AN EXTENDED MILES' THEORY FOR WAVE GENERATION BY WIND

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January 1981

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Management Perspective

Fresh water lakes require the stresses exerted by the wind and barometric gradients to mix and circulate the waters. Knowledge of these stresses is inadequate for the precise modelling of waves, wind wet up, circulation and mixing, all of which influence water quality. This paper tests an improved hypothesis for transfer of energy to the water by the wind and the subsequent generation of surface waves.

Although the theory does not provide answers, it improves the present theory on wave generation and may make an important step towards finding a correct and reliable model for wind stress on water.

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January 8, 1981

Perspective - Gestion

Il faut les contraintes exercées par le vent et par les gradients de pression pour mélanger et faire circuler les eaux des lacs d'eau douce. La connaissance que nous avons de ces contraintes ne permet pas d'établir avec précision des modèles des vagues, des montées de niveau dues au vent, de la circulation et du mélange, qui influencent tous la qualité de l'eau. Ce document avance une hypothèse améliorée sur la transmission de l'énergie du vent à l'eau et la formation consécutive de vagues à la surface.

Bien que la théorie n'apporte pas de réponse, il reste qu'elle améliore la théorie actuelle de la formation des vagues et qu'elle peut constituer un progrès vers la découverte d'un modèle exact et fiable des contraintes exercées par le vent sur l'eau.

Le chef de la Division de l'hydraulique

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Le 8 janvier 1981

Abstract

Miles' inviscid theory of surface wave generation by wind is (a) modified by replacing the logarithmic shear velocity profile with one which applies right down to the wave surface and which exhibits an explicit dependence on the roughness of the surface, and (b) extended to include the effects of the interaction of wave with air flow turbulence by considering the wave-modified mean flow as the mean of the actual turbulent air flow over water waves and using this in a mixing-length model.

The surface pressure is shown to depend significantly on the flow conditions being aerodynamically smooth or rough. Its component, in phase with the surface elevation, is practically unaffected by the wave-turbulence interaction. However, such interaction tends to increase the rate of energy input β from wind to waves travelling in the same direction, e.g., the increase is $2\kappa^2$ for aerodynamically rough flow, where κ is the Von Karman constant. It also provides damping of waves in an adverse wind which can be about 10% of the growth rate in a favourable wind.

Sommaire

La théorie de Miles relative à la formation par le vent de vagues de surface en écoulement non visqueux est (a) modifiée en remplaçant la courbe de vitesse logarithmique de cisaillement par une autre qui s'applique directement à la surface de la vague et présente une dépendance explicite avec l'inégalité de la surface et (b) étendue aux effets de l'interaction de la vague avec l'écoulement aérien turbulent, en considérant l'écoulement moyen modifié par les vagues comme s'il s'agissait en réalité de la moyenne de l'écoulement des turbulences aériennes au-dessus de vagues d'eau et en s'en servant dans un modèle de longueur du mélange.

On indique que la pression de surface dépend de façon importante des conditions laminaires ou turbulentes de l'écoulement. Quand la composante de la pression de surface est en phase avec le soulèvement de la surface, elle n'est pratiquement pas influencée par l'interaction entre la vague et la turbulence. Toutefois, cette interaction tend à accroître le taux de transmission d'énergie β du vent aux vagues qui se déplacent dans la même direction; par exemple, l'augmentation est de $2k^2$ pour un écoulement turbulent, où k est la constante de Von Karman. Elle occasionne aussi l'amortissement des vagues par vent contraire. L'amortissement peut représenter environ 10 p. 100 du taux d'augmentation par vent favorable.

§1. Introduction

Since its publication more than two decades ago, Miles' (1957) theory of surface wave generation by shear air flow has aroused great interest and considerable controversy. On the one hand it is a well-argued mathematical theory within the assumptions made, e.g. inviscid flow, small amplitude sinusoidal wave train, etc. (see also Miles 1959 and Benjamin 1959), and its prediction of exponential growth of wave energy is well confirmed by many field observations. On the other hand, field experiments, in particular Dobson (1971), Elliot (1972), and Snyder (1974) specifically designed to measure the rate of energy transfer from wind to wave, have generally yielded a value an order of magnitude larger than that predicted by Miles. This experimental data, which itself exhibits large scatter, has recently been reexamined by Snyder et al (1980) and corrected results now show an order of magnitude agreement with theory. However, Miles theory still predicts rates of energy transfer that are much smaller than measured values. Another deficiency of Miles' theory is that it predicts no damping of waves travelling against wind, which is evidently non-physical and in contradiction to the recent observations of Stewart and Teague (1980).

It is well recognised that in Miles' inviscid theory the air flow turbulence, which is necessarily present in the field, is largely neglected, except in its role in setting up the logarithmic shear flow. This neglect of turbulent effects in the theory has been ascribed (Miles 1967) to be the main reason for the very large differences between theory and experiments, and attempts were made by Davis (1969, 70, 72, 74) among others to extend

Miles' theory to include the effect numerically. It was found that the energy transfer rate so calculated depended sensitively on the closure model employed and on the details of the velocity profile assumed near the surface.

In this paper, Miles' inviscid theory will be improved in two ways. Firstly, the logarithmic form of the shear flow profile over a flat plate will be replaced by a form that is applicable right down to the surface and which exhibits an explicit dependence on the roughness of the surface. Secondly, the inviscid theory is extended to include the effects of air flow turbulence by assuming that Miles' perturbed flow (calculated with our profile) represents the mean of the actual turbulent flow over water waves, and by using this flow in a simple mixing-length model.

In §4 the results of this "extended Miles' theory" will be compared with the original Miles' theory and with field experiments of Snyder et al (1980). It will be seen that the extended theory predicts a growth rate of waves somewhat larger than that of Miles' original theory and that it predicts damping of waves in an adverse wind.

§2. Choice of Shear Flow Velocity Profile

According to Prandtl's mixing length model, the mean shear flow velocity $U_s(y)$ satisfies

$$\nu \frac{dU_s}{dy} + \ell^2 \left(\frac{dU_s}{dy}\right)^2 = U_*^2 \tag{1}$$

where ν is the kinematic viscosity of air, U_* the friction velocity, and ℓ Prandtl's mixing length.

The logarithmic profile

$$U_s(y) = \frac{U_*}{\kappa} \ln \frac{y}{y_0} \tag{2}$$

used by Miles follows by neglecting the viscous term in (1) and by taking the mixing length $\ell = \kappa y$, where $\kappa(= 0.4)$ is the Von Karman constant and y_0 is the roughness length. Experimental results have confirmed the validity of this logarithmic distribution of mean wind velocity away from the wave. However, Stewart (1970) found that near the water surface the effect of the viscous sublayer is of importance. Moreover, various numerical turbulent models, e.g. Davis (1970), have shown how sensitively the energy transfer between wind and wave is dependent on the assumed form of wind velocity in the sublayer. Further doubts concerning the use of the logarithmic profile are raised upon checking the self-consistency of Miles' model. For, if the ratio of the neglected Reynolds stresses to the pressure is estimated at $y = y_0$, the result indicates that the stresses are far from negligible for that particular profile. It is important, therefore, in using Miles' model for finding the wave-induced flow field to employ a profile which has the correct behaviour near the water surface.

This is further underlined by considering the position of Miles' critical height (height at which wind speed is equal to the wave phase speed c) with respect to the shape of the velocity profile. Figure 1 (Fig. 2 of Miles, 1957) shows the dependence of Miles' critical height y_c on c/U_1, where $U_1 = U_*/\kappa$. The critical height is normalized with wave number k and Miles' profile parameter $\Omega = gy_0/U_1{}^2$. We note that the velocity profile is influenced by molecular viscosity from the wall out to distances y such that $\frac{yU_*}{\nu} < 60$ (Van Driest, 1956). Therefore if the critical height should be below such values of y the assumption of a logarithmic profile would be unacceptable. For typical field conditions, during which direct input to waves has been explored, we refer to the recent Bight of Abaco experiments (Snyder et al. 1980 hereafter referred to as BOA) in which the average wind speed at 5 metres height U_5 was 644cm/sec (runs 15 to 20) and the average drag coefficient was 1.003×10^{-3}. These values correspond to $U_* = 20.4$ cm/sec, $y_0 = 1.64 \times 10^{-3}$ cm, $\Omega = 6.19 \times 10^{-4}$. Thus, in this case, the assumption of a logarithmic profile is valid only when the critical height y_c is such that

$$\frac{ky_c}{\Omega} \geq 234\ (c/U_1)^{-2} \tag{3}$$

The equality of (3) is shown dashed in Figure 1. It is clear that the assumption of a logarithmic profile for the conditions of the BOA experiment is not appropriate for values of c/U_1 less than 5. This is just the range in which the wave amplication is most pronounced (Miles, 1959).

In the absence of complete information, experimental or theoretical, of detailed velocity distribution near the wave surface the best hope of progress would seem to be in assuming that the mean flow behaves near the water wave in the same way as over a flat plate. We shall thus make this one of our assumptions in selecting a valid shear velocity profile.

