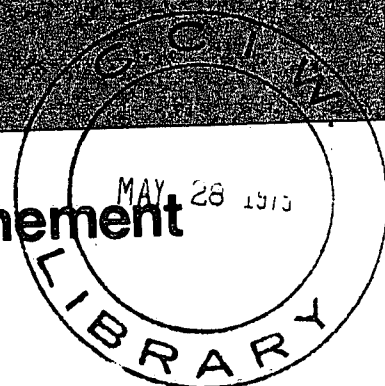




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TURBULENT SURFACE FILM THICKNESS FOR
OXYGEN ABSORPTION IN OPEN CHANNEL FLOWS

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TURBULENT SURFACE FILM THICKNESS FOR
OXYGEN ABSORPTION IN OPEN CHANNEL FLOWS

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ABSTRACT

The absorption of atmospheric oxygen into an open-channel flow is controlled by a small region near the free surface where the concentration gradient is large and the diffusivity is small. Data from reaeration experiments in an open-channel is used to calculate the thickness of the turbulent film, using some assumed values of diffusivity. It is shown that the dimensionless film thickness varied inversely as the shear Reynolds number and was also dependent on the channel roughness.

Résumé

L'absorption de l'oxygène de l'atmosphère dans l'écoulement d'un chenal dépend d'une petite zone, située près de la surface libre, où le gradient de concentration est élevé et la diffusivité faible. Pour calculer l'épaisseur de la pellicule turbulente, on utilise les données d'expériences de réaération dans un chenal, en prenant des valeurs hypothétiques de diffusivité. Il apparaît que la valeur sans dimension de l'épaisseur de la pellicule varie en sens contraire du nombre de Reynolds et qu'elle dépend aussi de la rugosité de chenal.

NOTATION

A	area
C	dissolved oxygen concentration
C_S	saturation concentration
D_m	molecular diffusivity
D_{TF}	diffusivity of the turbulent film
H	flow depth
K_L	liquid film coefficient
K_2	reaeration coefficient
k_2	$K_2/2.30$
q	mass transfer rate
R_e	$4UH/\nu$, flow Reynolds number
S	flume slope
T	temperature
U	flow velocity
U_*	\sqrt{gHS} , bottom shear velocity
y	vertical co-ordinate
α, β, λ	empirical constants in the stochastic model
δ	turbulent film thickness
ϵ	thermal diffusion coefficient
ν	kinematic, viscosity

INTRODUCTION

One of the first theoretical models of the process of oxygen absorption at an air-water interface was the two-film theory of Lewis and Whitman (1924) which assumed the existence of a stable laminar film at the interface. This model failed to recognize the fact that turbulence existed in the water even up to the free surface. Subsequent models all attempted to account for the effect of turbulence on the absorption rate. Such models were proposed by Higbie (1935), Danckwerts (1951), Dobbins (1956) and others. These models have all been compared and reviewed in the literature (Lau 1972, Bennett and Rathbun 1971).

A common criticism of all these conceptual models is that they all contain parameters which have not been related to the ordinary hydraulic variables and which are often most difficult to measure. The number of such parameters increased as models became more sophisticated. For example, Rudis and Machek (1972) introduced a stochastic model which postulated that the interface was at times in an eddying state and at other times in a stagnant film state. Their model gave the following expression for the liquid film coefficient K_L ,

$$K_L = \frac{\sqrt{D_m}}{1 + \beta/\alpha} \left[\sqrt{\beta + \frac{\beta}{\alpha}} \sqrt{(\alpha + \lambda)} \right] \quad (1)$$

where D_m is the molecular diffusivity, λ is the surface renewal rate and $\frac{1}{\beta}$ and $\frac{1}{\alpha}$ are the mean time duration of the eddy and film states respectively. To use this model, the three parameters λ , α and β must first be related to the mean flow parameters. However there is not even an

established method to measure λ , α and β . Such difficulties made it necessary to use empirically derived equations for K_L to calculate or predict the reaeration rate in rivers and streams.

