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DISPERSION OF GRANULAR MATERIAL DUMPED IN DEEP WATER

By

B. G. KRISHNAPPAN

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DISPERSION OF GRANULAR MATERIAL DUMPED IN DEEP WATER

By

B. G. KRISHNAPPAN

FINAL REPORT
ON
DISPERSION OF THE DREDGED MATERIAL WHEN DUMPED IN DEEP WATER

By

B. G. Krishnappan

ABSTRACT

In this research paper, the motion of dredged material when dumped near the surface of deep water is formulated using the principle of superposition. The dredged material is considered to consist of various fractions of uniform size particles and each fraction exerts an influence on the total behaviour of the dredged material in the same proportion as its negative buoyancy. The behaviour of uniform size particles has been formulated using the theory of dimensions and laboratory experiments. The results of the research show that the motion of the particles can be treated in two distinct phases, namely, the initial "entrainment" phase and the final "settling" phase. During the entrainment phase, the size of the "cloud" grows due to the incorporation of external fluid while the vertical downward velocity diminishes. During settling phase when the vertical downward velocity is the same as the fall velocity of the individual solid particles constituting the cloud, the increase in the cloud size is solely due to ambient turbulence.

The method developed in this work permits the evaluation of vertical height and horizontal size of the "mound" formed due to the deposition of the dredged material at the bottom of the deep water. It also indicates how the above characteristics of the mound depend on the volume of the dump, the size distribution of the dredged material and height of the deep water, thereby, providing the guidance for the selection of optimum dump size and the location for the disposal of the dredged material.

RAPPORT FINAL
SUR
LA DISPERSION DU MATÉRIEL DRAGUÉ DÉVERSÉ EN EAU PROFONDE

By

B. G. Krishnappan

RÉSUMÉ

Dans le présent exposé, nous expliquons, au moyen du principe de la superposition, le mouvement qu'adopte le matériel dragué lorsqu'il est déversé près de la surface d'une eau profonde. On considère que le matériel dragué se compose de divers fragments de particules de grosseur uniforme et que chaque fragment influe sur le comportement intégral du matériel dragué dans la même proportion que sa portance négative. Nous avons expliqué le comportement des particules de grosseur uniforme à l'aide de la théorie des dimensions et les essais de laboratoire. D'après les résultats de notre recherche, on peut distinguer deux phases dans la motion des particules, à savoir la phase initiale "d'entraînement" et la phase finale "de sédimentation". Durant la phase d'entraînement, le "nuage" s'accroît par l'intégration du fluide externe, alors que la vitesse de descente verticale diminue. Durant la phase de sédimentation, alors que la vitesse de chute verticale correspond à la vitesse de chute des différentes particules solides composant le nuage, l'accroissement du nuage est dû exclusivement à la turbulence ambiante.

La méthode développée dans la présente recherche permet de mesurer la hauteur et l'étendue horizontale du "monticule" ainsi formé par la sédimentation au fond de l'eau du matériel dragué. Cette méthode indique aussi comment ces caractéristiques du monticule dépendent du volume de la décharge, des différentes dimensions du matériel dragué et de la profondeur de l'eau; elle peut ainsi orienter la détermination du volume idéal de la décharge et le choix de l'emplacement pour le rejet du matériel dragué.

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LIST OF SYMBOLS

g: Acceleration due to gravity.

ρ: Average density of the receiving body of water.

μ: Average viscosity of the receiving body of water.

d: Depth of water at the location of the dump.

D: Diameter of the uniform size particles of the dredged material.

ρ_s: Density of the solid particles.

γ_s: $(\rho_s-\rho)g$: Submerged specific weight of the solid particles.

ω: Fall velocity of the solid particles

R: Radius of the cloud of the dumped material.

w: Vertical downward velocity of the solid particle cloud.

z: Vertical co-ordinate measured from the virtual origin.

ζ: Vertical co-ordinate measured from the dump level.

r: Radial co-ordinate.

R_o: Initial radius of the cloud or the dump size.

V_s: Volume of the dredged material dumped.

α,β: Dimensionless constants.

m: Shape factor of the solid particle cloud.

m_p: Shape factor of the solid particles

K: Turbulent diffusion coefficient in the horizontal direction.

σ: Standard deviation of the distribution of solid particles in the cloud.

U: The ambient current of the receiving body of water.

W: Weighting coefficients.

A: Dissipation parameter.

h: Height of the mound formed at the bottom of the receiving body of water.

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I. INTRODUCTION

Disposing of the dredged material by dumping it in deep water can be considered to be the most economical among the disposal methods. Indeed, it has been shown by a recent study conducted for disposing of 7 million cubic meters of dredged material resulting from the Chesapeake and Delaware Canal enlargement scheme that the method of dumping in deep water was 50% cheaper than the next cheapest alternate and it was six times cheaper than the method of storing in open water areas with confining dykes.[1]. However, this method may have its own adverse effects on the environment of the receiving body of water, especially if the amount of disposed material is enormous. Therefore, this method cannot be indiscriminately adopted in spite of its obvious economic advantage. The adoption of this method has to be always preceded by a proper assessment of its impact on the aquatic life and the quality of the water environment. For such assessments the knowledge of the physical behaviour of the dumped material is a prerequisite. Unfortunately, the physical behaviour of the dumped material is not very well known and only a very few attempts have been made so far to study such an important problem. In fact, there are only two studies made so far which would directly deal with the problem in hand. The first one is by R.C.Y. Koh and Y.C. Chang [2] and the second one is by B. L. Edge and B. C. Dysart [3]. In these two studies the dumped material is assumed to behave in the same manner as a denser liquid moving in a lighter liquid medium. However, the present experiments indicated that the spreading rate of the solid particles moving in a liquid medium is different from an equivalent denser liquid moving in the same medium. Therefore, the above two methods are not adequate to describe the behaviour of the dumped dredged material in a satisfactory manner, and hence there is a strong need for a new method which would correctly describe the behaviour of the dumped dredged material. The object of the present work is to develop such a method based on the systematic laboratory experiments, designed on the basis of the theory of dimensions.

II. REVIEW OF EXISTING METHODS

1. Method of Koh and Chang:

Koh and Chang considered three possible discharge methods, namely, a) dumping from a bottom opening, b) pumping through nozzles and c) discharging into the barge wake. In each of these operations the dumped material is assumed to undergo three different stages of motion, namely, a) convective descent, b) collapse and c) long-term dispersion. They considered the motion in three different parts because the governing mechanisms are different during the different stages of the motion. During the initial stages of motion the dumped material moved primarily because of its negative buoyancy. During this stage of the motion, because of the relative motion between the dumped material "cloud" and the ambient water entrainment of the ambient fluid into the dumped material cloud takes place which in turn reduces the negative buoyancy and at the end of this stage of motion the negative buoyancy is completely zero. The authors assume that at the end of this stage the cloud still undergoes spreading but only in the horizontal direction while the vertical position of the cloud does not change and they term this stage of motion the "collapse" stage. They assume that the governing mechanism during this stage is the non-equilibrium in the hydrostatic pressure between the cloud and the ambient fluid. At the end of the collapse stage when the horizontal spreading stops the cloud undergoes what the authors call the "long-term dispersion" when the motion of the cloud is caused by the local turbulence and the ambient current.

As indicated earlier, while treating the motion of the dredged material, the authors assume that the dredged material can be considered as consisting of only liquid medium whose density is equal to the equivalent density of the dredged material. However, the present laboratory experiments indicate that the behaviour of the solid particle cloud is very different from that of the liquid particle cloud and the difference is a function of the particle size. As the particle size decreases the difference between the behaviour of solid and liquid particle clouds also decreases tending to be zero in the limit. Therefore, treating the dredged material as liquid medium is not valid, especially if the dredged material consists of larger size particles.

The assumption of treating the dumped material as dense liquid has further consequences. For example, the need for a separate stage as collapse stage is a direct consequence of the above assumption. Indeed, the negative buoyancy of the dredged material consisting mainly of sand particles will not become zero with any amount of entrainment of the external fluid. The vertical movement of the cloud could be zero depending upon the vertical velocity of the cloud and the vertical turbulent velocities of the receiving medium. But this does not warrant a different treatment in a separate stage. Furthermore, in the analysis of the collapse stage, the authors have made several assumptions without any experimental support and it is doubtful whether they would correspond to reality.

The treatment of the long-term dispersion is based on the mass conservation equation and the boundary and initial conditions are provided from the results of the collapse stage. Because of this, the uncertainties of the collapse stage will also be reflected in the long-term dispersion results and any amount of refinement of the long-term dispersion analysis will not bear fruit.

2. Edge and Dysart Method:

The method of Edge and Dysart is very much similar to the method of Koh and Chang. Edge and Dysart have considered only one method of discharge namely pumping through nozzles. They treat the motion in two separate stages, namely, a) convective descent and b) settling with dispersion. In contrast to Koh and Chang they have not considered the collapse stage. In the treatment of the convective descent stage, Edge and Dysart also have assumed that the dredged material can be considered as consisting of liquid medium whose density is equal to the equivalent density of the dredged material and hence their method also suffers the same drawbacks as the method of Koh and Chang.

Their treatment of the settling and dispersion stage is based on the method proposed by R.C.Y. Koh [4]. According to this method, the solid particles settle with their fall velocities while undergoing horizontal spreading due to turbulent diffusion. The vertical turbulent diffusion is neglected. A 4/3 power law is used for the horizontal diffusion coefficient. The details of the treatment will be taken up while considering the theoretical formulation of the solid particle clouds in the next section.

III. BEHAVIOUR OF "SOLID PARTICLE-THERMALS"*

Even though a large number of experiments dealing with a liquid "thermal" moving in a liquid medium are available in the literature, [5], [6],[7],[8], the same cannot be said for the "Thermal" formed by the solid particles. Not even a single reference dealing with the solid particle thermals could be found in the literature in spite of a thorough search. Therefore, it is only logical to conduct experiments in the laboratory before any theoretical formulation could be attempted to deal with the behaviour of the solid particle motion in a liquid medium when they were dumped as a slug.

When a slug of uniform size solid particles is released in a homogeneous and stationary body of water without any initial momentum the particles moved as a cloud with distinct boundaries. The size of the cloud increased as it moved downwards until it attained a more or less constant size after which the cloud simply settled down without any appreciable change in the size. The downward vertical velocity, on the other hand decreased until it attained a constant value equal to the settling velocity of the individual particles. Figure 3.1 shows schematically the sequential motion of the solid particle cloud. Such a behaviour of the solid particle cloud is different from the liquid cloud in that the liquid cloud increases in its size throughout its motion, [5], [6], [7], [8].

The behaviour of the solid particle cloud can, therefore, be considered in two different stages, namely, the initial entrainment stage when the size of the cloud grew due to the incorporation of the external fluid and the settling phase when the cloud velocity attained the value equal to the fall velocity of the individual solid particle constituting the cloud.

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* the term thermal is used to mean the cloud of liquid or solid particles moving down in a liquid medium. See Ref. [5].

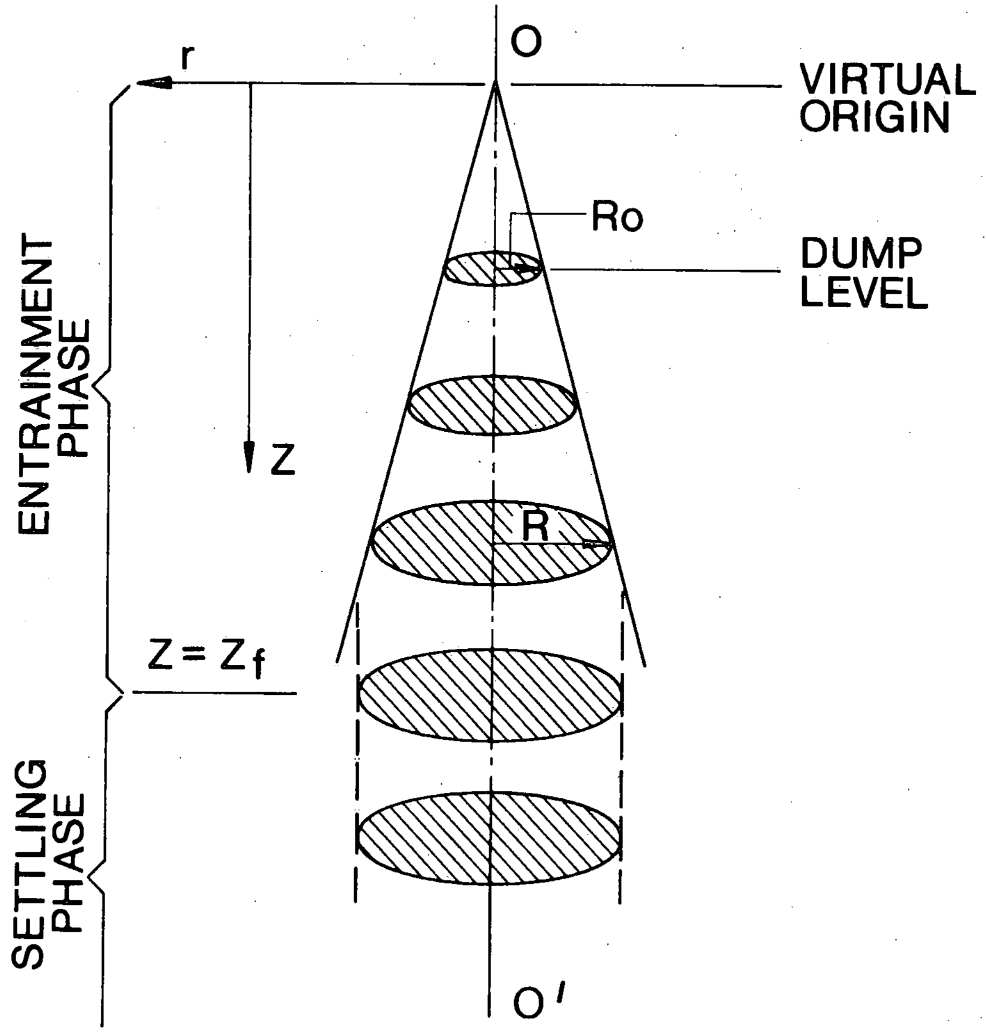


FIG.3.1 SCHEMATICAL REPRESENTATION OF THE PATTERN OF THE CLOUD OF SOLID PARTICLES

1. Entrainment Phase:

This phase of the particle motion can be formulated using the dimensional approach similar to that of G. K. Batchelor [9] who formulated the entrainment phase of the liquid particle motion. Accordingly, let V_s be the volume of solid particles dumped as a slug in a medium whose density is ρ and the viscosity is μ. Let γ_s be the submerged specific weight of the solid particles dumped in the medium:

$$\text{ie.} \quad \gamma_s = (\rho_s - \rho)g \tag{3.1}$$

where ρ_s is the density of the solid particles and g is the acceleration due to gravity. The motion of the solid particles is caused solely because of the excess submerged weight per unit mass of the fluid, ie. the total negative buoyancy F given by:

$$F = \frac{\rho_s - \rho}{\rho} g V_s \tag{3.2}$$

Because of the relative motion between the solid particles and the surrounding fluid, the entrainment of the ambient fluid into the solid particle cloud occurs and the size of the solid particle cloud increases as the cloud moves downwards. Therefore, both the size and the vertical downward velocity of the cloud are also a function of the position of the cloud. In order to define the position of the cloud let us adopt the co-ordinate system as shown in Fig. 3.1. The vertical downward distance is measured from the virtual origin 0 and the horizontal distances are measured from the vertical axis (00′) passing through 0. It is assumed that the cloud is symmetrical about the vertical axis and hence can be represented by a single length (R) measured from the vertical axis 00′. If w is the vertical downward velocity of the cloud then w and R can be expressed as follows:

$$\begin{aligned} w &= f_w(F;z) \\ R &= f_R(F;z) \end{aligned} \tag{3.3}$$

Applying the theory of dimensions the form of the functions f_w and f_R

can be evaluated and the relation (3.3) can be replaced by

$$w = \frac{\beta . F^{\frac{1}{2}}}{z} \qquad R = \alpha z \tag{3.4}$$

where β and α are the dimensionless constants. In order to verify the relation (3.4) experiments were conducted using three different values of F and for each case the dimensionless coefficients β and α were evaluated. The value of F was changed by changing only V_s (see Equation 3.2). The grainsize, specific weight and the properties of the receiving body of water were unaltered. It was found that the values of α and β remained constant irrespective of the value of F which indicates the validity of the dimensional approach adopted to treat the motion of the solid particles.

Even though α and β are treated as independent in the above dimensional approach, a relationship between them can be established as has been done by J. S. Turner [10] if the motion inside the cloud can be treated as that of a spherical vortex. The resulting relation between α and β is as follows:

$$2\, C_v m\, \alpha^3 \beta^2 = 1 \tag{3.5}$$

where m is the shape factor of the cloud which could be evaluated as

$$m = V/R^3 \tag{3.6}$$

where V is the volume of the cloud, C_v is the virtual mass coefficient which in turn is a function of the shape of the cloud.

For different values of the grain size D, the submerged specific weight γ_s and the properties of the receiving body of fluid (ρ and μ), the values of α and β and m in general, would be different and they can be expressed in terms of the above characteristic parameters as

$$\alpha = f_\alpha\ (\gamma_s;D;\rho;\mu) \qquad \beta = f_\beta\ (\gamma_s;D;\rho;\mu) \qquad m = f_m\ (\gamma_s;D;\rho;\mu) \tag{3.7}$$

Again, using the theory of dimensions the above relations can be brought into the following equivalent dimensionless versions:

$$\alpha = \Phi_\alpha \left(\frac{\gamma_s \rho D^3}{\mu^2}\right)$$
$$\beta = \Phi_\beta \left(\frac{\gamma_s \rho D^3}{\mu^2}\right) \qquad (3.8)$$
$$m = \Phi_m \left(\frac{\gamma_s \rho D^3}{\mu^2}\right)$$

Determination of any two of the above three functions are necessary and sufficient to describe the motion of solid particles of any size and specific weight in any medium of fluid during the entrainment phase. Since α and β determine the size and velocity of the cloud directly only Φ_α and Φ_β will be taken up after the consideration of the settling phase of the particle motion.

2. Settling Phase:

Settling phase is considered to start when the vertical downward velocity w of the cloud reaches the fall velocity (terminal velocity ω) of the individual particle forming the cloud. Now the fall velocity ω of the individual solid particle can be obtained as follows:

Let D be the grainsize of the solid particle and γ_s be the submerged specific weight.

During the uniform settling motion of the solid particle the submerged weight (G) of the solid particle balances exactly the fluid drag (F_d) exerted on the solid particle:

$$\text{ie. } \gamma_s . m_p . D^3 = C_D . \rho D^2 \omega^2 \qquad (3.9)$$

Where m_p is the shape factor of the solid particle and C_D is the coefficient of drag which is a function of the Reynolds number formed by the fall velocity of the particle:

$$\text{ie.} \quad C_D = \phi\left(\frac{\omega D\rho}{\mu}\right) \tag{3.10}$$

Substituting (3.10) in (3.9) and rearranging the relation (3.9) can be expressed as:

$$\frac{\rho\omega^2}{\gamma_s D} = \psi\left(\frac{\omega D\rho}{\mu}\right) \quad \text{where } \psi = m_p\phi^{-1} \tag{3.11}$$

Note that in Equation (3.11) the fall velocity ω appears on both sides and the determination of ω from such a relation has to be by trial and error. This can be overcome by eliminating ω from one of the sides of the Equation (3.11), ie. by dividing both sides of the equation (3.11) by $(\omega D\rho/\mu)^2$

$$\text{ie.} \frac{\left(\dfrac{\rho\omega^2}{\gamma_s D}\right)}{\left[\dfrac{\omega D\rho}{\mu}\right]^2} = \frac{\psi\left(\dfrac{\omega D\rho}{\mu}\right)}{\left[\dfrac{\omega D\rho}{\mu}\right]^2} \tag{3.12}$$

Equation (3.12) can be expressed by its equivalent form as:

$$\frac{\omega D\rho}{\mu} = \Phi_\omega\left(\frac{\gamma_s . \rho D^3}{\mu^2}\right) \tag{3.13}$$

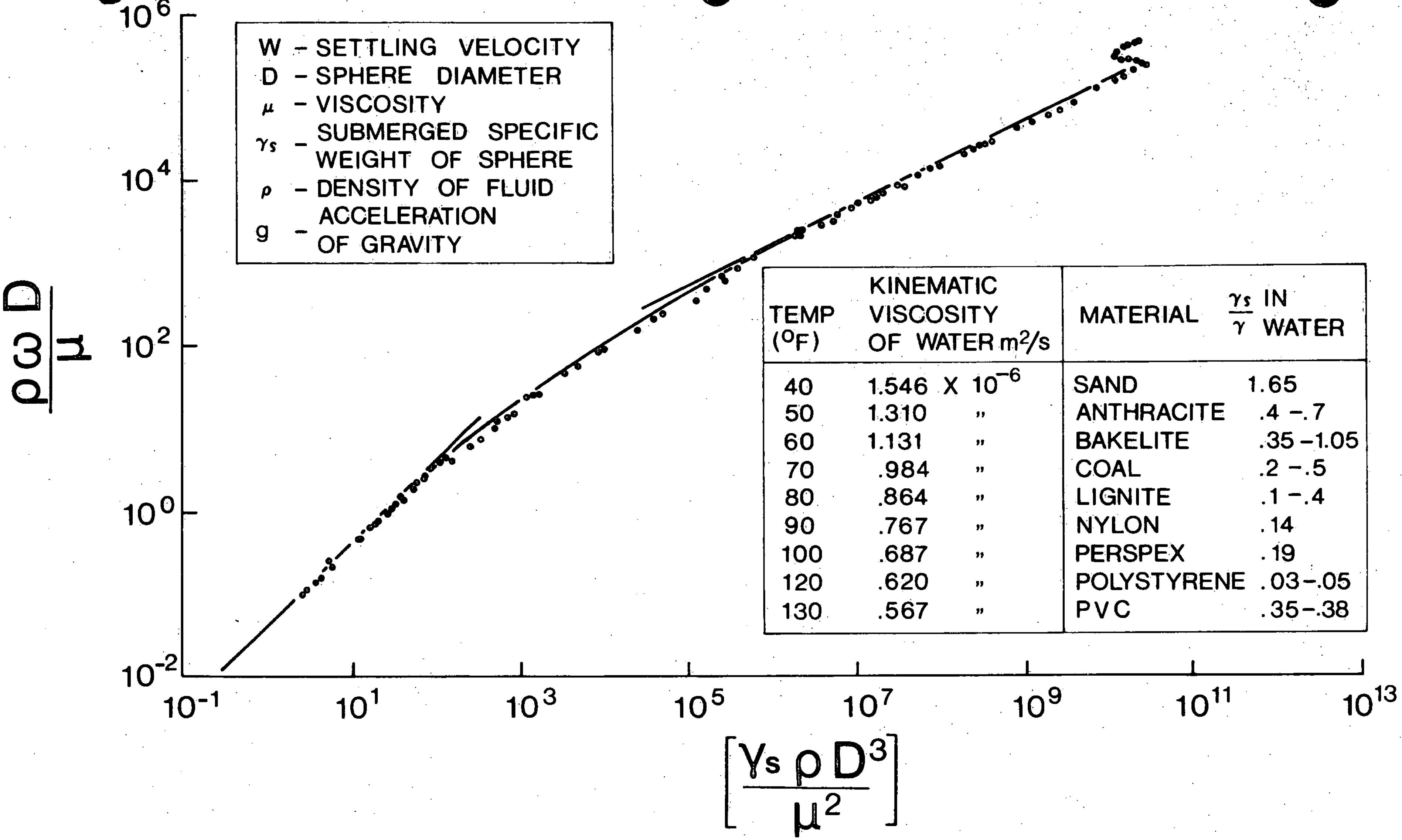
which gives the fall velocity of the solid particle explicitly in terms of the characteristics of the solid particle (γ_s;D) and the medium (ρ;μ). The form of the function Φ_ω is known only for spherical particles and can be obtained from the graph shown in Fig. (3.2).

Note that the same parameter ($\gamma_s \rho D^3/\mu^2$) which governs the coefficients of the entrainment phase (ie. α and β) also governs the fall velocity of an individual solid particle during the settling phase. Knowing the fall velocity, the distance from the virtual origin at which the settling phase begins can be obtained as:

$$z_f = \frac{\beta . F^{\frac{1}{2}}}{\omega} \tag{3.14}$$

and the size of the cloud at the begining of the settling phase is

$$R_f = \alpha . z_f \tag{3.15}$$



TEMP (^{o}F)	KINEMATIC VISCOSITY OF WATER m^2/s	MATERIAL	$\frac{\gamma_s}{\gamma}$ IN WATER
40	1.546×10^{-6}	SAND	1.65
50	1.310 "	ANTHRACITE	.4 -.7
60	1.131 "	BAKELITE	.35 -1.05
70	.984 "	COAL	.2 -.5
80	.864 "	LIGNITE	.1 -.4
90	.767 "	NYLON	.14
100	.687 "	PERSPEX	.19
120	.620 "	POLYSTYRENE	.03-.05
130	.567 "	PVC	.35-.38

FIG. 3.2 FALL VELOCITY OF SPHERICAL PARTICLES

There will be further increment in the size of the cloud during its settling phase due to the horizontal turbulent diffusion in the real situation. Such an increment is hardly noticeable in the present experimental set-up since the water is quiescent. Spreading during settling phase due to turbulent diffusion can be easily accounted for by adopting a turbulent diffusion coefficient K.

$$\text{ie.} \quad K = \frac{d}{dt}\left(\frac{\sigma^2}{2}\right) \tag{3.16}$$

where σ^2 is the variance of the concentration distribution of the solid particles, related to the size of the cloud. Knowing the initial variance, the variance at any time t can be obtained by equation (3.16), and hence the size of the cloud.

