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CAPITAL INVESTMENT AND THE COST OF CAPITAL: A DYNAMIC ANALYSIS

by

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A. INTRODUCTION

This study presents an empirical analysis of aggregate fixed investment in Canada, with emphasis upon measuring the effect of changes in the rate of interest (or the cost of capital) upon investment. This analysis is undertaken in order to shed some light on the possible investment effects of tax changes.

We find that Canadian investment behaviour can be predicted quite well by a dynamic equation incorporating changes in output and changes in interest rates. What distinguishes the model used from those of previous studies of investment in Canada is the particular specification of the lag structure linking investment decisions to investment expenditures. This particular specification, as well as the increasing richness of the monetary experience covered by the time series on interest rates, probably accounts for the significant negative interest rate coefficient obtained.

This study does not include an analysis of the various tax reforms recommended by the Royal Commission on Taxation; these have been examined in detail in Volume VI of the Report. 1/ However, the relevance of findings on the effect of the changes in the rate of interest for the effects of tax changes which affect the marginal rate of return and the marginal cost of capital is discussed briefly below and at greater length in the final section of this paper.

A major limitation of this study must be stated frankly at the outset. The models specified do not incorporate those features of the Canadian economy which identify it as an open economy. In particular, the effects of direct investment in Canada, of changes in export demand and of changes in the exchange rate are not taken into account directly. The models used here are correctly specified only if the effects of these variables work wholly through the independent variables included in the models— interest rates, output, retained earnings, and lagged investment itself.

Is this procedure valid? Elsewhere I have argued that the failure to incorporate such factors into investment models might help to explain the weaker predictive power of models fitted to industry data in Canada relative to those fitted to United States industry data. 2/

The same objections do not apply with equal force to aggregative models, however, because of the strong cyclical links between the Canadian and United States economies. As a consequence, domestic output in Canada may be a reasonable proxy for final demand for Canadian products, and gross domestic retentions a reasonable measure of available internal funds. 3/ As for exchange rate changes, unless these are both foreseen and acted upon, the omission of the exchange rate is not of serious consequence. Furthermore, it is difficult to specify the effects of large changes in exchange rates which occur in a crisis atmosphere. According to the residuals from the best equation estimated for this study, investment was depressed during 1962 when the exchange rate devaluation occurred. In this situation, the subsequent stimulation provided by devaluation may have been adequately picked up by the output variable. 4/

Many of the equations estimated in this paper do not include tax variables; in others only the overall effect of the corporate tax structure is allowed for. 5/ The effects of various changes in the tax structure may be inferred from the coefficients of other variables which affect the cost of capital, such as interest rates and retained earnings.

Such inferences will be accurate to the extent that the aggregate investment response to a tax induced change in the cost of capital is approximated by the response to a change brought about by interest rate movements. As will be discussed in the final section of this paper, inferences drawn in this fashion may be inaccurate, particularly for tax changes which lie outside the range of recent historical experience, such as those recommended by the Royal Commission on Taxation.

REFERENCES

1/ Report of the Royal Commission on Taxation, Ottawa, Queen's Printer, 1967.

2/ See my comment on N. H. Lithwick, G. Post and T. K. Rymes, "Postwar Production Relationships in Canada", N.B.E.R., The Theory and Empirical Analysis of Production, 1967, pp. 265-271.

3/ In an interesting but as yet unpublished study, Lupitz (Raymond Lupitz, United States Direct Investment in Canada and Canadian Capital Formation, unpublished Ph.D. thesis, Harvard University, 1966) has estimated a quarterly equation which incorporates a direct effect upon domestic investment of foreign direct investment in Canada. However, Lupitz makes no allowance for an interest rate effect and uses a lag structure quite different from the lag structure of the models in the present study. It would be interesting to examine the role of direct investment within the framework of the model used here.

4/ See the results of the forecasting tests reported in Section F below.

5/ In these equations, the interest rate is adjusted for relative prices of capital goods and the effective rate of taxation on corporate gross profits.

B. BACKGROUND

The traditional theory of investment is derived from the theory of the profit maximizing firm. The main proposition of this theory is that a firm maximizes its net worth when it equates the marginal rate of return on capital with the marginal cost of obtaining capital funds. Generally it has been assumed that the marginal cost of capital was adequately measured by the market rate of interest at which the firm could borrow. Recent theoretical and empirical developments have enlarged this concept to include the imputed costs of increased risk brought about by a rise in the debt/equity ratio. The inclusion of the effect of tax structure variables upon the cost of capital represents a further recent improvement.

Theory therefore provides the following guide to the empirical analysis of investment: seek those variables that affect either the marginal rate of return on capital or the cost of capital (or both).

The task of empirical analysis is therefore to measure the quantitative importance of the different variables and to assess whether some simplified theories, such as the accelerator theory which ignores cost of capital variables completely, or the accelerator-residual funds theory which ignores the cost of borrowed capital, are valid approximations for forecasting and, more important, for policy analysis.

Many empirical studies of the determinants of investment have been carried out since the pioneering explorations of Tinbergen. 1/ These studies have largely fallen into one of three classifications.

1. Time series analyses of investment by industries or larger aggregations (e.g., the manufacturing sector or the total economy).

2. Cross-section studies of individual firms based upon data from balance sheets and income statements, usually within industries.

3. Survey studies based on the responses of businessmen to questionnaires and interviews.

It is fair to say that, at first glance, the overwhelming majority of these studies does not provide encouragement for proponents of the traditional theory of investment. As Eisner and Strotz concluded in their 1963 survey of the existing investment literature:

> The interest rate has occasionally been found to be negatively related to capital expenditures, but such findings are not general. Coefficients are frequently uncertain, or, more important, so small in relation to the variations of the interest rate which have been allowed to occur as to deny that variable much historical role in influencing the rate of investment. 2/

Until recently, time series regressions have not detected a significant effect of the rate of interest. However, as recent studies have demonstrated, the vast majority of time series studies involved an inadequate specification of the investment process. In particular, the fact that an investment decision or appropriation gives rise to a time pattern of expenditures until the project is completed was not adequately taken into account.

On the basis of businessmen's responses to questions about the effect of the rate of interest, the questionnaire and interview studies have typically concluded that the cost of capital does not have much effect on investment. 3/ Many of these studies have been surveyed and criticized by White, 4/ who concludes that the defects in the surveys do not permit definite conclusions to be drawn. A major problem with many of the surveys is that the negative response to questions about the impact of interest rates simply reflects the fact that, in the period prior to each survey, fluctuations in interest rates have not been large enough to warrant them being uppermost in the minds of businessmen.

This objection does not apply to the recent survey study of the effects of monetary policy in Canada carried out by Young and Helliwell. 5/ This study was conducted in 1962-63, following upon three periods in which restrictive monetary policy was applied with vigour, with intervening periods of monetary expansion. Moreover, the design and execution of this study involved follow-up interviews as well as carefully worded questionnaires. Since Young and Helliwell reach the same negative conclusions as had most previous survey studies, a detailed discussion of their findings is presented in Section I below.

It should also be noted that the survey results may have more relevance for the short-run responses of business firms to changes in interest rates. However, such a result is consistent with a more substantial long-run effect if the longer range planning of business firms within which the year-to-year investment plans are made is influenced by cost of capital variables. For example, the expected "normal" interest rate and the various features of the tax structure may determine in part a "required rate of return" which is used as a guide to shorter term planning. Large changes in these financial variables could in the long run influence the required rate of return, although for any particular investment decision the firm may base its choice on a simple comparison of expected rate of return with the required rate of return.

The findings of recent time series studies which involve a more adequate specification of the lag structure of the investment process should be given greater weight than the earlier time series results. It is noteworthy that these studies have all detected a substantial investment response to changes in cost of capital variables. Before turning to a survey of these studies, however, we want to emphasize one aspect of the earlier results that has been somewhat neglected.

Many of the earlier studies, especially the cross section studies, found internal funds to be an important determinant of investment. 6/ Some of the recent studies have also incorporated internal funds variables. While often given prominence as an alternative to "pure accelerator" theories, these results can be interpreted within a more traditional framework. It is reasonable to suppose that the volume of internal funds relative to the volume of investment expenditures is related to the marginal cost of capital. 7/ Consequently, there is no need to suppose that the significance of the internal funds variables reflect self-imposed constraints upon the use of borrowed funds or funds obtained from new equity issues. The behaviour of a utility maximizing firm is consistent with these results, once the hidden cost of the increase in risk, brought about by increased borrowing, is taken into account. 8/ We therefore consider the significance of internal funds variables as evidence consistent with the view that the cost of capital is an important determinant of investment. 9/

Some of the more important recent quarterly time series studies of investment in the United States and Canada are briefly surveyed in the next section.

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1/ J. Tinbergen, Statistical Testing of Business Cycle Theories, Vol. 1: A Method and Its Application to Investment Activity, Geneva, League of Nations, 1939.

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3/ Less well known survey studies have reached opposite conclusions. See C. O. Hardy and J. Viner, Report on the Availability of Bank Credit in the Seventh Federal Reserve District, U.S. Government Printing Office, 1935. (Especially Appendix C.) See also the more recent study of withdrawals of security offerings by C. H. Schmidt ("Unsuccessful Post-War Security Financing") in Volume and Stability of Private Investment, U.S. Government Printing Office, 1950, p. 650.

4/ W. H. White, "Interest Inelasticity of Investment Demand: The Case from Business Attitude Surveys Re-examined", American Economic Review, Sept. 1956, pp. 565-587.

5/ J. H. Young and J. F. Helliwell, The Effects of Monetary Policy on Corporations, a study prepared for the Royal Commission on Banking and Finance, Ottawa, Queen's Printer, 1965.

6/ See especially J. R. Meyer and E. Kuh, The Investment Decision, Harvard University Press, 1957.

7/ See J. S. Duesenberry; Business Cycles and Economic Growth, McGraw-Hill, 1958, Ch. 5.

8/ Given that issue costs and tax disadvantages largely preclude reliance upon new stock issues.

9/ Furthermore, in time series equations which incorporate both internal funds and accelerator variables, the partial regression coefficient on retained earnings largely reflects the significance of changes in price-cost margins and changes in the tax treatment of profits.

C. SURVEY OF RECENT QUARTERLY TIME SERIES STUDIES

Studies of Investment in the United States

1. de Leeuw 1/

In his study of investment in United States manufacturing, de Leeuw experiments in detail with a variety of distributed lag relationships. He uses a model in which investment is determined by capacity requirements, internal funds, and the rate of interest. In his "best" equation, all three variables are statistically significant and quantitatively important. This equation predicts that an increase in gross retained earnings of $1 would raise investment demand by $1.27, and that a reduction in the rate of interest of one percentage point would lead to an increase in investment demand of $4,892 million; de Leeuw's results are, therefore, consistent with the hypothesis that the cost of capital is a determinant of the level of investment.

While he does not introduce tax structures variables explicitly into the analysis, de Leeuw's coefficient on the internal funds variables suggests that tax changes have been important. Because gross profits before taxes are related to the rate of utilization of capacity, 2/ changes in gross profits after taxes at a given level of utilization are largely determined by changes in tax rates, allowable capital consumption rates and other features of the structure of business income taxation. Changes in gross retentions reflect both these tax structure changes and changes in dividend payments. However, the latter are determined in turn as a distributed lag function of past profits. 3/ Consequently, the significance of the retained earnings variable suggests that these variables have an important influence upon investment.

2. Eckstein

In a recent note 4/ Eckstein has extended de Leeuw's results by introducing the change in unfilled orders into the analysis as an expectational variable. From a statistical standpoint, this results in some improvement.

For our purposes it is noteworthy that the significance of the rate of interest is unaltered by this improved specification. While the internal funds coefficient is reduced somewhat, it remains significant, and, as Eckstein points out, it represents "a long run marginal propensity to invest which is more acceptable on theoretical grounds". 5/ It is therefore clear that Eckstein's refinement of de Leeuw's model does not weaken de Leeuw's findings on the importance of the rate of interest.

3. Jorgenson

Jorgenson has developed a model of investment which is derived from the neo-classical theory of the firm but which, like de Leeuw's, takes into account the lag structure of the investment process. Furthermore, Jorgenson has incorporated several features of the United States corporate tax structure into this model.

The results of the first application of this model to the United States manufacturing sector were published in 1963. 6/ The elasticities of investment with respect to interest rates and tax structure variables presented there indicate that these variables have a substantial effect upon long-run investment demand. For example, a one percentage point rise in the rate of interest (which is a rise of about 25 per cent) would reduce investment by 9.5 per cent. A reduction in tax rates of 5 percentage points (or about 10 per cent) would raise investment by 5 per cent. 7/

In research in connection with the Brookings econometric model 8/ Jorgenson applied his investment model to several sectors of the United States economy. The results obtained confirm the proposition that the cost of capital is an important determinant of investment. 9/

4. Griliches and Wallace

Griliches and Wallace 10/ have applied a variant of the Grunfeld model to aggregative quarterly data for United States manufacturing. Of the various models fitted, the following one is selected:

$$NI_t = \underset{(.00324)}{.00879} V_{t-2} - \underset{(.1959)}{.8292} r_{t-2} + \underset{(.0027)}{.0123} O_{t-2} + \underset{(.0373)}{.7504} NI_{t-1}$$

$$R^2 = .937$$

$$Sest = .109$$

where

V_t is an index of current stock prices

r_t is the rate of interest

O_t is an index of manufacturing production

NI_t is net investment.

This model differs from the model estimated by Jorgenson in three ways:

a) A stock price variable is included which captures the essence of the original Grunfeld model. 11/

b) Whereas Jorgenson uses a composite exogenous variable which reflects both accelerator and cost of capital effects, Griliches and Wallace examine the role of these variables separately.

c) As Griliches and Wallace have shown, Jorgenson's model may be derived from their own by a first difference transformation.

As the authors have noted, the significance of Jorgenson's composite variable does not necessarily indicate that the cost of capital is itself important. Their finding that the partial regression coefficient for the rate of interest is statistically significant therefore provides additional support for the hypothesis that the cost of capital influences investment.

Let us now turn to a comparison of the distributed lag pattern implicit in the Jorgenson and Griliches-Wallace models. As is noted above, the lag structure of Jorgenson's estimating equation can be obtained by a first difference transformation of the Griliches-Wallace estimating equation. This has the implication that the correlogram of the true errors in the two models must necessarily be different. Ordinary least squares (OLS) will yield unbiased and efficient estimates only if the autocorrelation of the true errors is zero. This implies that if OLS yields unbiased estimates for the Griliches-Wallace model, it would yield biased estimates for the Jorgenson model and vice versa.

Hence a comparison of the Griliches-Wallace and Jorgenson results will reveal how sensitive are the estimated lag patterns to an autoregressive transformation of the model.

Table C-1 below provides the relevant comparison.

TABLE C-1

COMPARISON OF LAG PATTERN OF GRILICHES-WALLACE AND JORGENSON MODELS
(Net Investment Only)

	Griliches-Wallace	Jorgenson
1. Estimated Coefficients of Lagged Dependent Variables*		
First Lag	1.725	1.524
Second Lag	-.750	-.631
Sum	.975	.893
2. Per cent of Total Response Achieved by:		
2 quarters	2.5	10.1
4 quarters	12.4	44.4
6 quarters	25.6	76.6

* These are the estimates of the coefficients γ_1 and γ in the model:

$$I_t = \alpha + \beta \Delta X_t + \gamma_1 NI_{t-1} + \gamma_2 NI_{t-2}$$

This is the form of the equation as estimated by Jorgenson. For the Griliches-Wallace model, these estimates are derived via the first difference transformation.

Source: Griliches and Wallace, op. cit., pp. 320-321.

The sum of the coefficients of the lagged dependent variables are somewhat smaller in the Jorgenson model. However, as is shown in the second part of the table, these moderate differences between the estimated coefficients of the two models give rise to very different dynamic response patterns. 12/ For example, the Jorgenson estimates imply that 44 per cent of the ultimate response is attained 4 quarters after a change in investment determinants, whereas the Griliches-Wallace model implies that only 12.4 per cent of the ultimate response would be attained over such a period.

As Griliches and Wallace point out, these results suggest that when lag patterns are estimated from models with lagged dependent variables, the resulting dynamic response patterns are quite sensitive to the autocorrelation properties of the true errors.

5. Resek 13/

Resek uses the distributed lag weights estimated by Almon 14/ in the specification of a variety of empirical models. These models are fitted for individual industries as well as for all United States manufacturing. The results, while somewhat mixed, suggest that typically both of the cost of capital variables used by Resek (the rate of interest, and a variable based on the debt/asset ratio adjusted for retained earnings to take into account the risk cost of debt) are important.

Both the interest rate and the debt variable are statistically significant and of the expected sign for the majority of the industries, regardless of the particular variant of equation used. 15/ These results at the individual industry level therefore provide additional support for the hypothesis that investment is sensitive to the cost of funds.

6. Anderson 16/

In a recent monograph, Anderson has paid particular attention to the role of financial variables and interest rates in the determination of investment. While the examination of the lag structure is not as thorough as that of the previous studies already discussed, he did experiment with a variety of "decision lags" (i.e., the lag between the change in the causal factors and the making of the appropriation decision) and did make allowance for the lag between appropriations and expenditures. 17/ Regression equations applied to individual industries and to aggregate U.S. manufacturing indicate that the balance sheet position of the firm has an important influence on investment decisions. Anderson's conclusion in this respect is well worth quoting:

> A high rate of investment in fixed capital and current non-financial assets relative to retained earnings gradually creates conditions which make it difficult to justify the maintenance of this high rate, even if the investment is not successful in eliminating the capacity shortages which originally prompted it. 18/

He also finds that interest rates are of importance:

> Contrary to the findings of some other investigators, we show substantial responsiveness of borrowing and investment expenditure to changes in interest rates. 19/

However, the pattern of response to interest rate changes is complicated, and firms appear to react with a considerable lag.

The overall conclusion reached by Anderson is that both marginal rates of return which are largely determined by the state of demand as reflected in utilization rates, and the marginal cost of funds which includes both the market cost and the imputed risk cost of borrowing are important determinants of investment.

7. Meyer and Glauber

The Meyer-Glauber study Investment Decisions, Economic Forecasting and Public Policy 20/ is in part an extension and re-estimation of the cross-section work presented earlier in the pioneering study by Meyer and Kuh, but also includes the development of a "bifurcated" model for the analysis of investment in a time series context. The latter involves fitting different models to the upswing and downswing portions of the business cycle, and is the more interesting part of the study for our purposes. 21/ The results obtained indicate that during downswings, when the pressure of demand on capacity is weak, investment is reasonably well predicted by a distributed lag function of gross retentions and interest rates. During upswings, on the other hand, investment is explained by a distributed lag function of capacity requirements, stock prices, and interest rates. The interest rate variable is more important during upswings when firms need to rely more heavily upon borrowing than during downswings, but is statistically significant in both cases.

While Meyer and Glauber's specification of the lag structure of the investment process is simpler than those of de Leeuw and Jorgenson, the bifurcation of the time series analysis itself represents an improved specification of the structure of the investment process, because many of the causal variables have long been recognized to have asymmetrical effects. Their finding that interest rates and stock prices affect investments during booms and that interest rates and gross retentions determine investment during periods of slack demand therefore provides additional support of the hypothesis that the cost of capital influences the level of investment.

8. Bischoff

An important but as yet unpublished paper by Bischoff 22/ presents a model of investment behavior based upon flexible *ex ante* but fixed *ex post* capital output ratios. This model, like that of de Leeuw, has the interesting property that the time pattern of investment response to changes in the cost of capital differs from the time pattern in response to a change in output.

Bischoff uses the same cost of capital concept as Jorgenson, but experiments with alternative formulations of the discount rate. He finds that the cost of capital has a statistically a significant effect on investment. The final equation, which is estimated by an iterative procedure, has investment elasticities with respect to bond yields and dividend yields on equities of respectively -.21 and -.08. Combining these two elasticities yields an over-all elasticity of .29 with respect to a parallel relative increase in both yields, somewhat lower than the elasticity obtained by Jorgenson. However, Bischoff applies his model to investment in producers durable equipment, rather than total investment. The lower cost of capital elasticity is therefore not unreasonable, given the shorter life of equipment relative to plan.

9. Kareken and Solow

In their study of the lags in response to monetary policy, Kareken and Solow 23/ present an equation explaining new orders for machinery. Since these new orders are likely to be closely related to new capital appropriations for equipment, this obviates the need to specify the appropriation-expenditure lag. Hence the Kareken-Solow results provide a different kind of test of the hypothesis that the cost of capital is important.

Their findings provide support for the hypothesis. In all equations estimated, the rate of interest has a statistically significant negative impact on investment. The quantitative importance of this variable is indicated by the elasticity of -.40 obtained in their equation of best fit. 24/

Studies of Investment in Canada

A number of studies 25/ have appeared in recent years which include quantitative time series analyses of business fixed investment in Canada. In contrast to the findings of the recent studies of United States data, these analyses of investment in Canada do not appear to provide much support for the hypothesis that the cost of capital is a significant determinant of investment. Of the seven studies cited in reference 25/, only two 26/ find that cost of capital variables are significant.

However, two 27/ of the studies make no attempt to examine the role of the cost of capital; and in three 28/ studies the equations are based on annual data, with consequent misspecification of the lag structure. Of the remaining studies, Rhomberg finds the rate of interest to be significant, whereas Johnson and Winder do not. We shall briefly examine each of these studies.

10. Rhomberg

In developing a quarterly econometric model of the Canadian economy, Rhomberg estimates two equations explaining non-residential construction and machinery and equipment expenditures. He obtains statistically significant interest rate coefficients in both equations.

The estimated coefficients indicate, moreover, that interest rates have an important impact on aggregate fixed investment. An increase of one percentage point in the yield on government bonds leads to a reduction in investment of 204 million (1949) dollars per quarter, a substantial impact.

In contrast to the studies previously discussed, Rhomberg specifies a finite lag structure. However, he indicates that the significance of the interest rate coefficient does not depend on the particular finite lag selected. 29/

11. Johnson and Winder

The Johnson-Winder study of the lags in monetary policy includes regression equations for each of the two components of business fixed investment. 30/ In these equations, the independent variables are assumed to affect investment with a common distributed lag, which is incorporated into the investment model via the inclusion of one or two lagged values of the dependent variable on the right-hand side.

While this specification of the lag structure is superior to that of Rhomberg, the estimation technique used may account for the negative results. If the true error terms in their estimating equation are positively autocorrelated, the sum of the coefficients on the lagged dependent variable will be biased upward, and the coefficients on the other independent variables will tend to be biased toward zero. 31/ This phenomenon would help to account for the lack of statistical significance of the interest rate coefficients despite the fact that, for non-residential construction at least, the fitted equations imply a fairly large long term interest elasticity. 32/

Summary

In contrast to the findings of both the earlier econometric studies and the survey studies, recent econometric time series studies of quarterly investment in the United States have consistently detected a significant cost of capital effect. What distinguishes these time series studies from their predecessors is the specification of the lag structure. This, together with the increased richness of the basic data resulting from the vigorous use of monetary policy in the last decade, probably explains why the recent studies find significant interest rate coefficients.

The two recent studies of Canadian quarterly investment yield contradictory results. However, the positive results of the Rhomberg study are based on a finite rather than a distributed lag structure, and the negative results of the Johnson-Winder study may be a result of biases in the statistical estimates. We therefore feel that it is useful to construct and estimate an alternative model which incorporates a distributed lag relationship between investment and its determinants, but which avoids these statistical estimation problems. The selection and estimation of this model are described in the next two sections.

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3/ John Lintner, "Distribution of Incomes of Corporations Among Dividends, Returned Earnings, and Taxes", American Economic Review, May 1956, pp. 7-113. The more recent work of Brittain suggests that dividend payout ratios are affected by various elements of the tax structure as well. (John A. Brittain, Corporate Dividend Policy, The Brookings Institution, 1966.)

4/ Otto Eckstein, "Manufacturing Investment and Business Expectations: Extensions of de Leeuw's Results", Econometrica, April 1965, pp. 420-424.

5/ Ibid., p. 421.

6/ Dale W. Jorgenson, "Capital Theory and Investment Behavior", American Economic Review, May 1963, pp. 247-259.

7/ These estimates are the long term elasticities at the end of the estimation period reported by Jorgenson (op. cit., Table 3, p. 258).

8/ "Anticipations and Investment Behavior", in J. S. Duesenberry et al., The Brookings Quarterly Model of the United States, pp. 34-92.

