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PRACTICAL APPLICATION OF LINEAR PROGRAMMING AND NETWORK FLOWS — Proceedings of the Technology Transfer Workshop

PROGRAMMATION LINÉAIRE ET DÉBITS DU RÉSEAU: MISE EN PRATIQUE — Comptes rendus de l'atelier sur l'échange d'informations d'ordre technologique

TECHNICAL WORKSHOP SERIES NO. 3
ÉTUDE N° 3, SÉRIE DES ATELIERS TECHNIQUES

INLAND WATERS DIRECTORATE
OTTAWA, CANADA, 1982

DIRECTION GÉNÉRALE DES EAUX INTÉRIEURES
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Canada

PROCEEDINGS

of the

TECHNOLOGY TRANSFER WORKSHOP

PRACTICAL APPLICATION OF LINEAR PROGRAMMING AND NETWORK FLOWS

Hull, Quebec, September 15 and 16, 1981

TECHNICAL WORKSHOP SERIES No. 3

COMPTES RENDUS

de

L'ATELIER SUR L'ÉCHANGE D'INFORMATIONS D'ORDRE TECHNOLOGIQUE

PROGRAMMATION LINÉAIRE ET

DÉBITS DU RÉSEAU : MISE EN PRATIQUE

Hull (Québec), les 15 et 16 septembre 1981

ÉTUDE N° 3, SÉRIE DES ATELIERS TECHNIQUES

FOREWORD

The objective of this workshop was to improve the ability, efficiency and effectiveness of the Inland Waters Directorate to fulfill its responsibilities related to water management by:

(a) transferring to systems, engineering, management and scientific personnel the knowledge and experience acquired during recent years in the successful application of linear programming and network flows;

(b) correcting many misconceptions generally harbored by Canadian water resources specialists concerning systems engineering techniques such as linear programming and network flows; and

(c) identifying priorities for any further applications of systems engineering techniques.

About 50 pages of technical material describing linear programming, separable programming, and network flows were sent to workshop participants about five weeks in advance of the workshop. All of the participants who were novices in these optimization techniques were expected to have read this material before arrival at the workshop. Documentation manuals of the HCMS (Hydro-Configuration Modelling System) model and the Great Lakes SYSNET model were forwarded at the same time to participants. Participants with experience and training in optimization techniques were expected to review these manuals so that they could contribute positively to the workshop discussions.

Participants at the workshop were exposed to up-to-date optimization methodology to gain an insight into the appropriate application of this methodology to water resources management problems. Special emphasis was placed on the integration of optimization techniques, more specifically network flows and linear programming, with non-decision-making simulation models for increasing the effectiveness of contemporary mathematical modelling studies. Three documented models were discussed in detail to acquaint the attendees with the flexible application of recently developed large-scale modelling systems.

By means of a panel discussion during the final day of the workshop participants probed possibilities for further extending these optimization techniques to water resources problems.

The workshop Chairman was Lloyd Parker, Director of Management and Information Services, Hiram Walker Resources Ltd., Toronto. Lloyd is the current national Treasurer of the Canadian Operational Research Society (CORS), past president of the Ottawa Section of CORS, and a member of CIPS, ORSA and TIMS. He obtained his B.Sc. in Engineering Physics at Queen's University and his M.A.Sc. in Electrical Engineering at

the University of British Columbia. He has had many years of experience in applying systems engineering techniques to projects ranging from water resources to energy and transportation. He has recently assisted Energy, Mines and Resources in the revamping of the Emergency Petroleum Allocation Model. Lloyd has also been intimately involved in the development of the water resources package RRMS (River Regulation Modelling System), HCMS (Hydro-Configuration Modelling System), and SYSNET (network flows model for the Great Lakes).

The Proceedings are divided into three parts: the first part consists of a summary of the three documented models discussed at the workshop. The second part is a summary of the panel discussion and recommendations. The third part is an appendix containing the advance material for the workshop. This advance material was reviewed in lectures by Lloyd Parker with assistance from Don Farley during the workshop. A separate set of Mr. Parker's notes and visual illustrations were sent to the workshop attendees.

AVANT-PROPOS

Le but de l'atelier était d'améliorer l'habileté, l'efficacité, et la compétence de la Direction générale des eaux intérieures à remplir ses responsabilités reliées à la gestion des eaux en:

(a) transférant au personnel des systèmes, du génie, de la gérance et de la science les connaissances et l'expérience acquises durant les années récentes dans l'application réussie de la programmation linéaire et les débits du réseau;

(b) corrigeant plusieurs des malentendus généralement entretenus par les spécialistes canadiens des ressources en eau concernant les techniques de la recherche opérationnelle telles que la programmation linéaire et les débits du réseau; et

(c) identifiant les priorités pour les applications futures des techniques de la recherche opérationnelle.

A peu près 50 pages de matériel technique décrivant la programmation linéaire, la programmation séparable, et les débits du réseau étaient envoyées aux participants à l'atelier 5 semaines avant son déclenchement. On s'attendait à ce que tous les participants qui étaient des novices dans ces techniques d'optimisation lisent ce matériel avant d'arriver à l'atelier. Des manuels de documentation sur le modèle HCMS (Hydro-Configuration Modelling System) et le modèle des Grands lacs SYSNET étaient expédiés aux participants en même temps. Les participants expérimentés et entraînés dans les techniques d'optimisation auraient révisé ces manuels de façon à contribuer positivement aux discussions de l'atelier.

Les participants à l'atelier étaient exposés à une méthodologie d'optimisation de pointe, pour comprendre profondément l'application appropriée de cette méthodologie aux problèmes de la gestion des ressources en eau. Un accent spécial était placé sur l'intégration des techniques d'optimisation, tout spécialement les débits du réseau et la programmation linéaire, avec les modèles de simulation sans pouvoir de décision pour augmenter l'efficacité des études contemporaines relatives à la modélisation mathématique. Trois modèles documentés étaient examinés en détail pour familiariser les participants avec l'application flexible des modèles à grande échelle récemment développés.

Au moyen d'une discussion durant la journée finale de l'atelier, les participants ont exploré les possibilités pour des applications plus étendues des techniques d'optimisation aux problèmes de ressources en eau.

M. Lloyd Parker, directeur des services de gestion et d'information de la société Hiram Walker Resources Ltée (Toronto) a servi de président de l'atelier. Lloyd est présentement le trésorier de la

Société canadienne de la recherche opérationnelle et il est membre de plusieurs organismes en informatique. Il a obtenu son baccalauréat en génie physique à l'université Queen's, et sa maîtrise en génie électrique à l'université de la Colombie-Britannique. Il a plusieurs années d'expérience dans l'application des techniques de l'assistance technico-commerciale à des projets portant sur les ressources en eau, l'énergie et le transport. Tout récemment, il a aidé le ministère de l'Energie, des Mines et des Ressources à réorganiser le modèle d'allocation d'urgence de pétrole. Lloyd a été intimement impliqué dans le développement des programmes d'ordinateur RRMS (River Regulation Modelling System), HCMS (Hydro-Configuration Modelling System) et SYSNET (débits du réseau pour les Grands lacs).

Le compte rendu se divise en trois parties. La première consiste en trois modèles documentés discutés à l'atelier. La deuxième partie présente un sommaire de la discussion d'un groupe d'experts et de leurs recommandations. La troisième partie est un appendice qui contient du matériel technique décrivant la programmation linéaire, la programmation séparable, et les débits du réseau. Lloyd Parker, avec l'aide de Don Farley, a révisé ce matériel pendant l'atelier. Un ensemble séparé des documents et des illustrations visuelles employés par M. Parker était envoyé à chaque participant à l'atelier.

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SUMMARY OF DOCUMENTED MODELS DISCUSSED AT THE WORKSHOP

SOMMAIRE DES MODÈLES DOCUMENTÉS, DISCUTÉS À L'ATELIER

SUMMARY OF DOCUMENTED MODELS DISCUSSED AT THE WORKSHOP

Hydro-Configuration Modelling System (HCMS)

The Hydro-Configuration Modelling System (HCMS) is a generalized planning model which was developed during 1975/76. It has been used extensively in Ottawa River basin studies.

In essence, HCMS provides the capability to define a model of a given or proposed multireservoir river system with hydroelectric generators, and then to simulate the rational operation of the system over a long period of time. By carrying out this exercise for each of a number of proposed engineering/structural changes to a given system and/or a number of proposed sets of operating rules, the long-range impacts of these proposals can be assessed.

Specifically, a linear programming model of the river system is first generated by HCMS in such a way as to represent:

- physical characteristics of individual reservoirs, control structures, and river channels,
- linkage characteristics describing the river/reservoir/ hydrogenerator network,
- dynamic constraints relating to permissible operation of the system, and
- operating goals or targets.

The structure of the resulting LP model is as follows:

Variables

- reservoir storage/elevation,
- reservoir discharges, and
- artificial/relaxation variables (violations).

Constraints

- water balance equations,
- reservoir elevation-volume relationship,
- reservoir storage-discharge/maximum-discharge relationships,
- power generation relationships,
- operating bounds on variables due to flood protection, navigation, recreation and environmental considerations, and
- arbitrary or special constraints or targets.

The objective function is as follows:

- minimize weighted sum of violations of constraints,
- minimize weighted deviation from target values for certain variables, and

- maximize potential for satisfying future constraints (in the specific application described herein, the main component of the objective function to be maximized was the estimated *potential* hydroelectric energy remaining in the system at the end of each decision period).

In normal use, the model is applied to a series of individual time steps, each representing a simulated decision period. The model's outputs (optimal releases for the period and resulting end-of-period

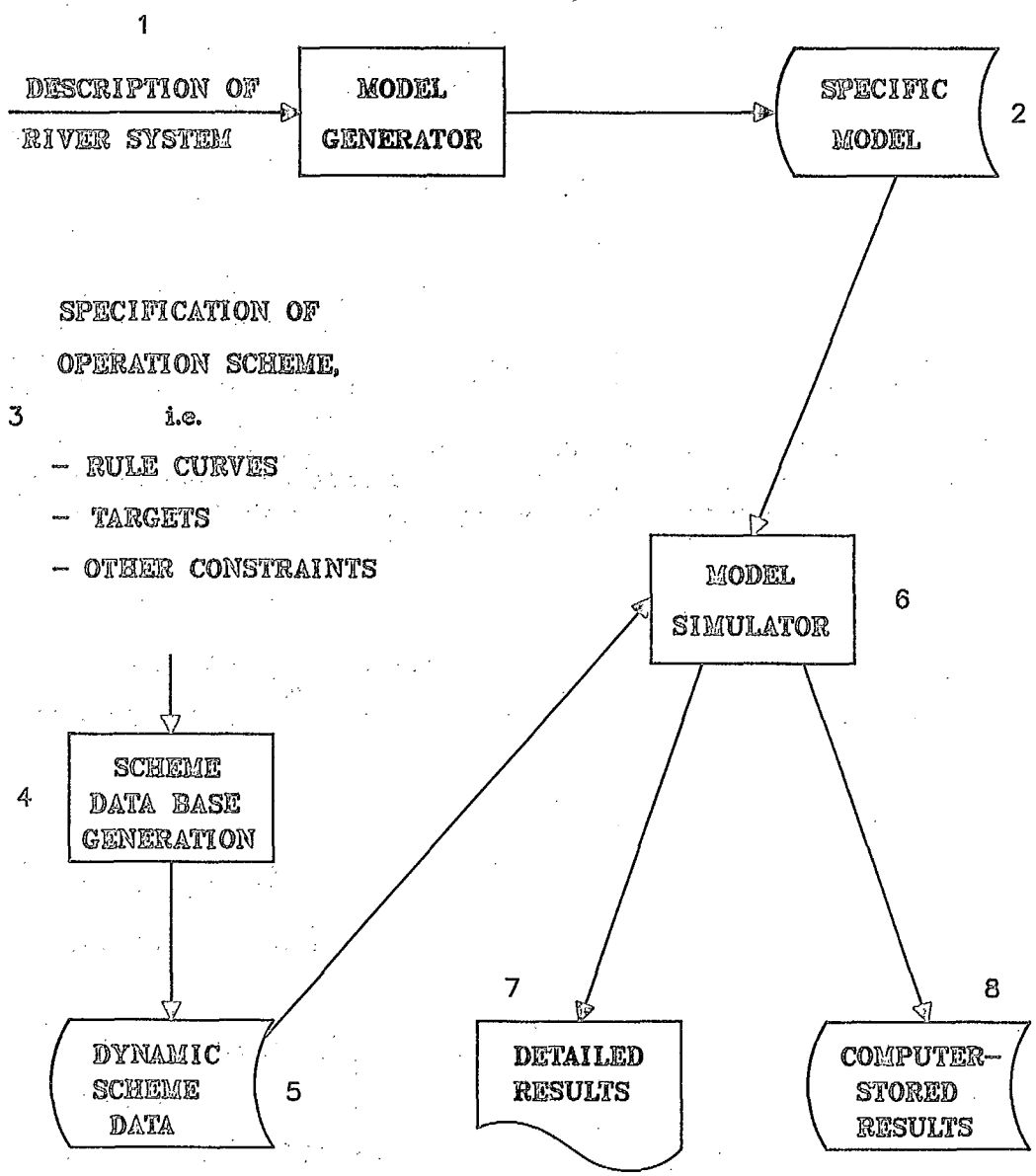


Figure 1. Overall model flow.

reservoir contents) are then used to provide starting values for the next period. At each stage, "optimal" releases are determined subject to current conditions (including a time series of natural inflows computed from the historical hydrometric records) and current constraints. This procedure is carried out for each time period in the simulated period of record, thereby simulating "rational behavior" (as modeled by optimal decision-making), given historical sequences of natural inflow and current knowledge of the system.

Through the repeated application of the model for various sets of operating rules and/or modifications to the physical system (as represented by storage capacity, channel capacity, control structure location, etc.), comparisons can be made between potential results for the various schemes proposed. The overall application of the model is shown pictorially in Figure 1.

First, a description of the river system (*1*) is used to generate a skeleton linear programming model of the system (*2*). Next, the specific scheme to be analyzed is represented in the form of data values for storage and discharge bounds, etc. (*3*). These are converted to machine-readable form (*4*) for use during the simulation. Finally, the model simulator (*6*) is used to solve the model repetitively for each time period of record to be simulated, and model outputs in the form of both printed results and a computer file are produced.

Figure 2 illustrates the overall logic of the model simulator phase.

The potential energy is described mathematically as follows:

$\Sigma P_i \, HR_i$ (units - MW·s)

where HR_i = reservoir level elevation of reservoir i and
$P_i = \Sigma CCP_j / DHRDV_i$
- j represents the power houses below i,
- CCP is the characteristic coefficient of the power house in megawatts per cubic foot per second, and
- $DHRDV_i$ is the rate of change of head with reservoir volume at reservoir i.

Auxiliary variables are employed in the model to permit violations of stated target constraints to avoid infeasible solutions which would otherwise occur. An example of the need for an auxiliary variable would be a specified reservoir level constraint being physically unachievable, given the maximum discharge capacity of the outlet and the relatively high inflow. Another example would be the inflow to a reservoir not being sufficient to satisfy a minimum level recreational constraint. The model is forced to try to keep these auxiliary variable

values at zero by applying large negative cost coefficients which severely penalize the objective function. Constraints that are never permitted to be violated are absolute maximum levels for reservoirs (otherwise, overtopping would occur) and maximum flow through turbines and spillways.

The model has the capability to maximize the primary power of the system, i.e., to produce the maximum continuous power generation that

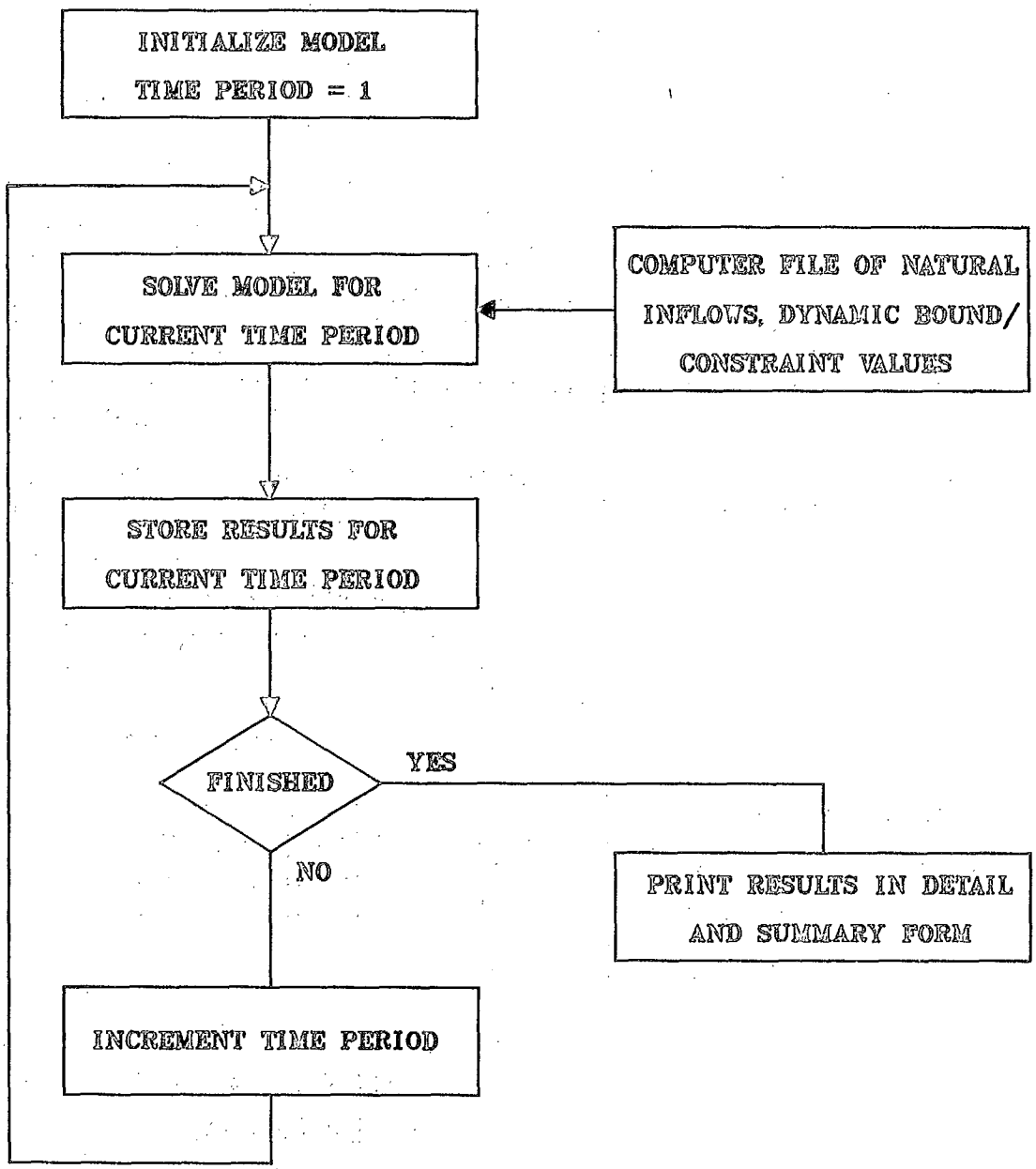


Figure 2. Model simulator logic.

can be guaranteed for the period of record under study. An option which is often used in the model is to satisfy the constraint of a stated primary power value for the system.

The model accepts the following three types of structures in the configuration:

Type 1 - Simple storage, i.e., a storage reservoir with a controlled outlet with no power generating units,
Type 2 - Power plant with reservoir storage, and
Type 3 - Run of river generating plants.

Any number of these plant types up to 99 can be linked together by river sections of varying lengths. Lag time for water to flow down these links is specified as a constant for each link with a maximum value equal to the unit time period.

Given the network configuration, the model generator automatically generates the LP model which consists of three sections:

(a) continuity equations,
(b) prime power achievement constraint, and
(c) objective function.

The model consists of two major elements - a "Driver" and a "Linear Programming Package." DOSPB, a sub-routine of the IBM SL-MATH package, is used for the linear programming (LP) solution. The model can use any unit time period (three days to one month), provided that the unit time period exceeds the longest lag time used in the routing calculations.

Because of the dimensions of the problem and the nonlinear nature of the relationships, the problem is modularized as follows:

(1) The total period of record (say 30 years) is broken down into the selected unit time periods.

(2) A guess is made concerning the achievable prime power over the 30-year period.

(3) For each unit time period, the system attempts to satisfy all constraints (including prime power) while maximizing a "future power potential" function. This is achieved through the LP model, which is updated with relinearized coefficients and bounds, and subsequently solved. The Driver analyses the LP output, makes adjustments to reservoir elevations which were computed from a linear relationship, and computes a new set of coefficients and bounds to be used in the LP solution for the next time period.

(4) An iteration control routine monitors the achieved power and adjusts the target in such a way that a good approximation to the maximum achievable prime power is reached after a small number of iterations (i.e., passes through the entire period of record).

The output from the last (i.e., "optimal") pass is stored on a file and can be printed or plotted by utility programs, as required. The output includes the following for each unit time period: reservoir elevations at the end of each period; mean discharges; mean power generation at each plant; specified bounds; violations of bounds; and mean spills at Type 2 and 3 plants.

HCMS Options

There are many options in using HCMS. For instance, primary power may not be an important consideration. In this case, the user could specify in the model input only one iteration for a given primary power value. Also, it might be found that primary power is governed for all schemes in a study by a particular period in the planning horizon. The iteration feature could then be employed for just the critical period. The value for primary power thus obtained could be used for the entire planning horizon with only one iteration specified.

The model assumes prior knowledge of basin supplies for only the unit time period under consideration. If a user felt that current and antecedent conditions, such as snow pack and soil moisture index, justified changes to the reservoir bounds, he could run the model on an annual basis.

HCMS Limitations

- Unit time period may vary from three days to 30 days.

- Reservoir level bounds may vary as specified for each unit period of a year; these values, however, remain constant throughout all the years of record unless the user decides to update the bounds for each year and run the model on an annual basis.

- Maximum discharge constraints pertaining to floods are not normally specified to the model. The usual procedure is to adjust the reservoir bounds by trial and error to improve the operating policy over the entire planning horizon. However, maximum discharge constraints can be considered at points in the network by simply specifying Type 1 plants with zero storage and a constant maximum discharge capacity. The user can expect infeasible solutions to occur with this approach during high flow incidents because of absolute level bounds on reservoirs conflicting with the specified maximum downstream discharges.

- The inclusion of naturally controlled reservoirs in the network introduces an error due to the assumption of constant discharge capacity during the unit time period. The discharge capacity for the time period is based on the reservoir level attained at the end of the previous time period. If this error was considered serious for the particular application, an iterative modification to the model would be required or the network flow technique described in SYSNET could be used as an alternative.

- Computer costs increase only linearly as the length of the time horizon simply because optimization is performed solely for the time period in question. A user, therefore, cannot consider more than one time period in optimization. If he wants to include several time periods within an optimization solution, he should consider the network flow technique in SYSNET as an alternative. If he wants to assume perfect hindsight, a controversial and costly approach, he should consider the ORRMS model as an alternative.

- Spill at power plants is assumed to have a completely linear effect on power production.

More detailed references for this model are:

Emery, A.K. Revised Documentation Report for Hydro Configuration Modelling System by L.E. Parker, 1975. Water Planning and Management Branch, Hull, Quebec, April 1979.

Parker, L. and D.W. Farley. The Hydro Configuration Modelling System: its application on the Ottawa River, *Interfaces*, Vol. 10, No. 4, August 1980.

Farley, D.W. Hydro Configuration Modelling System, Proceedings of the Technical Workshop Series No. 2, Modelling Activities Related to Flood Damage Reduction, Inland Waters Directorate, Environment Canada, 1980.

Ottawa River Regulation Modelling System (ORRMS)

Synopsis

A linear-separable programming model was developed to permit an integrated operation of the reservoirs located in the Ottawa River basin. The model can optimize the operation of the river for reducing flood damages only, or for energy purposes only, while respecting constraints on flows and levels imposed by other interests such as recreation, navigation and log driving. It also permits trade-offs between the sometimes contradictory objectives of flood protection and energy generation. The model was developed mainly to facilitate the operation of the Ottawa River. However, it can also be used for planning purposes such as determining the impact of new reservoirs and assessing the effect of new constraints, etc. Although the model was developed specifically for the Ottawa River, it is generalized and the computer program can be applied directly or with only minor modifications to other river systems.

Objective Functions

The two major objectives that were identified in the Ottawa River basin, energy generation and flood damage reduction, are rather contradictory during years with high spring runoffs. The approach of this model is therefore based on defining a composite objective function.

$$\max Z = C_1 Z_1 - C_2 Z_2$$

The first objective, Z_1, represents the value of energy generation, which is to be maximized, while the second objective, Z_2, represents flood damages, which is to be minimized.

