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# TRANSVERSE MIXING IN MEANDERING CHANNELS <br> WITH VARYING BOTTOM TOPOGRAPHY <br> by <br> B.G. Krishnappan and Y.L. Lau 

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Using the theory of dimensions, it has been established that the dimensionless dispersion coefficient in the transverse direction in meandering channels is a function of the friction factor, meander amplititude to width ratio and the width to hydraulic radius ratio. An attempt is made to establish the functional relationship by measuring the dispersion coefficients for various values of the above-mentioned parameters. The meandering channels used for this study had realistic bottom configurations which resulted from the scouring and deposition of the sand forming the bottom of the channels in contrast to the rigid bottom channels used in the previous studies. The dispersion coefficients were evaluated using the Generalized Change of Moment Method. Different assumptions for the behaviour of the dispersion coefficient were tested but none exhibited superiority over the others. A numerical method has been developed to predict the concentration distribution in a meandering channel.

## INTRODUCTION

When a pollutant is discharged into a natural stream, it gets transported by the flow velocities: while being transported it also spreads along the vertical, lateral and longitudinal direction simultaneously, due to the turbulence present in the natural stream. The rates of spreading in these directions depend upon the characteristics of the flow and the flow geometry. A quantitative knowledge on these rates of spreading of a pollutant in a natural stream is an essential management tool to decide upon the amount of pollutant that can be discharged into the stream without violating certain established water quality criteria. At present, such a knowledge is limited only to very simple flow geometries, and its application to field problems is often associated with many uncertainties. The object of the research outlined in the present paper. is to expand the knowledge on the spreading rate in the transverse direction in flow geometries more closer to those of the natural streams.

Natural streams usually meander, setting up transverse currents which move towards the inner bank at the bottom and towards the outer bank at the top. "As a consequence, erosion of sediments occur at the outside of the meander, and deposition on the inside. Therefore, the flow cross-sections vary along the length of the stream and the spreading rates in such flows will certainly be different from the simpler cases, wherein the flow geometry is the same throughout the length.

So far, there is no study found in the literature which dealt with such a flow geometry in the laboratory. Y. Chang (1), (1971); H.B. Fisher
(2), (1969); and Engman (3), (1974) studied lateral spreading processes in meandering channels in the laboratory; but the bottom of the channel is flat and hence the transverse currents are not stronger and are not representative of the real situation. Hence, the present research is undertaken to systematically study the lateral spreading processes in meandering channels having realistic bottom configurations. One of the main objectives is to investigate the effect of the meanders on the transverse dispersion coefficient.

## 1. Theoretical Background

a. General

The transport of a neutrally buoyant pollutant can be described by the eeneral diffusion equation (in Cartesian co-ordinate system).

$$
\begin{equation*}
\frac{\partial C}{\partial t}+\sum_{i=1}^{3} U_{i} \frac{\partial C}{\partial x_{i}}=\sum_{i=1}^{3} \frac{\partial}{\partial x_{i}}\left[\varepsilon_{i} \frac{\partial C}{\partial x_{i}}\right] \tag{1}
\end{equation*}
$$

where the dimensionless volunetric concentration $C$ is a function of space $\left(x_{i}, i=1,2,3\right)$ and time $t$. The above equation is derived by considering the conservation of mass principle for the pollutant. (See W.W. Sayre (4). The terms under the sigma sign on the left hand side of Equation (1) represent the convective transport of the pollutant due to the time average velocity components $U_{i}$, whereas, the terms with the square bracket on the right hand side give the transport of the pollutant due to the fluctuating velocity and concentration fields. The transport due to the
fluctuating velocity and concentration fields is assumed to be proportional to the concentration gradient ( $\partial \mathrm{C} / \partial \mathrm{x}_{\mathrm{i}}$ ), and the proportionality constant $\varepsilon_{i}$ is the turbulent diffusion coefficient in the ith direction. In other words, $\varepsilon_{i}$ is given by:

$$
\begin{equation*}
\varepsilon_{i}=\overline{-U_{i}^{\prime} c^{\prime}} / \partial C / \partial x_{i} \tag{2}
\end{equation*}
$$

where $U_{i}^{\prime}$ is the fluctuating component of the velocity in the ith direction and $C^{\prime}$ is the fluctuating part of the concentration $C$.

To describe the flow and concentration distributions in meandering channels, it is more convenient to have a co-ordinate system in which the longitudinal axis follows the meander. Such a co-ordinate system is defined as follows. Referring to Fig. $1, x$ is measured along the centre line of the meandering channel which is assumed to consist of circular bends alternating with the straight reaches. $y$ is measured along the vertical and $z$ is measured perpendicular to both $x$ and $y$, from the centre line. This system of co-ordinates is the same as the one used by Y . Chang (1), (1971). In this system of co-ordinates, Equation (1) becomes:

$$
\begin{aligned}
& \frac{\partial C}{\partial t}+\frac{1}{h_{1}} \frac{\partial}{\partial x}(c u)+\frac{\partial}{\partial y}(c v)+\frac{1}{h_{1}} \frac{\partial}{\partial z}\left(h_{1} c w\right)= \\
& \frac{1}{h_{1}^{2}} \frac{\partial}{\partial x}\left(\varepsilon_{x} \frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_{y} \frac{\partial C}{\partial y}\right)+\frac{1}{h_{1}} \frac{\partial}{\partial z}\left(h_{1} \varepsilon_{z} \frac{\partial C}{\partial z}\right)
\end{aligned}
$$



FIG. 1 CONFIGURATION OF MEANDERING CHANNEL AND THE MEANDERING CO-ORDINATES.
where $h_{l}$ is the metric coefficient given by,

$$
h_{1}= \begin{cases}1+z / r_{c} & \text { for the bend curving to the right }  \tag{4}\\ 1 & \text { for straight reach } \\ 1-z / r_{c} & \text { for the bend curving to the left }\end{cases}
$$

$u, v$ and $w$ are the velocity components in $x, y$ and $z$ directions, respectively, whereas $\varepsilon_{x}, \varepsilon_{y}$, and $\varepsilon_{z}$ are the turbulent diffusion coefficients along the same directions. When Equation (3) is integrated along y from the bottom to the free surface of the flow, the resulting depth average version of Equation (3) is:

$$
h_{i} \frac{\partial h \bar{C}}{\partial t} \frac{\partial}{\partial x}(h \bar{u} \bar{c})+\frac{\partial}{\partial z}\left(h_{1} h \bar{C} \bar{w}\right)=\frac{1}{h_{1}} \frac{\partial}{\partial x}\left(h \bar{\varepsilon}_{x} \frac{\partial \bar{C}}{\partial x}\right)+\frac{\partial}{\partial z}\left(h_{1} h \bar{\varepsilon}_{z} \frac{\partial \bar{C}}{\partial z}\right)
$$

where $\bar{u}, \bar{w}, \bar{c}, \bar{\varepsilon}_{x}$ and $\bar{\varepsilon}_{z}$ are the depth average quantities. The dispersion coefficients $\bar{\varepsilon}_{x}$ and $\bar{\varepsilon}_{z}$ now represent,

$$
\left.\begin{array}{l}
\bar{\varepsilon}_{x}=\left(\overline{u^{\prime \prime} C^{\prime \prime}}-\overline{u^{\prime} c^{\prime}}\right) / \frac{\partial \bar{C}}{\partial x}  \tag{6}\\
\bar{\varepsilon}_{z}=\left(\overline{w^{\prime \prime} C^{\prime}}-\overline{w^{\prime} C^{\prime}}\right) / \frac{\partial \bar{C}}{\partial z}
\end{array}\right\}
$$

where $u^{\prime \prime}, v^{\prime \prime}$ and w"are given by,

$$
\left.\begin{array}{rl}
u & =\vec{u}+u^{\prime \prime} \\
w & =\bar{w}+w^{\prime \prime} \\
\text { and } c & =\vec{c}+c^{\prime \prime}
\end{array}\right\}
$$

* The details of the derivation of Equation (5) from Equation (3) can be found in E.R. Holley (5), (1971).