9/ A word of caution is in order. In the empirical implementation of the model, Jorgenson estimates a distributed lag function in the changes in a composite variable $\frac{PX}{c}$, where X is output and $\frac{P}{c}$ is the ratio of the price of output to the user cost of capital. He therefore does not test whether the partial effect of $\frac{P}{c}$ is statistically significant. (See also the discussion of the paper by Griliches and Wallace below.)

10/ Z. Griliches and N. Wallace, "The Determinants of Investment Revisited", International Economic Review, September 1965, pp. 311-329.

11/ Y. Grunfeld, "The Determinants of Corporate Investment", in A. C. Harberger, ed., The Demand for Durable Goods, University of Chicago Press, 1960.

12/ As the specifications of the independent variables differ between the two models, this may also affect the estimated lag pattern.

13/ Robert W. Resek, "Investment in Manufacturing Firms: A Quarterly Time Series Analysis of Industry Data", Review of Economics and Statistics, August 1966, pp. 322-333.

14/ Shirley Almon, "The Distributed Lag Between Capital Appropriations and Expenditures", Econometrica, January 1965, pp. 178-196.

15/ Resek fitted three variants involving different specifications of the output variables and, in one case, the introduction of stock market prices.

16/ W. H. Locke Anderson, Corporate Finance and Fixed Investment, Graduate School of Business Administration, Harvard University, 1964.

17/ The lag pattern selected, however, is simpler and of shorter duration than that derived by Almon in her investigation of the relationships between capital appropriations and investment expenditures.

18/ Ibid., p. 118.

19/ Ibid., p. 121.

20/ John R. Meyer and Robert R. Glauber, Investment Decisions, Economic Forecasting and Public Policy, Graduate School of Business Administration, Harvard University, 1964.

21/ The authors indicate that the cross section results are somewhat mixed. In two years a capacity formulation of the acceleration theory works well, in one year a retained funds model works well, and in one year none of the models works well. Of course, it is impossible to examine the influence of the rate of interest in cross section analyses of this type.

22/ Charles W. Bischoff, "Elasticities of Substitution, Capital Malleability and Distributed Lag Investment Functions", a paper presented at the San Francisco Meetings of the Econometric Society, December 28, 1966. (Mimeo).

23/ John Kareken and Robert M. Solow, "Lags in Monetary Policy" in Commission on Money and Credit, Stabilization Policies, Prentice-Hall, 1963, pp. 14-96.

24/ It is also interesting to note that they achieve more meaningful results by ignoring the capacity term and simply using output as a measure of capacity requirements. Our own results are similar in this respect (see Section E below) especially Tables E-2 to E-6.

25/ H. G. Johnson and J. Winder, Lags in the Effects of Monetary Policy in Canada, a study prepared for the Royal Commission on Banking and Finance, Ottawa, Queen's Printer, 1962.

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Lawrence Officer, An Econometric Model of Canada Under the Fluctuating Exchange Rate, unpublished Ph.D. thesis, Harvard University, 1964; Raymond Lupitz, United States Direct Investment in Canada and Canadian Capital Formation, unpublished Ph.D. thesis, Harvard University, 1966.

See also the companion study by Wilson and Lithwick, op. cit., Ch. V.

26/ Rhomberg finds a significant interest rate effect; May includes stock price changes in the investment equation of Model XIV.

27/ Officer introduces a measure of bank credit availability instead of interest rates or other measures of the cost of capital, into each function, which are then estimated by a variety of estimation techniques. In none of the estimated equations is this variable significant (Officer, op. cit.). As noted in reference 3/ above, Lupitz makes no allowance for any interest rate effect.

28/ Lithwick, Post and Rymes, op. cit., May, op. cit., Wilson and Lithwick, op. cit.

29/ Rhomberg, op. cit., p. 10.

30/ Johnson and Winder, op. cit., pp. 208-220.

31/ Assuming that the independent variables are themselves positively autocorrelated.

32/ See the equation reported on p. 217 and the elasticities reported on p. 219 (ibid.).

D. THE BASIC MODEL

The important findings obtained in the United States under this new approach to investment suggest that it is worth applying such models to Canadian data. Work at the industry level is unfortunately precluded by the unavailability of quarterly investment data. Even at the aggregative level, where quarterly data are available, the reliability of the estimates is not above suspicion, as they are based on interpolative techniques. We decided to plunge ahead with the analysis at this level. Subsequent tests of the reliability of quarterly investment series, reported in Appendix A, show that systematic timing biases in the quarterly data are unimportant.

Specification of the Lag Structure

The determination of the lag structure between appropriation and expenditure is an important problem, but one that can be distinguished from other problems in the selection and estimation of an investment model. There are two possible approaches to the specification of this lag structure. Alternative lag structures may be specified prior to the statistical estimation of the models, or the estimation of the lag structure can be incorporated into the statistical estimation procedure.

A common variant of the latter approach is to include one or more lagged values of the dependent variable in the estimating equation. Of the studies previously discussed, this approach is used by Jorgenson, Griliches and Wallace, Meyer and Glauber, and Johnson and Winder. The danger in this procedure is that the estimates of the lag patterns obtained are quite sensitive to the autocorrelation properties of the true errors in the estimating equation. This danger is illustrated by the comparison of the lag patterns obtained by Jorgenson with those obtained by Griliches and Wallace discussed in the previous section.

As the estimating equations are typically derived by transformations of the postulated behavioral relationships, one must allow for the possibility that the true error has non-zero autocorrelation. Consequently, we have decided not to rely on this approach. Almon 1/ has developed a procedure for estimating lag structures which does not involve the use of lagged endogenous variables in the estimating process. She has applied this technique to an analysis of the distributed lag between capital appropriations and capital expenditures in United States manufacturing, obtaining results which indicate that the average lag is much shorter than that estimated by Jorgenson or by Griliches and Wallace.

Rather than use Almon's estimating procedure to derive lag structures in equations predicting investment from its determinants, we have chosen instead to use her estimated lag structure in the alternative approach of pre-specifying the lag structure. This approach recommends itself because the lag structure estimated by Almon is based on a model relating expenditures directly to appropriations. Consequently, the lag structure so obtained will represent what has been called the appropriations-expenditure lag—which is important in the specification of the investment model used—rather than a mixture of the appropriations-expenditure lag with the decision lag, or the delay between a change in an investment determinant and the appropriation decision made in response to it. 2/

It is not clear how appropriate for Canada are the weights for the lag structure obtained by Almon. Her analysis applies to United States manufacturing; the appropriation-expenditure lag is likely to be longer for the transportation, communications, and public utility industries, and shorter for agriculture and services. As the former sectors account for a substantial fraction of total investment, the mean lag between aggregate expenditures and appropriations in Canada may be somewhat longer than Almon's results indicate.

Consequently, a second lag structure is specified, an inverted "V" over twelve quarters. This is the lag structure which yields best results in de Leeuw's study. 3/

As mentioned previously, these lag structures imply a time pattern of response shorter than that obtained by Jorgenson, and much shorter than that obtained by Griliches and Wallace. The Almon lag pattern implies that the expenditure response to a change in appropriations is exhausted over an eight quarter period, with over one-half of the response occurring within one year following the appropriation. The de Leeuw pattern implies that about one quarter of the response occurs within one year and that the total response is completed within three years. To obtain some evidence on the validity of these patterns which is independent of the investment equations to be estimated, an analysis of the annual investment forecasts published in Private and Public Investment in Canada (PPIC) 4/ is carried out.

On the basis of a survey carried out each November, PPIC compiles investment intentions for the coming year. These intentions are aggregated by industry and by major sector to form the investment forecasts which are examined below. If these forecasts are not very accurate, this could suggest that a significant portion of the investment made during the year was not already in the backlog of unexpended appropriation at the start of the year, provided that the investment survey yields reliable information. On the other hand, accurate forecasts do not necessarily mean that a long lag structure is appropriate, since firms may be able to predict successfully their new capital appropriations for the coming year.

Tables D-1 and D-2 present an analysis of the forecasts of construction and machinery and equipment expenditures for selected major sectors and for industries within manufacturing. The results at the industry level support the hypothesis that the appropriation-expenditure lag is not very long—

TABLE D-1

FORECASTING FUNCTIONS FITTED TO SELECTED MAJOR SECTORS (1949-61)

Sector	$\bar{R}^2$ for Machinery + Equipment Forecast	$\bar{R}^2$ for Non-Residential Construction
Mining	.54	.92
Utilities	.81	.93
Trade	.14	.92
Fire	.06	.44
Comm. Services	.00	.38
Inst. Services	.00	.39
Manufacturing	.86	.83

Notes: 1. The $\bar{R}^2$ refer to forecasting functions predicting the change in investment from the predicted change and the change in the relevant prices of capital goods. The latter variable was included only if it improved the corrected $\bar{R}^2$.

2. Agriculture, fishing, and forestry sectors are omitted because forecasts are not based on surveys in these sectors. Public and Private Investment in Canada, 1946-1957, p. 7. Housing expenditures and capital expenditures of government departments are also excluded as they are not relevant for the analysis.

3. Manufacturing and mining figures are adjusted to include natural gas processing plants in the latter industry.

TABLE D-2

FORECASTING FUNCTIONS FITTED TO INDUSTRIES WITHIN MANUFACTURING (1949-61)

Industry	$\bar{R}^2$ for Machinery + Equipment	$\bar{R}^2$ for Non-Residential Construction
Food + Beverages	.33	.54
Textiles + Clothing	.32	.00
Wood Products	.01	.07
Paper Products	.12	.80
Printing	.20	.71
Trans. Equip.	.26	.79
Elect. Prods.	.15	.74
Non Metallic Minerals	.82	.78
Petroleum + Coal Prods.	.14	.51
Chemicals	.84	.83
Metals + Machinery	.80	.71
Misc.	.15	.14

Note: 1. See Note to Table 1 above.

2. Figures for petroleum and coal products industry are adjusted to exclude natural gas processing plants.

especially for machinery and equipment. For example, of the 11 manufacturing industries for which estimates are made, in only 3 are the R^2's of the forecasting equation for machinery and equipment over .50. As would be expected, the forecasts of non-residential construction do much better—in 9 of the 11 industries over 50 per cent of the actual change was explained by the forecasting functions. 5/

At the major sector level, the forecasts' performances are much improved, especially for construction. Forecasts of changes in these expenditures for the mining, utilities and trade sectors account for about 92 per cent of the variance of changes in construction expenditures; forecasts for the manufacturing sector account for 83 per cent of the variance. While the performance of the forecasts of machinery and equipment expenditures are also improved, in only two sectors is over 80 per cent of the variance explained by the forecasts.

Aggregation for all sectors 6/ improves matters further; the forecasts account for 92 and 77 per cent of the variance of the two types of capital expenditure, as is apparent from the first two equations represented in Table D-3. Finally, when machinery and equipment expenditures are grouped with construction expenditures, 92 per cent of the variance of this aggregate is accounted for by the forecasting equation. This improvement which results from aggregation indicates that the errors reported at the more disaggregated levels cancel out to a large extent. The standard error of estimate for the simple forecasting equation is 4.3 per cent and that of the equation incorporating capital goods price changes is 3.5 per cent.

To determine whether the differences between forecast and actual investment represent mere reporting noise, two realization functions are estimated. These equations relate these differences to changes in the

TABLE D-3

AGGREGATE INVESTMENT FORECASTING FUNCTIONS
(1948-63)

1. **Simple Forecasting Function**

$$\frac{I_t}{I_{t-1}} = -.2341 + \underset{(11.13)}{1.23} \frac{I_t^F}{I_{t-1}^P}$$

$$\bar{R}^2 = .88 \qquad \text{Sest} = .043 \qquad \text{D.W.} = 2.05$$

2. **Forecasts Modified for Capital Goods Price Changes**

$$\frac{I_t}{I_{t-1}} = .9727 + \underset{(9.01)}{1.03} \frac{I_t^F}{I_{t-1}^P} + \underset{(2.84)}{.92} \frac{P_t}{P_{t-1}}$$

$$\bar{R}^2 = .92 \qquad \text{Sest} = .035 \qquad \text{D.W.} = 2.65$$

3. **Forecasts of Non-Residential Construction and Machinery and Equipment Considered Separately**

$$\frac{I_t^C}{I_{t-1}^C} = -1.14 + \underset{(8.78)}{.97} \frac{I_t^{CF}}{I_{t-1}^{CP}} + \underset{(3.32)}{1.13} \frac{P_t^C}{P_{t-1}^C}$$

$$\bar{R}^2 = .92 \qquad \text{Sest} = .038 \qquad \text{D.W.} = 2.11$$

$$\frac{I_t^M}{I_{t-1}^M} = -.95 + \underset{(5.38)}{.90} \frac{I_t^{MF}}{I_{t-1}^{MP}} + \underset{(2.10)}{1.04} \frac{P_t^M}{P_{t-1}^M}$$

$$\bar{R}^2 = .77 \qquad \text{Sest} = .06 \qquad \text{D.W.} = 2.34$$

Notation for Tables D-3 and D-4.

$I_{(t)}$ - Investment (in current dollars).

I_t^P - Preliminary estimate of Investment.

I_t^F - Forecast value of Investment..

I_t^c , I_t^{cp} and I_t^{cF} are respectively actual, preliminary actual, and forecast values of non-residential construction.

I_t^m , I_t^{mp} and I_t^{mF} are respectively actual, preliminary actual and forecast values of Machinery and Equipment.

P_t , P_t^c and P_t^m are implicit deflations for business fixed investment and its two components.

Ret - Gross Corporate Retentions.

GDP - Gross Domestic Product Index.

TABLE D-4

REALIZATION OF AGGREGATE INVESTMENT ANTICIPATIONS
(1948-63)

1. Realization Function

$$\frac{I_t}{I_{t-1}} - \frac{I_t^F}{I_{t-1}^P} = -1.96 + \underset{(6.13)}{.92}\frac{P_t}{P_{t-1}} + \underset{(1.21)}{.08}\frac{Ret_t}{Ret_{t-1}}$$

$$+ \underset{(3.57)}{.90}\frac{GDP_t}{GDP_{t-1}}$$

$\bar{R}^2 = .81$ Sest = .020   D.W. = 2.23

Rate of interest insignificant (Partial correlation = +.13)

2. Realization Function Expressed in Real Terms

$$\frac{I_t}{P_t} - \frac{I_t^F}{P_{t-1}} = -4987 + \underset{(1.99)}{571}\frac{Ret_t}{Ret_{t-1}} + \underset{(3.75)}{4210}\frac{GDP_t}{GDP_{t-1}}$$

Rate of interest insignificant (Partial correlation = -.14)

$\bar{R}^2 = .68$ Sest = 91.13   D.W. = 1.66

Sest as % of constant dollar ($49) investment in 1963 = 2.6%.

logical determinants of investment between the year preceding the forecast and the year of the forecast. These functions reveal that there is a systematic relationship between the realization of intentions and changes in output, retentions and capital goods prices. The estimated equations explain 70 to 80 per cent of the deviation between actual and forecast investment. Hence we must reject the hypothesis that these deviations represent unsystematic reporting noise.

On the basis of the results of these analyses, a good case can be made for experimentation with lag structures that allow for at least a moderate response in the year following an appropriation. The weakness of the forecasts at the industry level, particularly for machinery and equipment, suggests that a substantial portion of capital expenditures made in a year are not already in the backlog of unexpended appropriations at the start of the year. This conclusion receives additional support from the estimated realization functions, which show that there is a systematic relationship at the aggregate level between forecasting errors and the logical determinants of investment.

Let us now turn to the discussion of the quarterly investment functions themselves. In the remainder of this section, we discuss the selection of the models used. The estimation of the equations is discussed in Section E.

<u>Selection of the Investment Model</u>

The model to be estimated is a modified version of the model estimated by de Leeuw. This model is selected for the following reasons:

1. In modified form it can be estimated with the aggregate data available for the Canadian economy.

2. Its statistical estimation, provided that the lag structure is predetermined, presents no unusual difficulties.

3. It implies a time pattern of investment response to changes in interest rates quite different from the time pattern of response to output changes.

This last property is inherent in the specification of this model, which treats capital deepening investment and capacity expanding investment differently. The latter is assumed to have a depressing effect on future investment whereas the former is not. The theoretical rationale for this distinction has been provided by Bischoff, 7/ who derives an investment model from production relationships with flexible ex ante but fixed ex post capital: capacity ratios. In contrast, in the models of Jorgenson and Griliches and Wallace, the two types of investment are treated symmetrically, which is appropriate if capital in place is as substitutable for other inputs as is new capital.

A unique characteristic of the de Leeuw model is the specification of investment demand as the backlog of unexpended appropriations (or uncompleted projects). Firms are assumed to adjust the backlog in response to changes in the determinants of investment. The alternative, used in several of the studies cited, is to assume that firms make new capital appropriations in response to changes in these determinants.

The selection of the "backlog" approach rather than the "appropriation" approach has two important implications. First, the stochastic specification of the backlog model implies something akin to error correction behavior by firms.

This is readily demonstrated

$$I_t^d = B_t = X_t + U_t$$

where I_t^d is investment demand (assumed equal to the backlog of projects B_t)

X_t is the effect of the determinants of investment, and

U_t is a stochastic variable.

$$N_t = \Delta B_t + I_t$$

where N_t is new capital appropriation and I_t is investment expenditures.

$$N_t = \Delta X_t + I_t + U_t - U_{t-1}.$$

Since U_{t-1} has a negative sign in the equation explaining new appropriations, an extraordinary large appropriation in one period is "corrected" in the following period.

Whereas the model developed by Bischoff implies that factor proportions become fixed at the time of appropriation, the de Leeuw backlog model implies that factor proportions are fixed only for completed projects. Projects which are in the backlog of unspent appropriations may be altered in their capital intensity by subsequent appropriations or cancellations.

Which of these various specifications of what are essentially the dynamics of investment behavior is correct is an empirical question, but one which we shall not examine in this study. While the evidence presented in the remainder of this paper does indicate that the model we estimate has approximated historical reality quite well, this is of course no indication that another model could not have done better.

The remainder of this section presents a precise derivation of the basic model to be estimated, and of the alternative measures used for each of the conceptual variables.

Although we select the de Leeuw model as the basic framework for the equations, these equations are not a straightforward replication of de Leeuw's equations with Canadian data. Various modifications are made, partly because of data limitations, but partly in order to improve upon the specification and estimation of the model.

These modifications are as follows:

1. In addition to the inverted "V" lag structure (across 12 quarters) which yields best results in de Leeuw's work, the estimated lag structure obtained by Almon for total manufacturing in the United States is used.

2. Whereas de Leeuw assumes that there is no lag between the change in one of the determinants of investment and the resulting capital appropriation, it is reasonable to make allowance for such a "decision lag". In addition to models with no decision lag, two types of decision lag are tried: a simple finite decision lag of two quarters, and a moving average distributed lag across the current and preceding three quarters.

3. de Leeuw constructed a capacity requirements variable which incorporates the difference between output and capacity projected into the future as well as replacement investment requirements. The lack of comparable data in Canada forces us to rely on simpler measures: Output (of which two variants were tried, real gross

national expenditure and the gross domestic product index) and the (inverse) gap between actual and potential output.

4. We, like de Leeuw, also find significant serial correlation to be present in the residuals when direct least squares estimation of the model is carried out. This is not unexpected, since the estimating equations are derived from the underlying relationships via moving average and first difference transformations. As a result, most models are re-estimated after an appropriate auto-regressive transformation.

Derivation of the Investment Equations to be Estimated 8/

1. $B_t = \alpha (Q_t^* - C_{p_t}) + \beta Ret_t^* + \gamma r_t^* + U_t$

where B_t is the backlog of investment projects at the end of period t, Q_t, Cp_t, Ret_t and r_t are respectively Output, Capacity, Gross retentions and interest rates in period t, and U_t represents a stochastic variable. A * indicates that the variable may be subject to a decision lag.

2. $N_t = \Delta B_t + I_t$

where N_t is capital appropriations in period t and I_t is investment in period t.

3. $I_t = \sum_{i=0}^{L} W(i)\, N_{t-1} + \epsilon_t$

where W(i) represents the distributed lag structure linking investment to capital appropriations ($\sum_{i=0}^{L} W_{(i)} = 1$), and ϵ_t is another stochastic variable.

Substituting from 1 into 2 and the result into 3 we obtain

4. $$I_t - \sum_{i=0}^{L} W(i)\, I_{t-1}$$

$$= \alpha \sum_{i=0}^{L} W(i)(\Delta Q^* - \Delta C_p)_{t-1}$$

$$+ \beta \sum_{i-0}^{L} W_{(i)}\, (\Delta Ret^*)_{t-i}$$

$$\gamma \sum_{i=0}^{L} W_{(i)}(\Delta r^*)_{t-i} + V_t$$

where $V_t = \sum_{i=0}^{L} W(i)\, \Delta U_{t-i} + \epsilon_t$.

Since we assume that capacity is a linear trend, and in order to make allowance for non-linearity and omitted variables with non-zero means, the following modification of equation 4 is the actual equation fitted.

5. $$I_t - \sum_{i=0}^{L} (W_i) I_{t-i}$$

$$= K_o + \alpha \sum_{i=0}^{L} W_i\, \Delta Q^*_{t-i}$$

$$+ \beta \sum_{i=0}^{L} W_i\, \Delta\, Ret^*_{t-i}$$

$$+ \gamma \sum_{i=0}^{L} W_i\, \Delta r^*_{t-i}$$

$$+ V_t.$$

$$K_o = \Delta C_p^T + E, \text{ where}$$

ΔC_p^T is the trend change in the capacity, and E is an arbitrary constant to allow for non-linearities and omitted variables. Note that V_t is likely to be positively autocorrelated. Where the residuals of equation 5 reveal positive autocorrelation, all variables will be transformed as follows:

$X_t^a = X_t - pX_{t-1}$, where p is based on the Durbin-Watson coefficient of the estimated residuals. If necessary, further autoregressive transformations could be carried out in the event that serial correlation is present in the transformed relationship.

Variables Used

To explain the dependent variable, real expenditure on business fixed capital formation, we introduce alternative measures of output or capacity requirements, the cost of capital, and the availability of internal funds. The specific variables used for each conceptual variable are as follows:

1. Output or capacity requirements:

   1a. Output minus potential output. (A-P)

   1b. Gross national expenditure in constant prices. (Y)

   1c. Index of gross domestic product. (Q)

2. The cost of capital:

   2a. The rate of interest on corporate bonds. rc

   2b. The rate of interest on long term government bonds. rg

   2c. The rate of interest on corporate bonds adjusted for effective corporate tax rates and the relative prices of capital goods ($C_c = rc \cdot \frac{q}{p} \cdot (1 - T)$)
   where q is the implicit deflator for investment, p is the implicit deflator for gross national expenditure and T is the effective tax rate on corporate gross profits.

   2d. The long term rate of interest on government bonds adjusted for effective corporate tax rates on the relative price of capital goods. ($C_g = rg \cdot \frac{q}{p} \cdot (1-T)$)

The rationale for the first variable is obvious. The second variable is introduced because the first is based on a fairly thin sample of corporate bonds. Given the covariation of different long term interest rates, the rate of interest on government bonds may be a better measure (in terms of changes, not, of course, in terms of levels) of the relevant private bond rate.

The last two variables represent attempts to construct a measure which better reflects the cost of capital to the firm.

3. Availability of internal funds:

   3a. Corporate retentions plus corporate depreciation. (Ret)

   3b. Corporate retentions plus corporate depreciation minus investment. (Ret - I)

The first variable is a common garden variety measure of residual funds. The second is a refinement based on the notion that, in a distributed lag model, the excess of past cash flows over past investment spending may be a better measure of the availability of internal funds for investment this period.

It should be noted that errors of measurement are particularly large for these cash flow variables, as the cash flow of unincorporated enterprises and government-owned corporations is not included although these types of firms account for a substantial portion of total capital expenditures.

REFERENCES

1/ Shirley Almon, op. cit., pp. 178-181.

2/ The direct estimation of the appropriation expenditure lag in Canada is ruled out by the lack of data on capital appropriations.

3/ de Leeuw, op. cit., Table 11, p. 419.

4/ Department of Trade and Commerce, Private and Public Investment in Canada, Ottawa, Queen's Printer.

5/ It should be noted that the regression analysis used makes allowance for systematic underprediction or overprediction of change, as the constant term is not constrained to be zero. In addition, changes in the prices of the relevant capital goods are included as an additional explanatory variable when the corrected R^2 is increased as a result.

