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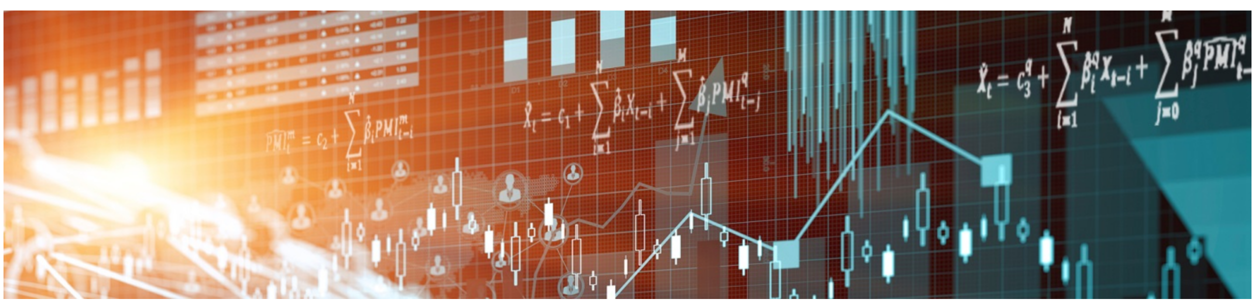
Monetary Policy and Government Debt Dynamics Without Commitment



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Monetary Policy and Government Debt Dynamics Without Commitment

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Abstract

I show that maturity considerations affect the optimal conduct of monetary and fiscal policy during a period of government debt reduction. I consider a New Keynesian model and study a dynamic game of monetary and fiscal policy authorities without commitment, characterizing the incentives that drive the choice of interest rate. The presence of long-term bonds makes government budgets less sensitive to changes in interest rates. As a result, a reduction of government debt induced by a lack of policy commitment is associated with tight monetary policy. Furthermore, the long maturity of bonds slows down the speed of debt reduction up to the rate consistent with existing empirical evidence on the persistence of government debt. Finally, the long maturity of bonds brings down the welfare loss associated with debt reduction.

Bank topics: Monetary policy; Fiscal policy
JEL codes: E52, E62, E63

Résumé

Je démontre que l'échéance des titres de dette publique influe sur la conduite optimale des politiques monétaire et budgétaire en période de réduction de la dette publique. À l'aide d'un modèle de type néokeynésien, j'étudie un jeu dynamique dans lequel il y a absence d'engagement des autorités monétaire et budgétaire, et dans lequel je caractérise les incitations qui déterminent le choix des taux d'intérêt. L'émission d'obligations à long terme rend les budgets du gouvernement moins sensibles aux fluctuations des taux d'intérêt. Par conséquent, une diminution de la dette publique attribuable à un manque d'engagement est associée à une politique monétaire restrictive. De plus, la longue échéance des obligations ralentit le rythme de réduction de la dette, qui atteint le taux conforme aux données empiriques existantes sur la persistance de la dette publique. Enfin, la longue échéance des obligations réduit la perte de bien-être associée à la diminution de la dette.

Sujets : Politique monétaire; Politique budgétaire
Codes JEL : E52, E62, E63

Non-Technical Summary

It is well known that the global economic downturn of 2008–2009 and subsequent fiscal stimulus left many advanced economies with unprecedented levels of government debt. It is also well known that monetary policy plays an important role in government debt dynamics. This creates concerns that central banks might use their influence over inflation and the price of government bonds to reduce the burden of government debt. Moreover, such policy bias is more likely to emerge when a central bank conducts policy period by period without binding itself to a preset course of actions. This paper examines the validity of such concerns.

I discuss the incentives of the monetary authority in the presence of elevated levels of government debt. Is there a reason for central banks to set the policy interest rate loose to accommodate the reduction of government debt? We address this question by studying a dynamic game of monetary and fiscal policy authorities without commitment in a model economy with costly price adjustment and distortionary labor income taxation.

The key innovation of this paper is to allow for government debt with long maturity, as observed in the data across advanced economies. The literature has focused mostly on short-term debt that matures every quarter. The key result of the current paper is that a reduction of government debt induced by a lack of policy commitment is associated with tight monetary policy. The long-term nature of government debt is important for this prescription. The revaluation of the outstanding debt makes government budget less sensitive to changes in interest rates. In turn, this affects the trade-off between inflation and output faced by the monetary authority and makes loose monetary policy less appealing.

I show that the trade-off faced by the monetary policy authority in the presence of government debt depends critically on its maturity. This paper suggests that debt maturity is an important factor for the design of both monetary and fiscal policy. A promising avenue for future research would be to re-examine conventional prescriptions of the monetary-fiscal policy mix depending on the maturity of government debt.

1 Introduction

The global economic downturn of 2008–2009 and subsequent fiscal stimulus have left many advanced economies with unprecedented levels of government debt. Consequently, these fiscal developments have intensified a discussion among policy-makers and economists regarding issues associated with stabilization and the reduction of government debt. Fiscal policy instruments, such as taxes and government spending, are natural candidates for controlling government debt dynamics, which are also driven by interest rates and inflation. This gives rise to the interaction between fiscal and monetary policies, and this paper addresses this interaction during a period of government debt reduction.

In particular, I use a baseline New Keynesian model where a benevolent government issues bonds and commits to repay but cannot commit to a path of policy instruments. The monetary policy instrument is the one-period nominal interest rate. The set of fiscal policy instruments consists of government spending and labor income tax. Because all available instruments are distortionary, the government faces a nontrivial problem of fiscal financing. The government acts in a discretionary manner—it balances the distortions it imposes today and the distortions it postpones by rolling debt over. Without commitment, government debt converges to a certain long-run steady-state level.[1] This paper focuses on the implications of government debt maturity for debt dynamics and underlying policies.

My main finding is that optimal monetary policy during the transition to the steady state is qualitatively different depending on whether the government issues one-period bonds or bonds of longer maturity. A government that starts off with debt above the steady-state level is likely to set a high tax rate. This in turn generates inflation by pushing up the marginal cost of production. If the cost-push inflationary pressure were exogenous, as in Clarida et al. (1999), this would unambiguously call for monetary tightening to contract demand and trade off a decline in output for a reduction in inflation. Here, however, loose monetary policy that stimulates demand could: (1) expand the tax base; (2) inflate away part of the outstanding nominal debt; and (3) improve the valuation of government debt. These fiscal effects of monetary policy alleviate the need to raise the tax rate.

The first two effects are independent of the maturity of government debt. In contrast, the third effect, which reflects the changing market value of government debt, becomes weaker with a longer debt maturity. Optimal monetary policy internalizes these fiscal effects and, therefore, depends on the debt maturity. Quantitative analysis shows that in an economy with one-period government debt, the valuation effect is strong enough to make monetary policy loose during the period of debt reduction. On the other hand, under a long debt maturity, the valuation effect significantly weakens, making tight monetary policy optimal. This key result, found in the baseline model, is reinforced by extending the analysis to a richer environment.

The quantitative analysis of the implications of debt maturity for monetary policy is also carried

[1] Steady-state distortions are eliminated with a lump-sum tax that finances a constant employment subsidy.

out in a medium-scale extension of the model. This extension is known to make the dynamic effects of monetary policy on key macroeconomic variables in the model quantitatively consistent with the data. It builds on the baseline model by adding working capital and wage-setting frictions, habit formation in consumption, and the accumulation of capital with the costs of utilization and investment adjustments. Using posterior mean parameter estimates of the U.S. economy by Christiano et al. (2010), I show that sustaining government debt above the steady-state level warrants loose monetary policy in the case of one-period debt, while lengthening the maturity beyond three periods reverses the monetary stance and makes it tight.

In addition, I show that the dependence of monetary policy on debt maturity has important implications for the optimal speed of debt reduction. The reduction of debt towards the steady state is driven by strategic incentives to manipulate future governments. Consider the case of one-period debt. Expectations of the government following loose monetary policy in the future create an indirect cost of maintaining debt above the steady-state level. An important component of this cost is an increase in current inflation due to expectations of high inflation in the future. Thus, it is optimal in every period for the government to leave a lower debt to its successor until the steady state is reached. As the debt maturity becomes long and monetary policy becomes tight, the indirect cost of maintaining debt falls and the reduction of debt becomes more gradual.

Accounting for long maturity aligns the speed of debt reduction with existing empirical evidence on the persistence of government debt. The quantitative assessment of the baseline model with one-period government debt shows that it is optimal to reduce debt by roughly half within a single quarter. Such a fast change of government debt is at odds with empirical evidence. Using postwar U.S. data, Friedman (2005), for instance, finds a half-life equal to 85 quarters for the debt-to-GDP ratio response to its own shock in a univariate autoregressive setting.[2] When the maturity of government debt is equal to four years, the half-life of debt reduction in the baseline model is equal to 92 quarters. The slowdown of the reduction of government debt with long maturity is also present in the medium-scale extension of the model.

Finally, this paper also contributes to a discussion regarding the relationship between the maturity of government debt and inflation. As the maturity of government debt affects the stance of monetary policy and the speed of debt reduction, it hence also affects the magnitude and persistence of inflation. In particular, tight monetary policy and the slowdown of debt reduction, brought about by long maturity, increase the persistence but reduce the magnitude of inflation. From the normative perspective, such intertemporal smoothing of inflation leads to a reduction in the welfare loss associated with debt reduction. From the positive perspective, this result provides a theoretical mechanism for the empirical finding of Rose (2014), who shows that the existence of a long-term government bonds market may help to keep inflation low and stable.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature.

[2] Marcet and Scott (2009) provide extensive evidence of high persistence of the market value of debt.

Section 3 highlights the main results analytically using a simple version of the model. Section 4 describes the baseline model and discusses parameterization and solution methods. Optimal policy and welfare in the baseline model are analyzed in Section 5. Section 6 discusses a medium-scale extension of the model. Section 7 concludes.

2 Related Literature

The current paper is related mostly to economies in which the risk of default can be abstracted such that the lack of commitment applies only to the path of policy instruments and not to a decision regarding debt repayment.[3] A number of papers study optimal discretionary policy with short-term (one-period) bonds. Debortoli and Nunes (2012) study the effects of lack of fiscal commitment on the steady-state level of debt in a real economy, showing that government debt must be reduced to zero. Gnocchi and Lambertini (2016) consider a nominal economy with sticky prices and show that commitment of a monetary authority to an inflation target can lead to a positive steady-state level of debt, despite a lack of fiscal commitment.[4] In a similar environment, Leith and Wren-Lewis (2013) focus on optimal policy under joint monetary and fiscal discretion while taking a positive steady-state level of debt as given.[5] They show that a reduction of debt towards the steady-state can be fast and accompanied by low real interest rates because the monetary policy stance is accommodative. The current paper demonstrates that, as debt maturity lengthens, the response of monetary policy to high levels of debt turns from accommodation to tightening.

In a related work, Bhattarai et al. (2019) and Leeper et al. (2019) demonstrate the importance of government debt maturity for the equilibrium outcome under optimal monetary-fiscal policy mix without commitment. While Bhattarai et al. (2019) show that longer maturity leads to a less accommodative monetary policy stance, I find that the change goes one step further to a qualitatively different stance of monetary policy. This change is due to the endogenous trade-off between inflation and output brought by the distortionary effect of labor income taxes. In contrast, the welfare cost of taxation in Bhattarai et al. (2019) is due to a collection cost of lump-sum taxes that do not directly affect the equilibrium. The current paper, therefore, shows that the allocation effects of taxes may have nontrivial implications when studying the role of debt maturity. Leeper et al. (2019) provide a detailed analysis of how the balance sheet incentive to inflate nominal government debt away interacts with the strategic incentives to manipulate future governments in the presence of long-term debt. The current paper adds a complementary emphasis on the incentive to increase the market price of newly issued debt. This incentive is an important driver of the monetary policy interest rate and is present independently of whether government debt is nominal or real. Finally, the current

[3] A review of the issues related to sovereign default and further references can be found in Aguiar et al. (2016).

[4] Ellison and Rankin (2007), Díaz-Giménez et al. (2008), Martin (2009) discuss the implications of a lack of monetary and fiscal commitment for the steady-state level of debt in a classical economy with cash-in-advance constraint.

[5] Other studies that use New Keynesian models to analyze optimal discretionary monetary and fiscal policy with one-period government debt include Burgert and Schmidt (2014), Eggertsson (2006), and Niemann et al. (2013).

paper complements both of these studies by extending the analysis of the optimal discretionary fiscal-monetary policy mix to a medium-scale version of the New Keynesian model.[6]

Faraglia et al. (2013) and Leeper and Zhou (2013) study the impact of government debt maturity on inflation under the assumption of joint monetary and fiscal commitment.[7] They provide a rationale for a longer average maturity leading to stronger and more persistent inflation. In these studies, however, there is no intrinsic incentive to bring debt towards a certain long-run level because government debt follows a near random-walk behavior. In the current paper, however, debt should be reduced in the absence of commitment until the steady state is reached.[8]

An alternative approach to questions related to government debt reduction is to abstract from the intrinsic reasons that induce debt reduction and impose it exogenously. Using this approach, Romei (2017) and Scheer (2019) analyze the performance of fiscal instruments for debt reduction in models of closed economies with nominal frictions. Both papers introduce monetary policy in the form of Taylor-type rules. Similarly, Andrés et al. (2019) study an exogenous fiscal consolidation using fiscal instruments amid private deleveraging in a model of a small open economy within a monetary union. Krause and Moyen (2016) examine the extent to which an exogenous change of the inflation target in a closed economy model may reduce government debt.[9] Thus, these studies abstract from the issues related to a lack of policy commitment in general and optimal monetary policy in particular, which are the focus of this paper.

This paper is broadly related to literature studying the optimal maturity structure of government debt. A number of papers incorporate a portfolio problem into the optimal policy problem of the government under the assumption of commitment—see, e.g., Angeletos (2002), Buera and Nicolini (2004), Faraglia et al. (2019), and Lustig et al. (2008). More closely related are those studies that introduce a lack of commitment and solve for optimal maturity structure in a real economy (see Debortoli et al. (2016)) and in a nominal economy with the cash-in-advance constraint (see Arellano et al. (2013)). In contrast, I treat maturity structure as determined by a separate debt management authority that is not explicitly modeled. Optimal fiscal and monetary policy is chosen assuming an average maturity of government debt that does not change over time, which is not much at odds

[6] In a recent study, Cantore et al. (2019) employ a medium-scale New Keynesian model with a sovereign risk premium to analyze optimal fiscal and monetary policy, including that under discretion, in times of a debt crisis. As was previously mentioned, the focus of the current paper is on the periods with a negligible sovereign risk.

[7] Optimal fiscal and monetary policy with commitment has been extensively studied under the assumption of one-period bonds. Seminal contributions include Lucas and Stokey (1983) and Aiyagari et al. (2002) in real economies with complete and incomplete markets, respectively, as well as Chari et al. (1991) and Schmitt-Grohé and Uribe (2004) in nominal economies with flexible and sticky prices, respectively.

[8] A number of studies under commitment do find incentives that make it optimal to reduce government debt. Adam (2011) and Bhandari et al. (2016) show that budget risk due to incomplete markets makes it optimal to reduce government debt over time, albeit very slowly. Horvath (2011) shows that assuming unconditional welfare objective also implies gradual adjustment of government debt towards a certain mean value. With complete markets, Ferrière and Karantounias (2019) show that ambiguity aversion makes it optimal to reduce government debt to zero if intertemporal elasticity of substitution is sufficiently low.

[9] Empirical estimates of the impact of inflation on the real value of government debt in the U.S. can be found in Hall and Sargent (2011) and Hilscher et al. (2014), which use backward- and forward-looking approaches, respectively.

with the stability of portfolio shares documented in Faraglia et al. (2019) using postwar U.S. data.

3 A Primer

This section develops an analytical intuition for the key mechanism and main results in a simplified version of the model. The analysis builds on a standard New Keynesian dynamic general equilibrium model of a closed economy with a monopolistically competitive intermediate-goods market and costly price adjustment. The model is augmented with fiscal policy *à la* Lucas and Stokey (1983): the government issues bonds and levies the labor income tax to finance government spending. There is no uncertainty; thus, financial markets are complete.

The analysis in the current section starts by postulating a linear-quadratic policy problem of the government subject to aggregate private-sector equilibrium conditions. A detailed micro-founded derivation of the model in a nonlinear setting is presented in the next section. The details on deriving the linear-quadratic approximation of the nonlinear policy problem are shown in Appendix A.6. The analysis in the current section is simplified by assuming that government spending is exogenous and constant over time and that the government issues bonds indexed to inflation.

Consider an economy with an efficient steady state, in which output is equal to $\bar{y}$, gross inflation $\bar{\pi}$ is equal to 1, and a positive (real) quantity of government debt $\bar{b} > 0$ is valued at a market price $\bar{q} > 0$. Moreover, in the steady state, the government taxes labor at a rate $\bar{\tau} > 0$ and receives labor tax revenue $\bar{\Upsilon} > 0$. In the first period, the government starts off with debt above the steady-state level; i.e., $\hat{b}_{-1} > 0$ (from now, a hat is used to denote percentage deviations from the steady state). The government can credibly promise to repay its debt but cannot commit to a transition path to the steady state and, therefore, optimizes sequentially. In each period t, the government solves the following constrained social planning problem:

$$\mathcal{U}(\hat{b}_{t-1}) = \max_{(\hat{y}_t, \hat{\pi}_t, \hat{q}_t, \hat{b}_t, \hat{\tau}_t)} -\frac{1}{2}(\vartheta \hat{y}_t^2 + \hat{\pi}_t^2) + \beta \mathcal{U}(\hat{b}_t) \tag{3.1}$$

subject to

$$\hat{\pi}_t = \kappa_y \hat{y}_t + \kappa_\tau \hat{\tau}_t + \beta \hat{\pi}_{t+1}, \tag{3.2}$$

$$\hat{y}_t = \hat{y}_{t+1} - \tilde{\gamma}_c^{-1} \left(\rho \beta \hat{q}_{t+1} - \hat{q}_t \right), \tag{3.3}$$

$$\bar{\Gamma}(\beta^{-1} \hat{b}_{t-1} + \rho \hat{q}_t) = \bar{\Gamma}(\hat{b}_t + \hat{q}_t) + \bar{\Upsilon}(\upsilon_\tau \hat{\tau}_t + \upsilon_y \hat{y}_t), \tag{3.4}$$

taking into account that the current choice of government bonds passed over into the next period, $\hat{b}_t$, affects optimal choice in the next period. In particular, next period output, inflation and the price of government bonds are determined by the corresponding linear equilibrium decision rules $\hat{y}_{t+1} = \mathcal{Y}_b \hat{b}_t$, $\hat{\pi}_{t+1} = \Pi_b \hat{b}_t$ and $\hat{q}_{t+1} = \mathcal{Q}_b \hat{b}_t$.[10]

[10] The analysis considers Markov perfect equilibria and abstracts from reputational equilibria.