The weighting coefficients, C_1 and C_2, were introduced especially to permit a trade-off between energy and flood damages which both have to be expressed in common terms (dollars). If $C_1=1$ and $C_2=0$, the model will maximize only the worth of energy. Conversely, if $C_1=0$ and $C_2=1$, the model will minimize only the flood damage without any consideration for energy. Clearly, there are an infinite number of alternatives between these two extremes. This technique, known as the weighting method, provides the decision-makers with a trade-off curve to be used further to identify the best compromise solution. The weighting method is considered the most efficient for the case when the composite objective function is made up of only two objectives.

Certain difficulties arise in the use of the objective function defined by the relationship above. The energy objective, for example, requires a one-year time horizon, since the energy demand and the elevation of water in most reservoirs have a one-year repetitive pattern. However, the flooding objective requires a different time horizon of only three to four months to cover the freshet season. In addition, the first objective (Z_1) depends on the time and space distribution of the annual volume of natural runoffs to be received by the system, while the second objective (Z_2) depends only on the peak flows at the flooding sites. These difficulties are resolved by using a hierarchical approach.

Hierarchical Approach

The energy objective (Z_1) requires, as already explained, a time horizon of one year. At the same time, the operation of the system, especially during the freshet season, requires daily guidelines on discharges from reservoirs. The utilization of an optimization technique to cover such a long time horizon with one-day time periods is not possible, since the dimensionality of the resulting problem exceeds the computational capabilities of the available computer technology. To overcome this difficulty, ORRMS employs the hierarchical approach which is based on time decomposition. Time decomposition entails replacing the initial large-size problem by a hierarchy of smaller problems which can be handled by present-day computers and software. The method also permits the optimization of objectives with different time scales and permits decision-makers to select a trade-off alternative solution acceptable to the involved interests.

The long-term model covers a one-year time horizon and is made up of time periods of variable length. On the example shown in Figure 3, time periods of two and four weeks are used. The model optimizes only

the worth of the energy generated by the system, and consequently the composite Objective Function is applied with $C_1=1$ and $C_2=0$. In addition to all system constraints, the state of the system (the amount of water in the reservoirs) at the end of the one-year time horizon is constrained to be equal to the average historical value. This constraint is based on the assumption that beyond the one-year time horizon, the best natural runoff forecast is given by the average historical runoff values and, therefore, the reservoirs should also be at their average historical level. The use of the long-term model permits: (a) taking into account the navigation, recreation, log driving, and low water augmentation constraints that occur after the end of the freshet; (b) providing a long-term view of the state of the system and its energy output; (c) calculating target storages to be used by the mid-term model, as shown in Figure 3.

The mid-term model covers a shorter time horizon of about 16 weeks to include the entire spring freshet (March to June) and it takes into account both objectives ($C_1=0$, $C_2=0$) of the composite objective function. The continuity between the models is assured by constraining the mid-term model to meet at the end of its time horizon the target storages provided by the long-term model. Usually the mid-term model has time periods of one week and can be used for various combinations of C_1 and C_2 coefficients. Its output will enable a decision on the trade-off between the two objectives (Z_1 and Z_2) and will provide target storages to be used by the short-term model.

The short-term model covers a time horizon of about ten days, with one-day time periods, and is constrained to meet the target storages calculated by the mid-term model. The model is run to provide guidelines for the daily operation of the reservoirs.

All three models are to be run sequentially with updated runoff forecasts. The entire process is rolled forward in time, as shown in Figure 3. The updating interval is variable and depends mainly on the changes in the natural runoff forecasts. The mid-term model is updated more often than the long-term model, while the short-term model is updated more often than the mid-term model. In the example shown in Figure 3, the updating interval is equal to four weeks for the long-term model, one week for the mid-term model, and one day for the short-term model. In general, the updating intervals are not fixed and are left at the user's discretion. All three models use the same computer program. The only difference between these three models is the input data, which define the length of the time horizon, the time periods, the target storages, and the objective function weights.

Optimization Technique

Large complex configurations such as the Ottawa River basin require the use of high-dimensional capacity linear programming software with separable programming capabilities. The MPSX/370 commercial

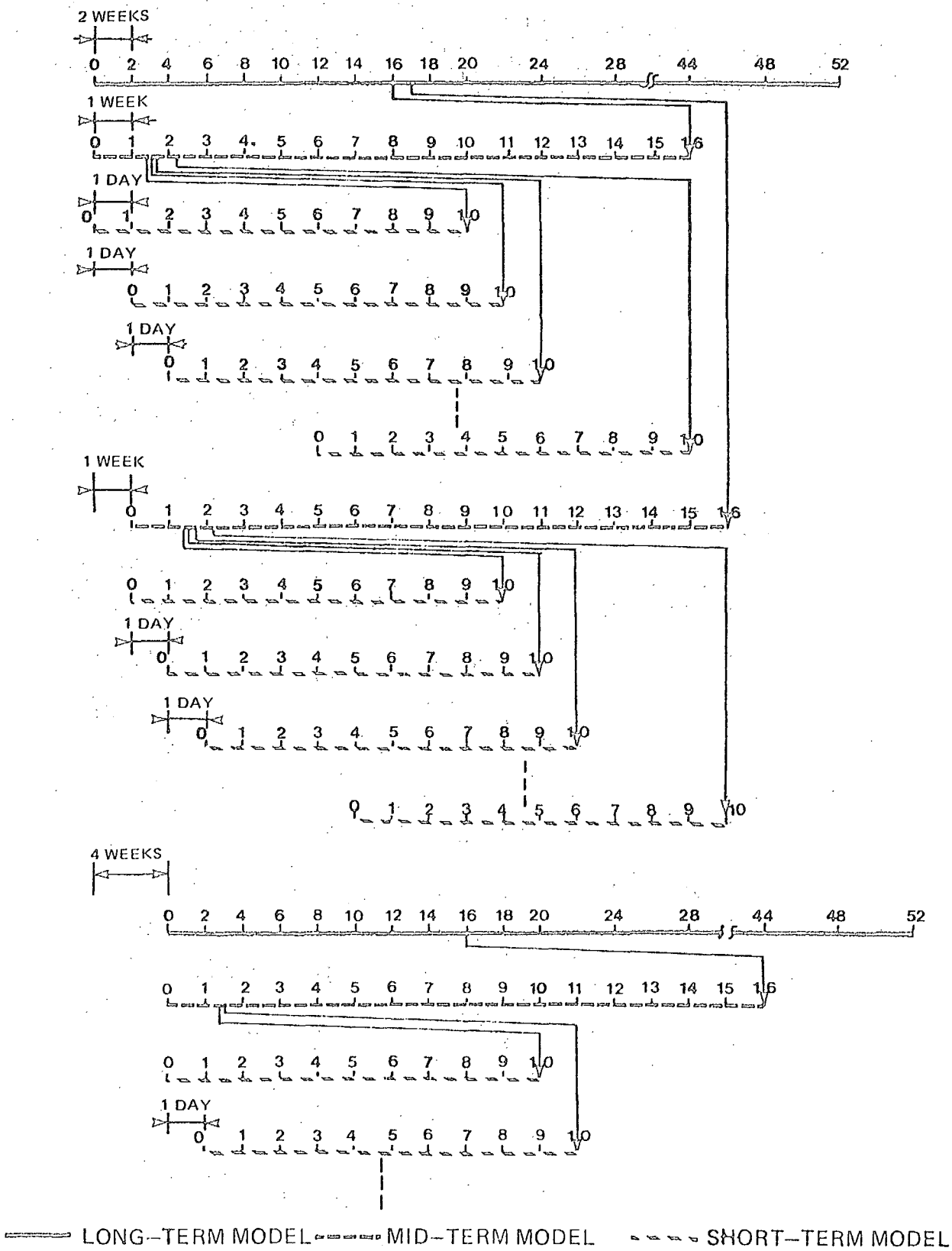


Figure 3. The forward rolling of the long-, mid- and short-term models.

software package developed by IBM in 1979 was selected because first, it represents the state of the art in linear and separable programming, and, secondly, IBM equipment was available among the agencies responsible for the development of ORRMS.

Model Configuration

The model configuration is made up of combinations of plants and channels. A Type 1 plant is a reservoir with one or several outlets. A Type 2 plant is made up of one or several adjacent run-of-the-river stations. The Type 2 plant may also be used as a logical (fictitious) station to represent flooding sites and/or constraints on river flows.

The river reaches are represented in the model by three types of channels, which are designated as controlled, free and generating. The controlled channel has at its upstream end a reservoir with a control structure that by means of gates and/or stop logs can continuously regulate the flow in the channel. The free channel has at its upper end a fixed control which is representative of a natural lake outlet or an overflow weir. The flow through such a channel is a function of the water level in the upstream reservoir. The generating channel has a generating station at its upstream end. The flow through such a channel is the turbine discharge and any spill which may occur.

The two types of plants and the three types of channels can be used to define the configuration of any river system including several flood danger sites. Application of the model to other river systems might require minor modifications. Irrigation interests and interbasin water transfers, for example, would require some modifications.

Inputs

The input data to the model may be classified as static, semi-static and dynamic.

The static data describe mainly the characteristics of the physical system of reservoirs and generating stations and need not be updated at each run of the model. They are composed of the following types of data: (a) number and length of the time periods considered in the model; (b) configuration information containing plant identification, plant type, number and type of channels, etc.; (c) lag times between plants; (d) coefficients of the stage-maximum discharge relationships; (e) coefficients of the stage-storage relationships; and (f) power-storage-discharge tables.

The semi-static data have to be updated only from time to time and include the following: (a) definition of agencies owning generating stations; (b) daily/weekly energy value weights; (c) agency constraints on power; (d) power patterns that describe maintenance schedules of the turbine/generator units, (e) discharge and storage/elevation bonds; (f) the average annual value of energy for each agency, and (g) gate settings.

The dynamic data describe the initial state of the system and the specific constraints that have to be satisfied during the next run of the model. Most of these data are normally changed each time the model is run and consist of the following: (a) model starting data; (b) initial states of the reservoirs throughout the system; (c) discharges prior to model starting date; (d) forecasted inflows for the time horizon to be modelled; (e) updated constraints on storages and flows; (f) target storages to be met at the end of the time horizon for which the model is run; (g) power constraints on groups of generating stations; (h) smoothing penalties on flows; and (i) relative weighting coefficients for the flood damage component of the objective function.

The critical dynamic input to the model is, of course, the natural runoff forecast. A conceptual watershed model or any other type of inflow prediction model may be employed to provide this input. Taking into consideration that quantitative meteorological forecasts are only available for two- to three-day periods, it is evident that historical climatological records are required for extending watershed forecasts several weeks or even months to meet the input needs of the mid-term and long-term models of ORRMS.

Power Characteristics

The general equation for hydraulic generating stations is as follows:

$$P = f\ (Q,H)$$

where P = power,
Q = total flow (turbine discharge and spill), and
H = head.

This is a nonlinear/non-separable function and, therefore, cannot be used directly with available linear programming software. This relationship is handled in the model by considering piece-wise linear segments in an iteration process. This relationship for run-of-the-river plants is expressed as:

$$P = f_1\ (Q) \qquad \text{(a)}$$

$$P = f_2\ (Q,DS) \qquad \text{(b)}$$

and for generating stations with a reservoir:

$$P = f_3\ (Q,S) \qquad \text{(c)}$$

$$P = f_4\ (Q,S,DS) \qquad \text{(d)}$$

where S = storage and
DS = downstream storage.

Expressions (b) and (d) apply when a downstream reservoir stage induces a backwater effect on the tailwater of the upstream generating station.

An iterative technique is employed to ensure that, subject to the constraints, power is being generated at each reservoir storage plant along the most efficient linear segment. This involves two or three iterations in which the problem is automatically re-submitted to MPSX. The iteration procedure is not costly, since the results of the previous MPSX solution are retained as the basic solution to the present iteration. Although the changes from the previous MPSX solution are usually minor, they are significantly worthwhile in terms of hydroelectric generation.

Constraints

The physical constraints considered in the model are water balance at each plant and between plants, and the maximum possible releases through controlled and free channels as a function of forebay elevation. Operating constraints include limitations on the energy generated by defined groups of generating stations and by each of the power utilities.

To represent the nonlinear relationships by piece-wise linear segments, it was necessary to specify special storage and discharge grid variables in the model. These grid variables represent additional constraints.

It is possible that a set of constraints provided to the system will conflict and result in an infeasible problem. To overcome the problem of MPSX terminating prematurely under these conditions, artificial variables were introduced to permit violating the bounds on storages and discharges as well as on agency/group power variables for a very high cost. Consequently, the user may receive, in addition to the optimum solution to his problem, warnings regarding any infeasibility.

Test results of the optimization model have shown highly variable discharges from one time period to the other. Additional constraints were implemented in the model to counter this effect that is undesirable in actual practice. Also, the objective function of the short- and mid-term models was extended to include smoothing penalties on these large discharge variations. The values of these smoothing penalties were calibrated such as to obtain practical results with a minimum loss of power.

Model Structure

The modelling system is made up of the following six modules: 1 - Skeleton Model Generator, 2 - Communication File 2 Generator, 3 - Skeleton Model Expansion, 4 - MPSX Optimizer, 5 - Analyze, 6 - Report Writer.

The purpose of the skeleton model generator is twofold. First, it processes all the static inputs to the model and generates Communication File 1 for later use by other parts of the model. Secondly, it automatically converts the static input data into a format consistent with the MPSX requirements. Unless there are structural or characteristic changes to the actual physical system, i.e., new generating stations and/or reservoirs, new generating units and/or control structures, etc., this module need not be run again.

The purpose of the Communication File 2 generator is to process all semi-static inputs to the model and to generate Communication File 2 for later use by other parts of the model.

The Skeleton model expansion module uses information from Communication Files 1 and 2 together with the dynamic input data and the natural runoff forecast data to generate Communication File 3 and other special files that contain additional coefficients in the MPSX format. The content of the files generated by this module changes from one run of the model to the next due to the dynamic nature of the input data and to the capability of the module to add or delete constraints.

The MPSX-optimizer is the IBM/370 linear programming package.

The Analyse module performs some of the calculations in the iterative process required by the non-separable relationships between power, storage, and total flow in accordance with the most recent solution from MPSX.

The Report Writer module accesses various files generated by the modules described above and generates a user-oriented report. The user has various options available to print only those sections of the report that would be of interest to a particular study.

A fundamental difference between this model, ORRMS, and the previously discussed planning model, HCMS, is that it considers all unit time periods simultaneously for optimization within the optimizer, MPSX. HCMS, however, considers only one unit time period within the optimizer, DOSPB, and steps through the problem by returning to the optimizer for each unit time period. HCMS, therefore, assumes no prior knowledge of runoff beyond the present unit-time period and for guidance relies on a trial and error choice of reservoir level and discharge bounds based on climatology.

The detailed reference documentation for this model follows:

Inland Waters Directorate, Environment Canada. Users Manual for the Ottawa River Regulation Modelling System, January 1982.

Great Lakes SYSNET Model

Synopsis

The SYSNET model is a specially designed optimal solution-seeking simulation model for regulation of the entire Great Lakes system in which Lake Nipigon, and the Ogoki, Long Lake, Chicago and Welland diversions are included in the model configuration. Ottawa River discharge above Montreal is also taken into consideration. Figure 4 is a schematic diagram of the Great Lakes system modelled by SYSNET.

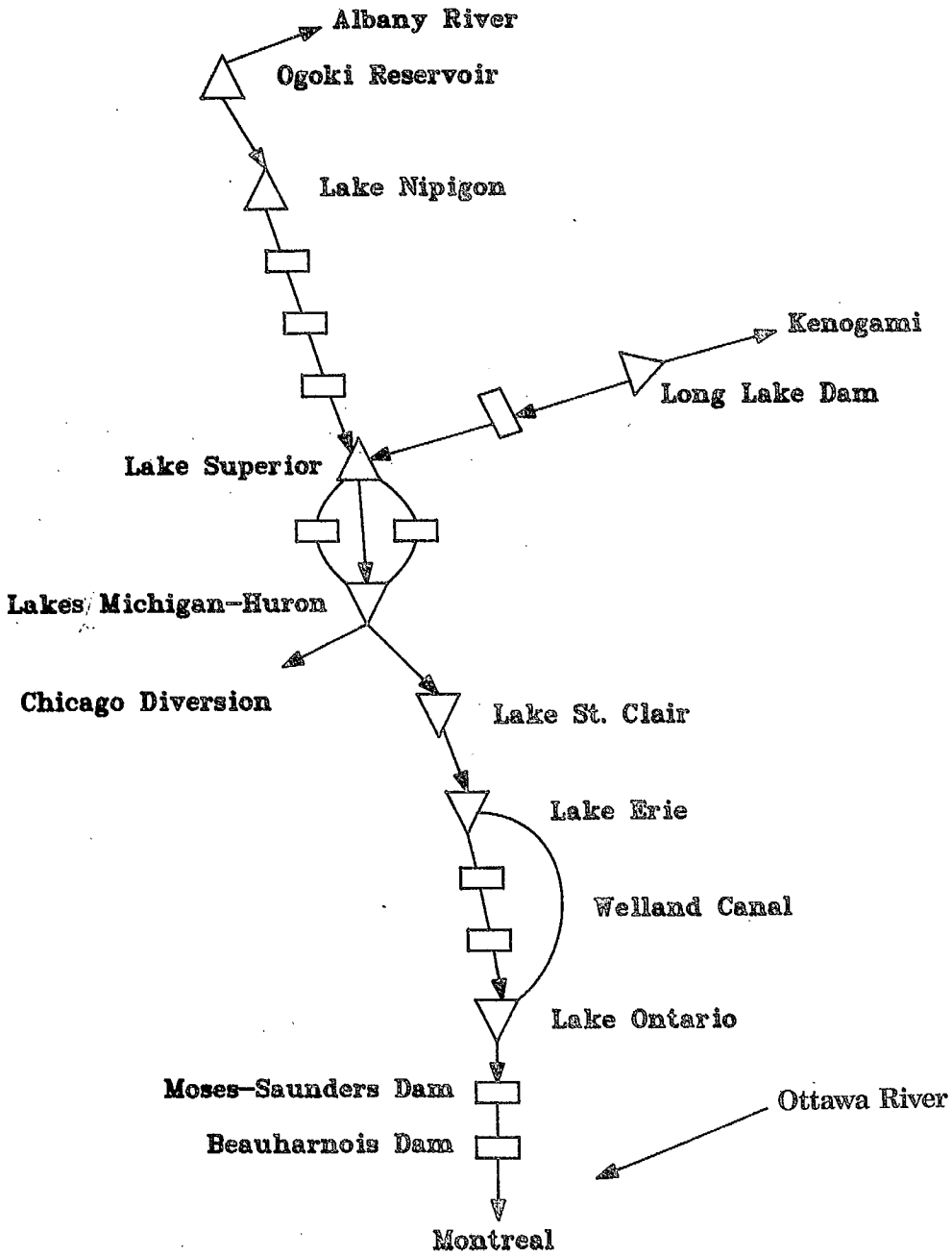


Figure 4. Great Lakes regulation schematic.

The channel routing relationships between the uncontrolled lakes remain the same as in the simulation model now used by the St. Lawrence Board of Control. However, solution convergence is achieved in a manner quite different from the Board's simulation model.

SYSNET permits a user to specify upper and lower constraining bounds on lake levels and discharges as a functon of time of the year divided into quarter-monthly periods. Most of these bounds are already specified in adopted criteria for regulation of the Great Lakes. However, to permit feasible solutions, this model employs auxiliary variables which, in effect, permit the specified level bounds to be violated to an ultimate maximum or minimum during periods of high stress, that is, in extreme dry and wet periods. The present models based on existing plans have difficulty coping with this situation and the "discretionary authority of the Board" must be employed. SYSNET thus has the advantage of more rationally simulating actual operating policy. It can be used to measure directly the impact of changing the level or flow constraints for any of the reservoirs or channels included in the configuration. The region of Montreal is considered by total flow (Ottawa River plus St. Lawrence) as opposed to simply Lake St. Louis discharge in the present model.

Another use of SYSNET would be to measure the impact of replacing the Lake Superior control structure with a modern structure which would permit continuously variable flows rather than the present discrete flows.

The objective function (optimization goal) essentially consists of keeping water upstream for future needs, as permitted within the specified flow and level constraints for each monthly period. The value of water in SYSNET is a function of its hydroelectric generating potential. High costs are incurred when auxiliary variables take on values other than zero. Thus, the first priority in SYSNET is to satisfy the specified constraints.

SYSNET assumes that forecasted supplies are available for each quarter of the month under consideration. Optimization is then performed for the month based on quarter-monthly calculations. Lake Superior outflows, in accordance with the existing operating plan (Plan 77), are governed by monthly gate settings, but the calculations to conform with the rest of the configuration are carried out for each quarter. The quarter-monthly inflows for the purpose of testing were obtained from the Great Lakes coordinated basic data files and were computed by the Water Planning and Management Branch of IWD for Lake Nipigon, Long Lake, Ogoki and the Ottawa River.

The network flow algorithm was found to be a very flexible technique and above all, at least one order of magnitude and probably two orders of magnitude cheaper in computer processing costs than commercially available linear programming algorithms.

Justification for Development of SYSNET

There has been a demonstrated need to consider the Great Lakes and their diversions as a single integrated system. The Board has frequently used its discretionary authority in the control of Lake Superior to the benefit of interests on the lower lakes and the region of Montreal. However, the presently approved regulation plan for Lake Superior does not specifically consider the situation on the lower lakes in determining the Lake Superior outflow. Recent diversion and consumptive use studies have indicated a further need for an integrated analysis in the consideration of variable diversion discharges.

Economic Optimization

There has been no attempt to introduce economic theory directly into the objective function. However, the present objective function represents a rational economic policy, that is, keep supplies upstream to meet later needs subject to constraints relating to power production, environment, navigation, recreation and flood protection. SYSNET was not intended to be an optimization model. Instead, the goal was to incorporate network flow optimization theory in a simulation model which would adequately meet the needs of the various study groups. As in the previously discussed HCMS and ORRMS models, SYSNET employs auxiliary variables and cost coefficients. Auxiliary variables are confined to zones between target levels and ultimate levels for storages and discharges. In Figure 5, the variable f (arc in network flow terminology) represents values between upper and lower bounds. The target upper level bound is referred to as TUL, and the target lower level bound, as TLL. Experience indicates that target bounds are frequently violated during periods of low runoff (1964) and during periods of high runoff (1973, 1974 and 1976). Thus, infeasibilities would occur if there were no auxiliary arcs to accommodate violations of the f arc. These auxiliary arcs have high cost coefficients, in the order of 100.00, in comparison with the end of the month storage arcs which have values in the vicinity of -.03. The coefficients are zero for the storage arcs of the first three quarters and for all f discharge arcs. It is evident that with cost coefficients for the auxiliary arcs several orders of magnitude greater than the coefficients for the storage arcs, SYSNET will strive to keep flow in these auxiliary arcs at a zero level.

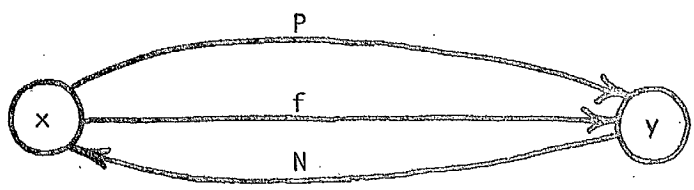


Figure 5

In Figure 5, nodes x and y are connected by arcs P, f and N where P represents the flow in the positive violation arc; f represents the flow between upper and lower target bounds; and N represents the flow in the negative violation arc. If the net flow between nodes x and y = NET, then NET = f + P - N.

The upper bound on arc P = upper ultimate bound - TUL. The lower bound for arc P is zero.

The upper bound on arc N = ultimate lower bound - TLL. The lower bound for arc N is zero.

All storages in the configuration have been assigned auxiliary arcs for both positive and negative violations of target bounds. The cost coefficient arbitrarily assigned to these arcs is 100.00. The Lake Nipigon positive violation arc has been assigned a greater cost coefficient of 150.00 because of a restricted outlet capacity which is required for protection of the main line of the Canadian National Railway. If SYSNET must violate storage arcs, it will generally store water upstream in the interest of maximizing potential energy in the system. Thus, when the cost coefficient for positive violations of Lake Nipigon storage is higher than those of the other lakes, SYSNET will move volumes of water downstream to Lake Superior in order to maintain a reservoir capacity to accommodate sudden flood flows into Lake Nipigon.

In the present network structure for SYSNET, auxiliary arcs for discharge arcs are employed only for positive violations of Lake Ontario outflows and total discharges into Montreal. The cost coefficient for these positive violations has been arbitrarily set at 150.00. Thus, for the upstream lakes, target bounds on discharges will not be permitted to be violated, and excesses or shortages of water will be accommodated by the storage auxiliary arcs.

Figure 6 illustrates the arcs for accommodating positive violations of Lake Ontario outflows. The numbers in each node represent the four quarters of each month being considered together in an iterative solution. The numbers on the arcs represent the flows for each quarter.

