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ABSTRACT

Measurements of transverse dispersion were made in two trapezoidal channels having different side slopes and using side injection as well as centreline injection. The dispersion coefficients were obtained from the change of moments and also from a numerical simulation. The dimensionless dispersion coefficients were compared with those for rectangular and triangular channels in an attempt to estimate the effect of cross section shape on the dispersion coefficient.

RESUME

Des mesures de la dispersion transversale one été faites dans deux canaux de forme trapézoidale symétrique; chacun des canaux avait une inclinaison latérale différente. Un traceur a été injecté sur le côté et au centre des canaux. Les coefficients de dispersion ont été obtenus à l'aide d'un changement de moments et d'une simulation numérique. Les coefficients de dispersion sans dimension ont été comparés à ceux de canaux rectangulaires et triangulaires afin d'évaluer l'effet de la section transversale sur le coefficient de dispersion.

1. INTRODUCTION

The transverse spreading of materials in rivers is frequently treated as a two-dimensional problem by considering variations in the longitudinal and transverse directions only and averaging all quantities over the depth. Holley [1] has shown that the depth-averaged transport in the transverse direction consists of a turbulent diffusive transport and a transport due to differential convection in the transverse direction. The total transport is normally represented by a Fickian type term - a dispersion coefficient times the concentration gradient.

Lau and Krishnappen [2] measured the dispersion coefficient in rectangular channels of varying aspect ratios and friction factors. Their results indicated that transport due to differential convection was dominant over that by turbulent diffusion. Therefore, it was suggested that secondary circulation had a strong influence on the transverse spreading in rivers. Because this secondary circulation is governed by the variation in transverse shear, it is very dependent on the shape of the cross section. Therefore, some investigation into the effect of cross-section shape on transverse dispersion is required.

The only laboratory measurement of transverse dispersion in non-rectangular channels was by Holly[3] who used a triangular cross section. In this paper the results of dispersion measurements in two trapezoidal channels with different side slopes are presented. These results are compared with published data to investigate the influence of cross-section shape on the dispersion coefficient.

2. EXPERIMENTAL SETUP AND PROCEDURE

The experiments were conducted in two flumes, each with a different trapezoidal cross section as shown in Figure 1. The first had a bottom width of 22 cm and a 1:1 slope for the sides. The second channel had a bottom width of 40 cm and 2:1 side slope. Both flumes were 30 metres in length. The flume slopes could be adjusted by a set of motorized screwjacks. Discharge measurements were made using a weirbox at the downstream end. A uniform roughness was achieved by gluing a layer of sand to the bed and sides of the flumes.

To set up a run, the required discharge was pumped into the flume and the tailgate and flume slope were adjusted until uniform flow at the required depth was established. Velocity traverses at different depths were then made using a Kent miniature current meter. From these velocity traverses, the depth averaged longitudinal velocities were computed.

For the dispersion measurements, a salt solution which was made neutrally buoyant with methanol was injected continuously into the flume. The injection rate was adjusted so that the solution issued from the discharge nozzle at the same speed as the ambient flow. Concentration measurements were then made at various stations downstream using a single electrode conductivity probe as described in Lau and Krishnappen [2].

Four different flow depths were used with the first trapezoidal channel and three flow depths were used with the second. The mean flow velocity was kept approximately constant in each case. The hydraulic data are summarized in Table 1.

For each flow condition, two dispersion experiments were made, one using salt injection at the centreline of the channel and the other using injection at the edge of the channel. Injections were made at mid-depth for that location.

