

Report Prepared for Department of the Environment  
Under Contract Serial No. OSS5 - 0299

RN: KL347-5-0761

FC: 8-12-5684-034-0431

THE DEVELOPMENT OF AUTOMATIC TECHNIQUES  
FOR THE REAL TIME PREDICTION OF STORM  
SURGES ON THE GREAT LAKES

F. Penicka

Hydraulics Research Division  
Canada Centre for Inland Waters  
February 1976

CONTENTS

	Page
Abstract	
1. Introduction . . . . .	1
2. Dynamic Equations. . . . .	4
3. Impulse Response and Transient Response. . . . .	7
4. Meteorological Input . . . . .	10
5. Computer Programs. . . . .	13
6. Results and Discussion . . . . .	15
7. Conclusions . . . . .	19
References . . . . .	20
Table 1: Input Data . . . . .	23
Table 2: Frequencies and periods of the six lowest seiches in Lake Ontario, Lake Erie and Lake St. Clair. . . .	25
Appendix A: Computer programs associated with the reported de- velopment of the storm surge prediction technique	
Appendix B: Computer programs associated with the reported devel- opment of the storm surge prediction technique. (The following programs, with the exception of VELOCITY, were neither written nor modified or used by the con- tractor).	
Appendix C: Forecast and observed storm surges for Lake St. Clair at Belle River.	
Appendix D: Forecast and observed storm surges for Lake Erie at Kingsville and Port Colborne, and for Lake Ontario at Burlington. (Present results superimposed on plots given in Venkatesh 1974).	
Appendix E: Regression equations for the prediction of the Great Lakes surface winds.	

LIST OF FIGURES

	<u>Page</u>
Figure 1 - Primitive Equation Model forecast grid and interpolation points . . . . .	26
Figure 2 - Calculated impulse and step response for Lake St. Clair at Belle River . . . . .	27
Figure 3 - Calculated impulse and step response for Lake Erie at Kingsville . . . . .	28
Figure 4 - Calculate impulse and step response for Lake Erie at Port Colborne . . . . .	29
Figure 5 - Calculated impulse and step response for Lake Ontario at Burlington . . . . .	30

## ABSTRACT

An automated technique is described, for the real time prediction of storm surges on Lake St. Clair, Lake Erie and Lake Ontario. The transient response is obtained as a convolution with wind stress, of an impulse response of a lake at a point of interest. The impulse response is a weighted sum of six lowest seiches for each lake. Surface winds predicted by regression technique from forecast elements of PE Model serve as meteorological input for Lake Erie and Lake Ontario while for Lake St. Clair it was necessary to use winds observed at Windsor Airport. Results of test runs for 17 storm episodes on Lake St. Clair and 2 storms on Lake Erie and Lake Ontario are included.

"La conception de techniques automatiques pour la prévision en temps réel des soulèvements de tempête dans les Grands lacs".

F. Penicka

#### RESUME

Voici l'exposé d'une technique automatique, pour la prévision en temps réel des soulèvements de tempête sur les lacs Ste-Claire, Erié et Ontario. La réaction transitoire résulte de la circonvolution, à un point précis, de l'impulsion du lac de concert avec la tension du vent. Cette réaction constitue la somme pondérée de six petites seiches, pour chaque lac. Les vents de surface, prévus par technique de régression à partir des éléments de prévision du modèle d'équations générales, servent de données météorologiques pour les lacs Erié et Ontario; dans le cas du lac Ste-Claire, l'observation des vents a dû se faire à l'aéroport de Windsor. Vous trouverez ci-joint les résultats d'expérience pour 17 épisodes de tempête survenus sur le lac Ste-Claire, et pour 2 tempêtes sur les lacs Erié et Ontario.

## 1. INTRODUCTION

The problem of storm surge prediction on the Great Lakes has been the subject of a number of investigations in the past. Lake Erie attracted the greatest attention (Henry 1902, Keulegan 1953, Hunt 1958, 59, Irish and Platzman 1962, Platzman 1963, Richardson and Pore 1969, 1972, and McClure 1970), mainly because of the large magnitude of water level setup observed during severe storms, occasionally exceeding 6 feet. Storm surges on other Great Lakes or similar large bodies of water have been studied by Garriett (1903), Harris and Angelo (1963), Jelesnianski (1966, 1967, 1970), Freeman and Murty (1972), Murty and Freeman (1973), Hamblin and Budgell (1973), Venkatesh (1974) and others.

Most of the techniques for the forecasting of storm surges are based either on statistical methods or on a numerical solution of simplified equations of motion for a particular basin.

The statistical technique depends on compilation of data on severe storms on the Great Lakes. Regression relationships are developed from observed water level fluctuations in terms of a set of parameters screened for the greatest effect. Recent examples are in Richardson and Pore (1969, 1972), Hamblin and Budgell (1973) and Venkatesh (1974). The method is efficient and easily adaptable to forecasting on an operation basis, its accuracy is, however, limited by the limited amount of data from which it was derived, and by the incompleteness of a small set of predictors chosen from all the possible parameters affecting the phenomenon. Periodic oscillations of the surface elevation (seiches) frequently observed during storm surge are seldom correctly predicted by statistical techniques.

The results obtained from dynamical models are in general sufficiently accurate for practical forecasting, their routine application is, however,

limited by the great amount of computer time required for the direct numerical solution of the time-dependent equations of motion. A general discussion of numerical prediction of storm surges has been given by Welander (1961); other relevant works are e.g. by Platzman (1963) and McClure (1970) who developed numerical models for Lake Erie, and Murty and Freeman (1973) for Lake Huron.

The present investigation is an attempt to develop a simple operational technique which would be comparable in accuracy to dynamical models, and in efficiency to statistical methods. Instead of the traditional integration of the governing differential equations in space and time by the use of finite differences, the transient response is represented by a linear superposition of free modes of oscillation of the basin which automatically satisfy the necessary boundary conditions. The eigenfunctions can be calculated once and for all for the particular lake, and the space dependent aspect of the forced solution can be removed this way from routine forecasting, reducing it to solving ordinary inhomogeneous differential equations of the first order in time for the expansion coefficients. Moreover, power spectra of recorded water-level fluctuations in any of the Great Lakes indicate that in most cases only a small number of the lowest modes need to be considered. A restricted theory (constant potential vorticity) of this spectral method has been presented by Rao (1973, 1974) who also applied it to some idealized enclosed basins of constant depth.

Further simplification can be achieved by assuming the forcing function (wind stress) to be approximately uniform over the whole lake area. The response of a lake to a uniform wind impulse can then be precalculated, and the transient response to any uniform time-dependent forcing can be obtained from simple convolution integral, as follows from the linearity of the governing equations. This is analogous to the influence method described by Welander

(1956, 1961) and applied with good results to the prediction of the sea level at Ystad in the Baltic Sea. It should be noted that the assumption of a uniform forcing was not necessary, however, area mean stresses acting over the Baltic were used for simplicity. Welander obtained the impulse response empirically, through numerical analysis of past records of the sea level elevations at the point of interest, and of the corresponding meteorological input.

The exclusion of subsynoptic meteorological processes (squall lines, atmospheric pressure jump lines), resulting from the assumption of uniform forcing, limits the generality and accuracy of the proposed scheme. The effect is not necessarily negligible. When an atmospheric disturbance moves at a speed close to the speed of long gravity waves, resonant coupling is likely to occur, amplifying the surge considerably. This problem has been investigated e.g. by Platzman (1958) in his study of a surge on Lake Michigan. Murty and Freeman (1973) conclude that both the synoptic and subsynoptic meteorological processes should be considered in any attempt at storm surge forecasting on the Great Lakes. However, the present approach is justified by the unavailability of suitable forecasts of surface winds over the lakes on subsynoptic scale.

## 2. DYNAMIC EQUATIONS

The flow and free surface field of a wind forced motion in a lake is described by a set of momentum and continuity equations. The simplified equations given below assume homogeneity, hydrostatic pressure and negligible non-linear effects and lateral friction:

$$\begin{aligned}\frac{\partial U}{\partial t} - fV &= -gh \frac{\partial \xi}{\partial x} + \frac{\tau_{wx}}{\rho} - \frac{\tau_{bx}}{\rho} \\ \frac{\partial V}{\partial t} + fU &= -gh \frac{\partial \xi}{\partial y} + \frac{\tau_{wy}}{\rho} - \frac{\tau_{by}}{\rho}\end{aligned}\quad (1)$$

$$\frac{\partial \xi}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

where  $U$  and  $V$  are horizontal transport components in the east and north direction respectively,  $\xi$  is the free surface displacement,  $h$  is the depth,  $f$  the Coriolis parameter,  $g$  the acceleration of gravity,  $\rho$  the density, and  $\tau_w$  and  $\tau_b$  are wind stress and bottom stress respectively.

Assuming that the spatial variation is discretized in a suitable way (e.g. by the finite element method), the governing set of equations (1) can be expressed in matrix form as

$$\frac{d}{dt} \{\eta\} + [A] \{\eta\} = \{\tau\} \quad (2)$$

where  $\{\eta\}$  is a vector representing the discretized unknowns  $U$ ,  $V$  and  $\xi$ ,  $[A]$  is the corresponding coefficient matrix which includes bottom friction, and vector  $\{\tau\}$  represents the discretized forcing function. For an initial state

$\{\eta(0)\}$ , equation (2) has a general solution

$$\{\eta(t)\} = [X(t)] \{\eta(0)\} + \int_0^t [X(t - \tau)] \{\tau(\tau)\} d\tau \quad (3)$$

where

$$[X(t)] = [C] \begin{bmatrix} e^{\sigma_1 t} & & & \\ & e^{\sigma_2 t} & & \\ & & \ddots & \\ & & & e^{\sigma_n t} \end{bmatrix} [C]^{-1} \quad (4)$$

is a matrix composed of free modes of oscillation of the lake, i.e. the solutions to the eigenvalue problem

$$([A] - \sigma[I]) \{C_i\} = 0 \quad (5)$$

For a suddenly imposed constant wind stress  $\{\tau(t)\} = K$  the transient response is given by

$$\begin{aligned} \{\eta(t)\}_{\tau=K} = & - [C] \begin{bmatrix} \frac{1}{\sigma_1} & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_n} \end{bmatrix} [C]^{-1} K + \\ & + [C] \begin{bmatrix} \frac{e^{\sigma_1 t} - 1}{\sigma_1} & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \frac{e^{\sigma_n t} - 1}{\sigma_n} \end{bmatrix} [C]^{-1} K \end{aligned} \quad (6)$$

The first term in the above expression

$$\{\eta_{SS}\} = [C] \begin{bmatrix} \frac{1}{\sigma_1} \\ \vdots \\ \frac{1}{\sigma_n} \end{bmatrix} [C]^{-1} K = A^{-1} K \quad (7)$$

is the solution to the steady state problem

$$[A] \{\eta_{SS}\} = K \quad (8)$$

The second term is a weighted sum of n free modes of oscillation

$$\sum_{i=1}^n W_i \{C_i\} e^{\sigma_i t} \quad (9)$$

where n is the number of parameters in the discretization process (e.g. the number of nodes in the finite element method). If the complete set of eigenvectors [C] and eigenvalues  $\sigma_i$  were known, the transient response to the suddenly applied constant wind stress could be assembled according to (6), providing, of course, hardly any advantage over a direct finite difference solution. However, analysis of storm surge records suggests that only a small number of the lowest modes are excited and therefore contribute significantly to the sum (9), so that

$$\eta_{\tau=K}(t) \approx \eta_{\tau=K}^l(t) = -\eta_{SS} + \sum_{i=1}^l W_i \{C_i\} e^{\sigma_i t} \quad (10)$$

where  $l \ll n$  is a subset of the total n eigenvectors of [A].

### 3. IMPULSE RESPONSE AND TRANSIENT RESPONSE

It is impossible to take an advantage of the rapid convergence of (9) directly in the calculation of transient response (6) since the truncated matrix of eigenvectors  $[c^L]$  is not square and consequently does not possess an inverse. This difficulty can be overcome using any of the weighted residual methods:

Approximating the exact forced solution (6) by a truncated expansion (10), and substituting it into (2), the resulting residual is in general not equal to zero:

$$R = \frac{\partial}{\partial t} \{\eta^L\} + [A] \{\eta^L\} - \{\tau\} \neq 0 \quad (11)$$

Minimization of the residual  $R$  over the whole space-time domain,  $D$ , provides a condition for the unknown parameters in (10),  $W_i$ , and an optimum approximation  $\{\eta^L\}$  to the exact solution  $\{\eta\}$ :

$$\int_D \delta_i R = 0 \quad (12)$$

where  $\delta_i$  is a suitable weighting function.

The same approach can be used for the calculation of the response to the uniform unit step forcing as well as to a general time and space dependent forcing  $\tau(x, y, t)$ .

Rather than using the above procedure Dr. Hamblin, the Scientific Authority for this contract, specified that the unit impulse response of each lake be calculated in the following manner:

Because in storm surge forecasting the free surface variable  $\xi$  is of

primary interest the velocity components will be deleted from the eigenvectors  $\{C_i\}$ . The vector of free surface displacements can be written analogously to (10) as

$$\{\xi(t)\} = -\xi_{SS} + \sum_i^{\ell} W_i \{C_i\} e^{\sigma_i t} \quad (13)$$

Assuming the lake to be at rest initially,  $\{\xi(0)\} = 0$ , it follows that

$$-\{\xi_{SS}\} + \sum_i^{\ell} W_i \{C_i\} = 0 \quad (14)$$

The fact that initially  $u$  and  $v$  are zero implies that

$$\left. \frac{\partial \xi}{\partial t} \right|_{t=0} = 0 \quad \text{and} \quad \left. \frac{\partial^2 \xi}{\partial t^2} \right|_{t=0} = 0$$

or in the discrete form

$$\sum_i^{\ell} \sigma_i W_i \{C_i\} = 0 \quad \text{and} \quad \sum_i^{\ell} \sigma_i^2 W_i \{C_i\} = 0 \quad (15)$$

The weighting coefficients,  $W_i$ , may be found from the complex equations (14) and (15) or, in matrix notation from:

$$\begin{bmatrix} [C_i^{\ell}] \\ \hline [\sigma_i C_i^{\ell}] \\ \hline [\sigma_i^2 C_i^{\ell}] \end{bmatrix} \{W\} = \begin{Bmatrix} \xi_{SS} \\ \hline 0 \\ \hline 0 \end{Bmatrix} \quad (16)$$

where e.g.  $[\sigma_i C_i^{\ell}]$  is a matrix composed from individual eigenvectors  $\{C_i\}$  multiplied by the corresponding eigenvalue  $\sigma_i$  as

$$[\sigma_1 \ C_1^L] = [\sigma_1 \{C_1^L\} \ ; \ \sigma_2 \{C_2^L\} \ ; \ \sigma_3 \{C_3^L\} \ ; \ \dots \text{etc.}]$$

Note, however, that the coefficient matrix in (16) has a dimension  $(6n \times 2L)$  and cannot be inverted. Therefore no exact solution can be found, instead  $\{W\}$  is determined as a least squares fit to (16).

Once the step function response,  $\xi_{ST}$ , is known at a point in the lake

$$\xi_{ST} = \sum_i^L W_i C_i e^{\sigma_i t} - \xi_{SS} \quad (17)$$

the response to any uniform time-varying wind stress and pressure gradient acting over the whole lake can be obtained by the summation integral

$$\xi(t) = \int_{T=0}^{\infty} \frac{d\tau}{dt} (t - T) \xi_{ST}(T) dT \quad (18)$$

or, alternatively, the unit impulse response,  $\xi_{imp.}$ , can be calculated by differentiation of the step function response

$$\xi_{imp}(t) = \frac{d\xi_{st}}{dt} \quad (19)$$

and the time-varying forcing response follows from the convolution integral

$$\xi(t) = \int_{T=0}^{\infty} \tau(t - T) \xi_{imp}(T) dT \quad (20)$$

since the input data is, in general, in discrete form, (20) becomes

$$\xi_K = \Delta T \sum_{j=0}^M (\xi_{imp})_j \tau_{K-j} \quad (21)$$

where  $M$  is a sufficiently large time interval.

In the above derivations it was assumed that the correct value of  $\sigma$ , the complex frequency of the damped modes of oscillation, is known. In fact only the inviscid seiches are available at present. Instead, the effect of friction is parameterized in terms of the  $Q$ -factor, which can be later adjusted for best agreement with the observed water level fluctuations. Hence the complex frequency  $\sigma = \sigma_R + i\sigma_I$  is taken as

$$\begin{aligned}\sigma_I &= \sigma_{\text{inviscid}} \sqrt{1 - \frac{1}{4Q^2}} \\ \sigma_R &= \frac{\sigma_{\text{inviscid}}}{2Q}\end{aligned}\tag{22}$$

The steady state solution and the free modes of oscillation used in (16) are obtained from finite element models. Both models used identical subdivision of a lake into quadratic elements and an identical transformation of boundary elements, and are therefore fully compatible. The details of the models will be omitted in this report. Instead, the reader is referred to Hamblin (1976) for particulars of the seiche model and to Penicka and Hamblin (1975) for the description of the steady state model.

This contractor has been opposed to the use of the method outlined above, for the calculation of step and impulse response (equation 16), and accepted Dr. Hamblin's judgement only on the basis of his longer experience.

#### 4. METEOROLOGICAL INPUT

To serve its purpose as an operational method for the prediction of storm surges the technique requires regular forecasts of surface winds over the Great Lakes as its input. At present there are no routine surface wind forecasts available, instead, regression relations developed by Feit and Barrientos (1974) have been used for the calculation of surface winds from various forecast elements computed by the U.S. National Meteorological Center Primitive Equation (PE) Model. The output from the Canadian PE model is at present not compatible with the published regression relations. A future modification of the PE model currently being developed at the Canada Meteorological Centre, Dorval, is expected to provide all predictors required for the application of the technique. The complete set of regression coefficients was kindly provided by Mr. D. M. Feit from NOAA, Silver Spring, Maryland, who also arranged for a retrieval of past PE outputs from NOAA archives for the test episodes. At the time of writing only one tape, covering the period from July 3, 1969 to April 17, 1970 has arrived. The format of this data is given in Table 1.

Figure 1 shows the grid points (of the total 255) used in the calculation of surface wind forecasts over the Great Lakes. The actual input in the regression equations are the predictor values at the centre of the lake obtained through biquadratic interpolation from their value at the six closest grid points (see Figure 1).

The Feit and Barrientos regression equations are not applicable to Lake St. Clair. Because of its small size and the resulting short response time of the order of 2 hours, the local hourly winds (as measured at the Windsor Airport weather station) had to be employed.