Objective function (3.1) describes a loss from deviations of inflation and output, where $\vartheta > 0$ is the weight of output deviation relative to inflation deviation and $\beta \in (0, 1)$ is the time discount factor. Equation (3.2) is a forward-looking Phillips curve that describes the relation between inflation dynamics and the real economic activity, where $\kappa_y, \kappa_\tau > 0$ are the elasticities of inflation with respect to aggregate demand and the tax rate. Investment-savings equation (3.3) describes the relation between demand dynamics and the real interest rate, where $\tilde{\gamma}_c$ is the inverse intertemporal elasticity of substitution of aggregate demand.

Equation (3.4) is the flow budget constraint of the government written in real terms, where $\bar{\Gamma} \equiv \bar{b}\bar{q}$ is the steady-state market value of government debt and $\upsilon_y, \upsilon_\tau > 0$ are the elasticities of tax revenue with respect to aggregate demand and the tax rate. Government debt is modeled as a portfolio of perpetual bonds with the structure of payoffs decaying at the exponential rate $\rho \in [0, 1]$. The larger ρ is, the longer the average maturity (duration) of government debt. Parameter ρ also captures the elasticity of the market value of outstanding unmatured government debt with respect to the price of newly issued government debt, $\hat{q}_t$.

One can show that a government that starts off with a tight budget chooses to tolerate positive inflation. The solution to the problem yields the following optimality conditions:

$$\hat{y}_t = \Phi_y \hat{\pi}_t, \tag{3.5}$$

$$\hat{\pi}_t = \hat{\pi}_{t+1} + \Phi_\pi \hat{\pi}_t. \tag{3.6}$$

where

$$\Phi_y \equiv \left[-\frac{\kappa_y}{\vartheta} + \left(\frac{\upsilon_y}{\upsilon_\tau} + \tilde{\gamma}_c \frac{(1-\rho)\bar{\Gamma}}{\upsilon_\tau \bar{\Upsilon}} \right) \frac{\kappa_\tau}{\vartheta} \right], \tag{3.7}$$

$$\Phi_\pi \equiv \left[\frac{\upsilon_\tau \bar{\Upsilon}}{\kappa_\tau \bar{\Gamma}} \beta \mathit{\Pi}_b + (1-\rho)\Big(\tilde{\gamma}_c \mathcal{Y}_b - \rho\beta \mathcal{Q}_b \Big) \right]. \tag{3.8}$$

Condition (3.5) is a targeting rule that describes the optimal way to balance the intratemporal trade-off between inflation and output. Coefficient Φ_y consists of three components that form the targeting rule. The first component reflects the incentive to contract demand whenever there is a positive deviation of inflation. The optimal scope of contraction is directly proportional to the gain in reduced inflation per unit of output deviation, κ_y, and inversely proportional to the relative weight placed on output deviation, ϑ. In the absence of endogenous fiscal policy, this component would be the sole determinant of the targeting rule. In what follows it is referred to as the monetary component of the targeting rule.

The two remaining components of the targeting rule are referred to as fiscal. The fiscal components reflect the incentive to stimulate demand whenever there is a positive deviation of inflation. Demand stimulus allows for the reduction of inflation deviation by reducing the tax rate while keeping the fiscal surplus constant. First, demand stimulus increases the tax base and allows for the reduction of the tax by ν_y/ν_τ per unit of output deviation. Second, demand stimulus increases the price of government debt by $\tilde{\gamma}_c$ per unit of output deviation. This allows for the tax to be reduced by $(1-\rho)\bar{\Gamma}/(\upsilon_\tau\bar{\Upsilon})$ per unit of debt price change. The overall incentive to reduce the tax by stimulating demand depends positively on the gain in reduced inflation per unit of tax rate change, κ_τ, and inversely on the relative weight of output deviation, ϑ.

Condition (3.6) is the generalized Euler equation that describes the optimal way to balance the intertemporal inflation trade-off. It implicitly determines government debt dynamics by equalizing the marginal benefits and marginal costs of changing government debt. The left-hand side of (3.6) shows the direct benefit of increasing debt derived from the lower tax and inflation in the current period. The debt increase creates the direct cost of higher tax and inflation in the next period, reflected by the first term on the right-hand side of (3.6). The coefficient Φ_π consists of two components that reflect the indirect costs of increasing government debt. First, anticipation of higher inflation in the next period leads to an increase in inflation in the current period. Second, falling demand for saving leads to a declining price of government bonds. This, in turn, leads to a higher tax rate and inflation in the current period. As a result of having these indirect costs, it is optimal to reduce government debt until the steady state is reached. The analysis below is restricted to monotone equilibria, i.e., such that $0 < \Phi_\pi < 1$.

Let $\hat{i}_t$ be the one-period nominal interest rate: the monetary policy instrument controlled by the government. The relation between the nominal and the real interest rate is described by the Fisher equation $\hat{i}_t = (\rho\beta\hat{q}_{t+1} - \hat{q}_t) + \hat{\pi}_{t+1}$, where $\rho\beta\hat{q}_{t+1} - \hat{q}_t$ is the real holding period return on government debt. The optimal choice of $\hat{i}_t$ is determined so as to be compatible with optimality conditions (3.2)–(3.6). Writing the optimal choice of $\hat{i}_t$ as a feedback to expected inflation $\hat{\pi}_{t+1}$ yields

$$\hat{i}_t = \gamma_\pi \hat{\pi}_{t+1}, \tag{3.9}$$

where

$$\gamma_\pi \equiv \left(1 - \tilde{\gamma}_c \Phi_y \frac{\Phi_\pi}{(1-\Phi_\pi)}\right).$$

The magnitude of the feedback coefficient on expected inflation, γ_π, relative to unity depends entirely on the sign of the targeting rule coefficient Φ_y. If the monetary component of the targeting rule is stronger, then $\Phi_y < 0$. In this case, the nominal interest rate reacts more than one-to-one to the expected inflation; i.e., $\gamma_\pi > 1$. As a result, the real interest rate exceeds the steady-state level and contracts demand along the transition path. In other words, monetary policy is tight during the period of government debt reduction. In contrast, when the fiscal components of the targeting rule are stronger, the nominal interest rate reacts less than one-to-one to the expected inflation, $\gamma_\pi < 1$, because $\Phi_y > 0$. In this case, the reduction of government debt is supported by loose monetary policy that keeps the real interest rate below the steady-state level and stimulates demand.

This characterization of optimal policy allows one to highlight the comparative effect of the maturity of government debt on the stance of monetary policy. The longer the maturity of government debt, the larger the elasticity of the market value of outstanding unmatured government debt with respect to the price of newly issued government debt. This makes demand stimulus less effective in allowing the government to reduce the tax rate and inflation. As a result, the relative strength of the fiscal components of the targeting rule declines. If the relative strength of the fiscal components varies enough, there is a threshold maturity that makes the targeting rule coefficient Φ_y switch its sign. In this case, monetary policy is loose if the maturity is shorter than the threshold and tight otherwise.

The maturity of government debt also affects the optimal speed of its reduction. It does so through the indirect costs of changing government debt, Φ_π. First, lengthening the maturity of government debt reduces the costs for a given next period decision rules $\mathit{\Pi}_b$, $\mathcal{Y}_b$, and $\mathcal{Q}_b$. Second, the decision rules are affected by the maturity of debt in equilibrium. These effects do not have a closed-form solution. Nevertheless, it is reasonable to expect that the coefficient of the decision rule for inflation, $\mathit{\Pi}_b$, is lower when monetary policy is tight than when it is loose. Therefore, lengthening the maturity is likely to be associated with slowing down the reduction of government debt.

Optimal discretionary monetary policy alone—without fiscal policy—is analyzed in Clarida et al. (1999). There the trade-off between inflation and output is created by an exogenous cost-push disturbance. In contrast, in the current paper, the inflation-output trade-off is a result of the endogenous choice of the labor income tax. The analysis above shows that optimal monetary policy internalizes the effects it has on the tax rate choice. This, in turn, influences the speed of the convergence of government debt to the steady state. Importantly, the strength of these fiscal effects of monetary policy depends on the maturity of government debt. The following sections assess the importance of the fiscal effects of monetary policy quantitatively in the model that relaxes the assumptions of debt indexation and fixed government spending.

4 The Baseline Model

This section provides a micro-founded derivation of the baseline model underlying the analysis in this paper. The baseline model described in this section consists of four types of economic agents: the representative household, the representative final-good producer, a continuum of intermediate-goods producers, and the government.

4.1 Households

The economy is populated by a continuum of ex-ante and ex-post identical infinitely lived households. The representative household dislikes labor and enjoys private consumption as well as consumption of public goods. Household preferences are represented by the following lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(G_t) - v(h_t) \right], \tag{4.1}$$

where $\beta \in (0,1)$ is the time discount factor, c_t is private consumption of the final (aggregate) good, h_t is labor supply (time endowment is normalized to one), G_t is (real) government spending on public provision of the final good, u and g are period utility functions of private consumption and consumption of public goods, and v is the period disutility function of labor. In what follows, functions u and g are assumed to be increasing and concave, and function v is assumed to be increasing and convex.

The household enters period t holding assets in the form of maturing nominal one-period (discount) government bonds B^s_{t-1} and a portfolio of long-term government bonds B_{t-1}. Supply of newly issued bonds in period t is determined by government policies, which are discussed later. One-period bonds issued in period t are purchased at a price R_t^{-1}, where R_t is a one-period nominal interest rate. As in Woodford (2001), the portfolio of long-term bonds is defined as a set of perpetual bonds with nominal payoffs that start from 1 and decay over time geometrically at the rate $\rho \in [0,1]$.[11] The outstanding portfolio B_{t-1} with remaining flow of payoffs is exchanged in period t for a new portfolio B_t at market prices. One can use the no-arbitrage argument to show that the price q^o_t of the former is equal to the price q_t of the latter scaled by the factor ρ; see Appendix A.1. Under the described market arrangement, the flow budget constraint of the household takes the following form:

$$P_t c_t + R_t^{-1} B^s_t + q_t B_t = (1-\tau_t) W_t h_t + B^s_{t-1} + (1+\rho q_t) B_{t-1} + \int_0^1 \Pi_{i,t} di - T_t, \tag{4.2}$$

where P_t is the unit price of the final good, W_t is the nominal wage, τ_t is the linear tax rate on labor income, $\Pi_{i,t}$ is the share of profits from sales of intermediate good of type i distributed in a lump-sum way, and T_t is the lump-sum tax collected by the government. To have a well-defined intertemporal budget constraint, an additional condition that rules out "Ponzi schemes" is implicitly imposed.

The household maximizes (4.1) by choosing consumption, labor and end-of-period bond holdings $\{c_t, h_t, B^S_t, B_t\}_{t=0}^{\infty}$ subject to the budget constraint (4.2) and a no-Ponzi condition, taking as given prices, policies and firms' profits $\{P_t, W_t, R_t, \tau_t, G_t, T_t, \Pi_t(i)\}_{t=0}^{\infty}$, as well as initial bond holdings B^S_{-1} and B_{-1}. The optimal plan of the household has to satisfy (4.2) and a standard transversality condition as well as the following optimality conditions:

[11] Another parsimonious way to model long-term debt is to assume bonds that mature probabilistically; see, e.g., Krause and Moyen (2016).

$$\lambda_t = \beta^t \frac{u'(c_t)}{P_t}, \tag{4.3}$$

$$R_t = \frac{1+\rho q_{t+1}}{q_t}, \tag{4.4}$$

$$1 = \beta \frac{R_t}{\pi_{t+1}} \frac{u'(c_{t+1})}{u'(c_t)}, \tag{4.5}$$

$$w_t = \frac{1}{(1-\tau_t)} \frac{v'(h_t)}{u'(c_t)}, \tag{4.6}$$

where λ_t is the Lagrange multiplier of the budget constraint (4.2), $\pi_{t+1} \equiv {}^{P_{t+1}}/{}_{P_t}$ is the gross one-period inflation rate, and $w_t \equiv {}^{W_t}/{}_{P_t}$ is the real wage. Equation (4.4) is the no-arbitrage condition between the one-period nominal interest rate and the price of long-term government bonds. Equation (4.5) is the Euler equation describing intertemporal allocation of consumption and savings. Equation (4.6) describes intratemporal trade-off between consumption and leisure.

When assuming indexation of long-term government bonds to inflation, as was done in a simplified version of the model studied in Section 3, the no-arbitrage condition (4.4) becomes:

$$\frac{R_t}{\pi_{t+1}} = \frac{1+\rho q_{t+1}}{q_t}. \tag{4.7}$$

4.2 Firms

Production of the final consumption good consists of two stages. On the upper stage of the production process, there is a (representative) perfectly competitive firm that assembles the final good y_t from a bundle of imperfectly substitutable intermediate goods $y_{i,t}$ indexed by $i \in [0,1]$ using the constant-returns-to-scale technology

$$y_t = \left(\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the intratemporal elasticity of substitution across different varieties of intermediate goods. Solving the profit maximization problem of the final-good producer results in the following demand schedule for every intermediate good i:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\theta} y_t, \tag{4.8}$$

where $P_{i,t}$ is the price of intermediate good i. From the zero-profit condition—brought by the absence of an entry cost—it follows that P_t can be written as a price index:

$$P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

On the lower stage of the production process, there is a continuum of firms of unit mass, each producing an intermediate good with a technology that is linear in labor:

$$y_{i,t} = h_{i,t},$$

where $h_{i,t}$ is the labor input of firm i. Imperfect price-elasticity of the final-good producer's demand (4.8) endows intermediate firms with market power to set prices, which in general distorts the economy. Another distortion in this economy is due to a nominal rigidity that makes price adjustment costly for a firm. It is modeled, following Rotemberg (1982), by introducing the quadratic cost of adjusting nominal prices (measured in terms of the final good) given by

$$\kappa_{i,t} \equiv \frac{\varphi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 y_t,$$

where $\varphi \geqslant 0$ measures the degree of nominal price rigidity. Higher values of φ indicate greater price stickiness, while $\varphi = 0$ corresponds to the case of perfectly flexible prices. The firm producing good i sets the price $P_{i,t}$ and hires, in a perfectly competitive labor market, the quantity of labor that is necessary to satisfy realized demand. The present discounted real value of profits received by this firm is given by:

$$\sum_{t=0}^{\infty} \lambda_t \left[P_{i,t} y_{i,t} - (1-s) W_t y_{i,t} - P_t \kappa_{i,t} \right], \tag{4.9}$$

where s is the time-invariant rate of labor (employment) subsidy provided by the government to eliminate steady-state distortions created by monopolistic competition and taxation of labor income.[12]

The pricing problem of the firm producing intermediate good i is dynamic, due to the presence of the price adjustment cost. The firm chooses a sequence of prices $\{P_{i,t}\}_{t=0}^{\infty}$ to maximize its profits (4.9) subject to demand function (4.8), taking as given nominal wage, aggregate index of prices, and aggregate demand $\{W_t, P_t, y_t\}_{t=0}^{\infty}$. In equilibrium all intermediate-goods producers behave symmetrically and charge identical prices $P_{i,t} = P_t$ for all $i \in [0, 1]$. Then, the optimizing behavior of intermediate-goods producers is characterized by the first-order condition of the pricing

[12] The specification follows Burgert and Schmidt (2014) and Leith and Wren-Lewis (2013). One can alternatively design a labor income subsidy, which works equivalently when the labor market is competitive.

problem, written as follows:

$$(1-s)w_t - \frac{(\theta-1)}{\theta} = \frac{\varphi}{\theta}\left((\pi_t - 1)\,\pi_t - \beta\frac{u_{c,t+1}}{u_{c,t}}\frac{y_{t+1}}{y_t}\,(\pi_{t+1}-1)\,\pi_{t+1}\right). \tag{4.10}$$

Equation (4.10) is a nonlinear version of the New Keynesian Phillips curve that describes evolution of inflation over time as driven by the real marginal cost of production, which is proportional to the real wage, w_t.

4.3 The Government

The government consists of a central bank and a treasury. The central bank controls the short-term nominal interest rate, R_t. The treasury chooses the amount of spending on public good provision to the household, G_t. To finance government spending, the treasury levies a labor income tax at the rate τ_t and participates in the bond market.

Assuming that one-period bonds are in zero net supply, the treasury can borrow on the bond market by issuing a portfolio of long-term government bonds in the form of perpetuities with decaying nominal payoffs. Modeled this way, government debt has a maturity structure with shares that decay with remaining maturity. The payoff decay factor, ρ, parametrizes the average maturity of government debt. Setting $\rho = 0$ makes the model identical to the case of one-period debt. Another extreme is the case of "consol" bonds obtained when $\rho = 1$. For a generic value of ρ, in a steady state with zero inflation, average maturity of government debt as measured by duration is equal to $(1-\beta\rho)^{-1}$ model periods; see Appendix A.2. The payoff decay factor is not a policy instrument of either the central bank or the treasury; it is assumed to be exogenously given and time invariant. The assumption of exogeneity captures the fact that management of the maturity structure of government debt is not part of conventional monetary and fiscal policy.[13]

One can think of $\{G_t, \tau_t, R_t\}_{t=0}^{\infty}$ as government policy instruments that pin down government debt as satisfying government budget. The consolidated flow budget constraint of the government is given by:

$$q_t B_t = (1+\rho q_t)B_{t-1} + P_t\left(G_t - (\tau_t - s)w_t h_t\right) - T_t.$$

Lump-sum tax T_t is restricted to be used for the sole purpose of transferring resources corresponding to the labor subsidy. Furthermore, because the goal of the labor subsidy is to correct only long-run distortions in the economy, the value of the lump-sum tax is set to be constant over time and equal to the steady-state value of the subsidy. The flow budget constraint of the government in real terms is then given by:

$$q_t b_t = (1+\rho q_t)b_{t-1}\pi_t^{-1} + \left(G_t + \varsigma_t - \tau_t w_t h_t\right), \tag{4.11}$$

[13] See Bhattarai et al. (2019) for a model with time-varying and endogenously determined payoff decay factor that captures quantitative easing, an unconventional monetary policy.

where $b_t \equiv B_t/P_t$ is the quantity of long-term government bonds in real terms, and $\varsigma_t \equiv s w_t h_t - s\bar{w}\bar{h}$ is the deviation of the labor subsidy from its steady-state level in real terms.

In a simplified version of the model studied in Section 3, government spending is assumed to be exogenously determined and constant over time, $G_t = \bar{G}$, long-term government bonds are assumed to be indexed to inflation, and the lump-sum tax is assumed to finance the labor subsidy also outside of the steady state, $\varsigma_t = 0$. In this case, the flow budget constraint of the government in real terms reads as:

$$q_t b_t = (1 + \rho q_t) b_{t-1} + \left(\bar{G} - \tau_t w_t h_t\right). \tag{4.12}$$

4.4 Private-Sector Equilibria

Given government policy, let the private-sector equilibrium be the economic outcome of the model consistent with optimal decisions of households and firms, where the markets are clear.

