BANK OF CANADA
BANQUE DU CANADA

Staff Working Paper/Document de travail du personnel — 2020-17

Last updated: May 8, 2020

Identifying Aggregate Shocks with Micro-level Heterogeneity: Financial Shocks and Investment Fluctuation

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ISSN 1701-9397

Acknowledgements

I would like to thank my advisors Julio Andres Blanco, Pablo Ottonello, Matthew Shapiro, and Toni Whited for their patient guidance and support. I also thank Jason Allen, Matteo Cacciatore, Tuo Chen, Javier Cravino, Katya Kartashova, John Leahy, Andrei A. Levchenko, Wenlan Luo, Sebastian Sotelo, and Wen Yao for their comments and suggestions at different stages of this research project, as well as the seminar participants at Tsinghua University, Bank of Canada, 2019 Asian Meeting of the Econometric Society, and Midwest Macroeconomics Meeting Fall 2019 for their useful comments. I would also like to thank Leanne Rancourt and Maren Hansen for their great editorial help. The views expressed herein are those of the author and not necessarily those of the Bank of Canada.

Abstract

This paper identifies the aggregate financial shocks and quantifies their effects on business investment based on an estimated DSGE model with firm-level heterogeneity. On average, financial shocks contribute only 1.1% of the variation in U.S. public firms' aggregate investment. The negligible aggregate relevance of financial shocks mainly results from the interaction between firm-level heterogeneity and general equilibrium effects. Following a contractionary financial shock, financially constrained firms are directly forced to cut investment, which dampens the aggregate investment demand and lowers the capital good price. The lower capital good price motivates the financially unconstrained firms to invest more, which largely cancels out the financial shock's direct effect in aggregation. If the firm-level heterogeneity is removed, the implied relevance of financial shocks to aggregate investment will be 50 times larger. This sharp difference indicates that representative firm models could overstate the relevance of financial shocks in driving the business cycle fluctuation and highlights the importance of micro-level heterogeneity in identifying the aggregate shocks.

Topics: Business fluctuation and cycles; Firm dynamics

JEL codes: E12, E22, G31, G32

Résumé

L'auteur met en évidence les chocs financiers globaux et en quantifie les effets sur les investissements des entreprises à partir d'un modèle d'équilibre général dynamique et stochastique (EGDS) estimé dans lequel les entreprises sont hétérogènes. En moyenne, les chocs financiers ne sont responsables que de 1,1 % des variations de l'investissement global des sociétés ouvertes américaines. Ce pourcentage négligeable est principalement attribuable à l'interaction entre l'hétérogénéité des entreprises et les effets d'équilibre général. En effet, après un choc financier restrictif, les entreprises soumises à des contraintes financières sont immédiatement forcées de réduire leurs dépenses en capital, ce qui freine la demande globale d'investissement et fait baisser le prix des biens d'équipement. Cette baisse de prix motive les autres entreprises à investir davantage, ce qui annule en grande partie l'effet direct du choc financier dans l'ensemble. Sans l'hétérogénéité des entreprises, l'incidence implicite des chocs financiers sur les investissements globaux serait 50 fois plus grande. Ce vaste écart indique que les modèles avec entreprises représentatives pourraient mener à une surestimation de l'incidence des chocs financiers sur les fluctuations du cycle économique et met en lumière l'importance de l'hétérogénéité microéconomique pour repérer les chocs globaux.

Sujets : Cycles et fluctuations économiques; Dynamique des entreprises

Codes JEL : E12, E22, G31, G32

Non-technical Summary

Business investment is the most volatile component of GDP. Fluctuations in business investment could be driven either by shocks to firms' investment profitability or by shocks to their financing conditions. It is important for policy makers to know which shock generates the changes in business investment to make the stabilization policies.

In previous studies, the models used for identifying financial shocks feature a representative firm whose marginal investment relies on external financing. However, cross-sectional evidence reveals that the investment decisions of a large fraction of firms do not depend on external financing. This paper contributes to the literature by incorporating this cross-sectional heterogeneity into the identification of financial shocks.

I estimate a DSGE model with firm-level heterogeneity and use it to evaluate the relevance of aggregate financial shocks for fluctuations in aggregate investment. I find that the average contribution of financial shocks to the fluctuations of U.S. public firms' aggregate investment is only 1/50 of the contribution implied by the comparable representative firm model. The main reason for this sharp difference is the interaction between firm-level heterogeneity and general equilibrium effects. Following a contractionary financial shock, firms whose marginal investment relies on external financing are directly affected. These firms cut their investment immediately, which lowers the aggregate demand for capital goods and dampens the price of capital goods. A lower capital good price motivates firms who have accumulated a large amount of internal financing capacity to invest more, which largely cancels out the financial shocks' direct effect on aggregate investment.

The results of this paper imply that representative firm models could have overstated the aggregate relevance of financial shocks in driving business investment fluctuations. The sharp difference in the quantitative implications highlights the importance of cross-sectional evidence in identifying aggregate shocks and, more broadly, the interaction between micro-level heterogeneity and general equilibrium effects in shaping the transmission of aggregate shocks.

1 Introduction

Shocks to firms' financing cost and capacity are often referred to as sources of business investment fluctuations, but how much they matter remains an open question. The observed variation in business investment is a joint result of the unobservable shocks to firms' financing conditions and those to firms' investment profitability. To quantify the relevance of financial shocks, we first need to identify them from the observed fluctuations. In previous studies (see, for example, Jermann and Quadrini, 2012; Christiano *et al.*, 2014), the identification is based on an estimated DSGE model with financially constrained representative firms. However, micro-level data reveal significant heterogeneity across firms in terms of how much their investment depends on external financing.

This paper enters this picture by incorporating firm-level heterogeneity into the identification of aggregate financial shocks. To generate both the cross-sectional and cross-time variations in firms' investment and financing behaviors, I build a general equilibrium model with three key components: a continuum of heterogeneous firms facing financial frictions, a group of representative agents featured with New Keynesian setups, and eight aggregate shocks.

The block of heterogeneous firms is designed to generate the cross-sectional variation in firms' investment and financing behaviors. In each period, firms make their choices based on the aggregate economic conditions and their idiosyncratic states, which include their size, leverage ratio, and idiosyncratic productivity. Their idiosyncratic productivity follows an exogenous mean-reverting process, and their production technology has decreasing returns to scale. These two features lead to a finite optimal target size for firms, and hence, small firms have higher investment demand than large firms. The firms can finance their investment from operational cash flows, debt, and equity. When they raise funding from external financing markets, there is a collateral constraint imposed on their debt issuance and a cost associated with their equity issuance. These two financial frictions generate a "pecking-order" in firms' financing choices.

The block of New Keynesian agents is designed to capture the endogenous variation in the aggregate economic conditions faced by the firms. This block is featured with sticky prices and wages, external habit formation in consumption, and adjustment costs in capital goods production. Within the eight aggregate shocks, there are two financial shocks: one captures the exogenous variation in the tightness of collateral constraints, and the other captures the exogenous variation in the cost of equity issuance. The other six aggregate shocks capture the exogenous variation in aggregate productivity, price markup, wage markup, the efficiency of transforming final goods to capital goods, households' inter-temporal substitution preference, and monetary policy. The combination of these New Keynesian frictions and aggregate shocks make the model able to generate the cyclical variations in the aggregate quantities and prices as faced by the firms in reality.

The model is quantified in two steps. First, I calibrate the parameters that exclude those governing the aggregate shock processes. This calibration makes the time-average of the moments about the distribution of firms' investment and financing behaviors match the micro-level evidence from Compustat. Then I use a Bayesian likelihood method to estimate the rest of the parameters to match the time variation in both the aggregate quantities and prices, as well as the U.S. public firms' financing choices on the disaggregate level.

With the estimated model, I quantify the effects of financial shocks on U.S. public firms' investment. On the disaggregate level, financial shocks are only essential to explain the variation in small firms' investment. On average, financial shocks contribute 30% of the small firms' investment variation, but only 6% of the large firms' investment variation. This difference directly results from the fact that within the group of small firms, there are more financially constrained firms whose investment relies on external financing. On the aggregate level, financial shocks contribute only 1.1% of the variation in U.S. public firms' investment.

The negligible aggregate relevance of financial shocks is mainly a result of the interaction between firm-level heterogeneity and general equilibrium effects. In the model, I divide the firm population into two groups: financially constrained and financially unconstrained firms, based on the reliance of their investment on external financing. In the episode with a contractionary financial shock, the shock hits the constrained firms directly, and they cut their investment immediately. Then the lower investment demand dampens the capital goods price, and the lower capital goods price motivates the unconstrained firms to increase their investment. On average, the unconstrained firms are larger and have ample financing capacity to seize profitable investment opportunities created by the dampened capital good price. Therefore, the direct effects of financial shocks are largely canceled out in aggregation.

Due to the absence of such an interaction mechanism, the representative firm model implies a more substantial aggregate relevance of the financial shocks. If I degenerate the heterogeneous firm model back to a representative firm model and repeat the same quantitative analysis, financial shocks would be implied to contribute 55% of the variation in U.S. public firms' aggregate investment, which is 50 times as large as the result from the heterogeneous firm model.

In the era of economics with richer micro-level data, it is inevitable to extend the quantitative analysis in macroeconomics to incorporate the micro-level evidence. However, it is challenging to handle the DSGE model with rich heterogeneity and many aggregate shocks, especially when we need to estimate the model. This paper contributes to the literature by incorporating firm-level heterogeneity into the identification of aggregate financial shocks. The pioneering practice in this paper shows that adding micro-level heterogeneity into the identification could lead to a significantly different implied relevance of a certain type of aggregate shocks, and this result highlights the important role of the interaction between micro-level heterogeneity and general equilibrium effects.

Related Literature This paper mainly contributes to two branches of literature.

First, this paper contributes to the literature that focuses on identifying the aggregate shocks to firms' financing conditions (see, for example, Justiniano *et al.*, 2011; Jermann and Quadrini, 2012; Christiano *et al.*, 2014; Eisfeldt and Muir, 2016). Compared with Justiniano *et al.* (2011), Jermann and Quadrini (2012), and Christiano *et al.* (2014), which identify the shocks based on a representative firm DSGE model and aggregate time-series, this paper incorporates firm-level heterogeneity and cross-sectional evidence into the identification. Eisfeldt and Muir (2016) also incorporate firm-level heterogeneity into the identification of aggregate financial shocks, but there is no general equilibrium feedback in their structural model. As illustrated in this paper, the interaction between firm-level heterogeneity and general equilibrium effects can lead to a significantly different implication about the aggregate relevance of financial shocks.

Second, this paper contributes to the literature that discusses how firm-level heterogeneity shapes the transmission of aggregate shocks (see, for example, Khan and Thomas, 2013; Zetlin-Jones and Shourideh, 2017; Ottonello and Winberry, 2018). Given that this paper focuses mostly on the financial shocks, it is closely related with Khan and Thomas (2013) and Zetlin-Jones and Shourideh (2017). In Khan and Thomas (2013), aggregate financial shocks have a large impact on aggregate quantities. However, in this paper, the financial shocks' aggregate effects are much weaker due to the dampening effects from the general equilibrium feedback through the capital good price. This mechanism and its result are aligned with those found in Zetlin-Jones and Shourideh (2017), where financial shocks' impact on aggregate investment is largely dampened by the general equilibrium feedback in the real interest rate.

Other than to the above literature, this paper is also related to the studies discussing the different cyclical patterns in small and large firms' choices (see, for example, Covas and Den Haan, 2011; Begenau and Salomao, 2018; Crouzet and Mehrotra, 2017). Given that this paper is based on a sample of U.S. public firms, Covas and Den Haan (2011) and Begenau and Salomao (2018) are the most closely related studies to this paper. Covas and Den Haan (2011) document that large firms are substituting between different financing sources over the business cycle, but small firms' financing flows do not have this feature. In Begenau and Salomao (2018), an important takeaway is that the variation in large firms' financing flows are mainly driven by their investment demand. Both of these studies motivate the choice of using firm size as the main dimension of heterogeneity for disciplining the model in this paper. At the same time, the mechanism in Begenau and Salomao (2018) also supports the necessity of cross-sectional-level data of firms' financing flows in identifying financial shocks because the aggregate financing flows are dominated by large firms' financing choices, and their choices are mostly driven by the shocks to firms' investment profitability, rather than the shocks to firms' financing conditions.

Road Map The remainder of this paper is organized as follows. Section 2 presents stylized facts about small firms' investment and financing, which guide the model setup. Section 3 introduces the model setup. Section 4 presents the calibration and estimation of the model. Section 5 discusses the main findings, and Section 6 concludes.

2 Guiding Facts

The most important quantitative objective to discipline in this paper is the cross-sectional variation across firms in terms of how much they are financially constrained. However, this objective cannot be directly measured in the data. During the past decades, there has been an active debate about the best proxy for the degree of financial constraint in corporate finance, but there is no consensus on choice, yet. This paper uses the most widely used proxy, firms' size, to help discipline the cross-sectional variation in firms' degree of being financially constrained. In the following section, I document the observed heterogeneity across U.S. public firms in terms of their investment and financing flows, as well as the reliance of their investment on different sources of external financing.