Holley (1973) suggested that a turbulent diffusion model could be used to represent the transport of oxygen downward from the free surface. The transport equation can be written as:

$$q = - A D_{TF} \frac{\partial C}{\partial y} \quad (2)$$

where y is the vertical co-ordinate, C is the dissolved oxygen concentration, A is the surface area perpendicular to y , q is the mass transport rate through A and D_{TF} is the diffusivity in the turbulent surface film. This model gave an expression for K_L which is analogous to that of the two-film theory

$$K_L = \frac{D_{TF}}{\delta} \quad (3)$$

where δ is the thickness of the turbulent surface film. To use this model as a tool for prediction, it is still necessary to relate D_{TF} and δ to mean flow parameters. In this article an attempt is made to relate the film thickness δ to mean flow parameters using the results of some experimental measurements of reaeration in open-channel flows.

TURBULENT FILM MODEL

Regardless of the assumptions used in the formulation of different models of oxygen absorption, it is generally recognized that,

as far as time-average concentration distribution is concerned, there is a small region near the surface which has a large concentration gradient while the distribution is essentially uniform below this region. The concentration decreases from saturation at the surface to the value of the bulk concentration through this small surface layer as shown in Figure 1.

Holley (1973) referred to this region as the oxygen boundary layer, analogous to the viscous sublayer in momentum transfer or similar temperature boundary layers in heat transfer. He reasoned that even though turbulence is present, the mass transport in this layer can be described by a gradient diffusion equation such as equation (2) with a turbulent diffusion co-efficient D_{TF} . Assuming a linear concentration distribution in this very thin layer of thickness δ , which he called the turbulent surface film and using the definition for K_L

$$q = K_L (C_s - C) \quad (4)$$

where C_s is the saturation concentration, equation (3) for K_L was obtained.

$$K_L = \frac{D_{TF}}{\delta} \quad (3)$$

Holley performed some experiments in heat diffusion in which the process was similar to that of oxygen absorption. The air over a mixing vessel filled with water was heated to a temperature higher than

the water temperature. Heat was thus absorbed by the water at the free surface and transported downward through the water and was lost through the bottom of the vessel. Various degrees of turbulence were created by oscillating a stack of grids in the water at different frequencies. Temperature distribution in the water was measured and it was found that the steady state temperature distribution had 2 linear regions. There was a small region near the surface which had a large temperature gradient and the rest of the fluid had a much smaller linear temperature gradient. Equating the heat flux in the two regions

$$\epsilon_1 \left(\frac{dT}{dy} \right)_1 = \epsilon_2 \left(\frac{dT}{dy} \right)_2 \quad (5)$$

where ϵ is the diffusion coefficient for heat, T is the temperature and subscripts 1 and 2 refer to the surface region and the main body of the water respectively. The diffusion coefficient in the turbulent field generated by these particular grids were known from previous studies and therefore Holley was able to use the measured temperature distributions to calculate the heat diffusion coefficient in the surface layer. The results showed that the diffusion coefficient in the turbulent surface film was not much larger than the molecular diffusivity. With the grid oscillating at 100 rev./min. ϵ was approximately equal to the molecular diffusivity. The value of ϵ increased with the speed of oscillation to about twice the molecular diffusivity at 250 rev./min. Even when the top screen actually came to the surface at its highest point of travel, the largest value of ϵ was only about six times the

molecular value.

Holley assumed that, for the same mixing conditions, the ratio of turbulent to molecular diffusivity was the same for oxygen absorption and heat diffusion. Using the results of some reaeration tests in the mixing vessel he was then able to calculate the thickness of the turbulent surface film which turned out to be much smaller than the turbulent film thickness for heat transfer. This difference in film thicknesses was shown to be caused by the difference between the Prandtl number for heat transfer and the Schmidt number for oxygen transfer.

REAERATION EXPERIMENTS

Using the result of Holley's measurements of diffusivity in the turbulent film it appears that one may be able to calculate the turbulent film thicknesses from measurements of reaeration in open-channel flow and in turn deduce the relationship between the film thickness and the mean flow parameters. The experimental work have been reported elsewhere (Lau, 1974) and will be described only briefly in this section.

The experiments were performed in a recirculating flume with a test section 60 cm. high and 30.5 meters long. Water was deoxygenated by the addition of sodium sulphite using cobalt chloride as catalyst. After the flume system was filled to the desired depth, the discharge was adjusted to the required amount and the flume slope was varied until uniform flow conditions were obtained. The system was kept

running to oxidize the remaining sulphite and to eliminate any abrupt concentration gradient in the water. Samples of water were then taken at intervals of time at two stations 30.5 meters apart. The dissolved oxygen concentrations were obtained from Winkler analysis using many of the special precautions suggested by Edington et al. (1970).