Note that the terms $\overline{u^{\prime \prime} C^{\prime \prime}}$ and $\overline{w^{\prime \prime} C^{" \prime}}$ are assumed to be proportional to the concentration gradients $\frac{\partial \bar{C}}{\partial x}$ and $\frac{\partial \bar{C}}{\partial z}$, respectively, analogous to the terms $\overline{u^{\prime} C^{\prime}}$ and $\overline{w^{\prime} C^{\prime}}$ (see Equation (2)).

For the case of a continuous injection of the pollutant, $\partial \bar{c} / \partial t$ becomes zero and the diffusion flux ( $\left.\bar{\varepsilon}_{x} \partial \bar{C} / \partial x\right)$ becomes negligible in comparison with the convective flux ( $\bar{u} \overline{\mathrm{C}}$ ) and hence the Equation (5) can be simplified as:

$$
\begin{equation*}
\frac{\partial}{\partial x}(h \bar{u} \bar{c})+\frac{\partial}{\partial z}\left(h_{2} h \bar{w} \bar{c}\right)=\frac{\partial}{\partial z}\left(h_{1} h \bar{\varepsilon}_{z} \frac{\partial \bar{c}}{\partial z}\right) \tag{8}
\end{equation*}
$$

using the depth average flow continuity equation

$$
\begin{equation*}
\frac{\partial}{\partial x}(h \bar{u})+\frac{\partial}{\partial z}(h h \bar{w})=0 \tag{9}
\end{equation*}
$$

Equation (8) can be written as:

$$
h\left[\bar{u} \frac{\partial \bar{C}}{\partial x}+h_{1} \bar{w} \frac{\partial \bar{c}}{\partial z}\right]=\frac{\partial}{\partial z}\left[h_{1} h \bar{\varepsilon} z \frac{\partial \bar{C}}{\partial z}\right]
$$

Equation (10) can be used to calculate concentration distributions provided that the depths, flow velocities and the dispersion coefficient $\bar{\varepsilon}_{z}$ are known. The first two items can be measured relatively easily but measurements of $\bar{\varepsilon}_{z}$ are usually time consuming and expensive. There are no methods for predicting $\bar{\varepsilon}_{z}$ for changing hydraulic conditions and no laboratory measurements of $\bar{\varepsilon}_{z}$ has been made for meandering channels in which there are variations in bed topography. In this paper, measurements of $\bar{\varepsilon}_{z}$ for such channel are reported and the effect of meander characteristics on $\bar{\varepsilon}_{z}$ is investigated. A numerical method to solve equation (10) is also outlined.

To aid in the planning of the experiments and the analysis of data, a dimensional analysis is performed to shed some light on the dependence of $\bar{\varepsilon}_{z}$ on the various hydraulic variables.

If the geometry of the flow boundaries is specified, then the flow structure and thus the $\bar{\varepsilon}_{z}$ can be defined by the following characteristic parameters:

$$
v_{夫}, h, \rho, \mu, k_{s}
$$

when $\rho$ and $\mu$ are the density and the absolute viscosity of the fluid respectively and $k_{s}$ is the height of the equivalent sand roughness of the boundaries.

In the case of meandering channels of various dimensions, the meander length $\lambda$, the meander amplititude $H$ and the channel width $B$ (Fig. 1) also affect $\bar{\varepsilon}_{z}$.

$$
\begin{equation*}
\therefore \bar{\varepsilon}_{z}=f\left(v_{*}, h, \rho, \mu, k_{s}, \lambda, H, B\right) \tag{11}
\end{equation*}
$$

The dimensionless relationship which can be formed is

$$
\begin{equation*}
\frac{\bar{\varepsilon}_{z}}{v_{*}: h}=\phi\left(\frac{\rho v_{x} h}{\mu}, \quad \frac{k_{S}}{h}, \quad \frac{\lambda}{h}, \frac{H}{h}, \quad \frac{B}{H}\right) \tag{12}
\end{equation*}
$$

In the present work, attention is focussed only on rough turbulent flows and for these flows the shear Reynolds number $\rho v_{*} h$ is no longer
length $\lambda$ and the width $B$ are interdependent. Figure 2, taken from Yalin (6) 1972 , presents data taken from various meanders in rivers, flumes, the Gulf Stream and in glacier ice and shows approximately linear relationship between $\lambda$ and $E$. Yalin (6), 1972, drawing the analogy with the wavelength of sandunes, reasoned that the line $\lambda=2 \pi B$ corresponds to rough turbulent flow conditions. Therefore, for the present investigation it is not unreasonable to assume that $\lambda$ and $B$ are interdependent and eliminate $\lambda / h$ from equation (12). The parameter $k_{s} / h$ can also be replaced by the friction factor $f$ when considering rough turbulent flows. Therefore equation (12) can Le simplified to

$$
\begin{equation*}
\frac{\frac{\overline{\varepsilon_{z}}}{v_{*} h}}{}=\psi \quad\left(f, \frac{H}{h}, \frac{B}{h}\right\rangle \tag{13}
\end{equation*}
$$

or $\quad \frac{\bar{\varepsilon}_{z}}{v_{*}}=\Psi \quad\left(f, \frac{H}{B}, \frac{B}{h}\right)$

In the experiments an attempt was made to keep the friction factor relatively constant. The channel width $B$ was kept constant and the meander amplititude and flow depth $h$ were varied to obtain variations in $H / B$ and $B / h$.

## 2. Experimental Set up and Procedure

The meandering configurations chosen for the present experiments are shown in Fig. 3. The width of the channel is kept constant at 30 cm . The centre line of the channels are sine curves which can be approximated by circular arcs and straight reaches as shown in Fig. 3. The amplitude


FIG. 2 Relation between the width (B) and the meander length ( $\lambda$ ) for natural streams.


$$
\frac{H}{B}=2.0
$$


$\frac{H}{B}=3.0$

$\frac{H}{B}=4.0$

$\frac{H}{B}=6.0$

FIG. 3 Various meandering patterns
of meander $H$, varied from 30 cm to 150 cm , whereas the wave length of meander $\lambda$, remained constant at 1.88 m , corresponding to $\lambda=2 \pi \mathrm{~B}$. These channels were built one at a time on a sloping table measuring $15.25 \mathrm{~m} \times$ 2.44 m . The slope of the top surface of the table in the longitudinal direction is $1 \%$ while the same in the transverse direction is $0 \%$. The general view of the whole set up with a meandering channel of amplititude 30 cm is schematically shown in Fig. 4.

