determining to which economy these additional productivity gains must be attributed. There is no question, however, that the countries productivity indices must somehow account for the productivity gains made on the imported inputs, contrary to Gollop's suggestion.

There are alternative ways of closing the economy on imported inputs. One that was suggested by Cas and Rymes (1991) is to trace back the primary inputs used in the trading partners' economies and integrate vertically as in the close economy model. That solution is inadequate, even if it were feasible, as it would attribute the productivity gains made in foreign economies to the domestic economy. Hence, Cas and Rymes' productivity estimates for Canada did not account for these productivity gains and, as they admit, were biased downward for that reason.

From an accounting point of view, the Cas-Rymes solution would amount to adding an import industry to the business sector that would use only primary inputs (those of the trading partners) and would produce an equivalent value of imported inputs delivered as intermediate inputs on the domestic market. An alternative approach is still to treat the imported inputs as the output of an import industry but to consider that these commodities are produced by intermediate inputs, themselves being computed as the import content of the final demand deliveries of the business sector. Such an import industry has no value added contrary to the previous case. Hence total value added remains the same as in the closed economy case in both nominal and real terms. The aggregate productivity is obtained by aggregating the productivity gains of all industries as in the closed economy model, which amounts to computing aggregate productivity on real value added and the domestic primary inputs of capital and labour.

Such productivity measures applied to all trading economies aggregate to the productivity gains of the integrated set of economies with weights equal to their value added shares of the total value added, that is with aggregation weights summing to one. Hence, that treatment accounts for all productivity gains. It also attributes correctly the productivity gains to the respective trading economies, as the imported inputs and their real contribution to the final demand deliveries of the business sector are removed from the domestic economy. In other words, the real value added of the final demand deliveries of the business sector is traced back and split between the domestic and the foreign inputs. The domestic part of the value added, which is equal to the business sector's value added, is related to the domestic primary inputs of capital and labour. This is the approach used in Statistics Canada's multifactor productivity accounts.

With respect to the second issue, namely the changes in the terms of trade, the question is whether the trade balance is or is not part of the final demand of the economy. If it is, then its double deflation is the proper measure to use, in which case the terms of trade effect would enter into a welfare index but would be excluded from the productivity index. As in Diewert and Morrison, the standard productivity index would be corrected by a terms of trade effect to give the overall welfare index[13].

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13. Welfare may be defined, following Ricardo, as the maximum consumption that can be reached for a given effort, that is, in a closed economy, by the productivity index.

Trade may alternatively be seen as an industrial activity by which the production possibility frontier of the economy may by pushed forward. Exports, indeed, are not domestically consumed and their sale only provide an income which may be applied to purchase other commodities either produced domestically or imported. The current account balance itself may be seen as a form of wealth accumulation similarly to net investment in the economy[14]. Hence, the trade balance may be deflated by weighting export and import prices by their shares of final demand rather than their shares of final domestic demand as in Diewert and Morrison. This gives the following Divisia index of final demand:

$$D(\boldsymbol{e}_{\mathrm{z}}, -\boldsymbol{m}; \boldsymbol{p}_{\mathrm{z}}, \boldsymbol{p}_{\mathrm{x}}, \boldsymbol{p}_{\mathrm{m}}) = \frac{\boldsymbol{p}_{\mathrm{z}}^{T}\hat{\boldsymbol{z}}\dot{\boldsymbol{z}} + \boldsymbol{p}_{\mathrm{x}}^{T}\hat{\boldsymbol{x}}\dot{\boldsymbol{x}} - \boldsymbol{p}_{\mathrm{m}}^{T}\hat{\boldsymbol{m}}\dot{\boldsymbol{m}}}{\boldsymbol{p}_{\mathrm{y}}^{T}\boldsymbol{e}_{\mathrm{ny}}} \tag{19}$$

where $\boldsymbol{e}_{\mathrm{z}}$ represents the final demand deliveries of the business sector (Gollop's measure of aggregate output), $\boldsymbol{m}$, the imports, $\boldsymbol{x}$, the exports, $\boldsymbol{z}$, final domestic demand ($\boldsymbol{z} = \boldsymbol{e}_{\mathrm{z}} - \boldsymbol{x}$) and $\boldsymbol{e}_{\mathrm{ny}}$, final demand obtained by netting out the imports of final goods and the *imported content*[15] of domestic production from final demand deliveries $\boldsymbol{e}_{\mathrm{z}}$. Corresponding price vectors are denoted by $\boldsymbol{p}$ with the appropriate subscript. The price equation corresponding to (19) is:

$$D(\boldsymbol{p}_{\mathrm{z}}, \boldsymbol{p}_{x}, \boldsymbol{p}_{\mathrm{m}}; \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{m}) = \frac{\boldsymbol{p}_{\mathrm{z}}^{T}\hat{\boldsymbol{z}}\dot{\boldsymbol{p}}_{\mathrm{z}} + \boldsymbol{p}_{\mathrm{x}}^{T}\hat{\boldsymbol{x}}\dot{\boldsymbol{p}}_{\mathrm{x}} - \boldsymbol{p}_{\mathrm{m}}^{T}\hat{\boldsymbol{m}}\dot{\boldsymbol{p}}_{\mathrm{m}}}{\boldsymbol{p}_{\mathrm{y}}^{T}\boldsymbol{e}_{\mathrm{ny}}} \tag{20}$$

Equation (20) indicates that the Divisia price index of the trade balance would be given by the difference in the weighted rates of growth of the export and import prices, the weights being the export and import shares respectively in the value of final demand. This result is similar to the Diewert - Morrison measure of the change in the terms of trade in which the same prices are weighted by their value shares in total final domestic demand.

Equation (19) could be used to define the aggregate output of the economy, in which case, the productivity index would include the terms of trade effect contrary to the case in which the trade balance is deflated by the double-deflation method. In the alternative framework in which the terms of trade effect would be incorporated to the estimate of productivity gains, the accounting framework would be similar to the one of the maintained alternative in the sense that imports would be treated as the intermediate inputs of the trading sector. These imports would be measured, by their final demand content equivalent (direct final demand import content and indirect import content), rather in the original commodity breakdown of imported inputs. However, the gross output of that industry would be given by exports rather than imported inputs. The trade surplus would be the value added of the trading industry. That value added would provide an estimate of the terms of trade effect if deflated similarly to the other industries' value added according the alternative methodology outline above and in Appendix 3 rather than using the double deflation technique. This requires that the final sales of the trading industry be defined. In the case of double deflation, the real value added of the industry would be equal to the traditional measure of the real trade balance and would exclude the terms of trade effect on real income. The traditional approach would require that the gross output of the trading industry be delivered directly to final demand. The alternative approach of Diewert - Morrison would have exports delivered indirectly to final demand in the form of a share of final domestic sales originating from the income generated by exports sales. A improvement in the terms of trade would generate additional income that would flow back into final sales, holding the trade deficit constant.