In searching for an appropriate profile, one must bear in mind that the flow is not aerodynamically smooth (see Stewart, loc cit). The research literature on this particular type of flow over a flat plate is somewhat sparse. However,there are two profiles in particular, which are worthy of consideration. The first was proposed by Van Driest (1956) and results from assuming that the mixing length ℓ in (1) is given by

$$\ell = \kappa y \left[1 - e^{-yU_*/\nu A_*}\right] \quad , \quad y \geq 0 \tag{4}$$

where A_* is a dimensionless parameter. Van Driest adjusted A_* to yield the best fit of his profile to Laufers' (1953) data from experiments with flow in pipes. With $A_* = 26$ the agreement between theory and experiment was excellent for Reynolds numbers of 5×10^4 and 5×10^5. In these experiments the flow is described as smooth; that is the characteristic size of the wall roughness elements d is such that $\frac{dU_*}{\nu} < 5$ (Rotta, 1962). Van Driest attempted to extend his profile to include transitional and rough flow by arguing that the roughnesses would act to destabilize the viscous sublayer and the net effect on the mean profile would be a reduction of the exponential factor, e^{-y_*/A_*} $(y_* = yU_*/\nu)$ in the mixing length (4). He added a "vortex generation factor" in the form $\exp(-60y_*/26d_*)$; where d_* is the normalized average roughness size $d_* = \frac{dU_*}{\nu}$. Evidently, to be consistent with the logarithmic law of the wall for fully rough flow, the exponential factor in Van Driests' mixing length (4) must disappear. This led to the factor 60 in his vortex generation term, since $d_* = 60$ corresponds to fully rough flow (roughness elements penetrating the viscous sub-layer) and then the vortex generation term exactly cancels the exponential term of (4).

Unfortunately, the generalization of Van Driests' profile to transitional and rough flow was not supported by experimental profiles. However, one can test the consequence of Van Driests' vortex generation idea against the observation that the roughness length y_o (virtual origin of logarithmic outer profile) bears a more or less constant relationship to the size of the roughness elements d for fully rough flow (see, for example, Businger et al. 1971). Figure 2 indicates that against this criterion Van Driests' extension to rough flow appears to be incorrect.

The second profile, proposed by Rotta (1962), follows by taking

$$\ell = \ell_o + \kappa y \tag{5}$$

where ℓ_o , a non-zero mixing length at the wall, is included to account for mixing due to wall roughness. It should be emphasized, perhaps, that Rotta's profile is only appropriate for rough flow since in the limit of smooth flow $(\ell_o \to o+)$, the resulting profile has a form near the wall which is in contradiction with the form implied by the equation of continuity (see Rotta 1962, pp. 58-59).

Both these profiles were, however, rejected: Rotta's profile leads to an inconsistent model for smooth flow, whereas Van Driest's is consistent only for smooth flow.

We were thus led to consider the profile resulting from setting

$$\ell = \ell_0 + \kappa y[1 - e^{-yU_*/\nu B_*}]^2 \qquad (6)$$

in (1), which suffers neither of these drawbacks.

The parameters ℓ_0 and B_* remain to be defined. ℓ_0 is best thought of as the mixing length at the surface, which for smooth flow is clearly zero. It accounts for the turbulent mixing which occurs right at the surface in rough flow, i.e. the turbulent stresses are directly communicated to the roughness elements which penetrate the viscous sub-layer.

For smooth flow ℓ_0 is zero, and $B_* = 13$ yields the best fit to Laufer's (1953) data used by Van Driest (1956). Integration of (1), using (6) with various values of ℓ_0, yields the roughness length y_0. Figure 3 illustrates the behaviour of the ratio of surface mixing length to roughness length ℓ_0/y_0 for various values of the normalized roughness length. Three regions are clearly defined: (1) $\ell_0 = 0$, smooth flow; (ii) ℓ_0 increases rapidly to a maximum and decreases again more slowly, transitional flow; (iii) ℓ_0 is proportional to y_0, rough flow. The corresponding ranges of the normalized roughness length $\frac{y_0 U_*}{\nu}$ are : (i) $\frac{y_0 U_*}{\nu} = 0.137$, smooth; (ii) $0.137 < \frac{y_0 U_*}{\nu} < 2.2$, transitional, (iii) $\frac{y_0 U_*}{\nu} > 2.2$, rough. We have expressed the roughness criterion in terms of normalized roughness length rather than normalized roughness size, because the latter is generally not known in flow over water waves. The relationship between d and y_0 is based on Rotta's (1962, Figure 11.14) summary of roughness effects on the velocity profile.

It is of interest to compare this behaviour with that of the ratio of average roughness size to roughness length d/y_0, since one would expect the mixing length at the surface ℓ_0 to be related to the average roughness size d. The dependence of average roughness size d

on roughness length may be extracted from Rotta's (1962) Figure 11.14 (in his notation the average roughness size is denoted by k_r), which is a summary of several different experiments with different roughness types. From experiment to experiment there is considerable variation in the ratio of d/y_0 but in all cases the behaviour is in qualitative agreement with Figure 3, having three regions: (i) $d = 0$; (ii) d increases rapidly and then decreases; (iii) d is proportional to y_0 .

It would appear then that the form of the mixing length (6) is consistent with both the constraints imposed by the continuity equation and the experimental results of others. The corresponding velocity profiles for smooth, transitional and rough flow are shown in Figure 4. The transitional profile plotted is that corresponding to the flow conditions of the BOA experiments.

The choice of a simple mixing length hypothesis, which ignores the "elasticity" effects, to deal with the turbulent stresses is justified by relatively short development time of the turbulence, $\tau_d \sim (\frac{dU_s}{dy})^{-1}$ compared to its advection time near the surface, $\tau_a \sim (\frac{ck}{2\pi})^{-1}$.

$$\frac{\tau_d}{\tau_a} \sim \frac{ck\,(\sqrt{\nu^2 + 4\ell_0 U_*^2} + \nu^2)}{4\pi U_*^2}$$

which is always very small.

§3. Effect of Air Flow Turbulence

As stated in the Introduction, Miles' theory of surface wave generation is a well-argued mathematical theory within the assumptions made, of which the most crucial is that the air flow is inviscid and the turbulence has no role to play except in setting up the shear flow. The question of its relevance arises only when an attempt is made to apply it to the field situation where the air flow is inevitably turbulent. Earlier attempts (Davis, Townsend (1972), Gent and Taylor (1976)) to calculate the turbulent effects numerically so as to account for the large difference between Miles' theory and field experiments,led to inconclusive results. It was even felt that (Phillips 1977),

> "The situation is not one in which firmly established methods lead to results that one might seek, with some confidence, to verify experimentally. On the contrary, because of sensitivity of the results to the assumptions made, the air flow over waves appears to provide an ideal context to test the theories of turbulent stress generation themselves."

Here we propose to extend Miles' theory to include the effects of air flow turbulence in a very simple manner. The idea is that if Miles' inviscid theory is at all relevant to turbulent air flow over water waves, the assumed shear flow together with the calculated wave-induced perturbation velocity must in some sense represent the mean of the actual turbulent flow. We shall thus specifically assume that the mean profile $U_s(y)$, resulting from (1) and (6) plus the wave-perturbation velocity as calculated from Miles' theory is the mean of the turbulent air flow over a train of small amplitude sinusoidal water waves. The Reynolds stresses, which were neglected in Miles' inviscid theory will then be re-introduced via a mixing length model with the mixing length given by (6). In this way the effects of the interaction of water waves with turbulence on the surface pressure distribution and on the rate of energy transfer from wind to wave may be calculated.

The equations governing the two-dimensional steady mean turbulent air flow in a frame moving at the phase velocity c of the sinusoidal wave train of small amplitude a are

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{7a}$$

$$(\bar{u} - c)\frac{\partial \bar{u}}{\partial x} + \bar{v}\,\frac{\partial \bar{u}}{\partial y} + \frac{1}{\rho}\,\frac{\partial \bar{p}}{\partial x} = \nu\,\nabla^2\bar{u} + \frac{\partial}{\partial x}\,(\overline{-u'^2}) + \frac{\partial}{\partial y}\,(-\overline{u'v'}) \tag{7b}$$

$$(\bar{u} - c)\frac{\partial \bar{v}}{\partial x} + \bar{v}\,\frac{\partial \bar{v}}{\partial y} + \frac{1}{\rho}\,\frac{\partial \bar{p}}{\partial y} = \nu\,\nabla^2\bar{v} + \frac{\partial}{\partial x}\,(-\overline{u'v'}) + \frac{\partial}{\partial y}\,(\overline{-v'^2}) \tag{7c}$$

In Equations (7) the coordinate x is taken along the propagation direction of the wave and y vertically upward measured from the mean water surface. $\bar{u}$ and $\bar{v}$ are the mean velocity components ρ is density of the air and $\bar{p}$ the mean pressure measured from its unperturbed value when there is no wave.