Water from the constant head tank of the laboratory flows through an inlet pipe and a diffuser into a head tank containing vertical baffles and smoothly enters into the meandering channel. At the downstream end of the channel, water passes through a tail gate down to a collecting chamber at the end of which there is a $V$ notch weir to measure the flow rate. Water flows through the notch to the sump from where it is pumped to the head tank again. The flow rate is controlled by a flow control valve at the inlet pipe and a rough estimate of the flow rate is made using an annubar flow meter mounted in the inlet pipe.

The vertical sides of the meandering channels are made up of galvanized sheet metal while the bottom of the channel is covered with loose ottawa sand. The sand is initially molded evenly across the channel to a desired longitudinal slope and the equilibrium bed form, corresponding to a particular flow rate, is achieved by letting the flow scour its own bed. A vibratory sand feeder is used to feed the sand at the upstream end to compensate for the amount of sediment transported by the flow to the downstream collecting chamber. With the flow rate kept constant, the equilibrium bed is formed usually after six to seven hours.


FIG. 4 General view of the experimental set up (not to scale)

Equilibrium bed is said to have formed when the configurations of the flow cross sections at corresponding sections of the various meandering cycles are nearly identical. Once the equilibrium bed is formed, water is drained away carefully without disturbing the bed forms and then the bed is solidified using the procedure suggested by M.B. Khalil (7) (1972). According to this procedure, the sand bed is allowed to dry until its water content is about $10 \%$ of the total volume and a resin commercially known as "Aerolite" mixed with equal weight of water is sprayed evenly followed by a coat of dilute solution of formic acid, known as the "Gardener G.P.X.". The resin and the formic acid react chemically and a hard crust is formed on the top surface of the sand bed without altering the roughness characteristics of the sand bed. For full details of this solidifying procedure, reference should be made to the original paper by Khalil (7) (1972).

The presence of the secondary currents due to the bends in the channel causes the erosion of sand on the outside of the bend and deposition on the inside and hence the flow cross-sections exhibit a large variation of flow depth across the channel. The variation of flow depths at certain sections (near the bends) is as high as $700 \%$ and this in turn causes an enhanced secondary current. Such a variation of flow depths in a crosssection is not noticed in the existing works on the dispersion studies in meandering channels.

As indicated in the introduction, experimental flumes of $Y$. Chang (1) (1971), H.B. Fisher (2) (1969) and P. Engman (3) (1974) had flat rigid bottoms and hence, the variation of flow depth across' a section is only due to superelevation, which is only of the order of a few percent and

Chang completely neglected it. Hence, the secondary currents present in these channels are not realistic and hence dispersion processes governed by these secondary currents do not correspond to those in nature. To the authors' knowledge, the present experimental set up is the only one so far to properly reproduce the variation of depth in a laboratory flume designed for dispersion studies.

The bottom profile and the water level elevations at various locations across the flume at a particular section were measured using a point gauge which facilitated the determination of the flow cross-sectional area $A$, wetted perimeter $P$, the hydraulic radius $R$ and the flow depth $h$ as a function of $z$ at that section. The longitudinal velocity component was measured by aligning a pitot tube tip parallel to the x-axis at a particular section and at various locations along $y$ - and $z$-axes, respectively, which allowed the determination of the depth average longitudinal velocity component $\bar{u}$ as a function $z$. Knowing $h$ and $\bar{u}$, the depth average transverse velocity component $\bar{w}$, is computed using the finite difference form of Equation (9). The hydraulic conditions of the flows tested are summarized in Table 1.

For the concentration measurements, salt solution was used as the tracer. It was mixed with methonal to make it neutrally buoyant. The concentration of the tracer solution is $62.5 \mathrm{gm} / 1 \mathrm{itre}$. It was injected continuously at a point in the flow with the same velocity as the flow in the channel.

The concentration of the salt-methonal mixture was measured using a single. electrode conductivity probe of the type used by R.S. McQuivey $\varepsilon$

| Run No. | H/B | Average Hydraulic Radius $R$ in cm | Average flow velocity $U$ in $\mathrm{cm} / \mathrm{s}$ | Average shear velocity $v_{*}$ in $\mathrm{cm} / \mathrm{s}$ | $B / R$ | $f=\frac{8 v_{t}^{2}}{u^{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.0 | 2.85 | 26.3 | 3.75 | 10.5 | 0.162 |
| 2 | 2.0 | 1.89 | 26.8 | 3.07 | 15.9 | 0.105 |
| 3 | 2.0 | 3.95 | 31.1 | 4.44 | 7.6 | 0.163 |
| 4 | 2.0 | 2.94 | 30.1 | 4.85 | 10.2 | 0.208 |
| 5 | 3.0 | 3.34 | 27.8 | 5.12 | 9.0 | 0.271 |
| 6 | 3.0 | 2.59 | 22.5 | 3.14 | 11.6 | 0.156 |
| 7 | 5.0 | 3.01 | 32.1 | 3.51 | 10.0 | 0.101 |

T.N. Keefer (8) (1972). This probe operates on the principles that when a large and a small electrode are immersed in an electrolyte solution, the resistance between the two will be governed by the volume elements adjacent to the small electrode. This theory was proposed by C.H. Gibson $\varepsilon$ W.H. Schwarz (9) (1963). The construction details, the bridge circuit to be used in conjunction with this probe, the calibration and the operational details are explained in a laboratory report by F. Dunnett (10) (1975). Concentration measurements were made at the same locations where the longitudinal velocity component was measured and an average value over the depth was computed.

The dispersion coefficient $\bar{\varepsilon}_{z}$ is evaluated using the "generalized change of moment" method proposed by the Holley (5), 1971. Equation (8) is multiplied by $z^{2}$ and is integrated across the width of the channel. i.e.

$$
\int_{-B / 2}^{+B / 2} \frac{\partial}{\partial x}(h \bar{u} \bar{c}) z^{2} d z+\int_{-B / 2}^{+B / 2} \frac{\partial}{\partial z}\left(h_{1} h \bar{w} \bar{C}\right) z^{2} d z
$$

$$
=\int_{-B / 2}^{+B / 2} \frac{\partial}{\partial z}\left(h_{2} h \bar{\varepsilon}_{z} \frac{\partial \bar{C}}{\partial z}\right) z^{2} d z
$$

Interchanging the order of integration and differentiation using Liebnitz Rule, carrying out some of the integration, and dividing throughout by the total flux of the tracer $\int_{-B / 2}^{B / 2} h \bar{u} \bar{C} d z$, we get:

$$
\begin{aligned}
& \frac{d}{d x}\left[\begin{array}{cccc|}
\int_{-B / 2}^{+B / 2} & h \bar{u} \bar{c} z^{2} d z \\
\int_{-B / 2}^{+B / 2} & h \bar{u} \bar{c} & d z
\end{array}\right]-2 \int_{-B / 2}^{+B / 2} h_{1} h \bar{w} \bar{c} z d z \\
& +B / 2 \\
& \int \cdot h_{2} h \bar{\varepsilon}_{z} \frac{\partial \bar{C}}{\partial z} z d z \\
& -2-B / 2 \\
& +B / 2 \\
& \int_{-B / 2} h \bar{u} \bar{c} d z
\end{aligned}
$$