Symmetric pricing behavior of all the firms in equilibrium makes them produce the same amount of output and hire the same amount of labor; hence, $y_{i,t} = y_t$ and $h_{i,t} = h_t$ for all $i \in [0, 1]$. Therefore, one can write the aggregate production function of the form as follows:

$$y_t = h_t, \tag{4.13}$$

and the aggregate resource constraint resulting from the clearing of the goods market as follows:

$$h_t = c_t + G_t + \frac{\varphi}{2} (\pi_t - 1)^2 h_t. \tag{4.14}$$

The aggregate resource constraint shows that nominal rigidity creates a wedge between the output and the aggregate consumption because a fraction of the output is allocated to paying the price adjustment cost.

Formally, the private-sector equilibrium is defined as follows.

Definition 1. *Given initial government debt b_{-1} and policy $\{G_t, \tau_t, R_t\}_{t=0}^{\infty}$, the private-sector equilibrium is a sequence $\{c_t, y_t, h_t, \pi_t, w_t, q_t, b_t\}_{t=0}^{\infty}$ satisfying equations (4.4)–(4.6), (4.10), (4.11), (4.13), (4.14) for $t \geqslant 0$ and the transversality condition.*

4.5 The First-Best Allocation

Before introducing the government's problem of policy choice, it is instructive to characterize the first-best allocation of private consumption, consumption of public good, and labor. This first-best allocation is meant to serve as the efficiency benchmark for the private-sector equilibrium under optimal government policy.

The first-best allocation is defined as a solution of the social planner's problem. The social planner does not allow monopoly power in the production of intermediate goods and allocates resources efficiently across varieties. Therefore, the planner maximizes the household's lifetime utility subject to the sequence of aggregate resource constraints of the form:

$$h_t = c_t + G_t. \tag{4.15}$$

The first-order conditions of the planner's problem imply (see Appendix A.3) that the period marginal utilities of private and public consumption are set equal to the marginal disutility of labor:

$$0 = g'(G_t) - u'(c_t), \tag{4.16}$$

$$0 = g'(G_t) - v'(h_t). \tag{4.17}$$

Optimality conditions (4.15)–(4.17) are static; thus, the first-best allocation (c_t, h_t, G_t) is constant over time. In other words, it is optimal to allocate a fixed amount of labor to production of output and then allocate fixed shares of output to private and public consumption.

4.6 The Policy Problem

The central bank and the treasury act benevolently with to maximize the lifetime utility (4.1) of the representative household in the private-sector equilibrium. Both authorities are modeled as not being able to commit to its future policy choices. Instead, they act in a discretionary way in every period of time. Government debt is assumed to be always repaid due to a high enough implicit cost of default. The optimal policy in this environment is modeled as an outcome of a dynamic game between successive selves of the consolidated government, which consists of the central bank and the treasury, as if these were separate policy-makers in every period of time. The analysis in this paper abstracts from reputation effects of policy choices and solves for a stationary Markov perfect equilibrium of this game, defined along the lines of Klein et al. (2008).

In the Markov perfect equilibrium, the choices of the government depend on the minimal payoff-relevant state of the economy. This state in every period t is entirely described by the quantity of outstanding government debt, b_{t-1}. In each period t, the government maximizes the utility of the representative household starting from its incumbent period onwards. When choosing period-t policy, the government takes into account how the private-sector equilibrium is affected by this choice, given anticipated future policy.

Formally, the Markov optimization problem of the discretionary government in any period t can be written as choosing $(c_t, y_t, h_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t, R_t)$ that maximize:

$$u(c_t) + g(G_t) - v(h_t) + \beta \mathcal{V}(b_t)$$

subject to:

$$0 = \mathbf{\Lambda}\big(b_{t-1}; c_t, y_t, h_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t, R_t; \mathcal{C}(b_t), \mathcal{Y}(b_t), \mathit{\Pi}(b_t), \mathcal{Q}(b_t)\big),$$

given outstanding debt, b_{t-1}, and anticipated future policy together with implied allocation and prices in the private-sector equilibrium, as described by functions $(\mathcal{C}, \mathcal{Y}, \mathcal{H}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{B}, \mathcal{G}, \mathcal{T}, \mathcal{R})$ that provide continuation utility $\mathcal{V}$. For brevity, the vector-function $\mathbf{\Lambda}$ is used to summarize the set of private-sector equilibrium conditions from Definition 1. For optimal policy to be time consistent, the government should find no incentives to deviate from the anticipated rules. This idea is captured in the formal definition of the Markov perfect equilibrium.

Definition 2. *The Markov perfect equilibrium is a function $\mathcal{V}$ and a tuple of decision rules $(\mathcal{C}, \mathcal{Y}, \mathcal{H}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{B}, \mathcal{G}, \mathcal{T}, \mathcal{R})$, each being a function of b_{t-1}, such that for all b_{t-1}:*

1. *Given $\mathcal{V}$, the tuple of rules solves the Markov problem of the government,*

2. *$\mathcal{V}$ is the value function of the government*

$$\mathcal{V}\Big(b_{t-1}\Big) = u\Big(\mathcal{C}(b_{t-1})\Big) + g\Big(\mathcal{G}(b_{t-1})\Big) - v\Big(\mathcal{H}(b_{t-1})\Big) + \beta\mathcal{V}\Big(\mathcal{B}(b_{t-1})\Big).$$

The analysis is restricted to equilibria with differentiable value function and equilibrium decision rules. Assuming that such an equilibrium exists, it can be characterized by the first-order conditions of the policy problem.[14] A detailed formulation of the policy problem and derivation of the corresponding first-order conditions is shown in Appendix A.4.

The first-order conditions imply that the Markov perfect equilibrium is characterized by the following optimality conditions:

$$\Delta_{c,t}\Omega_t g'(G_t) = g'(G_t) - u'(c_t), \tag{4.18}$$

$$\left(\frac{\varphi}{2}\left(\pi_t - 1\right)^2 + \Delta_{h,t}\Omega_t\right) g'(G_t) = g'(G_t) - v'(h_t), \tag{4.19}$$

$$\left(1 + \Delta_{b,t}\right)\Omega_t \frac{g'(G_t)}{u'(c_t)} = \Omega_{t+1}\frac{g'(G_{t+1})}{u'(c_{t+1})}, \tag{4.20}$$

where Ω_t is referred to as an inflation cost factor and is defined as follows:

$$\Omega_t \equiv \frac{\varphi\left(\pi_t - 1\right) y_t}{\varphi(\pi_t - 1)y_t + \frac{\varphi}{\theta}\left(2\pi_t - 1\right) y_t + \left(1 + \rho q_t\right)\frac{b_{t-1}}{\pi_t^2}},$$

[14] I refrain from a formal general proof of equilibrium existence and uniqueness. For a parameterized model, numeric results in the subsequent sections demonstrate existence and local uniqueness.

with the numerator equal to the marginal resource cost of inflation and the denominator equal to the sum of the marginal resource cost and marginal benefits of inflation due to the implicit profit tax and the real liability effects. The real liability effect of inflation refers to a decline in the real value of outstanding government debt. The implicit profit tax effect of inflation refers to a decline of monopolistic markup charged by the firms due to the corresponding increase of labor cost. The larger the marginal resource cost of inflation compared with its marginal benefits, the larger Ω_t. The remaining auxiliary variables $\Delta_{b,t}$, $\Delta_{c,t}$ and $\Delta_{h,t}$ are defined in Appendix A.4.

Equations (4.18) and (4.19) are the targeting rules that can be compared with the optimality conditions of the planner's problem (4.16) and (4.17). Both sets of equations establish an intratemporal relation between the marginal utility components. As has been previously discussed, efficiency dictates that marginal utilities of private consumption, government spending and disutility of work be made equal. In the case of optimal policy, however, there are nontrivial wedges between these marginal utility terms. These wedges arise due to the discretionary nature of government policy layered over the distortions present in the economy. The generalized Euler equation (GEE) (4.20) determines the optimal way to trade off distortions intertemporally. This equation describes the dynamics of the wedges of the targeting rules. The GEE is therefore crucial in the determining dynamic properties of the allocation and underlying prices and policies.

The inflation cost factor, Ω_t, enters optimality conditions (4.18)–(4.20). It allows one to draw two intuitive conclusions. First, there is a relation between inflation and inefficiency. If the Markov perfect allocation in a given period of time differs from the first-best, then inflation is different from zero. It also works in reverse; if inflation in the Markov perfect equilibrium differs from zero in a given period of time, then the allocation is different from the first-best. Second, the Markov perfect equilibrium features the efficient steady state, that is, the steady state consistent with the first-best allocation.

One can solve for the efficient steady state independently of the optimal dynamic policy; see Appendix A.5. The employment subsidy rate, s, determines the quantity of government debt in the efficient steady state. Following Leith and Wren-Lewis (2013) and Burgert and Schmidt (2014), it is assumed that this subsidy is used by the government to offset permanent distortions and make the efficient steady state be supported by a positive amount of government debt. The remainder of this section comments on how to solve for the optimal dynamics away from the steady state and also describes the baseline parameterization of the model.

4.7 Parameterization and Solution Methods

Solution for the Markov perfect equilibrium as defined above can be computed numerically by means of a global nonlinear approximation method. The solution method employed in this paper is based on a projection method described in Debortoli and Nunes (2012). In a nutshell, the method is based on approximating equilibrium decision rules with cubic splines and solving a system of the first-order conditions of the policy problem by looking for a fixed point of equilibrium decision rules. While accurate, results computed with this method lack analytical tractability. This is why, in addition to the nonlinear method, I rely on a local linear-quadratic approximation of the policy problem in the vicinity of the steady state in order to derive a number of analytical results.

To solve the model, I assume the functional form of the household utility components is $u(c_t) \equiv c_t^{(1-\gamma_c)}/(1-\gamma_c)$ for private consumption, $g(G_t) \equiv \nu_g G_t^{(1-\gamma_g)}/(1-\gamma_g)$ for consumption of public good, and $v(h_t) \equiv \nu_h h_t^{(1+\gamma_h)}/(1+\gamma_h)$ for the disutility from work.

Baseline parameter values used for computations are summarized in Table 1. Each time period in the model represents one quarter of a year. The time discount factor, β, is set to match the annual real interest rate of around 4 percent in the steady state. Values chosen for the utility weights ν_h and ν_g imply that in the steady state, households spend one quarter of their unitary time endowment working, and government spending amounts to 20 percent of the output.

The monopoly power of the firms is described by the elasticity of substitution between intermediate goods, θ, which is set to match desired markup of the price over the marginal cost of 10 percent. Given the value for θ, the parameter of price adjustment cost, φ, is set to match the slope of the Phillips curve consistent, up to the first order of approximation around the efficient deterministic steady state, with a Calvo (1983) price-setting specification in which the average price duration is equal to one year.

With respect to government debt characteristics, the model is parameterized as follows. The baseline target for the market value of government debt in the steady state is set equal to 40 percent of annual GDP. This number is consistent with the pre-crisis U.S. data available from the Federal Reserve Bank of Dallas. Parameter ρ is set to match certain values of the average maturity of government debt. Two baseline cases are considered: one-period bonds and bonds with the average maturity equal to four years. The latter is consistent with the pre-crisis duration of government debt in the U.S. as reported in Greenwood et al. (2014). Equilibrium characterization that relies on the local approximation method also looks into wider ranges of values for both the maturity and the market value of government debt. Simulations of government debt reduction that rely on the global nonlinear approximation method use an initial value such that the market value of government debt at the beginning of transition is 30 percent higher than in the steady state, which is consistent with the post-crisis U.S. data available from the Federal Reserve Bank of Dallas.

5 Optimal Policy

The government is assumed to start off with an outstanding debt above the steady-state level. The main purpose of this section is to characterize the Markov perfect equilibrium with the focus on comparative dynamic effects of the maturity of government debt. The focus is on the implications of government debt maturity for policy choices and the dynamic path of government debt.

Parameter	Description	Value
β	Discount factor	0.99
γ_c	Intertemporal elasticity for C	1
γ_g	Intertemporal elasticity for G	1
γ_h	Inverse Frisch elasticity	1
ν_g	Utility weight on govt spending	0.25
ν_h	Utility weight on labor	20
φ	Price adjustment cost	116.505
θ	Elasticity of substitution among goods	11

Table 1 – Parameter values

5.1 The Linear-Quadratic Approach

It is convenient to demonstrate the key mechanisms by studying a linear-quadratic approximation of the policy problem. This way, the analysis builds on and extends the discussion provided in the context of a simplified version of the model in Section 3. In order to obtain the linear-quadratic approximation of the policy problem, the objective function of the government is approximated up to the second order, whereas private-sector equilibrium conditions are approximated linearly.[15] The approximation is done in logarithmic deviations around the efficient steady state with zero inflation and a positive amount of government debt; see Appendix A.6 for derivations.

The first-order conditions of the linear-quadratic policy problem are used to derive a tractable analytical characterization of the Markov perfect equilibrium. The equilibrium decision rules are linear functions of $\hat{b}_{t-1}$ described by constant coefficients $(\mathcal{C}_b, \mathcal{Y}_b, \mathcal{H}_b, \Pi_b, \mathcal{W}_b, \mathcal{Q}_b, \mathcal{B}_b, \mathcal{G}_b, \mathcal{T}_b, \mathcal{I}_b)$, such that for any $\hat{b}_{t-1}$, the quantities, prices, and policies generated by these rules $(\hat{c}_t = \mathcal{C}_b\hat{b}_{t-1}, \hat{y}_t = \mathcal{Y}_b\hat{b}_{t-1}, \ldots, \hat{i}_t = \mathcal{I}_b\hat{b}_{t-1})$ satisfy optimality conditions that include private-sector equilibrium conditions (4.4)–(4.6), (4.10), (4.11), (4.13), and (4.14) approximated up to the first order:

$$\hat{y}_t = \hat{h}_t,$$

$$\hat{i}_t = \beta\rho\hat{q}_{t+1} - \hat{q}_t, \tag{5.1}$$

$$\hat{\omega}_t = \bar{\tau}\bar{w}\hat{\tau}_t + \gamma_c\hat{c}_t + \gamma_h\hat{y}_t, \tag{5.2}$$

$$\hat{\pi}_t = \varphi^{-1}(\theta - 1)\hat{w}_t + \beta\hat{\pi}_{t+1}, \tag{5.3}$$

$$\hat{q}_t = \gamma_c(\hat{c}_t - \hat{c}_{t+1}) + \beta\rho\hat{q}_{t+1} - \hat{\pi}_{t+1}, \tag{5.4}$$

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - (1-\rho)\hat{q}_t - \beta^{-1}\hat{\pi}_t + \bar{\Gamma}^{-1}\bar{G}\hat{G}_t - \bar{\Gamma}^{-1}\bar{w}\bar{y}\left((\bar{\tau} - s)(\hat{y}_t + \hat{w}_t) + \bar{\tau}\hat{\tau}_t\right), \tag{5.5}$$

[15] See Woodford (2003) for a detailed discussion of the linear-quadratic approach. One could alternatively derive a (log-)linear approximation of the first-order optimality conditions of the nonlinear Markov perfect equilibrium. It is possible to show that these two approaches would deliver identical results.

where variables with a bar denote steady-state values and variables with a hat denote percentage deviations from the steady state. Additionally, $\bar{\Gamma} \equiv \bar{b}\bar{q}$ is the market value of government debt in the steady state; $\hat{i}_t$ denotes the percentage deviation of the short-term nominal interest rate, R_t. The remaining optimality conditions read as

$$\hat{c}_t = \Phi_c \hat{\pi}_t, \tag{5.6}$$

$$\hat{y}_t = \Phi_y \hat{\pi}_t, \tag{5.7}$$

$$\hat{G}_t = \Phi_g \hat{\pi}_t, \tag{5.8}$$

$$0 = [\hat{\pi}_{t+1} - \hat{\pi}_t] + \beta \frac{\bar{y}}{\bar{\Gamma}} \frac{\varphi}{\theta} \mathit{\Pi}_b \hat{\pi}_t + (1-\rho)(\gamma_c \mathcal{C}_b + \mathit{\Pi}_b - \beta\rho Q_b)\hat{\pi}_t, \tag{5.9}$$

where the coefficients Φ_c, Φ_y and Φ_g depend, among other things, on parameter ρ governing the average maturity of government debt as well as on the market value of government debt in the steady state, $\bar{\Gamma}$. Expressions defining coefficients Φ_c, Φ_y and Φ_g are shown in Appendix A.6.

The analysis below is restricted to equilibria where government debt converges to the steady state monotonously or, formally, where $0 < \mathcal{B}_b < 1$. The comparative dynamic effects of the maturity of government debt are studied by changing parameter ρ. A plain *ceteris paribus* change of parameter ρ would also affect the total payoff and the steady-state market price $\bar{q}$ of government bonds. This, in turn, would affect the market value of government debt in the steady state, $\bar{\Gamma}$. In contrast, the effects discussed below are associated with a variation of parameter ρ conditional on sterilizing the change of the steady-state market value of government debt.

The analysis starts by providing a first glance at the behavior of government debt and inflation under optimal policy. The method of undetermined coefficients can be applied to optimality conditions (5.1)–(5.9). Finding the Markov perfect equilibrium is then reduced to solving a system of two nonlinear equations in terms of the decision rule coefficients for government debt and inflation, $\mathcal{B}_b$ and $\mathit{\Pi}_b$; see Appendix A.6 for details.[16]

Consider the economy with one-period government debt. Figure 1 plots coefficients $\mathcal{B}_b$ and $\mathit{\Pi}_b$ as functions of the market value of government debt in the steady state, while other parameters are set equal to their baseline values. There are two key takeaways from this graph. First, the monotone equilibrium exists only when the steady-state market value of debt is below 60 percent of annual GDP. Higher steady-state indebtedness makes government debt dynamics less persistent, which speeds up the reduction of debt. Second, a positive deviation of government debt from the steady-state level leads to a positive inflation.[17] The contemporaneous response of inflation increases

[16] Despite tractability and linearity of decision rules, discretionary equilibria in the linear-quadratic models in general are not immune to multiplicity; see Blake and Kirsanova (2012). Numerical checks show that under baseline parameter values, the approximate model does not feature multiple equilibria.

[17] This result holds in general as long as an outstanding amount of debt above the steady-state level makes the government budget constraint tighter for an optimizing government; see Appendix A.6.

with the steady-state market value of government debt.

[Figure 1 about here.]

Figure 2 plots the same graph for the economies where the average maturity of government debt is equal to four years. Compared with the previous graph, it shows that longer maturity corresponds to a weaker contemporaneous response of inflation to the deviation of government debt from the steady state. The long maturity also makes government debt dynamics more persistent, which slows down the reduction of government debt. Moreover, in the case of long-term government debt, decision rule coefficients $\mathcal{B}_b$ and Π_b vary less with the steady-state market value of debt than in the case of one-period debt.

