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Managing GDP Tail Risk

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Abstract

We propose a novel framework to analyze how policy-makers can manage risks to the median projection and risks specific to the tail of gross domestic product (GDP) growth. By combining a quantile regression of GDP growth with a vector autoregression, we show that monetary and macroprudential policy shocks can reduce credit growth and thus GDP tail risk. So policy-makers concerned about GDP tail risk would choose a tighter policy stance at the expense of macroeconomic stability. Using Canadian data, we show how our framework can add tail event information to projection models that ignore them and give policy-makers a tool to communicate the trade-offs they face.

Topics: Central bank research; Economic models; Financial stability; Financial system regulation and policies; Interest rates; Monetary policy; Monetary policy framework

JEL codes: E44, E52, E58, D8, G01

1 Introduction

Most standard projection models used for policy analysis focus on normal times without taking the risk of a crisis into account, defined as a sudden large decline in gross domestic product (GDP). Yet, policy-makers are worried about GDP tail risks.[1] We analyze how GDP tail risk can affect policy-makers' decisions, and how monetary and macroprudential policy can help to manage GDP tail risk. We provide a framework to explicitly weigh risks to GDP growth related to both macroeconomic and financial stability. Here, financial stability is proxied by GDP tail risk, defined as changes in the fifth percentile of the distribution of future GDP growth that are independent from changes in the central moment of the distribution. We assess the extent to which a policy-maker concerned about risks to financial stability can trade risks to the GDP growth outlook (or central risks) for GDP tail risks.

In a simple general equilibrium model, we show how risk shifting due to low interest rates or loose regulation can worsen GDP tail risk even if expected GDP increases. We obtain a policy choice set whereby tightening monetary and macroprudential policies can improve both central and tail risks to the distribution of future GDP growth by limiting risk shifting. Very strong tightening, however, generates a trade-off between worsening risks to the central path and improving GDP tail risks. A policy-maker concerned about financial stability, and aware of the build-up of tail risk, would prefer a tighter policy stance than otherwise.

To provide empirical support for these predictions, we introduce a new approach that sequentially combines quantile regressions with a structural vector autoregression (VAR). It combines the shock identification of a structural VAR with the projection of risks to GDP growth from quantile regressions. We track the impact of a policy shock that changes the shape of the distribution of future GDP growth and quantify risks to financial stability. Our analysis focuses on a simple summary statistic of GDP tail risks, the difference between the median and the fifth percentile (growth at risk) of future GDP growth. From Adrian, Boyarchenko and Giannone (2019), we know that the 95$^{\text{th}}$ percentile is relatively stable. With our definition, we focus on the additional risk in the left tail of the distribution of GDP growth that is not already contained in the central projection of GDP growth. We apply our approach to Canada, since it is one of the few countries with a long history of household-related macroprudential changes. Canada has implemented 31 household-related macroprudential measures since 1992,[2] while most

[1] "Central banks had won the war against inflation during the Great Moderation only to lose the peace as vulnerabilities built inexorably." (Carney, 2017) "These financial vulnerabilities have made monetary policy more complicated." (Poloz, 2018) "Macroprudential policy, addressing financial imbalances, can complement the long-run objective of monetary policy." (Draghi, 2017) "How should monetary and other policymakers balance macroprudential approaches and monetary policy in the pursuit of financial stability?" (Yellen, 2014)

[2] The IMF macroprudential database of Alam et al. (2019) reports that among the 36 advanced

other countries started to use macroprudential policies more recently. Combining those measures into a discrete index allows us to capture the effect of an average macroprudential change. We then compare the impact of monetary and macroprudential policies on credit growth and eventually on GDP tail risk.

We obtain three key results. First, both credit growth and financial market stress impact the fifth percentile of GDP growth more than the median, thus shaping the distribution of future GDP growth. We find that financial market stress is more relevant in the short run while credit growth has a larger impact in the medium run. Thus policymakers can manage GDP tail risk by using tools that target credit growth (in line with Schularick and Taylor, 2012; Jorda, Schularick and Taylor, 2011), while financial market stress is harder to influence before it materializes. Adrian, Boyarchenko and Giannone (2019) combine both indicators into a single composite that they use for their analysis of growth at risk, mainly finding short-run effects. But Ranciere, Tornell and Westermann (2008) point to credit skewness being a driver of subsequent crises.

Second, we find that both monetary and macroprudential policy shocks can reduce credit growth and influence the fifth percentile of future GDP growth more than the median, thus reducing GDP tail risk. An unexpected 100-basis-point increase in the policy rate narrows the distribution of GDP growth by 20 basis points. Similarly, tightening household-related macroprudential policy by a historical average amount narrows the distribution of GDP growth by 20 basis points. We are confident that our measure of macroprudential policy shocks reflects the effect of macroprudential changes. First, the effect disappears when we focus on business rather than household credit growth, since most macroprudential changes in Canada target households. Second, the effect is weaker when one includes macroprudential measures not targeting households. Finally, the effect does not survive a placebo analysis that tests for expectations of macroprudential measures.

Third, we conduct a policy experiment to show how our framework can be used in practice to introduce financial stability concerns into standard projections that abstract from the risk of a crisis. We illustrate the approach using Canadian projections conducted by commercial banks in early 2018 for the period 2018Q1 to 2021Q4. We construct the choice set in terms of central and tail risks for alternative policy profiles.[3] Our framework suggests the existence of a trade-off in early 2018 between raising rates fast to fight household indebtedness or raising rates slowly to support GDP growth. Our framework also suggests that some tightening of macroprudential policy rules—which eventually took place—supports both the macroeconomy and financial stability, but sizable tightening would generate a trade-off. A crisis-conscious policy-maker would have chosen a higher

economies they cover, less than 10 percent of them had macroprudential measures in place by the end of 1992, and more than 10 percent only by the turn of the century.

[3]Similarly, Figure 1 in Poloz (2014) suggests contrasting some measure of financial stability risks against a measure of inflation-target risks.

interest rate than commercial banks whose projection appears to focus only on central risk.

By combining a quantile model and a VAR, our new framework makes four contributions. First, we provide support for the risk-taking channel of monetary policy. Second, we identify macroprudential policy shocks and show that demand-related interventions are effective at reducing GDP tail risk. Third, we show how our framework can be used to map the monetary or macroprudential policy-makers' choice sets into measurable central and tail risk space, and how this can be used to adjust projections that do not take GDP tail risk into account. Fourth, we provide a simple communication tool that policy-makers can use to present the choices they face when weighing macroeconomic and financial stability goals.

The rest of the paper is organized as follows. Section 2 highlights three strands of the literature to which we contribute. Section 3 provides some theoretical foundation of our framework. Section 4 explains our empirical strategy, while Section 5 displays the results obtained for Canada. Section 6 discusses an application of our framework to introduce financial stability concerns into policy projections. Section 7 presents robustness checks, and Section 8 concludes.

2 Relevant literature review

Our work relates to three strands of the literature.

2.1 GDP tail risk and density forecasts

The first strand of the literature analyzes the density forecasts of future GDP growth and emphasizes its skewness.

Some models can generate asymmetry and skewness by moving away from the normality assumption on the distribution of shocks. Bayesian VAR with stochastic volatility (Cogley, Morozov and Sargent, 2005; Primiceri, 2005), Student's t-distributed disturbances (Chiu, Mumtaz and Pinter, 2017) or Gaussian copula with skewed marginal distributions (Smith and Vahey, 2016) provide better density forecasts, especially around events like the Great Recession. However, by changing the distributional assumptions, those models do not allow for a direct link between GDP tail risk and policy actions and are thus not the focus of this paper.

Instead, we explicitly link the skewness in the distribution of macroeconomic variables to different financial market conditions. This can be achieved with different degrees of granularity using Markov-switching models, threshold VAR or quantile regressions. For instance, Brave and Lopez (2017) use the Markov-switching framework extended to time-varying probabilities (Filardo, 1994; Diebold, Lee and Weinbach, 1994) and make the switch to a recession conditional on financial market conditions. Hubrich and Tetlow (2015) use the Markov-switching VAR framework to investigate the role of financial market conditions as a driver for the change in regimes. Adrian, Boyarchenko and Giannone (2019) use the quantile regression method of Koenker and Bassett (1978) and find that growth at risk, the fifth percentile of the distribution of future GDP growth, is largely driven by financial market conditions.[4] The International Monetary Fund (2017*a*) introduced the concept of growth at risk into the policy debate, and Aikman et al. (2018) study how a wider set of composite measures that impact GDP tail risks can describe the UK financial cycle and be used to communicate policy actions.

If growth at risk moves with a relatively high frequency as in Adrian, Boyarchenko and Giannone (2019), this leaves little room for policy actions, as the deterioration of financial market conditions ahead of a crisis event can be very close to the start of the crisis event. Instead, we emphasize the importance of credit accumulation for GDP tail risk. Thus we can operationalize growth at risk and test the ability of policy institutions to maintain control over GDP tail risk by targeting credit growth.

2.2 Systemic risks and banking crises

The second strand of the literature related to our study focuses on the identification of rising risks and derives early warnings of looming banking crises.

As suggested by Borio (2014), the financial cycle "can be most parsimoniously described in terms of credit and property prices." Jorda, Schularick and Taylor (2011) and Schularick and Taylor (2012) argue that over a century, credit growth is the best predictor of financial instability, defined as systemic banking crises similar to Leaven and Valencia (2013). Drehmann, Borio and Tsatsaronis (2011) and Drehmann and Tsatsaronis (2014) instead advocate the use of the credit-to-GDP gap for horizons between two to five years ahead, but the precise measure of credit can sometimes alter the risk assessment (Duprey, Grieder and Hogg, 2017). Most work on early warning models focuses on total credit, in part due to its wider availability and its broad and robust definition. However, financial stability is increasingly linked to real estate lending booms (Jorda, Schularick and Taylor, 2016). Mian, Sufi and Verner (2017) show that, across countries, an increase in the household debt to GDP ratio predicts lower GDP growth, and low mortgage spreads tend to enhance this effect. Similarly, households' debt service ratios can predict future crises (Drehmann and Juselius, 2012).[5]

[4]Growth at risk is coined by analogy to the value at risk concept. Cecchetti and Li (2008) initiated this literature by looking at the impact of equity and property booms on GDP at risk.

[5]Other sources of crisis risks could be the build-up of corporate debt (Grieder and Lipsitz, 2018), bank leverage (Hahm, Shin and Kwanho, 2013), banking sector capital flows (Bruno and Shin, 2015), current account deficits (Lo Duca and Peltonen, 2013) or exchange rate developments (Gourinchas and Obstfeld, 2012). To that extent, a composite measure of financial system vulnerabilities could better capture those different aspects (Ng, 2011; Schuler, Hiebert and Peltonen, 2017; Aikman et al., 2017;

The quantile regressions we perform can be understood as a generalization of binary logit models widely used in the systemic risk literature. Instead of predicting banking crisis dummies that occur with a relatively low frequency, we investigate how credit can predict the materialization of different rates of GDP growth that occur with a given probability.

2.3 Leaning against the wind

Third, we contribute to the literature on leaning against the wind and the risk-taking channel of monetary policy.

We assess how monetary policy or macroprudential policies can influence different percentiles of the GDP growth distribution. Thus we study how the implementation of different policies can harm GDP growth at the median but provide stability benefits in the tail by reducing financial imbalances. The trade-off exercise we perform, while less structural, is similar in spirit to Svensson (2017) and its extension to regulatory policy (Svensson, 2018). Two papers closely linked to ours are Angeloni and Faia (2013) and Alpanda and Ueberfeldt (2016). In both papers, GDP tail risk is endogenous to the model, and monetary policy, as well as regulatory policy, can influence that risk. However, in all those papers, the trade-off that monetary policy faces between standard macroeconomic stability and the risk of a crisis depends on the calibration of the crisis risk that arrives with a logit distributed probability. Instead, we obtain direct estimates of how credit influences moments of the GDP growth distribution and allow for a wider range of GDP tail events without distributional assumptions.

We find support for the view that monetary policy influences the aggregate risk in the economy, which is consistent with the risk-taking channel of monetary policy as outlined in Borio and Zhu (2012) and Martinez-Miera and Repullo (2017) and the micro-level evidence of Delis and Kouretas (2011) and Jiménez et al. (2014). Consistent with Coimbra and Rey (2017), we empirically find that the monetary authority faces a trade-off between stimulating the economy and promoting financial stability when the interest rate is too low and encourages risk shifting.

3 A simple model of macroeconomic and financial stability

We develop a simple model to illustrate the link between banks' asset portfolio choice, the risk of a crisis and policy decisions in line with our empirical analysis later on. At the core of our model are two market imperfections: a principal-agent problem whereby

Duprey and Roberts, 2017).

financial intermediaries maximize profits without taking shareholder welfare into account, and the combination of limited liability and a mispriced deposit insurance. These two features create a risk-shifting problem, with banks investing in too many high-risk projects.[6] Policy-makers can alleviate the risk-shifting problem by raising the policy rate or tightening the capital adequacy constraint of banks. The policy-makers' decisions are mapped into a central and tail risk choice set.

3.1 Model setup

The model has two periods and three actors: households that save, banks with risky and risk-free projects, and the government setting the risk-free interest rate as well as a capital adequacy requirement for banks.[7]

There is one source of aggregate risk in the model that renders high-risk projects unproductive. To keep things simple, we assume that the high-risk projects' productivity is given by a two state $S = \{B, G\}$ distribution:

$$z_r = \left\{ \begin{array}{ccc} \bar{z}_r & \text{with probability} & \begin{array}{cc} 1-p & \text{in the good state G} \\ p & \text{in the bad state B} \end{array} \\ 0 & & \end{array} \right\}.$$

For the low-risk productivity, we assume that $z_s = \bar{z}_s$, where $0 < \bar{z}_s < \bar{z}_r$. This is a simplification. The empirical distribution of GDP growth over the last few decades would look like a bimodal distribution with a fat left tail: see dashed line in Figure 1. To maintain analytic tractability, our theoretical model focuses on a degenerated version of this distribution with only two states and a low likelihood p of the bad state.

With this in mind, the typical banking problem becomes

$$\begin{aligned} \max_{d,x\geq 0} \quad & E\left[\max\left(0, z_r\left(S\right)\left(xd\right)^{\theta_r} - Rxd\right) + \max\left(0, \bar{z}_s\left(\left(1-x\right)d\right)^{\theta_s} - R\left(1-x\right)d\right)\right] \\ s.t. \quad & \\ x \quad & \leq \quad \eta. \end{aligned}$$

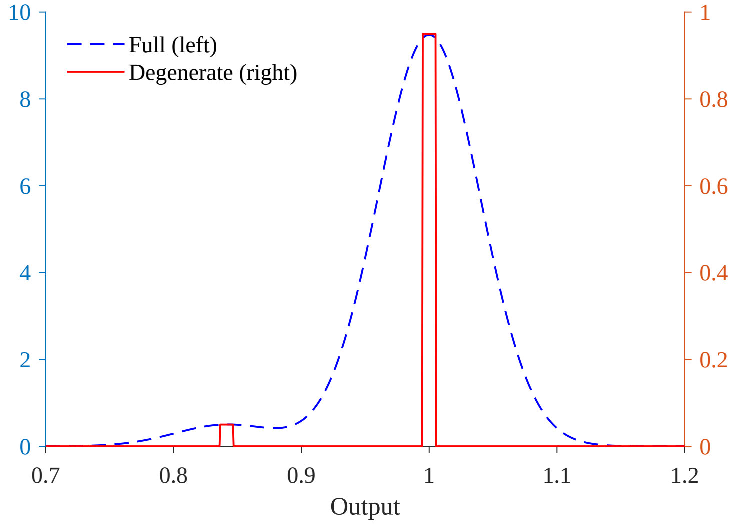
Here d is the total amount of deposits, x is the share of deposits invested in high-risk projects, and R is the interest rate set by the government. To contain their risk exposure, financial intermediaries create special-purpose vehicles. This allows them to contain their exposure to the high-risk projects. To curb the risk taking of banks, the government has imposed a capital adequacy constraint η.

Throughout this section, we impose a restriction on the relative capital shares of the

[6] We could also follow the crisis risk literature as implemented in Alpanda and Ueberfeldt (2016) to find similar results to what is in the text.

[7] We assume that the projects are efficiently run by the banks, abstracting from borrower-lender information problems. This simplifies our analysis without qualitatively changing the results, albeit with a weaker amplification channel.

Figure 1: Simplified distribution based on two normal distributions



two technologies.

Assumption 1. *The capital income share is higher for the risky technology than for the safe one, i.e.* $\theta_r > \theta_s$*.*

The first-order conditions summarized below imply that regulation and monetary policy interact, though not necessarily in a linear way:

$$x = \frac{\left(\frac{\bar{z}_r\theta_r}{R}\right)^{\frac{1}{1-\theta_r}}}{\left(\frac{\bar{z}_s\theta_s}{R}\right)^{\frac{1}{1-\theta_s}} + \left(\frac{\bar{z}_r\theta_r}{R}\right)^{\frac{1}{1-\theta_r}}} \leq \eta$$

$$\left(\begin{array}{c} d = \left(\frac{\bar{z}_s\theta_s}{R}\right)^{\frac{1}{1-\theta_s}} \frac{1}{(1-x)} \\ (d)^{\theta_r-1} p\bar{z}_r\theta_r\eta^{\theta_r} + (d)^{\theta_s-1} \bar{z}_s\theta_s (1-\eta)^{\theta_s} = (1-(1-p)\eta) R \end{array} \quad \text{for} \quad x \begin{pmatrix} < \\ = \end{pmatrix} \eta \right).$$

Proposition 1. *The share x of high-risk investments is decreasing in the interest rate.*

Proof. To show this point, we take the derivative with respect to the interest rate:

$$\frac{\partial x}{\partial R} = \left(\begin{array}{c} \frac{-1\frac{\theta_r-\theta_s}{(1-\theta_s)(1-\theta_r)}(\bar{z}_r\theta_r)^{\frac{-1}{1-\theta_r}}(\bar{z}_s\theta_s)^{\frac{1}{1-\theta_s}}(R)^{\frac{\theta_r-\theta_s}{(1-\theta_s)(1-\theta_r)}-1}}{\left(1+(\bar{z}_r\theta_r)^{\frac{-1}{1-\theta_r}}(\bar{z}_s\theta_s)^{\frac{1}{1-\theta_s}}(R)^{\frac{\theta_r-\theta_s}{(1-\theta_s)(1-\theta_r)}}\right)^2} < 0 \\ 0 \end{array} \quad \text{for} \quad x \begin{pmatrix} < \\ = \end{pmatrix} \eta \right).$$

This result holds given Assumption 1. □

Here we capture the risk-shifting idea, as suggested in the literature, according to which lower rates lead to more risk taking by financial intermediaries, see, for example, Jiménez et al. (2014) and Ioannidou, Ongena and Peydró (2015), with a stronger effect for banks with less capital. Note that risk might still increase even if x does not respond to lower rates.

Proposition 2. *For a given interest rate, the share of high-risk investment only increases in the regulatory requirement up to some optimal level and is constant afterwards.*

Proof. For a given interest rate, $\tilde{R}$, there exists a bank problem optimal share of high-risk investment:

$$\tilde{x} = \left(1 + \left(\bar{z}_r \theta_r\right)^{\frac{-1}{1-\theta_r}} \left(\bar{z}_s \theta_s\right)^{\frac{1}{1-\theta_s}} \left(\tilde{R}\right)^{\frac{\theta_r - \theta_s}{(1-\theta_s)(1-\theta_r)}}\right)^{-1}.$$

If the regulatory requirement is binding, i.e. $\eta < \tilde{x}$, then the share of high-risk investment moves one for one with the regulatory requirement. For any $\eta > \tilde{x}$, the intermediary will not increase its share of high-risk projects any further. □

Proposition 3. *The demand for deposits is decreasing in the interest rate.*

Proof. This result can be shown by considering the derivative of deposit demand with respect to the interest rate using the implicit function theorem:

$$\frac{\partial d}{\partial R} = \left(\begin{array}{c} -\left(\bar{z}_s \theta_s\right)^{\frac{1}{1-\theta_s}} (1-x)^{-2} (R)^{-\frac{1}{1-\theta_s}-1} \left(\frac{1}{1-\theta_s}(1-x) + \left(-\frac{\partial x}{\partial R}\right) R\right) \\ -\frac{(1-(1-p)\eta)}{\left((1-\theta_r)(d)^{\theta_r - 2} p \bar{z}_r \theta_r \eta^{\theta_r} + (1-\theta_s)(d)^{\theta_s - 2} \bar{z}_s \theta_s (1-\eta)^{\theta_s}\right)} \end{array} \right) \leq 0 \quad for \quad x \left(\begin{array}{c} < \\ = \end{array} \right) \eta .$$

□

The proposition suggests that economic risk increases as interest rates go down even when the regulatory constrained is binding. This is due to the fact that the risk exposure has an extensive channel, x, and an intensive channel, d. It is worthwhile noting that the level of regulatory requirement has an impact on monetary policy effectiveness.

The remainder of the model is standard. There are households that save before uncertainy is revealed and whose deposits are insured:

$$\begin{aligned} &\max_{C_0, C_1, D} U(C_0) + \beta E\left(U\left(C_1(S)\right)\right) \\ s.t.& \\ &C_0 + D + A \leq W + T_0 \\ &C_1 \leq RD + R^a A + T_1, \end{aligned}$$

where C_t is consumption in period t, D are the deposits and A are government bonds. T_t captures government transfers, profits from banks and covers losses in case the deposit insurance has to step in. Households are endowed with wealth W. In equilibrium, the return to government bonds R^a and those to deposits R need to be equal.

There are three markets to clear: consumption (in the second period), the deposit and the bond market.

$$\begin{aligned} C_1(S) &= z_r(S)(xd)^{\theta_r} + \bar{z}_s((1-x)d)^{\theta_s} \\ D &= d \\ A &= 0 \end{aligned}$$

Monetary policy is implemented via the risk-free interest rate R^a on bonds and regulatory policy via the capital adequacy parameter η.

Next, we explore the consequence of policy actions on macroeconomic and financial stability. In the context of the model, we define those concepts as follows.