TABLE 1 SUMMARY OF HYDRAULIC DATA

Channel No.	Q m^3/s	H_c cm	U cm/s	W cm	R cm	S	U_* cm/s	f	H cm	$\frac{W}{H}$
T-1	0.00796	12.01	20.01	44.60	7.25	5.0×10^{-4}	1.885	0.071	8.94	4.98
T-1	0.00625	10.01	19.99	40.80	6.34	5.9×10^{-4}	1.916	0.074	7.67	5.32
T-1	0.00469	8.01	19.97	36.92	5.36	7.43×10^{-4}	1.977	0.078	6.36	5.81
T-1	0.00329	6.01	19.98	33.20	4.30	10.1×10^{-4}	2.064	0.085	4.97	6.68
T-2	0.00804	9.00	20.02	49.0	6.66	7.10×10^{-4}	2.15	0.092	8.22	5.96
T-2	0.00629	8.00	17.82	48.0	6.08	6.2×10^{-4}	1.923	0.093	7.37	6.51
T-2	0.00447	6.00	17.29	46.0	4.83	8.0×10^{-4}	1.940	0.1		

Q = discharge; H_c = centreline depth; U = mean velocity; W = top width;
R = hydraulic radius; S = slope; U_* = shear velocity; f = friction factor;
H = average depth (area/top width)

3. EVALUATION OF THE TRANSVERSE DISPERSION COEFFICIENT

The transverse dispersion coefficient was evaluated using two different methods, namely

1) change of moment method
2) numerical simulation method

A brief description of these methods is given below:

1) Change of Moment Method

This follows the method suggested by Holley [1].

The depth-average mass-conservation equation which describes the spread of a conservative substance in a non-rectangular channel under steady state condition is

$$\frac{\partial}{\partial x}(huc) = \frac{\partial}{\partial z}\left(e_z h \frac{\partial C}{\partial z}\right) \tag{1}$$

where C is the depth-average value of the concentration, u is the depth-average longitudinal velocity component, h is the local depth, y is the dispersion coefficient, and x and z are the longitudinal and transverse coordinates respectively.

Multiplying both sides of Equation (1) by $\bar{z}$ and integrating across the width (W) of the channel and assuming that e_z=KUH in which K is a constant, one gets the following equation

$$\frac{d}{dx}\left[\frac{\int_{w/2}^{w/2} hucz^2\,dz}{\int_{w/2}^{w/2} hucdz}\right] = 2K\left[\frac{\int_{w/2} hU_* H \frac{\partial C}{\partial z} z\,dz}{\int_{-w/2}^{w/2} hucdz}\right] \tag{2}$$

or

$$\frac{d\sigma^2}{dx} = -2K\,f(x) \tag{3}$$

The definitions for σ^2 and f(x) can be seen by comparing Equations (2) and (3). These two terms were computed for each measuring station, using the measured concentration distributions and velocity profiles.

To obtain the value for K, Equation (3) was integrated to give

$$\sigma^2(x) - \sigma_o^2 = -2K \int_{x_o}^{x} f(x)\,dx = 2K\,F(x) \qquad (4)$$

where $\sigma_o^2 = \sigma^2(x_o)$ x_o being the location of an initial measuring station. The slope of the plot of $(\sigma^2 - \sigma_o^2)$ versus F(x) was then equal to 2K.

2) Numerical Simulation Method

In the numerical simulation method, the governing Equation (1) was solved numerically using as input the measured concentration distribution at the first station. e_z was assumed to be equal to KU_*H. Different values of K were tried and the resulting concentration distributions were then compared with the measured concentration distribution. The value of e_z for which the predicted concentration distribution gave the best agreement with measurement was considered to be the correct value of the dispersion coefficient.

A finite difference approximation was used to solve Equation (1) using a discretization procedure recommended by H. L. Stone and P.L.T. Brian [4]. The resulting tridiagonal matrix was solved using Gaus-Seidal technique. Details of this scheme can be found in Krishnappen & Lau [5].

4. RESULTS AND DISCUSSION

The values for the dimensionless dispersion coefficient e_z/U_*H obtained from the simulation and from the change of moment method are listed in Table 2. Also listed are the values for the coefficient e_z/U_*W which are derived from the values of e_z/U_*H obtained from simulation.

