CANADA CENTRE FOR INLAND WATERS

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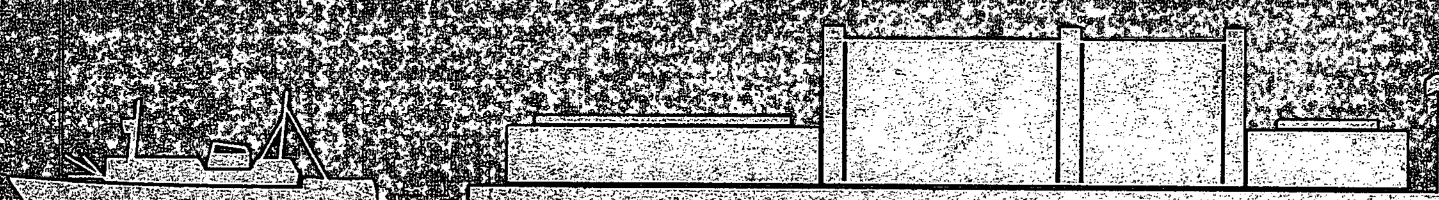

WIND-INDUCED WATER LEVEL CHANGES ON THE SOUTH-EASTERN SHORELINE OF LAKE ST. CLAIR

by

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WIND-INDUCED WATER LEVEL CHANGES ON THE SOUTH EASTERN SHORELINE OF LAKE ST. CLAIR

A prolonged period of abnormally high water levels now being experienced in Lake St. Clair coupled with a large area of potential inundation due to the low shoreline relief of the eastern shore has prompted the requirement for a practical guide to the forecasting of water levels by the Toronto International Weather Office. This somewhat limited investigation attempts to establish a relation between wind speed and water levels on the eastern shoreline of Lake St. Clair. The approach is statistical in nature and is based upon a compilation of data on severe storms on Lake St. Clair for the years 1969 to 1971. Because of the limited statistics available the resulting regression relations are compared to more comprehensive studies in Lake Erie, as well as a simple mathematical formula for the steady state set-up.

Wind data and lake level data

Three years of water level records (1969 to 1971) at the Belle River, Ontario gauging station were examined visually for changes over periods less than several days, of 0.4 ft. or more beyond the background level. For these episodes hourly values of wind speed and direction at the Windsor airport were obtained. The surge as defined is the difference between the instantaneous water level and the background level, which is taken as the average of daily means two days preceding and two days following the storm.

Other studies, for example, Keulegan (1953) have found significant correlations between water level and wind speed squared resolved along the major axis of the lake. For this reason wind speed squared resolved along the three directions of approximately maximum fetch, see Figure 1, north, north-northwest and northwest were chosen as the three predictors of a linear regression model of water level fluctuation.

The well known effect of thermal stability of the atmosphere on the wind stress is included in the analysis. Because of the limited statistics instead of seasonal classification, data are divided into two classes, the unstable period from September 1 to December 31 and the stable period comprising the remaining months.

As far as potential inundation and shoreline inundation is concerned it is of greater concern to forecast the magnitude of the set-up than its time of arrival. Therefore, the peak wind tide was regressed against the wind stress component squared.

Visual inspection of the wind and water level records suggested a further refinement. For the same storms an effective wind speed squared component, $\overline{W^2_{i-1}}$, was determined at one hour preceding a reference time by smoothing the wind squared component according to the following formula:

$$\overline{W^2_{i-1}} = .25W^2_i + .5W^2_{i-1} + .25W^2_{i-2}$$

where the subscript, i, denotes the reference time in hours. In the effective wind regression analysis each hour during the storm period is used instead of only the peak hour. Hence there appears to be many more points in the regression analysis than in the correlation between the peak values.

Results

Scatter diagrams of water level versus effective wind speed squared components for the cases of stable and unstable atmospheric stability are presented in Figures 2 and 3. These plots have higher correlation coefficients, 0.64 and 0.80 respectively than similar plots of components resolved along the northwest and north direction. The largest correlations of all cases where obtained by the correlation of peak wind surge and the corresponding wind speed squared resolved in the north direction which are shown in Figures 4 and 5. Table 1 summarizes the regression coefficients and correlation coefficients for the various cases studied. Several cases are shown in Table 1, in which the peak surge is correlated with the arithmetic average of preceding five hours of wind.