- In order to evaluate $\bar{\varepsilon}_{z}$ from Equation (16), its dependence on $z$ has to be known. For example, if $\bar{\varepsilon}_{z}$ is independent of $z$, then it can come out of the integral sign in Equation (16) and can be computed by knowing $h, \bar{u}$, $\bar{C}$, and $\bar{w}$. Since the dispersion coefficient can be normalized with a velocity and a length scale, the following relations are possible for the dispersion coefficient $\bar{E}_{z}$ :

$$
\begin{gather*}
\left.\left.\bar{\varepsilon}_{z} \quad=\quad \begin{array}{ccc}
\alpha_{2} & h & v_{\star} \\
\alpha_{2} & h & \bar{u} \\
\alpha_{3} & H & v_{\star} \\
\alpha_{4} & H & \bar{u}
\end{array}\right\}, ~\right\} \quad, ~ \tag{17}
\end{gather*}
$$

Each one of the relationships listed in equation (17) assumes a different behaviour for $\bar{\varepsilon}_{z}$. For example the first relation states that the transverse variation of $\bar{\varepsilon}_{z}$ is the same as that of the local depth $h$, whereas the third relation assumes that $\bar{\varepsilon}_{z}$ is constant across the width of the channel.

Choosing, for instance, the first relation of (17): (i.e. $\bar{\varepsilon}_{z}=\alpha_{1} h v_{i=}$ ), the Equation (16) can be expressed as:

$$
\frac{d}{d x} \sigma^{2}(x)-g(x)=2 \alpha_{1} f_{1}(x)
$$

where

$$
\begin{equation*}
g(x)=2 \frac{\int_{-B / 2}^{+B / 2} h_{1} h \bar{w} \bar{c} z d z}{\int_{-B / 2}^{+B / 2} h \bar{u} \bar{c} d z} \tag{19b}
\end{equation*}
$$

$$
\begin{equation*}
\sigma^{2}=\frac{\int_{-B / 2}^{+B / 2} h \bar{u} \bar{c} z^{2} d z}{\int_{-B / 2}^{+B / 2} h \bar{u} \vec{c} d z} \tag{18}
\end{equation*}
$$

and $\quad f_{1}(x)=-\frac{\int_{-B / 2}^{+B / 2} h_{1} h^{2} v_{2} \frac{\partial \overrightarrow{\mathrm{C}}}{\partial z} \cdot z \mathrm{dz}}{\int_{-B / 2}^{+B / 2} h_{-18-} \bar{u} \overline{\mathrm{c}} \mathrm{dz}}$

Therefore, knowing $\sigma^{2}(x), g(x)$ and $f_{1}(x)$, the dimensionless dispersion coefficient $\alpha_{1}$ can be evaluated from Equation (18). The term $\sigma^{2}$ is the variance of the distribution of the flux of the pollutant and it represents the total spread due to both the diffusion and the transverse velocity. The term $g(x)$ accounts for the spreading due to the transverse velocity and hence the difference $\left(d \sigma^{2} / d x-g(x)\right)$ represents the effects of just dispersion.

The application of Equation (18) to evaluate $\alpha_{1}$ requires the evaluation of the derivative of $\sigma^{2}$ with respect to $x$, and if the variation of $\sigma^{2}$ with $x$ is not linear, which is perfectly possible due to the presence of the transverse velocity $w$, it is difficult to get an accurate evaluation of ( $d \sigma^{2} / d x$ ). This difficulty can be overcome by considering the integral version of Equation (18), namely,

$$
\begin{equation*}
\sigma^{2}(x)-G(x)+A_{2}=2 \alpha_{1} F_{1}(x) \tag{20}
\end{equation*}
$$

where

$$
\begin{align*}
G(x) & =\int_{x_{0}}^{x} g(x) d x  \tag{21a}\\
F_{2}(x) & =\int_{x_{0}}^{x} f_{1}(x) d x \tag{21b}
\end{align*}
$$

and $A_{1}$ is the constant of integration. $x_{0}$ in Equation (21) is the location of the injection point.

Therefore, by plotting a graph between $\left\{\sigma^{2}(x)-G(x)\right\}$ vs $F(x)$ and measuring the slope of the line the dimensionless dispersion coefficient $\alpha_{1}$
can be obtained. It should be noted that if the variation of the dispersion coefficient is not correctly represented, then the plot between $\left\{\sigma^{2}(x)-G(x)\right\}$ and $F_{1}(x)$ will not be a straight line. But it is not certain whether the converse is necessarily true.

The values of $\sigma^{2}(x)$ and $G(x)$ as given by Equations (19a) and (21a), respectively, remain the same for all the assumptions regarding the variation of $\bar{\varepsilon}$ with $z$. The values of $F_{i}(x)$, on the other hand, depend on the assumption used to describe $\bar{\varepsilon}_{z}$. Equation (19c) gives the value of $f_{i}(x)$ when $\bar{\varepsilon}_{z}$ is expressed by the first of the four possibilities expressed by Equation (17) and Equation (20) then gives the value of the dimensionless dispersion coefficient $\alpha_{1}$. The values of $f_{i}(x)$ for the remaining expressions of $\bar{\varepsilon}_{z}$ become:

$$
\begin{align*}
& f_{2}(x)=-\frac{\int_{-B / 2}^{+B / 2} h_{1} h^{2} \bar{u} \frac{\partial \bar{C}}{\partial z} z d z}{\int_{-B / 2}^{+B / 2} h \bar{u} \overline{\bar{C}} d z}  \tag{22}\\
& f_{3}(x) \int_{-B / 2}^{+B / 2} h_{1} h H v_{\div} \frac{\partial \bar{C} z}{\partial z} d z  \tag{23}\\
&+B / 2 \\
& \int_{-B / 2} h \bar{u} \bar{C} d z
\end{align*}
$$

$$
\begin{equation*}
f_{4}(x)=\frac{\int_{-B / 2}^{+B / 2} h_{1} h H \bar{u} \frac{\partial \bar{C}}{\partial z} z d z}{\int_{-B / 2}^{+B / 2} h \bar{u} \vec{c} d z} \tag{24}
\end{equation*}
$$

and the corresponding dimensionless dispersion coefficients ( $\alpha_{i}, i=2,3,4$ ) are evaluated using Equation (20) with appropriate values of $\mathrm{F}_{\mathbf{i}}(\mathrm{x})$. A computer programme was written to evaluate $\sigma^{2}(x), G(x)$ and $F_{i}(x)$ using the trapezoidal rule for the integrations. The values of $\sigma^{2}(x), G(x)$ and $F_{i}(x)$ for all the runs are tabulated in an Appendix.

## 3. Discussion of Results

a. Flow cross-sections:

Figs. 5a to lla depict the flow cross-sections at various locations along the channel for all the runs. It can be seen from these figures that the flow cross-sections vary along the length of the channel in a cyclic manner. As indicated earlier, the channel is deeper in the outside of the bends and shallower inside, and hence the deeper and shallower portions interchange gradually. Because of this gradual interchange of the deeper and shallower portions, there are sections along the channel where the bottom is almost flat. (See Sections 1 and 9 in Fig. 5a). It can also be noted from the cross-sections for various values of $H / B$, the meander amplititude to channel width ratio, that as $H / B$ increases, the deeper portions are confined to sections close to the curved portions of the channel only and the length of the channel where the bottom is more or less flat becomes considerably larger compared to the channels with lower H/B ratios.