TABLE 2 SUMMARY OF EXPERIMENTAL DATA

Channel	H_c cm	Injection Location	$K=e_z/U_*H$ Simulation	$K=e_z/U_*H$ Moment Method	e_z/U_*W Simulation
T-1	12.01	centreline	0.11	0.09	22.0×10^{-3}
	12.01	side	0.12	0.23	24.0×10^{-3}
	10.01	centreline	0.11	0.095	20.6×10^{-3}
	10.01	side	0.09	0.165	16.8×10^{-3}
	8.01	centreline			
	8.01	side	0.115	0.116	19.7×10^{-3}
	6.01	centreline	0.11	0.092	16.6×10^{-3}
	6.01	side	0.15	0.152	22.5×10^{-3}
T-2	9.0	centreline	0.13	0.118	21.7×10^{-3}
	9.0	side	0.07	0.107	12.3×10^{-3}
	8.0	centreline	0.13	0.128	19.9×10^{-3}
	8.0	side	0.09	0.115	13.7×10^{-3}
	6.0	centreline	0.15	0.136	18.4×10^{-3}
	6.0	side	0.13	0.144	15.4×10^{-3}

It can be seen in Table 2 that the coefficients obtained from the change of moment method and from the numerical simulation generally agree reasonably well. However, there are a few cases in which the values differ substantially. In two instances the values obtained from the moment method were almost double those obtained from the simulation. After some reviewing of the data, it was concluded that the discrepancy resulted from the extreme sensitivity of the moment method to the concentration values of the tail end of the distribution. A slight error in those concentration values can result in substantial errors in the second moment, leading to erroneous values for the dispersion coefficient. As an example, the application of the change of moment method to obtain e_z/U_*H from the plot of ($^{2}-{}_{o}^{2}$) versus F(x) is shown in Figure 2. A value of e_z equal to 0.165 U_*H was obtained. However, the numerical simualtion for the same experiment showed that a value of e_z=0.09 U_*H gave the best results. The experimental and simulated concentration distributions are shown in Figure 3. For the sake of clarity, only the initial input distribution and three downstream stations are shown. It can be seen that the simulated profiles agree quite well with the measured profiles except at the outer edge where the measured concentrations have slightly higher values. As a comparison, e_z=0.165 U_*H was also used in the simulation and, as shown in Figure 3, it produced concentration profiles vastly different from the measured ones. The value of e_z equal to 0.165 U_*H is obviously incorrect. It can be concluded that the change of moment method is not as reliable as the simulation for estimating the value of the dispersion coefficient. The results from the simulations are used in all subsequent comparisons with published data.

The values of e_z/U_*H, given in Table 2, vary between 0.07 and 0.15 and are in the same range as published values for laboratory channels. It was shown by Lau and Krishnappen [2] that U_*H is actually not a good parameter for the representation of e_z. Published data for e_z/U_*H could not be correlated very well with friction factor and width to depth ratio, the two bulk parameters which e_z/U_*H depend on. It was also shown that e_z/U_*W was a better dimensionless dispersion coefficient to use than e_z/U_*H. All the published data could be collapsed on to one curve of e_z/U_*W versus W/H. Therefore, the values of e_z/U_*W from the trapezoidal channels are plotted against W/H in Figure 4 in order to compare with rectangular channel data. Figure 4 includes all the published data on rectangular channels given in Lau and Krishnappen [2]. It can be seen that the present data fit very well into the rectangular channel data, which indicates that the dispersion in the two trapezoidal channels is practically the same as for rectangular channels of the same W/H ratio. However, the values of e_z/U_*W for the triangular channels reported by Holly [3] are considerably larger and do not fit on the same general curve. The accuracy of those values may be questionable because, as mentioned by Holly, his flume slopes were very small and were subject to large errors. However, even if the values of the slopes were doubled, the dispersion coefficients would still be much larger than rectangular channels of the same W/H ratio. The same discrepancy exists when the comparison is made using the more familiar dispersion coefficient e_z/U_*H. Since the friction factors for Holly's runs were very small, about 0.02 and 0.04, the increase in dispersion can only be attributed to increases in the secondary circulation. It is worth noting that the data point with the larger values for e_z/U_*W was for the case in which bottom roughness was removed from the centre of the channel. This would have increased the variation in shear across the bottom which would have increased the secondary circulation.