6/ This aggregate excludes housing expenditures and the capital expenditures of government departments.

7/ Bischoff, op. cit.

8/ This derivation follows that presented by de Leeuw (op. cit., pp. 408-510).

E. ESTIMATION OF ALTERNATIVE INVESTMENT EQUATIONS

The basic model is estimated in a variety of specifications involving:

a) Alternative measures of the three conceptual variables;

b) Two alternative specifications of the appropriation-expenditure lag; and

c) Three alternative specifications of the decision lag.

The results indicate that models incorporating some kind of decision lag are superior to models with no decision lag, and that the appropriation-expenditure lag pattern using the Almon weights over eight quarters are superior to the inverted "V" distribution across twelve quarters. This is demonstrated in Table E-1, which compares the results under the alternative lag specifications for a particular specification of the three variables.

In addition, the results show that the excess retention variable is generally statistically insignificant. We shall therefore limit the more detailed discussion to models with Almon weights, and with the retention variable specified as gross corporate retentions.

The results of direct estimation by least squares of the 48 equations are presented in Tables E-2 through E-4. Table E-2 presents results for models with a zero decision lag.

As is apparent, with this specification the cost of capital is typically insignificant and frequently has an incorrect sign. The retention variable is of marginal significance in most formulations. Only the capacity requirements variables and the seasonal dummies have a statistically significant impact.

TABLE E-1

COMPARISON OF MODELS WITH DIFFERENT LAG STRUCTURES

Appropriations Expenditure Lag Structure	Decision Lag	Dependent Variable	Regression Coefficients* (t Values in Parentheses)								Goodness of Fit & Autocorrelation Statistics		
			Constant	Seasonal Dummies Q_1	Q_2	Q_3	Output**	Gross Retentions	Corporate Bond Yields		R^2	Sest***	D.W.
Almon Weights	0	Raw	-200.35	-32.27 (1.05)	314.13 (8.69)	170.08 (6.39)	7.59 (6.07)	0.825 (0.83)	141.25 (1.24)		.868	63.40	0.82
Almon Weights	2	Raw	7.77	-315.42 (9.50)	-49.74 (1.33)	87.93 (3.07)	7.43 (5.57)	0.699 (0.67)	-494.52 (4.19)		.865	64.86	0.80
Almon Weights	4 Quarter Moving Average	S.A.A.R.	-350.59	--	--	--	8.353 (7.41)	1.853 (2.05)	-372.58 (3.76)		.803	192.65	0.54
12 Quarter Inverted V	0	Raw	-150.78	-303.93 (7.84)	141.64 (4.08)	269.30 (7.19)	11.35 (4.71)	6.618 (3.63)	-710.71 (3.69)		.892	75.78	0.74
12 Quarter Inverted V	4 Quarter Moving Average	S.A.A.R.	-98.96	--	--	--	4.42 (1.82)	8.725 (4.99)	-1384.46 (7.02)		.838	277.67	0.48

* Since the appropriations expenditure lags are different between the first 3 and last 2 equations presented, the magnitude of the coefficients is affected. The Almon weights pattern implies an equilibrium investment flow equal to .31 times the backlog. The 12 quarter inverted V implies an equilibrium flow that is .15 times the backlog.

** Output is measured by the GDP index, base 1949 = 1000.

*** For comparibility with the equations predicting seasonally adjusted dependent variable, Sest for the equations with raw dependent variables must be multiplied by 4. (The regression coefficients do not need to be adjusted as the independent variables for the equations predicting seasonally adjusted investment (at annual rates) are moving sums rather than moving averages.)

TABLE E-2

INVESTMENT MODELS WITH ALMON LAG STRUCTURE AND ZERO DECISION LAG

SEASONALLY ADJUSTED DEPENDENT VARIABLE

Equation No.	Constant	D_1	D_2	D_3	Capacity Requirements	Internal Funds	Cost of Capital	R^2	Sest	D.W.
1	43.28	-157.430 (-5.32)	138.883 (4.84)	135.189 (4.88)	0.776 PNF GAP (5.19)	0.742 RET (0.66)	174.369 r_c (1.44)	0.849	67.86	0.71
2	-186.77	-41.357 (-1.19)	299.837 (7.22)	172.160 (5.71)	1.157 GNE (4.63)	2.253 RET (2.25)	190.780 r_c (1.51)	0.836	70.78	0.84
3	-200.35	-32.274 (-1.05)	314.134 (8.69)	170.084 (6.39)	7.586 GDP (6.07)	0.825 RET (0.83)	141.245 r_c (1.24)	0.868	63.40	0.82
4	46.52	-159.912 (-5.47)	137.457 (4.84)	135.135 (4.91)	0.818 PNF GAP (5.92)	0.469 RET (0.45)	205.294 r_g (1.63)	0.851	67.42	0.71
5	-194.68	-33.575 (-0.94)	313.784 (7.32)	176.579 (5.70)	1.282 GNE (4.95)	1.754 RET (1.73)	44.262 r_g (0.30)	0.827	72.65	0.82
6	-210.45	-21.241 (-0.67)	333.075 (8.91)	175.308 (6.47)	8.593 GDP (6.52)	0.060 RET (0.06)	-69.301 r_g (-0.51)	0.864	64.38	0.79
7	33.44	-153.425 (-5.32)	141.390 (5.07)	133.668 (4.97)	0.713 PNF GAP (4.77)	1.232 RET (1.09)	163.585 C_c (2.17)	0.858	65.87	0.76
8	-175.01	-47.764 (-1.36)	287.939 (6.68)	167.505 (5.54)	1.054 GNE (3.88)	2.568 RET (2.43)	147.245 C_c (1.72)	0.838	70.23	0.83
9	-194.82	-34.455 (-1.09)	310.023 (8.05)	168.484 (6.22)	7.382 GDP (5.19)	0.932 RET (0.84)	82.168 C_c (1.02)	0.867	63.78	0.80
10	42.28	-157.646 (-5.48)	138.641 (4.96)	133.965 (4.95)	0.785 PNF GAP (5.69)	0.718 RET (0.69)	155.006 C_g (2.07)	0.856	66.20	0.73
11	-190.91	-36.083 (-1.00)	309.240 (6.98)	174.895 (5.60)	1.243 GNE (4.49)	1.874 RET (1.78)	45.668 C_g (0.49)	0.828	72.52	0.81
12	-215.27	-18.358 (-0.57)	338.315 (8.62)	177.044 (6.48)	8.861 GDP (6.07)	-0.112 RET (-0.10)	-59.146 C_g (-0.66)	0.865	64.24	0.80

TABLE E-3

INVESTMENT MODELS WITH ALMON LAG STRUCTURE AND A TWO QUARTER DECISION LAG

SEASONALLY ADJUSTED DEPENDENT VARIABLE

Equation No.	Constant	D_1	D_2	D_3	Capacity Requirements	Internal Funds	Cost of Capital	R^2	Sest	D.W.
1	32.90	-198.831 (-5.49)	104.485 (2.90)	142.378 (4.17)	0.352 GAP^{PNF} (1.90)	2.819 RET (2.03)	-316.127 r_c (-2.12)	0.778	83.17	0.61
2	7.30	-305.397 (-8.58)	-42.794 (-1.04)	90.997 (2.95)	1.175 GNE (4.68)	2.005 RET (2.02)	-451.964 r_c (-3.61)	0.845	69.55	0.62
3	7.77	-315.415 (-9.50)	-49.739 (-1.33)	87.934 (3.07)	7.428 GDP (5.57)	0.699 RET (0.67)	-494.523 r_c (-4.19)	0.865	64.86	0.80
4	18.48	-204.409 (-5.48)	99.111 (2.67)	144.496 (4.09)	0.234 GAP^{PNF} (1.31)	3.678 RET (2.76)	-207.033 r_g (-1.28)	0.762	86.04	0.59
5	8.69	-303.650 (-8.21)	-40.317 (-0.94)	93.440 (2.92)	1.149 GNE (4.37)	2.314 RET (2.28)	-456.339 r_g (-3.08)	0.833	72.06	0.60
6	12.45	-321.610 (-9.48)	-58.575 (-1.52)	85.810 (2.96)	7.875 GDP (5.60)	0.696 RET (0.66)	-572.283 r_g (-4.07)	0.862	65.45	0.80
7	18.93	-202.235 (-5.34)	100.548 (2.67)	142.990 (4.02)	0.280 GAP^{PNF} (1.41)	3.373 RET (2.26)	-101.772 C_c (-1.02)	0.759	86.68	0.58
8	2.70	-310.737 (-8.13)	-53.123 (-1.18)	86.046 (2.61)	1.259 GNE (4.35)	1.819 RET (1.64)	-259.593 C_c (-2.89)	0.829	72.90	0.57
9	5.27	-330.534 (-9.41)	-74.048 (-1.82)	76.693 (2.56)	8.679 GDP (5.58)	-0.061 RET (-0.05)	-334.168 C_c (-3.90)	0.859	66.26	0.77
10	9.07	-206.465 (-5.43)	97.063 (2.56)	144.388 (4.01)	0.204 GAP^{PNF} (1.11)	3.945 RET (2.85)	-41.000 C_g (-0.41)	0.753	87.64	0.59
11	1.96	-303.906 (-7.71)	-42.319 (-0.91)	91.752 (2.70)	1.157 GNE (3.91)	2.315 RET (2.09)	-226.531 C_g (-2.29)	0.817	75.43	0.57
12	7.10	-329.696 (-9.03)	-72.486 (-1.71)	78.729 (2.55)	8.554 GDP (5.22)	0.293 RET (0.24)	-337.762 C_g (-3.47)	0.850	68.28	0.75

TABLE E-4

INVESTMENT MODELS WITH ALMON LAG STRUCTURE AND FOUR QUARTER MOVING AVERAGE DECISION LAG

SEASONALLY ADJUSTED DEPENDENT VARIABLE

Equation No.	Constant	Capacity Requirements	Internal Funds	Cost of Capital	R^2	Sest	D.W.
1	151.28	0.468 GAP^{PNF} (2.91)	4.060 RET (3.27)	-187.243 r_c (-1.41)	0.618	268.31	0.40
2	-338.10	1.333 GNE (6.02)	3.453 RET (3.91)	-338.171 r_c (-3.07)	0.755	214.73	0.50
3	-350.59	8.353 GDP (7.41)	1.853 RET (2.05)	-372.575 r_c (-3.76)	0.803	192.65	0.54
4	93.36	0.389 GAP^{PNF} (2.53)	4.692 RET (3.92)	-41.997 r_g (-0.30)	0.601	274.42	0.41
5	-324.56	1.266 GNE (5.51)	3.753 RET (4.13)	-295.483 r_g (-2.34)	0.735	223.61	0.49
6	-345.20	8.518 GDP (7.22)	1.853 RET (1.99)	-403.066 r_g (-3.52)	0.797	195.77	0.55
7	96.69	0.403 GAP^{PNF} (2.39)	4.609 RET (3.54)	-25.145 C_c (-0.29)	0.600	274.45	0.41
8	-369.63	1.398 GNE (5.48)	3.332 RET (3.40)	-194.818 C_c (-2.44)	0.737	222.50	0.49
9	-396.76	9.271 GDP (7.12)	1.295 RET (1.27)	-252.933 C_c (-3.51)	0.796	195.93	0.53
10	63.65	0.365 GAP^{PNF} (2.35)	4.965 RET (4.10)	44.646 C_g (0.52)	0.602	273.81	0.42
11	-345.97	1.267 GNE (4.98)	3.811 RET (3.90)	-142.384 C_g (-1.70)	0.719	230.16	0.49
12	-381.16	8.959 GDP (6.66)	1.619 RET (1.57)	-236.499 C_g (-3.01)	0.783	202.13	0.54

When either a finite decision lag of two quarters or a four quarter moving average decision lag is incorporated into the model, the situation is changed markedly. (See Tables E-3 and E-4.) The rate of interest or cost of capital variables are now statistically significant in most formulations and have the correct sign in all except one. Capital requirements based directly on output variables are highly significant while the gap between actual and potential output is generally of weaker significance.

The retentions variable is also typically significant and always has the expected sign. However, the significance of both the cost of capital and of retentions appears to be affected by the choice of capacity requirements variable. The cost of capital variable is typically insignificant when capacity requirements are measured by actual minus potential PNF output; the retention variable is occasionally insignificant when capacity requirements are measured by the GDP output index.

The equation with the lowest standard error of estimate 1/ is equation E-4.3 in Table E-4. In this equation the coefficients for all three independent variables are significant with correct signs. However, as is typical of all the equations presented in these tables, the Durbin-Watson coefficient indicates that significant positive serial correlation is present in the residuals. This is not surprising, since the derivation of the estimating equation would introduce positive serial correlation in the absence of serial correlation in the errors of the postulated behavioral relationships. 2/

In order to obtain more efficient estimates and more reliable tests of the various hypotheses, the models are transformed using an autoregression coefficient suggested by the observed Durbin-Watson coefficients. As the latter are typically near 0.50, all variables are transformed as follows:

$$X^*_t = X_t - 0.75\, X_{t-1}$$

All equations involving either moving average or two quarter decision lags are re-estimated with the transformed variables. The results are tabulated in Tables E-5 and E-6. As is apparent, this transformation eliminates the positive serial correlation in the residuals. While the R^2 in each equation drops, the standard error of estimate drops as well, indicating that the predictive power of the models fitted to transformed data is superior to that of models fitted to original data.

The equation of best fit remains number 3 with a moving average decision lag. (Equation E-6.3 in Table E-6.) However, while the coefficients for the interest rate and capacity requirements remain statistically significant, the coefficients of the retentions variable is no longer significant in the transformed model. Looking across the set of equations fitted to transformed data with the moving average decision lag (Table E-6), it is clear that the significance of the retentions variable is sensitive to the choice of output or capacity requirements variable. Where capacity requirements are measured by the gap between actual and potential output, retentions tend to be statistically significant (and capacity requirements insignificant); where the capacity requirements variable is measured by either the gross domestic product index or by real gross national expenditure, the retentions variable is not statistically significant (and the capacity requirements variable is statistically significant). In contrast, the significance of the cost of capital is not

TABLE E-5

INVESTMENT MODELS WITH ALMON LAG STRUCTURE AND A TWO QUARTER DECISION LAG AFTER AUTOREGRESSIVE TRANSFORMATION
(P =0.75)

Equation No.	Constant	D_1	D_2	D_3	Capacity Requirements	Internal Funds	Cost of Capital	R^2	Sest	D.W.
1	-98.14	-84.155 (-3.08)	360.105 (14.52)	157.628 (4.81)	0.232 GAP^{PNF} (0.70)	2.093 RET (0.89)	-419.029 r_c (-1.68)	0.901	59.37	1.54
2	-54.390	-252.96 (-5.65)	235.155 (6.43)	172.508 (7.28)	1.383 GNE (4.16)	0.132 RET (0.09)	-559.786 r_c (-2.97)	0.931	49.53	1.65
3	-57.88	-245.822 (-4.68)	248.051 (6.12)	175.405 (6.95)	7.704 GDP (3.31)	-1.001 RET (-0.53)	-537.022 r_c (-2.67)	0.923	52.65	1.77
4	-105.33	-88.500 (-3.18)	360.633 (14.17)	168.537 (5.15)	0.066 GAP^{PNF} (0.20)	3.268 RET (1.44)	-269.444 r_g (-0.88)	0.896	60.91	1.46
5	-53.30	-260.317 (-5.49)	230.993 (6.03)	179.295 (7.31)	1.440 GNE (4.07)	0.645 RET (0.42)	-641.796 r_g (-2.53)	0.928	50.88	1.55
6	-52.34	-266.908 (-4.73)	234.494 (5.47)	183.371 (7.11)	8.710 GDP (3.44)	-0.882 RET (-0.46)	-687.335 r_g (-2.48)	0.921	53.21	1.70
7	-100.92	-85.230 (-3.05)	358.219 (14.15)	158.576 (4.65)	0.181 GAP^{PNF} (0.51)	2.485 RET (1.03)	-206.482 C_c (-1.16)	0.898	60.46	1.48
8	-50.85	-265.745 (-5.71)	221.695 (5.80)	169.946 (7.13)	1.493 GNE (4.28)	0.074 RET (0.05)	-381.102 C_c (-2.88)	0.931	49.83	1.60
9	-52.87	-263.510 (4.78)	231.763 (5.42)	173.362 (6.84)	8.591 GDP (3.47)	-1.307 RET (-0.68)	-372.750 C_c (-2.60)	0.922	52.86	1.74
10	-108.13	-89.795 (-3.19)	359.689 (14.00)	169.208 (5.07)	0.004 GAP^{PNF} (0.01)	3.469 RET (1.51)	-70.303 C_g (-0.36)	0.946	61.43	1.44
11	-51.88	-266.176 (-5.35)	222.844 (5.49)	175.403 (7.09)	1.490 GNE (3.98)	0.593 RET (0.38)	-380.256 C_g (-2.26)	0.926	51.62	1.50
12	-49.67	-276.916 (-4.59)	223.331 (4.82)	179.573 (6.92)	9.207 GDP (3.37)	-1.072 RET (-0.55)	-419.881 C_g (-2.25)	0.919	53.90	1.66

TABLE E-6

INVESTMENT MODELS WITH ALMON LAG STRUCTURE AND A FOUR QUARTER MOVING AVERAGE DECISION LAG AFTER AUTOREGRESSIVE TRANSFORMATIONS
(P =0.75)

SEASONALLY ADJUSTED DEPENDENT VARIABLE

Equation No.	Constant	Capacity Requirements	Internal Funds	Cost of Capital	R^2	Sest	D.W.
1	25.06	0.401 GAP^{PNF} (1.56)	4.221 RET (2.18)	-418.506 r_c (-2.19)	0.460	150.59	1.51
2	-85.57	1.324 GNE (3.65)	2.579 RET (1.67)	-498.934 r_c (-3.13)	0.570	134.36	1.82
3	-89.97	8.890 GDP (4.41)	0.435 RET (0.25)	-530.032 r_c (-3.52)	0.615	127.25	2.02
4	-3.50	0.194 GAP^{PNF} (0.80)	5.737 RET (3.10)	-226.932 r_g (-1.02)	0.411	157.35	1.43
5	-83.63	1.262 GNE (3.23)	3.345 RET (2.11)	-473.747 r_g (-2.25)	0.525	141.27	1.70
6	-92.70	9.379 GDP (4.22)	0.807 RET (0.45)	-602.601 r_g (-2.96)	0.586	131.90	1.92
7	5.85	0.301 GAP^{PNF} (1.10)	4.955 RET (2.48)	-170.442 C_c (-1.28)	0.419	156.23	1.43
8	-97.63	1.412 GNE (3.52)	2.530 RET (1.55)	-302.621 C_c (-2.61)	0.543	138.61	1.73
9	-106.60	10.028 GDP (4.47)	-0.111 RET (-0.06)	-358.631 C_c (-3.25)	0.601	129.52	1.97
10	-15.27	0.135 GAP^{PNF} (0.54)	5.943 RET (3.17)	-39.183 C_g (-0.28)	0.396	159.22	1.41
11	-90.185	1.247 GNE (2.93)	3.380 RET (2.05)	-235.734 C_g (-1.66)	0.499	145.02	1.63
12	-104.88	9.877 GDP (3.97)	0.552 RET (0.29)	-354.436 C_g (-2.51)	0.564	135.34	1.85

TABLE E-7

SIMPLE CORRELATION MATRIX FOR ALTERNATIVE MEASURES OF THE COST OF CAPITAL*

	r_c^*	r_g^*	C_c^*	C_g^*
r_c^*	1.00	.84	.97	.77
r_g^*		1.00	.89	.98
C_c^*			1.00	.87
C_g^*				1.00

* These are the variables used in the equation reported in Table E-6. They are obtained from the basic data by the following transformation:

$$X^* = \sum_{i=c}^{>} w(i) : \Delta \overline{X}_t - 0.75 \sum_{i=c}^{>} w(i) \Delta \overline{X}_{c-1}$$

where $\overline{X}_t$ is a four quarter moving sum of the relevant basic variable and $w(i)$ are the Almon weights.

The simple correlation matrix for the variables used in the equations reported in Table E-5 is very similar.

affected by the choice of capacity requirements variable in the equations fitted to transformed data.

As the capacity requirements variables and particularly the retention variables are crude measures at best, these results serve more to illustrate the difficulty of measuring the separate effects of these factors than to enable us to accept or reject a role for retained earnings in the determination of investment.

The cost of capital variable which generally achieves best results is the rate of interest on corporate bonds. However, neither the goodness of fit nor the statistical significance of the regression coefficients is affected much by which cost of capital variable is selected. For example, a comparison of equations 3, 6, 9, and 12 in Table E-6 reveals that the maximum difference between the R^2 is only 0.051.

This result reflects the strong correlations between the four cost of capital variables, as shown in Table E-7. Evidently the variances of the two composite cost of capital variables (C_c and C_g) are largely accounted for by variations in the respective interest rates (r_c and r_g). The underlying data are therefore not rich enough to permit meaningful tests of the separate effects of tax changes. 3/

Because it has the lowest standard error of estimate, equation E-6.3 is selected for further examination. The standard error of estimate obtained with this equation indicates that 95 per cent of the variation in the original dependent variable is explained by this regression model. Actual and predicted investment are plotted in Chart E-1. While the equation appears to be somewhat sluggish at the turning points, the fit is remarkably tight.