2.1 Data Sample

The primary data source is the 2016 Compustat North America annual dataset. Firms from financial sectors (SIC 6000-6999), regulated utility sectors (SIC 4900-4999), and quasi-governmental sectors (SIC 9000-9999) are removed from the sample. To avoid the impact from the change of accounting rules in 1988 and to ensure that the tax environment faced by firms is stable, the sample period starts from 1989. All the nominal values are converted to real values by the PPI with 2010 as the base year. Besides the standard data cleaning procedure (see Appendix A for more details), I also discard the observations with mergers and acquisitions (M&A) larger than 5% of their book value assets since M&A can significantly change the capital structure of firms.

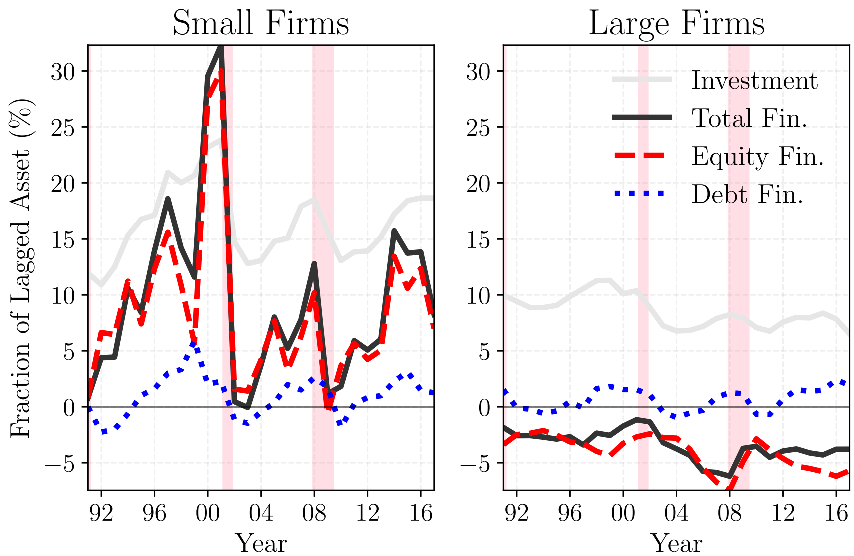
2.2 Investment and Financing Flows

Measurement The investment flow is measured as the sum of capital expenditure and research and development (R&D) expenditure. The debt financing flow is measured as the sum of net long-term debt issuance and the net change of current debt. The equity financing flow is measured as the difference between the issuance of common and preferred stocks and the sum of dividend and stock repurchase. The total external financing is measured as the sum of equity financing and debt financing.

In this paper, the issuance of common and preferred stocks is not directly measured by the item `sstk` reported in Compustat because a large part of the stock issuance reported in this item actually comes from employees exercising their stock options. These options are typically viewed as compensation, with years of delay between being granted and being exercised. To be consistent with the model where financing flows are determined by the managerial decisions in the current period, I eliminate this employee-driven equity issuance by merging the data with the equity offerings in the Security Data Company (SDC) database (see Appendix A for more details).

The size of each firm is measured by its lagged book value asset. In each year, the *small firms* are defined as the firms whose size is below the population median size in that year, and the *large*

Figure 1: Investment and Financing Flows of U.S. Public Firms



Note: In each period, the firm population is split into small and large firms by the population median size. The relative flow of investment or financing is measured as the total flow of each firm group normalized by the total size, which is measured by the firms' lagged asset of the corresponding firm group. The shaded bars indicate the NBER recession periods during the sample period.

firms are defined as the firms larger than the median size. Within each size group, I aggregate the firms' investment and financing flows and normalize them by the aggregate size of the corresponding size group. The time-series of these normalized investment and financing flows for each of the firm groups are depicted in Figure 1, and the mean of these time-series are summarized in Table 1.

Table 1: Time-series Average Investment and Financing Flows

	Small	Large
Investment	0.13	0.05
Total External Financing	0.07	-0.04
Equity Financing	0.06	-0.04
Debt Financing	0.005	0.003

Note: This table reports the mean of the flows shown in Figure 1 across the sample period 1989–2016.

Facts The investment rate of small firms stays both higher and more volatile than that of large firms. In terms of the external financing flows, small and large firms are different in terms of both the magnitude and the composition. As for the magnitude of the financing flow, small firms raise funding from the rest of the economy, but large firms pay out over the whole sample period. In terms of the composition of financing flow, most of the financing of small firms comes from the equity market, but the financing of large firms, if they have any, mostly comes from debt financing.

2.3 Reliance of Investment on Equity and Debt Financing

Measurement An immediate candidate for measuring this reliance is the ratio between the aggregate financing flows and the investment flow of a specific firm subgroup. But under this measure, some individual firm observations with large negative financing flows (e.g., a large fraction of large firms have large equity payout flows) could affect the measure and make the measure hard to be interpreted in the designed way. To deal with this problem, I construct a conceptual measure following the one used in Zetlin-Jones and Shourideh (2017), which is based on the truncated ratio between financing flow and investment on the individual level.

In period t, the fraction of the investment of an individual firm i financed from the funding source $\mathcal{F}$ is measured as

$$\texttt{Frac}_{i,t}^{\mathcal{F}} = \begin{cases} \frac{\mathcal{F}_{i,t}}{I_{i,t}} & \text{if } \mathcal{F}_{i,t} > 0 \text{ and } I_{i,t} > 0 \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

where $I_{i,t}$ and $\mathcal{F}_{i,t}$ denote the investment and the funding raised from source $\mathcal{F}$, respectively. For a firm group $\mathcal{I}_t$[1], the fraction of their investment financed from financing source $\mathcal{F}$ is calculated as the weighted average of the individual measure:

$$\texttt{Frac}_{\mathcal{I}_t,t}^{\mathcal{F}} = \sum_{i \in \mathcal{I}_t} \texttt{Frac}_{i,t}^{\mathcal{F}} \cdot \omega_i, \text{ where } \omega_i \equiv \frac{I_{i,t} \cdot \mathbb{1}^+(I_{i,t})}{\sum_{i \in \mathcal{I}_t} I_{i,t} \cdot \mathbb{1}^+(I_{i,t}).} \tag{2}$$

I first measure the fraction of investment financed from the equity and debt markets for small and large firms in each period by (2). Then I calculate the average of the time-series of these measures, which are summarized in Table 2.

Facts The results in Table 2 show how much external financing matters for the investment of small and large firms on average. For small firms, half of their investment is financed from the equity market, but only 7% is financed from the debt market. This implies the significant importance

[1] Under the classification criteria used in this paper, the sample of small and large firms changes over time, so the group set is indexed by the time. The sets of small and large firms in t are denoted as $\texttt{Small}_t$ and $\texttt{Large}_t$.

Table 2: Fraction of Investment from Different Financing Sources

	Small	Large
Equity	0.50	0.03
Debt	0.07	0.13

Note: This table reports the mean of $\texttt{Frac}^{\mathcal{F}}_{\text{Small}_t,t}$ and $\texttt{Frac}^{\mathcal{F}}_{\text{Large}_t,t}$ over the sample period 1989–2016.

of external financing, especially the equity financing, for small firms to finance their investment. For large firms, most of their investment is financed internally, i.e., from their retained earnings. Conditional on raising external funding, the debt market is relatively more important than the equity market for large firms to finance their investment.

3 Model

The model is designed to generate both the cross-sectional and cross-time variations in firms' investment and financing behaviors as documented in Section 2. There are three components in this model: a block of heterogeneous firms, a block of New Keynesian representative agents (see, e.g., Justiniano *et al.*, 2010), and eight aggregate shocks. The heterogeneous firm block endogenously generates the cross-sectional variation, the New Keynesian block endogenously determines the aggregate quantities and prices faced by the firms, and the aggregate shocks capture the exogenous variation in firms' financing conditions and investment profitability.

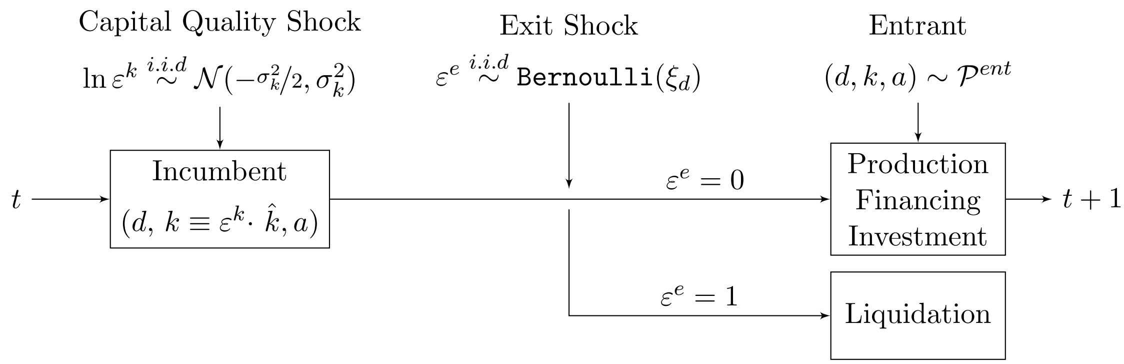
3.1 Heterogeneous Firm Block

3.1.1 Setup

In this economy, there is a continuum of heterogeneous firms indexed by $j \in [0, 1]$. They produce homogeneous intermediate goods and sell them in a competitive market.

Idiosyncratic State For incumbent firm i, there are three individual state variables revealed at the beginning of each period t: $(d_{j,t}, k_{j,t} \equiv \epsilon^k_{j,t} \cdot \hat{k}_{j,t}, a_{j,t})$. Here, the capital stock $\hat{k}_{j,t}$ and the nominal debt stock $d_{j,t}$ are inherited from period $t-1$. The capital quality shock $\epsilon^k_{j,t}$ and

Figure 2: Decision Timing for Heterogeneous Firms



idiosyncratic productivity $a_{j,t}$ are exogenous. The evolution of idiosyncratic productivity follows

$$\ln a_{j,t} = \rho_a \cdot \ln a_{j,t-1} + \sigma_a \cdot \varepsilon^a_{j,t},\ \varepsilon^a_{j,t} \overset{i.i.d}{\sim} \mathcal{N}(0,1). \tag{3}$$

The quality shock $\varepsilon^k_{j,t}$ transforms the predetermined capital stock $\hat{k}_{j,t}$ into the effective capital stock $k_{j,t}$[2]. $\varepsilon^k_{j,t}$ is i.i.d. and its distribution follows $\texttt{LogNormal}(-\frac{\sigma_k^2}{2}, \sigma_k^2)$.

Exit and Entry After the realization of their individual states, firm i will receive an i.i.d. exogenous exit shock $\varepsilon^e_{j,t} \sim \texttt{Bernoulli}(\xi_d)$. If $\varepsilon^e_{j,t} = 1$, firm i has to be liquidated and the shareholders exit the market with nominal liquidation value $\mathcal{LV}_{j,t} \equiv (1-\bar{\delta}) \cdot k_{j,t} \cdot Q_t - d_{j,t} \cdot R_{t-1}$, where R denotes the gross nominal interest rate and $\bar{\delta}$ is the adjustment cost when a firm fully installs its capital. If $\varepsilon^e_{j,t} = 0$, firm i can stay in operation. Firms in operation can produce, raise funding from financial markets, and invest in physical capital. Right after the exit of incumbents, a group of entrants enter the market and operate the same as the surviving incumbents do. The number of entrants is assumed to be equal to the number of exiting incumbents, so the firm population keeps constant over time, and the distribution of entrants is exogenously given as $\mathcal{P}^{ent}_t(d, k, a)$. After the exit and entry, the distribution of operating firms is denoted as $\mathcal{P}_t(d, k, a)$.

[2]The capital quality shock is just a technical assumption made for numerical purposes. Without this shock, there would be many mass points in the distribution, and the accuracy of numerical approximation will be affected. In the quantification part, this shock will be calibrated to be small, so it will not really affect the conclusion in this paper.

Production and Internal Financing Firm i in operation can produce intermediate goods $y_{j,t}$ with a decreasing-return-to-scale technology:

$$y_{j,t} = \bar{Z} \cdot \exp\left(\eta_{z,t}\right) \cdot a_{j,t} \cdot \left(k_{j,t}^{\alpha} \cdot l_{j,t}^{1-\alpha}\right)^{\theta}, \ \alpha \in (0,1), \ \theta \in (0,1), \tag{4}$$

where $\eta_{z,t}$ denotes the exogenous variation in the aggregate productivity and $l_{j,t}$ is the labor input of firm j. Firms' profit will be taxed at the constant tax rate of τ^c, and the total source of internal financing available for firm i is

$$u_{j,t} = (1-\tau^c) \cdot \underbrace{\left[y_{j,t} \cdot P_t - l_{j,t} \cdot W_t - d_{j,t} \cdot (R_{t-1} - 1)\right]}_{\text{Pre-tax Profit}} + \underbrace{\tau^c \cdot \delta \cdot k_{j,t} \cdot Q_t}_{\text{Tax Rebate from Depreciation}}, \tag{5}$$

where P, W, R, and Q are, respectively, the nominal intermediate good price, wage, gross interest rate, and capital good price.