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14. Contrary to the Diewert-Morrison view that it constitute an additional input. The Diewert-Morrison view could be considered for that reason as a short run view that opposes the long run view adopted here.
15. Hence $\boldsymbol{e}_{\mathrm{ny}}$ contains only positive elements since the import content of the business sector final demand deliveries must always be smaller or equal to the deliveries themselves. On the other hand, the final demand vector $\boldsymbol{e}$ may have negative elements when some imported commodities are used mainly as intermediate inputs. The total value of final demand, however, is the same in both cases. It is only the commodity shares that change.

The choice of including or not the terms of trade effect in the estimate of productivity gains is perhaps a matter of preference. There would be one advantage of including it in that the productivity index would then coincide with the welfare index as in the dynamic framework.

3 - Measuring Productivity Within the Dynamic Input-Output Framework

The dynamic framework is based on the view that capital goods are produced inputs, though different from current intermediate inputs because of their lasting nature extending over many periods. Vertical integration of industrial activities is therefore carried through time. Industries are interrelated across time periods by their exchange of capital services as they are interrelated within the same period by their exchange of intermediate inputs. Integration through time over capital goods is naturally accompanied by integration over all intermediate inputs so that the dynamic indices by industry are attached to value added only.

Since capital goods are produced inputs, this excludes them from the list of primary inputs. The question that this immediately raises is which inputs are primary in the dynamic framework? One possible answer to the question is to consider labour as the sole remaining primary input[16]. Capital goods would become direct and indirect labour in the classical Ricardian and Neo-Ricardian sense. Instead, we admit here that time is a real input into the production process. Production necessitates that the capital stock be held and carried over from one year to the next. This involves postponing present consumption for future consumption, that is waiting. Time in the form of waiting is not only required for production to occur but waiting is itself a form of input in that it involves a basic human effort like working. Interest is the income associated with the effort of waiting. Waiting is considered as a primary input: it is not produced and cannot be affected by technical progress contrary to depreciation which is considered as an intermediate input. Work and waiting are therefore considered as the two fundamental primary inputs in the dynamic input-output framework in contrast to work and capital goods in the static framework.

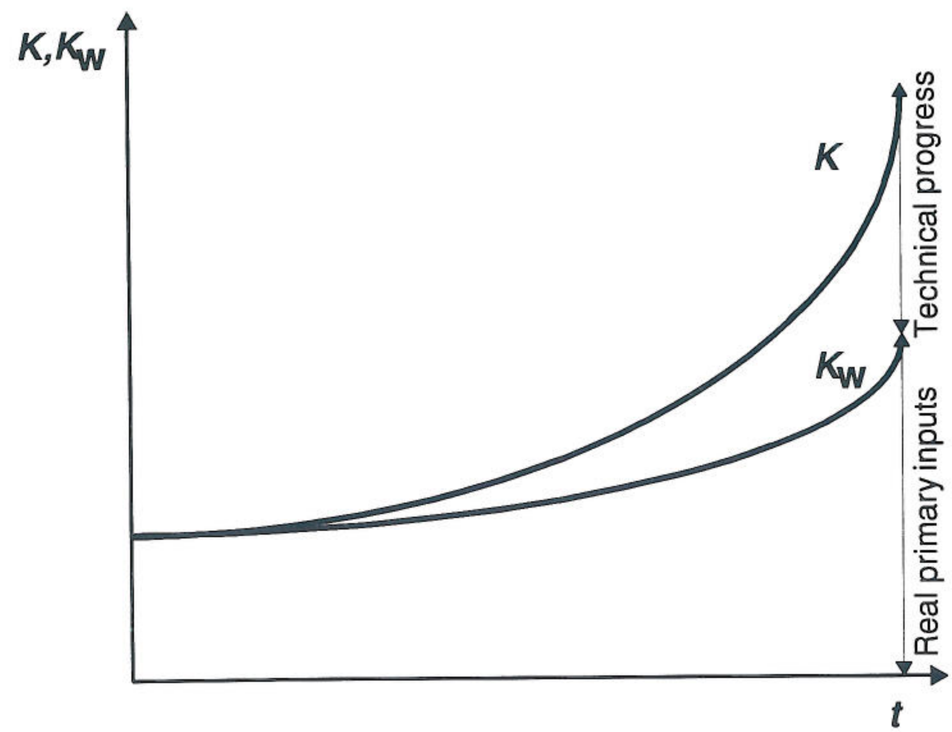
Looking at the supply side of the coin is insufficient. Individuals may save and dissave in different periods of their life in order to distribute more evenly their consumption through the various phases of their life cycle without society as a whole accumulating any capital goods. In order for society to do so, some gains must result from the process, that is waiting must somehow be productive. How that comes about is related to the existence of roundabout methods of production that requires the use of capital goods. These methods are themselves the result of the growing stock of knowledge of society. Technical progress thus appears as an essential ingredient to the very process of capital accumulation in a behavioral sense.

We are nevertheless more concerned in what follows by the relationship that exists between technical progress and the stock of capital from a measurement perspective rather than from a behavioral perspective. More precisely, we intend to break down the growth in the capital stock between that part that comes from the increasing supply of waiting services and that part that comes from the impact that technical progress has on the production of capital goods as depicted in figure 1 below. Indeed, with technical progress, more capital goods can be produced through time with the use of the same resource inputs so that the capital stock grows at a faster rate than the resources needed to support that growth.

We define the *accumulation flow* of waiting as *a process by which productive resources are diverted from the production of current consumption goods and used in the production of capital goods*. Therefore, it is possible to account for these resources in a manner similar to the one used for capital goods and obtain the amount of resources that have been put aside, accumulated and depreciated through time jointly with the associated capital goods. This will, following Cas and Rymes (1991), be called the *stock* of waiting. The value of the cumulative primary inputs of the waiting stock is just the value of the cumulative net capital stock. Thus, the basic difference between waiting and capital goods is that one is measured in input units[17] and the other in output units. Alternatively, capital goods may be seen as vehicles that move inputs into the

16. For a multisectoral labour value theory of economic growth and the associated accounting framework, see Passinetti (1981).
17. Or, equivalently, in consumption units that could have been produced by these resources, given the fixed technology of any base year.