Chart E-1

ACTUAL AND PREDICTED VALUES OF INVESTMENT FOR ESTIMATION PERIOD

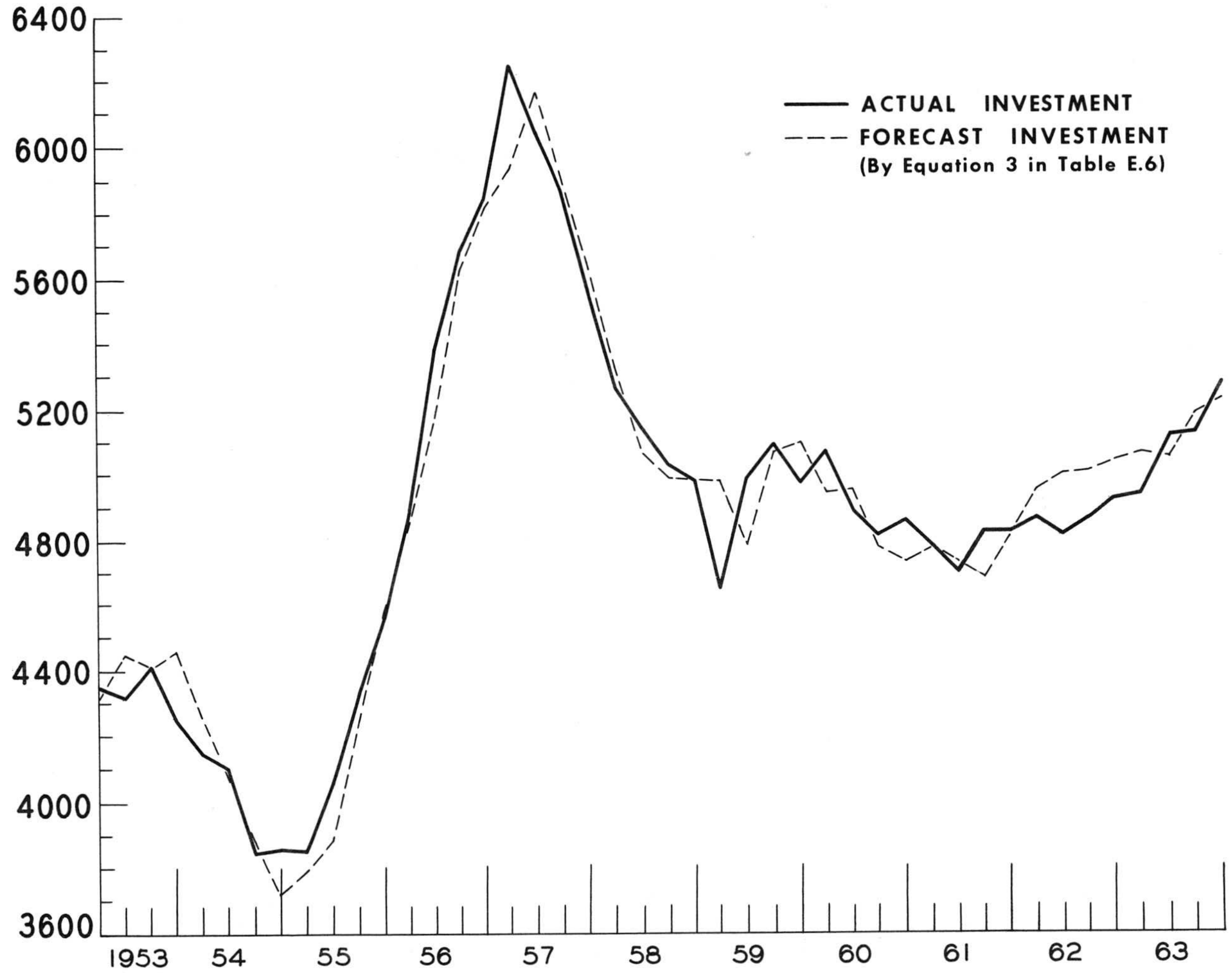
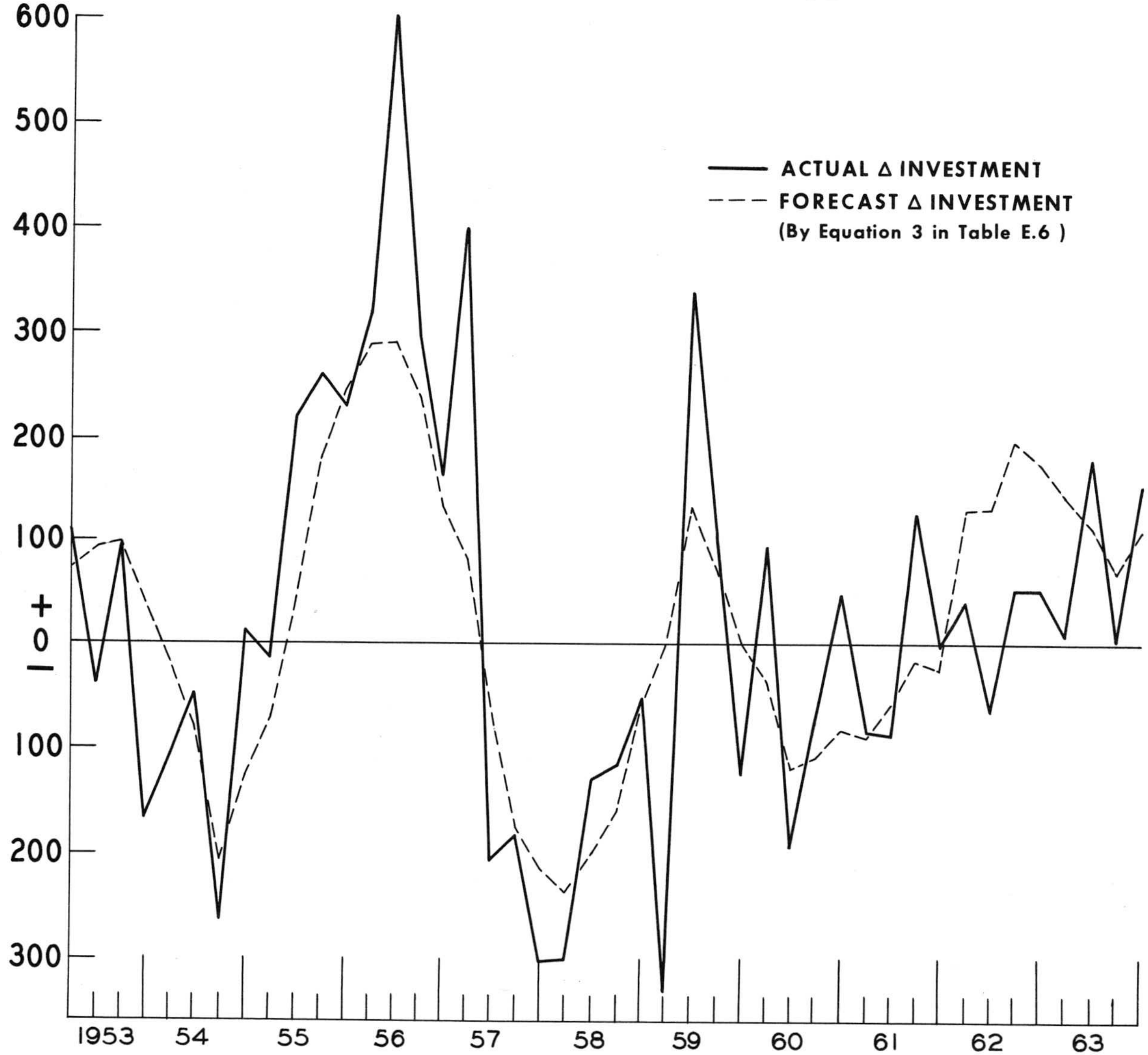


Chart E-2

ACTUAL AND PREDICTED CHANGES IN INVESTMENT FOR THE ESTIMATION PERIOD



Predicted and actual changes in investment are plotted in Chart E-2. This chart demonstrates that the regression equation predicts changes in investment quite well, thereby indicating that its goodness of fit is not wholly a matter of picking up the trend and the smoother cyclical variations in the original series.

In view of the sluggishness of the model near turning points, and as an additional historical check on the validity of this equation, an analysis of turning points and turning point errors is carried out. 4/ These results, tabulated in Table E-8, show that the equation predicts slightly over one half of the actual turning points correctly. The majority of these turning points represent minor fluctuations in the series which are reversed in the following quarter. When the 6 major turning points corresponding to the major cyclical movements in the economy are examined, the equation correctly forecasts two thirds of these. However, while the direction of change is correctly predicted for 4 of the 6 major turning points, the magnitude of the change is understated. In only one instance did the equation predict a false turn—it predicted a reversal of the 1958 downward swing of investment one quarter too soon.

Table E-9 summarizes the qualitative performance of the model in predicting the direction of change in investment. Of the 44 observations in the estimation period the model correctly predicts the direction of change 36 times, and successfully predicts positive and negative changes with about the same relative frequency.

A final test of this equation using historical data involves a comparison with a naive model. As is shown in Table E-10, of the 44 observations for the estimation period, this equation yielded predictions superior to a simple extrapolation of the previous level for 28 cases. The mean absolute forecasting error is 102 for the regression model, lower than the mean absolute error of 154 obtained with the naive model.

These results suggest that the model is able to reproduce historical experience quite well. A more stringent test is provided by the predictive power of the model for quarters subsequent to the estimation period. These "forecasting tests" are described in the next section.

TABLE E-8

ANALYSIS OF TURNING POINTS

Date	Turning Point Type	Actual Values: Turning Point	Actual Values: Quarter After	Actual Values: Change	Forecast Values: Quarter After	Forecast Values: Change	Comment
5301	U	4360	4324	-36	4463	+103	TP error
5302	L	4324	4424	+100	4421	+97	Called TP understated change
5303	U*	4424	4260	-164	4481	+57	TP error
5403	L	3852	3868	+16	3717	-135	TP error
5404	U	3868	3856	-12	3794	-74	Called TP overstated change
5501	L*	3856	4076	+220	3884	+28	Called TP understated change
5701	U*	6244	6040	-204	6177	-67	Called TP understated change
5804	F	4980	4652	-328	5005	+25	Predicted false turning point
5901	L*	4652	4992	+340	4772	+120	Called TP understated change
5903	U	5096	4976	-120	5109	+13	TP error
5904	L	4976	5072	+96	4935	-41	TP error
6001	U*	5072	4884	-188	4963	-109	Called TP understated change
6003	L	4812	4864	+52	4724	-88	TP error
6004	U	4864	4784	-80	4779	-85	Called TP overstated change
6102	L*	4700	4828	+128	4677	-23	TP error
6103	U	4828	4822	-1	4805	-23	Called TP overstated change
6104	L	4827	4872	+45	4965	+138	Called TP overstated change
6201	U	4872	4812	-60	5019	+192	TP error
6202	L	4812	4868	+56	5022	+210	Called TP overstated change

Symbols:
U = Upper Turning Point
L = Lower Turning Point
* Indicates a major turning point in relation to the general business cycle
F Indicates a false turning point

Number of Actual Turning Points = 18

TP error = 8
TP called with understated change = 5
TP called with overstated change = 5

Number of Major Turning Points = 6

TP error = 2
TP called with understated change = 4

Number of False Turning Points Predicted = 1.

TABLE E-9

ACCURACY OF THE INVESTMENT FORECASTS IN PREDICTING DIRECTION OF CHANGE

Predicted Change*	Actual Change		
	Positive	Negative	Totals
Positive	19	5	24
Negative	4	16	20
Totals	23	21	44

* Predicted $\text{Change}_{(t)}$ = Predicted Value (t) - Actual Value (t-1).

TABLE E-10

A COMPARISON WITH A NAIVE MODEL

Time	Actual Investment ($57)	Prediction Errors in Investment Based on Equation 3-6.2	Prediction Error of the Model After Adjustment	Predicted Investment	Error in prediction of naive model	Comparison (B - Better W - Worse)
	1	2	3	4	5	6
5301	4360	38	41	4319	112	B
5302	4324	-129	-139	4463	-36	W
5303	4424	3	3	4421	100	B
5304	4260	-205	-221	4481	-164	W
5401	4156	-87	-94	4250	-104	B
5402	4112	35	38	4074	-44	B
5403	3852	-57	-62	3914	-260	B
5404	3868	140	151	3717	16	W
5501	3856	57	62	3794	-12	W
5502	4076	178	192	3884	220	B
5503	4336	78	84	4252	260	B
5504	4564	-18	-19	4583	22	B
5601	4884	30	32	4852	320	B
5602	5384	208	225	5159	600	B
5603	5680	58	63	5617	296	B
5604	5844	30	32	5812	164	B
5701	6244	314	339	5905	400	B
5702	6040	-127	-137	6177	-204	B
5703	5860	-8	-9	5869	-180	B
5704	5560	-87	-94	5654	-300	B
5801	5264	-63	-68	5332	-29	B
5802	5140	72	78	5062	-124	B
5803	5028	41	44	4987	-112	B
5804	4980	3	3	4977	-48	B
5901	4652	-327	-353	5005	-328	W
5902	4992	204	220	4772	340	B
5903	5096	28	30	5066	104	B
5904	4976	-123	-133	5109	-120	W
6001	5072	127	137	4935	96	W
6002	4884	-73	-79	4963	-188	B
6003	4812	31	33	4779	-72	B
6004	4864	130	140	4724	52	W
6101	4784	5	5	4779	-80	B
6102	4700	-33	-36	4736	-84	B
6103	4824	140	151	4677	128	W
6104	4827	20	22	4805	-1	W
6201	4872	-86	-93	4965	45	W
6202	4812	-192	-207	5019	-60	W
6203	4868	-143	-154	5022	56	W
6204	4924	-120	-130	5054	56	W
6301	4936	-132	-143	5079	12	W
6302	5116	64	69	5047	180	B
6303	5120	-69	-75	5195	4	W
6304	5276	45	49	5227	156	B

REFERENCES

1/ As the dependent variable in equations with zero or two quarter decision lag is raw at quarterly rates, and the dependent variable in equations with a four quarter moving average decision lag is seasonally adjusted at usual rates, the standard error of the former must be multiplied by 4 for comparative purposes.

2/ See the derivation of the estimating equation in Section C above.

3/ Hence, as noted in Section A, and discussed further in Section J below, the effects of tax rate changes must be inferred from the effects of changes in interest rates.

4/ In each of these analyses, the prediction errors have been adjusted to eliminate the small spurious increase in accuracy which results from inclusion of current investment in the Almon weighting scheme. The adjusted residual or forecasting error is obtained by multiplying the unadjusted error by $\frac{1}{1-w_0}$, where w_0 is the weight attached to current investment in the Almon weighting scheme.

F. PREDICTIVE POWER OF THE INVESTMENT EQUATION

Equation E-6.3 is re-estimated for the period 1953-63 with revised data, using the same autoregressive transformation. The results, presented in the first part of Table F-1, show that the coefficients or output and interest rate are not changed very much, whereas the previously insignificant coefficient on retentions becomes negative. Consequently the equation is re-estimated with the retention variable omitted. This equation, which is the one used in the forecast tests, 1/ is presented in the second part of the table.

Short-run quarterly forecasts 2/ are generated for each quarter in the succeeding three-year period 1964-66. As the actual values of lagged investment (rather than values generated by the equation in preceding quarters) are used, these are short-run forecasting tests of the equation.

The results, tabulated in Table F-2 and graphed in Chart F-1, indicate that this equation forecasts remarkably well. While the mean square error of forecast is larger than the standard error of estimate over the observations used to estimate the equation, the variance of investment is also larger. As a result, the equation accounts for 93 per cent of the variance in investment during the forecasting period.

As the graph illustrates, the equation tracks the path of investment over the forecasting period quite well, being somewhat sluggish in the two periods of rapid upsurge. The forecasts based on the equations are superior to a naive extrapolation of investment in the previous quarter in 9 of the 12 quarters. The mean absolute error of forecast

TABLE F-1

RE-ESTIMATION OF EQUATION E-6.3 WITH REVISED DATA FOR THE 1953-63 PERIOD

Equation F-1.1 (All Independent Variables included.)

$$I_t^a - \sum_{i=0}^{7} W_i I_{t-i}^a = -101.37 + \underset{(4.18)}{9.87} \sum_{i=0}^{7} W_i \Delta \bar{Q}_{t-i}^a$$

$$- \underset{(0.38)}{0.78} \sum_{i=0}^{7} W_i \Delta \overline{Ret}_{t-i}^a - \underset{(3.42)}{641.31} \sum_{i=0}^{7} W_i \Delta \bar{r}_{c\,t-i}^a$$

$R^2 = .585$ Sest = 134.92 DW = 1.98

Equation F-1.2 (Retentions Variable omitted)

$$I_t^a - \sum_{i=0}^{7} W_i I_{t-i}^a = -97.97 + \underset{(7.26)}{9.11} \sum_{i=0}^{7} W_i \Delta \bar{Q}_{t-i}^a$$

$$- \underset{(4.01)}{598.87} \sum_{i=0}^{7} W_i \Delta \bar{r}c_{t-i}^a$$

$R^2 = .554$ Sest = 133.51 DW = 1.97

Notation

I = Business Fixed Investment (seasonally adjusted at annual rates is constant (1957) dollars).

$\bar{Q}$ = 4 Quarter Moving sum of Index of Gross Domestic Product (1949 = 1000.)

$\bar{r}_c$ = 4 Quarter Moving sum of Yield on corporate bonds (Percentage Points).

$\overline{Ret}$ = 4 Quarter Moving sum of Deflated Gross Private Corporate Retentions (at quarterly rates).

Superscript "a" indicates that the variable has been transformed as follows:

$$X_t^a = X_t - 0.75\, X_{t-1}$$

R^2 = Coefficient of multiple determination.

Sest = Standard error of estimate adjusted for degrees of freedom.

D.W. = Durbin Watson coefficient.

t values are in parentheses under the regression coefficients.

TABLE F-2

FORECASTS OF INVESTMENT, 1964-66 (BASED ON REGRESSION EQUATION F-1.1 PRESENTED IN TABLE F-1)

Year/62	Investment ($57, SAAR)	Forecast Investment 1/	Forecasting Error 1/	Comparison with Naive Model 2/
6401	5856	5502	354	B
6402	5944	5870	74	B
6403	5920	6144	-224	W
6404	6140	6161	-21	B
6501	6340	6359	-19	B
6502	6500	6530	-30	B
6503	7166	6697	469	B
6504	7548	7325	223	B
6601	7752	7723	29	B
6602	7824	7980	-156	W
6603	7708	8087	-379	W
6604	8016	7945	71	B

Root Mean Square Error 3/ = 226.6

Mean Absolute Error = 170.8

Mean Absolute Error, Naive Model = 246.7

Notes:

1/ Adjusted to exclude spurious increase in accuracy due to inclusion of current investment in the Almon weighting scheme.

2/ Simple extrapolation of Investment in previous period.

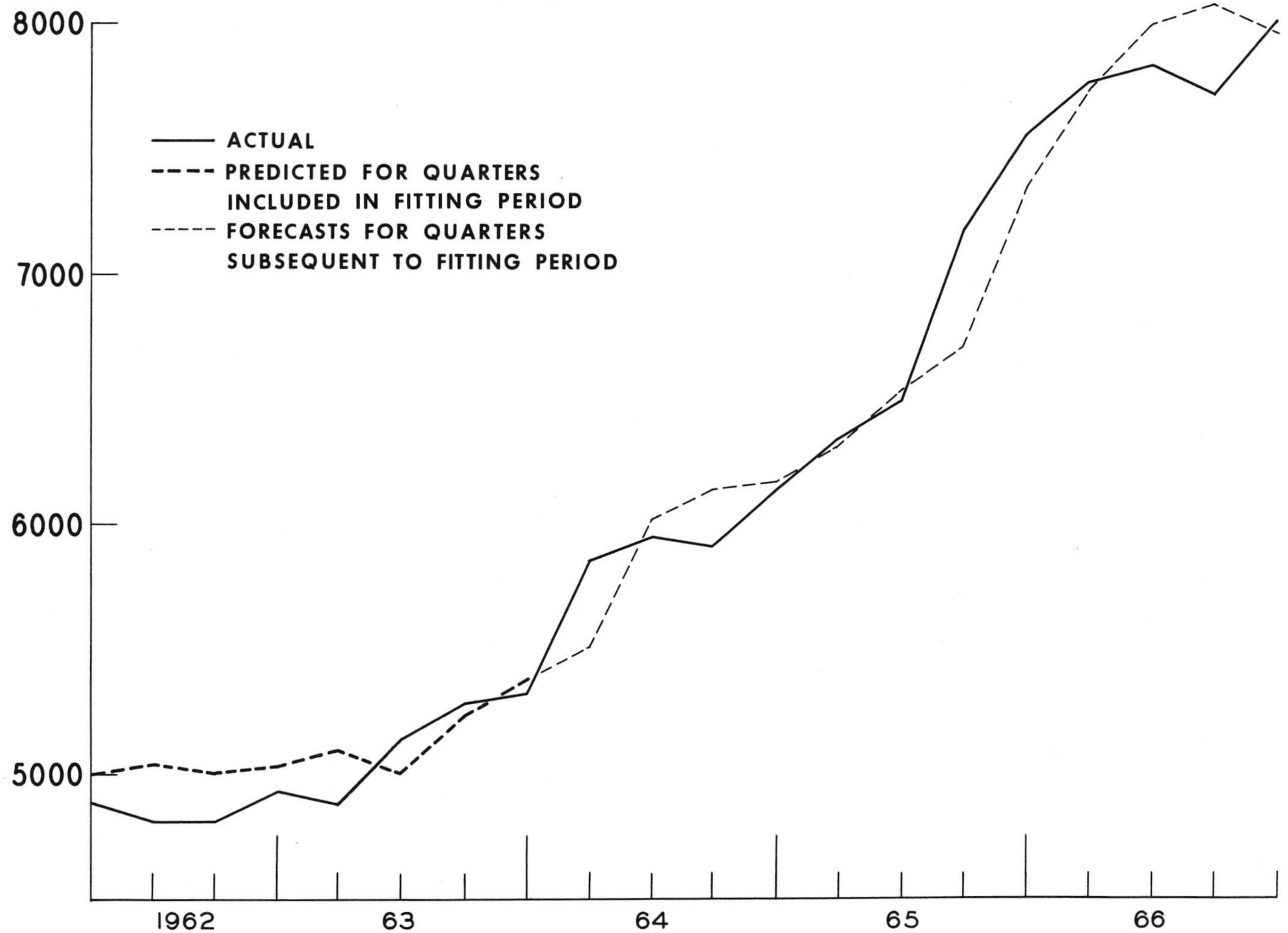
3/ Root Mean Square error before adjustment described in footnote 1/ = 246.7.

Chart F-1

INVESTMENT: ACTUAL AND FORECAST VALUES; 1962-1966

BUSINESS FIXED INVESTMENT

Seasonally Adjusted at Annual Rates
1957 Dollars



Notes 1. Based on Regression Equation fitted to revised data

2. Predictions and Forecasts are adjusted to eliminate the small spurious increased accuracy which results from the weight of .074 attached to current investment by the Almon weighting scheme

with the equation was 171, smaller than the mean absolute forecast error of 210 obtained with the naive model.

If the naive model is adjusted to take into account the historical (1950-63) growth of investment, 3/ its performance is improved somewhat, but the forecasts based on the investment equations are superior in 8 of 12 extrapolations.

These results are particularly impressive in the light of investment behavior during the forecasting period. In contrast to the preceding years of sluggish growth, investment experienced a remarkable upsurge between the end of 1963 and the beginning of 1966, rising by more than 50 per cent in real terms.

In order to provide additional tests of the validity of the model, annual forecasts are generated from the fourth quarter of each year. In contrast to the preceding results, predicted rather than actual values of lagged investment are used beyond the starting quarter during each forecasting sequence. The resulting forecasts are then aggregated to obtain annual predictions which are compared with two kinds of naive extrapolation as well as with the annual investment forecasts published in Public and Private Investment in Canada (PPIC). The results of this forecasting exercise are presented in Table F-3 and graphed in Chart F-2. The comparison of these results with the performance of naive models and the performance of the PPIC forecasts are presented in Table F-4.

These tests reveal that the equation has a marked superiority over both sets of naive extrapolations. More important, the predictions based on the investment equations are also superior to the PPIC forecasts. This

TABLE F-3

PREDICTED VALUES FOR ONE YEAR AHEAD

	Actual	Predicted	Error
A. Starting from 6304			
6401	5856	5502	354
6402	5944	5679	265
6403	5920	5854	66
6404	6140	6032	108
Year 1964	5965	5767	198
B. Starting from 6404			
6501	6340	6358	-19
6502	6500	6500	nil
6503	7166	6660	506
6504	7548	6836	734
Year 1965	6889	6589	300
C. Starting from 6504			
6601	7752	7719	33
6602	7824	7904	-80
6603	7708	8074	-366
6604	8016	8185	-169
Year 1966	7825	7971	-146

TABLE F-4

COMPARISON OF THE ANNUAL PREDICTION WITH PREDICTIONS BASED ON NAIVE MODELS AND WITH THE NOVEMBER PUBLIC AND PRIVATE INVESTMENT FORECASTS

	Year 1964	1965	1966	Root Mean Square Per cent Error
Actual Investment (National Accounts)	5965	6889	7825	
1. Forecasts based on Equation F-1.2	5767	6589	7971	
Per cent error	3.3%	4.4%	-1.9%	3.4%
2. Forecasts based on Naive Models				
Extrapolation of Previous Year's Investment	5157	5965	6889	
Per cent error	13.5%	13.4%	12.0%	13.0%
Extrapolation based on 1950-63 Trend 1/ applied to previous year's investment	5332	6168	7123	
Per cent error	10.5%	10.5%	9.0%	10.0%
3. Public and Private Investment Forecasts 2/				
Actual	7456	9064	10694 P	
Forecast	6678	8349	10345	
Per cent error	10.4%	7.9%	3.3%	7.8%
Forecast Modified to include change in Capital Goods Prices 3/	6945	8708	10717	
Per cent error	6.9%	3.9%	-0.2%	4.6%

Notes:

1/ This trend is 3.4 per cent per year over the 1950-63 period.

2/ Excluding Housing and Capital Expenditures by Government Departments.

3/ The published forecast is multiplied by the ratio of the current to the preceding year of the implicit deflator for business fixed investment.

P indicates preliminary estimate of actual investment.