It is difficult to compare channels with different cross sections and say which ones should correspond. Three properties have been used here to characterize the sections, namely, the top width, the average depth and the side slope. Rectangular channels can be considered as the limit with side slope equal to infinity. The trapezoidals used for this study had side slopes of 1.0 and 2.0 respectively while Holly's [3] channel had side slope of 0.3. Although the present data cannot be regarded as conclusive, it seems that for channels with side slopes equal to 1.0 or larger the dispersion coefficient can be estimated from rectangular channel data. For channels with smaller side slopes, the secondary circulation may be increased sufficiently to increase the dispersion over that for rectangular channels. If this is the case, Figure 4 should consist of curves for e_z/U_*W versus W/H, with side slope as a third parameter. However, there are no data available to construct such a set of curves.

There is no significant difference between the dispersion coefficients for the centreline injection and the side injection and the side injection cases. This is similar to the results of Holly [3]. It may be possible that the variations in the turbulence scale across the channel were offset by the variations in the secondary circulation.

5. SUMMARY

The transverse dispersion coefficient was measured in two channels with trapezoidal cross sections and with side slopes equal to 2.0 and 1.0 respectively. The dimensionless dispersion coefficient e_z/U_*W was found to be practically the same as that for rectangular channels of the same aspect ratio. However, data from Holly [3] in which the dispersion coefficient was measured in a triangular channel with side slope equal to 0.3 showed values at least twice as large as those for rectangular channels. This suggests that for side slopes the secondary circulation may increase enough to produce significantly larger dispersion over that for rectangular channels. The dispersion coefficients for centreline injection were about the same as those for side injection.

The dispersion coefficients were obtained from numerical simulation as well as from the usual change of moment method. It was discovered that the change of moment method could sometimes give results which were considerably in error. Therefore, in order to use the moment method, one must be very sure that the concentration values at the outer edge of the concentration distribution are very accurate since small errors there can affect the second moment of the distribution significantly.

6. REFERENCES

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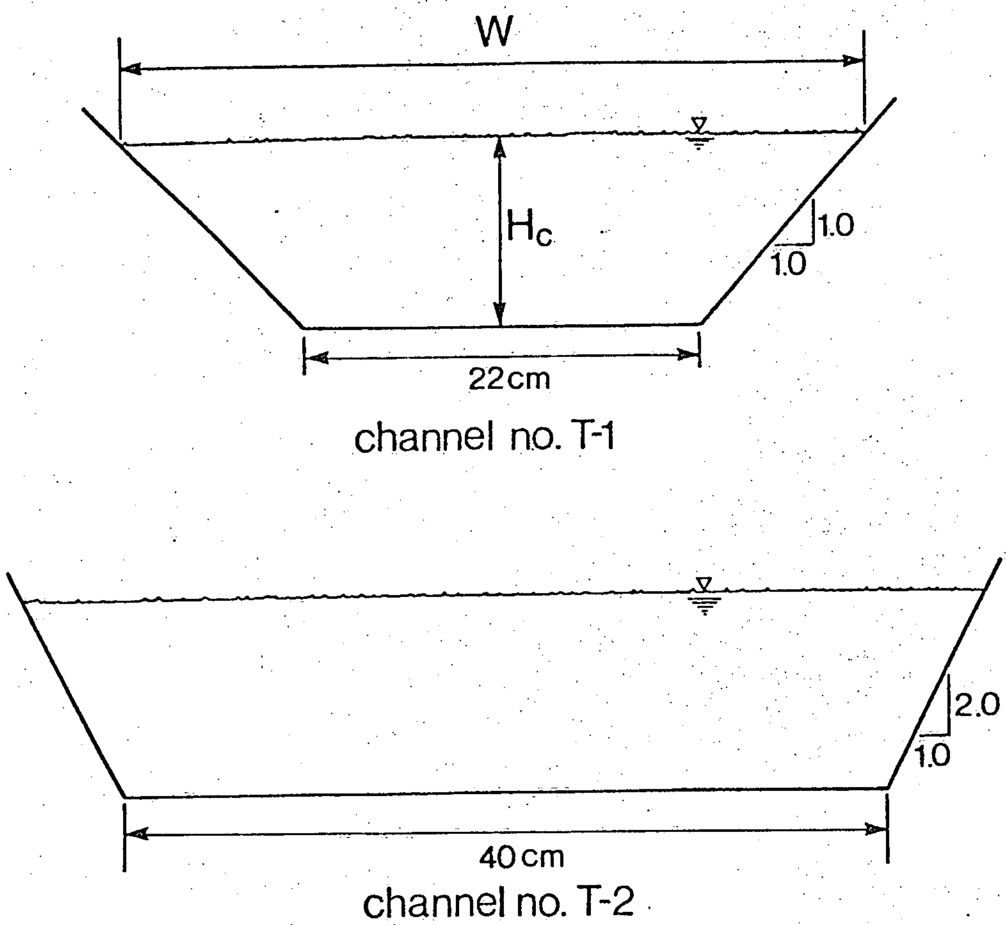

