

Nutrients in the Great Lakes:

a New Comparison

by

Hugh Dobson

Canada Centre for Inland Waters

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Perhaps it is true that a lake in order to be happy ought to be fertilized a little, but even if that is correct, we need to have an idea of what is a well-fertilized lake, or an over-fertilized or under-fertilized lake. The result of too much fertilizer is too much algae or too little oxygen or both.

Recently I have been working towards a description of the nutrient conditions in the Great Lakes, and today I am sharing with you some of my progress.

In this description it has been helpful to compare the lakes with each other, with respect to their seasonal nutrient cycles in surface waters and bottom waters.

This aspect of the Great Lakes has some importance because it involves a human influence on these large lakes.

There have been and will be major studies of this aspect involving the International Joint Commission, for the lower Great Lakes and the upper Great Lakes.

We need to try to learn the history of nutrient conditions in recent decades when the human influence has been increasing.

There are various measures of the standing stock of planktonic life in lakes, and it seems necessary now to establish approximate relationships among these various indicators, and to set up continuous scales that are consistent with the old classification of lakes into the three categories of oligotrophic, mesotrophic and eutrophic, meaning poorly fertilized, medium fertilized and well fertilized or even over fertilized.

The nutrient aspect of lakes is tied to the other aspects of thermal structure and the seasonal stratification, to the plankton biology and bacteriology and to sedimentation processes, and all of these connections must be understood and clarified.

We may find in our studies that the kinds of measurements are too few for complete understanding and that we need other data in the future. For instance, I encourage the measurement of dissolved organic nitrogen and carbon, and particulate organic nitrogen and carbon.

For a measure of algal abundance we rely heavily on chlorophyll analyses. We need now to clear up their relationship to other standing stock parameters such as particulate phosphorus.

Some of these goals are already being approached in my talk today, while others such as the chlorophyll problem await further work.

One of my strategies has been to search for relationships among the various parameters for the standing stock of life in lakes. Certain values of each parameter can then be assigned to the primitive classification scheme of old limnology, namely the threefold scheme oligotrophic mesotrophic and eutrophic. These words I will associate definitely with the standing stock of plankton, sometimes with detritus - together making up the suspended particulate material in lakes.

Some of the parameters to be studied and related are: transparency, particulate elemental concentrations of major nutrients, chlorophyll concentrations, zooplankton biomass estimates, oxygen depletion rates in the deep water during summer, and winter-time inorganic nutrient concentrations.

The concentrations of particulate organic carbon or nitrogen or phosphorus give an indication of the standing stock of algae plus zooplankton plus detritus. For the Great Lakes we lack data on particulate carbon or nitrogen, but we have data on particulate phosphorus.

I have labelled a certain range of particulate phosphorus concentrations, 5 to 10 micrograms phosphorus per litre, to be "mesotrophic", based on long-standing suggestions in the literature of limnology. Then less than 5 $\mu\text{g P/l}$ is "oligotrophic" and greater than 10 $\mu\text{g P/l}$ is "eutrophic", and we have a continuous trophic scale, not merely threefold categories.

I have begun to relate this scale to other parameters that indicate standing stock, such as Secchi disc transparency, and oxygen depletion in the deep water during summer.

Slide 1. Four important papers

Now I will mention four important papers which have influenced my work.

Edmondson, in 1970, published a paper on phosphorus, nitrogen and algae in Lake Washington after diversion of sewage. The diversion of sewage from Lake Washington caused a large decrease in the winter values of dissolved inorganic phosphate and a decrease in summertime chlorophyll concentrations. The winter nitrate values did not decrease much. The lesson from Edmondson's work is that the amount of algae in lakes is manageable through phosphorus control.

Slide 1

Important papers.

1. Edmondson, W. T. 1970. Phosphorus, nitrogen, and algae in Lake Washington after diversion of sewage. *Science*, 169: 690-691.
2. Rainey, R. H. 1967. Natural displacement of pollution from the Great Lakes. *Science*, 155: 1242-1243.
3. Schelske, C. L., and E. F. Stoermer. 1971. Eutrophication, silica depletion, and predicted changes in algal quality in Lake Michigan. *Science*, 173: 423-424.
4. Schindler, D. W. 1971. Carbon, nitrogen, and phosphorus and the eutrophication of freshwater lakes. *J. Phycol.* 7(4): 321-329.

Rainey published a short paper on the time response of the Great Lakes to changes in loading, which I will discuss in a moment.

Schelske and Stoermer, in 1971, published a paper titled "Eutrophication, silica depletion and predicted changes in algal quality in Lake Michigan". Increasing fertilization with phosphorus and increased sedimentation rates for diatoms, without any increase in the supply of silica to the lake, causes long-term depletion of dissolved silicate. Very low dissolved silicate values occur each summer, which result in replacement of diatoms by green algae and blue-green algae. The same situation is developing in Lake Huron and has already reached a more extreme stage in Lake Erie and Lake Ontario.

Schindler's 1971 paper dealt with carbon, nitrogen and phosphorus and the eutrophication of lakes. Lake 227 near Kenora was fertilized with nitrogen and phosphorus. The standing stock of algae increased about 40 times, whereas the carbon to nitrogen to phosphorus ratios in particulate matter did not change from ratios found in unfertilized lakes. The average C/N/P ratio by weight for the particulate matter was 130: 13: 1. The absence of carbon in the added fertilizer did not restrict the algal standing stock.

Slide 2. Equation for time response

Rainey, in his 1967 paper, provided a theory of the response of a well-mixed lake to a step-wise change in the loading of an unreactive substance such as chloride.

Slide 2

Equation for the time-response of a lake-concentration to a step-wise change in the loading:

Concentration at time t

$$= \left(\frac{\text{old net loading}}{\text{outflow rate}} \right) \times 2.718^{\frac{-F \cdot \Delta t}{V}}$$
$$+ \left(\frac{\text{new net loading}}{\text{outflow rate}} \right) \times \left(1 - 2.718^{\frac{-F \cdot \Delta t}{V}} \right)$$

Symbols: F = flow rate

V = lake volume

The residence time, which is the lake's volume divided by the outflow rate of water not including evaporation, was shown to correspond to 63% response, and 2.3 times the residence time was shown to correspond to 90% response.

A biologically reactive substance such as phosphorus requires more complex theory and observations in order to predict responses to changes in the loading, and the subject needs more work.

