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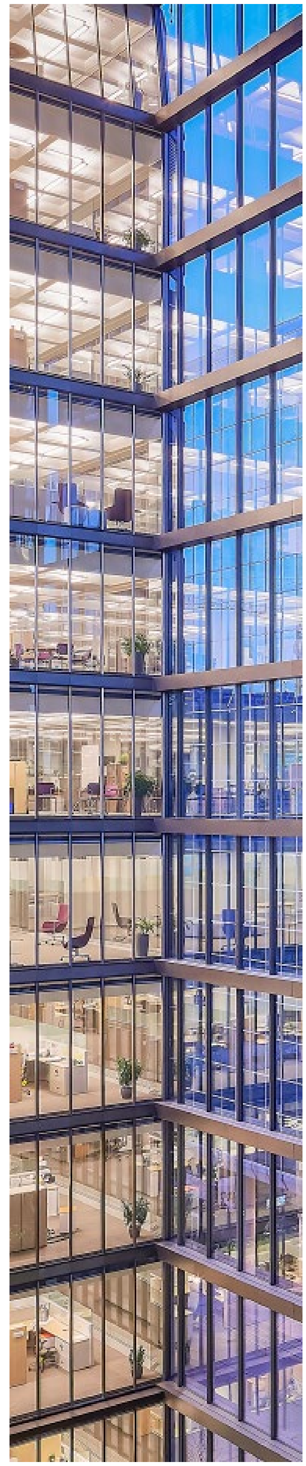

Strategic Uncertainty in Financial Markets: Evidence from a Consensus Pricing Service

by Lerby M. Ergun and Andreas Uthemann

Financial Markets Department
Bank of Canada, Ottawa, Ontario, Canada K1A 0G9

lergun@bank-banque-canada.ca, AUthemann@bank-banque-canada.ca

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Abstract

This paper measures valuation and strategic uncertainty in an over-the-counter market. The analysis uses a novel data set of price estimates that major financial institutions provide to a consensus pricing service. We model these institutions as Bayesian agents that learn from consensus prices about market conditions. Our uncertainty measures are derived from their beliefs through a structural estimation. The main contribution of the consensus pricing service is to reduce strategic uncertainty in the most opaque market segments. This stresses the importance of public data, such as financial benchmarks, for a shared understanding of market conditions in markets with limited price transparency.

Topics: Financial institutions, Financial markets, Market structure and pricing
JEL codes: C58, D53, D83, G12, G14

1 Introduction

Prices serve a dual purpose. They aggregate dispersed information about gains from trade. At the same time, they help to coordinate actions among market participants. Empirical work on the informational value of prices typically focuses on the former. However, the ability of prices to reduce uncertainty about market participants' actions and beliefs, that is, strategic uncertainty, can be of equal importance. This is especially true in markets with strong coordination motives (Angeletos and Pavan (2007); Morris and Shin (2002)). Intermediated markets are an important example. Intermediaries not only have to form beliefs about the gain from trades that they intermediate, but also have to take into account the behaviour of other intermediaries. The aggregate level of intermediation activity influences the number of trading parties that can be linked. It also determines the ability to share the risk that comes with the holding of inventory during the intermediation process. Therefore, an intermediary needs to form beliefs about both valuations and other intermediaries' behavior.

Public price data can help to coordinate these beliefs by establishing important reference points for trading and managing risk. A lack of price transparency, a common problem in intermediated markets, threatens the common understanding of market conditions and can lead to costly coordination failures (Morris and Shin 2012). Episodes of financial market freezes provide ample evidence for this fact.[1] In response to the temporary or permanent shortage of trading-based price data, so-called consensus pricing services have emerged. These services collect price estimates from market participants and aggregate the estimates into a consensus price. A consensus price is supposed to give the current market value of a specific asset, for example, a 2015 Honda Civic LX with 50,000 km mileage in the used car market or the interest rate for an unsecured 6-month US dollar loan in the London inter-bank market at 11 a.m. London time. A major advantage of consensus prices is that they can be generated irrespective of whether the asset has been recently traded. As the last example shows, consensus prices are an important mechanism in financial markets to calculate benchmark prices. It is thus crucially important to understand whether, and how, the consensus pricing mechanism works in practice.

In this paper, we study the informational value of the consensus pricing mechanism for market participants. To tackle this question, this paper develops and structurally estimates a model of learning from prices. The empirical analysis

[1] Lowenstein (2000) (p. 159), for example, gives a vivid account of the bond market at the height of the LTCM crisis on August 31, 1998: "It was as if a bomb had hit; traders looked at their screens, and the screens stared blankly back. [...] So few issues traded, you had to guess where they were."

is based on a new data set of price submissions that large dealer banks, highly sophisticated market participants, make to the main consensus pricing service in the over-the-counter (OTC) options market. We use the structural estimation to obtain empirical measures of uncertainty that are based on their model-implied beliefs. We also measure how efficient the consensus price is in aggregating dispersed information. In the model, dealer banks learn about a latent and time-varying option value from two types of signals: a noisy private signal and a consensus price. This consensus price is modelled as a noisy endogenous signal of dealer banks' average expectation about the option value. Each dealer bank is uncertain about the current value and other dealers' expectations thereof. We use the variance of a dealer bank's posterior beliefs about option value and competing dealers' average expectations to measure these two dimensions of uncertainty. Having access to a panel of individual dealer banks' price submissions allows us to identify the structural parameters that determine dealers' beliefs. To gauge the informational value of consensus prices, we perform counterfactual experiments on the option market's information structure.

We find that dealer banks' uncertainty about option values and strategic uncertainty varies substantially across the different segments of the options market. We find higher uncertainty for option contracts that are not listed on centralized exchanges but are exclusively traded over the counter. Dealer banks do not appear to rely heavily on the consensus price feedback to reduce their uncertainty about option values. This reduction in valuation uncertainty is at most 4.6 percent in the most opaque market segment. We find that the information the consensus price contains is most important for reducing strategic uncertainty, i.e., their uncertainty about competitors' valuation. This effect is strongest, up to 37.8 percent, in the most opaque segments of the options market and reflects the scarcity of public valuation information in these market segments. It stresses the importance of publicly observed market data, such as financial benchmarks, to establish a shared understanding of market conditions in markets with limited price transparency.

The estimation framework developed in this paper makes a methodological contribution by showing how to structurally identify the informativeness and informational efficiency of prices. The modern theoretical framework for these questions dates back to the early 1980s, with seminal contributions by Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981).[2] For example, Vives (1997) highlights the importance of the mix between public and private infor-

[2] The modern literature on information aggregation is too large to do justice to here. Important contributions have focused on auctions (Pesendorfer and Swinkels (1997); Kremer (2002)), decentralized trading (Gale (1986), Golosov et al. (2014)), asset design (Ostrovsky (2012)) or the trade-off between market size and information heterogeneity (Rostek and Weretka (2012)).

mation for the speed of information aggregation. However, as pointed out by Townsend (1983), determining the informational content of the price process in a dynamic equilibrium context poses significant technical challenges. Most structural empirical analyses of price formation assume that the asset value becomes at some point common knowledge (e.g., following Easley et al. (1996) information structure). This prevents long-lasting belief heterogeneity. The evolving source of uncertainty, paired with privately informed market participants, is a key source of belief heterogeneity and, hence, strategic uncertainty in our model. To solve the dynamic signal extraction problem and structurally estimate the model, we adopt an iterative algorithm previously used in Nimark (2014) and Barillas and Nimark (2017). We show that observing belief updating dynamics at the level of the individual institution is key for identifying the structural parameters of the model. Modelling the consensus price as an endogenous public signal allows us to conduct counterfactual experiments on the market's information structure to evaluate the strength of informational externalities caused by public information (Amador and Weill (2012)).

There is a sizeable literature that uses the cross-sectional dispersion among professional forecasters to study informational frictions (Coibion and Gorodnichenko (2012), Andrade et al. (2016)). However, the focus of these papers is on understanding the expectation formation process. The insights gained are then used to discriminate between alternative models, extrapolate to a wider group of market participants than just professional forecasters, and study macroeconomic consequences. In this paper we have a different objective, namely to measure uncertainty among market participants in an opaque market structure. For this, we assume that our units of observation, highly sophisticated financial market participants, are fully rational Bayesian agents. We then use this structural assumption to derive measurement devices for uncertainty based on their model-implied beliefs. An additional novel aspect of our empirical approach is the focus on strategic uncertainty. Here, the structural approach is particularly useful as data on market participants' higher-order beliefs are typically not available. Hortaçsu and Kastl (2012) and Hortaçsu et al. (2018), for example, use model-implied beliefs derived from a structural estimation to gauge the strategic value dealers derive from being able to observe client demand in Treasury auctions. Similarly, Boyarchenko et al. (2019) use a calibrated model to perform counterfactual informational experiments in the US Treasury market to evaluate the welfare implications of different order flow information-sharing arrangements among dealers and clients. However, the source of strategic information in these models is order flow information rather than price data. More generally, we see the counterfactual experiments we perform on the market's information structure as an illustration of the usefulness of a structural approach for empirical work on information design (Bergemann and Morris (2019)).

This paper contributes towards understanding the role of benchmarks in the well functioning of markets. Duffie et al. (2017) show how benchmarks can reduce informational asymmetries in search markets and thereby increase the participation of less-informed agents. Our approach differs, in that we focus on the informational properties of the benchmark itself. Furthermore, we model the benchmark as the outcome of an equilibrium process among symmetrically informed agents. This also contributes more widely to understanding of the informational value of non–transaction based price information. Previous work in this area has focused on different information aggregation mechanisms, in particular pre-opening prices (Biais et al. (1999), Cao et al. (2000)) and opening auctions (Madhavan and Panchapagesan (2000)) in stock markets. This paper is, to our knowledge, the first to provide an empirical evaluation of the informational properties of a consensus pricing mechanism.[3] While the importance of benchmarks for financial markets is widely appreciated, the attempted manipulation of major interest rate benchmarks has led to a regulatory push to base benchmarks on transaction prices or firm quotes rather than expert judgment (IOSCO (2013)). However, in illiquid markets and during crisis times, this might not always be possible. During the COVID-19 turmoil in March 2020, for example, three out of four candidate forward-looking term rate benchmark providers were unable to publish benchmarks during a three-day period due to the lack of transactions data (Risk.net).

The plan of the paper is as follows. Section 2 provides a brief explanation of the option market structure and the Totem consensus pricing service and presents the data. Section 3 develops the structural model of learning from consensus prices from which our empirical uncertainty measures are derived. Section 4 presents the structural estimation of the model and discusses identification and robustness. Section 5 illustrates our approach to measuring valuation and strategic uncertainty and presents results. Section 6 concludes.

[3]Many important financial benchmarks are consensus prices. It is also employed by information providers, such as Bloomberg, to calculate "generic prices" for a wide range of financial products.

2 Market structure and data

2.1 Options market structure

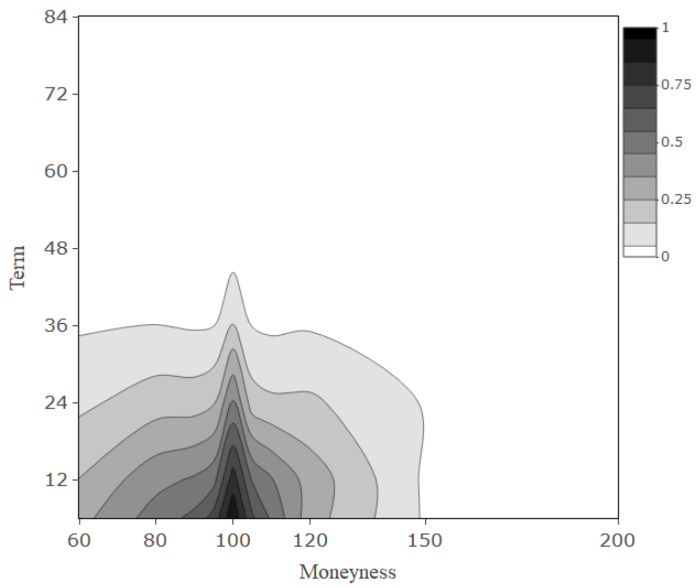
Options on the S&P 500 index are arguably the central derivatives contracts for the stock market. Their prices contain rich information on market participants' beliefs about future US stock market movements and risk premia. The VIX index, a popular measure of risk perception in financial markets, is based on S&P 500 option prices. The dominant market structure for options trading depends on the terms of the contract. Option contracts with times-to-expiration of less than 6 months and strike prices close to the current index value (moneyness close to 100) are typically traded on options exchanges, such as the Chicago Board Options Exchange (CBOE), via limit order books.[4] Price quotes, transaction prices and volumes are fully transparent and available to all market participants. For options with more extreme contract terms, the dominant market structure is OTC trading. Figure 1 displays the average on-exchange trading activity for S&P 500 index option over the period 2002 to 2015 for contract terms covered in this paper. On-exchange trading activity is decreasing with the time-to-expiration and the extremeness of the strike price of an option. For option contracts with times-to-expiration of more than 3 years, trading is almost exclusively OTC.[5]

The OTC segment of the options market is centred on a network of dealers. These are typically large investment banks that act as market-makers and trade with each other and with so-called clients: hedge funds, asset managers, insurance companies, and pension funds that want to manage portfolio risk or establish speculative positions. In terms of clientele, the market segment with times-to-expiration below one year is typically dominated by hedge funds trading short-term stock market volatility. The one- to three-year segment tends to be the domain of "real money," asset managers such as BlackRock that use options in their portfolio insurance strategies. Client demand in the market segment with times-to-expiration beyond 3 years tends to come from pension funds and life insurance companies that have long-term commitments linked to the evolution of the stock market. In the OTC market, trades are negotiated bilaterally, often over

[4] An option contract on an asset gives the owner the right (but not the obligation) to buy (call option) or sell (put option) the asset. The time-to-expiration of (European style) contracts specifies the date at which the option can be exercised; the strike price specifies the price at which the asset can be bought or sold. The strike price is often expressed as a ratio to the current price of the asset times 100. This is also called the moneyness of the option.

[5] Clearing house data (see, e.g., OOC) shows that in overlapping regions, traders often choose an OTC over an on-exchange trade. This can be because of greater flexibility in contract terms, lower trading fees, or market impact considerations. Large trades can be difficult to hedge if the trade is publicized. Here the transparency that comes with on-exchange trading is undesirable.

Figure 1: On-exchange option trading activity



This figure displays the trading activity on exchanges for the different put and call option contracts on the S&P500. We use daily aggregated data, provided by OptionMetrics, to display the percentage of days a particular contract is traded. We count the number of days with a contract-specific total trading volume of 10 or more. Due to the coarse grid of the options reported to the Totem service, exchange-traded contracts in the proximity of a Totem contract are aggregated to a single moneyness and maturity combination. Moneyness of the contracts are (60, 80, 90, 95, 100, 105, 110, 120, 150, 200) and times-to-expiration are (6, 12, 24, 36, 48, 60, 72, 84). Here, proximity is defined as less than half the distance to the next Totem contract in terms of moneyness and time-to-expiration. We use the trading volume for both call and put options between 2002 to 2015.

the phone or by email or instant messaging. Both transaction price and volume remain proprietary information of the two parties involved in the trade.[6] Rather than having to rely on pricing models to hedge option exposures, dealer banks typically prefer to conduct offsetting trades with each other in the inter-dealer segment of the options market. Hence, when trading with clients, a dealer bank

[6]Some dealers run proprietary electronic trading platforms on which they post price quotes. In 2010, various electronic trading platforms introduced request-for-quote systems to further increase pre-trade price transparency. The regulatory reforms after the financial crisis have also introduced mandatory post-trade reporting to trade repositories for all OTC derivatives transactions. But these are regulatory data and not available to market participants. Another source for price and volume information in OTC markets is central counterparties (CCPs). However, unlike for interest rate and credit derivatives, OTC equity derivatives trades are currently not subject to a central clearing mandate. The current proportion of OTC equity derivatives trades that is centrally cleared is negligible (see Financial Stability Board (2018)).

not only has to form a view on the fundamental drivers of option values. It also has to consider the valuations of other dealer banks with whom it can enter into offsetting trades. Appendix 7.3 develops a stylized model of this market structure to illustrate the value of information for dealer banks.