Different bed roughnesses were also used including the smooth flume bottom, a 3 mm mean diameter sand and gravel of 6.7 mm mean diameter. It is believed that the values of the reaeration coefficient obtained in these experiments are more reliable than those obtained in previous flume measurements because the effect of inaccuracies in DO measurements have been reduced and the errors in DO measurements have also been minimized (Lau, 1974).

The reaeration coefficient K_2 was obtained from the usual graphical method. The logarithm of the DO deficits were plotted against time for both the upstream and downstream stations. The slope of the line joining a point on the upstream line to a point on the downstream line at a time lag equal to the travel time between the two stations is equal to K_2 , which is related to the liquid film coefficient K_L by the equation:

$$K_L = K_2 H \quad (6)$$

where H is the mean flow depth.

The experimental results are presented in Figure 2 and they show that the dimensionless variable $k_2 H/U$ is a function of the Reynolds number UH/ν as well as the dimensionless friction velocity

U_*/U , where U_* is the bottom shear velocity, U is the mean flow velocity, ν is the kinematic viscosity of water and $k_2 = K_2/2.3$. These results have been corrected for temperature using the formula given by Elmore and West (1961) and so the Schmidt number effect has been eliminated. From Figure 2 one can see that it is impossible to use one single equation for k_2 to cover all ranges of flow but that for very large Reynolds numbers $k_2 H/U$ depends upon U_*/U only.

TURBULENT FILM THICKNESS

For a particular open-channel flow the oxygen sublayer thickness should depend on the flow velocity, mean depth, bottom shear velocity, viscosity of the water and the molecular diffusivity of oxygen in water.

$$\delta = f(U, H, U_*, \nu, D_m) \quad (7)$$

These independent variables form a minimum independent set which influence the oxygen sublayer thickness. Using dimensional analysis it can be shown that

$$\frac{\delta}{H} = f\left(\frac{U_* H}{\nu}, \frac{U_*}{U}, \frac{\nu}{D_m}\right) \quad (8)$$

For the constant temperature case in which the Schmidt number is constant,

$$\frac{\delta}{H} = f\left(\frac{U_* H}{\nu}, \frac{U_*}{U}\right) \quad (9)$$

Equation (9) shows that the dimensionless sublayer thickness is a function of the shear Reynolds number as well as the roughness of the channel.

The thickness of the turbulent surface film can be calculated from the reaeration coefficient K_2 and the diffusivity in the turbulent film D_{TF} . From equation (3)

$$\delta = \frac{D_{TF}}{K_L} = \frac{D_{TF}}{K_2 H} \quad (10)$$

Values for K_2 have been obtained from the reaeration experiments described previously but there are no corresponding measurements of D_{TF} . However, Holley's measurements have indicated that the diffusivity of heat in the turbulent surface films is of the same order as the molecular diffusivity. The ratio of turbulent to molecular diffusivity varied from 1.0 at slow stirring rates to about 2.0 at faster stirring rates. This ratio can be expected to be valid for gas transfer as well. Therefore it is not unreasonable to assume that D_{TF} varied from being nearly equal to the molecular diffusivity D_m for the smooth bed, low friction factor experiments, to a value of $2D_m$ for the gravel bed experiments. The sand bed experiments had friction factor values approximately half way in between and therefore D_{TF} was assumed to be equal to $1.5 D_m$ for those experiments. The value of D_m at 20°C was taken to be equal to $2.037 \times 10^{-5} \text{ cm}^2 \text{ per second}$.

Using these assumed values for D_{TF} , values for δ were then calculated from equation (10). These are presented in Table I together with other pertinent dimensionless variables and data from the reaeration experiments.