Figure 1 - Capital and Waiting Accumulation Through Time



future rather than the inputs themselves. Hence, if, for an arbitrarily chosen industry, p_W is the price of waiting units, p_K, the price of capital goods, K_W the stock of waiting units and K, the conventional capital stock, one has:

$$p_w K_w = p_k K \tag{21}$$

The above identity provides a benchmark value for the real waiting stock in any arbitrarily chosen base year in which price indices are set equal to one. Real values for other years may be derived if the rate of growth of K_W can be computed. In order to do so, one has to consider capital goods as produced commodities delivered to final demand. Their production requires the application of direct and indirect labour and waiting inputs, the weighted rate of growth of which, say $\dot{I}_w$, may be computed by deflating the rate of growth in real investment expenditure $\dot{I}$, by the productivity gains associated with their production τ_I:

$$\dot{I}_w = \dot{I} - \tau_I \tag{22}$$

As a starting point, the static commodity productivity index formula (9) may be used to compute the rate of growth of the waiting flow, $\dot{I}_w$. One must note, however, that, in the formula of the static model, the direct and indirect use of primary inputs is expressed in terms of labour and conventional capital rather than labour and waiting. Since waiting, in an economy with positive productivity growth, is growing at a lower rate than capital, productivity growth associated with capital goods would be underestimated in the above formula and the resulting estimate of primary input growth would be on the high side. However, this problem can be solved through a convergent iterative process.

Equation (22) can be considered as providing the dynamic measure of productivity in the dynamic model as it applies to consumption goods as well. More generally, for any final output Q, that equation may be written as:

$$\tau_W = \dot{Q} - \omega_L \dot{L} - \omega_K \dot{K}_W \tag{23}$$

where τ_W is the dynamic rate of technical progress and K_W the corresponding stock of waiting, ω_L and ω_K are the labour and capital cost shares. Assuming that, using (22), the rates of technical progress can be estimated in each industry, then one has the following identities:

$$I_{W,t} = I_{W,o} e^{\int_{t_0}^{t} \dot{I}_W(\tau) d\tau} \tag{24}$$

$$\dot{K}_W = \frac{I_W}{K_W} - \delta \tag{25}$$

where δ is the depreciation rate. Given the base year values of the capital stock and the waiting flows of industries, the above equations allow the computation of the waiting stock in each industry across time.

We defined above the accumulation flow of waiting as the flow of resources diverted from the production of consumption goods towards the production of capital goods. That process does not in itself constitute an input used in the production process but rather a process of input accumulation. The flow of waiting as an input into the production process, on the contrary, bears on the whole stock of waiting units rather than the marginal units produced in any given year. *It is the process of diverting the whole stock of the accumulated primary inputs away from consumption to carry it over to the following year.*

On the demand side, waiting must be productive for the stock to be maintained and carried over from one period to the next. That productivity originates itself from technical progress. The stock of capital per unit of labour input is indeed a function of the state of technology. Were it not for technical progress, society would not withdraw any advantage of postponing any further present consumption for future consumption: the future consumption that could be obtained from the accumulated productive resources would indeed be equal to the actual consumption that could be obtained by using the same resources now.

Hence capital accumulation per unit of labour must occur on the ground that the future use of the resources that are saved will produce more consumption goods than if used now. Accumulation will occur up to the point where the present value of the future delayed consumption is equal to the consumption that could be obtained now with the same resources[18]. Hence, waiting is productive only because of the very existence of technical progress itself and capital accumulation depends on technical progress in that behavioral sense. It follows that the growth in the stock of capital depends not only on the rate of growth of technical progress in capital goods producing industries that increases the amount of capital that can be produced from a given amount of resources as times passes (a measurement issue) but also on the fact that technical progress will increase the efficiency of both capital and labour in the future thereby inducing a positive saving rate (a behavioral issue).

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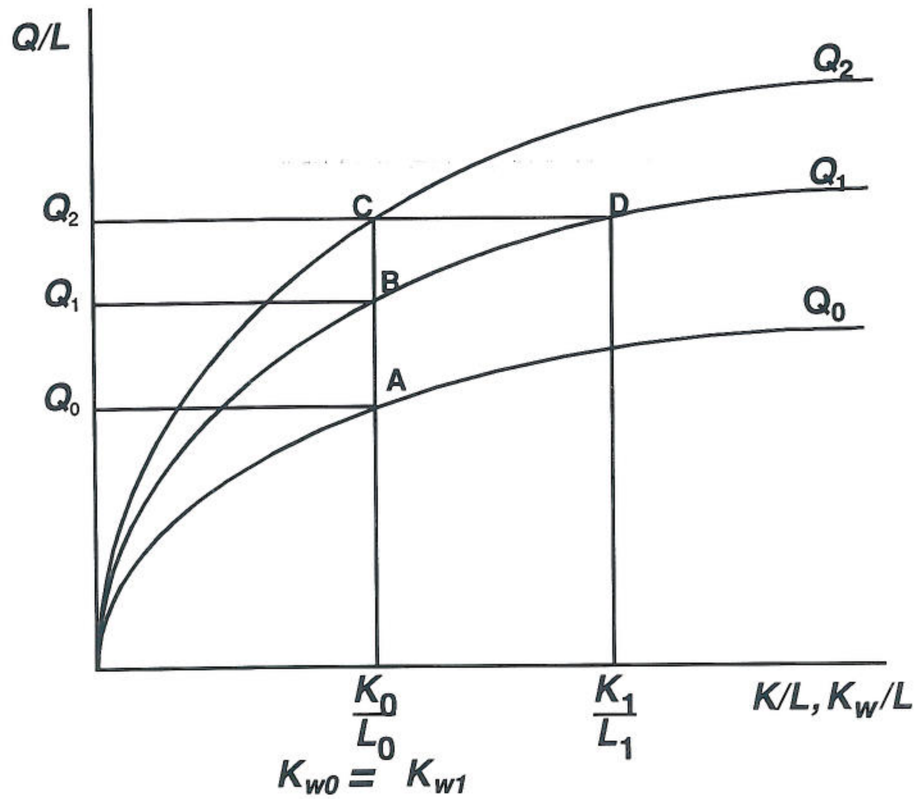
18. On a steady state growth path in which all profits are saved that leads to $i = \tau_W + \nu$ where ν is the rate of growth of labour.