2.2 Consensus Price Data

The empirical analysis is based on data from the main consensus pricing service for the OTC derivatives market, IHS Markit's Totem service. The service started in February 1997 with 6 major OTC derivatives dealers. Since then, Totem has become the leading platform for OTC consensus price data, with around 120 participants and a coverage of all major asset classes and types of derivatives contracts. In this paper, we focus on the consensus prices for call and put options on the S&P 500 index. We have access to the full history of Totem contributors' price submissions. The individual institutions are anonymized, but we can track each institution's submissions over time and across contracts. We restrict our sample to the period December 2002 to February 2015 to achieve a consistent set of option contracts and a stable group of submitting institutions. Table 1 reports the set of option contracts we consider (by time-to-expiration and moneyness) as well as the average number of institutions submitting price estimates for a given contract over our sample period.[7] Note that some participating institutions do not submit to the more extreme contracts.

The consensus pricing process

The Totem pricing service typically operates at a monthly frequency. At the end of a month, all submitters are asked to provide their best estimate of the mid-quotes (at a specific time on the so-called valuation day) for the set of derivatives contracts to which they contribute estimates. In addition to their estimate of the contract price itself, it includes other data used in the pricing of the contract, such as discount factors, dividend yields, and the price of the underlying asset.

Manipulation incentives for consensus prices of OTC derivatives are generally weaker than for benchmark interest rates, such as LIBOR, that are compiled using a similar method. Unlike in the benchmark interest rate case, no financial products in the OTC derivatives markets are indexed to consensus prices. Hence, changes in consensus prices do not immediately impact an institution's profits and

[7]For contracts with time-to-expiration of 6 and 12 months, we exclude the contracts with a moneyness of 200. For these contracts, prices are close to zero and crucially depend on the numerical precision used by Totem submitters when reporting prices. Additionally, the inversion of the prices to Black-Scholes implied volatilities can become numerically unstable.

Table 1: Average number of submitters

	moneyness										
term	60	80	90	95	100	105	110	120	130	150	200
6	27	31	31	31	31	31	31	31	29	27	·
9	26	29	29	29	29	29	29	29	29	26	·
12	27	30	30	30	30	30	30	30	28	27	19
24	27	30	30	30	30	30	30	30	28	26	19
36	26	29	29	29	29	29	29	29	27	26	18
48	26	29	29	29	29	29	29	29	26	25	18
60	25	28	28	28	28	28	28	28	26	25	18
84	24	25	25	25	25	25	25	25	24	23	17

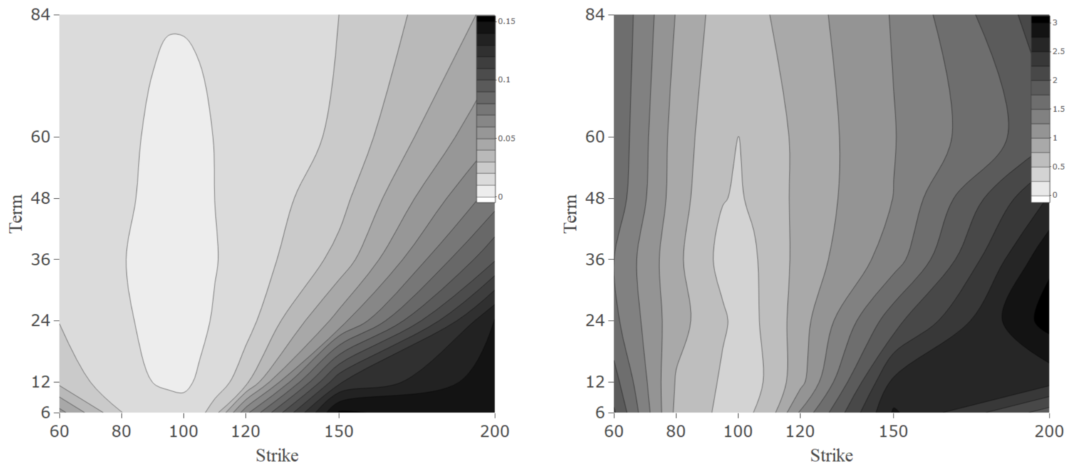
This table provides the average number of submitters for the specific options on the S&P 500 Index. These are the accepted prices per contract for the dates that the contract is polled. In our analysis we ignore submissions with a price of 0. The data sample is from December 2002 to February 2015.

losses. Furthermore, the Totem consensus pricing service has significantly more submitters than interest rate benchmarks, on average 30 per contract, which makes strategic manipulation of the consensus price more difficult. Nevertheless, Totem uses a range of formal and informal procedures to discourage price manipulation and incentivize high-quality price submissions. For the S&P 500 option service, each submitter is obligated to contribute to the contracts with time-to-expiration of 6 months and moneyness, expressed as the ratio of the option's strike price to the current index level, between 80 and 120. 'No-arbitrage" arguments between contracts allow for consistency checks between contracts. Comparison between submitters forms the basis for additional quality audits. In case of doubt, additional private conversation between submitters and IHS Markit employees (often ex-market participants) can take place to gather additional information on individual prices and market conditions. Price submissions that are deemed of low quality do not enter the consensus price calculation, and the submitting institution does not receive the consensus price for the submission period. This serves as a formal punishment mechanism for low-quality submissions.

Accepted price submissions are then used to calculate consensus prices, one for each derivatives contract. The highest and lowest accepted price are dropped from the sample before the mean is calculated. Totem provides contributors with the new consensus prices within 5 hours of their initial price submissions. The consensus price for a given options contract is the arithmetic mean of the accepted price estimates. For a detailed description of the submission process and the quantities submitted, see Appendix 7.4.

Valuation differences among dealers

To provide a sense of the cross-sectional dispersion in Totem submitters' option valuations, the left panel of Figure 2 depicts the cross-sectional standard deviation of price submissions, expressed in terms of implied volatility's,[8] averaged over the sample period. There is considerable variation in the dispersion in submitters' prices across the contract space. It is highest for short-dated options with extreme strike prices. For a given time-to-expiration, the dispersion is lowest for strike prices close to the current index level, that is a moneyness of 100. The price dispersion across submitters tends to decrease with time-to-expiration. These cross-sectional differences are economically meaningful; they are of similar magnitude to bid-ask spreads observed on option exchanges in regions where OTC and on-exchange trading overlaps, but they display a low level of correlation with these bid-ask spreads over time, as can be seen in Figure 6 of the Appendix.



(a) Cross-sectional dispersion (b) Half-lifes

Figure 2: The **left figure** displays the time-series average of the cross-sectional standard deviation of submitters' implied volatility estimates to a particular contract. The **right figure** present half-lives estimates of the individual deviations from the contemporaneous consensus price. The half-lives in months are transformations from an AR(1) regression. The estimates are from a pooled ordinary least squares regression. The y-axis of each figures gives the time-to-expiration and the x-axis the moneyness of the options contract under consideration. The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P 500 index.

[8]Throughout the paper, we express option prices in terms of Black-Scholes implied volatilities (IVs). This is the market convention for quoting option prices. It facilitates the comparison of option prices across times-to-expiration and strike prices.

The right panel of Figure 2 shows the persistence of individual submitters' deviations from the consensus price. For each option contract, we estimate the following AR(1) regressions to quantify this persistence:

$$p_{i,t}^{c} - p_{t}^{c} = \beta^{c}\left(p_{i,t-1}^{c} - p_{t-1}^{c}\right) + \epsilon_{i,t}^{c}.$$

Here $p_{i,t}^{c}$ is institution i's price submission for contract c in period t and p_{t}^{c} is the corresponding consensus price. The right panel of Figure 2 reports the estimated β coefficients expressed as half-lives, i.e., the number of months it takes to close half of the gap between an individual dealer's price estimate and the consensus price. Dealers' deviations from consensus are persistent for all contracts. The U-shaped persistence pattern in moneyness partially mirrors the cross-sectional dispersion in the left panel of Figure 2.

From these results, we draw some preliminary observations that guide our structural modelling. First, all submitters are asked to provide their best estimate for the mid-quote of a given contract, i.e. a market-wide price. If all dealers had access to the same information and used the same models, they should all provide the same price estimate. In this paper we abstract from model disagreement or model uncertainty and assume that submitters form expectations by updating beliefs using a model that itself is common knowledge. Under this interpretation of the data, the observed cross-sectional dispersion necessarily reveals informational frictions in the OTC market. Furthermore, these frictions vary across market segments.

Second, these informational frictions have to derive from dealer banks' private information. Imperfect information that is observed by all dealer banks does not induce cross-sectional dispersion. However, the cross-sectional dispersion alone cannot identify the precision of private valuation information, as both very precise and very imprecise private information imply low cross-sectional dispersion. This illustrates the conceptual problem of using the cross-sectional dispersion of the raw data for the measurement of informational frictions.

Last, if the consensus price perfectly aggregated dispersed information, all bank dealers should have the same expectation about the current mid-quote after observing the current consensus price. Any deviation from the consensus price has to be driven by new private information. If not, deviations from consensus could not be persistent; a dealer's past relative position to the consensus price would have no predictive power for its future relative position. This is clearly rejected by the data. The positive persistence points to imperfect information aggregation and, consequently, long-lived private information at the level of individual dealers.

3 A Model of Consensus Pricing

To model the key feature of the consensus pricing process, we set up the model as a social learning problem. The submission process and therefore the consensus price's informational content is an equilibrium outcome of the model.

3.1 The model

A large number of dealers, modelled as a continuum indexed by $i \in (0, 1)$, participate in a consensus pricing service. At discrete submission dates, indexed by $t = \{..., -1, 0, 1, ...\}$, a dealer i submits its best estimate for the current value of an option, which we denote by θ_t, to the service. θ_t is latent and follows an AR(1) process,

$$\theta_t = \rho\, \theta_{t-1} + \sigma_u\, u_t \;\text{ with }\; u_t \sim N\,(0, 1)\,, \tag{1}$$

and $-1 < \rho < 1$.[9]

At each submission date t, dealers observe two signals. Each dealer receives a noisy private signal $s_{i,t}$ about θ_t,

$$s_{i,t} = \theta_t + \sigma_\eta\, \eta_{i,t} \;\text{ with }\; \eta_{i,t} \sim N\,(0, 1)\,, \tag{2}$$

where $1/\sigma_\eta^2$ measures the precision of the private signal. All dealers receive signals of equal quality.

Additionally, each institution observes last period's consensus price p_{t-1}. This timing is a key difference to standard rational expectations equilibrium (REE) models. This feature allows us to avoid certain technical difficulties that arise in the REE literature. Given that it is a signal of the past state of the market, it can never fully reveal the current state. The consensus price p_t is a noisy average of submitters' best estimates of θ_t. Submitter i's information set at the time of period t's price submission consists of the (infinite) history of previous consensus prices and the private signals that i has observed up to period t, that is,

$$\Omega_{i,t} = \{s_{i,t}, p_{t-1}, \Omega_{i,t-1}\}.$$

All dealers submit their best estimate of θ_t. For each dealer, we take this to mean its conditional expectation of θ_t given $\Omega_{i,t}$. We denote this conditional expectation

[9] We do not explicitly model the economic forces responsible for the variation in fundamental values. A possible interpretation is based on demand-based option pricing models (see, e.g., Gârleanu et al. (2009)). Changes in fundamental values derive from time-varying client demand that is satisfied by risk-averse broker-dealers. Under this interpretation, u_t is an aggregate demand shock for options with a specific strike price and maturity combination.

by

$$\theta_{i,t} = \mathbb{E}\left(\theta_t | \Omega_{i,t}\right),$$

and the corresponding cross-sectional average across submitters by

$$\bar{\theta}_t = \int_0^1 \theta_{i,t}\, di.$$

We do not specify submitters' preferences, which would determine why they value the consensus price information. Certain preference specifications could create an incentive to strategically manipulate the consensus price, for example in order to experiment or to gain a competitive advantage (see Brancaccio et al. (2017) for experimentation motives in OTC markets). However, given the assumption of a continuum of submitters (and mild technical restrictions on admissible submissions), no single submitter can influence the consensus price. Hence, asking submitters to submit their best estimate of θ_t is compatible with their incentives.[10]

The consensus price itself is a noisy signal of the average expectation across submitting dealers, that is,

$$p_t = \bar{\theta}_t + \sigma_\varepsilon\, \varepsilon_t \;\text{ with }\; \varepsilon_t \sim N\left(0, 1\right). \tag{3}$$

Modelling the consensus price as a noisy public signal of average expectations is motivated by two considerations. First, as previously discussed, Totem eliminates the lowest, the highest, and problematic price submissions from the consensus price calculations. Hence, the consensus price itself does not exactly correspond to the average submission. Second, while we assume that there is a continuum of submitters, we want to allow for the possibility that the consensus price does not fully reveal the average expectation and, consequently, last period's fundamental value. Knowing past period's fundamental value rules out long-lived private information. But such long-lived private information is needed to capture the persistence of the deviations of individual price submissions from the consensus price, a feature we observe in the data.

3.2 Learning from consensus prices

In order to characterize dealer i's submission to the consensus pricing service, we need to calculate the dealer's conditional expectation $\mathbb{E}\left(\theta_t | \Omega_{i,t}\right)$. Its information

[10] Raith (1996) gives a theoretical analysis of the incentives for competitive firms to participate in (truthful) information-sharing arrangements. Appendix 7.3 provides a stylized model that shows how to approach such questions relating to the value of information in the context of consensus pricing.

set $\Omega_{i,t}$ depends on all other dealers' submissions via the consensus price process p_t. This information set is endogenous, as p_t is both an input and an output of the joint learning process of the consensus pricing participants. As first pointed out by Townsend (1983), signal extraction problems in which signals are equilibrium variables, such as prices, typically do not admit representations in which a finite number of variables can summarizes the current state of the system. For very restrictive settings, frequency domain techniques have been successfully employed to obtain exact finite state space representations, e.g. Kasa (2000). However, a popular direction of attack is truncation, i.e. to show that the original problem is well approximated by a finite state space even if the actual solution requires an infinite number of states (e.g. Sargent (1991), Lorenzoni (2009), Huo and Pedroni (2020)).

This is the approach taken here. We adopt an iterative algorithm previously used in Nimark (2014) and Barillas and Nimark (2017) to solve our filtering problem. The algorithm works as follows:

1. Start with any covariance-stationary process (p_t^0) that lies in the space spanned by linear combinations of current and past aggregate shocks (u_t) and (ε_t).

2. This consensus price process (p_t^0) yields information sets for all i and t defined recursively by $\Omega_{i,t}^0 = \{s_{i,t}, p_{t-1}^0, \Omega_{i,t-1}^0\}$.

3. Given information set $\Omega_{i,t}^0$, dealer i can compute the conditional expectation $\mathbb{E}(\theta_t|\Omega_{i,t}^0)$ for period t under the assumed stochastic process for (p_t^0).

4. Averaging the expectations across submitters yields a new consensus price process
$$p_t^1 = \int_0^1 \mathbb{E}(\theta_t|\Omega_{i,t}^0)\, di + \sigma_\varepsilon\, \varepsilon_t \;\text{ for all } t.$$

5. If the distance (in m.s.e.) between (p_t^0) and (p_t^1) is smaller than some prespecified stopping criterion, stop. Otherwise, go to step 2 with (p_t^1) as the new consensus price process and so on.

For any initial choice of (p_t^0), the sequence of price processes $\{(p_t^n)\}_n$ converges (in m.s.e.) to a unique limit process (p_t), the solution of the original filtering problem, when the integral in step 4 is a contraction on the space of covariance-stationary price processes. Starting with the initial guess $p_t^0 = \theta_t + \sigma_\varepsilon\, \varepsilon_t$ allows the problem to be solved by a sequential application of the Kalman filter. It also provides an upper bound on the approximation error if the algorithm is stopped after a finite number of steps.

After n steps, the equilibrium learning dynamics are approximated by a linear state-space system with an $n+1$ dimensional state vector x_t. The first and second element of x_t are the fundamental value θ_t and the cross-sectional average expectation $\bar{\theta}_t$, respectively.[11] The state evolves according to

$$x_t = Mx_{t-1} + Nv_t \quad \text{with} \quad v_t = (u_t, \varepsilon_{t-1})^{\mathsf{T}}, \quad v_t \sim N(0, I_2).$$

The matrices M and N are known functions of the model parameters, namely $\{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$. A dealer's signals in period t can be expressed as noisy observations of the state,[12]

$$s_{i,t} = e_1^{\mathsf{T}}\, x_t + \sigma_\eta\, \eta_{i,t} = \theta_t + \sigma_\eta\, \eta_{i,t},$$
$$p_{t-1} = e_2^{\mathsf{T}}\, x_{t-1} + \sigma_\varepsilon\, \varepsilon_{t-1} = \bar{\theta}_{t-1} + \sigma_\varepsilon\, \varepsilon_{t-1}.$$

The two signals can be written in vector form as

$$y_{i,t} = D_1\, x_t + D_2\, x_{t-1} + B\, \epsilon_{i,t},$$

with $y_{i,t} = (s_{i,t},\, p_{t-1})^{\mathsf{T}}$ and $\epsilon_{i,t} = (v_t^{\mathsf{T}},\, \eta_{i,t})^{\mathsf{T}}$.