The wind stress can be estimated from the forecast wind according to the formula

$$\tau = C \rho_a W_a^2 = \bar{C} W^2 \quad (23)$$

where  $W_a$  is the predicted wind,  $\rho_a$  the density of the air, and  $C$  a drag coefficient with a value around  $2.5 \times 10^{-3}$ . The true value of  $C$  (or  $\bar{C}$ ) can be adjusted for best agreement with observed surge levels.

The momentum exchange between the atmosphere and the water, and consequently the magnitude of the surge, are considerably increased during unstable conditions when the water is warmer than the air above it. According to McClure (1970) this effect can be accounted for by modifying the drag coefficient  $C$  in (23) by the air-water temperature difference as

$$C = A + B (T_A - T_W)$$

where  $A$  and  $B$  are constants.

Certain measure of the atmospheric stability is, however, inherent in the wind forecast equations due to the fact that the data, from which these equations were derived, were stratified by seasons. Because of the limited time available, no additional stability criterion was incorporated. For the same reason no attempt was made in the present investigation to include the effect of the atmospheric pressure gradient on the water level. (It can be argued that to some extent the pressure gradient is already implied in the wind forecasts).

## 5. COMPUTER PROGRAMS

A number of computer programs have been developed and used directly or indirectly in testing the method. The first group of programs is used for the actual storm surge forecasting either from observed hourly winds (Program RDTAPE), as is the case of Lake St. Clair, or from the forecast elements of the Primitive Equation Model (Program WINDS) in the case of the Great Lakes.

The second group is associated with the Finite Element Models (F.E.M.). In principle these programs are run only once for each lake, in order to obtain the impulse response of the lake at the point (or points) of interest. The impulse response at a point is a property of the lake and can be calculated once and for all.

Generating the division of a lake into finite elements is prone to error and the input data requires careful checking in order to avoid sometimes unrecognized errors in the results. Programs CHECK, FELM, and plotting the lake depth contours with DISPLAY assist with the input data checking, additional check is frequently provided by plots of model outputs.

Program STEADY calculates the steady state circulation while SEICHMO or the twin programs MATRIX and MODES compute the free modes of oscillation of a lake to be combined according to (16), (17) and (19) in program IMPULS (or XYKLM) to obtain responses of the lake to unit impulses in the direction of the co-ordinate axes. Program DISPLAY provides a convenient display of the F.E.M. model results.

Programs developed or modified by this contractor are listed and briefly described in Appendix A. Most of these programs are sufficiently general to be used without change for any lake providing that appropriate

input data is employed. Exceptions are mentioned in the program descriptions. Input data reading in IMPULS (or XYKLM) has to be modified for each lake individually in order to reduce, to acceptable level, the size of the coefficient matrix of the least squares problem by eliminating some nodal points.

At the request of Dr. Hamblin other programs in which development this contractor had not taken part but which are associated with the storm surge project are listed in Appendix B for the sake of completeness.

## 6. RESULTS AND DISCUSSION

The amount of verification of the present technique was severely restricted by the limited time available. Furthermore, due to some unforeseen difficulties, only one computer tape with PE outputs from NOAA, Silver Spring, Maryland arrived by the end of this contract. Consequently only 4 storm episodes could be calculated for the Great Lakes. The free modes of oscillation of Lake Huron and Georgian Bay are also not available at the present, and the storm surges on these lakes could not be modelled by the present technique.

### Lake St. Clair

The Lake St. Clair was treated separately because observed rather than predicted winds had to be used as input. This is a consequence of the considerably smaller size and shorter response time of this lake compared to other Great Lakes. The relatively round shape of Lake St. Clair also distinguishes it from other Great Lakes. While in the case of an elongated basin the impulse response can be expected to show a distinct dominance of the fundamental seiche this is not necessarily so in the case of Lake St. Clair where some higher modes may be of equal importance as the fundamental.

The response of Lake St. Clair to unit impulse forcing in the SSE-NNW and the WSW-ENE directions calculated from (16), (17) and (19) and the corresponding step function response (17) are shown in Fig. 2. Obviously the non-zero value of the calculated impulse responses (and the slope of the step responses) at zero time do not correspond to physical reality. Possibly a consequence of the employed approximation, it is difficult to estimate the effect it has on the calculated storm surges.

A comparison with a step impulse response for Lake St. Clair obtained by some alternative means would be desirable. For an elliptical basin, calculations undertaken by Dr. Hamblin have shown good agreement between an analytical impulse response and the impulse response obtained by the present technique. The responses shown in Fig. 2 were calculated for Q-factor equal 1. This represents a relatively strong damping, however, impulse responses with higher Q gave less favourable comparison between observed and computed water level fluctuations.

The same storm episodes (if available) as those used by Dr. Venkatesh (1974) for the development of his regression technique have been hindcast by the present model in order to facilitate comparison. The plots in Appendix C show observed water level fluctuations at Belle River (30 minute values extracted from 5 minute digitized values at the Belle River water level station), together with 30 minutes elevations obtained by the present model and hourly elevations calculated from

$$\text{elev.}_t = 0.189 + 0.0007511 V^2 \quad (25)$$

where  $V^2$  is the component of "effective" wind speed in the north-south direction. (For details see Venkatesh (1974)). The above expression is equivalent to the first 2 terms of the regression relation given by Venkatesh (1974) for Belle River and serves as a rough estimate for comparison with the present work. The neglected 2 terms are functions of atmospheric stability and in most cases will be small; it was unfortunately impossible to obtain the water temperature information at short notice. There is, however, no doubt that some failures of the forecasts by both the truncated regression and the present techniques can be explained by the neglect of the instability effects. The storm of November 1 to 4, 1966

is a likely example.

Local hourly winds (as measured at Windsor Airport) were used in (25). For the convolution sum (21) 30 minutes values of wind stress were linearly interpolated from hourly values.

Six modes of oscillation (see Table 2) were used for the calculation of step response (17). It can be seen that 30 minutes interval  $\Delta T$  in (21) does not provide sufficient resolution for the last 3 modes, and will be changed in future runs. 15 minutes interval should be adequate.

#### Lake Erie and Lake Ontario

Only one tape of PE outputs containing 5 storm episodes was available by the end of the present contract. It is impossible to make any meaningful conclusions from such a small number of forecasts, the results are included only for the sake of completeness.

The calculated response to unit impulse forcing along the longitudinal and transverse axes of the lake is shown in Fig. 3 and Fig. 4 for 2 locations on Lake Erie (Kingsville and Port Colborne), and in Fig. 5 for one location on Lake Ontario (Burlington). Step function responses for these 3 locations are also included in the figures. The angular frequencies and periods of modes used for the computation are given in Table 2. All the responses were calculated for  $Q=1$ . The same discrepancy as that for Lake St. Clair, large value of impulse response at zero time, can be observed also in the plots for Lake Erie and Lake Ontario.

Convolution sums (21) were calculated for available episodes and are given in Appendix D. Values of the various PE forecast elements were interpolated to the approximate centre of each lake\* from six closest NMC grid points (Fig. 1) and the resulting predictors were used with the

appropriate regression equations to obtain surface wind forecasts uniform over the lake area. Details of the method can be found in Feit and Barrientos (1974). The surface wind forecasts are spaced at 6 hours intervals and for the present purpose are linearly interpolated to 30 minutes intervals. In the calculation of wind stress a generally accepted value of drag coefficient,  $2.8 \times 10^{-3}$ , was found to be sufficient for lakes Erie and Ontario. However, for Lake St. Clair about twice as large drag coefficient,  $5.5 \times 10^{-3}$ , was required for good results.

\* This is somewhat different from the original forecast procedure where both Lake Erie and Lake Ontario are divided into two halves and the predictors are interpolated to forecast locations at the centre of each half.

## 7. CONCLUSIONS

Reasonable agreement between computed and observed water level fluctuations has been obtained for a number of storms on Lake St. Clair. Some discrepancies can be explained by the neglect of atmospheric instability effects, particularly in the case of episodes occurring between September and December. Neglecting the atmospheric pressure gradient and small scale atmospheric disturbances is not likely to have had any significant effect on the forecasts because of the small size of Lake St. Clair.

No meaningful conclusions can be drawn from the small number of episodes on lake Erie and Lake Ontario. Since the water level forecasts are made on the basis of predicted rather than observed winds, less favourable results than for Lake St. Clair may be expected.

In conclusion it will be necessary before this technique can be implemented on a routine basis on the CMC weather model to evaluate the hindcasts more thoroughly on Lake Ontario and Lake Erie.

REFERENCES

- FEIT, D.M. and (1974) Great Lakes Wind Forecasts Based on  
BARRIENTOS, C. S. MDS. Proc. 17th Conf. Great Lakes Res.  
pp 725-32
- FREEMAN, N.G. and (1972) A study of a Storm Surge on Lake Huron.  
MURTY, T.S. Proceedings 15th Conference Great Lakes  
Research pp 562-82.
- GARRIOTT, E.B. (1903) Storms of the Great Lakes, U.S. Weather  
Bureau, Bulletin K, Washington, D.C.
- HAMBLIN, P.F. and (1973) Wind-induced Water Level Changes on the  
BUDGELL, N.P. Southeastern Shore of Lake St. Clair  
CCIW Paper No. 12.
- HAMBLIN, P.F. (1976) Seiches, Circulation and Storm Surges of  
an Ice-free Lake Winnipeg, Unpublished  
manuscript, CCIW.
- HARRIS, D.L. and (1963) A Regression Model for Storm Surge Pre-  
ANGELO, A. diction. Monthly Weather Review Vo. 21,  
pp 701-726.
- HENRY, A.J. (1902) Wind Velocity and Fluctuations of Water  
Level on Lake Erie, U.S. Weather Bureau,  
Bulletin J, Washington, D.C.
- HUNT, I.A. (1958,59) Winds, Wind-Setups, and Seiches on Lake  
Erie, Parts 1, 2. U.S. Corps of Engineers,  
Lake Survey.
- IRISH, S.M., and (1962) An Investigation of the Meteorological  
PLATZMAN, G.W. Conditions Associated with Extreme Wind  
Tide on Lake Erie, Monthly Weather Review,  
Vol. 90, No. 2, pp 39-47
- JELESNIANSKI, (1967) Numerical Computations of Storm Surges  
With Bottom Stress. Monthly Weather Re-  
view Vol. 95, No. 11, pp 740-756.
- JELESNIANSKI, (1970) "Bottom Stress Time-History" in Linearized  
Equations of Motion for Storm Surges.  
Monthly Weather Review, Vol. 98, No. 6,  
462-478.
- KEULEGAN, G.H. (1953) Hydrodynamic Effects of Gales on Lake Erie:  
Journal of Research, National Bureau of  
Standards 50 pp 99-109.

- McCLURE, D.J. (1970) Dynamic Forecasting of Lake Erie Water Levels Report No. 70-250-H, Hydro-Electric Power Commission of Ontario. Research Division Report.
- MURTY, T.S. and FREEMAN, N.G. (1973) Storm Surge Models of Lake Huron. Proceedings 16th Conference on Great Lake Res. pp 533-548.
- PENICKA, F. and HAMBLIN, P.F. (1975) Finite Element Model of Steady State Wind-driven Circulation in a Lake, Proc. Symp. on Modeling of Transport Mechanisms in Oceans and Lakes, CCIW.
- PLATZMAN, G.W. (1958) A Numerical Computation of the Surge of 26 June 1954 on Lake Michigan, Geophysics, 6, pp 407-438
- PLATZMAN, G. W. (1963) The Dynamical Prediction of Wind Tides on Lake Erie. Meteorological Monographs Vol. 4, No. 26 pp 1-44.
- PORE, N. A., PERROTTI, H.P., and RICHARDSON, W.S. (1975) Climatology of Lake Erie Storm Surges at Buffalo and Toledo, NOAA Technical Memorandum NWS TDL-54
- RAO, D.B. (1973) Spectral Methods of Forecasting Storm Surges, Hydrological Sciences Bulletin, XVIII, 3, pp 311-316
- RAO, D.B. (1974) Transient Response of Shallow Enclosed Basins Using the Method of Normal Modes, Inland Waters Directorate Scientific Series No. 38.
- RICHARDSON, W.S. and PORE, N.A. (1969) Aug. A Lake Erie Storm Surge Forecasting Technique. ESSA Technical Memorandum WBTM, TDL 24.
- RICHARDSON, W. S. and PORE, N.A. (1972) Sept. Weather Service Program in Lake Erie Storm Surge Forecasting. T.D.L. Report.
- VENKATESH, S. (1974) The Development of Manual Techniques for the Real Time Prediction of Storm Surges on the Great Lakes, Report prepared for Dept. of Environment under Contract No. OSP3-0248.

- WELANDER, P. (1956) On the Wind Action on Shallow Seas with Particular Reference to the Problem of Numerical Sea Level Prediction, Report Inst. Meteorol., University of Stockholm.
- WELANDER, P. (1961) Numerical Prediction of Storm Surges, Advances in Geophysics 8, pp 315-379, Academic Press, New York.

TABLE 1: Input Data

Lake St. Clair:

WATER LEVEL:

Tape No: 19347

Format: (I5, 1X, 4I2, 1X, A1, 12I4)

Data: (Station No., Year, Month, Day, Hour, Units (B),  
12 five-minute water levels)

Station No: 11965 (Belle River)

Units: "B" for level in 0.01 feet

Note: The first record for each station gives the  
number of storms, followed by comments. Format  
(I4, 19A4). The end of data for each station is  
indicated by EOF mark.

WINDS:

Tape No: THC 704

Format: (I5, 4I2, I4, I2, I3, I3)

Data: (Station No., Year, Month, Day, Hour, Sea level  
pressure, Wind direction, Wind speed, Dry bulb  
temperature)

Station No: 94810 (Windsor Airport)

Units: Wind speed in miles per hour. For the wind direction  
code see #1 card format documentation published by the  
Climatology Division of the Atmospheric Environment  
Service, Toronto.

TABLE 1 . . . continued

Lake Erie:

WATER LEVEL:

Tape No: 19347

Format: (I5, 1X, 4I2 1X, A1, I2I4)

Data: (Station No., Year, Month, Day, Hour, Units (B),  
12 five-minute water levels)

Units: "B" for level in 0.01 feet

Station 12065 (Kingsville)  
No: 12865 (Port Colborne)

PE-OUTPUT:

Tape No: PEOUT 1 (period from July 3, 1969 to April 17, 1970)

Format: (I2, 2I3, I4, 4I2, 255F10.4)

Data:	Characters	Parameter
	1-2	Cycle (00Z to 12Z)
	3-5	Type Field } *
	6-8	Type Surface }
	9-12	Level
	13-14	Year
	15-16	Month
	17-18	Day
	19-20	Projection
	21-2570	255 grid point values

\* For code see Office Note 84 as below

Units: See Office Note 84, U.S. National Meteorological  
Center, June 1975.

Interpolation 172, 204, 174, 188, 189, 173  
Grid Points:

Lake Ontario

As for Lake Erie except:

WATER LEVEL:

Station 13150 (Burlington)  
No.:

PE-OUTPUT:

Interpolation grid points: 173, 203, 205, 188, 204, 189

**TABLE 2:** Frequencies and periods of the six lowest seiches in Lake Ontario, Lake Erie and Lake St. Clair.

Mode No.	Angular Frequency (1/ sec)	Period (hrs)
LAKE ONTARIO		
1	$3.4327 \times 10^{-4}$	5.084
2	$5.3925 \times 10^{-4}$	3.236
3	$7.3913 \times 10^{-4}$	2.361
4	$10.058 \times 10^{-4}$	1.735
5	$10.514 \times 10^{-4}$	1.660
6	$12.078 \times 10^{-4}$	1.445
LAKE ERIE		
1	$1.2346 \times 10^{-4}$	14.137
2	$1.9508 \times 10^{-4}$	8.947
3	$2.9883 \times 10^{-4}$	5.840
4	$4.1532 \times 10^{-4}$	4.202
5	$4.6829 \times 10^{-4}$	3.727
6	$5.2602 \times 10^{-4}$	3.318
LAKE ST. CLAIR		
1	$7.8248 \times 10^{-4}$	2.230
2	$10.541 \times 10^{-4}$	1.656
3	$13.355 \times 10^{-4}$	1.307
4	$18.611 \times 10^{-4}$	0.938
5	$21.701 \times 10^{-4}$	0.804
6	$22.123 \times 10^{-4}$	0.789



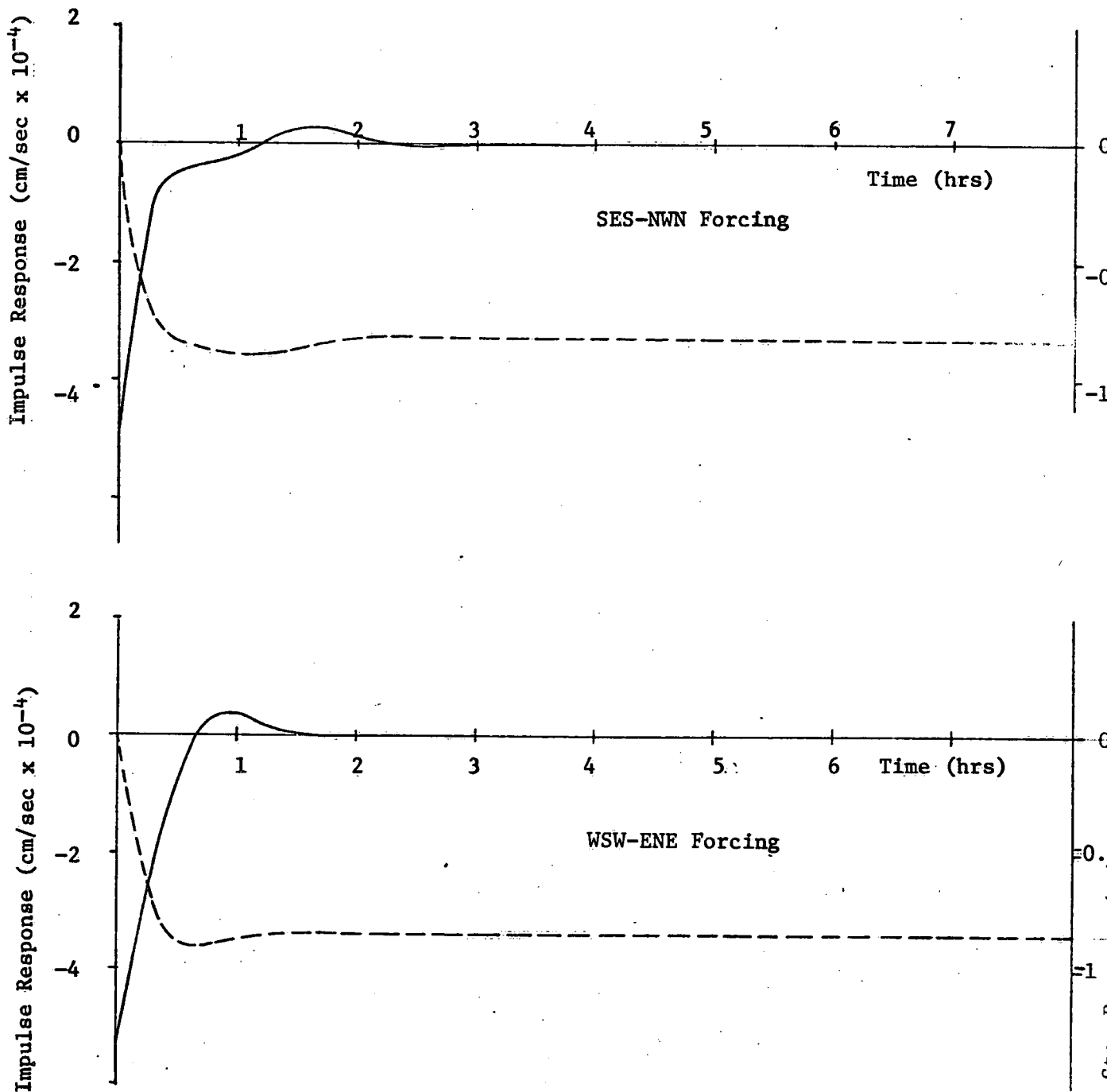


Fig. 2: Calculated impulse response (—) and step response (---) for Lake St. Clair at Belle River