According to the idea mentioned above, the mean velocity and pressure are taken to be

$$\bar{u} = U_s(y) + u_w \tag{8a}$$

$$\bar{v} = v_w \tag{8b}$$

$$\bar{p} = p_w + p_T \tag{8c}$$

where the wave-induced velocity components u_w and v_w and the pressure p_w are given by Miles' inviscid theory with the shear flow velocity defined by (1) and (6). Thus

$$u_w = -\frac{U_*}{\kappa}\,k\frac{d\phi(\xi)}{d\xi}\eta \tag{9a}$$

$$v_w = i\frac{U_*}{\kappa} k\phi(\xi)\eta \tag{9b}$$

$$p_w\Big|_{y=0} = \rho\frac{U_*^2}{\kappa^2} k\eta \left[w(\xi)\frac{d\phi(\xi)}{d\xi} - \frac{dw}{d\xi}\phi(\xi)\right]_{\xi=0} \equiv \rho\frac{U_*^2}{\kappa^2} k\eta(\alpha_w + i\beta_w) \tag{9c}$$

$$\xi = ky \ , \quad \eta = ae^{i(kx-ct)} \quad , \quad w = \frac{\kappa}{U_*}[U_s(y) \pm c] \tag{9d}$$

where the "-" and "+" sign corresponds, respectively, to the case of wave travelling in the same and opposite direction as wind. The function $\phi(\xi)$ satisfies the Miles' equation (Miles, 1957, Eq. (3.6)) with $U_s(y)$ given by (1) and (6). The pressure p_T represents that due to the interaction of wave with viscosity and turbulence. The Reynolds stress $-\overline{u'v'}$ is related to the mean flow via a mixing length model, viz

$$-\overline{u'v'} = \ell^2 \left(\frac{\partial\bar{u}}{\partial y}\right)^2 \tag{10}$$

with ℓ given by (6). The Reynolds stress $-\overline{u'^2}$ is taken to be proportional to $-\overline{u'v'}$, (Davis, 1970)#

$$-\overline{u'^2} = b\ \overline{u'v'} \quad , \quad b = 2.4 \ . \tag{11}$$

Substituting (8), (10) and (11) into (7), neglecting terms $O(a^2)$, we get

$$\frac{1}{\rho}\frac{\partial p_T}{\partial x} = \frac{\partial}{\partial y}\left[\nu\frac{\partial u_w}{\partial y} + 2\ell^2\frac{dU_s}{\partial y}\frac{\partial u_w}{\partial y}\right]$$

$$+ \frac{\partial}{\partial x}\left[\nu\frac{\partial u_w}{\partial x} + 2b\ell^2\frac{dU_s}{\partial y}\frac{\partial u_w}{\partial y}\right] \tag{12}$$

Using (1) and (6) for $U_s(y)$ and (9) for u_w, we obtain the following expression for the pressure correction‡) p_T due to interaction of wave with air

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#) Although the assumption that b is real may not be justifiable, it can be seen from equations (13) and (14) that because of the dominance of $\phi'(0)$ over $\phi(0)$ the term $\overline{u'^2}$ has negligible contribution to p_T.

‡) Such a correction would be enormously large had the logarithmic shear flow profile (2) been used.

flow turbulence

$$p_T\Big|_{y=0} = \rho \frac{U_*^2}{\kappa^2} k\eta(\alpha_T + i\beta_T) \tag{13}$$

where

$$\alpha_T = -\kappa\left[\sqrt{\left(\frac{k\nu}{U_*}\right)^2 + (2k\ell_o)^2} - \frac{k\nu}{U_*}\right]\left[b\phi(\xi) + I_m\left(\frac{d\phi}{d\xi}\right)\right]_{\xi=0} \tag{14a}$$

$$\beta_T = \kappa\left[\sqrt{\left(\frac{k\nu}{U_*}\right)^2 + (2k\ell_o)^2} - \frac{k\nu}{U_*}\right] R_e\left(\frac{d\phi}{d\xi}\right)_{\xi=0} \tag{14b}$$

To sum up, with the interaction between wave and turbulence included, the total value of the component of the surface pressure in-phase with the wave elevation is equal to $\rho\frac{U_*^2}{\kappa^2}k\eta\alpha$, where $\alpha = \alpha_W + \alpha_T$. On the other hand, the total value of the component of the surface pressure in quadrature with wave-elevation is $\rho\frac{U_*^2}{\kappa^2}k\eta\beta$, where $\beta = \beta_W + \beta_T$. Numerical results for α and β will be presented in §4.

It can be seen from (14) and (9) that the effect of wave-turbulence interaction on the in-phase pressure, as represented by α_T/α_W, is $O((k\ell_o)^2)$, which is very small under typical field conditions. On the other hand, such effect on the energy transfer from wind to wave can be significant. Thus for rough flow

$$\beta_T = 2\kappa^2 = 0.32 \quad , \tag{15}$$

a constant for waves in favourable and adverse wind conditions.

§4 Numerical Results and Comparison

The numerical computation is straightforward and proceeds in much the same as in Conte and Miles (1959), but velocities are made non-dimensional with respect to c rather than U_*/κ. One advantage of this is clear from an examination of the form of the pressure correction in the two cases. In Conte and Miles, the calculation of $\alpha + i\beta$ involves the difference of two large numbers, $w'(0)$ and $\phi'(0)$, and which results in a severe difficulty with loss of significance. However with our scaling, these numbers are scaled down considerably over much of the parameter range of interest. For example, the velocity gradient $w'(0)$ appearing in Miles' formulation is scaled down by a factor $U_*/\kappa c$ in ours, and in the practical cases $U_*/\kappa c \leq 1$. Of course, if a case where $U_*/\kappa c > 1$ needs to be considered, Miles' formulation is to be preferred.

In comparing our results with Conte and Miles (1959) we shall consider smooth, transitional and rough ranges to be represented by the values of $\frac{y_0 U_*}{\nu}$: 0.137, 0.26 and 160. respectively. The appropriate values of ℓ_0/y_0 are obtained from Figure 3 and y_0 itself is set by Ω, Miles profile parameter.

Figure 5 shows the dependence of the normalized critical height on c/U_1. It is seen that the rough case -flow fully turbulent down to the surface - corresponds with Conte and Miles, but in the region of greatest energy transfer from wind to wave ($\frac{c}{U_1} < 5$) the critical height is very dependent on the aerodynamic roughness.

Figures 6 and 7 compare Miles results (from Conte and Miles, 1959) with the calculations of this paper for all three ranges of aerodynamic roughness. The in phase (with surface elevation) component of surface pressure α (Figure 6) calculated by Conte and Miles is reproduced by our calculations for the rough case. Thus the inclusion of turbulent effects, through our mixing length hypothesis, leaves the in phase pressure component unchanged. However, the quadrature component β (in phase with surface slope) is increased by about 10% over Conte and Miles for aerodynamically rough flows in the region of most active wave generation $c/U_1 < 5$ (Figure 7). The transitional and smooth flow cases, which as we shall show later may be more appropriate for comparison with existing experiments, are reduced relative to Miles results. Note that the fractional rate of energy increase per radian, ζ is related to β by:

$$\zeta = \frac{\dot{\bar{E}}}{\omega\bar{E}} = \frac{\rho}{\rho_w}\left(\frac{U_1}{c}\right)^2 \beta \equiv \frac{\rho}{\rho_w}\gamma_i \qquad (16)$$

This underlines the significance of the region of small c/U_1 and the importance of profile shape (smooth, transitional or rough) in the phenomenon of wave generation by wind.

Comparison with the Bight of Abaco experiment

We have attempted in the above to extend Miles theory of wave generation by wind to include effects of realistic wind profiles and to model direct effects of turbulence through a mixing length concept. Our results are sufficiently different from Miles' computations (Miles, 1959 and Conte and Miles, 1959) that it is of interest to see how they compare with the latest reports of the actual wave generation process in nature. The Bight of Abaco experiments (Snyder et al. 1980) brought together several independent researchers in this topic and the published results therefrom (Snyder et al. 1980) provide excellent material for comparison with our computations. In order to compare their results with our model the value of Miles' profile parameter Ω and of the aerodynamic roughness for the BOA experiment must be established from Table 2 of Snyder et al. (1980). We determine that the average wind speed at 5 metres height for runs 15 to 20 was 644 cm/s and the average friction velocity was 20.4 cm/s. These correspond to roughness length $y_0 = 1.64 \times 10^{-3}$ cm, Miles' profile parameter $\Omega = \dfrac{gy_o}{U_1^2} = 6.19 \times 10^{-4}$, normalized roughness length $\dfrac{y_0 U_*}{\nu} = 0.26$, surface mixing length $\ell_0 = 2.0 \times 10^{-3}$ cm. We see, therefore, that the flow is transitional and the Miles' profile parameter is almost a factor of 5 smaller than the smallest of the values he published: M_1 in the notation of Snyder et al. (1980). Runs 15 to 20, which play a large part in Snyder et al. (1980), are drawn from a relatively narrow set of conditions and are not closely modelled by the published results of Miles (1959) both because of the difference in profile parameter values and the aerodynamic flow conditions. Using the same method, as that used by Snyder et al. (1980) to relate U_1/c to U_5/c (R.B. Long, Personal communication), we compare Figure 22 of Snyder et al. (1980) with our computations appropriate to those conditions: (i) $\Omega = 6.19 \times 10^{-4}$ and (ii) transitional flow with $\ell_0 = 2.0 \times 10^{-3}$ cm.