We can now use the Kalman filter to obtain dealer i's beliefs about θ_t and $\bar{\theta}_t$, the first two elements of x_t, given the information in $\Omega_{i,t}$. The linear-normal structure of the state-space system implies that dealer i's beliefs are normally distributed,

$$x_t \mid \Omega_{i,t} \sim N\left(x_{i,t}, \Sigma^p\right),$$

where the conditional expectations about the state evolve according to

$$x_{i,t} = Mx_{i,t-1} + K\left(y_{i,t} - \left(D_1 M + D_2\right) x_{i,t-1}\right), \tag{4}$$

and K is a $(n+1)\times 2$ dimensional matrix of Kalman gains. Here K and the covariance matrix of dealers' beliefs Σ^p are known functions of the model parameters.[13]

[11] The kth element of x_t is the cross-sectional average of submitters' k^{th}-order expectation of θ_t given their information in period t. Appendix 7.5 provides a definition of these higher-order expectations and a detailed description of the solution algorithm.

[12] Here, e_n^{T} is a vector with 1 in the n^{th} position, 0 otherwise.

[13] Given the infinite history of past signals, the covariance matrix Σ^p and the matrix of Kalman gains K are not time dependent. Also, Σ^p and K are not dealer-specific as dealers are symmetrically informed. They all receive signals of the same quality. Superscripts, here p, are used to index information structures. In the counterfactual experiments we modify the information structure.

4 Estimation

To estimate the model parameters, namely $\phi = \{\rho, \sigma_u, \sigma_\varepsilon, \sigma_\eta\}$, for each contract we opt to estimate the parameters for each contract separately. In an alternative approach one could jointly estimate the contracts and impose a correlation structure on fundamental shocks u_t across contracts. This approach would also require a change in the signal structure that allows dealers to learn from options in close proximity. We have opted for the more parsimonious approach. First, we can prove that the structural parameters of the simpler model are identified. Second, not all participating dealer bank submit to all contracts. To allow for this compositional effect in a joint framework would further complicate the model structure and its estimation. Last, contract-by-contract estimation allows for the examination of the stability of the coefficients estimates between contracts and thereby provides a check of our estimation results.

For a given contract our data consists of the panel of Totem price submissions of individual dealers and the corresponding consensus price. We denote by $\iota_t \subset \{1, 2, .., S\}$ the set of dealers active in t where S is the total number of distinct dealers that have submitted to Totem over the course of our sample period. The time series of submissions is given by $(\mathbf{p}_t)_{t=1}^T$, where $\mathbf{p}_t = (p_{j,t})_{j\in\iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. Our data set for a given contract, $(\mathbf{y})_{t=1}^T$, then consists of the time series of dealers' price submissions for this contract and the corresponding consensus price, i.e. $\mathbf{y}_t = (p_t \, , \mathbf{p}_t)^\intercal$.[14]

4.1 Likelihood function and estimation

To estimate the model for a given contract, we cast it into state-space form. The panel of individual price submissions and the time series of consensus prices constitute the available observations of the system.

Based on Section 3, the latent state space has the following dynamics:

$$x_t = M(\phi)\, x_{t-1} + N(\phi)\, v_t \quad , \quad v_t \sim N(\mathbf{0}, I_2),$$

where $v_t = (u_t \ \varepsilon_t)^\intercal$. $M(\phi)$ and $N(\phi)$ are obtained employing the previously explained solution algorithm for a given parameter vector ϕ. We assume that dealer i's price submission for period t is its conditional expectation of θ_t, i.e. $p_{i,t} = \theta_{i,t}$.

[14]To be precise, $p_{j,t}$ and p_t are the time t submitter j's and by IHSMarkit calculated consensus implied volatility, respectively. These series are demeaned by the time-series average of p_t.

Dealer i's conditional expectations of the current state x_t are updated follows,

$$x_{i,t} = M(\phi)\, x_{i,t-1} + K(\phi) \left[\begin{pmatrix} s_{i,t} \\ p_{t-1} \end{pmatrix} - (D_1\, M(\phi) + D_2)\, x_{i,t-1} \right]. \qquad (5)$$

Dealer i's private signal $s_{i,t}$, given in (2), is treated as a latent variable in the estimation. It is observed by the dealer but not by the econometrician. The noise in the private signal is assumed to be uncorrelated across submitting dealers and time. The consensus price, p_t in (3), is observed by both the dealers and the econometrician. The econometrician only observes the first element of dealer i's conditional expectations $x_{i,t}$, namely $\theta_{i,t}$, which is a dealer first-order expectation of the current state. We assume that the dealer submits this expectation to the Totem service. Equation (5) provides us with a disciplined way of modelling the belief updating dynamics at the level of the individual dealer. This illustrates both the necessity of the structural model and the importance of observing a time series of price submissions at the level of individual dealers to estimate dynamic social learning models. More naive approaches that estimate belief-updating equations using only first-order expectations without taking into account the importance of higher-order expectations in the filtering problem will suffer from omitted variable biases.

Given the linearity of the above system and the joint normality of all shocks, the likelihood function for the observed data $(\mathbf{y})_{t=1}^{T}$ can be derived using the Kalman filter. We obtain maximum-likelihood estimates for the parameter vector ϕ using MCMC methods with diffuse priors. Appendix 7.6 provides a detailed derivation of the filter for the above model and discusses the estimation technique.[15] Appendix 7.1 reports parameter estimates and standard errors for $\rho, \sigma_u, \sigma_\varepsilon$, and σ_η for all contracts.

4.2 Identification

Appendix 7.7 provides a formal proof of identification for the model. Here, we give a short summary of which moments of the data help us to identify the structural parameters of the model. The time-series variance of the differences between p_t and cross-sectional average of submission identifies σ_ε. The speed at which individual deviations from the average submission mean-revert determines the weight submitters put on their prior expectations (as opposed to weight put on news in the current signal and consensus price). Knowing this weight allows us to isolate changes in price submissions that are due to new information a submitter

[15] We constrain $\sigma_u, \sigma_\varepsilon$, and σ_η to be positive and $0 < \rho < 1$. For each contract we run chains of length 100.000 with the Metropolis-Hastings algorithm and disregard the first 10.000 draws. We subsequently pick every 10^{th} draw to construct the posterior distribution of the parameters.

received in a given period. As these news are linked to the current fundamental, the autocorrelation of these expectation updates that have been "cleaned" of prior expectations allow us to identify ρ, the persistence in the fundamental value process. The weight submitters put on their prior depends on how persistent the fundamental is and how high the quality of their new information is, i.e. the signal-to-noise ratio of their signals. Having identified ρ, we can now identify this signal-to-noise ratio from the weight submitters put on their prior expectations. The signal-to-noise ratio depends on the variance of the fundamental shocks, σ_u^2, and the precision of private signals and the consensus price as determined by σ_η and σ_ε. We have already identified σ_ε. The relative weight submitters put on the consensus price as opposed to the private signal depends on the relative precision of these two signals. This allows us to identify σ_η and, finally, σ_u from the signal-to-noise ratio.

4.3 Model fit and robustness checks

To judge how well the model fits the data, we compare the model-implied cross-sectional dispersion of price submissions and the time-series volatility of the consensus price to their empirical analogues. The upper panel in Table 3 in the Appendix displays the ratio of the model-implied cross-sectional standard deviation and the empirical cross-sectional standard deviation. This ratio for the different contracts is between 0.909 and 1.125, which implies that the model is able to reproduce the size of the cross-sectional dispersion for the different contracts. We can neither reject that the model-implied and empirical consensus price volatility differ from one another.

Furthermore, the sample period covers two peculiar time periods: the low volatility period from 2002 to 2006 and the Great Recession from 2007 to 2010. The estimated values may be driven solely by the dynamics in one of these periods. We find that our results do not change if we consider these two subsample periods separately. Another potential split is that of submitters that participate for a limited time frame and routine submitters. We therefore re-estimate the model excluding submitters who have submitted less than 40% of the total sample period. The parameter estimates are comparable to the whole sample period. Including only the 'routine' submitters makes the contrast between the at-the-money (ATM) and the out-of-the-money (OTM) options slightly larger. The estimation results for these three additional data treatments are reported in the online appendix.

5 Results

The contract-by-contract estimation of the structural parameters allows us to determine how the relative weight dealers put on private and consensus price information varies across segments of the options market. Of particular interest is the contrast between market segments that overlap with exchange-based trading and those that are fully OTC. In these segments, we extract how informative and how efficient the consensus price is.

5.1 Valuation and strategic uncertainty

Our uncertainty measures are based on the covariance matrix of bank dealers' beliefs, Σ^p. In particular, we focus on a dealer's beliefs about the current fundamental value of a contract, θ_t, and the cross-sectional average expectation of this value, $\bar{\theta}_t$,

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} \mid \Omega_{i,t} \sim N\left(\begin{pmatrix} \theta_{i,t} \\ \bar{\theta}_{i,t} \end{pmatrix}, \begin{pmatrix} \sigma_{11}^p & \sigma_{12}^p \\ \sigma_{12}^p & \sigma_{22}^p \end{pmatrix} \right), \tag{6}$$

where dealer i's conditional expectations about θ_t and $\bar{\theta}_t$ are updated according to

$$\theta_{i,t} = \rho\, \theta_{i,t-1} + k_s\, (s_{i,t} - \rho\, \theta_{i,t-1}) + k_p\, \left(p_{t-1} - \bar{\theta}_{i,t-1}\right), \tag{7}$$

$$\bar{\theta}_{i,t} = m_2 \cdot x_{i,t-1} + \bar{k}_s\, (s_{i,t} - \rho\, \theta_{i,t-1}) + \bar{k}_p\, \left(p_{t-1} - \bar{\theta}_{i,t-1}\right). \tag{8}$$

The above covariance matrix of beliefs corresponds to the top left 2×2 sub-matrix of Σ^p given in (3.2). The parameters k_s and k_p are the Kalman gains for private signal and the consensus price, respectively. The Kalman gains give the weight a dealer puts on the "news" contained in the signals when updating expectations about the fundamental value θ_t. They correspond to the first row of K in (4). Similarly, $\bar{k}_s$ and $\bar{k}_p$ are the Kalman gains for the private signal and consensus price, respectively, for the average expectation $\bar{\theta}_t$. They correspond to the second row of K.

Our measures of valuation and strategic uncertainty are based on the variances of posterior beliefs about θ_t and $\bar{\theta}_t$ as given by σ_{11}^p and σ_{22}^p. They are also the variance of a dealer's forecast errors, $\theta_{i,t} - \theta_t$ and $\bar{\theta}_{i,t} - \bar{\theta}_t$, at the time of its consensus price submission. The correlation between these two forecast errors, $\rho_{12} = \sigma_{12}^p / \sqrt{\sigma_{11}^p \sigma_{22}^p}$, is a natural measure for the commonality of information (Angeletos and Pavan (2007)); higher correlations imply that different consensus price submitters interpret new valuation information in a similar way.

To develop an intuition for the relationship between valuation uncertainty, strategic and informational commonality, it is best to split up the expectation updating for θ_t into two steps: (i) First, the dealer updates its expectations about θ_{t-1} after observing the consensus price p_{t-1}. Call this updated expectation $\theta^+_{i,t-1}$.(ii) Next, after having observed private information $s_{i,t}$ in period t, the dealer submits its updated expectation about θ_t to the consensus pricing service. We have

$$\theta_{i,t} = (1-k_s)\rho \underbrace{\left[\theta_{i,t-1} + \frac{k_p}{\rho(1-k_s)}(p_{t-1} - \bar{\theta}_{i,t-1})\right]}_{\theta^+_{i,t-1}} + k_s\, s_{i,t}\ ,$$

where the term in the square brackets gives the first updating step. The average expectation can then be expressed as

$$\bar{\theta}_t = (1-k_s)\rho\, \bar{\theta}^+_{t-1} + k_s\, \theta_t.$$

This allows us to link the forecast errors for θ_t and $\bar{\theta}_t$ as follows:

$$\bar{\theta}_{i,t} - \bar{\theta}_t = (1-k_s)\left[\mathbb{E}(\rho\, \bar{\theta}^+_{t-1}|\Omega_{i,t}) - \rho\, \bar{\theta}^+_{t-1}\right] + k_s\,(\theta_{i,t} - \theta_t)\,.$$

The forecast error for $\bar{\theta}_t$ is a weighted sum of the forecast error for θ_t and the forecast error for the average prior expectation about θ_t before observing the private signal in period t. Submitter i's forecast errors for $\bar{\theta}_t$ and θ_t are perfectly correlated if the submitter knows the average expectation $\bar{\theta}^+_{t-1}$. In our model, where the only exogenous source of information is the private signals, this can only happen if the consensus price perfectly aggregates all dispersed information. In that case, all submitters have a common posterior expectation $\bar{\theta}^+_{t-1} = p_{t-1}$ and the average expectation is given by $\bar{\theta}_t = (1-k_s)\rho\, p_{t-1} + k_s\, \theta_t$. As a result, they are less uncertain about $\bar{\theta}_t$ than about θ_t, i.e. $\sigma^p_{22} = k_s^2\, \sigma^p_{11}$.

If submitters are uncertain about the average expectation $\bar{\theta}^+_{t-1}$, this simple relationship no longer holds. For a submitter whose prior expectations for fundamental and average valuation are equal, we can rewrite (7) into

$$\theta_{i,t} = (1-k)\rho\, \theta_{i,t-1} + k_s\, s_{i,t} + k_p\, p_{t-1} + k_p\, \left(\theta_{i,t-1} - \bar{\theta}_{i,t-1}\right), \tag{9}$$

where $k = k_s + {}^{k_p}/_{\rho}$. If submitters put $(1-k)$ weight on their prior expectations, individual perceptions are partially dependent on the individual history of private signals. Hence, submitters do not return to a common market perception after observing the consensus price. The lack of a common perspective on past market conditions partially feeds into current uncertainty about $\bar{\theta}_t$.[16]

[16]See Sethi and Yildiz (2016) for a related discussion on well-informed and well-understood information sources and the implications for information segregation in markets.

5.1.1 Price versus private information

A key structural parameter for understanding the variation in the Kalman gains, stated in (7) and (8), across market segments is $1/\sigma_\eta$, the precision of the private signal. The estimates for σ_η, given in the fourth row of Table 2, show that dealers receive very precise private signals for contracts that overlap with active exchange-based trading activity.[17] Consequently, the implied Kalman gains for these contracts in Figure 3 show that submitters put essentially full weight on their private signal and largely ignore the information contained in the consensus price when updating expectations about θ_t. For option contracts with low exchange-based trading activity, the private signals are estimated to be noisier. Therefore, increasingly more weight is given to the consensus price. When updating expectations about $\bar{\theta}_t$, the consensus price receives relatively higher weight for all contracts. This highlights the scarcity of information in these market segments, but it also illustrates the strategic value of the consensus price as a focal public signal. The estimates with $1-k>0$, reported in row five of Table 2, show that dealers do not put all weight on new information for exclusively OTC-traded contracts. Furthermore, the $\rho_{12}<1$ estimates for these contracts convey that dealers' forecast errors are not perfectly correlated. As explained in Section 5.1, this leads to long-lived private information and, consequently, dispersed priors among submitters.

5.1.2 The uncertainty "smile"

The previous discussion on the parameter estimates and model-implied objects derived from these estimates sheds light on the nature and size of valuation and strategic uncertainty in the options market. As dealers' beliefs are normal distributions, we can display these uncertainties in the form of 95% posterior intervals centered around the time-series average of the consensus price. The two top panels in Figure 4 show the well-known "smile" of the implied volatility curve. OTM put options (moneyness below 100) tend to be relatively more expensive than ATM put options or OTM call options reflecting market participants' demand for insurance against drops in the S&P 500 index. The width of the posterior intervals shows that for options with more extreme strike prices (further away from moneyness 100), valuation and strategic uncertainty are higher. These areas correspond to market segments in which trading is predominantly or exclusively OTC, as was previously shown in Figure 1. For options with moneyness 150 and time-to-expiration of 12 months, for example, the posterior intervals are on the order of 8 volatility points.

[17] Table 2 provides the parameter estimates for the contracts with time-to-expiration of 12 months. In the Appendix we provide the estimates for the complete contract space. Table 4 provides estimates for ρ and σ_u; Table 5 for σ_ε and σ_η, and Table 6 for k and ρ_{12}.