Figure 2 - Alternative Formulations of Technical Progress



Our formulation of the productivity gains looks as if it would basically amounts to that of Hulten (1992) but, in fact, it entails many differences. First, it has the advantage - that it shares with Rymes and Cas' formulation - over Hulten's formulation of expressing technical progress as a pure shift of the production function (from point **A** to **C** on figure 2 above), thus maintaining a single notion of technical progress, namely Solow's residual. In Hulten's model, technical progress shifts the production function in the conventional way (from point **A** to point **B**) but it also induces a move along the production function (from point **B** to point **D**) as technical progress also fosters the growth of the capital stock. Secondly, our formulation gives an explicit expression of waiting as being distinct from capital goods while for Hulten, capital goods are considered as intermediate goods with no primary input counterpart. Indeed, Hulten clearly states that labour is the sole primary production factor in the economy and this brings a fundamental difference between his model and ours. In particular, Hulten cannot recognize the impact of technical progress on the price of capital services[19].

Concerning the latter, it is usually defined as a *usage* or rental price, often called the "user" cost of capital since its insertion in the economic literature by Jorgenson (1963). In the absence of taxation, the cost of capital services, r, is then given by:

$$r = p_k(i+\delta) \tag{26}$$

19. Joan Robinson's (1956-1969) suggestion that the stock of capital be deflated by the wage rate, although related to the idea that capital is a form of indirect labour, nevertheless recognizes that capital (waiting) as an input must be valued such that technical progress increases its price through time. In addition, if the relative price of labour and waiting remain constant through time, this would yield results identical to ours. Keynes in *The General Theory* also measured capital in wage units.

where i is the real rate of interest (the nominal rate of interest net of capital gains). Hence, the cost of using capital goods per unit is given by the interest carrying charges (the charge for waiting) and the replacement or depreciation cost minus the capital gain resulting from the appreciation of the assets represented by the percentage time variation in the capital goods price deflator. Since waiting and capital income are the same we have:

$$r_w K_w = rK = Y_K \tag{27}$$

where r_w is the price of waiting services which remains to be defined. The waiting/capital income equation (27) is similar to equation (21) relating the price of waiting units to the price of capital goods. Solving (27) in terms of stock prices using (21) gives:

$$r_w = r\frac{p_w}{p_k} \tag{28}$$

Substituting for r, using (26), gives the defining equation for r_w:

$$r_w = p_w(i+\delta) \tag{29}$$

Equations (26) and (29) give the price of acquiring the services of additional capital goods/waiting units at the margin given the prevailing rate of interest. Note that these user costs are per unit of capital stock or waiting stock rather than per unit of service. They also apply to service units only if the latter are proportional to their respective stocks, which will be criticized shortly. Since the relative price of waiting units with respect to capital goods is related to technical progress, it follows from (29) that the real price of waiting services is growing through time as a result of technical progress, contrary to the user cost of capital which only follows the fluctuations in the rate of interest and the rate of inflation without any long term trend.

Hulten's measure of technical progress, since it involves only labour would be similar to Rymes and Passinetti's measures[20]. In general and contrary to the particular case depicted on figure 2, his measure of the total impact of technical progress, that is both the shift of and the move along the production function, will give results different than ours except when the economy is on a steady state growth path. This comes from a different assessment of the share of the growth in the capital stock that result from technical progress as opposed to the share of that growth originating from more waiting.

Our dynamic formulation differs from Rymes' static measure of waiting in an important respect. Rymes suggests indeed to define the stock of waiting by the following relationship:

$$\dot{K} = \dot{K}_R + \tau_R \tag{30}$$

where τ_R is Rymes' measure of technical progress associated with his measure of waiting, K_R. That amounts to deflating the stock of capital rather than the flow of investment by the rate of technical progress as in (22) above. It can be shown that this amounts to replacing the stock of capital by indirect labour rather than by a stock of waiting except under equilibrium steady state growth. In that case, however, multifactor productivity is equal to labour productivity and the ratio of the waiting stock to labour (capital stock to output)

20. His measure would be identical to theirs were it not for using gross domestic product as a measure of output rather than net domestic product as in the case of Rymes and, perhaps also, although it is not quite clear, Passinetti. But using gross domestic product as a measure of output is inconsistent with his treatment of capital as being intermediate. See Appendix 4 for more discussion of this aspect.

remains constant. On the dual side, the relative prices of waiting and labour remain constant. That bundles waiting and labour into a single input so that waiting becomes trivial. In non steady state growth, however, the waiting/labour ratio must be allowed to change and, as well, as their relative prices which is not the case in Rymes framework. Indeed, replacing the stock of waiting by the stock of capital in the aggregate productivity equation (23) using Rymes measures for technical progress and the stock of waiting defined in equation (30) and solving for the rate of technical progress, one obtains:

$$\tau_R = \frac{\tau}{\omega_L} \tag{31}$$

Rymes' measure of technical progress is therefore equal to the traditional measure of technical progress of the static framework of Section 2 divided by the labour income share. Considering the stock of capital as an intermediate input and applying the integration rule of Appendix 2 to the aggregate static productivity index, one obtains the same result. Indeed, in that case, the "gross" output is provided by value added y while the net output is given by the wage bill $\omega_L y$. Their ratio, $1/\omega_L$, is the integration factor. Hence Rymes' measure of multifactor productivity, similarly to Hulten and Passinetti's measures, is a "total" labour measure of productivity for an economy having only one primary input, namely labour. However, this difference from our measure is a technical one as Rymes would deny intending to reduce all inputs to labour. The fundamental notion of waiting which is adopted here is otherwise all identical to the one developed by Rymes and from which we borrowed that breakthrough concept.

Waiting units are not produced and are identical to one another whatever type of capital goods are produced. Therefore, these units are homogeneous through time and apply equally to all capital vintages and types of capital goods. Aggregation does not depend on any stringent assumptions as is the case for capital goods which are produced goods subject to qualitative changes through time and, therefore, heterogeneous.

The capital goods are intermediate-like inputs in our dynamic framework rather than final outputs consumed. Primary inputs of working and waiting are transformed into capital goods, which are further transformed into final goods. In that perspective, the stock of capital goods can be seen as an inventory of final goods in process that will become available only in future time periods or as a measure of future delayed consumption. This strongly supports Denison's suggestion to deflate capital goods, as an output of the production process, by the consumption price deflator[21].