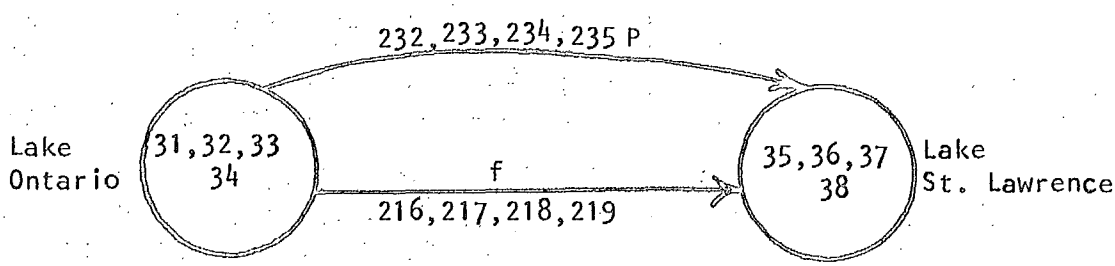


Figure 6

It is common to use integers rather than real numbers in network flow applications. In SYSNET real numbers are used partly because of the need to balance flows and levels through the naturally controlled lakes. In Section 4.2 of the Appendix it is stated that conservation of flow must be maintained. The use of real numbers may sometimes result in small round-off errors and thus disturb the conservation of flow principle when proceeding from one time period to the next. To ensure that this does not become a serious problem, all arcs are set to zero at the end of each year in the simulation.

SYSNET is similar to HCMS in that it considers a short time period (four quarter-months) within the optimizer, and steps through the problem for each month of record. During discussion at this workshop, it was argued that the objective function of tending to keep supplies upstream, subject to constraints, would reserve inadequate upstream storage for flood protection. The response to this criticism was that the defined upper storage bounds for each time period have been derived from climatology. These bounds take precedence over the policy of keeping supplies upstream. Consequently, upstream storage requirements have been defined in advance to meet flood protection needs.

It would be relatively easy to introduce to this model economic costs of flooding, power production benefits, etc. These values, however, are often subjective and not readily acceptable to all agencies. More than one set of auxiliary arcs could be used for each variable to represent the different magnitudes of flood damages associated with stage.

Diversions

The diversion rates vary for each quarter based on Ontario Hydro operational criteria. However, the diversion rates vary as a continuous function. In the case of the Ogoki reservoir, the two control structures, namely Waboose controlling flows to the Albany and Summit controlling diversion flows to Lake Nipigon, are stop log structures and it is probably not appropriate to treat outflows from these structures as a continuous function during the winter months. Fortunately, it is simple to introduce sub-routines in the driver portion of the model which would compute the discharge during winter quarter-months as a function of reservoir level and stop log setting. The Welland Canal is also treated as a variable diversion subject to the specified constraints. Chicago diversions are also variable between the minimum and maximum outflows as a function of the quarter-month of the year. These limiting values were supplied by the U.S. Corps of Engineers. It would be simple to compute dynamically these outflow bounds within the driver as a function of some variable, say weekly rainfall or flow in the Illinois River.

SYSNET Structure

The structure and design of SYSNET can be described in two logical parts:

(1) the overall simulation and control system and

(2) the internal "optimizer."

The purpose of the overall simulation and control system is to

- access historical values of Net Basin Supplies,
- access historical constraints and targets,
- impose these historical values upon the month-by-month optimizer in the form of constraints, bounds and incentives,
- drive the internal optimizer, and
- log output from the optimizer for later reporting and analysis.

The purpose of the internal optimizer is to simulate rational behaviour in the operation of the system in a way that is both orderly and realistic. This is done through the month-by-month solution of a derived mathematical programming problem by means of iterative application of a network flow optimizer algorithm, plus a number of significant refinements.

The sub-problem to be solved by the optimizer is significantly more complex than a classical network flow problem. In addition to the usual capacitated network flow formulation (minimize cost of upper and lower bounded arc flows in a circulating network of arcs), there are additional model formulation components due to

- nonlinear stage-discharge relationships for the "natural reservoirs" of Lakes Michigan/Huron, St. Clair and Erie,

- discrete nonlinear gate setting-stage-discharge relationships for Lake Superior, and

- limits on the magnitude of week-to-week changes in discharge from Lake Ontario.

In the existing situation there are implicit functional relationships between storage and discharge. In the one direction, the network flow algorithm handles the change in storage as a function of flow (simple conversion of flow). However, in the reverse direction, i.e., flow as a function of storage (or, more properly, elevation), the network algorithm cannot handle the relationship.

To overcome these nonlinear components of the problem, several approaches were considered:

- separable programming using a commercially available Mathematical Programming System such as MPSX or APEX (such as was used successfully in ORRMS),

- decomposition using a network flow sub-problem, and

- successive "linearization" and network flow optimization.

In an attempt to evaluate the relative performance advantages of network flow algorithms over more general mathematical programming software (such as has been widely reported recently in *Management Science, Operations Research, Mathematical Programming* and other such journals), the third alternative was chosen.

In essence, the approach can be summarized as follows:

(1) Analyse the current solution for a given time period (or, to start, the final solution obtained for the previous period), and, using all the appropriate nonlinear relationships, compute upper and/or lower bounds on the discharges from the system reservoirs (compute flow as function of storage).

(2) Update the current problem to reflect these new bounds and submit the resulting simple network flow problem for optimization (compute optimal storage as a function of flow).

(3) Measure the difference between the new solution and the previous solution (in fact, compute the sum of the absolute values of differences between corresponding arc flows in successive solutions).

(4) If this difference is within a previously defined tolerance, accept the current solution as the "converged optimal" solution. Otherwise, return to step (1).

This procedure was found to be quite reliable and very cost-effective (in terms of computer time). However, several refinements were required to

(a) overcome a tendency for the optimizer to "flip-flop" endlessly between solutions that were almost equivalent from the point of view of objective function, but quite different in terms of arc flows (the measure of convergence) and

(b) take advantage of a special structure within the problem which permitted the application of a tight inner loop to accelerate the convergence of the iterative approach (by a factor of 5-6).

In the first of these problems, several special features were added "to smooth", in effect, the successive solutions. These features included

(a) exponential smoothing of successive bound sets,

(b) restriction of the magnitude of arc flow change from one iteration to the next, and

(c) recognition and acceptance of a good feasible solution after an appropriate number of iterations.

In the second situation, it was found that significantly better performance (in terms of computer time) could be achieved by iteratively computing discharges from the natural reservoirs and then immediately adjusting the resulting storages in an inner loop between executions of the major optimizer.

Reference

Environment Canada and TEE Consulting Services Inc. SYSNET Model for Great Lakes - Documentation Report, November 5, 1980.

SOMMAIRE DES MODÈLES DOCUMENTÉS, DISCUTÉS À L'ATELIER

Système de modélisation hydro-configuration (SMHC)

Le Système de modélisation hydro-configuration (SMHC) est un modèle généralisé de planification qui a été développé pendant les années 1975/76. On s'est servi beaucoup de ce modèle pour des études sur le bassin de la rivière des Outaouais.

Par essence, SMHC fournit de la capacité de définir un réseau des cours d'eau multi-réservoirs existant ou projetés, et par la suite, la simulation rationale du système pendant longtemps. En effectuant cet exercice pour chacun de plusieurs changements hydrauliques projetés pour un système sous considération, le choc à longue échéance peut être évalué.

Spécifiquement, SMHC engendre premièrement un modèle de programmation linéaire (PL) de tel façon pour représenter:

- caractéristiques physiques des réservoirs individuels, et les chenaux des rivières,
- le réseau des cours d'eau, des barrages, réservoirs et centrales hydroélectriques,
- les contraintes dynamiques en ce qui concerne l'opération admissible du système, et
- buts ou cibles d'opération.

La structure du modèle PL engendré est comme suit.

Variables

- emmagasinement/niveau des réservoirs,
- débits des réservoirs, et
- variables artificielles de relâchement (violations).

Contraintes

- équations de bilan hydrique,
- relation entre les variables élévations et volume pour chaque réservoir,
- relations entre les variables l'emmagasinement d'un réservoir et la décharge actuelle ou la décharge maximum possible,
- relations d'engendrement de puissance aux centrales,
- les limites d'opération sur les variables déterminées par la protection contre les inondations, et les intérêts récréatifs et écologiques, et
- les contraintes ou les cibles spéciales ou arbitraires.

La fonction objective est comme suit:

- réduire au minimum la somme pondérée des violations des contraintes,

- réduire au minimum l'écart pondéré des valeurs cibles pour certaines variables, et
- maximiser le potentiel pour satisfaire les contraintes futures. (Dans l'application décrite ci-inclus, le composant à être maximisé était l'énergie hydroélectrique restant dans le système à la fin de chaque période de décision.)

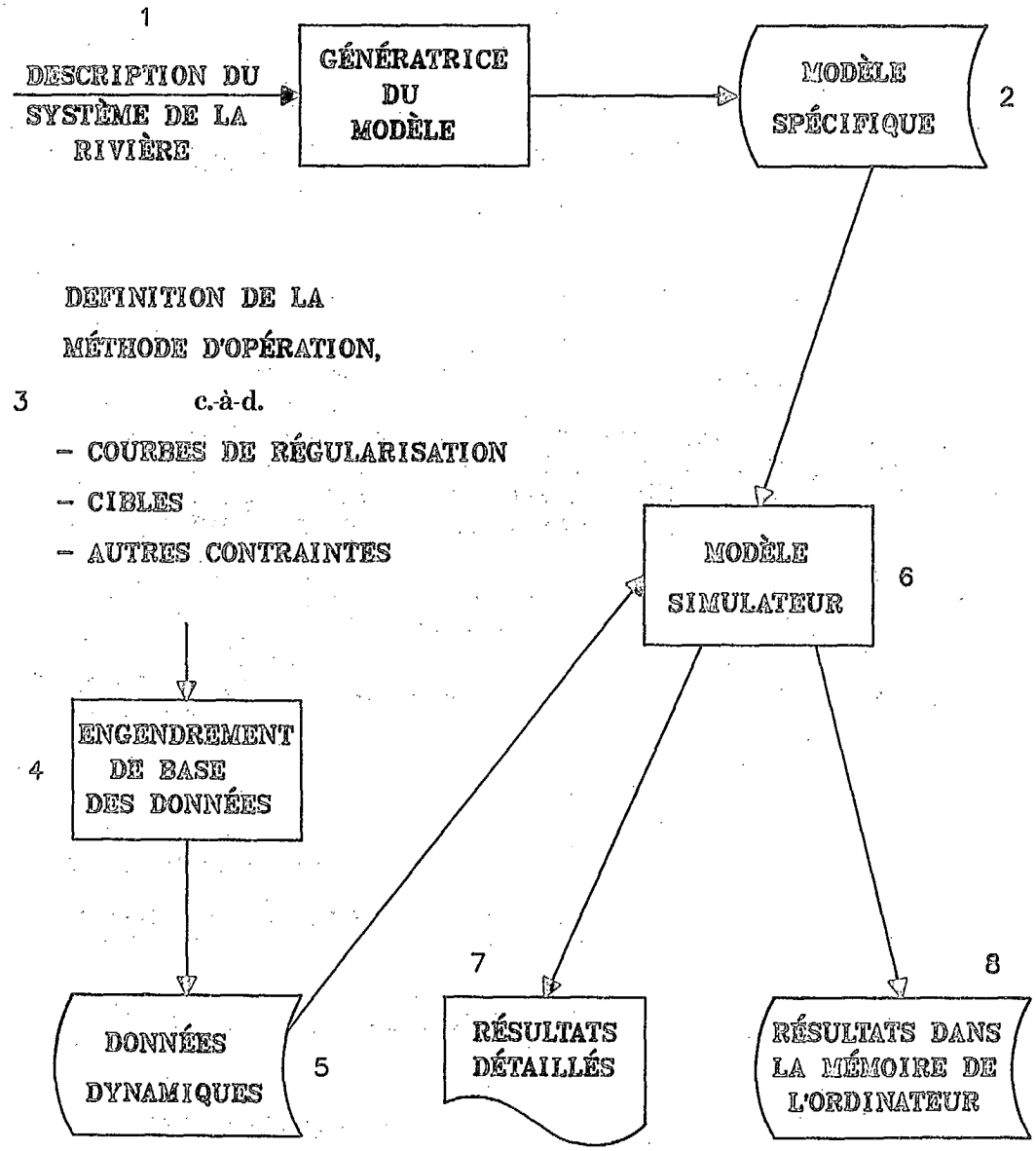


Figure 1. Débit général du modèle.

Pendant l'usage normal, le modèle est appliqué à une série d'unités de temps, chacune représentant une période simulée de décision. La sortie du modèle (les décharges optimums pour la période et le contenu résultant de réservoir à la fin de la période) sont employées au début de la prochaine période. À chaque période, les décharges optimums sont déterminées assujetties aux conditions actuelles (incluant une série de temps des apports naturels calculés des relevés hydrométriques

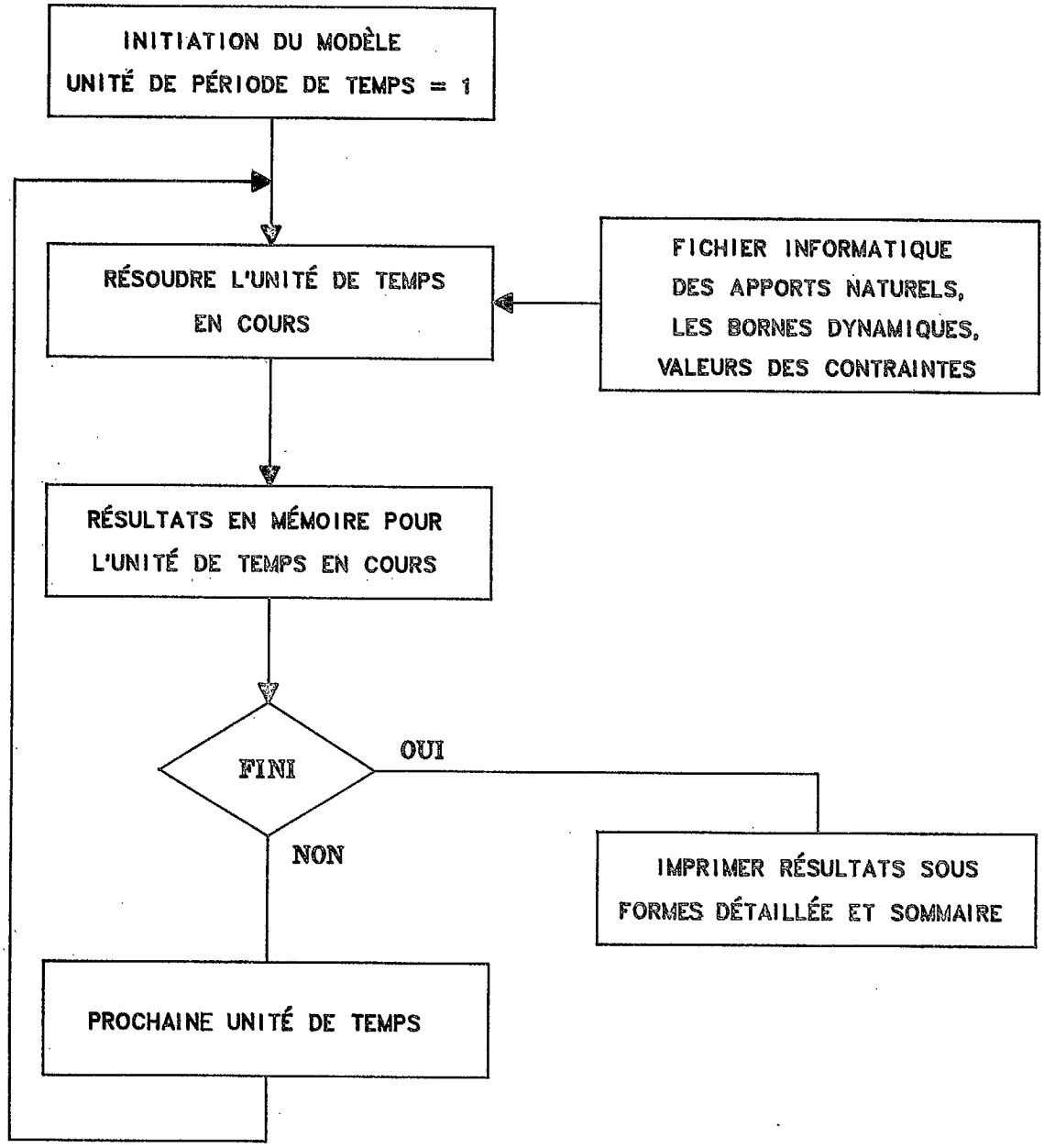


Figure 2. Logique du modèle simulateur.

historiques) et aux contraintes actuelles. Ce procédé est effectué pour chaque unité de temps dans la période des relevés hydrométriques simulés par ce moyen, la conduite raisonnable est simulée (comme modelé par le prenant optimal des décisions) données les séquences historiques des apports naturels et la connaissance actuelle du système.

Par moyen de l'application répétée du modèle pour des ensembles variés des règlements d'exploitation des réservoirs et/ou des modifications au système physique (comme représenté par la capacité d'emmagasinement, la capacité des chenaux, le réseau des structures de contrôle, etcetera) on peut faire des comparaisons entre les résultats anticipés des plans divers projetés. L'application globale du modèle est illustrée à la figure 1.

Premièrement, une description du système de la rivière (*1*) est utilisée pour engendrer un modèle squelettique de la programmation linéaire du système (*2*). Ensuite, le plan spécifique à être analysé est représenté sous forme des valeurs des bornes d'emmagasinement et débits, etcetera (*3*). Celles-ci sont transformées sous forme lisible par machine (*4*) pour l'emploi pendant la simulation. Enfin, le simulateur du modèle (*6*) est employé par séquences pour chaque unité de temps qui doit être résolu et la sortie du modèle consiste en des résultats imprimés et un fichier sur mémoire à disque.

La figure 2 illustre la logique globale de la phase du simulateur du modèle.

L'énergie potentielle est décrite mathématiquement comme suit:

$\Sigma P_i HR_i$ (unités - MW∘s)

ou HR_i = l'élévation du niveau du réservoir i

$P_i = \Sigma CCP_j/DHRDV_i$

- j représente les centrales en aval de i,
- CCP est le coefficient caractéristique de la centrale en MW/pcs, et
- $DHRDV_i$ est le taux de changement de charge avec volume au réservoir i.

Des variables auxiliaires sont employées dans le modèle pour permettre la violation des contraintes cibles spécifiées afin d'éviter des solutions infaisables qui autrement arriveraient. Un exemple du besoin pour une variable auxiliaire serait une contrainte sur le niveau d'un réservoir étant physiquement inaccessible donnés la capacité maximum de décharge de l'éxutoire et le taux haut des apports. Un autre exemple serait les apports à un réservoir étant insuffisants à satisfaire une contrainte récréative du niveau minimum. On force le modèle à essayer de garder ces valeurs des variables auxiliaires à zéro par l'application des grands coefficients négatifs de coût qui sanctionnent sévèrement la fonction objective du modèle. Ce modèle ne viole jamais les contraintes

des niveaux maximals des réservoirs (autrement le débordement des barrages aurait lieu), et débits maximals aux déversions et turbines.

Le modèle a la capacité de maximiser la puissance hydroélectrique primaire du système. C'est à dire, il peut déterminer la puissance maximale continue qui peut être garantie disponible pendant la période de relevés historiques sous étude. On se sert souvent d'une option qui est de satisfaire une valeur spécifiée de la puissance primaire pour le système sous étude.

Ce modèle accepte 3 types de structures dans la disposition:

Type 1 - l'emmagasinement simple, c.-à-d. un réservoir d'emmagasinement avec un déversoir de contrôle sans turbines pour produire de la puissance,

Type 2 - centrale avec emmagasinement, et

Type 3 - centrale au fil de l'eau.

N'importe quel nombre de ces types de structures jusqu'à 99 peuvent être liées ensembles par des biefs de longueurs différentes. Le temps de retard pour écouler l'eau le long de ces biefs est spécifié comme constant pour chaque bief avec une valeur maximale égale à l'unité de temps choisie.

Étant donnée la configuration du réseau, la génératrice engendre automatiquement le modèle squelettique PL qui consiste en trois composants:

(a) équations de continuité,
(b) contrainte sur la réalisation de la puissance primaire, et
(c) fonction objective.

Le modèle consiste en deux parties majeures - un <<conducteur>> et le logiciel <<PL>>. DOSPB, une sous-routine du paquet d'IBM SL-MATH, est employée pour résoudre le problème PL. Le modèle peut s'occuper de n'importe quelle unité de temps (trois jours à un mois) pourvu que l'unité de temps n'excède pas le plus long retard d'écoulement dans le laminage.

À cause des dimensions du problème et du comportement non-linéaire des variables, le problème est segmenté comme suit:

(1) La période de relevés (par exemple, 30 ans) est divisée en unités égales de temps sélectionnées pour l'étude.

(2) On conjecture la puissance primaire qui est accessible pendant la période de relevés.

(3) Le modèle essaye de satisfaire toutes les contraintes (la puissance primaire incluse) pour chaque unité de temps pendant qu'il maximise une fonction du <<potentiel de puissance future>>. On accomplit ce dernier par moyen du modèle PL qui est mis à date avec les coefficients linéaires et limites et ensuite résolu. Le <<conducteur>> analyse les données de sortie PL, ajuste les élévations des réservoirs qui ont été calculées par moyen des fonctions linéaires, et calcule un nouvel ensemble de coefficients et de limites à être utilisés dans la résolution PL de la prochaine unité de temps.

(4) Une routine de contrôle d'itération contrôle la puissance réalisée et ajuste la cible de telle façon qu'une bonne approximation de la puissance primaire maximum réalisable est obtenue après peu d'itérations de la période entière de relevés.

Les données de sortie de la dernière itération (c'est-à-dire la solution optimale) sont emmagasinées dans la mémoire à disque, et celles-ci peuvent être imprimées ou tracées sous demande. La sortie pour chaque unité de temps consiste en: les élévations des réservoirs à la fin de chaque période; les débits moyens; la puissance moyenne à chaque centrale; les limites spécifiées; violations des limites, et déversements moyens aux centrales.

Options de SMHC

Il y a beaucoup d'options dans l'emploi de SMHC. Par exemple, il se peut que la puissance primaire ne soit pas une considération importante. Dans ce cas, l'utilisateur pourrait spécifier dans les données d'entrée une seule itération pour une valeur fournie de la puissance primaire. De plus, il arrive souvent que la puissance primaire soit gouvernée pour tous les projets d'étude par une période particulière dans les relevés historiques. Donc, on pourrait employer la capacité d'itération pour seulement cette période critique. Ensuite, la valeur obtenue pour la puissance primaire pourrait être utilisée pour l'horizon entier de planification sous l'énoncé d'une seule itération.

Le modèle ne présume qu'une connaissance antérieure d'une unité de temps pour les apports hydriques. Si un utilisateur pensait que les conditions actuelles et antérieures telles que la couverture de neige et l'humidité du sol justifiaient des changements aux limites des réservoirs il pourrait simuler un an à la fois.

Limitations de SMHC

- Les unités de temps peuvent varier de 3 jours à 30 jours.

- Les limites des niveaux des réservoirs peuvent varier comme spécifiées pour chaque unité de temps d'une année, cependant ces valeurs restent constantes pendant toutes les années incluses dans le fichier d'entrée à moins que l'utilisateur prenne la décision de mettre à date les limites pour chaque année et de simuler avec le modèle sur une base d'un an à la fois.

- Les contraintes de décharge maximale en ce qui concerne la protection contre les inondations ne sont pas normalement spécifiées au modèle. Le procédé normal est d'ajuster les limites des réservoirs par plusieurs essais de simulation afin d'améliorer la politique d'exploitation pour l'horizon entier de planification. Cependant, on peut spécifier les contraintes de décharges maximales aux points d'intérêt dans le réseau simplement en indiquant les structures de contrôle de Type 1 à ces points avec une décharge maximale constante et une capacité d'emmagasinement de zéro. En utilisant cette technique, l'utilisateur peut s'attendre à des solutions infaisables pendant les crues à cause des conflits entre les niveaux maximals des réservoirs et ces contraintes de décharge maximale spécifiée en aval.

- L'inclusion des lacs et des réservoirs avec des exutoires naturels introduit une erreur par suite de la présomption d'une capacité constante de décharge pendant chaque unité de temps. La capacité de décharge pour l'unité particulière de temps est fondée sur le niveau du réservoir atteint à la fin de l'unité de temps précédente. Si on pensait que cette erreur était significative pour l'application, on aurait dû apporter des modification d'itération au modèle, ou on pourrait utiliser la technique de <<débits du réseau>> décrite dans l'article sur SYSNET comme alternative.