Figure 8 contains several sets of results. Our results (dashed) show very good agreement with respect to the in-phase component of surface pressure α ($\gamma_r = (U_1/c)^2\alpha$). However, the quadrature component $\beta(\gamma_i = (\frac{U_1}{c})^2\beta)$ tells an altogether different story. Our computations for transitional flow, which corresponds to the conditions of the BOA runs 15 to 20, are about one third as large as the lowest of the curves of Snyder et al. (1980), even though the turbulent correction is included in our results. The computations for rough flow agree more closely with the BOA results but this does not help since the BOA conditions were not aerodynamically rough during runs 15 to 20. (The same is true of run 14). Our computations for negative U_5/c are likewise much smaller than the BoA results, but it is significant that they are non-zero corresponding to some wave damping and roughly 10% of the values for U_5/c positive. The recent measurements of Stewart and Teague (1980) of wave decay in an adverse wind suggest values of β about an order of magnitude smaller than for wave growth in a favourable wind. All three (Snyder et.al., Stewart and Teague and this paper are consistent in the sense that the magnitude of β for wave damping ($U_5/c < 0$) is about 10% of that for wave growth. However Stewart's and Teague's growth rate (Figure 8) is about twice the mean of the BOA estimates of energy input to the waves. It should be pointed out that the growth rate, as measured by Stewart and Teague, includes the effects of wave-wave interaction and dissipation and therefore can be larger than the wind input for waves on the forward face of the spectrum.

A reliable estimate of the growth rate of a spectrum of waves may be deduced from fetch-limited studies such as the JONSWAP (Hasselmann et.al., 1973) experiment. Since the total energy in the spectrum is considered, wave-wave interaction has no effect on the computation. However dissipation is ignored and, in this sense, the estimate of growth rate as derived should be regarded as a lower limit to the wind input.

Hasselmann et. al. (1973) include laboratory data in their similarity scaling of wave parameters. We agree with Phillips (1977) that it is more appropriate to consider field and laboratory data separately. Using Phillips' modification* of the JONSWAP results:

$$\gamma_i = 2.4 \times 10^{-5} \frac{\rho_w}{\rho} \left(\frac{U_5}{c}\right)^2 , \quad 1.3 < \frac{U_5}{c} < 4.7 \qquad (17)$$

This curve has been added to Figure 8 and it is reassuring to note that our extended theory produces growth rates comparable with this lower limit.

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* Note that although Phillips (1977) presented the JONSWAP data in terms of U_*, the friction velocity was not measured but deduced from $U_* = \sqrt{C_{10}}\, U_{10}$, and C_{10} was taken to be a constant $= 1 \times 10^{-3}$ for the JONSWAP data.

§5 Concluding Remarks

In this paper we have shown that the surface pressure very much depends on the flow condition being aerodynamically smooth or rough. Typical field data corresponds to transitional flow conditions. We have also shown that there is practically no effect of interaction of wave with air flow turbulence on the in-phase (with surface elevation) component of surface pressure but that such interaction tends to increase the energy input from wind to wave travelling with the wind by about 10% for aerodynamically rough flow. Furthermore, the interaction of waves with air flow turbulence is shown to provide damping of waves in an adverse wind, which is typically an order of magnitude smaller than the growth rate in favourable wind conditions. This is consistent with the recent field observation of Snyder et. al. (1980) and Stewart and Teague (1980).

The extended theory predicts, however, a rate of energy input by wind that is still a factor of 2-3 too small compared with the best known field data of Snyder et. al. (1980), although it is not inconsistent with the lower limit provided by the JONSWAP fetch-limited wave growth observation.

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Figure Captions

Figure 1 Normalized critical height as calculated by Miles (1957) using the logarithmic wind profile. The dashed line denotes the height below which the profile is modified from its logarithmic form by molecular viscosity.

Figure 2 The ratio of roughness size d to roughness length y_0 for Van Driest's profile versus normalized roughness length.

Figure 3 The ratio of surface mixing length ℓ_0 to roughness length y_0 for the profile resulting from (6). The borders between smooth to transitional and transitional to rough are at normalized roughness lengths of 0.137 and 60 respectively.

Figure 4 Velocity profiles from Van Driest (——) for smooth flow; from integration of (1) using (6) for smooth flow, $\frac{y_0 U_*}{\nu} = 0.137$ (---); for rough flow, $\frac{y_0 U_*}{\nu} = 60$ (—·—) and for transitional flow, $\frac{y_0 U_*}{\nu} = 0.26$ (.....). The profile shown for transitional flow corresponds to the conditions of the Bight of Abaco experiment. The circles are from the pipe flow experiments of Laufer (1953) at Reynolds numbers of 50,000 (open circles) and 500,000 (solid circles).

Figure 5 Normalized critical height versus c/U_1 . (——) Conte and Miles (1959). The other curves correspond to the mixing length (6) for smooth flow (....), transitional flow (—·—) and rough flow (— — —).

Figure 6 The in phase component of surface pressure α for smooth (....), transitional (—·—) and rough flow (— — —). Figure 6a is for $\Omega = 0.003$ and 6b is for $\Omega = 0.003$. The calculations of Miles and Conte (1959) coincide with the rough case.

Figure 7 The quadrature component of surface pressure β for smooth flow (....), transitional flow without turbulent correction (—··—··), transitional flow with turbulent correction (—·—); rough flow without turbulent (——), rough flow with turbulent correction (– – –). Figure 7a is for Ω = .003 and 7b is for Ω = 0.02. The calculations of Miles and Conte (1959) coincide with the rough case without turbulent correction.

Figure 8 Comparison of our calculations with the BoA results. The various calculations of Snyder et al. (1980) from the BoA experiment fall within the shaded areas for $U_5/c > 1$. Present calculations for transitional flow (—·—), and rough flow (– – –) are shown as are the calculations of Conte and Miles (1959) (——). The growth rate deduced from the JONSWAP experiment (—··—··) includes dissipation and so implies a lower limit to the wind input. The growth rate of Stewart and Teague, 1980 (·) includes dissipation and wave-wave interaction.