Investment and External Financing Besides the internal financing source, firm i can also finance its investment $I_{j,t} \cdot Q_t$ from the debt and equity markets. The budget constraint for firm i is

$$i_{j,t} \cdot Q_t = \underbrace{u_{j,t}}_{\text{internal financing}} + \underbrace{e_{j,t}}_{\text{equity financing}} + \underbrace{d_{j,t+1} - d_{j,t}}_{\text{debt financing}}. \tag{6}$$

The investment will be used to build up the capital stock, and the accumulation of capital follows

$$\hat{k}_{j,t+1} = (1-\delta) \cdot k_{j,t} + i_{j,t} - \Phi(\hat{k}_{j,t+1}, k_{j,t}), \tag{7}$$

where $\Phi(\hat{k}_{j,t+1}, k_{j,t})$ is the capital adjustment cost, which captures the extra managerial effort required for adjusting the scale of production. The capital adjustment cost is constructed as

$$\Phi\left(\hat{k}', k\right) = \frac{\phi^k}{2} \cdot \left(\frac{\hat{k}'}{k} - 1\right)^2 \cdot k. \tag{8}$$

The definition of equity financing here is consistent with the measurement in Section 2. When $E_{j,t} \leq 0$, it is counted as a dividend payment; when $E_{j,t} > 0$, it is counted as equity issuance. The equity issuance is costly, and the costs mainly include both the explicit floatation cost (Altinkilic and Hansen, 2000) and the implicit adverse selection premium (Myers and Majluf, 1984) as well as the market misvaluation (Warusawitharana and Whited, 2016). Given the quantitative purpose of this paper, I abstract from the micro-foundation of the equity issuance cost and model the equity issuance cost in the reduced-form way: there is a proportional cost $\phi_t^e \cdot E_{j,t}$ associated with the issuance, and this cost is paid by the existing shareholders[3]. The equity issuance cost is time-varying, which mainly reflects the potential time variation in the adverse selection premium (Choe *et al.*, 1993) and the market misvaluation (Baker and Wurgler, 2007).

The debt issuance of firm i is subject to a collateral constraint:

$$d_{j,t+1} \leq \phi_t^d \cdot k_{j,t} \cdot Q_t. \tag{9}$$

The construction of the collateral constraint follows Kiyotaki and Moore (1997), and a similar setup is widely used in the literature (e.g., Jermann and Quadrini, 2012; Khan and Thomas, 2013). The tightness of collateral constraint ϕ_t^d reflects the supply condition in the debt financing market. The time variation in ϕ_t^d reflects the time-varying supply condition in the debt financing market (Becker and Ivashina, 2014).

The financial frictions are parameterized as

$$\phi_t^x = \bar{\phi}^x \cdot \exp\left(\eta_{x,t}/\bar{\phi}^x\right), \ \forall x \in \{e, d\}. \tag{10}$$

Here, $\bar{\phi}^x$ denotes the level of financial frictions in the steady state, and $\eta_{x,t}$ denotes the exogenous time variation in the financial frictions.

3.1.2 Decisions

In each period, the firms in operation have to make their decisions about labor hiring, investment, debt financing, and equity financing to maximize the total net present value of their future dividend

[3]This construction of equity issuance cost follows Gomes (2001), and similar setups can be found in many other works in structural corporate finance (e.g., Hennessy and Whited, 2007; Warusawitharana and Whited, 2016). This type of equity financing cost is typically micro-founded by the information asymmetry. Due to the information advantage of internal managers, equity is typically issued to external investors with a price discount, and this price discount is isomorphic to a proportional issuance cost.

payment. The recursive representation of the decision problem is

$$V_t(d,k,a) = \max_{l,i,d',e} -e \cdot [1 + \phi_t^e \cdot \mathbb{1}_{e>0}] \\ + \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \frac{\bar{P}_t}{\bar{P}_{t+1}} \cdot \left[\xi_d \cdot \mathcal{LV}_{t+1} + (1-\xi_d) \cdot V_{t+1} \left(d', e^{\varepsilon^{k'}} \cdot \hat{k}', a' \right) \right] \right] \tag{11}$$

s.t. : production technology (4), internal financing source (5),

budget constraint (6), capital accumulation (7),

collateral constraint (9),

where $V_t(\cdot)$ is the real cum-dividend value of the firm in period t, and Λ denotes the real discounting factor (SDF) that will be determined by households' preference. The subscript t indicates the dependence of firms' value on the aggregate economic conditions, which will be endogenously determined by the New Keynesian block. The firms' value comes from two parts: the flow from the dividend payment or equity issuance, and the discounted future value. The future value is the weighted average of the liquidation value and the continuation value from continuing operation in the next period.

3.2 New Keynesian Block

3.2.1 Setup

Retailers There is a continuum of retailers indexed by $\iota \in [0,1]$. Retailers produce differentiated retail goods $\hat{y}_{\iota,t}$ from the homogeneous intermediate goods $y_{\iota,t}$ with the technology specified as

$$\hat{y}_{\iota,t} = y_{\iota,t}. \tag{12}$$

Each retailer ι has monopolistic power, and following Calvo (1983), there is a probability of $1-\xi_p$ for retailer ι to get the opportunity to reset its nominal prices in each period.

Final Good Producers There is a representative final good producer who produces final good Y_t by packing retail goods $\{\hat{y}_{\iota,t}\}_{\iota\in[0,1]}$ through a Dixit-Stiglitz aggregator:

$$Y_t = \left(\int \hat{y}_{\iota,t}^{\frac{1}{\gamma_p+\eta_{p,t}}} dh\right)^{\gamma_p+\eta_{p,t}}, \tag{13}$$

where γ_p is the price markup in the steady state and $\eta_{p,t}$ is the exogenous variation of the price mark-up. The final good market is perfectly competitive, and the nominal price of the final goods is denoted as $\bar{P}_t$.

Households There is a representative household who consumes final good C_t, supplies labor N_t, owns all the firms, and saves in one-period nominal bonds $\frac{B_{t+1}}{R_t}$. The utility function of the household is specified as

$$\sum_{t=0}^{\infty} \beta^t \cdot \exp\left(\eta_{u,t}\right) \cdot \left[\log\left(C_t - h \cdot C_{t-1}\right) - \Psi \cdot N_t\right], \tag{14}$$

where β is the discounting factor, $\eta_{u,t}$ is the exogenous variation in the households' inter-temporal substitution decision, and h is the parameter controlling the external consumption habit formation. The budget constraint for the household is

$$C_t \cdot \bar{P}_t + \frac{B_{t+1}}{R_t} = N_t \cdot W_t^N + B_t + T_t, \tag{15}$$

where W_t^N is the nominal wage to the household's labor, and T_t is the lump-sum transfer such that the bond market clears.

Labor Union There is a continuum of labor unions, indexed by $s \in [0,1]$, which purchase the homogeneous labor supply $n_{s,t}$ from the representative household and transform it as heterogeneous intermediate labor service $\hat{n}_{s,t}$ with the technology as follows:

$$\hat{n}_{s,t} = n_{s,t}. \tag{16}$$

Each union s has monopolistic power, and there is a probability of $1 - \xi_w$ for union s to get the opportunity to reset the nominal wage of its specialized labor service.

Labor Packer There is a representative labor packer that packages the heterogeneous types of labor supply $\{\hat{n}_{s,t}\}_{s\in[0,1]}$ as the final labor service L_t with the technology

$$L_t = \left(\int \hat{n}_{s,t}^{\frac{1}{\gamma_w + \eta_{w,t}}} \right)^{\gamma_w + \eta_{w,t}},$$

where γ_w is the wage markup in the steady state and $\eta_{w,t}$ is the exogenous variation in the wage markup. The market for the final labor service is perfectly competitive, and the nominal wage of the final labor service is denoted as W_t.

Investment Good Producers There is a representative investment good producer that produces investment good $\hat{I}_t$ from final good Y_t^I with the technology

$$\hat{I}_t = \exp\left(-\eta_{q,t}\right) \cdot Y_t^I, \tag{17}$$

where $\eta_{q,t}$ is the exogenous variation in the efficiency of transforming final goods into investment goods. The investment good market is competitive, and the nominal investment good price is Q_t^I.

Capital Good Producers There is a representative capital good producer that produces capital good I_t from investment good $\hat{I}_t$ with the technology specified as

$$I_t = \left[1 - S\left(\frac{\hat{I}_t}{\hat{I}_{t-1}}\right)\right] \cdot \hat{I}_t, \tag{18}$$

where $S(\cdot)$ is the function characterizing the adjustment cost, and the adjustment cost function is assumed to satisfy $S(1) = S'(1) = 0$ and $S''(1) > 0$. The investment good market is perfectly competitive, and the nominal price of the capital good is denoted as Q_t.

Monetary Authority The monetary policy is assumed to follow

$$\ln R_t - \ln\frac{1}{\beta} = \lambda_R \cdot \left[\ln R_{t-1} - \ln\frac{1}{\beta}\right] + \lambda_\pi \cdot \ln \pi_t + \eta_{m,t}, \tag{19}$$

where $\pi_t \equiv \frac{\bar{P}_t}{\bar{P}_{t-1}}$ denotes the gross inflation rate and $\eta_{m,t}$ is the exogenous variation in the nominal interest rate.

3.2.2 Decisions

Given that the decision problems in this block are close to the ones specified in Justiniano *et al.* (2010), I skip the detailed derivation and directly present the derived decisions in Appendix B. The decisions in this block play two roles in this paper. First, they model the general equilibrium feedbacks, which matters for the transmission of the financial shocks. Second, they provide a structure to build in both the exogenous and endogenous variation in the aggregate quantities and prices, so we can quantitatively match the variation in firms' investment profitability, which is important for identifying the financial shocks.

3.3 Aggregate Shocks

There are eight exogenous variables in this model. Their evolution follows the AR(1) process:

$$\eta_{x,t} = \rho_x \cdot \eta_{x,t-1} + \sigma_x \cdot \varepsilon_{x,t}, \ \forall x \in \{e, d, z, p, w, q, m, u\}, \tag{20}$$

where the independent exogenous variations $\varepsilon_{x,t} \overset{i.i.d}{\sim} \mathcal{N}(0,1)$ are the aggregate shocks to this economy. Within these eight aggregate shocks, two of them ($\varepsilon_{e,t}$ and $\varepsilon_{d,t}$) come from financial markets and capture the exogenous variation in firms' financing conditions. The remaining six aggregate shocks directly or indirectly capture the exogenous variation in the firms' investment profitability by affecting the production efficiency, prices, or preferences in this economy.

3.4 Equilibrium

An equilibrium of this model is a collection of

1. value function $V_t(d,k,a)$ and the associated policy functions for hiring, production, investment, debt issuance, equity financing, and capital holding: respectively $l_t(d,k,a)$, $y_t(d,k,a)$, $i_t(d,k,a)$, $d'_t(d,k,a)$, $e_t(d,k,a)$, and $\hat{k}'_t(d,k,a)$;

2. distribution $\mathcal{P}_t(k,D,a)$; and

3. aggregate quantities and prices Y_t, C_t, Y^I_t, I_t, $\hat{I}_t$, N_t, L_t, p_t, w_t, w^N_t, q_t, q^I_t, R_t, π_t, $\Lambda_{t,t+1}$

such that, given the exogenous process of $\eta_{x,t}$, $\forall x \in \{e, d, z, p, w, q, m, u\}$,

1. value function $V_t(d,k,a)$ solves the firm's problem in (11) with the associated policy functions;
2. distribution $\mathcal{P}_t(d,k,a)$ evolves as

$$\begin{aligned}\mathcal{P}_t(d,k,a) =&(1-\xi_d)\cdot\int \mathbf{1}\left\{\exp\left(\varepsilon^k\right)\cdot\hat{k}'_{t-1}(d_-,k_-,a_-)=k\right\}\\ &\times\mathbf{1}\left\{d'_{t-1}(d_-,k_-,a_-)=d\right\}\\ &\times\mathbf{1}\left\{\rho_a\cdot\ln a_-+\sigma_a\cdot\varepsilon^a=\ln a\right\}\\ &\times\phi\left(\frac{\varepsilon^k+\frac{\sigma_k^2}{2}}{\sigma_k}\right)\phi\left(\varepsilon^a\right)d\varepsilon^k d\varepsilon^a d\mathcal{P}_{t-1}(d_-,k_-,a_-)\\ &+\xi_d\cdot\mathcal{P}_t^{ent}(d,k,a),\end{aligned} \tag{21}$$

   where $\phi(\cdot)$ is the density of the standard normal distribution;

3. the aggregate quantities and prices satisfy the monetary policy specified in (19) and the conditions specified in the New Keynesian block; and
4. the markets for final goods, intermediate goods, capital goods, and labor all clear

$$Y_t = C_t + Y_t^I \tag{22}$$

$$y_t = \int y_t(d,k,a)d\mathcal{P}_t(d,k,a) \tag{23}$$

$$I_t = \int i_t(d,k,a)d\mathcal{P}_t(d,k,a) \tag{24}$$

$$L_t = \int l_t(d,k,a)d\mathcal{P}_t(d,k,a). \tag{25}$$

4 Quantification

The model can be cast into the standard form of the rational expectations model:

$$\mathbb{E}_t\left[\mathcal{F}\left(\mathbb{Y}_{t+1},\mathbb{X}_{t+1},\mathbb{Y}_t,\mathbb{X}_t|\Theta_{HF},\Theta_{NK},\Theta_{SH}\right)\right]=0, \tag{26}$$

where $\mathbb{X}$ is the collection of the state variables, which include the exogenous variables $\eta_{x,t},\ \forall x\in\{e,d,z,p,w,q,m,u\}$ and the endogenous firm distribution $\mathcal{P}_t$; and $\mathbb{Y}$ is the collection of the control variables, which include the endogenous aggregate quantities, prices, firms' value functions, and policy functions. Corresponding to the three components of this model, there are three groups of parameters in this model: Θ_{HF} is the collection of parameters governing the heterogeneous firm block, Θ_{NK} is the collection of parameters in the New Keynesian block, and Θ_{SH} collects the persistence and on-impact response to aggregate shocks of the exogenous processes, i.e., ρ_x and σ_x for $x \in \{e, d, z, p, w, q, m, u\}$.