Discussion

Correlation coefficients as high as 0.94 between wind and water level surges provides encouragement for the practical predictability of water level based on a single simply measured or forecast parameter, namely the longitudinal or north-south component of wind speed squared at Windsor airport.

Correlation of wind versus water level is greater in all cases studied for unstable conditions than for stable conditions. This may be due to the fact that the influence of wind is weaker for stable conditions so that the effects of other factors such as surface barometric pressure may be relatively greater for this case.

Analysis shows that the response of water level to applied wind is nearly immediate. Correlations with instantaneous wind are as large as or larger than correlations with wind lagged by one hour.

Wind resolved along the north-northwest direction accounts for a greater portion of the variance for the case of the effective wind speed squared whereas for the peak instantaneous surge versus the corresponding wind speed squared the largest correlations occurred along the north axis which is also the direction of maximum fetch at the Belle River station.

Because of the relatively limited amount of data on which the statistics are based a comparison of the slopes of the regression equations obtained in the study is made both with a similar analysis in Lake Erie and with a simple mathematical expression.

The downwind displacement of the free surface from mean level, Z, resulting from a steady frictionless balance with the imposed wind stress requires that

$$Z = \frac{L}{2} \frac{\rho a \; C_D \; W^2}{\rho_W \; g \; H}$$

where ρ_W is the density of the water, ρa is the density of the air = 1.2×10^{-3} g/cc, L, the length of the lake, here 46 km, g, the acceleration of gravity, H, the depth, which is taken as 6 m for Lake St. Clair, W the wind speed and C_D the drag coefficient which itself is dependent on atmospheric stability. From McClure, 1970, who studied the dynamic response of the free surface in Lake Erie to wind forcing the values of C_D of 1.3×10^{-3} was taken appropriate to the stable conditions (spring and summer) and 4.7×10^{-3} appropriate to the fall or unstable conditions.

Using McClure's values for drag coefficients and the mathematical formula we arrive at the following relations for Lake St. Clair:

$Z = 4.0 \times 10^{-4}$ W stable condition

$Z = 1.44 \times 10^{-3}$ W unstable

where Z is expressed in feet and W in mph.

When these formulas are compared with the regression equation the agreement is reasonable considering that effects such as friction and correction of anemometer height are not taken into account. The considerably smaller range between the coefficients for unstable and stable conditions observed in Lake St. Clair suggest that the water temperature in Lake St. Clair may be closer to atmospheric temperatures than is the case for Lake Erie. Hence, it would appear that stability of the atmosphere is somewhat less important for the prediction of water levels in Lake St. Clair than Lake Erie.

The above formula may also be used to compare surges observed in Lake Erie with those observed in Lake St. Clair. Assuming that drag coefficients are the same but that the length of Lake Erie may be taken as 360 km and that an average depth is 25 m then Keulegan's (1953) relation for set-up and longitudinal component of wind speed squared for mainly unstable conditions in Lake Erie is given by

$$Z_{Erie} = 3.5 \times 10^{-3}\ W^2$$

and when scaled according to the mathematic formula

$$Z_{StCLAIR} = 3.5 \times 10^{-3} \times \frac{46}{360} \times \frac{25}{6} W^2 = 1.86 \times 10^{-3}\ W^2$$

which is close to the theoretically predicted relation and within the uncertainty of the observed regression coefficients of 2.2×10^{-3} for the peak north component correlations and 1.4×10^{-3} for the effective wind stress correlations (see Figures 5 and 3, respectively).

Conclusions and Recommendations

In spite of the relatively limited amount of statistical data available for this study the results of the analysis of the relation between wind and water level in Lake St. Clair agree reasonably well with those of another study in Lake Erie and with a simple mathematical relation. It is recommended that the effect of stability of the atmosphere be taken into account and that for the unstable case the following prognostic equation be used

$$Z = 2.2 \times 10^{-3}\ W^2$$

and for the stable case

$$Z = .9 \times 10^{-3}\ W^2 + 0.31$$

where Z in the instantaneous water level displacement at Belle River, Ontario in feet and where W^2 is the corresponding wind speed in mph squared, resolved along a north-south direction.