Slide 3. Residence times

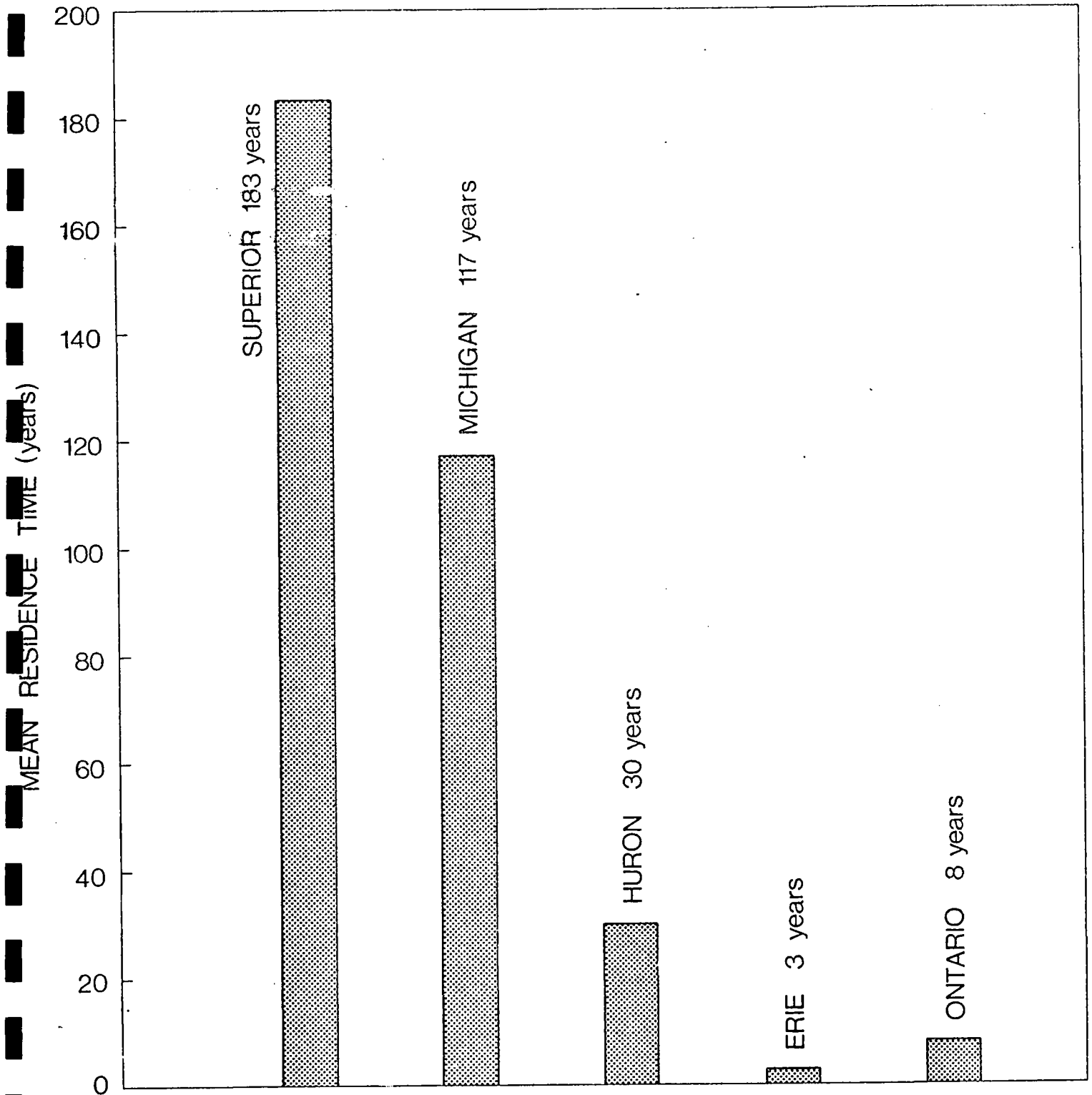
The calculated mean residence times range from 3 years for Lake Erie to 183 years for Lake Superior. The value for Lake Ontario is 8 years.

This time interval corresponds to a 63% response to a stepwise change in the loading.

These indicators of time response should influence the frequency of surveys of water quality in each lake. Lake Erie and Lake Ontario are likely to change most rapidly after a loading change which for phosphorus is due to take place in the next few years. The response of the lower lakes to phosphorus control measures should be observed by means of frequent cruises.

Slide 4. Steady state equation

Here is a steady state equation relating fluxes and concentrations of substances



SLIDE 3.

COMPARISON OF RESIDENCE TIMES OF WATER IN THE GREAT LAKES (VOLUME DIVIDED BY THE OUTFLOW NOT INCLUDING EVAPORATION).

Slide 4

Steady state equation for the concentration
of a substance in a lake:

Concentration in outflow

$$= \frac{\text{Net loading (mass/time)}}{\text{outflow of water (Vol/time)}}$$

$$= \frac{(\text{outside loading minus net sedimentation})}{\text{outflow of water}}$$

in a lake. Later on I will use the equation to derive the net sedimentation term for Lake Erie and Lake Ontario.

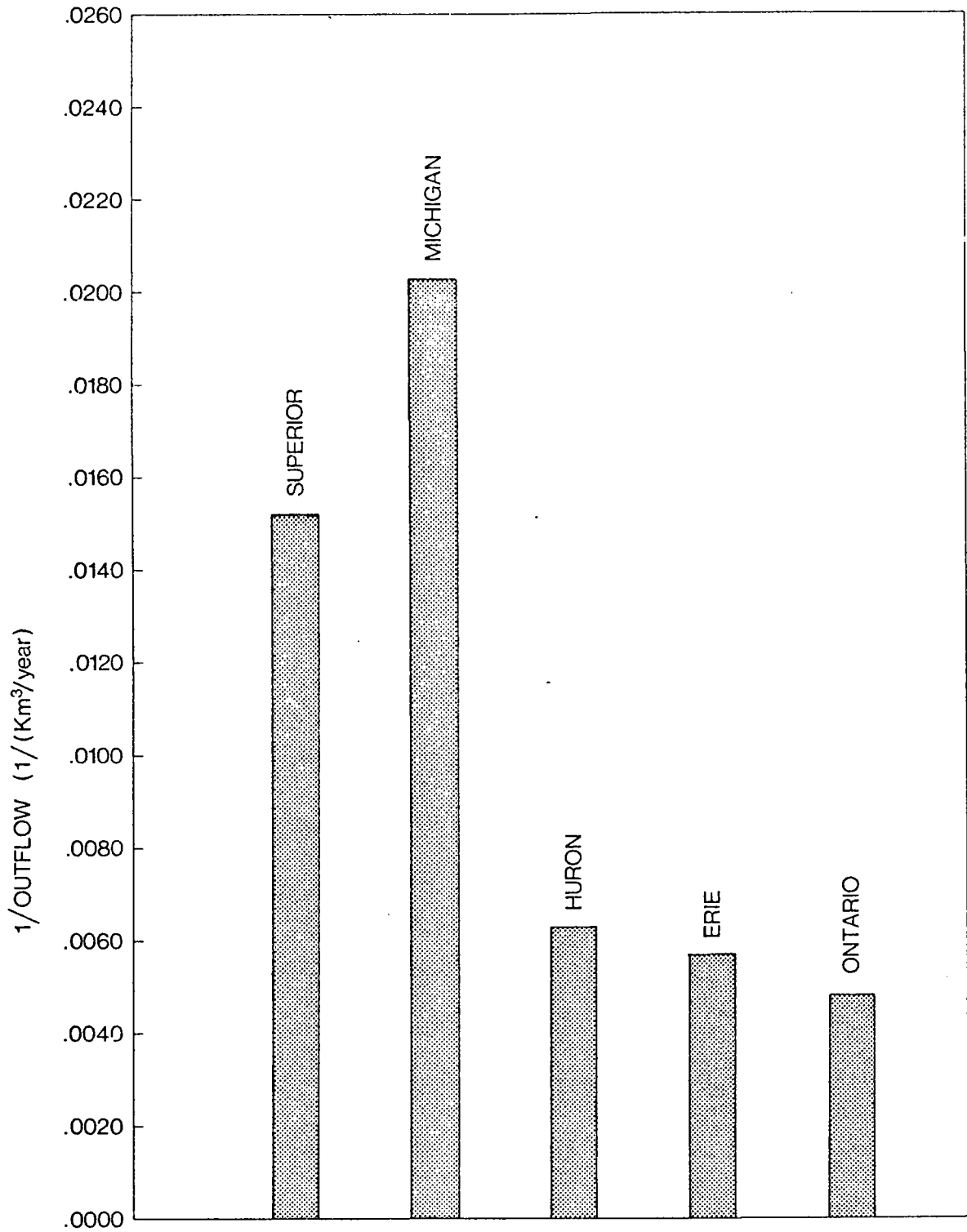
Notice from the equation that the term $[1/\text{outflow}]$ is the factor that the net loading for a substance has to be multiplied by to calculate the concentration at the outlet of the lake. This dilution factor should be considered in design criteria for managing the lakes by phosphorus control. This factor quantifies the idea that a small creek is more easily polluted than the Amazon River.