Definition 1. ***Central risk*** *(cr) is defined as the expected squared deviation of output from the welfare optimal level of output conditional on no crisis:*

$$cr = E\left[(Y - E(Y^*|\, p{=}0))^2\right].$$

The reference point is selected to reflect the common practice in policy institutions, whereby the projection model is focusing on normal times and tail risk is largely ignored.

Definition 2. ***Tail risk*** *(tr) is defined as the gap between expected output and the output given the realization of a bad state* $Y_B = \bar{z}_s((1-x)d)^{\theta_s}$*:*

$$tr = E(Y) - Y_B.$$

Definition 2 reflects financial stability concerns via the risk-taking channel of monetary policy or too loose regulatory requirements. Our tail risk metric is complementary to the central risk metric, since the former nets out changes in the central part of the distribution, while the latter abstracts from crisis risks. Thus a policy-maker concerned about tail risks will likely be more restrictive trading central against tail risk. In a more abstract sense, our definitions capture different aspects of the distribution with a limited overlap in information content.[8]

In Appendix A, we consider the shape of the indifference curves in the central and tail risk space. In Appendix B, we consider the equilibria in a simultaneous move game between the monetary and the regulatory authority. We find a substitution effect between policies across Nash equilibria: when policy rates are low (high), capital adequacy ratios would be tightened (relaxed).

[8] An alternative definition of tail risk could only focus on the output realization in a crisis, $t\tilde{r} = Y_b$, or instead of the mean use the median of the distribution as the reference point. While this would have quantitative consequences, it would not change the qualitative results and the trade-off we are emphasizing.

3.2 How does monetary policy influence central and tail risks?

In our setup, we assess the consequences of varying the policy rate (see Figure 2) for a given level of regulation.

As expected given our theoretical results, we find that lower interest rates lead both to more savings and to a higher share of risky investments by the banking sector. These, in turn, raise expected output. However, the higher expected output is exposed to significant risk and lower consumption today (implicit in the higher savings). Next, we consider the consequences of lower rates on welfare, i.e. expected utility. Here we find that too low rates lead to less welfare and that the welfare optimal interest rate is lower when we abstract from the possibility of a crisis (red circle). Thus a monetary authority that worries about crisis risk will set interest rates slightly higher than one that does not take it into account.

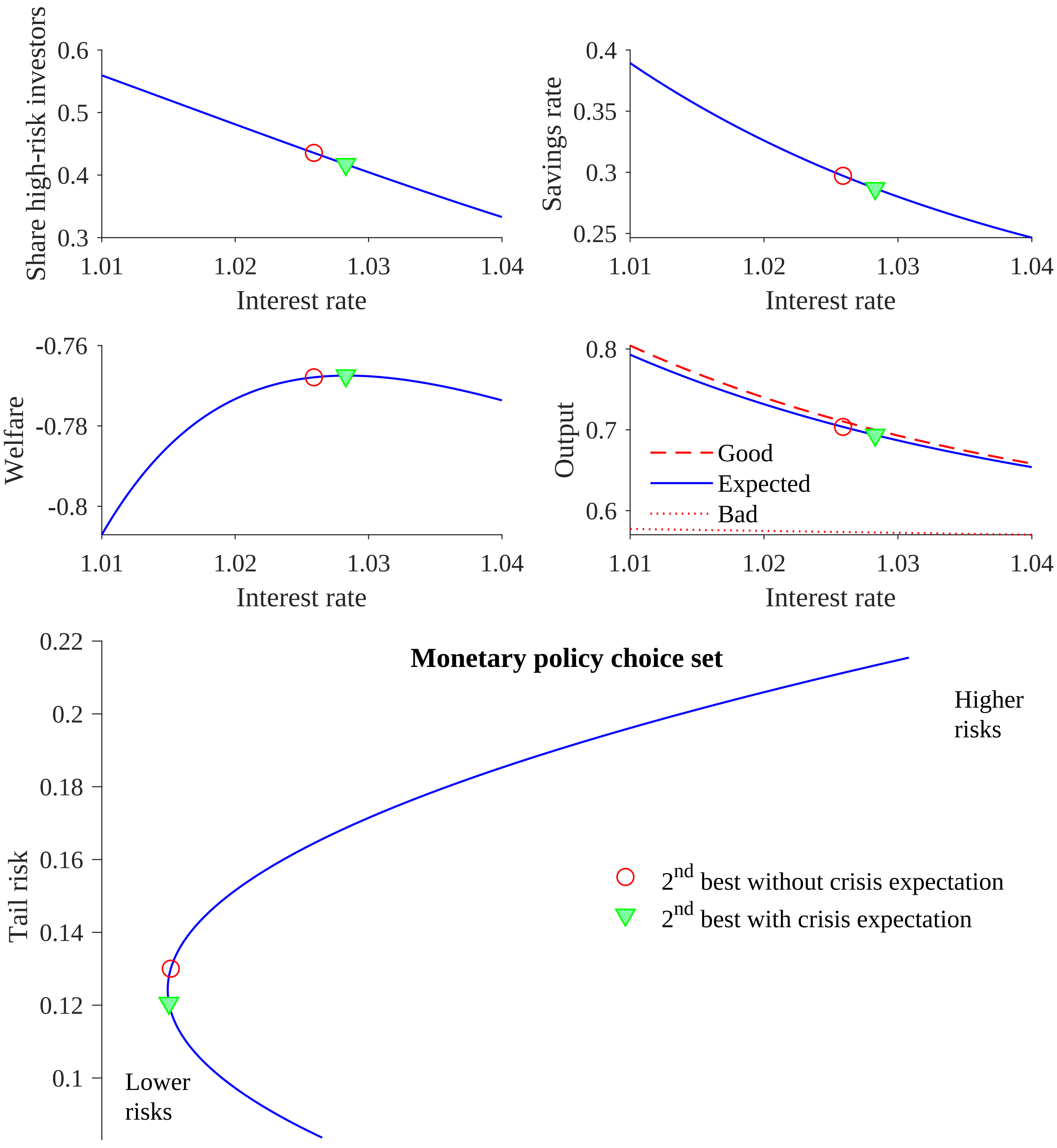
Finally, we show the choice set between central risk (horizontal axis) and tail risk (vertical axis) for various levels of the policy rate. This choice set resembles a parabola. The two arms are relative to the output that is optimal when there is no crisis risk (red circle). The upper arm is inefficient since both central and tail risks go up as the policy rate eases. There is too much risky investment and the economy is facing a larger decline in initial and expected consumption, due to the possibility of a bad shock realization. The bottom arm captures the key trade-off monetary policy faces. As the interest rate increases, central risk increases as output deviates from the no-crisis risk reference point, while tail risk decreases as the crisis risk goes down. A small amount of tightening is welfare increasing, from the red circle up to the green triangle. When the interest rate is too high, then welfare also decreases as the demand for deposits and the expected output starts to decline too much with negative implications for central risk.

So, policy-makers will face the trade-off on the lower arm with the optimal policy rate depending on the current economic conditions and tail risks. The trade-off between central and tail risks should be explicitly taken into account by policy-makers and lead to a slightly higher rate than what a no-crises projection suggests.

3.3 How does regulation influence central and tail risks?

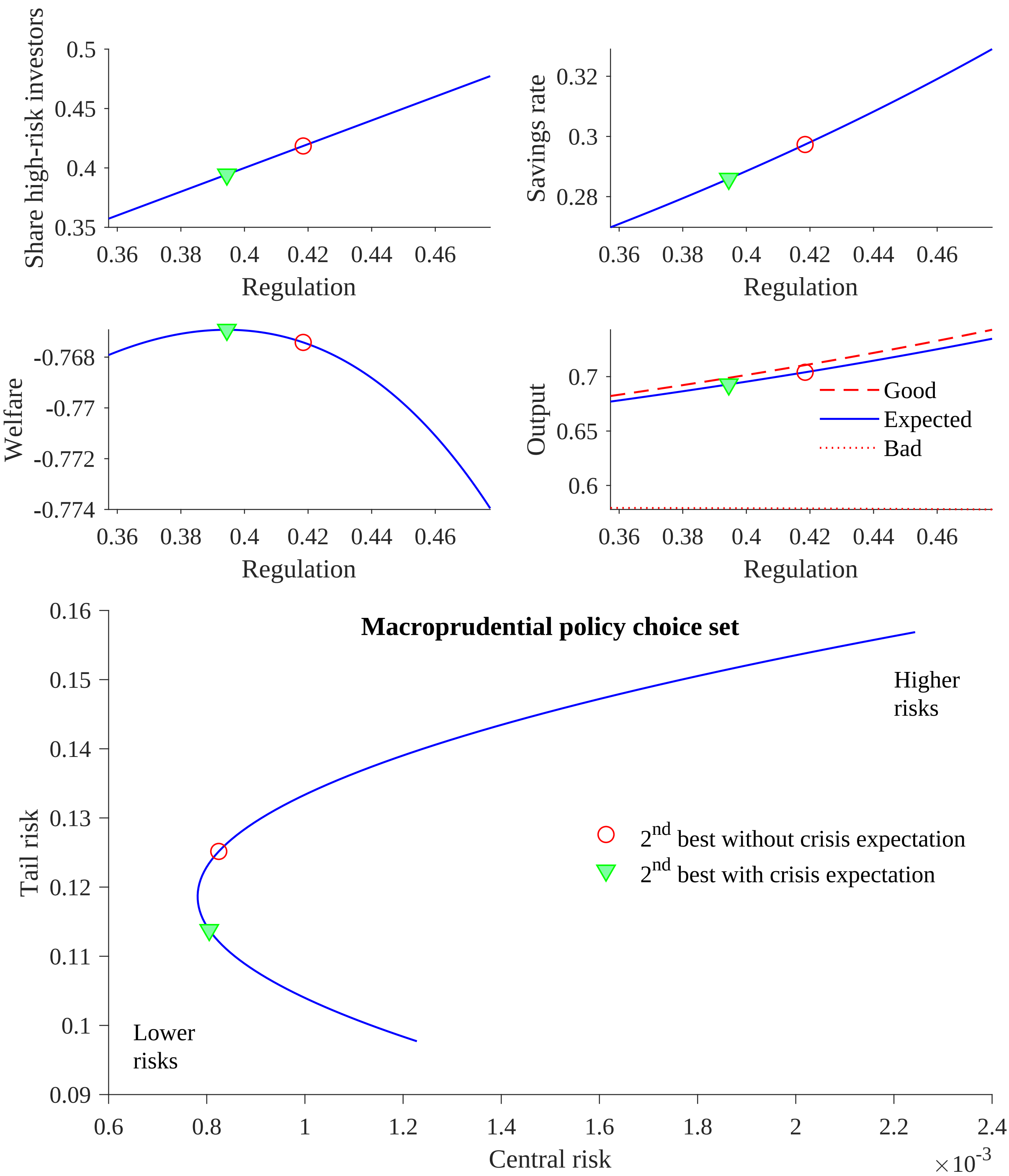
Next, we tighten regulation and consider the impact taking the monetary policy rate as given: see Figure 3. Not surprisingly, we find that restricting the share of high-risk investments also makes saving less attractive and thus leads to lower savings and lower expected output. Based on households' utility as our welfare criterion, we find that an optimal regulation exists and is tighter when taking the crisis risk into account. We also find a parabola-shaped policy choice set. Tighter regulation reduces tail risk by reducing the resources allocated to risky projects. But beyond a certain level, tighter regulation generates a trade-off, as it also reduces expected output below the welfare optimum.

Figure 2: Outcome for various levels of the interest rate



Notes: Simulations use CRRA preferences and are for the parameter values $\sigma = 1.5; \beta = 0.99; \bar{z}_r = 0.9947; \bar{z}_s = 0.98; p = 0.05; W = 1; \sigma_r = 0.97; \sigma_s = 0.3$. The policy rate R^a varies while the capital adequacy ratio $\eta = 1$ is fixed.

Figure 3: Outcome for various levels of the macroprudential regulation



Notes: Simulations use CRRA preferences and are for the parameter values $\sigma = 1.5; \beta = 0.99; \bar{z}_r = 0.9947; \bar{z}_s = 0.98; p = 0.05; W = 1; \sigma_r = 0.97; \sigma_s = 0.3$. Regulation, in the form of the capital adequacy ratio η, is varying. A tighter regulation means a lower fraction η of deposits allocated to risky projects. The policy rate $R^a = 1.0283$ is fixed.

The next sections present an empirical framework to analyze central and tail risks and lead to a policy-maker choice set comparable to the one in the simple model.

4 An empirical framework to analyze macroeconomic and financial stability

Our empirical investigation follows two steps, outlined in Figure 4 and applied to quarterly Canadian data over 1982-2018.

First, we estimate a VAR to generate the mean response to structural shocks. We focus on the impact of monetary and macroprudential shocks on the most likely path of future GDP growth. We are able to capture macroprudential policy shocks since Canada is one of the few countries with a long history of household-related macroprudential policy changes. In particular, both shocks can also have an indirect effect on GDP growth via their influence on financial vulnerabilities.

Second, we estimate a quantile regression on the same dataset to recover the distribution of future GDP growth. In particular, we focus on potential differences between the movement of the median and the lower tail of the GDP growth distribution.

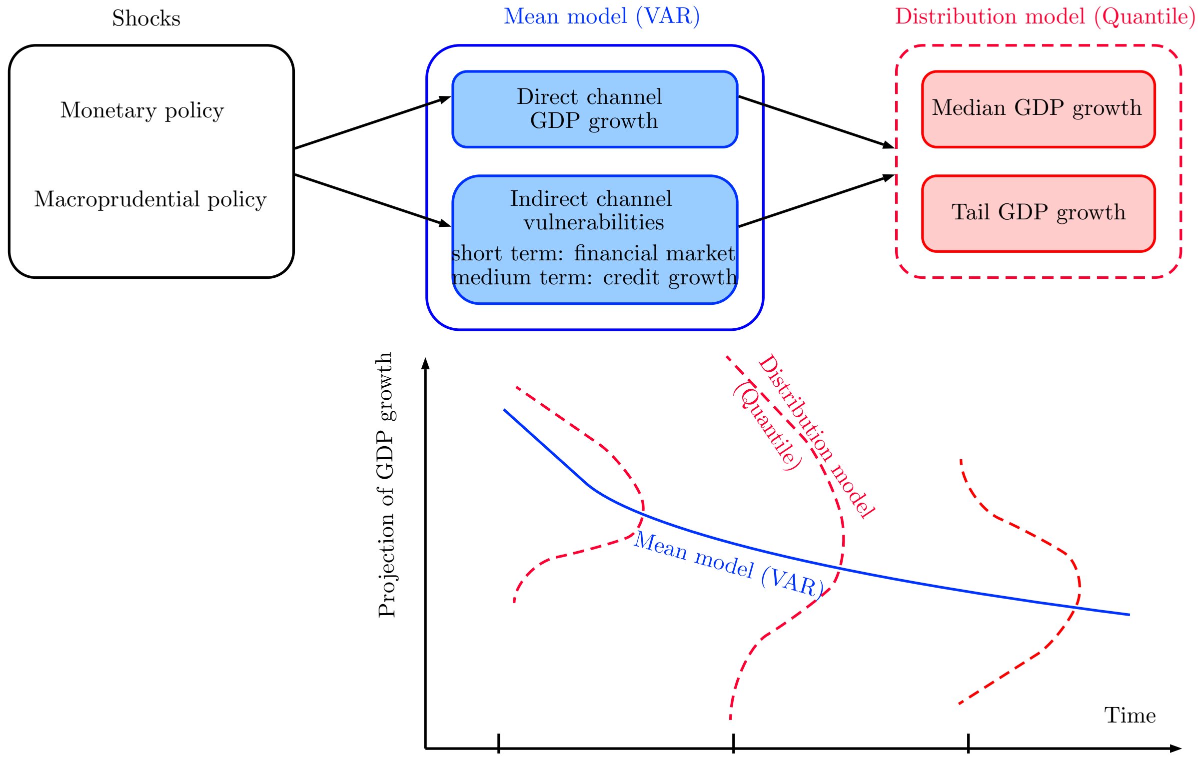
By inserting the impulse response of the VAR into the quantile regression, we can track the impact of a monetary or macroprudential shock on the distribution of future GDP growth (dashed line in the chart of Figure 4). This allows us to test for the existence of a potential indirect benefit of policy tightening through the reduction of financial vulnerabilities that could, in turn, alleviate downside risk to future GDP growth.

We estimate both models separately, acknowledging that Chavleishvili and Manganelli (2019) provide a framework that combines the auto-regression with the quantile estimation. Their approach allows for shocks to specific quantiles of the distribution of future GDP growth. However, it does not allow for the identification of structural shocks, which is important in our context. Hence, we identify structural shocks at the cost of separately estimating the mean and the distribution around it.

4.1 Projection of the mean: structural VAR

We project the most likely state of the economy using the VAR of Equation (1) that includes the year-over-year growth of real GDP (GDP), the year-over-year growth of the consumer price index (CPI), the year-over-year change in the policy rate ($Rate$), the year-over-year growth of real household credit from banks ($Credit$) and the average intensity of the country-level index of financial stress (FSI) of Duprey, Klaus and Peltonen (2017). We estimate the model with year-over-year growth rates to follow existing empirical evidence suggesting that credit growth accumulated over multiple periods is a

Figure 4: Sequential integration of a structural vector autoregression with a quantile model



stronger signal of tail GDP risks (Schularick and Taylor, 2012).[9] Note, in our benchmark specification, we use credit growth rather than the ratio of credit to GDP, since the latter tends to be procyclical: the numerator is slower moving than the denominator, such that short- and medium-run dynamics are entangled (Bauer and Granziera, 2017).

The VAR has two exogenous variables.[10] First, the sequence of monetary policy shocks (MP) is directly taken from Champagne and Sekkel (2018), who rely on a narrative approach.[11] Second, we use an index of macroprudential policy changes (MAP) whose construction, based on narrative evidence, is discussed in the next section.

$$\begin{bmatrix} GDP_t \\ CPI_t \\ Rate_t \\ Credit_t \\ FSI_t \end{bmatrix} = \alpha + \sum_{p=1}^{P} \beta_p \begin{bmatrix} GDP_{t-p} \\ CPI_{t-p} \\ Rate_{t-p} \\ Credit_{t-p} \\ FSI_{t-p} \end{bmatrix} + \xi MP_t + \sum_{l=0}^{L} \zeta_l MAP_{t-l} + \epsilon_t \tag{1}$$

The choice of the lag order of the VAR(P, L) is informed by standard information criterion (Table 1). As expected, the Akaike favors many more lags P as the metric

[9] They show that two-year credit growth is an excellent predictor of a future banking crisis.

[10] Given that Canada is a small open economy, we control for the US federal funds rate: see Appendix E.

[11] Alternative identifications (sign restrictions and Cholesky) are left as robustness tests: see Appendix E. As expected, ξ in Equation (1) is close to one in the equation for $Rate_t$.

continues to fall when the lag order increases. Instead, alternative metrics that put more weight on model parsimony favor $P = 1$. As a benchmark, we use $P = 4$ to strike a balance between flexibility and parsimony and to be consistent with the choice of year-over-year growth rates.[12]

Table 1: Information criterion to guide the choice of lag order in the VAR

VAP(P,L)	(1,0)	(2,0)	(3,0)	(4,0)	(1,1)	(2,1)	(3,1)	(4,1)	(1,2)	(2,2)	(3,2)	(4,2)
Akaike	1.1	1.0	1.1	**0.9**	1.2	1.0	1.2	**0.9**	1.2	1.0	1.2	**0.9**
Hannan-Quinn	**1.4**	1.5	1.9	1.9	1.5	1.6	1.9	1.9	1.6	1.7	2.0	2.0
Schwarz	**1.9**	2.3	3.0	3.3	2.0	2.4	3.1	3.4	2.2	2.5	3.2	3.5
Final Prediction Error	3.1	2.7	3.1	**2.5**	3.3	2.8	3.2	**2.5**	3.3	2.8	3.3	2.6

Note: Information criteria are used as a guide to choose the lag order of the autoregressive part of the VAR (P) and the persistence in the macroprudential policy index (L). The numbers in bold show the lag order that is best according to each criterion.

4.2 Identification of macroprudential shocks

Macroprudential policy shocks are notoriously harder to identify than monetary policy shocks. We rely on the narrative approach and build our own index that records the number of macroprudential policy changes implemented in each quarter (Equation (2)). When several measures are implemented in the same quarter, our measures focus on the net effect. However, historically, we see that in such cases all measures went in the same direction of either all easing or all tightening.

$$MAP_t = \# \text{ of tightening measures at time t} - \# \text{ of easing measures at time t} \quad (2)$$

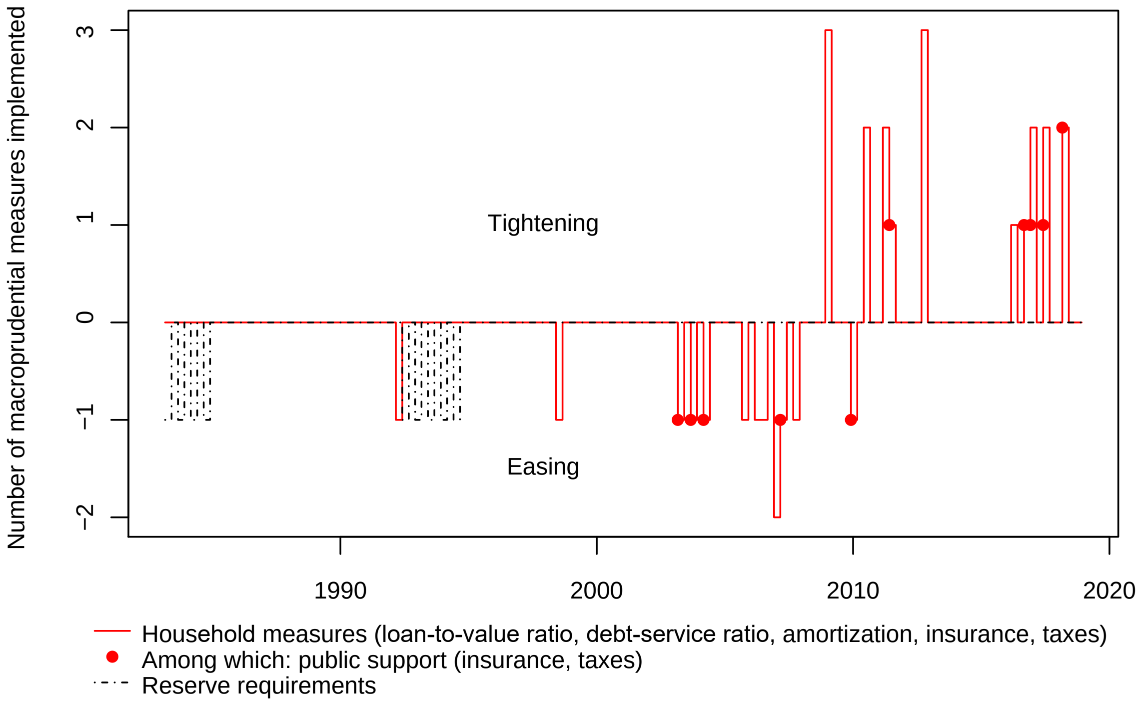
One macroprudential tightening (easing) increases (decreases) the macroprudential index by one unit: see Figure 5. The index combines the main macroprudential instruments used in Canada, the mortgage loan-to-value ratio, the debt-service ratio, rules regarding the mortgage amortization period and access to government-sponsored mortgage insurance.[13] We also include federal or provincial tax changes that directly target the housing market. Macroprudential measures covered by our index are listed in Appendix C.