  Le coût de traitement informatique augmente linéairement comme fonction de longueur de l'horizon de temps parce qu'on n'optimise qu'une unité de temps à la fois. Donc, un utilisateur ne considère qu'une unité de temps en faisant l'optimisation. S'il veut inclure quelques unités de temps dans une analyse d'optimisation, il doit considérer la méthode décrite en SYSNET comme alternative. S'il veut présumer une sagesse parfaite en rétrospective, une présomption coûteuse et controversable, il doit considérer le modèle MORRO comme alternatif.

- Le déversement aux centrales est présumé avoir un effet linéaire sur la production de puissance.

Des références plus détaillées pour ce modèle sont:

Emery, A.K. <<Revised Documentation Report for Hydro Configuration Modelling System>>, par L.E. Parker, 1975. Direction de la planification et de la gestion (eaux), Hull, Québec, avril 1979.

Parker, L. et D.W. Farley. <<The Hydro Configuration Modelling System. Its Application on the Ottawa River>>, *Interfaces*, Vol. 10, N° 4, août 1980.

Farley, D.W. <<Hydro-Configuration Modelling System>>. Étude N° 2, Série des ateliers techniques, activités de modélisation reliées à la réduction des dommages dus aux inondations, Direction générale des eaux intérieures, Environnement Canada, 1980.

Modèle d'optimisation de la régularisation de la rivière des Outaouais (MORRO)

Résumé

On a développé un modèle de la programmation linéaire-séparable afin de permettre une exploitation intégrée des réservoirs qui sont situés dans le bassin de la rivière des Outaouais. Le modèle peut optimiser l'exploitation de la rivière afin de réduire seulement les dommages d'inondation, ou pour l'engendrement de l'énergie hydroélectrique, pendant qu'il respecte les contraintes sur les niveaux et les débits imposées par d'autres intérêts tels que récréation, navigation, et flottage. Il permet de troquer les bénéfices entre les objectifs, plusieurs fois contradictoires, de la protection contre les inondations et l'engendrement de l'énergie. On a développé le modèle surtout pour l'exploitation en temps réel de la rivière des Outaouais. Cependant, on peut l'utiliser pour des études de planification telles que la détermination du choc de nouveaux réservoirs projetés, et l'évaluation des effets de nouvelles contraintes projetées. Quoique le modèle ait été développé spécifiquement pour la rivière des Outaouais, il a des caractéristiques généralisées, et il peut être appliqué avec seulement des modifications mineures à n'importe quel bassin de rivière.

Fonctions objectives

Les deux objectifs majeurs du modèle pour la rivière des Outaouais sont l'engendrement d'énergie et la diminution des dommages d'inondation. Ces objectifs sont contradictoires pendant les années des crues excessives de printemps. Ce modèle est fondé, par conséquent, sur une fonction objective composite.

$$\max Z = C_1 Z_1 - C_2 Z_2$$

Le premier objectif, Z_1, représente la valeur de l'engendrement d'énergie qui doit être maximisé, pendant que le deuxième objectif, Z_2, représente les dommages d'inondation qui doivent être minimisés.

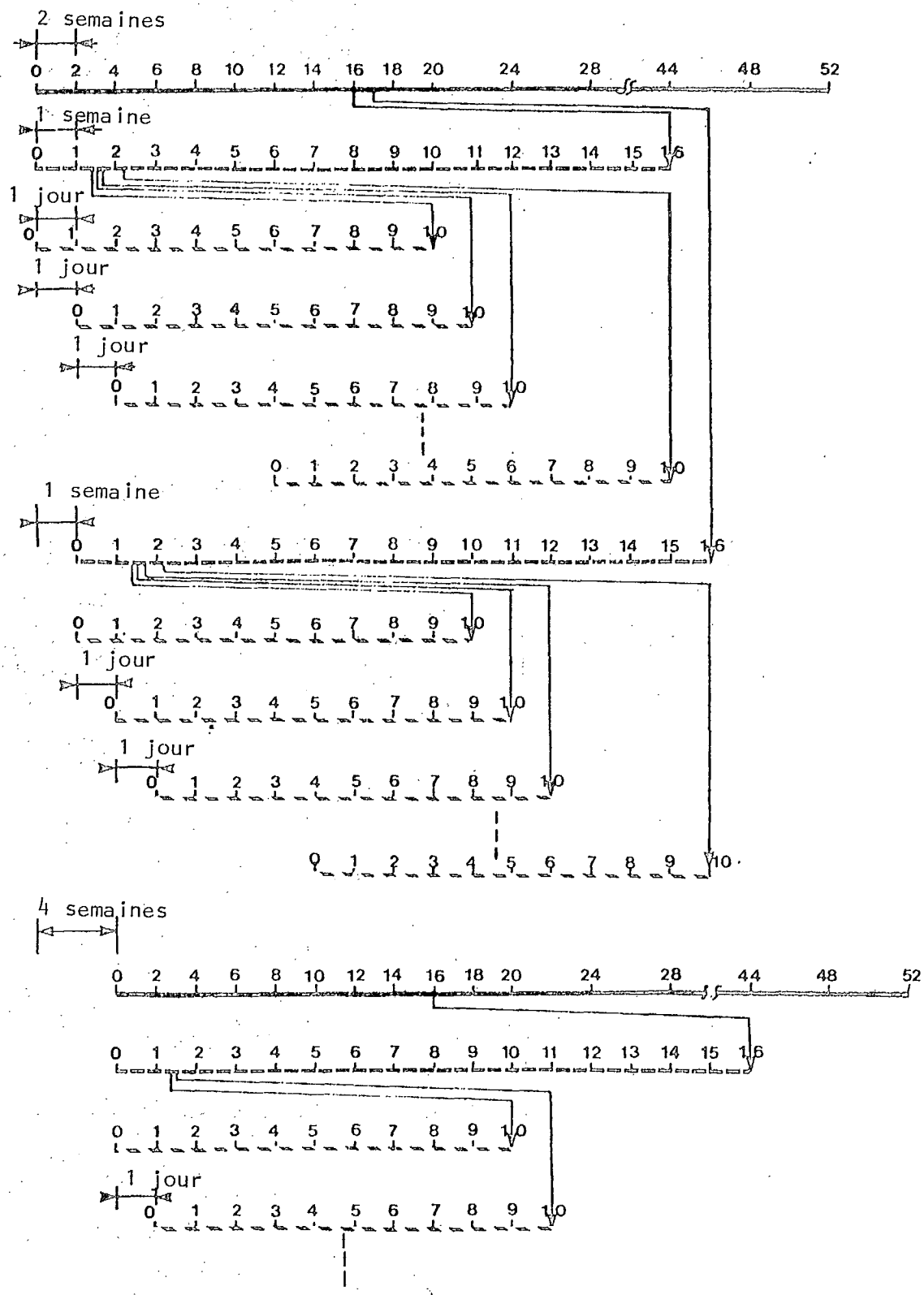
Les coefficients de pondération, C_1 et C_2, étaient introduits spécifiquement afin de permettre de troquer les bénifices de l'engendrement d'énergie avec les dommages d'inondation. Ces deux items sont exprimés en termes communs de dollars. Si $C_1 = 1$ et $C_2 = 0$, le modèle ne maximisera que la valeur d'énergie. Inversement, si $C_1 = 0$ et $C_2 = 1$, le modèle ne minimisera que les dommages d'inondation sans aucune considération pour l'énergie. C'est évident qu'il y a un nombre infini d'alternatifs entre ces deux extrêmes. Cette technique connue comme la méthode de pondération fournit une courbe de comparaison pour identifier le meilleur compromis. La méthode de pondération est considérée la plus efficace pour le cas où la fonction objective composite n'inclut que deux objectifs.

On rencontre des difficultés certaines en utilisant cette fonction objective composite. Par exemple, l'objectif d'énergie requiert un horizon de temps d'un an puisque la demande d'énergie et l'emmagasinement des réservoirs ont un cycle d'un an. Cependant, l'objectif pour la diminution des dommages d'inondation ne requiert qu'un horizon de trois à quatre mois pour couvrir la saison de crue. De plus, le premier objectif, Z_1, dépend de la répartition en espace et temps des apports naturels annuels reçus par le réseau de chenaux et réservoirs, tandis qu'au contraire le deuxième objectif, Z_2, ne dépend que des débits à pointe aux sites d'inondation. On résoud ces difficultés en utilisant une méthode hiérarchique.

Méthode hiérarchique

L'objectif d'énergie, Z_1, requiert, comme expliqué au-dessus, un horizon de temps d'un an. En même temps, l'exploitation du système hydraulique, surtout pendant la saison de crue, requiert la direction journalière en ce qui concerne la régularisation des réservoirs. Le succès d'une technique d'optimisation pour analyser un horizon d'un an divisé en unités de temps d'un jour est inabordable parce que l'encombrement de variables de décision dépasse les capacités disponibles du matériel et logiciel informatique. Pour surmonter ce problème, MORRO emploie la méthode hiérarchique qui consiste en la décomposition de temps. La décomposition de temps entraîne le remplacement du gros problème initial par une hiérarchie de plus petit problèmes qu'on peut résoudre avec le matériel et logiciel actuel. De plus, la méthode permet l'optimisation des objectifs avec les échelles de temps différentes et permet aux planificateurs de trouver directement une solution de compromis.

Le modèle à long terme tient compte d'un horizon d'un an qui est divisé en unités de temps variables. Dans l'exemple illustré par la figure 3, les unités de temps de deux et quatre semaines sont employées. Le modèle n'optimise que la valeur de l'énergie engendrée par le système, et par conséquent on applique la fonction objective avec $C_1 = 1$ et $C_2 = 0$. En sus de toutes contraintes du système, l'état du système (l'emmagasinement d'eau dans les réservoirs) à la fin de l'horizon d'un an est restreint à être égal à la valeur moyenne historique. Cette contrainte est fondée sur la présomption qu'au delà de l'horizon d'un an, la meilleure prévision des apports naturels est la moyenne historique des valeurs d'apports, et donc, les réservoirs devront être à leurs niveaux historiques. L'utilisation du modèle à long terme permet: (a) de tenir compte des contraintes de la navigation, de la récréation, du flottage et de la protection contre les étiages qui deviennent importantes après la fin de la crue; (b) de fournir une prévision à long terme de l'état du système et de la production d'énergie; (c) de calculer les cibles d'emmagasinement à être employées par le modèle à mi-terme illustré à la figure 3.



— MODÈLE À LONG TERME---MODÈLE À MI-TERME---MODÈLE À COURT TERME

Figure 3. Progression en avant des modèles à long, mi- et à court termes.

Le modèle à mi-terme considère un horizon de temps plus court d'à peu près 16 semaines afin d'inclure la crue printanière entière (mars à juin) et tient compte des deux objectifs ($C_1 = 0$, $C_2 = 0$) de la fonction objective composite. On assure l'enchaînement entre le modèle à long terme et le modèle à mi-terme en établissant les contraintes d'emmagasinement à la fin de l'horizon de 16 semaines égales à celles calculées par le modèle à long terme. D'habitude le modèle à mi-terme a des unités de temps d'une semaine et il peut simuler les combinaisons variées des coefficients C_1 et C_2. Les sorties de ces simulations permettront aux planificateurs de troquer les bénéfices entre les deux objectifs Z_1 et Z_2 et de fixer les cibles d'emmagasinement des réservoirs que le modèle doit employer à court terme.

Le modèle à court terme tient compte d'un horizon de temps d'à peu près 10 jours, avec des unités de temps d'un jour. Encore, l'enchaînement entre le modèle à mi-terme et ce modèle demande qu'on atteigne les cibles d'emmagasinement calculées précédemment par le modèle à mi-terme. Ce modèle simule l'exploitation journalière optimum de tous les réservoirs sujet aux contraintes spécifiées.

On fait fonctionner tous les trois modèles en ordre séquentiel avec les prévisions mises à dates des apports. Le processus entier avance en temps comme illustré à la figure 3. L'intervalle pour mettre à date les trois sous-modèles est variable et il depend principalement des changements de prévisions des apports naturels. Le modèle à mi-terme est mis à date plus souvent que le modèle à long terme, pendant que le modèle à court terme est mis à date plus souvent que le modèle à mi-terme. Dans l'exemple illustré à la figure 3, l'intervalle est de quatre semaines pour le modèle à long terme, une semaine pour le modèle à mi-terme, et un jour pour le modèle à court terme. L'utilisateur peut choisir avec discrétion les intervalles d'après l'application et des circonstances particulières. Il faut noter que tous les trois modèles se servent du même programme. La seule différence entre les trois modèles est la donnée d'entrée qui définit la longueur de l'horizon de temps, les unités de temps, les cibles d'emmagasinement, et la pondération des deux objectifs de la fonction objective.

Technique d'optimisation

Les grosses configurations complexes telles que le bassin de la rivière des Outaouais requièrent le logiciel de la programmation linéaire doté d'une grosse capacité dimensionnelle et de la programmation séparable. Le logiciel commercial MPSX/370 développé par IBM en 1979 a été choisi, premièrement parce qu'il représente l'état de l'art dans la programmation linéaire et séparable, et deuxièmement parce que le matériel IBM était disponible parmi les organismes responsables pour le développement de MORRO.

Configuration du modèle

La configuration du modèle se compose des combinaisons des installations et chenaux. Une installation de Type 1 est un réservoir qui a un ou plusieurs exutoires. Une installation de Type 2 consiste d'une ou plusieurs centrales au fil de l'eau. L'installation de Type 2 peut servir aussi comme un noeud logique (fictif) afin de représenter les sites d'inondation et/ou les contraintes sur les débits des rivières.

Les biefs sont représentés dans le modèle par trois types de chenaux qui sont désignés comme chenaux régularisés, libres, et génératrices. Le chenal régularisé a un réservoir à son bout à l'amont avec une structure de contrôle qui peut régulariser continûment le débit du chenal par moyen des vannes ou des batardeaux. Le chenal libre a un contrôle hydraulique fixé à son but à l'amont qui représente un exutoire naturel d'un lac ou le débordement d'un déversoir ou un seuil. La décharge à un tel chenal est une fonction du niveau d'eau du réservoir en amont. Le chenal de génératrice a à son bout en amont un ensemble de turbines. Le débit dans un tel chenal se compose de la décharge des turbines et aucun déversement qui peut arriver.

Les deux types d'installation et les trois types de chenaux peuvent être utilisés pour définir toute configuration d'un système d'une rivière comprenant plusieurs sites subissant des inondations. L'application du modèle à d'autres systèmes pourra exiger des modifications mineures. Par exemple, un intérêt d'irrigation et des dérivations d'eau à un autre bassin exigeraient des modifications.

Donnée d'entrée

On peut classifier la donnée d'entrée comme statique, semi-statique, et dynamique. Les données statiques décrivent principalement les caractéristiques physiques du système de réservoirs et centrales et donc, on n'a pas besoin de mettre à date ces données pour chaque essai du modèle. Les données incluent: (a) le nombre et la durée des unités de temps considérées par le modèle; (b) les renseignements sur la configuration, c'est-à-dire l'identification numérique et type des installations, chenaux, et réservoirs; (c) le temps de retard de laminage entre chaque installation; (d) les coefficients de régression pour la relation entre le niveau et la décharge maximum à chaque exutoire; (e) les coefficients de régression pour la relation entre le niveau et l'emmagasinement pour chaque réservoir; et (f) les tables de puissance-emmagasinement-décharge.

On doit mettre à date les données semi-statiques seulement de temps à temps pour les paramètres qui suivent: (a) identification des organismes qui ont des centrales; (b) facteurs de pondération pour les valeurs d'énergie journalières et hebdomadaires; (c) contraintes de chaque organisme pour la puissance; (d) l'horaire d'entretien des turbines/génératrices; (e) les limites de décharge et d'emmagasinement;

(f) la valeur annuelle moyenne d'énergie pour chaque organisme; et (g) la pose des vannes.

Les données dynamiques décrivent l'état initial du système et les contraintes spécifiques qui doivent être satisfaites pendant le prochain essai du modèle. La plupart de ces données sont normalement changée après chaque essai du modèle. Ces données consistent en: (a) la date de mise en marche pour l'essai du modèle; (b) l'emmagasinement initial de chaque réservoir du système; (c) les débits avant la date de mise en marche pour l'essai du modèle; (d) les apports prévus pour l'horizon de temps qui doit être simulé; (e) les contraintes mises à date sur les emmagasinements et les décharges; (f) les cibles d'emmagasinement qui doivent être atteintes à la fin de l'horizon de temps simulé; (g) les contraintes de puissance sur les groupes de centrales; (h) facteurs de punition pour amortir les changements journaliers de décharge; et (i) et les coefficients de pondération pour la composante du dommage d'inondation de la fonction objective.

Les données d'entrée critiques sont surtout les prévisions d'apports naturels. On peut employer un modèle conceptuel d'écoulement quotidien pour n'importe quel type de modèle de prévisions des apports afin de fournir ces données d'entrée. En tenant compte que les prévisions quantitatives météorologiques ne sont que disponibles pour les périodes de deux à trois jours, il est évident qu'on doit se servir des relevés climatologiques historiques pour prolonger les prévisions d'apports de plusieurs semaines ou même de mois pour satisfaire les besoins des modèles mi-terme et long terme de MORRO.

Fonctions de puissance

L'équation générale pour les centrales est comme suit:

$$P = f\ (Q,H)$$

où P = puissance,
Q = décharge totale (décharge du turbine et le déversement), et
H = la charge.

Celle-ci est une fonction non-linéaire/inséparable, et donc on ne peut pas l'employer directement avec le logiciel disponible pour la programmation linéaire. On s'occupe de cette relation dans le modèle en présumant les segments linéaires dans un procédé itératif. La relation ci-dessus pour centrales au fil de l'eau est exprimée comme suit:

$$P = f_1\ (Q) \qquad \text{(a)}$$

$$P = f_2\ (Q,DS) \qquad \text{(b)}$$

et pour les centrales avec un réservoir en amont:

$$P = f_3\ (Q,S) \qquad (c)$$

$$P = f_4\ (Q,S,DS) \qquad (d)$$

où S = l'emmagasinement du réservoir et

DS = l'emmagasinement d'un réservoir en aval.

Les expressions (b) et (d) s'appliquent quand le niveau d'un réservoir en aval produit une courbe d'exhaussement sur le canal de fuite de la centrale sous considération.

Une technique itérative est employée, sujette aux contraintes, pour trouver l'ensemble le plus efficace des segments linéaires pour l'engendrement de puissance. Ceci nécessite deux ou trois itérations dans lesquelles le problème doit être soumis à MPSX chaque fois. Ce procédé itératif ne coûte pas cher puisqu'on garde les résultats de la solution antérieure de MPSX pour la solution de base pour l'itération actuelle. D'habitude, les changements sont mineurs entre la solution antérieure et la solution actuelle. Toutefois, ces changements valent la peine du côté de la production d'énergie hydroélectrique.

Contraintes

Les contraintes physiques constituent le bilan hydrique à chaque installation et entre les installations, et les débits maximum possibles comme fonction du niveau du bief d'amont pour les chenaux contrôlés et libres. Les contraintes d'exploitation incluent des limites sur l'énergie engendrée par des groupes de centrales aussi bien que des corporations hydroélectriques.

Afin de représenter les relations non-linéaires par moyen des segments linéaires il faut introduire au modèle des variables quadrillées. Ces variables quadrillées représentent des contraintes supplémentaires.

Il se peut qu'un ensemble de contraintes fournies au système entrent en conflit l'une avec l'autre. Ceci présenterait un problème infaisable. Afin de surmonter le problème de MPSX se terminant prématurement sous ces conditions, on a introduit des variables artificielles, afin de permettre les violations à coût élevé des limites sur les emmagasinements et les décharges aussi bien que la puissance pour un groupe de centrales ou pour une corporation. Par conséquent, l'utilisateur peut recevoir en sus de la solution optimum, des avertissements, des violations de ces limites spécifiées.

Les résultats de plusieurs essais de ce modèle d'optimisation ont démontré des décharges trop variables d'une unité de temps à une autre. Conséquemment, on a dû ajouter des contraintes d'ammortissement afin de réduire la variation journalière. De plus, la fonction objective du modèle à court terme et à mi-terme était modifiée par l'addition de facteurs à coût élevé pour amortir les variations de décharge. On est arrivé à ces facteurs après quelques essais de calibrage.

Structure du modèle

Le modèle se compose des six modèles suivants: 1 - le générateur du squelettique, 2 - le générateur du fichier 2 de communications, 3 - le développement du modèle squelettique, 4 - le logiciel d'optimisation, 5 - l'analyseur, 6 - le rédacteur des rapports.

Il y a deux objets du générateur du modèle squelettique. Premièrement, il traite toutes les données statiques d'entrée au modèle et il produit le fichier 1 de communications pour l'utilisation par les autres composantes du modèle. Deuxièmement, il convertit automatiquement les données statiques d'entrée à une structure acceptable à MPSX. On ne se sert de ce module qu'une fois au début de l'étude ou quand il y a des changements physiques du système en ce qui concerne des centrales, des génératrices, des réservoirs, des déversoirs et des biefs.

Le but du générateur du fichier 2 de communications est de traiter toutes les données semi-statiques d'entrée et de générer le fichier 2 de communications pour l'utilisation plus tard par les autres composantes du modèle.

Le développement du modèle squelettique se sert d'information des fichiers 1 et 2 de communications aussi bien que des données dynamiques d'entrée et les prévisions des apports naturels afin de générer le fichier 3 de communications et des autres fichiers spéciaux qui contiennent des coefficients additionnels dans la disposition requise par MPSX. Le contenu de ces fichiers change d'un essai du modèle à l'essai suivant à cause du caractère dynamique des données d'entrée et la capacité du modèle d'adjoindre ou d'effacer des contraintes.

Le logiciel spécifique de MPSX est la version IBM/370.

Le module de l'analyseur exécute plusieurs des calculs itératifs requis par les relations inséparables entre la puissance, l'emmagasinement, et le débit total d'après la dernière solution de MPSX.

Le module du rédacteur des rapports accède aux fichiers générés par les autres modules et ensuite il génère un rapport d'utilisateur. L'utilisateur a plusieurs options d'imprimer seulement les sections qui l'intéressent du rapport détaillé afin de remplir les besoins de son étude particulière.

Une différence fondamentale entre ce modèle, MORRO, et le modèle de planification discuté préalablement, SMHC, est que ce modèle considère simultanément dans le logiciel d'optimisation, c'est-à-dire MPSX, toutes les unités de temps. Cependant, SMHC ne considère qu'une unité de temps dans le logiciel d'optimisation, c'est-à-dire DOSPB, et il marche pas à pas dans le problème en revenant à DOSPB pour chaque unité de temps. Donc, SMHC ne présume aucune connaissance des prévisions des apports naturels au-delà de l'unité de temps qu'il considère à présent, et il compte pour la direction sur les tâtonnements des limites de niveau et décharge des réservoirs fondés sur la climatologie.

La référence plus détaillée pour ce modèle est:

Direction générale des eaux intérieures, Environnement Canada. <<Users Manual for the Ottawa River Regulation Modelling System>>, janvier 1982.

Modèle SYSNET pour les Grands lacs

Résumé

Le modèle SYSNET est un modèle de simulation qui recherche une solution optimale. Ce modèle a été développé spécifiquement pour la régularisation du système entier des Grands lacs dans lequel sont inclus: lac Nipigon; les dérivations Ogoki, Long Lake, Chicago et Welland; et le débit de la rivière des Outaouais en amont de Montréal. La figure 4 est un diagramme schématique du système des Grands lacs qui est simulé par SYSNET.

Les relations de laminage des chenaux entre les lacs non-régularisés restent les mêmes que dans le modèle de simulation d'usage courant par le Bureau international de la régularisation du fleuve St-Laurent. Cependant, on accomplit la convergence de la solution d'une manière tout à fait différente du modèle d'usage courant.