Slide 5. Pollution Susceptibility Index

I call the factor $1/\text{outflow}$ in the steady state equation the pollution susceptibility index. For the Great Lakes, it indicates that in steady state conditions Lake Superior and Lake Michigan are more fragile than Huron, Erie or Ontario, and that Superior and Michigan may require smaller loadings to produce any desired concentration within the lakes.

Now we will look at the evidence for the nutrient status of the Great Lakes, and we will examine mainly transparency, dissolved oxygen, nitrogen, silicate, and phosphorus.

The following material is a compressing of our CCIW monitor cruise data, using mean values for surface waters and mean values for cold bottom waters on numerous cruises, to show seasonal cycles and in some cases the long-term trends.



SLIDE 5. COMPARISON OF THE FACTOR (1/OUTFLOW), THE POLLUTION SUSCEPTIBILITY INDEX OF THE GREAT LAKES.

I am by-passing the details that would be contained in horizontal maps and in vertical sections, in order to give a quantitative but simplified picture using a rather small number of graphs.

Slide 6. Lake Huron surface temperatures

Nutrient cycles are related to the thermal structure which I will illustrate briefly.

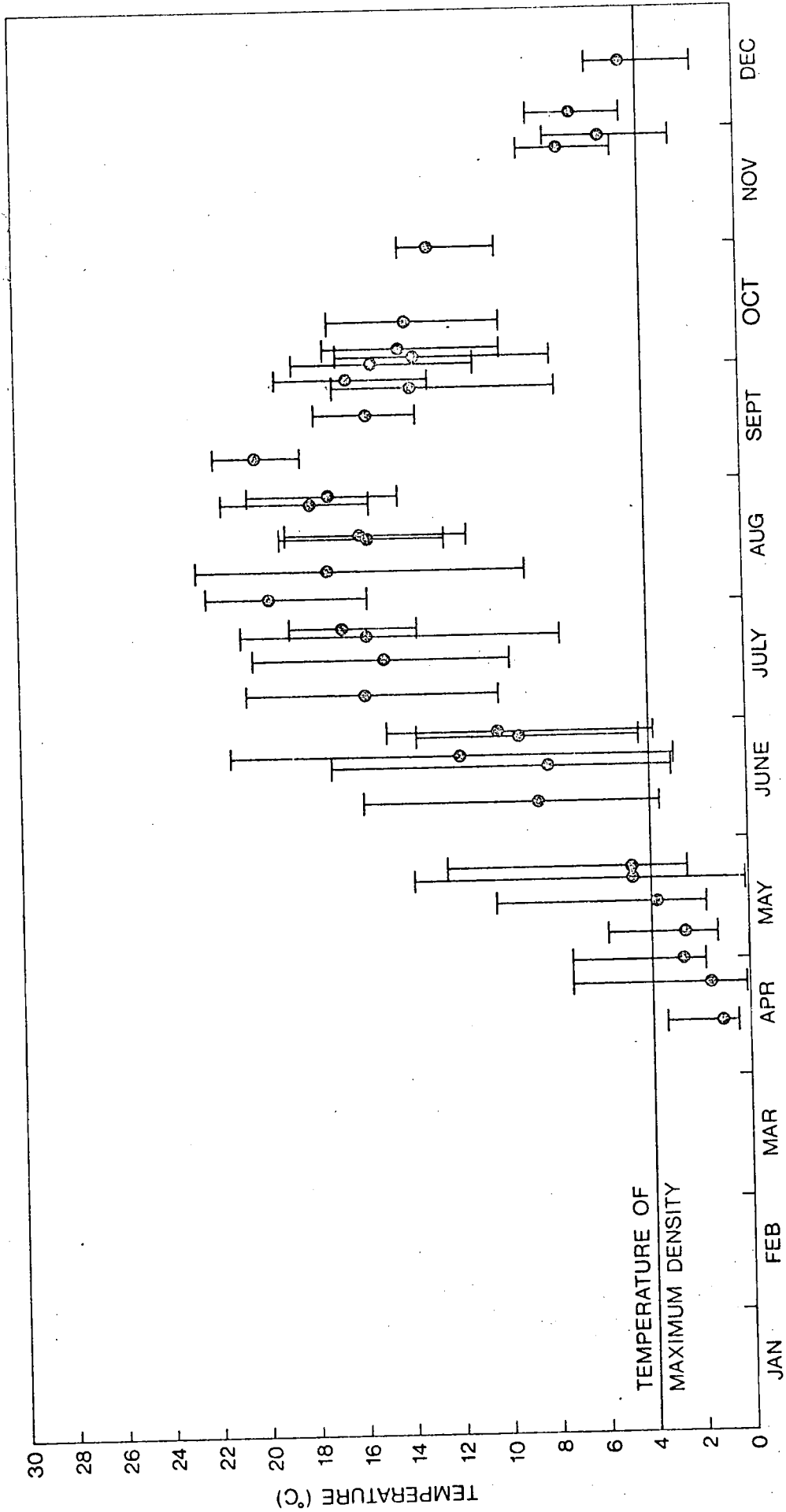
Surface temperatures go through the well-known seasonal cycle shown here for the case of Lake Huron. In summer the lakes have cold water in their deeper parts and a boundary region called the thermocline.