The index relies on four sources: Cheung (2014), Kuttner and Shim (2016), Allen

[12]Our results are robust to alternative lag orders: see Appendix D.

[13]Macroprudential policy in Canada mostly channels through the mortgage insurance provided by the federal government. The Crown corporation Canada Mortgage and Housing Corporation (CMHC) is the main provider of mortgage insurance with 100 percent public backing, while other private mortgage insurers benefit from a 90 percent public guarantee. Loans with a loan-to-value ratio larger than 80 percent must be insured and, as such, follow strict issuance rules on debt servicing and amortization.

Figure 5: Index of macroprudential policy changes



Notes: The index relies on four sources: Cheung (2014), Kuttner and Shim (2016), Allen et al. (2017) and Bank of Canada (2017). The list of macroprudential events included in the index are listed in Appendix C. A positive (negative) change by one unit in a given quarter consists in the implementation of a tightening (easing) of one macroprudential measure.

et al. (2017) and Bank of Canada (2017).[14] Canada implemented 31 policy measures that qualify as household-related macroprudential policy changes,[15] with easing mostly taking place from 1992 to 2008 and tightening from 2008 to 2018. Given the discrete nature of the index, a one-unit macroprudential policy shock should be understood as the historical average impact of one macroprudential change. We cannot assess the magnitude of the impact of specific macroprudential policy measures.

Similar to most of the existing empirical literature on the effect of macroprudential changes (Galati and Moessner, 2018), we cannot properly account for the endogeneity of macroprudential decisions and recover the unexpected macroprudential policy changes. Existing research either uses micro-level data (individual behaviors are unlikely to trigger a policy change) or relies on a panel estimation across countries. In a panel, Cerutti, Claessens and Laeven (2017) use lagged macroeconomic variables as instruments to partly control for the expected macroprudential policy change. This suggests that we should not have more lags for the macroprudential index than other macroeconomic variables ($L \leq P$). In Section 7.3, we use the part of the index that cannot be predicted by other macro variables and perform placebo tests. In both cases we confirm that even if macroprudential measures were partly anticipated, it would not change our results.

Our estimation strategy further relies on the assumption that macroprudential measures do not respond to contemporaneous shocks (demand, supply, monetary). This is consistent with the longer decision process that reflects the slow build-up of vulnerabilities. Also, in Canada, macroprudential policy is influenced by several policy-makers (central bank, regulatory authority, Ministry of Finance) who may each have different reaction functions, such that decisions cannot be fully endogenous within a quarter.[16]

From Table 1, all information criteria suggest $L = 0$, i.e. only a contemporaneous impact of the macroprudential change. But Kuncl (2016) shows in the Canadian context that the effect of macroprudential changes is not always immediate. Thus we prefer to allow for a delayed effect of macroprudential changes. As expected, when $L > 0$, GDP and credit growth fall when there is a macroprudential tightening, while there is no significant impact if $L = 0$, as the macroprudential shock is no longer allowed to be persistent. For simplicity, we pick $L = P > 0$ in all our experiments, unless otherwise specified.

[14] At the time of writing this article, the cross-country IMF database on macroprudential measures (Alam et al., 2019) did not have the same granularity for Canada as our compilation.

[15] Kuttner and Shim (2016) also include changes to bank reserve requirements as macroprudential policy measures for Canada, but we leave these for robustness checks (Section 7.2) and instead focus on household-related measures.

[16] Treating the macroprudential index as an endogenous variable would not change our qualitative results. But treating a sequence of dummy variables as endogenous would create additional econometric challenges. Conversely, a macroprudential measure like the recently introduced countercyclical capital buffer should be modeled as an endogenous continuous variable.

4.3 Projection of the tail: quantile regression

Once we are equipped with a mean forecast following a monetary or macroprudential policy shock, we project the associated distribution of future GDP growth using quantile regressions (Koenker and Bassett, 1978). Equation (3) is estimated with the same data as the VAR.[17] We focus on the predicted median and fifth percentiles ($\tau = .50$ or $\tau = .05$) of real GDP growth one year ahead:[18]

$$\mathbb{Q}_{GDP,t}(\tau) = \alpha(\tau) + \gamma(\tau) \begin{bmatrix} GDP_{t-q} \\ CPI_{t-q} \\ Rate_{t-q} \\ Credit_{t-q} \\ FSI_{t-q} \end{bmatrix} \text{ for } \tau = \{.50, .05\}. \quad (3)$$

The choice of the lag order of the quantile regression (q) is not irrelevant to our goal: see Figure 6 and Table 2. Financial stress, which is harder to influence with policy actions, is the main explanatory variable driving tail GDP growth for projections up to one year ahead. The impact of financial stress on tail GDP growth diminishes and disappears beyond one year. Credit growth, a variable that policy-makers can influence, is more important for projections one year ahead or more.[19] We choose a lag of $q = 4$ quarters to strike a balance between the short- and medium-term drivers of the tail of future GDP growth.[20]

Growth at risk (Adrian, Boyarchenko and Giannone, 2019; International Monetary Fund, 2017*a*) is defined as $\mathbb{Q}_{GDP,t+4}(\tau{=}.05)$. It relies on the fifth percentile of the distribution of future GDP growth, compromising between estimation challenges and the need to quantify tail risks.[21] The empirical counterpart to Definition 2 of financial stability is as follows.

[17] We use the R package `quantreg` developed by Koenker (2013). For simplicity, we do not use the CAViaR method of Engle and Manganelli (2004), who introduce autoregressive unobserved quantiles as explanatory variables. This would make the computation of the impulse response derived from the combination of the VAR and the quantile regression very complex. For a recent effort dedicated to the issue of quantile impulse response function, see Chavleishvili and Manganelli (2017).

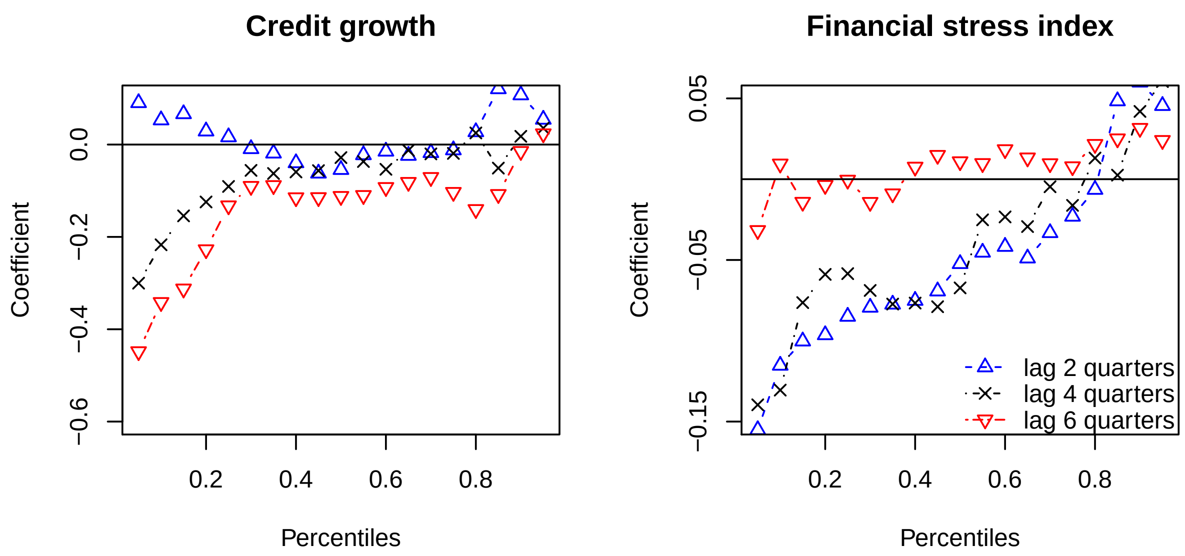
[18] To gauge the validity of our setup, we computed the expectation over the projection of the whole distribution of future GDP growth. We find that, compared to actual realizations since 1997, forecasts have, on average, the correct sign of the change in GDP growth two thirds of the time, similar to the ratio obtained with forecasts from the *Monetary Policy Reports* of the Bank of Canada (Binette and Tchebotarev, 2017). McDonald et al. (2016) found that density forecasts published by the Bank of England in its quarterly *Inflation Report* since 1996 outperform conventional autoregressive models in the context of inflation targeting.

[19] Although one may argue that Canada did not experience many crises, similar patterns for financial stress and credit growth are observed in a cross-country quantile regression: see Appendix F.

[20] However, our results would be stronger with $q = 6$. We leave it as a robustness test: see Appendix D.

[21] As a robustness test, we instead use the tenth percentile: see Appendix D. Throughout the paper, we also report growth at risk separately, which can be thought of as an alternative tail risk measure that does not net out the effect of the median.

Figure 6: Choice of lag order for the quantile regression



Note: The figure displays the coefficients of Equation (3) for credit growth and financial stress (vertical axis) by quantile (horizontal axis) for a lag order q of the quantile regression of 2, 4 or 6. This figure uses Canadian data, but a similar figure using a cross-country dataset can be found in Appendix F.

Table 2: Median and fifth percentile coefficients per lag order q

q	1	2	3	4	5	6	7	8
Coefficient on credit growth								
$\tau = .50$	0.013	-0.054*	-0.083	-0.028	-0.087**	-0.112**	-0.094	-0.075
$\tau = .05$	0.090**	0.090	-0.034	-0.300***	-0.473***	-0.448***	-0.343***	-0.365**
Coefficient on financial stress								
$\tau = .50$	-0.046***	-0.052**	-0.064**	-0.067*	-0.022	0.011	0.025	0.014
$\tau = .05$	-0.053***	-0.155***	-0.179***	-0.140***	-0.052	-0.032	0.024	0.069*

Note: Coefficients of the median ($\tau = .50$) and the fifth percentile ($\tau = .05$) obtained by running the quantile regression of Equation (3) eight times for eight different lag orders q. Significance levels are computed using the wild bootstrap of re-sampled residuals of Feng, He and Hu (2011).
* p-value<0.1; ** p-value<0.05; *** p-value<0.01.

Definition 3. *Empirically, **tail risk** is defined as the gap between the median and the fifth percentile of GDP growth four quarters ahead:*

$$\mathbb{Q}_{GDP,t+4}(\tau{=}.50) - \mathbb{Q}_{GDP,t+4}(\tau{=}.05).$$

4.4 Computation of confidence bands for impulse responses

Impulse responses to structural shocks are obtained in the usual way. But the impulse response of the median and fifth percentiles of future GDP growth cannot be directly obtained from the quantile regressions. The structural shocks originate in the VAR decomposition, not in the quantile regression. In order to compute the impulse response of the median and fifth percentiles of future GDP growth, we combine the point estimate of the VAR impulse responses for all the variables with the associated estimated parameters from the quantile regression.

Our modeling strategy aims at capturing non-Gaussian distributions and potential changes in the skewness of the distribution of future GDP growth. Therefore we cannot rely on standard distributional assumptions to compute the confidence bands. The quantile regressions allow for a non-Gaussian distribution of future GDP growth. In the VAR, once credit growth and financial stress are controlled for, we do not find strong evidence of skewness in the residual for GDP growth.[22]

To preserve the correlation over time, we re-sample the residuals (rather than the observations) of the VAR and the quantile regressions. The algorithm to produce the confidence bands of both models is described below:

1. Re-sample the residuals of Equation (1) associated with N random draws from a uniform distribution $U \sim [1; N]$ and create a new dataset given the initial estimates $\{\widehat{\alpha}, \widehat{\beta_P}, \widehat{\xi}, \widehat{\zeta_l}\}$.

2. Re-estimate the VAR of Equation (1) on the new dataset and obtain new parameter estimates $\{\widetilde{\alpha}, \widetilde{\beta_P}, \widetilde{\xi}, \widetilde{\zeta_l}\}$. We implement the bootstrap-based small sample error correction method of Kilian (1998). If the new estimated VAR model is not stable, go back to Step 1.

3. Produce the impulse response $\widetilde{IRF}_{VAR}$ using the new parameter estimates $\{\widetilde{\alpha}, \widetilde{\beta_P}, \widetilde{\xi}, \widetilde{\zeta_l}\}$.

4. For each $\tau = \{5, 50\}$, re-sample the residuals of Equations (3) using the method of Feng, He and Hu (2011) and create a new dataset given the initial estimates $\{\widehat{\alpha(\tau)}, \widehat{\gamma(\tau)}\}$.

[22] Quantile-quantile plots fail to reject the absence of skewness in the residual of GDP growth, but for other variables a rejection is possible.

5. For each $\tau = \{5, 50\}$, re-estimate the quantile regressions of Equation (3) on the new dataset and obtain new parameter estimates $\{\widetilde{\alpha(\tau)}, \widetilde{\gamma(\tau)}\}$.

6. For each $\tau = \{5, 50\}$, combine the impulse responses $\widetilde{IRF}_{VAR}$ with the new parameter estimates of the quantile regression $\{\widetilde{\alpha(\tau)}, \widetilde{\gamma(\tau)}\}$ to obtain the impulse response of GDP growth $\widetilde{IRF}_{GDP(\tau)}$.

7. Compute the difference of the impulse responses of the median $\widetilde{IRF}_{GDP(\tau=50)}$ and the impulse responses of the fifth percentile $\widetilde{IRF}_{GDP(\tau=5)}$ to obtain the impulse response of the difference between the median and the tail $\widetilde{IRF}_{GDP(\Delta\tau)}$.

8. Repeat Steps 1-7 B=5000 times and compute the 80th and 90th percentile confidence intervals of the impulse responses $\{\widetilde{IRF}_{VAR};\ \widetilde{IRF}_{GDP(\tau)};\ \widetilde{IRF}_{GDP(\Delta\tau)}\}$, which correspond to a one-sided test at the 10 and 5 percent significance level.

Note that the confidence bands of the impulse response generated for the median and the fifth percentile of future GDP growth reflect the uncertainty around two models: the VAR and the quantile regressions. Similarly, the confidence bands of the difference between the median and the fifth percentile reflect the uncertainty around three models: the VAR and both quantile regressions for the median and the fifth percentile. Because of this cumulated uncertainty across multiple models, we may want to use a looser significance level for the impulse response that includes quantile estimates. Therefore, all our impulse response analyses report two significance levels that correspond to the one-sided test at 10 and 5 percent.

5 Empirical results for Canada

5.1 Drivers of the distribution of GDP growth

We analyze how the distribution of GDP growth moves given initial conditions by looking at the estimated coefficients of the quantile regression: see Figure 7. The parameter for GDP growth is positive and significant, reflecting some momentum four quarters ahead. But the quantile estimates (dotted black line) lie within the confidence bands of the ordinary least square (OLS) estimate (dashed red horizontal lines): we cannot differentiate both estimates, such that the momentum in GDP does not change the shape of the distribution.

The parameter for the policy rate is negative and significant, suggesting a persistent impact of a policy rate change on the future path of GDP growth. Here again, the quantile estimates lie within the confidence bands of the OLS estimate, except for the higher percentiles. This suggests that the direct impact of the policy rate on GDP growth at a one-year horizon is mostly a location shift, except for the upper tail, where monetary policy is somewhat less effective. If a policy rate shock has a non-linear effect on the lower part of the distribution of future GDP growth, it will be mostly via its indirect impact through other variables.

Variables that impact the shape of the distribution of future GDP growth are either credit growth or financial market stress.[23] Household bank credit growth has a stronger negative impact on GDP growth at lower percentiles with no significant impact on the middle and upper part of the distribution. In the tail, the negative relationship between credit and GDP growth is understated when compared to the linear OLS estimator. Financial stress also has a large negative impact on the left tail of future GDP growth, consistent with the results of Adrian, Boyarchenko and Giannone (2019), with no significant impact on the highest deciles.

5.2 Monetary policy shock

5.2.1 Mean effect of the shock

To determine the effect of a monetary policy tightening, we analyze the impulse response function following an unexpected 100-basis-point tightening: see Figure 8. As expected, GDP growth is reduced and inflation is weaker one year after the shock. Notably, credit growth slows, while financial market stress barely increases in the first year.

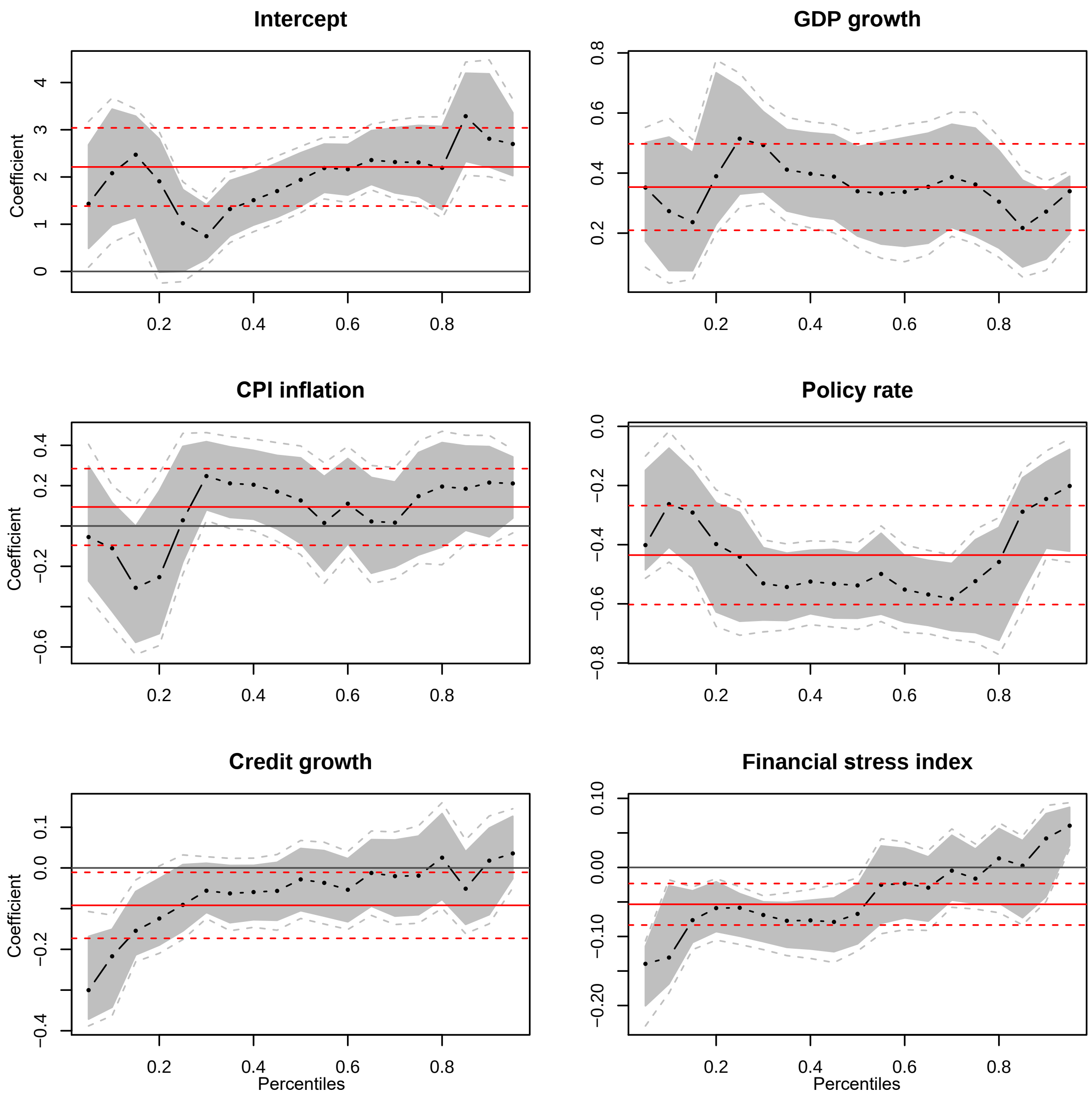
5.2.2 Distributional effect of the shock

Combining the results of the VAR and those of the quantile regression, we can construct the response of the lower fifth percentile and the median of the four-quarter-ahead GDP growth. The bottom of Figure 8 shows that both the median and the fifth percentile of GDP growth four quarters ahead decrease on impact. The magnitude is consistent with the expected GDP growth after four quarters obtained from the VAR (top left of Figure 8 from period 4 onwards). Tail risk, i.e. the distance between the median and the fifth percentile four quarters ahead, is not significant at conventional levels except for the fourth quarter, i.e. the expectation formed at quarter t+4 about GDP growth at t+8.

We now decompose the drivers of the movement in tail risk: see Figure 9. The distance between the center and the tail of the distribution comes from the negative impact of monetary policy tightening on credit growth. After an unexpected 100-basis-point increase in the policy rate, the slower credit growth alleviates the drag on the tail of future GDP growth, which significantly narrows the distribution of future GDP growth by 20 basis points (middle left). Because the effect of financial market stress goes in the opposite direction (middle right) and the other drivers of future GDP growth are

[23]For both variables, the fifth percentile is statistically different from the median at the 95th percentile confidence band.