ACKNOWLEDGEMENTS

The assistance of J. McCulloch of the Atmospheric Environment Service in his prompt response to requests for surface wind data and his helpful suggestions concerning the study is gratefully acknowledged. Mr. G. Dohler of Tides and Water Levels Section of M.S.D. has kindly supplied water level data. Finally, N. Freeman of the Geotechnology Section, M.S.D. has provided many helpful suggestions during the course of the study.

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McCLURE, D. J. 1970. Dynamic forecasting of Lake Erie water levels. Hydro-Electric Power Commission of Ontario. R. No. 70-250-H.

STABLE CONDITIONS

STORM SURGE VALUE	$(\text{WIND SPEED})^2$ COMPONENT	AXIS	SLOPE	INTERCEPT	CORRELATION COEFFICIENT
Hourly Average	Hourly Instantaneous	N	5.07×10^{-4}	+0.40	0.591
		NW	3.46×10^{-4}	+0.48	0.528
		NNW	4.39×10^{-4}	+0.42	0.612
Peak Hourly Average	Hourly Instantaneous	N	8.88×10^{-4}	+0.31	0.877
		NNW	4.41×10^{-4}	+0.53	0.649
Hourly Average	Weighted Average on Preceding 2 hours	N	5.58×10^{-4}	+0.40	0.592
		NNW	4.85×10^{-4}	+0.42	0.646
		NNW	8.38×10^{-4}*	+0.13*	0.636*
Peak Hourly Average	Weighted Average on Preceding 2 hours	N			
		NNW	4.12×10^{-4}	+0.62	0.569
Hourly Average	Averaged over Preceding 5 hours	N	0.0×10^{-3}	+0.43	0.507
		NNW	2.0×10^{-4}	+0.54	0.194
Peak Hourly Average	Averaged over Preceding 5 hours	N	1.0×10^{-3}	+0.58	0.735
		NNW			

TABLE 1A

Data used: $(\text{Wind Speed})^2$ component >0, storm surge >0.4 ft.

*For these values $(\text{Wind Speed})^2$ component >0, storm surge >0.

UNSTABLE CONDITIONS

STORM SURGE VALUE	$(\text{WIND SPEED})^2$ COMPONENT	AXIS	SLOPE	INTERCEPT	CORRELATION COEFFICIENT
Hourly Average	Hourly Instantaneous	N	1.00×10^{-3}	0.38	0.680
		NNW	1.11×10^{-3}	0.34	0.707
Peak Hourly Average	Hourly Instantaneous	N	2.16×10^{-3}	-0.05	0.942
		NNW	2.3×10^{-3}	-0.17	0.916
Hourly Average	Weighted Average on Preceding 2 hours	N	1.04×10^{-3}	0.35	0.701
		NNW	1.28×10^{-3}	0.27	0.794
		NNW	1.38×10^{-3}*	0.12*	0.800*
Peak Hourly Average	Weighted Average on Preceding 2 hours	N	1.16×10^{-3}	0.51	0.649
		NNW	1.50×10^{-3}	0.33	0.748
Hourly Average	Averaged over Preceding 5 hours	N	1.05×10^{-3}	0.34	0.512
		NNW			
Peak Hourly Average	Averaged over Preceding 5 hours	N	1.0×10^{-3}	0.41	0.711
		NNW			

TABLE 1B

Data used: $(\text{Wind Speed})^2$ component >0, storm surge >0.4 ft.

*For these values $(\text{Wind Speed})^2$ component >0, storm surge >0.