The calculated turbulent film thicknesses, non-dimensionlised by flow depth H , were plotted against U_*H/ν in a log-log plot as shown in Figure 3. It can be seen that each of the three sets of data, corresponding to the three different bottom roughness conditions, fits very well on a straight line of slope equal to minus one. The effect of roughness is also evident, although for the sand and gravel bed cases the difference is slight. Thus the data confirms the results of the dimensional analysis that δ/H is a function of U_*H/ν and U_*/U . The dependence indicated by the data is that

$$\frac{\delta}{H} \sim \left(\frac{U_*H}{\nu} \right)^{-1}$$

with the constant of proportionality varying with U_*/U . For the smooth wall data the following equation is obtained:

$$\frac{\delta}{H} = 2.28 \frac{\nu}{U_*H} \quad \text{Smooth bed data (11)}$$

SUMMARY

Using the measured oxygen absorption rates in open-channel flows and the assumed values of diffusivity in the surface film, the thicknesses of the surface film layer were calculated. It was found that for each bed roughness condition, the thickness δ is proportional

to ν/U_* . This dependence is exactly similar to that of the sublayer thickness in momentum transfer in which the viscous sublayer thickness is equal to $5.5 \nu/U_*$, (Schlichting, 1960). These results lend further support to the validity of the turbulent surface film model and to the concept of the free surface region being an oxygen boundary layer.

The data also shows that the dimensionless oxygen sublayer thickness does depend on U_*H/ν and U_*/U as shown in equation (9). Although there is little doubt that δ/H varies with $(U_*H/\nu)^{-1}$ for constant U_*/U , the degree of dependence on U_*/U is not yet well established because of uncertainties in the values of D_{TF} . However, the proportionality constant of 2.28 which was obtained for the smooth wall data should be quite accurate because the assumption that $D_{TF} = D_m$ ought to be valid for smooth flows. More accurate values of δ for other cases cannot be obtained until instrumentation which is capable of measuring the DO concentration in the surface film region is available.

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TABLE 1. Data From Reaeration Experiments and Calculated Turbulent Film Thicknesses

Bottom Roughness	H (cm)	U (cm sec ⁻¹)	U _* /U	Re	(k ₂ H/U) 20°C	δ (cm)	δ/H	U _* H/U
Smooth Channel	3.0	17.62	0.0617	21140	2.28×10 ⁻⁵	0.0220	0.00733	326.1
	3.0	24.08	0.0610	28890	2.28 " "	0.0161	0.00537	440.6
	3.0	29.90	0.0596	35880	1.97 " "	0.0150	0.00500	534.6
	3.0	44.32	0.0558	53180	1.80 " "	0.0111	0.00370	741.9
	3.0	29.90	0.0535	35880	2.15 " "	0.0138	0.00459	479.9
	3.0	24.32	0.0553	29180	2.06 " "	0.0177	0.00590	403.5
	5.0	17.95	0.0523	35900	2.06 " "	0.0244	0.00488	469.4
	5.0	26.64	0.0510	53280	1.91 " "	0.0174	0.00348	679.3
	5.0	28.32	0.0488	56640	2.18 " "	0.0143	0.00286	691.0
	5.0	30.29	0.0498	60580	1.897 " "	0.0154	0.00308	754.2
	5.0	47.24	0.040	94480	2.187 " "	0.0086	0.00172	944.8
	3.0	4.18	0.0675	5016	3.12 " "	0.068	0.02267	84.6
	3.0	5.30	0.0663	6360	2.45 " "	0.068	0.02267	105.4
	3.0	3.66	0.064	4392	3.40 " "	0.071	0.02367	70.3
3mm sand	3.0	17.47	0.0942	20960	2.44×10 ⁻⁵	0.0312	0.01040	493.7
	3.0	24.08	0.0944	28890	2.33 " "	0.0237	0.00790	681.9
	3.0	29.90	0.0939	35880	2.35 " "	0.0225	0.00750	842.3
	3.0	36.67	0.0934	44000	2.00 " "	0.0166	0.00553	1027.5
	3.0	44.32	0.0949	53180	2.40 " "	0.0124	0.00413	1261.8
	5.0	14.44	0.0798	28880	1.83 " "	0.0501	0.01002	576.1
	5.0	17.95	0.0779	35900	2.18 " "	0.0339	0.00678	699.1
	5.0	28.32	0.0782	56640	1.56 " "	0.030	0.00600	1107.3
	5.0	30.30	0.0794	60600	2.03 " "	0.0216	0.00432	1202.9
	5.0	47.24	0.0809	94480	2.25 " "	0.0124	0.00248	1910.8
6.7mm Gravel	3.0	17.47	0.117	20960	3.57×10 ⁻⁵	0.0284	0.00947	613.2
	3.0	24.08	0.122	28890	3.67 " "	0.020	0.00667	881.3
	3.0	29.90	0.119	35880	3.87 " "	0.0153	0.00510	1067.4
	3.0	36.67	0.121	44000	3.65 " "	0.0132	0.0044	1331.1
	5.0	14.44	0.108	28880	2.78 " "	0.044	0.0088	779.8
	5.0	17.95	0.100	35900	3.075 " "	0.032	0.0064	897.5
	5.0	26.64	0.100	53280	2.95 " "	0.0224	0.0044	1332.0
	5.0	28.32	0.099	56640	3.22 " "	0.0194	0.0039	1401.8
	5.0	30.30	0.098	60600	2.65 " "	0.0220	0.0044	1484.7
	5.0	47.24	0.101	94480	3.08 " "	0.0121	0.0024	2385.6
	2.5	21.15	0.130	21150	4.09 " "	0.0204	0.0082	687.4
	2.5	19.08	0.127	19080	3.85 " "	0.0250	0.00100	605.8
	2.5	28.89	0.128	28890	4.17 " "	0.0147	0.0059	924.5
	2.5	33.62	0.137	33620	3.59 " "	0.0147	0.0059	1151.5
	2.0	23.59	0.146	18872	4.47 " "	0.0168	0.0084	688.8
	2.0	26.42	0.143	21136	4.58 " "	0.0146	0.0073	755.6