SYSNET permet à l'utilisateur de spécifier les limites de contraintes sur les niveaux et décharges pour chaque quartier de mois. La plupart de ces limites sont déjà spécifiées dans les critériums adoptés pour la régularisation des Grands lacs. Cependant, pour permettre des solutions faisables, ce modèle se sert des variables auxiliaires qui, en réalité, permettent la violation des limites spécifiées à un maximum final ou à un minimum final pendant les périodes extrêmes hydrologiques, c'est-à-dire pendant les étiages ou les crues. Les modèles présents ont des difficultés à simuler la régularisation rationnelle pendant ces périodes de gros temps, et le Bureau international doit se servir de son autorité discrétionnaire. Donc, SYSNET a l'avantage de simuler d'une façon rationnelle la politique actuelle de régularisation. On peut l'utiliser directement pour prévoir le choc sur l'environnement et sur les secteurs économiques dû aux

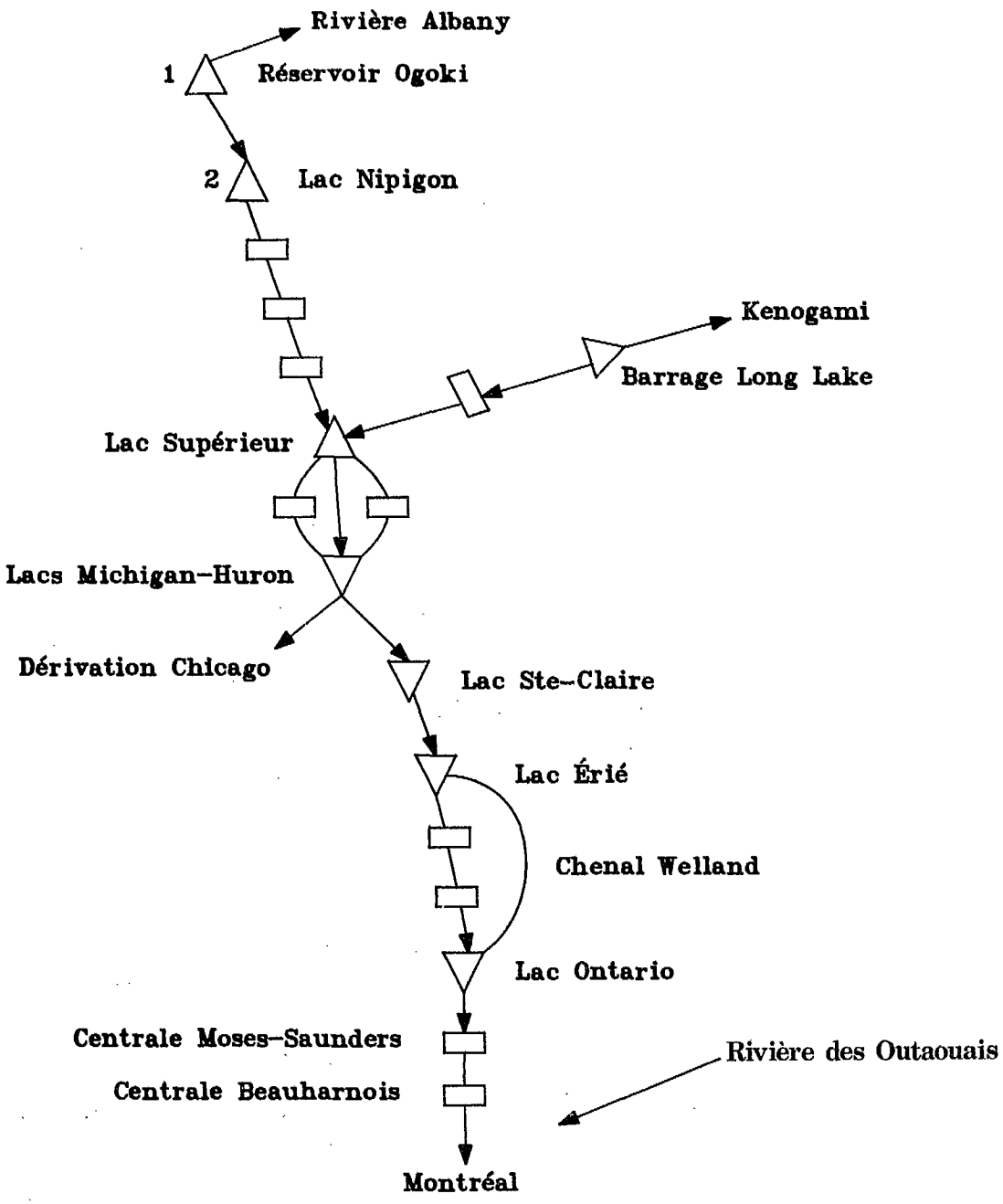


Figure 4. Régularisation sur les Grands lacs - schématique.

changements des contraintes sur les niveaux et débits pour n'importe quel chenal ou réservoir inclus dans la configuration. On considère la région de Montréal par le débit total (la rivière des Outaouais plus le fleuve St-Laurent) en comparaison avec simplement les décharges du lac St-Louis comme dans le modèle présent.

De plus, on pourrait utiliser SYSNET pour prévoir le choc du remplacement de la structure de contrôle du lac Supérieur avec une structure moderne qui permettrait des décharges continûment variables plutôt que les décharges discrètes actuelles.

La fonction objective (but d'optimisation) consiste essentiellement à garder l'eau en amont pour les besoins futurs sujets aux contraintes de niveaux et de débits pour chaque période d'un mois. La valeur d'eau dans SYSNET est une fonction de l'engendrement potentiel hydroélectrique. On encourt des coûts plus élevés quand les variables auxiliaires prennent des valeurs autres que zéro. Donc, la priorité primaire de SYSNET est de satisfaire les contraintes spécifiées.

SYSNET présume que les apports prévus sont disponibles pour chaque quart du mois sous considération. Ensuite, on optimise l'exploitation pour le mois fondée sur les calculs quartier-mois. Les décharges du lac Supérieur en conformité avec le plan actuel d'exploitation (Plan 77) sont gouvernées par la disposition mensuelle des vannes, mais les calculs sont faits pour chaque quartier-mois afin d'être consistant partout dans la configuration. Les apports quartier-mensuelles dans le but de vérification étaient obtenus des fichiers des données coordonnées de base des Grands lacs, et ils étaient calculés pour lac Nipigon, Long Lake, Ogoki, et la rivière des Outaouais par la Direction de la planification et de la gestion (eaux).

On a trouvé que l'algorithme des débits de réseau est une technique très flexible, et surtout, au moins un ordre de magnitude, et probablement deux ordres de magnitude moins cher en traitement informatique que par moyen du logiciel commercial de la programmation linéaire.

Justification pour le développement du SYSNET

Il reste un besoin démontré de considérer les Grands lacs et leurs dérivations comme un seul système intégré. Le Bureau international a employé plusieurs fois son autorité discrétionnaire dans l'exploitation du lac Supérieur au profit des intérêts des lacs en aval et la région de Montréal. Cependant, le plan du lac Supérieur approuvé maintenant par le Bureau international ne considère pas spécifiquement la situation en aval en déterminant l'exploitation du lac Supérieur. Les études récentes sur les dérivations et la consommation d'eau ont démontré de plus le besoin d'une analyse intégrée en ce qui concerne les dérivations variables.

L'optimisation économique

On n'a pas tenté d'introduire directement la théorie économique dans la fonction objective. Toutefois, la fonction objective actuelle représente une politique économique rationnelle, c'est-à-dire, garder l'eau supplémentaire en amont sujet aux contraintes de la production d'énergie, de la navigation, de l'environnement et de la protection contre les inondations. Au lieu d'avoir développé SYSNET comme un modèle pur d'optimisation, le but était d'incorporer la théorie des débits de réseaux dans un modèle de simulation qui ferait face suffisamment aux besoins des groupes divers d'étude. Comme on a discuté auparavant en rapport avec les modèles SMHC et MORRO, SYSNET aussi se sert des variables auxiliaires et des coefficients de coût. Les variables auxiliaires sont limitées aux zones entre les niveaux cibles et les niveaux extrêmes pour les emmagasinements et les décharges. À la figure 5, la variable f (arc en terminologie des débits de réseau) représente des valeurs entre les limites supérieures et inférieures. La limite cible supérieure est spécifiée comme <<TUL>>, et la limite cible inférieure est spécifiée comme <<TLL>>. L'expérience indique que les limites cibles sont fréquemment violées pendant les étiages (1964) et pendant les périodes d'écoulement excessif (1973, 1974, et 1976). Donc, les solutions infaisables arriveraient s'il n'y avait pas d'arcs auxiliaires pour accommoder les violations de l'arc f. Ces arcs auxiliaires ont des coefficients élevés de coût, dans l'ordre de 100.00, en comparaison avec les arcs d'emmagasinement à la fin du mois qui ont des valeurs à proximité de -.03. Les coefficients sont zéro pour les arcs d'emmagasinement des premiers trois quartiers et pour tous les arcs f de décharge. Il est évident avec les coefficients de coût plusieurs ordres de magnitude plus hauts pour les arcs auxiliaires que pour les arcs d'emmagasinement que SYSNET tentera de garder le débit à zéro dans ces arcs auxiliaires.

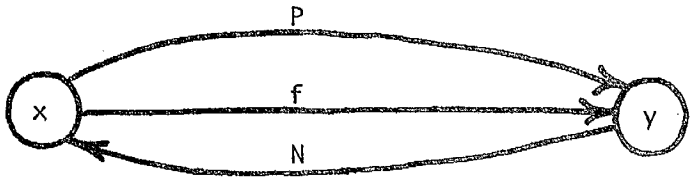


Figure 5

À la figure 5, les noeuds x et y sont reliés par les arcs P, f, et N ou P représente le débit dans l'arc de violation positive; f représente le débit entre les limites cibles supérieures et inférieures;

et N représente le débit dans l'arc de violation négative. Si le débit entre les noeuds x et y = NET, donc, NET = f + P - N.

La limite supérieure sur l'arc P = la limite supérieure extrême - TUL. La limite inférieure pour l'arc P est zéro.

La limite supérieure sur l'arc N = la limite inférieure extrême - TLL. La limite inférieure sur l'arc N est zéro.

On a assigné des arcs auxiliaires à tous les emmagasinements dans la configuration pour couvrir toutes les deux violations positives et négatives des limites cibles. On a assigné un coefficient arbitraire de 100.00 à ces arcs. L'arc de violation positive du lac Nipigon a un coefficient de coût plus élevé que 150.00 à cause de l'exutoire avec une capacité de décharge très limitée afin de protéger le chemin de fer Canadien National. Si SYSNET doit violer les arcs d'emmagasinement il emmagasinera ordinairement de l'eau en amont dans l'intérêt de maximiser le potentiel d'énergie. Donc, quand le coefficient de coût pour des violations positives de l'emmagasinement du lac Nipigon est plus élevé que pour ceux des autres lacs en aval, SYSNET enverra des volumes d'eau en aval au lac Supérieur afin d'accommoder les apports soudains de crue au lac Nipigon.

Dans la structure actuelle du réseau pour SYSNET, les arcs auxiliaires sont employés pour les violations positives des décharges du lac Ontario et les décharges totales dans la région de Montréal. Le coefficient de coût pour ces violations positives avait été posé arbitrairement à 150.00. Donc, pour les lacs en amont, le modèle ne permet pas les violations des limites cibles sur les décharges, et les arcs auxiliaires d'emmagasinement accommoderont les excès ou les déficits d'eau.

La figure 6 illustre les arcs pour accommoder des violations positives des décharges du lac Ontario. Les chiffres dans chaque noeud représentent les 4 quartiers qui sont considérés ensemble dans une solution itérative. Les chiffres sur les arcs représentent les débits pour chaque quartier.

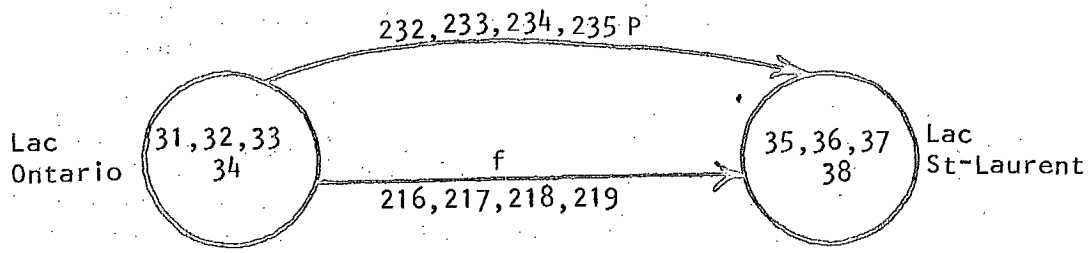


Figure 6

D'habitude dans l'application des débits du réseau on utilise des nombres entiers au lieu des nombres réels. On utilise les nombres réels en SYSNET partiellement dû au besoin de balancer les décharges et les niveaux des lacs avec les contrôles naturels aux exutoires. On dit au section 4.2 de l'annexe qu'on doit maintenir la conservation de débit. Il se peut que l'emploi des nombres réels mène aux erreurs d'arrondissement des chiffres et donc, perturbe le principe de la conservation de débit quand le modèle passe d'une unité de temps à la prochaine. Afin d'assurer que ceci ne devienne pas un problème sérieux, tous les arcs sont fixés à zéro à la fin de chaque année de simulation.

SYSNET est semblable à SMHC où il considère une période de temps courte (4 quartier-mois) dans le logiciel d'optimisation, et il passe à travers le problème d'un mois de relevés à la prochaine. Pendant la discussion à cet atelier, on a argumenté sur la fonction objective qui a tendance à garder l'excès d'eau en amont, assujetti aux contraintes. On a soutenu que cette politique ne réservera pas assez d'emmagasinement en amont pour la protection contre les inondations. On a répondu à cette critique en disant que les limites supérieures d'emmagasinement pour chaque unité de temps sont provenues de la climatologie. Ces limites prennent de la précédence sur la politique de garder de l'eau en amont. Par conséquent, l'emmagasinement en amont a été précisé en avance pour la protection contre les inondations.

On pourrait facilement introduire à ce modèle le coût économique d'inondation, les bénéfices de la production d'énergie, le coût des étiages et des autres variables, toutefois, ces valeurs sont souvent subjectives et non-acceptables à tous les organismes. Plus d'un ensemble d'arcs auxiliaires pourrait être utilisés pour chaque variable afin de représenter les magnitudes différentes des dommages d'inondation associés aux niveaux d'eau.

Dérivations d'eau

Les taux des dérivations d'eau varient pour chaque quartier-mois d'après des critères opérationnels de l'Hydro Ontario. Cependant, les taux des dérivations varient comme des fonctions continues. Dans le cas du réservoir Ogoki, les deux structures de contrôle, c'est-à-dire Waboose qui régularise les débits à la rivière Albany et Summit qui régularise les débits au lac Nipigon, sont les structures des batardeaux. Par conséquent, il se peut que la présomption de débit continu pendant les mois d'hiver ne soit pas appropriée. Heureusement, il est facile d'introduire des sous-routines dans le composant dirigeant qui calculerait les décharges pendant les quartiers-mois d'hiver comme une fonction du niveau du réservoir et la disposition des batardeaux. On traite aussi du canal Welland comme une dérivation variable d'eau assujettie aux contraintes spécifiées. La dérivation d'eau à Chicago est aussi variable entre le débit maximum et minimum comme fonction du quartier-mois de l'année. Le <<U.S. Corps of Engineers>> a fourni ces valeurs. On pourrait facilement calculer ces limites de façon

dynamique comme fonction d'un paramètre, soit la précipitation hebdomadaire ou le débit dans la rivière Illinois.

Structure de SYSNET

On peut décrire la structure de SYSNET en deux parties logiques:

(1) la simulation d'ensemble et le système de contrôle et
(2) le logiciel interne d'optimisation.

Le but de la simulation d'ensemble et le système de contrôle est de:

- accéder aux valeurs historiques des apports naturels,
- accéder aux contraintes et cibles historiques,
- imposer ces valeurs historiques sur l'optimisation de mois par mois en forme des contraintes, des limites, et des aiguillons,
- conduire le logiciel interne d'optimisation, et
- noter sur le registre les données de sortie du logiciel d'optimisation pour l'analyse et les rapports qui suivent.

Le but du logiciel interne d'optimisation est de simuler la conduite rationnelle dans l'opération du système de telle façon qu'elle est à la fois méthodique et réaliste. On accomplit ceci par la solution mois par mois d'un problème dérivé de la programmation mathématique par moyen de l'application itérative d'un logiciel d'optimisation des débits du réseau, en sus de plusieurs raffinements significatifs.

Le sous-problème résolu par le logiciel d'optimisation est plus complexe qu'un problème classique des débits du réseau. En plus de la formulation ordinaire qualifiée des débits du réseau (minimiser le coût des débits d'arcs avec limites supérieures et inférieures dans un réseau circulant d'arcs), il y a des composantes supplémentaires de la formulation du modèle dû aux:

- relations non-linéaires de débit par rapport à la décharge pour les lacs avec les exutoires naturels, c'est-à-dire les lacs Michigan/Huron, Ste-Clair et Érié,

- relations discrètes non-linéaires de débit par rapport à la décharge pour la disposition des vannes du lac Supérieur, et

- limites sur les changements hebdomadaire des décharges du lac Ontario.

Dans la situation actuelle il y a des relations implicites entre l'emmagasinement et la décharge. Dans une seule direction, l'algorithme des débits du réseau considère le changement en emmagasinement comme une fonction de débit (conservation simple de la masse). Cependant, dans la direction inverse, c'est-à-dire le débit comme fonction de

l'emmagasinement ou de niveau, l'algorithme des débits du réseau ne peut pas s'occuper de cette relation. Afin de surmonter ces composantes non-linéaires du problème, on a considéré les approches suivantes:

- l'emploi du logiciel commercial de la programme mathématique avec la capacité de la programmation séparable telle que MPSX ou APEX qu'on a employée dans le modèle MORRO,

- la décomposition par l'emploi d'un sous-problème des débits du réseau, et

- la représentation linéaire successive et l'optimisation des débits du réseau.

On a choisi la troisième alternative afin d'évaluer les avantages relatifs (tels que décrits récemment dans *Management Science, Operations Research, Mathematical Programming*, et d'autres revues du genre) des algorithmes des débits du réseau par rapport au logiciel plus général de la programmation mathématique.

Un résumé de l'approche est comme suit:

(1) Analyser la solution courante pour l'unité donnée de temps (ou, au début, la solution finale obtenue à la fin de l'unité de temps antérieure), et en employant toutes les relations appropriées non-linéaires, calculer les limites supérieures et/ou inférieures sur les décharges des réservoirs (calculer le débit comme fonction de l'emmagasinement).

(2) Mettre à date le problème courant pour réfléchir ces limites nouvelles, et soumettre le problème simple des débits du réseau qui en résulte pour l'optimisation. (Calculer l'emmagasinement optimal comme fonction de débit.)

(3) Mesurer la différence entre la solution nouvelle et la solution antérieure (de fait, calculer la somme des valeurs absolues des différences entre les débits correspondants des arcs dans les solutions successives).

(4) Si cette différence est dans une tolérance définie antérieurement, accepter la solution courante comme la solution convergée optimale. Autrement, retourner à la première démarche.

On a trouvé ce procédé tout à fait sûr et bon marché en ce qui concerne le traitement informatique. Toutefois, on a exécuté quelques rehaussements pour:

(a) surmonter une tendance dans quelques cas à ne pas converger sur une solution finale même s'il n'y avait pas grande différence entre les valeurs successives de la fonction objective. Dans ces cas il y avait des différences entre les débits d'arc (la mesure de convergence) et

(b) mettre à profit la structure spéciale dans le problème qui a permis l'application d'une boucle intérieure serrée pour accélérer la convergence de la solution par un facteur de 5-6.

Dans le premier de ces problèmes, on a ajouté plusieurs rehaussements pour aplanir les différences entre les solutions successives. Ces rehaussements ont inclu:

(a) l'aplanissement exponentiel des ensembles successifs de limites,

(b) les magnitudes restreintes des changements de débit d'arc entre chaque itération, et

(c) la capacité de reconnaître une solution bonne et faisable après un nombre approprié d'itérations.

En ce qui concerne le deuxième problème on a accompli une meilleure performance dans le traitement informatique en calculant de façon itérative le débit de chaque réservoir naturel et immédiatement ensuite en ajustant les emmagasinements résultants dans une boucle intérieure entre chaque exécution du logiciel d'optimisation.

Référence

Environnement Canada et TEE Consulting Services Inc. <<SYSNET Model for Great Lakes - Documentation Report>>, le 5 novembre, 1980.

SUMMARY OF PANEL DISCUSSION AND RECOMMENDATIONS

SOMMAIRE DE LA DISCUSSION ET RECOMMANDATIONS DU GROUPE DES EXPERTS

SUMMARY OF PANEL DISCUSSION AND RECOMMENDATIONS

Panel Chaired by Vit Klemes of NHRI

This Summary was based on Minutes prepared by Don Farley on September 21, 1981.

Regional Experience

The regional representatives on the panel presented the following reports on regional experience in the application of optimization techniques such as linear programming and network flows.

Neil Lyons of the *Pacific and Yukon Region* reported that in his region IWD had not used any optimization techniques whatsoever since about 1974. In 1973 and 1974, a linear programming algorithm had been developed for planning the operation of six proposed reservoirs on the upper Fraser River. The six reservoirs were considered together on a daily basis over a three-month period for three different years of records. The regulation policy suggested by the linear programming model was then assessed in a simulation model which was run without hindsight, and instead, forecasted inflows were assumed which would be equivalent to those produced during a rational real-time operation. The simulation model was run again to assess the impact of various environmental constraints. The objective function in the linear programming model was formulated to minimize downstream peak flows. Prior to 1972 there had been various optimization models used in Columbia River studies. However, optimization techniques are not used by the Columbia River Board in real-time regulation of the Columbia River reservoirs.

Ron Hofer of the *Western and Northern Region* reported that in his region IWD had not resorted to optimization techniques since the Saskatchewan-Nelson Basin Study Board had used linear programming to a limited extent in 1972. The objective function in this application involved minimizing the deviation between target flows and actual flows at specified jurisdictional boundary points in the basin. One of the major weaknesses in this model was the assumption of perfect hindsight over the period being modelled. In Churchill River and Reindeer Lake studies concerning proposed hydroelectric development, a simulation model without any optimization techniques was developed for studying Saskatchewan Power Commission regulation schemes. Maximum and minimum reservoir levels were imposed in the model along with power production requirements. A trial and error method of analysis was employed to determine the most appropriate regulation plan.

Future applications of optimization techniques should be considered for the Mackenzie Basin where proposals for interbasin diversions and power development will require considerable analysis. Minimization of environmental impact is seen as an important objective.

Syed Moins of the *Ontario Region* stated that in his region IWD had not used any optimization techniques since regional offices were established in Burlington and Toronto at the beginning of the last decade. *Don Farley* pointed out that prior to establishing the regional offices, headquarters had applied a dynamic programming algorithm to Great Lakes regulation studies. The complexity of the objective function, which considered perhaps non-commensurate items such as recreation and coastline damages, led to a general dissatisfaction with the model.

Mr. Moins reported that the Hydro-Configuration Modelling System (HCMS) developed for planning studies of the Ottawa River Basin had been considered by a consulting engineering firm in March 1980 for application to the Sturgeon River/Lake Nipissing/French River system. However, the firm decided to reject HCMS and any other optimization technique, and instead chose to use the simulation model "HEC5" of the U.S. Corps of Engineers. The consultant's report stated that the dimensionality requirement of not more than ten control points in the HEC5 model generally restricts its use to basins that have less than five reservoirs to be considered on a daily basis.

Dave Smith of the *Atlantic Region* stated that in his region IWD is not employing linear programming or network techniques. He was not aware of any other optimization techniques which had been attempted since the Saint John River water quality study terminated in 1974. A linear programming model had been used for the Saint John River water quality study. The objective function of the model was to minimize the total cost of effluent treatment throughout the basin subject to specified minimum dissolved oxygen concentrations at selected quality points. The optimization model solutions were not employed in the planning process to establish targets and priorities on treatment for the Saint John basin. The results were disappointing in that they did not provide insights beyond what had been known even without the model. Consequently, the model had no effect on the decisions made. The lack of success with the model was blamed on the following: inadequate water quality data, both in the spatial and temporal sense, to define adequately the transfer coefficients used in the biological reaction processes; one plant, namely Fraser's pulp mills above Grand Falls, was responsible for over 90% of the pollutant loading, thereby making irrelevant the consideration of all treatment plants in the basin; and the treatment costs per unit loading that were used in the model were believed to be inaccurate.

Tom Dafoe of the *Quebec Region* reported that in his region IWD had relied fully on headquarters units to do modelling whether it be pure simulation or of a variety requiring optimization techniques. The Quebec Region is anticipating a report on the Ottawa River Regulation Modelling System (ORRMS) which will give the results of tests with the model and outline both negative and positive experience with the system to date. This workshop has highlighted some of these results and experience which would likely be included in the report.

Mr. Dafoe expressed disappointment that construction of dykes in the region of Montreal had been carried out prior to any extensive modelling studies. However, he pointed out that construction of the dykes was considered a priority item and that planning was still required for the proposed increased regulation upstream from Montreal and the construction of major control dams in the region of Montreal.

Mr. Dafoe stated that the discretionary powers of the Great Lakes Board of Control with respect to Lake Ontario regulation are often a contentious issue in Quebec. However, alternative modelling schemes to replace this discretionary authority of the Board have not been proposed.

Finally, Mr. Dafoe expressed the opinion that all resources in a modelling study should be considered as economic resources and thus assigned monetary values. For example, hydroelectric energy and wildlife such as muskrats and ducks should be assigned a dollar value per unit of production.

Appraisal of Workshop Achievements

There was a consensus among the panel members that the workshop had achieved the goal of informing participants of successes and failures in the application of optimization techniques to water resources modelling.

Participants gained an appreciation of how optimization techniques could be efficiently incorporated in simulation modelling without resorting to attempts of global optimization.

Participants now have a better understanding of what is involved in linear programming and network flow applications and are more fully aware of the strengths of these techniques as well as the pitfalls.

The advance material was generally useful, but care should be exercised in completely editing the notes for future reference by the participants.