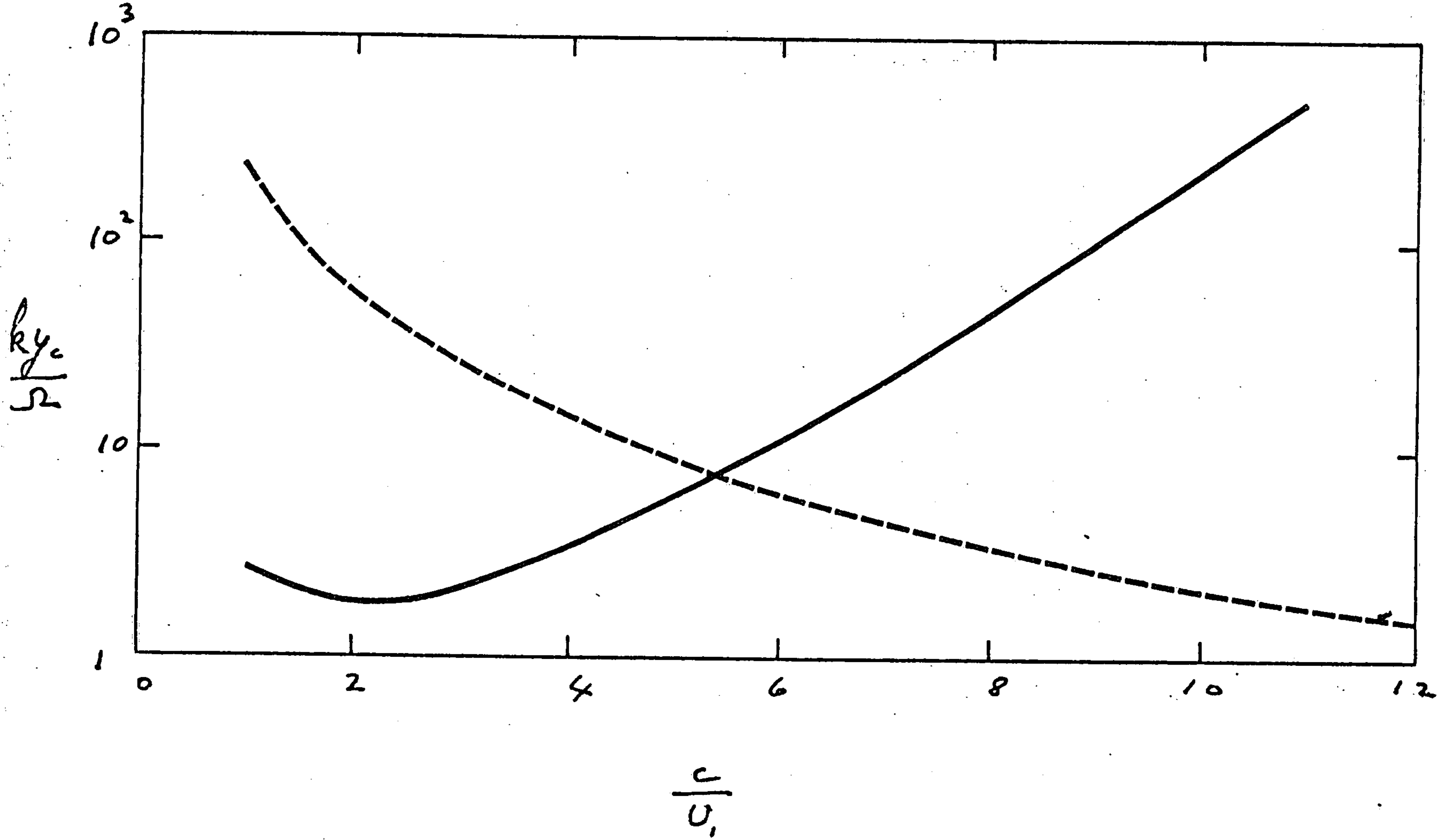
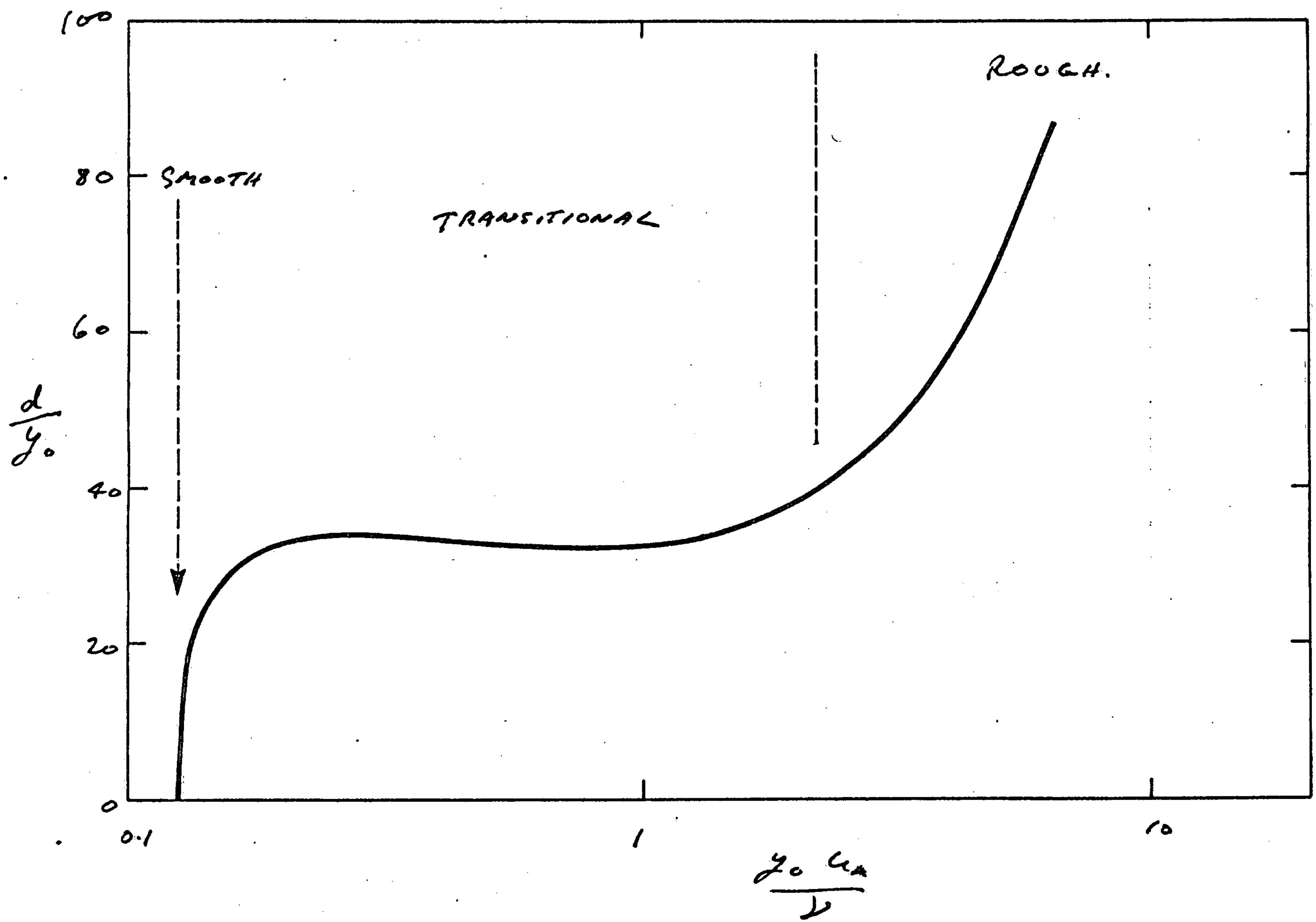


FIGURE 1



ROUGH.
SMOOTH
TRANSITIONAL
100
80
60
40