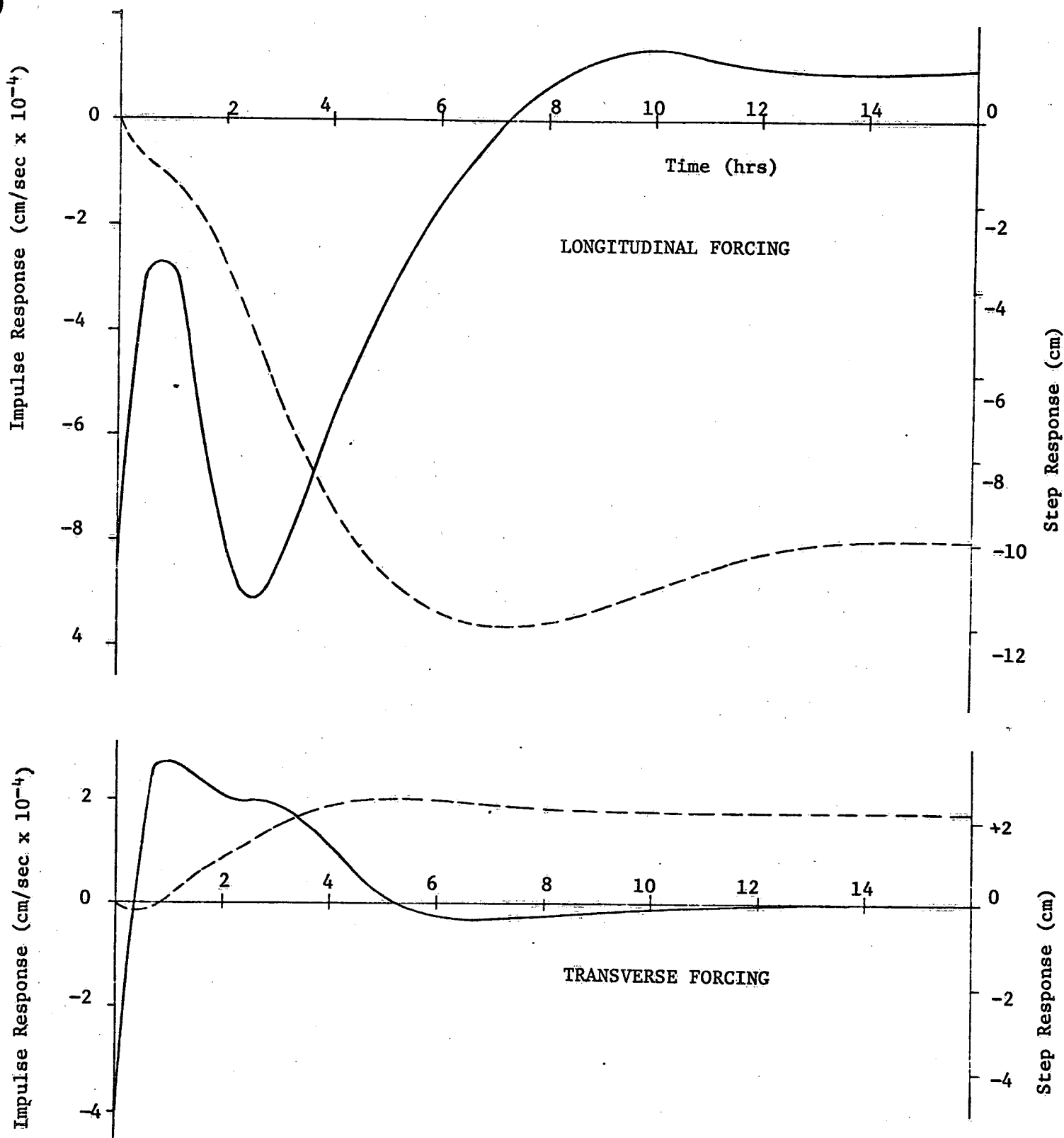


Fig. 3: Calculated Impulse response ( — ) and step response ( - - - ) for lake Erie at Kingsville

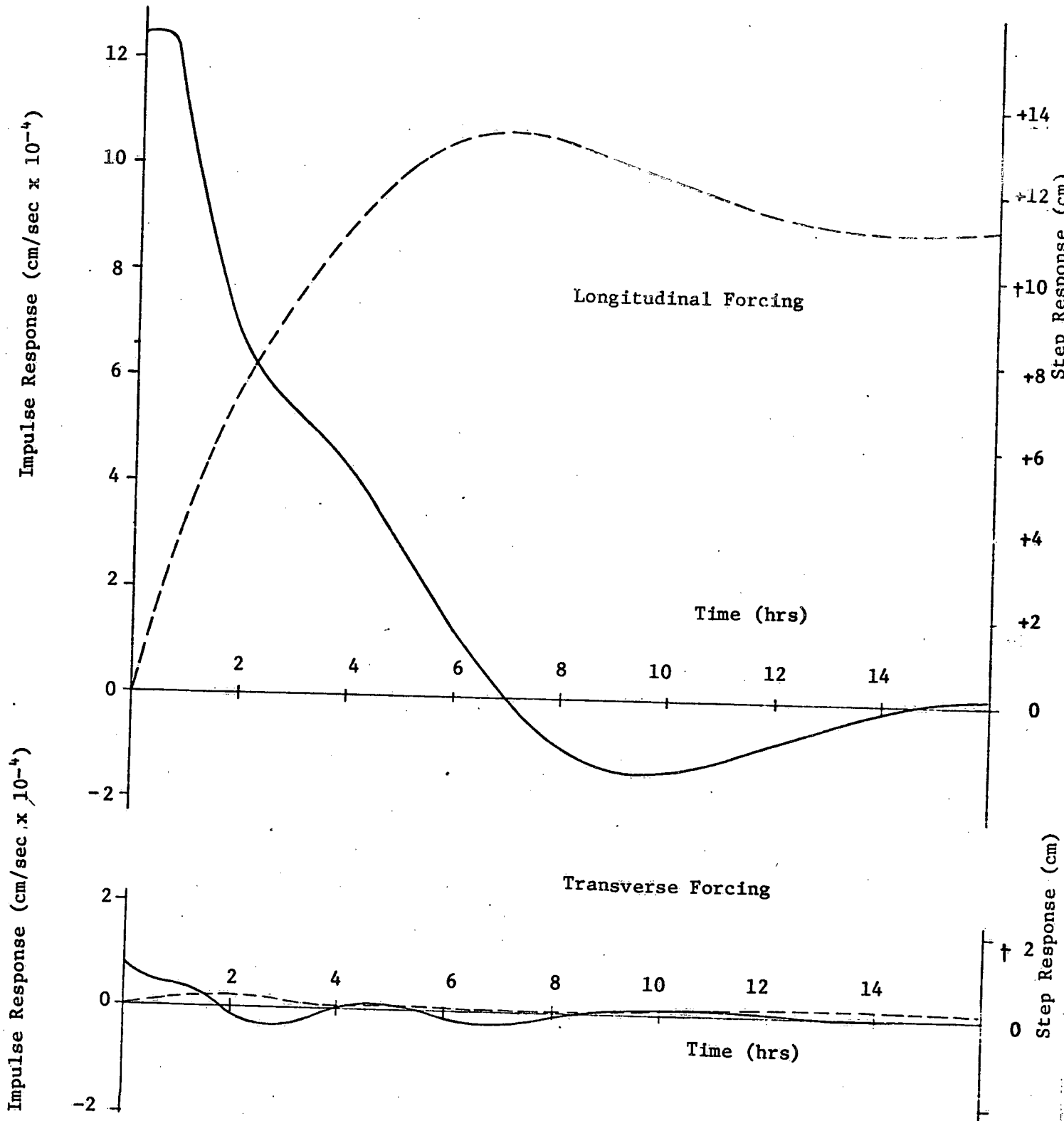


Fig. 4: Calculated impulse response (—) and step response (---) for Lake Erie at Port Colborne.

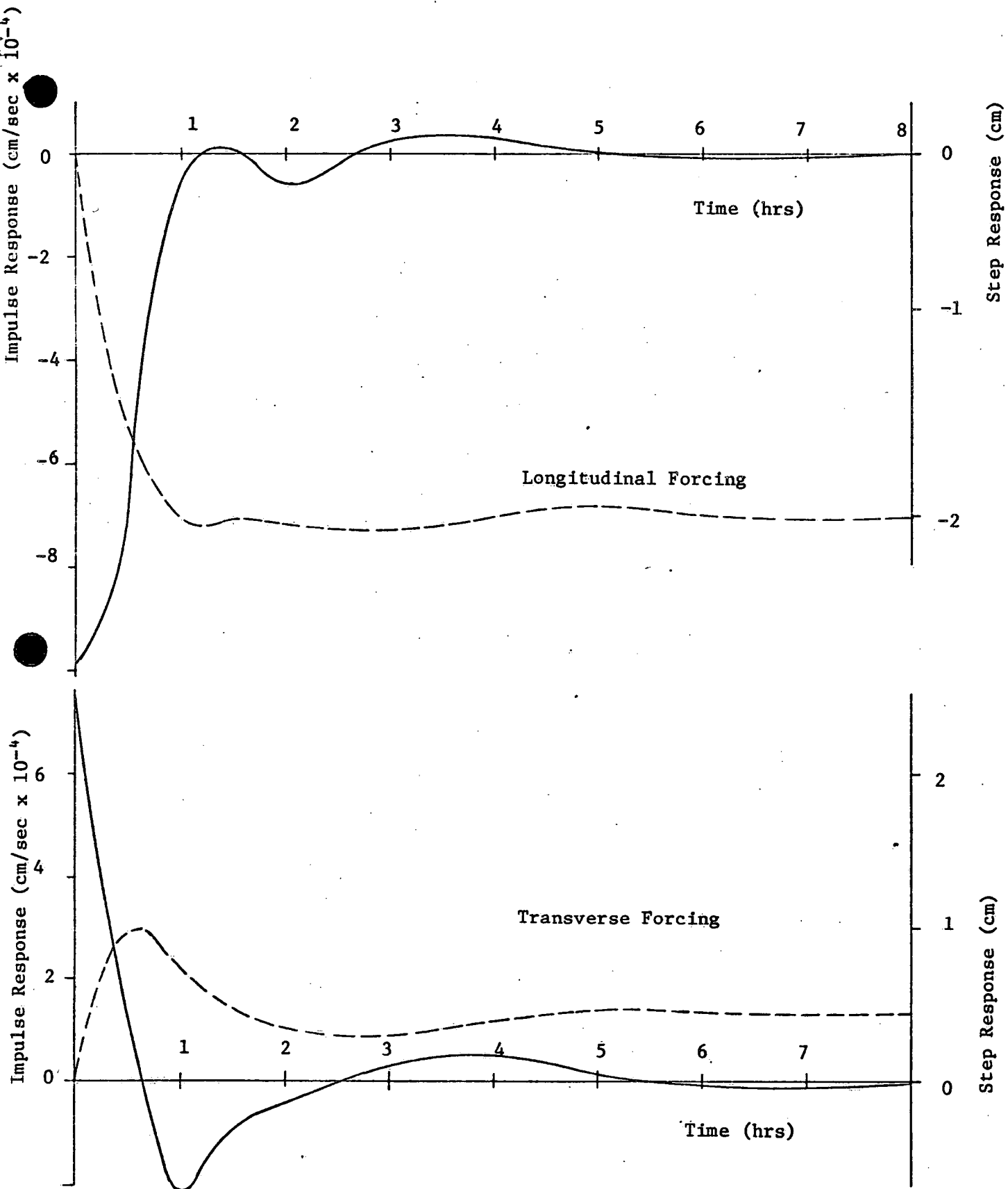


Fig. 5: Calculated impulse response (—) and step response (---) for Lake Ontario at Burlington.

APPENDIX - A

Computer programs associated with the reported development  
of the storm surge prediction technique.

NAME	CHECK
PURPOSE	To facilitate checking the division of a lake into finite elements.
DESCRIPTION	<ul style="list-style-type: none"> <li>- Checks element connections for error. If an inconsistency is found, correct coordinates are calculated and a node number is suggested if a suitable node is available (for mid-edge nodes).</li> <li>- Prints outline of each element on the line printer.</li> <li>- Finds elements with zero depth everywhere, elements with zero area, and nodes with negative depth.</li> <li>- Finds the maximum and minimum values of X and Y coordinates.</li> </ul>
USAGE	-
INPUT	Cards: 1) No. of elements, nodes 2) Element connections 3) Nodal coordinates
OUTPUT	Line printer: - element and node numbers - element outlines - appropriate messages
SUBROUTINES USED	CONECT, WHERE, BNDRY, SHORE, SIDE, GRAPH.
STORAGE REQUIREMENTS	CORE = 40 qp.
SOURCE LANGUAGE	FORTRAN
MACHINE	CDC 6600
PROGRAMMER	F. Penicka
REFERENCES	-

NAME	FELM
PURPOSE	To plot the outline of a lake together with its sub-division into triangular elements.
DESCRIPTION	<ul style="list-style-type: none"><li>- Plots the outline of a lake as represented by the finite element model, plots the boundaries between elements and numbers each element according to the connecting card.</li><li>- Plots the north - south direction and the scale of the plot.</li><li>- The plotting of the element boundaries can be bypassed to obtain a plot of the shoreline.</li></ul>
USAGE	An appropriate scale for 12" chart paper is calculated in subroutine WHERE. The resulting plot size is in constant ratio to corresponding line-printer plots in programs ELAKE, DISPLAY etc. in order to facilitate drafting. The scale factor can be overwritten if plots of other size are required.
INPUT	Cards: 1) Title card (Lake name, Coriolis parameter, Eddy coef., Angle of north-south direction with X-axis). 2) No. of elements, nodes. 3) Element connections 4) Nodal coordinates.
OUTPUT	CALCOMP Plotter: Outline and element subdivision of a lake.

SUBROUTINES USED      CONECT, WHERE, SETUP, TRANSF, SHORE, DRAWIT

STORAGE REQUIREMENTS      CORE = 55 qp.

SOURCE LANGUAGE      FORTRAN

MACHINE      CDC 6600

PROGRAMMER      F. Penicka

REFERENCES      -

NAME	STEADY
PURPOSE	To calculate steady state circulation in an enclosed basin using finite element method.
DESCRIPTION	<ul style="list-style-type: none"> <li>- Forms a FEM equivalent of steady state equations of motion;</li> <li>- Solves the resulting matrix equation for free surface elevation;</li> <li>- Calculates corresponding velocities and plots the vectors on calcomp plotter;</li> <li>- Contours the free surface elevation and plots it on a line printer.</li> </ul>
SUBROUTINE DESCRIPTIONS	
CONECT	- Reads from cards and prints out the element connections.
WHERE	<ul style="list-style-type: none"> <li>- Reads from cards nodal coordinates and depths, converts them to centimeters and prints them out.</li> <li>- Finds the minimum and maximum values of X and Y coordinates, if required rotates the coordinate system.</li> <li>- Calculates scales for display such that the sizes of the calcomp plot and the line printer plot are in constant ratio (after suitable reduction the plots can be superimposed)</li> </ul>
SETUP	<ul style="list-style-type: none"> <li>- Calculates elements of coefficient matrix and right hand side vector as per (1).</li> <li>- Assembles the complete system matrix from individual element matrices and rearranges the system matrix into a one-dimensional array.</li> <li>- Writes the system coeffs. matrix on disc.</li> </ul>

- TRANSF - Performs the boundary element transformation of coordinates.
- Function P - Calculates the value of a variable at a selected position inside an element from its values at six nodes.
- Function DER - Calculates the partial derivative with respect to X or Y of a shape function at a selected position inside an element.
- SOLVE - Banded matrix solver based on Gaussian elimination. This subroutine is an adaptation for intermediate disc storage of the coefficient matrix, of the library subroutine MAS 008. Part of the coef. matrix is read from disc into an auxiliary array U1. If any element in a row being processed is not within the dimension of U1 either subroutine SCRATCH (during element reduction) or subroutine SUPPLY (during back substitution) is called to read or write an additional section of the coeff. matrix into U1
- SCRATCH - Writes and reads sections of coeff. matrix on/from disc from/into auxiliary array U1, shifts elements in U1 forward to make room for new section, zeros the vacated positions.
- SUPPLY - Shifts elements in U1 backward, locates a new section on disc and reads it into V1.
- VELOCITY - Calculates velocity components at all nodal points and at centroid points of boundary elements, from free surface elevations at the nodes of each element.