The workshop clearly identified linear programming and network flows as tools for consideration in modelling applications which, in appropriate cases, could increase the efficiency of both the manpower and the data processing components.

The emphasis of the workshop on linear programming and network flows may have introduced an element of bias in favour of the two techniques. To provide a balanced view of systems analysis applications, similar workshops on other techniques should be considered in the future.

A very positive aspect of the workshop was that the organizers had emphasized simulation approaches rather than having tried to demonstrate the scientific quest for the nebulous global optimum of a

water resources problem. This approach tended to dispel the myth that there is a single optimum scientific solution to each problem. In practical situations, elements involving political decisions, morals and common law must be considered in harmony with the other elements of a problem which can be analyzed scientifically and economically. This dichotomy was well illustrated in the discussion on the ORRMS model relating to the choice of a C_1 coefficient in the objective function for hydroelectric production and a somewhat competing coefficient, C_2, also in the objective function for flood damages.

Specific Comments Raised in Discussion

The workshop organizers emphasized that they did not try to impose on the attendees the idea that linear programming and network flows were the ultimate means to reach decisions in water resources projects. Instead, an attempt was made to convey to the attendees the problems encountered in adapting the technology to water resources problems, specifically for measuring the economic impact of trade-offs.

Several members of the panel and at least half of the audience requested the computer solutions for the example problems given in the precourse material. It was agreed that the input data for the three problems in the precourse material would be assembled for analysis by the IBM package MPSX and forwarded with an example solution to workshop attendees. Interested attendees could experiment with the input parameters of the problems in order to gain an in-depth understanding of linear programming. MPSX is available at all major IBM computer service bureaus. However, members were cautioned that the example simplistic solutions in the precourse material are not representative of typical water resources problems. The models discussed at the workshop are, in the opinion of the workshop organizers, practical applications of the technology. These models with documentation are available upon demand. It was emphasized that successful application would require the user to familiarize himself with the techniques at a deeper level than that presented in the workshop materials.

Panel members believed that headquarters should maintain a centre of expertise in systems techniques. This was also the viewpoint of the panel chairman, who pointed out that the systems coordination function was conceived at the IWD mathematical modelling workshop in Winnipeg in November 1976. Headquarters personnel stressed that such a centre of expertise at headquarters must rely to a significant extent on experience and technological advances in the regions and, in many cases, headquarters main function should be systems coordination. Some members felt that the Systems Evaluation Section at headquarters, which has the responsibility for coordination, is inadequately staffed for providing satisfactory service to the regions. The Quebec Region representative expressed satisfaction with the coordination service provided to date. At least one region was not even aware of this systems coordination function at headquarters.

One problem highlighted by the panel was the obvious mismatch between the capability of an optimal seeking simulation model like ORRMS and the myopic capabilities of a watershed hydrologic forecasting model which provides the necessary input to ORRMS. It was pointed out that it would be rather useless to attempt to apply significant sophistication involving optimization techniques to flow regulation problems before the forecasting models had become more reliable. Thus, more effort should be expended in reducing this mismatch between the theoretical possibilities of systems techniques for optimization and simulation, and the practical capabilities of the current flow forecasting models.

Recommendations Concerning the Application of Linear Programming and Network Flows

(1) The analyst must clearly demonstrate that the modelling approach employing linear programming and network flows represents reality.

(2) For successful adaptation of linear programming and network flows to planning studies and real time operation, the analyst must ensure that the inherent techniques, assumptions and data input are acceptable to users and operators.

(3) Political and nontechnical obstacles should be recognized and identified in the conception of the model and in its effective application.

SOMMAIRE DE LA DISCUSSION ET RECOMMANDATIONS DU GROUPE DES EXPERTS

Président du groupe - Vit Klemes de l'INRH

Ce sommaire est fondé sur le compte rendu préparé par Don Farley le 21 septembre 1981.

Expérience régionale

Les représentants régionaux du groupe d'experts ont présenté les rapports suivants portant sur l'expérience régionale dans l'application des techniques d'optimisation telles que la programmation linéaire et les débits du réseau.

Neil Lyons de la *Région du Pacifique et du Yukon* a précisé que la Direction générale des eaux intérieures de sa région n'a pas employé de techniques quelconques d'optimisation depuis à peu près 1974. On a développé un algorithme de la programmation linéaire en 1973 et 1974 pour la planification et l'exploitation de six réservoirs projetés en amont du fleuve Fraser. On a considéré les six réservoirs ensemble sur une base journalière sur une période de trois mois pour trois années de relevés historiques différents. La politique de régularisation suggérée par le modèle de la programmation linéaire a été évaluée au moyen d'un modèle de simulation qu'on a conduit sans sagesse en rétrospective, et plutôt, on a présumé les prévisions d'apports naturels correspondant à celles disponibles en temps réel. On a encore conduit le modèle de simulation pour évaluer le choc de plusieurs contraintes de l'environnement. On a formulé la fonction objective dans le modèle de la programmation linéaire pour minimiser les débits de pointe pendant la crue. Antérieurement à 1972 on a essayé quelques modèles d'optimisation sur les études du fleuve Colombie. Cependant, le Bureau international du fleuve Colombie n'utilise pas de techniques d'optimisation pour l'exploitation en temps réel des réservoirs du fleuve Colombie.

Ron Hofer de la *Région de l'Ouest et du Nord* a énoncé que sa région n'a pas profité des techniques d'optimisation depuis 1972 quand le bureau de l'étude sur le bassin Saskatchewan-Nelson s'en est servi d'une façon limitée. La fonction objective dans cette application a comporté la minimisation de la déviation entre les débits de cible et les débits actuels aux pointes sur les frontières juridictionnelles dans le bassin. Une faiblesse majeure de ce modèle entraîne la présomption de la sagesse parfaite en rétrospective sur la période des relevés historiques. On a développé un modèle de simulation qui n'a eu aucune capacité d'optimisation pour étudier les schémas de régularisation pour la rivière Churchill et lac Reindeer par la Commission de l'énergie de Saskatchewan. Des niveaux maximum et minimum des réservoirs ont été imposés dans le modèle avec des contraintes sur la production d'énergie. On a utilisé une analyse tâtonnante pour trouver le plan d'exploitation le plus approprié.

On devra considérer des applications futures des techniques d'optimisation sur les bassins du fleuve Mackenzie où les propositions de développement pour des dérivations entre des bassins et la production d'énergie nécessiteraient une analyse minutieuse et étendue. On prévoit que l'objectif de la minimisation du choc sur l'environnement est très important.

Syed Moins de la *Région de l'Ontario* a précisé que la Direction générale des eaux intérieures dans sa région n'avait pas utilisé de techniques d'optimisation depuis que le bureau régional a été établi à Burlington au début de la dernière décennie. *Don Farley* a fait ressortir le fait que préablement à l'établissement des bureaux régionaux, le quartier-général appliquait un algorithme de la programmation dynamique aux études de régularisation des Grands lacs. La complexité de la fonction objective qui considérait peut-être des items sans valeurs connues telles que les dommages à la recréation et au littoral a conduit à un mécontentement général à propos du modèle.

M. Moins a rapporté que le modèle SMHC développé pour planifier les études du bassin de la rivière des Outaouais avait été pris en considération par une compagnie de conseillers en génie en mars 1980 pour être appliqué au système de la rivière des Esturgeons/le lac Nipissing/et la rivière des Français. Toutefois, la compagnie des conseillers en génie a décidé de rejeter SMHC ou toute autre technique d'optimisation, et plutôt a choisi d'utiliser le modèle de simulation <<HEC5>> du Corps des Ingénieurs américains. Le rapport du conseiller a précisé que la contrainte dimensionnelle de pas plus de dix points de contrôle dans le modèle <<HEC5>> limite généralement son usage aux bassins qui ont moins de cinq réservoirs à considérer sur une base journalière.

Dave Smith de la *Région de l'Atlantique* a dit que DEI dans sa région n'emploie pas la programmation linéaire ou des techniques de réseau. Il n'était pas au courant de l'utilisation de toute autre technique d'optimisation depuis la fin de l'étude de la qualité de l'eau de la rivière St-Jean en 1974. Un modèle de programmation linéaire a été utilisé pour l'étude de la qualité d'eau de la rivière St-Jean. La fonction objective du modèle était de minimiser le coût total de l'épuration de l'effluent à travers le bassin assujetti à un minimum spécifié de concentration d'oxygène dissous à des points choisis de qualité. Les solutions du modèle d'optimisation n'ont pas été utilisées dans le procédé de planification pour établir les cibles et priorités d'épuration du bassin de la St-Jean. Les résultats ont été désappointants dans le fait qu'ils n'ont pas donné de prévisions plus avancées que celles qui étaient déjà connues sans le modèle. Par conséquent, le modèle n'a eu aucun effet sur les décisions prises. Le manque de succès du modèle a été blâmé sur les suivants: des données insuffisantes de la qualité d'eau, au sens spatial aussi bien que temporel, pour définir adéquatement les coefficients de transfert utilisés dans les procédés de réactions biologiques; une usine, nommément le moulin à papier Fraser en amont de Grand Falls, a été responsable pour plus de 90% de la pollution, rendant négligible l'effet de toutes autres

usines d'épuration d'effluent dans le bassin' on a cru que les coûts par unité de charge de DBO qu'on a utilisés dans le modèle étaient imprécis.

Tom Dafoe de la *Région du Québec* a dit que DEI dans sa région a compté entièrement sur les sections au quartier-général pour faire la modélisation que ce soit une simulation pure ou une variété réquérant des techniques d'optimisation. La Région du Québec anticipe un rapport sur le système du Modèle d'optimisation de la régularisation de la rivière des Outaouais (MORRO) qui va donner les résultats des tests du modèle et exposer l'expérience négative et positive du système jusqu'à date. Cet atelier a illustré quelques-uns de ces résultats et expériences qui seront probablement inclus dans le rapport.

M. Dafoe a exprimé son désappointement que la construction de digues dans la région de Montréal ait été faite avant les études extensives de modélisation. Cependant, il a indiqué que la construction de digues était considéré comme un item prioritaire et que la planification était toujours requise pour l'augmentation de la régularisation projetée en amont de Montréal et la construction des barrages de contrôle dans la région de Montréal.

M. Dafoe a affirmé que les pouvoirs discrétionnaires du Bureau international de contrôle des Grands lacs en ce qui concerne la régularisation du lac Ontario sont souvent une issue contestée au Québec. Toutefois, des schémas alternatifs de modélisation pour remplacer cette autorité discrétionnaire du Bureau international n'ont pas été présentés.

M. Dafoe a exprimé l'opinion que toutes les ressources dans une étude de modélisation devraient être présumées comme une ressource économique et par conséquent avoir une valeur monétaire. Par exemple, l'énergie hydroélectrique et la faune telle que les rats musqués et les canards devraient avoir une valeur en dollars par unité de production.

Appréciation des réalisations de l'atelier

Les membres du groupe d'experts étaient d'accord que l'atelier a accompli le but d'informer les participants des succès et des faillites dans l'application des techniques d'optimisation de modélisation des ressources en eau.

Les participants ont gagné une appréciation de la façon dont les techniques d'optimisation pourraient être efficacement incorporées dans la modélisation de simulation sans ressortir à des tentatives d'optimisation globale.

Les participants ont une meilleur connaissance de ce qui est impliqué dans l'application de la programmation linéaire et les débits du réseau, et sont maintenant pleinement conscients des avantages de ces techniques ainsi que des faiblesses.

Les informations préliminaires étaient généralement utiles, mais on devrait prendre soin de faire la rédaction complète des notes pour référence future pour les participants.

L'atelier a identifié clairement la programmation linéaire et les débits du réseau comme des outils possibles dans des applications de modélisation qui, dans les cas appropriés, peuvent ajouter de l'efficacité à la main d'oeuvre et au traitement informatique.

L'emphase de l'atelier sur la programmation linéaire et les débits du réseau peut avoir introduit un élément de préjugé en faveur des deux techniques. Pour fournir une vue équilibrée des applications des analyses de systèmes, des ateliers semblables sur d'autres techniques devraient être planifiés dans le futur.

Un aspect très positif de l'atelier est que les organisateurs ont appuyé sur les approches de simulation plutôt que d'essayer de démontrer la recherche scientifique pour l'optimisation globale nébuleuse d'un problème en ressources d'eau. Cet approche a dissipé le mythe qu'il y a une seule solution optimum scientifique pour chaque problème. Dans des situations pratiques, les éléments qui impliquent des décisions politiques, morales, et le droit commun doivent être considérés en harmonie avec les autres éléments d'un problème qui peut être analysé scientifiquement et économiquement. Cette dichtomie était bien illustrée dans la discussion sur le modèle MORRO en ce qui concerne le choix du coefficient C_1 dans la fonction objective de la production hydroélectrique et un coefficient C_2 quelque peu en conflit avec la fonction objective pour les dommages d'inondation.

Commentaires soulevés pendant la discussion

Les organisateurs de l'atelier ont appuyé sur le fait qu'ils n'ont pas essayé d'imposer aux participants l'idée que la programmation linéaire et les débits du réseau étaient les moyens suprêmes pour arriver à des décisions dans des projets de ressources en eau. Contrairement, une tentative a été faite pour faire comprendre aux participants les problèmes rencontrés dans l'adaptation de la technologie aux problèmes de ressources en eau, spécifiquement pour mesurer le choix économique d'échanges de buts.

Plusieurs membres du groupe d'experts et au moins la moitié de l'audience ont demandé les solutions informatiques pour les problèmes exemples donnés dans les informations préliminaires. On a été d'accord que les données d'entrée pour les trois problèmes seraient assemblées pour analyse par le logiciel MPSX de IBM, et envoyées avec une solution exemple aux participants de l'atelier. Les participants intéressés pourraient expérimenter avec les paramètres d'entrée des problèmes de façon à obtenir une compréhension approfondie de la programmation linéaire. MPSX est disponible dans tous les bureaux majeurs de service informatique IBM. Toutefois, on a prévenu les participants que les

solutions des exemples simplistiques dans les informations préliminaires ne sont pas représentantes de problèmes typiques de ressources en eau. Les modèles discutés à l'atelier sont, dans l'opinion des organisateurs de l'atelier, des applications pratiques de la technologie. Ces modèles avec documentation sont disponibles sur demande. On a appuyé sur le fait que l'application réussie demanderait que l'usager se faméliarise avec les techniques à un niveau plus approfondi que celui présenté à l'atelier.

Les membres du groupe d'experts ont cru que le quartier-général devrait maintenir un centre d'expertise dans les techniques de systèmes. C'était aussi le point de vue du président du groupe d'experts qui a indiqué que la fonction de coordination de systèmes a été conçue à un atelier de modélisation mathématique à Winnipeg en novembre 1976. Le personnel du quartier-général a insisté qu'un tel centre d'expertise au quartier-général doit dépendre d'une façon significative sur l'expérience et le progrès technologique dans les régions et, dans beaucoup de cas, la fonction principale du quartier-général devrait être la coordination de systèmes. Quelques membres ont pensé que la Section d'évaluation de systèmes au quartier-général, qui a la responsabilité pour la coordination, n'a pas le personnel adéquat pour fournir un service satisfaisant aux régions. Le représentant de la Région du Québec a exprimé sa satisfaction du service de coordination fourni jusqu'à date. Au moins, une région n'était même pas au courant de la fonction de coordination de systèmes au quartier-général.

Un problème mis en lumière par le groupe d'experts a été le désaccord évident entre le capacité d'un modèle de simulation qui cherche l'optimum comme MORRO, et les capacités myopiques d'un modèle de prévision des apports naturels qui fournit les données d'entrée nécessaires à MORRO. On a indiqué qu'il serait plutôt inutile d'essayer d'appliquer une sophistication significative impliquant les techniques d'optimisation aux problèmes de régularisation d'eau avant que les modèles de prévisions deviennent plus fiables. Par conséquent, plus d'efforts devraient être exercés pour réduire ce désaccord entre les possibilités théoriques des techniques de systèmes pour l'optimisation et la simulation, et les capacités pratiques des modèles actuels de prévision de débit.

Recommandations en ce qui concerne l'application de la programmation linéaire et débits du réseau

(1) L'analyste doit démontrer clairement que l'approche de modélisation employant la programmation linéaire et les débits du réseau représente la réalité.

(2) Pour une adaptation réussie de la programmation linéaire et des débits du réseau aux études de planification et l'opération en temps réel, l'analyste doit s'assurer que les techniques inhérentes, les présomptions, et les données d'entrée sont acceptables aux usagers et aux opérateurs.

(3) Les obstacles politiques et non-techniques devraient être reconnus et identifiés dans la conception du modèle et son application effective.

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LISTE DES PARTICIPANTS

TECHNOLOGY TRANSFER WORKSHOP

PRACTICAL APPLICATION OF LINEAR PROGRAMMING AND NETWORK FLOWS

Hull, P.Q.
September 15 and 16, 1981

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APPENDIX

ADVANCE MATERIAL FOR TECHNICAL WORKSHOP

ANNEXE

DOCUMENTS DE RÉFÉRENCE POUR L'ATELIER TECHNIQUE

FOREWORD

The notes in this appendix are for the benefit of workshop attendees who have had little or no exposure to linear programming and network flows. Novices should have little difficulty in reading these notes and gaining an adequate level of knowledge prior to the workshop. Example problems in these notes are purposefully simplistic in order to demonstrate concepts. The workshop organizers will strive to show how these techniques are employed effectively for the usually much more complex problems which occur in water resources planning and management. If a novice wishes to gain more insight than these notes permit, he is referred to Chiang (1974), Bazaraa and Jarvis (1977) and IBM (1979) in the list of references at the end of this appendix.

Don Farley

Head, Systems Operations

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1. Optimization Terminology

Systems analysis is fundamentally a science of choice. When any kind of a project is to be implemented, such as the production of a specified level of output, there are usually a number of alternative ways of accomplishing it. One (or more) of these alternatives will, however, be more desirable than the others from the standpoint of some criterion, and it is the essence of the optimization problem to choose, on the basis of that specified criterion, the best alternative available.

The most common criterion of choice among alternatives, in systems analysis, is the goal of maximizing something (such as maximizing a firm's profit, maximizing the number of full-time jobs created or maximizing the rate of growth of a firm or of a country's economy) or of minimizing something (such as minimizing the cost of producing a given output). Economically, such maximization and minimization processes can be categorized under the general heading of *optimization*, meaning "the quest for the best." From a purely mathematical point of view, however, the terms "maximum" and "minimum" do not have any connotation of optimality. Therefore, the collective term for maximum and minimum, as mathematical concepts, is the more matter-of-fact designation *extremum*, meaning an extreme value. In the practical examples in water resources discussed in Section 2 of this appendix, there is little or no attempt to optimize. Instead, the emphasis is placed on following a rational policy in which deviations beyond desirable limits are minimized. Undesirable

events might include high or low water levels, extensive and sudden fluctuations in levels or flows, and dissolved oxygen levels falling below 5 ppm. For example, the rational policy might be to keep water supplies upstream as far as possible when given the choice.

In working out an optimization problem, the first order of business is to delineate an *objective function* in which the dependent variable represents the object of maximization or minimization and in which the set of independent variables indicates the objects whose magnitudes can be chosen with a view to optimizing. We shall therefore refer to the independent variables as *decision variables*. The essence of the optimization process is simply to find the set of values of the decision variables that will yield the desired extremum of the objective function.

Commonly used optimization techniques are:

(a) calculus without constraints,
(b) calculus with constraints,
(c) gradient method,
(d) linear programming and network flows, and
(e) dynamic programming.

Linear programming is by far the most popular technique used in the planning and management aspects of water resources. The gradient

technique is being used to some extent in calibrating conceptual watershed models where the objective function is to minimize the sum of the squares of the deviations between observed and simulated values. Some valiant attempts have been made in applying dynamic programming to practical water resources problems. However, dynamic programming suffers from the severe limitation that it cannot cope with more than one or two decision variables within any unit time period of the particular time series being studied. Hence multiple reservoir and/or multiple use problems which prevail in water resources preclude the application of dynamic programming. Network flow techniques have been rapidly gaining in popularity due to the computing efficiency of presently available algorithms. The assumptions which normally apply to linear programming also apply to network flows. More work appears to be involved in setting up a network flow model as compared with a linear programming model for an identical problem.

When each decision variable is assigned a particular value, the resulting set of decisions is called a *policy* or an *assignment*. In general, there will be *constraints* which will limit the possible policies. A policy which does not violate any constraints is a *feasible policy*. The subset consisting of all possible feasible policies is termed the *policy space*. The policy space may be quite variable over time and physical space. A future policy space may be greatly enlarged or, conversely, seriously restricted by the particular decision set selected today.

The policy space may be *convex* or *nonconvex*. The general idea of convexity may be illustrated geometrically in two-dimensional space by a closed curve which has the property that a straight line connecting any two points on the boundary includes only boundary and/or interior points. Figure A-1 illustrates examples of convex and nonconvex two-dimensional policy space.

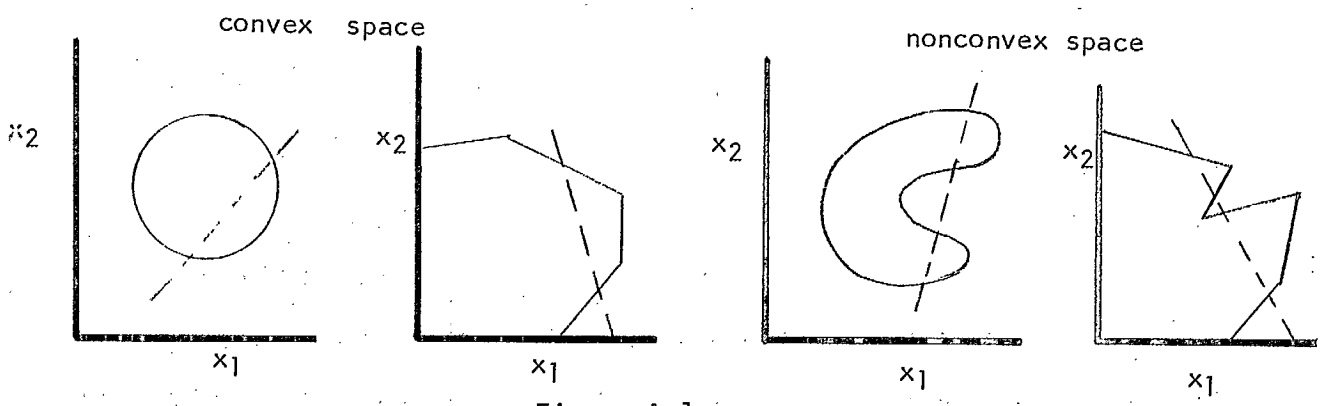


Figure A-1

The conditions of the system proper at any time and place are represented by variables known as *state variables*. Supplementing the state variables are the *system parameters*, which essentially can be considered *constraints* or *bounds* which are either constraints or vary in a prescribed manner for purposes of the study.

The concept of a "best" policy or assignment implies criteria whereby the effect of any feasible policy on the desirable and undersirable outputs can be measured. This criterion is called the *objective* of the model. The objective in most instances consists of many component objectives, some of which are *quantitative objectives*, measurable with some degree of accuracy, and *nonquantitative*

objectives, measurable at most in an order of ranking or in a qualitative sense. If two objectives can be measured or described in the same units or terms (e.g. dollars, kilowatt hours) with the same general degree of accuracy, they are *commensurate objectives*. *Noncommensurate objectives* either cannot be expressed in common units or the order of magnitude of the errors inherent in the evaluation of one masks the significance of the magnitude of the other. The *objective function* which has already been defined is limited in scope to the determination of those quantitative objectives that are fully commensurate. This usage often obscures the true nature of systems analysis, but it is necessary in quantitative analysis, since there is no logical mathematical procedure by which an objective function consisting of more than one noncommensurate quantitative objective can be optimized.

Since the objective function is the crux of systems analysis, it is important to emphasize again the limitations of such a simplified statement of objectives. The objective function for most problems, especially those occurring in water resources, only represents a partial set of the objectives. For instance, the units in the objective function might be dollars representing the value of hydroelectric energy, irrigation supplies, and water treatment costs. However, there are usually many other project objectives which might include maintaining lake levels within a certain narrow range during the summer months to satisfy recreational demands; limiting variation in discharges to protect a fishery resource; and maintaining minimum discharges for aesthetic reasons. None of these objectives is commensurate with the dollars

associated with the objective function. However, the effect of constraints related to these objectives can be measured in dollars by comparing the results of one scheme with those of another. This concept will be demonstrated later in the section on linear programming.

2. Mathematical Programming

"Mathematical programming" is a general term covering linear programming as well as nonlinear programming and game theory. Mathematical programming differs from classical optimization in that it seeks to tackle problems in which the optimizer faces *inequality* constraints - constraints in the form of, say, $g(x,y) \leq c$ rather than $g(x,y) = c$. As a specific illustration, instead of requiring a consumer to spend the exact amount of $250, the mathematical programming framework would permit him the freedom of spending either $250 or less if he chooses. By thus liberalizing the constraint requirement, this new framework of optimization makes the problem at once more interesting and more realistic. However, it also calls for the development of new methods of solution, since inequality constraints cannot be handled by the classical techniques of calculus.