A method to study the spreading of the particles during settling due to turbulent diffusion was formulated by R.C.Y. Koh [4] as indicated earlier. According to this method, the distribution of the concentration of the solid particles is assumed to be Gaussian and the turbulent diffusion coefficient K is assumed to follow the 4/3 power law commonly used in ocean turbulence studies. Accordingly, if the initial variance at the beginning of the settling phase is σ_o^2 the variance of the distribution at any time t after the beginning of the settling phase is σ_t^2, given by:

$$\sigma_t^2 = \sigma_o^2 \left[1 + 4^{4/3} \cdot \frac{2}{3} \cdot \frac{A}{\sigma_o^{2/3}} \cdot t\right]^{\frac{3}{2}} \tag{3.17}$$

where A is the dissipation parameter which appears in the 4/3 power law for diffusion coefficient. ie:

$$K = A.(4\sigma)^{4/3} \tag{3.18}$$

The numerical value of A is usually taken as .000068 $m^{2/3}$/sec. The assumption of the Gaussian distribution for the concentration of the solid particles facilitates the determination of the initial variance

σ_o^2 or the standard deviation σ_o knowing the radius of the cloud at the beginning of the settling phase, ie. R_f. Indeed for 99.994% of the particles to be within the radius R_f the standard deviation σ_o has to be 8 times smaller than R_f, ie.

$$\sigma_o = R_f/8 \tag{3.19}$$

Because of the same reasoning the size of the cloud at any time t after the beginning of the settling phase R_{ft} will be 8 times bigger than σ_t,

$$\text{ie.} \quad R_{ft} = 8\sigma_t \tag{3.20}$$

3. Experimental Set-up and Procedure to Determine Φ_α and Φ_β:

Even though the fall velocity of solid particles is a function of the shape of the particle the dependence between the two are not very well known at the present time. Several attempts have already been made to establish the effect of the shape of the solid particle on the fall velocity; but without much success. Until a better method is devised which would correctly include the effect of shape of the particle, the curve given in Figure (3.2) which corresponds to the spherical particles, will be used to predict the fall velocity of the solid particles. Therefore the quantities which have to be determined, in order to predict the motion of the solid particles when dumped as a slug, are the dimensionless functions Φ_α and Φ_β and they can be determined only by experimental measurements. In this subsection the experimental set-up and the procedure to determine Φ_α and Φ_β will be outlined.

a) Experimental Set-up:

The experimental set-up consists of a large tank which measures 3.75m x 3.75m x 3.75m and has two adjacent transparent sides. A dumping device is mounted on the top of the tank which facilitates releasing, without initial momentum, three different volumes of the solid particles as a slug. The vertical descent of the solid particle clouds were photographed at regular intervals of time using a Hasselblad still camera. A grid system with 5cm x 5cm openings is placed at the outside of the tank against the viewing side of the tank and is used to measure the size and location of the cloud. Appropriate corrections were applied for not measuring at the vertical axis of the cloud and for the refractions at the plexiglass wall and the water medium. A schematic view of the experimental set-up is shown in Fig. (3.3). Descent of a typical cloud of solid particles is shown in Fig. (3.4).

b) Determination of α:

The location and the size of the cloud at various instants of time from the release of the particles are measured from the sequence of photographs similar to the ones shown in Fig. (3.4) are obtained and are plotted as shown in Fig. (3.5). A smooth line is drawn to represent the outline of the cloud. During the entrainment phase the growth of the cloud is linear and hence the line representing the outline of the cloud is extrapolated as shown in Fig. (3.5) until it meets the vertical axis of the cloud at the point 0 which is the virtual origin. The slope of the line representing the outline of the cloud at the virtual origin gives the value of α.

c) Determination of β:

β appears in the equation which gives the vertical downward velocity w of the cloud, ie. equation (3.4) which can be written as

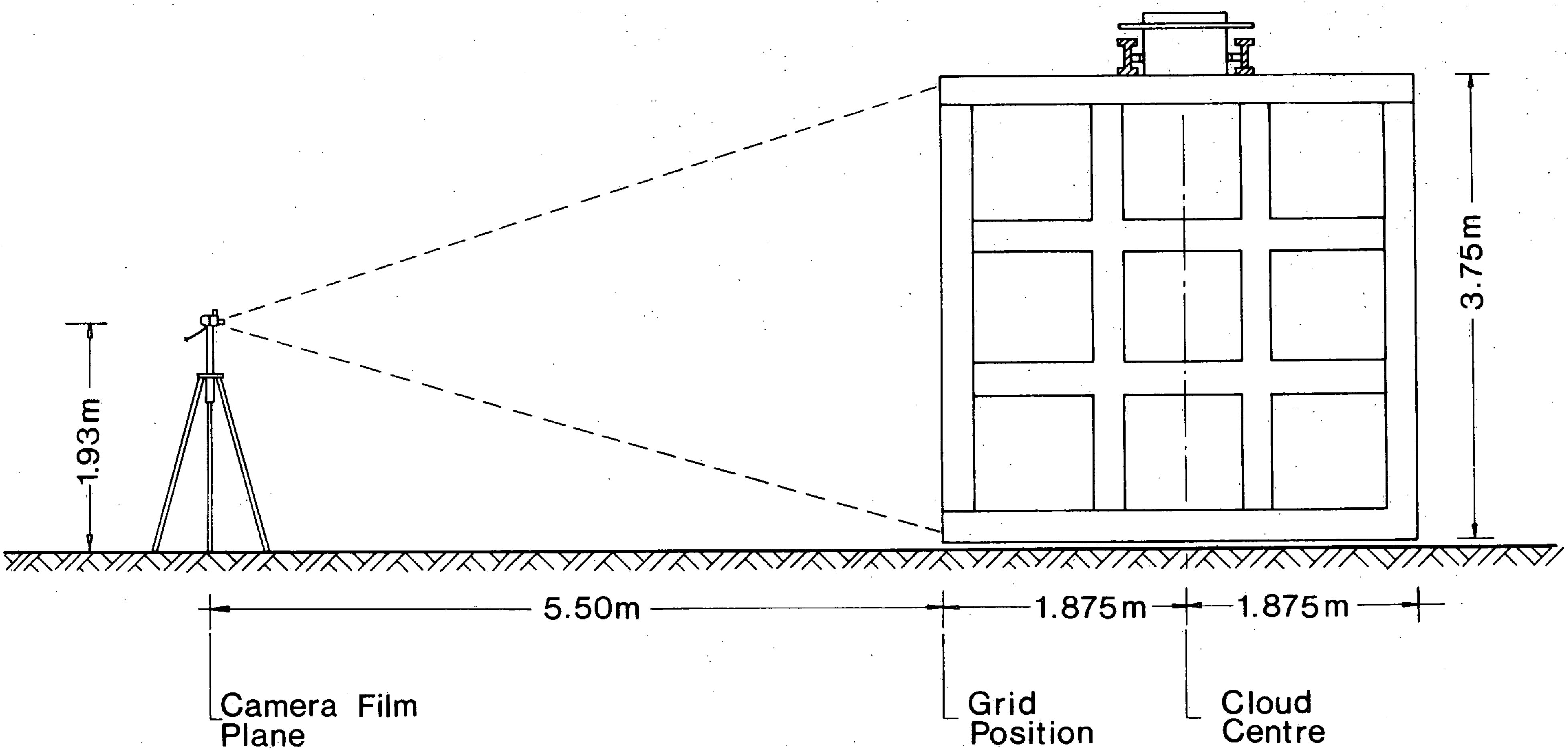
$$z\,dz = \beta F^{\frac{1}{2}} dt, \qquad (3.21)$$

since

$$w = \frac{dz}{dt} .$$

Integrating both sides:

$$\frac{z^2}{2} = \beta F^{\frac{1}{2}} t + C \qquad (3.22)$$



Tank ; 3.75m x 3.75m x 3.75m

Camera ; Hasselblad Model “500- Elm”

FIG. 3.3 SCHEMATICAL VIEW OF EXPERIMENTAL SET-UP

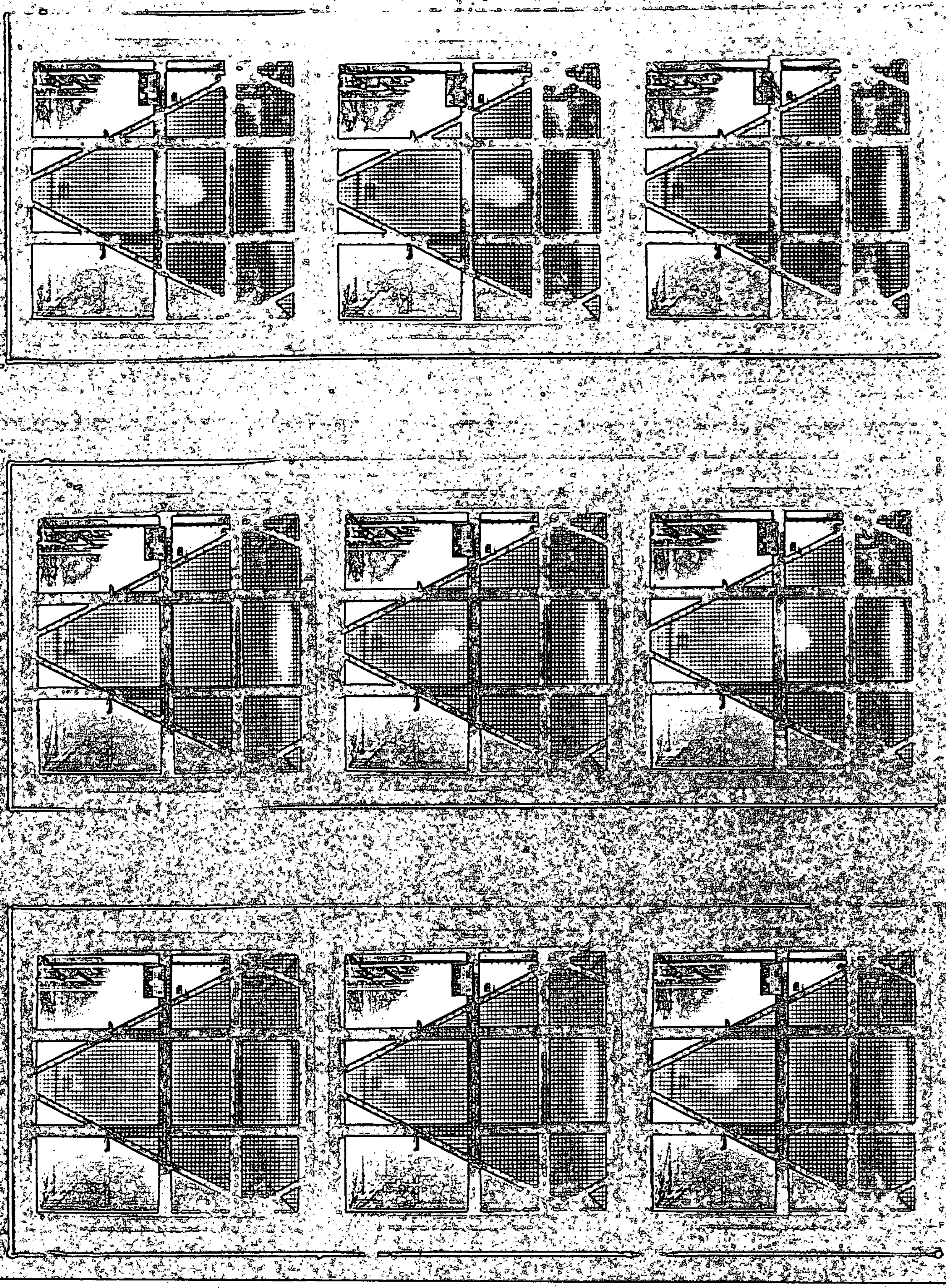


Fig. (3.4) Sequential motion of the cloud

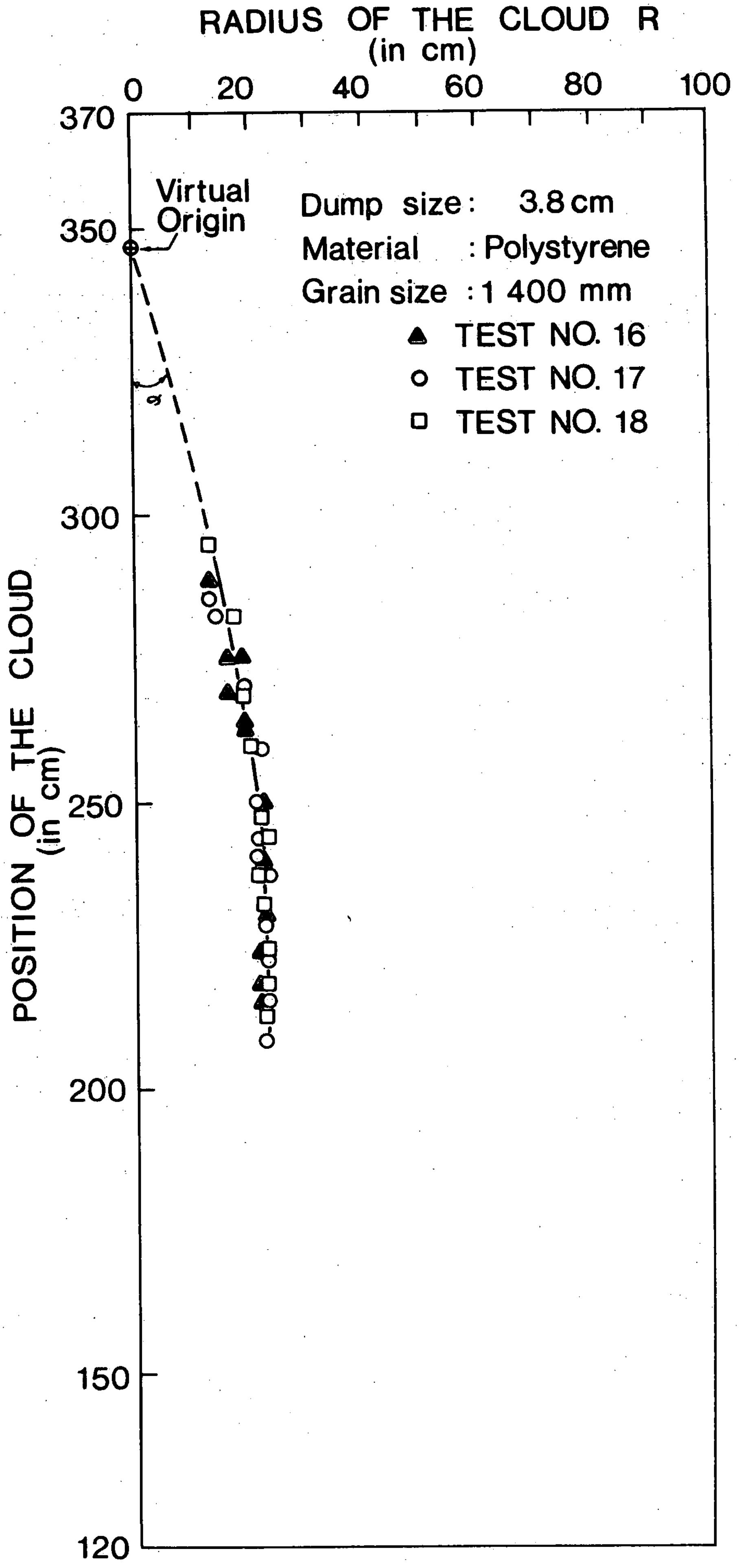


FIG. 3.5 A TYPICAL GRAPH BETWEEN THE POSITION AND THE SIZE OF THE CLOUD

where C is an integration constant.

Using the condition that $z = z_o$ when $t = 0$ the integration constant C can be evaluated as

$$C = \frac{z_o^2}{2}$$

Substituting in (3.22) the relation becomes:

$$z^2 - z_o^2 = 2\beta F^{\frac{1}{2}} t \qquad (3.23)$$

Hence, by plotting a graph between $(z^2 - z_o^2)$ and t and measuring the slope of the line which represents the product $2\beta F^{\frac{1}{2}}$ the value of β can be evaluated since F is known. A typical graph between $(z^2 - z_o^2)$ and t is shown in Fig. (3.6).

d) Determination of Φ_α and Φ_β:

The determination of the form of the functions Φ_α and Φ_β involves the measurement of α and β for various values of the parameter $(\gamma_s \rho D^3/\mu^2)$ as implied by the equation (3.8). The parameter $(\gamma_s \rho D^3/\mu^2)$ can be varied by either varying the characteristics of the solid particles and/or changing the properties of the receiving body of fluid. In the present experiments only the properties of the solid particles (γ_s and D) are varied. The range of the parameter $(\gamma_s \rho D^3/\mu^2)$ tested in the present experiment is 1.25 to 5552 which would cover a majority of the constituents of the dredged material. The various specific weights and the grain sizes of the solid particles used in the present experiments are given in Table (3.1). This table also contains the measured values of α and β. For each value of the parameter $(\gamma_s \rho D^3/\mu^2)$ at least 3 runs were made and the values of α and β are evaluated for each run. The values appearing in Table (3.1) are the average values of all the runs having the same value of $\gamma_s \rho D^3/\mu^2$. Altogether 51 runs were measured and all the data are given in the Appendix. The forms of the

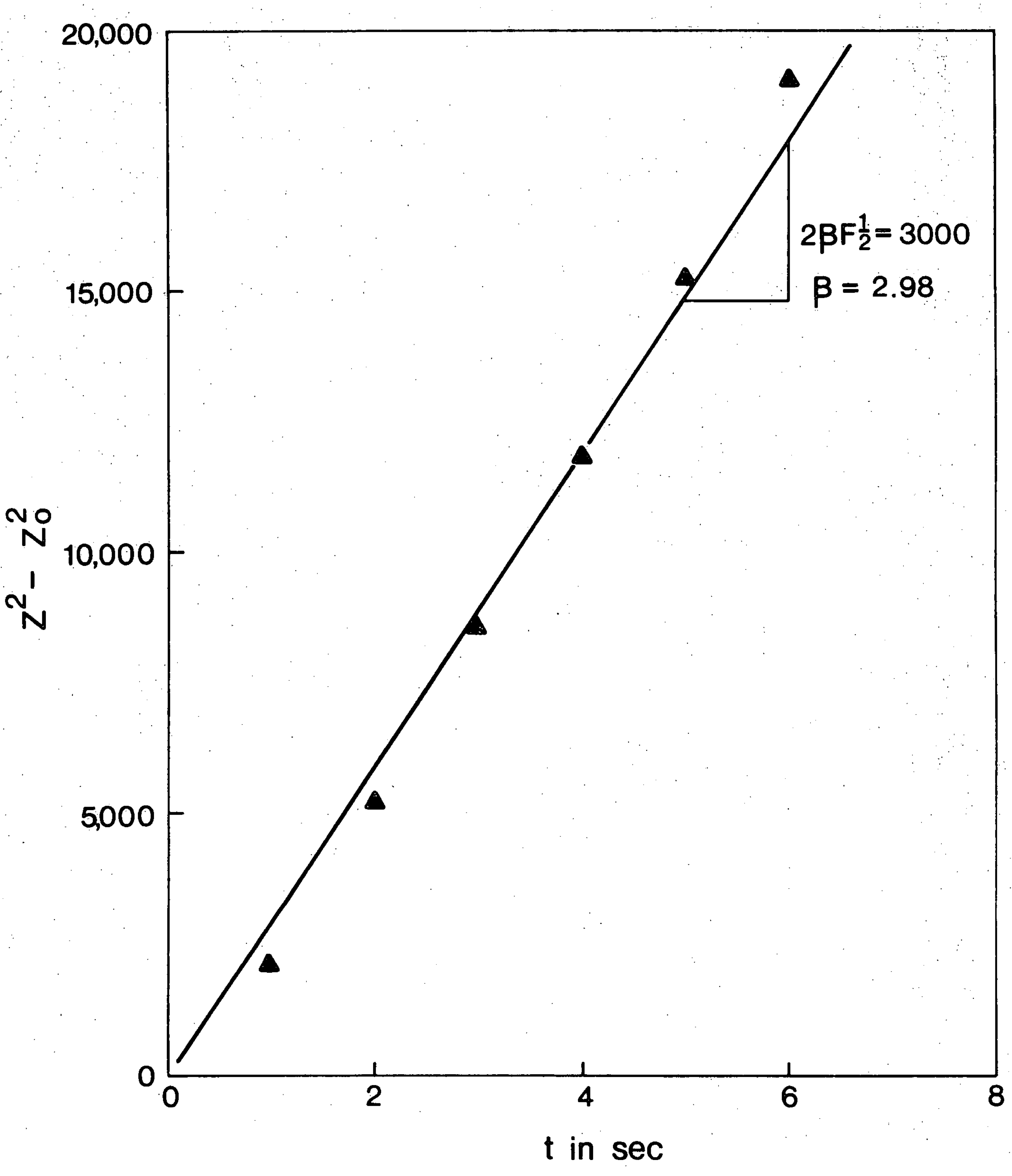


FIG. 3.6 A TYPICAL GRAPH BETWEEN $(Z^2 - Z_o^2)$ and t

TABLE 3.1

Material / Properties	Ottawa Sand		Polystyrene	Glass Beads					
Submerged Specific Weight	1.65×10^3 kg/m^3		$.05 \times 10^3$ kg/m^3	1.50×10^3 kg/m^3					
Grain size in mm	0.700	0.400	1.400	0.235	0.213	0.180	0.151	0.084	.044
$\gamma_s \rho D^3/\mu^2$	5552	1036	1346	238	142	86	51	8.7	1.25
α	0.232	0.251	0.240	0.271	0.286	0.299	0.310	0.312	0.312
β	3.54	1.99	2.25	1.69	1.44	1.23	1.22	1.11	1.13

functions Φ_α and Φ_β are plotted in Figs. (3.7) and (3.8) respectively. It can be seen from these figures that both the dimensionless constants α and β do vary with the parameter $\gamma_s \rho D^3/\mu^2$ approaching to constant values as the parameter decreases. Such a tendency can, indeed, be expected since as the particle size decreases (ie. the parameter $\gamma_s \rho D^3/\mu^2$ decreases) the behaviour of the solid particle cloud should approach to that of the liquid particle cloud, thereby becoming independent of the particle size. The limiting value of α measured in the present experiments is 0.312 which is very close to the value of α for liquid clouds measured by R. S. Scorer [5] and P. J. Sullivan [8]. They obtained a value of 0.310 for the liquid clouds. The limiting value of β is 1.10 while the same measured for the liquid cloud by the above authors is 1.54. The difference between these two values could be due to the difference in shape between the solid particle cloud and liquid cloud.