[Figure 2 about here.]

When analyzing fiscal and monetary policy in separate models (without nominal rigidity and with lump-sum taxes, respectively), looking at the dynamics of government debt or inflation reveals a lot about the underlying policy. Analysis of the model in this paper is less straightforward because the two variables are intertwined and determined by the fiscal-monetary policy mix. The analysis below takes advantage of the linearity of the first-order conditions (5.1)–(5.9) so as to disentangle the underlying policy. The analysis is split into two parts. The first part studies government debt dynamics in more detail and discusses incentives to reduce government debt when it is above the steady-state level. It explicitly shows that the path of government reduction towards the steady state depends on both fiscal and monetary policy. The second part of the analysis looks in more detail at the policy mix that implements debt reduction in the equilibrium.

5.2 Dynamics of Government Debt Reduction

Using the linear equilibrium decision rule for inflation, the GEE (5.9) can be written in terms of government debt as follows:

$$0 = [\hat{b}_t - \hat{b}_{t-1}] + \beta \frac{\bar{y}}{\bar{\Gamma}} \frac{\varphi}{\theta} \Pi_b \hat{b}_{t-1} + (1-\rho)\big(\gamma_c \mathcal{C}_b + \Pi_b - \beta\rho \mathcal{Q}_b\big)\hat{b}_{t-1}. \tag{5.10}$$

This representation of the GEE shows explicitly that if the government has outstanding debt different from the steady-state level, then the optimal amount of newly issued bonds balances direct gains from smoothing debt and corresponding distortions over time against indirect losses coming from the anticipated effects of doing so on the decision of the government in the following period.

Think of the government that starts off with an outstanding debt above the steady-state level. A higher level of debt can be supported by raising the tax rate. An increase of the tax rate is costly because it creates a trade-off between inflation and output by pushing up the marginal cost of production. The first term in the GEE reflects direct gains from smoothing adjustment of the tax rate intertemporally. This term introduces a permanent component into the dynamic behavior of the economy. The government in the following period, however, does not internalize the effects of its policy choice on trade-offs faced by the government in the current period. As a result, sustainment of a high debt level in the long run ceases to be optimal because it brings along indirect costs.

The second term in the GEE reflects the indirect cost from leaving the debt level unchanged that comes from the marginal effect of having extra inflation in the successive period. The expectation of inflation works its way through the Phillips curve, (5.3), and makes firms more willing to raise prices further up in the current period.[18] The third term in the GEE reflects the indirect cost from leaving the debt level unchanged that comes from the marginal effects on the consumption, inflation, and the price of government bonds in the following period. Joint expectations regarding these variables affect the consumption-savings decision of the household, as described by the Euler equation, (5.4), and make agents less willing to save in the current period. Note that inflation expectations enter this term because the household saves in nominal bonds, which is different from the analysis in Section 3, where bonds are indexed to inflation. Such a real interest rate effect makes it more expensive for the government to borrow today, which requires a stronger tax rate increase and higher inflation.[19] These two GEE terms introduce a transitory component and pin down the persistence of the dynamic behavior of the economy.

In particular, it follows directly from (5.10) that the equilibrium dynamics of government debt follow an autoregressive process with coefficient $\left(1 - \beta \frac{\bar{y}}{\bar{\Gamma}} \frac{\varphi}{\theta} \mathit{\Pi}_b - (1-\rho)(\gamma_c \mathcal{C}_b + \mathit{\Pi}_b - \beta\rho\mathcal{Q}_b)\right)$. The larger the indirect losses, the stronger debt reduction intended to strategically manipulate future policy and the faster convergence to the steady state. Furthermore, the linear structure of the decision rules implies that the remaining economic variables inherit the persistence of government debt. Thus, one can see that economies with different market value of government debt in the steady state, $\bar{\Gamma}$, and parameter ρ governing the maturity of government bonds are going to have different dynamics. First, the difference comes through the change in the sensitivity of government budget in a given period to marginal effects of newly issued debt on the equilibrium decision rules in the following period. Second, the difference comes through changes in the equilibrium decision rules themselves. Equilibrium decision rules depend on the entire policy mix that includes the tax instrument as well as government spending and monetary policy interest rate. The next part of this section looks at the optimal policy mix in detail.

5.3 Policy Instruments

Equations (5.6)–(5.8) constitute a set of targeting rules. Targeting rules in general offer a convenient way to describe the outcome of solving optimal policy problems. The set of targeting rules in

[18] This is a distinct loss that emerges due to the presence of nominal rigidity in the model. It is absent in a model with competitive goods market and flexible prices; see Debortoli and Nunes (2012).

[19] This loss is nil in the case of consol bonds, $\rho = 1$, due to the flat payoff structure of such bonds as discussed by Debortoli et al. (2016) in the context of a model with competitive goods market and flexible prices.

this model describes optimal intratemporal relations between deviations (gaps) of target variables, namely private consumption, $\hat{c}_t$, output, $\hat{y}_t$, government spending, $\hat{G}_t$, and inflation, $\hat{\pi}_t$, that the government seeks to maintain in every period t. Compared with the simplified version of the model discussed in Section 3, the baseline model features two more target variables and, therefore, two more targeting rules. The targeting rules allow one to characterize equilibrium paths of monetary and fiscal policy instruments during the transition period of government debt reduction. The analysis below shows that the signs of targeting coefficients Φ_g, Φ_c, and Φ_y are crucial in characterizing the dynamics of policy instruments qualitatively.

The targeting rule (5.8) for government spending has a negative coefficient Φ_g; hence, the sign of the government spending gap is the opposite of inflation. This result is robust to variations of the steady-state market value and the maturity of government debt.[20] Recall that numerical results in the beginning of this section show how government debt above the steady-state level leads to an upward pressure on prices, $\Pi_b > 0$. Targeting rule (5.8) then implies that a reduction of government debt is accompanied by a relatively low government spending compared with its steady-state level. The optimal reduction of government spending balances the cost of providing an inefficiently low amount of public goods against direct and indirect benefits that allow for the reduction of the adverse cost-push effect of raising the tax rate. The direct benefit stems from an increase in the primary surplus, whereas the indirect benefit comes from an offsetting effect of a corresponding reduction in demand on the willingness of firms to raise prices.

The signs of the remaining targeting coefficients, Φ_c, and Φ_y, depend on the steady-state market value and average maturity of government debt. As a result, the transition behavior of interest and tax rates also varies depending on these characteristics of government debt. One can combine the Euler equation, (5.4), the decision rule for government debt, the targeting rule (5.6) and the no-arbitrage condition (5.1), to derive an equilibrium relation between the one-period nominal interest rate and expected inflation

$$\hat{i}_t = \left(1 - \gamma_c \Phi_c \frac{(1 - \mathcal{B}_b)}{\mathcal{B}_b}\right) \hat{\pi}_{t+1}. \tag{5.11}$$

Equation (5.11) provides a clear criterion of the equilibrium stance of monetary policy. Monetary policy raises the nominal interest rate less than one to one in response to expected inflation—a loose stance—if and only if targeting coefficient Φ_c is positive. As in Section 3, the sign of the targeting coefficient Φ_c is determined by opposing incentives to reduce private demand so as to mitigate the adverse cost-push effect of raising the tax rate and to stimulate private demand so as to reduce the need to raise the tax rate in the first place. Relaxing the assumption of inflation-indexed government bonds in the baseline model creates an additional incentive to stimulate demand because it leads to a higher inflation that reduces the real value of outstanding government debt.

[20] Using the analytical expression defining Φ_g in Appendix A.6, one can see that its sign is unambiguously negative, given the previously made assumption of a positive amount of government debt in the steady state.

It is straightforward to see that the stance of monetary policy has a direct implication for the path of the real interest rate during the transition of the economy towards the steady state. One can characterize equilibrium dynamics of the real interest rates by rewriting (5.11) using decision rules for inflation and government debt:

$$\hat{r}_t = -\gamma_c \Phi_c \mathit{\Pi}_b (1 - \mathcal{B}_b) \hat{b}_{t-1}, \tag{5.12}$$

where $\hat{r}_t = \hat{\imath}_t - \hat{\pi}_{t+1}$ is the one-period real interest rate. Thus, whenever monetary policy is loose, a reduction of government debt is accompanied by real interest rates that are below the long-run level. The opposite happens when monetary policy is tight.

The equilibrium tax rate can be characterized by using the decision rule for inflation and government debt, targeting rules (5.6)–(5.7), and equation (5.2), which describes the optimal intratemporal choice of the household between consumption and leisure, in order to rewrite the Phillips curve, (5.3), as follows:

$$\frac{\bar{\tau}}{\mathit{\Pi}_b (1 - \bar{\tau})} \hat{\tau}_t = \frac{\varphi}{\theta - 1} (1 - \beta \mathcal{B}_b) \hat{b}_{t-1} - \left(\gamma_c \Phi_c + \gamma_h \Phi_y\right) \hat{b}_{t-1}. \tag{5.13}$$

For convenience, let tax policy be referred to as tight when reduction of government debt in equilibrium is implemented with the labor tax rate above the long-run level, and referred to as loose otherwise. A sufficient condition for tax policy to be tight is to have a linear combination of targeting coefficients $\left(\gamma_c \Phi_c + \gamma_h \Phi_y\right)$ with a negative sign. Having the same linear combination with a positive sign is a necessary condition for tax policy to be loose. It is not a sufficient condition because if the equilibrium speed of government debt reduction is fast enough, then the tax policy implementing it has to be tight.

[Figure 3 about here.]

Figure 3 depicts the signs of targeting coefficients Φ_c and Φ_y for various combinations of the steady-state market value and average maturity of government debt while other parameters are set equal to their baseline values. The signs of these two targeting coefficients coincide in most cases. Moreover, the sign of targeting coefficient Φ_y is always negative in the region where the sign of targeting coefficient Φ_c is negative. Hence, tight monetary policy always goes together with tight tax policy. Loose monetary policy, however, potentially can co-occur with both tight and loose tax policy. It is natural to continue by drawing on the previous characterization and discuss comparative dynamic effects of government debt characteristics.

The mix of tight monetary and tight tax policy is observed in the region where the steady-state market value of government debt is low enough and/or where the average maturity of government debt is long enough. In economies with debt characteristics from this region, it is optimal to reduce excessive government debt with the policy that sets high tax rates, low government spending, and high real interest rates. Inflation observed during the transition in these economies is due to the cost-push effect of high tax rates. A tight stance of monetary policy that results in high real interest rates is driven primarily by the incentive to reduce private demand so as to mitigate cost-push inflation. In other words, incentives that call for loose monetary policy to stimulate private demand so as to reduce the need to raise the tax rate are relatively weak in these economies.

Shortening the average maturity of government debt increases the relative strength of the incentives that call for a loose stance of monetary policy. In particular, shorter maturity creates a stronger incentive to benefit from a higher price of newly issued debt by keeping the real interest rate low because of a smaller corresponding capital loss on outstanding debt. Thus, short enough maturity may reverse the optimal stance of monetary policy and make it loose in equilibrium. Changes in the optimal stance of monetary policy depending on the maturity of government bonds also require a high enough steady-state value of government debt. Figure 3 shows that in the economy with a steady-state market value of debt around 40 percent of annual GDP, the optimal stance of monetary policy turns loose with maturity of government bonds only as short as one quarter. Importantly, increasing the steady-state market value of debt still requires a very short average maturity of government debt to make it optimal for monetary policy to take a loose stance. Looking at the economies with a steady-state market value of government debt as high as 160 percent of annual GDP, monetary policy turns loose with an average maturity of government debt shorter than one year.

5.4 Transition Simulations

The analysis above provided tractable equilibrium characterization by using a linear-quadratic approximation of the policy problem. This section presents transition simulations that provide illustrative examples supporting that characterization. It also provides a quantitative assessment of the comparative dynamic effects of the maturity of government bonds. Reported simulations are produced from a fully nonlinear solution of the policy problem. Using a nonlinear solution shows that the characterization provides valid insights into dynamics of the model even when moving away from vicinity of the steady state.

Figure 4 shows the transition dynamics of two economies towards the steady state. Red dotted lines correspond to the economy in which the government issues one-period bonds. Solid black lines correspond to the economy in which the government issues bonds with an average maturity equal to four years. Both economies start off with an initial quantity of bonds that makes the market value of outstanding government debt 30 percent higher than in the steady state. The graphs report dynamics of private consumption, government spending, aggregate output, and real market value of outstanding government debt in percentage deviation from the steady state. Inflation, the (net) nominal and real interest rates are reported in annualized percentages. The labor tax rate is reported in percentages of labor income.

[Figure 4 about here.]

First, consider the case of one-period bonds. Debt reduction is performed fast, with a half-life of roughly one-quarter, which means that every period excess debt is cut by half. The tax rate is initially set almost twice as high as the long-run level and declines over time. Also, initially there is a large negative gap of more than 30 percent in government spending, which declines during the transition. Such dynamics of government spending mitigate the adverse cost-push effect of elevated tax rates and help to avoid further increases of the tax rate. Low government spending pulls down aggregate demand and results in the negative output gap along the transition despite a relative boom in private consumption. The latter is due to a loose stance of monetary policy, which manifests itself in the real interest rates below the long-run level. Monetary policy is driven primarily by incentives to expand the tax base, increase the price of newly issued government bonds, and inflate away outstanding government bonds. Naturally, the observed monetary policy jointly with tax policy lead to a surge in inflation that goes up to 10 percent initially and then declines over time.

An economy in which the government issues bonds with an average maturity equal to four years exhibits notably different transition dynamics. While the behavior of fiscal policy instruments is qualitatively unchanged compared with the case with one-period bonds, there is a change in the optimal stance of monetary policy. Longer maturity reduces the incentive to use interest rate policy in order to manipulate the price of government bonds. The incentive to reduce private demand so as to mitigate the adverse cost-push effect of elevated tax rates becomes relatively more important, which makes the optimal stance of monetary policy tight. As a consequence, the transition path features a negative consumption gap and high real interest rates. A switch in the stance of monetary policy yields a decline in the indirect cost of smoothing reduction of government debt over time. Therefore, debt reduction with long-term bonds is more gradual and has a half-life of approximately 23 years, which is a slowdown by a factor of 92 compared with one-period bonds. The slowdown comes with a sizable reduction in amplitudes of changes in other variables. Inflation, in particular, runs only as high as 15 basis points in the first period and then declines.

5.5 Welfare Analysis

Given that maturity of government debt affects the dynamic properties of the economy during transition towards the steady state, it is of interest to analyze and compare corresponding changes in welfare. Using the baseline parameterization of the model, Figure 5 shows welfare losses incurred because of the reduction of government debt depending on its average maturity under optimal discretionary policy. Losses are measured using welfare-equivalent permanent reduction of consumption in the steady state. The computation is done by solving for nonlinear transitions towards the steady state with a length of 1000 years, starting from the quantity of bonds that makes the market value of outstanding government liabilities in real terms 30 percent higher than in the steady state. The absolute size of welfare losses depends on the initial condition that determines the amount of debt in excess of the steady-state level. Naturally, the further away from the steady-state the initial condition is, the larger the accruing loss. Therefore, the reported losses are scaled relative to the welfare loss of the transition in the economy with consol bonds.

As shown in the left panel of Figure 5, under the standard assumption of one-period government bonds the consumption loss is larger by a factor of 80 than the analogous loss in the economy with consol bonds. As the maturity of government bonds lengthens to one year, the relative loss declines 60 times and is equal to 1.34. The right panel of Figure 5 shows that the decline of the relative loss continues as the maturity of government debt increases further, albeit at a slower speed.

[Figure 5 about here.]

These results show that welfare losses of government debt reduction decline notably if the maturity of government debt is longer than one period. An important reason is the underlying change of monetary policy stance from loose to tight. As was previously discussed, this change leads to a decline of the indirect cost of smoothing reduction of government debt over time and an effective slowdown of the speed of government debt reduction. A smoother reduction of government debt then translates into an improvement of welfare.

6 The Medium-Scale Model

This section extends the analysis to a richer environment that is known to capture the inertial behavior of aggregate dynamics in response to variation of the monetary policy interest rate. In particular, the analysis in this section builds on a workhorse medium-scale New Keynesian model as described, for instance, in Christiano et al. (2010). The model includes monopolistic competition not only in the goods market but also in the labor market. Following Erceg et al. (2000), both markets feature nominal rigidity in the form of sticky prices and wages. Real rigidities include working capital friction, habit formation in consumption, and accumulation of capital with the costs of utilization and investment adjustment.

The workhorse medium-scale model is extended by introducing fiscal sector in a way identical to the earlier analysis of the baseline model in the current paper. The model is then used to solve for optimal discretionary monetary and fiscal policy under fixed parameter values. Most of the parameter values are set to be equal to the posterior mean parameter estimates of the U.S. economy by Christiano et al. (2010). The model is solved by deriving the linear-quadratic approximation and then using the method described by Debortoli et al. (2014). A more detailed description of the parameterization and the private-sector equilibrium conditions can be found in Appendix B.

The discussion of optimal policy below focuses on the implications of the government debt maturity for monetary policy and debt dynamics. First, consider monetary policy. Figure 6 displays the gap between the real interest rate and its steady-state counterpart in a single period of time where government debt is above the steady-state level by 1 percent while other state variables are equal to their steady-state values. This gap is plotted against the average maturity of bonds issued by the government.

[Figure 6 about here.]

A negative gap implies that government debt above the steady-state level makes it optimal for monetary policy to stimulate the economy. In other words, monetary policy is loose in such a case. A positive gap implies the opposite: the stance of monetary policy is tight if government debt is above the steady-state level.[21] The graph shows that the stance of monetary policy in the medium-scale model exhibits a switch from loose to tight when the maturity of government debt becomes long enough. Moreover, similar to the baseline model, the switch occurs at a very short average maturity of around three quarters.

This analysis isolates the effects of debt maturity on monetary policy when government debt is above the steady-state level by keeping the remaining endogenous state variables at their steady-state levels. The real interest rate gap would differ in more general cases with different initial conditions of other state variables such as, for instance, capital. To put it differently, the overall optimal stance of monetary policy in equilibrium depends on all states of the economy. Nevertheless, linearity of the decision rules implies that the analysis presented here would be valid to the extent that it represents how an amount of government debt above the steady-state level affects the stance of monetary policy *ceteris paribus*.

Next, consider the effects of debt maturity on debt dynamics. Figure 6 displays the coefficient of the autoregressive component in the decision rule for government debt. The coefficient is plotted against the average maturity of bonds. This coefficient affects the speed of government debt reduction. Let the initial conditions be the same as for the real interest rate response in Figure 6. Specifically, the initial government debt is above the steady-state level by 1 percent while other state variables are equal to their steady-state values. In this case, the autoregressive coefficient can be interpreted as the percentage deviation of government debt from the steady-state level in one period.