Slide 7. Lake Erie temperature section

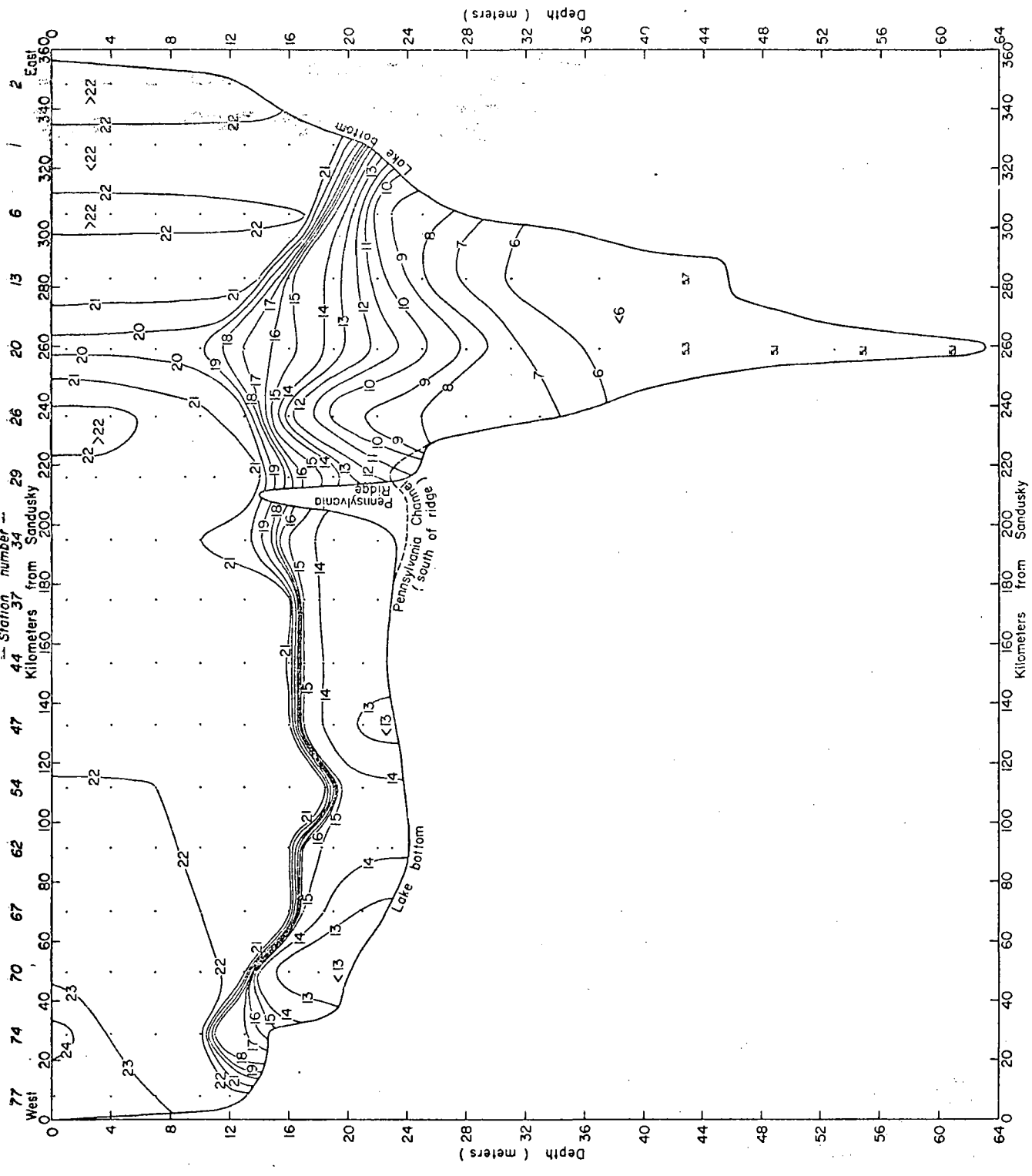
Here is a vertical section through Lake Erie in summer, illustrating thermal stratification. Later I am going to characterize the warm surface water and the cool bottom water of each basin by means of single mean nutrient values observed on each cruise.

Slide 8. Lake Erie thermocline tilting

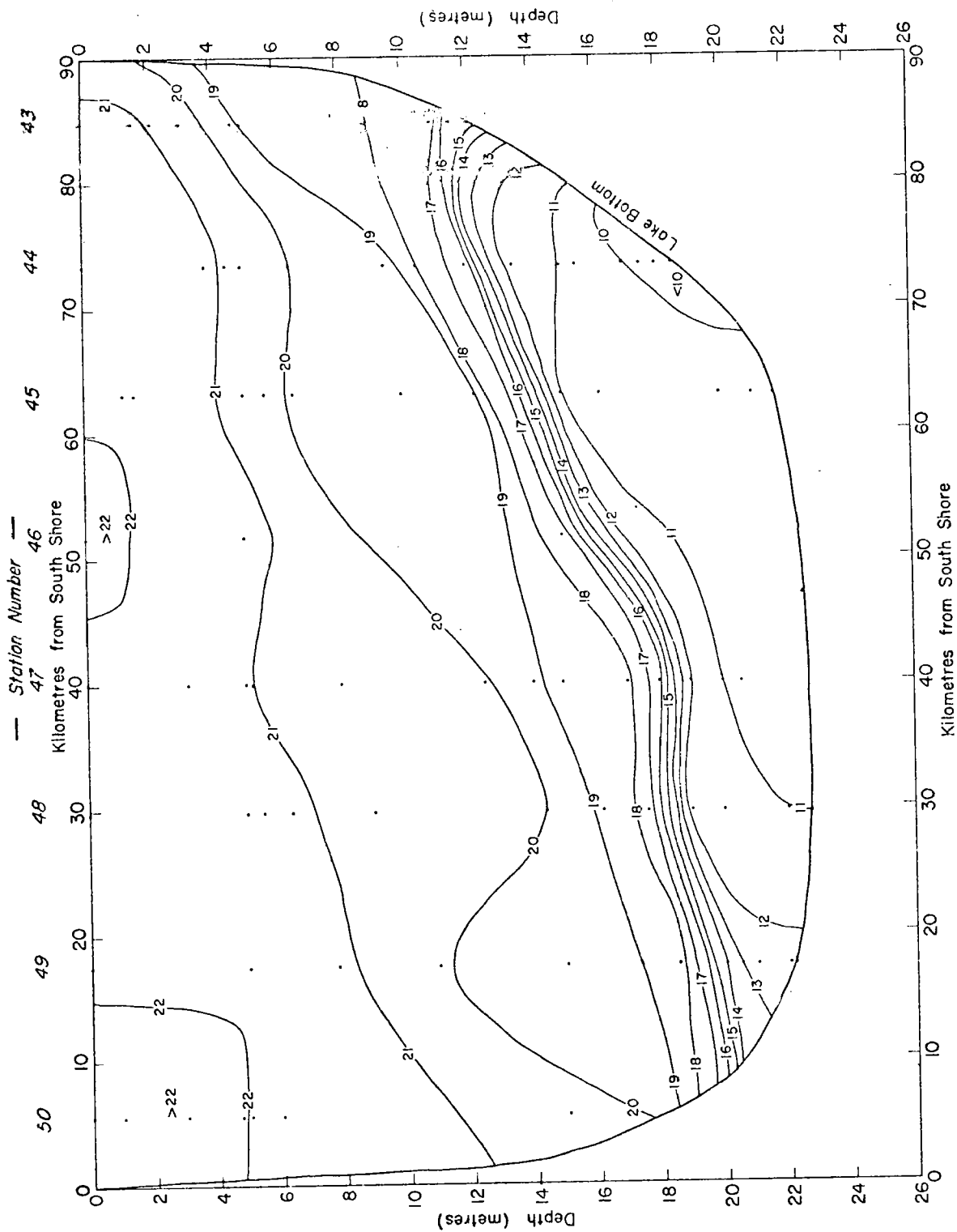
The thermocline can be tilted by the wind stress as illustrated here for Lake



SLIDE 6. MEAN AND EXTREME TEMPERATURES AT A DEPTH OF 1 METRE IN LAKE HURON, FROM SYNOPTIC CRUISES, 1960 TO 1971.