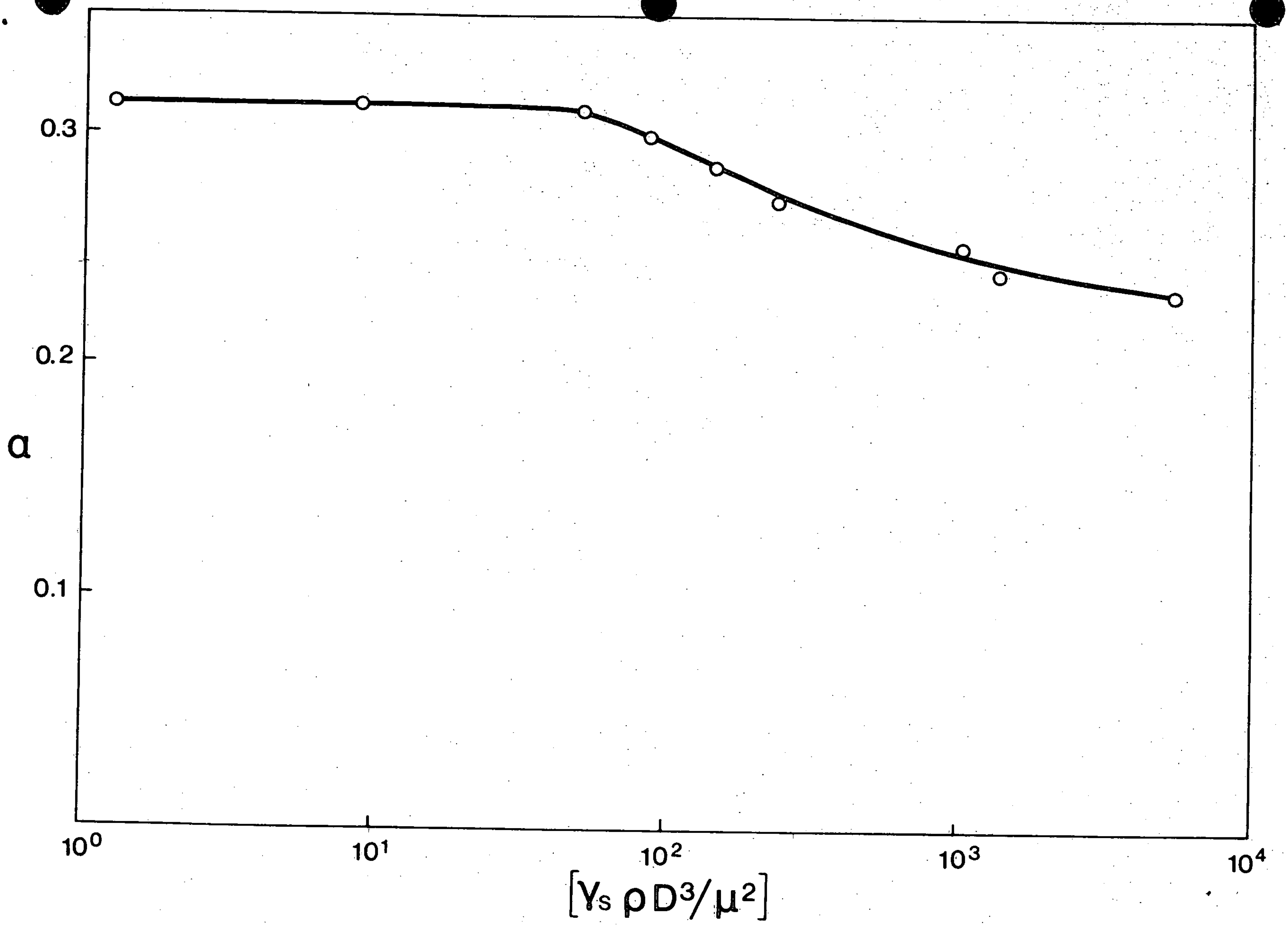


FIG. 3.7

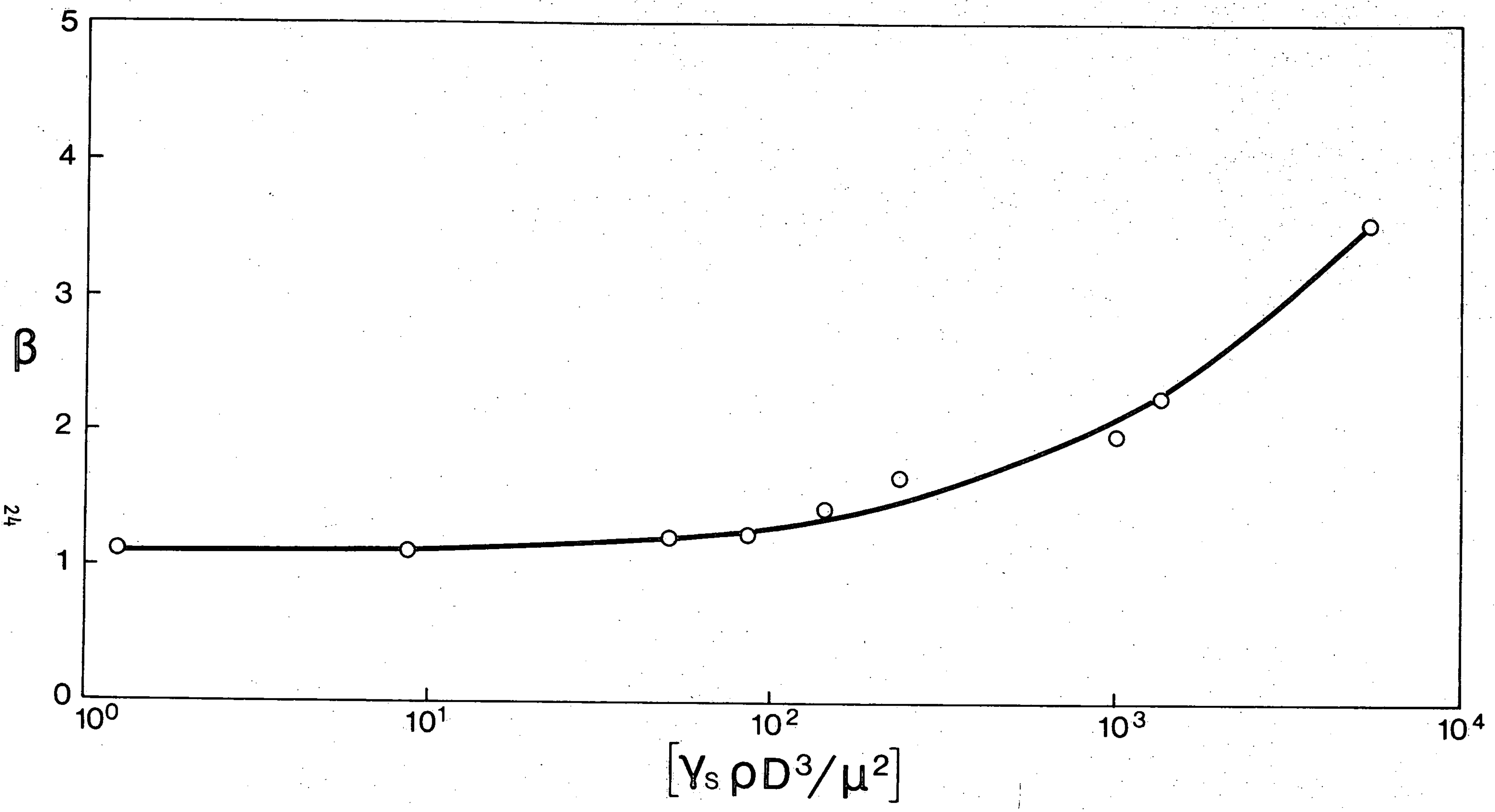


FIG. 3.8

IV. FORMULATION OF THE PRESENT METHOD

In contrast to the solid particle cloud considered in the previous section, the dredged material consists of solid particles of different sizes and the receiving body of water is turbulent with ambient current and density gradient. These complications, however, should not pose any great difficulty for formulating the method to describe the motion of the dredged material when dumped in deep water. Indeed, in this section, a method based on the principle of superposition will be formulated as follows:

Let V_s be the total volume of the dredged material dumped as a slug without any initial momentum in a deep water where the depth is d. Let the dumped material consist of particles of different specific weights and grain sizes, and let V_{sij} be the volume of a fraction of the dumped material whose submerged specific weight is γ_{si} and the mean grain size D_j. Since, during the entrainment phase of the solid particle cloud motion, the dominant force is the negative buoyancy, the effect of turbulence on the entrainment phase is comparatively small and can be neglected. However, turbulence is the dominant factor for spreading during the settling phase and hence the effect of turbulence will be considered only during settling phase of the motion of the dredged material.

The ambient current is considered as uniform and hence its effect is simply to translate the cloud along with it in the horizontal direction. Let U be the magnitude of the ambient current. The effect of the density gradient of the receiving body of water is not considered in the present formulation since a majority of the dredged material is sand and the density differential between the particles and the water is large compared to the variation of density of water over the depth. Let ρ_s be the average density of the receiving body of water.

1. Entrainment Phase:

Unlike in the previous section the position of the dump is taken as the origin and the vertical distance is measured from the level of the dump, and let the vertical co-ordinate be ζ. The horizontal distance is measured from the vertical axis passing through the origin and the cloud is assumed to be symmetrical about the vertical axis passing through the point of maximum concentration of the solid particles. (See Fig. 4.1) If the ambient current is zero then the maximum concentration is along the vertical axis passing through the origin. The size of the dump is equated to the initial size of the cloud and hence the relation between z and ζ becomes:

$$z = \zeta + \frac{R_o}{\alpha_m} \tag{4.1}$$

Where Ro is the radius of the dump and α_m is the entrainment coefficient α corresponding to the mixture of different size particles i.e. the dredged material.

When the mixture of different specific weight and different grain size materials are moving together as a cloud it is hypothesized that each fraction exerts influence on the total behaviour of the cloud in the same proportion as its buoyancy. In other words, if Fij is the buoyancy of the fraction (ij) and F the total buoyancy, then the influence of the fraction (ij) on the total behaviour of the main cloud (or on the coefficients describing the total behaviour) is proportional to the ratio Fij/F. The ratios Fij/F are the weighting coefficients which would determine the behaviour of the total cloud from the

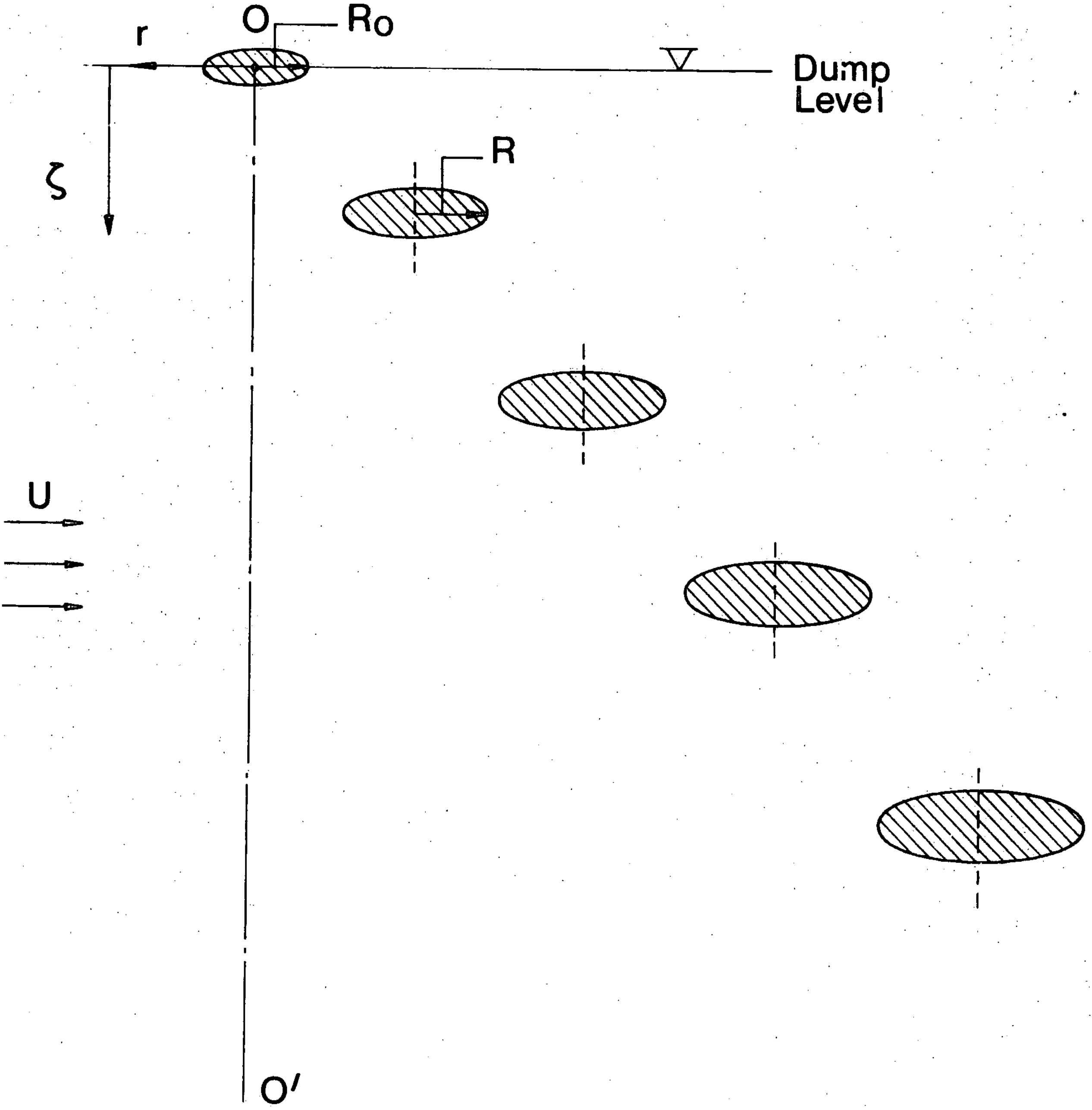


FIG. 4.1 DESCRIPTION OF THE CO-ORDINATE SYSTEM

behaviour of the individual fractions. For example if α_m and β_m are the dimensionless coefficients governing the motion of the total cloud then they can be evaluated using the weighting coefficients as follows:

$$\text{ie} \qquad \alpha_m = \sum_i \sum_j \frac{F_{ij}}{F} \alpha_{ij}$$
$$\text{and} \qquad \beta_m = \sum_i \sum_j \frac{F_{ij}}{F} \beta_{ij} \tag{4.2}$$

$$\text{where} \qquad F_{ij} = \frac{\gamma_{si} \; V_{sij}}{\rho}$$
$$\text{and} \qquad F = \sum_i \sum_j F_{ij}. \tag{4.3}$$

α_{ij} and β_{ij} are the dimensionless coefficients which could be obtained from the graphs (3.7) and (3.8) corresponding to the parameter $\gamma_{si} \; \rho \; D_j^3/\mu^2$.

The behaviour of the cloud of mixture of the particles can, therefore, be expressed as

$$r = R_o + \alpha_m \zeta$$
$$\text{and} \qquad w = \frac{\beta_m F^{1/2}}{(\zeta + \frac{R_o}{\alpha_m})} \tag{4.4}$$

As the cloud of the dredged material moves down the downward vertical velocity decreases and it might become less than the fall velocity of one of its constituents in which case the fraction having fall velocity greater than the cloud velocity would separate out of the main cloud and it would undergo the second phase of the motion, namely, settling phase with spreading due to turbulent diffusion in the horizontal direction. The main cloud, on the other hand, would still undergo entrainment phase of the motion but now the total buoyancy F would have been reduced by the amount of the buoyancy of the separated fraction, say, $F_{k,l}$. Therefore, the total buoyancy of the main cloud now is F' and is given by

$$F' = F - F_{k,l} \tag{4.5}$$

and hence the weighting coefficients would also be altered as follows:

$$W'_{ij} = \frac{F_{ij}}{F'} \quad \text{where} \quad i \neq k \text{ and } j \neq l \tag{4.6}$$

(W'_{ij} are the symbols used for the weighting coefficients)

and

consequently the parameters defining the motion of the new cloud would become

$$\begin{aligned} \alpha_m' &= \sum_{i\neq k} \sum_{j\neq l} W'_{ij} \, \alpha_{ij} \\ \beta_m' &= \sum_{i\neq k} \sum_{j\neq l} W'_{ij} \, \beta_{ij} \end{aligned} \tag{4.7}$$

The level at which such a separation occurs can be calculated knowing the fall velocity of the fraction separated, i.e. $\omega_{k,l}$. Denoting this level by $\zeta_{f\ k,l}$ it can be evaluated as:

$$\zeta_{f\ k,l} = \left(\frac{\beta_m F^{1/2}}{\omega_{k,l}} - \frac{R_o}{\alpha_m} \right) \tag{4.8}$$

using equation (4.4). The size of the main cloud, which is also the same as the size of the separated cloud can be calculated as:

$$R_{f\,k,l} = R_o + \alpha_m \zeta_{f\,k,l} \tag{4.9}$$

again using equation (4.4). The time elapsed $[t_{f\,k,l}]$ from the instant the material is dumped and the instant the fraction (k,l) separated can be calculated using equation (4.4) as follows:

$$w = \frac{d\zeta}{dt} = \frac{\beta_m F^{1/2}}{\zeta + \frac{R_o}{\gamma_m}}$$

or $$\left(\zeta + \frac{R_o}{\alpha_m}\right) d\zeta = \beta_m F^{1/2} dt$$

Integrating both sides and using the initial condition that

$\zeta = o$ when $t = 0$, $t_{f\,k,l}$ becomes:

$$t_{f\,k,l} = \frac{\left(\frac{\zeta_{f\,k,l}}{2} + \frac{R_o}{\alpha_m}\right)}{\beta_m F^{1/2}} \cdot \zeta_{f\,k,l} \tag{4.10}$$

where $\zeta_{f\,k,l}$ is given by equation (4.8)

The lateral distance travelled by the cloud at the time of separation due to the ambient current U is $L_{k,l}$ and is given by

$$L_{k,l} = U t_{f\,k,l} \tag{4.11}$$

After the separation of the fraction (k,l) the behaviour of the main cloud will be described by

$$r = R_{f\,k,l} + \alpha_m' [\zeta - \zeta_{f\,k,l}]$$

$$w = \frac{\beta_m' F'^{\,1/2}}{\left(\zeta + \frac{R_o}{\alpha_m}\right)} \qquad \text{for } \zeta > \zeta_{f\,k,l} \tag{4.12}$$

Again, as can be seen from the equation (4.12) the downward velocity of the cloud decreases as the cloud moves downward and it could reach a value equal to the fall velocity of another fraction, say, (pq). In that case, the fraction (pq) will settle out of the cloud and would undergo settling phase while the main cloud would undergo entrainment phase. The new buoyancy is F'' given by

$$F'' = F - (F_{k,l} + F_{p,q}) \tag{4.13}$$

and the weighting coefficients would be

$$W''_{ij} = \frac{F_{ij}}{F''} \quad \text{for } i \neq k \text{ and } p \text{ and } j \neq l \text{ and } q \tag{4.14}$$

and α_m'' and β_m'' are given as follows:

$$\begin{aligned} \alpha_m'' &= \sum_{i \neq k,p} \sum_{j \neq l,q} W''_{ij}\, \alpha_{ij} \\ \beta_m'' &= \sum_{l \neq k,p} \sum_{j \neq l,q} W_{ij}''\, \beta_{ij} \end{aligned} \tag{4.15}$$

The level at which the second separation occurs is given by:

$$\zeta_{f\ p,q} = \left[\frac{\beta_m' \, F'^{\frac{1}{2}}}{\omega_{pq}} - \frac{R_o}{\alpha_m} \right] \tag{4.16}$$

The size of the cloud at the time of second separation is:

$$R_{f\ pq} = R_{f\ k,l} + \alpha_m'' \, [\zeta_{f\ p,q} - \zeta_{f\ k,l}] \tag{4.17}$$

and the time at which the second separation occurs $t_{f\ p,q}$ can be obtained from

$$\left(t_{f\ pq} - t_{f\ k,l} \right) = \frac{\left[\dfrac{\zeta_{f\ pq}^{\ 2} - \zeta_{f\ k,l}^{\ 2}}{2} \right] - \dfrac{R_o}{\alpha_m} \left(\zeta_{f\ pq} - \zeta_{f\ k,l} \right)}{\beta_m' \, F'^{\frac{1}{2}}} \tag{4.18}$$

The lateral distance travelled is L_{pq} given by

$$L_{pq} = U t_{f\ pq} \tag{4.19}$$

The behaviour of the main cloud after $\zeta_{f\ pq}$ is given by:

$$r = R_{f\ pq} + \alpha_m'' [\zeta - \zeta_{f\ pq}] \quad \text{for } \zeta > \zeta_{f\ pq}$$

$$\text{and} \quad \omega = \frac{\beta_m'' F''^{\frac{1}{2}}}{\zeta + \frac{R_o}{\alpha_m}} \tag{4.19}$$

and the process continues until all the fractions are settled out of the cloud and/or bottom of the deep water is reached.

2. <u>Settling Phase</u>: Let us consider the fraction which is undergoing settling phase and let it be denoted by the subscript (ij). This fraction, while settling with the fall velocity of ω'_{ij}, would also undergo spreading due to turbulent diffusion. Only the horizontal turbulence is considered. The effect of the vertical turbulence is to spread the particle in the vertical direction thereby slowing down the settling particles and it is assumed here that the slowdown is negligible in comparison with the fall velocity of the particles. This is true only when the particle size is bigger; otherwise for smaller size particles the fall velocity will be of the same order of magnitude as the vertical turbulent velocities and the vertical turbulence cannot be neglected. The inclusion of the effect of the vertical turbulence, however, will not pose any problem. For simplicity the vertical turbulence is not considered in the present derivation.

Let us assume that the fraction (ij) separated from the main cloud at a distance of $\zeta_{f\ ij}$ from the dump level and the size at the instant of separation is $R_{f\ ij}$. The time at which the separation occurs is $t_{f\ ij}$. Adopting the method described in section III the initial concentration distribution of the solid particles in the cloud can be expressed as:

$$C = C_{max} \exp\left[- \frac{(r - Ut_{f\,ij})^2}{2\sigma_{o\,ij}^{\;2}} \right] \tag{4.21}$$

where $\sigma_{o\;ij}$ is given by

$$\sigma_{o\;ij} = R_{f\;ij}/8 \tag{4.22}$$

The concentration distribution of the cloud just prior to hitting the bottom would be given by:

$$C = C_{max} \exp\left[- \frac{\left\{ r - U(t_{f\,ij} + \frac{d - \zeta_{f\,ij}}{\omega_{ij}}) \right\}^2}{2\sigma_{ij}^{\;2}} \right] \tag{4.23}$$

where σ_{ij} is given by:

$$\sigma_{ij} = \sigma_{oij} \left[1 + 4^{\;4/3} \cdot \frac{2}{3} \cdot \frac{A}{\sigma_{oij}^{\;\;2/3}} \cdot \left\{ \frac{d - \zeta_{f\,ij}}{\omega_{ij}} \right\} \right]^{3/2} \tag{4.24}$$

and the radius of the cloud at the bottom will be

$$R_{ij} = 8.\ \sigma_{ij} \tag{4.25}$$

When the particles settle at the bottom, they would produce a mound and the distribution of the height of the mound can also be described by a gaussian curve as has been done by R. C. Y. Koh [4].

$$\text{i.e.} \quad h = h_{max} \exp\left[- \frac{(r - Ut)^2}{2\sigma^2} \right] \tag{4.26}$$

Where σ is the standard deviation of the concentration distribution of the particles just prior to hitting the bottom and t is the total duration of the travel of the cloud.

For the settling fraction (ij) the distribution of the height of the mound formed, therefore, can be given by:

$$h_{ij} = h_{\max\ ij} \exp\left[- \frac{\left\{ r - U \left(t_{fij} + \frac{d - \zeta_{fij}}{\omega_{ij}} \right) \right\}^2}{2\ \sigma_{ij}{}^2} \right] \tag{4.27}$$

The value of h_{maxij} can be computed knowing the volume of the fraction V_{sij} and the porosity (n) of the mound. Indeed, by knowing the height distribution the volume of the mound can be evaluated as:

$$V = \int_0^{2\pi} 2\pi r dr . h \tag{4.28}$$

and introducing $V_{sij} = (1-n)V$

the value of hmax becomes:

$$hmax_{ij} = \frac{V_{sij}}{(1-n)\ 2\pi\sigma_{ij}{}^2} \tag{4.29}$$

Similar expressions can be derived for each of the fractions settling out of the main cloud and the total height formed by all the fractions can be obtained by simply adding them. i.e.:

$$h = \sum_i \sum_j h_{ij} \tag{4.30}$$

When the cloud reaches the bottom even before the separation of any of the fractions which is perfectly possible, especially, if the depth of the deep water is not too great as in the case of Great Lakes, the same approach can be used to calculate the height of the mound formed at the bottom. In this case, the assumption of the gaussion distribution for the concentration of the solid particles is extended even to the entrainment phase which is a perfectly valid assumption. In fact, L.M. Brush Jr. [11] have measured the concentration distribution in a jet and concludes that the distribution can be approximated by a gaussion curve.

The method presented above can be best understood by solving the following example:

3. Illustrative Examples;

Example 1: Let us assume that $8m^3$ of dredged materials were dumped as a slug in a deep water area where the depth is 150 metres. Let the constituents of the dump be as follows:

	Grain Size in mm	Specific Weight in Water Kg/m^3	% by Volume
1)	0.700	1650	10
2)	0.253	1650	20
3)	0.180	1650	30
4)	0.044	1650	40

Let us assume that the vertical turbulence is negligible and there is no ambient current.

Since the specific weight of all the constituents is the same the subscript i will be dropped and j varies from 1 to 4.

Evaluation of the Weighting Coefficients: F_j/F

$$F = \gamma s \quad Vs/\rho = 1.65 \times 9.81 \times 8 = 129.49 \ m^4/sec^2$$

$$F_1 = 1.65 \times 9.81 \times 0.8 = 12.949 \ m^4/sec^2$$

$$F_2 = 1.65 \times 9.81 \times 1.6 = 25.898 \text{ '' ''}$$

$$F_3 = 1.65 \times 9.81 \times 2.4 = 38.847 \text{ '' ''}$$

$$F_4 = 1.65 \times 9.81 \times 3.2 = 51.796 \text{ '' ''}$$

$$\therefore \quad W_1 = \frac{F_1}{F} = 0.1$$

$$W_2 = \frac{F_2}{F} = 0.2$$

$$W_3 = \frac{F_3}{F} = 0.3$$

$$W_4 = \frac{F_4}{F} = 0.4$$

Determination of α_j, β_j and ω_j

The parameters $\gamma s \ \rho D_j^3/\mu^2$ for each fractions and the values of α_j and β_j obtained from the graphs (3.7) and (3.8) are as follows:

Grain Size (mm)	$\gamma s \ \rho \ D_j^3/\mu^2$	α_j	β_j	ω_j
0.700	5552	0.232	3.54	10 cm/sec.
0.253	262	0.272	1.52	2.8 cm/sec.
0.180	94.6	0.295	1.30	
0.044	1.38	0.312	1.10	

$$\alpha_m = \sum_{j=1}^{4} W_j\alpha_j = (0.1 \times 0.232) + (0.2 \times 0.272) + (0.3 \times 0.295) + (0.4 \times 0.312)$$

$$= \underline{\underline{0.2910}}$$

$$\beta_m = \sum_{j=k}^{4} W_j\beta_j = (0.1 \times 3.54) + (0.2 \times 1.52) + (0.3 \times 1.30) + (0.4 \times 1.10)$$

$$= \underline{\underline{1.488}}$$

The equations describing the behaviour of the dumped material are:

$$r = 2 + 0.291\zeta \text{ (taking } R_o \text{ as 2 metres)}$$

$$w = \frac{1.488 \times 129.49\ 1/2}{\zeta + 6.87} = \frac{16.93}{\zeta + 6.87}$$

<u>To check whether the fraction (1) would separate before the cloud reaches the bottom.</u>

The distance needed for w to become ω_1 is:

$$\zeta_{f_1} = \frac{16.93}{0.1} - 6.87$$

$$+ 169.3 - 6.87 = 162.46 \text{ metres.}$$

Since the depth is only 150 metres the separation will not occur and the whole cloud undergoes entrainment phase until it hits the bottom.