The core aim of this paper is to decompose the observed time variation in firms' investment into the contribution of different aggregate shocks. To make reasonable decomposition results, I need to estimate the model to match the observed time variation first. To estimate the model, I have to solve the model fast enough. But the firm-level heterogeneity significantly increases the dimension of this model, and it is technically challenging to estimate it.

In this section, I first sketch out the algorithm used to solve the model, which is crucial to make the estimation of the model feasible. Then I present how to pin down the parameter values by calibration and estimation. Within these three groups of parameters, all of the parameters in Θ_{NK} and a part of the parameters in Θ_{HF} are fixed at the values from literature or directly from data. The rest of the parameters in Θ_{HF} are calibrated to match the average investment and financing flows as documented in the guiding facts. I discuss the firms' policy functions and life-cycle dynamics in the steady state to provide intuitions behind the calibration. The parameters in Θ_{SH} are estimated with the Bayesian method to match the observed time variation in both the quantities and prices on the macro-level and the firms' financing choices on the disaggregate level. In the estimation part, I discuss the intuition behind the identification of these different aggregate shocks.

4.1 Numerical Solution

I use a hybrid method (Reiter, 2009) to solve this model sufficiently fast such that the estimation becomes feasible. This method combines the projection method applied on the micro-level and the perturbation method applied on the aggregate level[4]. The implementation includes two steps:

1. I solve the steady state with the aggregate shocks shut off. This steady state includes the firms'

[4]The backbone method was initially proposed by Reiter (2009). In this paper, to further reduce the dimension of the system, the distribution is approximated as in Algan *et al.* (2008) and Winberry (2018). Under the current numerical approximation scheme, the dimension of the system is close to 3000. See Appendix C for more details about the computation.

value functions, policy functions, and distributions when the aggregate economic quantities and prices are fixed at the steady-state levels. The firms' distribution and the curvature in their policy functions are generated by the combination of the mean-reverting idiosyncratic productivity process, decreasing-return-to-scale technology, and frictions on their financing and investment.

2. I solve the first-order perturbation solution around the steady state. The solved dynamics characterize the response of firms' policy function, value function, and distribution, as well as the aggregate quantities and prices to various aggregate shocks.

4.2 Fixed Parameters

Parameters in Θ_{HF} The upper panel of Table 3 is a collection of the parameters that control firms' operation flow and life-cycle dynamics. The corporate tax rate τ^c is set at 35%, which is the median tax rate as reported in Graham (2000). The share of capital α is set at the standard value 0.30. The return to scale, persistence, and conditional standard deviation of the idiosyncratic productivity process are calibrated to match the values used in Begenau and Salomao (2018). The exogenous exit probability is set at 3% to match the average fraction of entrants in Compustat. The capital quality shock in this model only serves a numerical purpose: without this shock, there will be a lot of mass points in the firms' distribution, which will significantly decrease the accuracy of the numerical approximation. To minimize its impact on the quantitative results, I set its standard deviation at 0.1%.

The middle panel of Table 3 is a collection of the parameters that control the distribution of entrants. The distribution of entrants is assumed to have independent normal marginal distributions over their leverage ratio, log of size, and log of idiosyncratic productivity:

$$\mathcal{P}_t^{ent}(d,k,a) = \phi\left(\frac{D/k \cdot Q_t - \mu_{lev}^{ent}}{\sigma_{lev}^{ent}}\right) \cdot \phi\left(\frac{\ln k - \mu_k^{ent}}{\sigma_k^{ent}}\right) \cdot \phi\left(\frac{\ln a - \mu_a^{ent}}{\sigma_a^{ent}}\right), \tag{27}$$

where $\phi(\cdot)$ is the probability density function (p.d.f.) of standard normal distribution and $(\mu_v^{ent}, \sigma_v^{ent})$, $\forall v \in \{lev, k, a\}$ are the mean and standard deviation of the corresponding marginal distributions. μ_k^{ent} is set to match the gap between the 90% quantile and the 10% quantile of the firm size in Compustat.

μ_a^{ent} is set at $-0.5 \times \frac{\sigma_a}{\sqrt{1-\rho_a^2}}$ to be consistent with the fact documented by Foster *et al.* (2016) that young firms have lower measured TFP during their early life periods[5]. Since most of the entrant firms have little debt in the data, μ_{lev}^{ent} is set at 0. For a similar numerical reason, the standard deviations of these three marginal distributions are set at 1%, which is non-zero but small enough to have a negligible impact on the quantitative results.

Parameters in Θ_{NK} The lower panel of Table 3 collects the parameters that control the dynamics in the New Keynesian block[6]. The discount factor is set at 0.98 to match the average real interest rate at 2%. The elasticity of the capital good price, the average price and wage markups, the probability of price and wage adjustment, as well as the coefficients of the Taylor rule are set to be consistent with the values estimated in Justiniano *et al.* (2011).

4.3 Calibration and the Steady State

Four parameters in Θ_{HF} are calibrated to match the firms' choices in the steady state with the corresponding moments in the data. To understand the intuition behind the calibration of these parameters, I will first present and interpret the firms' policies, distribution, and life-cycle dynamics in the steady state.

Investment and Financing Policy Functions To illustrate firms' policy functions, I choose $\exp\left(\pm 1.5 \times \sigma_a / \sqrt{1-\rho_a^2}\right)$ as the representative high and low idiosyncratic productivity levels, choose the median size of physical capital and median leverage of the population distribution as the representative levels for physical capital and leverage, and depict how firms' investment and financing vary along a specific dimension of their idiosyncratic states when holding the other dimensions at the representative level in the four panels in Figure 3. The graphs in each row share the same level of representative productivity. In each row, the leverage ratio is fixed at the representative level in the left graph and the size is fixed at the representative level in the right graph. To inform the relevance of different states, I also plot the marginal distribution of the incumbents' size and

[5] Given the small fraction of entrants in the whole population, the quantitative results of interest are robust to different choices of this parameter.

[6] In each round of calibration, the labor dis-utility parameter Ψ is always calibrated to generate a steady-state employment rate of 60%.

Table 3: Fixed Parameters

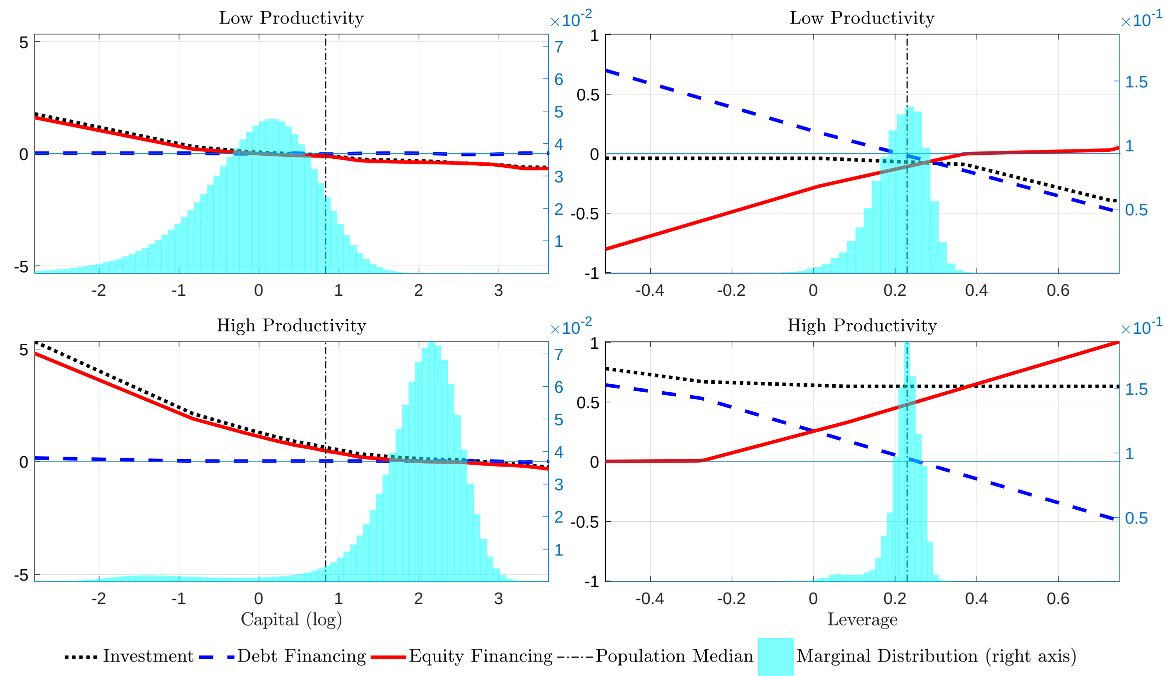
Parameters		Value	Source
Panel 1: Parameters in Θ_{HF} Controlling Firms' Technology and Life-cycle Dynamics			
τ^c	Corporate tax rate	0.35	Graham (2000)
α	Capital share	0.30	
θ	Return to scale	0.88	Begenau and Salomao (2018)
ρ_a	Persistence of idio. TFP	0.90	Begenau and Salomao (2018)
σ_a	Std. of idio. TFP shock	0.06	Begenau and Salomao (2018)
ξ_d	Prob. of firm exit	3.0%	% of new firms in Compustat (CS)
σ_k	Std. of capital quality shock	0.1%	Numerical purpose, small
Panel 2: Parameters in Θ_{HF} Controlling the Distribution of Entrants			
μ_k^{ent}	Mean of size	-1.91	Avg. $\mathtt{Q}_{0.9} - \mathtt{Q}_{0.1}$ of size dist. in CS
μ_a^{ent}	Mean of idio. TFP	-0.07	Foster *et al.* (2016)
μ_{lev}^{ent}	Mean of leverage	0	Leverage of entrants in CS
σ_k^{ent}	Std. of size	0.01	Numerical purpose, small
σ_a^{ent}	Std. of idio. TFP	0.01	Numerical purpose, small
σ_{lev}^{ent}	Std. of leverage	0.01	Numerical purpose, small
Panel 3: Parameters in Θ_{NK}			
β	Discount factor	0.98	Average real interest rate at 2%
$S''(1)$	Elasticity of capital good price	0.65	Justiniano *et al.* (2010)
γ_p	Avg. price markup	1.1	Justiniano *et al.* (2010)
γ_w	Avg. wage markup	1.3	Justiniano *et al.* (2010)
$1 - \xi_p$	Prob. of price adjustment	0.59	Justiniano *et al.* (2010)
$1 - \xi_w$	Prob. of wage adjustment	0.59	Justiniano *et al.* (2010)
λ_R	Taylor rule coefficient	0.85	Justiniano *et al.* (2010)
λ_π	Taylor rule coefficient	1.5	Justiniano *et al.* (2010)

leverage ratio[7] conditional on the corresponding productivity levels.

Along the size dimension, there is a significant variation in the investment and equity financing policy, but little variation in the debt financing policy. Given the decreasing-return-to-scale technology and mean-reverting productivity process, at a given productivity, firms have the finite optimal target size. For small firms, they are further away from the optimal size and have higher demand of investment; for large firms, they are already around the optimal size and their investment demand is small or even negative. This is the main reason why the investment policy decreases with the size. Given the tax benefit of debt in this model, most of the firms bind their collateral constraint. So the firms with a median leverage ratio have little capacity to issue new debt because the median

[7] Given that only 3% of firms are entrants and their size is much smaller than the incumbent, I do not show their distribution here.

Figure 3: Policy Function and Conditional Distribution



Note: The distributions are of incumbents' size and leverage after the realization of a capital quality shock and exit shock, but before production, and conditional on each level of idiosyncratic productivity. At each given productivity, the leverage ratio underlying the policy functions along the dimension of firm size is fixed at the median leverage of the firm population; and the size underlying the policy function along the dimension of the leverage ratio is fixed at the median size of the firm population. The investment and financing presented here are corresponding flows normalized by the firm size.

leverage ratio almost reaches the limit. Therefore, the debt financing of the firms in the left two graphs is very close to 0, and their equity financing closely follows the investment policy. The most important features in the left two panels in Figure 3 are that small firms have higher investment rates and their investment is mainly financed by equity financing, which is consistent with the facts documented in Section 2.

Along the dimension of the leverage ratio, there is not much variation in the investment policy, but a significant variation in financing policies. As in the previous analysis, the investment demand is mostly fixed at the given level of productivity and size. With the increase in the leverage ratio, the residual capacity of debt financing decreases, so the debt financing is lower and the equity financing increases correspondingly. Due to the existence of equity issuance costs, there is an inactive area where the equity financing is zero and investment decreases with the leverage ratio.