20
0
d/y_0
0.1
1
10
$\frac{y_0 u_*}{\nu}$

FIGURE 2

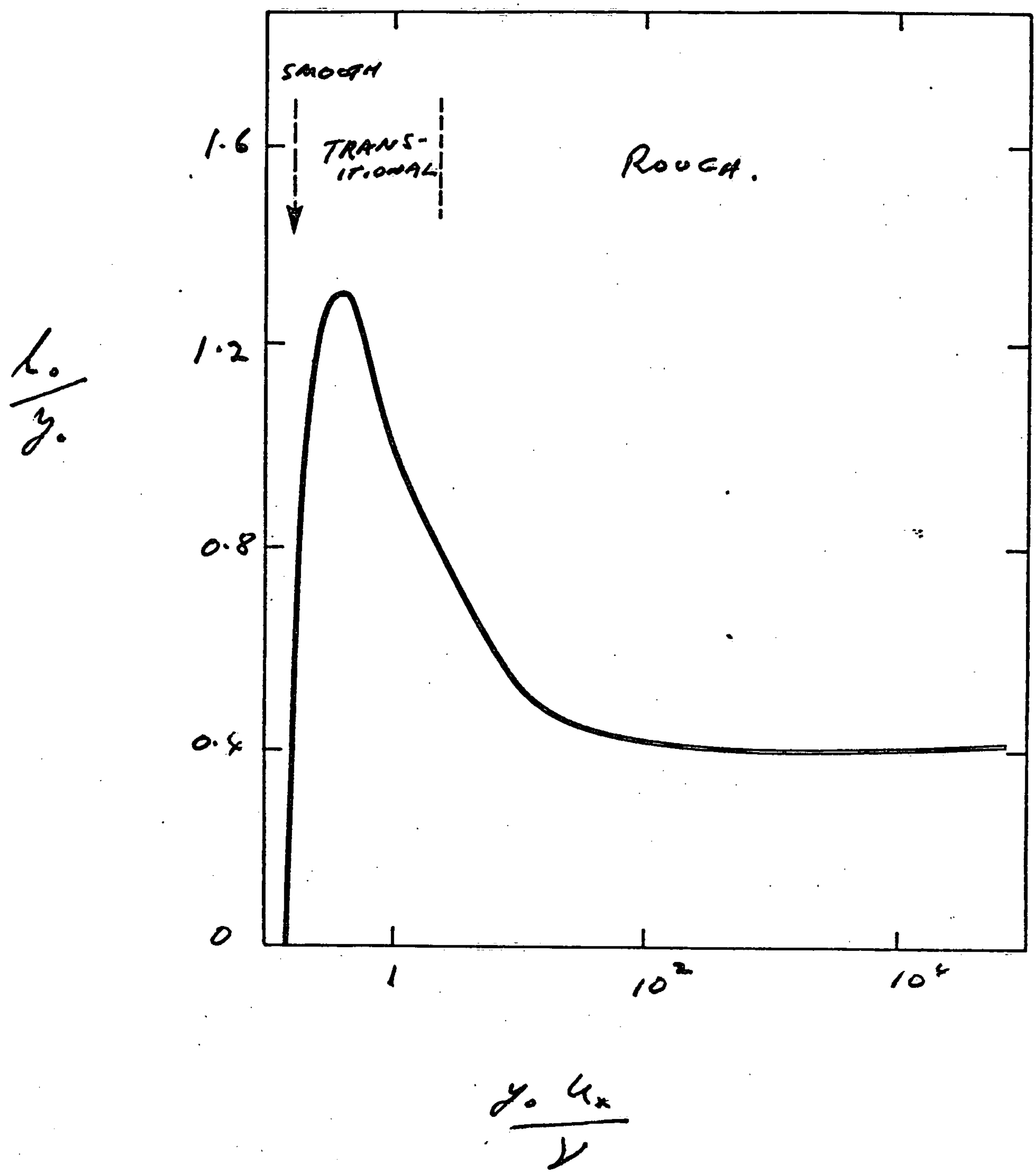
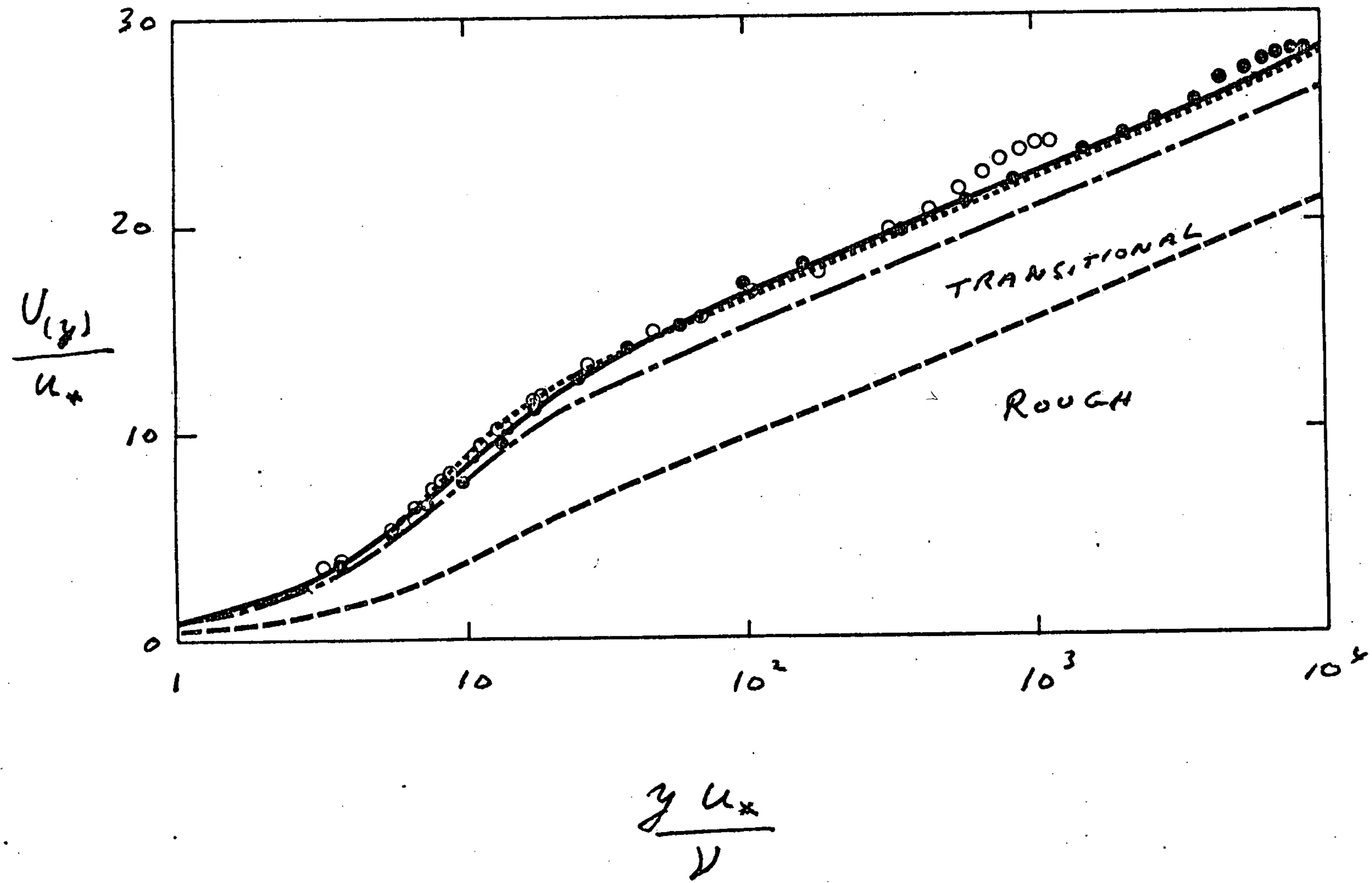