The radius of the cloud when it hits the bottom is:

$$R_f = 2 + 0.291 \times 150 = \underline{\underline{45.65}} \text{ metres.}$$

The standard deviation σ of the concentration or the height of the mound formed at the bottom is, therefore:

$$\sigma = R_f/8 = 45.65/8 = 5.71 \ \underline{\underline{\text{metres}}}.$$

and the maximum height of the mound formed at the bottom is:

$$h_{max} = \frac{Vs}{(1-n)2\pi\sigma^2} = \frac{8}{(1-n)204.75}$$

choosing a value of 0.33 for n, hmax becomes:

$$h_{max} = .\underline{\underline{0586}} \text{ metres}$$

Example 2: Let us assume that the total volume of the dump is only $1m^3$ with the same proportion of the same constituents. As before there is no ambient current and the effect of vertical turbulence is neglected.

The weighting coefficients are:

$W_1 = 0.1$

$W_2 = 0.2$

$W_3 = 0.3$

$W_4 = 0.4$

and $\alpha_m = 0.291$ and $\beta_m = 1.488$ since the constituents and their proportions are the same.

The equations describing the motion of the cloud becomes;

$$r = 1 + 0.291\ \zeta \text{ (taking the initial radius as 1 metre)}$$

and

$$w = \frac{1.488 \times (1.65 \times 9.81 \times 1)\ 1/2}{\zeta + (^1/o.291)} = \frac{5.986}{\zeta + 3.44}$$

To check whether the separation of fraction (1) will occur:

The depth required for separation is:

$$\zeta_{f_1} = \frac{5.986}{0.1} - 3.44$$

$$= 59.86 - 3.44$$

$$= \underline{\underline{56.42}} \text{ metres}$$

Since the total depth is 150 metres the separation of 0.7mm fraction is possible. Therefore, at the depth of 56.42 metres the 0.7mm fraction would settle out of the main cloud and would undergo settling phase.

The radius of the cloud at the instant of separation $\Big\}$ $R_{f_1} = 1 + 0.291 \times 56.42$

$$= \underline{\underline{17.41}} \text{ metres}$$

The time required for the first separation is: $\Big\}$

$$t_{f_1} = \frac{\left[\frac{\zeta_{f_1}}{2} + \frac{R_o}{\alpha_m}\right]\zeta_{f_1}}{\beta_m \, F\ 1/2} = \frac{\left(\frac{56.42}{2} + 3.44\right) 56.42}{5.986}$$

$$= 298 \text{ sec} = \underline{\underline{4.97}} \text{ min.}$$

Consider the main cloud undergoing entrainment:

$$F' = F - F_1$$

$$= 1.65 \times 9.81 \times 1 - 1.65 \times 9.81 \times 0.1$$

$$= 14.567 \text{ m}^4/\text{sec}^2$$

$$F_2 = 1.65 \times 9.81 \times 0.2 = 3.237 \text{ m}^4/\text{sec}^2$$

$$F_3 = 1.65 \times 9.81 \times 0.3 = 4.856 \quad '' \quad ''$$

$$F_4 = 1.65 \times 9.81 \times 0.4 = 6.475 \quad '' \quad ''$$

$$\therefore \quad W_2 = \frac{F_2}{F^1} = 0.222$$

$$W_3 = \frac{F_3}{F^1} = 0.333$$

$$W_4 = \frac{F_4}{F^1} = 0.444$$

$$\alpha'_m = \sum_{j=2}^{4} \alpha_j W_j = (0.222 \times 0.272) + (0.333 \times 0.295) + (0.444 \times 0.312)$$

$$= \underline{\underline{0.297}}$$

$$\beta'_m = \sum_{j=2}^{4} \beta_j W_j = (0.222 \times 1.52) + (0.33 \times 1.30) + (0.444 \times 1.10)$$

$$= \underline{\underline{1.260}}$$

The equations describing the behaviour of the cloud becomes:

$$r = 17.41 + 0.297\,(\zeta - 56.42) \text{ for } \zeta > 56.42 \text{ metres}$$

and

$$w = \frac{1.260 \times (14.567)^{1/2}}{\zeta + 3.44} = \frac{4.809}{\zeta + 3.44}$$

<u>To check whether the separation of the fraction (2) will occur:</u>

The depth required for w to become ω_2 is:

$$\zeta_{f_2} = \frac{4.809}{.028} - 3.44$$

$$= \underline{\underline{170.36}} \text{ metres}$$

Since the depth is only 150 metres the separation of the fraction (2) will not occur and the cloud reaches the bottom while undergoing entrainment phase.

The radius of the cloud when it hits the bottom is:

$$R_f = 17.41 + 0.297\,(150 - 56.42)$$

$$= \underline{\underline{45.20}} \text{ metres}$$

The time required from the first separation until it hits the bottom can be evaluated as follows:

$$w = \frac{4.809}{\zeta+3.44}$$

or $$\frac{d\zeta}{dt} = \frac{4.809}{\zeta+3.44}$$

$$(\zeta+3.44)\, d\zeta = 4.809\, dt.$$

integrating:

$$\zeta^2 + 3.44\zeta = 4.809 + C$$

when $t = t_{f_1}$ $\zeta = \zeta_{f_1} = 56.42$

$$\therefore \quad C = 352.7$$

$$\therefore \quad \frac{\zeta^2}{2} + 3.44\zeta = 4.809t + 352.7$$

or $$t = \frac{1}{4.809} \left[\frac{\zeta^2}{2} + 3.44\,\zeta - 352.7 \right]$$

when $\zeta = 150$ metres $t = t_f$

or $t_f = 2127$ sec $= \underline{\underline{35.45}}$ min.

The maximum height of the mound formed at the bottom due to this cloud.

$$\text{hmax} = \frac{V_s - V_{s1}}{(1-n)2\pi\sigma^2}$$

$$= \frac{0.9 \times 64}{(1-n)2\pi(45.20)^2}$$

$$= \underline{\underline{6.7}} \text{ mm}$$

<u>Consider the fraction (1) undergoing settling phase:</u>

The standard deviation σ_1 of the initial distribution of the settling cloud is:

$$\sigma_1 = \frac{R_{f1}}{8} = 17.41/8 = 2.176 \text{ metres}$$

The time required for the settling particles to reach the bottom is t_1 given by:

$$t_1 = \frac{d - \zeta_{f1}}{\omega_1} = \frac{150 - 56.42}{0.1}$$

$$= \underline{\underline{935.8}} \text{ sec}$$

.. The standard deviation of the distribution of the solid particles just prior to reaching the bottom is:

$$\sigma = 2.176 \left[1 + 4^{4/3} \frac{2}{3} \frac{0.000068}{(2.176)^{2/3}} 935.8 \right]^{1.5}$$

$$= \underline{\underline{2.72}} \text{ metres}$$

.. The radius of the cloud while hitting the bottom is:

$$R_{f1} = 2.72 \times 8 = \underline{\underline{21.76}} \text{ metres}$$

The maximum height of the mount formed due to the settling of this fraction is:

$$hmax_1 = \frac{V_{s1}}{(1-n)2\pi\sigma_1^2} = \frac{0.1}{\frac{2}{3}\ 2\pi\ (2.72)^2}$$

$$= \underline{\underline{3.2}} \text{ mm}$$

Therefore, the total height of the mound formed by dumping $1 m^3$ of the dredged material is: $6.7 + 3.2 = 9.9$ mm $\approx \underline{\underline{1 \text{ cm}}}$

If 8 such dumps are made then the maximum height of the mound created would be 8 cm.

Note that if all $8 m^3$ of the dredged material were dumped at once the maximum height of the mound formed would only be ≃ 6 cm. The radius of the cloud in both cases is more or less the same.

Example 3:

Let an ambient current of 0.10 metre/sec be introduced in Example 2. The vertical turbulence is still assumed to be negligible.

In this case, when the separation of the fraction (1) i.e. 0.700 mm grains occurs the cloud would have displaced in the horizontal direction by an amount L_1 given by:

$$L_1 = Ut_{f1} = 0.1 \times 298 = \underline{\underline{29.8}} \text{ metres}$$

After the first separation the main cloud would have moved horizontally a distance of L_2 before hitting the ground. L_2 is given by:

$$L_2 = U\,[tf - t_{f1}] = \underline{\underline{182.9}} \text{ metres}$$

The total horizontal distance moved by the cloud undergoing entrainment $L = 182.9 + 29.8 = 212.7$ metres.

The height distribution of the mound formed at the bottom due to this cloud is:

$$h = \text{hmax}\ e^{\frac{-(r-212.7)^2}{2 \times 5.652}}$$

where hmax = 0.0067 metres

Consider the fraction undergoing settling phase:

The lateral distance travelled during settling is L_s given by:

$$L_s = Ut = 0.10 \times 935.8 = 93.58 \text{ metres.}$$

The total horizontal displacement is 93.58 + 29.8 = 123.38. Therefore, the distribution of the height of the mound formed by the fraction undergoing settling is:

$$h_1 = \text{hmax}_1\ e^{-\frac{(r-123.38)^2}{2 \times (2.72)^2}}$$

where hmax = 0.0032 metres.

The distribution of the net height of the mound formed by both the clouds is:

$$h = h + h_1 = .0067\ e^{-\frac{(r-212.7)^2}{63.85}} + .0032\ e^{-\frac{(r-123.38)^2}{14.80}}$$

$$h_{212.7} = .0067 + .0032\ e^{-539} = .0067 \text{ metres}$$

$$h_{123.38} = .0067\ e^{-124.95} + .0032 = .0032 \text{ metres}$$

and hence the maximum height formed because of such dumping is 6.7mm. Note that the cloud in this case has been broken up into two pieces and they are deposited on the bottom, far apart from each other. If there were 8 such dumps then the maximum height formed would be only 5.36cm and it occurs at a distance of 213 metres from the location of the dump.

4. Conclusions:

From the above examples, the following observations can be made. First of all, it can be seen that the settling phase will occur only if the amount of material dumped at one time is less or the depth at the location of the dump is of the order of thousands of metres as in the case of ocean dumping. However, in the case of lake dumping where the depth is only hundreds of metres the possibility for the occurrence of settling phase is rare and the entrainment phase will predominate. Consequently, methods, such as the method of Koh and Chang, which devote considerable attention for the long term dispersion phase, will be unnecessarily elaborate.

The present method treats the entrainment phase elaborately and the input parameters required for the entrainment phase were obtained from the laboratory measurements and therefore, it is more suitable for lake dumping problems.

Furthermore, the method is simple and it doesn't require the use of computers for its application.

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4.	Robert C.Y. Koh	"Ocean Sludge Disposal by Barges" Water Resources Research, Vol.7 No.6 Dec. 1971
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7.	B.R. Morton et al	"Turbulent Gravitational Convection from Maintained and Instantaneous Sources", Royal Society of London Vol. A234, pp1-23, 1956
8.	P.J. Sullivan	"The Penetration of a Density Interface by Heavy Vortex Rings", Water, Air and Soil Pollution 1, pp326-336 1972
9.	G.K. Batchelor	"Heat Convection and Buoyancy effects in Fluids", Quarterly Journal Royal Meteorological Society, Vol. 80, pp339-358, 1954.
10.	J.S. Turner	"The Dynamics of Spheroidal Masses of Buoyant Fluid", Journal of Fluid Mechanics, 1964.
11.	L.M. Brush, Jr.	"Exploratory Study of Sediment Diffusion," Journal of Geophysical Research Vol. 67, No. 4, April 1962.

APPENDIX

TEST NO: 1

MATERIAL: Ottawa Sand

DUMP SIZE: 5.72 cm

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{kg/m}^3$

D = 0.700 mm

VIRTUAL ORIGIN IS AT Y* = 362

z_o = z** at t=0 = 74

α = 0.232

β = 2.98

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	264	10	288	13	0	74	0
2	254	18	275	22	1	87	2063
3	239	21	259	26	2	103	5247
4	229	25	244	31	3	118	8646
5	218	28	231	35	4	131	11866
6	208	28	218	35	5	144	15260
7	198	28	205	35	6	157	19173
11	188	31	192	38	10	170	23424
12	152	32	148	40			
13	142	32	135	40			
14	127	32	116	40			
15	117	32	103	40			
16	107	33	91	41			
19	81	32	59	40			
20	71	32	46	40			
21	61	33	33	41			
22	60	33	27	41			
23	48	33	14	41			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 2

MATERIAL: Ottawa Sand

DUMP SIZE: 5.72 cm

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{ kg/m}^3$

$D = 0.700$ mm

VIRTUAL ORIGIN IS AT Y* = 362

$z_o = z^{**}$ at $t=0 = 68$

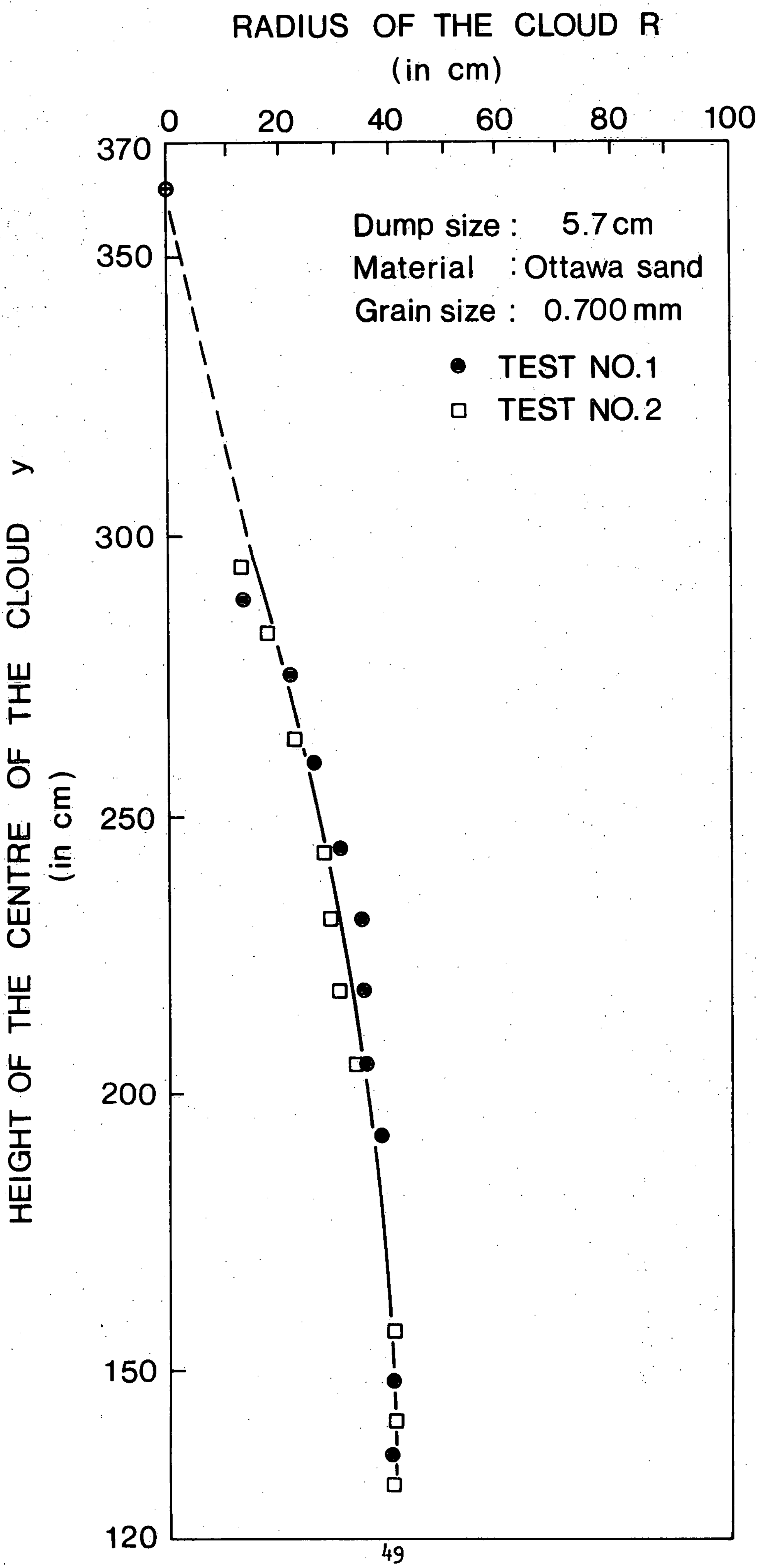
$\alpha = 0.232$

$\beta = 3.25$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13	0	68	0
2	259	14	282	18	1	80	1870
3	243	18	263	23	2	99	5291
4	228	22	243	28	3	119	9441
5	218	23	231	29	4	130	12605
6	208	24	218	31	5	143	16120
7	198	27	205	34	6	156	19712
11	160	32	157	41			
12	143	33	141	41			
13	137	33	129	41			
14	127	33	116	42			
15	116	34	103	44			
16	111	35	97	45			
19	81	38	59	48			
20	71	38	46	48			
21	61	38	33	48			
22	50	38	21	49			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7 cm
Material : Ottawa sand
Grain size : 0.700 mm
● TEST NO.1
□ TEST NO. 2

TEST NO: 3

MATERIAL: Ottawa Sand

DUMP SIZE: 3.8 cm

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \mathrm{kg/m}^3$

D = 0.700 mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = **z** at t=0** = 59

α = 0.232

β = 4.13

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
2	254	10	275	13	0	59	0
3	243	12	263	16	1	71	1644
4	233	15	250	19	2	84	3628
5	223	15	237	19	3	97	5921
6	213	15	224	20	4	110	8535
7	203	17	212	22	5	122	11497
8	193	17	199	22	6	135	14744
9	182	19	186	24	7	148	18423
12	157	20	154	25	10	160	22119
13	147	21	141	26	11	173	26448
14	137	21	129	27			
15	129	22	119	28			
16	122	22	110	29			
17	111	23	97	29			
18	101	23	84	29			
21	76	24	52	30			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 4

MATERIAL: Ottawa Sand

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \, kg/m^3$

D = 0.700 mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = z** at t=0 = 46

α = 0.232

β = 3.73

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
2	264	8	288	11	0	46	0
3	251	10	272	13	1	62	1706
4	241	12	259	15	2	75	3432
5	231	13	247	17	3	87	5480
6	218	14	231	17	4	103	8520
7	210	15	221	19	5	113	10653
8	193	17	199	22	6	125	13509
9	185	17	189	22	7	135	16109
12	162	18	161	23	10	163	24453
13	152	18	148	23			
14	139	19	132	24			
15	132	20	122	25			
16	121	20	110	25			
17	111	21	97	27			
18	101	22	84	28			
21	73	22	49	29			
22	66	22	40	29			
23	55	22	27	29			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 5

MATERIAL: Ottawa Sand

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65\text{x}10^3 \text{kg/m}^3$

D = 0.700 mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = z** at t=0 = 46

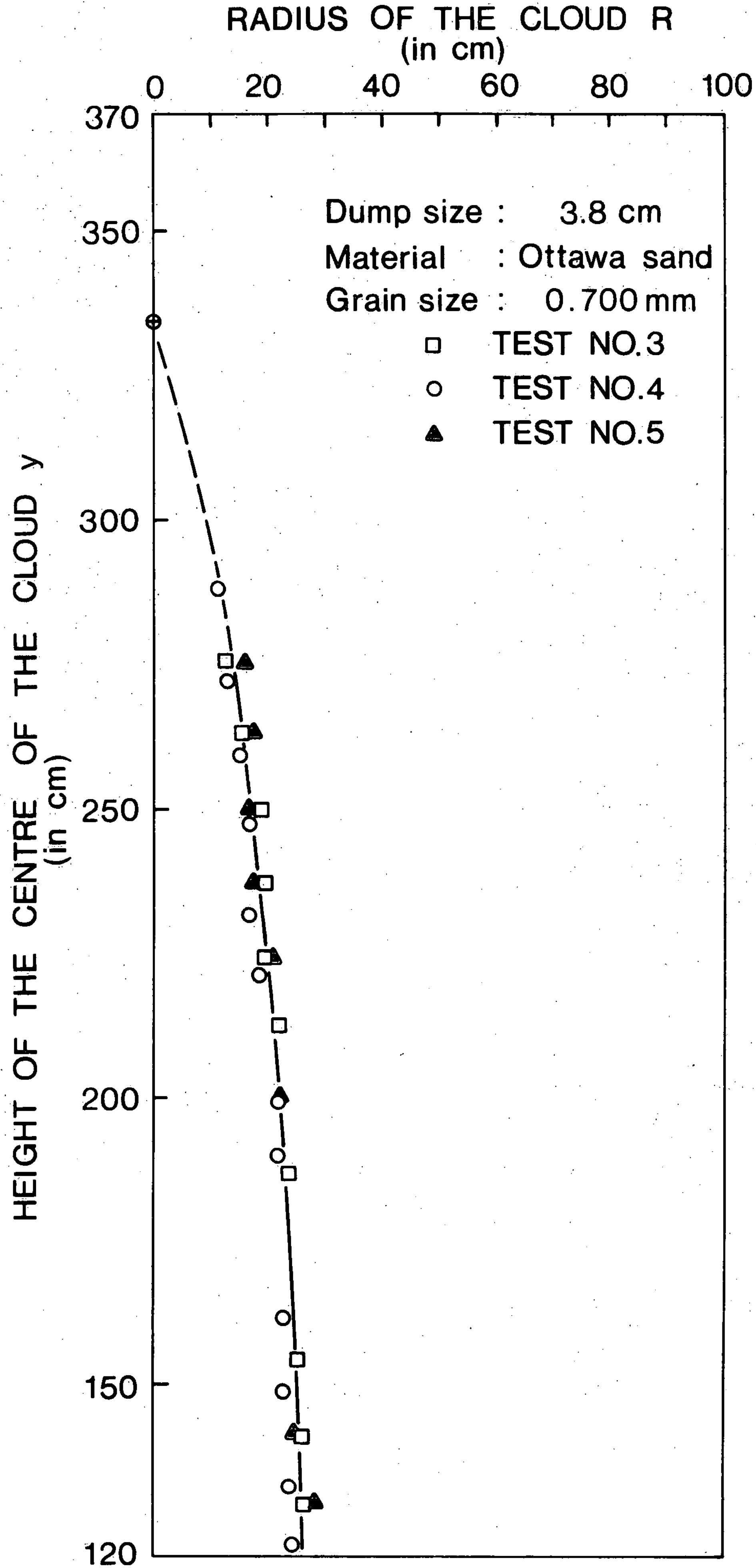
α = 0.232

β = 3.60

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
2	264	8	288	11	0	46	0
3	254	12	275	16	1	59	1322
4	243	13	263	17	2	71	2966
5	233	14	250	17	3	84	4950
6	223	14	237	18	4	97	7243
7	213	16	224	21	5	110	9858
8	203	17	212	22	6	122	12768
9	193	17	199	22	7	135	16109
13	157	20	154	25	11	150	20384
14	147	21	141	25			
15	137	22	129	28			
16	127	22	116	29			
17	119	22	106	29			
18	106	22	91	29			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8 cm
Material : Ottawa sand
Grain size : 0.700 mm
□ TEST NO.3
○ TEST NO.4
▲ TEST NO.5

TEST NO: 6

MATERIAL: Ottawa Sand

DUMP SIZE: 7.6 mm

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{kg/m}^3$

$D = 0.400$ mm

VIRTUAL ORIGIN IS AT Y* = 338

z_o = z** at t=0 = 53

$\alpha = 0.253$

$\beta = 1.88$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	261	11	285	14	0	53	0
2	248	16	269	21	1	69	1947
3	231	17	247	22	2	91	5531
4	218	20	231	26	3	107	8693
5	208	24	218	30	4	120	11607
6	198	27	205	33	5	133	14821
7	193	30	199	38	6	139	16563
8	188	33	192	41	7	146	18353
14	152	49	148	62	13	190	33373
15	147	50	141	64	14	197	35850

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 7

MATERIAL: Ottawa Sand

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{kg/m}^3$

$D = 0.400$ mm

VIRTUAL ORIGIN IS AT Y* = 338

z_o = z** at t=0 = 56

$\alpha = 0.253$

$\beta = 2.10$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	259	10	282	13	0	56	0
2	236	15	253	19	1	85	4158
3	226	20	240	25	2	98	6529
4	213	21	224	27	3	114	13383
5	198	22	205	29	4	133	14760
6	193	25	199	32	5	139	16526
7	185	30	189	38	6	149	19299
11	154	38	151	48	10	187	32310
12	147	43	141	54	11	197	36031
13	142	44	135	56	12	203	38573
14	137	48	129	61	13	209	41237
15	132	49	122	62	14	216	43982