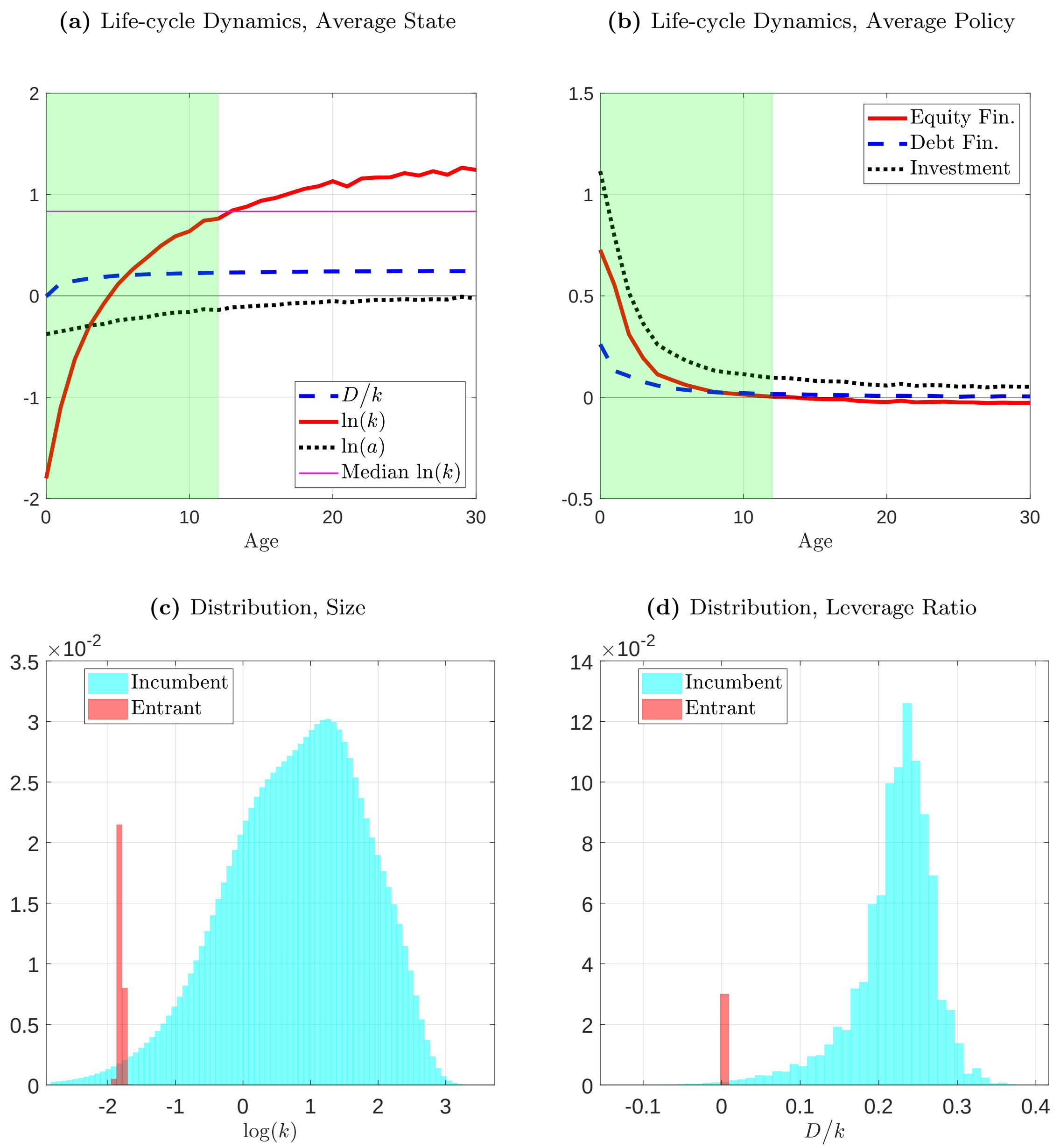
Along the productivity dimension, there are two points worth highlighting. First, the firms with the higher productivity level have higher investment and, correspondingly, higher equity financing flows. The key assumption that leads to this difference is that the idiosyncratic productivity is persistent. The firms with higher current productivity also hold a better perspective about their future investment profitability, so they want to invest more. Given that their debt financing capacity is limited, their equity financing increases with their investment demand. Second, the firms with a higher productivity level are larger on average than the firms with lower productivity. This is also due to the persistence of the idiosyncratic productivity process. The reasoning can be illustrated from two different perspectives. Due to the persistence, firms with higher productivity will have a relatively higher target size, so their marginal distribution of size is centered at a higher level. Also, firms with a higher current productivity level tend to have a history with more positive productivity shocks; therefore, they are more likely to accumulate more financing resources and have invested more, both of which can lead to larger capital stock.

Firms' Life-cycle Dynamics The age profile of firms' average states and policies are summarized in Figures 4a and 4b. To compare the entrants and incumbents, the marginal distributions of their size and leverage ratio are summarized in 4c and 4d.

As shown in Figure 4a, firms start their life with a smaller size, lower productivity, and little debt, which is consistent with the setup of entrants. With ongoing time, they gradually grow up. On average, firms become large at the age of 12 years, which is close to the cutoff age to classify the firms as mature firms[8]. The age profile of firm size highlights the large overlapping area between "being small" and "being young" within this setup. As the firms grow up, their leverage ratio and productivity level also gradually increase. The increase of the leverage ratio is mainly because entrants hold little debt. Due to the tax benefit, firms will try to issue as much debt as they can, but the total debt issuance is limited by their current size. With the firms becoming closer and closer to their target size, their size becomes stable, and their leverage ratio gets closer and closer to the ratio between their debt issuance and their current size, i.e., the tightness of collateral constraint. The increase of the productivity level is a direct result of the mean reverting feature of the idiosyncratic productivity process. The productivity gradually converges to its unconditional

[8] In Haltiwanger *et al.* (2013), 10 is set as the cutoff age between young firms and mature firms.

Figure 4: Firms' Life-cycle Dynamics and Distribution in Steady State



(a) Life-cycle Dynamics, Average State

(b) Life-cycle Dynamics, Average Policy

(c) Distribution, Size

(d) Distribution, Leverage Ratio

Note: The life-cycle dynamics are based on a simulated panel with 10,000 firms over 100 years with entry and exit. 4a plots the average state D/k, $\ln(k)$, and $\ln(a)$ within each age group. 4b plots the weighted average financing and investment flows within each age group. The firm size is used as the weight for calculating the average policy, so it is consistent with the measurement underlying the empirical facts documented in Section 2. The shaded area is the life stage when an average firm is classified as a "small firm". The distributions in 4c and 4d are the ergodic distribution in the steady state with the aggregate shocks shut off. The leverage ratio is measured at the time after the realization of capital quality shocks but before making operating decisions.

mean.

The age profile of the policies in Figure 4b is consistent with the age profile of the states in Figure 4a. The investment rate decreases because the firms get larger when they grow up and they are closer to their target size. After the firms become "large firms", their size becomes stable and their investment rate is very close to the depreciation rate. The debt financing decreases because the firms' investment demand is decreasing and the increment of their debt capacity is decreasing with the growth in their size. The equity financing decreases mostly because the investment demand is decreasing. When the firms become large, they have little incentive to invest, and they start paying dividends to investors.

Calibration There are 4 parameters to be calibrated: the steady-state level of equity issuance cost $\bar{\phi}^e$ and collateral constraint tightness $\bar{\phi}^d$, the depreciation rate δ, and the capital adjustment cost ϕ^k. The value of the calibrated parameters and their corresponding moments are summarized in Table 4.

Table 4: Calibrated Parameters and Target Moments

Parameter	Value	Mean	Sample	Data	Model
δ	0.0374	Investment Rate	Large	0.06	0.05
ϕ^k	0.0682	Investment Rate	Small	0.13	0.14
$\bar{\phi}^d$	0.2566	Leverage Ratio	Large	0.27	0.24
$\bar{\phi}^e$	0.0800	Equity Fin. Rate	Small	0.05	0.05

Note: In each period, the firm population is split into small and large firms by the population median size. The investment rate or financing rate is measured as the total flow of each firm group normalized by the total size of the corresponding firm group. The leverage ratio is measured as the total liability after debt issuance of each firm group normalized by the total size of the corresponding firm group. This table reports the average of these measured rates over the sample period. The moments generated from the model are constructed in the same way.

The steady-state level of financial frictions are calibrated to match the firms' average financing choices over the business cycle. As in the analysis of firms' policies, it is mainly the small firms who are issuing equity, so $\bar{\phi}^e$ is calibrated to match the cross-time average of the small firms' equity financing flow. As in the analysis of firms' life-cycle dynamics, the large firms grow little in size, and their leverage ratio is close to the collateral constraint, so $\bar{\phi}^d$ is calibrated to match the cross-time average of the large firms' leverage ratio.

For a similar reason, large firms' investment is mainly to replenish their depreciated capital, so δ is calibrated to match the cross-time average of large firms' investment rate. For small firms, there is significant growth in their size, and their investment choice is subjected much more to the capital adjustment cost, so ϕ^k is calibrated to match the cross-time average of the small firms' investment.

Another eight non-targeted moments are chosen to check the calibration. The comparison between the moments in the data and the model is summarized in Table 5. Overall, the moments about firms' financing choices are well matched. In terms of the fraction of investment from equity and debt financing, the moments generated from the model match the differential results as documented: small firms' investment mostly relies on equity financing and large firms' investment depends relatively more on debt financing. But the model overshoots the fraction of equity financing for large firms and the fraction of debt financing for both small and large firms. Therefore, the model might overstate the importance of debt financing for both firm groups and the importance of equity financing for the large firms.

Table 5: Non-Target Moments in Steady State

Statistics	Sample	Data	Model
Leverage Ratio	Small	0.18	0.21
Equity Financing Rate	Large	-0.04	-0.03
Debt Financing Rate	Small	0.005	0.023
Debt Financing Rate	Large	0.003	0.003
Fraction of Investment from Equity	Small	0.50	0.47
Fraction of Investment from Equity	Large	0.03	0.16
Fraction of Investment from Debt	Small	0.07	0.20
Fraction of Investment from Debt	Large	0.13	0.20

Note: The leverage ratio and financing flows are measured in the same way as in Table 4. The fractions of investment from equity and debt are measured in the same way as illustrated in Section 2.3.

4.4 Estimation and the Cyclical Dynamics

The parameters in Θ_{HF} determine the sensitivity of different firms' choices to aggregate prices, and those in Θ_{NK} determine the sensitivity of aggregate prices in the general equilibrium, so by pinning down the value of parameters in Θ_{HF} and Θ_{NK}, I fix the sensitivity of firms' choices to the aggregate shocks within the general equilibrium. With these parameters calibrated, Θ_{SH} can be estimated to match the observed variations in firms' investment and financing choices, as well as the macroeconomic quantities and prices.

In terms of identifying these shocks, the intuition can be elaborated as follows. The observed financing flows are jointly determined by firms' financing demand and their financing capacity. The firms' financing demands are driven by these six non-financial shocks, which are identified by the non-financial aggregate time-series. With the financing demand controlled by these non-financial aggregate time-series, the variation in firms' financing capacity is identified by the residual part of observed variation in firms' financing flows that cannot be explained by the financing demand variation. In the following parts, I illustrate in detail how to identify these non-financial shocks from aggregate time-series and how to separately identify different financial shocks using the time variation in the financing flows of different subgroups of firms.

Observable Time-series for Estimation There are eight aggregate shocks in this model, and eight observable time-series are chosen as the input for the Bayesian estimation:

$$\left\{ \frac{\sum_{j\in \texttt{Small}_t} e_j}{\sum_{j\in \texttt{Small}_t} k_j \cdot Q_t}, \frac{\sum_{j\in[0,1]} D_{j,t+1}}{\sum_{j\in[0,1]} k_{j,t+1} \cdot Q_t}, \Delta \ln Y_t, \ln \pi_t, \Delta \ln w_t, \Delta \ln q_t^I, \ln R_t, \Delta \ln C_t \right\}. \tag{28}$$

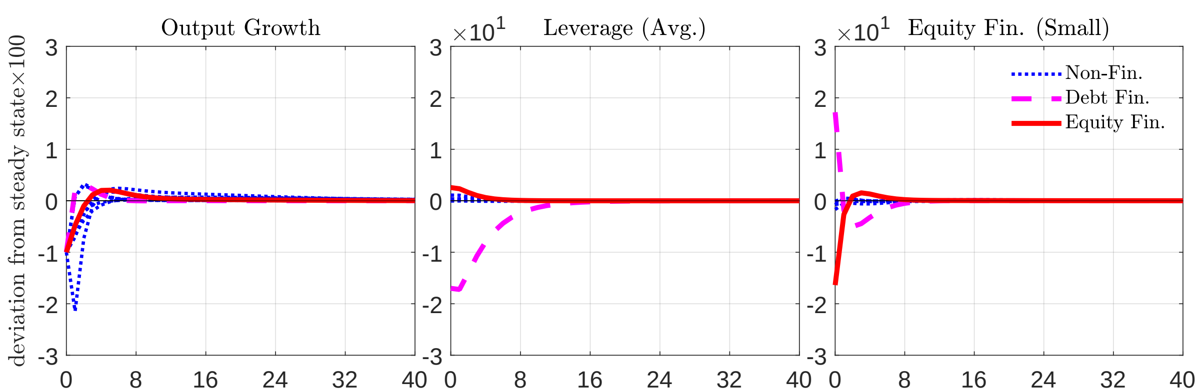
Here, $\frac{\sum_{j\in \texttt{Small}_t} e_j}{\sum_{j\in \texttt{Small}_t} k_{j,t} \cdot Q_t}$ is the equity financing of small firms measured in the same way as in Section 2. $\frac{\sum_{j\in[0,1]} D_{j,t+1}}{\sum_{j\in[0,1]} k_{j,t+1} \cdot Q_t}$ is the weighted average leverage ratio at the end of each period. Output Y_t is measured by the real gross value added by the non-financial corporate sector in Flow of Funds. Consumption C_t is the consumption of non-durable goods and services in real terms. The nominal price $\bar{P}_t$ is measured as the deflator for the non-durable goods and services[9] and $\pi_t \equiv \bar{P}_t/\bar{P}_{t-1}$. The nominal wage W_t is measured as the hourly earnings in the U.S. manufacturing sector and the real wage w_t is measured as $W_t/\bar{P}_t$. The investment good price Q_t^I is measured by the deflator of the durable-good and private investment, and $q_t^I \equiv Q_t^I/P_t$. R_t is measured by the federal funds rate. For both Y_t and C_t, the conversion between nominal values and real values is based on the price index $\bar{P}_t$. The frequency is annual and all the observable time-series are linear-detrended before the estimation.