- SELECT** - For each element finds which surface elevation contours pass through the element.
- CONTOUR** - For each element finds the X and Y coordinates of a sufficient number of points lying on the selected contours, by calculating roots of a biquadratic polynomial (in triangular area coordinates)
- SHORE** - For each boundary element finds the X and Y coordinates of a sufficient number of points on the lake boundary using the relations for isoparametric transformation of boundary elements. Plots the resulting shoreline on Calcomp plotter.
- PLOTD** - Plots the velocity vectors on Calcomp plotter.
- LEGEND** - Plots the legend for the velocity plot on the Calcomp plotter. (name of the lake, distance and velocity scales, important parameters, north-south direction, direction of wind stress)
- PRINTD** - Prints out the distribution of free surface elevation on the line printer.
- GRAPH** - Prints out the shoreline and surface elevation contours on the line printer.

**USAGE**

- INPUT CARDS:**
- 1) Title card (Lake name, Coriolis parameter, Eddy coef., Angle of north-south direction.
  - 2) No. of elements, nodes; matrix band-width.
  - 3) Element connections.
  - 4) Nodal coordinates.

## OUTPUT

Line printer: - input data

- numerical results (surface elevation, vertically averaged velocity)

- plots of surface elevation contours as per  
DISPLAY

Card punch - output data (surface elevation and  
velocity at nodal points)

Calcomp Plotter - Velocity vectors at nodal points with lake  
shoreline and scales (12" chart paper)

SUBROUTINES  
USED

CONECT, WHERE, SETUP, TRANSF, SCRATCH, SUPPLY, SOLVE,  
VELOCITY, SELECT, CONTOUR, SHORE, PLOTD, LEGEND, PRINTD,  
GRAPH; Functions: SINH, COSH, P, DER

STORAGE  
REQUIREMENTS

CORE = 113, SCR = 8

SOURCE  
LANGUAGE

FORTRAN

## MACHINE

CDC 6600

## PROGRAMMER

F. Penicka

## REFERENCES

(1) Penicka, F. and Hamblin, P. F., Finite Element Model of  
Steady State Wind-driven Circulation in a lake; Proc. Symp.  
on Modeling of Transport Mechanism in Oceans and Lakes,  
CCIW, 1975.

NAME	MATRIX
PURPOSE	To form the system matrices for the FEM seiche model and to write them on permanent file (disc)
DESCRIPTION	Adaptation of program SEICHMO * for complex arithmetic and for disc storage of the system matrices. The banded matrices are stored row-wise (band only).
USAGE	To be used with program MODES.
INPUT	CARDS: 1) Title card (lake name, coriolis parameter); 2) No. of elements, No. of nodes, matrix band width, no. of modes; 3) Element connections; 4) Nodal coordinates
OUTPUT	Line printer: - input data - Total number of coefficients written on disc (MS1)  Disc: system matrices.
SUBROUTINES USED	CONECT, WHERE, SETUP, BNDY, PSETA, PSETB, PSETC, SCRATCH; Functions: P, Q, TG
STORAGE REQUIREMENTS	CORE = 100 qp
SOURCE LANGUAGE	FORTRAN
MACHINE	CDC 6600
PROGRAMMER	F. Penicka
REFERENCES	*) Program SEICHMO developed by Dr. P. F. Hamblin

NAME	MODES
PURPOSE	To compute free modes of oscillation of a lake using Rayleigh iteration scheme, from system matrices written on disc.
DESCRIPTION	Adaptation of program SEICHMO* , for complex arithmetic and for disc storage of input and intermediate matrices. Complex solver CSOLVE and associated subroutines are analogous to real solver SOLVE and associated subroutines in STEADY. READ reads from disc and shifts elements in system matrices. Other subroutines are analogous to corresponding subroutines in SEICHMO.
USAGE	To be used with program MATRIX.
INPUT	<p>Cards: 1) Title card (lake name, number of coefficients written on disc)</p> <p>2) No. of elements, no. of nodes, matrix bandwidth, no. of modes</p> <p>3) Initial guesses for frequencies.</p> <p>Disc: System matrices.</p>
OUTPUT	<p>Line printer: - intermediate iteration results;</p> <p>- final frequencies and surface displacements.</p> <p>Card punch: - mode card (mode number, mode frequency)</p> <p>- complex displacements at nodal positions.</p>
SUBROUTINES USED	RAYQUO, RGHTHN, READ, PACK, CSOLVE, SCRATCH, SUPPLY, NORMAL.
STORAGE REQUIREMENTS	CORE = 128 qp SCR = 10

SOURCE  
LANGUAGE

FORTRAN

MACHINE

CDC 6600

PROGRAMMER

F. Penicka

REFERENCES

\*) Program SEICHMO developed by Dr. P. F. Hamblin

NAME	DISPLAY
PURPOSE	To display data obtained by finite element models in the form of contours.
DESCRIPTION	<ul style="list-style-type: none"> <li>- Finds a suitable scale for line-printer plots (subroutine WHERE)</li> <li>- Prints out the scaled and truncated data values at their appropriate node positions (subroutine PRINTD)</li> <li>- Calculates coordinates for the shoreline from information provided by element connections cards and by zero depth nodes (subroutines TRANSF and SHORE)</li> <li>- Calculates coordinates for contours (subroutines SELECT and CONTOUR)</li> <li>- Prints out the shoreline and the contours on line printer (subroutine GRAPH)</li> </ul>
USAGE	<p>In its present form the program processes the complex numbers obtained from seiche models. The amplitudes and phases are calculated and both are contoured. Cophase lines are incremented by <math>90^{\circ}</math>. The main program has to be modified for plotting of other finite element data (e.g. depth contours and surface elevations, by bypassing the phase and amplitude calculation and the phase plotting.</p>
INPUT	<p>Cards: 1) Title card (lake name, coriolis parameter, Eddy coef., Angle of north-south direction).</p> <p>2) No. of elements, nodes</p> <p>3) Element connections</p> <p>4) Nodal coordinates</p>

5) Data title card (must contain contour increment)

6) Data to be contoured.

OUTPUT	Line printer: - data values printed at nodal positions - contours and shoreline plots
SUBROUTINES USED	CONNECT, WHERE, PRINTD, SETUP, TRANSF, SELECT, CONTOUR, SHORE, GRAPH; Function P
STORAGE REQUIREMENTS	CORE = 100 qp., SCR = 4
SOURCE LANGUAGE	FORTRAN
MACHINE	CDC 6600
PROGRAMMER	F. Penicka

NAME WINDS

PURPOSE To calculate storm surge from surface wind forecasts and compare it with observed water level fluctuations.

DESCRIPTION

- Reads water level fluctuations and PE output from tapes;
- Calculates surface winds from PE outputs;
- Calculates wind stress and its convolution integral.
- Prints out resulting response together with observed water level fluctuations as functions of time.

SUBROUTINE  
DESCRIPTION

Main Program - Searches appropriate tape and reads water levels for specified station and specified date.

- Searches appropriate tape and reads PE outputs for specified date.
- Chooses relevant grid values of PE elements and assigns them to their appropriate positions in regression variable array, computes additional regression variables.
- Fills in intermediate (1/2 hour) values of wind stress.

REGCOEF - Reads from cards regression coefficients and variable subscripts.

Forms appropriate regression equations and prints them out.

Function P - Biguadratic interpolation

Function SPEED - Calculates speed from U and V components

REGRES - Using regression equations calculates surface wind speed and U and V components.

**CONVOLVE** - Calculates Wind stress in the appropriate coordinate system and plots it on line printer as a function of time.

- Calculates the convolution integral and plots it together with the observed water level fluctuations on line printer as a function of time.

**USAGE** Water level station number and the number of storms must be specified in the main program.

**INPUT** Cards: 1) X - impuls response of lake  
 2) Y - impuls response of lake  
 3) Triangular area coordinates of a centre of lake w.r.t. the closest 6 PE - model grid points.  
 4) Gridpoint numbers as above  
 5) Regression coef. and variable subscripts  
 6) PE output control card  
 7) Storm dates

Tapes: 1) Observed water levels  
 2) PE output

**OUTPUT** Line printer:- set of regression equations for the calculation of surface velocities.

- plot of X and Y components of wind stress vs. time

- plot of calculated and observed water level fluctuations vs. time

SUBROUTINES USED	REGCOEF, REGRES, CONVOLVE, Functions: P, SPEED
STORAGE REQUIREMENTS	CORE = 55 qp
SOURCE LANGUAGE	FORTRAN
MACHINE	CDC 6600
PROGRAMMER	F. Penicka

NAME	RDTAPE
PURPOSE	To calculate storm surge from observed winds and compare the results with observed water level fluctuations.
DESCRIPTION	<p>-Searches appropriate tape and reads water levels for specified station and given dates (read from data cards). If a storm is found, which was not listed on the input data cards, the additional storm is processed.</p> <p>-Searches appropriate tape and reads wind speeds and directions for specified date.</p> <p>-Calculates wind stress and its convolution integral.</p> <p>-Prints out resulting response together with observed water level fluctuations as functions of time.</p>
USAGE	<p>In its present form the program processes the Lake St. Clair data. Slight modification would make it applicable to other lakes.</p> <p>Number of storms to be processed is specified in the main program.</p>
INPUT	<p>Cards: 1) X - impulse response of lake. 2) Y - impulse response of lake. 3) Storm dates (beginning and end)</p> <p>Tapes: 1) Observed water levels 2) Observed winds</p>

## OUTPUT

Line printer - plot of X and Y components of wind  
stress vs. time.

- plot of calculated and observed water  
level fluctuations vs. time.

SUBROUTINES  
USED

CONVOLVE

STORAGE  
REQUIREMENTS

CORE = 55 qp.

SOURCE  
LANGUAGE

FORTRAN

## MACHINE

CDC 6600

## PROGRAMMER

F. Penicka

NAME	IMPULS
PURPOSE	To compute Impulse Response function at 1 or more points in lake from steady state and 6 seiches
DESCRIPTION	Computes weighting factors of free modes of oscillation for unit step forcing response from steady state and zero initial velocity and acceleration conditions. Least squares solver is used. Unit impuls forcing response is obtained by differentiation of the step response.
USAGE	Some nodal points may have to be decimated in order to decrease the size of coeff. matrix.  Frictional damping must be specified in the main program by defining Q.
INPUT	Cards: 1) Steady state 2) Free surface elevation modes.
OUTPUT	Line Printer: Plots of impulse and step forcing responses Card punch: Impulse response values (100)
SUBROUTINES USED	Least squares solver
STORAGE REQUIREMENTS	95 gp.
SOURCE LANGUAGE	FORTRAN
MACHINE	CDC 6600
PROGRAMMER	P. F. Hamblin

#### APPENDIX - B

Computer programs associated with the reported development of the storm surge prediction technique. (The following programs, with the exception of VELOCITY, were neither written nor modified or used by the contractor).

## DESK SET-UP ORDER

PROGRAM: Read off Digitized Tape

DATE:

MACHINE: CDC 3170

PROGRAMMER:

NO.	CONTROL CARD(S)	COMMENT(S)
1	\$JOB, 845, 080/READ, 1, 2000, 500	
2	\$SCHED, CLASS=C, SCR=3, 607=1, CORE=60	
3	\$*DEF(T,,1, 607, FE999, 0,,,,,H,,I)	
4	\$FTNU (X,S,L)	
5	PROGRAM	
6	\$X, LGO	
7	DATA CARD: NO. OF ELEMENTS - COLS. 1-3 NAME OF LAKE - COL 5 - N	
8	E-O-F	

## DESK SET-UP ORDER

PROGRAM: ELEMENT PLOT ROUTINE

DATE:

MACHINE: CDC 3170

PROGRAMMER: Lakshmi

NO.	CONTROL CARD(S)	COMMENT(S)
1	\$JOB, 845, 080ELMTS, 10, 5000	
2	\$ SCHED, CLASS=C, CORE=44, TIME=02, SCR=3, 607=1	
3	\$FILE 5 = INP	
4	\$FILE 6 = OUT	
5	\$FTNU (L, S, X)	
6	FORTTRAN PROGRAM	
7	\$ X, LGO	
8	DATA DECK (a) SPECIFICATION CARD (b) ELEMENT - NODE LIST (c) NODES,, X, Y - COORDS, DEPTH - CARDS (d) HEADER CARDS	
9	E-O-F	

## DESK SET-UP ORDER

PROGRAM: CONTOURING PROGRAM PRODUCING NODE NOS, &amp; DEPTHS

DATE:

MACHINE: CDC 3170

PROGRAMMER:

NO.	CONTROL CARD(S)	COMMENT(S)
1	\$ JOB, 845, 080WLCNT 030, 20000, 0000,,	
2	\$ SCHED, TIME = 010, CLASS = C, CORE = 065, SCR = 002, 607 = 1	
3	\$*DEF(0,, XQT, 001/017, GPCP - ABS, 01)	
4	\$GPCP, XQT	
5	JOB	
6	FLEX	
7	SIZE CARD	
8	3 CNTL CARDS	
9	DATA DECK	
	NODE NOS, X, Y - COORDS, DEPTHS	
10	BLANK CARD	
11	PHS4	
12	BRDR	
13	PLOT	
14	END	
15	STOP	
16	E-O-F	

NAME XYKLM

PURPOSE Computes Response function at 2 or more points in lake from steady state and 1-6-Seiches

DESCRIPTION Computes weighting functions of seiches modes from steady state and zero initial velocity and acceleration condition for least squares solver. May have decimation of nodal points.

USAGE

INPUT Steady state free surface modes. Note that frictional damping is added by specifying a

$$Q, \sigma R = \frac{\sigma}{2Q}$$

value  $\sigma I = \frac{-\sigma}{8Q} + \sigma$

OUTPUT Line Printer Plots of Impulse and Step Function Responses and punched deck of 100 filter weights

SUBROUTINES USED Least squares solver

NAME	FINL2
PURPOSE	Compute and Plot Time History of Water Level from Response function and Wind Time Series
DESCRIPTION	At two or more points the wind-induced response is plotted as well as wind stress components and observed water levels
USAGE	Convolution must be multiplied by $\Delta t$ and drag coefficient specified $3 \times 10^{-3}$ also orientation of wind axis
INPUT	longitudinal response $p^t$ 1 " " $p^t$ 2 transverse " $p^t$ 1 transverse " $p^t$ 2 speed and direction of winds for a number of storms observed water levels
OUTPUT	line printer plots of wind stress Observed water level Predicted water level

NAME	FUNDM or VELOCITY
PURPOSE	Compute velocities, current ellipses, plot on calcomp
DESCRIPTION	<p>Computes U and V complex velocities from complex free surface, in M. Ellipses, major and minor axis, orientation IB (X) X specifies points at which velocities to be plotted if not at interior vertex, velocities at <math>t=0</math> denoted by <math>\Delta</math> and at <math>ot = \pi/2</math>, by <math>*</math>. Final velocities are in units/S and consistent with a normalized free surface to a maximum amplitude of 100 units</p>
USAGE	<p>Specify f, g, orientation of the lake (ROT) mid-edge points to be plotted,</p> <p>takes into account isoparametric triangles</p> <p>interpolates currents at mid-edges from vertices</p>
INPUT	<p>Connect - reads in data.</p> <p>Where: scales nodal data</p> <p style="padding-left: 40px;">rotates coordinates</p> <p>Bndy - isoparametric</p> <p>TRANSF transforms bndy element</p> <p>Curlips - computes component of ellipse</p> <p>Vector Sets up 0 and 90° currents</p>
OUTPUT	Calcomp Plot with Shoreline
SUBROUTINES USED	<p>Draw - Plot major and minor axes</p> <p>Shore Plots continuous</p> <p style="padding-left: 40px;">Shoreline based on zero depth</p>

NAME

FUNDM

Input

Lake -

ANorth - angle s-N to X axis

Nelement's

N Nodes

Freq. Card -

Freq. Surface

Lake

Skip Card - 1

Skip Card - 0

REFERENCES

Drawit Plots

Legend- legends - Identifies Lake

- North Axis

- Velocity & Distance Scales

NAME	SEICHMO
PURPOSE	Compute free surface and frequencies of free modes of oscillation
DESCRIPTION	Calculates matrix coefficients from lake data and finds frequencies for NMODES initial estimates of frequencies. Based on finite Element Method - punches frequency and free surface solution on cards
USAGE	CDC 6600 entirely in core,
INPUT	lake Data - element cards <ul style="list-style-type: none"> <li>- nodal cards</li> <li>- Blank for Plotter</li> <li>- approximate frequencies</li> </ul>
OUTPUT	- line printer plot amplitude and phase of free surface
SUBROUTINES USED	Connect, Locate, Set-up, Rayquo, Pack, Righthan, Decompa, Normal, Plotlk
STORAGE REQUIREMENTS	320000 Koctal
MACHINE	CDC 6600
REFERENCES	Multiple Access