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 8

MATERIAL: Ottawa Sand

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{kg/m}^3$

D = 0.400 mm

VIRTUAL ORIGIN IS AT Y* = 338

z_o = z** at t=0 = 63

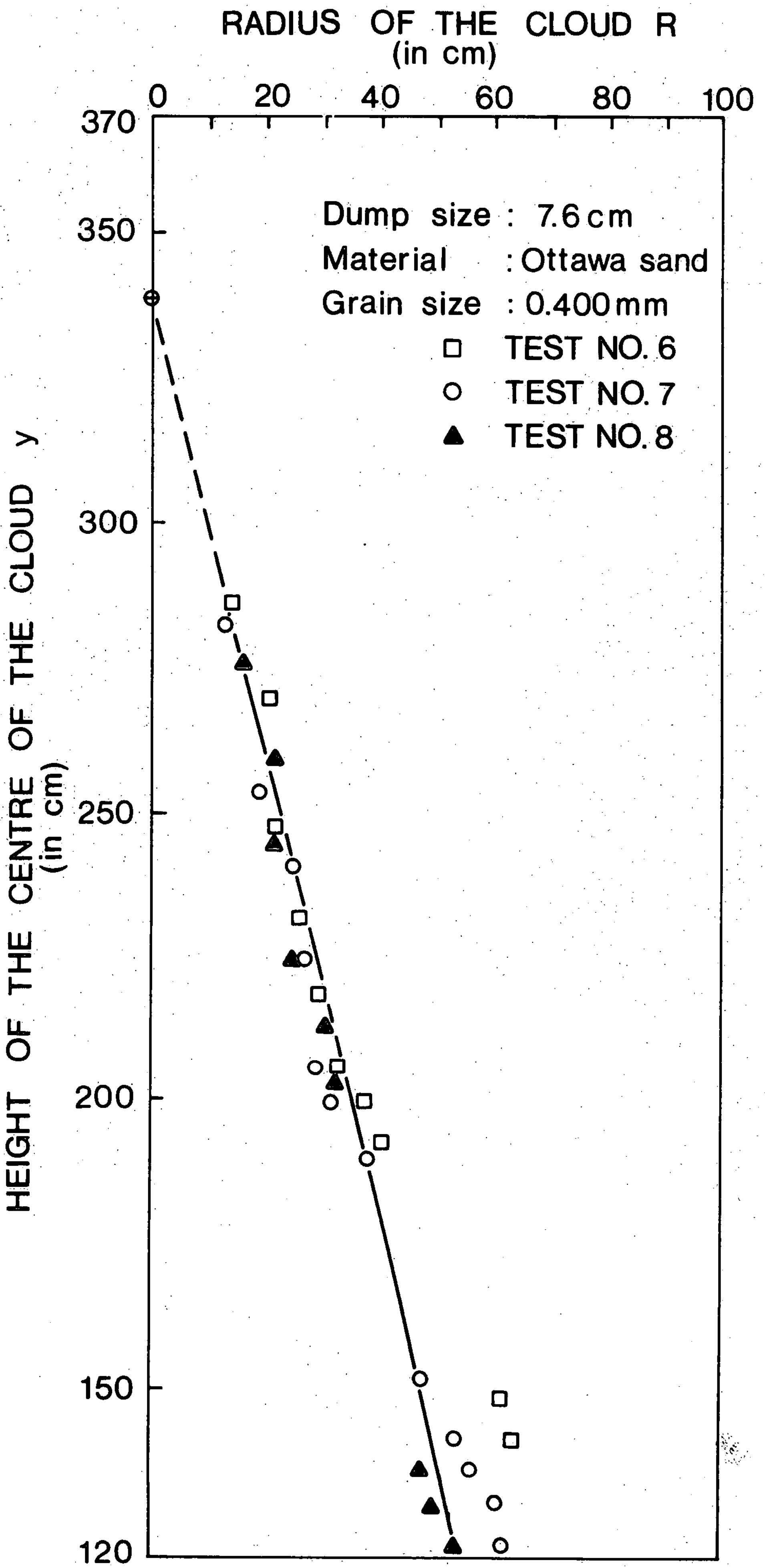
α = 0.253

β = 2.00

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	254	12	275	16	0	63	0
2	238	17	259	22	1	79	2367
3	228	17	244	22	2	94	5049
4	213	20	224	25	3	114	9055
5	203	24	212	31	4	126	12147
6	195	26	202	33	5	136	14654
13	142	38	135	48	12	203	37461
14	137	40	129	50	13	209	40067
15	132	43	122	54	14	216	42796
16	127	43	116	54	15	222	45607

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 7.6 cm
Material : Ottawa sand
Grain size : 0.400 mm
□ TEST NO. 6
○ TEST NO. 7
▲ TEST NO. 8

TEST NO: 9

MATERIAL: Ottawa Sand

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.65×10^4 kg/m^3

D = 0.400 mm

VIRTUAL ORIGIN IS AT Y* = 327

z_o = z** at t=0 = 55

α = 0.250

β = 2.00

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13			
2	251	12	272	16	0	55	0
3	241	15	259	19	1	68	1548
4	231	17	247	22	2	80	3434
5	221	19	234	24	3	93	5630
6	213	22	224	29	4	103	7504
7	203	25	212	32	5	115	10266
8	198	29	205	37	6	122	11781
9	193	33	199	41	7	128	13402
15	157	38	154	48			
16	152	44	148	56			
17	147	44	141	56			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 10

MATERIAL: Ottawa Sand

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.65 \times 10^3 \text{kg/m}^3$

$D = 0.400$ mm

VIRTUAL ORIGIN IS AT Y* = 327

z_o = z** at t=0 = 48

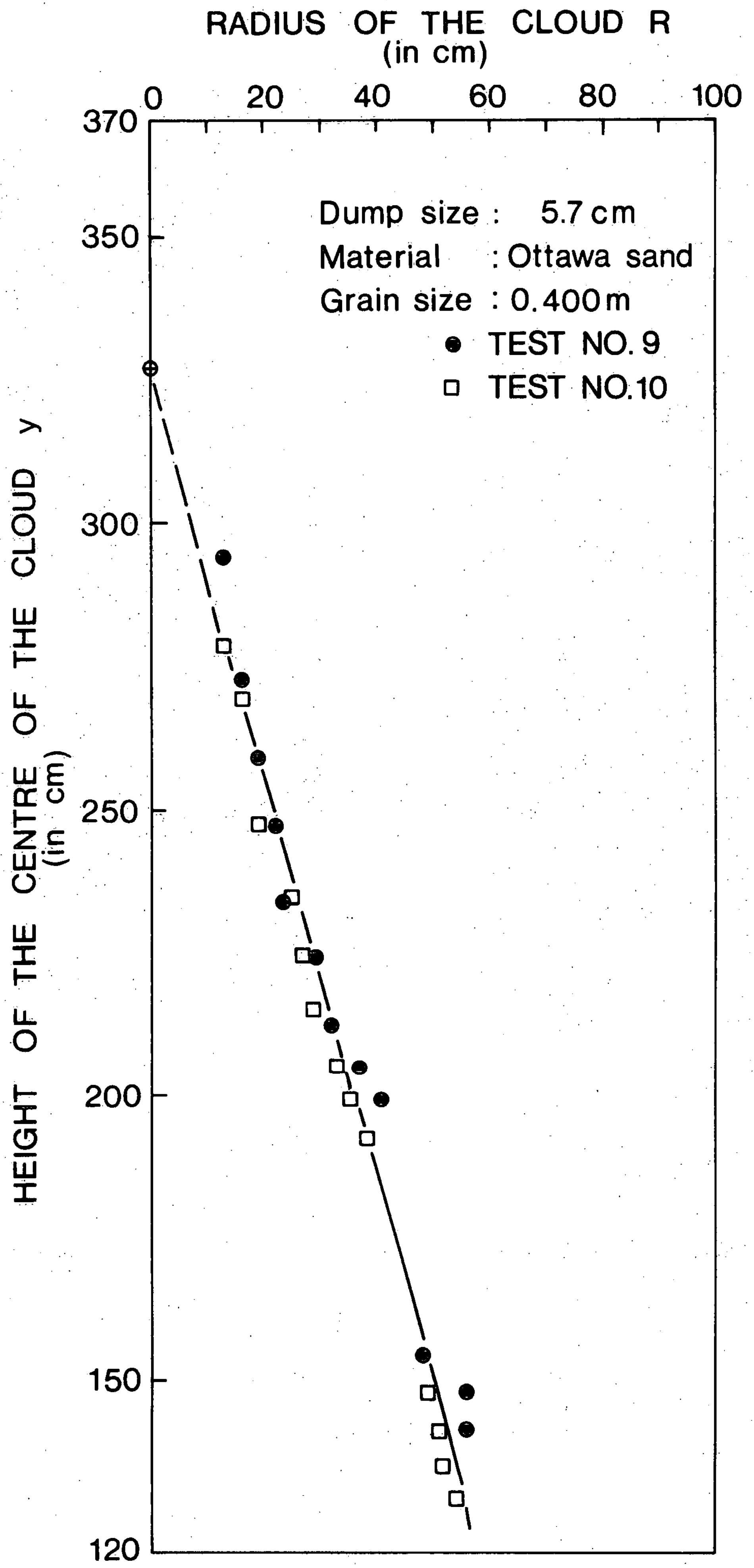
$\alpha = 0.250$

$\beta = 1.83$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	256	10	278	13	0	49	0
2	241	12	259	16	1	68	2196
3	231	15	247	19	2	80	4083
4	221	20	234	25	3	93	6279
5	213	21	224	27	4	102	8152
6	205	22	215	29	5	112	10188
7	193	26	205	33	6	122	12429
8	188	27	199	35	7	128	13999
9	152	30	192	38	8	135	15676
15	76	39	148	49			
16	73	40	141	51			
17	71	41	135	52			
18	68	43	129	54			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7 cm
Material : Ottawa sand
Grain size : 0.400 m
● TEST NO. 9
□ TEST NO.10

TEST NO: 11

MATERIAL: Ottawa Sand

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.65 x 10^3kg/m^3

D = 0.400 mm

VIRTUAL ORIGIN IS AT Y* = 342

z_o = z** at t=0 = 45

α = 0.250

β = 2.18

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	271	8	297	11	0	45	0
2	264	10	288	13	1	54	935
3	256	15	278	19	2	64	2052
4	248	15	269	20	3	73	3348
5	241	16	259	21	4	83	4842
6	236	17	253	22	5	89	5896
7	228	19	243	25	6	99	7776
8	223	20	237	25			
9	218	21	231	26			
10	213	21	224	27			
11	208	22	218	28			
12	203	23	212	29			
13	198	24	205	30			
14	193	25	199	32			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 12

MATERIAL: Ottawa Sand

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.65 x 10^3 kg/m^3

D = 0.400 mm

VIRTUAL ORIGIN IS AT Y* = 342

z_o = z** at t=0 = 45

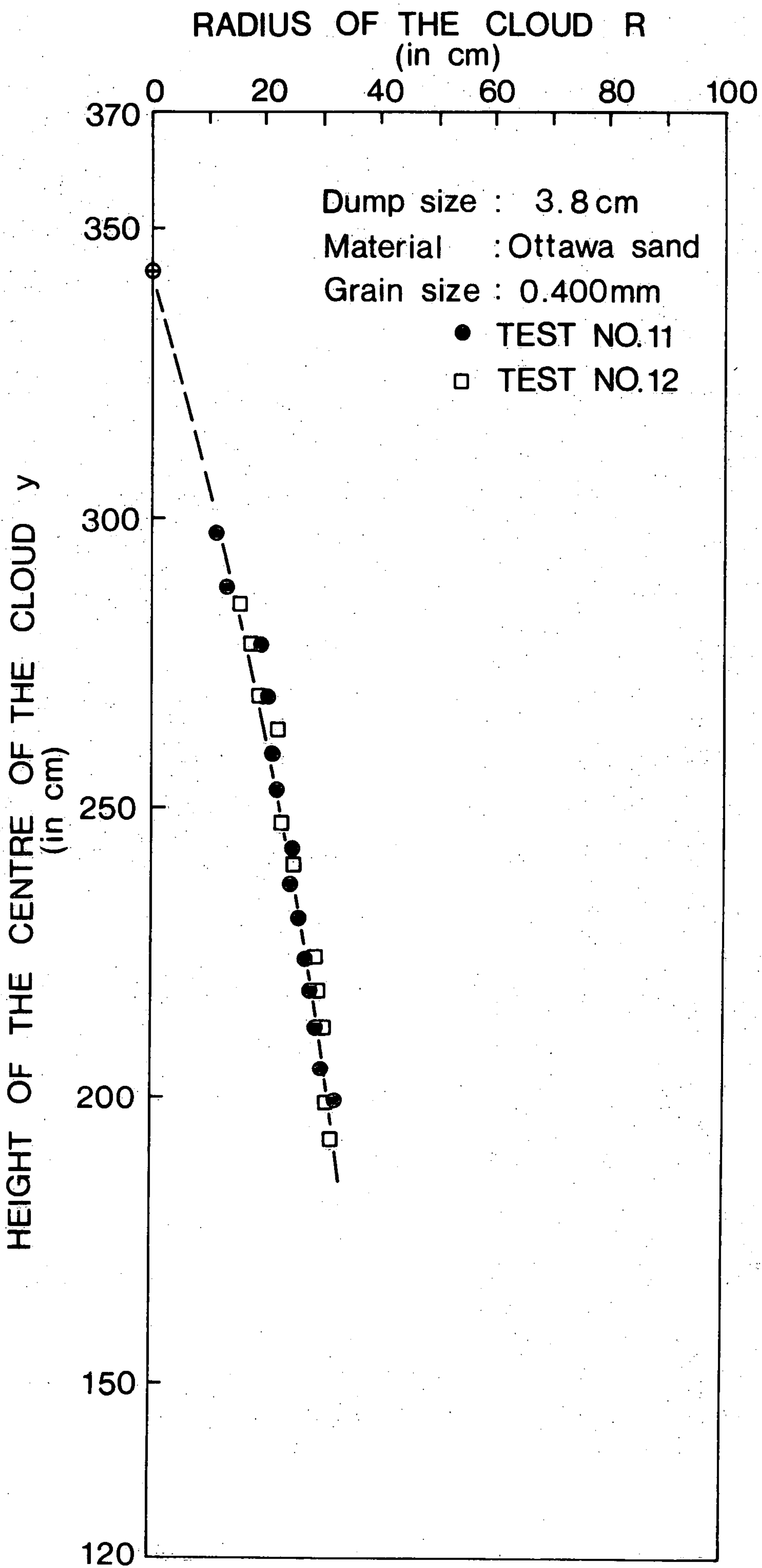
α = 0.250

β = 1.96

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	271	8	297	11	0	45	0
2	261	12	285	15	1	57	1322
3	256	14	278	17	2	64	2110
4	248	15	269	19	3	73	3432
5	243	17	263	22	4	79	4424
6	236	17	253	22	5	89	6047
7	231	18	247	23	6	95	7000
8	226	20	240	25	7	102	8379
9	218	21	231	26	8	111	10296
10	213	22	224	29			
11	208	23	218	29			
12	203	24	212	30			
13	198	24	205	30			
14	193	24	199	31			
15	188	25	192	32			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8cm
Material : Ottawa sand
Grain size : 0.400mm
● TEST NO.11
□ TEST NO.12

TEST NO: 13

MATERIAL: Polystyrene

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 3 sec.

γ_s = .05 × 10^3 kg/m^3

D = 1.4 mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 58

α = 0.240

β = 1.82

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	264	8	288	11	0	58	0
2	254	11	275	14	3	71	1644
3	251	12	272	16	6	74	2109
4	246	15	266	19	9	80	3101
5	241	15	259	19	12	87	4158
6	233	15	250	19	15	96	5852
7	228	16	243	21	18	103	7245
8	223	17	237	22			
9	218	17	231	22			
10	210	17	221	22			
11	203	17	212	22			
12	198	20	205	25			
13	195	20	202	25			
14	193	20	199	25			
15	190	20	196	25			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 14

MATERIAL: Polystyrene

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 3 secs.

γ_s = .05 x 10^3kg/m^3

D = 1.40 mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 52

α = 0.240

β = 2.07

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	12	294	16	0	52	0
2	259	15	282	19	3	64	1652
3	254	15	275	19	6	71	2537
4	246	15	266	20	9	80	4003
5	243	16	263	21	12	83	4538
6	238	17	259	22	15	87	4865
7	233	18	250	23	18	96	6512
8	228	20	243	25			
9	223	20	237	25			
10	218	20	231	25			
11	213	20	224	25			
12	208	20	218	25			
13	203	20	212	25			
14	198	20	205	25			
15	193	20	199	25			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 15

MATERIAL: Polystyrene

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 3 secs.

γ_s = $0.05 \times 10^3 \text{kg/m}^3$

D = 1.40 mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 61

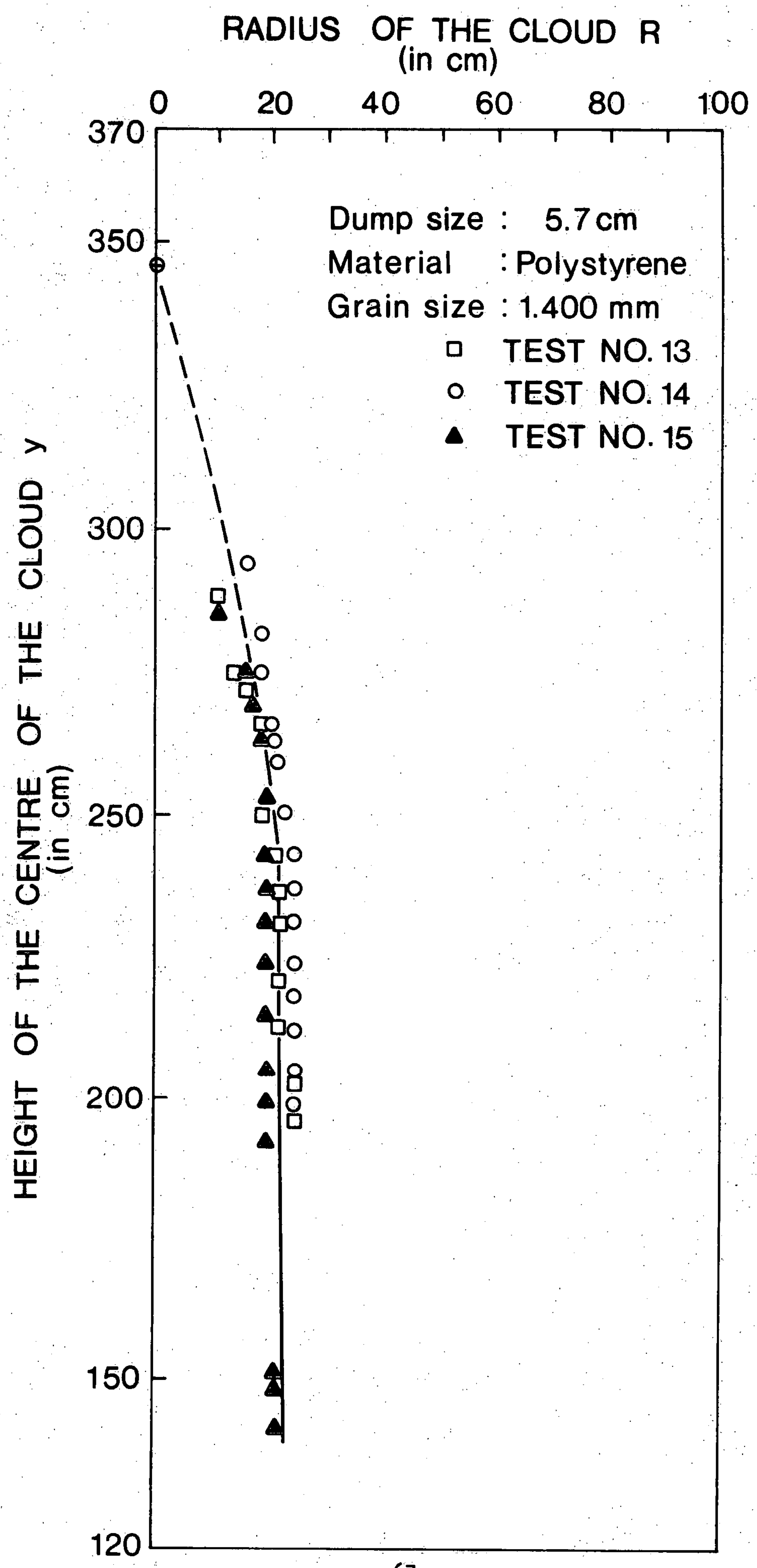
α = 0.240

β = 2.09

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	261	8	285	11	0	61	0
2	254	12	275	16	3	71	1360
3	248	13	269	17	6	77	2305
4	243	15	263	19	9	83	3345
5	236	15	253	20	12	93	5041
6	228	15	243	20			
7	223	15	237	20			
8	218	15	231	20			
9	213	15	224	20			
10	205	15	215	20			
11	198	15	205	20			
12	193	15	199	20			
13	188	15	192	20			
19	154	17	151	22			
20	152	17	148	22			
21	147	17	141	22			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7 cm
Material : Polystyrene
Grain size : 1.400 mm
□ TEST NO. 13
○ TEST NO. 14
▲ TEST NO. 15

TEST NO: 16

MATERIAL: Polystyrene

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 4 secs.

γ_s = 0.05 x 10^3 kg/m^3

D = 1.40 mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 58

α = 0.240

β = 2.45

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	264	10	288	12	0	58	0
2	259	12	282	16	4	64	811
3	254	12	275	16	8	71	1668
4	248	12	269	16	12	77	2619
5	243	15	263	19	16	83	3635
6	238	16	259	21	20	87	4304
7	233	17	250	22			
8	228	17	243	22			
9	226	17	240	22			
10	223	17	237	22			
11	218	17	231	22			
12	213	17	224	22			
13	208	17	218	22			
14	205	17	215	22			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 17

MATERIAL: Polystyrene

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 4 secs.

$\gamma_s = 0.05 \times 10^3 kg/m^3$

$D = 1.400$ mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 61

$\alpha = 0.240$

$\beta = 2.50$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	261	10	285	13	0	61	0
2	259	11	282	14	4	64	678
3	254	15	275	19	8	71	1559
4	248	15	269	19	12	77	2534
5	243	15	263	19	16	83	3591
6	238	16	259	21	20	87	4712
7	233	17	250	22			
8	228	17	243	22			
9	226	17	240	22			
10	223	18	237	23			
11	215	18	228	23			
12	210	18	221	23			
13	205	18	215	23			
14	200	18	208	23			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 18

MATERIAL: Polystyrene

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 4 secs.