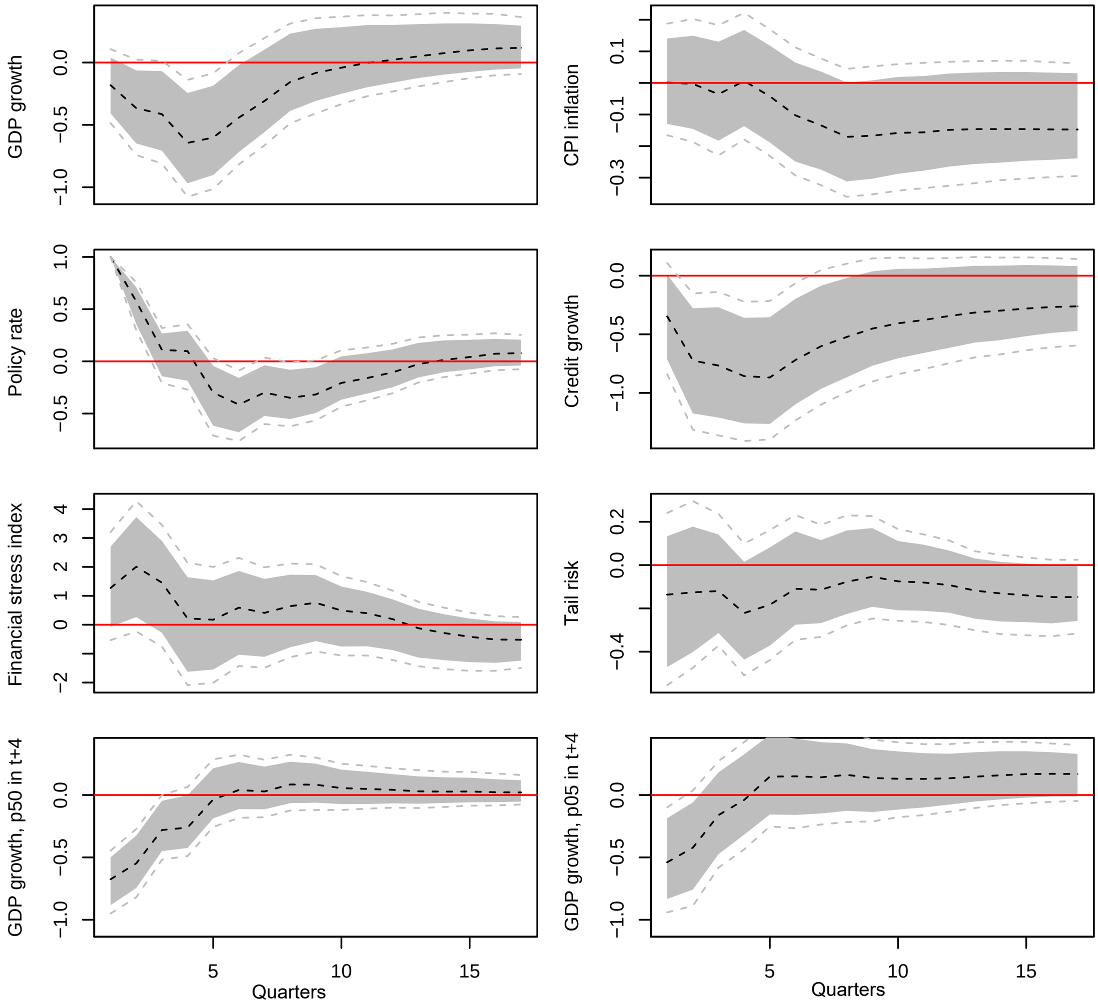
Figure 7: Quantile regression coefficients



Note: The figure displays the coefficient of Equation (3) for each variable (vertical axis) for each percentile of the distribution of GDP growth four quarters ahead (horizontal axis). The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals. The three horizontal plain and dashed red lines correspond to the equivalent ordinary least square model with the one-sided significance level of 0.05.

not significant, the overall indirect effect captured by the tail risk metric is often not significant (top left).

Figure 8: Response after a 100-basis-point monetary policy tightening



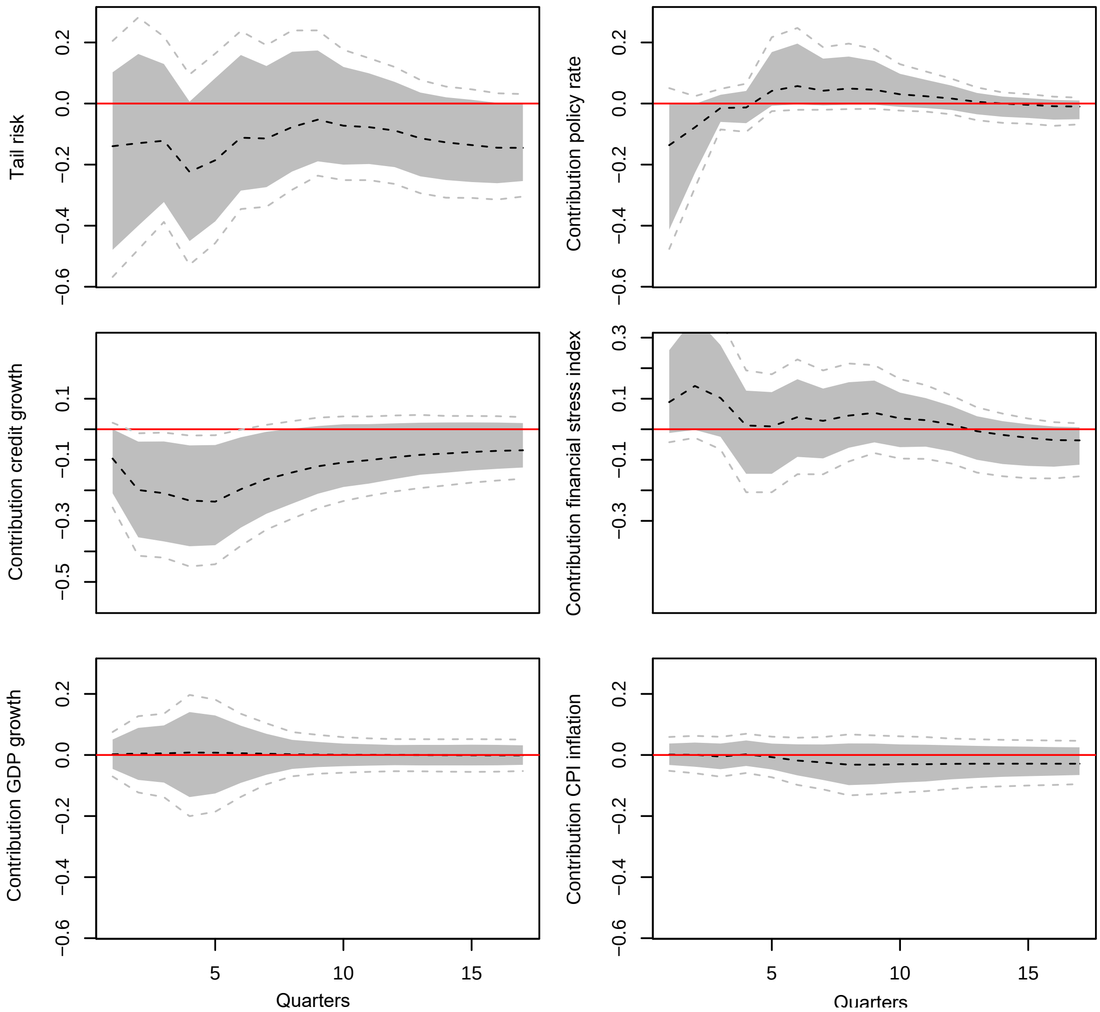
Note: The figure displays the impulse response function over 16 quarters following a 100-basis-point increase in the monetary policy rate. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals. GDP growth, inflation, policy rate, credit growth and financial stress index are the variables from the structural VAR. The VAR uses four lags, and shocks are identified with the narrative method. Tail risk is the difference between the median (p50) and the fifth percentile (p05) of the distribution of GDP growth four quarters ahead, expressed in percentage points of real GDP growth. Tail risk, the fifth percentile and the median are derived by combining the VAR and quantile regressions, such that the confidence bands capture the uncertainty of both models.

5.3 Macroprudential policy shock

5.3.1 Mean effect of the shock

Turning to the consequences of macroprudential measures, we consider the impulse response following the tightening of one regulatory policy, i.e. the historical average impact

Figure 9: Decomposition of the impact of a 100-basis-point monetary policy tightening on how narrow the distribution of future GDP growth is



Note: The figure displays the impulse response function over 16 quarters following a 100-basis-point increase in the monetary policy rate. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals. Tail risk is the difference between the median and the fifth percentile of the distribution of GDP growth four quarters ahead. Policy rate, credit growth and financial stress are represented as their contribution to the tail risk, expressed in percentage points of real GDP growth. They reflect the joint effect of the shock on the VAR and the quantile model, such that the confidence bands capture the uncertainty of both models. When the contribution is different from zero, it means that the variable has a differential impact on the median and the fifth percentile of the distribution of future GDP growth.

of one macroprudential change: see Figure 10. The implementation of the macroprudential measure increases financial market stress and decreases GDP growth in the first year, while credit growth decreases after one year. From Figures 8 and 10, one macroprudential measure reduces credit growth by 40 basis points, while the same reduction in credit growth requires a 50-basis-point unexpected monetary policy tightening.

5.3.2 Distributional effect of the shock

From the quantile regression coefficients, Figure 7, we know that lower credit growth alleviates risks to future GDP growth while higher financial market stress magnifies it. The bottom panel of Figure 10 shows the net impact on the median and the fifth percentile of the distribution of GDP growth four quarters ahead. The two effects via credit and financial stress are small and cancel out for the median. Conversely, the credit channel dominates for the fifth percentile. There is a significant reduction in the risk to future GDP growth at a two-year horizon, which leads to a more narrow distribution of future GDP growth: the tail risk metric is reduced.

This can be observed more clearly in the decomposition of the shift in the distribution: see Figure 11. Slower credit growth contributes to a more narrow distribution of future GDP growth by 15 basis points (middle left). Conversely heightened financial stress initially widens the distribution of future GDP growth by 15 basis points (middle right). This suggests that the benefits of tighter macroprudential policy regarding the distribution of future GDP growth can be delayed depending on the perception of the financial market participants.

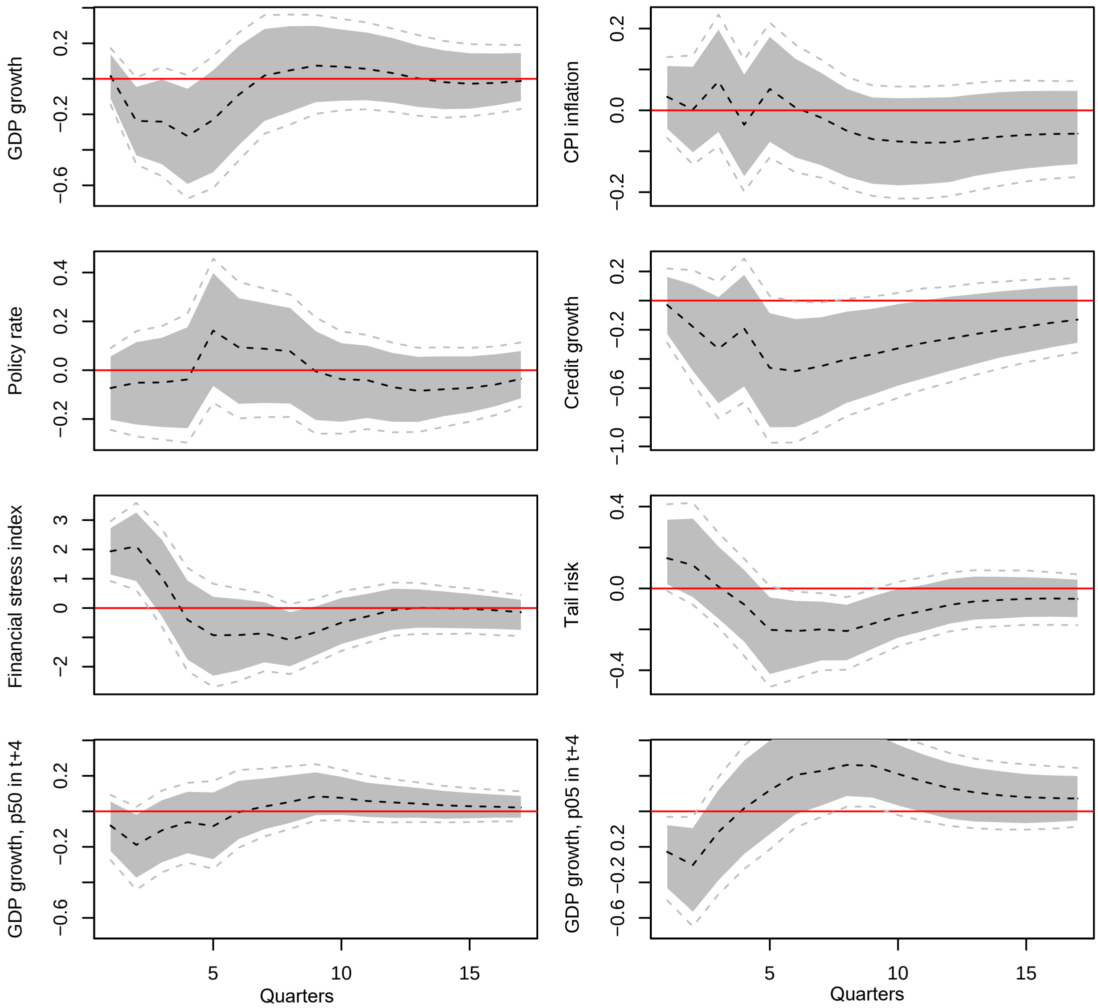
Compared to the impact of monetary policy, the gain of a macroprudential intervention via slower credit growth is driven by improvements specific to the tail of future GDP growth more than by changes to the median. The tightening of one macroprudential measure by the historical average amount narrows the distribution of future GDP growth by 20 basis points after one year. Comparing Figures 8 and 10, the distributional impact of a macroprudential intervention corresponds roughly to the benefits of 100 basis points of unexpected tightening of monetary policy, although it materializes more slowly.

6 Application to policy making

The previous section discussed the impact of a shock on the response of mean, median and tail GDP growth. This can be expressed in terms of the impact on central risk (Definition 4)[24] and tail risk (Definition 3). We now outline how simulations can be used

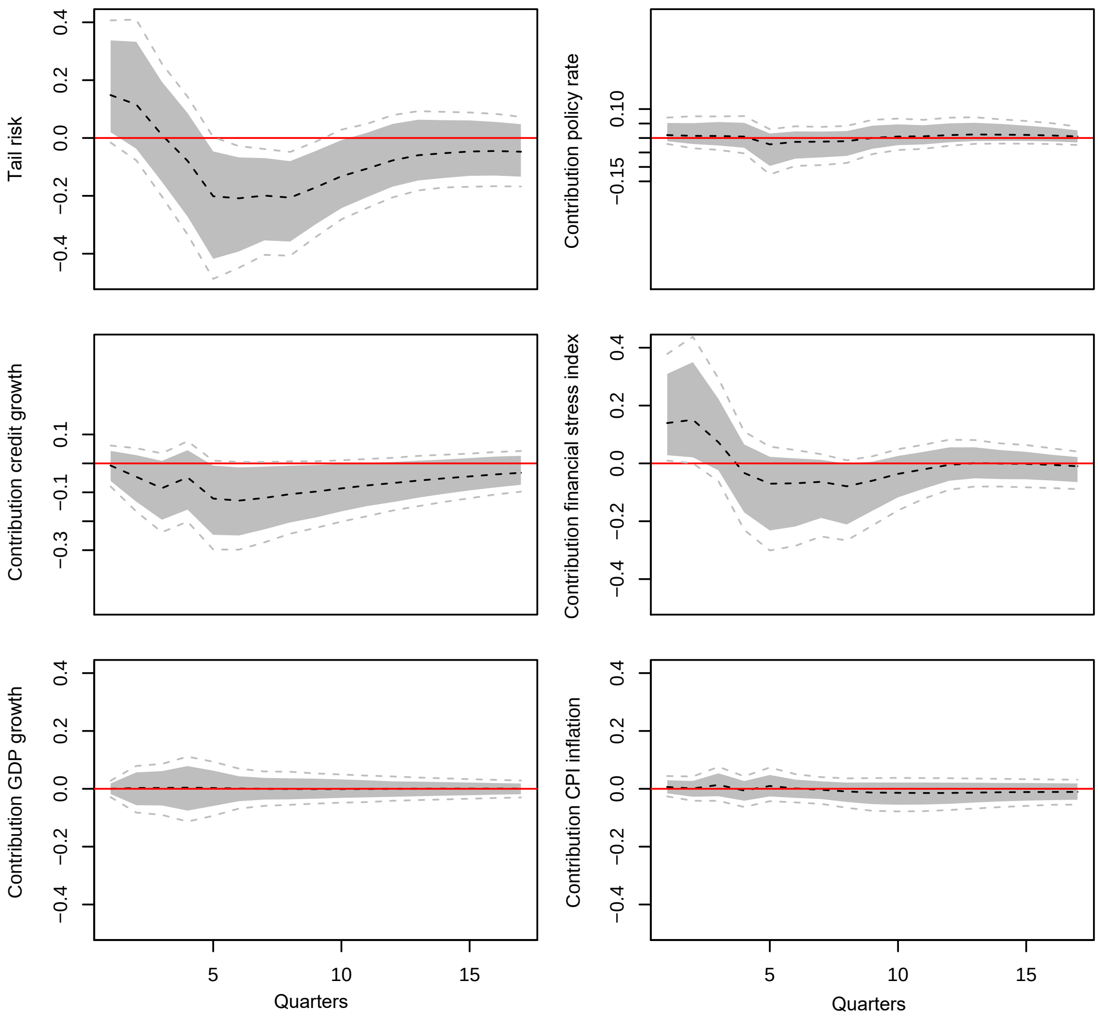
[24]For simplicity, we use the difference of GDP growth from the estimated potential GDP growth published in the relevant *Monetary Policy Report* of the Bank of Canada. Alternative measures of central risk could be the output gap or a loss function between the output and inflation gaps. We would obtain similar results in our specific application, given that the output gap was close to zero to start

Figure 10: Response after one macroprudential policy tightening



Note: The figure displays the impulse response function over 16 quarters following the tightening of one household-related macroprudential policy instrument, i.e. the historical average impact of one macroprudential tightening. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals. GDP growth, inflation, policy rate, credit growth and financial stress index are the variables from the structural VAR. The VAR uses four lags, and shocks are identified with the narrative method. Tail risk is the difference between the median (p50) and the fifth percentile (p05) of the distribution of GDP growth four quarters ahead, expressed in percentage points of real GDP growth. Tail risk, the fifth percentile and the median are derived by combining the VAR and quantile regressions, such that the confidence bands capture the uncertainty of both models.

Figure 11: Decomposition of the impact of one macroprudential policy tightening on how narrow the distribution of future GDP growth is



Note: The figure displays the impulse response function over 16 quarters following the tightening of one household-related macroprudential policy instrument, i.e. the historical average impact of one macroprudential tightening. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals. Tail risk is the difference between the median and the fifth percentile of the distribution of GDP growth four quarters ahead. GDP, inflation, policy rate, credit growth and financial stress are represented as their contribution to the tail risk, expressed in percentage points of real GDP growth. They reflect the joint effect of the shock on the VAR and the quantile model, such that the confidence bands capture the uncertainty of both models. When the contribution is different from zero, it means that the variable has a differential impact on the median and the fifth percentile of the distribution of future GDP growth.

to enhance the ability of policy-makers to take financial stability issues into account when making their decision. Our reduced-form approach augments standard macroeconomic projection models by adding tail event relevant information.

Definition 4. *Empirically,* ***central risk*** *is defined as the squared deviation of the projected GDP growth from potential GDP growth:*

$$\left(\widehat{GDP}_t - GDP_t^*\right)^2.$$

We apply the framework to Canada for the projection horizon 2018Q1 to 2021Q4. The cycle of monetary policy tightening that started in June 2017 makes it particularly relevant to analyze the trade-off between central risk and tail risk related to financial stability as of 2017Q4. Low interest rates encouraged household borrowing that potentially reinforced risks to future GDP growth. This led to a sequence of macroprudential policy tightening and discussions of the pace of monetary policy tightening in the context of rising financial stability concerns.

6.1 Construction of the policy-maker choice set

We anchor the projection of our model around a baseline forecast. In practice, this would be the projection created by a policy institution. Typically, those policy models do not capture the expectation of a future crisis, and policy decisions are focused only on macroeconomic stability. Once added to a policy model, our framework provides the policy-maker with guidance regarding the optimal policy interest rate path when taking tail risks into account.

In this example, our baseline is the projections of the main Canadian commercial banks at the beginning of 2018. We use the residuals of our VAR to align the projected path for GDP growth, inflation and the policy rate. To match the desired path of three variables in the VAR, we require three shocks. In addition to the monetary policy shock already identified, we decompose the residuals from the VAR regression into a demand and a supply shock using standard sign restrictions.[25] For a given baseline forecast, we recover the associated central and tail risk through the lens of our framework.

Then, we use the monetary or macroprudential policy shocks to simulate alternative policy scenarios around the baseline forecast. Each policy scenario is then mapped into average central and tail risk. For simplicity, we display the average of each metric over

with. In our example, we prefer to stay in the growth space for consistency with the measure of tail risk that is growth-based and focused on GDP.