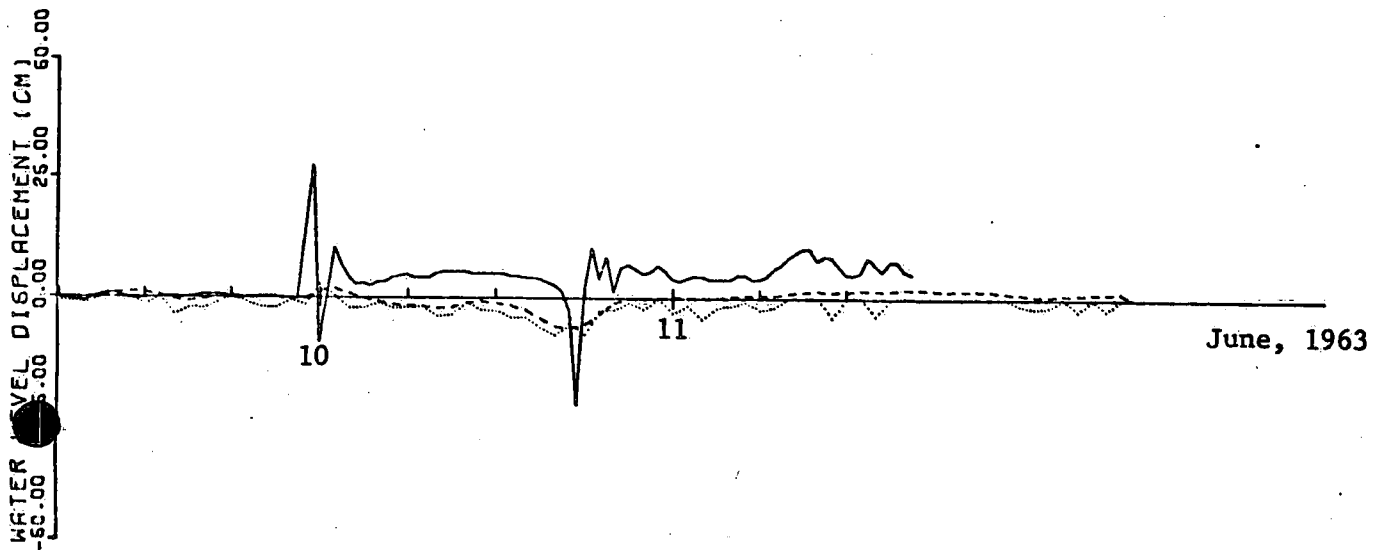
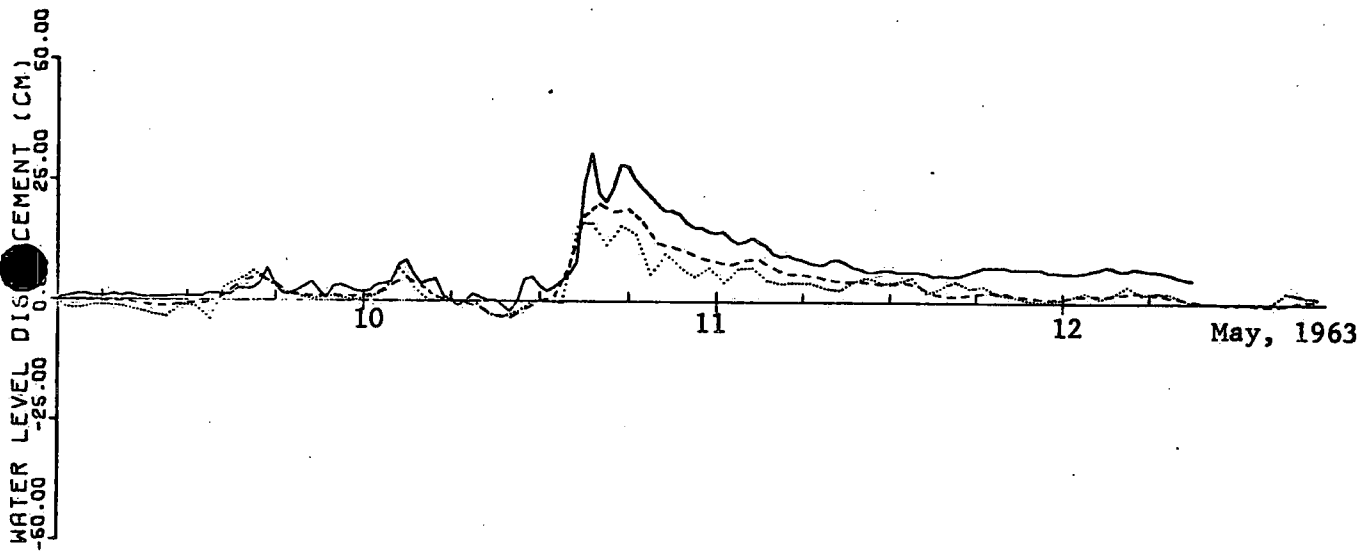
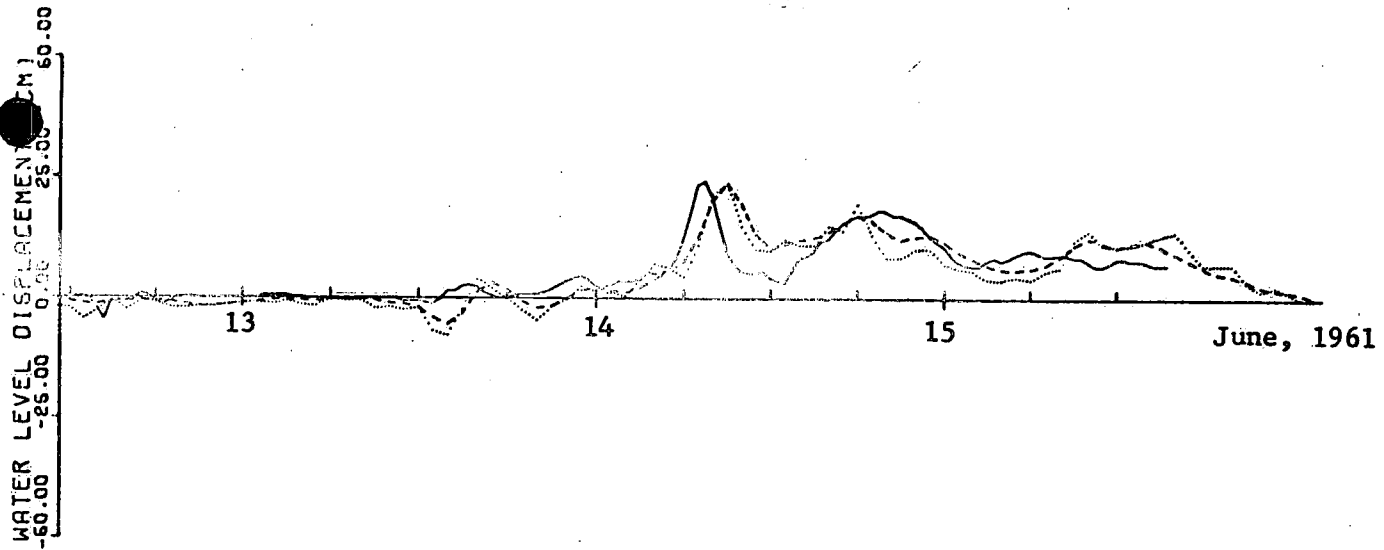
NAME	YINLT
PURPOSE	Calculates the maximum band width from elements data
DESCRIPTION	Node numbers of each element are compared element by element for the $\frac{1}{2}$ band width. Maximum band width is found. In the second half the compressed band width is computed and maximum located.
USAGE	
INPUT	Total number elements NM " " nodes MM Element data
OUTPUT	List
SUBROUTINES USED	-
CALLED BY	-
PROGRAMMER	P. Hamblin

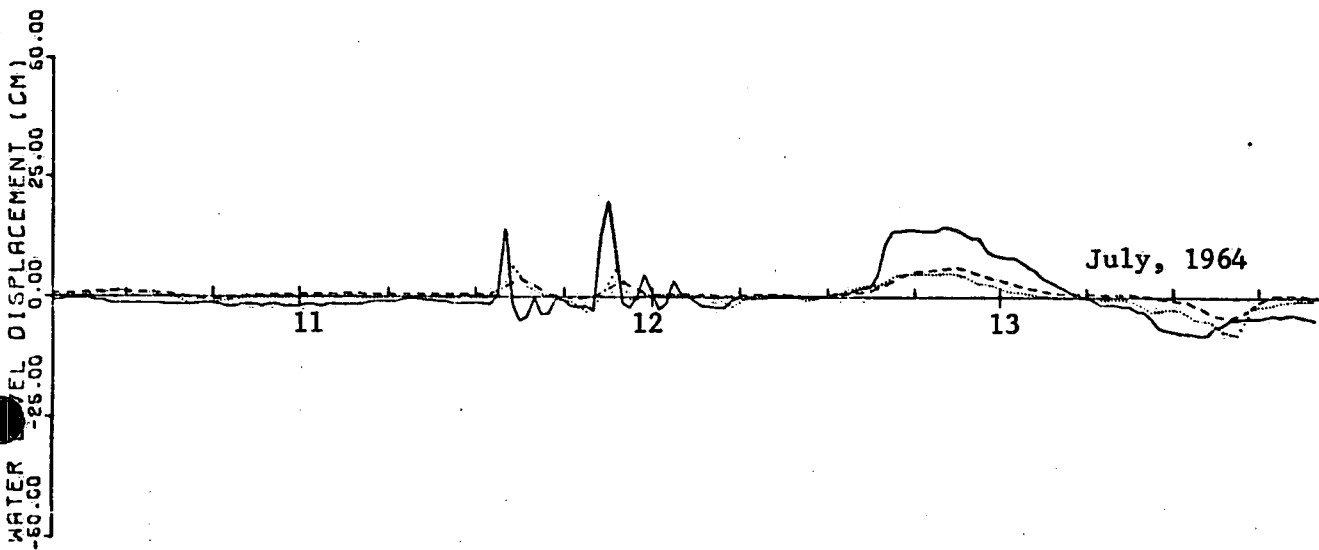
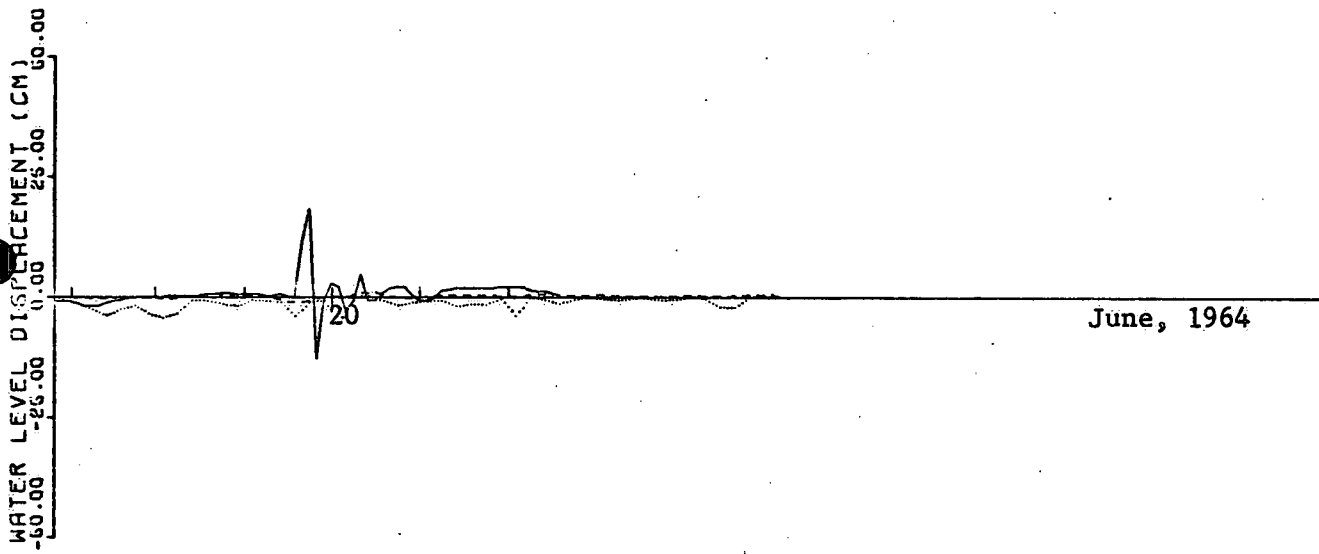
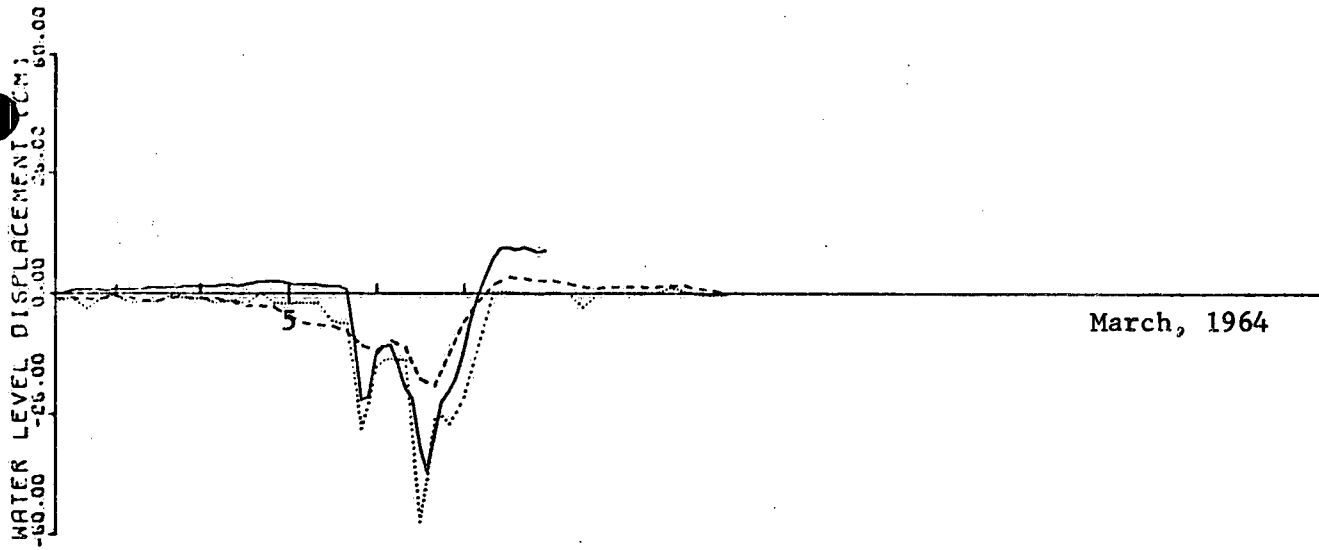
APPENDIX - C

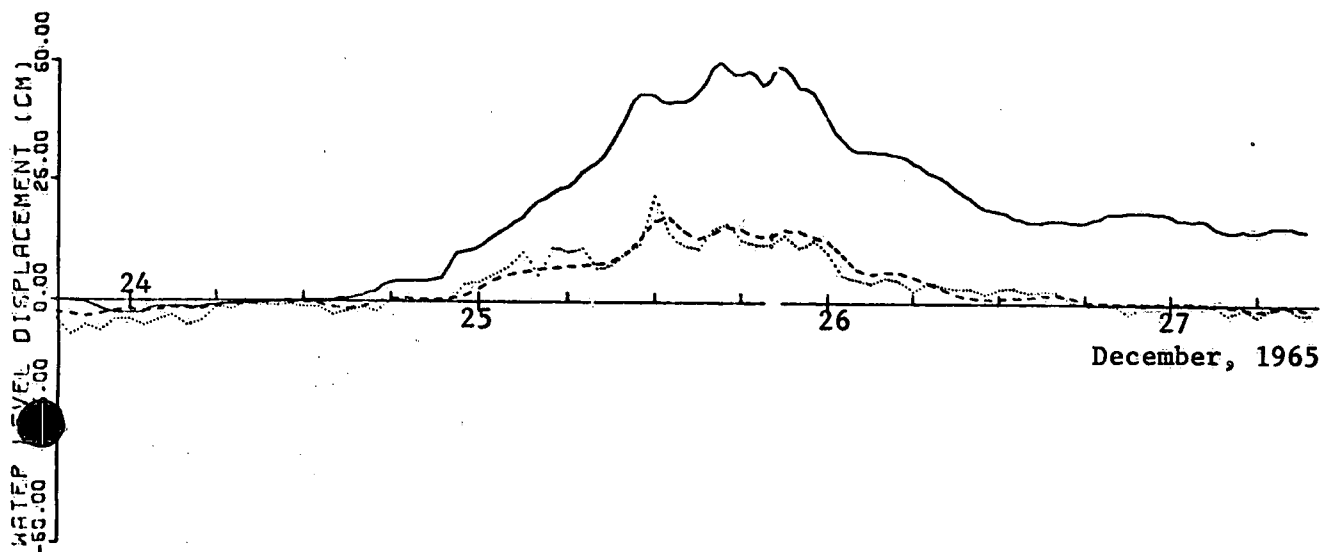
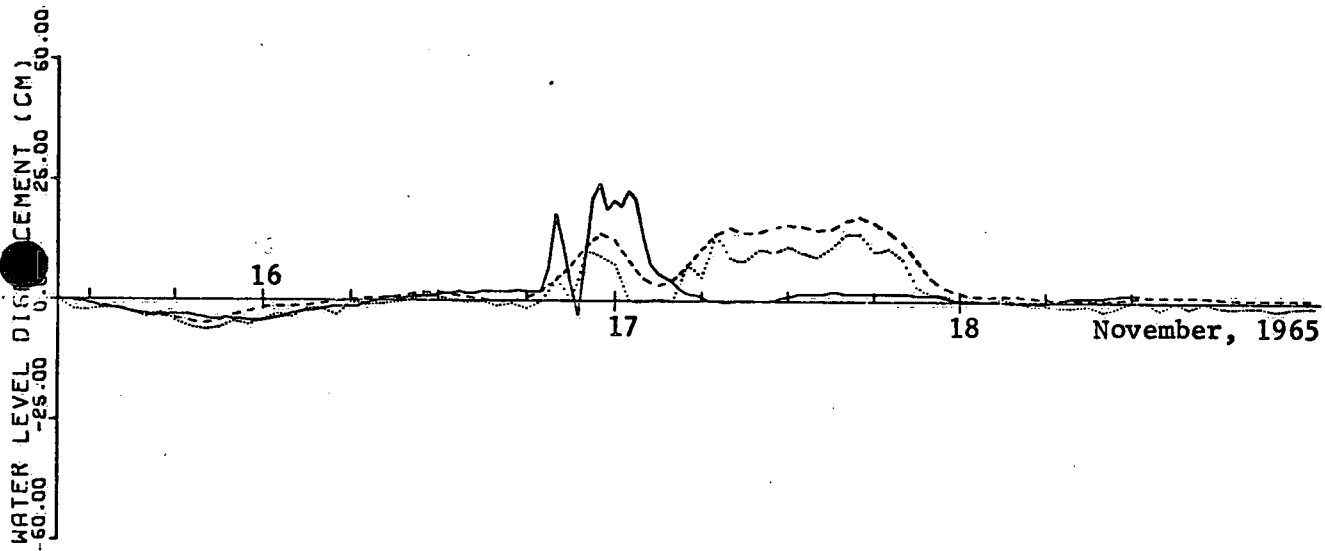
Forecast and observed storm surges for Lake St. Clair  
at Belle River.