$\gamma_s = 0.05 \times 10^3 kg/m^3$

$D = 1.400$ mm

VIRTUAL ORIGIN IS AT Y* = 346

z_o = z** at t=0 = 52

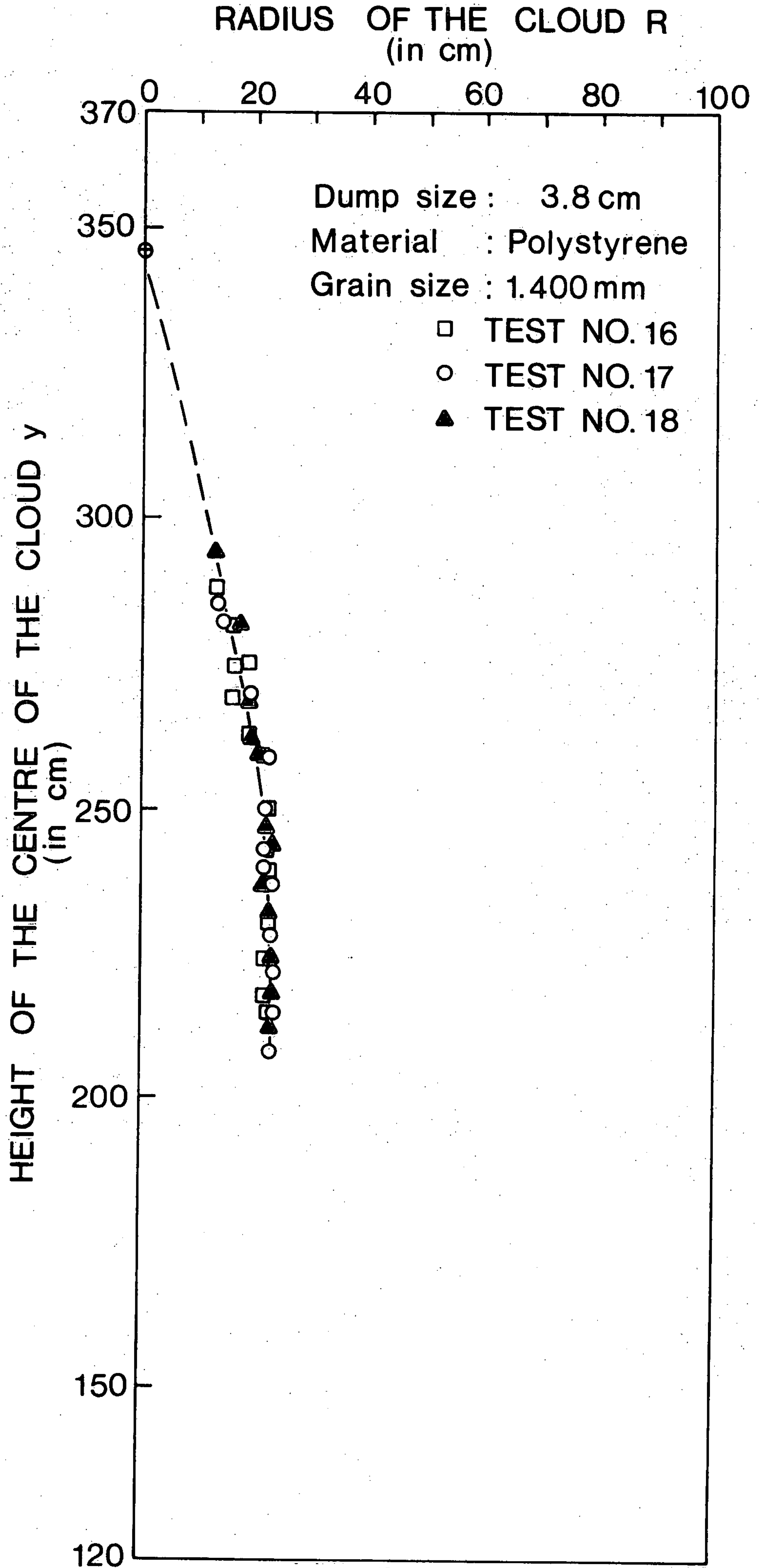
$\alpha = 0.240$

$\beta = 2.58$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13	0	52	0
2	259	12	282	16	4	64	1515
3	254	15	275	19	8	71	2372
4	248	15	269	19	12	77	2867
5	243	15	263	19	16	83	4339
6	238	16	259	21	20	87	4865
7	233	17	250	22			
8	231	17	247	22			
9	228	17	243	22			
10	223	17	237	22			
11	218	17	231	22			
12	213	17	224	22			
13	208	17	218	22			
14	203	18	212	23			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8 cm
Material : Polystyrene
Grain size : 1.400 mm
□ TEST NO. 16
○ TEST NO. 17
▲ TEST NO. 18

TEST NO: 19

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.253$ mm

VIRTUAL ORIGIN IS AT Y* = 353

$z_o = z^{**}$ at $t=0$ = 110

$\alpha = 0.258$

$\beta = 1.83$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	12	294	16			
2	228	15	243	19	0	110	0
3	203	25	212	32	2	141	7897
4	188	30	192	38	4	161	13651
5	172	33	173	41	6	180	20136
6	157	33	154	41	8	199	27390
7	142	34	135	43	10	218	35337
8	132	38	122	48	12	231	41025
9	121	43	110	54	14	243	47085
12	101	55	84	70	20	269	60133
13	91	58	71	73	22	282	67117
14	81	61	59	76	24	294	74423

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 20

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.50 x 10^3 kg/m^3

D = 0.253 mm

VIRTUAL ORIGIN IS AT Y* = 353

z_o = z** at t=0 = 84

α = 0.258

β = 2.04

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	248	15	269	19	0	84	0
2	238	17	259	22	1	94	1763
3	223	19	237	24	2	116	6290
4	213	20	224	25	3	129	9413
5	198	24	205	30	4	148	14683
6	188	27	192	35	5	161	18591
7	182	30	186	30	6	167	20682
8	175	33	177	41	7	176	23973
9	167	34	167	43	8	186	27448
10	157	34	154	43	9	199	32329
11	152	35	148	45	10	205	34911
12	149	38	145	48	11	208	36232
13	144	40	138	51	12	215	38937
14	137	43	129	54	13	224	43103
15	132	44	122	56	14	231	46011
16	127	45	116	57	15	237	49001

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 21

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.50 x 10^3kg/m^3

D = 0.253 mm

VIRTUAL ORIGIN IS AT Y* = 353

z_o = z** at t=0 = 81

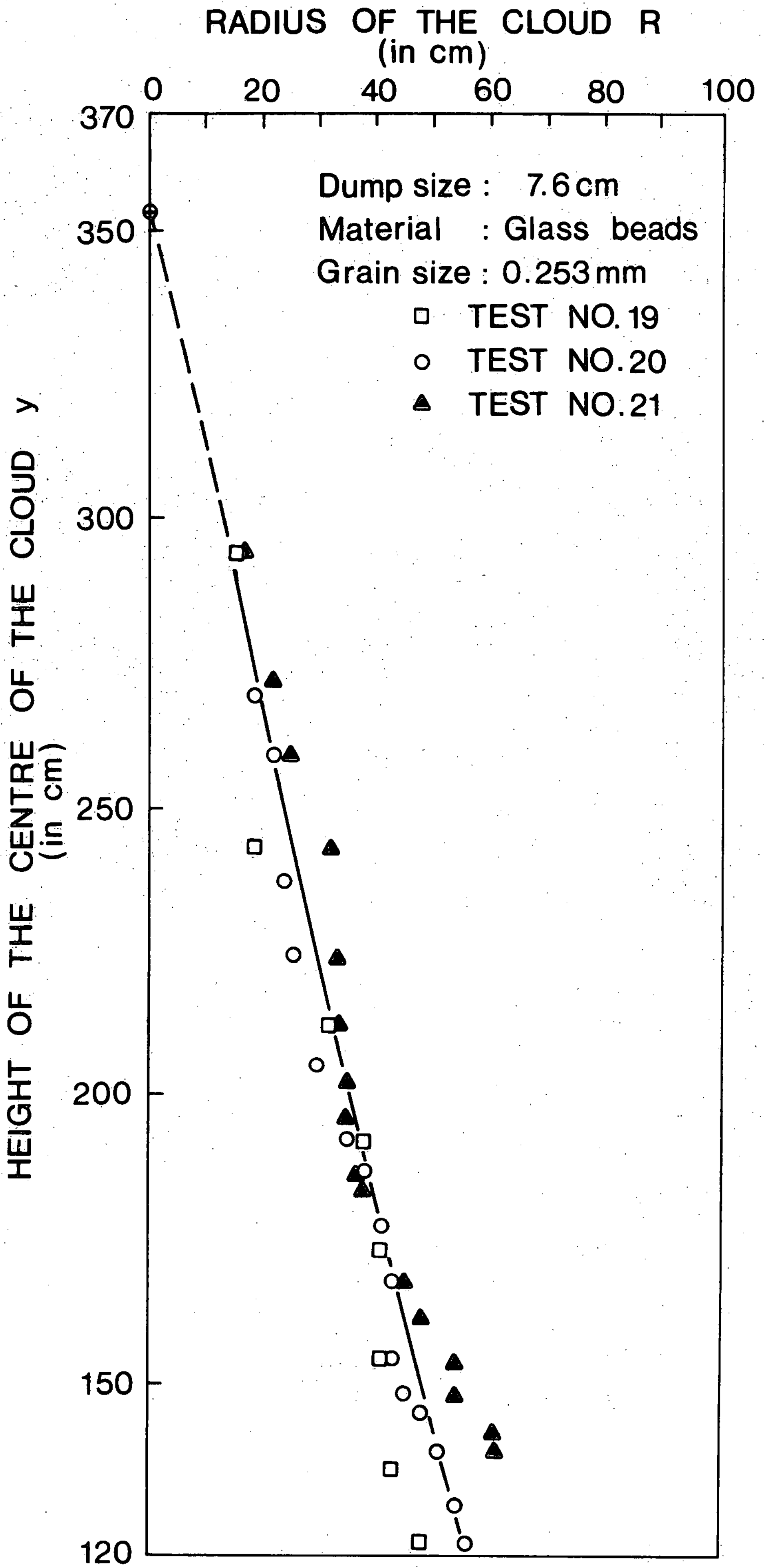
α = 0.258

β = 1.74

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	12	294	16			
2	251	17	272	22	0	81	0
3	238	20	259	25	1	94	2162
4	221	25	234	32	2	110	7410
5	213	26	224	33	3	129	9771
6	203	26	212	33	4	141	13179
7	195	27	202	35	5	151	15969
8	190	27	196	35	6	157	17900
9	182	29	186	37	7	167	20995
10	180	30	183	38	8	170	22067
11	167	35	167	45	9	186	27701
12	162	38	161	48	10	192	30111
13	157	43	154	54	11	199	32603
14	152	43	148	54	12	205	35177
15	147	48	141	61	13	212	37791
16	144	48	138	61	14	215	39064

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 7.6 cm
Material : Glass beads
Grain size : 0.253 mm
□ TEST NO. 19
○ TEST NO. 20
▲ TEST NO. 21

TEST NO: 22

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.253$ mm

VIRTUAL ORIGIN IS AT Y* = 365

z_o = z** at t=0 = 106

$\alpha = 0.288$

$\beta = 1.45$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	299	12	332	16			
2	241	20	259	25	0	106	0
3	228	29	243	37	2	122	3079
4	218	33	231	41	4	134	6315
5	208	38	218	48	6	147	9849
6	200	40	208	51	8	157	12735
7	195	43	202	54	10	163	14761
8	185	44	189	56	12	176	19025
9	180	48	183	61	14	182	21295

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 23

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \mathrm{kg/m}^3$

D = 0.253 mm

VIRTUAL ORIGIN IS AT Y* = 365

z_o = z** at t=0 = 96

α = 0.288

β = 1.72

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	266	11	291	14			
2	248	20	269	25	0	96	0
3	236	27	253	35	2	112	3264
4	223	33	237	41	4	128	7765
5	210	38	221	48	6	144	12180
6	203	39	212	49	8	153	15059
7	198	39	205	49	10	160	17101
8	188	40	192	51	12	173	21430
9	178	43	180	54	14	185	26049
14	144	55	138	70			
15	137	58	129	73			
16	134	61	126	76			
17	132	61	122	76			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 24

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.253$ mm

VIRTUAL ORIGIN IS AT Y* = 365

$z_o = z^{**}$ at $t=0$ = 87

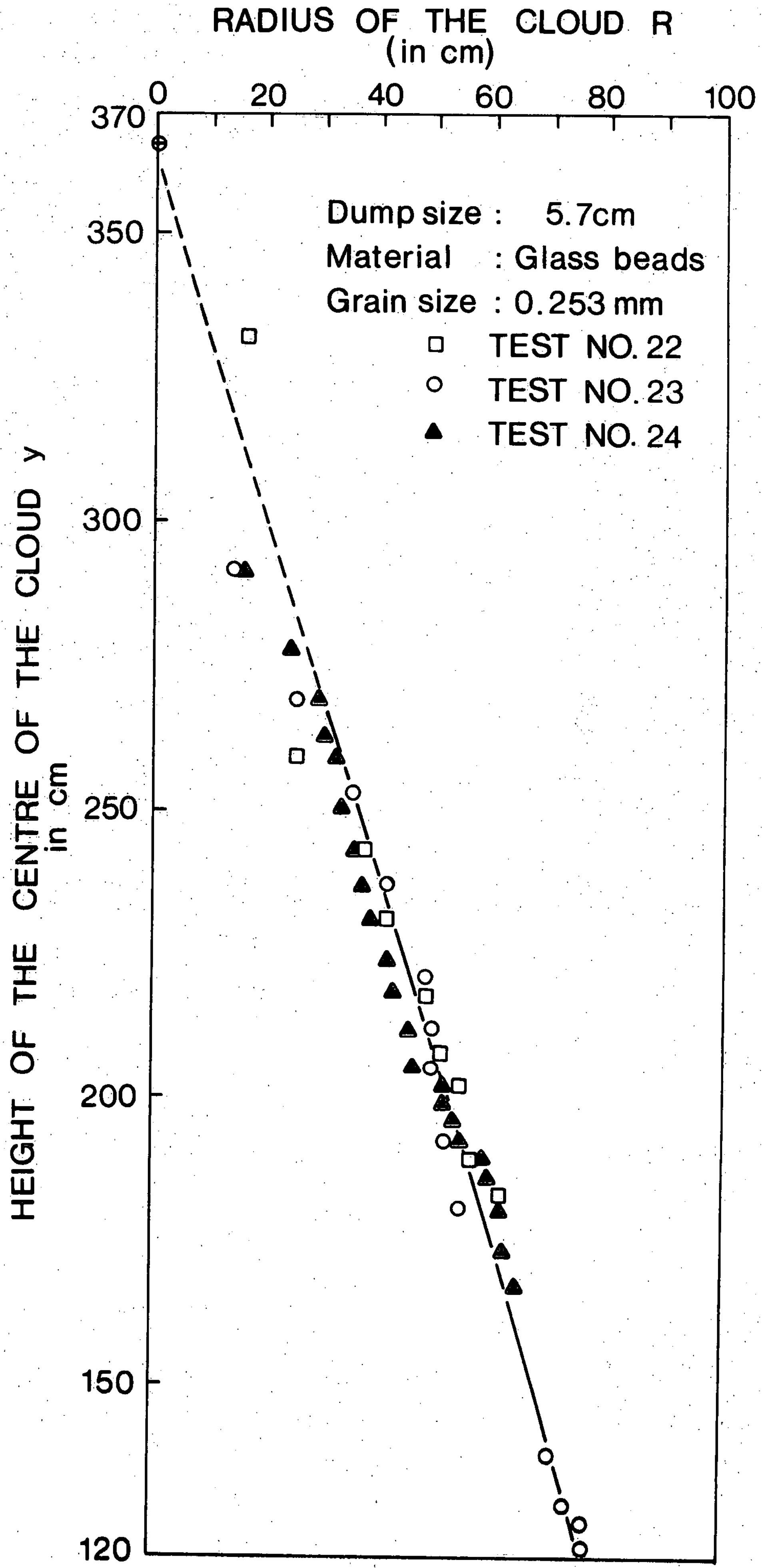
$\alpha = 0.288$

$\beta = 1.53$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	266	12	291	16	-	-	-
2	256	19	278	24	0	87	0
3	248	22	269	29	1	96	1865
4	243	24	263	30	2	102	3091
5	238	25	259	32	3	106	3892
6	233	26	250	33	4	115	5904
7	228	27	243	35	5	122	7409
8	223	29	237	37	6	129	9019
9	218	30	231	38	7	135	10712
10	213	33	224	41	8	142	12458
11	208	33	218	42	9	148	14312
12	203	35	212	45	10	154	16249
13	198	36	205	46	11	161	18235
14	195	40	202	51	12	164	19275
15	193	40	199	51	13	167	20335
16	190	41	196	53	14	170	21415
17	188	43	192	54			
18	185	45	189	57			
19	182	47	186	59			
20	180	48	183	61			
21	177	48	180	61			
22	172	49	173	62			
23	167	50	167	64			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
in cm
Dump size : 5.7cm
Material : Glass beads
Grain size : 0.253 mm
□ TEST NO. 22
○ TEST NO. 23
▲ TEST NO. 24

TEST NO: 25

MATERIAL: Glass beads $\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

DUMP SIZE: 3.8 cm cube D = 0.253 mm

TIME INTERVAL: 1 sec.

VIRTUAL ORIGIN IS AT Y* = 325

z_o = z** at t=0 = 37

α = 0.268

β = 1.41

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	7	301	10	-	-	-
2	264	10	288	13	0	37	0
3	259	10	282	13	1	43	490
4	251	12	272	16	2	53	1400
5	246	13	266	17	3	59	2116
6	243	15	263	19	4	62	2505
7	238	15	259	19	5	66	2993
8	233	17	250	22	6	75	4249
9	228	18	243	23	7	82	5233
10	226	18	240	23	8	85	5764
11	223	20	237	25	9	88	6315
12	218	21	231	27	10	94	7478
13	213	22	224	29	11	101	8724
14	210	22	221	29	12	104	9357
15	208	23	218	29	13	107	10030
16	205	25	215	32	14	110	10724
17	203	25	212	32	15	113	11438
18	198	27	205	35			
19	193	27	199	35			
20	188	27	192	35			
21	182	28	186	36			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 26

MATERIAL: Glass Beads $\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

DUMP SIZE: 3.8 cm cube $D = 0.253$ mm

TIME INTERVAL: 1 sec.

VIRTUAL ORIGIN IS AT Y* = 325

z_o = z** at t=0 = 66

$\alpha = 0.268$

$\beta = 1.67$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	248	10	269	13	-	-	-
2	238	12	259	16	0	66	0
3	233	15	250	19	1	75	1229
4	228	16	243	21	2	82	2214
5	223	17	237	22	3	88	3 296
6	221	19	234	24	4	91	3885
7	218	20	231	25	5	94	4440
8	213	21	224	26	6	101	5684
9	208	21	218	26	7	107	7011
10	205	21	215	26	8	110	7704
11	200	21	208	26	9	117	9153
12	198	22	205	29	10	120	9884
13	195	23	202	29	11	123	10660
14	193	25	199	32			
15	188	27	192	35			
16	182	27	186	35			
24	160	38	157	48			
25	157	38	154	48			
26	155	38	151	48			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 27

MATERIAL: Glass Beads $\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

DUMP SIZE: 3.8 cm cube D = 0.253 mm

TIME INTERVAL: 2 sec.

VIRTUAL ORIGIN IS AT Y* = 325

z_o = z** at t=0 = 56

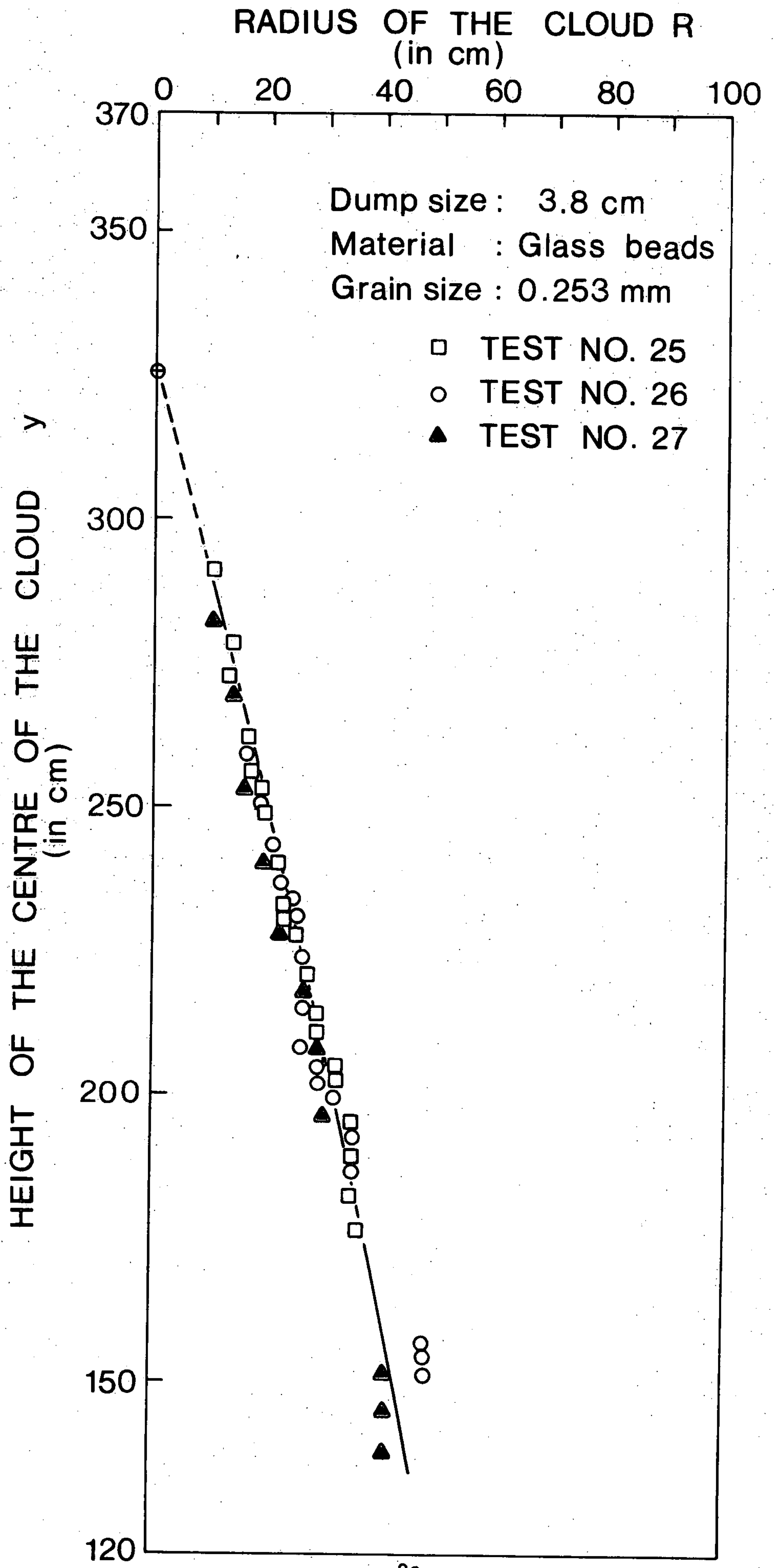
α = 0.268

β = 1.86

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	259	8	282	10	-	-	-
2	248	10	269	13	0	56	0
3	236	13	253	16	2	88	4608
4	226	15	240	19	4	101	7065
5	215	18	228	22	6	114	9860
6	208	20	218	26	8	123	11993
7	200	23	208	29	10	133	14553
8	198	24	205	30	12	136	15360
9	191	24	196	30	14	143	17313
15	155	33	151	41			
16	150	33	145	41			
17	145	34	138	41			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8 cm
Material : Glass beads
Grain size : 0.253 mm
□ TEST NO. 25
○ TEST NO. 26
▲ TEST NO. 27

TEST NO: 28

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.50 x 10^3 kg/m^3

D = 0.213 mm

VIRTUAL ORIGIN IS AT Y* = 349

z_o = z** at t=0 = 48

α = 0.288

β = 1.024

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	10	301	13	-	-	-
2	259	18	282	22	0	48	0
3	249	20	269	26	1	61	1417
4	239	27	259	33	2	71	2737
5	234	28	250	35	3	80	4096
6	224	30	237	38	4	93	6345
7	218	30	231	38	5	99	7497
8	211	31	221	39	6	109	9577
9	203	33	212	41	7	118	11620
10	201	36	208	45	8	122	12580
11	198	36	205	45	9	125	13321
12	196	37	202	46	10	128	14080
13	193	38	199	48	11	131	14857
14	191	38	196	48	12	134	15652
15	185	41	189	51	13	141	17577
16	178	42	180	53	14	150	20196
17	175	43	177	54	15	153	21105
18	168	48	167	61	16	163	24265
19	163	48	161	61			
20	157	51	154	64			
21	152	53	148	67			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 29

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.213$ mm

VIRTUAL ORIGIN IS AT Y* = 349

z_o = z** at t=0 = 80

$\alpha = 0.288$

$\beta = 1.540$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	264	13	288	16	-	-	-
2	249	18	269	22	0	80	0
3	239	20	259	26	1	90	1700
4	227	26	244	32	2	105	4625
5	224	28	237	35	3	112	6144
6	213	30	224	38	4	125	9225
7	203	36	212	45	5	137	12369
8	193	38	199	48	6	150	16100
9	188	38	193	48	7	156	17936
10	183	43	186	54	8	163	20169
11	178	46	180	57	9	169	22161
12	173	51	173	64	10	176	24576

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 30

MATERIAL: Glass Beads $\gamma_s = 1.50 \times 10^3 \mathrm{kg/m^3}$

DUMP SIZE: 7.6 cm cube $D = 0.213$ mm

TIME INTERVAL: 1 sec.