4 - Waiting and Capital Services Reconsidered[22]

In Section 2 and 3, we have adopted implicitly the traditional assumption that the services of capital or waiting are proportional to their respective stock. This assumption will presently be relaxed.

One major difficulty with the traditional proportionality assumption appears when one compares time path of waiting accumulation and productivity growth based on alternative (but otherwise fixed) depreciation rates. For two alternative growth scenarios with the same output path, the same allocation of output to gross investment and consumption, and the same initial stock of waiting but different depreciation rates, the stock of waiting will grow at a faster rate in the scenario with the lower depreciation rate. The converse is exactly true for productivity growth if waiting services are assumed to be proportional to their stock. Thus, in a one-sector-one-good model of economic growth, the economy having the largest stock of capital goods at some terminal date, all other things being equal, would be considered the least efficient despite the fact that capital goods are identical to consumption goods and could be consumed at that date. This appears, at face value, quite paradoxical[23]!

21. "In constant prices, this result is obtained by deflating investment by prices of consumer goods". Denison (1993)
22. That section is based on Durand (1994b).

If, on the contrary, one assumes that waiting services are proportional to real depreciation, then it follows that the economy having experienced the largest depreciation over time is the least efficient and the least wealthy at the terminal date. This appears to be much more reasonable than the traditional view. The new view also entails that one sees the production process as yielding both a current output and a one year older capital stock. Wealth accumulation as well as current output matters. In fact, in a dynamic perspective, one can show that the highest consumption path can be supported by maximizing net income, that is current consumption plus net wealth accumulation. Net domestic product rather than gross domestic product becomes the relevant output measure and depreciation becomes an intermediate input.

Bringing back an old debate, Hulten (1992) argued recently that the economy's output should be gross rather than net of capital depreciation for the purpose of measuring efficiency in production. That is, final demand output must be computed as including gross fixed capital formation contrary to the suggestion made earlier by Denison (1962)[24]that it include net investment and that aggregate output be measured by net national product.

Basically, Hulten's point is that even if utility depends on the intertemporal flow of consumption, this flow can only be sustained by producing capital goods replacement units. These units in a one-sector-one-good model are any units produced and there is no reason not to account for these units not used in consumption but used to replace the capital units consumed in the production process. In any case, the statistician observing production in the economy is observing the total amount of commodities produced whatever the eventual split of that production between consumption and investment.

Hulten's argumentation, although intuitively appealing, is nevertheless not quite fully convincing. In the first place, what the statistician would observe would be the output gross of both the intermediate use of the commodities and of their use to replace worn out capital units. Hence, following that argument, the same gross output measure as used at the industry level should be used for the aggregate business sector as well. In the second place, nothing prevents the statistician from doing an imputation of depreciation costs to any production in the economy and, consequently, to measure that production net of depreciation. The fundamental issue, therefore, is whether depreciation is an intermediate or a primary input.

The dynamic framework more naturally sees net production as the proper measure of maximum *sustainable* output in the economy when the measure of capital services accounts for possible changes in the depreciation rate. Consequently, Statistics Canada's value-added productivity indices at both the industry and the business sector levels, although presently defined on *real gross* value added, will eventually be estimated on output net of depreciation.

We conclude that if waiting or capital services are assumed to be proportional to the corresponding stocks, that proportion must be related to the depreciation rate. The higher the depreciation rate, the higher the quantity of services used and the less efficient is the production process. Note that depreciation here is taken as the loss in the real value of capital assets or what has been termed *economic depreciation*. The latter may be associated with physical wear and tear or *physical depreciation* but it may as well be related to obsolescence[25]. Applying a similar reasoning to the interest component of the service flow, one obtains the following service flow measure:

$$S_w = \frac{p_w(i+\delta)K_w}{p_w} = (i+\delta)K_w \tag{32}$$

23. Such a paradox was drawn to our attention recently when the estimated growth path of the capital stock in Canada was revised downward from a maintained base year benchmark value. Following that revision, we had to revise Canadian estimates of multifactor productivity growth upward, without any further historical change in the output, investment and consumption growth path. Hence, we had to admit that the Canadian economy had been more efficient than we thought earlier because it had accumulated less wealth!
24. See also Denison (1989), p.21.
25. For instance, Hulten and Wykoff (1981, p.370) define economic depreciation as "the rate of change of asset price with age at a point in time. In the absence of inflation, this definition corresponds to the widely accepted view that economic depreciation is the value of the capital stock which must be replaced in order to maintain initial investment".

The price of waiting services, therefore, reduces to the price of waiting units, p_W. Note that total waiting costs are the same as stated above. It is the split between the quantity and the price components of the waiting costs which is altered.

Interest is the second major reason for rejecting the traditional assumption of proportionality. That interest enters into the quantity of waiting services is not as obvious as for depreciation and is now briefly discussed. First, one may note that, technically speaking, the rate of interest is a pure percentage per time period just like the depreciation rate. The habit that economists have acquired to consider the rate of interest as the price for waiting comes from the implicit assumption that this rate is multiplied by a price of capital goods set to one. Multiplying the rate of interest by the price of capital goods gives the fraction of the value of a unit of capital that must be charged to production cost. That transforms the percentage rate of interest into a money rate of interest. A symmetric consideration applies to depreciation. Multiplying those unit interest and depreciation costs by the number of capital units gives the total capital cost. But the calculations may just as well be done the other way around with equally sound logic. Multiplying the rate of depreciation by the stock of capital gives the number of capital units lost in the production process. Multiplying these by the price of a capital unit gives the total depreciation charge. Similarly, multiplying the rate of interest by the stock of capital gives the total waiting charge associated with the use of the stock of capital measured in capital units. Multiplying these by the price of capital units gives their value.

Upward movements in either the depreciation or the real interest rates can be qualified as technical regress or downward shifts in the production function as the stock of existing knowledge cannot support the same level of output *and* level of wealth. An increase in the depreciation rate decreases the rate of growth of the capital stock and the discounted future stream of output, thereby decreasing the discounted value of the stock of waiting. A similar reasoning applies to an increase in the real interest rate. For instance, if the interest rate shifts to a higher level following a permanent change in time preferences, this shift increases the waiting service flow input into production. Said differently, the present discounted value of the waiting stock carried over one year into the future is lower at a higher interest rate so that more interest waiting units have to be charged against current production.