LIST OF FIGURES

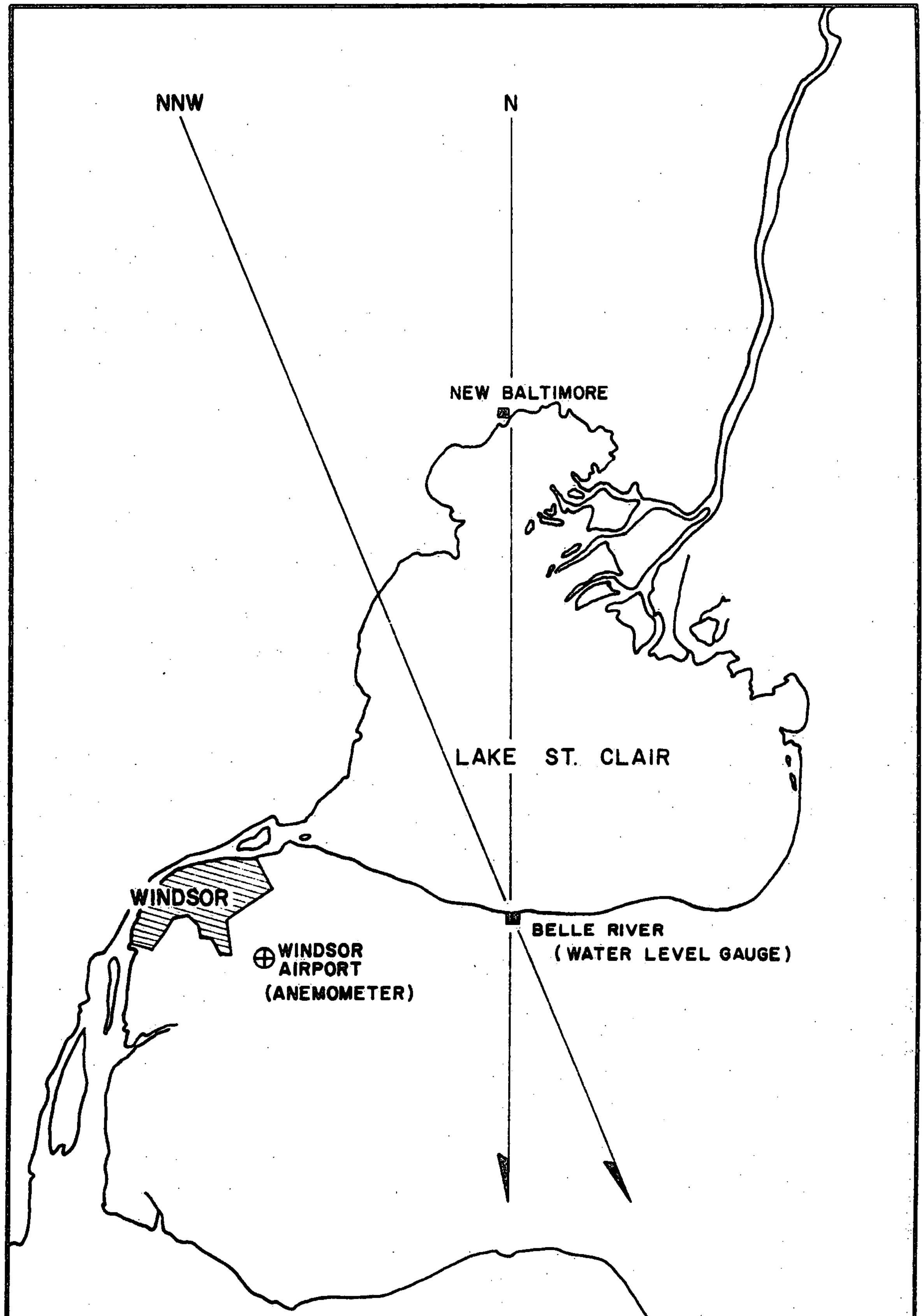

NNW
N
NEW BALTIMORE
LAKE ST. CLAIR
WINDSOR
BELLE RIVER
(WATER LEVEL GAUGE)
WINDSOR
AIRPORT
(ANEMOMETER)

Figure 1.