The convention adopted in the following plots is:

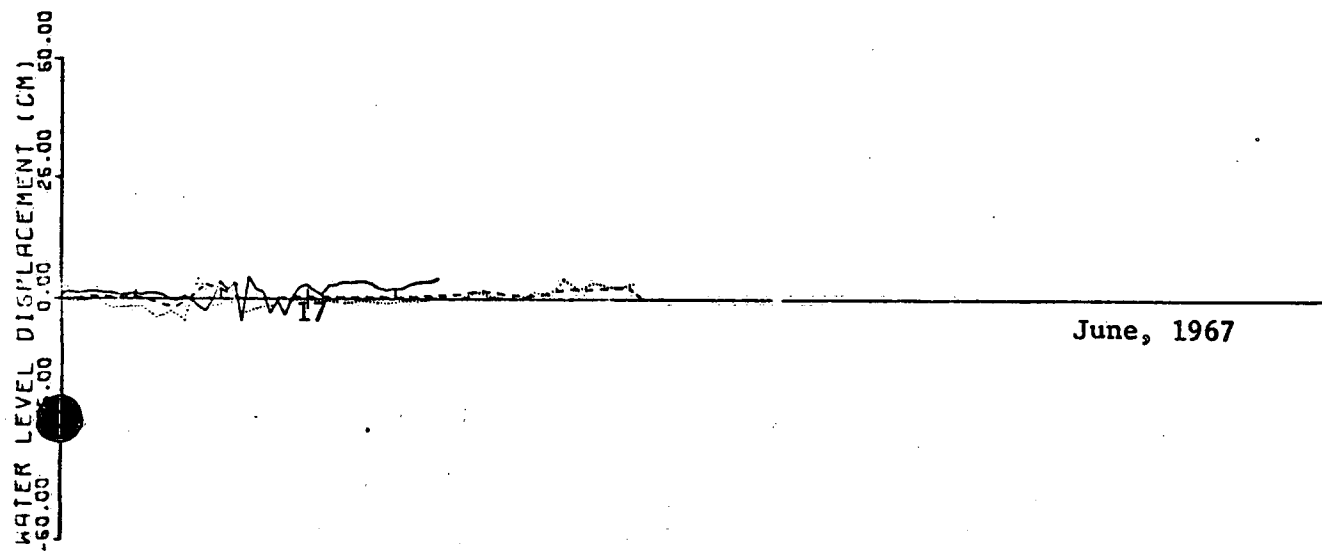
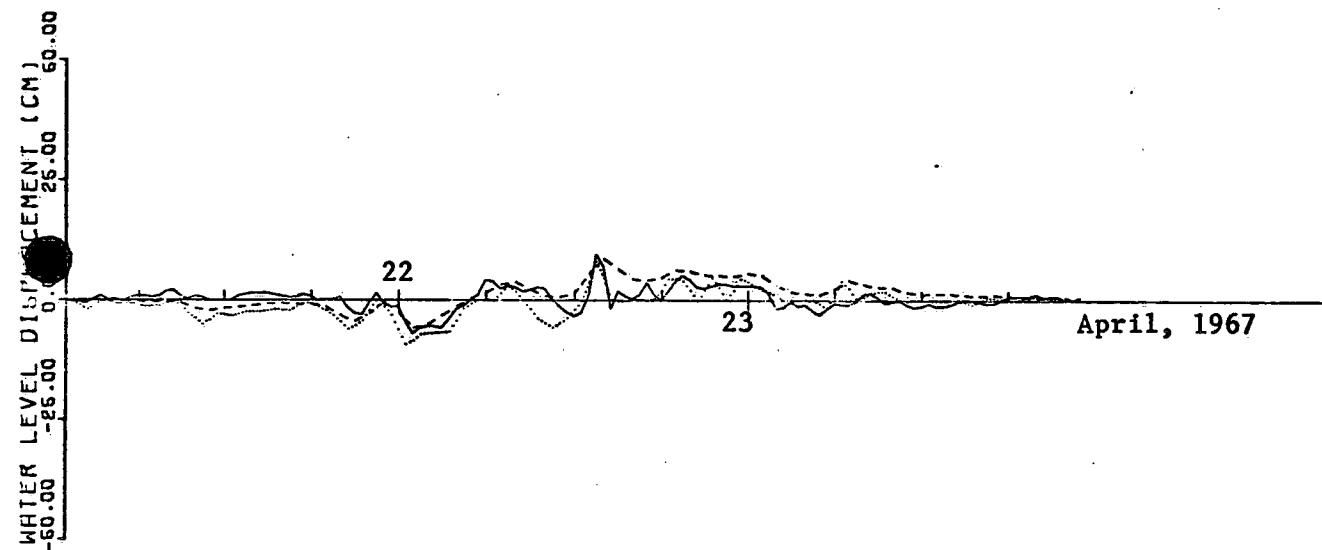
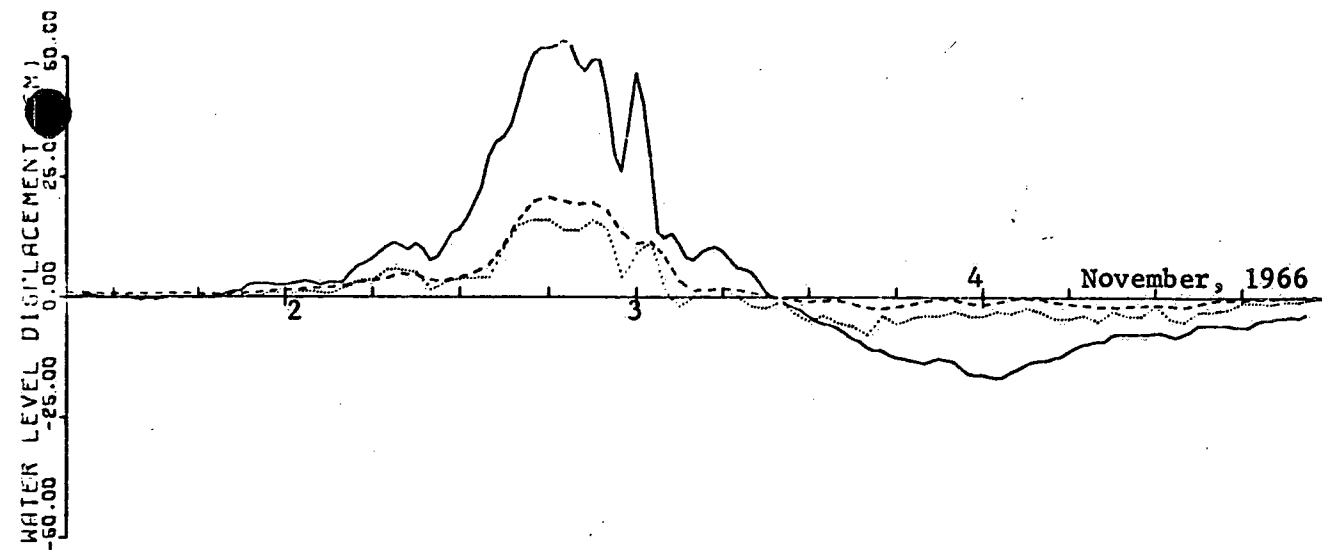
- \_\_\_\_\_ observed water level fluctuations
- water level fluctuations predicted by  
regression technique (without stability effects)
- . . . . . water level fluctuations predicted by the  
present technique.

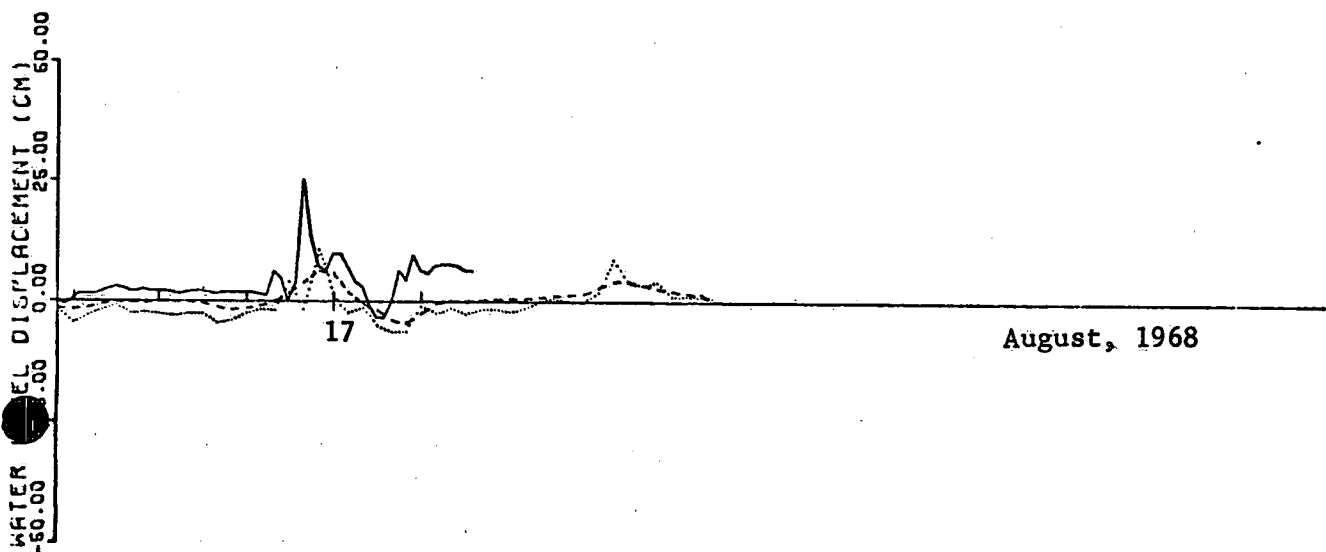
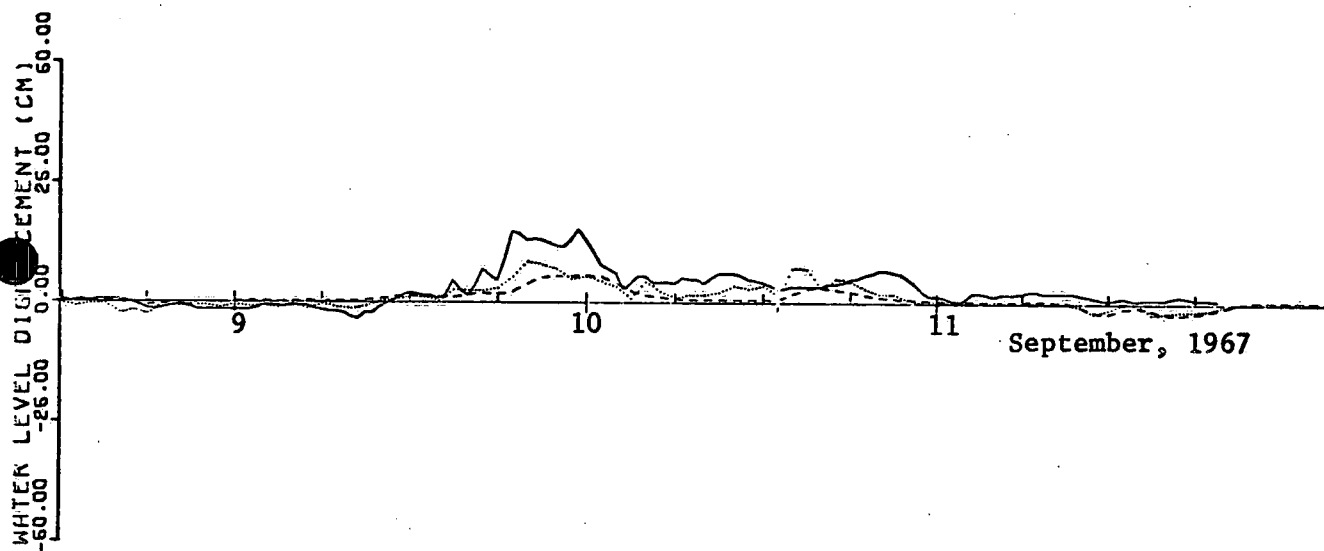
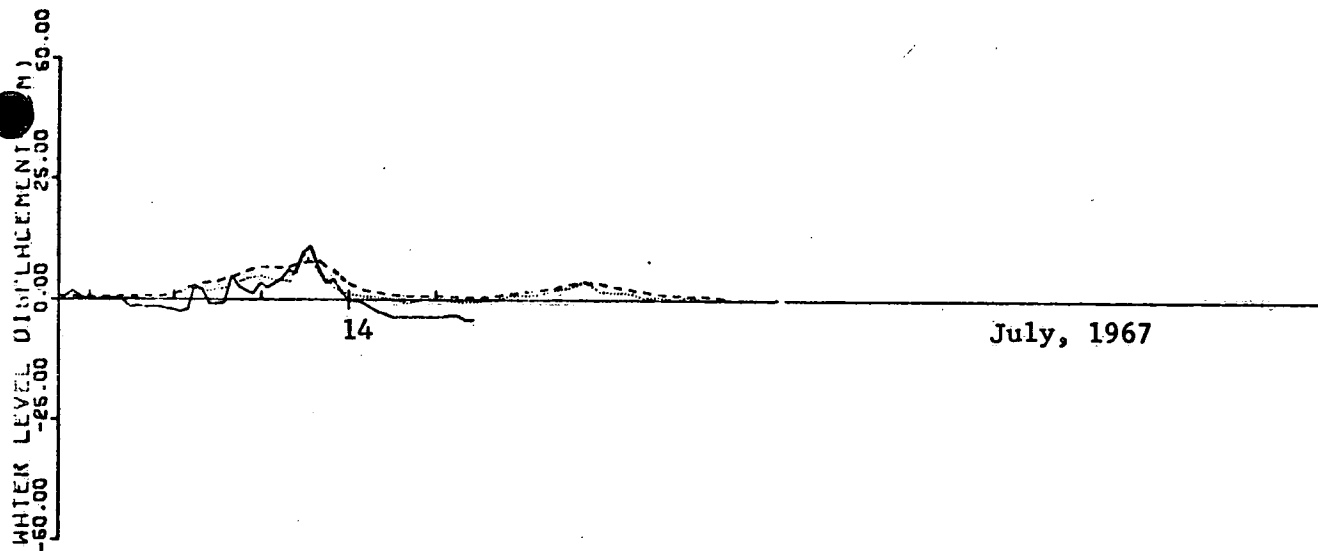


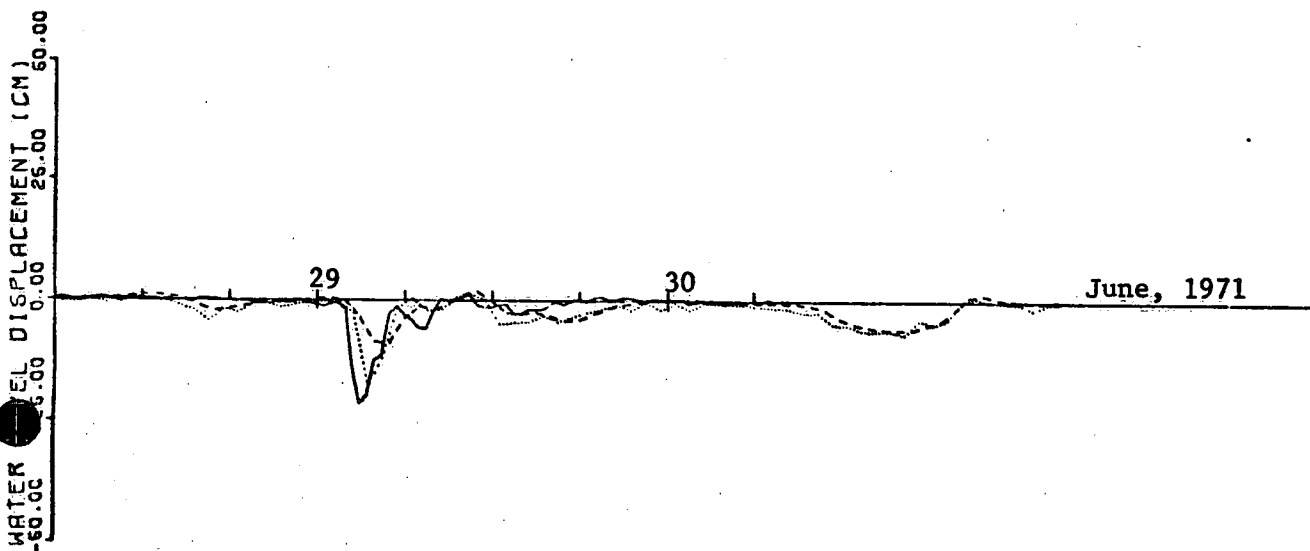
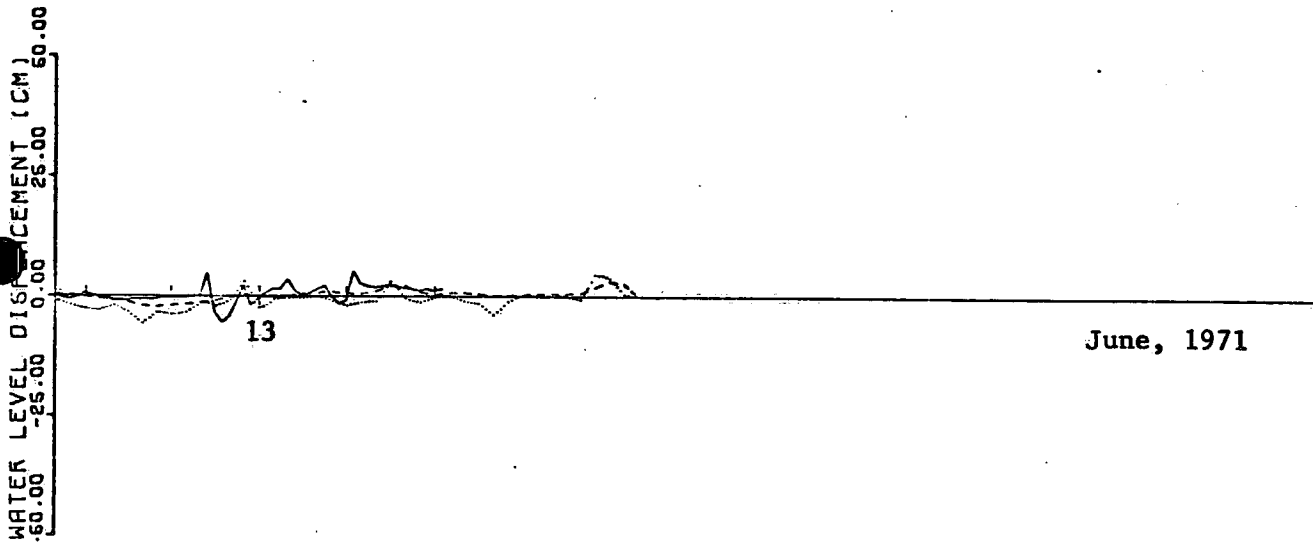
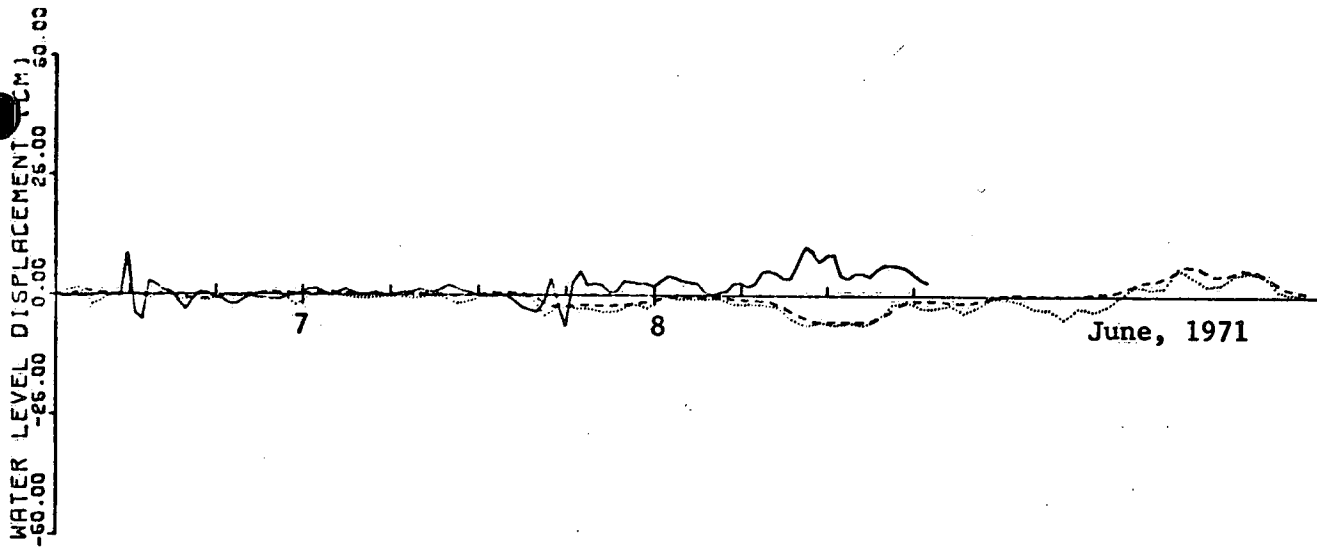