[9] The deflator is the weighted average of the deflator for the non-durable good and the deflator for the services. The values of non-durable goods and services are used as the weight. A similar construction also applies to the measure of the investment good price. Details about the underlying data source can be found in Appendix A.

Identification Within these eight time-series, these six non-financial macroeconomic time-series are standard, and they are used to identify the six non-financial aggregate shocks. The remaining two time-series about firms' financing choices are used to identify the financial shocks. To illustrate the intuition about how these different shocks are identified, I normalize these eight shocks such that they generate a 1% decrease in the output growth rate on impact. Then I plot the responses of the weighted average leverage ratio and the small firms' equity financing flow in the middle and right panels of Figure 5. It is clear that firms' financing choices are much more responsive to the financial shocks than to the non-financial shocks. Therefore, with the non-financial shocks controlled by these six macroeconomic non-financial time-series, these two financial shocks are mainly identified by these two financial time-series.

Within these two financial shocks, the debt financing shock has much larger effects on the leverage than the equity financing shock, so the debt financing shock is mainly identified by the time-series of the average leverage ratio. The small firms' equity financing flow is responsive to both financial shocks. Given that the debt financing shock is mostly identified by the time-series of average leverage ratio, the variation in small firms' equity financing flow will help identify the equity financing shock.

Figure 5: Impulse Response to Financial and Non-Financial Shocks



Note: The size of shocks is calibrated to generate a 1% on-impact decrease in the output growth. The impulse responses are the deviation of the corresponding variables from their steady-state level. The x-axis indicates the number of years after the shock.

Estimates and Fitting with Data The priors and the key statistics of the estimated posteriors are listed in Table 6. The moments generated by the model are summarized in Table 7. Given that the parameters in Θ_{NK} are fixed, the model does a reasonably good job in fitting the data overall. Relatively speaking, the model overshoots the volatility of the aggregate prices, and this indicates that the model might understate the relevance of financial shocks in driving the firms' investment dynamics.

Table 6: Estimates of the Aggregate Shock Process Parameters

Parameter	Shock Process	Prior			Posterior		
		Type	Mean	Std.	Mode	5%	95%
Ω: Persistentce							
ρ_e	Equity Financing	Beta	0.6	0.2	0.5440	0.3499	0.6845
ρ_d	Debt Financing	Beta	0.6	0.2	0.7370	0.4381	0.8734
ρ_z	Aggregate TFP	Beta	0.6	0.2	0.1601	0.0300	0.2719
ρ_w	Wage Markup	Beta	0.6	0.2	0.9369	0.5802	0.9610
ρ_p	Price Markup	Beta	0.6	0.2	0.9470	0.8695	0.9752
ρ_m	Monetary Policy	Beta	0.6	0.2	0.1930	0.0593	0.3226
ρ_q	Investment Good Price	Beta	0.6	0.2	0.7528	0.5269	0.9153
ρ_u	Preference	Beta	0.6	0.2	0.7124	0.3762	0.8331
Σ: Std. of the Shocks							
σ_e	Equity Financing	Exp.	0.02	0.02	0.0420	0.0322	0.0544
σ_d	Debt Financing	Exp.	0.02	0.02	0.0297	0.0243	0.0398
σ_z	Aggregate TFP	Exp.	0.02	0.02	0.0279	0.0238	0.0402
σ_w	Wage Markup	Exp.	0.02	0.02	0.0395	0.0316	0.0488
σ_p	Price Markup	Exp.	0.02	0.02	0.0476	0.0408	0.0654
σ_m	Monetary Policy	Exp.	0.02	0.02	0.0154	0.0127	0.0190
σ_q	Investment Good Price	Exp.	0.02	0.02	0.0080	0.0068	0.0110
σ_u	Preference	Exp.	0.02	0.02	0.0343	0.0287	0.0493

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 10,000 draws.

Recovered Unobservable Shocks Based on the mode of the posterior, I recover the history of the unobserved financial shocks by the Kalman smoother, which is presented in Figure 6b. For the equity financing shock history, there were significant increases in the equity financing cost during the two recessions in the sample periods, and the increase during the burst of the "dot com bubble" was much larger than the increase during the recession in 2008. This feature is consistent with the standard perception that the financial crisis in 2001 was mainly marked by the collapse of the stock

Table 7: Moments of the Observable Variables

Variable	Standard Deviation				Autocorrelation			
	Data	Model			Data	Model		
		Mode	5%	95%		Mode	5%	95%
Equity Fin. (Small)	0.049	0.056	0.049	0.069	0.243	0.177	0.029	0.283
Leverage (Avg.)	0.020	0.035	0.027	0.049	0.719	0.832	0.685	0.906
Output Growth	0.027	0.040	0.035	0.050	0.414	0.452	0.362	0.478
Consumption Growth	0.011	0.019	0.016	0.023	0.538	0.506	0.335	0.550
Wage Growth	0.010	0.031	0.029	0.041	0.356	-0.015	-0.122	0.028
Inv. Price Growth	0.009	0.009	0.007	0.012	0.107	-0.124	-0.237	-0.042
Inflation	0.009	0.017	0.015	0.022	0.271	-0.053	-0.176	0.026
Nominal Rate	0.015	0.022	0.020	0.030	0.588	0.397	0.327	0.459
	Relative Std.				Cyclicality			
	Data	Model			Data	Model		
		Mode	5%	95%		Mode	5%	95%
Equity Fin. (Small)	1.834	1.422	1.105	1.744	0.460	0.227	0.199	0.289
Leverage (Avg.)	0.762	0.884	0.593	1.224	-0.342	-0.099	-0.218	-0.060
Consumption Growth	0.411	0.468	0.435	0.485	0.715	0.870	0.812	0.908
Wage Growth	0.378	0.795	0.759	0.907	-0.247	0.607	0.512	0.734
Inv. Price Growth	0.332	0.216	0.163	0.311	0.099	-0.006	-0.007	0.002
Inflation	0.326	0.419	0.359	0.529	-0.034	-0.386	-0.541	-0.323
Nominal Rate	0.575	0.552	0.495	0.692	0.148	-0.619	-0.739	-0.559

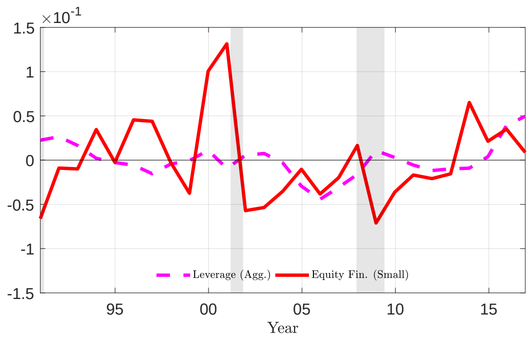
Note: Here the relative std. refers to the standard deviation normalized by the standard deviation of output growth. The cyclicality refers to the correlation with the output growth. The posterior distribution is obtained using the Metropolis-Hastings algorithm with 10,000 draws.

market. In Eisfeldt and Muir (2016), they also estimate an external financing cost that is similar to the equity financing cost in this paper. I depict their estimated time-series in Figure 6b as a comparison. The correlation between their estimates and my estimates is as high as 0.73. This high correlation can serve as an external validation of my results.

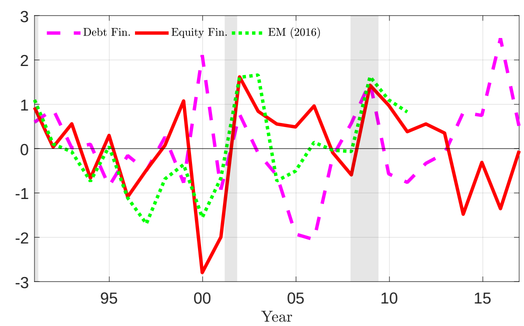
As for the recovered history of debt financing shock, the model implies that the collateral constraint becomes looser during the recessions. This result is mainly driven by the time-series of the public firms' leverage ratio, which is shown in Figure 6a. In the data, the average leverage ratio of the U.S. public firms did increase during the recessions. Similar findings are also found by the micro-level study of Halling *et al.* (2016). The economic mechanism behind this data feature is beyond the scope of this paper, but this feature reflects that the public firms, especially the large public firms, are a special sample of firms in the economy, and they have not experienced extremely tough financing conditions during the last recession on average.

Figure 6: History of Observables and Unobservables, and the Decomposition of Observed History

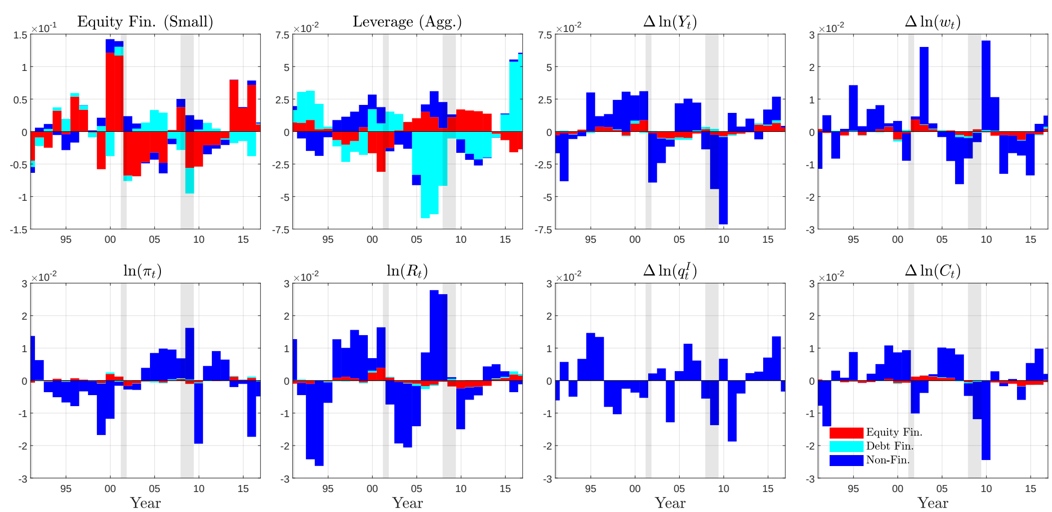
(a) Observed Financial Time-Series



(b) Recovered Unobservable Financial Shocks



(c) Decomposition of the Observed History, Financial Shocks vs. Non-financial Shocks



Note: In panel 6a, the time-series is the deviation of the observed time-series from their linear trend. In panel 6b, the units are the standard deviation of the corresponding shocks. The units in panel 6c are the same as the data.

History Decomposition Based on the mode of the posterior, I also decompose the observable history into the contribution of different aggregate shocks, which are summarized in Figure 6c. The history decomposition results confirm the intuition that the financial shock processes are mostly identified by the financial time-series. The macroeconomic non-financial time-series are almost totally driven by the non-financial shocks, so the time variation in these variables will identify the parameters governing the non-financial shock processes. Conditional on the variation in the aggregate quantities and prices, the financial shocks are identified by the time variation in firms' financial choices.

5 Results and Analysis

The research question of this paper is: How much of the observed variation in firms' investment is driven by financial shocks? To answer this question, I decompose the forecast error variance of firms' investment, on both the disaggregate and aggregate level, into the contribution of different aggregate shocks based on the estimated model, and summarize the results in Table 8.

Table 8: Variance Decomposition of Investment

	On Impact			Average		
Shocks	Aggregate	Small	Large	Aggregate	Small	Large
Financial	2.1	97.3	31.8	**1.1**	**29.1**	**6.5**
Equity	1.9	93.5	28.8	1.1	27.3	6
Debt	0.2	3.8	3	0	1.8	0.5
Non-Financial	97.9	2.7	68.2	98.9	70.9	93.5
Price Markup	67.9	1.8	43.7	77.1	57.6	73.6
Wage Markup	7.8	0.3	4.7	12.3	9.2	11.2
TFP	10.8	0.2	10.9	1.7	0.7	1.9
Preference	8.6	0.1	6.1	6.7	3	5.7
Monetary	1.7	0.2	2.3	0.3	0.2	0.4
Investment Good Price	1.2	0.1	0.6	0.8	0.2	0.7

Note: Here, the variance of the forecast error is decomposed at two different horizons: on impact and over the whole sample history. The variance decomposition is based on the mode of the estimated posterior.

On the disaggregate level, financial shocks play a much more important role in explaining the small firms' investment variation regardless of whether we are looking at on-impact or long-run results. This result is relatively intuitive because small firms' investment depends more on external financing on average, as shown in Tables 2 and 5. Another feature worth mentioning is that financial shocks are the dominant source of the variation in small firms' investment in the short run, but their importance quickly decays over time. This is consistent with the estimates in Table 6: the process of the equity financing shock, which is the major financial shock, has a large standard deviation for its innovation but a low persistence for its propagation.