Figure 1. Cross section of trapezoidal channels.

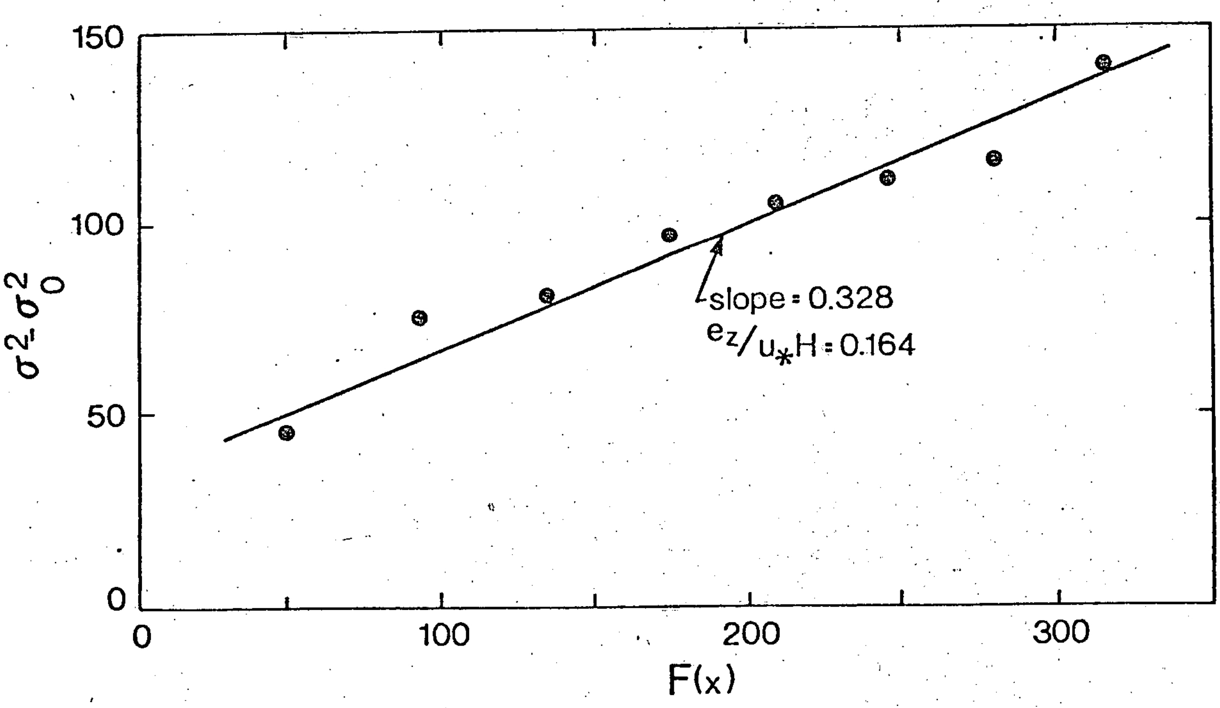

Figure 2. Change of moment analysis. Channel T-1, H_c=10cm., side injection