[Figure 7 about here.]

Negative values of the coefficient imply that during one period, government debt is reduced below the steady-state level. Positive values less than 1 imply that government debt is reduced but stays

[21] As in the baseline model, the real interest rate depends on the intratemporal and intertemporal optimality conditions. Qualitatively, the stance of monetary policy in the baseline model was shown to be determined exclusively by the intratemporal trade-off. Complexity of the medium-scale model does not permit these two dimensions to be disentangled.

above the steady-state level. The closer this coefficient to 1, the slower the debt reduction. The graph shows that, except for a non-monotonic change at short maturities less than 1 year, the speed of government debt reduction slows down as the maturity of government debt becomes longer. This pattern is consistent with the baseline model. Note, however, that the speed of debt reduction in the medium-scale model in general depends on the initial conditions of all the states and cannot be measured with a single half-life indicator.

7 Conclusion

In the aftermath of the Great Recession, governments of many countries have accumulated public debt that exceeds historic averages. A pressing policy concern is whether there is a need to reduce the stock of this debt and, if yes, how fast the adjustment should be performed and, equally important, using which policies. The topic that spurs a lot of controversies is a capacity of central banks to ease the burden of government debt. I examine the relative importance of the need for government debt reduction in shaping the optimal stance of monetary policy.

I focus on the case in which the reduction of government debt occurs because of a lack of commitment by the government to future policy choices. Analytical results demonstrate that the stance of monetary policy during the period of government debt reduction is driven by opposing incentives to reduce private demand so as to mitigate the adverse cost-push effect of raising the tax rate and to stimulate private demand so as to reduce the need to raise the tax rate in the first place. Quantitative analysis demonstrates strong support for the dominance of the former incentive and the resulting tight stance of monetary policy. The crucial feature of the analysis is that it takes into account the long-term nature of government debt. In can be optimal to switch the stance of monetary policy and make it loose only if the government were to issue bonds with the average maturity less than 1 year. Government debt in the form of such short-term bonds also has to be reduced quickly. Under the long maturity of bonds, the optimal speed of debt reduction is more gradual.

The analysis in this paper is built under the assumption that the government can always adjust either monetary or fiscal policy instruments to maintain fiscal sustainability. Clearly, there are cases when keeping public debt on a sustainable path requires governments to default. Hence, abstracting from the risk of default may be not without loss of generality and is a relevant direction for future research.

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A Appendix: The Baseline Model

A.1 Recursive Payoff Structure and Pricing of Government Debt

Consider two portfolios of long-term government bonds, B_{t-1} and B_t, issued in the two consecutive periods $t-1$ and t, respectively. The former portfolio pays 1 in period t, ρ in period $t+1$, ρ^2 in period $t+2$, etc. The latter portfolio pays 1 in period $t+1$, ρ in period $t+2$, ρ^2 in period $t+3$, etc.

Let q_t^{o} and q_t denote period-t prices of B_{t-1} and B_t, respectively. Taking into account the structure of payoffs, these two prices have to satisfy the following asset pricing equations:

$$q_t^{\mathrm{o}} = \sum_{j=1}^{\infty} \prod_{i=1}^{j} \frac{\rho^j}{R_{t+i-1}},$$

$$q_t = \sum_{j=1}^{\infty} \prod_{i=1}^{j} \frac{\rho^{j-1}}{R_{t+i-1}},$$

where the one-period nominal interest rate is used for discounting the payoffs.[22] It is then straightforward to see that market prices of the two portfolios satisfy:

$$q_t^{\mathrm{o}} = \rho q_t.$$

A.2 Duration of Government Debt

Weighted average maturity of the portfolio of government bonds issued in period t is measured as the Macaulay duration of perpetuities entering this portfolio.

To compute the duration, it is convenient to define the yield-to-maturity R_t^m on a perpetual bond as an implicit constant one-period interest rate at which the discounted value of its payoffs equals its price:

$$q_t = \sum_{j=1}^{\infty} \frac{\rho^{j-1}}{(R_t^m)^j} = \frac{1}{R_t^m - \rho}. \tag{A.1}$$

The Macaulay duration is defined as the weighted average of the time until each payoff, with the weights determined by discounted payoffs as a fraction of the bond's price:

[22] In a model with uncertainty, one would have to use the stochastic discount factor.

$$d_t \equiv \frac{1}{q_t}\sum_{j=1}^{\infty} j\frac{\rho^{j-1}}{(R_t^m)^j} = \frac{1}{q_t R_t^m}\sum_{j=1}^{\infty} j\left(\frac{\rho}{R_t^m}\right)^{j-1}$$

$$= \frac{1}{q_t R_t^m}\frac{\partial\left(\sum_{j=1}^{\infty}\left(\frac{\rho}{R_t^m}\right)^j\right)}{\partial\left(\frac{\rho}{R_t^m}\right)} = \frac{1}{q_t R_t^m}\frac{\partial\left(\frac{\rho}{R_t^m}\frac{1}{1-\frac{\rho}{R_t^m}}\right)}{\partial\left(\frac{\rho}{R_t^m}\right)}$$

$$= \frac{1}{q_t R_t^m}\left(\frac{R_t^m}{R_t^m-\rho}\right)^2 = \frac{R_t^m}{R_t^m-\rho}, \quad \text{(A.2)}$$

where the last step uses equation (A.1). The duration as defined above is measured in units of a time period of the model.

Note further that the Euler equation (4.5) at the steady state with zero inflation takes the form:

$$\bar{q} = \frac{1}{\beta^{-1}-\rho}, \quad \text{(A.3)}$$

where the bar over variables is used to denote their values at the steady state. A comparison of (A.1) and (A.3) shows that the yield-to-maturity of any long-term portfolio of government bonds at the steady state is equal to the inverse of the time discount factor$\bar{R^m} = \beta^{-1}$. It is then straightforward to see, using (A.2), that the corresponding steady-state duration is determined as follows:

$$\bar{d} = (1-\beta\rho)^{-1}.$$

A.3 The First-Best Allocation

Lagrangian corresponding to the planner's problem is:

$$\mathcal{L} \equiv \sum_{t=0}^{\infty}\beta^t\left[u(c_t) + g(G_t) - v(h_t) + \gamma_t\left(h_t - c_t - G_t\right)\right].$$

First-order conditions with respect to (c_t, G_t, h_t) read as:

$$u'(c_t) = \gamma_t, \qquad g'(G_t) = \gamma_t, \qquad v'(h_t) = \gamma_t.$$

Eliminating Lagrange multiplier γ_t produces the system of two equations:

$$0 = g'(G_t) - u'(c_t), \qquad 0 = g'(G_t) - v'(h_t),$$

which together with the resource constraint $h_t = c_t + G_t$ characterize the first-best allocation as a solution of the planner's problem.

A.4 The Markov Perfect Equilibrium

The Markov perfect equilibrium is associated with a solution to the following Bellman equation:

$$\mathcal{V}(b_{t-1}) = \max_{(c_t, y_t, \pi_t, w_t, q_t, b_t, G_t, \tau_t)} u(c_t) + g(G_t) - v(h_t) + \beta \mathcal{V}(b_t)$$

subject to

$$0 = c_t + G_t - y_t \left(1 - \frac{1}{2}\varphi\left(\pi_t - 1\right)^2\right), \tag{A.4}$$

$$0 = \left(1 - \tau_t\right) w_t - \frac{v'(y_t)}{u'(c_t)}, \tag{A.5}$$

$$0 = q_t - \beta \frac{\left(1 + \rho Q(b_t)\right)}{\mathit{\Pi}(b_t)} \frac{u'(\mathcal{C}(b_t))}{u'(c_t)}, \tag{A.6}$$

$$0 = w_t - \frac{\theta - 1}{(1 - s)\theta} - \frac{\varphi}{\theta(1 - s)} \left(\pi_t \left(\pi_t - 1\right) - \beta \frac{u'(\mathcal{C}(b_t))}{u'(c_t)} \frac{\mathcal{Y}(b_t)}{y_t} \mathit{\Pi}(b_t) \left(\mathit{\Pi}(b_t) - 1\right)\right), \tag{A.7}$$

$$0 = q_t b_t - \left(\frac{1 + \rho q_t}{\pi_t}\right) b_{t-1} - G_t + \tau_t w_t y_t - s\left(w_t y_t - \bar{w}\bar{y}\right). \tag{A.8}$$

This is a dynamic functional problem whose solution consists of a value function $\mathcal{V}$ and decision rules $(\mathcal{C}, \mathcal{Y}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{B}, \mathcal{G}, \mathcal{T})$ that determine private-sector equilibrium in a given period of time as a function of outstanding debt, b_{t-1}, as in $c_t = \mathcal{C}(b_{t-1})$, $y_t = \mathcal{Y}(b_{t-1})$, etc. To ease the exposition, two variables have been eliminated from the system of private-sector equilibrium conditions using two of its equations. These variables are employment, h_t, and the short-term nominal interest rate, R_t, whereas the corresponding equations are the aggregate production function (4.13) and the no-arbitrage condition (4.4). Decision rules for the former, $\mathcal{H}(b_{t-1})$ and $\mathcal{R}(b_{t-1})$, are recovered using the latter, given the solution of the Bellman equation above.[23]

I consider equilibria with differentiable decision rules, which makes it possible to characterize these equilibria with the first-order conditions. Let $(\lambda_{r,t}, \lambda_{h,t}, \lambda_{q,t}, \lambda_{\pi,t}, \lambda_{b,t})$ be Lagrange multipliers corresponding to private-sector equilibrium conditions (A.4)–(A.8). The first-order condition with respect to b_t reads as:

[23] Treating the short-term nominal interest rate in such a "residual" way is innocuous as long as the Zero Lower Bound on nominal interest rates is ignored or is never binding.

$$
\begin{aligned}
0 = {} & \beta\mathcal{V}'(b_t) - q_t\lambda_{b,t} \\
& - \beta\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\left[\left(\frac{1+\rho\mathcal{Q}(b_t)}{\Pi(b_t)}\right)\left(\frac{\Pi'(b_t)}{\Pi(b_t)} - \frac{\mathcal{C}'(b_t)u''(\mathcal{C}(b_t))}{u'(\mathcal{C}(b_t))}\right) - \rho\frac{\mathcal{Q}'(b_t)}{\Pi(b_t)}\right]\lambda_{q,t} \\
& + \beta\frac{\varphi}{(1-s)\theta y_t}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\Bigg[-\Pi(b_t)\left(\Pi(b_t)-1\right)\left(\frac{\mathcal{C}'(b_t)u''(\mathcal{C}(b_t))}{u'(\mathcal{C}(b_t))}\mathcal{Y}(b_t) + \mathcal{Y}'(b_t)\right) \\
& - \Pi'(b_t)\left(2\Pi(b_t)-1\right)\mathcal{Y}(b_t)\Bigg]\lambda_{\pi,t} \qquad \text{(A.9)}
\end{aligned}
$$

The first-order conditions with respect to $(c_t, y_t, \pi_t, w_t, q_t, G_t, \tau_t)$ read as:

$$
\begin{aligned}
0 = {} & u'(c_t) - \lambda_{r,t} - \left(\frac{u''(c_t)}{u'(c_t)}\frac{v'(y_t)}{u'(c_t)}\right)\lambda_{h,t} - \beta\left(\frac{u''(c_t)}{u'(c_t)}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\frac{(1+\rho\mathcal{Q}(b_t))}{\Pi(b_t)}\right)\lambda_{q,t} \\
& + \beta\left(\frac{\varphi}{\theta}\frac{u''(c_t)}{u'(c_t)}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\frac{\mathcal{Y}(b_t)}{y_t}\frac{\Pi(b_t)\left(\Pi(b_t)-1\right)}{(1-s)}\right)\lambda_{\pi,t}, \qquad \text{(A.10)} \\
0 = {} & -v'(y_t) + \left(1 - \frac{1}{2}\varphi\left(\pi_t - 1\right)^2\right)\lambda_{r,t} + \left(\frac{v''(y_t)}{u'(c_t)}\right)\lambda_{h,t} \\
& + \beta\left(\frac{\varphi}{\theta}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\frac{\mathcal{Y}(b_t)}{y_t^2}\frac{\Pi(b_t)\left(\Pi(b_t)-1\right)}{(1-s)}\right)\lambda_{\pi,t} - \left(\tau_t - s\right)w_t\lambda_{b,t}, \qquad \text{(A.11)} \\
0 = {} & -\varphi\left(\pi_t - 1\right)y_t\lambda_{r,t} + \frac{\varphi}{\theta}\left(\frac{2\pi_t - 1}{1-s}\right)\lambda_{\pi,t} - \frac{\left(1+\rho q_t\right)b_{t-1}}{\pi_t^2}\lambda_{b,t}, \qquad \text{(A.12)} \\
0 = {} & -\left(1-\tau_t\right)\lambda_{h,t} - \lambda_{\pi,t} - \left(\tau_t y_t - s y_t\right)\lambda_{b,t} \qquad \text{(A.13)} \\
0 = {} & \lambda_{q,t} + \left(b_t - \frac{\rho b_{t-1}}{\pi_t}\right)\lambda_{b,t}, \qquad \text{(A.14)} \\
0 = {} & g'(G_t) - \lambda_{r,t} + \lambda_{b,t}, \qquad \text{(A.15)} \\
0 = {} & w_t\lambda_{h,t} - y_t w_t\lambda_{b,t}, \qquad \text{(A.16)}
\end{aligned}
$$

One can use equations (A.12)–(A.16) to solve for Lagrange multipliers

$$
\begin{aligned}
\lambda_{r,t} &= \left(1-\Omega_t\right)g'(G_t), \qquad \text{(A.17)} \\
\lambda_{h,t} &= -y_t\Omega_t g'(G_t), \qquad \text{(A.18)} \\
\lambda_{q,t} &= \left(b_t - \rho\,\pi_t^{-1}b_{t-1}\right)\Omega_t g'(G_t), \qquad \text{(A.19)} \\
\lambda_{\pi,t} &= (1-s)y_t\Omega_t g'(G_t), \qquad \text{(A.20)} \\
\lambda_{b,t} &= -\Omega_t g'(G_t), \qquad \text{(A.21)}
\end{aligned}
$$

where Ω_t is an auxiliary variable referred to as an inflation cost factor and defined as follows:

$$\Omega_t \equiv \frac{\varphi\,(\pi_t - 1)\,y_t}{\varphi(\pi_t-1)y_t + \frac{\varphi}{\theta}\,(2\pi_t - 1)\,y_t + (1+\rho q_t)\,\frac{b_{t-1}}{\pi_t^2}}.$$

It is straightforward to see that $\Omega_t < 1$. Moreover, Ω_t is unambiguously positive when debt level and inflation are positive. The larger the marginal resource cost of inflation compared with its marginal benefits is, the larger Ω_t and the tighter the constraints (A.5)–(A.8). If there were to be no inflation, then Ω_t would be equal to zero and (A.4) would be the only binding constraint, which would make the policy problem isomorphic to the social planner's problem. It is the inability to fully contain inflation at zero at all states that makes the policy problem nondegenerate.

The Envelope condition:

$$\mathcal{V}'(b_t) = \left(\frac{1+\rho\mathcal{Q}(b_t)}{\mathit{\Pi}(b_t)}\right)\lambda_{b,t+1}$$

allows one to substitute for the derivative of the value function in equation (A.9). Consequently, equations (A.17)–(A.21) allow one to substitute for Lagrange multipliers in equations (A.9)–(A.11), which turn into

$$\left(1+\Delta_{b,t}\right)\Omega_t \frac{g'(G_t)}{u'(c_t)} = \Omega_{t+1}\frac{g'(\mathcal{G}(b_t))}{u'(\mathcal{C}(b_t))}, \tag{A.22}$$

$$\Delta_{c,t}\Omega_t g'(G_t) = g'(G_t) - u'(c_t), \tag{A.23}$$

$$\left(\frac{\varphi}{2}\,(\pi_t-1)^{\,2} + \Delta_{h,t}\Omega_t\right) g'(G_t) = g'(G_t) - v'(y_t), \tag{A.24}$$

where $\Omega_{t+1} = \Omega(b_t)$ and the auxiliary variables $\Delta_{b,t}$, $\Delta_{c,t}$ and $\Delta_{h,t}$ are defined as follows:

$$\begin{aligned}
\Delta_{b,t} \equiv\ & \frac{\varphi}{\theta}\left(\frac{\mathit{\Pi}(b_t)}{1+\rho Q(b_t)}\right)\left[-\,\mathit{\Pi}(b_t)\left(\mathit{\Pi}(b_t)-1\right)\left(\frac{\mathcal{C}'(b_t)u''(\mathcal{C}(b_t))}{u'(\mathcal{C}(b_t))}\mathcal{Y}(b_t) + \mathcal{Y}'(b_t)\right) - \mathit{\Pi}'(b_t)\left(2\mathit{\Pi}(b_t)-1\right)\mathcal{Y}(b_t)\right] \\
& - \left[\frac{\mathit{\Pi}'(b_t)}{\mathit{\Pi}(b_t)} - \frac{\mathcal{C}'(b_t)u''(\mathcal{C}(b_t))}{u'(\mathcal{C}(b_t))} - \frac{\rho\mathcal{Q}'(b_t)}{1+\rho\mathcal{Q}(b_t)}\right]\left(b_t - \rho\,\pi_t^{-1}b_{t-1}\right), \\
\Delta_{c,t} \equiv\ & 1 - \beta\frac{u''(c_t)}{u'(c_t)}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\left[\left(\frac{1+\rho\mathcal{Q}(b_t)}{\mathit{\Pi}(b_t)}\right)\left(b_t - \rho\,\pi_t^{-1}b_{t-1}\right) - \frac{\varphi}{\theta}\mathcal{Y}(b_t)\mathit{\Pi}(b_t)\left(\mathit{\Pi}(b_t)-1\right)\right] \\
& + \left(\frac{u''(c_t)}{u'(c_t)}\frac{v'(y_t)}{u'(c_t)}\right), \\
\Delta_{h,t} \equiv\ & \left(1 - \frac{\varphi}{2}\,(\pi_t-1)^{\,2}\right) + \left(\frac{v''(y_t)}{u'(c_t)}\right) - \beta\frac{\varphi}{\theta}\frac{u'(\mathcal{C}(b_t))}{u'(c_t)}\mathcal{Y}(b_t)\mathit{\Pi}(b_t)\left(\mathit{\Pi}(b_t)-1\right) - (\tau_t - s)\,\omega_t.
\end{aligned}$$

Equations (A.4)–(A.8), (A.22)–(A.24) constitute a dynamic system of eight functional equations in eight unknowns $(\mathcal{C}, \mathcal{Y}, \mathit{\Pi}, \mathcal{W}, \mathcal{Q}, \mathcal{B}, \mathcal{G}, \mathcal{T})$.[24] The presence of derivatives of the policy functions in (A.9) complicates the use of this system for the purposes of characterizing and solving for the Markov perfect equilibrium because solving for unknown policy functions requires taking into account how these functions react to variations in the state of the economy.