VIRTUAL ORIGIN IS AT Y* = 349

z_o = z** at t=0 = 86

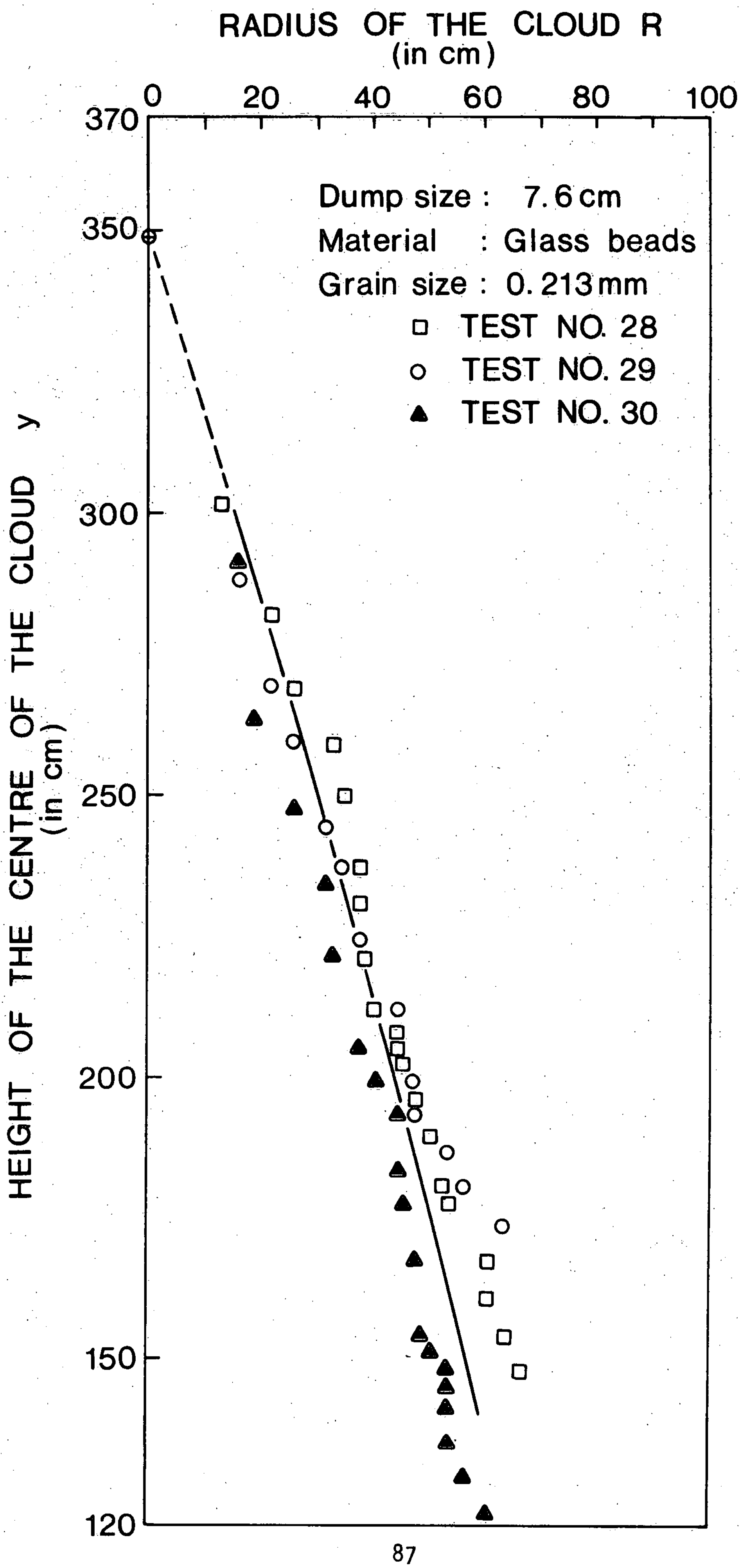
$\alpha = 0.288$

$\beta = 1.710$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	267	13	291	16	-	-	-
2	244	15	263	19	0	86	0
3	231	20	247	26	1	102	3008
4	221	25	234	32	2	115	5829
5	211	27	221	33	3	128	8988
6	198	30	205	38	4	144	13340
7	193	33	199	41	5	150	15104
8	188	36	193	45	6	156	16940
9	180	36	183	45	7	166	20160
10	175	37	177	46	8	172	22188
11	168	38	167	48	9	182	25728
12	157	39	154	49	10	195	30629
13	155	41	151	51	11	198	31808
14	152	43	148	54			
15	150	43	145	54			
16	147	43	141	54			
17	142	43	135	54			
18	137	46	129	57			
19	132	48	122	61			
20	122	51	110	64			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 7.6 cm
Material : Glass beads
Grain size : 0.213 mm
□ TEST NO. 28
○ TEST NO. 29
▲ TEST NO. 30

TEST NO: 31

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3 kg/m^3

D = 0.213 mm

VIRTUAL ORIGIN IS AT Y* = 340

z_o = z** at t=0 = 58

α = 0.283

β = 1.313

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	272	10	298	13	-	-	-
2	259	13	282	16	0	58	0
3	249	15	269	19	2	71	1677
4	241	18	259	22	4	81	3197
5	234	20	250	26	6	90	4736
6	229	23	244	29	8	96	5852
7	226	24	240	30	10	100	6636
8	218	25	231	32	12	109	8517
9	213	28	224	35	14	116	10092
10	208	29	218	37	16	122	11520
11	203	30	212	38	18	128	13020
12	196	33	202	41	20	138	15680
13	191	34	196	43			
14	185	34	189	43			
15	178	36	180	45			
16	175	36	177	45			
17	170	37	170	46			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 32

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.213$ mm

VIRTUAL ORIGIN IS AT Y* = 340

z_o = z** at t=0 = 62

$\alpha = 0.283$

$\beta = 1.570$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13	-	-	-
2	257	13	278	16	0	62	0
3	246	15	266	19	2	74	1632
4	239	16	259	20	4	81	2717
5	226	20	240	26	6	100	6156
6	224	20	237	26	8	103	6765
7	218	23	231	24	10	109	8037
8	211	25	221	32	12	119	10317
9	203	30	212	38	14	128	12540
10	198	32	205	40	16	135	14381
11	196	33	202	41	18	138	15200
12	191	36	196	45	20	144	16892
13	178	38	180	48	22	160	21756

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 33

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.213$

VIRTUAL ORIGIN IS AT Y* = 340

z_o = z** at t=0 = 58

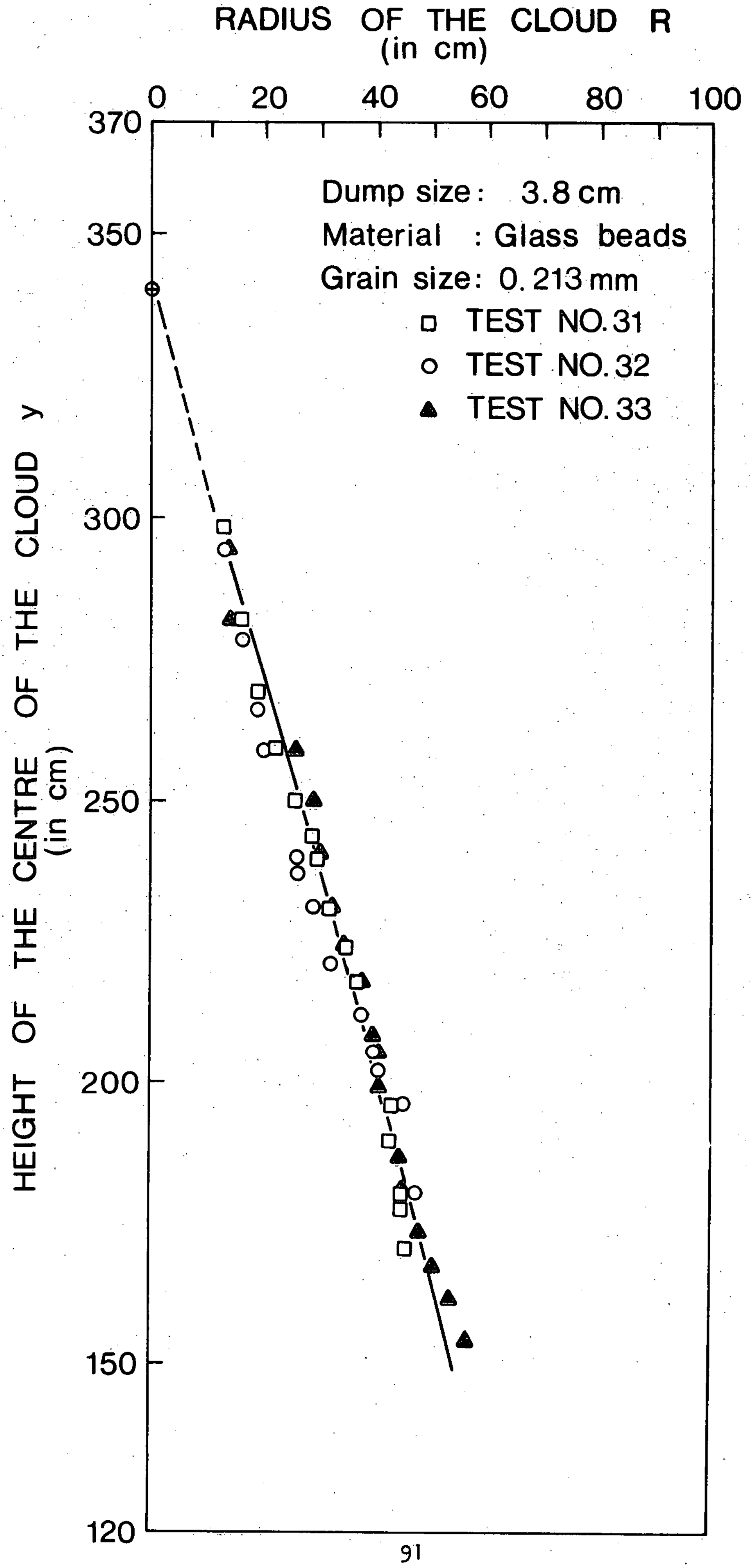
$\alpha = 0.283$

$\beta = 1.460$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13	-	-	-
2	259	11	282	14	0	58	0
3	246	15	266	19	2	74	2112
4	239	20	259	26	4	81	3197
5	234	23	250	29	6	90	4736
6	226	24	240	30	8	100	6636
7	218	25	231	32	10	109	8517
8	213	28	224	35	12	116	10092
9	208	30	218	38	14	122	11520
10	201	32	208	40	16	132	14060
11	198	33	205	41	18	135	14861
12	193	33	199	41			
13	183	33	186	41			
14	178	36	180	45			
15	173	38	173	48			
16	168	40	167	51			
17	163	43	161	54			
18	157	46	154	57			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size: 3.8 cm
Material : Glass beads
Grain size: 0.213 mm
□ TEST NO. 31
○ TEST NO. 32
▲ TEST NO. 33

TEST NO: 34

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.50 x 10^3 kg/m^3

D = 0.180 mm

VIRTUAL ORIGIN IS AT Y* = 359

z_o = z** at t=0 = 77

α = 0.298

β = 1.133

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	272	10	298	13	-	-	-
2	259	18	282	22	0	77	0
3	254	23	275	29	1	84	1127
4	239	25	259	32	2	100	4071
5	231	30	247	38	3	112	6615
6	224	36	237	45	4	122	8955
7	221	39	234	49	5	125	9696
8	216	42	228	53	6	131	11232
9	211	46	221	57	7	138	13115
10	206	46	215	57	8	144	14807

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 35

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

D = 0.180 mm

VIRTUAL ORIGIN IS AT Y* = 359

z_o = z** at t=0 = 84

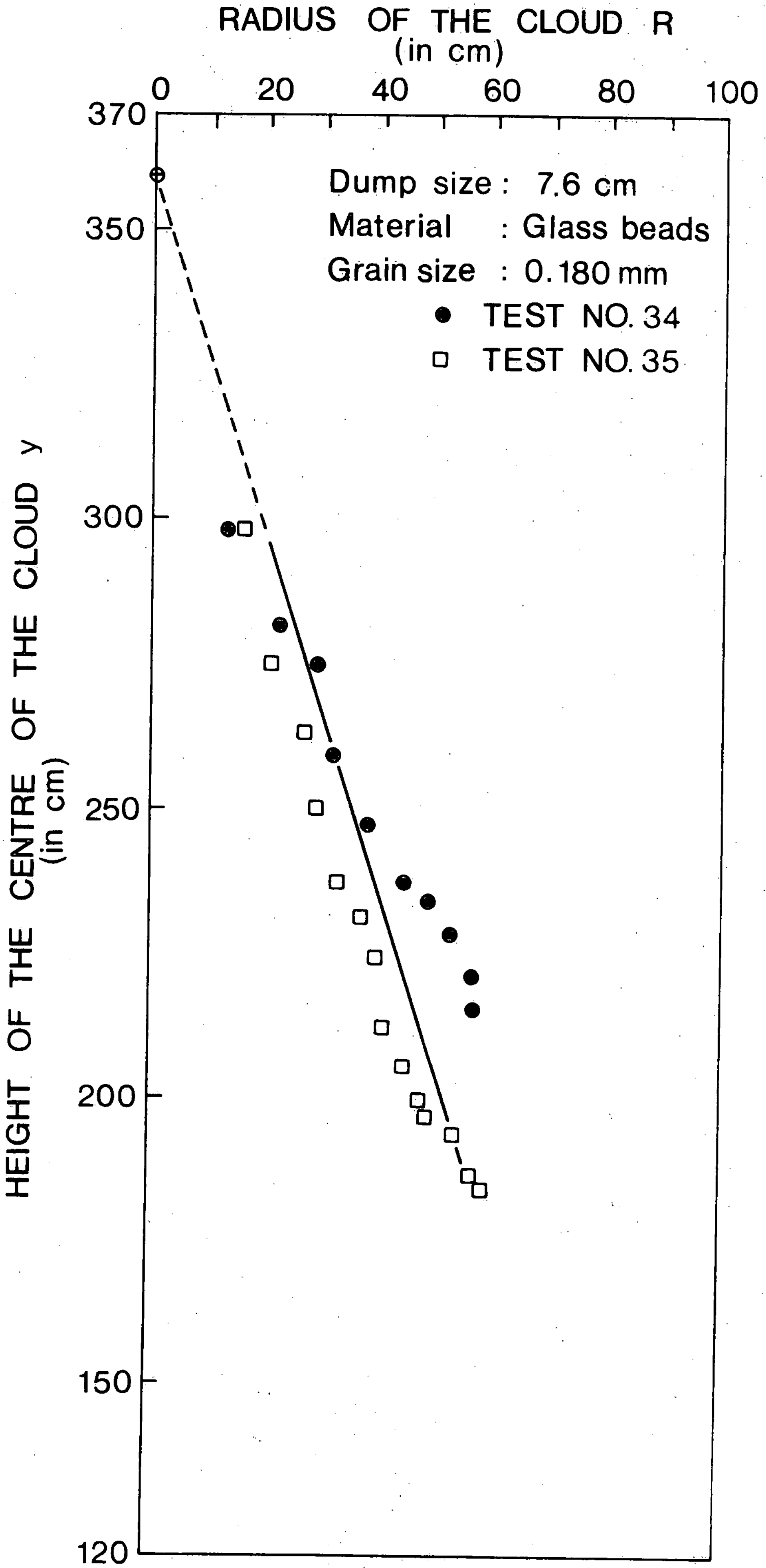
α = 0.298

β = 1.290

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	272	13	298	16	-	-	-
2	254	17	275	21	0	84	0
3	244	22	263	27	1	96	2160
4	234	23	250	29	2	109	4825
5	224	27	237	33	3	122	7828
6	218	29	231	37	4	128	9328
7	213	32	224	40	5	135	11169
8	203	33	212	41	6	147	14553
9	198	36	205	45	7	154	16660
10	193	38	199	48	8	160	18544
11	191	39	196	49	9	166	20500
12	188	43	193	54	10	169	21505
13	183	46	186	57	11	176	23920
14	180	47	183	59	12	183	26433

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 7.6 cm
Material : Glass beads
Grain size : 0.180 mm
● TEST NO. 34
□ TEST NO. 35

TEST NO: 36

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \mathrm{kg/m}^3$

$D = 0.180$ mm

VIRTUAL ORIGIN IS AT Y* = 337

z_o = z** at t=0 = 78

$\alpha = 0.300$

$\beta = 1.000$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	269	10	294	13	-	-	-
2	239	14	259	18	0	78	0
3	234	18	250	22	2	87	1485
4	226	23	240	29	4	97	3325
5	213	28	224	35	6	113	6685
6	206	30	215	38	8	122	8800
7	198	30	205	38	10	132	11340
8	196	30	202	38	12	135	12141
9	193	36	199	45	14	138	12960
10	183	37	186	46	16	151	16717
11	178	39	180	49	18	157	18565
12	173	43	173	54	20	164	20812
13	168	46	167	57	22	170	22816
14	157	48	154	61	24	183	27405
15	152	51	148	64	26	189	29637

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 37

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3kg/m^3

D = -.180 mm

VIRTUAL ORIGIN IS AT Y* = 337

z_o = z** at t=0 = 62

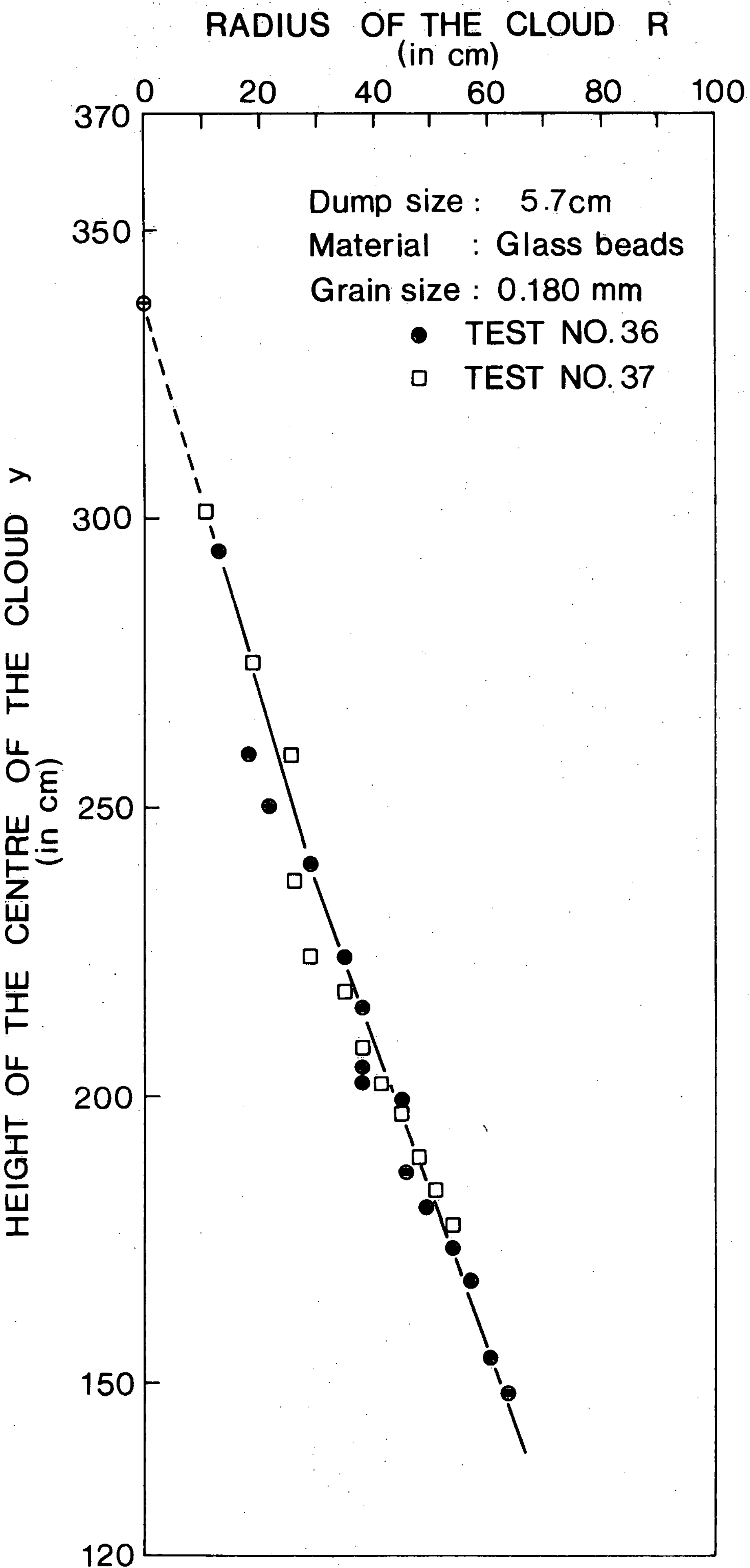
α = 0.300

β = 0.980

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	9	301	11	-	-	-
2	254	15	275	19	0	62	0
3	239	20	259	26	2	78	2240
4	224	21	237	26	4	100	6156
5	213	23	224	29	6	113	8925
6	208	28	218	35	8	119	10317
7	201	30	208	38	10	129	12797
8	196	33	202	41	12	135	14381
9	191	36	196	45	14	141	16037
10	185	38	189	48	16	148	18060
11	180	41	183	51	18	154	19872
12	175	43	177	54	20	160	21756

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7cm
Material : Glass beads
Grain size : 0.180 mm
● TEST NO. 36
□ TEST NO. 37

TEST NO: 38

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

D = 0.180 mm

VIRTUAL ORIGIN IS AT Y* = 337

z_o = z** at t=0 = 65

α = 0.300

β = 1.360

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	262	10	284	13	-	-	-
2	251	15	272	19	0	65	0
3	241	20	259	26	2	78	1859
4	234	24	250	30	4	87	3344
5	229	28	244	35	6	93	4424
6	224	28	237	35	8	100	5775
7	216	30	228	38	10	109	7656
8	211	30	221	38	12	116	9231
9	203	33	212	41	14	125	11400
10	198	33	205	41			
11	193	34	199	43			
12	191	34	196	43			
13	185	36	189	45			
14	180	37	183	46			
15	175	38	177	48			
16	170	39	170	49			
17	165	41	164	51			
18	157	42	154	53			
19	155	43	151	54			
20	152	44	148	56			
21	150	44	145	56			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 39

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3kg/m^3

D = 0.180 mm

VIRTUAL ORIGIN IS AT Y* = 337

z_o = z** at t=0 = 65

α = 0.300

β = 1.393

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	262	13	285	16	-	-	-
2	251	16	272	20	0	65	0
3	241	19	259	24	2	78	1859
4	236	20	253	26	4	84	2831
5	229	25	244	32	6	93	4424
6	221	28	234	35	8	103	6384
7	216	29	228	36	10	109	7656
8	211	30	221	38	12	116	9231
9	203	33	212	41	14	125	11400
10	198	34	205	43	16	132	13199
11	196	36	202	45	18	135	14000
12	191	36	196	45			
13	185	37	189	46			
14	180	37	183	46			
15	175	38	177	48			
16	165	39	164	49			
17	160	41	157	51			
18	157	41	154	51			
19	152	41	148	51			
20	150	43	145	54			
21	147	43	142	54			
22	145	46	138	57			
23	142	47	135	59			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 40

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50×10^3 kg/m^3

D = 0.180 mm

VIRTUAL ORIGIN IS AT Y* = 337

z_o = z** at t=0 = 74

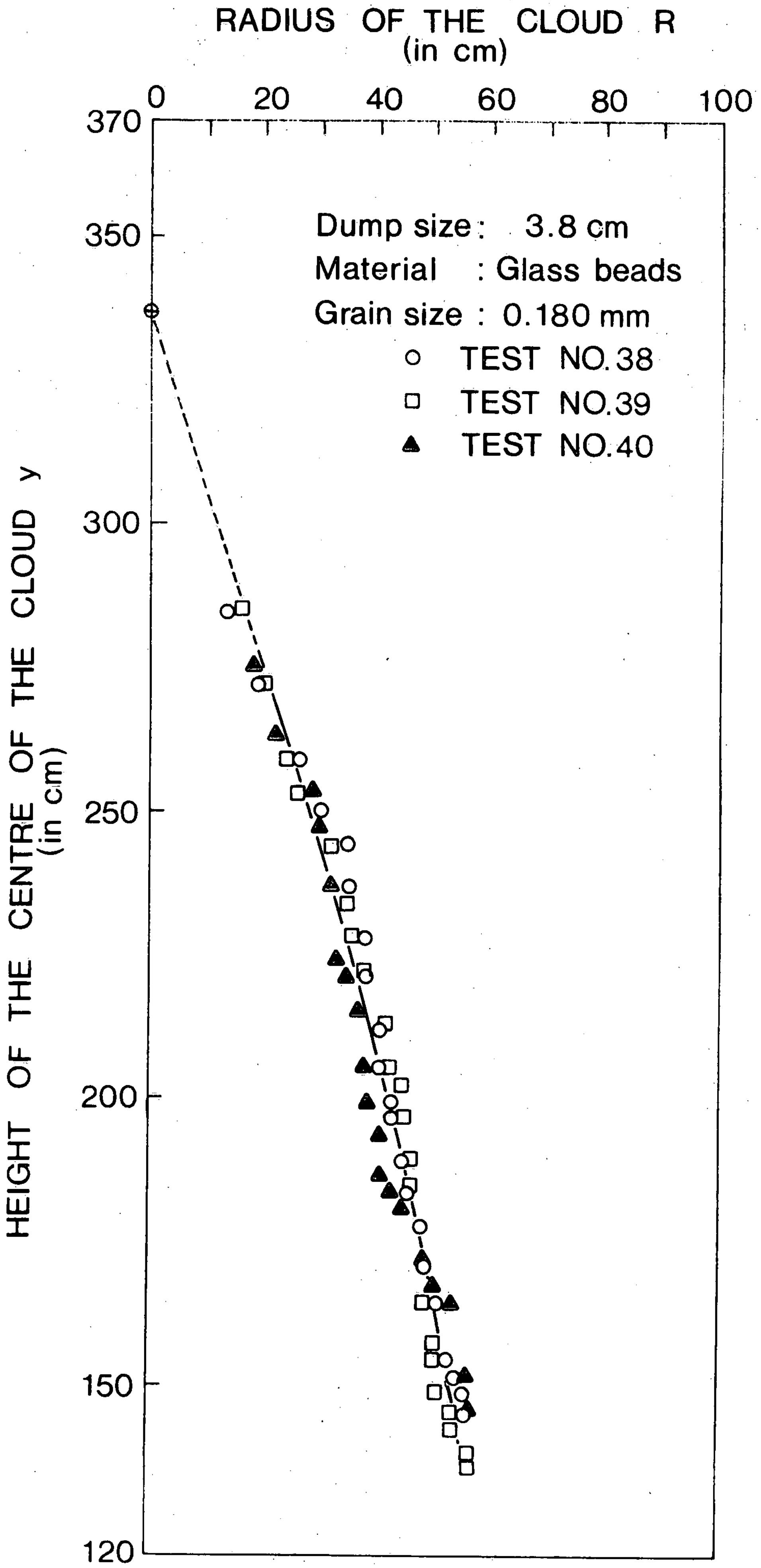
α = 0.300

β = 1.487

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	254	14	275	18	-	-	-
2	244	18	263	22	0	74	0
3	236	23	253	29	2	84	1580
4	231	24	247	30	4	90	2624
5	224	25	237	32	6	100	4524
6	213	26	224	33	8	113	7293
7	211	28	221	35	10	116	7980
8	206	29	215	37	12	122	9408
9	198	30	205	38	14	132	11948
10	193	31	199	39	16	138	13568
11	188	33	193	41	18	144	15260
12	183	33	186	41			
13	180	34	183	43			
14	178	36	180	45			
15	170	39	170	49			
16	168	41	167	51			
17	165	43	164	54			
18	157	43	154	54			
19	155	46	151	57			
20	150	46	145	57			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8 cm
Material : Glass beads
Grain size : 0.180 mm
○ TEST NO. 38
□ TEST NO.39
▲ TEST NO.40

TEST NO: 41

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = 1.50 x 10^3 kg/m^3

D = 0.151 mm

VIRTUAL ORIGIN IS AT Y* = 356

z_o = z** at t=0 = 93

α = 0.310

β = 1.320

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	257	17	278	21	-	-	-
2	244	19	263	24	0	93	0
3	234	23	250	29	1	106	2587
4	229	23	244	29	2	112	3895
5	221	25	234	32	3	122	6235
6	213	27	224	33	4	132	8775
7	208	30	218	38	5	138	10395
8	203	33	212	41	6	144	12087
9	198	37	205	46	7	151	14152
10	191	38	196	48	8	160	16951
11	183	41	186	51	9	170	20251
12	180	43	183	54	10	173	21280

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 42

MATERIAL: Glass Beads

DUMP SIZE: 7.6 cm cube

TIME INTERVAL: 1 sec.