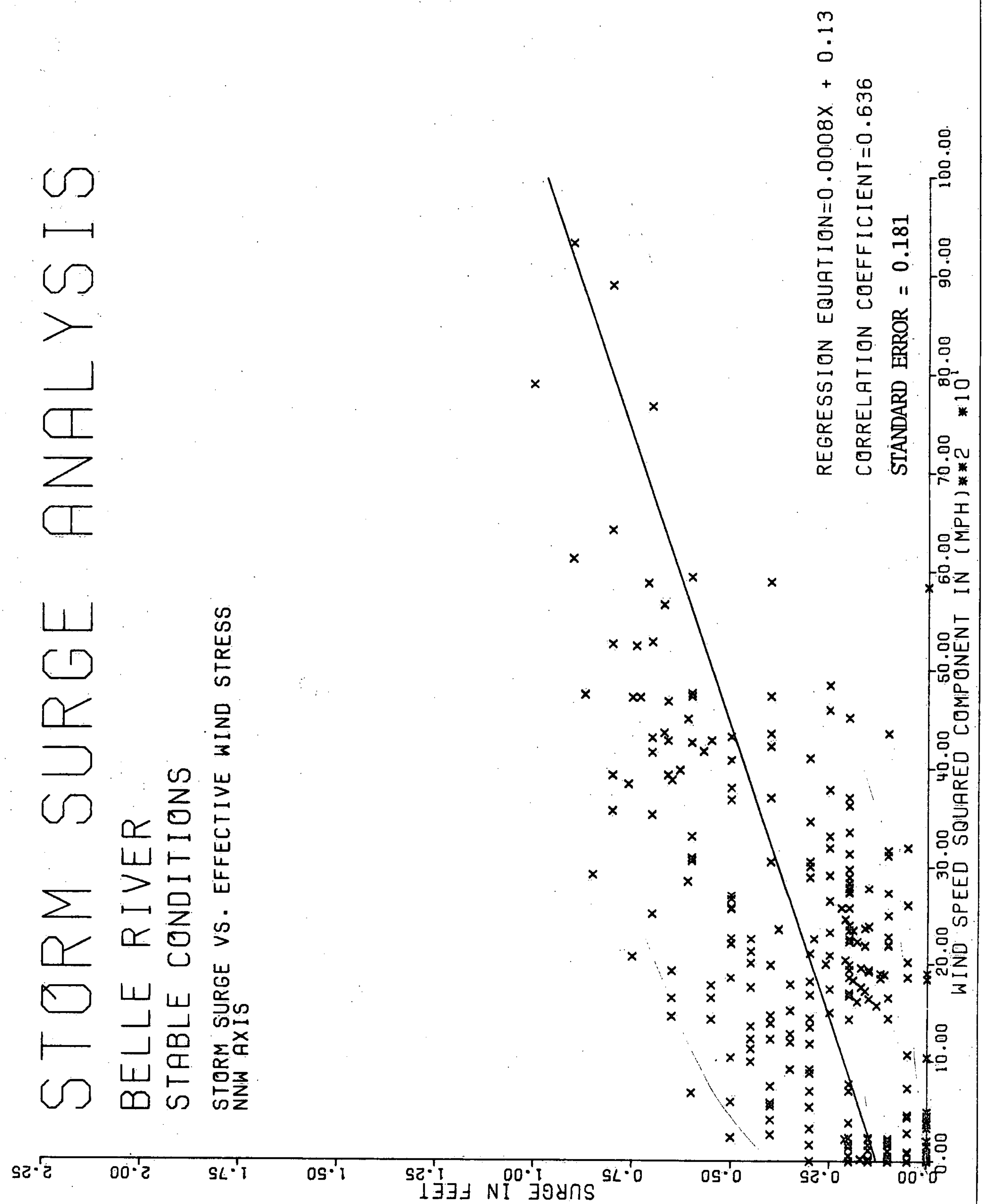

STORM SURGE ANALYSIS
BELLE RIVER
STABLE CONDITIONS
STORM SURGE VS. EFFECTIVE WIND STRESS
NNW AXIS
REGRESSION EQUATION=0.0008X + 0.13
CORRELATION COEFFICIENT=0.636
STANDARD ERROR = 0.181
SURGE IN FEET
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
WIND SPEED SQUARED COMPONENT IN (MPH)**2 *10¹
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

Figure 2.