A.5 The Steady State

Let bars denote steady-state values. Then equation (A.22) at the steady-state becomes:

$$\bar{\Delta}_b \bar{\Omega} = 0, \tag{A.25}$$

where:

$$\begin{aligned}
\bar{\Omega} &= \frac{\varphi\left(\bar{\pi}-1\right)\bar{y}}{\varphi(\bar{\pi}-1)\bar{y} + \frac{\varphi}{\theta}\left(2\bar{\pi}-1\right)\bar{y} + \left(1+\rho q_t\right)\frac{\bar{b}}{\bar{\pi}^2}}, \\
\bar{\Delta}_b &= \frac{\varphi}{\theta}\left(\frac{\bar{\pi}}{1+\rho\bar{q}}\right)\left(-\bar{\pi}\left(\bar{\pi}-1\right)\left(\frac{\mathcal{C}'(\bar{b})u''(\bar{c})}{u'(\bar{c})}\bar{y} + \mathcal{Y}'(\bar{b})\right) - \mathit{\Pi}'(\bar{b})\left(2\bar{\pi}-1\right)\bar{y}\right) \\
&\qquad - \left(\frac{\mathit{\Pi}'(\bar{b})}{\bar{\pi}} - \frac{\mathcal{C}'(\bar{b})u''(\bar{c})}{u'(\bar{c})} - \frac{\rho\mathcal{Q}'(\bar{b})}{1+\rho\bar{q}}\right)\left(1-\frac{\rho}{\bar{\pi}}\right)\bar{b}.
\end{aligned}$$

It implies that there are two types of steady states. In the first steady state, where $\bar{\Omega} = 0$, inflation is zero; i.e. $\bar{\pi} = 1$. In the second steady state, where $\bar{\Delta}_b = 0$, a marginal change of government debt does not provide any gains from affecting next period decisions. The remainder of this section further characterizes the deterministic steady state of the first type.

The remaining Markov perfect equilibrium conditions, (A.4)–(A.8), (A.23) and (A.24), at the steady state with zero inflation read as:

$$0 = \bar{c} + \bar{G} - \bar{y}, \tag{A.26}$$

$$0 = \left(1-\bar{\tau}\right)\bar{w} - \frac{v'(\bar{y})}{u'(\bar{c})}, \tag{A.27}$$

$$0 = \bar{q} - \beta\left(1+\rho\bar{q}\right), \tag{A.28}$$

$$0 = \bar{w} - \frac{\theta-1}{(1-s)\theta}, \tag{A.29}$$

$$0 = \bar{q}\bar{b} - \left(1+\rho\bar{q}\right)\bar{b} - \bar{G} + \bar{\tau}\bar{w}\bar{y}, \tag{A.30}$$

$$0 = g'(\bar{G}) - u'(\bar{c}), \tag{A.31}$$

$$0 = g'(\bar{G}) - v'(\bar{y}). \tag{A.32}$$

[24] In the main text, (A.24) is reported after substituting for output using the aggregate production function (4.13).

Equations (A.26)–(A.32) implicitly determine steady-state values $(\bar{c}, \bar{y}, \bar{w}, \bar{q}, \bar{b}, \bar{G}, \bar{\tau})$. Furthermore, the steady-state values of employment and the nominal interest rate are recovered by evaluating at the steady state the aggregate production function (4.13) and the no-arbitrage condition (4.4):

$$\bar{h} = \bar{y}, \tag{A.33}$$

$$\bar{R} = \beta^{-1}. \tag{A.34}$$

Equations (A.26), (A.31), (A.32) and (A.33) imply that this steady state is efficient; that is, consistent with the first-best allocation.

Furthermore, combining (A.31) and (A.32) results in:

$$0 = 1 - \frac{v'(\bar{y})}{u'(\bar{c})},$$

which, together with (A.27), yields condition $1 = (1 - \bar{\tau})\,\bar{\omega}$, where the tax rate and the real wage can be further substituted for using equations (A.28)–(A.30) so as to get:

$$1 = \frac{\theta - 1}{(1-s)\theta} - \frac{\bar{G}}{\bar{y}} - \left(\frac{1-\beta}{1-\beta\rho}\right)\frac{\bar{b}}{\bar{y}}. \tag{A.35}$$

The steady-state levels of government spending and output, $\bar{G}$ and $\bar{y}$, are determined independently of equation (A.35). Therefore, equation (A.35) implicitly determines the steady-state level of government debt, $\bar{b}$, as a function of the rate of employment subsidy, s.

Note that a simplified version of the model with inflation-indexed government debt features an identical efficient steady state. This follows directly from the fact that conditions of private-sector equilibrium with nominal and inflation-indexed debt are isomorphic whenever inflation is zero.

A.6 The Linear-Quadratic Approximation

A.6.1 Linear Constraints

The first step in deriving a linear-quadratic approximation of the policy problem is to log-linearize policy constraints (A.4)–(A.8) around the efficient steady state with a nonzero amount of government debt:

$$0 = \bar{c}\hat{c}_t - \bar{y}\hat{y}_t + \bar{G}\hat{G}_t, \tag{A.36}$$

$$0 = \hat{w}_t - \bar{\tau}\bar{w}\hat{\tau}_t - \gamma_c\hat{c}_t - \gamma_h\hat{y}_t, \tag{A.37}$$

$$0 = \hat{q}_t - \rho\beta\hat{q}_{t+1} + \hat{\pi}_{t+1} - \gamma_c\left(\hat{c}_t - \hat{c}_{t+1}\right), \tag{A.38}$$

$$0 = \hat{\omega}_t - \frac{\varphi}{\theta - 1}\hat{\pi}_t + \beta\frac{\varphi}{\theta - 1}\hat{\pi}_{t+1}, \tag{A.39}$$

$$0 = \bar{b}\bar{q}\hat{b}_t - \bar{b}\bar{q}\frac{\hat{b}_{t-1}}{\beta} + (1-\rho)\,\bar{b}\bar{q}\hat{q}_t + \bar{b}\bar{q}\frac{\hat{\pi}_t}{\beta} - \bar{G}\hat{G}_t + \bar{w}\bar{y}\left(\left(\bar{\tau} - s\right)\left(\hat{y}_t + \hat{w}_t\right) + \bar{\tau}\hat{\tau}_t\right), \tag{A.40}$$

where a hat is used to denote the log-deviation of the corresponding variable from its steady-state value.

Values of the one-period nominal interest rate and employment can be recovered from the log-linearized versions of the aggregate production function (4.13) and the no-arbitrage condition (4.4):

$$\hat{h}_t = \hat{y}_t, \tag{A.41}$$

$$\hat{i}_t = \rho\beta\hat{q}_{t+1} - \hat{q}_t, \tag{A.42}$$

where $\hat{i}_t$ conventionally denotes the log-deviation of the short-term nominal interest rate, R_t.

In a simplified version of the model used in Section 3, equations (A.36), (A.38), (A.40), and (A.42) should be interchanged with:

$$0 = \bar{c}\hat{c}_t - \bar{y}\hat{y}_t, \tag{A.43}$$

$$0 = \hat{q}_t - \rho\beta\hat{q}_{t+1} - \gamma_c\left(\hat{c}_t - \hat{c}_{t+1}\right), \tag{A.44}$$

$$0 = \bar{b}\bar{q}\hat{b}_t - \bar{b}\bar{q}\frac{\hat{b}_{t-1}}{\beta} + (1-\rho)\,\bar{b}\bar{q}\hat{q}_t + \bar{\tau}\bar{w}\bar{y}\left(\hat{y}_t + \hat{w}_t + \hat{\tau}_t\right), \tag{A.45}$$

$$\hat{i}_t = \rho\beta\hat{q}_{t+1} - \hat{q}_t + \hat{\pi}_{t+1}. \tag{A.46}$$

Reducing the set of constraints in this simplified version of the model to make it look like in the main text of the paper requires eliminating consumption, $\hat{c}_t$, and the real wage, $\hat{w}_t$, using resource constraint (A.43) and consumption-leisure trade-off equation (A.37), respectively.

A.6.2 Quadratic Objective Function

The second step in deriving a linear-quadratic approximation of the policy problem is to derive a quadratic approximation of the household's welfare. Let the household's period t utility be defined as:

$$U_t \equiv \frac{c_t^{1-\gamma_c}}{1-\gamma_c} + \nu_g\frac{G_t^{1-\gamma_g}}{1-\gamma_g} - \nu_h\frac{h_t^{1+\gamma_h}}{1+\gamma_h}.$$

In order to derive an accurate approximation of welfare that preserves the ranking of government policy alternatives when maximizing subject to the (log-)linearized private-sector equilibrium conditions, one can substitute for consumption in the period t utility using the resource constraint (A.4) and substitute employment with output using the aggregate production function (4.13).[25] The resulting function reads as:

[25] Alternatively, one can approximate the original function up to the second order and then use the second-order approximation of the resource constraint to eliminate the linear terms and introduce the remaining quadratic terms.

$$\tilde{U}_t \equiv \frac{\left(y_t\left(1-\frac{\varphi}{2}\left(\pi_t-1\right)^2\right)-G_t\right)^{1-\gamma_c}}{1-\gamma_c}+\nu_g\frac{G_t^{1-\gamma_g}}{1-\gamma_g}-\nu_h\frac{y_t^{\gamma_h+1}}{1+\gamma_h}. \tag{A.47}$$

Next, make a variable change in (A.47) by substituting the original variables with their log-deviations from the efficient deterministic steady state, where hats are used to denote corresponding log-deviations. Formally, use the following identity for a generic variable X_t:

$$X_t = \bar{X}e^{\hat{X}_t}, \tag{A.48}$$

where $\hat{X}_t \equiv \ln X_t - \ln \bar{X}$. The resulting function is then approximated to the second order using Taylor expansion around the efficient steady state as follows:

$$\begin{aligned}
\tilde{U}_t \simeq -\frac{1}{2}\left[\bar{y}\gamma_c\left(\bar{y}-\bar{G}\right)^{-\gamma_c-1}-\left(\bar{y}-\bar{G}\right)^{-\gamma_c}+\left(\gamma_h+1\right)\nu_h\bar{y}^{\gamma_h}\right]\bar{y}\hat{y}_t^2 \\
-\frac{1}{2}\left[\left(\bar{y}-\bar{G}\right)^{-\gamma_c-1}\left(\bar{G}\left(\gamma_c-\gamma_g\right)+\bar{y}\gamma_g\right)\right]\bar{G}\hat{G}_t^2 \\
+\left[\gamma_c\left(\bar{y}-\bar{G}\right)^{-\gamma_c-1}\right]\bar{G}\bar{y}\hat{G}_t\hat{y}_t-\frac{1}{2}\left[\varphi\bar{y}\left(\bar{y}-\bar{G}\right)^{-\gamma_c}\right]\hat{\pi}_t^2+\text{t.i.p.},
\end{aligned} \tag{A.49}$$

where t.i.p. stands for "terms independent of policy," and policy-dependent linear terms have been eliminated using steady-state conditions (A.26), (A.31) and (A.32).

One can further transform (A.49) by reintroducing consumption, $\hat{c}_t$, using linearized versions of the resource constraint (A.36) and the aggregate production function (A.41). As a result, the household's welfare can be approximated (up to additive terms independent of policy) by

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\gamma_c\bar{c}\hat{c}_t^2+\gamma_g\bar{G}\hat{G}_t^2+\gamma_h\bar{y}\hat{y}_t^2+\varphi\bar{y}\hat{\pi}_t^2\right). \tag{A.50}$$

It is straightforward to modify (A.50) for the simplified version of the model in which government spending is assumed to be equal to the constant first-best level. Note that this assumption implies $\hat{G}_t = 0$. Using the linearized resource constraint (A.43) to substitute output for consumption, the household's welfare approximation can be transformed to read as:

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\frac{\gamma_h+\gamma_c(\bar{y}/\bar{c})}{\varphi}\hat{y}_t^2+\hat{\pi}_t^2\right).$$

A.6.3 Equilibrium Characterization

The approximated Markov perfect equilibrium is associated with a recursive optimization problem where the benevolent government maximizes the approximated lifetime welfare of the household (A.50) by choosing $(\hat{c}_t, \hat{y}_t, \hat{\pi}_t, \hat{w}_t, \hat{q}_t, \hat{b}_t, \hat{\tau}_t, \hat{G}_t)$ subject to the approximated constraints (A.36)–(A.40) taking as given choices in the future periods. Formally, the Bellman equation reads as:

$$\mathcal{U}\left(\hat{b}_{t-1}\right) = \max_{(\hat{c}_t,\hat{y}_t,\hat{\pi}_t,\hat{w}_t,\hat{q}_t,\hat{b}_t,\hat{\tau}_t,\hat{G}_t)} \left[-\frac{1}{2}\left(\gamma_c\bar{c}\hat{c}_t^2 + \gamma_g\bar{G}\hat{G}_t^2 + \gamma_h\bar{y}\hat{y}_t^2 + \varphi\bar{y}\hat{\pi}_t^2\right) + \beta\mathcal{U}\left(\hat{b}_t\right)\right]$$

subject to:

$$0 = \bar{c}\hat{c}_t - \bar{y}\hat{y}_t + \bar{G}\hat{G}_t, \tag{A.51}$$

$$0 = \hat{w}_t - \bar{\tau}\bar{w}\hat{\tau}_t - \gamma_c\hat{c}_t - \gamma_h\hat{y}_t, \tag{A.52}$$

$$0 = \hat{q}_t + \hat{\mathit{\Pi}}(\hat{b}_t) - \beta\rho\hat{\mathcal{Q}}(\hat{b}_t) - \gamma_c\left(\hat{c}_t - \hat{\mathcal{C}}(\hat{b}_t)\right), \tag{A.53}$$

$$0 = \hat{\omega}_t - \frac{\varphi}{\theta-1}\hat{\pi}_t + \frac{\varphi}{\theta-1}\beta\hat{\mathit{\Pi}}(\hat{b}_t), \tag{A.54}$$

$$0 = \bar{b}\bar{q}\hat{b}_t - \bar{b}\bar{q}\frac{\hat{b}_{t-1}}{\beta} + (1-\rho)\,\bar{b}\bar{q}\hat{q}_t + \bar{b}\bar{q}\frac{\hat{\pi}_t}{\beta} - \bar{G}\hat{G}_t + \bar{w}\bar{y}\left((\bar{\tau}-s)\left(\hat{y}_t + \hat{w}_t\right) + \bar{\tau}\hat{\tau}_t\right), \tag{A.55}$$

where $\mathcal{U}(\hat{b}_{t-1})$ is the value function, and functions $\hat{\mathit{\Pi}}(\hat{b}_t), \hat{\mathcal{Q}}(\hat{b}_t), \hat{\mathcal{C}}(\hat{b}_t)$ determine inflation, government bond price and consumption in period $t+1$ as functions of the debt level outstanding at the beginning of period $t+1$, $\hat{b}_t$.

The approximated Markov perfect equilibrium can be characterized with the first-order conditions. Let $(\alpha_{r,t}, \alpha_{h,t}, \alpha_{q,t}, \alpha_{\pi,t}, \alpha_{b,t})$ be Lagrange multipliers corresponding to the constraints (A.51)–(A.55). The first-order conditions with respect to $(\hat{\pi}_t, \hat{w}_t, \hat{q}_t, \hat{G}_t, \hat{\tau}_t)$ can be used to solve for Lagrange multipliers:

$$\alpha_{r,t} = -\left(\gamma_g\hat{G}_t + \Psi\hat{\pi}_t\right), \tag{A.56}$$

$$\alpha_{h,t} = -\bar{y}\Psi\hat{\pi}_t, \tag{A.57}$$

$$\alpha_{q,t} = (1-\rho)\bar{b}\Psi\hat{\pi}_t, \tag{A.58}$$

$$\alpha_{\pi,t} = (1-s)\bar{y}\Psi\hat{\pi}_t, \tag{A.59}$$

$$\alpha_{b,t} = -\Psi\hat{\pi}_t, \tag{A.60}$$

where the coefficient Ψ scales the effect that inflation has on tightness of the constraints and is defined as follows:

$$\Psi \equiv \frac{\varphi\bar{y}}{\frac{\varphi}{\theta}\bar{y} + \beta^{-1}\bar{\Gamma}},$$

where $\bar{\Gamma} \equiv \bar{b}\bar{q}$ is the market value of government debt in the steady state. It is straightforward to see that Ψ is unambiguously positive when the steady-state market value of debt is positive. The coefficient Ψ has an interpretation similar to the inflation cost factor, Ω_t, in the nonlinear analysis: the larger the marginal resource cost of inflation compared with its marginal benefits, the larger is Ψ is. Also note that if an outstanding amount of debt in excess of its level in the steady state,

$\hat{b}_{t-1} > 0$, makes the government budget constraint tight, $\alpha_{b,t} < 0$, then condition (A.60) implies that the corresponding inflation has to be positive, $\hat{\pi}_t > 0$.