Table 2: Sample parameter estimates and implied quantities

	60	80	90	95	100	105	110	120	150
ρ	0.967	0.930	0.939	0.949	0.941	0.930	0.949	0.967	0.969
	(0.015)	(0.024)	(0.027)	(0.028)	(0.022)	(0.025)	(0.022)	(0.013)	(0.017)
σ_u	0.047	0.076	0.082	0.086	0.091	0.095	0.095	0.073	0.135
	(0.001)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)
σ_ε	0.055	0.004	0.006	0.007	0.009	0.011	0.014	0.036	0.262
	(0.004)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.016)
σ_η	0.041	0.014	0.010	0.010	0.010	0.011	0.015	0.036	0.281
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)
k	0.733	0.998	0.996	0.996	0.995	0.993	0.989	0.901	0.490
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.010)	(0.014)
ρ_{12}	0.974	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)

This table presents the mean and standard deviation of the model parameter estimates and implied quantities for the contracts with 12 months' time-to-expiration. The first and second rows contain the estimate of the persistence of the process for the fundamental, ρ, and the variance of the shock to the fundamental, σ_u. The third and fourth rows contain the estimate of the noise of the public signal, σ_ε, and the noise of the private signal submitter i receives, σ_η. The fifth and sixth rows contain the weight submitter put on new information, kalman gain k, and the correlation between the forecast error for asset value and average valuations, ρ_{12}. The header for each column denotes the moneyness of the contract under consideration. The structural model is estimated with Bayesian analysis through MCMC methods. The standard deviation of the posterior distribution of the parameter is given in parentheses below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P 500 index.

This is substantial given that the time-series average and standard deviation of the consensus price for this contract are 13 and 3.8 volatility points, respectively. It reflects the low precision of the private signal for this contract and, consequently, a higher weight put on prior expectations. This, in turn, is the source of "long-lived" private information and sizable strategic uncertainty. It contrasts with posterior intervals well below one volatility point for options in market segments which likely see more trading activity. Here, private signals are estimated to be precise, which implies lower values of σ_{11}^p. As all submitters are symmetrically informed and receive private signals from the same distribution, strategic uncertainty is small as well. This difference in results illustrates that for the exclusively OTC-traded areas of the option market, dealers are not only relatively uncertain about the correctness of their own option valuations, but also face substantial uncertainty about the relative position of their valuation to the average market valuation.

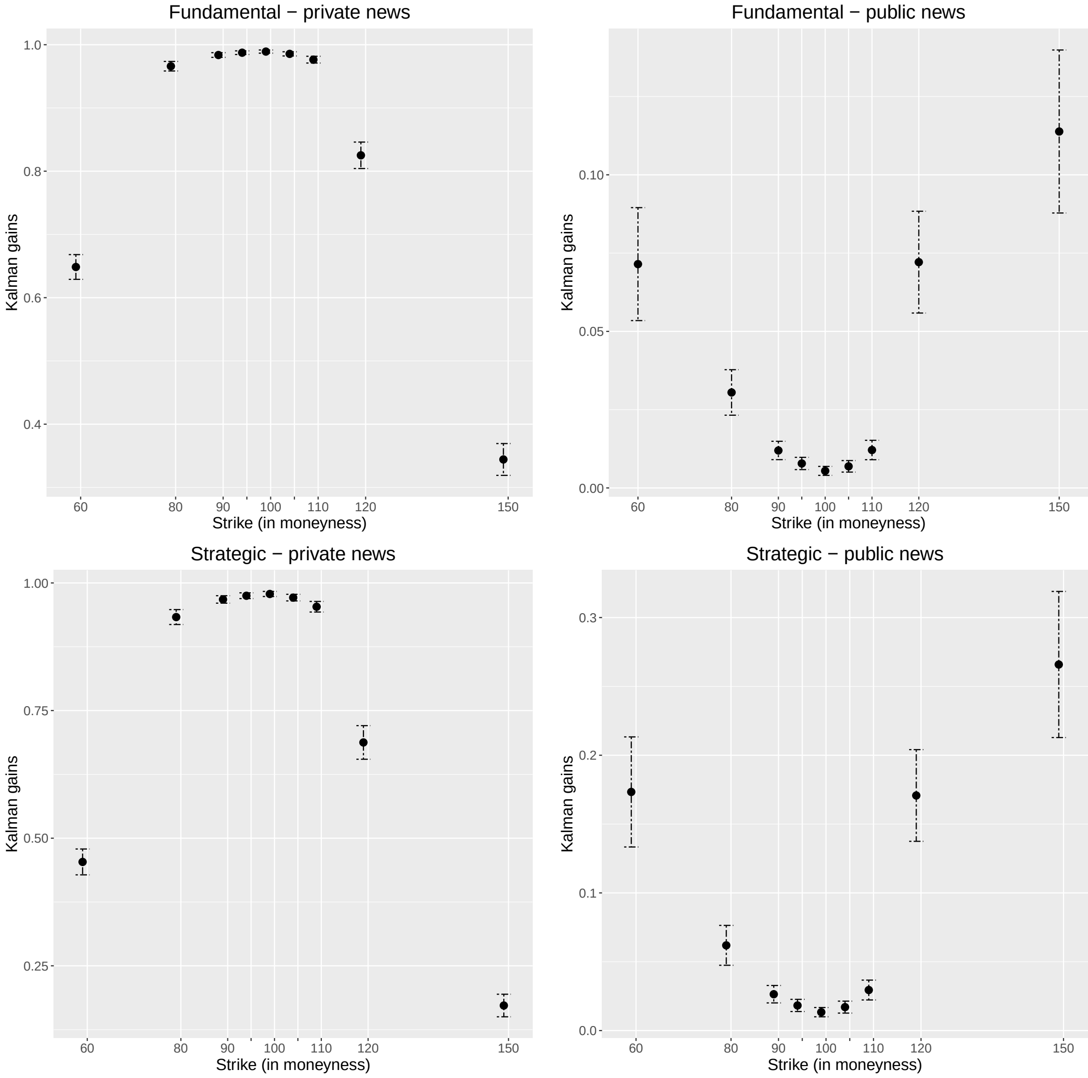


Figure 3: These figures present the Kalman gains with respect to their private signals and consensus feedback for their first- and second-order posterior believes. The horizontal axis denotes the moneyness of the option contracts. The black dots in the figures represent the Kalman gain extracted from the K matrix in Equation (6). The *top* figures depict the k_s and k_p elements. From left to right, these are the weights put on the private signal and public signal in updating the posterior believe about the fundamental. The *bottom* figures depict the $\bar{k}_s$ and $\bar{k}_p$ elements. From left to right, these are the weights put on the private signal and public signal in updating the posterior believe about the average believe. The 95% centred interval of the posterior distribution of Kalman gain estimates are given by the dotted lines surrounding the dots. The Kalman gains are for the option contracts with a **time-to-expiration** of **12 months**. The sample period is December 2002 to February 2015 for the option contracts on the S&P 500 index.

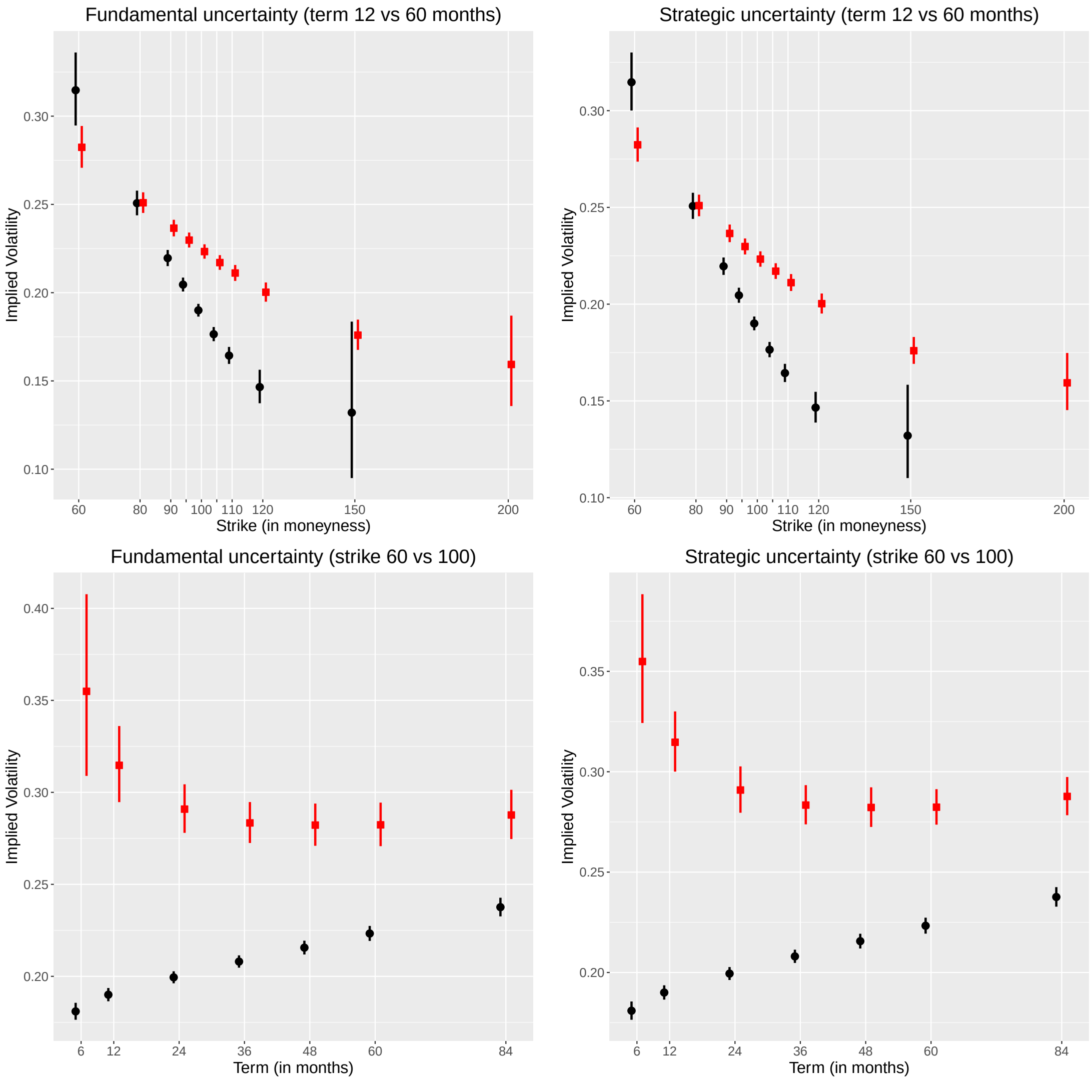


Figure 4: These figures present the variance of submitters' posterior beliefs expressed in terms of posterior intervals centred on the sample mean of the corresponding consensus price. The *left* figures depict the 95% posterior intervals for first-order beliefs, $[\bar{p} \pm 1.96 \cdot \sigma_{11}]$, as given in (6). The figures on the *right* display the posterior intervals for second-order beliefs, $[\bar{p} \pm 1.96 \cdot \sigma_{22}]$. The two *top* panels depict the variances along the different levels of moneyness for the option contracts with a **time-to-expiration** of **12 months** and **60 months**. The two *bottom* panels show the term structure of the uncertainties for ATM options with moneyness **100** and OTM options with moneyness of **60**. The sample period is December 2002 to February 2015 for the option contracts on the S&P 500 index.

5.2 The informational properties of consensus prices

We now consider two counterfactual information structures for the options markets and study how these alternative informational settings would influence uncertainty among dealer banks. We assume that the structural parameters of our model are invariant to these informational experiments. In particular, we assume that dealer banks do not adjust their information acquisition strategy, which would likely influence the precision of the private signal.

To study the informativeness of the consensus price for dealer banks, we consider an information structure under which dealers only have access to their private signals. Denote by Σ^s the covariance matrix of dealer i's posterior beliefs under this counterfactual information set, namely $\Omega^s_{i,t} = \{s_{i,t}, \Omega^s_{i,t-1}\}$. This covariance matrix can be obtained by solving a standard single-agent learning problem using parameter estimates for $\{\rho, \sigma_u, \sigma_\eta\}$.[18] We use the percentage reduction in uncertainty that results from having access to the consensus price as a measure of price informativeness,

$$\Delta^p_i = \frac{(\sigma^s_{ii} - \sigma^p_{ii})}{\sigma^s_{ii}},$$

where $i = 1$ corresponds to valuation uncertainty and $i = 2$ to strategic uncertainty.

To elicit the efficiency of the consensus price mechanism in aggregating dispersed information, we introduce a counterfactual setting with a fully efficient price that perfectly reveals last period's fundamental value, i.e. θ_{t-1}. As the price reveals θ_{t-1}, it provides submitters with a common prior before receiving new private signals. In addition to providing a benchmark for efficiency, this counterfactual also helps us understand how big an impediment the lack of a common prior is for creating a common understanding of market conditions. Denote by Σ^θ the counterfactual covariance matrix of posterior beliefs for a dealer who receives a fully efficient consensus price in the above sense. We measure the inefficiency of the consensus price by the increase in posterior variance in moving from a fully efficient price to the current consensus price expressed as a ratio to the posterior variance without consensus price,

$$\Delta^\theta_i = \frac{\left(\sigma^p_{ii} - \sigma^\theta_{ii}\right)}{\sigma^s_{ii}}.$$

We use this somewhat unnatural seeming definition of inefficiency to obtain the following decomposition,

$$1 = \underbrace{\frac{(\sigma^s_{ii} - \sigma^p_{ii})}{\sigma^s_{ii}}}_{\substack{\Delta^p_i \\ \text{Price informativeness}}} + \underbrace{\frac{\left(\sigma^p_{ii} - \sigma^\theta_{ii}\right)}{\sigma^s_{ii}}}_{\substack{\Delta^\theta_i \\ \text{Price inefficiency}}} + \underbrace{\frac{\sigma^\theta_{ii}}{\sigma^s_{ii}}}_{\substack{\Delta^R_i \\ \text{Residual informational} \\ \text{friction}}} \tag{10}$$

[18] Appendix 7.8 derives the stationary posterior covariance matrices for first- and second-order beliefs for all counterfactual informational scenarios.

Given the lagged nature of the consensus price, even a fully efficient price does not eliminate all uncertainty about asset values. We quantify the potential for further uncertainty reduction outside the scope of this consensus price mechanism, i.e. residual informational friction. The potential uncertainty reduction for a market structure without an information aggregation mechanism can thus be decomposed into price informativeness, price inefficiency, and residual informational frictions.

The influence of information structure on uncertainty

Figure 5 displays the percentage reductions in uncertainty under the different informational settings for contracts with a fixed time-to-expiration of 12 months. The dark gray region, i.e. Δ_1^p, displays the informativeness of the price for the different contracts. The lack of uncertainty reduction in the moneyness range from 80 to 110 is to be expected as submitters solely rely on their precise private signal, as previously revealed by the estimates of the Kalman gain k_s. In the more opaque market segments, the consensus price is more informative about θ_t; its Kalman gain k_p is higher. Table 7 in the Appendix shows similar patterns for other times-to-expiration. For all the contracts under consideration, the reduction in valuation uncertainty is between 0% for the ATM contracts to 4.6% for the more extreme contracts. The comparison between the upper and lower panel in Figure 5 shows that the consensus price signal is much more informative about $\bar{\theta}_t$. The reduction in strategic uncertainty ranges from 0.02% to 37.75% (see Table 7). The relative larger decrease in strategic uncertainty in comparison to valuation uncertainty points to the importance of the consensus price for learning about strategic aspects of the market. This is also echoed in the difference between k_p and $\bar{k}_p$. Given the scarcity of shared trade data in market segments that are dominated by OTC trading, the ability of the consensus price to significantly reduce strategic uncertainty is both intuitive and important.

The light gray area in Figure 5 corresponds to Δ_1^θ, the additional reduction in uncertainty that could be achieved by a price that perfectly reveals θ_{t-1}. Knowing previous period's option value eliminates two sources of uncertainty. First, it eliminates the uncertainty that originates from the noise in the consensus price. Second, it gets rid of the uncertainty that emanates from the dispersion in dealers' prior expectations. In the top and bottom panel of Figure 5, the lack of uncertainty reduction in the moneyness range from 80 to 110 is mainly due to the precision of private information. The signal-to-noise ratio ${}^{\sigma_u}/{}_{\sigma_\eta}$ puts an upper bound on the weight the consensus price can receive when updating expectations. The consensus price can at most reveal the past value, while the private signal is a signal about the current value. This limits the potential impact of a fully efficient price on valuation uncertainty. For contracts with intermediate moneyness, little weight is put on prior expectations, thus limiting the potential of a fully efficient price to reduce uncertainty by providing a common prior. For contracts with extreme moneyness, the relative imprecision of the private signal shifts weight towards the consensus price and the prior. This explains the up to 33.46% drop in valuation uncertainty and 62.10% drop in strategic uncertainty for the deep OTM call options, as seen in Table 8 in the Appendix. Sethi and Yildiz (2016) highlight that dispersion in priors can lead market participants to search out other participants with similar priors, leading to informational segmentation of markets. A focal point, such as a consensus price, helps to reduce dispersion in priors, reduces strategic uncertainty and creates a common understanding of market conditions.