Game theory, the other development, departs in a radical way from the methods of optimization to be discussed in this workshop. Instead of seeking a maximum or minimum value of a variable, the optimizer will be assumed to be looking for a so-called "minimax" or "maximin" value. Game theory bears a close relationship to linear

programming and network flows even though these methods were developed independently of one another.

3. Linear Programming

In linear programming, the objective function as well as the constraint inequalities are all linear. The policy space must be convex to ensure that any local maximum of a linear objective function is a global maximum.

The general problem in linear programming may be stated by the following set of equations.

Maximize

$$V = c_1x_1 + c_2x_2 + c_3x_3 + \dots + c_nx_n \qquad \text{(A-1)}$$

subject to the following constraints:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots a_{2n}x_n &\leq b_2 \\ a_{r1}x_1 + a_{r2}x_2 + \dots a_{rn}x_n &\leq b_r \end{aligned} \qquad \text{(A-2)}$$

where x = the decision variables,

c = the profit/unit of x, and

b = the constraint coefficients.

For convenience in notation let us write (A-1) and (A-2) in summation notation:

$$\text{Max} \sum_{j=1}^{n} c_j x_j \tag{A-3}$$

$$\text{s.t.} \sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad i = 1,2,\ldots r \tag{A-4}$$

Since in many cases it is not reasonable to have negative operating levels, there is often a further natural constraint that

$$x \geq 0 \qquad j = 1,2,\ldots n \tag{A-5}$$

Statement (A-3) is then the statement of objective of the model, the objective being to maximize profit. The constraint statements summarize the conditions under which the profit is to be maximized. When maximizing profits from a water resources project, the constraints may include the following:

- maximum capital available,
- minimum hydroelectric power requirements, and
- minimum downstream flow requirements.

A *minimization* problem may be converted to a maximization problem by changing the sign in the objective function. Specifically,

$$\text{Minimize } V = \sum_{j=1}^{n} c_j x_j \text{ can be treated as}$$

$$\text{Maximize } -V = \sum_{j=1}^{n} (-c_j)\, x_j \text{ and vice versa.}$$

The sense of the inequality in constraints may be changed utilizing the following fact:

$$\sum_{j=1}^{n} a_j x_j \leq b \quad \text{can be written as} \quad (-a_j)\, x_j \geq -b$$

Before the solution algorithms can be utilized, it is generally necessary to have the constraints expressed as strict equalities. This can be achieved by introducing slack variables for $\leq$ and surplus variables for $\geq$. As an example, $x_1 + x_2 + x_3 \leq 4$ can be changed to $x_1 + x_2 + x_3 + x_4 = 4$ $(x_4 \geq 0)$ where x_4 is called the slack variable. If $x_4 \geq 0$ in the final solution, this indicates the resources or process described in the constraint has some slack ("left over" resource).

3.1 *Feasible Policy Space*

For a solution to be feasible it must satisfy the constraints set. In simple problems of two dimensions, it is possible to plot all the constraint sets. This simple problem will give considerable insight into the more general "n" dimensional problem, since the theory remains

the same. The computation soon becomes very burdensome, and with more than three or four variables becomes impossible to do by hand.

3.2 *Assumptions of Linear Programming*

To represent an optimization problem as a linear program, several assumptions that are implicit in the linear programming formulation discussed above are needed. A brief discussion of these assumptions is given below.

(1) *Proportionality*. Given a variable x_j, its contribution to cost is $c_j x_j$ and its contribution to the ith constraint is $a_{ij} x_j$. This means that if x_j is doubled, for example, so is its contribution to cost and to each of the constraints. As an illustration, let us suppose that x_j is the amount of activity j used. For instance, if $x_j = 10$, then the cost of this activity is $10c_j$. If $x_j = 20$, then the cost is $20c_j$, and so on. This means that no savings (or extra costs) are realized by using more of activity j. Also no setup cost for starting the activity is realized.

(2) *Additivity*. This assumption guarantees that the total cost is the sum of the individual costs and that the total contribution to the ith restriction is the sum of the individual contributions of the individual activities.

(3) *Divisibility*. This assumption ensures that the decision variables can be divided into any fractional levels so that non-integer values for the decision variables are permitted.

In summation, an optimization problem can be cast as a linear program only if the aforementioned assumptions hold. This precludes situations in which economies of scale exist, for example, when the unit cost decreases as the amount produced is increased. In these situations, one must resort to nonlinear programs. It should also be noted that the parameters c_j, a_{ij}, and b_i must be known or estimated.

3.3 *Graphical Solutions to Simple Linear Programming Problems*

$$\text{Max } v = 5x_1 + 3x_2$$

$$\text{s.t.} \quad 3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

Plotting the constraint set will tell us where in the x_1 x_2 plane the optimal solution can exist. Thus, the constraints define a feasible solution space. Figure A-2 illustrates this feasible solution space for the example problem.

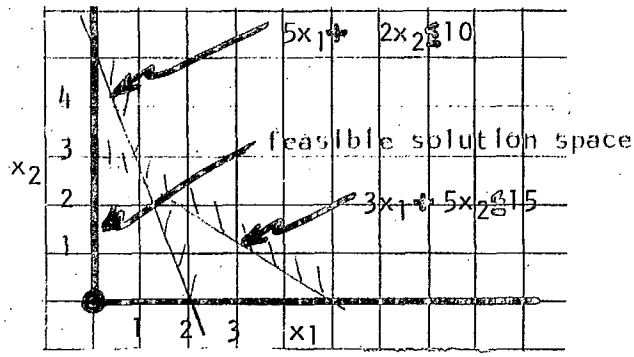


Figure A-2

Plotting the objective function on the same graph makes the answer to the problem quite evident. The greatest value of $5x_1 + 3x_2$ is obtained at $x_2 = 2.4$ $x_1 = 1.1$ at which $V = 12.7$. The solution is illustrated in Figure A-3. Check that at no other point in the feasible solution space is V larger than 12.7.

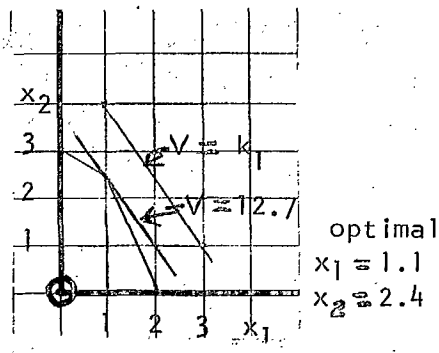


Figure A-3

In this case we notice that the optimal solution occurs at a point of intersection in the feasible solution space. This is true in general. The optimal solution is always at an extreme point if it exists.

Occasionally, there are two optimal points. In such a case any convex combination of these two points is also optimal (i.e., any point

on the line joining these two points is also optimal). An example of such a case is illustrated in the following problem.

$$\text{Max. } V = 2.6\,x_1 + x_2$$

$$\text{s.t.} \quad 3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1 \geq 0 \quad x_2 \geq 0$$

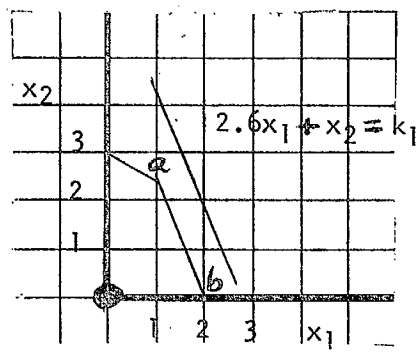


Figure A-4

The optimal solution includes $x_1 = 1.1$, $x_2 = 2.4$ and $x_2 = 0$, $x_1 = 2$, $V = 5.2$ (Fig. A-4). Any point on the line between (1.1, 2.4) and (2,0) is also an optimal solution. This is therefore an example of a multiple optimal solution.

3.4 *Simplex Method*

The method of solution most frequently used in linear programming problems is the simplex algorithm which was developed in 1959 by George B. Dantzig, who is now at Stanford University. The solution is of interest when there are fewer equations than unknowns. Only in such cases is there more than one alternative from which to choose. The simplex algorithm starts with a basic solution which represents any

extreme point (sometimes it becomes necessary to introduce an artificial variable to obtain an easy extreme point). The simplex algorithm then permits one to test to see whether this basic feasible solution is optimal (which is not likely) and, if it is not, it guides one to another extreme point in the feasible solution space. It does so by introducing one of the (n-r) variables not in the initial basis, in such a way as to make the greatest possible improvement in the objective function. This process of testing for optimality and improving the solution is repeated until no further improvement is possible in the value of the objective function. At the time when the solution is optimal, the coefficients of the variables indicate the optimal level of operation.

The simplex method has proved extremely efficient in solving complex linear programming problems with thousands of constraints. For theoreticians, however, its speed in solving such problems has been somewhat puzzling. There is no definitive explanation of why the method is so efficient. Indeed, there are problems devised by mathematicians for which the simplex method is intolerably slow. For reasons that are not entirely clear, such problems do not seem to arise in practice.

The following discussion on the simplex method was taken from Bland (1981). Even when an efficient algorithm is employed, the setup costs of solving a large linear programming problem can be considerable. Expressing a practical set of circumstances in terms of linear programming is not a trivial enterprise and neither is the collection and

organization of the data describing the circumstances. Moreover, solving the problem is feasible only with the aid of a high-speed computer.

Nevertheless, the economic benefits of linear programming are often substantial. In the mid-1950's, when linear programming methods were developed to guide the blending of gasoline, the Exxon Corporation began saving from two to three percent of the cost of its blending operations. The application soon spread within the petroleum industry to the control of additional refinery operations, including catalytic cracking, distillation and polymerization. At about the same time other industries, notably paper products, food distribution, agriculture, steel and metalworking, began to adopt linear programming. Charles Boudrye of Linear Programming, Inc., of Silver Spring, Md., estimates that a paper manufacturer increased its profits by $15 million in a single year by employing linear programming to determine its assortment of products.

Today, "packages" of computer programs based on the simplex algorithm are offered commercially by some ten companies. Roughly 1000 customers make use of the packages under licenses from the developers. Since each customer pays a sizable monthly fee, it is likely that each makes use of the method regularly. Many more organizations have access to the packages through consultants. In addition, special-purpose programs for solving problems of flows in networks have been developed, and they may be in even wider service than the general-purpose linear programming algorithms. (These methods are considered under "network flows" in this workshop.)

In the water resources applications discussed at this workshop, we will be referring to the IBM package MPSX which can handle over 5000 equations. A smaller IBM package, DOSPB, which can accommodate up to 150 equations in an efficient manner, will also be discussed. A comparable package to MPSX on CDC equipment is APEX III.

The broad applicability of linear programming can be illustrated even within a single organization. Exxon currently applies linear programming to the scheduling of drilling operations, to the allocation of crude oil among refineries, to the setting of refinery operating conditions, to the distribution of products and to the planning of business strategy. David Smith of the Communication and Computer Sciences Department of Exxon reports that linear programming and its extensions account for from five to ten percent of the company's total computing load. This share has kept pace for the past 20 years with rapidly expanding general applications of information processing.

Although the simplex method is a powerful tool, it is founded on elementary ideas. To understand some of these ideas, it is useful to examine a specially structured allocation problem called the assignment problem. Let us consider the situation of a government relocation committee with three buildings available for renovation and three functions the buildings are to serve. Let us suppose that the functions are those of a laboratory, a library and an equipment maintenance centre, and that there can be only one function to a building. The tables in Figure A-5 indicate the cost of renovation for each of the nine possible

	Laboratory	Library	Maintenance Centre
Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

6 + 6 + 4 =16

	Laboratory	Library	Maintenance Centre
Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

1 + 7 + 4 = 12

	Laboratory	Library	Maintenance Centre
Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

2 + 7 + 2 = 11

Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

6 + 5 + 2 = 13

Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

1 + 5 + 6 = 12

Building A	6	1	2
Building B	7	6	5
Building C	6	2	4

2 + 6 + 6 = 14

Assignment Problem: The above matrix of numbers represents costs in millions of dollars

Figure A-5

matches of a building with a function. How can the committee minimize the renovation cost?

The problem can be solved by marking three of the squares in the 3 by 3 table. Precisely, one square must be marked in each row and one must be marked in each column so that each building has a function and all the functions are accommodated. The solution being sought is the one in which the sum of the costs in the marked squares is as small as possible. Finding an optimum solution is not difficult because there are only a few possible ways of marking the squares. Once one of the three squares in the first row is marked, only two squares in the second row remain available for marking. In the third row, the choice is forced because only one square appears in a heretofore unmarked column. Hence there are 3 x 2 x 1, or six, ways of making the assignment. It is an easy matter to enumerate all six possibilities, evaluate the total cost of each one and select the least expensive assignment.

This enumerative approach solves the problem of a 3 by 3 matrix, but it becomes impractical for larger problems. Suppose there are four buildings and four functions; the number of possible assignments is then 4 x 3 x 2 x 1, or 24. In the general statement of the problem there are n buildings and n functions, and the number of assignments is n factorial (written n!), which signifies n multiplied by all the integers from 1 to n - 1. For n = 10 there are 10!, or more than 3.6 million, distinct assignments.

The rapid growth of n! dispels any enthusiasm one might have for the enumerative method. Suppose one had to solve a 35 by 35 assignment problem by enumeration and one had a computer that could sort through the possible assignments, evaluate the cost of each one and compare it with the lowest-cost assignments encountered up to that point at a rate of a billion assignments per second. (A computer capable of this speed would be much faster than any available now.) Even if the task of enumerating the 35! assignments were to be shared by a billion such computers, only an insignificant fraction of the required computations would be completed after a billion years.

The 35 by 35 assignment problem is not large. If the problem were one of assigning personnel to jobs to minimize the total cost of job training, n might well be equal to 35. There are numerous other assignment problems in which n is equal to 1000 or more. Clearly, such problems require a more efficient procedure than enumeration.

The burden of enumeration might be greatly reduced if one could avoid examining assignments that are costlier than the ones already checked. This effect would be achieved if there were a stopping rule or optimality criterion that would permit an optimum assignment to be recognized quickly once it was encountered. Any algorithm that incorporates such a criterion offers important collateral benefits. The benefits are summarized by what Jack Edmonds of the University of Waterloo in Ontario calls "the principle of the absolute supervisor," or what might also be called "the problem of the skeptical boss."

Let us suppose that after tedious enumeration one has solved the 10 by 10 assignment problem indicated in Figure A-6 by examining all 3 628 800 possible assignments. And the optimum assignment, it is maintained, is the shaded one in Figure A-6. It would seem that the only way to demonstrate the merits of your solution is to repeat the examination of all 3 628 800 possibilities.

A stopping rule offers concise proof of optimality. Suppose that along with the optimum assignment you also have a set of numbers to be subtracted from the entries in each row and column. Setting forth how these numbers are obtained would require a detailed discussion of the assignment problem. It will suffice to point out that the set of numbers can be specified by an efficient computer algorithm. The utility of the numbers, once they have been found, is readily appreciated. Note that subtracting the same number from every entry in a given row or column is equivalent to subtracting this amount from the total cost of every possible assignment. The reason is that every feasible assignment must choose one and only one entry from each row and each column. For example, if 5 is subtracted from every entry in the sixth row, every possible assignment will include exactly one entry that is 5 less than the corresponding assignment made with the original array of costs. The relative costs of all the assignments will therefore remain unchanged. Such subtraction can be done repeatedly, provided it is always applied uniformly to every entry in a given row or column.

	-3	-4	-8	-2	-9	-7	-0	-5	-1	-6
-2	6	6	12	4	14	11	3	9	4	10
-5	8	12	17	8	16	16	6	13	7	13
-6	11	13	18	9	17	16	8	14	8	14
-4	8	10	15	10	16	13	7	9	7	11
-8	13	15	16	11	19	19	10	16	12	18
-5	9	12	17	8	16	15	7	13	7	11
-3	8	8	13	8	13	12	6	8	4	11
-1	6	8	11	5	14	10	2	8	3	9
-0	5	5	12	2	12	7	2	9	6	12
-7	15	14	17	13	17	18	7	18	10	16

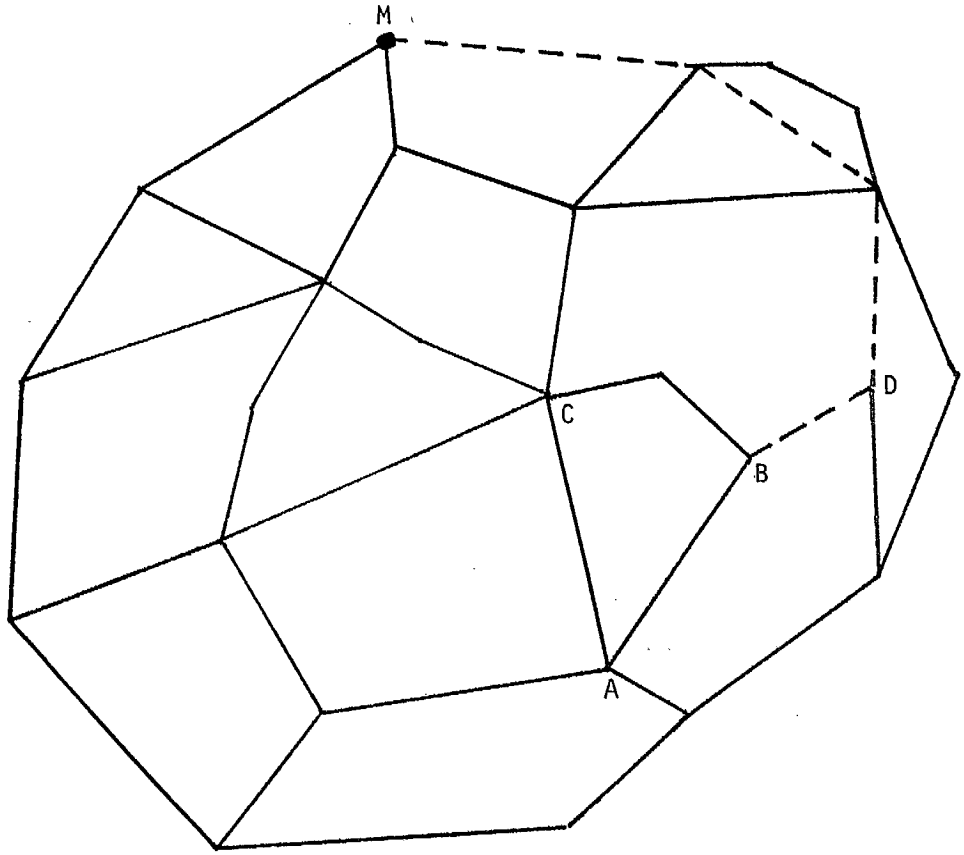
1	0	2	0	3	2	1	2	1	2
0	3	4	1	2	4	1	3	1	2
2	3	4	1	0	3	2	3	1	2
1	2	3	4	3	2	3	0	2	1
2	3	0	1	2	4	2	3	3	4
1	3	4	1	2	3	2	3	1	0
2	1	2	3	1	2	3	0	0	2
2	3	2	0	4	2	1	2	1	2
2	1	4	0	3	0	2	4	5	6
5	3	2	4	1	4	0	6	2	3

Figure A-6

By means of repeated subtraction you can transform the original cost matrix into the matrix shown in Figure A-6. The latter array of costs has a remarkable property. The costs corresponding to the squares selected by your assignment are all zero and no entries in the matrix are less than zero. Since the sum of the costs marked by your assignment is zero and there are no negative costs, no other possible assignment can have a lower cost. In short, you have indicated that, with a few hundred calculations rather than tens of millions, no assignment can be less expensive than the one you have chosen.

Although it has not been demonstrated here how to solve an assignment problem or how to find the row and column numbers to subtract, the assignment problem does illustrate the necessity of avoiding enumeration and the possibility of doing so by means of a stopping rule that recognizes an optimum solution. These are design characteristics of algorithms that can be applied not merely to the assignment problem but to linear programming problems in general.

Figure A-7 illustrates the application of the simplex method to a problem of more than two dimensions. In this case, the feasible policy space is represented by a polytope, a three-dimensional solid whose faces are polygons. When the relation between resources and benefits or costs is linear, the maximum and minimum values must lie at one of the vertices of the polytope. The simplex algorithm examines the vertices selectively, tracing a path (A,B,C,...M) along the edges of the polytope. At each step along the path the measure of benefits or costs



Simplex method applied to polytope

Figure A-7

is improved until at the point M the maximum or minimum is reached. Often there are many paths from A, the starting vertex, to M. Linear programming problems commonly have thousands of dimensions. A detailed mathematical treatment of the simplex method is beyond the scope of this workshop and is considered of low priority in comparison with the practical application of linear programming. Those workshop attendees who wish to study the simplex algorithm at a more rigorous level are referred to Bazaraa and Jarvis (1977), who give a smooth and concise treatment of the method.

Any solution that corresponds to one of the extreme points of the feasible region is termed a "basic feasible solution" (BFS) of the linear program. It is feasible because the solution is in the feasible region. It satisfies the constraints as well as the non-negativity restrictions. It is basic because the availability of this solution is contingent upon the presence of "n" linearly independent coefficient vectors, which together form the basis for an n-space. This latter n-space, referred to as the *requirement space*, is defined by the constraints, and the dimension "n" is equal to the number of constraint equations in the problem under consideration.

3.5 *Duality*

Associated with each linear programming problem there is another linear programming problem called the *dual*. Its variables provide

extremely useful information about the optimal solution to the original linear program, referred to as the *primal*.

The dual linear program possesses many important properties relative to the original primal linear program. There are two important forms (definitions) of duality: the *canonical* form of duality and the *standard* form of duality. These two forms are completely equivalent.

3.5.1 *Canonical form of duality*

Let us suppose that the primal linear program is given in the form:

P: Minimize cx

Subject to $Ax \geq b$

$x \geq 0$

Then the *dual* linear program is defined by:

D: Maximize wb

Subject to $wA \leq c$

$w \geq 0$

Note that there is exactly one dual variable for each primal constraint and exactly one dual constraint for each primal variable. We shall say more about this later.

Example:

Let us consider the following linear program and its dual.

P: Minimize $6x_1 + 8x_2$

$$\begin{aligned} \text{Subject to } 3x_1 + x_2 &\geq 4 \\ 5x_1 + 2x_2 &\geq 7 \\ x_1, \quad x_2 &\geq 0 \end{aligned}$$

D: Maximize $4w_1 + 7w_2$

$$\begin{aligned} \text{Subject to } 3w_1 + 5w_2 &\leq 6 \\ w_1 + 2w_2 &\leq 8 \\ w_1, \quad w_2 &\geq 0 \end{aligned}$$

In the canonical definition of duality it is important for problem P to have a "minimization" objective with all "greater than or equal to" constraints and all "non-negative" variables.

3.5.2 *Standard form of duality*

Another equivalent definition of duality applies when the constraints are equalities. Suppose that the primal linear program is given in the form:

P: Minimize cx

Subject to $Ax = b$

$x \geq 0$

Then the *dual* linear program is defined by:

D: Maximize wb

Subject to $wA \leq c$

w unrestricted

Example:

Let us consider the following linear program and its dual.

P: Minimize $6x_1 + 8x_2$

Subject to $3x_1 + x_2 - x_3 = 4$

$5x_1 + 2x_2 - x_4 = 7$

$x_1, x_2, x_3, x_4 \geq 0$

D: Maximize $4w_1 + 7w_2$

$$\begin{aligned} \text{Subject to } 3w_1 + 5w_2 &\leq 6 \\ w_1 + 2w_2 &\leq 8 \\ -w_1 \quad &\leq 0 \\ -w_2 \quad &\leq 0 \\ w_1, w_2 &\text{ unrestricted} \end{aligned}$$

The optimal values of the objective functions in the primal and in the dual are always identical. Thus, we have the option of picking the easier of the two programs with which to work. The *dual simplex* method of solving a linear programming problem involves iterating from a basic feasible solution of the dual problem to an improved basic feasible solution until optimality of the dual (and also the primal) is achieved, or else until it is concluded that the dual is unbounded and that the primal is infeasible.

3.6 *Examples of Linear Programming Problems*

Although the following three examples are simplistic, they illustrate the concepts involved in setting up a linear programming problem. A realistic appreciation of the power of linear programming will be gained after reading the material in Section 2. These simple examples could be solved graphically, but for demonstration purposes, the simplex method in the IBM package MPSX was employed.