SLIDE 7.
Lake Erie: Temperature (°C) in a vertical section
from Sandusky to Buffalo, July 29 to August 2, 1968.
Vessel "Theron" (Canada Centre for Inland Waters).
Cruise 68-1-08. Vertical exaggeration: X 5,000.



SLIDE 8.
Lake Erie: Temperature (°C) in a vertical section from South to North Shore near Ashtabula, July 24, 1963.
"Porie Dauphine" (University of Toronto) Cruise E-63-7.
Vertical exaggeration: X2,500.

Erie. I use temperature data to learn which water-mass is being observed in each nutrient analysis. Temperature, and not the location, is the reliable key to water-mass identification.

Slide 9. Secchi depth versus particulate phosphorus, log scales

It is important to learn relationships between Secchi disc transparency and other standing stock indicators. Secchi depth is a simple measurement for which there is much historical data.

Here are some cruise-mean values in each basin for Secchi depth and particulate phosphorus plotted on logarithmic scales. Most of the data are for Lake Erie;

The dotted line is a least squares line of the form

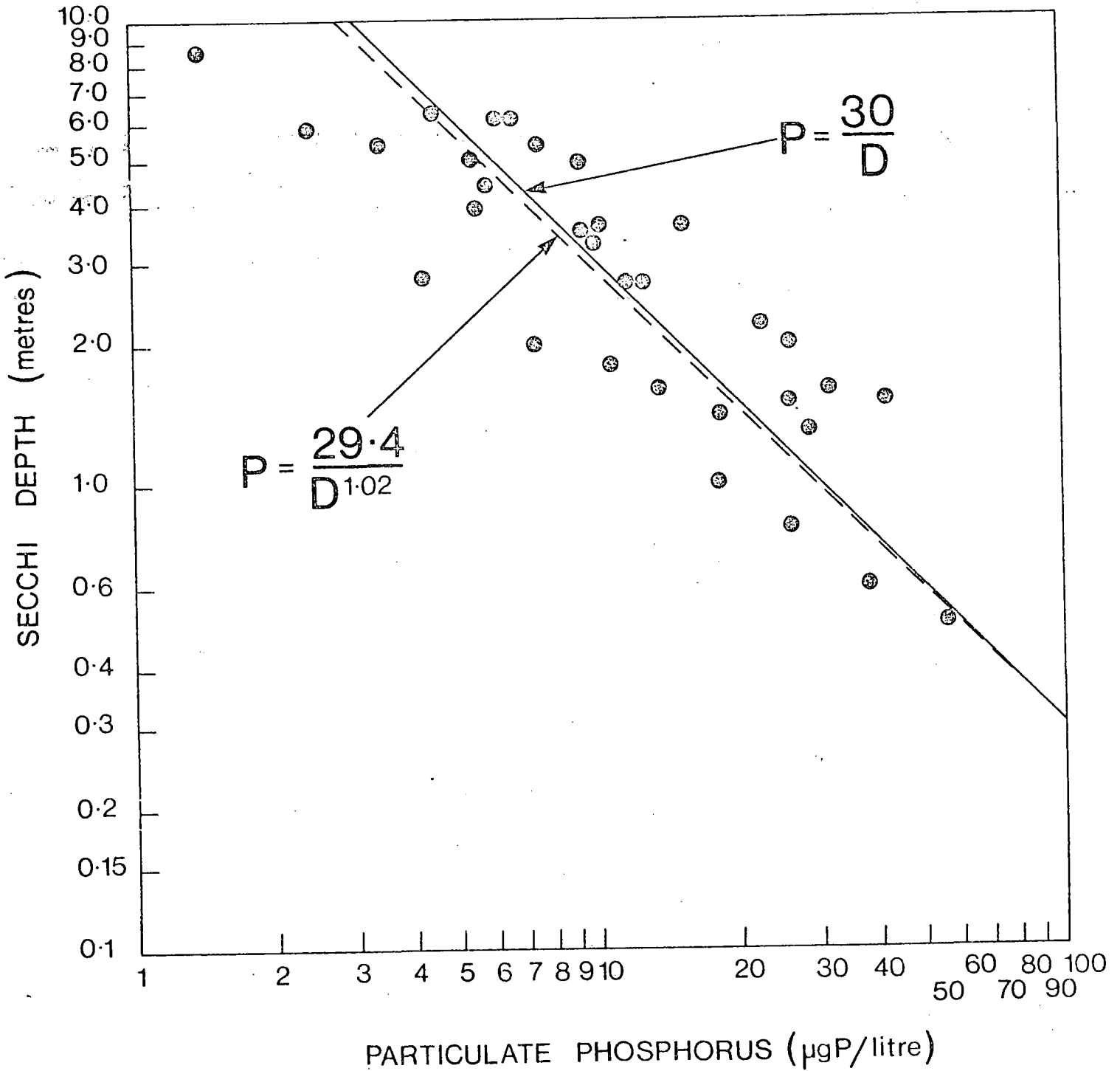
$$\text{Particulate Phosphorus } (\mu\text{g P/l}) = \frac{a}{\text{SD}^b},$$

and the solid line is a simple but adequate approximation:

$$\text{Particulate Phosphorus} = \frac{30.}{\text{SD}}$$

Slide 10. Secchi depth versus particulate phosphorus, linear scales

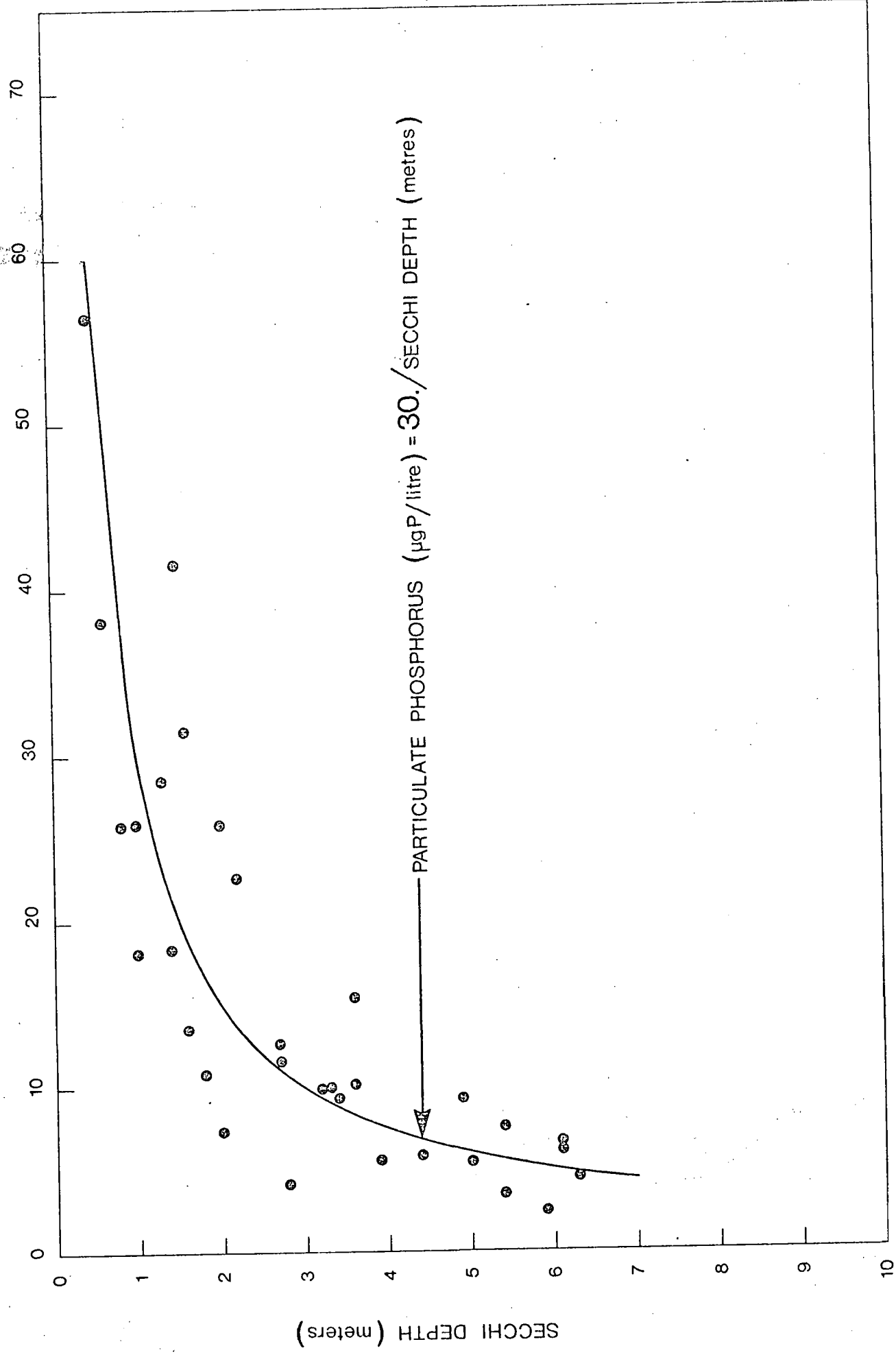
The same points on linear scales look like this. Secchi depth is a non-linear approximate indicator of the standing stock of particulate phosphorus.



SLIDE 9.

SECCHI DEPTH VS. PARTICULATE PHOSPHORUS, FOR GREAT LAKES SURFACE WATERS. MOST OF THE DATA ARE FOR LAKE ERIE.

PARTICULATE PHOSPHORUS ($\mu\text{g P/litre}$)



SLIDE 10. RELATION BETWEEN PARTICULATE PHOSPHORUS AND SECCHI DEPTH IN SURFACE WATERS OF THE GREAT LAKES.

Notice that

$5 \mu\text{g P/l} = 6 \text{ metres}$

and

$10 \mu\text{g P/l} = 3 \text{ metres.}$

Thus

<3 metres corresponds to eutrophic

and

3 to 6 metre readings are mesotrophic

and

>6 metres corresponds to oligotrophic

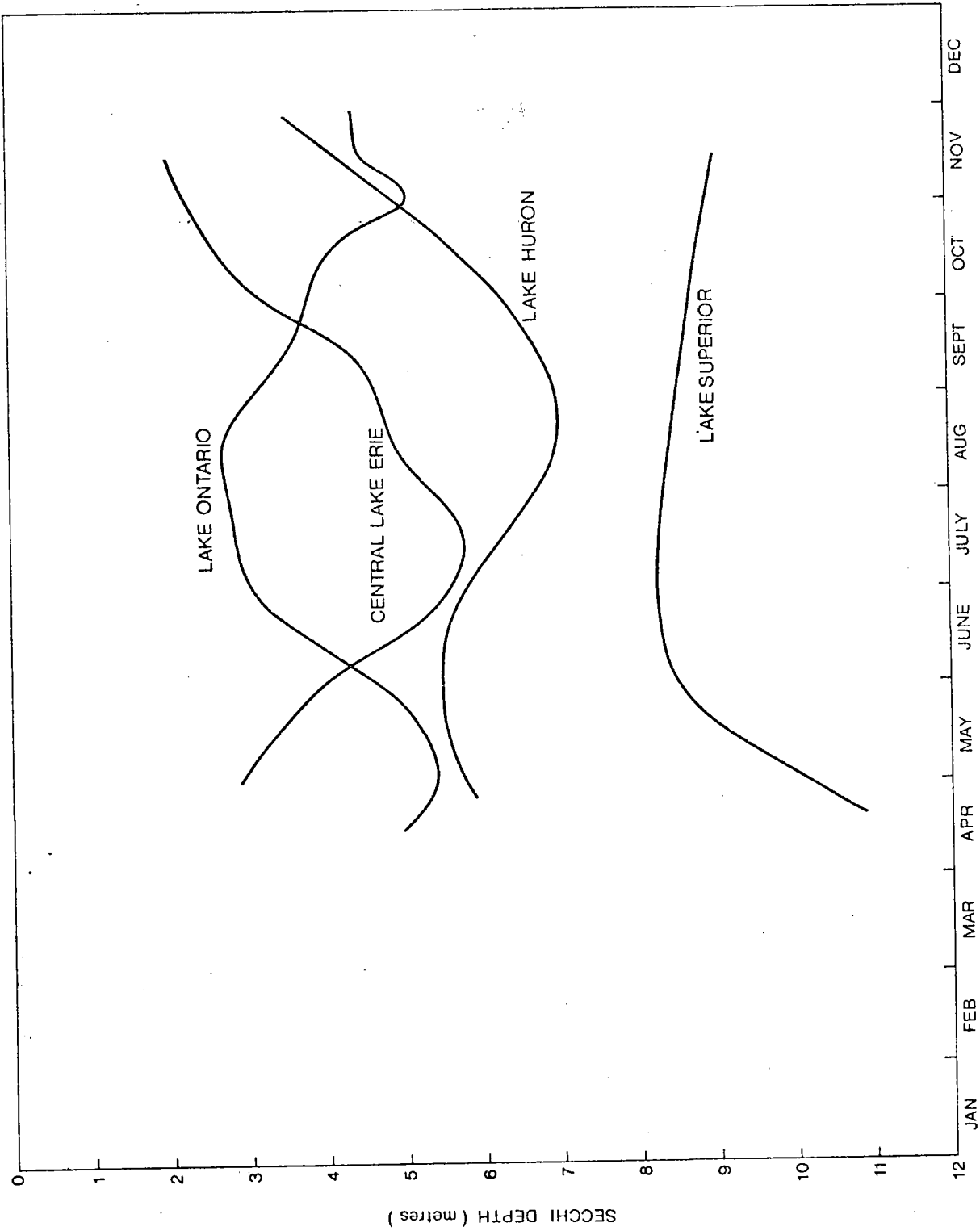
Slide 11. Secchi depths in the Great Lakes

This graph shows the seasonal cycle of Secchi disc transparencies in the Great Lakes. Notice low readings or high standing stocks are towards the top of the graph.