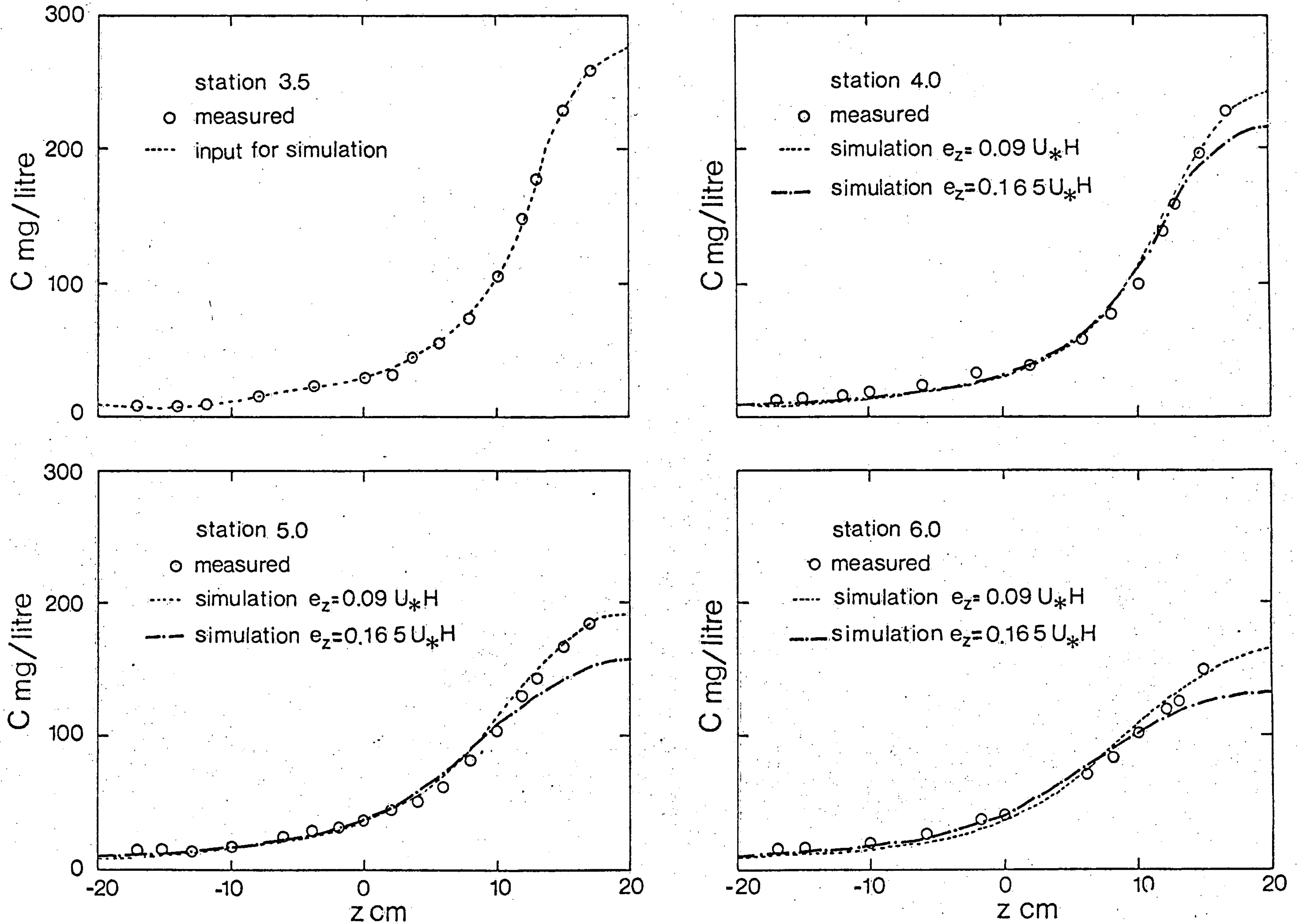

Figure 3. Measured and simulated concentration profiles. Channel T-1, H_c= 10cm, side injection