SMOOTH
TRANS-
ITIONAL
ROUGH.
1.6
1.2
0.8
0.4
0
1
10^2
10^4
$\frac{l_0}{y_0}$
$\frac{y_0 u_*}{\nu}$

FIGURE 3.



30
20
10
0
1
10
10^2
10^3
10^4
TRANSITIONAL
ROUGH
$\frac{U_{(y)}}{u_*}$
$\frac{y\,u_*}{\nu}$

FIGURE 4

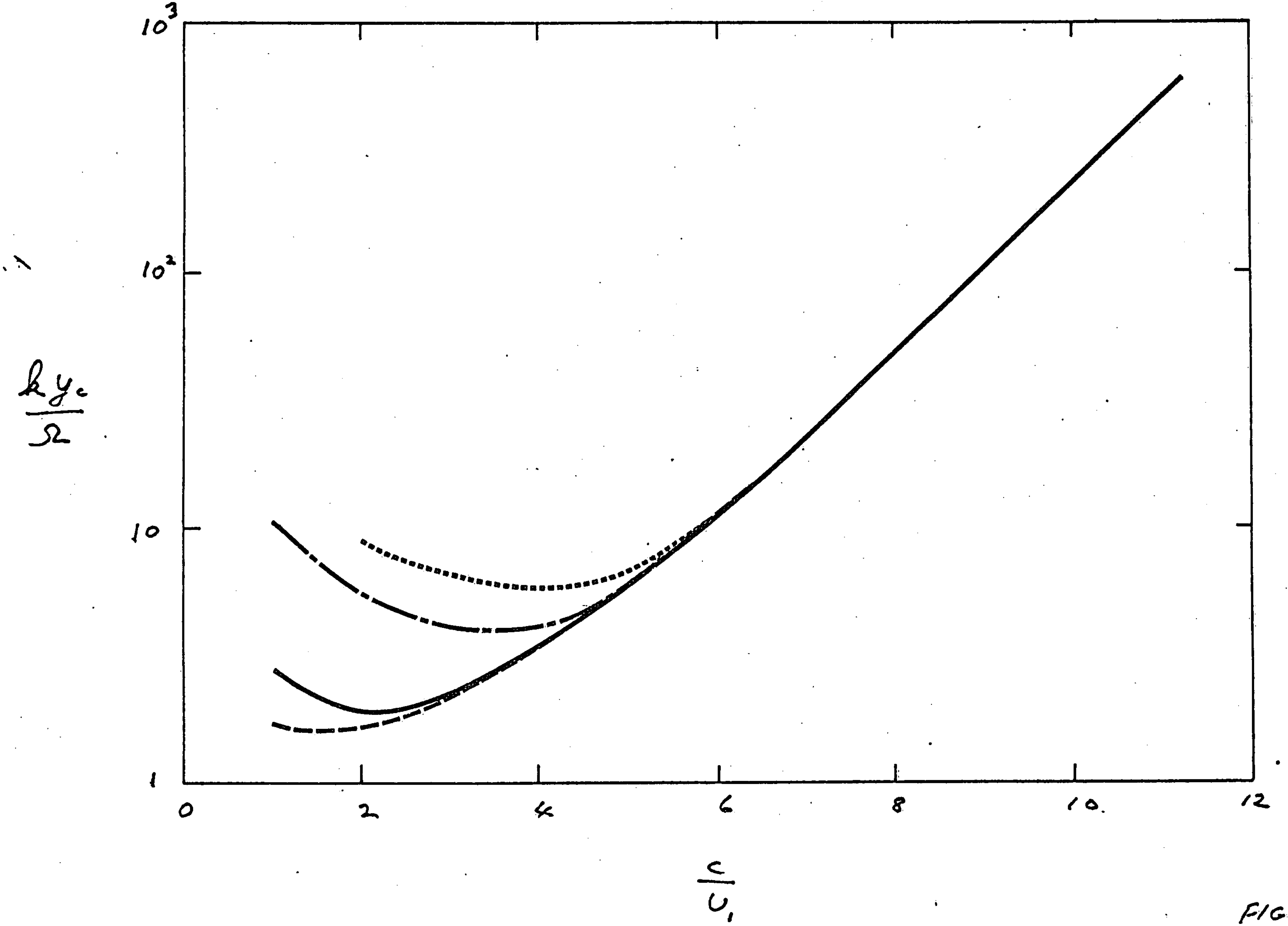


FIGURE 5

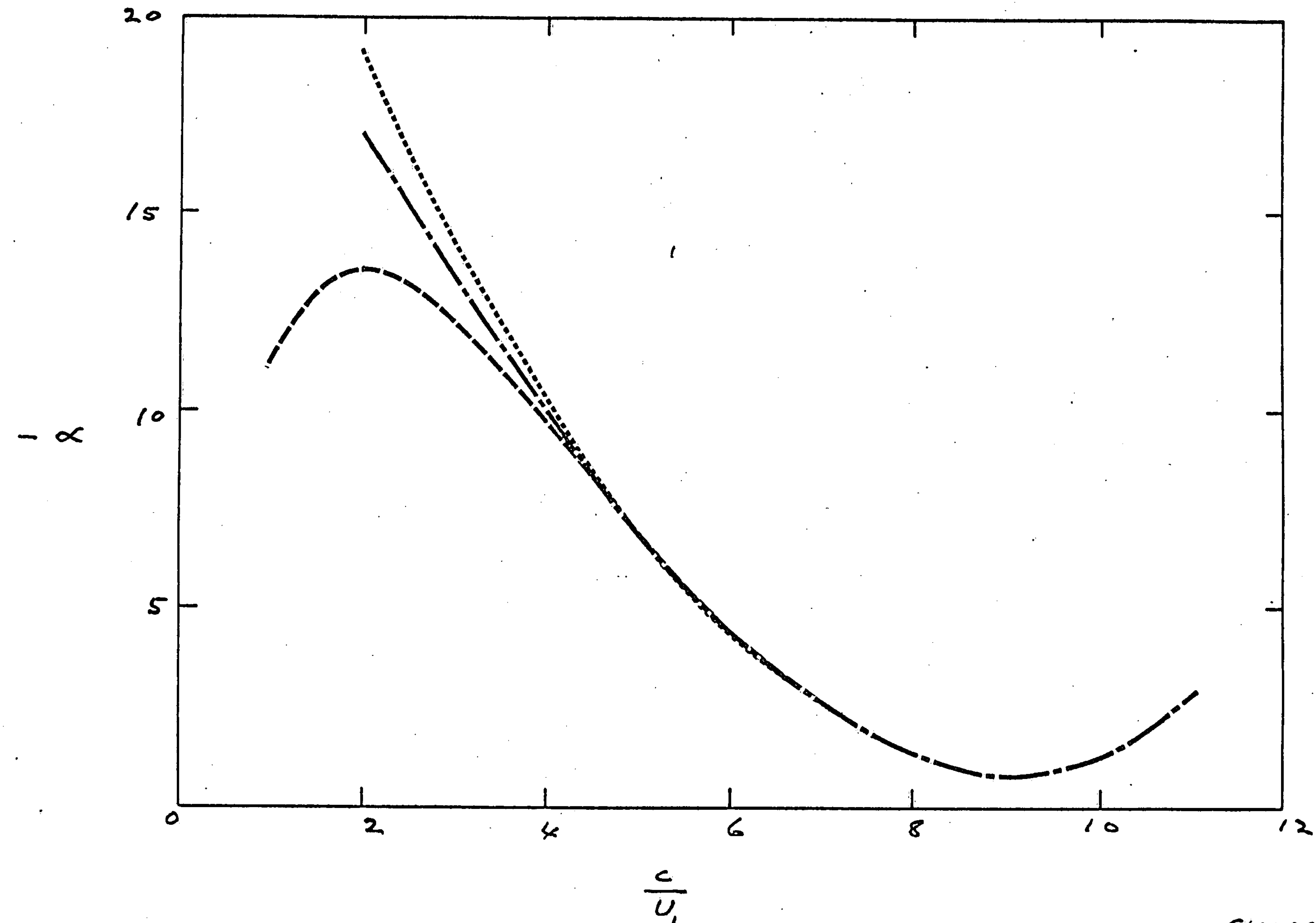
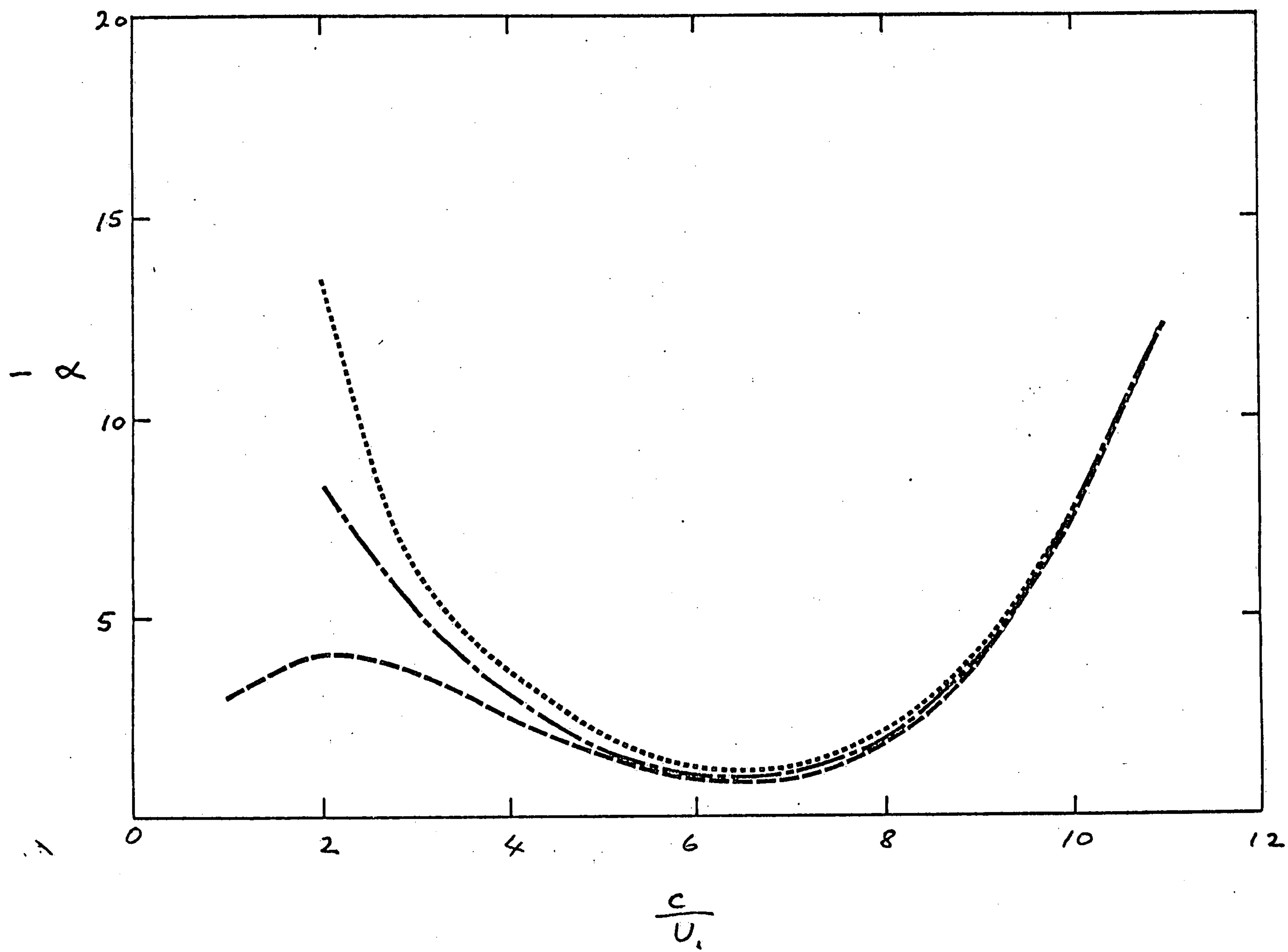