γ_s = $1.50 \times 10^3 kg/m^3$

D = 0.151 mm

VIRTUAL ORIGIN IS AT Y* = 356

z_o = z** at t=0 = 87

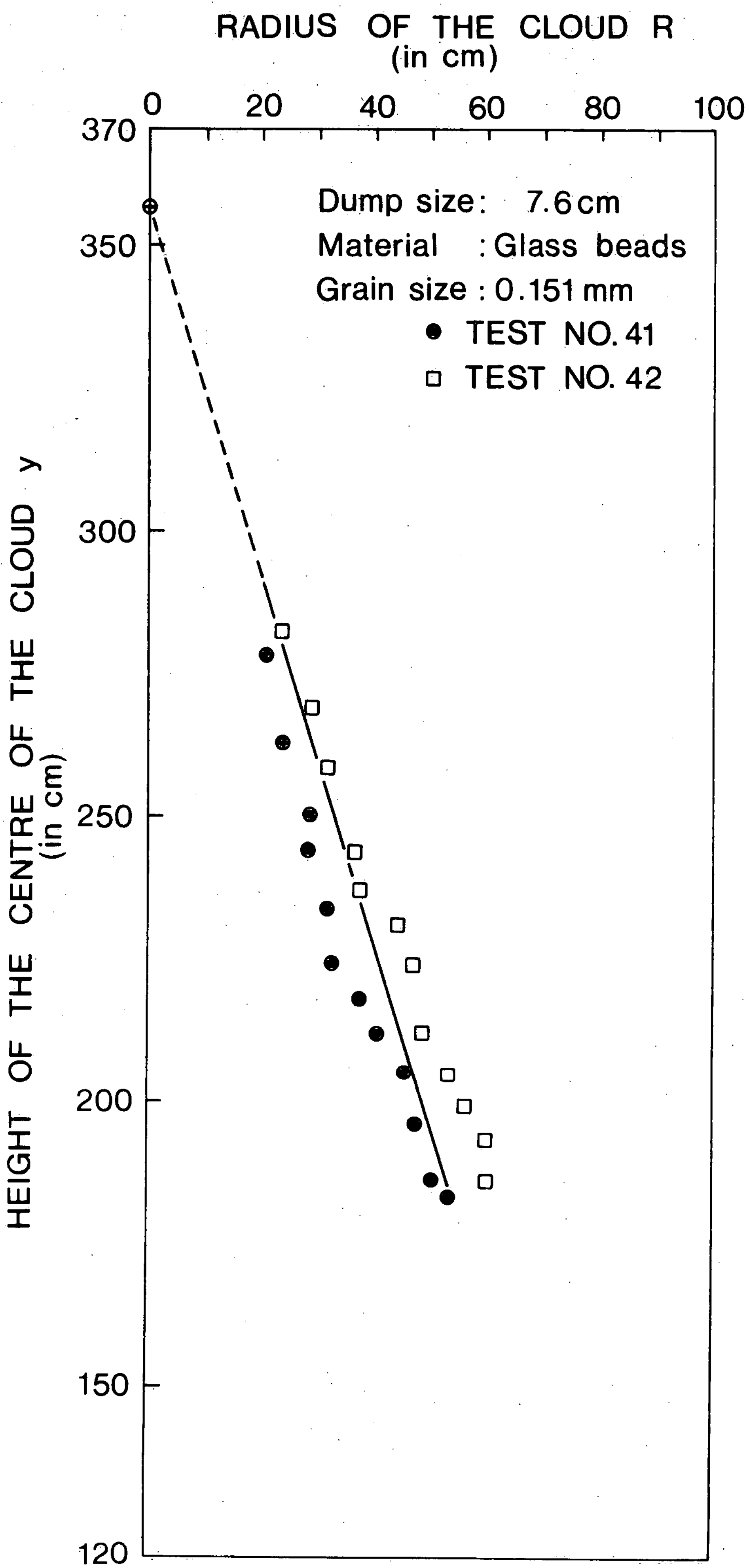
α = 0.310

β = 1.306

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	259	19	282	24	-	-	-
2	249	23	269	29	0	87	0
3	239	25	259	32	1	97	1840
4	229	29	244	37	2	112	4975
5	224	30	237	38	3	119	6592
6	218	36	231	45	4	125	8056
7	213	38	224	48	5	132	9855
8	203	39	212	49	6	144	13167
9	198	43	205	54	7	151	15232
10	193	46	199	57	8	157	17080
11	188	48	193	61	9	163	19000
12	183	48	186	61	10	169	20992

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size: 7.6 cm
Material : Glass beads
Grain size : 0.151 mm
● TEST NO. 41
□ TEST NO. 42

TEST NO: 43

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.151$ mm

VIRTUAL ORIGIN IS AT Y* = 328

z_o = z** at t=0 = 69

$\alpha = 0.310$

$\beta = 1.299$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	10	301	13	-	-	-
2	239	13	259	16	0	69	0
3	224	18	237	22	2	91	3520
4	218	23	231	29	4	97	4648
5	198	25	205	32	6	123	10368
6	193	28	199	35	8	129	11880
7	188	30	193	38	10	135	13464
8	178	32	180	40	12	148	17143
9	168	33	167	41	14	161	21160
10	157	33	154	41	16	174	25515
11	147	36	142	45	18	186	29835
12	142	36	135	45	20	193	32488

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 44

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 × 10^3 kg/m^3

D = 0.151 mm

VIRTUAL ORIGIN IS AT Y* = 328

z_o = z** at t=0 = 59

α = 0.310

β = 0.864

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	10	301	13	-	-	-
2	249	13	269	16	0	59	0
3	234	20	250	26	2	78	2603
4	229	25	244	32	4	84	3575
5	218	28	231	35	6	97	5928
6	211	30	221	38	8	107	7968
7	203	33	212	41	10	116	9975
8	198	36	205	45	12	123	11648
9	196	41	202	51	14	126	12395
10	188	43	193	54	16	135	14744
11	183	46	186	57	18	142	16688
12	178	47	180	59	20	148	18423
13	173	51	173	64	22	155	20544

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 45

VIRTUAL ORIGIN IS AT Y* = 328

MATERIAL: Glass Beads

γ_s = $1.50 \times 10^3 kg/m^3$

z_o = z** at t=0 = 69

DUMP SIZE: 5.7 cm cube

D = 0.151 mm

α = 0.310

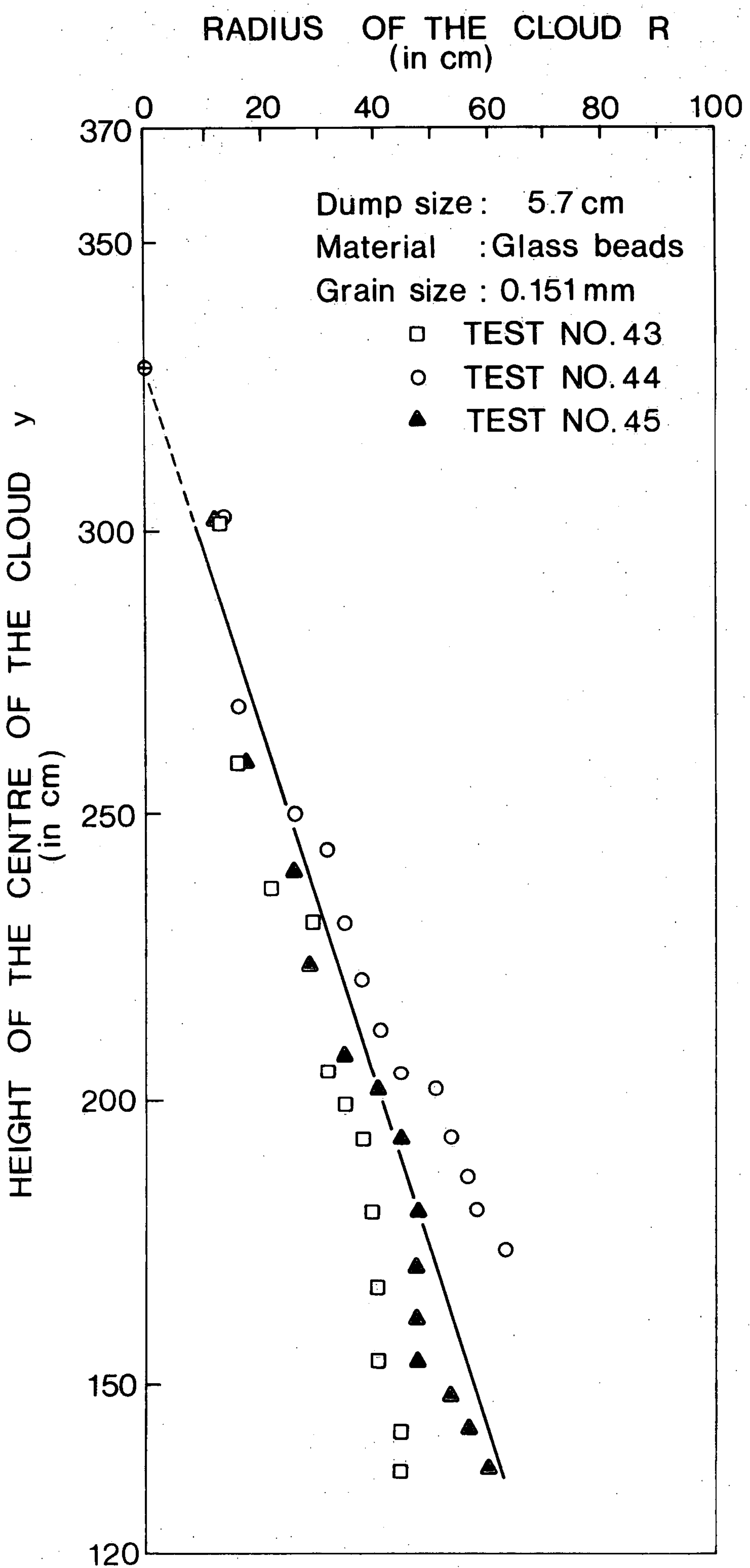
TIME INTERVAL: 2 secs.

β = 1.330

Frame No.	Apparent dimensions(cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	10	301	13	-	-	-
2	241	13	259	16	0	69	0
3	226	20	240	26	2	88	2983
4	213	23	224	29	4	104	6055
5	201	28	208	35	6	120	9639
6	196	33	202	41	8	126	11115
7	188	36	193	45	10	135	13464
8	178	38	180	48	12	148	17143
9	170	38	170	48	14	158	20203
10	163	38	161	48			
11	157	38	154	48			
12	152	43	148	54			
13	147	46	142	57			
14	142	48	135	61			
15	135	51	126	64			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7 cm
Material : Glass beads
Grain size : 0.151 mm
□ TEST NO. 43
○ TEST NO. 44
▲ TEST NO. 45

TEST NO: 46

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50×10^3 kg/m^3

D = 0.084 mm

VIRTUAL ORIGIN IS AT Y* = 328

z_o = z** at t=0 = 43

α = 0.312

β = 0.900

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	272	8	298	10	-	-	-
2	262	13	285	16	0	43	0
3	251	18	272	22	2	56	1287
4	246	21	266	26	4	62	1995
5	241	23	259	29	6	69	2912
6	234	24	250	30	8	78	4235
7	229	25	244	32	10	84	5207
8	226	28	240	35	12	88	5895
9	224	28	237	35	14	91	6432
10	213	30	224	38	16	104	8967
11	208	30	218	38	18	110	10251
12	203	32	212	41			
13	198	33	205	41			
14	193	34	199	42			
15	188	34	193	43			
16	183	36	186	44			
17	180	36	183	45			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 47

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \text{kg/m}^3$

$D = 0.084$ mm

VIRTUAL ORIGIN IS AT Y* = 328

z_o = z** at t=0 = 46

$\alpha = 0.312$

$\beta = 1.050$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	274	8	301	10	-	-	-
2	259	10	282	13	0	46	0
3	246	13	266	16	2	62	1728
4	236	13	253	16	4	75	3509
5	231	15	247	19	6	81	4445
6	226	18	240	22	8	88	5628
7	221	23	234	29	10	94	6720
8	216	23	228	29	12	100	7884
9	211	25	221	32	14	107	9334
10	208	28	218	35	16	110	9984
11	203	29	212	36	18	116	11340
12	198	30	205	37	20	123	13013
13	196	30	202	38			
14	191	30	196	38			
15	185	33	189	41			
16	183	36	186	45			
17	180	36	183	45			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 48

MATERIAL: Glass Beads

DUMP SIZE: 3.8 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3kg/m^3

D = 0.084 mm

VIRTUAL ORIGIN IS AT Y* = 328

z_c = z** at t=0 = 50

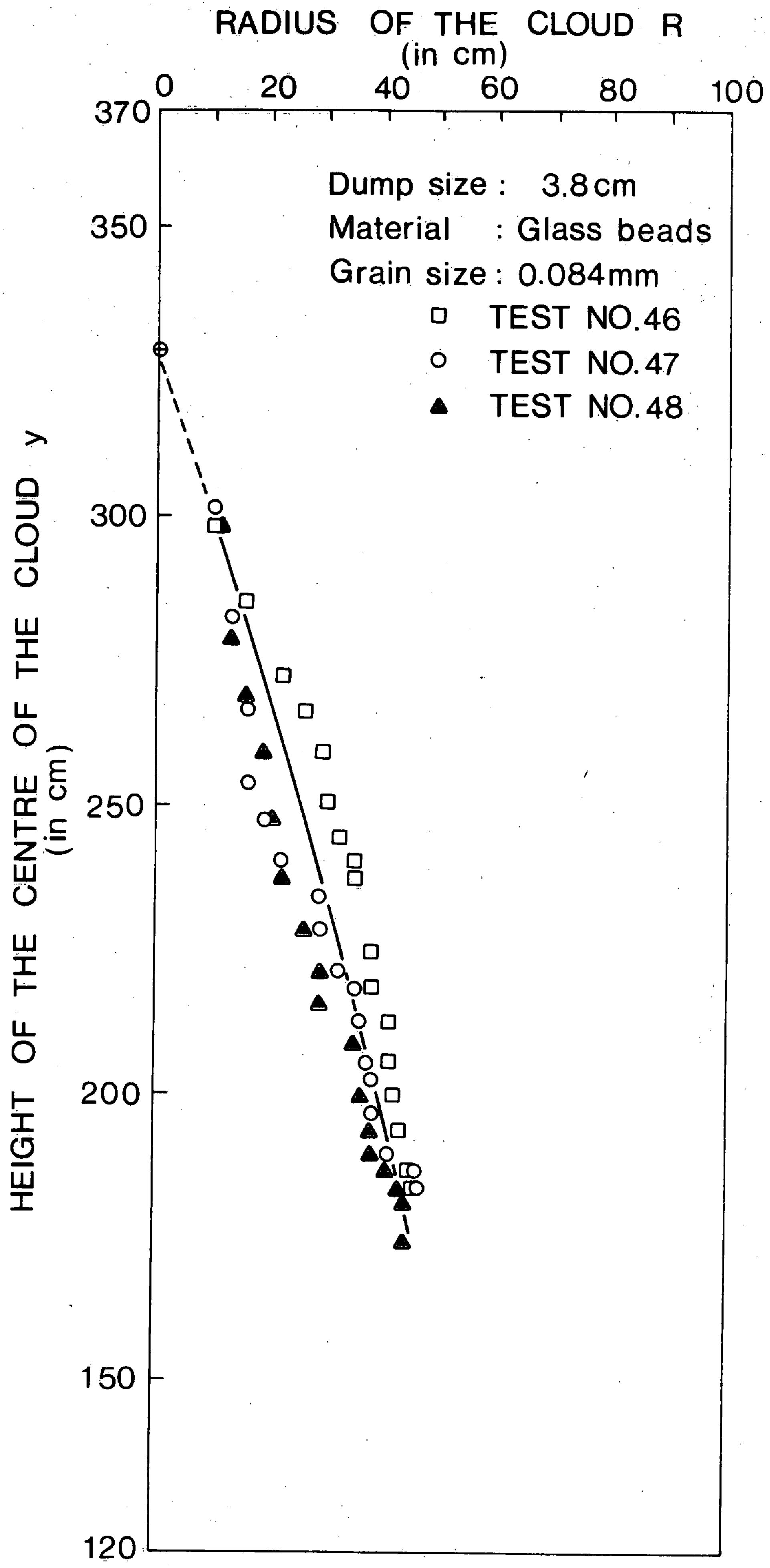
α = 0.312

β = 1.394

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	272	9	298	11	-	-	-
2	257	10	278	13	0	50	0
3	249	13	269	16	2	59	981
4	239	15	259	19	4	69	2261
5	231	16	247	20	6	81	4061
6	224	18	237	22	8	91	5781
7	216	21	228	26	10	100	7500
8	211	23	221	29	12	107	8949
9	206	23	215	29	14	113	10269
10	201	28	208	35	16	120	11900
11	193	29	199	36	18	129	14141
12	188	30	193	38			
13	185	30	189	38			
14	183	33	186	41			
15	180	34	183	43			
16	178	36	180	44			
17	173	36	173	44			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)



RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 3.8cm
Material : Glass beads
Grain size : 0.084mm
□ TEST NO.46
○ TEST NO.47
▲ TEST NO.48

TEST NO: 49

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3kg/m^3

D = 0.044 mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = z** at t=0 = 46

α = 0.312

β = 1.023

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	279	8	307	10	-	-	-
2	264	15	288	19	0	46	0
3	254	17	275	21	2	59	1365
4	244	20	263	26	4	71	2925
5	234	22	250	28	6	84	4940
6	226	23	240	29	8	94	6720
7	218	23	231	29	10	103	8493
8	211	25	221	32	12	113	10653
9	203	28	212	35	14	122	12768
10	198	29	205	37	16	129	14525
11	188	29	193	37	18	141	17765
12	178	30	180	38	20	154	21600
13	163	32	161	40			
14	152	33	148	41			
15	147	36	142	44			
16	142	36	135	44			
17	137	36	129	44			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 50

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

γ_s = 1.50 x 10^3kg/m^3

D = 0.044 mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = z** at t=0 = 65

α = 0.312

β = 1.280

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	262	15	285	19	-	-	-
2	249	15	269	19	0	65	0
3	239	19	259	24	2	75	1400
4	229	20	244	26	4	90	3875
5	218	23	231	29	6	103	6384
6	208	25	218	32	8	116	9231
7	203	25	212	32	10	122	10659
8	198	28	205	35	12	129	12416
9	188	32	193	40	14	141	15656
10	178	32	180	40			
11	163	33	161	41			
12	157	33	154	41			
13	152	36	148	45			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)

TEST NO: 51

MATERIAL: Glass Beads

DUMP SIZE: 5.7 cm cube

TIME INTERVAL: 2 secs.

$\gamma_s = 1.50 \times 10^3 \mathrm{kg/m}^3$

$D = 0.044$ mm

VIRTUAL ORIGIN IS AT Y* = 334

z_o = z** at t=0 = 59

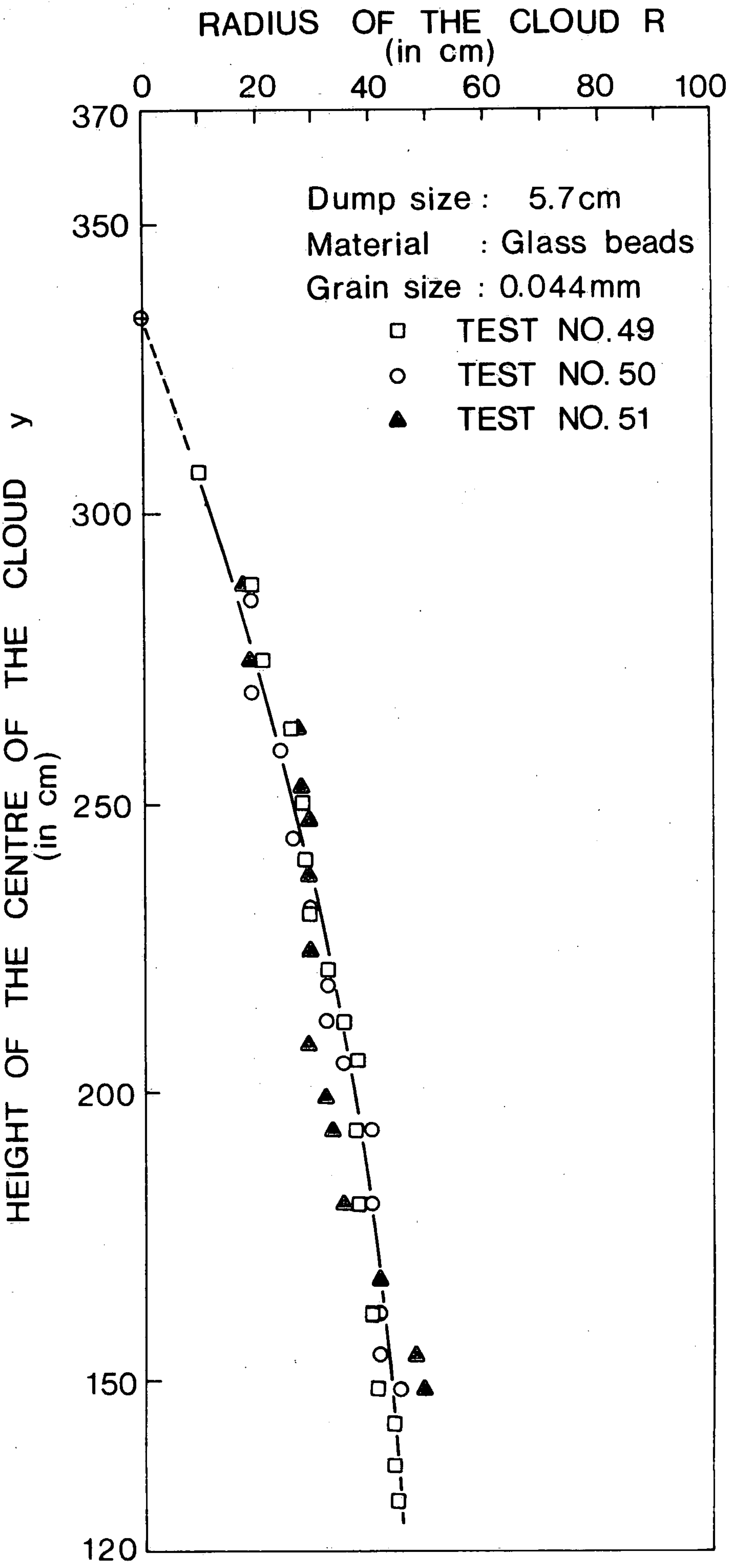
$\alpha = 0.312$

$\beta = 1.100$

Frame No.	Apparent dimensions (cm)		Actual dimensions (cm)		time t in sec.	z in cm	$z^2 - z_o^2$
	Height y	Radius R	Height y	Radius R			
1	264	14	288	18	-	-	-
2	254	15	275	19	0	59	0
3	244	20	263	26	2	71	1560
4	236	22	253	28	4	81	3080
5	231	23	247	29	6	87	4088
6	224	23	237	29	8	97	5928
7	213	23	224	29	10	110	8619
8	201	23	208	29	12	126	12395
9	193	25	199	32	14	135	14744
10	188	27	193	33			
11	178	28	180	35			
12	168	33	167	41			
13	157	38	154	48			
14	152	39	148	49			

* distance measured from the ground level (cm)

** distance measured from the virtual origin (cm)


RADIUS OF THE CLOUD R
(in cm)
0
20
40
60
80
100
370
350
300
250
200
150
120
HEIGHT OF THE CENTRE OF THE CLOUD y
(in cm)
Dump size : 5.7cm
Material : Glass beads
Grain size : 0.044mm
□ TEST NO. 49
○ TEST NO. 50
▲ TEST NO. 51