[25] On impact, we require the demand shock to be positively correlated with GDP growth, inflation and the policy rate. On impact, the supply shock is positively correlated with GDP growth and negatively correlated with inflation and the policy rate.

the projection horizon.[26]

Eventually, policy-makers can choose the level of central and tail risk that best corresponds to their preferences. They can back out the associated policy path that delivers their desired outcome. With our reduced-form model, we do not explicitly model the policy-maker's preferences. We only identify the choice set of the policy-maker.[27]

6.2 Choice set of the monetary policy authority

We construct a monetary policy choice set by creating alternative scenarios to the baseline policy rate forecast. Concretely, we create two extreme hypothetical monetary policy scenarios on the left side of Figure 12: an upper bound (the red triangles) and a lower bound (the blue squares) of plus and minus 100 basis points around the baseline policy rate forecast. We assume that the alternative policy rate paths converge back at the end of the projection horizon. Those two extreme paths are summarized in the average central and tail risk space with the red triangle and the blue square on the right side of Figure 12. The choice set available to the monetary policy authority when contemplating all alternative monetary policy paths in between the two extreme scenarios is given by the black line on the right side of Figure 12.

For policy rate paths that remain low for long (closer to the blue squares), there is no trade-off: both central and tail risks increase and, in the choice set, one moves north-east towards the blue square.

For policy rate paths that increase earlier and faster (closer to the red triangles), there is a trade-off: with the rate rising faster, tail risk decreases by curbing credit growth, but the economy slows down and deviates from potential growth from below. In the choice set, the economy moves south-east towards the red triangle.

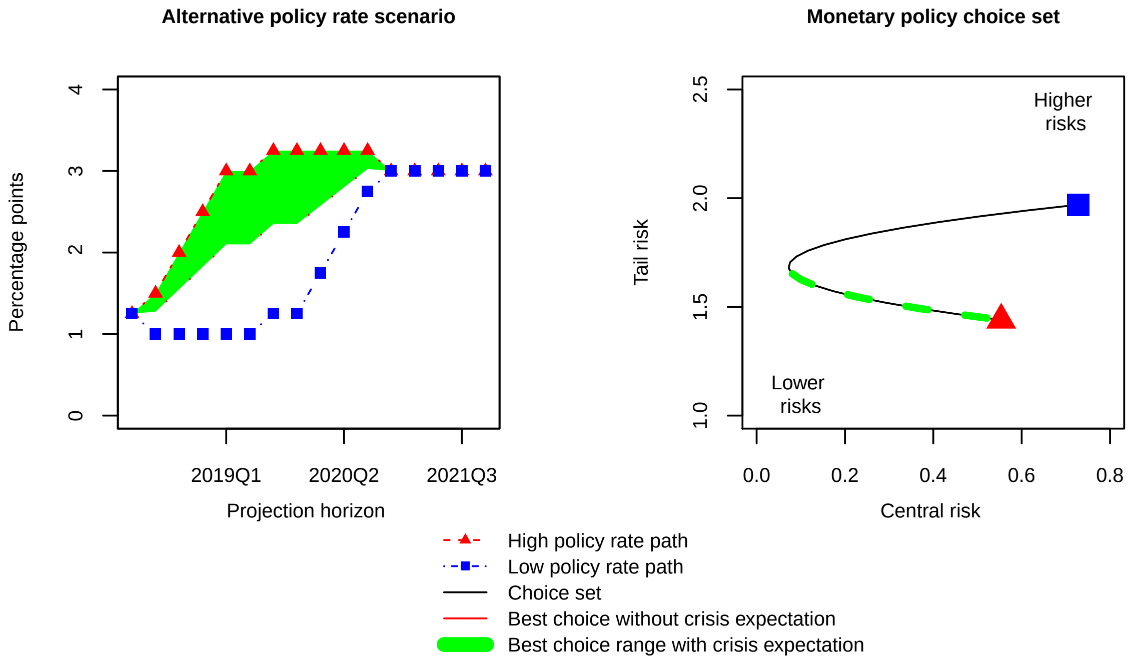
A monetary policy-maker unconcerned about tail risk would only focus on central risk and pick the tip of the choice set (the red circle on the right panel). This is associated with a policy rate path very close to the commercial banks' baseline forecast (the red circles on the left panel).

If instead policy-makers were concerned about tail risk generated by excessive credit growth, they would choose a point on the lower arm of the choice set (thick dashed green line on the right panel). This is equivalent to a tighter monetary policy path going through the green area on the left panel.

[26] In the robustness Section 7.1, we consider short- and medium-term horizons. This confirms that the gain of macroprudential policy in terms of reducing future tail risk is not immediate.

[27] The choice set could be reduced to one dimension by minimizing an augmented loss function of a generic form $f(\mathbb{E}_{GDP} - \mathbb{E}_{GDP^*}) + o(\mathbb{Q}_{GDP}(\tau{=}.50) - \mathbb{Q}_{GDP}(\tau{=}.05))$. Empirically, this welfare criterion cannot be obtained. Thus we only display the choice the policy-maker is facing. A theory-based preference of a policy-maker that maximizes household welfare given market forces can be found in Appendix A. Although beyond the scope of this paper, historical preferences could be backed out by comparing historical choices against baseline forecasts available at each point in time, for instance, using the vintage macroeconomic forecasts of Champagne, Poulin-Bellisle and Sekkel (2018).

Figure 12: Choice set of the monetary policy authority, projection 2018Q1-2021Q4



Notes: The interest rate paths on the left panel are hypothetical scenarios anchored around the average path expected by the main Canadian banks in early 2018. Central and tail risks (see Definition 4 and 3) in the right panel are the average over the 2018Q1-2021Q4 projection horizon. Moving along the black line in the right panel corresponds to picking an interest rate path closer to the red triangles or to the blue squares in the left panel.

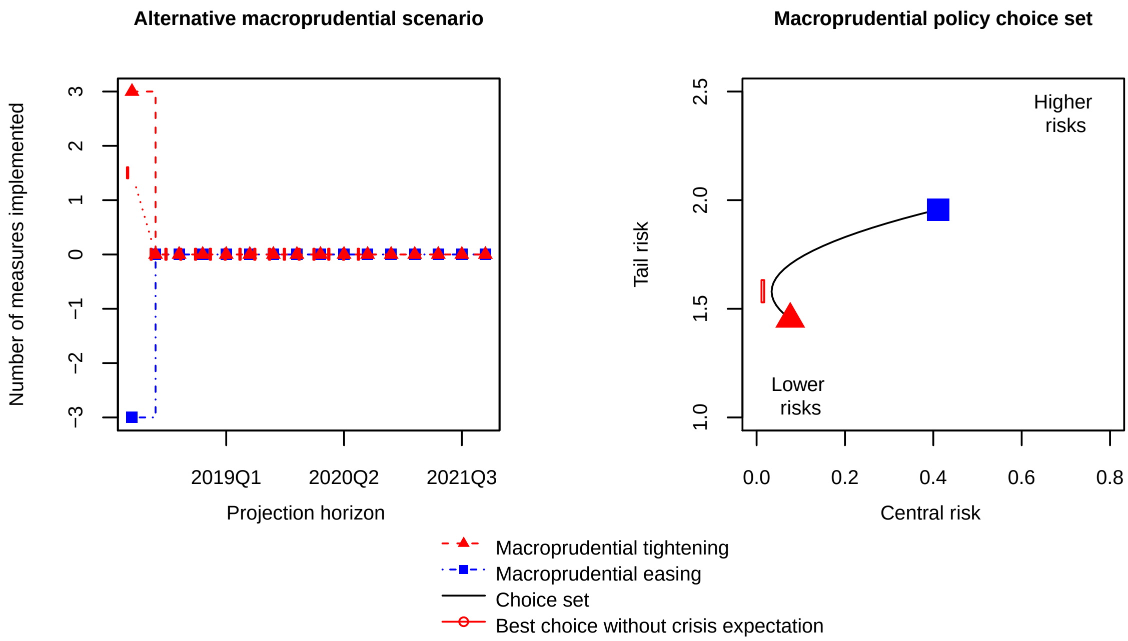
6.3 Choice set of the macroprudential policy authority

The choice set of a regulatory authority is created by varying our macroprudential index: see left side of Figure 13. Focusing on 2018Q1, we change the number of implemented measures between an upper bound (the red triangles) and a lower bound (the blue squares) with plus or minus three measures at the extremes. Those bounds are summarized in the average central and tail risk space with the red triangle and the blue square on the right side of Figure 13. The choice set available to the macroprudential policy authority when contemplating all alternative macroprudential policy paths in between the two bounds is given by the black line on the right side of Figure 13.

Easing macroprudential policies in 2018 (getting closer to the blue squares) increases both central and tail risk. The policy-maker would be better off with some macroprudential policy tightening. Indeed, in early 2018, Canada implemented more restrictive mortgage origination guidelines by requiring banks to stress-test potential borrowers before they can qualify for an uninsured mortgage. A tightening of one macroprudential measure (red circles) leads mainly to a reduction of central risk (tip of the choice set). This would be the best choice absent concerns about tail risk.

There is a trade-off if the regulator decides to implement more than one macroprudential measure of average impact. It would further decrease tail risk by further curbing

Figure 13: Choice set of the macroprudential policy authority, projection 2018Q1-2021Q4



Notes: The macroprudential policy changes on the left panel are hypothetical scenarios anchored around the average economic forecasts of the main Canadian banks in early 2018. Central and tail risks (see Definition 4 and 3) in the right panel are the average over the 2018Q1-2021Q4 projection horizon. Moving along the black line corresponds to picking a number of macroprudential policy changes closer to the red triangles or to the blue squares in the left panel.

credit growth, but at the cost of a weaker economy with increased central risks.

6.4 Choice set if authorities coordinate

Should the monetary and regulatory authorities coordinate their efforts, then the choice set becomes richer. Figure 14 shows the entire choice set when any combination of the previously discussed policy paths can be selected.[28] The choice set is now wider than before, reflecting a larger range of policy options.

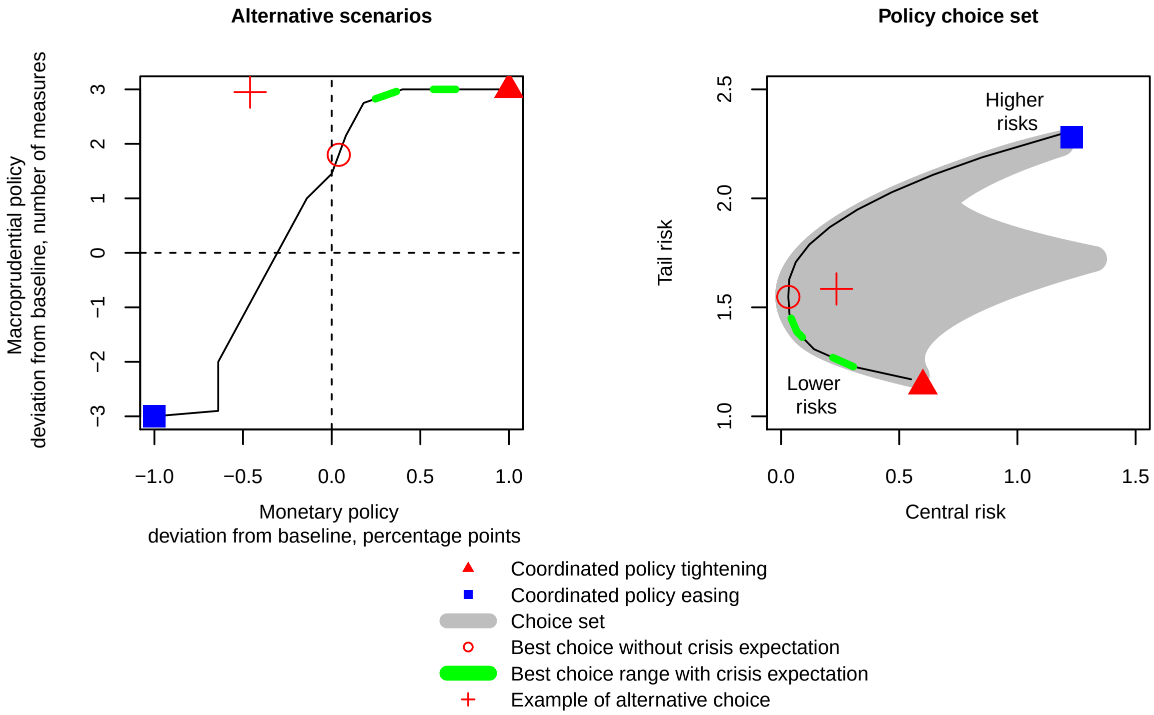
If policy-makers were unconcerned about GDP tail risk and able to coordinate, as of 2017Q4, the best policy mix to minimize the central risk would be to tighten macroprudential policy by about one time the historical average amount and follow the baseline policy rate path (red circle).

If instead policy-makers were equally concerned about GDP tail risk and able to coordinate, then they would choose a point on the lower arm of the choice set, with both tighter monetary and macroprudential policies (thick dashed green line on both panels).

In the absence of coordination, or if the monetary and macroprudential policy-makers have different objectives regarding the GDP growth distribution, then the outcome would

[28]We limit the simulations to up to plus or minus three macroprudential measures, in line with historical decisions. This artificially creates the kinks on the left panel of Figure 14.

Figure 14: Choice set if authorities coordinate, projection over 2018Q1 to 2021Q4



Notes: The monetary and macroprudential policy changes on the left panel are hypothetical scenarios anchored around the average economic forecasts of the main Canadian banks in early 2018. The scenarios reflect policy paths over time shown in Figures 12 and 13. The macroprudential policy scenarios correspond to various intensities of macroprudential decisions in 2018Q1. The monetary policy scenarios correspond to a peak difference of plus or minus one percentage point around the baseline in 2019Q1. Central and tail risks (see Definition 4 and 3) in the right panel are the average over the 2018Q1-2021Q4 projection horizon. Moving along the black line corresponds to picking a policy scenario closer to the red triangle or to the blue square in the left panel.

be within the choice frontier. For instance, if the monetary policy authority sets the rate 0.5 percentage point lower, the regulatory authority would have to tighten macroprudential measures twice as much (red cross versus red circle). This suggests some degree of substitution between the policy instruments.

7 Robustness

We highlight here three main robustness tests regarding the inter-temporal trade-off at play when computing the policy choice sets, the effectiveness of supply- versus demand-driven macroprudential changes and the endogeneity of macroprudential measures. Other robustness tests are left for the appendices where we analyze alternative model specifications (Appendix D), alternative monetary policy shock identifications (Appendix E) and the cross-country correlation between credit and GDP growth across percentiles of the GDP growth distribution (Appendix F).

7.1 Intertemporal trade-off and the policy choice set

The choice of the forecast horizon matters for the assessment of financial and macroeconomic stability. Figure 15 shows the choice set for alternative monetary policy paths averaged over different horizons. In the short run, given the small economic boom that Canada experienced at the end of 2017, raising rates slightly faster also minimizes central risk by cooling down the economy, with less tail risk benefit. After some time, this same path would lower tail risks by slowing credit growth.

Not surprisingly, the extension of the horizon to three or four years leads to a wider choice set and larger tail benefits from raising rates. In our model, the difference between a policy-maker that focuses on the short run versus one that focuses on the medium run is a tightening bias of 25 basis points (the difference between the crossed and empty red circles). A policy-maker concerned about GDP tail risk should consider a longer projection horizon, as benefits for the tail of future GDP growth are likely to be delayed.

Instead of our tail risk measure that nets out the variation in central risk (Definition 3), it is also possible to focus on the overall tail of the GDP growth distribution directly by using growth at risk, the fifth percentile of future GDP growth. We obtain the alternative choice set of Figure 16. Qualitative results are similar to the ones obtained above, although the movement of growth at risk now reflects both a change specific to the tail *and* a change in the location of the median of the distribution. Growth at risk does not immediately improve because in the short run, growth at risk also reflects the negative impact on *median* GDP growth, while this effect is netted out in Definition 3.

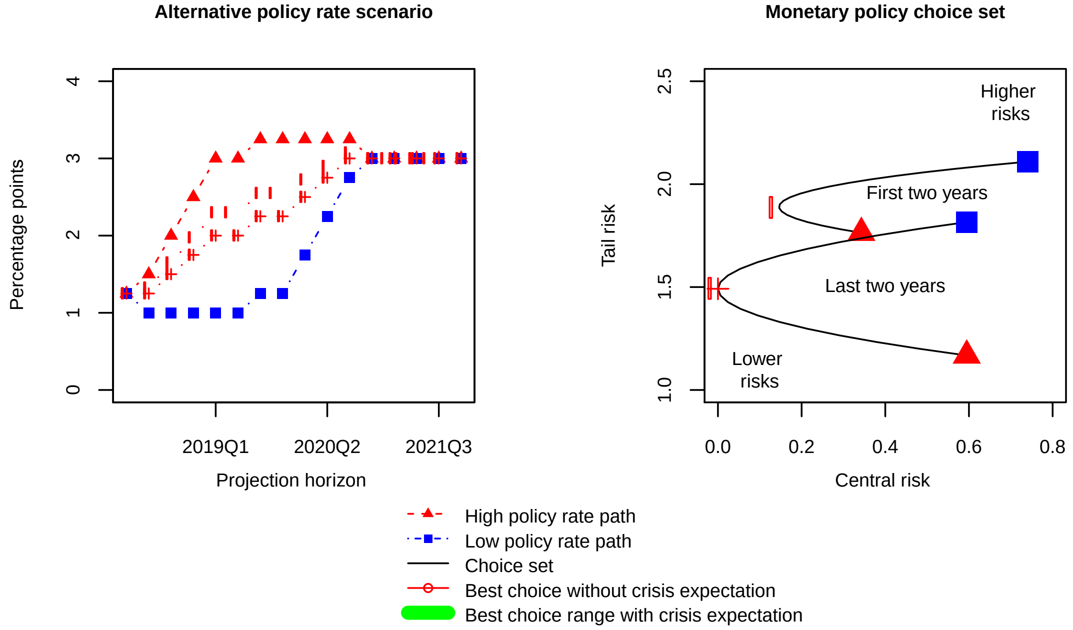
7.2 Supply versus demand macroprudential policy changes

In our benchmark, macroprudential policy changes are defined by combining all the household-related macroprudential events (loan-to-value ratio, debt-service ratio, amortization, mortgage insurance by government-backed agencies, housing-related taxes). Here we consider how effective different instruments are in reducing tail risk.

We assess an alternative loan demand-side macroprudential index by focusing on the rules that directly restrict the demand for loans (loan-to-value ratio, debt-service ratio, amortization): see Figure 17, first column. Then we look at a loan supply-side macroprudential index that covers both changes to the reserve requirements of the 1980s and early 1990s as well as mortgage insurance rules that take credit risk away from banks' balance sheets: see Figure 17, second column. Note that supply-side measures are less frequent in Canada and do not capture changes to bank equity requirements.

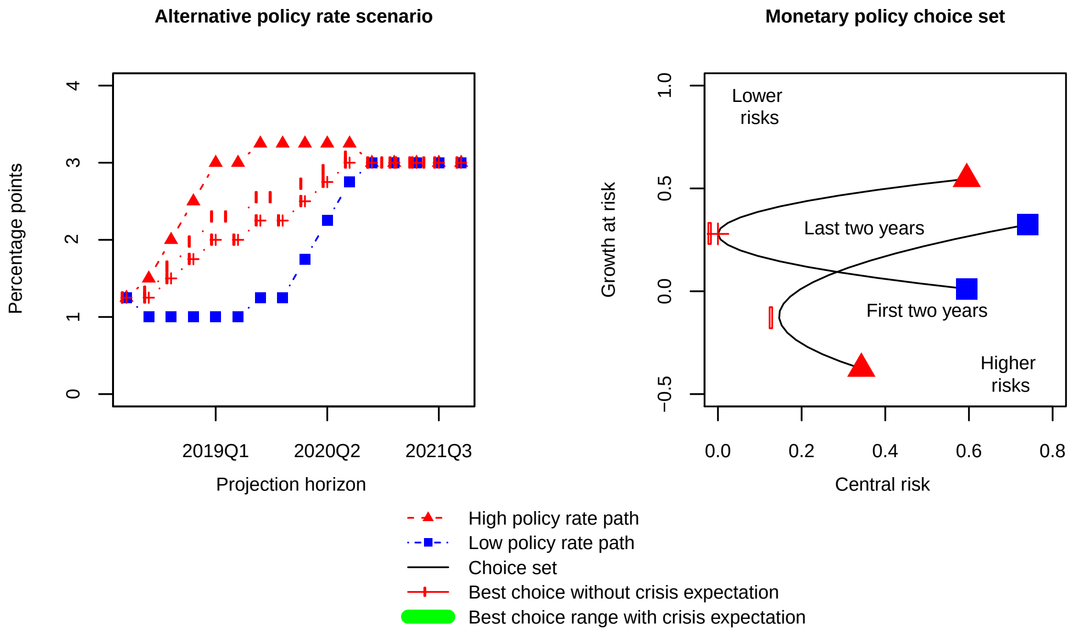
The response of tail risk is most similar for household-related and demand-related measures, while supply-side measures do not significantly impact tail risk. This is consistent with macroprudential policies in Canada being mostly implemented through household demand-side restrictions on mortgages (Allen et al., 2017) and demand-related measures

Figure 15: Trade-off for monetary policy: the horizon matters



Notes: The interest rate paths on the left panel are hypothetical scenarios anchored around the average path expected by the main Canadian banks in early 2018. Central and tail risks (see Definition 4 and 3) in the right figure are the average over the projection horizon, either over the first two years (2018Q1-2019Q4) or the subsequent two years (2020Q1-2021Q4). Moving along the black line in the right figure corresponds to picking an interest rate path closer to the red triangles or to the blue squares in the left figure.

Figure 16: Trade-off for monetary policy: robustness using growth at risk



Notes: The interest rate paths on the left panel are hypothetical scenarios anchored around the average path expected by the main Canadian banks in early 2018. Central risk (see Definition 4) and growth at risk (the fifth percentile of the one-year ahead GDP growth) in the right figure are the average over the projection horizon, either over the first two years (2018Q1-2019Q4) or the subsequent two years (2020Q1-2021Q4). Moving along the black line in the right figure corresponds to picking an interest rate path closer to the red triangles or to the blue squares in the left figure.

being more effective (Damar and Molico, 2016).

7.3 Addressing the endogeneity of macroprudential policies

So far we have only used an exogenous index of macroprudential changes to proxy for macroprudential policy scenarios. We now use the unexpected portion of the macroprudential index and perform placebo analyses to assess the extent to which we capture macroprudential shocks, i.e. if expected macroprudential events affect our results.