The white area in the figures marks Δ_i^R, the potential uncertainty reduction outside of the scope of this consensus pricing mechanism. The reduced size of this area for more extreme contracts illustrates the importance of public information in opaque parts of the market, especially in providing information about other dealers' valuations. For contracts with moneyness between 80 and 110, informational frictions that could not be remedied by a perfectly efficient consensus pricing mechanism dominate. Reducing the remaining uncertainty would require changing the design parameters of the consensus pricing service. Increasing the frequency of the consensus service, for example, can be thought of as lowering the variance of the fundamental shocks, σ_u^2, in our model. However, given the labour-intensive nature of the consensus pricing process, running a more frequent service is costly. It appears that the marginal cost of increasing the frequency of the service exceeds the dealers' willingness to pay for a marginal reduction in uncertainty.

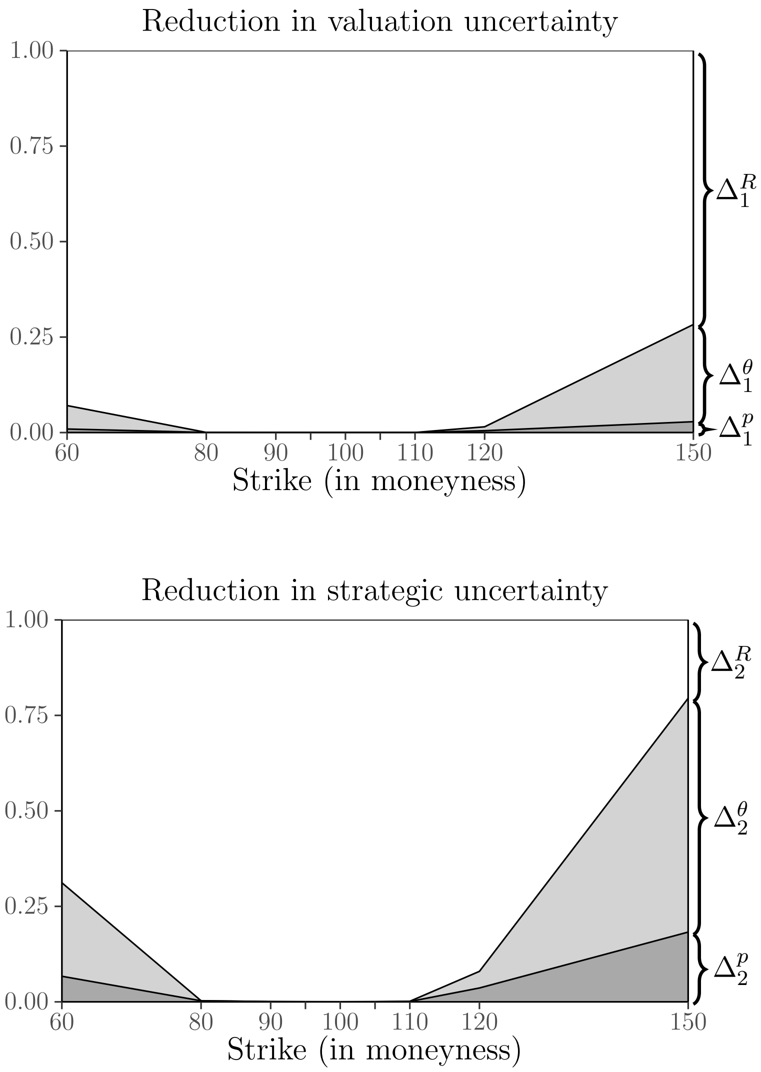


Figure 5: These two figures present the percentage reductions in valuation and strategic uncertainty under different informational settings. The top figure presents the results for the percentage reduction in **valuation uncertainty** and the bottom figure presents the reductions in **strategic uncertainty**. The figures depict the percentage uncertainty reductions (y-axis) along the different levels of moneyness (x-axis) for the option contracts with a **time-to-expiration** of **12 months**. In the base case setting, submitters only observes their private signal. This is indicated by the lower horizontal axis. The dark grey area is the percentage reduction in uncertainty due to observing the consensus price, i.e., Δ^p in (10). The light grey area indicates the further reduction in uncertainty due to observing the past state, i.e., Δ^θ. The white area is the further reduction in uncertainty that can be achieved from an information structure that eliminates informational frictions, i.e., Δ^R. The sample period is from December 2002 to February 2015.

6 Conclusion

In this paper we provide empirical evidence on the ability of consensus prices to reduce valuation uncertainty among major dealer banks in the over-the-counter market for S&P 500 index options. This evidence is based on a structural model of learning from prices. The estimation is based on a proprietary panel of price estimates that large broker-dealers have provided to a consensus pricing service for OTC derivatives. The structural model allows us to address three questions. First, how large is the valuation uncertainty of dealer banks participating in the OTC market for S&P 500 index options? Here we consider two dimensions of uncertainty: a dealer bank's uncertainty about fundamental values and uncertainty about its valuations in relation to other market participants' valuations. Second, does the consensus price feedback help to reduce market participants' valuation uncertainty? Last, how well does the consensus pricing mechanism aggregate dispersed information?

Both valuation and strategic valuation uncertainty vary substantially across the different market segments. We find higher uncertainty for option contracts with strike prices that correspond to more extreme index moves; these contracts are typically traded in the OTC segment of the market. Dealer banks do not appear to rely heavily on the consensus price feedback to reduce valuation uncertainty. The consensus price feedback is found to be most important for reducing strategic uncertainty, and particularly so for extreme option contracts. This result is consistent with the scarcity of shared valuation information for such extreme contracts. It stresses the importance of publicly observable valuation data, such as benchmarks, to establish a shared understanding of market conditions in OTC markets. Such a shared understanding can be particularly valuable during episodes of market stress where high levels of strategic uncertainty might cause derivatives markets to freeze up.

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7 Appendix

7.1 Tables

Table 3: Matching **cross-sectional dispersion** and **consensus price volatility**

	60	80	90	95	100	105	110	120	150	200
6	1.021	1.043	1.045	1.050	1.052	1.058	1.115	1.125	0.961	·
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.012)	(0.014)	·
12	1.026	1.050	1.039	1.047	1.056	1.049	1.058	1.103	0.988	·
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.013)	·
24	1.079	1.052	1.047	1.053	1.064	1.063	1.164	1.082	0.999	0.909
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.011)	(0.012)	(0.016)
36	1.058	1.040	1.043	1.048	1.052	1.059	1.060	1.061	1.013	0.928
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.014)
48	1.036	1.033	1.038	1.038	1.033	1.034	1.035	1.041	1.019	0.960
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.014)
60	0.992	1.028	1.041	1.039	1.038	1.037	1.040	1.043	1.014	1.003
	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.012)	(0.015)
84	0.986	1.026	1.021	1.019	1.015	1.012	1.015	1.023	1.005	0.913
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.021)	(0.012)	(0.012)	(0.012)	(0.014)

(a) Matching cross-sectional dispersion

	60	80	90	95	100	105	110	120	150	200
6	1.437	1.058	1.138	1.053	1.045	1.401	1.155	1.242	2.595	·
	(0.271)	(0.243)	(0.294)	(0.224)	(0.277)	(0.780)	(0.315)	(0.213)	(0.629)	·
12	1.110	1.028	1.117	1.320	1.104	0.990	1.120	1.014	2.573	·
	(0.264)	(0.201)	(0.313)	(0.553)	(0.252)	(0.219)	(0.263)	(0.228)	(0.755)	·
24	1.086	1.030	1.050	1.413	1.213	1.033	1.095	1.183	1.483	3.635
	(0.286)	(0.203)	(0.214)	(0.733)	(0.484)	(0.290)	(0.324)	(0.394)	(0.476)	(2.366)
36	1.019	1.158	1.049	2.062	1.199	0.931	1.383	1.017	1.061	1.761
	(0.214)	(0.593)	(0.336)	(1.594)	(0.365)	(0.158)	(0.644)	(0.254)	(0.347)	(0.379)
48	0.987	1.203	1.090	0.985	1.086	1.027	1.346	1.011	0.968	1.490
	(0.203)	(0.561)	(0.327)	(0.203)	(0.339)	(0.229)	(0.838)	(0.244)	(0.246)	(0.420)
60	1.296	1.373	1.157	1.009	1.106	1.043	1.070	1.300	0.980	1.324
	(0.418)	(0.642)	(0.409)	(0.247)	(0.338)	(0.299)	(0.306)	(0.579)	(0.254)	(0.433)
84	0.942	1.149	1.142	1.098	0.968	1.029	0.989	1.054	1.045	1.055
	(0.179)	(0.526)	(0.379)	(0.341)	(0.181)	(0.222)	(0.209)	(0.265)	(0.265)	(0.234)

(b) Matching volatility consensus price

These two tables present the mean and standard deviation of the ratio of the raw moments of the data versus the model implied moments for each contract. The upper table displays the ratio of the model-implied cross-sectional dispersion versus the average cross-sectional standard deviation in the data. The lower table displays the ratio of the model-implied volatility of the consensus price to the empirical counterpart from the data. The model-implied volatility is given by the unconditional volatility of the average first-order belief plus σ_ϵ. The unconditional variance of $\bar{\theta}_t$ is the solution to a Lyapunov equation that defines the unconditional variance of the state. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the ratios is given in parentheses below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is January 2002 to December 2015.

Table 4: Model parameter estimates ρ and σ_u

	60	80	90	95	100	105	110	120	150	200
6	0.950	0.911	0.930	0.923	0.920	0.945	0.949	0.956	0.950	·
	(0.019)	(0.031)	(0.028)	(0.026)	(0.028)	(0.030)	(0.022)	(0.015)	(0.021)	·
12	0.967	0.930	0.939	0.949	0.941	0.930	0.949	0.967	0.969	·
	(0.015)	(0.024)	(0.027)	(0.028)	(0.022)	(0.025)	(0.022)	(0.013)	(0.017)	·
24	0.935	0.940	0.943	0.956	0.947	0.938	0.945	0.962	0.970	0.971
	(0.025)	(0.021)	(0.020)	(0.026)	(0.028)	(0.024)	(0.023)	(0.020)	(0.015)	(0.017)
36	0.939	0.943	0.941	0.969	0.952	0.932	0.958	0.948	0.963	0.946
	(0.022)	(0.026)	(0.023)	(0.026)	(0.023)	(0.021)	(0.025)	(0.021)	(0.017)	(0.021)
48	0.935	0.949	0.947	0.938	0.944	0.942	0.953	0.945	0.959	0.943
	(0.022)	(0.025)	(0.022)	(0.021)	(0.024)	(0.022)	(0.026)	(0.020)	(0.017)	(0.022)
60	0.982	0.956	0.951	0.941	0.948	0.941	0.945	0.956	0.963	0.937
	(0.012)	(0.028)	(0.022)	(0.021)	(0.021)	(0.025)	(0.024)	(0.027)	(0.016)	(0.026)
84	0.968	0.947	0.949	0.947	0.939	0.945	0.940	0.945	0.941	0.954
	(0.011)	(0.025)	(0.026)	(0.023)	(0.019)	(0.019)	(0.022)	(0.024)	(0.023)	(0.017)

(a) Mean and standard deviation ρ

	60	80	90	95	100	105	110	120	150	200
6	0.079	0.092	0.099	0.106	0.116	0.120	0.115	0.111	0.166	·
	(0.001)	(0.005)	(0.005)	(0.006)	(0.006)	(0.007)	(0.006)	(0.002)	(0.005)	·
12	0.047	0.076	0.082	0.086	0.091	0.095	0.095	0.073	0.135	·
	(0.001)	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.002)	(0.003)	·
24	0.064	0.065	0.070	0.072	0.075	0.078	0.080	0.073	0.078	0.135
	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.005)
36	0.056	0.059	0.063	0.065	0.067	0.069	0.070	0.068	0.061	0.110
	(0.002)	(0.003)	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.001)	(0.003)
48	0.054	0.056	0.059	0.061	0.062	0.064	0.065	0.065	0.057	0.097
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)
60	0.033	0.055	0.055	0.057	0.058	0.059	0.060	0.061	0.051	0.089
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.002)	(0.002)
84	0.033	0.050	0.051	0.052	0.053	0.054	0.055	0.056	0.062	0.058
	(0.001)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)	(0.003)	(0.003)	(0.003)	(0.001)

(b) Mean and standard deviation σ_u

These two tables present the mean and standard deviation of the estimate of the persistence of the process for the fundamental, ρ, and the variance of the shock to the fundamental, σ_u. The structural model is estimated with Bayesian analysis through MCMC methods. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P 500 index.

Table 5: Model parameter estimates σ_ε and σ_η

	60	80	90	95	100	105	110	120	150	200
6	0.121	0.004	0.007	0.009	0.011	0.016	0.021	0.151	0.328	·
	(0.008)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.002)	(0.010)	(0.013)	·
12	0.055	0.004	0.006	0.007	0.009	0.011	0.014	0.036	0.262	·
	(0.004)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.003)	(0.016)	·
24	0.002	0.004	0.006	0.007	0.008	0.009	0.010	0.015	0.114	0.270
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.008)	(0.016)
36	0.003	0.004	0.005	0.006	0.007	0.008	0.008	0.011	0.054	0.183
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)	(0.012)
48	0.003	0.004	0.005	0.006	0.006	0.007	0.007	0.009	0.034	0.121
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	(0.009)
60	0.032	0.002	0.004	0.005	0.005	0.006	0.006	0.007	0.024	0.083
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.006)
84	0.034	0.003	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.069
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)

(a) Mean and standard deviation σ_ε

	60	80	90	95	100	105	110	120	150	200
6	0.096	0.022	0.015	0.013	0.013	0.018	0.030	0.131	0.380	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.020)	·
12	0.041	0.014	0.010	0.010	0.010	0.011	0.015	0.036	0.281	·
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)	·
24	0.024	0.012	0.009	0.008	0.008	0.009	0.013	0.018	0.093	0.395
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.023)
36	0.021	0.011	0.009	0.008	0.008	0.009	0.010	0.015	0.049	0.231
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.011)
48	0.022	0.012	0.010	0.009	0.009	0.009	0.011	0.014	0.035	0.155
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.006)
60	0.025	0.012	0.010	0.009	0.009	0.010	0.011	0.014	0.027	0.116
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.004)
84	0.029	0.014	0.011	0.011	0.011	0.011	0.012	0.014	0.023	0.069
	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)

(b) Mean and standard deviation σ_η

These tables presents the mean and standard deviation of the estimate of the noise of the public signal, σ_ε, and the noise of the private signal submitter i receives, σ_η. The structural model is estimated with Bayesian analysis through MCMC methods. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.000 signifies standard deviations below 0.0005). The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P 500 index.

Table 6: Weight on new information estimates and belief correlation

	60	80	90	95	100	105	110	120	150	200
6	0.625	0.998	0.996	0.995	0.995	0.990	0.978	0.649	0.465	·
	(0.012)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.012)	·
12	0.733	0.998	0.996	0.996	0.995	0.993	0.989	0.901	0.490	·
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.010)	(0.014)	·
24	0.999	0.997	0.995	0.995	0.995	0.993	0.990	0.976	0.639	0.440
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.013)	(0.017)
36	0.997	0.995	0.995	0.995	0.994	0.993	0.991	0.984	0.781	0.513
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.012)	(0.017)
48	0.996	0.995	0.994	0.994	0.993	0.993	0.991	0.986	0.860	0.613
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.013)	(0.019)
60	0.778	0.999	0.994	0.994	0.994	0.993	0.992	0.989	0.900	0.703
	(0.011)	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.015)	(0.018)
84	0.747	0.997	0.996	0.996	0.995	0.995	0.994	0.993	0.994	0.671
	(0.012)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.016)

(a) k

	60	80	90	95	100	105	110	120	150	200
6	0.954	1.000	1.000	1.000	1.000	1.000	1.000	0.958	0.915	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.003)	·
12	0.974	1.000	1.000	1.000	1.000	1.000	1.000	0.997	0.922	·
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)	·
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.956	0.909
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
36	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.982	0.925
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.004)
48	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.947
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
60	0.981	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.996	0.964
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.003)
84	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.962
	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)

(b) ρ_{12}

This table presents the mean and standard errors of the Kalman gain, k, and the correlation between the beliefs about θ_t and $\bar{\theta}_t$. The Kalman gain gives the weight submitters put on new information and $1-k$ shows how much weight is put on the prior. ρ_{12} is the correlation between the forecast error for asset value and average valuations. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The sample period of the data is December 2002 to February 2015 for the option contracts on the S&P 500 index.