The first-order conditions with respect to $\hat{c}_t$ and $\hat{y}_t$ are as follows:

$$0 = \gamma_c \bar{c}\hat{c}_t + \bar{c}\alpha_{r,t} - \gamma_c \alpha_{h,t} - \gamma_c \frac{\beta}{1-\beta\rho}\alpha_{q,t} \tag{A.61}$$

$$0 = \gamma_h \bar{y}\hat{y}_t - \bar{y}\alpha_{r,t} - \gamma_h \alpha_{h,t} + \bar{y}\bar{w}\left(\bar{\tau} - s\right)\alpha_{b,t} \tag{A.62}$$

Lagrange multipliers in these two equations can be eliminated using equations (A.56)–(A.60). Combining the resulting equations with the aggregate resource constraint yields three targeting rules:

$$\hat{c}_t = \Phi_c \hat{\pi}_t, \qquad \hat{y}_t = \Phi_y \hat{\pi}_t, \qquad \hat{G}_t = \Phi_g \hat{\pi}_t,$$

where

$$\begin{aligned}
\Phi_c &\equiv -\Psi \frac{\bar{c}\left(\gamma_g\left(\gamma_h + \theta^{-1}\right)\bar{y} - \gamma_h \bar{G}\right) + \left(\bar{y} - (1-\rho)\bar{\Gamma}\right)\left(\gamma_h\gamma_c\bar{G} + \gamma_c\gamma_g\bar{y}\right)}{\bar{c}\left(\gamma_h\gamma_g\bar{c} + \gamma_h\gamma_c\bar{G} + \gamma_c\gamma_g\bar{y}\right)}, \\
\Phi_y &\equiv -\Psi \frac{\gamma_g\left(\gamma_h + \theta^{-1}\right)\bar{c} + \gamma_c\left(1 + \gamma_h + \theta^{-1}\right)\bar{G} + \gamma_c\gamma_g\bar{y} - \gamma_g\gamma_c(1-\rho)\bar{\Gamma}}{\gamma_h\gamma_g\bar{c} + \gamma_h\gamma_c\bar{G} + \gamma_c\gamma_g\bar{y}}, \\
\Phi_g &\equiv -\Psi \frac{\gamma_h\bar{c} + \gamma_c\left(1 + \theta^{-1}\right)\bar{y} + \gamma_h\gamma_c(1-\rho)\bar{\Gamma}}{\gamma_h\gamma_g\bar{c} + \gamma_h\gamma_c\bar{G} + \gamma_c\gamma_g\bar{y}}.
\end{aligned}$$

The first-order condition with respect to $\hat{b}_t$ reads as:

$$0 = \beta\mathcal{U}'(\hat{b}_t) - \bar{\Gamma}\alpha_{b,t} - \bar{q}\left(\gamma_c\hat{\mathcal{C}}'(\hat{b}_t) + \hat{\Pi}'(\hat{b}_t) - \beta\rho\hat{\mathcal{Q}}'(\hat{b}_t)\right)\alpha_{q,t} - \frac{\varphi\beta}{\theta\left(1-s\right)}\hat{\Pi}'(\hat{b}_t)\alpha_{\pi,t}, \tag{A.63}$$

The derivative of the value function, $\mathcal{U}'(\hat{b}_t)$, can be expressed in terms of Lagrange multiplier $\alpha_{b,t+1}$ using the Envelope condition

$$\mathcal{U}'(\hat{b}_t) = \beta^{-1}\bar{\Gamma}\alpha_{b,t+1}.$$

Furthermore, Lagrange multipliers $\alpha_{q,t}$, $\alpha_{\pi,t}$ and $\alpha_{b,t}$ have been previously solved for in terms of inflation as described by (A.58)–(A.60). Then, (A.63) can be rewritten as follows:

$$0 = \hat{\pi}_{t+1} - \hat{\pi}_t + (1-\rho)\left(\gamma_c \hat{\mathcal{C}}'(\hat{b}_t) + \hat{\mathit{\Pi}}'(\hat{b}_t) - \beta\rho\hat{\mathcal{Q}}'(\hat{b}_t)\right)\hat{\pi}_t + \frac{\varphi}{\theta}\frac{\bar{y}}{\bar{\Gamma}}\beta\hat{\mathit{\Pi}}'(\hat{b}_t)\hat{\pi}_t. \tag{A.64}$$

The linear-quadratic structure of the optimization problem underlying the approximated Markov perfect equilibrium is known for its convenient property that solutions belong to the class of linear functions of state variables. In other words, the approximated Markov perfect equilibrium is completely represented by the constant coefficients $(\mathcal{C}_b, \mathcal{Y}_b, \mathcal{H}_b, \mathit{\Pi}_b, \mathcal{W}_b, \mathcal{Q}_b, \mathcal{B}_b, \mathcal{G}_b, \mathcal{T}_b, \mathcal{I}_b)$ that determine economic outcome in a given period as proportional to the outstanding level of government debt:

$$\begin{aligned}
&\hat{c}_t = \mathcal{C}_b\hat{b}_{t-1}, && \hat{y}_t = \mathcal{Y}_b\hat{b}_{t-1}, && \hat{h}_t = \mathcal{H}_b\hat{b}_{t-1}, && \hat{\pi}_t = \mathit{\Pi}_b\hat{b}_{t-1}, && \hat{w}_t = \mathcal{W}_b\hat{b}_{t-1},\\
&\hat{q}_t = \mathcal{Q}_b\hat{b}_{t-1}, && \hat{b}_t = \mathcal{B}_b\hat{b}_{t-1}, && \hat{G}_t = \mathcal{G}_b\hat{b}_{t-1}, && \hat{\tau}_t = \mathcal{T}_b\hat{b}_{t-1}, && \hat{i}_t = \mathcal{I}_b\hat{b}_{t-1}.
\end{aligned}$$

Imposing a linear solution structure on the system of the first-order conditions described above allows for the method of undetermined coefficients to be used in order to solve for the equilibrium. To do so, one can start by rewriting equation (A.64) and the government budget constraint (A.55) as follows:

$$0 = (\mathcal{B}_b - 1) + (1-\rho)\left(\gamma_c\mathcal{C}_b + \mathit{\Pi}_b - \beta\rho\mathcal{Q}_b\right) + \frac{\varphi}{\theta}\frac{\bar{y}}{\bar{\Gamma}}\beta\mathit{\Pi}_b, \tag{A.65}$$

$$0 = (\mathcal{B}_b - \beta^{-1}) + (1-\rho)\,\mathcal{Q}_b + \frac{\mathit{\Pi}_b}{\beta} - \frac{\bar{G}}{\bar{\Gamma}}\mathcal{G}_b + \frac{\bar{\omega}\bar{y}}{\bar{\Gamma}}\left((\bar{\tau} - s)\left(\mathcal{Y}_b + \mathcal{W}_b\right) + \bar{\tau}\mathcal{T}_b\right). \tag{A.66}$$

The remaining first-order conditions, namely the three targeting rules, and equations (A.37)–(A.39), (A.41), and (A.42) can be rearranged to deliver the following equilibrium conditions:

$$\begin{aligned}
\mathcal{C}_b &= \Phi_c\mathit{\Pi}_b,\\
\mathcal{Y}_b &= \Phi_y\mathit{\Pi}_b,\\
\mathcal{G}_b &= \Phi_g\mathit{\Pi}_b,\\
\mathcal{H}_b &= \Phi_y\mathit{\Pi}_b,\\
\mathcal{W}_b &= \frac{\varphi(1-\beta\mathcal{B}_b)}{\theta-1}\mathit{\Pi}_b,\\
\mathcal{Q}_b &= \frac{(\gamma_c\Phi_c(1-\mathcal{B}_b) - \mathcal{B}_b)}{1-\beta\rho\mathcal{B}_b}\mathit{\Pi}_b,\\
\mathcal{I}_b &= -(\gamma_c\Phi_c(1-\mathcal{B}_b) - \mathcal{B}_b)\mathit{\Pi}_b,\\
\mathcal{T}_b &= \frac{(1-\bar{\tau})}{\bar{\tau}}\left(\frac{\varphi}{(\theta-1)}(1-\beta\mathcal{B}_b) - (\gamma_c\Phi_c + \gamma_h\Phi_y)\right)\mathit{\Pi}_b.
\end{aligned}$$

It is then straightforward to use the last set of equations to substitute for all the coefficients but $\mathcal{B}_b$ and $\mathit{\Pi}_b$ in (A.65)–(A.66). The last step is to solve the resulting system of two nonlinear equations for the pair of coefficients $(\mathcal{B}_b, \mathit{\Pi}_b)$. The analysis of this paper is restricted to stationary and monotone solutions with $0 < \mathcal{B}_b < 1$. Checking the existence and uniqueness of the $(\mathcal{B}_b, \mathit{\Pi}_b)$ solution is equivalent to checking the existence and uniqueness of the approximated Markov perfect equilibrium. It is not possible to derive analytical criteria of either existence or uniqueness for generic values of structural parameters. When checking numerically, the model with baseline parameter values does not display multiplicity of equilibria for $\rho \in [0, 1]$.

B Appendix: The Medium-Scale Model

B.1 List of Variables

Allocation and related variables:

- X_t - aggregate output
- L_t - aggregate labor
- C_t - aggregate private consumption
- I_t - investment
- K_t - physical capital stock
- Z_t - level of capital utilization
- $\mathcal{K}_t$ - capital services
- v_t - real marginal cost

Prices and related variables:

- r_t^k - real rental price of capital services
- i_t - net nominal interest rate (monetary policy rate)
- q_t - price of government bonds portfolio
- π_t - gross inflation rate
- p_t^* - reset price
- s_t - price dispersion
- ω_t - real wage
- ω_t^* - reset real wage
- s_t^ω - real wage dispersion

Fiscal variables:

- τ_t - labor income tax rate
- G_t - government spending
- b_t - portfolio of government bonds

Auxiliary variables: $\lambda_t, \mu_t, f_{1,t}, f_{2,t}, x_{1,t}, x_{2,t}$.

B.2 Summary of the Private-Sector Equilibrium Conditions

Utility of the representative household:

$$\sum_{t=0}^{\infty} \beta^t \left[\log C_t + \nu_g \log G_t - \eta L_t^{1+\chi}/(1+\chi)\right] \tag{B.1}$$

Private-sector equilibrium conditions:

$$\lambda_t = (C_t - bC_{t-1})^{-1} - b\beta(C_{t+1} - bC_t)^{-1}, \tag{B.2}$$

$$\mu_t = \beta(1-\delta)\mu_{t+1} + \beta\lambda_{t+1}(r_{t+1}^k Z_{t+1} - \gamma_1(Z_{t+1}-1) - (\gamma_2/2)(Z_{t+1}-1)^2), \tag{B.3}$$

$$\lambda_t = \mu_t\left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}} - 1\right)\right) + \beta\kappa\mu_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{I_{t+1}}{I_t} - 1\right), \tag{B.4}$$

$$r_t^k = \gamma_1 + \gamma_2(Z_t - 1), \tag{B.5}$$

$$\lambda_t = \beta(1+i_t)\lambda_{t+1}\pi_{t+1}^{-1}, \tag{B.6}$$

$$\lambda_t = \beta(1+\rho q_{t+1})q_t^{-1}\lambda_{t+1}\pi_{t+1}^{-1}, \tag{B.7}$$

$$K_t = (1-\delta)K_{t-1} + \left(1 - (\kappa/2)(I_t/I_{t-1} - 1)^2\right)I_t, \tag{B.8}$$

$$\mathcal{K}_t = Z_t K_{t-1}, \tag{B.9}$$

$$\omega_t^* = \sigma f_{1,t}((\sigma-1)f_{2,t})^{-1}, \tag{B.10}$$

$$f_{1,t} = \eta\left(\omega_t^*\omega_t^{-1}\right)^{-\sigma(1+\chi)}L_t^{1+\chi} + \beta\xi_\omega\left(\pi_t^{\zeta_\omega}\pi_{t+1}^{-1}\omega_t^*\right)^{-\sigma(1+\chi)}\left(\omega_{t+1}^*\right)^{\sigma(1+\chi)}f_{1,t+1}, \tag{B.11}$$

$$f_{2,t} = \lambda_t(1-\tau_t)\left(\omega_t^*\omega_t^{-1}\right)^{-\sigma}L_t + \beta\xi_\omega\left(\pi_t^{\zeta_\omega}\pi_{t+1}^{-1}\right)^{1-\sigma}\left(\omega_t^*\right)^{-\sigma}\left(\omega_{t+1}^*\right)^{\sigma}f_{2,t+1}, \tag{B.12}$$

$$1 = (1-\xi_\omega)\left(\omega_t^*\omega_t^{-1}\right)^{1-\sigma} + \xi_\omega\left(\pi_{t-1}^{\zeta_\omega}\pi_t^{-1}\omega_{t-1}\omega_t^{-1}\right)^{1-\sigma}, \tag{B.13}$$

$$s_t^\omega = (1-\xi_\omega)\left(\omega_t^*\omega_t^{-1}\right)^{-\sigma(1+\chi)} + \xi_\omega\left(\pi_{t-1}^{\zeta_\omega}\pi_t^{-1}\omega_{t-1}\omega_t^{-1}\right)^{-\sigma(1+\chi)}s_{t-1}^\omega \tag{B.14}$$

$$\mathcal{K}_t = \alpha\frac{\upsilon_t}{(1-s_{\mathcal{K}})r_t^k}\left(s_t X_t + F\right), \tag{B.15}$$

$$L_t = (1-\alpha)\frac{\upsilon_t}{(1-s_L)(1+i_t)\omega_t}\left(s_t X_t + F\right), \tag{B.16}$$

$$s_t X_t = \mathcal{K}_t^\alpha L_t^{1-\alpha} - F, \tag{B.17}$$

$$p_t^* = \theta x_{1,t}((\theta-1)x_{2,t})^{-1}, \tag{B.18}$$

$$x_{1,t} = \lambda_t X_t \upsilon_t + \beta\xi_p\left(\pi_t^{\zeta_p}\pi_{t+1}^{-1}\right)^{-\theta}x_{1,t+1}, \tag{B.19}$$

$$x_{2,t} = \lambda_t X_t + \beta\xi_p\left(\pi_t^{\zeta_p}\pi_{t+1}^{-1}\right)^{1-\theta}x_{2,t+1}, \tag{B.20}$$

$$1 = (1-\xi_p)(p_t^*)^{1-\theta} + \xi_p\left(\pi_{t-1}^{\zeta_p}\pi_t^{-1}\right)^{1-\theta}, \tag{B.21}$$

$$s_t = (1-\xi_p)(p_t^*)^{-\theta} + \xi_p\left(\pi_{t-1}^{\zeta_p}\pi_t^{-1}\right)^{-\theta}s_{t-1}, \tag{B.22}$$

$$X_t = C_t + G_t + I_t + \left(\gamma_1(Z_t-1) + (\gamma_2/2)(Z_t-1)^2\right)K_{t-1}, \tag{B.23}$$

$$q_t b_t = (1+\rho q_t)b_{t-1}\pi_t^{-1} + (G_t + \varsigma_t - \tau_t w_t L_t)\,. \tag{B.24}$$

B.3 The Policy Problem

As in the baseline model, the policy problem of the government consists of maximizing the welfare of the representative household (B.1) subject to private-sector equilibrium conditions (B.2)–(B.24) while lacking commitment to its own future choices. The policy problem can be solved using the linear-quadratic approach. To do so, one can follow the steps described earlier in relation to the baseline model. This section makes a number of remarks specific to the medium-scale version without repeating these steps in detail.

First, note that the approximation of the medium-scale model is done around the efficient steady state supported by a positive government debt. As in the baseline model, all the static distortions are eliminated by designing appropriate production subsidies. In the medium-scale model, one can do so using subsidies $s_{\mathcal{K}}$ and s_L, one for each factor of production.

Second, the approximation requires the derivation of a quadratic approximation of the utility-based welfare function. Let U_t be a period-t utility component in (B.1). Its quadratic approximation in the vicinity of the steady state can then be written as follows:

$$
\begin{aligned}
U_t \simeq -\frac{1}{2}\bar{\mu}\Bigg(& \bar{C}\hat{C}_t^2 + \frac{b\bar{C}}{(1-b)(1-b\beta)}\tilde{C}_t^2 + \bar{G}\hat{G}_t^2 + \kappa\bar{I}\tilde{I}_t^2 + \gamma_1\bar{K}\hat{\mathcal{K}}_t^2 \\
& + \gamma_2\bar{K}\hat{Z}_t^2 + (1-\alpha)(1+\chi)(1-\phi)(\bar{X}+F)\hat{L}_t^2 - \frac{\bar{X}^2}{\bar{X}+F}\hat{X}_t^2 \\
& + \frac{\sigma\xi_\omega(1-\alpha)(1-\phi)(\sigma\chi+1)(\bar{X}+F)}{(1-\beta\xi_\omega)(1-\xi_\omega)}\tilde{\omega}_t^2 + \frac{\theta\xi_p\bar{X}}{(1-\beta\xi_p)(1-\xi_p)}\tilde{\pi}_t^2\Bigg) + \text{t.i.p.},
\end{aligned}
$$

where:

$$
\begin{aligned}
\tilde{C}_t &\equiv \hat{C}_t - \hat{C}_{t-1}, \\
\tilde{I}_t &\equiv \hat{I}_t - \hat{I}_{t-1}, \\
\tilde{\pi}_t &\equiv \hat{\pi}_t - \xi_p\hat{\pi}_{t-1}, \\
\tilde{\omega}_t &\equiv \hat{\omega}_t - \hat{\omega}_{t-1} + \hat{\pi}_t - \zeta_\omega\hat{\pi}_{t-1},
\end{aligned}
$$

and a bar denotes the steady-state value of the corresponding variable, whereas a hat is used to denote the percentage deviation from this steady-state value.

Parameter	Description	Value
β	Discount factor	0.994
b	Internal habit formation	0.77
ν_g	Government spending utility weight	0.36
η	Labor disutility weight	2.22
χ	Inverse labor supply elasticity	0.12
κ	Investment adjustment cost	14.3
δ	Depreciation rate	0.025
γ_1	Utilization adjustment cost linear term	$\bar{Z} = 1$
γ_2	Utilization adjustment cost squared term	0.0093
ξ_p	Calvo price stickiness	0.62
ξ_ω	Calvo wage stickiness	0.75
ζ_p	Price indexation	0
ζ_ω	Wage indexation	1
θ	Elasticity of substitution: goods	6
σ	Elasticity of substitution: labor	101
α	Capital share	0.25
F	Fixed cost of production	$\bar{\Pi} = 0$

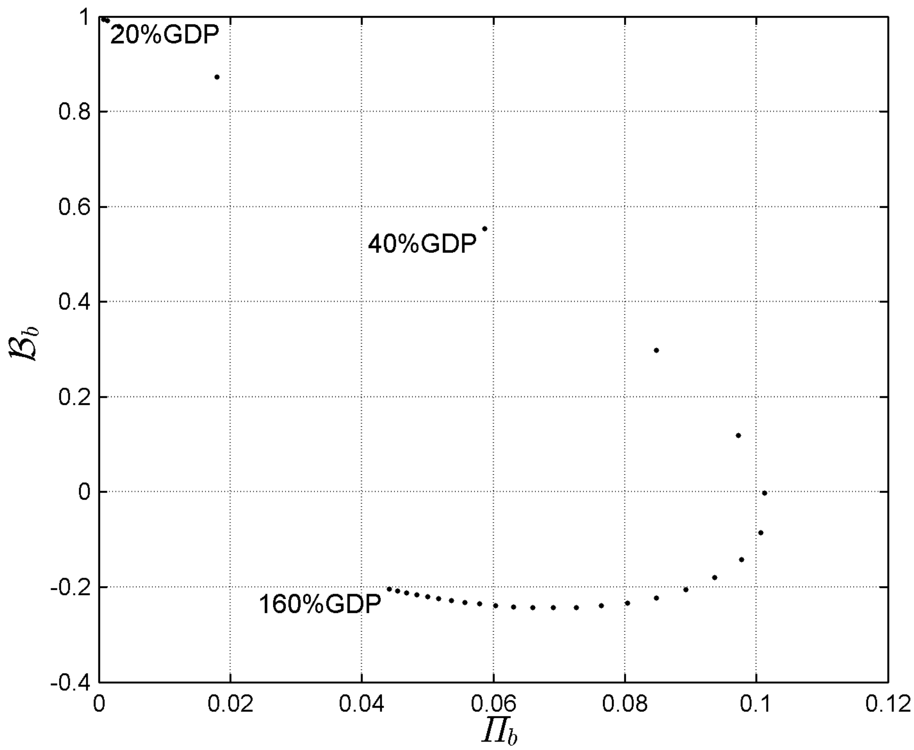
Table 2 – Parameterization of the Medium-scale Model

B.4 Parameterization

Most of the parameter values displayed in Table 2 are based on fixed parameters and posterior mean estimates of the U.S. economy by Christiano et al. (2010). Parametric differences with respect to the model estimates in Christiano et al. (2010) are as follows. The medium-scale model in this paper abstracts from the drift in productivity terms that generate trend growth in output. Moreover, the model abstracts from all stationary disturbances. The model has optimal zero inflation in the steady state.