Chart F-2

INVESTMENT: ACTUAL AND PREDICTED VALUES ONE YEAR AHEAD; QUARTERLY SEQUENCES

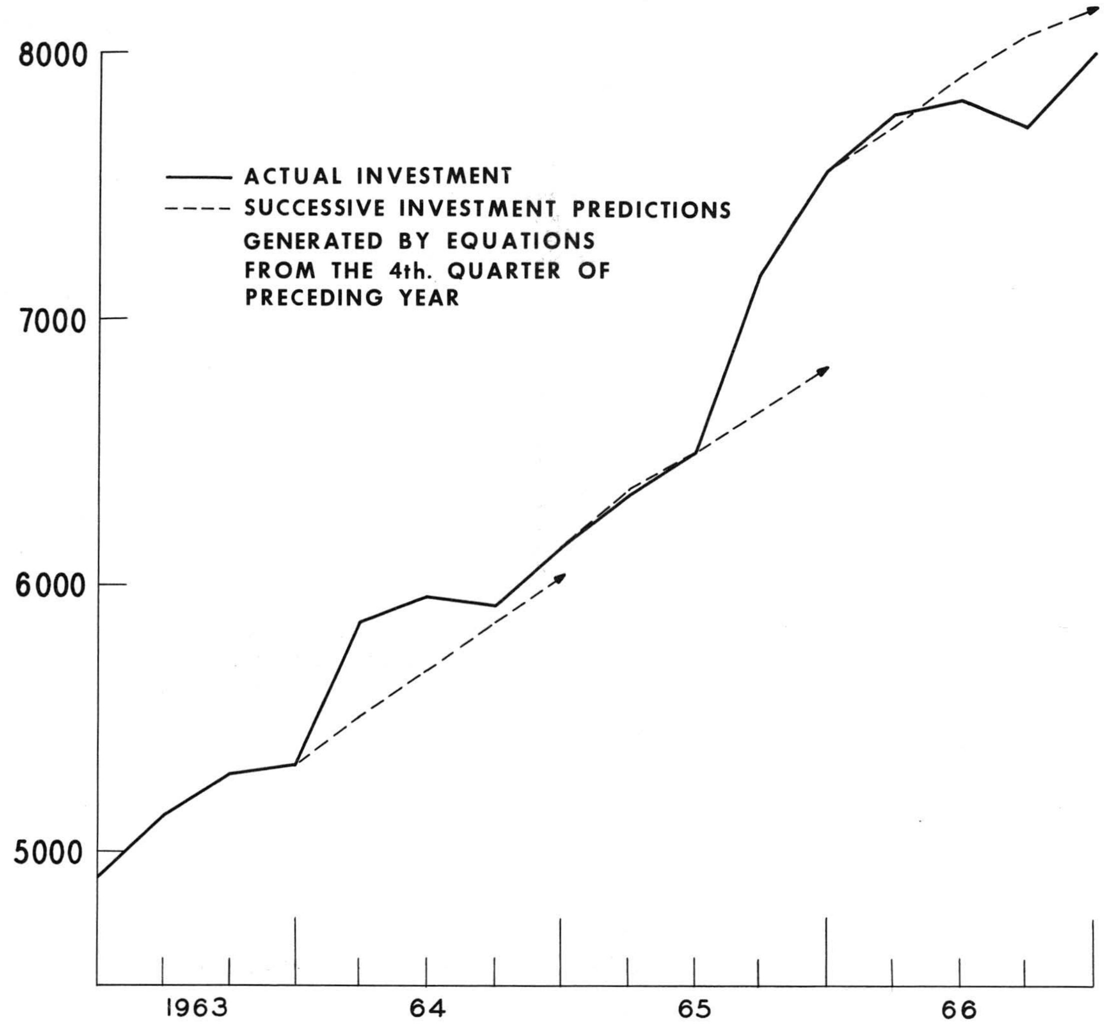
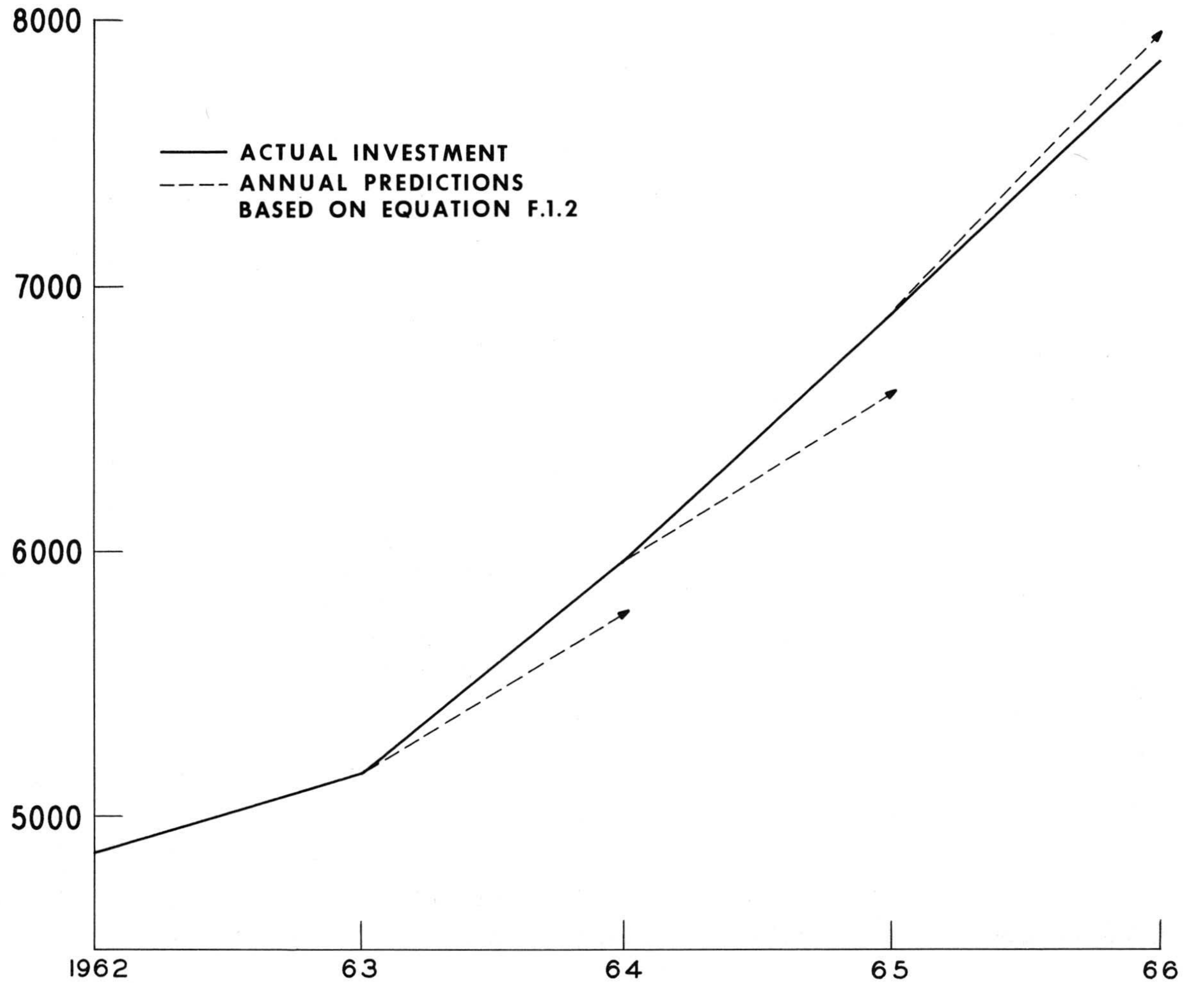


Chart F-2A

INVESTMENT: ACTUAL AND PREDICTED VALUES ONE YEAR AHEAD; ANNUAL DATA



in part reflects the distorting effect of changes in capital goods prices, however. When the PPIC forecasts are adjusted to allow for the effect of these price changes, their performance is improved substantially.

A final test involved generating alternative forecasting sequences over the 1964-66 period. In each sequence, forecast instead of actual values of investment subsequent to the starting point are used to generate succeeding forecasts.

Such long-run forecasting with this dynamic model will be sensitive to the initial conditions specified at the start of the sequence, because the model is autoregressive in both the dependent variable and in the errors.

As Houthakker and Taylor have point out, 4/ it is better to use actual rather than predicted values of the lagged endogenous variables to initialize the model at the start of a forecasting sequence. However, the use of this procedure means that the forecasting performance of the model will likely be sensitive to the particular initialization period selected. Good luck or bad luck in the first few forecasts would give a misleading impression of the overall long-run forecasting performance of the model.

In order to mitigate this difficulty, four alternative forecasting sequences are generated. The first sequence starts from the fourth quarter of 1963, the last observation of the estimation period. The rest of the sequences start from each of the first three observations of the extrapolation period (6401-6403).

The results of this exercise are presented in Table F-5 and graphed in Chart F-3. As is apparent, initialization at the end of the estimation period yields the worst results, and initialization at the first quarter of the extrapolation period yields the best results.

The longest sequence is for twelve quarters generated from the 6304 starting point. Over these twelve quarters, the equation predicts a growth rate of investment of 11.1 per cent. While this is an under-estimate of the strong growth rate of 14.5 per cent actually achieved, it is a much closer estimate than is the 1950-63 trend of 3.4 per cent. However, the model clearly fails to pick up the two episodes of strong growth (6401 and 6503). This sluggishness may be due to the absence of any true forward looking variables (such as new orders) in the equation, a problem which is discussed further below.

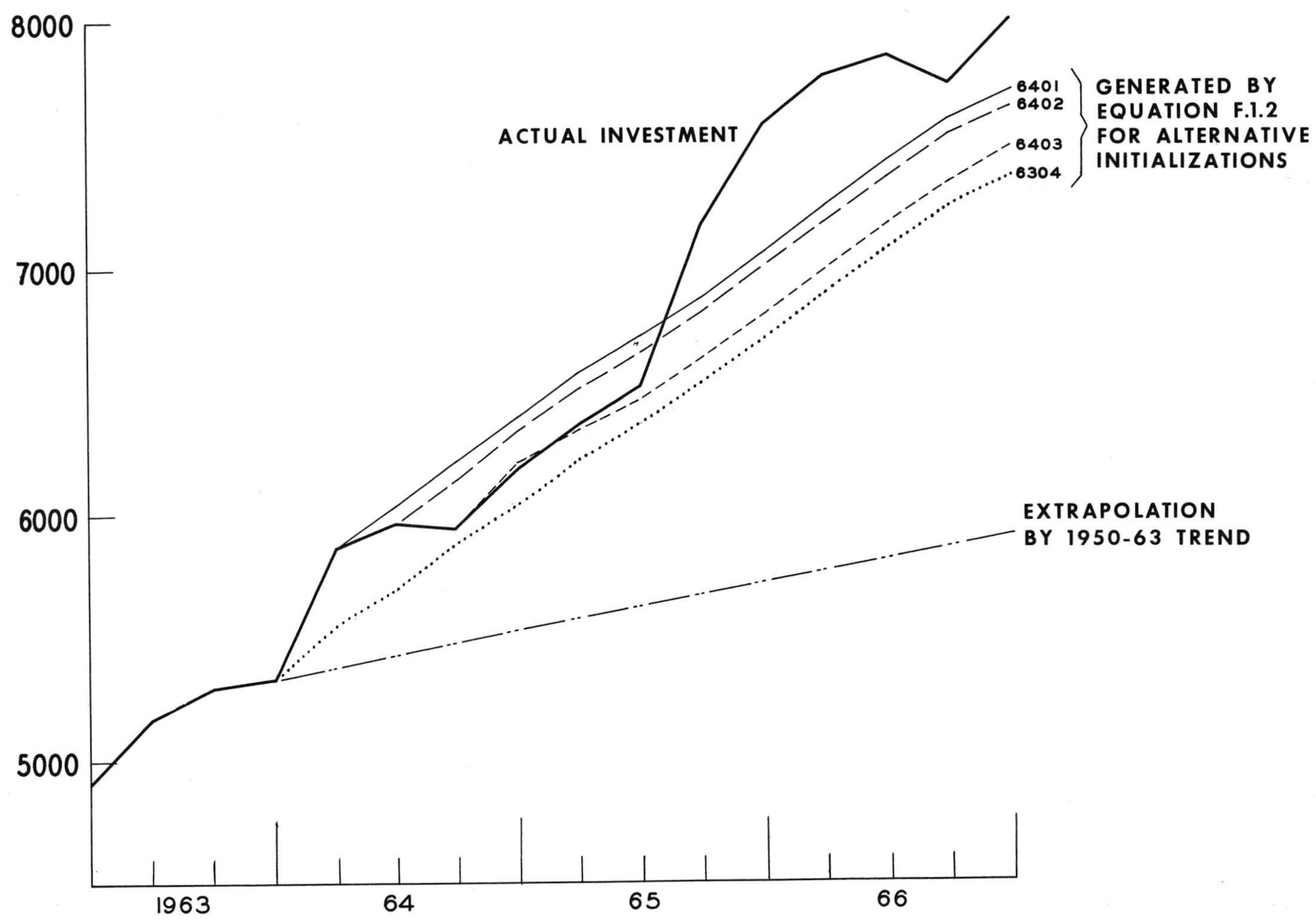
TABLE F-5

ALTERNATIVE LONG-RUN FORECASTING SEQUENCES

Year/ Quarter	Actual Investment	Forecasts Generated by Equation F-1.2 for Alternative Initializations:			
		6304	6401	6402	6403
6401	5856	5502	—	—	—
6402	5944	5679	6019	—	—
6403	5920	5854	6195	6144	—
6404	6140	6032	6373	6320	6161
6501	6340	6196	6537	6484	6318
6502	0500	6344	6685	6631	6467
6503	7166	6503	6844	6791	6626
6504	7548	6679	7020	6966	6801
6601	7752	6860	7201	7148	6983
6602	7824	7044	7386	7332	7167
6603	7708	7215	7556	7503	7337
6604	8016	7325	7667	7613	7448
Root Mean Square Forecasting Error		546	335	376	509

Chart F-3

INVESTMENT: LONG RUN PREDICTIONS GENERATED BY SUCCESSIVE INTERATIONS OF EQUATION F.1.2. COMPARED WITH SIMPLE EXTRAPOLATION OF 1950-63 TREND



REFERENCES

1/ The equation fitted to revised data is used only in this section. The interest elasticities reported in Section G, and the simulation experiments in Section H are based on equation E-6.3.

2/ Throughout this section we use the terms "forecasting" and "prediction" in a special sense. As actual values of the current independent variables are used, and as revisions in the data subsequent to each quarter predicted are incorporated in all series, these tests are not representative of the short-run forecasts that could have been made for each quarter or year with the data available at the time. Hence, all the tests described in this section represent tests of the extrapolation of the estimated relationship rather than tests of the usefulness of the model for pure forecasting purposes.

3/ In this comparison, the naive model forecast is as follows:

$$\text{Forecast Investment}_t = \text{Actual Investment}_{t-1} \cdot (1.034).$$

The growth rate of 3.4 per cent is the annual average rate of growth of investment between 1950 and 1963.

4/ H. S. Houthakker and L. D. Taylor, Consumer Demand in the United States 1929-70, Harvard University Press, 1966, p. 37.

G. THE INTEREST ELASTICITY OF INVESTMENT DEMAND

The long run interest elasticity of investment demand derived from equation E-6.3 evaluated at the end of the estimation period is -.67. This means that an increase of one percentage point in the rate of interest payable on corporate bonds would eventually reduce the flow of investment by 165 million dollars per quarter, which is a substantial impact.

Is this substantial interest elasticity reasonable? This estimate may be compared with the elasticity with respect to the cost of capital implicit in the aggregate production function estimated in the companion study. 1/ As this production function is a Cobb-Douglas, the elasticity of the demand for capital with respect to the cost of capital at a given level of output (i.e., along a production isoquant) is readily determined as $1 - \alpha$, where α is the exponent for capital input. The estimate of .315 for the capital exponent used in the companion study therefore implies a cost of capital elasticity which is very close to the long-run interest rate elasticity obtained in this paper. 2/

Nevertheless, it might be argued that this elasticity is on the high side for two reasons. First, the user cost of capital includes depreciation expense as well as the cost of capital. Consequently a given relative change in interest rates leads to a smaller relative change in the user cost of capital. Second, in the presence of uncertainty a firm may base its investment on a comparison of expected rates of return with a required rate of return. As the latter incorporates a margin to allow for risk, given relative changes in interest rates should lead to smaller relative changes in required rates of return.

This argument neglects the fact that an increase in interest rates will, in the long run, alter optimal debt/equity ratios and raise the risk premium appropriate at a given debt/equity ratio, since fixed interest charges would rise in relation to the cash flow of the firm.

Similarly, changes in interest rates will affect both the optimal intensity of use of capital and the optimum durability of new capital put in place. Lower interest rates will be associated in the long run with lower true depreciation rates.

In the shorter run, there are two additional factors to consider. First, interest rate changes may be associated with credit rationing. Second, changes in yields on seasoned bonds move more sluggishly than changes in yields on new issues, which are relevant to a firm planning to finance investment with borrowed funds. An analysis of data on yields of new and seasoned securities in the United States reveals that a change in the yield on seasoned issues is associated with a larger change in the yield on new issues. 3/ It would follow that the elasticity of investment with respect to changes in yields on new issues is lower than the estimates obtained from functions fitted with data on seasoned issues. 4/

For comparative purposes, Table G-1 presents the estimated interest elasticities reported by the authors of several of the United States studies discussed in section C above, for which elasticities are either published or readily derived from data referenced in the study. As is apparent, there is a great range to the available estimates. However, if we confine our attention to those studies which involve the estimation of aggregative models to which our own study is most comparable—the studies of Jorgenson, Griliches and Wallace, de Leeuw, and Bischoff—the elasticities

TABLE G-1

LONG-TERM INTEREST ELASTICITIES OF INVESTMENT DEMAND DERIVED FROM RECENT TIME SERIES STUDIES

Study	Coverage	Elasticity	Year of Evaluation
de Leeuw 1/	U.S. Mfg.	-.29	1962
Jorgenson (1963) 2/	U.S. Mfg.	-.38*	1959
Griliches and Wallace 3/	U.S. Mfg.	-.37*	1962
Meyer and Glauber 4/	U.S. Mfg.	a) - .91 b) - .51	mean upswings mean downswings
Eckstein 5/	U.S. Mfg.	- .22	1962
Anderson 6/	U.S. Mfg.	a) - .40 b) - 1.08	means means
Resek 7/	U.S. Mfg.	a) - .96* b) - 1.32* c) - 1.38*	means means means
Bischoff 8/	U.S. Private purchases of producers durable equipment	- .23*	1965
Kareken and Solow 9/	U.S. new orders of producers durable equipment	- .40*	means
Present Study 1/, 10/	Canadian aggregate investment	- .67	1963

* The elasticity is published in the study.

Notes:

1/ The calculation of long-term investment response for backlog models involves dividing the estimated regression coefficient which reflects the impact of interest rates on the backlog of projects, a stock variable, by the equilibrium stock-flow ratio. The latter depends on the postulated investment appropriations lag, as follows:

$$\sum_{i=1}^{L} 1 - \left(\sum_{j=i}^{L} W_j \right)$$

where F is the equilibrium stock-flow ratio and W_j are the weights in the distributed lag relating expenditures to appropriations.

For the 12 quarter inverted V used by de Leeuw (1/F) = .15.
For the Almon lag structure used in this paper (1/F) = .31.

The calculation of the elasticities for the de Leeuw equations are based on data made available to the author by Otto Eckstein.

2/ Jorgenson, op. cit., p. 258.

3/ Griliches and Wallace, op cit., p. 324.

4/ The bifurcated model presented by Meyer and Glauber is used. The equations used for this result appear on p. 163. Meyer and Glauber themselves report an interest elasticity of -.165 for the non-bifurcated model. These are short-run elasticities. We do not present the derived long term elasticities since Meyer and Glauber imply that these are not reliable (op. cit., p. 156, note 7).

5/ The equation selected from the Eckstein note is equation 3 (Table 1, p. 422). It includes changes in the order backlog in addition to the variables used by de Leeuw. Otherwise the method is that used for the de Leeuw results.

6/ These are elasticities with respect to the bill rate and the yield on long term industrial bonds, respectively. They are based on line 1, table 7-1, p. 10 and on line 1, table 5-6, p. 81. The second elasticity is short term as Anderson does not report the coefficient for the lagged dependent variable.

7/ The three estimates correspond to the three alternative models presented by Resek (Tables 2, 3, and 4, pp. 330-332).

8/ This is the sum of the interest rate and stock yield elasticities reported by Bischoff.

9/ Kareken and Solow, op. cit., p. 36.

10/ This is based on equation E-6-3. Alternative elasticities based on alternative cost of capital variables in Table E-6 are as follows:

Equation	Cost of Capital Variable	Long-term elasticity (calculated for 1963)
E-6.6	r_g	- .72
E-6.9	c_c	- .65
E-6.12	c_g	- .61

reported are consistently below that estimated from our model. 5/ These differences may be explained in part by the different coverage of the models. Jorgenson, Griliches-Wallace, and de Leeuw estimate models for manufacturing, which may be expected to have a lower interest elasticity of investment demand than the economy as a whole. 6/ Bischoff's analysis covers the total private economy, but explains only investment in producers durable equipment, which, as noted earlier, may be expected to have a lower interest elasticity than total investment because of the shorter life of equipment relative to plant.

Finally, we note that there is reason to believe that the interest elasticity of total investment demand may be higher in Canada than in the United States. Investment in communications, public utilities and transportation account for a higher share of investment, and manufacturing for a lower share than is the case in the United States. In the aggregate, internal funds finance a smaller percentage of the investment in Canada. Many Canadian firms have access to funds from private corporations abroad; however, even after allowance is made for this, external funds requirements remain relatively higher in Canada. 7/

On the basis of these considerations, the interest elasticity obtained in equation E-6.3 is not unreasonable. Additional checks on the reasonableness of this estimate are provided by the simulation experiments discussed in the next section.

REFERENCES

1/ T. A. Wilson and N. H. Lithwick, The Sources of Economic Growth, a study prepared for the Royal Commission on Taxation, Ottawa, Queen's Printer, 1967.

2/ The alternative capital coefficient estimates by regression techniques presented by Wilson and Lithwick. This would imply a somewhat higher cost of capital elasticity.

3/ The U.S. Treasury Department made available a quarterly series on the yields of new corporate issues for the 1959-63 period. A regression analysis of the relationship between changes in these yields and changes in yields on seasoned corporated issues yielded the following equation:

$$\Delta r_n = -.02 + 1.59 \Delta r_s \quad (R^2 = .64)$$

Where Δr_n is the change in the yield on new issues and Δr_s is the change in the yield on AAA corporate bonds, reported by Moody's.

4/ As yields on new issues of Canadian securities are not published, the alternative of using such data in the regression analysis is not possible at this time.

5/ However, the differences between the estimated elasticity derived from Equation E-6.3 and those reported by these four authors are not statistically significant.

6/ The longer life of capital and the greater reliance upon external sources of funds in the public utilities, communication and transportation sectors should result in an interest elasticity greater than in manufacturing. In addition, small enterprises in the agriculture, services, and trade sectors are more likely to be affected by any credit rationing which accompanies periods of rising interest rates.

7/ In 1965, gross corporate retentions plus net direct foreign investment in Canada amounted to 69 per cent of business fixed investment in Canada. In the same year, gross corporate retentions amounted to 88 per cent of business fixed investment in the United States.
Sources: D.B.S., National Accounts, 1965, Tables 2, 50 and 51; Bank of Canada, Statistical Supplement, 1965, p. 151; The National Income and Product Accounts of the United States, 1929-65, Tables 1.1 and 1.14.

H. RESULTS OF SIMULATION EXPERIMENTS

The equations estimated in this paper have a sufficiently complex lag structure that it is not immediately apparent what the response patterns are to changes in the independent variables. In order to determine the response pattern to changes in interest rates and output, and to investigate the effects of alternative fiscal and monetary policies upon investment, a number of simulation experiments using different investment equations together with a simple capacity feedback equation are carried out. These results, while interesting in themselves, also provide a check on the reasonableness of the model.

The best equation obtained is equation E-6.3 which uses changes in output alone as the capacity requirements variable. In the context of the capacity adjustment model this is appropriate if capacity may be approximated by a linear trend. The large negative constant term obtained is largely accounted for by the omission of the capacity trend variable—capacity growth at the same rate as the growth of output over the period, would affect the constant term by -115 in the fitted equation (after autoregressive transformation). 1/

While we have effectively ignored the effect of capacity expanding investment upon capacity (except in so far as both are approximated by a trend) in the estimation of equation E-6.3, it would be misleading to simulate the response of investment to assumed output changes without making some allowances for the effect of capacity expanding investment upon capacity, which in turn will affect investment in future periods.

The procedure used is straightforward. The change in capacity is assumed to be equal to the output: net capital stock ratio multiplied by capacity expanding investment. Since the simulations are either hypothetical or else pertain to the period of the late 1950's or early 1960's we use an estimate of the output: net capital ratio appropriate for those years.

Based on data made available by N. H. Lithwick, together with the recent estimates through the manufacturing sector published by D.B.S. 2/ a ratio of the GDP index to the net stock of capital of 0.0035 is used. 3/ Since this estimate is itself very rough, the simulation results for changes in output are useful mainly as illustrations of the response pattern to such changes.