We compute the probability of facing an unexpected change in the macroprudential index as part of the probability of facing a tightening/easing in the household-related MAP that is not explained by the lagged endogenous variables: see Equation (4).[29] This is in the spirit of Alam et al. (2019), who use a propensity score method in a cross-country setup to compute the exogenous part of loan-to-value decisions not correlated with past macroeconomic variables.

$$\widetilde{MAP_t} = (\mathbb{1}_{MAP_t>0} - \mathbb{1}_{MAP_t<0}) - (\Pr(MAP_t > 0) - \Pr(MAP_t < 0)) \tag{4}$$

$$\text{with } \Pr(MAP_t > 0) = \Phi\left(\hat{c} + \sum_{p=1}^{P} \hat{\rho}_p \begin{bmatrix} GDP_{t-p} \\ CPI_{t-p} \\ Rate_{t-p} \\ Credit_{t-p} \\ FSI_{t-p} \end{bmatrix}\right)$$

and a similar equation for $\Pr(MAP_t < 0)$

We compute the response to a completely unexpected macroprudential policy tightening: see Figure 17, column three. All results remain as before, which suggests that the potential feedback from previous macroeconomic conditions onto subsequent macroprudential policy does not severely bias our results. We confirm this with placebo analyses.

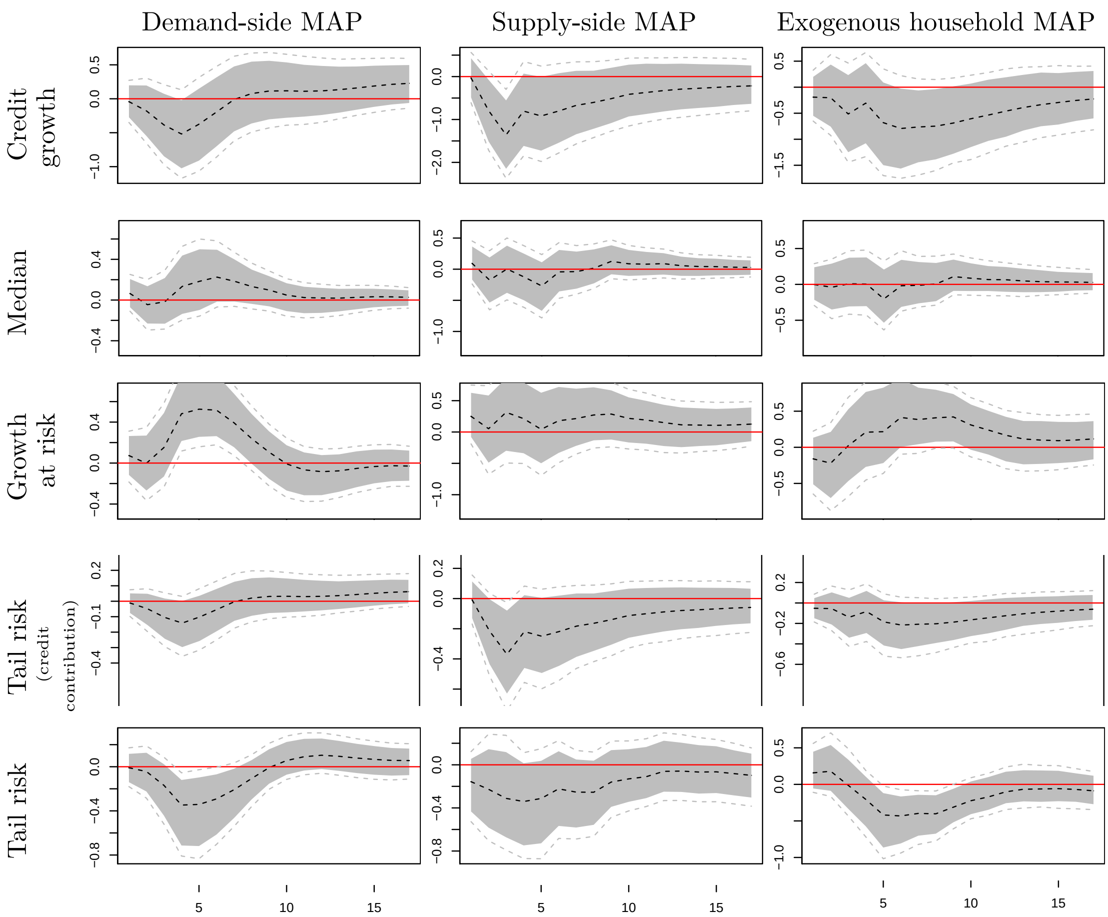
By changing the timing of our macroprudential index, we can further test if the effect of macroprudential policy is partly anticipated or not. For instance, a pull-forward behavior could take place after a macroprudential tightening is announced and before it is implemented.[30] We run a placebo test by lagging or forwarding the macroprudential index by one or two years in the VAR. Results are reported in Figure 18.

When using the lagged index, credit growth is still decreasing in the short run. This

[29] In this case, we lose the information regarding the number of tightenings/easings occurring in a given quarter. Alternatively, we can compute the unexpected macroprudential change as the difference between the macroprudential index and the index fitted on all endogenous variables with P lags, using an OLS estimator. Results are then even closer to our benchmark findings.

[30] In Equation (4), we also used as a robustness check the date the macroprudential change was announced, instead of the implementation date. As identified with a star in Appendix C, this makes little difference for most macroprudential decisions. Similar results would be obtained.

Figure 17: Response after different types of macroprudential policy tightening



Note: The figure displays the response of the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The columns refer to different indices of macroprudential policy (MAP). The benchmark household-related MAP index includes changes to loan-to-value ratio, debt-service ratio, amortization, insurance and tax rules. The demand-side MAP index includes only changes to loan-to-value ratio, debt-service ratio and amortization rules. The supply-side MAP index focuses on reserve requirements and mortgage insurance rules. The exogenous household MAP corresponds to Equation (4) and is computed as the variation in MAP that is not predicted by past endogenous variables. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

suggests that macroprudential changes can have some weaker delayed impact even after two years. When using the forwarded index by one or two years, credit growth, growth at risk and the tail risk metric are no longer significant, suggesting little anticipation effect.

Figure 19 shows the peak response for credit and tail risk for each placebo model. It confirms that credit growth (tail risk) is not significantly reduced when one assumes that macroprudential measures are anticipated by two (one) quarters or more. Conversely, lagged macroprudential policy indices still have a significant impact up to one year and a half after the policy change. Credit growth is reduced most during the first year, while tail risk is reduced most after one year, when the short-run impact of financial stress dissipates. This confirms the importance of allowing for some persistence by including both contemporaneous and lagged macroprudential policy indices in the VAR ($L > 0$) of Equation (1).[31]

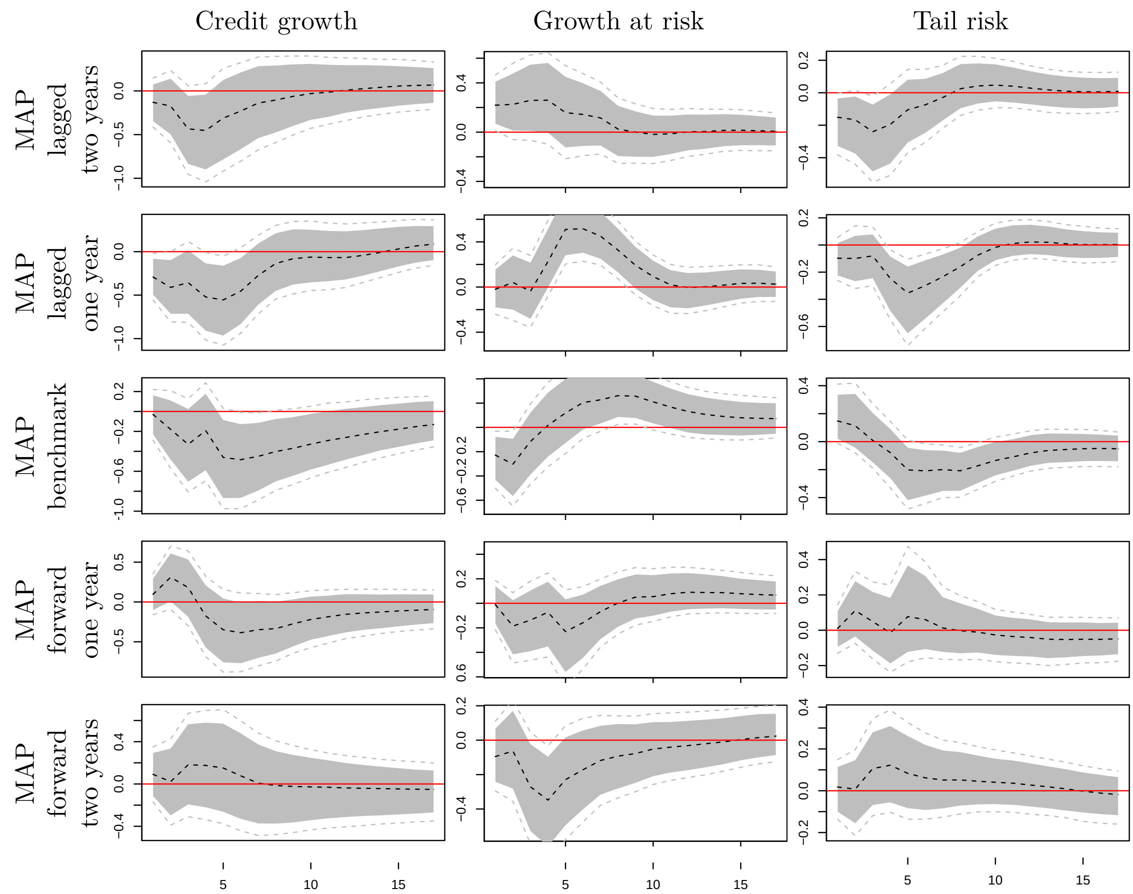
8 Conclusion

We propose a framework that allows policy-makers to easily see existing trade-offs between central risk, i.e. deviation from the expected GDP growth, and tail risk, i.e. risks to the tail of GDP growth relative to the median. Consistent with a simple risk-shifting model, we show that credit growth is the main driver of tail risk beyond a one-year horizon. Tighter monetary and macroprudential policies can both reduce tail risks by targeting credit growth. So the impact of low rates on risks to financial stability can be partly compensated by tighter macroprudential rules.

Our novel sequential integration of a quantile model with a VAR can be used by policy-makers as a simple communication tool to anchor their narrative and show the trade-off they may face between macroeconomic and financial stability. Our work supports the view that policy-makers should consider the distribution—and not only the expectation—of future GDP growth when taking decisions. This calls for more research efforts to develop macroeconomic models able to analyze GDP tail risks. While this agenda is partly underway (e.g. Brunnermeier and Sannikov, 2014; Coimbra and Rey, 2017), it faces significant computational and modeling challenges. In the meantime, we provide a viable alternative to integrate tail risks into an otherwise standard projection environment.

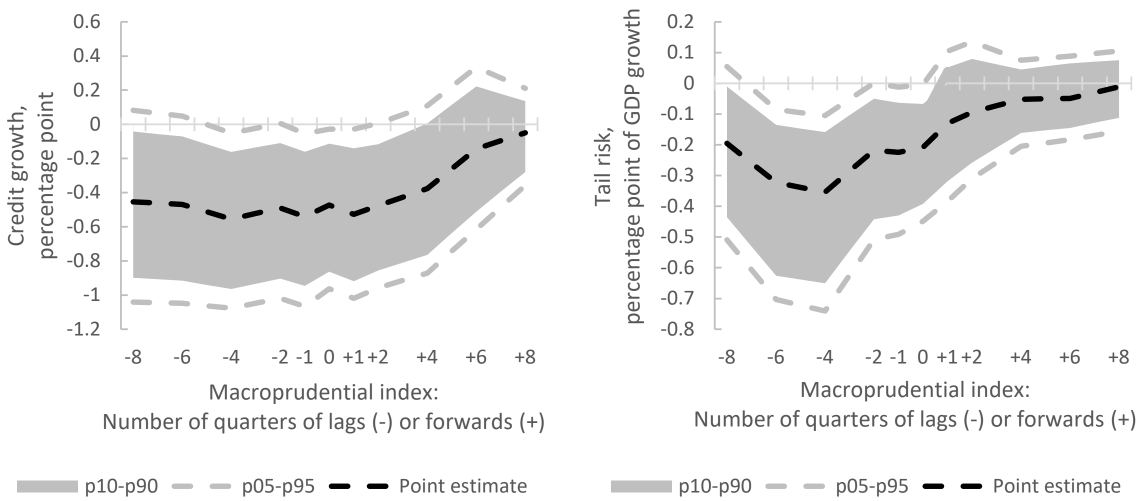
[31] This is consistent with Cerutti, Claessens and Laeven (2017) trying to address the endogeneity problem by using more lags in a univariate cross-country generalized method of moments estimation.

Figure 18: Placebo response after one macroprudential policy tightening



Note: The figure shows the response of the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening. The columns show the response of credit growth, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The rows show the response for the benchmark household-related macroprudential tightening and four placebo exercises where the macroprudential policy (MAP) index is shifted forward or lagged by one or two years. The MAP index counts the number of changes implemented in each quarter regarding the loan-to-value ratio, debt-service ratio, amortization, insurance and tax rules. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

Figure 19: Peak response to one macroprudential policy tightening: placebo response using lags/forwards of macroprudential index



Note: The figure shows the peak response after the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening. Each point corresponds to a different estimation where the benchmark household-related macroprudential index is shifted forward or lagged by one to eight quarters. The left panel shows the response of credit growth and the right panel the response of the tail risk measure (see Definition 3). Lower tail risk corresponds to a reduction in the gap between the median and growth at risk (the fifth percentile), expressed in percentage points of real GDP growth four quarters ahead. The macroprudential index counts the number of changes implemented in each quarter regarding the loan-to-value ratio, debt-service ratio, amortization, insurance and tax rules. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

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Appendices

A Preferences in the policy choice set

Applying Definitions 1 and 2 for central and tail risks, we can obtain relationships between the state of the economy and the risk measures:

$$\begin{aligned} cr &= p\left(Y_G - E\left(Y^*|\, p{=}0\right)\right)^2 + (1-p)\left(Y_B - E\left(Y^*|\, p{=}0\right)\right)^2 \\ tr &= pY_G + (1-p)\,Y_B - Y_B, \end{aligned}$$

where

$$\begin{aligned} Y_G &= \bar{z}_r\,(xd)^{\theta_r} + \bar{z}_s\left((1-x)\,d\right)^{\theta_s} \\ Y_B &= \bar{z}_s\left((1-x)\,d\right)^{\theta_s}. \end{aligned}$$

From this we are able to deduce relationships between (Y_G, Y_B) and (cr, tr):

$$\begin{aligned} Y_G &= E\left(Y^*|\, p{=}0\right) + \frac{(1-p)}{p}tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}\,(tr)^2} \\ Y_B &= E\left(Y^*|\, p{=}0\right) - tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}\,(tr)^2}. \end{aligned}$$

Next, we use the first-order conditions from the banking problem to map the choices of (x, d) onto (cr, tr):

$$\frac{\theta_r p Y_{r,G} + \theta_s Y_s}{(1-(1-p)\,x)} = Rd$$

$$Rd + \frac{\eta}{(1-p)}\mu = \frac{\theta_s Y_s}{(1-p)\,(1-x)} - \frac{p\theta_r Y_{r,G}}{(1-p)\,x}, \mu \geq 0, (\eta - x)\,\mu = 0,$$

focusing on the case of an unconstrained solution:

$$\begin{aligned} x &= \frac{\theta_r Y_{r,G}}{\theta_s Y_s + \theta_r Y_{r,G}} \\ d &= \frac{(\theta_s Y_s + \theta_r Y_{r,G})}{R}. \end{aligned}$$

Next, for the mapping into the welfare space, we have to use

$$\begin{aligned} Y_G &= Y_{r,G} + Y_s \\ Y_B &= Y_s \end{aligned}$$

to obtain

$$\begin{aligned} Y_s &= Y_B = E\left(Y^*|\, p{=}0\right) - tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}\,(tr)^2} \\ Y_{r,G} &= Y_G - Y_s = Y_G - Y_B = \frac{tr}{p}. \end{aligned}$$

Next, we replace realized output by these functions in (cr, tr) as we evaluate welfare:

$$V = U(W-d) + \beta\left[pU(Y_G) + (1-p)U(Y_B)\right],$$

where we have to replace d in $W-d$ by filling in d as a function of (cr, tr):

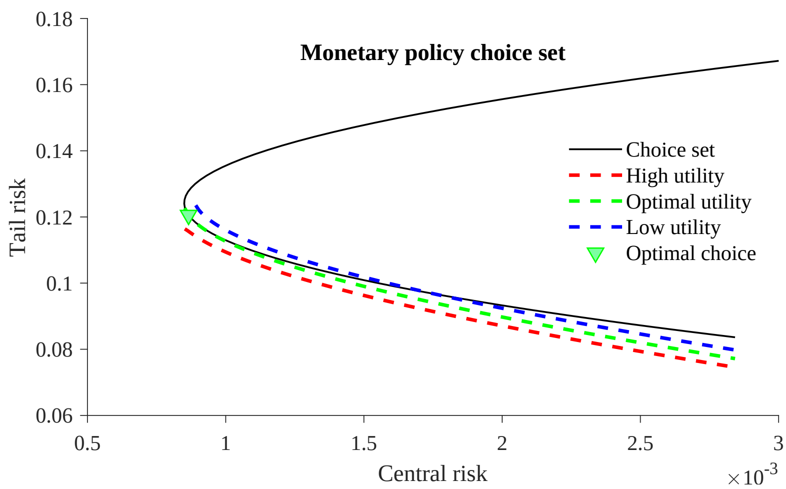
$$V = U\left(W - \frac{\left(\theta_s\left(E\left(Y^*\right|p{=}0) - tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}(tr)^2}\right) + \theta_r\left(\frac{tr}{p}\right)\right)}{R}\right)$$
$$+\beta\left[\begin{array}{c} pU\left(E\left(Y^*\right|p{=}0) + \frac{(1-p)}{p}tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}(tr)^2}\right) \\ +(1-p)U\left(E\left(Y^*\right|p{=}0) - tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}(tr)^2}\right)\end{array}\right]$$

s.t.

$$R = (\bar{z}_s)^{\frac{1}{\theta_s}}(\theta_s)\left(E\left(Y^*\right|p{=}0) - tr \pm \sqrt[2]{cr - \frac{(1-p)}{p}(tr)^2}\right)^{\frac{\theta_s-1}{\theta_s}}$$

Using the numerical example from above, we can obtain a graphic representation of the indifference curves and show that, at the optimal level, the indifference curves touch the trade-off frontier: see Figure 20.

Figure 20: Choice set with preference curves



Notes: Simulations use CRRA preferences and are for the parameter values $\sigma = 1.5; \beta = 0.99; \bar{z}_r = 0.9947; \bar{z}_s = 0.98; p = 0.05; W = 1; \sigma_r = 0.97; \sigma_s = 0.3$. The policy rate R^a varies while the capital adequacy ratio $\eta = 1$ is fixed.

B The Ramsey problem and policy choices

To understand how policy can counteract the two imperfections, we first take a look at the Ramsey problem:

$$
\begin{aligned}
V(R,\eta;p) \;&=\; U(W-d)+\beta E\left[U\left(z_r(S)(xd)^{\theta_r}+\bar{z}_s((1-x)d)^{\theta_s}\right)\right]\\
&\quad s.t.\\
&\quad x=\min\left(\eta,\left(1+(\bar{z}_r\theta_r)^{\frac{-1}{1-\theta_r}}(\bar{z}_s\theta_s)^{\frac{1}{1-\theta_s}}(R)^{\frac{\theta_r-\theta_s}{(1-\theta_s)(1-\theta_r)}}\right)^{-1}\right)\\
&\quad \begin{pmatrix} d=\left(\frac{\bar{z}_r\theta_r}{R}\right)^{\frac{1}{1-\theta_r}}+\left(\frac{\bar{z}_s\theta_s}{R}\right)^{\frac{1}{1-\theta_s}} \\ (d)^{\theta_r-1}p\bar{z}_r\theta_r\eta^{\theta_r}+(d)^{\theta_s-1}\bar{z}_s\theta_s(1-\eta)^{\theta_s}=(1-(1-p)\eta)R \end{pmatrix} \text{ for } x\begin{pmatrix} < \\ = \end{pmatrix}\eta \Bigg),
\end{aligned}
$$

which can be further reduced to

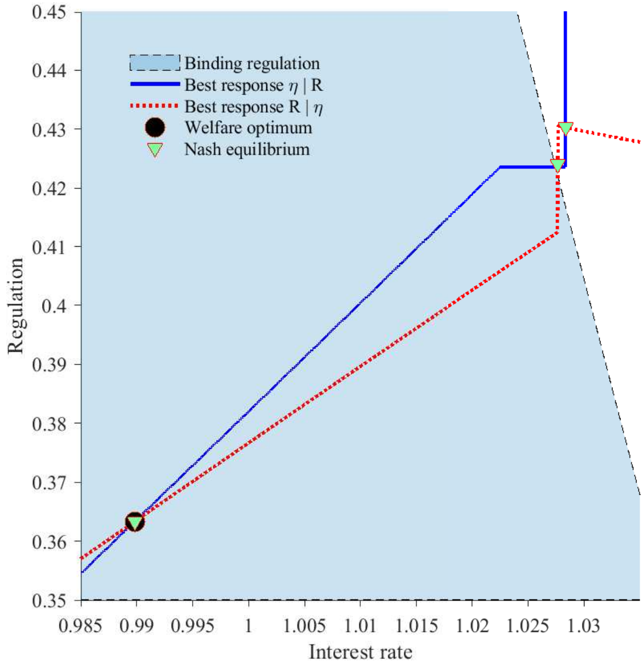
$$
V(R,\eta;p)=\left(\begin{array}{c}
\begin{pmatrix} U\left(W-(\bar{z}_r\theta_r)^{\frac{1}{1-\theta_r}}(R)^{\frac{-1}{1-\theta_r}}-(\bar{z}_s\theta_s)^{\frac{1}{1-\theta_s}}(R)^{\frac{-1}{1-\theta_s}}\right) \\ +\beta E\left[U\left(z_r(S)(\bar{z}_r\theta_r)^{\frac{\theta_r}{1-\theta_r}}(R)^{\frac{-\theta_r}{(1-\theta_r)}}+\bar{z}_s(\bar{z}_s\theta_s)^{\frac{\theta_s}{1-\theta_s}}(R)^{\frac{-\theta_s}{1-\theta_s}}\right)\right] \end{pmatrix} \\
\begin{pmatrix} U(W-d)+\beta E\left[U\left(z_r(S)(\eta d)^{\theta_r}+\bar{z}_s((1-\eta)d)^{\theta_s}\right)\right] \\ s.t.\;\;(1-(1-p)\eta)R=p\bar{z}_r(d)^{\theta_r-1}\theta_r\eta^{\theta_r}+(d)^{\theta_s-1}\bar{z}_s\theta_s(1-\eta)^{\theta_s} \end{pmatrix}
\end{array}\right) \text{ for } x\begin{pmatrix} < \\ = \end{pmatrix}\eta
$$

The simplified formulation makes it clear that regulation is good at controlling the portfolio choice, if it is tight enough. The interest rate policy mainly influences the savings behavior of households. The interaction between the two policies is also directly visible in the constraint's term $(1-(1-p)\eta)R$.