Table 7: Counterfactual experiment – **No consensus price**

	60	80	90	95	100	105	110	120	150	200
6	1.38	0.12	0.02	0.01	0.00*	0.01	0.11	1.58	2.91	·
	(0.17)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.18)	(0.29)	·
12	0.93	0.05	0.01	0.00*	0.00*	0.00*	0.01	0.50	2.85	·
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.07)	(0.32)	·
24	0.62	0.04	0.01	0.01	0.00*	0.00*	0.02	0.07	1.52	4.36
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.18)	(0.50)
36	0.59	0.04	0.01	0.01	0.01	0.01	0.01	0.06	1.14	3.20
	(0.12)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.14)	(0.38)
48	0.76	0.07	0.02	0.01	0.01	0.01	0.02	0.06	0.92	3.06
	(0.15)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.12)	(0.36)
60	0.93	0.09	0.04	0.02	0.02	0.02	0.03	0.09	0.76	3.06
	(0.12)	(0.02)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.02)	(0.10)	(0.34)
84	1.21	0.20	0.09	0.07	0.06	0.07	0.08	0.14	0.53	1.90
	(0.15)	(0.04)	(0.02)	(0.02)	(0.01)	(0.10)	(0.02)	(0.03)	(0.11)	(0.24)

(a) *Decrease in valuation uncertainty*: Δ_1^p

	60	80	90	95	100	105	110	120	150	200
6	9.65	0.70	0.12	0.05	0.03	0.09	0.77	10.82	19.17	·
	(1.02)	(0.16)	(0.03)	(0.01)	(0.01)	(0.02)	(0.16)	(1.08)	(1.40)	·
12	6.70	0.28	0.06	0.03	0.02	0.04	0.10	3.63	18.27	·
	(0.75)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.42)	(1.66)	·
24	3.48	0.26	0.06	0.04	0.03	0.03	0.12	0.54	10.37	27.33
	(0.66)	(0.06)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.11)	(1.09)	(2.33)
36	3.33	0.28	0.08	0.05	0.04	0.05	0.09	0.40	7.98	20.69
	(0.63)	(0.06)	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.09)	(0.83)	(1.93)
48	4.25	0.44	0.15	0.10	0.08	0.10	0.15	0.44	6.50	19.34
	(0.79)	(0.10)	(0.03)	(0.02)	(0.02)	(0.02)	(0.03)	(0.10)	(0.73)	(1.84)
60	6.63	0.53	0.25	0.16	0.14	0.15	0.22	0.56	5.29	19.08
	(0.75)	(0.12)	(0.06)	(0.04)	(0.03)	(0.03)	(0.05)	(0.12)	(0.61)	(1.70)
84	8.45	1.18	0.53	0.43	0.39	0.44	0.49	0.87	3.08	12.67
	(0.93)	(0.26)	(0.12)	(0.10)	(0.09)	(0.52)	(0.11)	(0.19)	(0.60)	(1.35)

(b) *Decrease in strategic uncertainty*: Δ_2^p

These two tables present the counterfactual results of the percentage decrease in valuation and strategic uncertainty when moving from a setting without consensus price to a setting with consensus price feedback. The upper table presents the results for the percentage decrease in **valuation uncertainty**, Δ_1^p in (10). The lower table presents the results for the percentage increase in **strategic uncertainty**, Δ_2^p. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.00 signifies standard deviations below 0.005). The sample period is from December 2002 to February 2015.

Table 8: Counterfactual experiment – **Perfect Consensus price**

	60	80	90	95	100	105	110	120	150	200
6	12.02	0.01	0.00*	0.00*	0.00*	0.01	0.07	11.35	27.01	·
	(0.52)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.03)	·
12	6.15	0.00*	0.00*	0.00*	0.00*	0.00*	0.01	1.02	25.45	·
	(0.35)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.16)	(0.84)	·
24	0.01	0.01	0.00*	0.00*	0.00*	0.00*	0.01	0.07	11.99	33.46
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.52)	(1.21)
36	0.02	0.01	0.00*	0.00*	0.00*	0.01	0.01	0.03	4.71	24.14
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.34)	(1.00)
48	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.03	2.19	16.80
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.27)	(0.82)
60	4.60	0.00*	0.01	0.01	0.01	0.01	0.01	0.03	1.24	11.55
	(0.33)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.24)	(0.74)
84	6.13	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.04	10.88
	(0.39)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.63)

(a) *Reduction in valuation Uncertainty*: Δ_1^θ

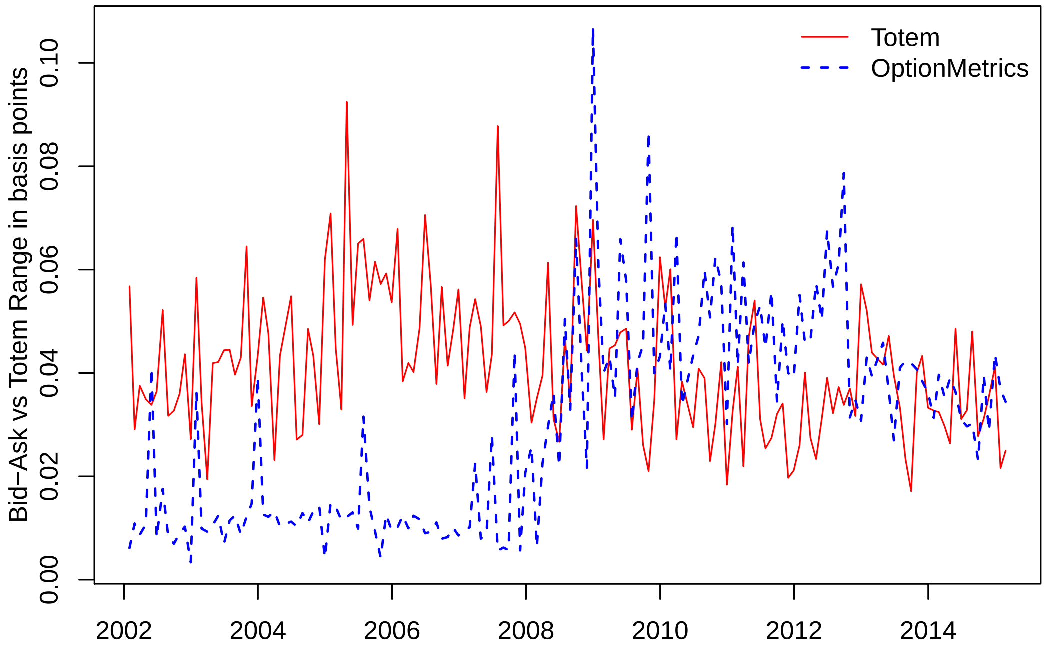
	60	80	90	95	100	105	110	120	150	200
6	41.31	0.02	0.01	0.01	0.01	0.04	0.27	38.80	62.69	·
	(1.43)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.49)	(1.09)	·
12	24.49	0.01	0.01	0.01	0.01	0.02	0.05	4.35	61.23	·
	(1.28)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.74)	(1.52)	·
24	0.03	0.02	0.01	0.01	0.01	0.02	0.05	0.27	40.68	62.10
	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.07)	(1.48)	(1.95)
36	0.07	0.03	0.02	0.02	0.02	0.02	0.04	0.13	18.68	57.87
	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.04)	(1.31)	(1.75)
48	0.09	0.03	0.03	0.02	0.02	0.03	0.04	0.12	8.94	46.67
	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.16)	(1.89)
60	18.72	0.01	0.03	0.02	0.02	0.03	0.04	0.10	5.06	35.14
	(1.29)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(1.05)	(1.94)
84	23.65	0.04	0.03	0.03	0.03	0.03	0.04	0.07	0.13	36.56
	(1.41)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.04)	(1.82)

(b) *Reduction in Strategic Uncertainty*: Δ_2^θ

These two tables present the counterfactual results of the percentage reduction in valuation and strategic uncertainty when moving from the current information structure to an information structure where the consensus price perfectly reveals last period's state (0.00* means below 0.005). The upper table presents the results for the percentage reduction in **valuation uncertainty**, Δ_1^θ in (10). The lower table presents the results for the percentage reduction in **strategic uncertainty**, Δ_2^θ. The first row and first column of each table denote the moneyness and time-to-expiration, respectively, of the options under consideration. The standard deviation of the posterior distribution of the parameter is given in parenthesis below its mean (0.00 signifies standard deviations below 0.005). The sample period is from December 2002 to February 2015.

7.2 Additional figures

Figure 6: Bid-Ask spread vs Submission Range IV (S&P 500)



The figure above displays the range of the price submissions to the IHS Markit's Totem service and bid-ask spread on traded options from OptionMetrics. This is for a contract with time-to-expiration of 6 months and moneyness 100. The bid-ask spread is given by the difference between the best closing bid price and best closing ask price across all US option exchanges. The options in the Totem service are matched to the traded options in the OptionMetrics database. On a given Totem valuation date we match OptionMetrics option contracts that are a close proxy for the totem option contracts. We search for contracts with a ± 10 days-to-maturity and a ±1 moneyness value. When multiple options match the criteria an average is taken of their bid-ask spread.

7.3 The value of information in the OTC options market

This is a simple one-period model to illustrate the value of the consensus price information for dealers in the OTC options market. It shows that dealers that use an interdealer market to share risk are naturally concerned about both fundamental asset values and other dealers' valuation. A dealer is willing to pay for information that reduces its uncertainty in any of these two dimensions.

7.3.1 The model

Before entering the market, every dealer $i \in [0,1]$ observes a private signal about the fundamental value of an option, given by the random variable θ. She can also pay to receive a public signal about that value. For now, the exact form of these signals is not important. The game proceeds in three steps:

1. Dealer $i \in [0,1]$ decides whether to buy the public signal at cost f.
2. After observing signal(s), the dealer enters the market and is matched with a client. A client is a buyer or seller of one option contract with equal probability. The dealer can credibly communicate her valuation of the option to the client. The client is willing to pay (receive) at most Δ in excess of (below) the dealer's valuation.
3. After buying or selling the option from the client, dealer i enters the interdealer market. She is matched with a dealer with opposite option inventory with probability $0 \leq \gamma \leq 1$. If matched, dealers trade at the average expectation of fundamental values among active dealers denoted by $\bar{\theta}$.
4. If a dealer has not been matched in the interdealer market (probability $1-\gamma$) she hedges the option herself. At expiry, she receives the fundamental value θ but hedging physically creates a cost of $c > 0$.

7.3.2 Pricing after entry

Suppose dealer i is matched with a client that wants to buy. The dealer charges a price a_i to the client. If the dealer is matched in the interdealer market, her profit is $a_i - \bar{\theta}$. Otherwise her profit is $a_i - \theta - c$. We assume that the dealer minimizes a loss function that is quadratic in losses.[19] The pricing problem is then

$$\mathcal{L}_i^s = \min_a \mathbb{E}_i \left\{ \gamma(a - \bar{\theta} - \pi)^2 + (1-\gamma)(a - \theta - c - \pi)^2 \right\},$$

[19] This captures the idea that dealers' institutions prefer smooth profits with target level π.

where the expectation is taken over dealer i's information set when she is interacting with the client, that is after entry and having observed signals, but before entering the interdealer market. The first-order condition for a yields the optimal price,

$$a_i^* = \pi + \gamma\,\mathbb{E}_i\,\bar{\theta} + (1-\gamma)\mathbb{E}_i\,(\theta + c).$$

We assume that dealer i can credibly communicate the "fair value" of the option, namely $\gamma\,\mathbb{E}_i\,\bar{\theta} + (1-\gamma)\mathbb{E}_i\,(\theta + c)$, to her client. For the client to buy, we further assume that the markup in the optimal price is smaller than the client's maximal willingness to pay, that is $\pi \leq \Delta$.

Substituting a_i^* back into the loss function we find

$$\mathcal{L}_i^s = \gamma\,\mathbb{E}_i\left(\bar{\theta} - \bar{\theta}_i\right)^2 + (1-\gamma)\,\mathbb{E}_i\,(\theta - \theta_i)^2 + \gamma(1-\gamma)(\delta_i + c)^2,$$

where $\delta_i = \theta_i - \bar{\theta}_i$.

The case for a dealer buying from a client at price b is symmetric with loss function

$$\mathcal{L}_i^b = \min_b \mathbb{E}_i\left\{\gamma(\bar{\theta} - b - \pi)^2 + (1-\gamma)(\theta - b - c - \pi)^2\right\}.$$

It yields a nearly identical loss function to the case of buying from a client, namely,

$$\mathcal{L}_i^b = \gamma\,\mathbb{E}_i\left(\bar{\theta} - \bar{\theta}_i\right)^2 + (1-\gamma)\,\mathbb{E}_i\,(\theta - \theta_i)^2 + \gamma(1-\gamma)(\delta_i - c)^2.$$

7.3.3 Participation decision

The ex-ante expected loss of dealer i with signals $\mathbf{s}_i$ is

$$\begin{aligned}-\,\mathbb{E}\left(\frac{1}{2}\mathcal{L}_i^s + \frac{1}{2}\mathcal{L}_i^b \mid \mathbf{s}_i\right) = &-\gamma\,\mathrm{Var}(\bar{\theta} \mid \mathbf{s}_i) - (1-\gamma)\mathrm{Var}(\theta \mid \mathbf{s}_i)\\ &- \gamma(1-\gamma)\,\mathbb{E}(\,\delta_i^2 \mid \mathbf{s}_i) - \gamma(1-\gamma)\,c^2.\end{aligned}$$

The dealer buys the public signal if the reduction in expected loss exceeds the price of the signal, which is f.

The public signal is valued as it allows for better pricing decisions. Its ability to reduce strategic uncertainty is valued as it helps to predict prices in the interdealer market.

7.4 IHS Markit's Totem submission process

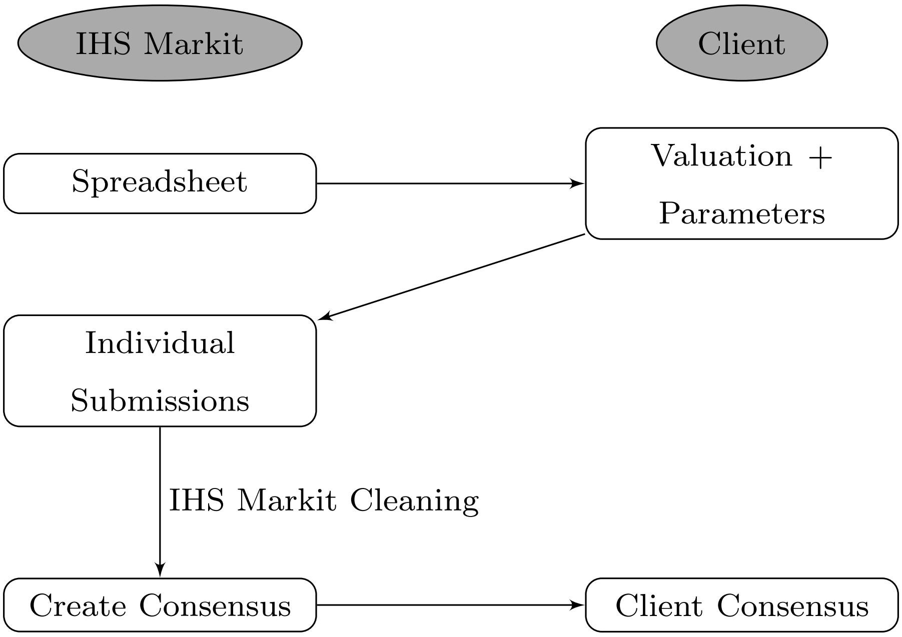


Figure 7: Diagram – Submission process

Figure 7 depicts a diagram of the submission process to IHS Markit's Totem service for plain vanilla index options.[20] Totem issues on the last trading day of each month a spreadsheet to $N_{K,T}$ submitters. Here K is the moneyness of the contract defined as the strike price divided by the spot price multiplied by 100, and T is the time-to-expiration of the contract in months. Participating submitters are required to submit their mid-price estimate for a range of put options with a moneyness between 80 and 100 and a range of call option with a moneyness ranging from 100 to 120 with a time-to-expiration of 6 months. Submitters which want to submit to any other contracts with a different maturity or/and different moneyness are

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required to submit to all the available strike price and time-to-expiration combinations which lie in between the required contracts and the additionally demanded contracts.