FIG. 6 Measured depth (5a) velocity (5b) and concentration (5c) distributions


Fig. 6 MEASURED DEPTH (6a), VELOCITY (6) AND CONCENTRATION (6:) DISTRIBUTIONS FOR RUN NO. 2


Fig. 7 MEASURED DEPTH (7a), VELOCITY (7b) AND CONCENTRATION :7c) DISTRIBUTIONS FOR RUN NO. 3


* Predicted concentrations
$X$ injection point

FIG. 8 MEASURED DEPTH (8a), VELOCITY (8b) AND CONCENTRATION (8c) DISTRIBUTIONS FOR RUN NO. 4


- Predicted concentrations
$X$ injection point
fig. 9 measured depth (9a). velocity (9b) and concentration (Uo) distributions for run no. 5

- Predicted concentrations

Xinjection point
FIG.16 MEASURED DEPTH ( 100 ), VELOCITY ( 100 ) AND CONCENTRATION (HOC)
DISTRIBUTIONS FOR RUN NO
DISTRIBUTIONS FOR RUN NO. 6


Fig. 11 measureo depth (11a), velocity fifib) and concentration fila

## b. Velocity Distribution:

The depth average velocity vectors at various locations along the channel for all the runs are shown in Figs. 5b to llb. The depth average transverse velocity components were determined from the depth average continuity equation (9) using the measured values of the longitudinal velocity components and the depths. It can be seen from these figures that the transverse velocity components are larger in sections near the bends, and hence, in channels with higher values of $H / B$, there are larger variations in the convective transport due to the transverse velocity components between different sections.

It should be pointed out here that the measurements were carried out in stations, four or five meandering cycles downstream, thereby providing enough distance for the flow to stabilize. It was noticed that the velocity and cross-sectional distributions were similar in sections seperated by one meandering wavelength, when the flow is fully stabilized.

## c. Concentration distributions:

The measured concentration distributions are shown by the solid lines in Fig. 5c to llc for all the runs. The location of the injection of the tracer solution is shown by a cross $X$ in these figures. The tip of the injection unit is aligned with the direction of the flow at the injection point in order to minimize the initial mixing. It can be seen from these figures that the concentration distributions become very nearly uniform within one meandering cycle.

The large convective transport due to the transverse velocity components in the bends is responsible for such rapid mixing in meandering channels. Therefore the variation of the cross-sectional shapes along the length of the stream which gives rise to the transverse velocity components is a major governing factor for the mixing of the pollutants in natural streams.

## d. The dispersion coefficients

Figs. 12 to 18 give the graphs between $\left[\sigma^{2}(x)-G(x)\right]$ and $F_{i}(x)$ for all runs. The slopes of the straight lines drawn through the experimental points are equal to twice the dimensionless dispersion coefficients $\alpha_{i}$. For each run, there are four lines, each resulting from a particular assumption for the variation of the dispersion coefficient with $z$. It has been mentioned earlier that if the variation of $\bar{\varepsilon}_{z}$ with $z$ is correctly represented, then the graph between $\left[\sigma^{2}(x)-G(x)\right]$ and $F_{i}(x)$ should be a straight line, and this fact can be used as an indicator for establishing the superiority of one assumption over the other. By looking at Figs. 12 to 18 , it can be seen that the scatter of the experimental points from the straight lines drawn through the points is of the same order of magnitude for all the graphs and hence, it is not possible to conclude from these figures which expression $i s$ the best representation for $\bar{\varepsilon}_{z}$.

One of the main objects of the present work is to investigate the dependence of the dispersion coefficient on the flow characteristics and flow geometry. Referring to equation (14), the dimensionless dispersion coefficient $\left(\bar{\varepsilon}_{z} / \nu_{\neq} h\right)$ is expressed as a function of the friction factor $f$, the amplitude to width ratio $H / B$ and the width to depth ratio $B / H$.


Fig. 12 Plots between $\left[\sigma^{2}(x)-G(x)\right]$ and $F i(x)$ for Run No. 1


Fig. 13 Plots between $\left[\sigma^{2}(x)-\mathrm{G}(x)\right]$ and $\mathrm{Fi}(x)$ for Run No. 2

$$
-32-
$$



Fig. 44 Plots between $\left[\sigma^{2}(x)-\mathrm{G}(x)\right]$ and $\mathrm{Fi}(x)$ for Run No. 3


Fig. 15 Plots between $\left[\sigma^{2}(b)-G(x)\right]$ and $F i(x)$ for Run No. 4


Fig. 16 Plots between $\left[\sigma^{2}(x)-G(x)\right]$ and $F i(x)$ for Run No. 5


Fig. 17 Plots between $\left[\sigma^{2}(x)-G(x)\right]$ and $\mathrm{Fi}(x)$ for Run No. 6


Fig. 18 Plots between $\left[\sigma^{2}(x)-\mathrm{G}(x)\right]$ and $\mathrm{Fi}(x)$ for Run No. 7

Considering the different assumptions which are made for $\bar{\varepsilon}_{z}$ in equation (17) a general form of the dimensionless relationship can be written as

$$
\begin{equation*}
\alpha_{i}=\psi_{i}\left(f, \frac{H}{B}, \frac{B}{h}\right) \tag{25}
\end{equation*}
$$

A summary of the values of $\alpha_{1}$ to $\alpha_{4}$ which were obtained in the seven experimental runs are listed in Table 2, together with the other dimensionless parameters in equation (25). The values of $B / h$ are not listed because the depth varies both across the channel and along the channel and it is not easy to define one representative $h$. Instead, an average hydraulic radius, $R$, was calculated by considering all the sections within one meandering cycle and the parameter $B / R$ is listed. It should also be noted that since the cross sectional area also varies in the downstream direction, the average velocity $\bar{u}$ also varies from section to section. For the calculation of the average friction factor $f$, the friction factors of all the sections within one meandering cycles were considered.