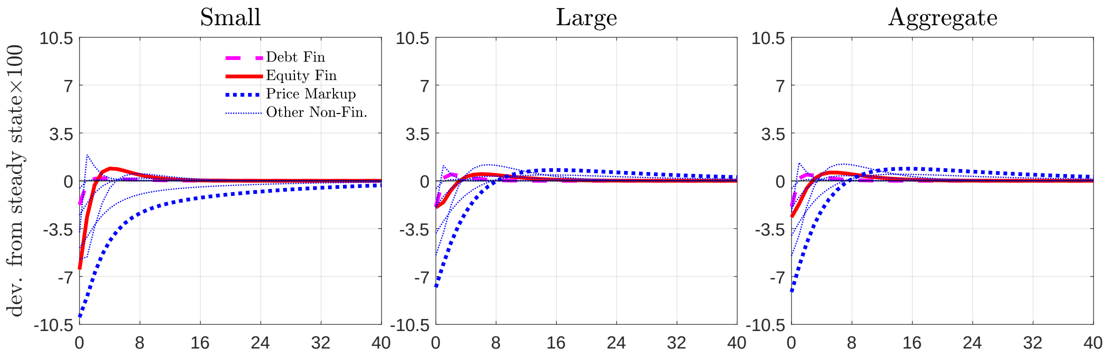
On the aggregate level, financial shocks contribute little to the aggregate investment variation regardless of whether we are looking at on-impact or long-run results. In the standard partial equilibrium with heterogeneous firms, aggregate financial shocks have weaker effects due to the size composition: financial shocks only affect the financially constrained firms, and on average, the financially constrained firms are smaller. Under most parameterization, the total investment of the constrained firms count for only a small fraction of the aggregate investment, even if their investment rate is much higher. The size-composition effect is also presented in this paper, and it is a part of the reason for the small aggregate relevance of financial shocks. But if it were the only reason in this paper, the relative relevance of financial shocks to the aggregate investment should be a weighted average of their relevance to small and large firms, i.e., it should be close and a little bit higher than their relevance to the large firms' investment. However, as shown in Table 8, the aggregate relevance of financial shocks is even smaller than their relevance to the large firms' investment. This negligible aggregate relevance highlights an important economic mechanism in this paper: the interaction between heterogeneity and general equilibrium effects.

This interaction features the reallocation of investment across different firms after the financial shocks. To explain and highlight the importance of this reallocation channel, I first shut off the general equilibrium feedbacks and present the impulse response of firms' investment to different aggregate shocks within the partial equilibrium setup in Figure 7a. Then I switch on the general equilibrium feedbacks, present them through the responses of the aggregate prices in Figure 7c, and discuss how they change the responses of different firms' investment to these shocks in Figure 7b. The input shocks underlying these impulse responses are all contractionary, and their sizes are all calibrated to one estimated standard deviation as the posterior modes in Table 6.

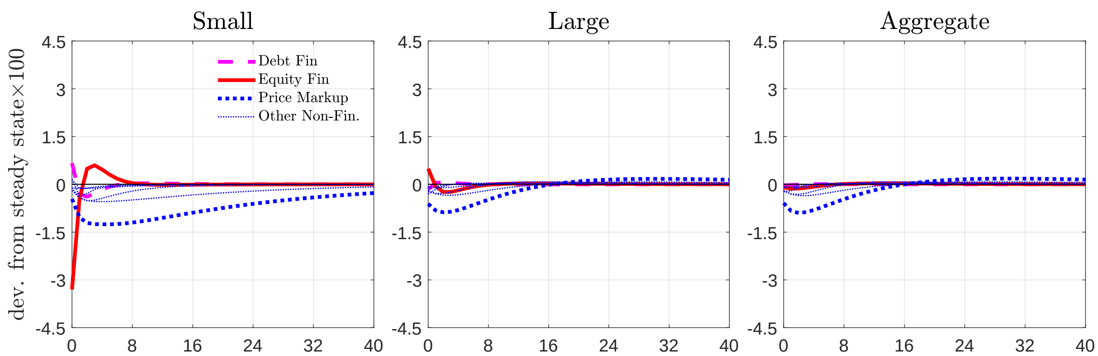
Direct Effects Without the general equilibrium feedbacks, both small and large firms' investment decreases following the contractionary financial shocks. There are two features worth emphasizing here. First, the responses of small firms' investment is larger than the responses of large firms' investment for almost any type of aggregate shock. Small firms are more responsive to the equity financing shock because they rely more on equity financing. The difference in their responses to the non-financial shocks are mainly due to the amplification effect of the financial frictions. Second, the responses of the aggregate investment are very close to the responses of the large firms' investment,

Figure 7: Impulse Response to Different Aggregate Shocks

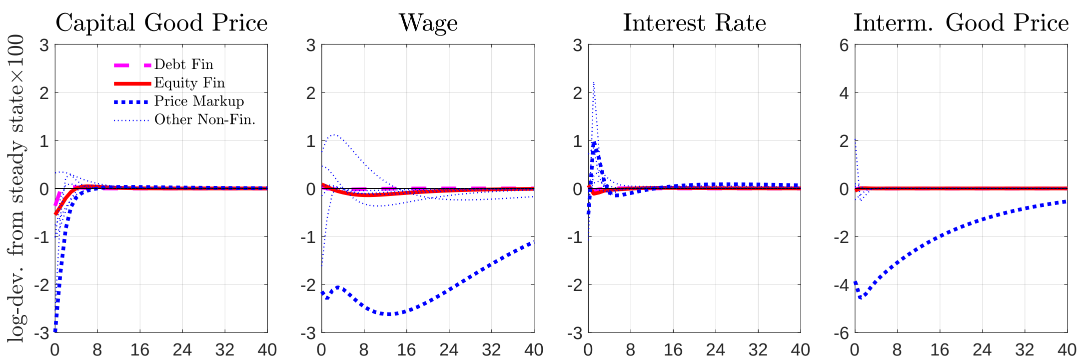
(a) Investment of Different Firm Groups, without General Equilibrium Feedbacks



(b) Investment of Different Firm Groups, with General Equilibrium Feedbacks



(c) Aggregate Prices, with General Equilibrium Feedbacks



Note: The impulse responses are to the one-standard-deviation contractionary shocks. The impulse responses of investment are the deviation of the measured investment from their steady-state level. The investment is measured as $\sum_{j\in\mathcal{J}} i_j / \sum_{j\in\mathcal{J}} k_j$, where $\mathcal{J}$ is the indicator for different firm groups and $\mathcal{J} \in \{\text{small, large, population}\}$. The impulse responses of the aggregate prices are the log-deviation of the real capital good price, real wage, real gross interest rate, and real intermediate good price from their corresponding steady-state levels. The x-axis indicates the number of years after the shock.

which is due to the dominance of large firms' total size[10].

General Equilibrium Feedbacks There are four aggregate prices whose dynamics feature the general equilibrium feedbacks: capital good price (q_t), wage (w_t), real interest rate ($\frac{R_{t-1}}{\pi_t}$), and intermediate good price (p_t). Their responses to different aggregate shocks within the general equilibrium are summarized in Figure 7c. By comparing the responses of different aggregate prices to financial shocks, we find that financial shocks trigger significant responses only in the capital good price. In the following part, I focus on the general equilibrium effect through the response of the capital good price when I explain the indirect effects of financial shocks.

Indirect Effects By comparing the impulse responses in Figures 7b and 7a, we can tell that general equilibrium feedbacks significantly dampen the effects of the non-financial shocks, and they even alter the direction of firms' investment responses to the financial shocks on the disaggregate level. The composition of small and large firms is the key to understanding the direction of their responses to financial shocks. There are three endogenously generated types of firms in this model, financially unconstrained firms (i.e., the firms who are paying dividends), financially constrained and debt-dependent firms (i.e., the firms who are neither paying dividends nor issuing equity), and financially constrained and equity-dependent firms (i.e., the firms who are issuing equity). The fraction of each types' total size within the small and large firms in the steady state are summarized in Table 9.

Table 9: Size Composition of Small and Large Firms in Steady State (%)

	Constrained		Unconstained
	Equity Dependent	Debt Dependent	
Small	51.43	6.57	41.99
Large	20.52	23.04	56.44

Following an equity financing shock that increases the equity issuance cost, the equity-dependent firms are hit directly and will have to cut their investment. The lower investment demand dampens the capital good price, and the lower capital good price motivates the unconstrained firms to

[10]In the data, the total investment from the small firms counts for 5% of the aggregate investment. In the model, the fraction of small firms' investment within the aggregate investment is 25%. But this does not change the conclusion of this paper that the negligible aggregate effects of financial shocks are mainly due to the reallocation of investment across different firms.

increase their investment. Given that the equity-dependent firms are mainly concentrated in small firms and the unconstrained firms are mainly concentrated in the large firms, the response of the small firms' investment is negative and the response of the large firms' investment is positive. In the aggregation, their responses largely cancel each other.

Following a debt financing shock that tightens the collateral constraint, the debt-dependent firms are hit directly, so they cut their investment and induce the decrease in the capital good price. At this time, the lower capital good price motivates both the unconstrained firms and the equity-dependent firms to increase their investment[11]. Given that there are quite a few debt-dependent firms among the small firms, the indirect effects of the debt financing shock are more loaded on the small firms and the response of their investment is positive. Within the large firms, there is a significant fraction of debt-dependent firms, so the direct effects of the debt financing shock are more loaded on the large firms and there is a negative response in their investment. Because there is also a significant fraction of unconstrained firms within the large firms, the magnitude of large firms' investment response is not very large. But given the size dominance of the large firms, their slight negative response is able to eat away the large positive response of the small firms' investment, so the aggregate effects of the debt financing shock are also negative but negligible.

Representative Firm Model vs. Heterogeneous Firm Model As discussed above, the negligible aggregate relevance of financial shocks is mainly a result of the interaction between firm-level heterogeneity and general equilibrium feedbacks, which is absent in the representative firm model (e.g., Jermann and Quadrini, 2012). By construction, the representative firm model will imply a larger aggregate relevance of financial shocks because the only firm in the economy is constrained and other firms dampen the direct effects from financial shocks. In this paper, I evaluate how large this difference could be quantitatively.

To make a reasonable comparison, I degenerate the heterogeneous firm model in this paper back to a representative model by removing the idiosyncratic productivity shocks. Then I estimate the model to match the time-variation in both aggregate quantities and prices, as well as the aggregate financing choices of U.S. public firms. Based on the estimates, I quantify the aggregate effects of financial shocks on the aggregate investment (see details in Appendix D). As summarized

[11] The increase in equity-dependent firms' investment is mainly because the issuance cost is proportional.

in Table 10, the representative firm model implies a much larger aggregate effect of financial shocks: financial shocks contribute 55.4% of the variation in the aggregate investment, and they become the dominant driving force of the aggregate investment.

Table 10: Variance Decomposition of Aggregate Investment

	On Impact		Historical Average	
Shock	Rep. Firm	Hete. Firm	Rep. Firm	Hete. Firm
Financial	11.8	2.1	55.4	1.1
Non-Financial	88.2	97.9	44.6	98.9

6 Concluding Remarks

This paper incorporates firm-level heterogeneity into the identification of aggregate financial shocks. The identification is based on an estimated DSGE model that includes a continuum of heterogeneous firms to match the cross-sectional evidence about firms' investment and financing, and eight aggregate shocks to separately capture the exogenous variation in firms' financing conditions and investment profitability. Based on the estimated model, financial shocks only explain a negligible fraction of the fluctuations in U.S. public firms' aggregate investment on average. This negligible aggregate relevance of financial shocks is mainly a result from the interaction between firm-level heterogeneity and general equilibrium effects. Due to the absence of such an interaction mechanism, a comparable representative firm model implies a 50 times larger aggregate relevance of financial shocks. This large difference indicates the importance of micro-level heterogeneity for us to correctly understand the sources of business cycle fluctuations.

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Appendix A Data Appendix

Data from Compustat The Compustat annual dataset is downloaded from WRDS. The variables used in the empirical evidence are listed in Table A.1. The data cleaning procedure has the following steps:

1. Only keep U.S. firms: `fic`='USA'.
2. Discard the observations from financial, utility, and quasi-governmental sectors, i.e., the observations with `sic` in $6000-6999$, $4900-4999$, and $9000-9999$.
3. Discard the four giant firms that were affected most by the 1988 accounting rule change: GE (`gvkey`==005047), Ford (`gvkey`==004839), Chrysler (`gvkey`==003022), GM (`gvkey`==005073).
4. Keep only the records with standard format, i.e., `datafmt`='STD'.
5. Keep only the records with SCF format code 7, i.e., `scf`=7.
6. Keep only the observations listed on U.S. stock markets, i.e., `exchg` in $0-4$ and $11-20$.
7. Discard the observation with M&A larger than 5% of its book value assets.
8. Drop observations with missing values for the book value assets.
9. Discard observations with real book value assets smaller than \$10 million (1982 \$).

Table A.1: Data Source of Micro-level Evidence

Variable Name	Compustat Variable
M&A	`aqc`
Book value asset	`at`
Capital expenditure	`capx`
R&D expenditure	`xrd`
Net long-term debt issuance	`dltis`−`dltr`
Net change of current debt	`dlcch`
Issuance of common and preferred stocks	`sstk`
Dividend	`dv`
Stock repurchase	`prstkc`

Data in SDC Used to Refine Stock Issuance (sstk) I extract the IPO and SEO deals in SDC issued between 1989 and 2016. Only the deals that were issued in U.S. markets are kept. The deals in SDC are merged with Compustat data mainly by the first six digits of the CUSIP code. For the remaining unmatched deals, I use their ticker symbol to match them with the firms in Compustat.