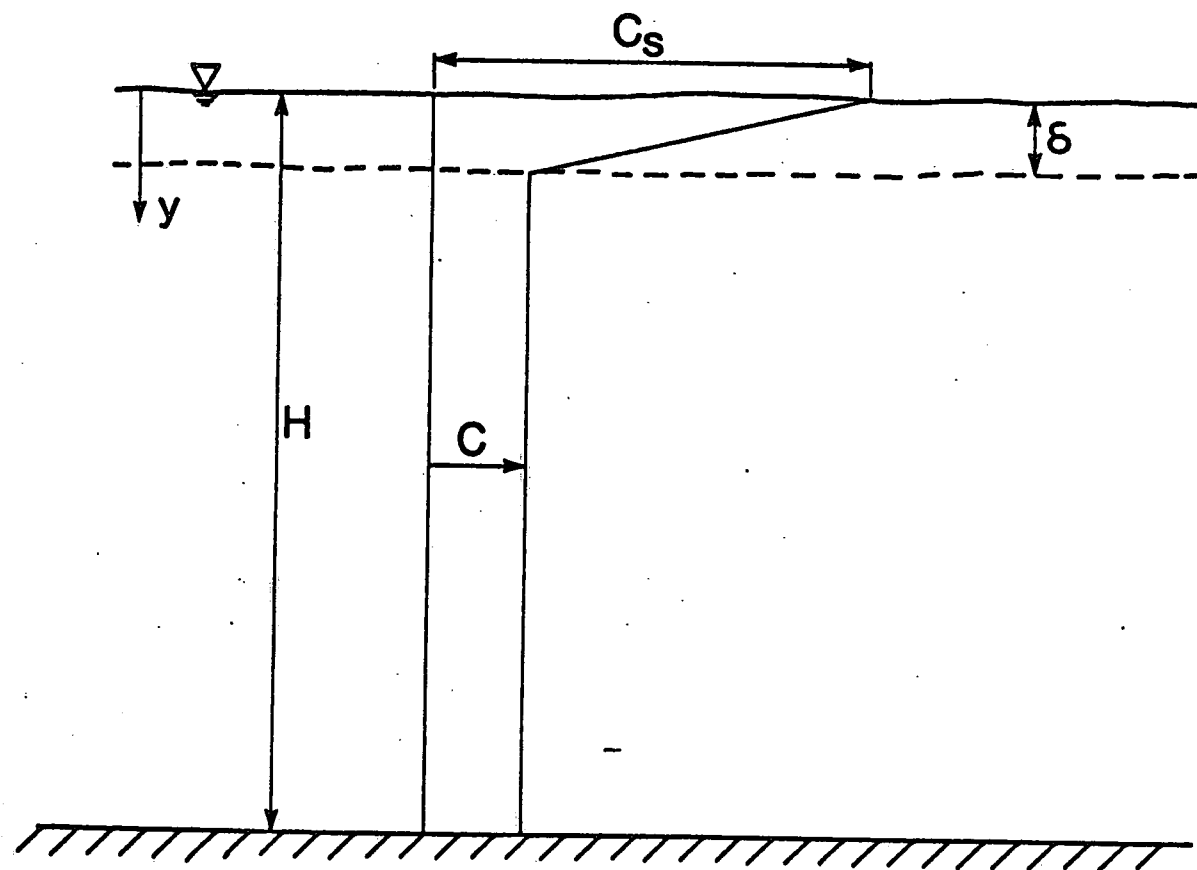


Figure 1

Schematic sketch of dissolved oxygen concentration distribution.