In Figs. 19 to 22 , the values of $\alpha_{i}$ are plotted against H/B. $\alpha_{1}$ appears to increase slightly with $H / B$, whereas, $\alpha_{3}$ and $\alpha_{4}$ are decreasing as $H / B$ increases. Values for $\dot{\alpha}_{2}$ exhibit rather large scatter and it is difficult to infer how it should vary with H/B. The absolute values of $\bar{\varepsilon}_{z}$ in all cases increases with increasing values of $H$, even though the rate of increase decreases as $H$ becomes larger and larger. No attempt has been made to establish the effect of $f$ and $B / R$ on $\alpha_{i}$ 's since there are not enough experimental points. $B / R$ values for the experimental data vary from 7.6 to 15.9 , whereas the $f$ values range from

TABLE 2
SUMMARY OF MEASURED DIMENSIONLESS DISPERSION COEFFICIENTS

| Run No. | H/B | $B / R$ | f | $\alpha_{1}=\frac{\bar{\varepsilon}_{z}}{\bar{h}^{v_{*}}}$ | $\alpha_{2}=\frac{\bar{\varepsilon}_{z}}{\overline{h \bar{u}}}$ | $\alpha_{3}=\frac{\bar{\varepsilon}_{z}}{\bar{H} v_{i}}$ | $\alpha_{4}=\frac{\bar{\varepsilon}_{z}}{H \bar{u}}$ | $\frac{\bar{\varepsilon}_{z}}{\overline{\nu_{z}}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1.0 | 10.5 | 0.162 | 0.075 | 0.020 | 0.0211 | 0.0035 | 0.222 |
| 2 | 2.0 | 15.9 | 0.105 | 0.075 | 0.028 | 0.0131 | 0.0018 | 0.416 |
| 3 | 2.0 | 7.6 | 0.163 | 0.100 | 0.025 | 0.0140 | 0.0035 | 0.213 |
| 4 | 2.0 | 10.2 | 0.208 | 0.163 | 0.055 | 0.0186 | 0.0060 | 0.379 |
| 5 | 3.0 | 9.0 | 0.271 | 0.225 | 0.056 | 0.0125 | 0.0032 | 0.338 |
| 6 | 3.0 | 11.6 | 0.156 | 0.200 | 0.045 | 0.0093 | 0.0010 | 0.324 |
| 7 | 5.0 | 10.0 | 0.101 | 0.131 | 0.028 | 0.0062 | 0.0008 | 0.310 |



Fig. 19 Variation of the dimensionless dispersion coefficient $\alpha_{1}$ with the amplitude to width ratio $H / B$


Fig. 20 Variation of $\alpha_{2}$ with $\mathrm{H} / \mathrm{B}$


Fig. 21 Variation of $\alpha_{3}$ with H/B


Fig. 22 Variation of $\alpha_{4}$ with $\mathrm{H} / \mathrm{B}$
0.101 to 0.271 . Even though both $f$ and $B / R$ have influence over the mixing processes, it was felt that the effects of $H / B$ would be considerably larger than those of $f$ and $B / R$. Indeed, the variation of $H / B$ alters the distribution of the transverse velocity components in the vertical, which alters the differential convection and consequently, the dispersion coefficient.

Referring to Figs. 19 to $22, \alpha_{3}$ exhibits the least amount of scatter of the experimental points. It should be noted that $\alpha_{3}$ results from the assumption that the transverse dispersion coefficient $\bar{\varepsilon}_{z}$ is constant across the width of the channel. Since it is difficult to establish the variations of $\bar{\varepsilon}_{z}$ with $z$ from the Figs. 12 to 18 , and since $\alpha_{3}$ yields a better corelation with $H / B$ than the other dimensionless dispersion coefficients, it can be concluded that for all practical purposes, the transverse dispersion coefficient $\bar{\varepsilon}_{z}$ can be assumed to be constant across the width of the meandering channel. At this point it is interesting to refer to the work of F.M. Holly Jr. and D.B. Simons (11), 1975, who measured dispersion in trapezoidal channels, and obtained values of $\bar{\varepsilon}_{z}$ by simulation. They also found best agreement between simulated and measured concentrations when $\bar{\varepsilon}_{z}$ was assumed to be constant across the channel.

In most of the existing studies on the measurement of the transverse dispersion coefficient $\bar{\varepsilon}_{z}$, it is customary to lump the convective transport due to the time average transverse velocity in together with the dispersive transport and calculate an overall dispersion coefficient.

An inspection of the values of $G(x)$ in tables in the Appendix $A$ which represent the cumulative effect of transverse velocity $\bar{w}$ along the length of the channel, indicates that both $\sigma^{2}(x)$ and $G(x)$ are of the same order of magnitude. This implies that the magnitude of the convective transport is of the same order of magnitude as the dispersive transport and is one reason why field measurements all reported much larger values of transverse dispersion than flume values. Lumping the convective transport and dispersive transport together has the following disadvantages. First of all, by lumping the convective transport with the dispersion transport and calculating the dispersion coefficient, it is implicitly assumed that the convective transport can also be given by a gradient type expression similar to Equation (6). At present, it is not possible to say whether it is so. Secondly, the convective transport changes its direction along the length of the channel, sometimes acting in the direction of the dispersion transport and sometimes in the opposite direction. Inclusion of convective transport with the dispersive transport, therefore, would result in a larger scatter in figures 12 to 18 , thereby making it difficult to evaluate a reliable value for the dispersion coefficient. In order to overcome these disadvantages, in the present research, the effects of transverse velocity is subtracted from the total spread and hence the dispersion coefficient accounts for only the turbulent diffusion and the differential convection, which at the present state of knowledge we are not able to predict.

The existing values for the transverse dispersion coefficients are usually expressed in terms of the shear velocity and the mean depth or the hydraulic radius. Hence, in order to compare the results of the present study with the existing data, the transverse dispersion coefficient
$\bar{\varepsilon}_{z}$ was made dimensionless using the average shear velocity $\nu_{\%}$ and the average hydraulic radius $R$ and are shown in the last column of Table 2.

The values for $\bar{E}_{z} / \nu_{*} R$ vary between 0.213 and 0.416 and are larger than published values for straight open channels which generally vary between 0.1 and 0.2 . This is not surprising since the differential convection in straight channels is due to secondary circulation resulting from transverse variations in shear stress while in curved channels there exists in addition secondary currents caused by variations in centrifugal forces. However, the present values of $\bar{\varepsilon}_{z} / v_{*} R$ are considerably smaller than those given by Chang (1) who obtained $\bar{\varepsilon}_{z}$ by comparisons of numerical simulation and laboratory measurement of concentration distribution in meandering channels. Chang (1) allowed $\bar{\varepsilon}_{z}$ to vary with distance along the channel but the average values of $\bar{\varepsilon}_{z} / \nu_{*} R$ were between 0.62 and 1.23 . In the next section it will be seen that simulation of the present observed concentration distributions using values from Table 2 were also quite successful. Therefore it is suspected that the big difference in the mixing coefficients may be caused by the fact that Chang's measurements were made in channels with rectangular cross sections and flat bottoms whereas the present experiments were performed in channels with large transverse variation in depth as well as variations of cross section in the downstream direction.

## 4. Numerical Prediction of Concentration Distribution

Knowing the values of the $\bar{\varepsilon} \bar{z}_{z}$ as a function of amplitude of meander to width ratio $H / B$ and using the measured values of $\bar{u}$ and $h$ it is possible to predict the concentration distribution of a pollutant in a meandering channel by solving Equation (10). However, it is not possible to obtain an analytical solution and therefore a numerical procedure is outlined below.