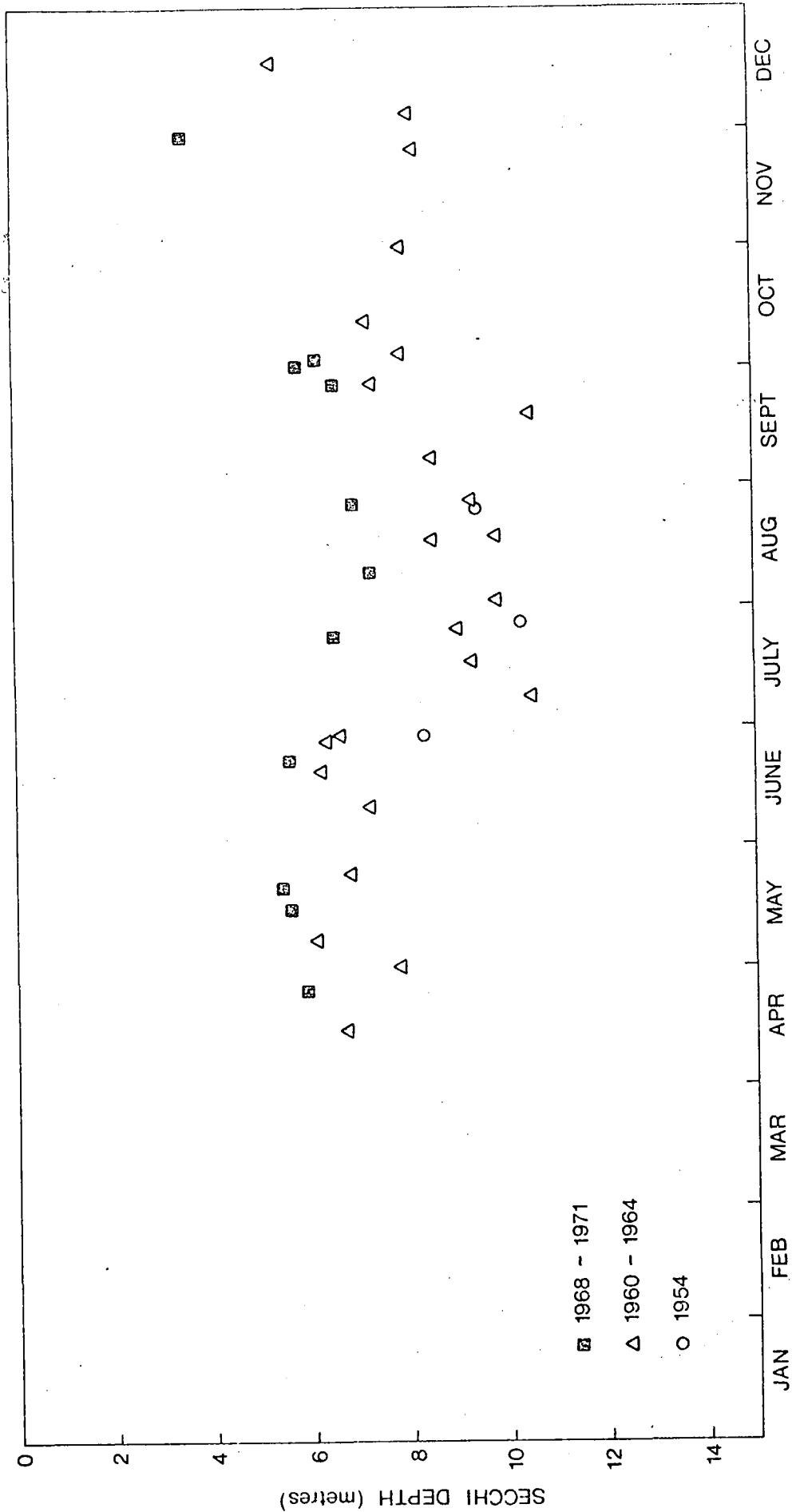
Lake Ontario becomes eutrophic in mid-summer. Central Erie becomes more transparent in midsummer. Lake Superior has the least standing stock of algae.

Slide 12. Lake Huron Secchi depths

Some changes have been occurring in the transparency of Lake Huron. In the period around 1962, readings averaged near 8 or 9 metres, in the oligotrophic range.



SLIDE 11. LAKE-WIDE MEAN SECCHI DEPTH VERSUS TIME OF YEAR, FOR LAKES SUPERIOR, HURON AND ONTARIO, AND FOR THE OFFSHORE PART OF CENTRAL LAKE ERIE. THE CURVES APPLY APPROXIMATELY TO THE YEARS 1967 TO 1971.



SLIDE 12. LAKE HURON SECCHI DEPTHS: LAKE-WIDE MEAN VALUES FROM SYNOPTIC CRUISES, 1954 TO 1971.

These older data are shown by the white symbols. In 1971, shown by the black dots, readings averaged near 6 metres, on the boundary between oligotrophic and mesotrophic values.

Probably the cause of the change in transparency is an increase in algal abundance due to an increase in fertilization with phosphorus.