Next, we analyze the reaction functions of the monetary and the regulatory authority. Assuming both are maximizing households' welfare, we are able to find the best response for each taking the other decision maker's action as given: see Figure 21. We find three pure strategy Nash equilibria. The highest welfare is achieved by an equilibrium that has a tight regulation and low interest rates. Then there is a middle equilibrium with marginally binding regulation, but much tighter monetary policy. Finally, there is a laissez-faire equilibrium regarding regulation and an even tighter monetary policy. This highlights that monetary policy at times compensates for loose regulation.[32]

[32] Note that the monetary and regulatory examples in the main text are not taken from the Nash equilibria.

Figure 21: Reaction functions of the monetary and macroprudential authorities



Notes: Simulations use CRRA preferences and are for the parameter values $\sigma = 1.5; \beta = 0.99; \bar{z}_r = 0.9947; \bar{z}_s = 0.98; p = 0.05; W = 1; \sigma_r = 0.97; \sigma_s = 0.3$. Both η and R^a vary.

C Macroprudential events in Canada since 1980

Table 3: Macroprudential events in Canada since 1980

Implementation		Type	Description
1980Q1	easing	reserves	lower requirements for Canadian dollar notice deposits
1980Q1	tightening	reserves	introduce 3% on foreign currency deposits
1981Q1	easing	reserves	lower requirements for Canadian dollar demand deposits
1981Q3	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1982Q1	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1982Q3	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1983Q1	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1983Q3	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1984Q1	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1984Q3	easing	reserves	lower requirements for Canadian dollar demand and notice deposits
1992Q1	easing	LTV	maximum LTV from 90 to 95% for first-time home buyers
1992Q2	easing	reserves	gradual phase-out of reserve requirements
1992Q4	easing	reserves	gradual phase-out of reserve requirements
1993Q2	easing	reserves	gradual phase-out of reserve requirements
1993Q4	easing	reserves	gradual phase-out of reserve requirements
1994Q2	easing	reserves	reserve requirements completely eliminated
1998Q2+	easing	LTV	maximum LTV from 90 to 95% to all home buyers within regional house price limits
2003Q1	easing	insurance access	minimum down payment of 5% can be borrowed (Genworth)
2003Q3	easing	insurance access	removal of regional house-price caps on mortgage insurance access
2004Q1	easing	insurance access	minimum down payment of 5% can be borrowed (CMHC)
2005Q3	easing	LTV	maximum LTV from 90 to 95% for variable rate mortgages
2006Q1	easing	amortization	maximum amortization from 25 to 30 years
2006Q2	easing	amortization	maximum amortization from 30 to 35 years
2006Q4	easing	amortization	maximum amortization from 35 to 40 years
2006Q4	easing	LTV	maximum LTV from 95 to 100%
2007Q1	easing	insurance access	insured mortgages for self-employed by CMHC
2007Q3	easing	LTV	maximum LTV from 90 to 95% for refinancing
2008Q4*	tightening	LTV	maximum LTV from 100 to 95%
2008Q4*	tightening	amortization	maximum amortization from 40 to 35 years
2008Q4*	tightening	DSR	maximum total debt service ratio of 45%
2009Q4	easing	taxes	tax credit for first-time home buyers and renovations
2010Q2*	tightening	LTV	maximum LTV from 95 to 90% for refinancing and from 95 to 80% for investment properties
2010Q2*	tightening	DSR	maximum total debt service ratio of 45% based on five-year fixed posted rate of the Big 6 banks
2011Q1	tightening	amortization	maximum amortization from 35 to 30 years
2011Q1	tightening	LTV	maximum LTV from 90 to 85% for refinancing
2011Q2*	tightening	insurance access	no insurance for non-amortizing lines of credit secured by homes
2012Q3+	tightening	LTV	maximum LTV from 95 to 80% for house prices over $1 million and from 85 to 80% for refinancing
2012Q3+	tightening	amortization	maximum amortization from 30 to 25 years
2012Q3+	tightening	DSR	maximum gross debt service ratio of 39% and total debt service ratio of 44%
2016Q1*	tightening	LTV	maximum LTV from 95 to 90% for house prices between $0.5 and $1 million
2016Q3	tightening	taxes	foreign buyer tax in Vancouver of 15%
2016Q4	tightening	DSR	DSR limit computed with the higher of the contractual mortgage rate or the Bank of Canada conventional five-year posted mortgage rate
2016Q4	tightening	insurance access	current rules for access to government insurance of mortgages with high LTV ratios applied to mortgages with low LTV ratios
2017Q2	tightening	taxes	foreign buyer tax in the Greater Golden Horseshoe area (around Toronto) of 15%
2018Q1	tightening	taxes	foreign buyer tax in Vancouver from 15 to 20% with extended geographical coverage
2018Q1*	tightening	insurance access	non-insured high LTV ratio mortgages qualify at the greater of the contractual rate plus 2 percentage points or the benchmark five-year posted rate

Notes: For more details on the changes, refer to Cheung (2014), Kuttner and Shim (2016), Allen et al. (2017) or Bank of Canada (2017) for the latest measures. Kuttner and Shim (2016) also classify reserve requirements as macroprudential measures but fail to capture the rest of the macroprudential easing prior to 2008. Most other macroprudential changes concern government-backed mortgage insurance rules, except the last macroprudential measure of 2018Q1 and the federal or provincial taxes. In our benchmark household-related macroprudential index, we do not include the changes to the reserve requirements that directly impact banks and focus instead on the other measures that impact households' borrowing. * If the measure was announced in the quarter prior to its implementation. + If the measure was announced in the previous quarter, but just a few days prior to its implementation.

D Alternative specification of the empirical model

We first show how alternative lag orders impact the response to monetary and macroprudential policy tightening, respectively Figures 22 and 23. Column one has a shorter horizon for the VAR of Equation (1), two lags ($P = L = 2$) instead of four in the benchmark. Consistent with the stronger elasticity of credit shown in Table 2, column three uses a longer horizon for the quantile regressions of Equation (3), six quarters ($q = 6$) instead of four in the benchmark. Column two combines a shorter horizon for the VAR and a longer horizon for the quantile regressions. In each case, credit growth is significantly reduced (first row). The variations of the median are usually more muted (second row), while growth at risk usually deteriorates at first and then significantly improves (third row). This implies a significant contribution to tail risk (fourth row) because credit growth has a larger negative impact on the tail of future GDP growth than the median. For a monetary policy tightening, the response of tail risk is mostly significant at the 10 percent level (last row of Figure 22). But this level of significance is not always met in the case of macroprudential policy tightening (last row of Figure 23), depending on the behavior of other variables that contribute to movements in the tail.

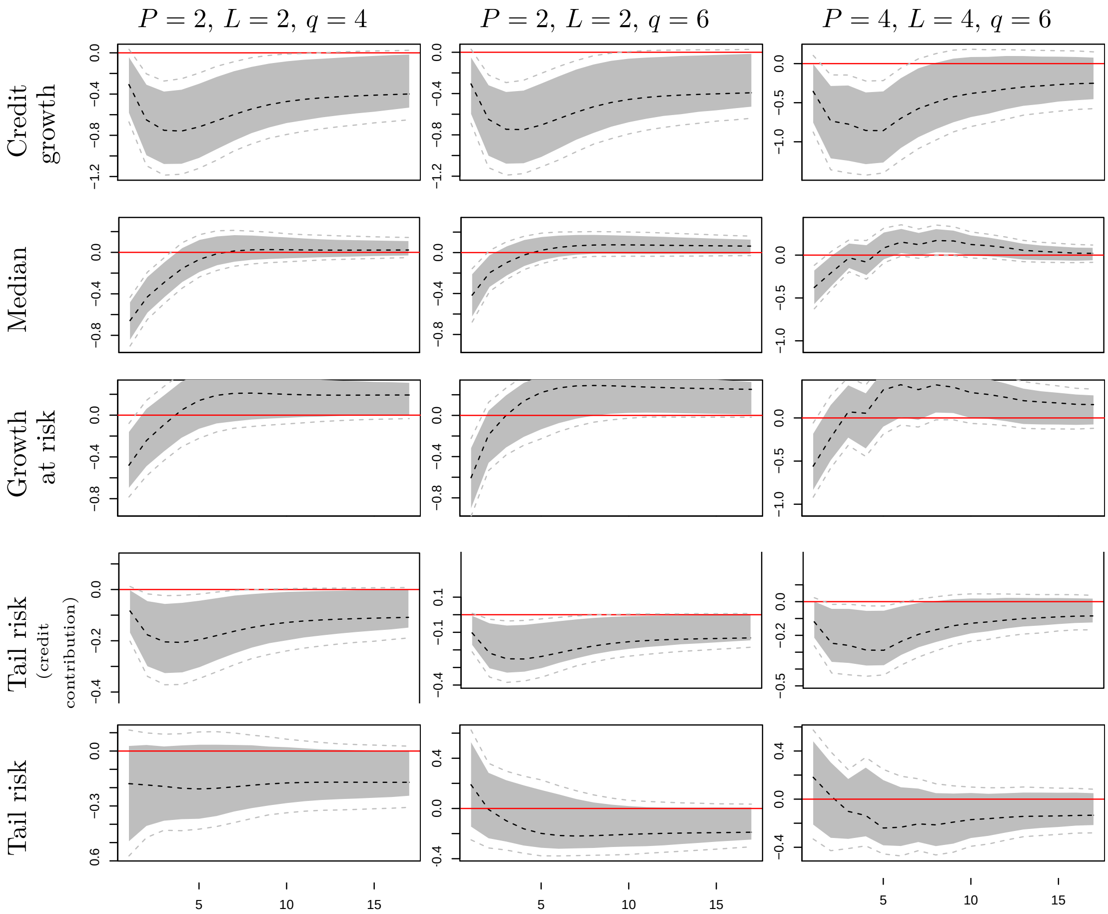
Second, we consider different proxies for the build-up of vulnerabilities when computing the responses to monetary and macroprudential policy tightening, respectively Figures 24 and 25. In column one, instead of the year-over-year growth (our benchmark), we use quarter-over-quarter growth that does not capture credit growth persistence anymore (Schularick and Taylor, 2012). As expected, quarter-over-quarter changes reduce the correlation with future tail risk. In the case of a monetary policy tightening, the contribution of credit growth to tail risk (fourth row) is not strong enough to generate a significant response of tail risk (fifth column). Results remain significant at the 10 percent level for macroprudential policy tightening. Turning to columns two and three, instead of household bank credit growth (our benchmark), we use broad credit (households and non-financial firms) or mortgage credit (International Monetary Fund, 2017*b*; Allen et al., 2017), both in year-over-year growth.[33] The results are weaker when using broad credit instead of household credit, which suggests that household and business credit may not have the same sensitivity to monetary policy changes. The results are even stronger when using the narrowly defined mortgage credit growth. This is especially true for macroprudential tightening. This confirms that the macroprudential policy index captures what it is intended to capture, given that we focus on household-related macroprudential measures, many of which target the mortgage market.[34]

Lastly, we investigate alternative specifications of model variables on the response to monetary and macroprudential policy tightening, respectively Figures 26 and 27. Column one defines growth at risk as the tenth instead of the fifth percentile of future GDP growth. In the case of a monetary policy tightening, the contribution of credit to tail risk (the difference between the tenth percentile and the median GDP growth four quarters ahead) remains significant, but the overall response of tail risk is significant only in the short term

[33] Column three uses the year-over-year growth of mortgage credit as well as the five-year government bond rate that is the usual anchor for mortgage lending in Canada.

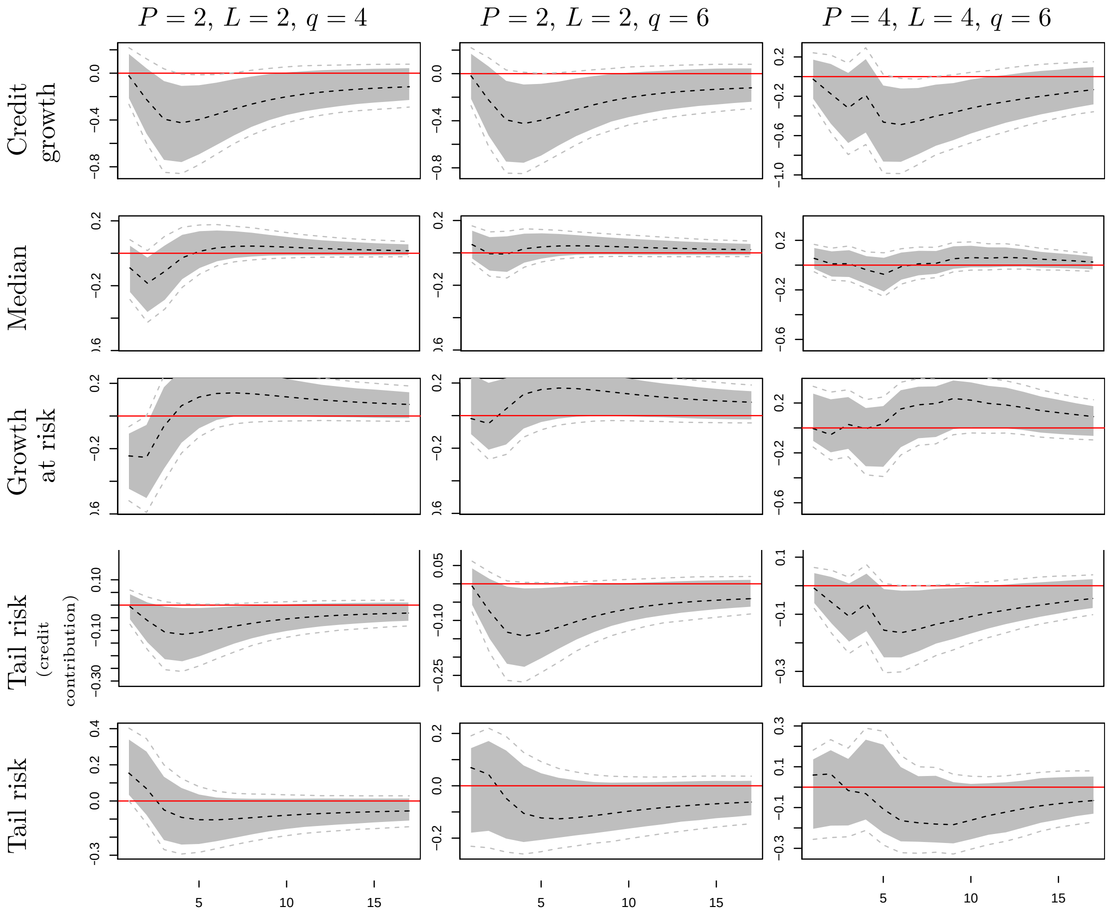
[34] One additional robustness test could be to use the difference of credit growth from the long-term rolling mean or the filtered trend (Drehmann, Borio and Tsatsaronis, 2011; Borio, 2014). Compared to the median, the elasticity of GDP growth to credit at the fifth percentile, although more negative, tends to be not significant anymore. Similar movements of tail risks can still be obtained, but mostly because of variations in the financial stress index. This possibly shows the challenges associated with abstracting from the policy impacts on trend credit growth.

Figure 22: Response after 100-basis-point monetary policy tightening - robustness test, various lags



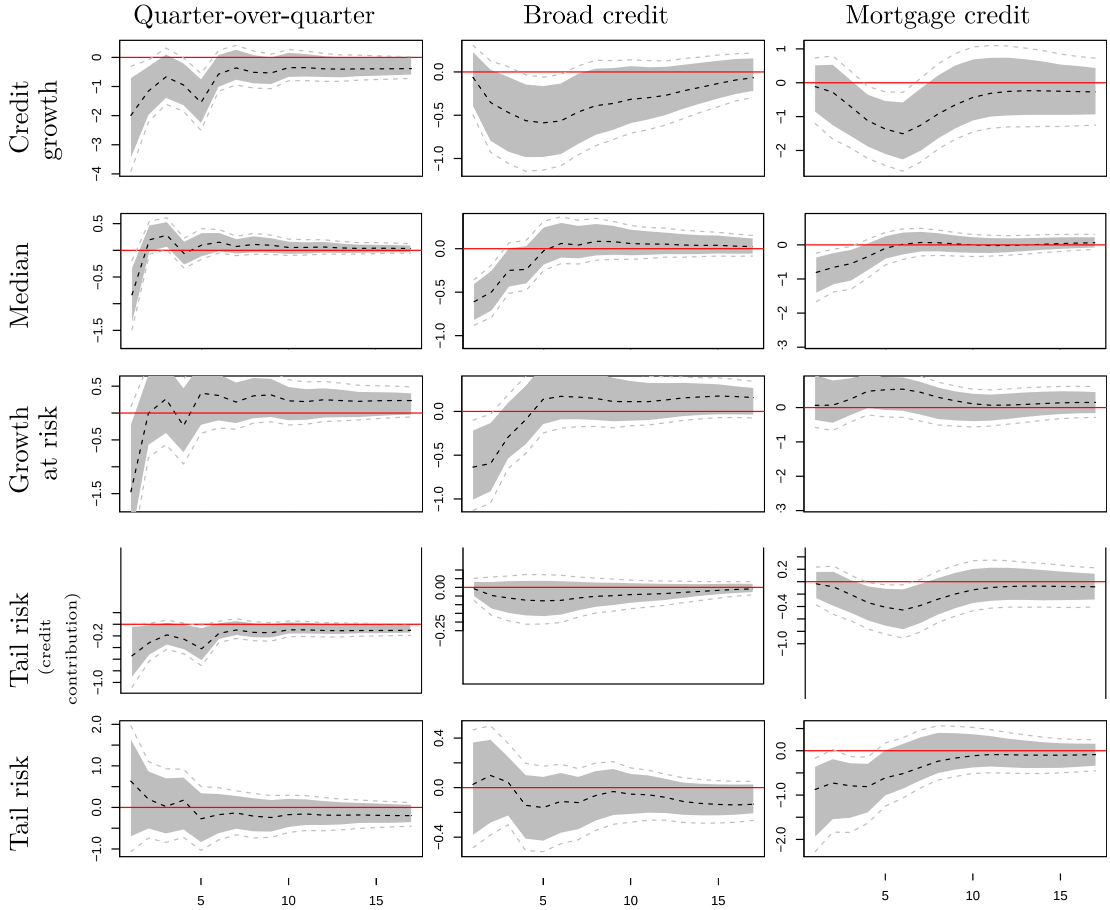
Note: The figure displays the response of the tightening of monetary policy by 100 basis points for different lag order specifications of Equations (1) and (3). The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

Figure 23: Response after macroprudential policy tightening - robustness test, various lags



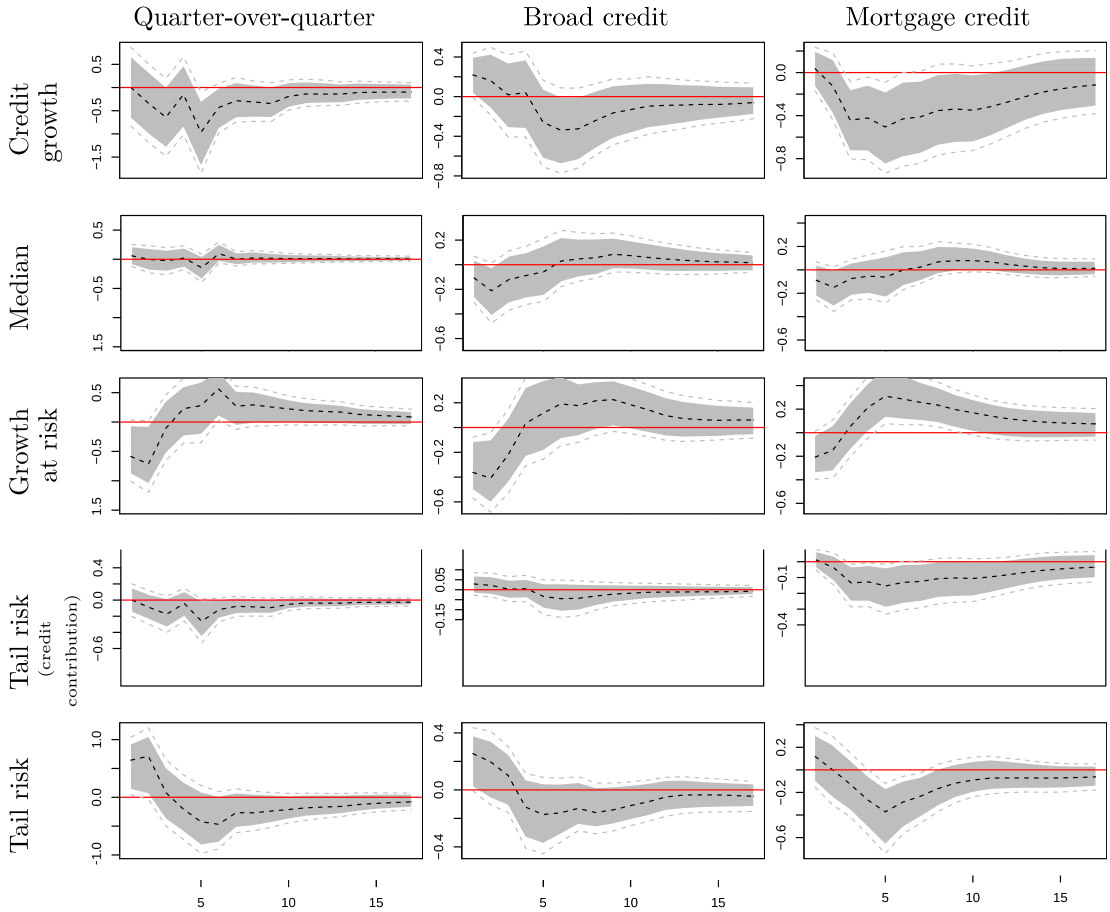
Note: The figure displays the response of the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening, for different lag order specifications of Equations (1) and (3). The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