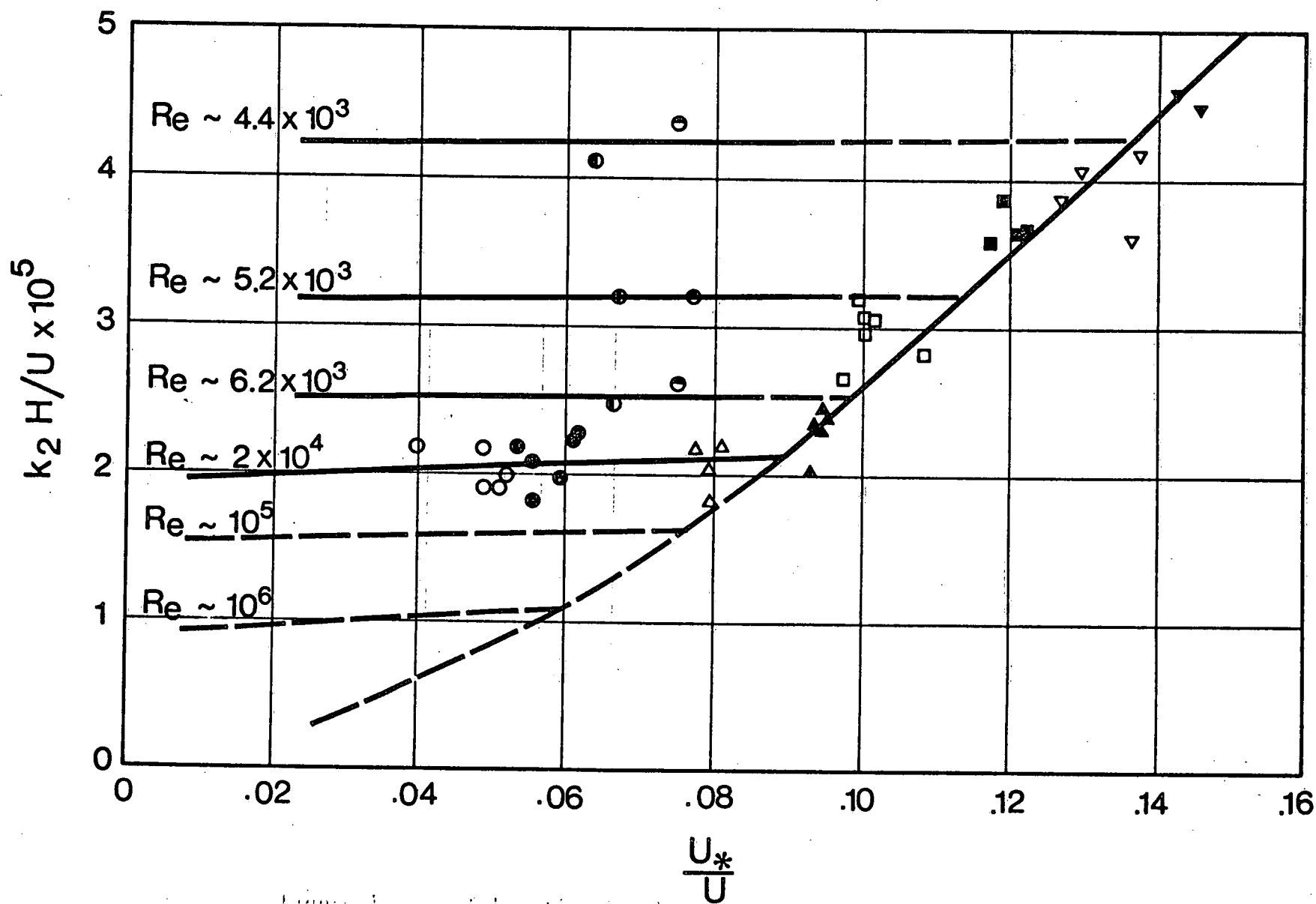


Figure 2

Dependence of the reaeration variable on Reynolds

Number and Dimensionless Shear Velocity (Lau 1974).