C-4







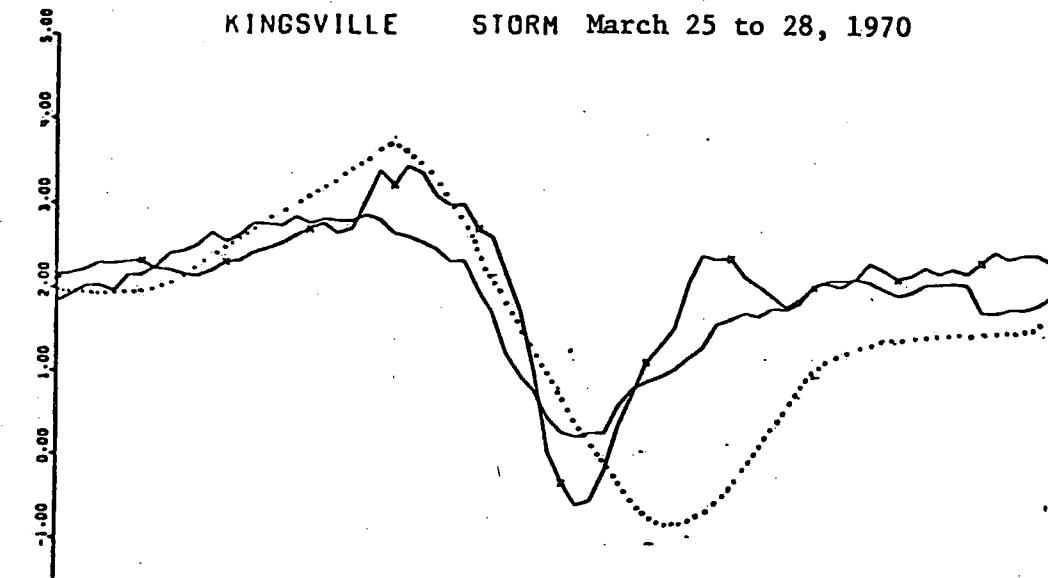
APPENDIX - D

Forecast and observed storm surges for Lake Erie at Kingsville and Port Colborne, and for Lake Ontario at Burlington. (Present results superimposed on plots given in Venkatesh 1974)

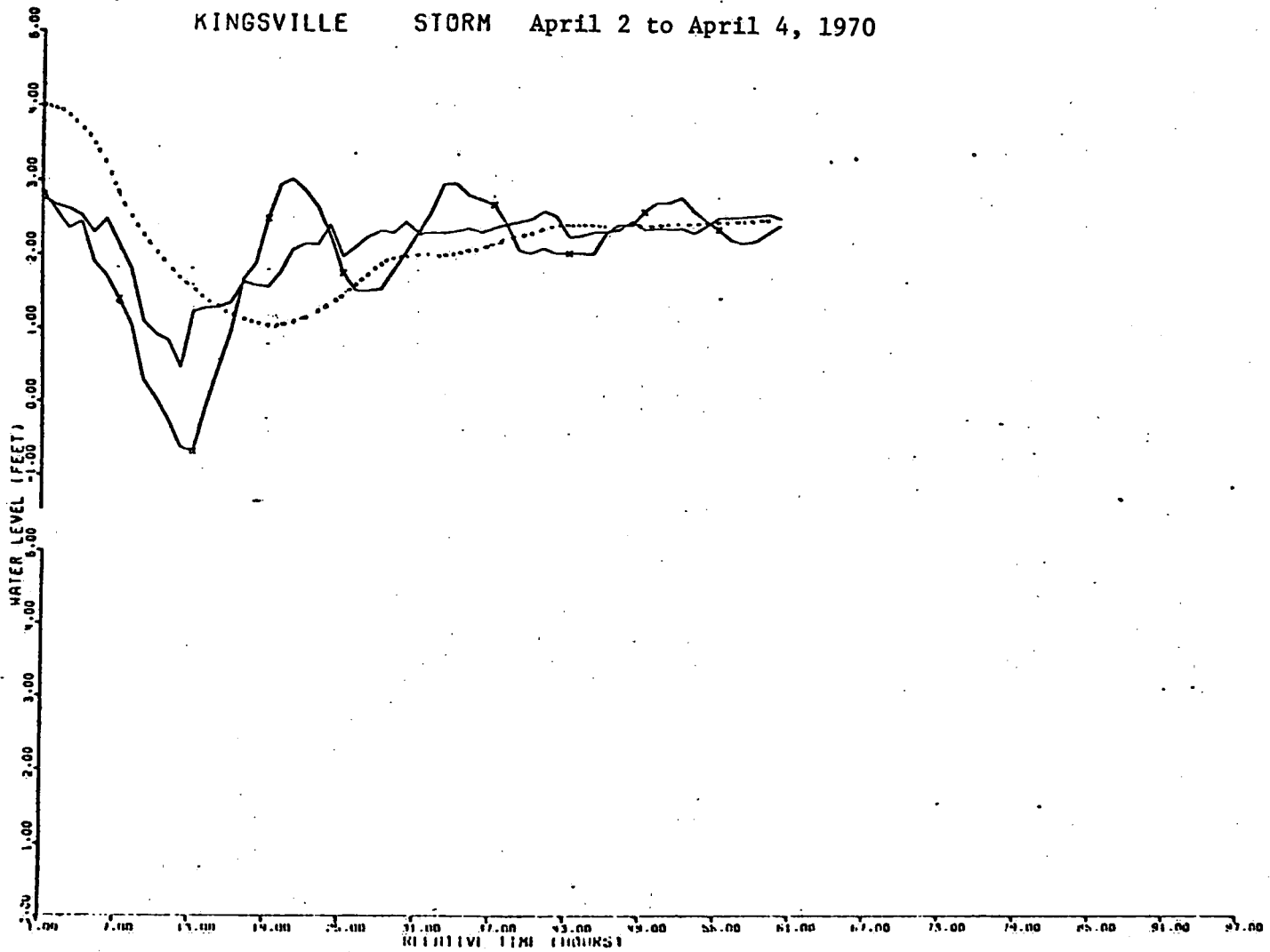
The convention adopted in the following plots is:

- ~~— x — x — x — x —~~ observed water level fluctuations
- water level fluctuations predicted by regression technique
- . . . . . water level fluctuations predicted by the present technique

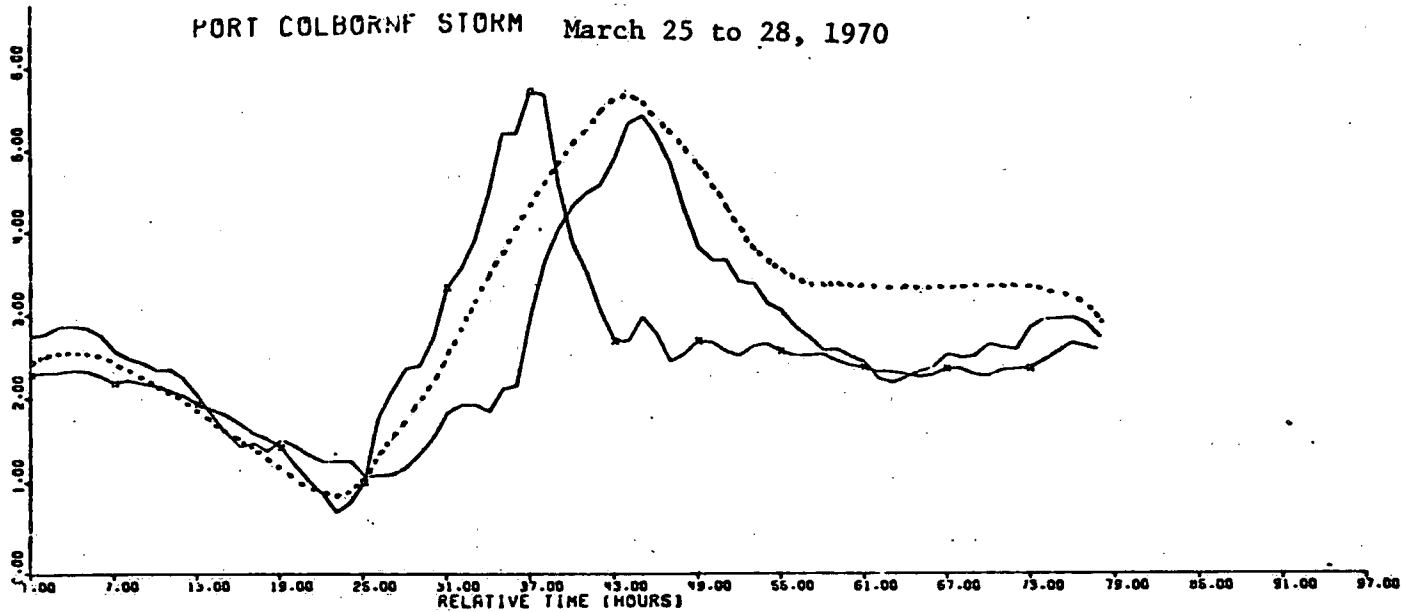
## KINGSVILLE STORM March 25 to 28, 1970



## KINGSVILLE STORM April 2 to April 4, 1970

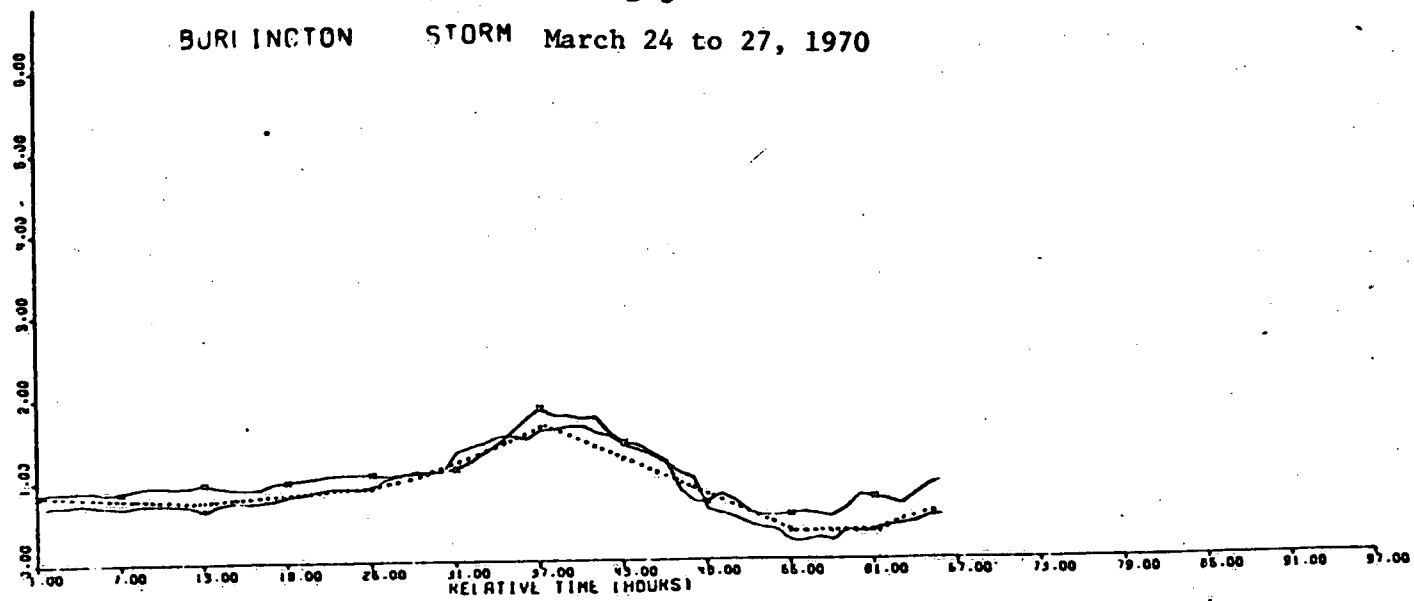


## PORT COLBORNE STORM March 25 to 28, 1970



BURLINGTON

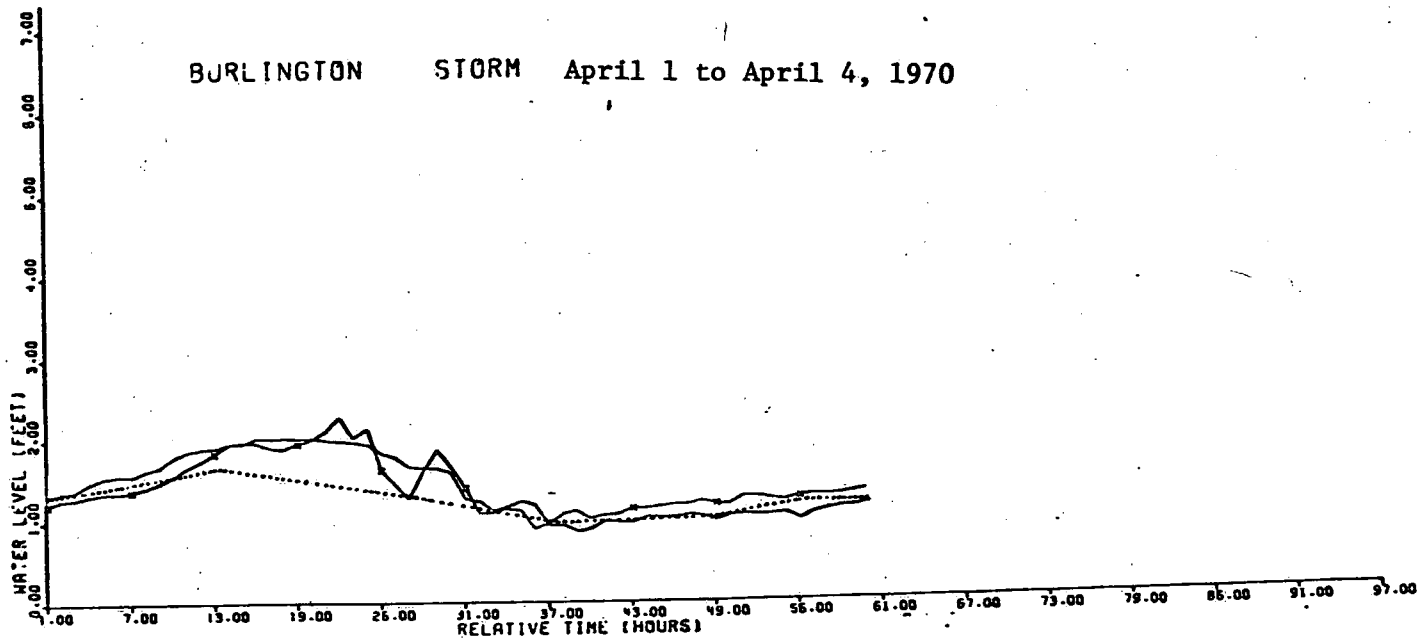
STORM March 24 to 27, 1970



BURLINGTON

STORM

April 1 to April 4, 1970



APPENDIX - E

Regression equations for the prediction of the Great

Lakes surface winds:

Winter set (October to March).....page E1, E2

Summer set (April to September).....page E 3, E4

Corresponding set of predictors with variable  
subscripts.....page E5

Provided courtesy of Mr. D. M. Feit and Dr. C. S. Barrientos,

Techniques Development Laboratory, NOAA, Silver Spring, Maryland.