According to the traditional view, nothing happens to the level of productivity at the time of an upward movement in the depreciation or the interest rate since capital services do not depend directly on depreciation or time preferences. The potential of the economy remains the same despite the loss of real wealth. Again, this is highly counterintuitive. The alternative measure of waiting or capital services introduced here also has the advantage of reconciling different notions of capital and, in particular, the notion of capital as an input with the notion of capital as a stock of wealth: an increase in either the depreciation or the interest rate decreases the discounted value of the future income stream associated with the capital stock, that is its present value, just as it decreases its efficiency in production as an input.

Hence, the proportionality of stocks and services for both capital and waiting holds provided that both the interest and the depreciation rates remain fixed along a given time path. As a consequence, the price of capital/waiting services reduces to the price of capital/waiting units, p_K, p_{W}.. This seems far more reasonable than the traditional measures that includes the rate of depreciation and the real rate of interest as changes in the depreciation or real interest rate could hardly be associated with pure inflation.

5 - Conclusion

This article has provided a general overview of most aspects of Statistics Canada's Multifactor Productivity Accounts. Although largely using standard methods proposed in the literature and used by other statistical agencies, Statistics Canada is also departing from those methods in important ways. The interindustry/commodity index is certainly one illustration of that departure. Statistics Canada is presently the only statistical agency producing this type of productivity index. The distinction between primary or non-produced and intermediate or produced inputs initially suggested by Rymes and which extends to capital goods in the dynamic input-output framework is presently specific to the program conducted at Statistics Canada, although no dynamic indices have yet been produced. These new conceptual developments were all introduced using the notion of integration. This notion proved to be useful in understanding the various indices and their relationships to one another.

Opening the economy is not a trivial issue. First, one must determine if the imported inputs are primary or intermediate and secondly, one has to deal with the terms of trade issue: is the change in the terms of trade a pure consumption effect that should be excluded from the computation of productivity gains or is it a production effect that must be accounted for by the productivity index. We have rejected Gollop's approach and considered imported inputs as intermediate inputs rather than primary inputs. On the terms of trade issued, our solution is close to the Diewert-Morrison approach except that the trade balance is considered as part of savings rather than as an input.

More importantly, this article has introduced a dynamic framework to illustrate that capital goods are produced inputs over which a process of vertical integration is possible. In that process, industries were viewed as exchanging capital services across time periods. This provided a mean of estimating the productivity gains associated with the production of capital goods and to better disentangle input growth from technical progress.

The measure of waiting that has been proposed is an alternative to Rymes' measure of waiting. But Rymes' underlying concept of waiting has been taken unaltered. It means that Rymes' conclusions with respect to the implications of the new concept of capital services in steady state growth (see Rymes, 1983), also apply when our alternative measure of waiting services is substituted for his. In fact, the two measures of waiting are identical under steady state growth, when the real price of waiting services grows at the same rate as the real wage rate. Under steady state growth, a wage deflator would be just as suitable as our proposed deflator, and Passinetti would not be wrong to treat capital goods as indirect labour [26].

This article has also introduced the notion that the depreciation rate and the interest rate must been taken to be part of the quantity of service flows or capital or waiting rather than being part of their prices contrary to Jorgenson's user cost formula. Services are measured in stock units consumed in the production process either in the form of economic depreciation or interest foregone. The price of services is the same as the price of stock units.

This article has finally advocated the view that economic efficiency must be related to both the flow of current output and the stock of wealth. Wealth accumulation stands for discounted future consumption, so that it disappears from the scene only when considering an infinite time horizon. Efficiency is then defined with respect to the infinite future consumption stream while the traditional view of the static framework considers only current period output. Although this was not shown here, that consumption stream is delimited by the net domestic product rather than the gross domestic product. That includes net wealth accumulation resulting from net domestic investment and the current account surplus. This is consistent with the view that depreciation should be considered as an intermediate input. Capital services can therefore be subdivided into two major components: depreciation, considered as an intermediate input, and waiting services, considered as primary inputs.

The productivity of waiting would originate itself from the state of technology. Without technical progress, societies would have no advantage in accumulating additional capital goods per unit of labour input. Primitive societies do not accumulate productive capital although, in many cases, they could save part of their production if they wished to do so.

26. "Labour emerges from the very logic of the present analysis as the only ultimate factor of production", (Passinetti 1981, p.133).

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Appendix 1: The Computation of Intra-industry Sales

In input-output tables, the industries' gross output is generally defined as the sum of the total production of goods and services of their establishments. It is measured as their total sales in constant prices corrected for inventory changes[27]. The commodity inputs of industries include the purchase of all goods and services originating from all industries including intra-industry sales as well as imported goods and services and other leakages such as government supply of goods and services. This Appendix discusses the methodology used to estimate intra-industry sales and to compute gross outputs net of these sales.

One simple way of doing this would be to look at the commodity composition of industries' outputs and inputs and to match goods and services that appear in both commodity sets. Matching inputs would then be subtracted from the corresponding outputs to calculate gross output net of intra-industry sales. This is the correct answer for a closed economy in a square input-output framework. As our productivity measures apply to an open economy within a rectangular input-output framework, two modifications must be made to the above simple methodology to take account of the many potential origins of commodities.

First, in a rectangular input-output framework, commodities may be produced by more than one industry. Consequently, an industry may use a commodity that it produces, sourcing its input partly from its own production, but also from the other industries producing the same commodity. However, no statistics exist on the exact origin and destination of commodities so that some assumption must be made on their origin and destination. One such assumption corresponds to taking the market shares of industries in the total supply of goods and services of the business sector. Under that assumption, the market share matrix $\boldsymbol{D}$ would be applied to input uses of industries, net of leakages, to determine their origin[28]:

$$U_0 = DU_n \tag{A1.1}$$

where U_n is the matrix of intermediate inputs of industries net of all leakages and U_0 is the matrix of intermediate inputs by industry of origin (U_0 is square). Inputs net of leakages are computed as follows:

$$U_n = U - F_U \bullet U \tag{A1.2}$$

where U is the matrix of intermediate inputs and F_U, the matrix of leakage parameters for intermediate imports, changes in inventories, government supply of goods and services, etc. Equations (A1.1) and (A1.2) provide estimates of own use outputs along the diagonal of U_0. Indeed, the first column of U_0 gives the vector of gross output (aggregated over commodities) of all industries associated with the net input uses of the first industry. The first element of that vector gives the gross output of the first industry associated with its own use of inputs. Similarly, the second element of the second column vector of U_0 will give own use of outputs of industry 2, etc. Own use of outputs of industries must be subtracted from their gross output g to give their output net of intra-industry sales g_n:

$$\begin{aligned} g_n &= g - (I \bullet U_0)i \\ &= g - I \varnothing U_0 \end{aligned} \tag{A1.3}$$

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27. An exception to that rule is given by the output of the construction industry, which excludes intra-industry sales. Other exceptions can be found in many service industries in which output is defined by some margin like retail and wholesale trade.