Figure 24: Response after 100-basis-point monetary policy tightening - robustness test, alternative credit transformations



Note: The figure displays the response of the tightening of monetary policy by 100 basis points for different specifications of credit growth. When taking credit growth quarter over quarter, the other variables are also in quarter-over-quarter growth. Broad credit is defined as household and business credit. When using mortgage credit, we also use the five-year government bond interest rate used to anchor the mortgage credit rate in Canada. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

Figure 25: Response after macroprudential policy tightening - robustness test, alternative credit transformations

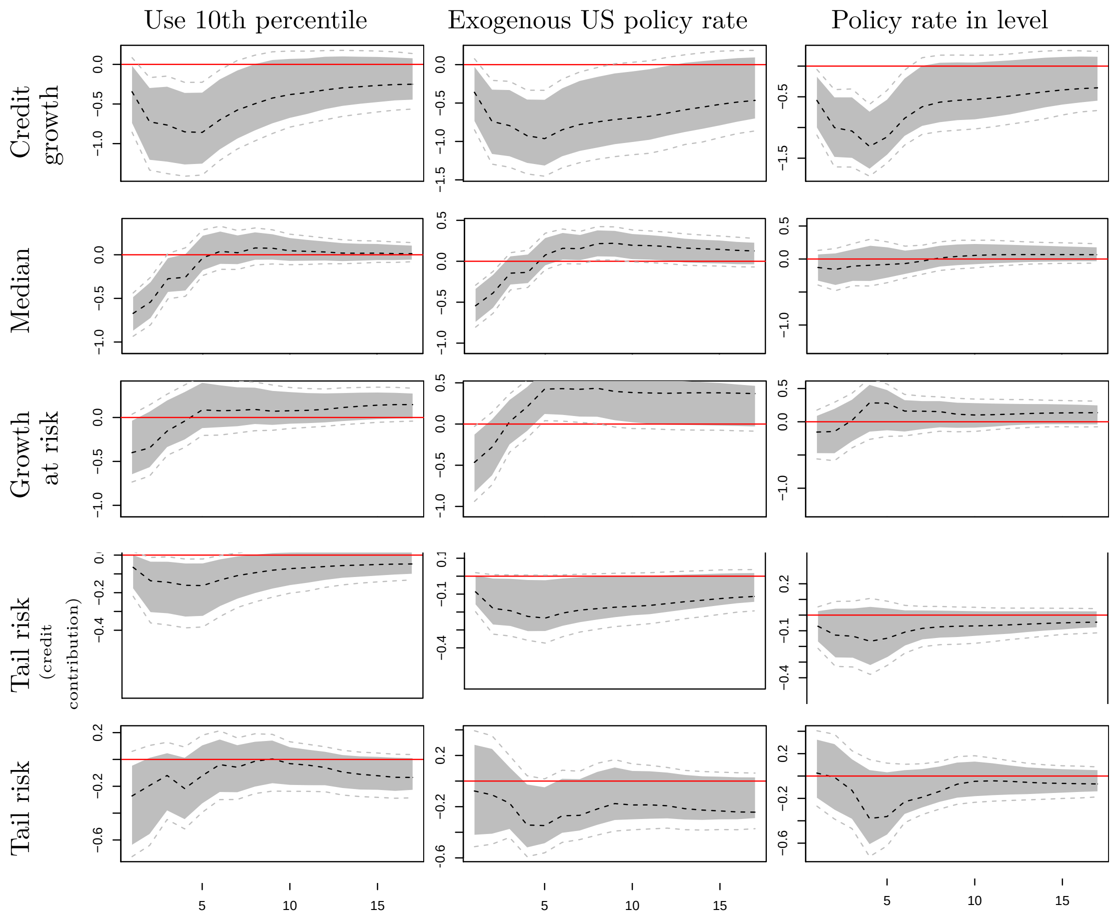


Note: The figure displays the response of the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening, for different specifications of credit growth. When taking credit growth quarter over quarter, the other variables are also in quarter-over-quarter growth. Broad credit is defined as household and business credit. When using mortgage credit, we also use the five-year government bond interest rate used to anchor the mortgage credit rate in Canada. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk (the fifth percentile of the one-year-ahead real GDP growth) and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

due to counteracting responses of other variables. In the case of a macroprudential policy tightening, the results stay intact. Column two introduces the year-over-year change in the US policy in both the VAR (as an exogenous variable) and the quantile regression, in order to take into account the small open economy nature of Canada vis-à-vis the US. All results remain as before. Column three uses the level of the policy rate instead of the year-over-year change, in order to better capture the intensity of policy stimulus.[35] The impact on tail risk is not significant at conventional levels, because introducing a non-stationary variable in the quantile regression changes the tail correlations. Although the impulse response of credit remains strongly significant (first row), the contribution of credit to tail risk driven by the differential response at the fifth percentile and the median is no longer significant (fourth row). Still, for the case of a macroprudential policy tightening, growth at risk significantly increases (third row), although the increase is not significantly different from that of the median (second row).

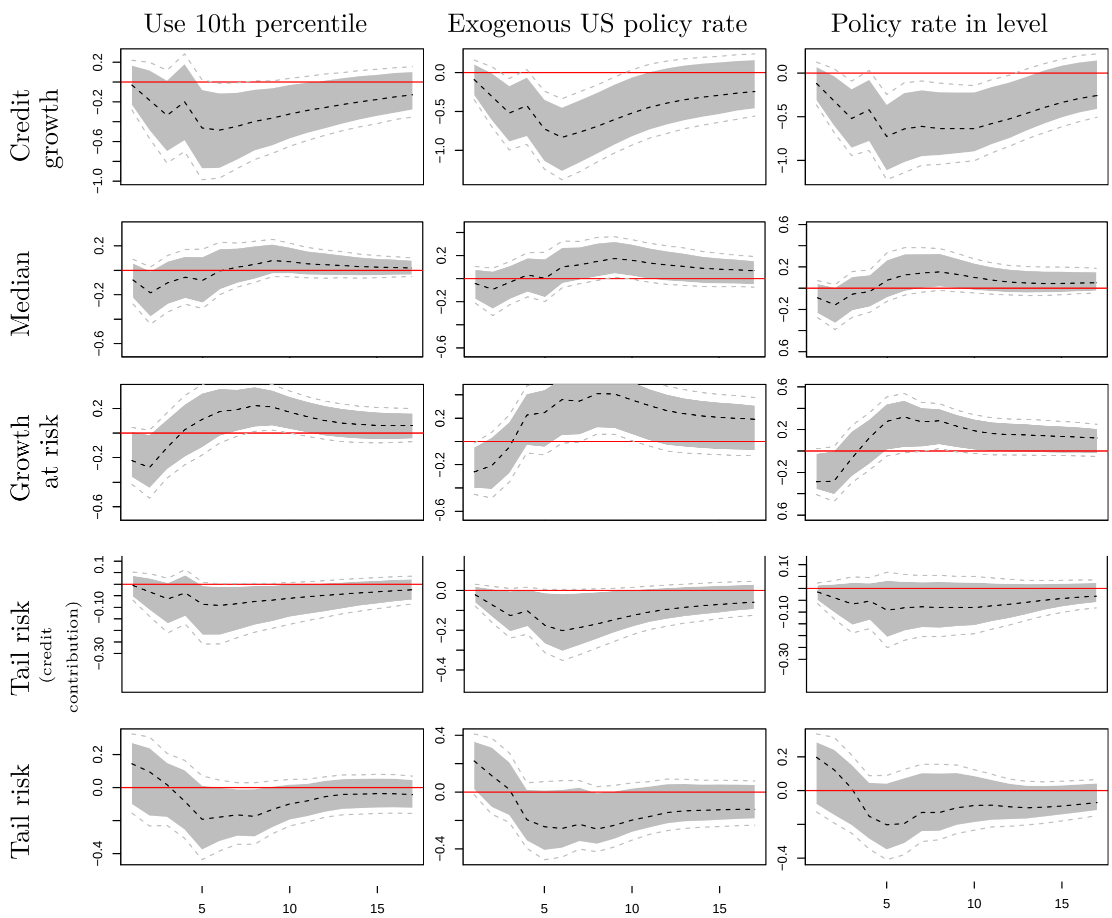
[35]However, as the policy rate since the early 1980s is non-stationary, we also introduce the level of the US policy rate as an exogenous control in the VAR to capture a potential trend in the world neutral rate.

Figure 26: Response after 100-basis-point monetary policy tightening - robustness text, alternative specifications



Note: The figure displays the response of the tightening of monetary policy by 100 basis points for different specifications of credit growth. The first column corresponds to the results where growth at risk is defined as the tenth instead of the fifth percentile. The second column adds an exogenous control in all equations for the year-over-year change in the US policy rate. The third column uses the level of the Canadian policy rate and controls for the exogenous level of the US policy rate to capture the trend in the level of the policy rate. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

Figure 27: Response after macroprudential policy tightening - robustness test, alternative specifications



Note: The figure displays the response of the tightening of macroprudential policy by one measure, i.e. the historical average impact of one macroprudential tightening, for different specifications of credit growth. The first column corresponds to the results where growth at risk is defined as the tenth instead of the fifth percentile. The second column adds an exogenous control in all equations for the year-over-year change in the US policy rate. The third column uses the level of the Canadian policy rate and controls for the exogenous level of the US policy rate to capture the trend in the level of the policy rate. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

E Alternative monetary policy shock identification

We investigate alternative identifications of monetary policy shocks: see Figure 28. Instead of using the time series of monetary policy shocks of Champagne and Sekkel (2018) as an exogenous variable in the VAR, we used a standard Cholesky decomposition (column one). We rank the credit and financial stress variables last (same order as Equation (1)). All results still hold.

We also investigate the use of contemporaneous sign restrictions: see Table 4. Either we use a restricted identification and focus only on three variables–GDP, inflation and the policy rate–for the identification of supply, demand and monetary policy shocks (the first three rows and columns). Or we extend the restrictions to account for a potentially different impact of monetary policy on credit growth. Indeed, if the policy rate increases, one would expect credit growth to be reduced (fourth row). But credit growth can also increase if agents pull forward their demand for credit when the policy rate is expected to increase further (fourth column). We also impose a positive sign restriction on the policy rate for four quarters: given that we use the year-over-year change of the policy rate, a policy rate shock should increase the policy rate for at least up to one year.

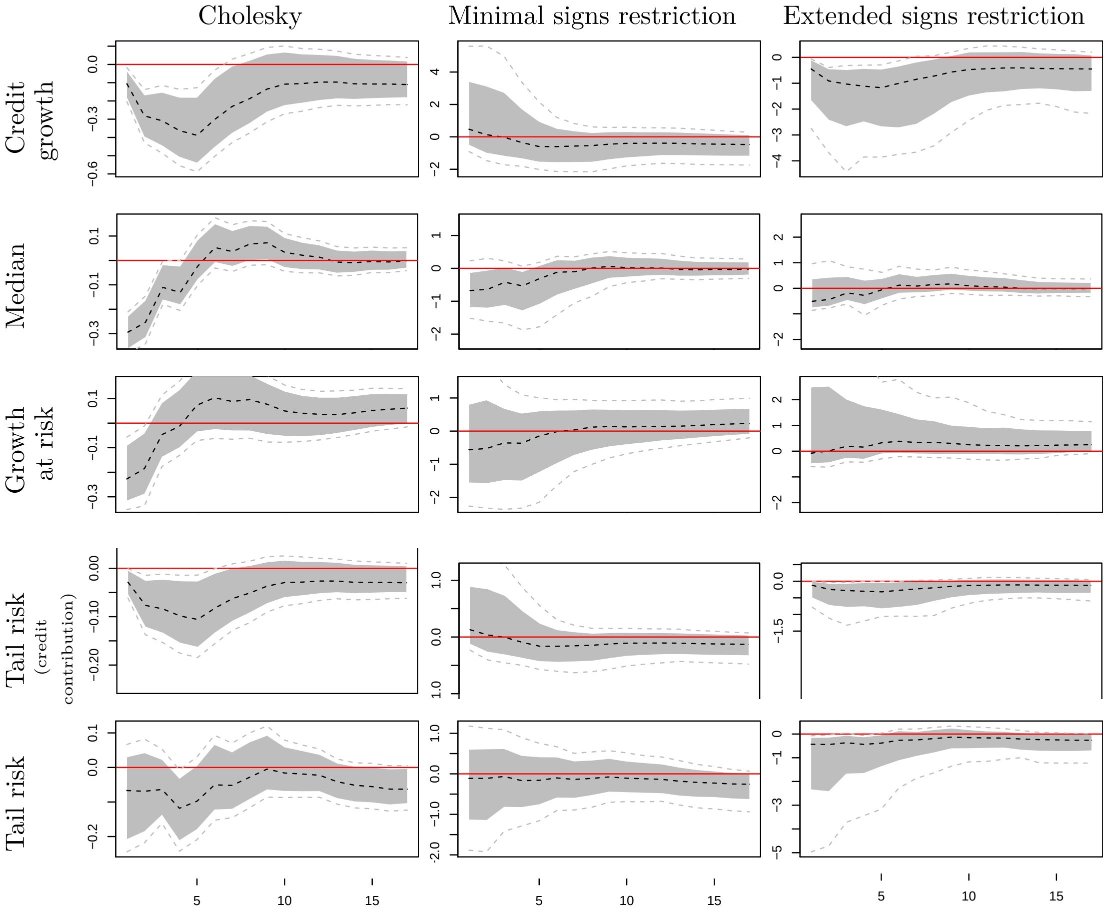
Table 4: Alternative monetary policy shock identification in the VAR with sign restriction

	Demand shock	Inflation shock	Monetary shock	Monetary shock (alt)	other shock
GDP	+	-	-	-	
CPI	+	+	-	-	
Rate	+	+	+ (4)	+	
Credit			-	+	
FSI					

Notes: Sign restriction is used in columns two and three of Figure 28. Restrictions that last for several quarters are identified with the number of quarters in parentheses. Other restrictions are only contemporaneous. Sign restrictions in gray correspond to additional restrictions (column three of Figure 28) in addition to the minimal set of restrictions (column two of Figure 28).

When using the minimal set of sign restrictions (second column of Figure 28), tail risk does not significantly respond to a monetary policy tightening, because monetary policy shocks in the VAR are not associated with a significant reduction in credit growth. This could be because households expect the increase in the policy rate to continue and pull forward some of their demand for credit. This effect disappears when using the extended set of sign restrictions (third column of Figure 28). In this case tail risk significantly decreases, but with wider confidence bands, due to the potential amplification of the response from financial stress.

Figure 28: Response after 100-basis-point monetary policy tightening - robustness test, alternative identification



Note: The figure displays the response of the tightening of monetary policy by 100 basis points for different shock identifications. The first column uses Cholesky. The second column uses sign restrictions on impact to identify demand, supply and monetary policy shocks, leaving the other two shocks unidentified in the five-variable VAR. The third column uses sign restrictions on impact to identify demand and supply shocks, as well as monetary policy shocks that are either associated with a decrease or an increase of credit growth, leaving only one other shock unidentified in the the five-variable VAR. Both the second and third columns use a long-run restriction of the policy rate shock on itself, to ensure that positive policy rate shocks in the year-over-year change of the policy rate keep the policy rate positive for one year. The figure shows the response of credit growth, median GDP growth four quarters ahead, growth at risk and the tail risk measure of Definition 3. Lower tail risk corresponds to a reduction in the gap between the median and growth at risk, expressed in percentage points of real GDP growth. The dashed black line is the point estimate, the gray area reflects a one-sided significance level of 0.1, and the dashed gray lines reflect a one-sided significance level of 0.05. Bootstrapped confidence bands are performed by re-sampling residuals.

F Cross-country quantile regressions

The choice set between central and tail risk matters only to the extent that our measure of tail risk captures a distinct movement in the tail of the distribution of future GDP growth coming from credit growth. This relies on the existence of a significant correlation between episodes of negative credit growth and subsequent negative GDP growth. In other words, it relies on the existence of enough severe episodes of corrections in GDP growth. Given the limited number of Canadian financial downturns, we test the robustness of our results in a cross-country analysis equivalent to Equation (3), now estimated on $C = 16$ developed economies.[36]

$$\mathbb{Q}_{GDP,c,t}(\tau) = \alpha(\tau) + \sum^{C} \alpha_c(\tau)\mathbb{1}_c + \gamma(\tau) \begin{bmatrix} GDP_{c,t-4} \\ CPI_{c,t-4} \\ Rate_{c,t-4} \\ Credit_{c,t-4} \\ FSI_{c,t-4} \end{bmatrix} \tag{5}$$

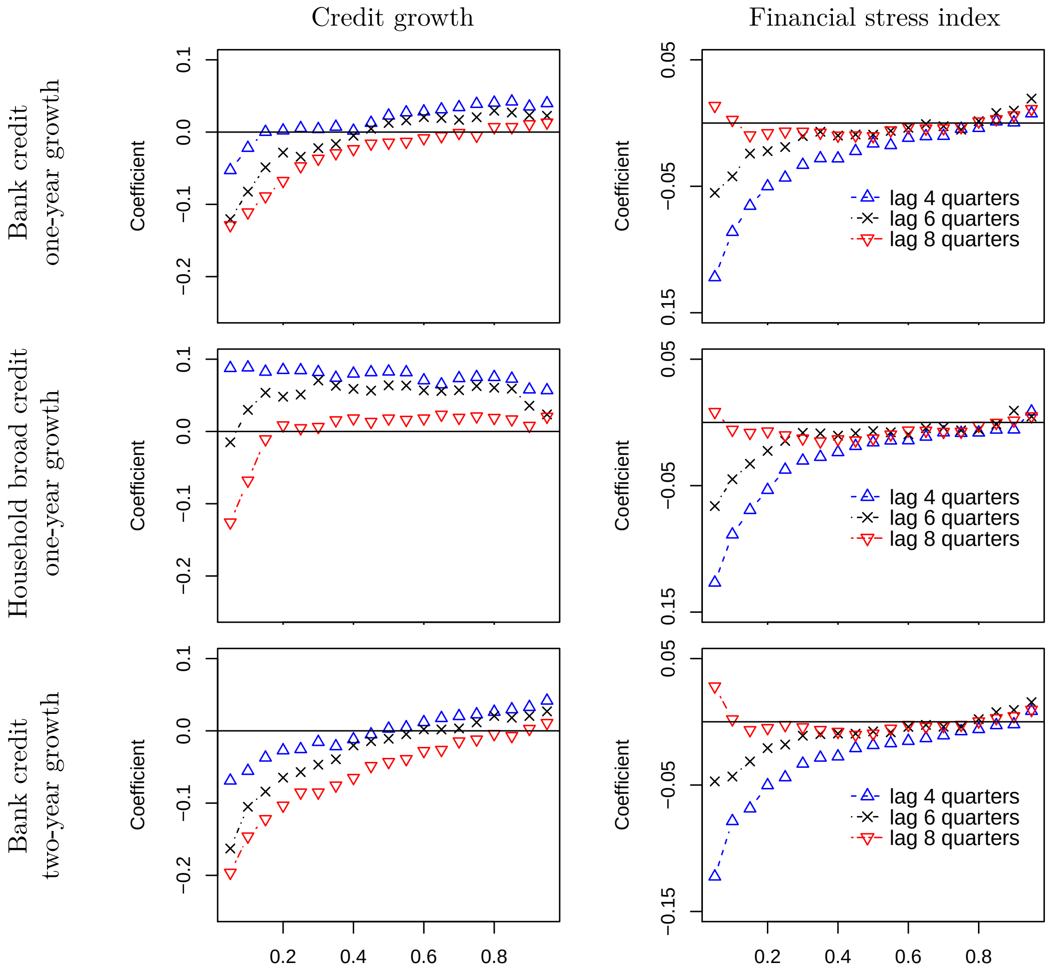
Credit growth comes from the BIS, but since household bank credit growth is not available in a consistent manner across countries, we look at both households' broad (bank and non-bank) credit growth and bank (households and corporate) credit growth.[37]

We obtain results, Figure 29, that are similar to those for Canada only: see Figure 6. First, more (bank or household) credit growth is associated with a worsening of the lower tail of future GDP growth, while it increases the upper tail. Second, slower-moving measures of vulnerabilities like the accumulation of credit are better able to forecast economic downturns two years or more ahead. For shorter-term forecasts, financial market stress is the main driver of risks to future GDP growth. Third, measures of bank credit appear to be more strongly associated with risks to future GDP growth for all projection horizons. This is historically consistent with most crises being associated with banking crises (Leaven and Valencia, 2013). Finally, sustained credit growth over two years is more strongly associated with future corrections of GDP growth, which is consistent with the findings of Schularick and Taylor (2012).

[36] Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom and the United States.

[37] Note that the financial stress index of Duprey, Klaus and Peltonen (2017) is adjusted to make the highest level of financial stress within a country comparable across countries. To ensure cross-country comparability, each input of the FSI is normalized against previous observations within but also across countries.

Figure 29: Cross-country quantile regressions for alternative lag orders



Note: The figure displays the coefficients of Equation (5) for credit growth and financial stress (vertical axis) by quantile (horizontal axis) for different lag orders of the cross-country quantile regression. The top panel uses average bank credit over one year. The middle panel uses average household broad (bank and non-bank) credit over one year. The bottom panel uses average bank credit over two years. Country-fixed effects are computed with the two-step method of Canay (2011).