The capacity feedback equation used is as follows:

$$\Delta C_{P_{t+1}} = .035\ I_t^w$$

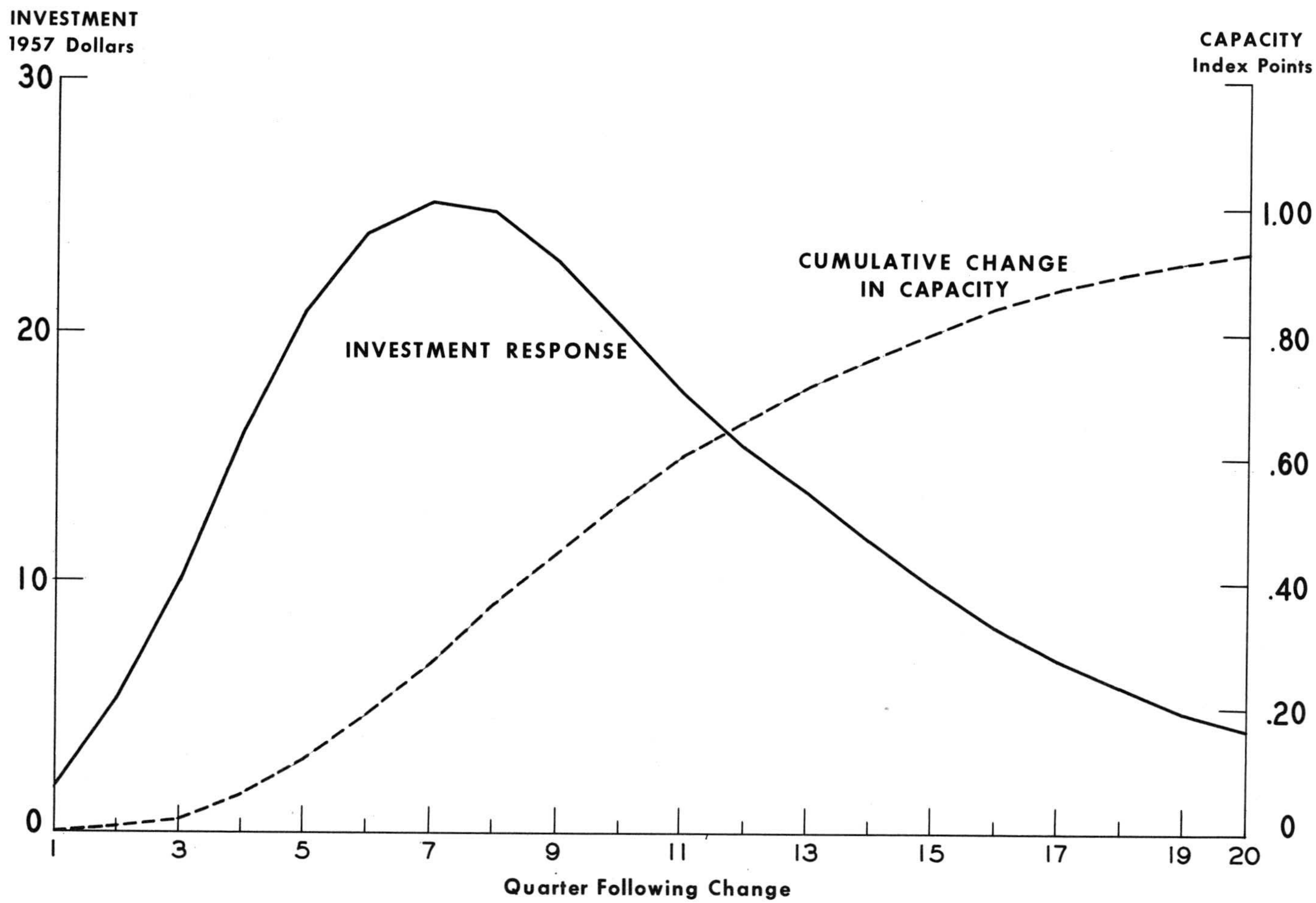
where $\Delta C_{P_{t+1}}$ is the change in capacity during period t+1 and I_t^w is capacity expanding investment occurring in period t.

It is assumed that all investment occurring in response to output changes is capacity expanding and all investment in response to changes in interest rates is capital deepening. 4/

The first two experiments involve predicting the response of investment to once-and-for-all changes in output and interest rates. As can be seen in Chart H-1, an increase in output brings about a rapid build-up

Chart H-1

INVESTMENT RESPONSE TO AN INCREASE IN OUTPUT OF ONE INDEX POINT



in investment over a period of seven quarters, followed by a more gradual decline subsequently as the depressing effects of the cumulative growth in capacity are felt.

As Chart H-2 illustrates, a reduction in interest rates leads to a different pattern—investment builds up rapidly for 7 or 8 quarters and then levels off. This results from the assumption that all investment induced by the interest rate reduction is capital deepening, and hence no capacity feedback effects occur. 5/

The dotted line in Chart H-2 represents the situation in which other policies are used to increase output in order to prevent the rise in unemployment that would otherwise occur with capital deepening investment. 6/

In the next experiment, the change in output grows at a rate of one per cent per quarter. Hence this experiment simulates the effect of a "growth spurt", with the growth rate gradually declining to a rate of one per cent per quarter following the spurt. In contrast to the strong oscillatory response depicted in Chart H-1, Chart H-3 shows that investment adapts to the growth spurt with reasonably steady growth. This suggests that a step-up in the rate of growth of output would not have the destabilizing effect of a sharp change in output levels.

Experiment 4 examines the consequences of a restrictive monetary policy followed by a subsequent expansionary monetary policy. In this experiment, interest rates rise by .10 points per quarter for five quarters, followed by a symmetrical decline. As Chart H-4 demonstrates, this policy achieves its maximum restrictive effect on investment three quarters after the policy is reversed, and the depressing effect of the restrictive policy remains strong for several additional quarters. This experiment indicates

Chart H-2

INVESTMENT RESPONSE TO AN INTEREST RATE REDUCTION OF .10 PERCENTAGE POINTS

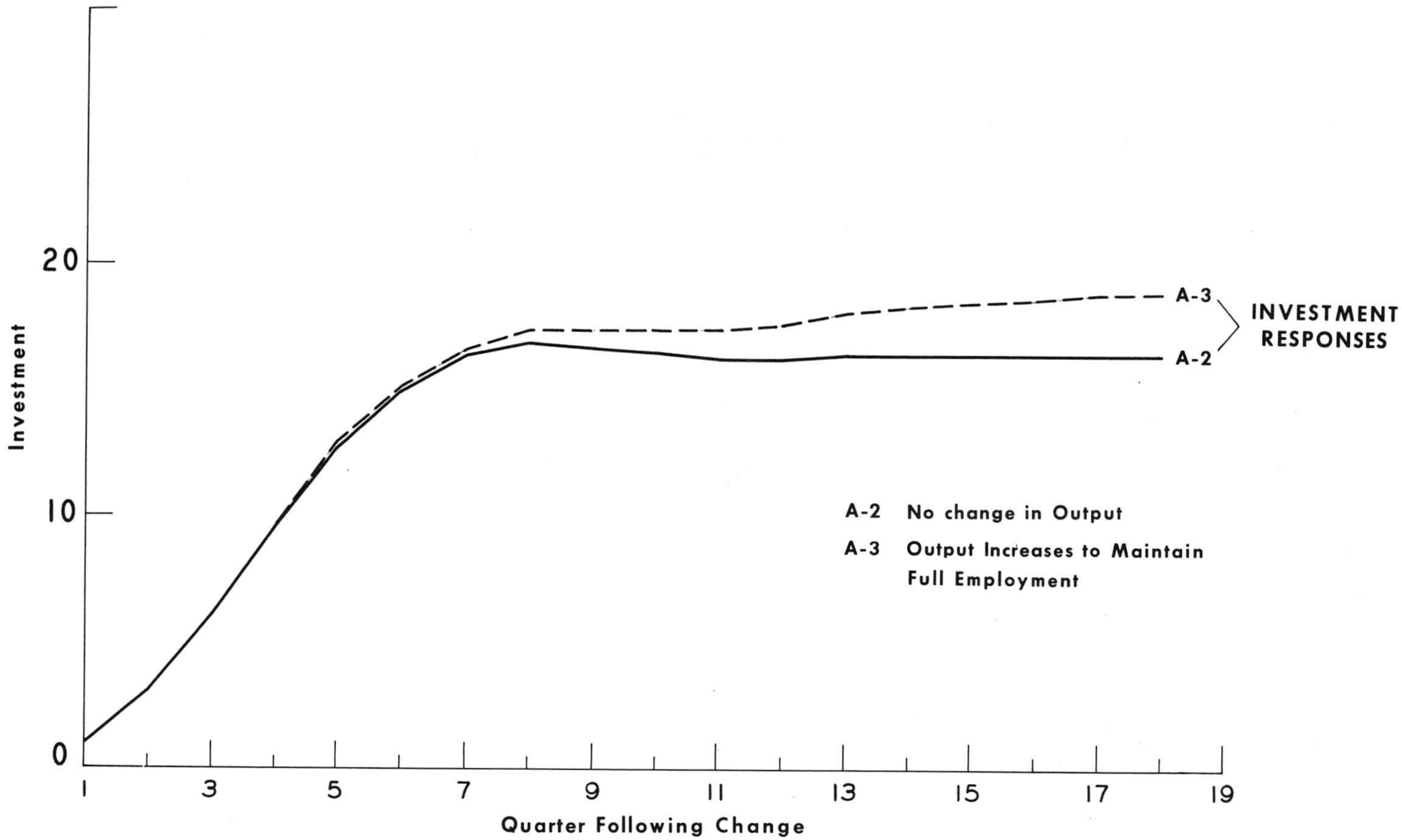


Chart H-3

INVESTMENT EFFECTS OF A GROWTH SPURT

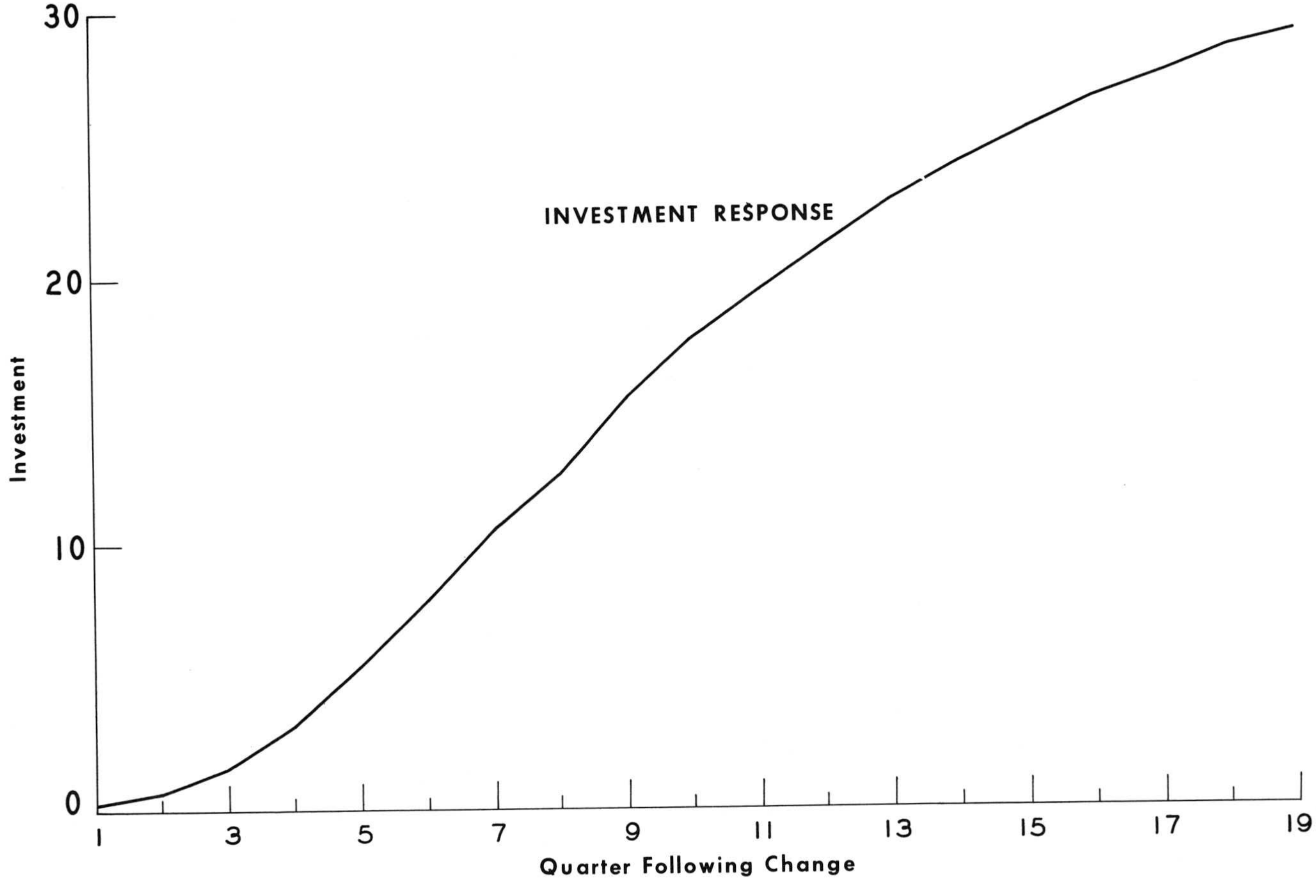
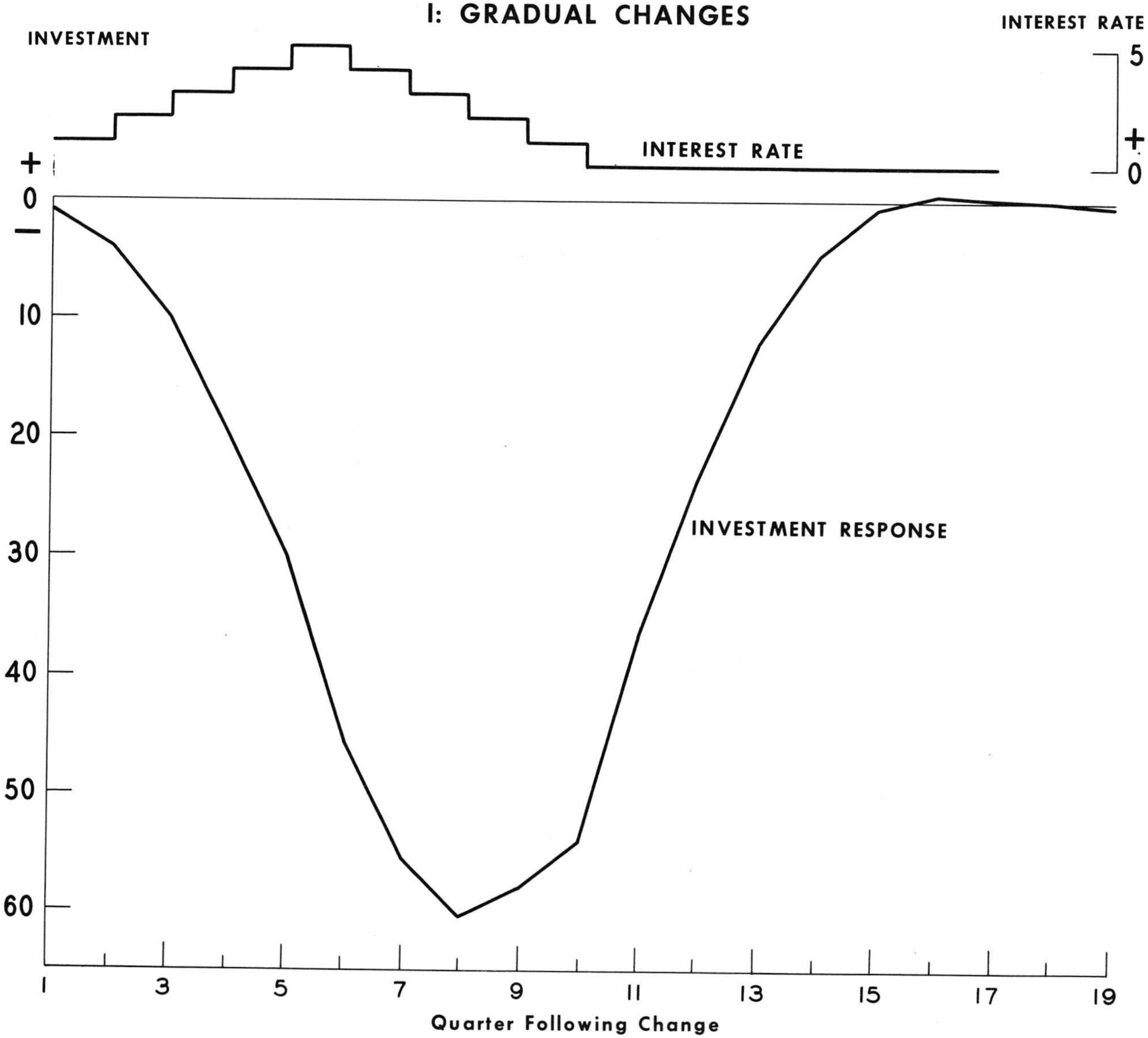


Chart H-4

INVESTMENT EFFECTS OF MONETARY RESTRICTION FOLLOWED BY MONETARY EXPANSION
I: GRADUAL CHANGES



that considerable momentum is built up by a period of tightening (or easing) of monetary policy, so that a reversal of the policy does not lead to a very rapid investment response in the desired direction.

Experiment 5 presents a minor variation on experiment 4; a large increase in interest rates is effective for four quarters, followed by an immediate reduction to the previous level. Such sharp changes in interest rates lead to a more rapid investment response. Following the reversal of the policy, investment is virtually halted after one quarter and begins to rise after two quarters.

Taken together, experiments 4 and 5 indicate that a change in the apparent lead-lag relationship between changes in interest rates and investment may be brought about by sharper monetary changes. However, as the underlying structure is given, such an apparent change does not, of course, mean that a change in the lag structure of investment response has occurred.

Each of the remaining experiments examines different pseudo-realistic situations, in the sense that comparisons with actual patterns observed in recent years are made. These experiments may be conveniently classified into:

a) Aggregate demand experiments, which examine the effect of deviations from assumed growth paths of output.

b) Monetary experiments involving variations in interest rate patterns in relation to patterns observed in recent years.

We shall discuss each set in turn.

The first aggregate demand experiment—experiment 6—examines the effect upon investment of departures from potential output during the 1954-63 period. The excess of actual over potential output in late 1955 and 1956 gives rise to an investment simulus of $40 million per quarter during the last quarter of 1956 and the first half of 1957, contributing to the inflationary pressures of that period.

With the sharp recession of 1957-58, the abortive recovery of 1950-60, and the recession of 1960-61, the situation is drastically reversed. The large gap between actual and potential GNP itself depresses investment by 225 to 260 million dollars per quarter from the middle of 1959 to the end of 1961.

Furthermore, this large investment shortfall has cumulative effects on capacity. As a result, by the end of 1963, capacity is 14 points lower than it otherwise would have been. Although a substantial gap between actual and potential output remains, investment has nearly reached its full employment level and is rising rapidly.

This experiment puts in perspective the 1964-66 investment surge, a surge which contributed to the re-emergence of premature inflation during 1965, and is in part responsible for the stronger inflationary pressures of 1966. This investment surge may be attributable in part to the foregone investment opportunities of the preceding six years. The low level of investment in those years shifted the relationship between capacity utilization and full employment, so that inflationary pressures arising from high utilization rates are felt before full employment is achieved.

Experiment 7 examines the consequences of sustained growth coming out of the 1958 recession. In this experiment, growth proceeds at 1.25

per cent per quarter until full employment output is achieved, and proceeds at the slow growth of potential output thereafter. Had policies to achieve such a result been adopted, investment would have grown rapidly during 1960 and 1961, instead of leveling off.

However, the support for aggregate demand achieved in this way is short-lived—in 1962 and 1963, investment would have declined, thereby necessitating further offsetting expansionary policies. Experiment 7 therefore provides a more concrete illustration of the stabilization problem revealed in experiment 1—stimulating output leads to a period of investment growth followed by a period of investment decline.

Three experiments designed to shed light on the effects of monetary policies are carried out. Before proceeding to a discussion of these, we should emphasize that we base the analysis wholly on interest rate movements and leave aside the question of whether the interest rate movements would in fact have been achievable by domestic monetary measures alone. 7/ Because of this problem, we carried out one experiment designed to show the effects of the widening of the Canadian-United States interest rate differential that occurred in the 1959-61 period.

Experiment 8 examines the extent to which the monetary policies of 1955-56 and 1959-60 differed in their impact on investment. It is widely recognized that the policies adopted in these two cyclical expansions differed greatly. Whereas monetary expansion continued throughout the year 1955, monetary policy tightened very sharply in 1959. This difference is revealed graphically in Chart H-8, where the differences between the growth of interest rates (from the levels prevailing at the start of each period) is plotted. Both corporate and government bond yields rose much more sharply in the first 5 or 6 quarters of the 1959-60 recovery than they did during 1955 and 1956.

To what extent did these sharp differences in monetary policy affect investment? The implication of two of the models fitted are graphed in the bottom part of Chart H-8. The results obtained with either model are similar. Each of the models shows investment being about $100 million per quarter higher in the peak quarter of 1956 than would have been the case had the more stringent monetary policy of 1959-60 been adopted during the earlier period.

These results imply that monetary expansion in 1955 contributed significantly to the investment boom in 1956 and conversely that restriction in 1959 contributed significantly to the abortion of the recovery in 1960.

Because of the general concern about the role of monetary policy in 1959-60, two additional experiments are run for that period. Experiment 9 examines the effects of the increase in interest rates over the level prevailing at the end of 1958. As is shown in Chart H-9, this policy depressed investment by 115 million per quarter during the last half of 1960 and the first half of 1961, a period which contained the beginning of a recession.

The final experiment examines the effect of a widening of the differential between yields on Canadian and United States corporate securities which occurred in 1959-60. As the graph in the top half of Chart H-10 shows, this interest rate differential increased by 0.65 points between the fourth quarter of 1958 and the first quarter of 1960.