3.6.1 *Example 1*

A manufacturing plant can produce finished goods that sell for a unit price of 10.0; they cost 2.7 per unit to produce. For each unit produced, 3.0 units of waste are generated. The manufacturer has the option of treating up to a maximum of 9.0 units of waste per day (waste load reduced by 85%) at a cost of 0.5 per unit of influent waste, or discharging the waste directly into the watercourse and paying an "effluent tax" of 1.76 per unit. In any case, the water pollution control agency has specified an upper limit on the amount of waste any manufacturer may discharge per day (2.25 units of waste). The manufacturer must decide how many units of finished goods to produce per day (X_1), how much of the total waste ($3X_1$) he should discharge untreated (X_2) into a nearby watercourse, and how much he should pass through the waste treatment plant to maximize profits minus the cost of manufacture, waste treatment, and effluent taxes (while not exceeding the limitations imposed by the regulatory agency on the amount of waste discharged, or the capacity of the existing waste treatment plant). Formulate a linear programming solution to this problem and solve for X_1 and X_2.

Solution:

Objective function:

$$10x_1 - 2.7x_1 - 1.76x_2 - (3X_1 - X_2)\ .5 - .15\ (3X_1 - X_2)\ 1.76$$

Simplifying and rounding off to 3 significant figures, the objective function is:

$$\text{Maximize } f = 5X_1 - X_2$$

Constraints:

(1) $.15\,(3X_1 - X_2) + X_2 \leq 2.25$

or $.45X_1 + .85X_2 \leq 2.25$

(2) $3X_1 - X_2 \geq 0$ (waste treated cannot be less than zero)

(3) $x_2 \geq 0$ (not required - redundant)

After three iterations by the simplex method, the following values were found:

$$X_1 = 3.3,\ X_2 = .90,\ X_3 = 9.0,\ f = 15.6$$

where X_3 is the waste passing through the treatment plant per day.

3.6.2 *Example 2*

A manufacturing company requires the daily consumption of 4 million gallons of water of a quality such that the concentration of magnesium must be kept below 100 mg/L. The water can be supplied from three sources: (1) as a purchase from a local company at the cost of

$100 per million gallons (concentration of magnesium = 50 mg/L); (2) as a purchase from the municipality at $220 per million gallons (concentration of magnesium = 5 mg/L); and (3) by pumping from a nearby stream at a cost of $50 per million gallons (concentration of magnesium = 200 mg/L). The water from the three sources is completely mixed before it is used. Now the company plans a new expansion which will at least double the daily consumption of water of the same quality. The local company (1) can further supply up to 10 million gallons daily, but the additional quantity to be pumped from the nearby stream (3) cannot exceed 2 million gallons daily. The manufacturing company must pay the municipality (2) for 0.2 million gallons per day regardless of whether the company has need of the water. Determine by linear programming the additional quantities of water to be obtained from these three sources to meet the need of the new expansion such that the total cost will be a minimum. Write the objective function and the constraints *only* in algebraic form for solution by the simplex procedure in the MPS package.

Solution:

Write the objective function and constraints for existing demand of 4×10^6 gallons/day

where x_1 = supply from local company,

x_2 = supply from the municipality, and

x_3 = supply from the stream.

$$\text{Minimize } f = 100x_1 + 220x_2 + 50x_3$$

subject to the constraints:

$$x_1 + x_2 + x_3 \geq 4 \times 10^6$$

$$(50x_1 + 5x_2 + 200x_3)(4 \times 10^6)^{-1} \leq 100$$

$$x_3 \leq 2{,}000{,}000$$
$$x_2 \geq 200{,}000$$
$$x_1 \geq 0$$
$$x_3 \geq 0$$

This problem is solved by the simplex method, and then is repeated with only the first constraint being changed, i.e.,

$$x_1 + x_2 + x_3 \geq 8 \times 10^6$$

3.6.3 *Evaluation of incremental changes in values of constraints*

It is frequently useful to include objectives which are difficult to quantify as constraints in a linear programming model. The impact of these objectives, usually in economic terms, can be determined by varying constraint levels and measuring the "costs" of incremental changes. This information is then incorporated into the decision-making

process. The following example demonstrates the approach for a water quality planning problem.

Consider the case presented in Figure A-8.[1]

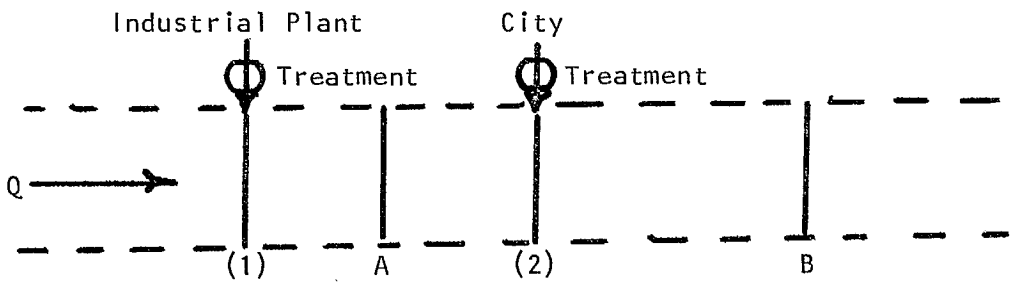


Figure A-8

A large industrial plant (at site *1*) dumps waste into a river upstream from a city and upstream from two stream quality control points (*A* and *B*). The city discharges waste (at site *2*) upstream from quality control point *B*.

A measure of the organic pollution load in a river is the biochemical oxygen demand (BOD), which indicates the rate at which the dissolved oxygen in the river is depleted. The total daily BOD waste production at the plant is 10 000 g per day and 8000 g per day at the city. The river flow, Q, is 4 million litres per day and constant, and the dissolved oxygen content of the river above point A is 6 mg/L.

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[1] This example problem is taken from Meta Systems Inc., 1975. Errors found in the text, however, have been corrected in these notes.

It is assumed that the dissolved oxygen concentration in the stream is the appropriate surrogate measure for quality, the flows containing the waste of both the plant and city are negligible, and the wastes dumped into the river mix perfectly with the river water.

In addition, it is assumed that a set of "transfer coefficients," d_{1A}, d_{1B}, and d_{2B} can be defined. These coefficients specify the effect on dissolved oxygen content at a quality control point resulting from a unit change in BOD waste discharged into the stream by the city or the industry. The numbered subscript represents the waste source and the lettered subscript represents the quality control point.

Waste treatment (BOD reduction) is possible at both discharge points. The percent reduction in BOD, x_i $(i = 1,2)$, has an annual total cost at the plant and city of 3 and 2 mega dollars per unit percent removal, respectively.

The environmental quality objective is expressed in the statement: let the quality at points A and B be at least T mg/L dissolved oxygen. Algebraically, this is written as:

$$6-d_{1A} \cdot \frac{10\ 000 \times 10^3}{Q} (1-x_1) \geq T$$

and

$$6-d_{1B} \frac{10\ 000 \times 10^3}{Q} (1-x_1) - d_{2B} \frac{8000 \times 10^3}{Q} (1-x_2) \geq T$$

Let us consider the following problem - design a system to minimize the cost of meeting the quality objective. If d_{1A}, d_{1B}, and d_{2B} equal .3, .6 and .3, respectively, and T equals 5 mg/L, the following linear programming model is obtained:

Minimize $3x_1 + 2x_2$

subject to: $1.5x_1 \geq -.5$ (eliminate this constraint as lower one governs)

$3x_1 + 1.2x_2 \geq 2.2$

$x_1 \leq 1$ (% of treatment cannot be greater

$x_2 \leq 1$ than 100%)

$x_1, x_2 \geq 0$

From which x_1 = .73333, x_2 = 0 and V or cost = 2.2 mega dollars.

When the water quality objective "T" is increased to 5.5, the L.P. problem is formulated as follows:

Minimize $3x_1 + 2x_2$

subject to: $1.5\ x_1 \geq .5$

$3x_1 + 1.2x_2 \geq 3.2$

$x_1 \leq 1$

$x_2 \leq 1$

$x_1, x_2 \geq 0$

from which $x_1 = 1.0$, $x_2 = .167$ and V or cost = 3.333 mega dollars.

Similarly, decreasing the water quality objective to 4.5 mg/L, the L.P. problem is formulated as follows:

$$\text{Minimize } 3x_1 + 2x_2$$

$$\text{subject to: } 3x_1 + 1.2x \geq 1.2$$
$$x_1 \leq 1$$
$$x_2 \leq 1$$
$$x_1, x_2 \geq 0$$

From which $x_1 = 0.4$, $x_2 = 0$ and V or cost = 1.2 mega dollars.

3.7 *Overview of Separable Programming*

Separable Programming was developed by Miller in 1963 for handling certain nonlinearities which occur in programming problems. Many large-scale MP packages include SP as an option, thereby facilitating the use of all the capabilities of matrix generation, post optimal analysis, etc., which are normally used with LP. Separable programming is employed in the ORRMS model which is discussed in Section 2 of this advance workshop material.

One disadvantage of SP is that it will not necessarily terminate on a global optimum but may terminate on a local optimum (for non-convex

problems). In spite of this difficulty, certain procedures can be utilized which can provide "better" solutions, even if not optimal.

The basic ideas of SP are described in IBM (1979). Basically, SP involves a piece-wise linearization of nonlinear functions and the definition of grid variables in such a way that variables are permitted into the solution only in a prescribed sequence, ensuring that the piece-wise linear approximation remains valid. This is illustrated by the following example (Fig. A-9).

Let us suppose the cost function for a given variable is as shown:

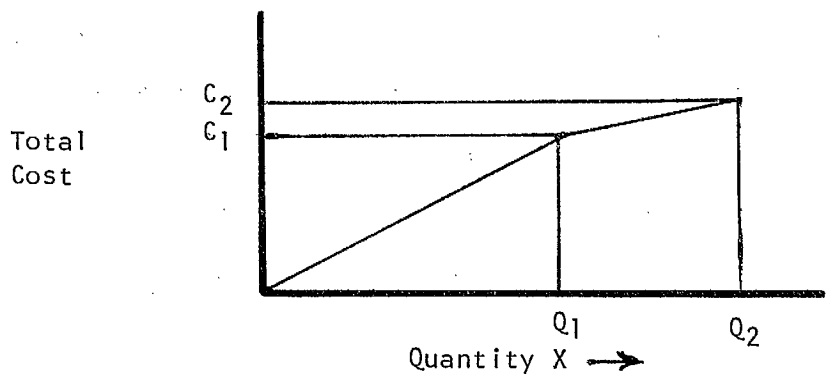


Figure A-9

If the usual expedient of "piece-wise linearization" were to be applied, the total quantity of the variable would be defined as the sum of two variables:

$$X = X_1 + X_2$$

$$X_1 \leq Q_1$$

$$X_2 \leq Q_2 - Q_1$$

$$\text{Total cost} = (C_1/Q_1)X_1 + ((C_2 - C_1)/(Q_2 - Q_1))X_2$$

Under this arrangement, the sub-variable designated by X_2 would be "more attractive" to the LP routine because of its lower unit cost (as seen in the cost curve). Therefore, it would be utilized first. As can be seen from the curve, however, the unit cost for X_2 is not available until X_1 is used at its maximum. Separable programming, in essence, provides the necessary extensions to the LP algorithm to permit the user to define X_1 and X_2 as members of a "separable set," and then permits X_2 to be used only when "all variables before it in the set" - in this case X_1 - are at their maximum values.

3.8 *Overview of Mixed Integer Programming*

Recent advances in mathematical programming have provided numerous improvements in basic algorithms as well as significant new capabilities. One of these capabilities is mixed integer programming, which is essentially linear programming with the additional capability that some, but not necessarily all, of the variables may be restricted to

integer values. This requirement may arise in two types of problems in which:

(1) variables representing the number of discrete items (e.g. trucks, men, trunk lines, etc.) are defined and

(2) variables representing decisions (e.g. to build a warehouse at a specific location) are defined.

As in the case of separable programming, most major computer codes for LP include MIP capabilities (e.g. IBM's MPSX).

The use of MIP to model systems requiring discrete quantities of some variables is fairly straightforward; the capability to define "fixed-charge" and logical type situations, however, is much more subtle and adds very significantly to the "state of the art." For example, consider the hypothetical problem in which there are four gates in a reservoir system and an operational decision must be made by the model as to which of the gates is to be opened. Let us define GA, GB, GC, GD to be the 0-1 variables representing the gates (i.e., GA=0 means gate A is closed; GA=1 means gate A is open). Also define FA, FB, FC, FD to be the flows through gates A,B,C, and D, respectively.

It is possible to restrict flow through a gate to zero if the gate is closed and to let it assume any positive value (subject only to other constraints) if the gate is open.

$M.GA \geq FA$, where M is a very large number.

Suppose we wish to permit a maximum of three gates to be open in any time period.

$$GA + GB + GC + GD \leq 3$$

Suppose gate D cannot be open unless gate A is open.

$$GA \geq GD$$

These examples illustrate a few of the types of practical constraints which can easily be modelled with MIP.

3.9 *Summary*

3.9.1 *Postulates*

(a) Objective function and constraints can be expressed as simple algebraic expressions.

(b) Objective function must be convex and the contraints must form a convex policy space.

(c) All decision variables must be positive.

3.9.2 *Advantages and limitations*

(1) Use standard programs.

(2) User does not have to know details of method, only its philosophy and limitations.

(3) Can handle a large number of state variables.

(4) Can handle fewer decision variables than dynamic programming on a sequential basis, but can handle a large dimension of decision variables for one time period. (Dynamic programming must keep the dimensionality of decision variables as small as possible, preferably less than two within each unit time period.)

(5) Constraints tend to increase the computational cost.

(6) Requires convex systems (convex policy space).

(7) Requires linear relationships (nonlinear relationship can be linearized in several presently available packages).

(8) Optimum obtained is only a single set of decisions (nothing known about second or third best).

4. Network Flows

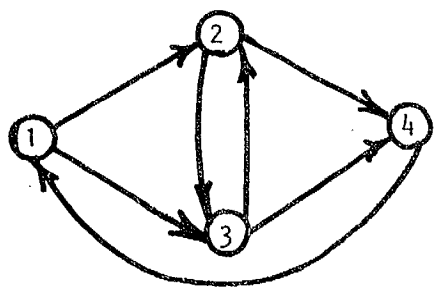
A generalized network (GN) problem is simply a type of LP problem and can thus be solved using any standard LP solution technique. However, none of the current LP systems is capable of fully exploiting the structure of generalized network problems. Modelers have begun to devote attention to determining whether an LP model is a GN problem and, more important, to devising formulations in which generalized networks play the role of critical components.

There are two compelling incentives for adopting a GN formulation whenever possible. The major advantage is the ability to solve GN problems, including a variety of problems with GN components, with a remarkable degree of efficiency. The second motivation for using GN models is that they can be conceptualized graphically as well as algebraically. The pictorial presentation of a generalized network is a useful device for communicating models to nonscientific users and for teaching others how to formulate problems.

Network flow problems possess an important special structure that permits the simplification of the (primal) simplex procedure to a point where it may be applied directly on the network without the need of a simplex tableau.

4.1 *The Minimal Cost Network Flow Problem*

Figure A-10 presents a network with four nodes and seven arcs. Refer to this directed network as G. With each node i in G we associate a number b_i that is the available supply of an item (if $b_i > 0$) or the required demand for the item (if $b_i < 0$). Nodes with $b_i > 0$ are sometimes called sources, and nodes with $b_i < 0$ are sometimes called sinks. If $b_i = 0$, then none of the item is available at node i and none is required. In this case node i is sometimes called an <u>intermediate</u> (or transhipment) node. Associated with each arc (i,j) we let x_{ij} be the amount of flow on the arc (we assume $0 \leq x_{ij}$) and c_{ij} be the unit shipping cost along the arc.



Arc	Flow
(1,2)	1
(1,3)	1
(2,3)	0
(2,4)	1
(3,2)	0
(3,4)	1
(4,1)	2

Figure A-10

We shall assume that the total supply equals the total demand within the network, that is, $\sum_{i=1}^{m} b_i = 0$. If this is not the case, that is, $\sum_{i=1}^{m} b_i > 0$, then add a dummy demand node, m+1, with $b_{m+1} = -\sum_{i=1}^{m} b_i$ and arcs with zero cost from each supply node to the new node.

The minimal cost network flow problem may be stated as follows. Ship the available supply through the network to satisfy demand at minimal cost. Mathematically, this problem becomes (where summations are taken over existing arcs)

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{m} C_{ij} X_{ij}$$

$$\text{Subject to} \quad \sum_{j=1}^{m} x_{ij} - \sum_{k=1}^{m} x_{ki} = b_i \qquad i = 1,\ldots m$$

$$x_{ij} \geq 0 \qquad i,j = 1,\ldots m$$

The constraints above are called the *flow conservation* or *Kirchoff* equations and indicate that the flow may be neither created nor destroyed in the network. In the conservation equations, $\sum_{j=1}^{m} x_{ij}$ represents the total flow out of node i while $\sum_{k=1}^{m} x_{ki}$ indicates the total flow into node i. These equations require that the net flow out of node i, $\sum_{j=1}^{m} x_{ij} - \sum_{n=1}^{m} x_{ki}$, should equal b_i. If $b_i < 0$, then there should be more flow into i than out of i.

Minimal cost network flow problems occur in the design and analysis of communication systems, oil and gas pipeline systems, tanker scheduling problems, and a variety of other areas. In this workshop we will show in the SYSNET model how this technology can be applied to water resources problems.

Clearly, the minimal cost flow program is a linear program and can be solved in any one of several ways. One way is to apply the ordinary simplex algorithm to the problem. However, we will show how a solution can be found without the need of a simplex tableau.

4.2 *The Out of Kilter Network Algorithm*

Network flow algorithms have been developed to handle a special class of linear programming problems. These problems may each be described as a network whose links carry flow. Network algorithms take advantage of this special structure to produce an optimal solution, much more quickly, with less core storage required, and with virtually no round-off error in comparison with general linear programming codes.

Network problems which do not necessarily represent physical networks are defined by the points, places or locations designated to be network "nodes" and the links or "arcs" that connect certain of these nodes. These arcs permit movement between the nodes which they connect. Movement of flow along each network arc is restricted by a maximum

capacity (upper bound) and a minimum required flow (lower bound) which is perhaps greater than zero. Flow along each arc is the flow per unit time. For example, with respect to the Great Lakes, the unit time period selected is 0.25 month which varies from 7 to 8 days. A discharge arc is the total flow in cubic feet during the time period as opposed to a flow rate of cubic feet per second.

A storage arc is simply the total storage in cubic feet for a particular lake above an arbitrarily assumed lake elevation of zero storage. The flow is not required to be fluid in the sense of a liquid, but must meet a physical restriction that one might normally expect of a flow. Roughly, flow cannot "stack up" at a node; total flow into a node must equal total flow out of that node. This property is called conservation of flow (Woolsey and Swanson, 1975).

In general, network problems may be characterized as falling into one of the three following categories:

(1) Find that maximum flow from a designated node to a different designated node subject to the capacity restrictions on flow in each arc and conservation of flow at each node.

(2) Find the maximum flow from a designated node to a different designated node that minimizes the cost incurred, subject to capacity restrictions on flow in each arc and conservation of flow at each node.

(3) Find the flow that minimizes the cost of shipping k units (k is a known integer that might be less than the maximum flow) from a designated node to a different designated node, subject to capacity restrictions on flows in each arc and conservation of flow at each node.

The out of kilter algorithm can solve problems in each of the above categories plus problems in some additional categories by proper selection of costs, capacities and network configuration. The particular category that is represented by the SYSNET model is referred to as the minimum cost circulation network flow problem.

To discuss the algorithm, a small amount of jargon is necessary. A path from node A to node B in a network is a set of sequential arcs that forms a route from A to B. Arcs (1,2), (2,3) and (3,4) form a path from node 1 to node 4 in the network of Figure A-10. A cycle in a network is a path that ultimately leads back to the starting node. Arc (4, 1), in addition to the previously mentioned path, forms a cycle.

A circulation in a network is a set of flows on the arcs of the network that preserves conservation of flow at each node. The flows in the table of Figure A-10 define a circulation for the accompanying network (note that there are many other possible circulations). The out of kilter algorithm operates in such a way as to maintain a circulation in the network while rerouting flows so as to minimize the sum of cost

times flow and satisfy capacity restrictions on each arc. This generates an optimal solution to the minimum cost circulation network problem, if such a solution exists. The theoretical aspects of this process arise from the primal dual theory of linear programming (complementary slackness). The brief discussion that follows is intended to give the reader an intuitive idea of how and why the algorithm works. A more detailed discussion may be found in Woolsey and Swanson (1975). To find the optimal flow in a network, the algorithm is initialized with a circulation flow of 0 in each arc (conservation is preserved at each node). Note that such a circulation might not satisfy capacity constraints (the problem is dual feasible). In addition, a number must be assigned to each node (these node numbers are actually values of the dual variables, and they may be initialized at value 0). The "economic" or intuitive interpretation of each node number is the "value" or price of a unit of flow at that node. Examine an arc of the network, say from node x to node y. The cost C_{xy} of flow on arc (x, y) is known, so we can compute $D_{xy} = P_x + C_{xy} - P_y$, where $P_x(P_y)$ is the node number at node x (node y). Note that the economic interpretation of the number D_{xy} is then the value of flow at node x plus the cost of moving from node x to node y along arc (x, y) minus the value of flow at node y. Then, intuitively:

(1) If $D_{xy} < 0$, the value of flow at node y is greater than the value of flow at node x plus the cost of getting the flow at node x to node y. This implies that the flow in arc (x, y) should be as great as possible, that is, equal to the upper bound on arc (x, y). $D_{xy} < 0 \times f_{xy} = U_{xy}$.

(2) If $D_{xy} > 0$, then, similarly, the flow in arc (x, y) should be as little as possible (the cost of moving from node x to node y is greater than the increase in value of moving from node x to node y); the flow should equal the lower bound, $D_{xy} > 0 \times f_{xy} = l_{xy}$.

(3) If $D_{xy} = 0$, the increase in value of moving from node x to node y exactly matches the cost of doing so; thus the flow in arc (x, y) need only satisfy the capacity restrictions on arc (x, y). $D_{xy} = 0 \times l_{xy} < f_{xy} < U_{xy}$.

If the flow in arc (x, y) is "appropriate" for the value of D_{xy}, then arc (x, y) is said to be "in kilter" (actually satisfies optimally conditions). Otherwise, an arc is said to be "out of kilter." If an arc (x, y) is out of kilter, the algorithm seeks to find a path from node y to node x that together with arc (x, y) forms a cycle along which the flow may be changed by a constant amount without disturbing conservation of flow at any node. Actually, a special kind of path must be found, one along which a change in flow would not worsen the "kilter condition" (cases 1-3 above) of any arc. In addition, the out of kilter algorithm has a routine to change flows once a cycle is found for an out of kilter arc, and a routine to change the node numbers (dual variables) when that becomes necessary. When each arc is in kilter the flow is optimal. To summarize, the algorithm proceeds as follows:

(0) Start with any circulation and any set of node numbers.

(1) Find an out of kilter arc, (s, t). If there is none, stop.

(2) Determine if the flow in arc (s, t) is to be increased or decreased to bring (s, t) into kilter. If the flow should be increased, go to step (3). If the flow should be decreased, go to step (4).

(3) Find a path from t to s along which the flow can be increased without causing any arc on the path to become more out of kilter. If a path is found, increase the flow along the path and also in (s, t). If (s, t) is in kilter, go to step (1). If (s, t) is still out of kilter, repeat step (3). If no path can be found go to step (5).

(4) Find a path from s to t along which the flow can be increased without causing any arc to become more out of kilter. If a path is found, increase the flow along the path and decrease the flow in (s, t). If (s, t) is in kilter, go to step (1). If (s, t) is still out of kilter, repeat step (4). If no path is found, go to step (5).

(5) Change the node numbers, If (s, t) is in kilter, go to step (1). If (s, t) is still out of kilter, repeat step (2).

A comprehensive description of the "out of Kilter" algorithm is found in Woolsey and Swanson (1975).

Glover *et al.* (1978) assessed the efficiency of the solution procedure of a network flow algorithm, NETG, by comparing results with the linear programming computer code APEX-III. APEX-III is maintained by

CDC and is operational on all CDC 6600 series and CYBER-70 series computers. (The equivalent linear programming computer code on IBM systems is MPSX.) The following table by Glover *et al.* (1978) reveals startling comparisons.

Table A-1. NETG vs. APEX-III

Problem	Number of nodes	Number of arcs	NETG SBU's[a]	NETG Cost[b]	APEX-III SBU's	APEX-III Cost
1	100	1000	7.51	$1.35	62.65	$ 11.28
2	100	1000	7.29	1.31	80.93	14.57
3	1000	1000	9.70	1.75	94.72	17.02
4	250	4000	16.65	3.00	453.02	81.54
5	250	4000	14.74	2.65	742.61	133.67
6	500	5000	22.55	4.06	1044.34	187.98[c]
7	1000	6000	50.22	9.04	1633.64	294.06[d]

a. CYBER-74 System Billing Unit

b. Computer at $0.18 per SBU

c. Stopped after 10 000 iterations.

Objective Function Value = 25 337 282

Optimal Objective Function Value = 3 354 927

d. Stopped after 10 000 iterations.

Objective Function Value = 1 340 958 349

Optimal Objective Function Value = 3 964 490.

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