FIGURE 6a



FIGURE

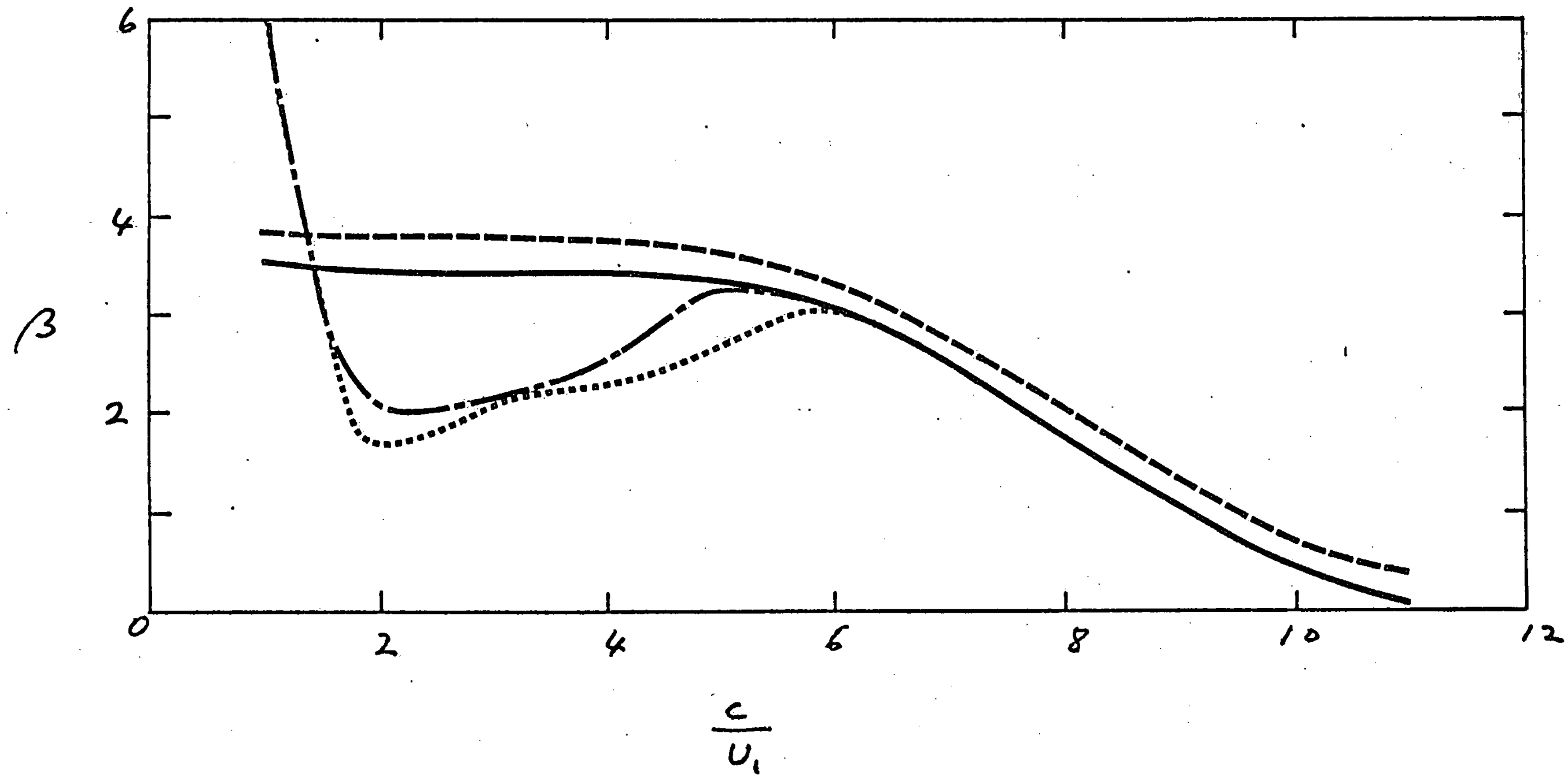


FIGURE 7a

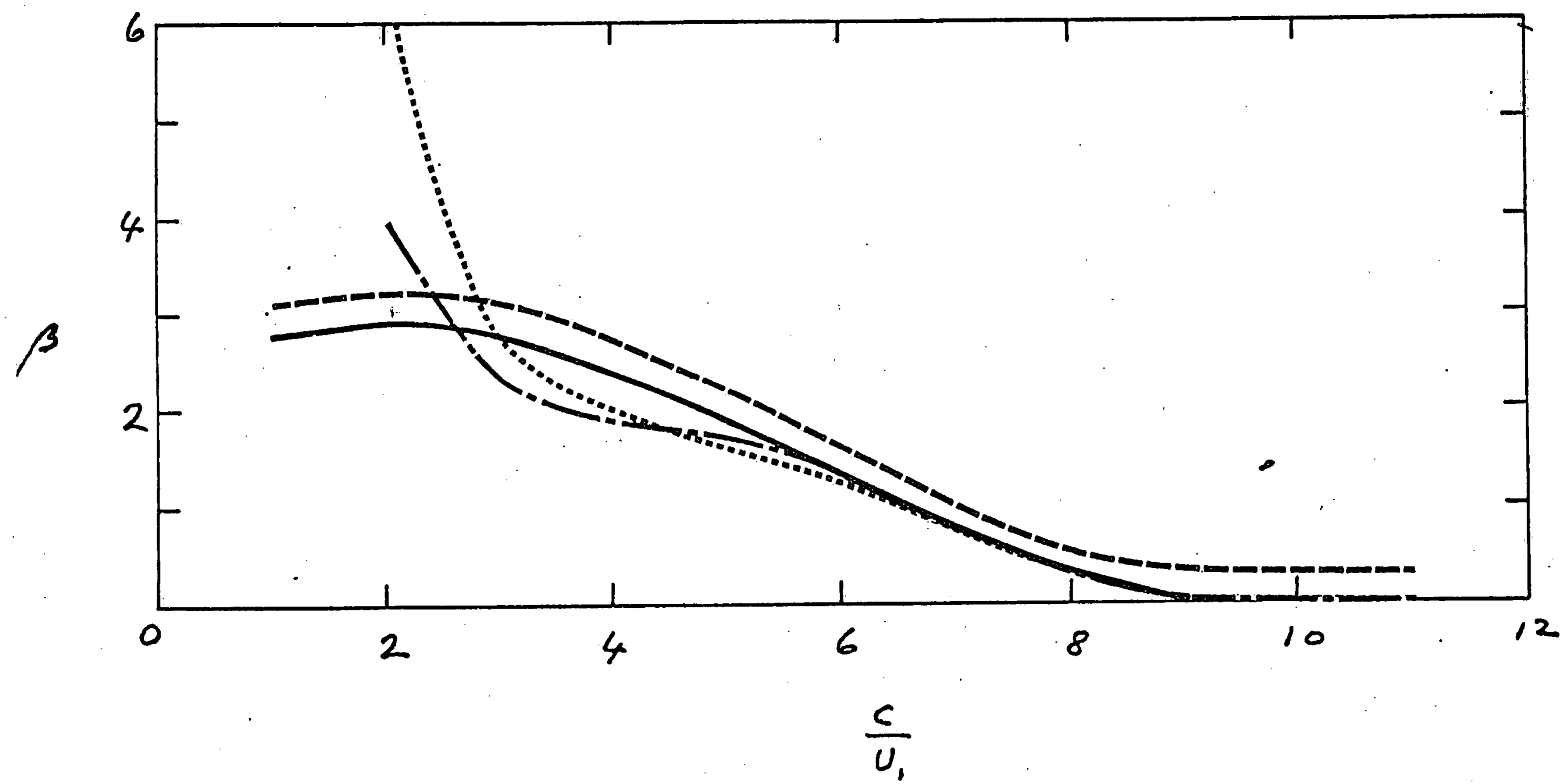


FIGURE 7b

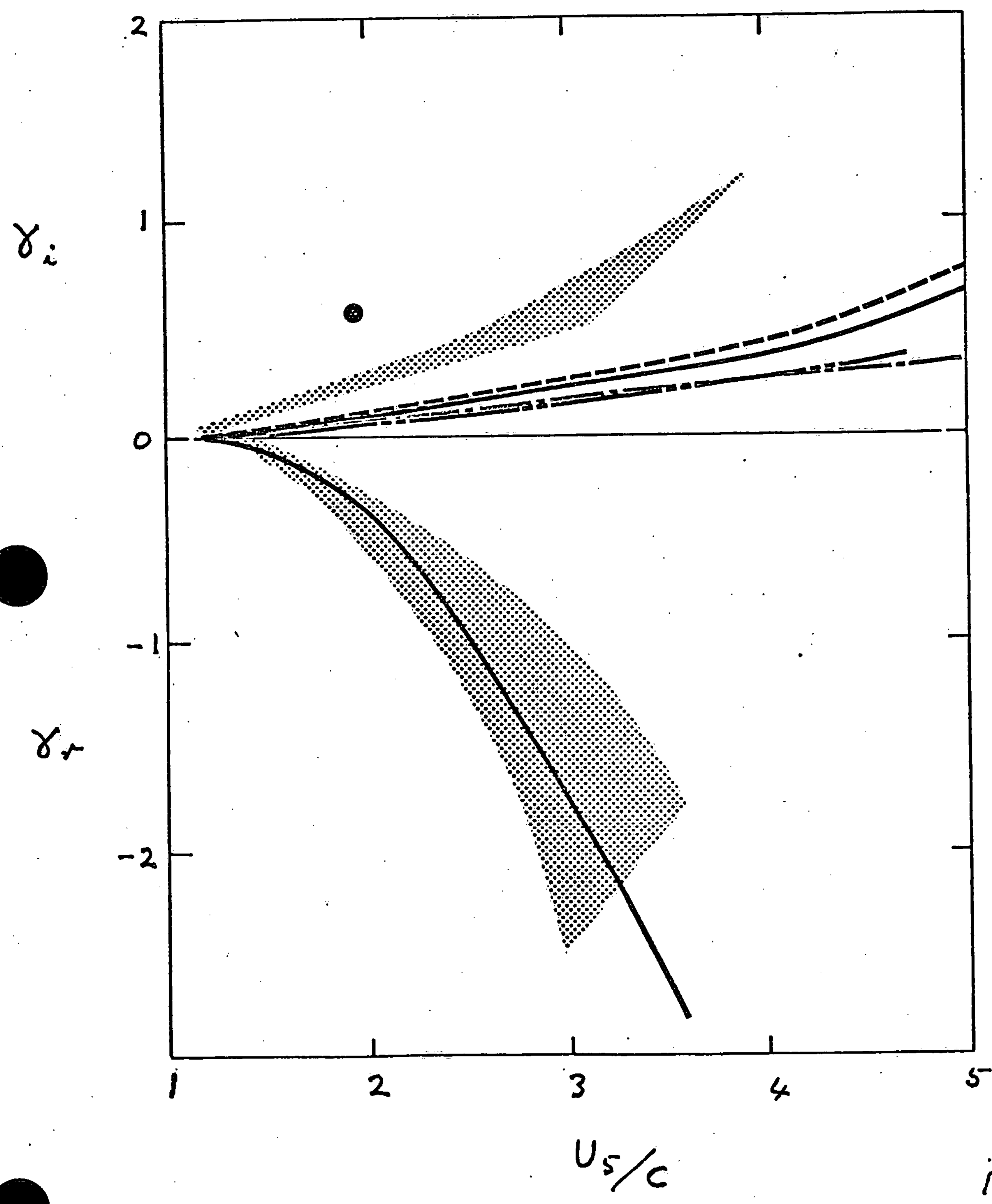


FIGURE 8

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