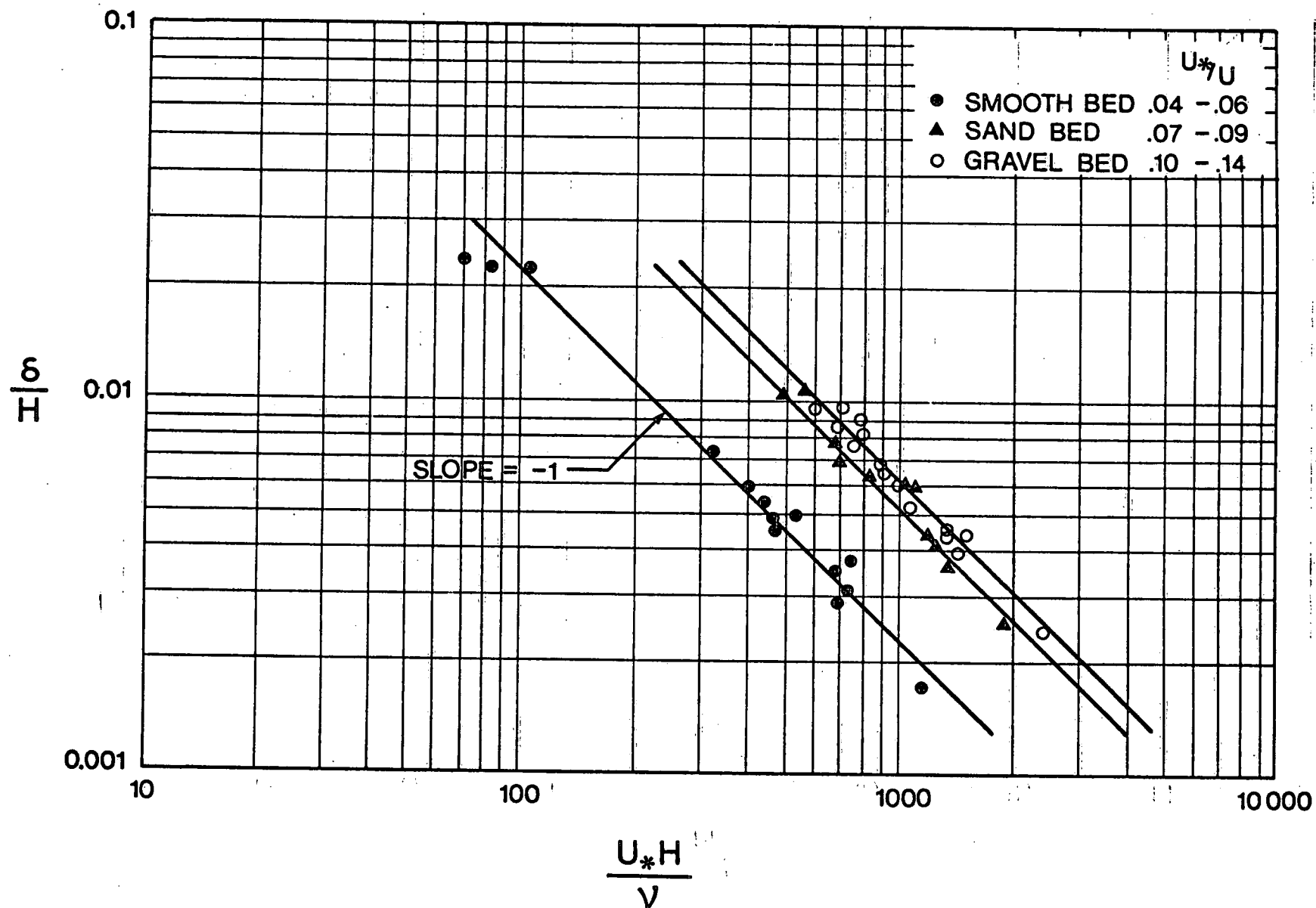


Figure 3

Variation of the dimensionless turbulent film thickness with shear Reynolds number and channel roughness.

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