## REGRESSION EQUATIONS FOR WINTER BASED ON OZ PE

S(6) =	8.142529+ 0.74133VAR(39)+-0.00022VAR(21)+ 0.72767VAR(43)+ 0.00019VAR(16)+ 0.22469VAR(44)+-0.10222VAR(43)+-0.00011VAR(33)
U(6) =	0.753329+ 1.56117VAR(3)+-0.25202VAR(7)+-0.01028VAR(30)+-0.20412VAR(53)+ 0.46625VAR(11)+-0.26925VAR(24)+ 0.17613VAR(26)
V(6) =	-5.355504+ 1.29637VAR(4)+ 0.69661VAR(11)+ 0.4333VAR(3)+-0.08619VAR(61)+ 1.89279VAR(69)+ 3.05835VAR(63)+ 0.35432VAR(36)
S(12) =	11.692364+ 0.73043VAR(43)+ 0.00072VAR(6)+ 0.45254VAR(45)+-0.29038VAR(46)+-0.00037VAR(33)+-0.33360VAR(8)+-0.000355VAR(32)
U(12) =	11.884782+ 0.37693VAR(10)+ 1.04204VAR(14)+-0.28244VAR(8)+ 0.46621VAR(3)+-0.27013VAR(43)+ 0.06965VAR(23)+-1.04567VAR(5)
V(12) =	-2.5310447+ 0.65991VAR(11)+ 0.43268VAR(15)+ 0.15871VAR(3)+ 0.04098VAR(31)+-0.25989VAR(38)+ 0.54327VAR(25)+ 0.32455VAR(4)
S(18) =	0.659143+ 0.67903VAR(49)+-0.00011VAR(18)+ 0.22128VAR(54)+ 0.20397VAR(7)+-0.17714VAR(14)+-0.37563VAR(22)+ 0.09176VAR(37)
U(18) =	31.396530+ 0.86935VAR(14)+-0.00038VAR(16)+ 0.42716VAR(26)+-0.17132VAR(27)+ 0.03033VAR(2)+-0.35725VAR(9)+-0.00015VAR(33)
V(18) =	0.3103673+ 0.62515VAR(15)+ 0.89855VAR(25)+ 0.11433VAR(31)+ 0.36895VAR(10)+ 1.09955VAR(42)+-0.06851VAR(34)+-0.20137VAR(39)
S(24) =	9.775930+ 0.35012VAR(53)+-0.00024VAR(18)+ 0.37479VAR(66)+ 0.11522VAR(7)+ 0.00025VAR(2)+-0.12422VAR(39)+ 0.14222VAR(54)
U(24) =	-2.0224154+ 0.17345VAR(24)+-0.00055VAR(18)+ 0.53772VAR(37)+ 0.87637VAR(25)+-0.40713VAR(35)+-0.40457VAR(23)+ 0.15630VAR(62)
V(24) =	3.343351+ 0.74329VAR(25)+ 0.69935VAR(38)+ 0.10800VAR(31)+-0.97476VAR(55)+ 0.36441VAR(11)+-0.30676VAR(43)+ 0.36211VAR(67)
S(30) =	12.000390+ 0.73753VAR(66)+-0.00061VAR(33)+-0.43822VAR(35)+-0.00004VAR(6)+ 0.23932VAR(19)+-0.17337VAR(10)+ 0.00045VAR(27)
U(30) =	16.131346+ 0.88221VAR(37)+-0.45172VAR(35)+ 0.43650VAR(26)+-0.188670VAR(36)+-0.19214VAR(30)+-0.31701VAR(3)+ 0.04110VAR(62)
V(30) =	2.5224195+ 1.29751VAR(38)+ 0.33065VAR(27)+-0.15531VAR(54)+-0.038073VAR(58)+-0.23570VAR(67)+ 0.26350VAR(24)+-0.23070VAR(35)
S(36) =	12.83+2211+ 0.55337VAR(66)+-0.00032VAR(33)+-0.26512VAR(4)+ 0.08465VAR(31)+ 0.28102VAR(44)+-0.11150VAR(37)+ 0.07193VAR(30)
U(36) =	40.050270+ 1.31443VAR(37)+-0.00036VAR(33)+ 0.00077VAR(16)+-0.34206VAR(3)+-0.51331VAR(9)+ 0.00000VAR(2)+-0.29302VAR(45)(25)
V(36) =	0.233949+ 1.42575VAR(36)+-0.46688VAR(37)+ 0.40160VAR(24)+-0.16953VAR(10)+-0.22016VAR(25)+ 0.15015VAR(22)+ 0.13253VAR(45)

2	S(1) =	4.543592+ 0.63763VAF(19)+-0.248130VAF(21)+ 0.33224VAF(23)+ 0.55417VAF(24)+-0.36358VAF(25)+ 0.07803VAF(26)+-1.55743VAF(28)	
3	U(1) =	4.6552939+ 1.202187AF( 3)+-0.283070AF( 8)+ 0.31438VAF(10)+-0.59108VAF( 4)+ 0.32556VAF(11)+-0.21613VAF(24)+-0.33902VAF( 5)	
4	V(1) =	-2.565578+ 1.21444VAF( 4)+ 0.60887VAF( 3)+ 0.85062VAF(11)+-0.52751VAF(13)+-0.20312VAF(25)+-0.10921VAF(22)+-0.18053VAF(25)	
5	S(12) =	5.6595317+ 0.72737VAF(43)+-0.22532VAF(21)+ 0.15488VAF(54)+-0.09750VAF(127)+-0.13877VAF(19)+ 0.24533VAF(144)+ 0.21935VAF(245)	
6	U(12) =	-1.5931305+ 0.48476VAF(11)+-0.36758VAF(20)+ 0.26495VAF(26)+-0.16970VAF(127)+ 0.18462VAF(38)+ 0.553373VAF(114)+ 0.41335VAF( 3)	
7	V(12) =	0.222347+ 0.787387VAF(11)+ 0.31607VAF(15)+ 0.35908VAF( 3)+-0.46524VAF(44)+ 0.15235VAF(31)+-0.17723VAF(29)+-0.15295VAF(156)	
8	S(13) =	1.0.320419+ 0.84176VAF(53)+-0.80315VAF(18)+ 0.33731VAF(156)+-0.10856VAF(138)+-0.35645VAF(26)+-0.15559VAF( 3)+-0.64445VAF(27)	
9	U(13) =	-0.158101+ 0.67314VAF(14)+ 0.53978VAF(28)+-0.31666VAF(20)+ 3.63141VAF(69)+-0.24399VAF(43)+-0.04343VAF(31)+-0.11792VAF(257)	
10	V(13) =	1.401320+ 0.71143VAF(25)+ 0.40547VAF(15)+ 0.27256VAF(127)+ 0.34227VAF(114)+-0.27776VAF(44)+-0.356572VAF(61)+ 1.18655VAF(246)	
11	S(14) =	11.021739+ 0.46473VAF(53)+-0.00084VAF(17)+ 0.228417VAF(66)+-0.25243VAF(25)+ 0.65663VAF(13)+ 0.17364VAF(54)+-0.14733VAF( 3)	
12	U(14) =	1.118316+ 0.60494VAF(24)+ 0.54856VAF(37)+-0.29637VAF(20)+ 0.48540VAF(126)+-0.34661VAF(53)+-0.32763VAF( 3)+-0.20675VAF(15)	
13	V(14) =	-3.991519+ 1.48014VAF(25)+ 0.65976VAF(36)+ 0.33185VAF(127)+-1.62739VAF(69)+-0.54235VAF(63)+-0.69025VAF(33)+ 0.00202VAF( 1)	
14	S(15) =	11.541367+ 0.62421VAF(66)+-0.00023VAF(17)+ 0.61685VAF(53)+-0.47536VAF(79)+-0.18092VAF(35)+ 0.04121VAF(31)+ 0.14055VAF(156)	
15	U(15) =	32.9345226+ 1.66333VAF(37)+-0.57452VAF(35)+ 0.44350VAF(126)+-0.42735VAF(15)+-0.356704VAF(36)+-0.39435VAF( 3)+ 0.00137VAF(23)	
16	V(15) =	5.446327+ 1.24761VAF(38)+ 0.51501VAF(127)+-0.99185VAF(63)+-0.42787VAF( 8)+-0.34684VAF(25)+ 0.19793VAF(144)+ 0.37653VAF(21)	
17	S(16) =	13.567787+0.00059VAF(33)+ 0.36342VAF(53)+ 0.52693VAF(55)+ 0.00020VAF(12)+-0.15267VAF(25)+ 0.66953VAF(28)+-0.10145VAF(10)	
18	U(16) =	-19.807321+ 0.98015VAF(37)+-0.00077VAF(13)+ 0.21812VAF(60)+ 1.17096VAF(36)+-0.33559VAF(21)+ 0.00567VAF( 2)+ 0.25733VAF(55)	
19	V(16) =	3.239960+ 0.92732VAF(27)+ 0.00100VAF(16)+-0.44669VAF(29)+-0.30091VAF( 5)+-0.56482VAF(36)+ 0.00078VAF(31)+-0.50034VAF(32)	

REGRESSION EQUATIONS FOR SUMMER BASED ON 02 PE

$$S(\hat{\beta}) = 17.25527 + 0.06673VAR(3) + 0.0014VAR(6) + 0.14231VAR(9) + 0.2402VAR(16) + 0.1929VAR(14) + 0.75625VAR(9) + 0.02957VAR(13)$$
[illegible]
$$V(\vec{z}) = -4.60 \pm 32 + 0.509 \times \text{GPA}(\text{K}) + 0.0775 \times \text{WARR}(\text{JIT}) + 0.3592 \times \text{KP}(\text{IXI}) + 0.3639 \times \text{VAR}(\text{JIT}) + 0.1255 \times \text{VAR}(\text{IXI}) + 0.0000 \times \text{VAR}(\text{JIT}) + 0.0000 \times \text{VAR}(\text{IXI})$$
$$S(2) = 1.525633 + .674244P(3) + .000089A(6) + 0.352747\text{VAR}(5) + 0.36132\text{VAR}(3) + 0.23795\text{VAR}(9) + .014323\text{VAR}(20) + .013727\text{VAR}(4)$$

```
J(2,2) = -0.368789 0.70551VAR(10) + 0.20159VAR(27) + 0.43714VAR(41) + 0.19109VAR(138) + 2.8+66VAR(169) + 6.25359VAR(3)
```

[illegible]
$$S(2,2) = (-0.593632 + 0.3356V_{\text{KZ}}(153) - 0.00059V_{\text{FIR}}(17) + 0.26892V_{\text{AR}}(96) + 0.0033V_{\text{AR}}(32) + 0.04223V_{\text{AR}}(32) + 0.00000V_{\text{AR}}(18) + 0.04595V_{\text{AR}}(12))$$
[illegible]
$$= 1.72 \text{ A}.$$
$$5(4) = 11.627355 + 0.1765 \text{VAR}(53) + 0.07008 \text{VAR}(33) + 0.24330 \text{VAR}(66) + 0.0620 \text{VAR}(31) + 0.14748 \text{VAR}(59) + 0.2332 \text{VAR}(67) + 0.000000 \text{VAR}(65)$$
$$J(2^*) = -0.32766 + 0.5277\text{VAR}(G9) + 0.3387\text{VAR}(Z7) + 0.3558\text{VAR}(Z3) + 0.0219\text{VAR}(Z2) + 0.0000\text{VAR}(Z1)$$
$$1/2 = (-2)^{-1} + (-3)^{-1} + \dots + (-n)^{-1} + (-n+1)^{-1} + \dots + (-1)^{-1}$$
$$S(0) = -0.002395 + 0.33813V_4 + 0.6617L + 0.00023W_2 + 0.3399VAP(37) + 0.0568VAR(29) + 0.0568VAR(54) + 0.1230VAR(33) + 0.1230VAR(54)$$
$$J(\cdot) = [0.0559 + 0.0425 \text{VAR}(37) + 0.5325 \text{VAR}(25) + 0.3003 \text{VAR}(16) + 0.7465 \text{VAR}(26) + 0.3340 \text{VAR}(28) + 0.3800 \text{VAR}(29)]$$
[illegible][illegible]
$$U(3) = -2.55 \pm 0.82 \pm 0.95 \text{ VAR}(3) + 0.005 \text{ VAR}(12) + 0.335 \text{ VAR}(15) + 0.111 \text{ VAR}(17) + 0.104 \text{ VAR}(18) + 0.076 \text{ VAR}(19) + 0.112 \text{ VAR}(18) + 0.076 \text{ VAR}(19)$$

$\frac{1}{2}(\rho_1 + \rho_2) = -2.24532$      $\frac{1}{2}(\rho_1 + \rho_2) = 1.06850$      $\frac{1}{2}(\rho_1 + \rho_2) = 0.36177$      $\frac{1}{2}(\rho_1 + \rho_2) = 0.00033$   
 $\frac{1}{2}(\rho_1 - \rho_2) = -0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$   
 $\frac{1}{2}(\rho_1 + \rho_2) = -2.24532$      $\frac{1}{2}(\rho_1 + \rho_2) = 1.06850$      $\frac{1}{2}(\rho_1 + \rho_2) = 0.36177$      $\frac{1}{2}(\rho_1 + \rho_2) = 0.00033$   
 $\frac{1}{2}(\rho_1 - \rho_2) = -0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$      $\frac{1}{2}(\rho_1 - \rho_2) = 0.00000$

[illegible]

1. The first step in the process of creating a new product is to identify a market need. This involves conducting market research to understand the target audience's preferences and pain points. Once a need is identified, the next step is to develop a concept that addresses this need. This often involves brainstorming sessions with a team of designers and engineers. The concept is then refined through prototyping and testing, ensuring it meets the market requirements. Finally, the product is launched into the market, with ongoing monitoring and feedback loops to ensure its success and make necessary improvements.

1

REGRESSION EQUATIONS FOR SUMMER BASED ON 122 PE

$$\begin{aligned}
 S(5) &= 7.629721 + 0.3662VAR(39) + 0.00015VAR(6) + 0.56576VAR(43) + 0.19911VAR(25) + 0.04757VAR(161) + 0.07056VAR(38) + 0.00006VAR(62) \\
 U(5) &= 13.339754 + 0.73005VAR(3) + 0.76229VAR(4) + 0.81491VAR(10) + 0.0036VAR(32) + 0.18862VAR(39) + 1.75046VAR(36) + 0.17137VAR(24) \\
 V(6) &= -2.655741 + 0.42584VAR(4) + 0.71694VAR(3) + 0.59410VAR(11) + 0.39755VAR(7) + 0.24642VAR(35) + 0.00645VAR(1) + 0.00035VAR(12) \\
 S(12) &= 3.512747 + 0.40845VAR(43) + 0.00021VAR(6) + 0.23064VAR(53) + 0.17483VAR(60) + 0.13650VAR(26) + 0.16645VAR(4) + 0.05721VAR(27) \\
 U(12) &= -3.177347 + 0.76174VAR(10) + 0.48354VAR(15) + 0.33874VAR(63) + 0.38232VAR(24) + 0.12940VAR(38) + 0.25363VAR(39) + 0.24266VAR(4) \\
 V(12) &= -4.443323 + 0.70954VAR(11) + 0.60133VAR(3) + 0.50108VAR(15) + 0.14060VAR(67) + 0.13021VAR(37) + 0.00613VAR(2) + 0.38535VAR(4) \\
 S(13) &= 9.740345 + 0.44959VAR(53) + 0.00035VAR(17) + 0.13065VAR(60) + 0.15100VAR(43) + 0.07428VAR(37) + 0.07593VAR(30) + 0.00015VAR(2) \\
 U(13) &= -1.222767 + 0.36471VAR(14) + 0.06064VAR(27) + 0.46371VAR(24) + 0.57282VAR(15) + 0.00019VAR(33) + 0.40622VAR(26) + 0.13254VAR(26) \\
 V(13) &= -1.657175 + 0.63527VAR(25) + 0.24432VAR(14) + 0.00024VAR(2) + 0.19284VAR(37) + 0.23159VAR(27) + 2.11654VAR(63) + 0.14025VAR(4) \\
 S(14) &= 13.499924 + 0.44551VAR(53) + 0.03020VAR(17) + 0.03922VAR(30) + 0.23467VAR(66) + 0.21770VAR(38) + 0.13543VAR(27) + 0.10489VAR(11) \\
 U(14) &= -3.539350 + 0.22356VAR(24) + 0.04252VAR(15) + 0.46535VAR(37) + 0.62143VAR(26) + 0.34806VAR(30) + 0.42069VAR(25) + 0.25149VAR(26) \\
 V(14) &= -3.233311 + 0.49360VAR(25) + 0.52382VAR(33) + 0.23215VAR(37) + 0.24465VAR(67) + 0.44300VAR(4) + 0.25861VAR(27) + 0.27359VAR(14) \\
 S(15) &= 13.715485 + 0.36813VAR(66) + 0.00021VAR(33) + 0.52844VAR(31) + 0.36866VAR(51) + 0.08325VAR(10) + 0.24441VAR(53) + 0.15751VAR(49) \\
 U(15) &= 9.440314 + 0.77796VAR(37) + 0.23864VAR(38) + 0.18904VAR(56) + 0.33180VAR(15) + 1.02116VAR(23) + 0.11753VAR(26) + 0.25136VAR(44) \\
 V(15) &= -2.112129 + 0.65073VAR(38) + 0.30872VAR(37) + 0.35170VAR(34) + 0.54316VAR(15) + 0.36210VAR(7) + 0.20740VAR(4) + 0.12333VAR(27) \\
 S(16) &= 14.667386 + 0.40532VAR(66) + 0.03028VAR(33) + 0.11411VAR(27) + 0.05955VAR(61) + 0.05493VAR(37) + 1.30112VAR(63) + 0.05910VAR(49) \\
 U(16) &= -2.289180 + 0.55241VAR(37) + 0.44368VAR(38) + 0.35660VAR(24) + 0.36236VAR(60) + 0.07740VAR(30) + 0.32363VAR(49) + 0.05552VAR(51) \\
 V(16) &= -2.647333 + 0.92883VAR(38) + 0.61115VAR(67) + 0.35261VAR(29) + 0.40613VAR(37) + 0.80344VAR(25) + 0.52843VAR(55) + 0.55156VAR(27)
 \end{aligned}$$

## SUBSCRIPTS FOR PE MOS PREDICTORS FOR THE GREAT LAKES WINDS

VARIABLE		VALID TIME (HOURS)					
		06	12	18	24	30	36
BASIC	1000 MB HEIGHTS	1	5	12	16		32
	850 MB HEIGHTS	2	6	13	17		33
	500 MB HEIGHTS				18		
	1000 MB TEMPERATURES		7		19		34
	850 MB TEMPERATURES		8		20		35
	700 MB TEMPERATURES				21		
	500 MB TEMPERATURES				22		
	P* SURFACE PRESSURE		9		23		36
	BOUNDARY LAYER U COMPONENT	3	10	14	24		37
	BOUNDARY LAYER V COMPONENT	4	11	15	25		38
	850 MB U COMPONENT				26		
	850 MB V COMPONENT				27		
	700 MB U COMPONENT				28		
	700 MB V COMPONENT				29		
	500 MB U COMPONENT				30		
	500 MB V COMPONENT				31		
COMPUTED	BOUNDARY LAYER WIND SPEED	39	43	49	53		66
	850 MB WIND SPEED				54		
	1000 MB - 850 MB TEMPERATURE		44		55		67
	850 MB - BOUNDARY LAYER WIND SPEED				56		
	$\frac{(1000 \text{ MB} - 850 \text{ MB}) \text{ TEMPERATURES}}{(850 \text{ MB} - \text{BOUNDARY LAYER}) \text{ WIND SPEED}}$				57		
	700 MB WIND SPEED				60		
	500 MB WIND SPEED				61		
	$\frac{(850-1000 \text{ MB}) \text{ HGTS} - (500-850 \text{ MB}) \text{ HGTS}}{(500 \text{ MB} - \text{BOUNDARY LAYER}) \text{ WIND SPEED}}$				62		
	$P_{12}^* - P_{36}^*$ (SURFACE PRESSURE CHANGE)						69
	$P_{12}^* - P_{24}^*$ (SURFACE PRESSURE CHANGE)				63		