Aggregate Data The aggregate variables are extracted from Flow of Funds and FRED, and the details are listed in Table A.2.

Table A.2: Data Source of Aggregate-level Evidence

Variable	Data Source	Variable ID
Gross value added of non-financial corporate sector	Flow of Funds	FA106902501.A
Consumption, non-durable good	FRED	PCNDA
Consumption, service	FRED	PCESVA
Consumption, durable good	FRED	PCDGA
Investment, private	FRED	GPDIA
Deflator, non-durable good	FRED	DNDGRD3A086NBEA
Deflator, service	FRED	DSERRD3A086NBEA
Deflator, durable good	FRED	DDURRD3A086NBEA
Deflator, private investment	FRED	A006RD3A086NBEA
Hourly earnings in the U.S. manufacturing sectors	FRED	USAHOUREAAISMEI
Federal fund rate (monthly)	FRED	FEDFUNDS

Appendix B Decisions in the New Keynesian Block

The decisions are directly presented in log-linearized forms in this section, where $\tilde{X}_t$ denotes the log deviation of X_t from its steady-state level.

Final Goods Supply and Inflation Dynamics The final good producers maximize their expected total discounted profits by choosing their input of retailed goods. Given the demand from final good producers, retailers maximize their expected total discounted profits by setting the nominal price of their goods. Following Calvo (1983), it is assumed that only a randomly chosen fraction $(1-\xi_p)$ of the retailers can reset their price in each period. The decisions of final good producers and retailers jointly determine the aggregate supply of final goods and inflation dynamics:

$$\tilde{Y}_t = \tilde{y}_t \tag{B.1}$$

$$\tilde{\pi}_t = \frac{1-\xi_p}{\xi_p} \cdot (1 - \xi_p \cdot \beta) \cdot (\tilde{p}_t + \eta_{p,t}) + \beta \cdot \mathbb{E}_t [\tilde{\pi}_{t+1}], \tag{B.2}$$

where Y_t is the total output of final goods, y_t is the total output of intermediate goods, and $p_t \equiv P_t/\bar{P}_t$ is the intermediate good price in real terms.

Labor Demand and Wage Dynamics The labor packer maximizes their expected total discounted profits by choosing their input of differentiated labor services. Given the demand from the labor packer, labor unions maximize their expected total discounted profits by setting the nominal wage of their differentiated labor service. It is also assumed that only a randomly chosen fraction $(1 - \xi_w)$ of the labor unions can reset their wages in each period. The decisions of the labor packer and labor unions jointly determine the aggregate demand for the households' labor and wage dynamics:

$$\tilde{N}_t = \tilde{L}_t \tag{B.3}$$

$$\begin{aligned} \tilde{w}_t =& \frac{\xi_w}{1 + \beta \cdot \xi_w^2 \cdot (1 - \xi_w)} \cdot (\tilde{w}_{t-1} - \tilde{\pi}_t) + \frac{(1 - \xi_w) \cdot (1 - \beta \cdot \xi_w)}{1 + \beta \cdot \xi_w^2 \cdot (1 - \xi_w)} \cdot \left(\tilde{w}_t^N + \eta_{w,t}\right) \\ &+ \frac{(1 - \xi_w) \cdot \beta \cdot \xi_w}{1 + \beta \cdot \xi_w^2 \cdot (1 - \xi_w)} \cdot \mathbb{E}_t [\tilde{w}_{t+1} + \tilde{\pi}_{t+1}], \end{aligned} \tag{B.4}$$

where N_t denotes the quantity of households' labor, L_t denotes the total final labor service used by the intermediate good firms, and $w_t \equiv W_t/\bar{P}_t$ and $w_t^N \equiv W_t^N/\bar{P}_t$ are the final labor service wage and household labor wage in real terms. In the equation of wage dynamics, the first term characterizes the backward-looking feature coming from nominal wage rigidity, the second term characterizes the effects from the aggregate wage markup shocks, and the last term characterizes the forward-looking feature coming from the rational expectations of labor unions.

Capital Good Supply and Capital Good Price Dynamics The investment good producer and the capital good producer maximize their expected total discounted profits by choosing their inputs Y_t^I and $\hat{I}_t$. Based on their optimal choice, the price of the investment good satisfies:

$$\tilde{q}_t^I = \eta_{q,t}. \tag{B.5}$$

The total supply of the capital good and the capital good price dynamics are:

$$\tilde{I}_t = \tilde{Y}_t^I - \eta_{q,t} \tag{B.6}$$

$$\tilde{q}_t = \tilde{q}_t^I + S''(1) \cdot \left[\left[\tilde{I}_t - \tilde{I}_{t-1}\right] - \beta \cdot \mathbb{E}_t \left[\tilde{I}_{t+1} - \tilde{I}_t\right]\right], \tag{B.7}$$

where $q_t^I \equiv Q_t^I/\bar{P}_t$ and $q_t = Q_t/\bar{P}_t$ denote the real price of the investment good and capital good, respectively.

Labor Supply and Stochastic Discounting Factor Dynamics The representative household maximizes its utility specified in (14) subject to their budget constraint in (15). The consumption Euler equation and labor supply are:

$$0 = \mathbb{E}_t \left[\tilde{\Lambda}_{t,t+1} + \tilde{R}_t - \tilde{\pi}_{t+1}\right] \tag{B.8}$$

$$\tilde{w}_t^N = \eta_{u,t} - \widetilde{MU}_t. \tag{B.9}$$

The dynamics of the real SDF is disciplined by:

$$\widetilde{MU}_t = \eta_{u,t} - \left[\frac{1}{1-h} \cdot \tilde{C}_t - \frac{h}{1-h} \cdot \tilde{C}_{t-1}\right] \tag{B.10}$$

$$\tilde{\Lambda}_{t,t+1} = \widetilde{MU}_{t+1} - \widetilde{MU}_t. \tag{B.11}$$

Appendix C Numerical Appendix

Value Function Approximation The Bellman equation system (11) is first transformed into the following form:

$$V_t(\tilde{x}, k, a) = \max_{\tilde{e}, ki, \tilde{d}', \tilde{k}'} - \tilde{e} \cdot k \cdot q_t \cdot \left(1 + \mathbb{1}_{\tilde{e}>0} \cdot \phi_t^E\right)$$
$$+ \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \left[(1-\xi_d) \cdot V_{t+1}\left(\tilde{x}', \varepsilon^{k'} \cdot \tilde{k}' \cdot k, a'\right) + \xi_d \cdot \tilde{lv}_{t+1}\left(\tilde{d}', \varepsilon^{k'} \cdot \tilde{k}', a'\right)\right]\right],$$

subject to

$$\begin{aligned}
\left[\tilde{k}' + \Phi^K(\tilde{k}' \cdot k, k)\frac{1}{k}\right] \cdot q_t =& \tilde{x} + \tilde{e} + \tilde{d}' \\
\tilde{d}' \leq& \phi_t^D \cdot q_t \\
\tilde{lv}_{t+1}(\tilde{d}', \varepsilon^{k'} \cdot \tilde{k}', a') =& (1-\delta) \cdot q_{t+1} \cdot \varepsilon^{k'} \cdot \tilde{k}' \cdot k - \frac{1+i_t}{1+\pi_{t+1}} \cdot \tilde{d}' \cdot k \cdot q_{t+1} \\
\tilde{x}' =& (1-\tau) \cdot \iota \cdot w_{t+1}^{\frac{-\zeta}{1-\zeta}} \cdot (p_{t+1} \cdot Z_{t+1})^{\frac{1}{1-\zeta}} \cdot (a')^{\frac{1}{1-\zeta}} \cdot \left(\varepsilon^{k'} \cdot \tilde{k}' \cdot k\right)^{\tilde{\theta}-1} \\
&+ (1-\delta) \cdot q_{t+1} - \frac{1+(1-\tau) \cdot i_t}{1+\pi_{t+1}} \cdot \frac{\tilde{d}'}{\tilde{k}' \cdot \varepsilon^{k'}},
\end{aligned}$$

where $\zeta \equiv \theta(1-\alpha)$, $\iota \equiv \zeta^{\frac{\zeta}{1-\zeta}} - \zeta^{\frac{1}{1-\zeta}}$, $\tilde{\theta} \equiv \frac{\alpha\theta}{1-\zeta}$, and $\tilde{x} \equiv \frac{x}{k}, \tilde{d}' \equiv \frac{d'}{k}, ki \equiv \frac{i}{k}, \tilde{k}' \equiv \frac{\hat{k}'}{k}, \tilde{e} \equiv \frac{e}{k}$.

Then the grid for the idiosyncratic productivity a is constructed based on Tauchen (1986): there are 3 grid points chosen over the scale $[-1.5 \times \sigma_a/\sqrt{1-\rho_a^2}, 1.5 \times \sigma_a/\sqrt{1-\rho_a^2}]$. For each grid point of idiosyncratic productivity state, $V_t(\tilde{x}, k, a)$ is approximated by the spline with the order of 2 on the dimension of $\tilde{x}$ and k. On the dimension of $\tilde{x}$, there are 10 grid points evenly spread between 0.01 and 2. On the dimension of k, there are 25 grid points spread between $\exp(-5) \times \left[\frac{A_{1,SS} \cdot \exp\left(1.5 \times \sigma_a/\sqrt{1-\rho_a^2} \cdot \frac{1}{1-\zeta}\right)}{\delta + \frac{1}{\beta} - 1}\right]^{\frac{1}{1-\tilde{\theta}}}$ and $1.1 \times \left[\frac{A_{1,SS} \cdot \exp\left(1.5 \times \sigma_a/\sqrt{1-\rho_a^2} \cdot \frac{1}{1-\zeta}\right)}{\delta + \frac{1}{\beta} - 1}\right]^{\frac{1}{1-\tilde{\theta}}}$, where $A_{1,SS} = (1-\tau) \cdot \iota \cdot \frac{1}{q_{SS}} \cdot w_{SS}^{\frac{-\zeta}{1-\zeta}} \cdot (p_{SS})^{\frac{1}{1-\zeta}}$. The grid for k is distributed such that $\frac{\log k}{0.5}$ are evenly spread. The collocation method is applied in solving the steady state.

Distribution Approximation Since the entrant distribution is fixed, I approximate the distribution of firms after the realization of capital quality shocks and exit shocks in two parts: a quadrature with fixed weights for the entrants and a quadrature with varying weights for the incumbents. Instead of approximating over $(d \equiv \frac{d}{k}, k, a)$, I approximate the distribution over a transformed equivalent space (dx, k, a) where $dx \equiv A_{1,SS} \cdot a^{\frac{1}{1-\varsigma}} \cdot k^{\tilde{\theta}-1} + (1-(1-\tau)\cdot\delta) - \frac{1+(1-\tau)\cdot i_{SS}}{1+\pi_{SS}} \cdot \cdot d$. By this transformation, the geometry of the distribution support at each grid point of a is more regular. At each grid of a, the approximation scheme follows the method used in Algan *et al.* (2008) and Winberry (2018) with the highest order of moments set at 4.

Appendix D The Comparable Representative Firm Model

Firms' Decision Compared to the setup of the heterogeneous firm model, there are only two changes in this representative firm model. First, after the incumbents exit, a group of entrants take over their capital and enter the operation. Second, equity financing friction is modeled as a time-varying wedge, which captures the marginal funding cost of the firms. The recursive representation of the firms' decision is

$$\begin{aligned} V_t(d,k) = \max_{l,i,d',e} & -e \cdot [1+\phi_t^e] \\ & + \mathbb{E}_t \left[\Lambda_{t,t+1} \cdot \frac{\bar{P}_t}{\bar{P}_{t+1}} \cdot \left[(1-\xi_d) \cdot V_{t+1}\left(d',k'\right) + \xi_d \cdot \mathcal{LV}(d',k')_{t+1} \right] \right] \end{aligned} \tag{D.12}$$

s.t. : production technology (4), internal financing source (5),

budget constraint (6), capital accumulation (7),

collateral constraint (9).

Quantification All of the non-estimated parameters are still fixed at the same level, except for the value of $\bar{\phi}^e$. $\bar{\phi}^e$ is calibrated to match the cross-time average level of the aggregate equity financing flow. Then the parameters in Θ_{SH} are estimated based on the new model. There is only one input observable, which is different from the benchmark exercise: the small firms' equity financing flow is replaced with the aggregate equity financing flow. Based on the mode of the posterior, the variation decomposition results are summarized in Table D.3.

Table D.3: Variance Decomposition of Aggregate Investment, Details

	On Impact		Historical Average	
Shock	Rep. Firm	Hete. Firm	Rep. Firm	Hete. Firm
Financial	11.8	2.1	55.4	1.1
Equity	11.8	0.2	55.4	0
Debt	0	1.9	0	1.1
Non-Financial	88.2	98	44.5	98.9
Price Markup	43.9	67.9	21.7	77.1
Wage Markup	0.7	7.8	1.8	12.3
TFP	42	10.8	19.7	1.7
Preference	1.4	8.6	1.1	6.7
Monetary	0.1	1.7	0	0.3
Investment Good Price	0.1	1.2	0.2	0.8