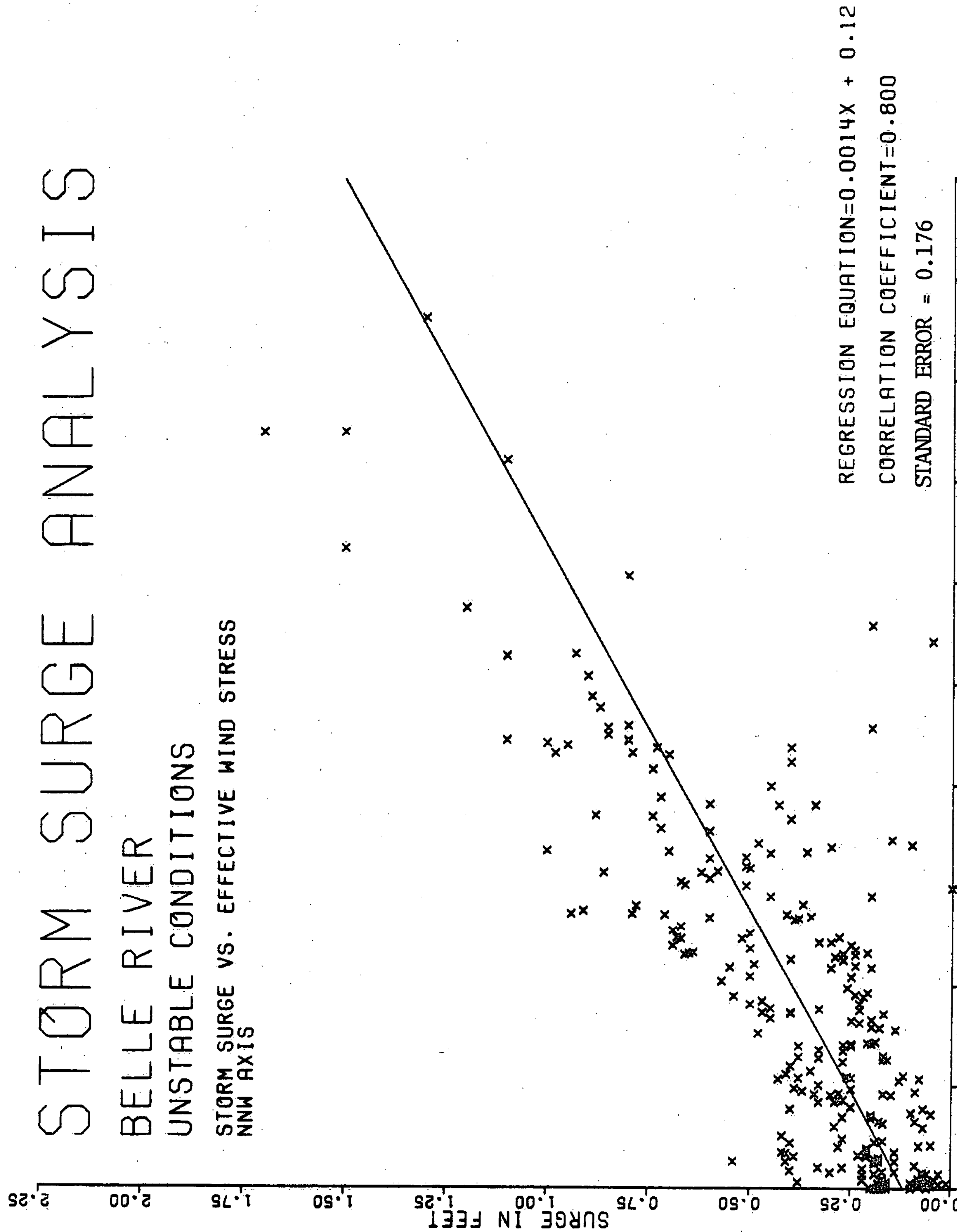

STORM SURGE ANALYSIS
BELLE RIVER
UNSTABLE CONDITIONS
STORM SURGE VS. EFFECTIVE WIND STRESS
NNW AXIS
REGRESSION EQUATION=0.0014X + 0.12
CORRELATION COEFFICIENT=0.800
STANDARD ERROR = 0.176
SURGE IN FEET
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
WIND SPEED SQUARED COMPONENT IN (MPH)**2 *10^1
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

Figure 3.