Expanding the RHS and rearranging terms, Equation (10) can be written as:

$$
\frac{\partial \bar{C}}{\partial x}+v \frac{\partial \bar{C}}{\partial z}=D \frac{\partial^{2} \bar{C}}{\partial z^{2}}
$$

where $V$ and $D$ are given by:

$$
\begin{aligned}
& V=h_{1} \frac{\bar{w}}{\bar{u}}-\frac{1}{\bar{u} h} \frac{\partial}{\partial z}\left(h_{1} h \bar{\varepsilon}_{z}\right) \\
& D=\frac{h_{1}}{\bar{u}} \bar{\varepsilon}_{z}
\end{aligned}
$$

Following the discretization procedure recommended by H.L. Stone and P.L.T. Brian (12) (1963), Equation (10) can be expressed in finite difference form as:

$$
\begin{aligned}
& \frac{1}{\Delta x}\left[g\left(\bar{c}_{i+1, j}-\bar{c}_{i, j}\right)+\frac{\theta}{2}\left(\bar{c}_{i+1, j-1}-\bar{c}_{i, j-1}\right)+m\left(\bar{c}_{i+1, j+1}-\dot{\bar{c}}_{i, j+1}\right)\right] \\
& +\frac{\dot{v}_{i, j}}{\Delta z}\left[a\left(\bar{c}_{i, j+1}-\bar{c}_{i, j}\right)+\frac{e}{2}\left(\bar{c}_{i, j}-\bar{c}_{i, j-1}\right)+\dot{c}\left(\bar{c}_{i+1, j+1}-\bar{c}_{i+1, j}\right)\right. \\
& +d\left(\bar{c}_{i+1, j}-\bar{c}_{i+1, j-1}\right)=\frac{D_{i, j}}{2(\Delta z)^{2}}\left[\bar{c}_{i+1, j+1}-2 \bar{c}_{i+1, j}+\bar{c}_{i+1, j-1}\right. \\
& \left.+\vec{c}_{i, j+1}-2 \bar{c}_{i, j}+\bar{c}_{i, j-1}\right]
\end{aligned}
$$

where the weighting coefficients $g, \theta, m, a, e, c$, and $d$ satisfy:

$$
\left.\begin{array}{l}
9+\frac{\theta}{2}+m \quad 1  \tag{29}\\
a+\frac{e}{2}+c+d=1
\end{array}\right\}
$$

For an optimum numerical solution without numerical oscillations and dispersion, Stone and Brian recommend the following values for $\theta$ and $e$, with $c=e / 2, a=d$ and $m=\theta / 2$.

$$
\left.\begin{array}{l}
\theta=\frac{1}{3}  \tag{30}\\
e=\frac{1}{2}
\end{array}\right\}
$$

Therefore, the numerical values of the weighting coefficients become:

$$
\left.\begin{array}{lc}
g=\frac{2}{3} ; & \frac{\theta}{2}=m=\frac{1}{6}  \tag{31}\\
a=c=d=\frac{1}{4} ; & e=\frac{1}{2}
\end{array}\right\}
$$

Knowing the concentration distribution at a station, say $i$, and using the no flux boundary conditions across the side walls, the expanded version of Equation (28) can be solved for the concentration distributions at subsequent stations $i+1, i+2$, etc., by solving the following matrix equations by Gauss' elimination method.

where

$$
\begin{aligned}
q_{j}= & \frac{g}{\Delta x}+D_{i, j} \frac{1}{\Delta z^{2}}+v_{i, j} \frac{1}{\Delta z}(d-c) \\
p_{j}= & -\frac{\theta}{2} \frac{1}{\Delta x}+D_{i, j} \frac{1}{2 \Delta z^{2}}+v_{i, j} \frac{1}{\Delta z} \cdot d \quad(j=3, m) \\
r_{j}= & -\frac{m}{\Delta x}+D_{i, j} \frac{1}{2 \Delta z^{2}}-v_{i, j} \frac{1}{z} \cdot c \\
s_{j}= & \bar{c}_{i, j-1}\left(p_{j}+\frac{\theta}{\Delta x}\right)+\bar{c}_{i, j}\left[q_{j}-\frac{2.0}{\Delta z^{2}} D_{i, j}+\frac{v_{i, j}}{\Delta z}(d-c)\right] \\
& +\bar{c}_{i, j+1}\left(v_{j}+2.0 \frac{m}{\Delta x}\right)
\end{aligned}
$$

The coefficients $V_{i j}$ and $D_{i j}$ (given by Equation (27). can be evaluated using the measured values of $\bar{u}, h$ and $\bar{\varepsilon}_{z}$ and the computed values of $\bar{w}_{1}$.

The listing of a computer programme to predict the concentration distributions of a tracer injected continuously in a meandering channel. using the measured values of $\bar{u}, h$ and $\bar{\varepsilon}_{z}$ is given in Krishnappan and La (13).

Using the above numerical scheme and the measured values of $\alpha_{3}$ the concentration distributions were predicted for all the runs and these prodiction are shown in Figures 5 c to 12 c . Note that a favourable agreement exists between the measurement and the prediction. In order to check the effect of the variable dispersion coefficient on the numerical scheme, the concentration distributions were also predicted using the dimensionless dispersion coefficient $\alpha_{2}$. The results indicated that there is no
significant difference between these predicted concentrations (not shown). This is only reasonable, because the dimensionless dispersion coefficients are evaluated using Equation (10) and the variances calculated from measured concentration distributions and hence the substitution of these coefficients back into the equation should yield concentration distributions not too different from each other.

Holly (11) also performed numerical simulations using experimentally derived dispersion coefficients and compared the simulated distributions with measured ones which were obtained in a straight channel of triangular cross section. He found that for the cases where the tracer was injected at the centre of the channel, different assumptions for $\alpha$ produced about the same result but for the cases where injections were made at the sides, only the assumption of constant $\bar{\varepsilon}_{z}$, i.e. $\alpha_{3}$, gave reson $\mathfrak{b}$ le comparison with measured data.

## 5. Conclusions

From the present experimental programme on the transverse dispersion in meandering channels, the following conclusions can be drawn.

1. The variation of the flow depth across the width of a meandering channel with movable bed is large and plays an important role in setting up transverse currents, thereby affecting the dispersion processes.
2. The convective transport in the transverse direction is of the same order of magnitude as the transport due to dispersion.
3. The transverse dispersion coefficient $\bar{\varepsilon}_{z}$ can be treated as being constant and independent of $z$.
4. The variation of the dimensionless dispersion coefficient $\alpha_{3}$ can be roughly represented by the curve shown in Fig. 21, although slight variations with $f$ values can be expected.
5. Prediction of concentration distribution of a pollutant in meandering channels can be achieved using the numerical method described in the report.