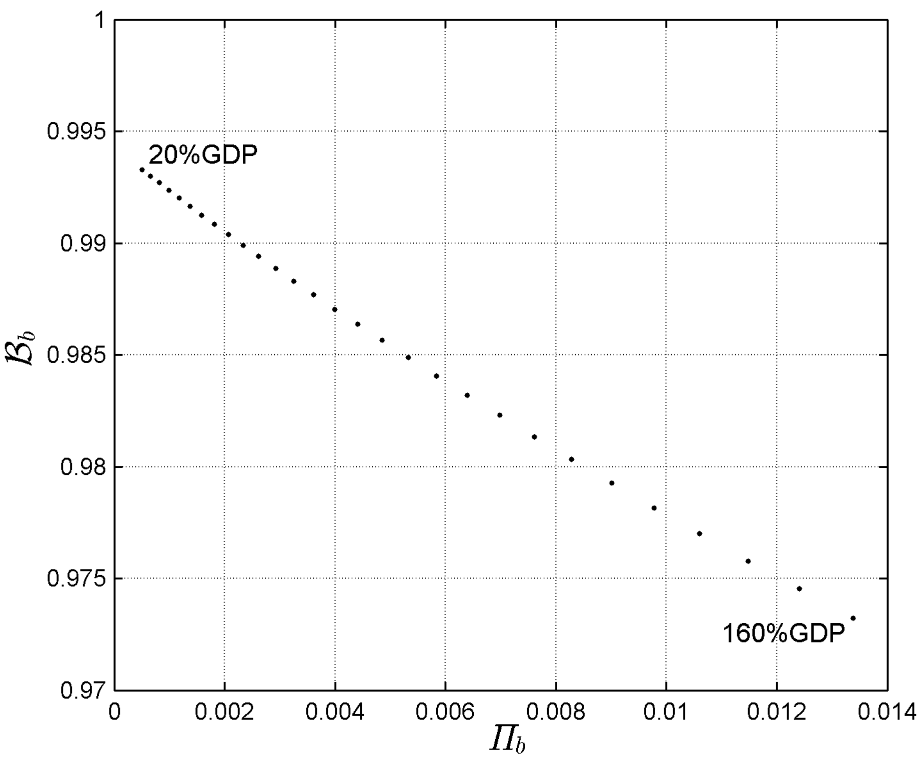
The fiscal sector of the model, which is absent in Christiano et al. (2010), is parameterized following the logic of the baseline model. The value for the weight on the public consumption utility component is chosen so that in the steady state, government spending amounts to 20 percent of the value added. The target for the market value of government debt in the steady state is set equal to 40 percent of annual GDP.

Figure 1 – Linear Equilibrium Decision Rules for Government Debt and Inflation: One-Period Bonds



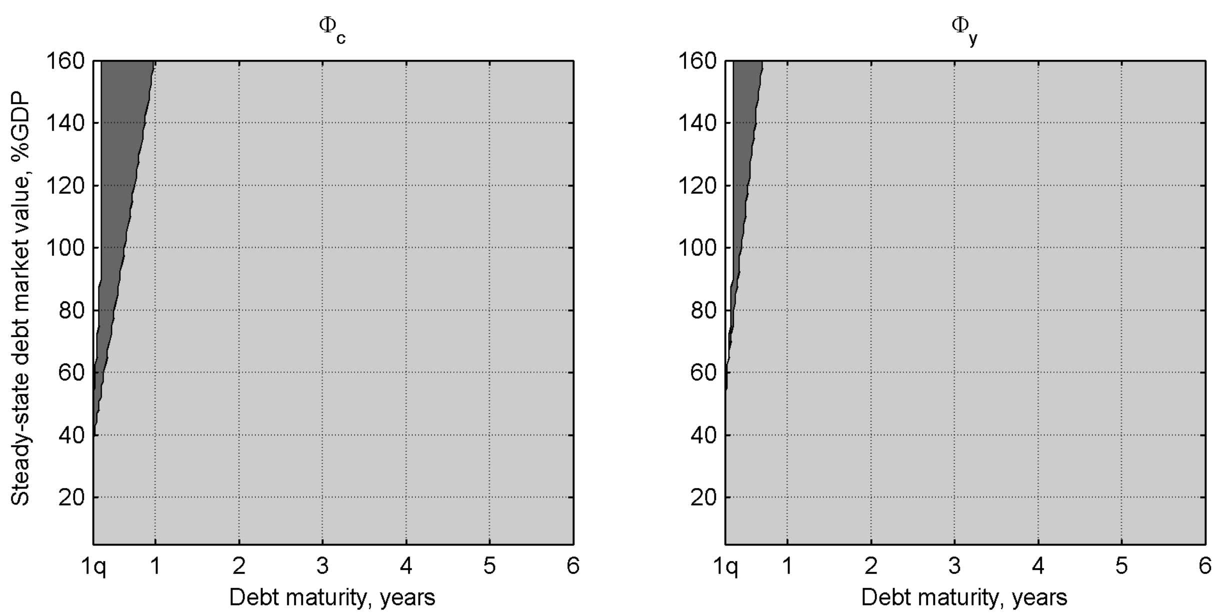
Notes: Parametric plot of coefficients in the linear equilibrium decision rules for government debt (vertical axis) and inflation (horizontal axis) as functions of the market value of government debt as a fraction of (annual) GDP in the steady state. Government debt in the form of one-period bonds. Data are plotted with an increment of 5 percent of GDP.

Figure 2 – Decision Rules for Government Debt and Inflation: Bonds with Four-Year Average Maturity



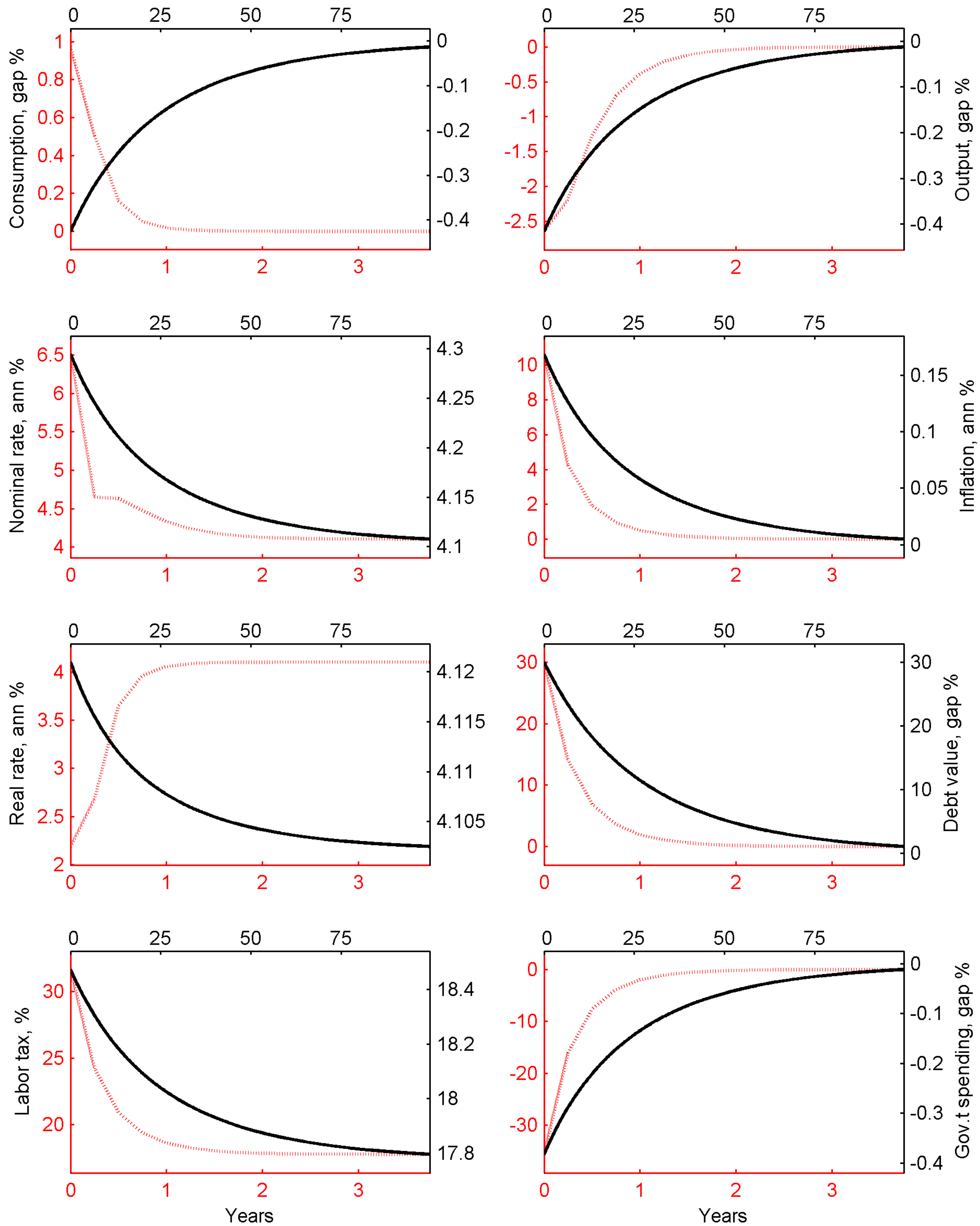
Notes: Parametric plot of coefficients in the linear equilibrium decision rules for government debt (vertical axis) and inflation (horizontal axis) as functions of the market value of government debt as a fraction of (annual) GDP in the steady state. The duration of government debt is four years. Data are plotted with an increment of 5 percent of GDP.

Figure 3 – Targeting Rules Coefficients



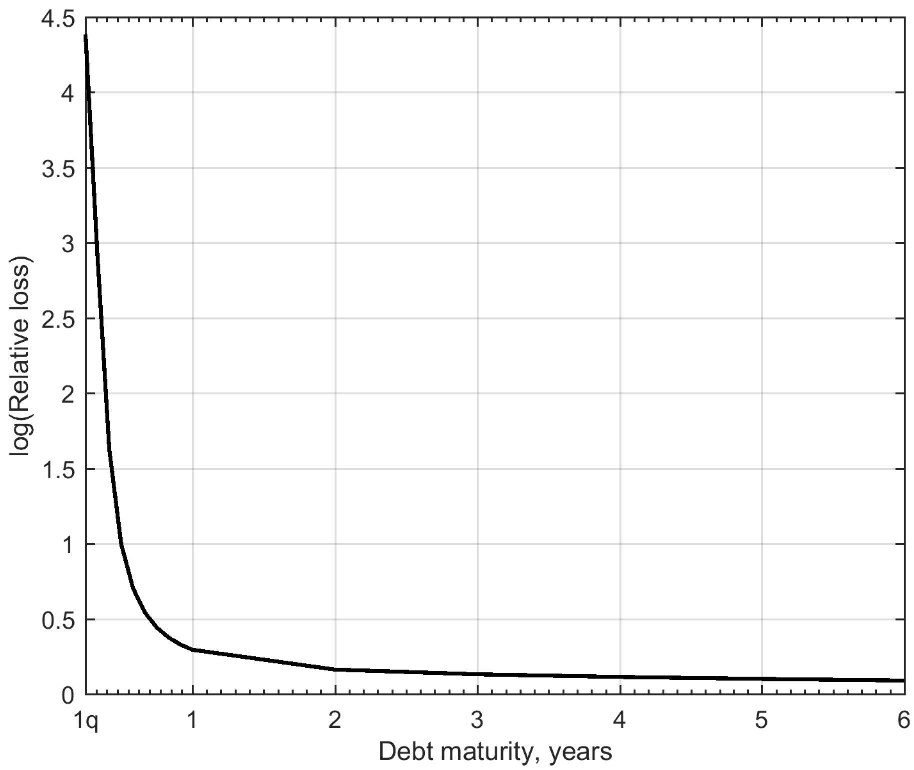
Notes: Contour plots of the coefficients in the targeting rules for consumption and output as functions of government debt characteristics. *Light gray area*: negative values of coefficients. *Dark gray area*: positive values of coefficients. *White area*: positive values of coefficients but a non-monotone equilibrium.

Figure 4 – Dynamics with One-Period vs Four-Year Average Maturity Bonds



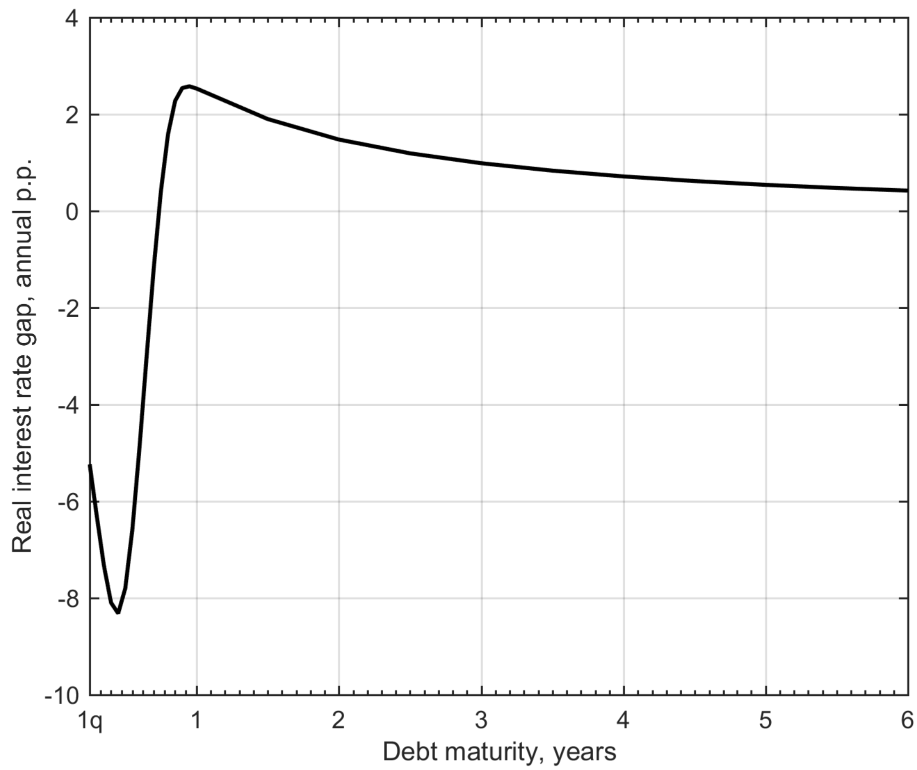
Notes: The figure plots dynamics of the equilibrium variables over time, given an initial quantity of government bonds such that the market value of outstanding government debt in real terms is 30 percent higher than the steady-state level. The steady-state market value of debt is 40 percent of annual GDP. *Solid black lines and black axes*: bonds with average maturity equal to four years. *Dotted red lines and red axes*: one-period bonds.

Figure 5 – Consumption Equivalent Losses of Debt Reduction



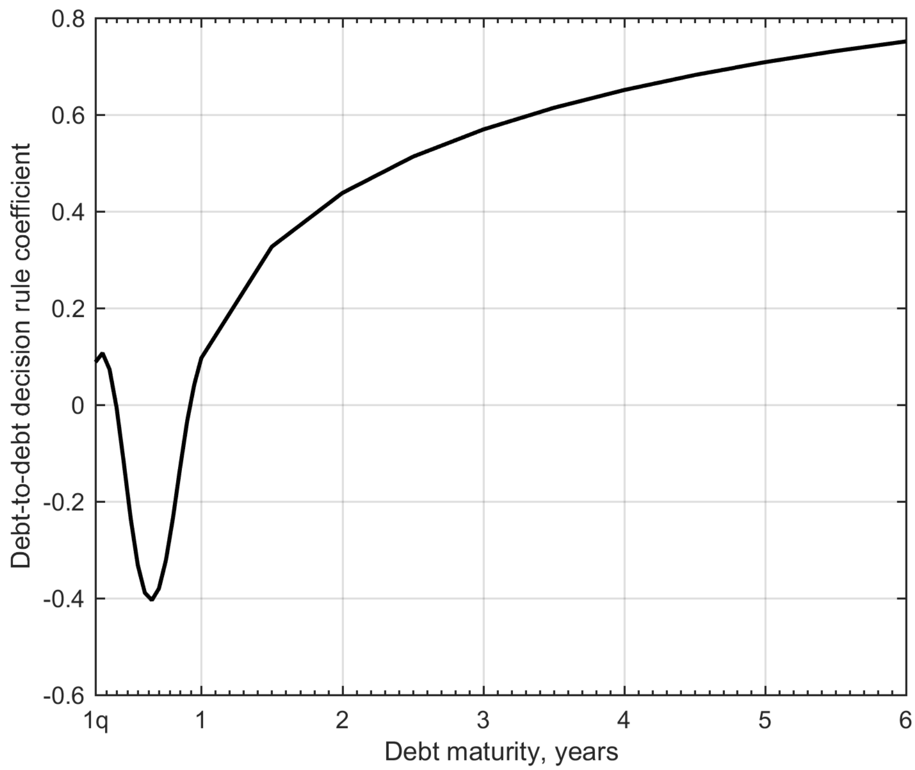
Notes: The steady-state market value of debt is 40 percent of annual GDP. Transition starts from an initial quantity of government bonds such that the market value of outstanding government debt in real terms is 30 percent higher than the steady-state level. Welfare losses are computed in terms of equivalent permanent steady-state consumption reduction and normalized by the welfare loss in the economy with consol bonds.

Figure 6 – Debt Maturity and Real Interest Rates in the Medium-Scale Model



Notes: The vertical axis shows the difference, measured in annualized percentage points, between the real interest rate in the period t where the stock of outstanding government debt, $\hat{b}_{t-1}$, is the only state variable that is different from the steady-state value and the real interest rate in the steady state. The outstanding stock of government debt exceeds the steady-state value by 1 percent. The real interest rate is given by $\hat{r}_t = \hat{i}_t - \hat{\pi}_{t+1}$, where $\hat{i}_t$ is the nominal interest rate and $\hat{\pi}_{t+1}$ is expected inflation.

Figure 7 – Debt Maturity and Debt Dynamics in the Medium-Scale Model



Note: The vertical axis shows the coefficient of the autoregressive component of the equilibrium decision rule for government debt.