28. To maintain comparability with the U.S. figures, import leakages are netted out. At the aggregate level, however, this leads to Gollop's measure of multifactor productivity based on the final demand deliveries of the business sector rather than our aggregate business sector measure of productivity which is based on value added as explained in Section 2.

Equation (A1.3) does not seem, at first glance, to provide the required answer as U_0 is giving the output by industry associated with intermediate input uses by industry rather than the output by industry *and* commodity associated with intermediate uses by industries. The matrix U_0 is indeed a square industry by industry matrix. The commodity classification was lost in the transformation process above even though it may seem that it has to be maintained in order to be able to subtract, commodity-wise, own-use commodity output from commodity outputs of industries.

In fact, things are a little simpler. The passage from the actual gross output to gross-output-minus-own-use of inputs productivity indices can be done by re-weighting the neoclassical productivity results by the ratio of these two output measures themselves in current prices[29]. To determine this ratio, it is not necessary to know the commodity breakdown of own use of outputs[30].

Hence, current price outputs of own use may be computed along the lines suggested above. In the above transformation, however, one has to note that inputs include taxes minus subsidies while outputs are at producers' prices with, in some cases, corrections for subsidies. This raises the question of whether own use of inputs must be transformed from input to output prices before being deducted from output. The answer to this question is negative. At the aggregate level, that is when considering the deliveries of the business sector to final demand, the output measure not only shifts from gross output to value added but from gross output at producers' prices to value added at factor prices. In general, whatever the level of aggregation, only the taxes paid by all producers on their inputs are considered in both costs and sales valuation. At the business sector level, these taxes comprise only the direct income taxes on capital and labour plus the indirect taxes on capital services (other indirect taxes). This is how "factor prices" are defined in the productivity database. At the disaggregated industry level, the taxes paid on intermediate inputs are included in the valuation of inputs and implicitly included in producers' prices on the output side. When removing some intermediate inputs from the input list, it then seems only logical to remove their associated taxes altogether. The residual inputs will include all taxes on primary inputs plus the taxes on remaining intermediate inputs.

Going from disaggregated industry levels to gradually higher levels of aggregation is therefore done by removing gradually more and more commodity indirect taxes. These taxes are completely removed at the business sector level. "Valuation" of transactions on the output side gradually shifts from producers' prices to factor prices in a smooth fashion as the gross output similarly converges smoothly to value added.

The ratio of the actual nominal gross output to the nominal gross output net of intra-industry sales must be computed at all levels of aggregation. Own use of outputs for an industry group is not the simple sum of own use of outputs of the component industries. That is, when aggregating industries together, some commodity flows that were in the interindustry category now fall into the intra-industry category. In order to compute the intra-industry flows at various levels of aggregation, one simply has to add up all of the non-diagonal elements of U_0. These elements give the interindustry flows. Both directional flows have to be aggregated. If K_a is an industry aggregation matrix at the *a* level of aggregation, then the intra-industry flows at that aggregation level are given by the diagonal of the following matrix[31]:

$$g_a - g_{na} = I \varnothing K_a U_0 K_a^T \qquad \text{(A1.4)}$$

29. For a direct demonstration, see Gullickson W. and M.J. Harper (1987). For a more general derivation, see Appendix 2.
30. The re-weighting of the productivity indices should occur before the computation of the Törnqvist indices to maintain the one-step aggregation procedure.
31. The rows of the aggregation matrix are composed of ones (included industries) and zeros (excluded industries for the corresponding aggregate.

Appendix 2: Integration and Industry Productivity Measures

Appendix 1 makes use of the result that productivity growth on gross output net of intra-industry sales can be obtained from productivity growth on gross output by weighting the latter. This result is achieved by subtracting from the outputs of an industry its own use of inputs commodity-wise, removing the same inputs from the input set and computing the productivity growth associated with the remaining set of outputs and inputs. However, the subtraction of real quantities of outputs and inputs is legitimate only if done commodity-wise. This process cannot, therefore, be used to derive equation (13) that relates productivity growth on gross output to productivity growth on value added. A more general result, which could be applied to any measure of net output and, in particular, to the measure of gross output net of intra-industry sales of Appendix 1, is necessary for that purpose.

Note that equation (13) is similar to equation (5) relating productivity growth on gross output to productivity growth on gross output net of intra-industry sales. In both cases, the productivity growth measure on net output is obtained by weighting the gross output productivity growth measure by the ratio of the value of gross output to the value of net output. Note also that it can easily be shown that addition (or equivalently subtraction) of real quantities commodity-wise does not affect the Divisia index of an aggregate. Aggregation of Divisia indices, as discussed in Section 2, amounts to the application of the Divisia index formula to an enlarged set of quantities. It amounts to the usual concept of aggregation as a summation process only when that summation is done commodity-wise. This immediately provides a clue as to how to derive indices of productivity on net measures of outputs. Indeed, instead of subtracting quantities of commodities from both the output and the input set, one would rather take the Divisia index of the net output and input set. If $\boldsymbol{v}$ is the vector of the outputs of an industry and $\boldsymbol{u}$ and $\boldsymbol{x}$ are two complementary input vectors such that:

$$\boldsymbol{p}^T\boldsymbol{v} = \rho^T\boldsymbol{u} + \boldsymbol{w}^T\boldsymbol{x} \tag{A2.1}$$

where $\boldsymbol{p}$, ρ and $\boldsymbol{w}$ are respectively the output and input price vectors and if the value of net output is given by:

$$y = \boldsymbol{w}^T\boldsymbol{x} = \boldsymbol{p}^T\boldsymbol{v} - \rho^T\boldsymbol{u} \tag{A2.2}$$

and if we define the Divisia index of the net output vector $[\boldsymbol{v}, -\boldsymbol{u}]$ by:

$$D(\boldsymbol{v}, -\boldsymbol{u}) = \frac{\boldsymbol{p}^T\hat{\boldsymbol{v}}\dot{\boldsymbol{v}} - \rho^T\hat{\boldsymbol{u}}\dot{\boldsymbol{u}}}{y} \tag{A2.3}$$

then, productivity growth on net output is given by:

$$\tau_y = \frac{\boldsymbol{p}^T\hat{\boldsymbol{v}}\dot{\boldsymbol{v}} - \rho^T\hat{\boldsymbol{u}}\dot{\boldsymbol{u}}}{y} - \frac{\boldsymbol{w}^T\hat{\boldsymbol{x}}\dot{\boldsymbol{x}}}{y} \tag{A2.4}$$