## LIST OF REFERENCES

Y. Chang, 'Lateral mixing in meandering channels', Ph.D. Thesis, Dept. of Mechanics and Hydraulics, University of lowa, May, 1971.
H.B. Fisher, "The effect of Bends on Dispersion in Streams", Water Resources Research, Vol. 5, No. 2, April, 1969.
E.0. Engmann, "Transverse mixing characteristics of open and icecovered channel flows", Ph.D. Thesis, Dept. of Civil Engineering, University of Alberta at Edmonton, Alberta, 1974.
W.W. Sayre, 'Natural Mixing Processes in Rivers', Chapter 6, Environmental Impact on Rivers, published by H.W. Shen, Colorado, 1973.
E.R. Holley, "Transverse Mixing in Rivers", Part 1, Delft Hydraulics Laboratory Report No. S132, 1971.
M.S. Yalin, "Mechanics of Sediment Transport", Program Press, 1972.
M.B. Khalil, "On preserving the sand patterns in river models", Journal of Hydraulic Research, Vol. 10, No. 3, 1972.
R.S. McQuivey $\varepsilon$ T.N. Keefer, 'Measurement of ${ }^{\text {velocity - concentration }}$ co-variance", Journal of the Hydraulics Division, ASCE, Vol. 98, No. HY9, 1972.
C.H. Gibson $\varepsilon$ W.H. Schwarz, 'Detection of Conductivity Fluctuations in a Turbulent Flow Field', Journal of Fluid Mechanics, Vol. 16, 1963, pp. 357-364.
F. Dunnett, "Concentration Measurement System for Diffusion Experiments in Laboratory Flumes', Unpublished Report, Hydraulics Research Division, CCIW, 1975.
F.M. Holly, Jr. \& D.B. Simons, "Transverse mixing of neutrally buoyant tracers in non rectangular channels", Proc. XVI Congress of IAHR, Sao Paulo, Brazil, 1975.
H.L. Stone $\varepsilon$ P.L.T. Brian, 'Numerical Solution of Convective Transport Problems', A.l. Ch. E. Journal, Vol. 9, pp. 681-688, 1963.
B.G. Krishnappan $\varepsilon$ Y.L. Lau, 'Transverse dispersion in meandering channels'", IWD publication, CCIW, Burlington (in press).
$z$ : co-ordinate along the transverse direction perpendicular to both $x$ and $y$
$h_{1} \quad$ : metric coefficient
h : flow depth

R : average hydraulic radius over one meandering cycle
$\lambda$. : meander wavelength

H : meander amplitude

B : channel width
$r_{c}$ : radius of curvature of circular arc
$\rho \quad: \quad$ density of fluid
$\mu \quad: \quad$ absolute viscosity of fluid

$\nu_{\perp} \quad: \quad$ average shear velocity over one meandering cycle

C : volumetric concentration of tracer
$\varepsilon_{i} \quad: \quad$ diffusion coefficient in the ith direction

$$
\begin{aligned}
& \alpha_{1}, \alpha_{2}, \alpha_{3} \text { and } \alpha_{4} \\
& \quad: \quad \text { dimensionless dispersion coefficients in the } z \text {-direction }
\end{aligned}
$$

## APPENDIXA

Tabulated values of $\sigma^{2}(x), G(x)$ and $F_{i}(x)$ for all the runs.

| SECTION | $\sigma^{2}(x)$ | $G(x)$ | $F_{1}(x)$ | $F_{2}(x)$ | $F_{3}(x)$ | $F_{4}(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 12.82 |  |  |  |  |  |
| 3 | 24.63 | -10.78 | 16.9 | .73 .5 | 79.9 | 447.9 |
| 4 | 47.38 | -16.81 | 33.5 | 145.7 | 148.4 | 855.2 |
| 5 | 57.18 | -5.76 | 49.1 | 222.6 | 216.3 | 1316.0 |
| 6 | 55.98 | -5.51 | 64.7 | 279.9 | 294.2 | 1798.0 |
| 7 | 47.73 | -13.64 | 74.8 | 336.5 | 381.3 | 2203.0 |
| 8 | 67.43 | -3.56 | .85 .8 | 392.9 | 473.6 | 2591.2 |


| SECTION | $\sigma^{2}(x)$ | $G(x)$ | $F_{1}(x)$ | $F_{2}(x)$ | $F_{3}(x)$ | $F_{4}(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 54.93 |  |  |  |  |  |
| 4 | 54.36 | 2.85 | 7.8 | 52.1 | 122.9 | 413.4 |
| 5 | 63.41 | 10.84 | 14.1 | 90.5 | 256.0 | 824.1 |
| 6 | 57.89 | -3.35 | 20.0 | 126.0 | 375.0 | 1211.5 |
| 7 | 17.51 | -22.80 | 26.4 | 162.9 | 467.8 | 1452.5 |
| 8 | 51.77 | -10.60 | 34.5 | 205.3 | 583.1 | 1696.4 |
| 9 | 66.07 | 2.62 | 43.4 | 248.3 | 731.1 | 2061.3 |
| 10 | 59.71 |  |  |  |  |  |



RUN NO: 4


RUN NO: 5

| SECTION | $\sigma^{2}(\mathrm{x})$ | $G(x)$ | $F_{1}(\mathrm{x})$ | $F_{2}(x)$ | $F_{3}(x)$ | $\mathrm{F}_{4}(\mathrm{x})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 15.87 |  |  |  |  |  |
| 6 | 15.34 | - 0.47 | 22.1 | 77.9 | 334.6 | 516.3 |
| 7 | 11.25 | - 7.58 | 45.8 | 140.1 | 596.6 | 781.9 |
| 8 | 22.43 | - 8.72 | 62.9 | 202.1 | 798.8 | 1085.0 |
| 9 | 34.72 | 0.68 | 81.9 | 276.1 | 1121.3 | 1535.3 |
| 10 | 47.86 | 3.98 | 100.5 | 355.6 | 1505.6 | 2066.9 |
| 11 | 52.41 | 4.59 | 116.3 | 426.1 | 1882.2 | 2561.6 |
| 12 | 26.12 | - 45.00 | 137.1 | 479.2 | 2261.9 | 2834.9 |

RUN NO: 6

| SECTION | $\sigma^{2}(x)$ | $G(x)$ | $F_{1}(x)$ | $F_{2}(x)$ | $F_{3}(x)$ | $F_{4}(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 5.90 |  |  |  |  |  |
| 5 | 10.61 | 1.05 | 5.6 | 61.1 | 201.1 | 660.9 |
| 6 | 14.56 | -0.18 | 13.5 | 130.6 | 393.0 | 1229.5 |
| 7 | 8.41 | - 3.84 | 22.7 | 199.8 | 540.3 | 1646.9 |
| 8 | 12.75 | - 4.82 | 31.9 | 277.5 | 686.5 | 2114.3 |
| 9 | 21.59 | - 0.60 | 40.5 | 357.6 | 868.3 | 2716.0 |
| 10 | 30.00 | 5.73 | 48.2 | 424.6 | 1069.6 | 3311.7 |
| 11 | 39.17 | 10.01 | 55.5 | 489.3 | 1282.0 | 3922.5 |

RUN NO: 7

| SECTION | $\sigma^{2}(x)$ | $G(x)$ | $F_{1}(x)$ | $F_{2}(x)$ | $F_{3}(x)$ | $F_{4}(x)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 | 16.06 |  |  |  |  |  |
| 6 | 23.20 | - 3.11 | 6.7 | 73.2 | 341.2 | 654.1 |
| 7 | 21.94 | - 11.76 | 13.9 | 142.8 | 696.6 | 1212.6 |
| 8 | 16.22 | - 17.18 | 41.8 | 309.9 | 1073.7 | 1823.5 |
| 9 | 26.33 | - 7.99 | 77.9 | 511.4 | 1487.9 | 2495.9 |
| 10 | 43.67 | 3.29 | 95.9 | 614.3 | 1871.7 | 3000.5 |
| 11 | 59.69 | 10.05 | 107.1 | 677.6 | 2209.7 | 3404.9 |
| 12 | 58.89 | 12.11 | 118.3 | 735.8 | 2593.8 | 3827.3 |
| 13 | 63.58 | 5.56 | 128.1 | 790.0 | 2964.4 | 4250.6 |