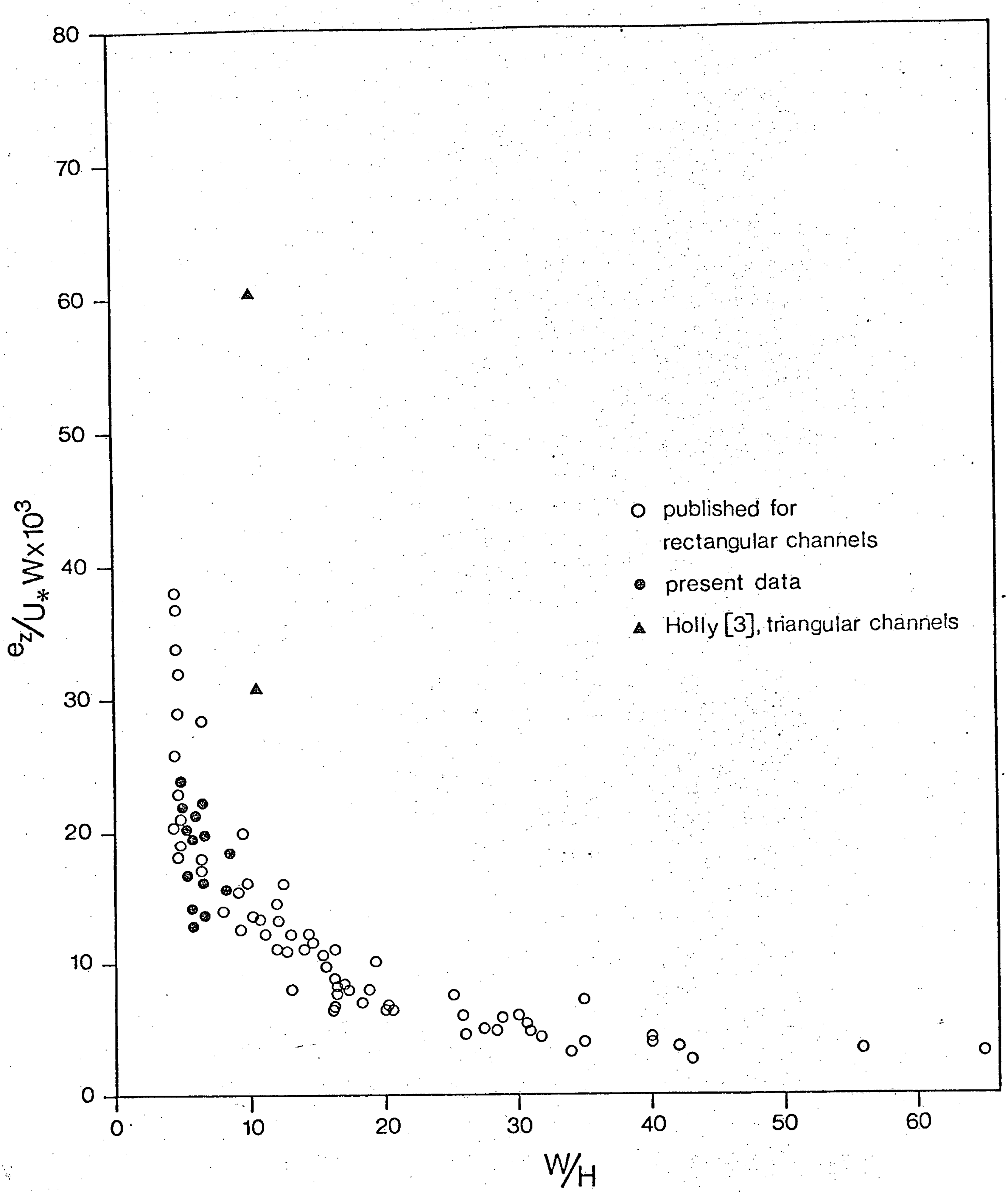

Figure 4. Variation of e_z/U_*W with W/H