Chart H-5

INVESTMENT EFFECTS OF MONETARY RESTRICTION FOLLOWED BY MONETARY EXPANSION
II: SHARP CHANGES

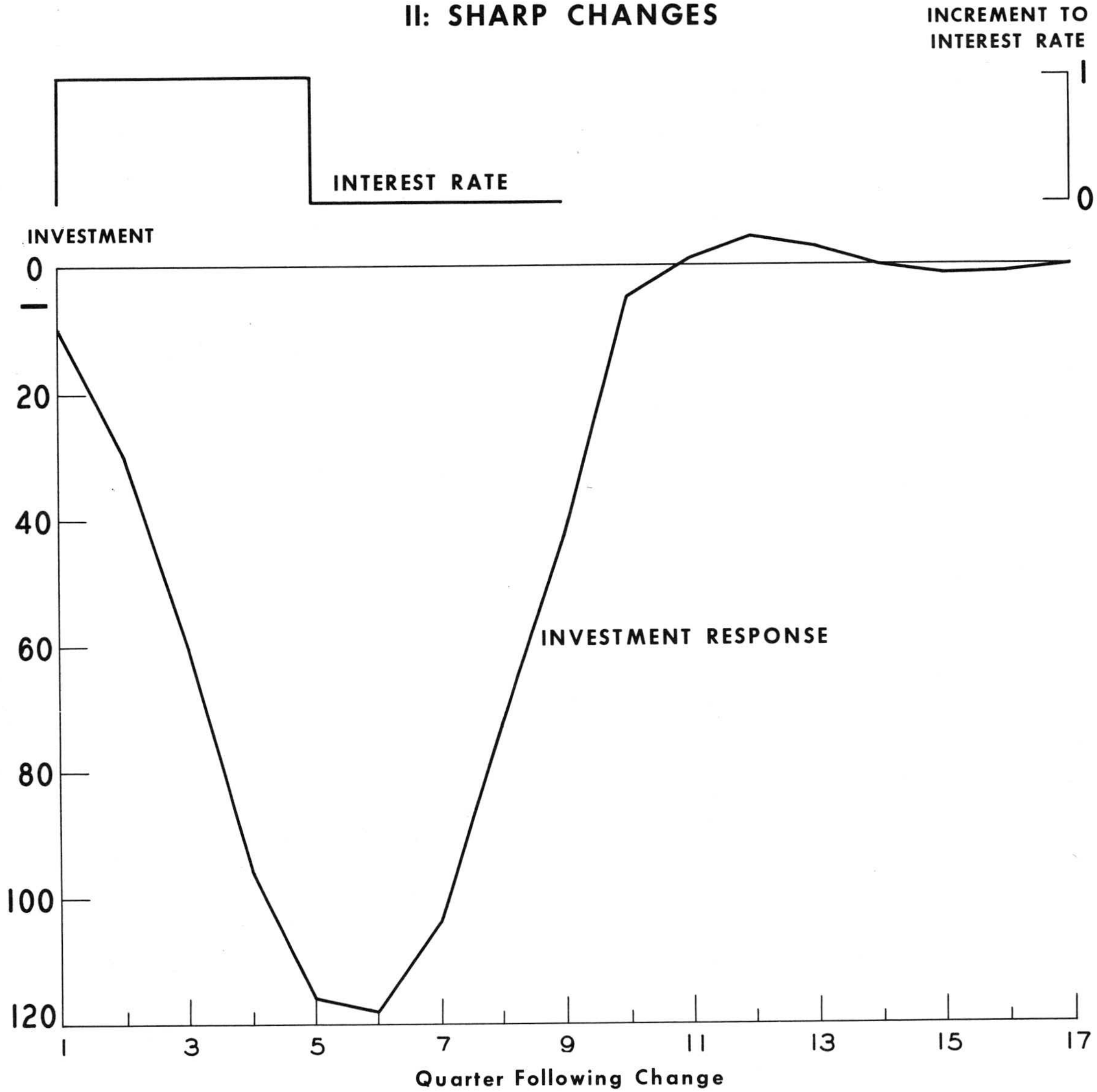


Chart H-6

INVESTMENT EFFECTS OF DEVIATION FROM FULL-EMPLOYMENT GROWTH, 1955-1963

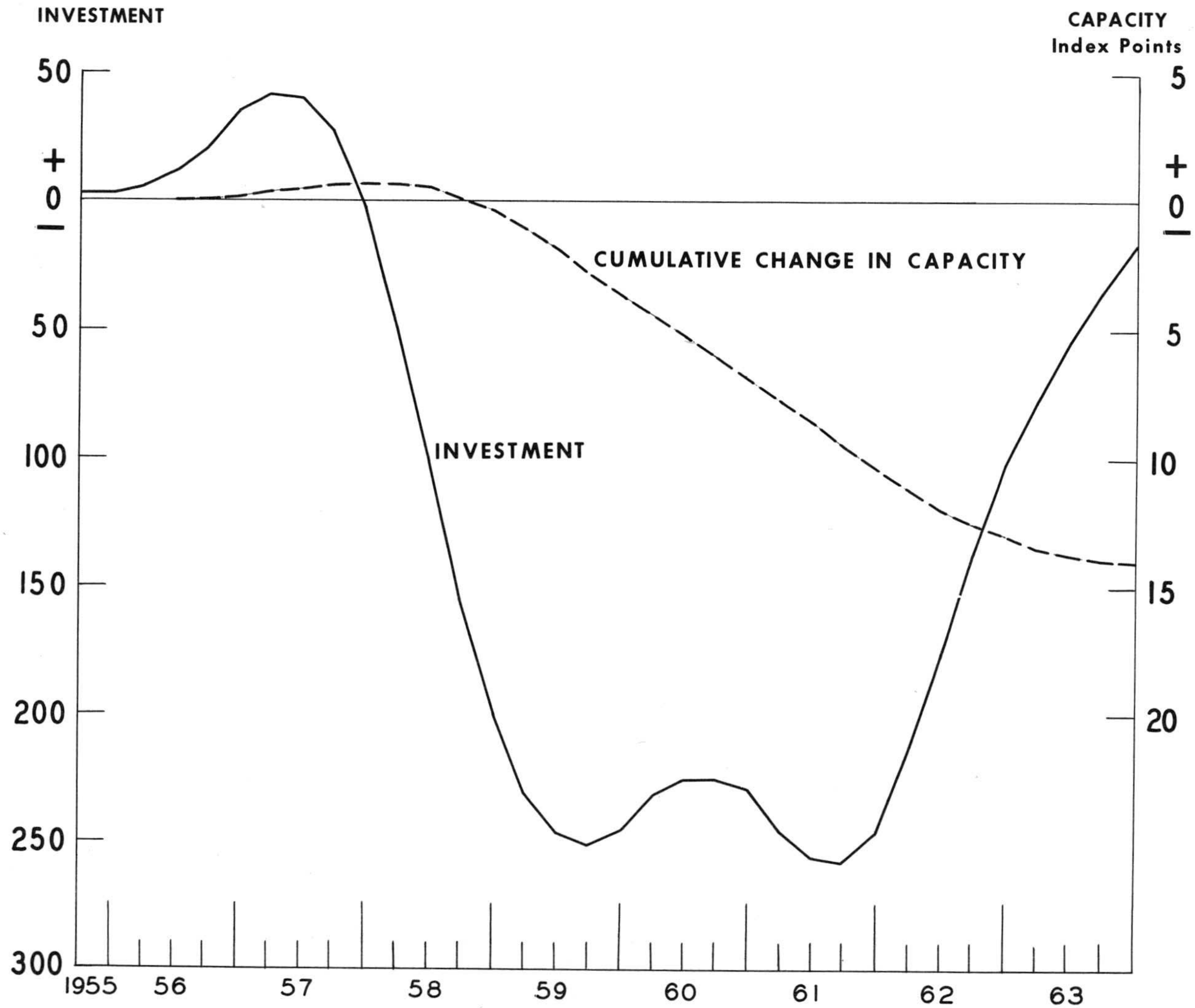


Chart H-7

INVESTMENT CONSEQUENCES OF SUSTAINED GROWTH COMING OUT OF 1957-58 RECESSION

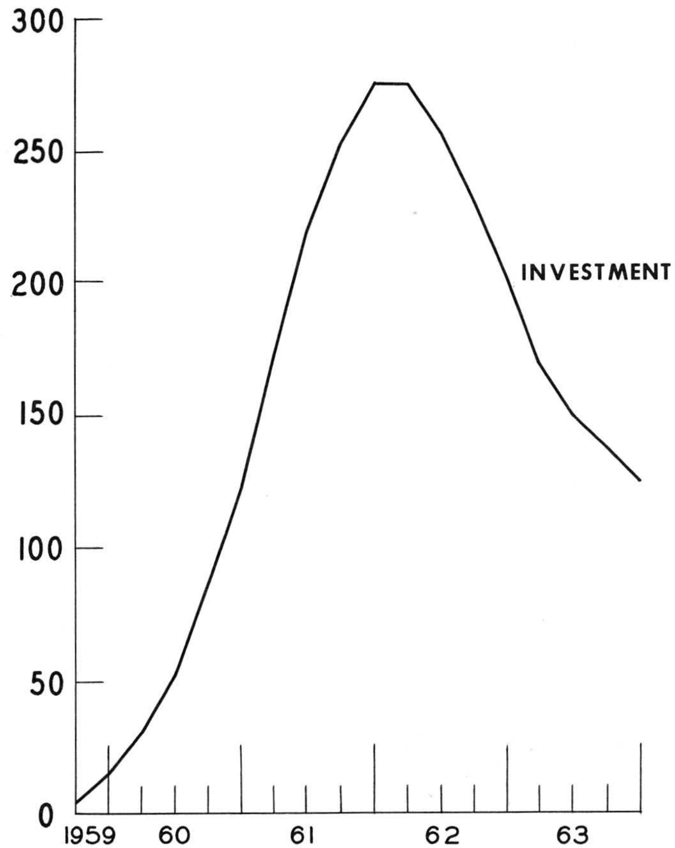


Chart H-8

INVESTMENT EFFECTS OF MONETARY POLICIES 1954-56 AND 1958-60 COMPARED

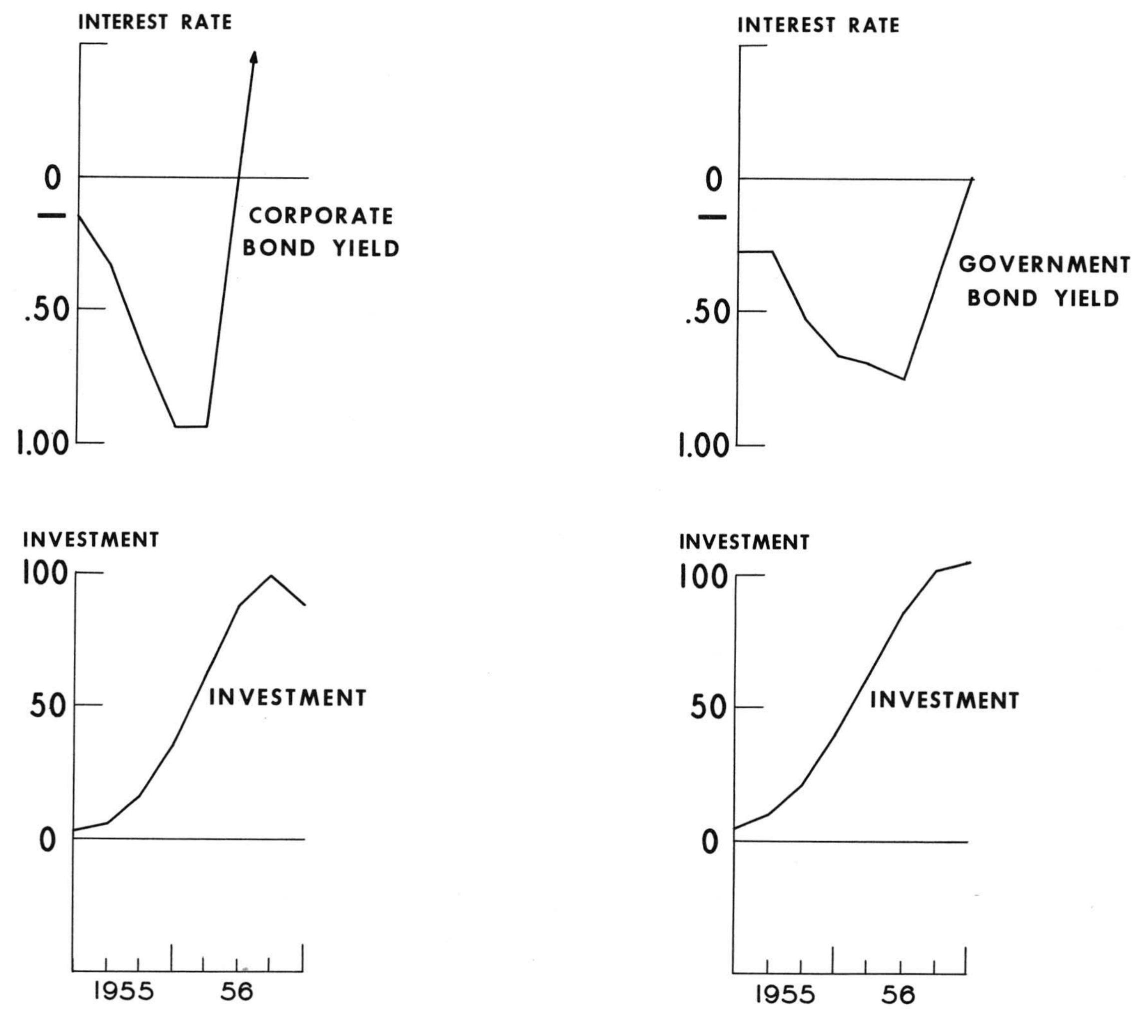


Chart H-9

INVESTMENT EFFECTS OF RISE IN INTEREST RATES AFTER 1958

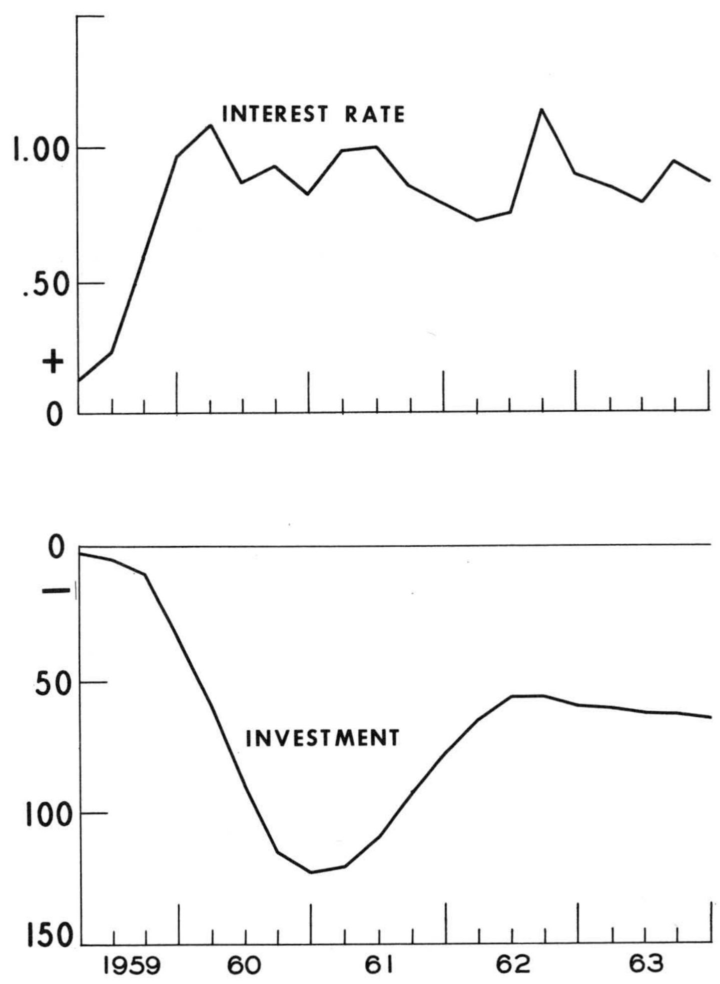
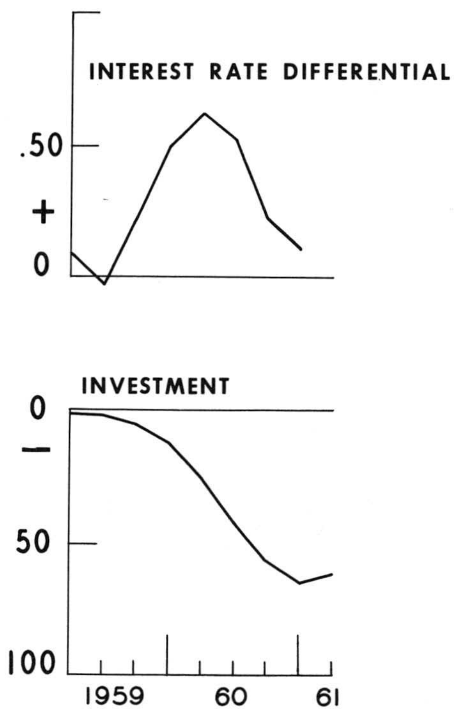


Chart H-10

INVESTMENT EFFECTS OF 1959-60 WIDENING OF CANADA-U.S. INTEREST RATE DIFFERENTIAL



The effect upon investment of the widening of the interest rate differential is of course more modest than the effect of the overall monetary restrictions shown in the previous experiment, since United States interest rates also rose during this period.

However, the effect remains substantial and is again felt at the time of the onset of the 1960-61 recession, when investment was $60 million per quarter smaller than it would have been with a more "neutral" monetary policy. Had domestic monetary policy changed the differential in the opposite direction, investment would have been stimulated to the same extent. It therefore follows that the effect of domestic monetary policy actions alone upon investment may be important substantial, being of the order of $120 million per quarter, or $480 million at annual rates. 8/

Summary of Simulation Results

It is worth summarizing the implications of the model made clear by these simulations.

1. Stimulating aggregate demand gives rise to an investment cycle— investment at first expands in response to the expansion of demand, and then contracts in response to the resulting growth of capacity. Such an oscillatory response complicates the task of maintaining the stability of aggregate demand.

2. Stimulating investment by monetary policies (or by tax structure changes which have an analogous effect) results in a strong growth in investment for a short period which then levels off near the peak level achieved. This result, which reflects the essential capital deepening character of investment made in response to a

lower cost of capital, is in accord with the empirical results for the United States recently obtained by Bischoff.

3. Sluggish growth, leading to a widening of the gap between actual and potential output, sows the seed of future sectoral bottlenecks and resulting premature inflation problems. During a period of slow growth, capacity expanding investment is cut back, with the result that when output moves back toward its full employment level capacity shortages are encountered before full employment is reached.

4. Finally, the model implies that there are important lags in the response to monetary and fiscal policies. These lags mean that the authorities should take into account the momentum built up by earlier policies in deciding upon the magnitude and timing of current policy changes.

One proviso is in order, one that we think is important enough to warrant a plea for additional research with existing data and perhaps for obtaining new types of data. None of the models presented in this paper incorporate forward-looking variables such as new orders, indices of leading indicators, contracts let, or investment appropriations themselves. Since Eckstein has found that incorporating new orders significantly improves the fit of the original de Leeuw model, it may be worth experimenting with these kinds of variables in Canada. The sluggishness of the best model obtained in this study near turning points and points of inflection suggests that there may be some mileage in this.

If it turns out that such forward-looking variables are important, the lags in the response to policy changes may be shorter, since policy measures may affect the forward-looking variables directly.

REFERENCES

1/ A regression of the GDP index on time and seasonal dummy variables yields a trend of 12.94 index points per quarter. As the moving sum and autoregressive transformations cancel out, the estimated effect on the constant term is simply obtained by multiplying the trend growth of 12.94 by the estimated capacity coefficient of -8.89.

2/ D.B.S., Fixed Capital Flows and Stocks Manufacturing, Ottawa, Queen's Printer, 1966.

3/ On the basis of the capital stock data for the economy as a whole made available by N.H. Lithwick, the 1959 ratio of the GDP index (1949 = 1000.) to the net capital stock (in constant 1957 dollars) is .0334. In order to make some allowance for the likely under-utilization of capacity in that year, the estimate of .035 is used.

A check on this coefficient is provided by the data for manufacturing published by D.B.S. (op. cit., Table 3, p. A.6). The ratio of constant dollar GDP to the net capital stock within that sector is multiplied by the ratio of the aggregate GDP index to constant dollar GDP. This procedure yields an estimate of .0359 for 1959.

4/ Any interaction between the two types of investment is neglected in these experiments, with the exception of experiment 2.

5/ See the discussion of alternative models in Section D above.

6/ This involves estimating a third equation predicting the effect of capital deepening investment upon output. This equation, based on the production function presented in Wilson and Lithwick, op. cit., is as follows:

$$\Delta F = .00647\ I^{d},$$

where ΔF is the change in the full-employment GDP index, and I^{d} is capital-deepening investment.

7/ See also the discussion in Section I below.

8/ Of course, under a fixed exchange rate, the monetary authorities could not vary the interest rate differential very much to achieve domestic policy objectives. However, it is important to know the consequences for investment of changes in the interest rate differential adopted for other policy purposes.

I. COMPARISON WITH THE YOUNG-HELLIWELL SURVEY RESULTS

The findings of this study may be contrasted to those of a recent questionnaire and interview study carried out for the Banking Commission by Young and Helliwell. They conclude:

> the evidence...suggests that the effects on capital expenditure of short-run changes in credit conditions over the range we have experienced in the last decade in Canada have in the aggregate been quite limited. 1/

This finding is disturbing, but hardly surprising, since most previous survey studies have yielded consistently negative findings on the importance of monetary policy, whereas, as is discussed in Section C above, recent econometric studies have shown consistently that interest rates or the cost of capital have an important effect on investment.

Many of the criticisms leveled at previous survey studies 2/ do not apply to the Young-Helliwell study. The questionnaire-interview survey is carefully planned and carried out; more important, the timing of this study is much more appropriate from the standpoint of the investigation of changes in monetary policy. However, White has argued 3/ that the Young-Helliwell results are in fact consistent with a larger short-run response to monetary policy, an argument which he supports by a detailed discussion of various small downward biases in the survey findings. As the majority of White's criticism is effectively rebutted by Young and Helliwell, 4/ his analysis cannot be used as a basis for rejecting their findings.

Rather than attempt to criticize their findings, let us inquire instead whether they are reconcilable with the results of the present study. Young and Helliwell emphasize the 1959-60 period of tight money since they feel that their survey results are more accurate for this period than for earlier or subsequent periods of monetary restraint. Consequently, we shall illustrate the discussion with references to that period alone.

Young and Helliwell estimate that the restrictive monetary policy of the 1959-60 period led to a reduction or a postponement of capital expenditures of between $50 million and $65 million in the peak three quarters of the cycle (last quarter of 1959 and first two quarters of 1960). 5/ With no definition of what is meant precisely by the degree of restriction during the period, we shall measure it by the difference between the interest rate prevailing during each quarter of the period and the interest rate that existed at the start of the period (last quarter of 1958). The results of simulation experiment 9 reported above can then be used for comparative purposes. The estimated time path of quarterly investment is shown in Table I-1. In the same peak three quarters investment is restricted by $185 million, a response considerably above that estimated by Young and Helliwell. 6/

One possible source of this discrepancy lies in the emphasis of the Young-Helliwell study upon the postponement or abandonment of projects already planned. Indeed, a strict interpretation of questions le and lf in their questionnaire would suggest that they confine their attention to whether capital expenditures on projects already planned are either postponed or reduced. 7/

This possible bias illustrates a weakness of the survey approach which is made clear by the careful wording of the key questions of the Young-Helliwell questionnaire. How is reliable evidence of projects abandoned in the very early planning stages—stages prior to the decision

TABLE I-1

EFFECTS OF 1959-60 RISE IN INTEREST RATES ABOVE LEVEL PREVAILING AT END OF 1958

Year/Quarter	Deviation of Interest Rate on Corporate Bonds From 5804 Level	Effect on Investment (Millions of Constant (57) Dollars)
5901	+.12	-1.3
5902	.23	-4.9
5903	.59	-14.5
5904	.97	-33.5
6001	1.08	-60.6
6002	.87	-90.9
6003	.43	-114.1
6004	.33	-123.9

Sum 5904-6002 Inclusive 184

See also Chart G-9 Above.

to commit funds (appropriate) for the project—to be obtained? The relevance of the Young-Helliwell and other survey findings may be largely confined to the postponement and abandonment of investment projects for which appropriations were previously made. If this were the case, the results of the survey studies could be consistent with the econometric findings of the time series studies, which assume that the cost of capital affects appropriations and that the appropriations-expenditure lag structure is predetermined.

Appropriations are defined as the final stage of approval for capital expenditures—a confirmation of plans previously made. 8/ In the United States manufacturing, where data on appropriation and cancellation are available, cancellations average only 6.7% of appropriations. 9/ Such a low rate of cancellations may in part account for the high negative response to the key questions in the Young-Helliwell questionnaire. The Young-Helliwell study refers to projects already planned, which would include projects planned but for which appropriations have not yet been made. However, the effect of monetary policy in the early planning stages would appear to be ruled out by questions 1e and 1f which refer to "projects" and request details about the projects. This emphasis is of course consistent with the aim of Young and Helliwell—mentioned at different points in their study—to measure the short-run effect of monetary policy.

A perusal of the rich detail of quotations provided by Young and Helliwell indicates, however, that some firms at least did not interpret this question to be restricted to the abandonment of projects already planned, since there are illustrations of projects tentatively examined and subsequently abandoned. However, having no information on the firms which responded negatively to the questionnaire, we do not know whether any of them interpreted this question to be so restricted.

Perhaps more important, is the fact that the Young-Helliwell study focuses on the effectiveness of domestic monetary policy whereas the present study attempts to measure the effect of Canadian interest rates, which reflect conditions in the United States capital markets as well as changes in Canadian monetary policy. This different emphasis suggests re-examining simulation experiment 10, which measures the impact of the widening of the differential between Canadian and the United States interest rates over the 1959-60 period.

The results, presented in Table I-2, show that the greater relative degree of restriction in Canada vis-a-vis the United States leads to a reduction in investment in the peak three quarters of $80 million, which is closer to the estimate published by Young and Helliwell. 10/ This simulation would appear at first glance to make perhaps too much allowance for the role of the United States interest rates, since it implicitly assumes that a neutral Canadian monetary policy is one that keeps interest rates moving in parallel with the United States rates. On the other hand, as our model does not distinguish a separate Canadian interest rate increase from a general North American interest rate increase, this calculation may overstate the impact of a change in interest rates which results from domestic monetary policy alone.