Submitter i submits its mid-price estimate for different out of the money put and call options, $P_t^i(p, K, T)$ and $P_t^i(c, K, T)$, respectively. The inputs which are required in addition to the mid-price estimates are

- Their discount factor $\beta_t^i(T)$
- Reference level $R_t^i(T)$ (This is the price of a futures contract with maturity date closest to the valuation date, i.e t.)
- Implied spot level $S_t^i(K, T)$ (Implied level of the underlying index of the futures contract)

There are strict instructions on the timing of the valuation of the contract and the reference level used. To address any issues which might still arise with respect to valuation timing and the effect it could have on the comparability of the prices, the various prices are aligned according to the following mechanism.

1. $\text{Basis}_i = R^i{}_t(T = 6) - S_t^i(K = 100, T = 6)$
2. $S_t^{i*}(K, T) = \underset{i}{\text{mode}}\left[R^i{}_t(T)\right] - \text{Basis}_i$
3. Remove from $S_t^{i*}(K, T)$ the lowest, highest, and erroneous adjusted spot levels.
4. $\bar{S}_t(K, T) = \underset{i}{\text{mode}}\left[R^i{}_t(T)\right] - \frac{1}{N^*(K,T)} \sum_{i=1}^{N^*(K,T)} S_t^i(K, T)$

This consensus-implied spot from the at-the-money 6-month option is used for all other combinations of K and T. The submitted prices are restated in terms of $\bar{S}_t(K, T)$, giving: $\widehat{p}_t^i(\{c, p\}, K, T) = {}^{P_t^i(\{c,p\},K,T)}/_{\bar{S}_t(K,T)}$.

Given the submitted quantities, a security analyst calculates various implied quantities to validate the individual submissions. The security analyst utilizes put-call parity on ATM options to retrieve the relative forward, i.e.,

$$f_t^i(K, T) = \frac{\widehat{p}_t^i(c, K, T) - \widehat{p}_t^i(p, K, T)}{\beta_t^i(T)} + 1$$

The above inputs are then used in the Black-Scholes model,

$$\widehat{p}_t^i(c, K, T) = \beta_t^i(T)\left[f_t^i(K, T)\, N(d_{1,i}) - KN(d_{2,i})\right]$$

$$d_1 = \frac{\ln\left(\frac{f}{K}\right) + \left(\frac{\sigma^2}{2}\right)\Delta T_t}{\sigma\sqrt{\Delta T_t}} \quad \text{, where} \quad \Delta T_t = \frac{\text{days(T)}}{365.25}$$

$$d_2 = d_1 - \sigma\sqrt{\Delta T_t}$$

to back-out the σ_i, i.e. the implied volatility $\sigma_i(K,T)$.

When reviewing submissions, security analysts compare the implied volatilities against other submitted prices and market conditions by taking the following points into consideration:

- The number of contributors
- Market activity & news
- Frequency of change of variables
- Market conventions
- In a "one way market," is the concept of a mid-market price clearly understood?
- The distribution and spread of contributed data.

In addition to these criteria, security analysts also visually inspect the ATM implied volatility term structure and the implied volatility along the moneyness for a given term, also referred to as the skew or smile. After the vetting process, the security analyst proceeds to the aggregation of the individual submissions into the consensus data.

Given the Black-Scholes model, they back out $\sigma_i(K,T)$ and aggregate it into the consensus-implied volatility.

$$\bar{\sigma}\left(K,T\right) = \frac{1}{n_{(K,T)} - n^r} \sum_{i=1}^{n_{K,T}-n^r} \sigma_i\left(K,T\right)$$

Here n^r are the number of excluded prices. The exclusions consist of the lowest, highest, and rejected prices. The highest and lowest acceptable $\sigma_i\left(K,T\right)$ are consistent and reasonable IVs, but are excluded to safeguard the stability of the consensus IV.[21] The same process takes place for the submitted prices.

The submitters of which the pricing information is not rejected receive from the security analyst the consensus information. The security analysis of Totem aims to return the consensus price to the eligible contributers within 5 hours of the submission deadline. The consensus data includes the average, standard deviation, skewness, and kurtosis of the distribution of accepted prices and implied volatilities. They also include the number of submitters to the consensus data. In our setup, the contributors are only uncertain about the mean of the cross-sectional distribution.

[21] If the number of acceptable prices is 6 or below the highest and lowest submissions are included in the consensus price calculations.

7.5 Solution algorithm

Here we show how to solve the consensus pricing problem of Section 3. We adopt the following standard notation for higher-order beliefs, defining recursively

$$\theta_t^{(0)} = \theta_t,$$
$$\theta_{i,t}^{(k+1)} = \mathbb{E}\left(\theta_t^{(k)} | \Omega_{i,t}\right) \quad \text{and} \quad \theta_t^{(k+1)} = \int_0^1 \theta_{i,t}^{(k+1)} \, di \text{ for all } k \geq 0.$$

We denote institution i's hierarchy of beliefs up to order k by

$$\theta_{i,t}^{(1:k)} = \left(\theta_{i,t}^{(1)}, ..., \theta_{i,t}^{(k)}\right)^{\mathsf{T}}$$

and for the hierarchy of average beliefs up to order k, including the fundamental value $\theta_t^{(0)}$ as first element,

$$\theta_t^{(0:k)} = \left(\theta_t^{(0)}, \theta_t^{(1)}, ..., \theta_t^{(k)}\right)^{\mathsf{T}}.$$

The solution procedure proceeds recursively. It starts with a fixed order of beliefs $k \geq 0$ and postulates that the dynamics of average beliefs $\theta_t^{(0:k)}$ are given by the VAR(1)

$$\theta_t^{(0:k)} = M_k \, \theta_{t-1}^{(0:k)} + N_k \, w_t, \tag{11}$$

with $w_t = (u_t, \varepsilon_t)^{\mathsf{T}}$ and $\theta_t^{(n)} = \theta_t^{(k)}$ for all $n \geq k$.

Institution i's signal can be expressed in terms of current and past average beliefs, $\theta_t^{(0:k)}$ and $\theta_{t-1}^{(0:k)}$, and the period t shocks w_t and $n_{i,t}$. The private signal can be written as

$$s_{i,t} = e_1^{\mathsf{T}} \, \theta_t^{(0:k)} + \sigma_\eta \, \eta_{i,t},$$

where e_j denotes a column vector of conformable length with a 1 in position j, all other elements being 0. Similarly, we can express the consensus price p_t as

$$p_t = \theta_{t-1}^{(1)} + \sigma_\varepsilon \, \varepsilon_t = e_2^{\mathsf{T}} \, \theta_{t-1}^{(0:k)} + \sigma_\varepsilon \, \varepsilon_t.$$

Denote the vector of signals by $y_{i,t} = (s_{i,t}, p_t)^{\mathsf{T}}$. We can now express the signals in terms of current average beliefs and shocks,

$$y_{i,t} = D_{k,1} \, \theta_t^{(0:k)} + D_{k,2} \, \theta_{t-1}^{(0:k)} + R_w \, w_t + R_\eta \, \eta_{i,t}, \tag{12}$$

where

$$D_{k,1} = \begin{bmatrix} e_1^{\mathsf{T}} \\ 0_{k+1}^{\mathsf{T}} \end{bmatrix}, \quad D_{k,2} = \begin{bmatrix} 0_{k+1}^{\mathsf{T}} \\ e_2^{\mathsf{T}} \end{bmatrix}, \quad R_\eta = \begin{bmatrix} \sigma_\eta \\ 0 \end{bmatrix} \quad \text{and} \quad R_w = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\varepsilon \end{bmatrix}.$$

We thus obtain a state space representation of the system from the perspective of institution i. Equation (11) describes the dynamics of the latent state variable $\theta_t^{(0:k)}$; Equation (12) is the observation equation that provides the link between the current state and i's signals. Using a Kalman filter that allows for lagged state variables (see Nimark (2015)) allows us to express institution i's beliefs conditional on the information contained in $\Omega_{i,t}$ as

$$\theta_{i,t}^{(1:k+1)} = M_k\, \theta_{i,t-1}^{(1:k+1)} + K_k \left[y_{i,t} - D_{1,k}\, M_k\, \theta_{i,t-1}^{(1:k+1)} - D_{2,k}\, \theta_{i,t-1}^{(1:k+1)} \right], \tag{13}$$

where K_k is the (stationary) Kalman gain. Substituting out the signal vector in terms of current state and shocks, this can equivalently be written as

$$\begin{aligned} \theta_{i,t}^{(1:k+1)} &= \left[M_k - K_k(D_{1,k}M_k + D_{2,k}) \right] \theta_{i,t-1}^{(1:k+1)} \\ &\qquad + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t + K_k R_\eta\, \eta_{i,t}. \end{aligned}$$

Averaging this expression across all submitters, assuming that by a law of large numbers $\int_0^1 \eta_{i,t}\, di = 0$, average beliefs are then given by

$$\begin{aligned} \theta_{t}^{(1:k+1)} &= \left[M_k - K_k(D_{1,k}M_k + D_{2,k}) \right] \theta_{t-1}^{(1:k+1)} \\ &\qquad + K_k(D_{1,k}M_k + D_{2,k})\theta_{t-1}^{(0:k)} + K_k(D_{1,k}N_k + R_w)w_t. \end{aligned}$$

Combined with the fact that $\theta_t^{(0)} = \rho\, \theta_{t-1}^{(0)} + \sigma_u\, u_t$, we now obtain a new law of motion for the state,

$$\theta_t^{(0:k+1)} = M_{k+1}\, \theta_{t-1}^{(0:k+1)} + N_{k+1}\, w_t,$$

with

$$M_{k+1} = \begin{bmatrix} \rho\, e_1^\intercal & 0 \\ K_k(D_{1,k}M_k + D_{2,k}) & 0_{k\times 1} \end{bmatrix} + \begin{bmatrix} 0 & 0_{1\times k} \\ 0_{k\times 1} & M_k - K_k(D_{1,k}M_k + D_{2,k}) \end{bmatrix} \tag{14}$$

and

$$N_{k+1} = \begin{bmatrix} \sigma_u\, e_1^\intercal \\ K_k(D_{1,k}N_k + R_w) \end{bmatrix}. \tag{15}$$

Note, however, that now the state space has increased by one dimension from $k+1$ to $k+2$. This is a consequence of the well-known infinite regress problem when filtering endogenous signals. When filtering average beliefs of order k, institutions have to form beliefs about average beliefs of order k. But this implies that equilibrium dynamics are influenced by average beliefs of order $k+1$, and so on for all orders $k \geq 0$.

In practice, the solution algorithm works as follows. We initialize the iteration at $k = 0$ with $M_0 = \rho$ and $N_0 = \sigma_u$, which implies that $\theta_t^{(1)} = \theta_t^{(0)}$ for all t. Consequently, the consensus price of the first iteration is given by[22]

$$p_t^{[1]} = \theta_{t-1}^{(0)} + \sigma_\varepsilon \, \varepsilon_t.$$

This yields a Kalman gain K_0 (here a two-dimensional vector) which can then be used to obtain M_1 and N_1 via equations (14) and (15) and so on until convergence of the process $p_t^{[n]}$ has been achieved according to a prespecified convergence criteria after n steps. The highest-order belief that is not trivially defined by lower-order beliefs is then of order n.

7.6 Kalman Filter for Estimation

For a given contract, that is, a given time-to-expiration, moneyness, and option type (put or call), our data consists of two time series. Let S be the total number of institutions that have submitted to Totem over the course of our sample and let $\iota_t \subset \{1, 2, .., S\}$ be the set of institutions active in t.[23] Our sample of submissions is then given by $(\mathbf{m}_t)_{t=1}^T$, where $\mathbf{m}_t = (m_{j,t})_{j \in \iota_t}$ is a $|\iota_t|$-dimensional vector consisting of the individual period t consensus price submissions. We assume that consensus price submissions are institution i's best estimate of θ_t plus uncorrelated measurement error[24]

$$m_{i,t} = \theta_{i,t}^{(1)} + \sigma_\psi \, \psi_{i,t} \;\text{ with }\; \psi_{i,t} \sim N(0, 1). \tag{16}$$

Following our model, we assume that the consensus price of period $t-1$, which we call p_t, equals the average first-order belief of period $t-1$ plus aggregate noise, that is,

$$p_t = \theta_{t-1}^{(1)} + \sigma_\varepsilon \, \varepsilon_t.$$

Our data set for a given contract, $(\mathbf{y})_{t=1}^T$, then consists of the time-series of institutions' price submissions for this contract and the corresponding consensus price, i.e. $\mathbf{y}_t = (p_t \, , \mathbf{m}_t)^\mathsf{T}$.[25]

To estimate the model, we fix the maximum order of beliefs at $\bar{k} = 4$ and assume that the system has reached its stationary limit.[26] Average beliefs then

[22]Superscripts in square brackets denote iterations of the algorithm.

[23]If an institution does not submit a price in t, we treat this as a missing value. However, it is assumed that this institution received both the consensus price and the private signal about the fundamental in that period.

[24]In our main specification we assume that there is not measurement error, i.e. $\sigma_\psi = 0$.

[25]To be precise, $m_{j,t}$ is the (demeaned) natural logarithm of the Black-Scholes implied volatility of submitter j's time t price submission, and p_t is the (demeaned) natural logarithm of the consensus Black-Scholes implied volatility calculated by Totem for the corresponding contract.

[26]Allowing $\bar{k}$ greater than 4 does not change the estimates noticeably.

evolve according to (11), namely,

$$\theta_t^{(0:\bar{k})} = M_{\bar{k}}\, \theta_{t-1}^{(0:\bar{k})} + N_{\bar{k}}\, w_t,$$

where $M_{\bar{k}}$ and $N_{\bar{k}}$ are functions of the parameters ϕ defined recursively by equations (14) and (15) and $w_t = (u_t\,,\, \varepsilon_t)^{\mathsf{T}} \sim N(\mathbf{0}_2, I_2)$.[27] The dynamics of institution i's conditional beliefs $\theta_{i,t}^{(1:\bar{k})}$ can be expressed in terms of deviations from average beliefs, $\hat{\theta}_{i,t}^{(1:\bar{k})} \equiv \theta_{i,t}^{(1:\bar{k})} - \theta_t^{(1:\bar{k})}$, as

$$\hat{\theta}_{i,t}^{(1:\bar{k})} = Q_{\bar{k}}\, \hat{\theta}_{i,t-1}^{(1:\bar{k})} + V_{\bar{k}}\, \eta_{i,t},$$

where

$$Q_{\bar{k}} = \left[M_{\bar{k}} - K_{\bar{k}}(D_{1,\bar{k}} M_{\bar{k}} + D_{2,\bar{k}})\right] \quad \text{and} \quad V_{\bar{k}} = K_{\bar{k}}\, R_\eta.$$

Given the linearity of the above system and the assumed normality of shocks, the likelihood function for the observed data $(\mathbf{y})_{t=1}^T$ with $\mathbf{y}_t = (p_t\,, \mathbf{m}_t)^{\mathsf{T}}$ can be derived using the Kalman filter. We define $\alpha_t = (\theta_t^{(0:\bar{k})}\,, \hat{\theta}_{1,t}^{(1:\bar{k})}\,, ..., \hat{\theta}_{S,t}^{(1:\bar{k})}\,, \varepsilon_t)^{\mathsf{T}}$ to be the state of the system in t.

The *transition equation* of the system in state space form is then given by

$$\alpha_t = T\alpha_{t-1} + R\,\epsilon_t,$$

where

$$T = \begin{pmatrix} M_{\bar{k}}\,,\, 0_{\bar{k}+1\times S\bar{k}+1} \\ 0_{S\,\bar{k}\times\bar{k}+1}\,,\, I_S \otimes Q_{\bar{k}}\,,\, 0_{S\,\bar{k}\times 1} \\ 0_{2\times\bar{k}+1+S\bar{k}+1} \end{pmatrix}\,,\; R = \begin{pmatrix} N_{\bar{k}}\,,\, 0_{\bar{k}+1\times S} \\ 0_{S\bar{k}\times 2}\,,\, I_S \otimes \sigma_\eta\, V_{\bar{k}} \\ I_2\,,\, 0_{2\times S} \end{pmatrix}$$

and $\epsilon_t = (u_t\,, \varepsilon_t\,, \eta_{1,t}\,, ..., \eta_{S,t})^{\mathsf{T}} \sim N\left(\mathbf{0}_{2+S}, I_{2+S}\right)$.