This is easily seen as equal to productivity growth on gross output multiplied by the value of gross output on the value of net output. This result applies to all industries. When the vector $\boldsymbol{u}$ includes only own use of inputs as in Appendix 1, then the Divisia index of the vector $[\boldsymbol{v}, -\boldsymbol{u}]$ is just equal to the difference $[\boldsymbol{v}-\boldsymbol{u}]$ so that (A2.4) proves the result used in Appendix 1. Considering y as the value added and $\boldsymbol{x}$ as the vector of primary inputs leads to equation (13) linking productivity on value added to productivity on gross output. Thus, equation (A2.4) provides a general integration rule of industry productivity growth indices. From that rule, it follows immediately that integrated industry indices of productivity growth have higher absolute values than less integrated indices.

Appendix 3: Linking Productivity Indices in the Input-Output Framework

We have seen that the interindustry index is linked to the industry index, in the static framework, through a simple linear relationship that involves the impact (Leontief inverse) matrix. That impact matrix also links the final demand commodity productivity index to the industry productivity index as shown in this Appendix. It follows that the final commodity productivity index is equal to the gross output based interindustry index. To show this, one has first to link the value final demand deliveries of industries to their nominal gross output, $\boldsymbol{g}$, as follows:

$$\boldsymbol{g} = [\boldsymbol{I} - \boldsymbol{DB}]^{-1}\boldsymbol{f} \tag{A3.1}$$

where the vector of nominal deliveries of industries, denoted by $\boldsymbol{f}$, are related to final commodity sales by:

$$\boldsymbol{f} = \boldsymbol{D}\hat{\boldsymbol{p}}\boldsymbol{e} \tag{A3.2}$$

The gross output column vectors associated with each element of $\boldsymbol{f}$ are given by:

$$\boldsymbol{G} = [\boldsymbol{I} - \boldsymbol{DB}]^{-1}\hat{\boldsymbol{f}} \tag{A3.3}$$

The productivity gains associated with the expenditure on any category of final demand deliveries f_i can be computed by computing the average productivity gains associated with the direct and indirect production of these deliveries. This is done according to Domar's rule by taking

$$\tau_{fi} = \beta_i^T \tau_g \tag{A3.4}$$

where the weights β_i are computed by using column i of $\boldsymbol{G}$ and dividing by the total value added associated with f_i that is f_i itself. That gives

$$\tau_{fi} = \boldsymbol{u}_i^T[\boldsymbol{I} - \boldsymbol{B}^T\boldsymbol{D}^T]^{-1}\tau_g \tag{A3.5}$$

where $\boldsymbol{u}_i$ is the i th column of the identity matrix. Taking all final demand deliveries together, one has:

$$\tau_f = [\boldsymbol{I} - \boldsymbol{B}^T\boldsymbol{D}^T]^{-1}\tau_g \tag{A3.6}$$

But this is the same equation that relates interindustry productivity indices to the gross output productivity indices. Equation (A3.6) establishes the equivalence between these two set of indices.

Using the impact matrix, the nominal value added of industries, $\boldsymbol{y}$, can be related to the final sales $\boldsymbol{f}$ by:

$$\boldsymbol{y} = \hat{\lambda}[\boldsymbol{I} - \boldsymbol{DB}]^{-1}\boldsymbol{f}$$

where the diagonal matrix, $\hat{\lambda}$ includes nominal value added to nominal gross output ratios of industries, $\hat{\boldsymbol{y}}\hat{\boldsymbol{g}}^{-1}$.

Replacing $\boldsymbol{f}$ by its diagonal gives an industry by commodity value added matrix $\boldsymbol{Y}$:

$$\boldsymbol{Y} = \hat{\lambda}[\boldsymbol{I} - \boldsymbol{DB}]^{-1}\hat{\boldsymbol{f}} \tag{A3.7}$$

Using (3.7) in place of (3.3) and making a similar reasoning leads to the following relationship between the productivity indices on final sales and the productivity indices of industries on their value added:

$$\tau_{\mathrm{f}} = [\boldsymbol{I} - \boldsymbol{B}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}]^{-1}\hat{\lambda}\tau_{\mathrm{y}} \tag{A3.8}$$

Given this relationship and productivity gains on gross output established by equation (A3.6), one may find a relationship between productivity gains associated with the value added of industries and the productivity gains associated with their gross output:

$$\tau_{\mathrm{y}} = \hat{\lambda}^{-1}\tau_{\mathrm{g}} \tag{A3.9}$$

The productivity gains on industries value added differ from the productivity gains on their gross output because they refer to different production processes. The first set integrates productivity gains downward to final markets while the second set does not integrate production activities: Intermediate inputs are considered as exogenous. Note also that the value added matrix $\boldsymbol{Y}$ may be used to compute the real value added of industries corresponding to the value added productivity measure just defined. One has simply to deflate $\boldsymbol{Y}$ by the commodity prices to get the constant prices value-added matrix $\boldsymbol{Y}_{\mathrm{k}}$:

$$\boldsymbol{Y}_{\mathrm{k}} = \boldsymbol{Y}\hat{\boldsymbol{p}}^{-1} \tag{A3.10}$$

The matrices $\boldsymbol{Y}$ and $\boldsymbol{Y}_{\mathrm{k}}$ may be used to compute the Divisia indices of real industry value added and any of their discrete time approximations as shown in Durand (1994a). Alternatively, real value added may be computed by adding the productivity gains on value added to the growth of industries' primary inputs. To conclude, productivity indices relating to alternative measures of output can be related together using the transpose of the nominal relationships relating the nominal value of these outputs. This is similar to the more familiar dual price relationships of input-output models.