These considerations suggest that the conflict between the Young-Helliwell survey results and the results of the econometric analysis of the present study is much less than would appear at first glance. Let us therefore compare the salient conclusions drawn from each analysis. Young

TABLE I-2

EFFECT OF 1959-60 WIDENING OF CANADA-UNITED STATES INTEREST RATE DIFFERENTIAL

A. Based on Differential Between Yields on Corporate Bonds

Year/Quarter	Deviation of Interest Rate Differential from 5804 Level	Effect on Investment
5901	+.09	-1.0
5902	-.02	-1.6
5903	+.22	-4.5
5904	+.50	-12.5
6001	+.63	-25.4
6002	+.52	-42.5
6003	+.22	-57.1
6004	+.11	-64.2
Sum 5904-6002 Inclusive		80.4

B. Based on Differential Between Yields on Government Long-Term Bonds*

Year/Quarter	Deviation of Interest Rate Differential from 5804 Level	Effect on Investment
5901	+.076	-0.9
5902	+.126	-3.3
5903	+.270	-8.7
5904	+.403	-18.5
6001	+.456	-31.7
6002	+.323	-45.4
6003	+.390	-57.9
6004	+.460	-67.7
Sum 5904-6002 Inclusive		95.6

See also Chart G-9 Above.

* Estimated investment effects of changes in the yield differential on government bonds are based on equation E-6.6.

and Helliwell conclude that the short-run impact upon business fixed investment of domestic monetary policy is weak. The findings of the present study neither confirm nor deny this conclusion. An important conclusion of the present study is that the cost of capital is an important determinant of investment, but that the effects of changes in this cost are spread out over a period of six to eight quarters subsequent to the initial change. This conclusion is in turn neither contradicted nor confirmed by the results of the Young-Helliwell survey. In sum, the two studies focus upon quite different problems, and additional research will be required to specify the links between domestic monetary policy and Canadian interest rates, to examine the effect of changes in Canadian interest rates brought about solely by domestic monetary policy, and to determine the stages in the decision process between the germination of an idea and the completion of a facility at which the cost of capital enters in critical fashion, before the findings of either the Young-Helliwell study or the present study (or both) can be regarded as confirmed.

REFERENCES

1/ J. H. Young and J. F. Helliwell, The Effects of Monetary Policy on Corporations, p. 371.

2/ See W. H. White, op. cit.

3/ W. H. White, "The Stronger Effects of Monetary Policy on Corporations," in Economic Council of Canada, Conference on Stabilization Policies, Ottawa, Queen's Printer, 1965, pp. 53-88.

4/ See their comment on White's paper in Conference on Stabilization Policies, pp. 89-97.

5/ Young and Helliwell, op. cit., p. 67.

6/ The model yields even greater estimated restrictive effects in subsequent quarters.

7/ See the replica of the corporate survey questionnaire in Young and Helliwell, op. cit., p. 431.

8/ National Industrial Conference Board, "A New Survey of Capital Appropriations," Conference Board Business Record, 1956, pp. 418-434.

9/ Almon, op. cit., p. 183.

10/ Once again, however, the model shows a greater degree of restriction in subsequent quarters.

J. SUMMARY: POLICY IMPLICATIONS OF THE ANALYSIS

1. The Determinants of Investment

This study demonstrates that the new approach to the analysis of investment can yield empirical dividends when applied to Canadian as well as United States data. In contrast to most previous econometric analyses, 1/ the rate of interest has an important impact on investment in most models estimated in this study.

In part this reflects the increasing richness of the basic data, as the period of pegged interest rates fades into the past, and as time series covering periods of vigorous use of monetary policy grow longer. However, the building-in of some kind of appropriation-expenditure lag into the models is also important. If these lags are neglected, 2/ the resulting misspecified models are seriously affected by simultaneous equations bias. Current interest rates will reflect simply the tug-of-war between fluctuations in investment demand (caused in part by past changes in interest rates) and current monetary policy. The simple correlation between interest rates and investment that results will be positive. It is doubtful, moreover, whether the inclusion of such investment equations within a multi-equation econometric model and the use of simultaneous estimation techniques would overcome this effect of misspecifying the lag structure. 3/

The important effect of output in these models is, of course, less of a surprise. The effect of output on investment has been detected by numerous other authors. In contrast, the findings for the retained earnings variable, are mixed. While the retention variable is unimportant in the best equation, and is dropped altogether for the equation re-estimated for the forecasts, it is statistically significant in several of the models estimated. Furthermore, the retention variable is subject to errors of measurement greater than those of the other variables. As a result, our findings must be regarded as inconclusive as to the role of this variable.

2. <u>Investment Dynamics</u>

It has become fashionable to analyze "lags in monetary policy", 4/ and some writers have asserted that this lag is long enough and/or variable enough to call into question the use of discretionary monetary policy for stabilization purposes. The simulation experiments that are described in Section H above have the following implications for this policy issue:

a) The response to a change in interest rates is spread out over a seven to eight quarter period. However, about sixty per cent a the ultimate response is attained by the fourth quarter following the change in interest rates. If reasonable short-run forecasts of aggregate demand may be made a year ahead, and provided that the monetary authorities take into account the momentum built up by past policies, the model implies that it would be feasible to use monetary policy for stabilization purposes. The reader should bear in mind, moreover, that the sluggish performance of the model near turning points and points of inflection suggest that these lags may be overstated. It would be interesting to determine whether the lag structure of the model would be affected by the inclusion of forward-looking variables.

b) The model is constructed on the assumption that the lag structure is fixed rather than variable. It is therefore interesting to note that the apparent lag between monetary policy and the resulting investment response is affected by the kind of policy changes adopted. This is apparent in a comparison of Charts H-4 and H-5, which demonstrates that it may be misleading to measure lags by the analysis of turning points in the series, as sharp increases in interest rates could cause a downturn in investment sooner than would a more moderate interest rate increase.

c) Because of the lag structure, it is important that the monetary authorities take into account the momentum built up (in one direction or the other) by their own past actions. If a clear signal for a turnabout in policy occurs, the lag structure necessitates greater rather than less vigour in monetary expansion or contraction during the period immediately after the policy turnaround. Timid policies at this point may contribute to instability. 5/

Once investment begins to respond, on the other hand, the resulting multiplier effects of aggregate demand (which are not incorporated into the simulations of investment response to interest rate changes) will tend to produce an additional oscillatory investment response, the consequences of which are discussed below.

3. The Degree of Independence of Canadian Monetary Policy

The extent to which domestic monetary policy can play an independent role is perhaps the most important issue in the analysis of Canadian Monetary

Policy. It is apparent that under a fixed exchange rate, this role is much more limited than under the flexible exchange rate which prevailed over most of the postwar period. 6/

What our results suggest (see especially simulation experiment 10 in Section H) is that the changes in the United States-Canadian interest differential that occurred during the fluctuating exchange rate period did have important effect on investment demand. This means that the restrictive effects of tight money were not confined to its exchange rate effects (which may be largely offset by stabilizing short-term capital movements).

4. Implications of the Findings for Fiscal Policy

We note briefly the implications of the analysis for fiscal policy 7/ and for other general policies to affect aggregate demand.

First, the significance of the output variables in the models implies that policies to stimulate aggregate demand will have important effects on investment after a few quarters have elapsed. Second, as simulation experiment 1 makes clear, this response is likely to be reversed in subsequent quarters as the capacity built in response to the initial stimulus is completed. A steady rate of expansion of demand would greatly mitigate this difficulty.

These findings, based as they are on a simple two-equation investment model, are suggestive rather than conclusive. It will be interesting to explore investment and output paths when the model is expanded to incorporate the multiplier effects of changes in investment expenditures.

5. Implications of the Findings for the Analysis of Tax Changes

Finally we return to our starting point. The analysis of the cost of capital effects of the tax reforms recommended by the Commission is included in Chapter 37 of their Report, which concludes that the net effect of the reforms will be to reduce the overall cost of capital to most resident-owned firms. This results mainly from the reduction in the cost of equity capital (retained earnings and new stock issues) brought about by the integration of the corporate and personal income taxes.

Our analysis suggests that for each one per cent reduction in the cost of capital achieved, investment demand would in the long run increase by two thirds of one per cent. 8/ This is a point estimate. Given the estimated variance of the regression coefficient of equation E-6.3, the range of the likely long-run investment response is from 0.29 to 1.05 per cent for each one per cent change in the cost of capital. 9/

However, we must caution the reader as to the highly tentative nature of these conclusions. There are a number of factors which could account for a response to these tax induced changes in the cost of equity capital which is different from the response to interest rate changes. 10/ These are as follows:

a) Possible errors of aggregation. The effect of the tax changes will vary across firms in a pattern different from the effect of interest rate changes. The effect of the tax changes will vary with the industry, capital structure, size and degree of foreign ownership of the firm. If the cost of capital elasticity of investment demand varies with these characteristics of firms, the aggregate response to the tax-induced change will differ from the aggregate response to an interest rate change.

b) The indeterminancy of behavior of firms in certain market situations. In regulated industries, the response to tax-induced changes will depend on the policies adopted by the regulatory authorities. Full forward-shifting of the tax in price changes may occur under some regulatory policies, in which case no stimulus to investment is provided by the tax changes. In unregulated but monopolistic or oligopolistic market situations, the tax changes will affect monopoly profits as well as the cost of capital. How firms respond to the tax changes will depend in part on the goals of these firms, which, since they sell in imperfectly competitive markets, need not be confined to profit maximization. It is worth noting, however, that extreme examples of non-maximizing behavior—such as limiting investment to available internally generated funds—imply an insensitivity both to interest rate changes and to tax changes which affect the marginal rate of return (at a given level of retentions).

c) The possibility that the indirect consequences of the tax changes may offset in large part the direct effects. For instance, the Report predicts that interest rates will rise moderately after the reforms. This increase in the cost of borrowed capital will act to offset in part the reduced cost of equity capital. A second example is provided by the balance-of-payments consequences of the reforms, which the *Report* argues will probably be moderately favorable, but recognizes that net adverse effects are possible. If the latter were to occur, even higher interest rates would be necessary to maintain the capital inflow in order to maintain balance-of-payments equilibrium. This would further erode any stimulus to investment provided by the reduction in the cost of equity capital.

These considerations limit the accuracy of long run tax response estimates derived from the models fitted in this study. However, they do not wholly vitiate such calculations. Rather the calculations should be interpreted as indicating the approximate orders of magnitude of the response and a wide variety of responses in different sectors of the economy may be expected.

The timing of the response to tax changes is likely to deviate substantially from the pattern derived from a mechanical application of the model developed in this paper. Aside from uncertainties following upon the enactment of the reforms, firms may take time to adapt to changes in the tax laws. In contrast to fluctuations in interest rates which lie within the historical experience of firms, tax reforms such as integration and capital gains taxation would represent novel experience. Making the necessary adaptation of a firm's financial, dividend and investment policies to such a fundamental set of tax changes may require more time than would adaptation to the more routine changes in long-term interest rates. We should therefore not be surprised to find that any investment response to the tax reform is felt more gradually than a response to a change in long-term interest rates.

REFERENCES

1/ See the studies of investment in Canada cited in reference 25/ in Section C above.

2/ We do not argue for the particular lag structure of models used in this paper; rather we emphasize that henceforth econometricians should pay explicit attention to the appropriation— expenditure lags in the design of their models. This approach may even be a valuable one for the analysis of annual data, although the simultaneity problem in models fitted to such data may remain acute.

3/ There is an element of simultaneity in some of the models estimated in the paper, since the independent variables are weighted averages of current and past changes in the basic variables. However, the weight of current changes in these variables is only .02 in the "best" equation (E-6.3). The resulting simultaneous equation bias will therefore be trivial, and ordinary least squares no doubt will yield estimates more efficient than those obtainable with the available simultaneous equation techniques.

4/ See M. Friedman, "The Lag in Effect of Monetary Policy," Journal of Political Economy, Oct. 1961, pp. 447-66, D. J. Daly, "The Scope for Monetary Policy - A Synthesis," in Conference on Stabilization Policies, pp. 1-51, and H. G. Johnson and J. Winder, op. cit.

5/ We leave aside the question of the importance of maintaining stability in the financial markets themselves, which is another goal of monetary policy.

6/ See R. A. Mundell, "Capital Mobility and Stabilization Policy Under Fixed and Flexible Exchange Rates," Canadian Journal of Economics, Nov. 1963, pp. 475-85.

7/ Fiscal policy, as distinct from tax structure changes, denotes changes in expenditures or overall tax levels.

8/ The likely long-run investment consequences of the enactment of the recommended reforms are analyzed in a companion study. (Wilson and Lithwick, op. cit., Ch. 7). These estimates are based on the estimated long-run interest elasticity from equation E-6.3 above and the estimated relative effect of these reforms upon the after-tax corporate rate of return. The latter estimates are based on the predicted revenue effects published in Volume 4 of the Report.

9/ This range is based on the elasticities estimated at the end of the estimation period calculated at the extreme ends of the 95 per cent confidence interval.

10/ In addition, the estimated cost of capital effects of the tax reform provided in Volume 4 of the Report are highly tentative. It is difficult to measure these effects and to determine what are equivalent interest rate changes.

APPENDIX A

THE RELIABILITY OF THE QUARTERLY INVESTMENT SERIES

As is made clear by the notes appended to the quarterly National Income Accounts, the quarterly estimates of investment expenditures are not based directly on expenditure data as are the annual estimates. Instead, quarterly series are derived by allocating the annual data on the basis of quarterly interpolative indices. For non-residential construction, the interpolative index is based on data from employment, hours worked, and prices in the construction industry. For machinery and equipment, the interpolator is based on shipments of these capital goods, after adjustment to include imports and to exclude exports and government purchases. 1/

Whether the quarterly series of investment expenditures so constructed incorporates systematic errors, which could affect our findings, is a question worth examining. Since systematic errors would likely affect the timing of movements in quarterly series, we shall investigate the lead-lag relationships revealed between the Canadian and the United States data. As the United States quarterly investment series is based directly upon expenditure data, it will be used as a bench mark to examine possible timing errors in the Canadian quarterly series.

The procedure used is as follows:

1. Lead-lag patterns between the Canadian annual series and alternative four-quarter groupings for the United States series are determined. Per cent changes as well as levels are examined. 2/

2. Lead-lag patterns between the Canadian quarterly series and the

United States quarterly series are examined. Four-quarter overlapping percentage changes as well as levels are examined.

If systematic timing errors are introduced as a result of the method of construction of the quarterly Canadian Investment series, one would expect that the lead-lag patterns between the two quarterly series would be shifted relative to that between the Canadian Annual Series and the Four-Quarter Groupings of the United States quarterly data. If the lead-lag pattern does not shift, this would suggest that such timing errors are relatively unimportant for quarterly data.

The results of the analysis of investment levels is presented in Table A-1 and that of the analysis of per cent changes in A-2. The comparison of absolute levels shows that maximum correlation occurs with a three-quarter lag in both annual and quarterly Canadian investment series. The analysis of four-quarter per cent changes reveals a maximum correlation with a one-quarter lag for both annual and quarterly Canadian investment series.

It is therefore apparent that very little, if any, change in the lead-lag pattern of the correlation coefficients occurred, whether the comparison is based upon levels or upon changes across four quarters. This indicates that serious systematic timing errors are not introduced by the interpolative procedures used.

A second test involves examining the lead-lag pattern between machinery and equipment expenditures and the output and shipments of capital equipment in the United States. As shipments form the basis of the quarterly interpolative series in Canada, it would be instructive to learn whether in the United States capital goods output and shipments lead, lag, or are coincident with capital expenditures. If the series are coincident, this would be an additional piece of evidence suggesting that the use of such interpolative indices does not introduce timing errors in the quarterly data.

The results of the analysis of the timing relationship between expenditures (in constant dollars) and output is presented in Table A-3, that of expenditures (in current dollars) and shipments in Table A-4. These analyses show that both shipments and output are nearly coincident with capital expenditures in the United States. The coincident comparisons yield the highest correlation in all trials with quarterly data although there is a hint that output and shipments may precede expenditure because the correlation obtained with a one-quarter lag is higher than the correlation with a one-quarter lead. The analysis of monthly lags confirms this result, and indicates that changes in production and shipments appear to precede expenditures by, at most, one month.

On the basis of these results and the results of the previous analysis of the lead-lag relationships between investment in Canada and the United States we conclude that systematic timing errors large enough to affect the analysis carried out in this paper have not been introduced by the interpolative methods used to construct the quarterly investment series. Unsystematic errors of observation in the dependent variable do not, of course, affect the validity of the positive statistical findings. Such errors simply increase the variance of the estimates and consequently make it more difficult to reject the null hypothesis.

TABLE A-1

ANALYSIS OF LEAD-LAG PATTERNS BETWEEN CANADIAN AND UNITED STATES INVESTMENT SERIES: ABSOLUTE LEVELS FOR 1951-65 PERIOD

	Correlations for Alternative Lags (in Quarters)** of the U.S. Series							
	Lag **							
Comparison	4	3	2	1	0	-1	-2	-3
Canadian Quarterly and United States Quarterly (Seasonally Adj.)	.9108	.9174	.9137	.9058	.8934	.8692	.8418	.8100
Canadian Annual and Alternative Four-Quarter sums of United States Quarterly*	.9031	.9110	.9076	.8957	.8676	.8359	.8056	.7754

* For example, the correlation for lag 1 is the correlation of annual Canadian Investment and United States investment summed from the last quarter of the preceding year to the third quarter of the current year.

** A + sign indicates that changes in the United States series preceded changes in the Canadian series, a - sign indicates that changes in the Canadian series precede changes in the United States series.

TABLE A-2

ANALYSIS OF LEAD-LAG PATTERNS BETWEEN CANADIAN AND UNITED STATES INVESTMENT SERIES: FOUR-QUARTER PER CENT CHANGES* FOR 1951-65 PERIOD

Comparison	Correlations for Alternative Lags (in Quarters) of the U.S. Series							
	Lag**							
	4	3	2	1	0	-1	-2	-3
Canadian Quarterly and United States Quarterly (Seas. Adj.)	.3806	.4987	.5544	.5684	.5540	.4265	.2714	.0623
Canadian Annual and Alternative Four-Quarter Groupings of United States Quarterly	.4536	.5932	.6947	.7275	.5716	.4237	.2529	.0478

* For the annual comparison, these are simply the correlation between the per cent changes in the two series. For the quarterly series they are the correlations of the per cent changes between the current quarter and the corresponding quarter in the preceding year. This approach is used for the comparisons since the annual change is simply an aggregation of the four-quarter changes in the quarterly series.

** A + sign indicates that changes in the United States series preceded changes in the Canadian series, a - sign indicates that changes in the Canadian series precede changes in the United States series.

TABLE A-3

LEAD-LAG PATTERNS BETWEEN EQUIPMENT EXPENDITURES (IN CONSTANT DOLLARS) AND PRODUCTION OF BUSINESS EQUIPMENT IN THE UNITED STATES (195301-6601 INCLUSIVE)

1. Correlation for Alternative Lags (in Quarters) of Production.

	Lag*						
Comparison	3	2	1	0	-1	-2	-3
Levels	.9535	.9535	.9687	.9737	.9617	.9327	.9053
% Changes	.0749	.2568	.6069	.6920	.3828	.0922	-.0761
Four-Quarter % Change	.1821	.5243	.7734	.8399	.6882	.3672	.0944

2. Correlations for Alternative Monthly Lags of Production.

	Lag*				
Comparison	2	1	0	-1	-2
Levels	.9775	.9799	.9809	.9806	.9792
% Changes	.6889	.7170	.6959	.6202	.5277
Four-Quarter % Changes	.8232	.8440	.8398	.8198	.7780

* A positive lag indicates that changes in production precede changes in expenditure; a negative lag indicates that expenditures precede production.

TABLE A-4

LEAD-LAG PATTERNS BETWEEN EQUIPMENT EXPENDITURES (IN CURRENT DOLLARS) AND SHIPMENTS OF EQUIPMENT IN THE UNITED STATES

1. Correlations for Alternative Lags (in Quarters) of Shipments.

	Lag*						
Comparison	3	2	1	0	-1	-2	-3
Levels	.9458	.9634	.9797	.9868	.9791	.9606	.9432
% Changes	.0253	.2877	.5449	.6076	.3380	.0879	-.1223
Four-Quarter % Changes	.1183	.5024	.7989	.9048	.7182	.3451	.0250

2. Correlation for Alternative Monthly Lags of Shipments.

	Lag*				
Comparison	2	1	0	-1	-2
Levels	.9814	.9837	.9848	.9823	.9795
% Change	.5636	.5722	.5743	.5139	.4433
Four-Quarter % Change	.8448	.8738	.8738	.8439	.7705

* A positive lag indicates that changes in shipments precede expenditures; a negative lag indicates that changes in expenditures precede shipments.

REFERENCES

1/ D.B.S., *National Accounts Income and Expenditure, by Quarters, 1947-61*, pp. 93-94.

2/ See the notes to Table A-1.

APPENDIX B

NOTES ON SOURCES OF DATA

For the convenience of the reader the descriptions of the series used are laid out in Table B-1. The following notes supplement this table.

1. With the exception of the series used in Section F, the data series analyzed in this paper were compiled by the Commission staff in the spring and summer of 1964, and incorporate the revisions in series made up to the end of 1963.

2. The series used in Section F incorporate the revisions in the published series available in June 1967.

3. All series used are based on readily available published information with the exception of the potential output series. These are based on linear interpolation between annual estimates of a preliminary series constructed for the use of the Commission staff.

In connection with the forecasting tests some equations incorporating the gap between actual and potential output as the "capacity requirements variables" are re-estimated using revised published data and a potential output series obtained by linear interpolation of the annual estimates published by Wilson and Lithwick. In no instance does this re-estimation yield improved results.

4. Inspection of the alternative long-term government bond yield series during the periods of overlap indicated that any linking adjustment would be quite arbitrary. As the difference between the series are quite small in any case, no linking adjustment is made.

5. The weights for the appropriations-expenditure lag structure are taken from the equation for total manufacturing reported by Almon (*op. cit.*, Table II, p. 188) after normalization to make the sum of the weights equal to unity.

TABLE B-1

SOURCES OF AND DETAILS OF CONSTRUCTION OF SERIES USED

Symbol	Name	Source	Notes
Q	GDP Index (1949=1000)	DBS Indexes of Real Domestic Product by Industry of Origin	Raw series are used throughout. The revised series used in Section F incorporate the fundamental revisions published in DBS, Annual Supplement to the Monthly Index and Industrial Production, May 1966.
Y	Gross National Expenditure in Constant (1957) Dollars	DBS National Accounts	Raw series are used throughout.
A-P	Inverse Gap between Actual and Potential Output	See Note 3	Potential output series is obtained by linear interpolation between annual estimates; annual output is seasonally adjusted at annual rates.
Ret	Deflated gross Retention of Corporations	DBS National Accounts and DBS Corporation Profits	Retained profits reported in the National Accounts plus depreciation reported in Corporation Profits deflated by the implicit deflator for business fixed investment (exc. housing) calculated from National Accounts data. Raw series are used throughout.
r_c	Corporate Yield Bond	McCleod Young Weir & Co.	Average of the yields on 10 industrial and 10 utility bonds. (quarterly averages of monthly data).
r_g	Long-term government bond yield	Bank of Canada	Prior to 1952 the 15-year theoretical yield is used; from 1952 to June 1958 the 20-year theoretical yield is used; after June 1958, an average of the yields on all direct Government of Canada securities with at least 5 years to maturity or earliest call date is used. (Quarterly averages of mid-month figures) See also Note 4.
$\frac{q}{p}$	Relative Price of Capital Goods	DBS National Accounts	This is the ratio of the implicit deflator for business fixed investment (exc. housing) to the implicit deflator for gross national expenditure.
t	Effective tax rate on Corporate Gross Profits	DBS Corporation Profits	Corporate tax accruals divided by the sum of before-tax profits and corporate depreciation.
I	Business fixed investment (exc. housing) in constant 1957 dollars	DBS National Accounts	The sum of expenditures on non-residential construction and machinery and equipment. For equations with a zero or two-quarter decision lag, raw data are used, and seasonal dummy variables incorporated to account for seasonal effects. For equations with a four-quarter moving average decision lag, seasonally adjusted data are used.