We now derive the *observation equation* for the system given by

$$\mathbf{y}_t = Z_{1,t}\, \alpha_t + Z_{2,t}\, \alpha_{t-1} + \phi_t.$$

First note that the consensus price p_t can be expressed in terms of the past state vector α_t as

$$p_t = e_2^{\mathsf{T}}\, \theta_{t-1}^{(1)} + \sigma_\varepsilon\, \varepsilon_t.$$

Next, note that we can write institution i's submission $m_{i,t}$ as

$$m_{i,t} = \theta_{i,t}^{(1)} + \sigma_\psi\, \psi_{i,t} = \theta_t^{(1)} + x_{i,t}^{(1)} + \sigma_\psi\, \psi_{i,t}.$$

[27] We use $0_{n\times m}$ to denote a $n \times m$ matrix of zeros, 1_n is a (column) vector containing n ones, and I_n is an n-dimensional identity matrix.

The above derivations allow us to write c_t, $Z_{1,t}$, and $Z_{2,t}$ in terms of the parameters of the model. We start by defining an auxiliary matrix J_t that allows us to deal with missing submissions by some institutions in period t. Recall that $\iota_t \subset \{1, 2, .., S\}$ is the set of institutions submitting in t. Let $\iota_{k,t}$ designate the k-th element of the index ι_t. J_t is a $(|\iota_t| \times S)$ matrix whose k-th row has a 1 in position $\iota_{k,t}$ and zeros otherwise.

We thus have

$$\phi_t = \begin{pmatrix} 0 \\ \sigma_\psi \, J_t \, (\psi_{1,t}, ..., \psi_{N,t})^\intercal \end{pmatrix} \text{ with } \Gamma_t = \mathbb{E}\left(\phi_t \, \phi_t^\intercal\right) = \begin{pmatrix} 0 & \mathbf{0}_{|\iota_t|}^\intercal \\ \mathbf{0}_{|\iota_t|} & \sigma_\psi^2 \, I_{|\iota_t|} \end{pmatrix}.$$

Furthermore, we have $Z_{1,t} = J_t \, Z_1$ and $Z_{2,t} = J_t \, Z_2$, where

$$Z_1 = \begin{pmatrix} 0_{1\times 1+\bar{k}+S\bar{k}} \, , \, \sigma_\varepsilon \\ 0 \, , \, 1 \, , \, e_1^\intercal \\ 0 \, , \, 1 \, , \, e_{\bar{k}+1}^\intercal \\ \vdots \\ 0 \, , \, 1 \, , \, e_{(S-1)\bar{k}+1}^\intercal \end{pmatrix}, \text{ and } Z_2 = \begin{pmatrix} 0, 1, 0_{1\times(\bar{k}-2)+S\bar{k}+1} \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \\ \vdots \\ 0_{1\times 1+\bar{k}+S\bar{k}+1} \end{pmatrix}.$$

Given a prior for the state of the system at $t = 1$, $\alpha_1 \sim N(\mathbf{a}_1, P_1)$, we can now apply the usual Kalman filter recursion to derive the likelihood function for our data $(\mathbf{y}_t)_{t=1}^T$ given the parameter vector ϕ denoted $L\left((\mathbf{y}_t)_{t=1}^T \mid \phi\right)$.

7.7 Proof of identification

Strategy of proof The proof of identification proceeds in two steps. First, we establish identification for the model under the assumption that submitting institutions take the consensus price to be an exogenous signal of the past state, i.e. $p_t = \theta_{t-1} + \varepsilon_t$. This is the model of the first step in Nimark (2014)'s solution algorithm. Second, once we have established identification of the first-step model, we proceed by induction. In particular, we argue that if the model is identified at step n of the algorithm, it is also identified at step $n+1$. This then establishes identification of the model at all steps of the algorithm.

A. Identification with exogenous consensus price signal

If submitters assume that the consensus price is an exogenous signal of the (past) state, then individual submitters' first-order beliefs are updated according to

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + (k_{11}\; k_{12}) \begin{pmatrix} \theta_t + \eta_{i,t} - \rho\,\theta_{i,t-1} \\ \theta_{t-1} + \varepsilon_t - \theta_{i,t-1} \end{pmatrix}.$$

We can write this as

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_t + k_{11}\,\eta_{i,t}, \tag{17}$$

where the Kalman gains k_{11} and k_{12} are given by

$$k_{11} = \frac{\zeta + \rho^2 k}{\zeta + \rho^2 + \psi/(1-\psi)} \quad \text{and} \quad k_{12} = \rho(k - k_{11}) \quad \text{with}$$

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2 + \xi\right]^{\frac{1}{2}}\left[(1+\rho)^2 + \xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\xi = \frac{\zeta}{\psi} \quad , \quad \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2 + \sigma_\eta^2} \quad \text{and} \quad \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

The average first-order belief is then

$$\bar{\theta}_t = (1-k)\rho\,\bar{\theta}_{t-1} + k\,\rho\,\theta_{t-1} + k_{11}\,u_t + k_{12}\,\varepsilon_t,$$

with corresponding (step 2) consensus price process

$$p_t = \bar{\theta}_{t-1} + \varepsilon_t.$$

This implies the following dynamics for the consensus price,

$$p_t = (1-k)\rho\,p_{t-1} + k\,\rho\,\theta_{t-2} + k_{11}\,u_{t-1} + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t. \tag{18}$$

Observed data We assume that our observed data consists of a panel of individual first-order beliefs for N submitting institutions $\{\{\theta_{i,t}\}_{i=1}^{N}\}_{t=1}^{T}$ that evolve according to (17), and the corresponding time-series of consensus prices $\{p_t\}_{t=1}^{T}$ generated by the process specified in (18).

We now show how the distribution of the above data identifies the model parameters of interest, namely $\{\rho, \sigma_\varepsilon^2, \sigma_\eta^2, \sigma_u^2\}$.

1. Deviations of the consensus price from average expectations identify σ_ε^2.
We obtain estimates for the error ε_t from the difference between the current consensus price and the past mean submission as

$$\varepsilon_t = p_t - \bar{\theta}_{t-1}.$$

We can thus identify σ_ε^2 from the time-series variance of the estimated errors.

2. Individual deviations from average expectations identify $(1-k)\rho$.
Individual deviations from the consensus, $\hat{\theta}_{i,t} = \theta_{i,t} - \bar{\theta}_t$ are given by

$$\hat{\theta}_{i,t} = (1-k)\rho\, \hat{\theta}_{i,t-1} + k_{11}\, \eta_{i,t}.$$

Individual deviations follow an AR(1) process. Deviations from consensus mean-revert more quickly if submitters put less weight on past information (higher k), or if the fundamental value process is less persistent (low ρ). We can therefore identify $(1-k)\rho$ from the auto-covariances of individual deviations from the current mean submission.

3. Persistence in consensus price updates identify ρ and hence k via $(1-k)\rho$.
Having identified $(1-k)\rho$ we can obtain $\omega_t = p_t - (1-k)\rho\, p_{t-1}$ from our data, where

$$\omega_t = k_{11}\, u_{t-1} + k\,\rho \left(\frac{u_{t-2}}{1-\rho L} \right) + (k_{12} - (1-k)\rho)\varepsilon_{t-1} + \varepsilon_t.$$

ω_t is a noisy measure of the fundamental news submitters receive in period t. By subtracting $(1-k)\rho\, p_{t-1}$ from p_t it "cleans out" their prior beliefs. For sufficiently long lags, ω_t's auto-correlation exclusively comes from its dependence on the fundamental process and not the aggregate noise, ε_t. Its auto-covariances thus allow us to identify the persistence in the process of θ_t. In particular, we can see that the auto-covariances of ω_t have to satisfy

$$Cov(\omega_t, \omega_{t-3}) = \rho\, Cov(\omega_t, \omega_{t-2}).$$

The ratio of these auto-covariances thus identify ρ,

$$\rho = Cov(\omega_t, \omega_{t-3}) / Cov(\omega_t, \omega_{t-2}),$$

which together with $(1-k)\rho$ then allow us to identify $1-k$, i.e. the persistence in individual expectations due to informational frictions.

4. The weight submitters put on the consensus price when updating expectations identifies σ_η^2 and hence σ_u^2 via k.
k determines how much weight submitters put on new information as opposed to their priors. It is given by

$$k = \frac{1}{2} + \frac{1}{2\rho^2}\left\{\left[(1-\rho)^2+\xi\right]^{\frac{1}{2}}\left[(1+\rho)^2+\xi\right]^{\frac{1}{2}} - (1+\xi)\right\},$$

$$\text{where } \ \xi = \frac{\zeta}{\psi} \ \text{ with } \ \psi = \frac{\sigma_\eta^2}{\sigma_\varepsilon^2+\sigma_\eta^2} \ \text{ and } \ \zeta = \frac{\sigma_u^2}{\sigma_\varepsilon^2}.$$

It is a function of ξ, which is a ratio of the variance of the shocks to the fundamental value to the variance of the signal noises and can thus be seen as a measure of the signal to noise ratio. k is monotonically increasing in ξ; a higher signal to noise ratio implies a higher weight on current signals. Hence, having already identified k, we can also identify ξ.

In turn, the weights submitters put on the private signal and the consensus price can be expressed in terms of k, ξ, and ψ, namely

$$k_{11} = \frac{\xi\,\psi + \rho^2\,k}{\xi\,\psi + \rho^2 + \psi/(1-\psi)} \ \text{ and } \ k_{12} = \rho(k-k_{11}).$$

It can be shown that, for a given k, the weight on the private signal, k_{11}, is monotonically decreasing and the weight on the consensus price, k_{12}, monotonically increasing in ψ for $\psi \in (0,1)$; a relatively more noisy private signal will lead submitters to shift weight from the private signal to the consensus price (given k). As we have already identify k and ξ, knowing either k_{11} or k_{12} will allow us to identify ψ. Given ψ we can then back out σ_η^2 and ζ, which yields σ_u^2.

We now proceed to show identification of k_{12}, which by the previous argument establishes identification of the model. To do so, we return to the individual expectation updating equation,

$$\theta_{i,t} = (1-k)\rho\,\theta_{i,t-1} + k_{11}\,\rho\,\theta_{t-1} + k_{12}\,p_t + k_{11}\,\eta_{i,t} + k_{11}\,u_t.$$

We also have

$$\theta_{i,t-1} = (1-k)\rho\,\theta_{i,t-2} + k_{11}\,\theta_{t-1} + k_{12}\,p_{t-1} + k_{11}\,\eta_{i,t-1}.$$

Multiplying the latter expression by ρ and subtracting from the former eliminates the unobservable θ_{t-1}. We obtain an expression in terms of observables and shocks,

$$\theta_{i,t} - \rho\,\theta_{i,t-1} = (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}) + k_{12}\,(p_t - \rho\,p_{t-1}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

Note that we have already identified $(1-k)\rho$. Define

$$y_{i,t} = \theta_{i,t} - \rho\,\theta_{i,t-1} - (1-k)\rho(\theta_{i,t-1} - \rho\,\theta_{i,t-2}).$$

We can then identify the coefficient k_{12} from the covariance of $y_{i,t}$ and $p_t - \rho\,p_{t-1}$ noting that

$$y_{i,t} = k_{12}\,(p_t - \rho\,p_{t-1}) + k_{11}(\eta_{i,t} - \rho\,\eta_{i,t-1}) + k_{11}\,u_t.$$

This is possible as p_t is a signal based on information available in $t-1$ plus ε_t. It is not correlated with the shock u_t. Furthermore, the idiosyncratic noise terms $\eta_{i,t}$ and $\eta_{i,t-1}$ are uncorrelated with the consensus price process by construction.

B. Establishing identification by induction

Suppose we have established identification of the model parameters by our observed data for step n of the algorithm. That is, any two distinct sets of parameters ϕ_1 and ϕ_2 imply distinct distributions of the observable data. In particular, the step n consensus price process that submitters will assume in step $n+1$ differs across the two parameter sets. This necessarily implies that the distribution of individual expectations will differ across the two parameter sets in step $n+1$. This then establishes identification of the model at step $n+1$ of the algorithm. ■

7.8 Covariance Matrices for Counterfactual Scenarios

7.8.1 Consensus price perfectly reveals past state

If the consensus price perfectly aggregates dispersed information, we have

$$p_t = \theta_{t-1}.$$

In this case all submitters start period t with a common prior about θ_t, namely $\rho\,\theta_{t-1}$, and there is no higher-order uncertainty before receiving new signals. This is because every submitter knows that every submitter knows (and so on ...) that the average expected value of θ_t before receiving period t signals is $\rho\,\theta_{t-1}$.

Submitter i's expectations about the fundamental given signal $s_{i,t} = \theta_t + \eta_{i,t}$ can be obtained by the standard updating formula as state θ_t and signal $s_{i,t}$ given θ_{t-1} are jointly normally distributed:

$$\mathbb{E}_{i,t}\,(\theta_t) = \theta_{i,t} = \rho\,\theta_{t-1} + k_1\,(s_{i,t} - \rho\,\theta_{t-1}) = \rho\,\theta_{t-1} + k_1(u_t + \eta_{i,t})\,,$$

where k_1 is the Kalman gain

$$k_1 = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\eta^2}.$$

It follows that the average expectation is

$$\bar{\theta}_t = \rho\,\theta_{t-1} + k_1\,u_t.$$

Now define the random vector

$$X_t = \left[\theta_t - \rho\,\theta_{t-1}\,,\, \bar{\theta}_t - \rho\,\theta_{t-1}\right] = \left[u_t\,,\, k_1\,u_t\right]$$

and

$$y_{i,t} = s_{i,t} - \rho\,\theta_{t-1} = u_t + \eta_{i,t}.$$

X_t and $y_{i,t}$ are jointly normally distributed. Thus, the covariance of X_t given $y_{i,t}$ is

$$Var\,(X_t|y_{i,t}) = \Sigma_{xx} - \Sigma_{xy}\left(\sigma_y^2\right)^{-1}\Sigma_{xy}^{\mathsf{T}},$$

where Σ_{xx} is the variance of X_t and Σ_{xy} is the covariance of X_t and $y_{i,t}$, namely,

$$\Sigma_{xx} = \begin{bmatrix} \sigma_u^2 & k_1\sigma_u^2 \\ k_1\sigma_u^2 & k_1^2\,\sigma_u^2, \end{bmatrix} \quad , \quad \Sigma_{xy} = \left[\sigma_u^2\;,\; k_1\,\sigma_u^2\right]^{\mathsf{T}}.$$

As $\rho\,\theta_{t-1}$ is known in t, $Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\theta_{t-1}, y_{i,t}) = Var\,(X_t|y_{i,t})$. It follows that

$$Var((\theta_t, \bar{\theta}_t)^{\mathsf{T}}|\Omega_{i,t}) = \begin{bmatrix} \frac{\sigma_u^2\,\sigma_\eta^2}{\sigma_u^2+\sigma_\eta^2} & \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} \\ \frac{\sigma_u^4\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^2} & \frac{\sigma_u^6\sigma_\eta^2}{(\sigma_u^2+\sigma_\eta^2)^3} \end{bmatrix}.$$

7.8.2 No consensus price feedback

Without consensus price feedback, the stationary expectation dynamics of submitter i are given by

$$\theta_{i,t} = \rho\,\theta_{i,t-1} + k_1\left(s_{i,t} - \rho\,\theta_{i,t-1}\right),$$

where k_1 is the stationary Kalman gain. k_1 is the solution to the system of two equations in two unknowns, k_1 and σ^2,

$$k_1 = \frac{\sigma^2}{\sigma^2 + \sigma_\eta^2}\;,\; \sigma^2 = \rho^2(1-k_1)\sigma^2 + \sigma_u^2.$$

The average stationary expectation then evolves according to

$$\bar{\theta}_t = (1-k_1)\rho\,\bar{\theta}_{t-1} + k_1\rho\,\theta_{t-1} + k_1\,u_t.$$

We can now write the dynamics for $(\theta_t, \bar{\theta}_t)^\mathsf{T}$ in state space form, with transition equation

$$\begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} = \begin{bmatrix} \rho & 0 \\ k_1\rho & (1-k_1)\rho \end{bmatrix} \begin{pmatrix} \theta_{t-1} \\ \bar{\theta}_{t-1} \end{pmatrix} + \begin{bmatrix} 1 \\ k_1 \end{bmatrix} u_t$$

and measurement equation

$$y_{i,t} = (1\ ,\ 0) \begin{pmatrix} \theta_t \\ \bar{\theta}_t \end{pmatrix} + \eta_{i,t}.$$

The stationary covariance matrix for the state given the history of signals up to t, $Var((\theta_t, \bar{\theta}_t)^\mathsf{T} | \{s_{i,t-j}\}_{j=0}^{\infty})$ can now be derived with a standard Kalman filter.