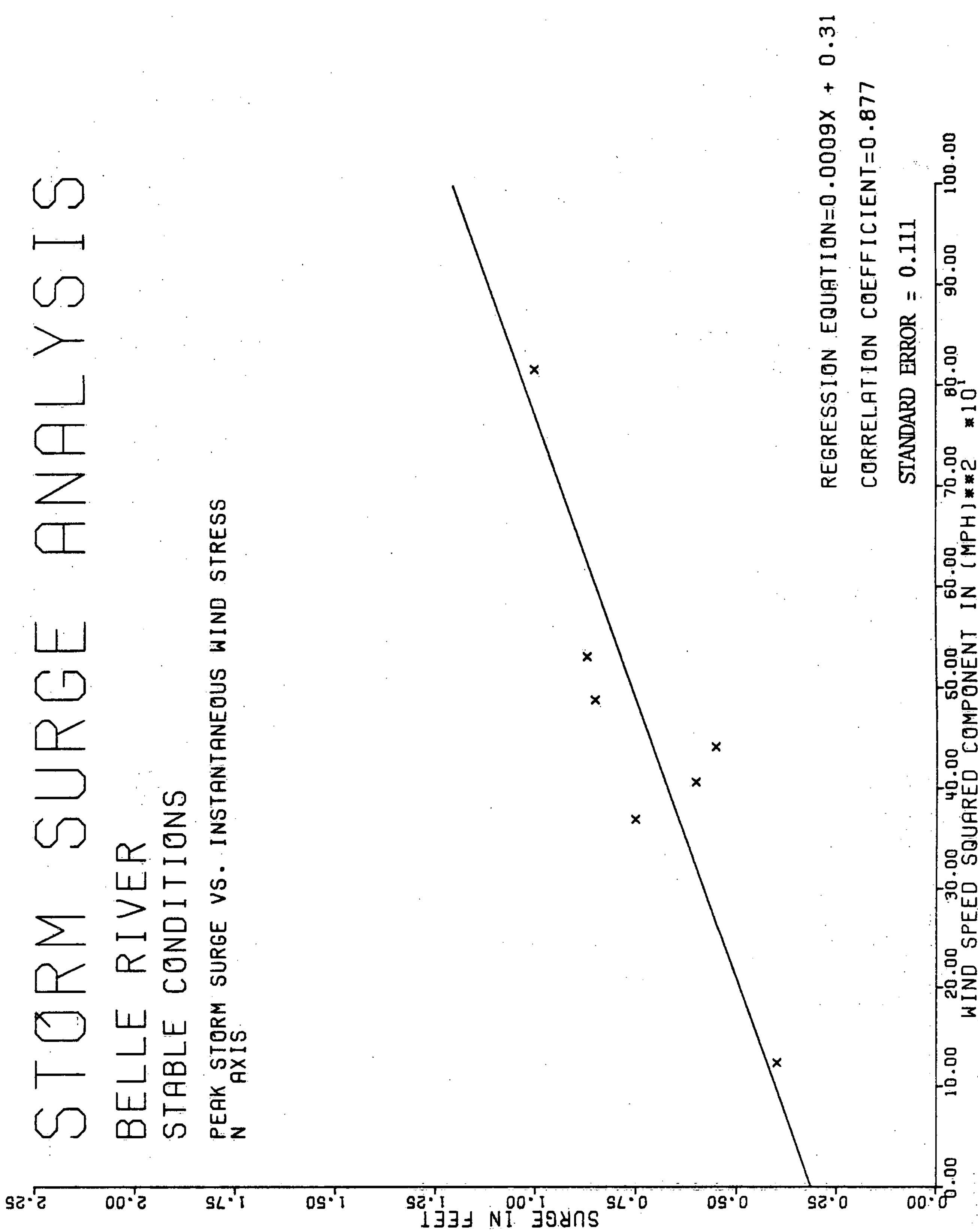
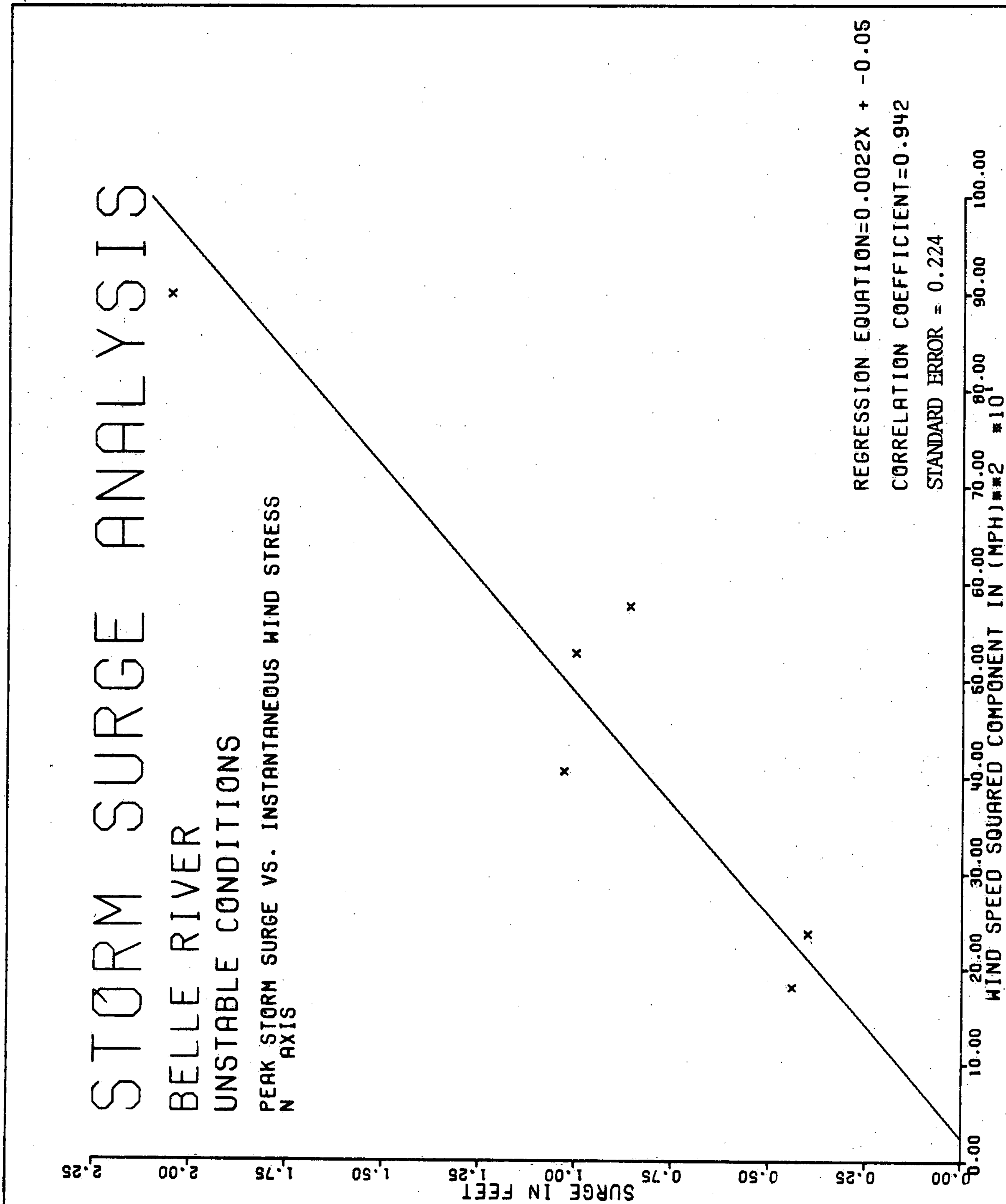

STØRM SURGE ANALYSIS
BELLE RIVER
STABLE CONDITIONS
PEAK STORM SURGE VS. INSTANTANEOUS WIND STRESS
N AXIS
REGRESSION EQUATION=0.0009X + 0.31
CORRELATION COEFFICIENT=0.877
STANDARD ERROR = 0.111
WIND SPEED SQUARED COMPONENT IN (MPH)**2 *10¹
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
SURGE IN FEET
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25

Figure 4.



STORM SURGE ANALYSIS
BELLE RIVER
UNSTABLE CONDITIONS
PEAK STORM SURGE VS. INSTANTANEOUS WIND STRESS
N AXIS
REGRESSION EQUATION=0.0022X + -0.05
CORRELATION COEFFICIENT=0.942
STANDARD ERROR = 0.224
SURGE IN FEET
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
WIND SPEED SQUARED COMPONENT IN (MPH)**2 *10
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

Figure 5.