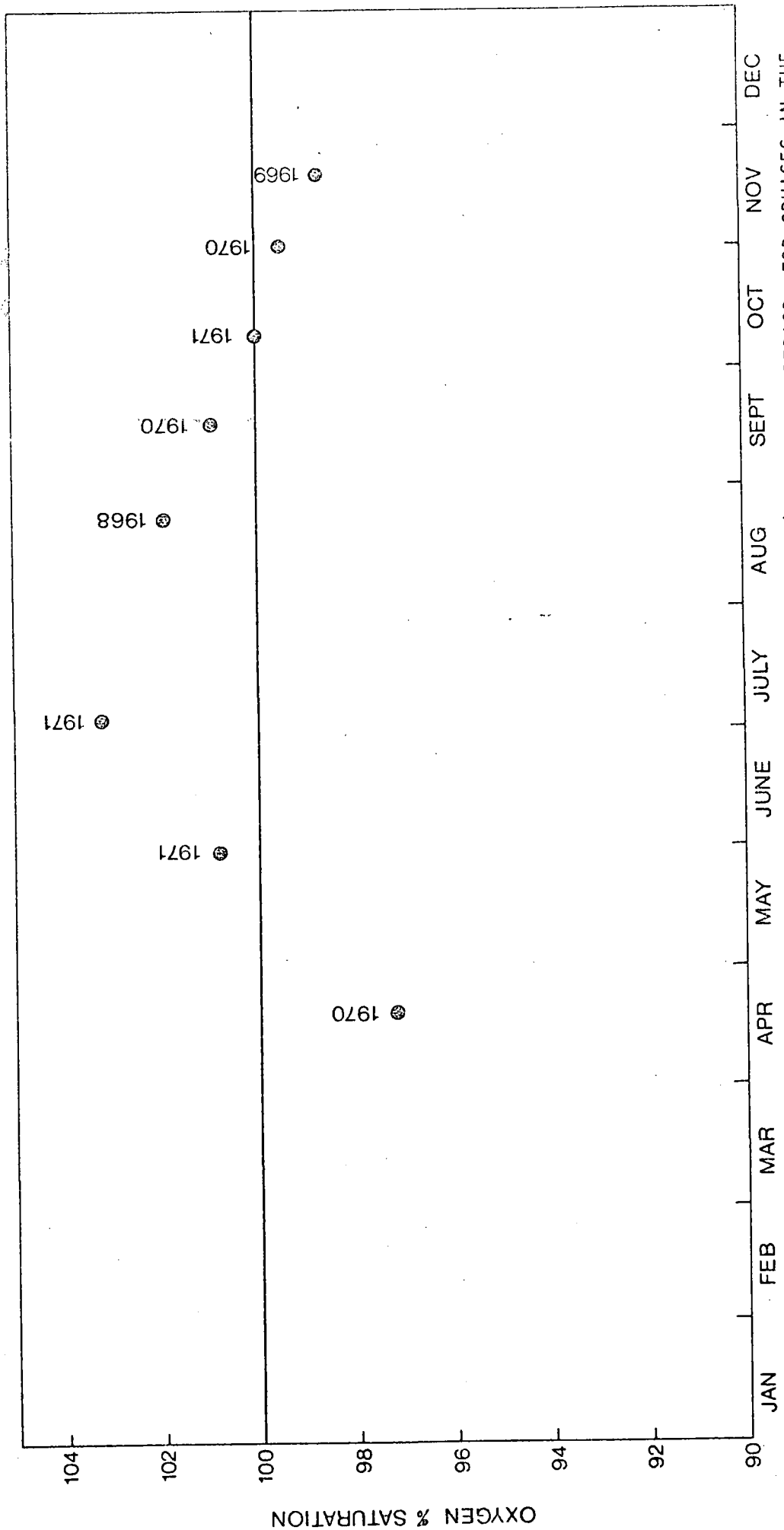
We need to decide whether this change is enough to require a reduction in phosphorus loading. Probably a levelling off of the annual loading at present values would be ideal for Lake Huron.

Slide 13. Oxygen % in Lake Superior

In lakes, dissolved oxygen declines in the hypolimnion, or the cold water below the thermocline, during summer, due to biological activities of respiration and decay. The oxygen depletion rate is a sign of metabolic intensity in the deep water, but also a reflection of the standing stock of algae in surface waters. Some of the algae sink and influence the deeper water.

Oxygen percent saturation in Lake Superior increased in the period before summer stratification, due not to an increase in oxygen but rather to an increase in temperature which changed the oxygen solubility value.

During summer stratification the oxygen values in the deep water of Lake Superior declined steadily but still remained near 99%.



SLIDE 13. MEAN OXYGEN % SATURATION VALUES OF THE COLD WATER-MASS (T<5°C.) IN LAKE SUPERIOR, FOR CRUISES IN THE YEARS 1968 TO 1971.

Slide 14. Oxygen concentration in Lake Superior

This next slide shows oxygen concentration instead of percent saturation, in the deep water of Lake Superior. The oxygen concentration remained steady in springtime, while percent saturation was increasing. The same thing happens in Lake Huron and in Lake Ontario.

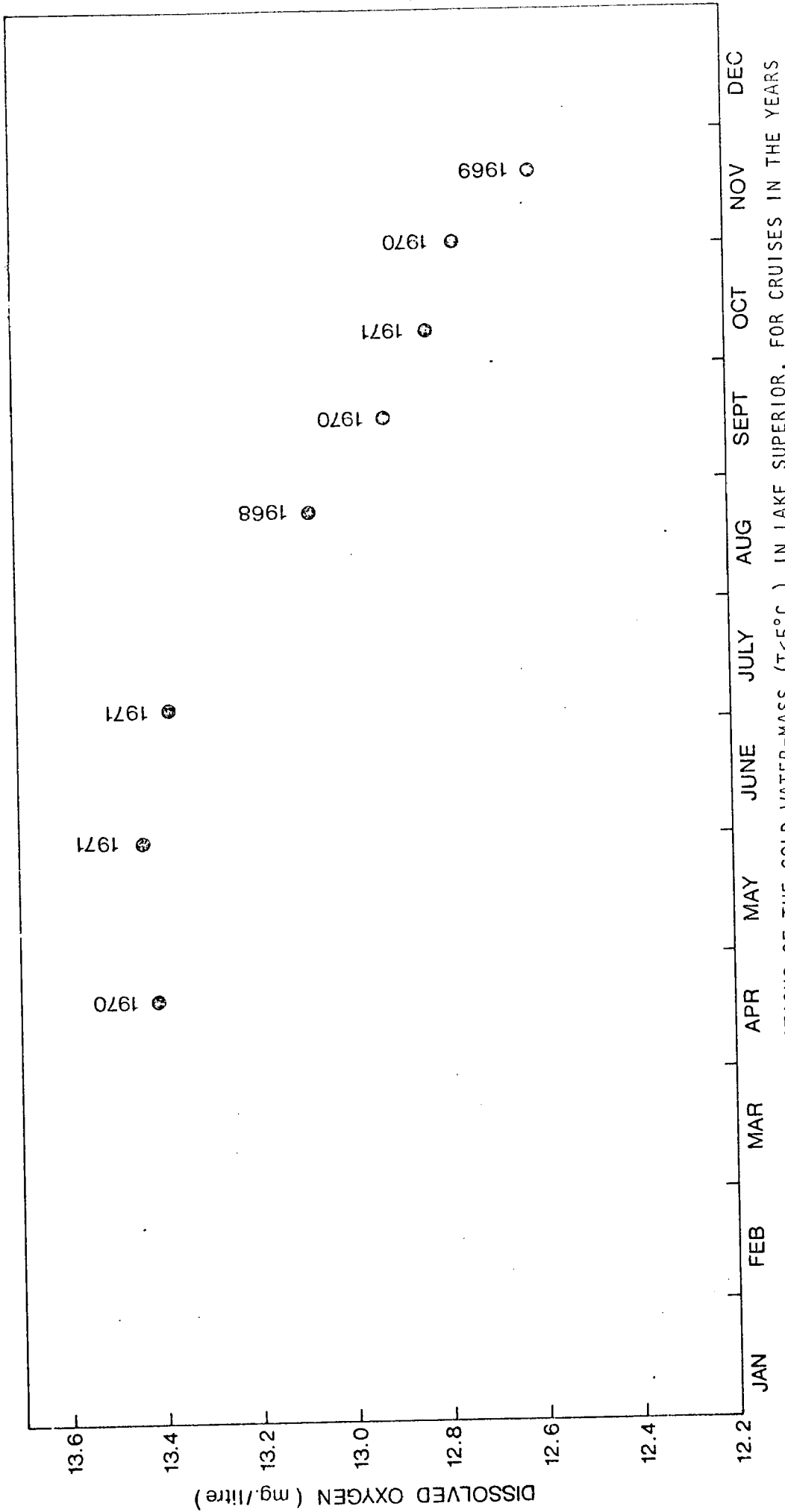
Slide 15. Temperatures in the deep water of Lake Superior

Here are mean temperatures for the same deep water in Lake Superior, showing the warming trend of early summer.

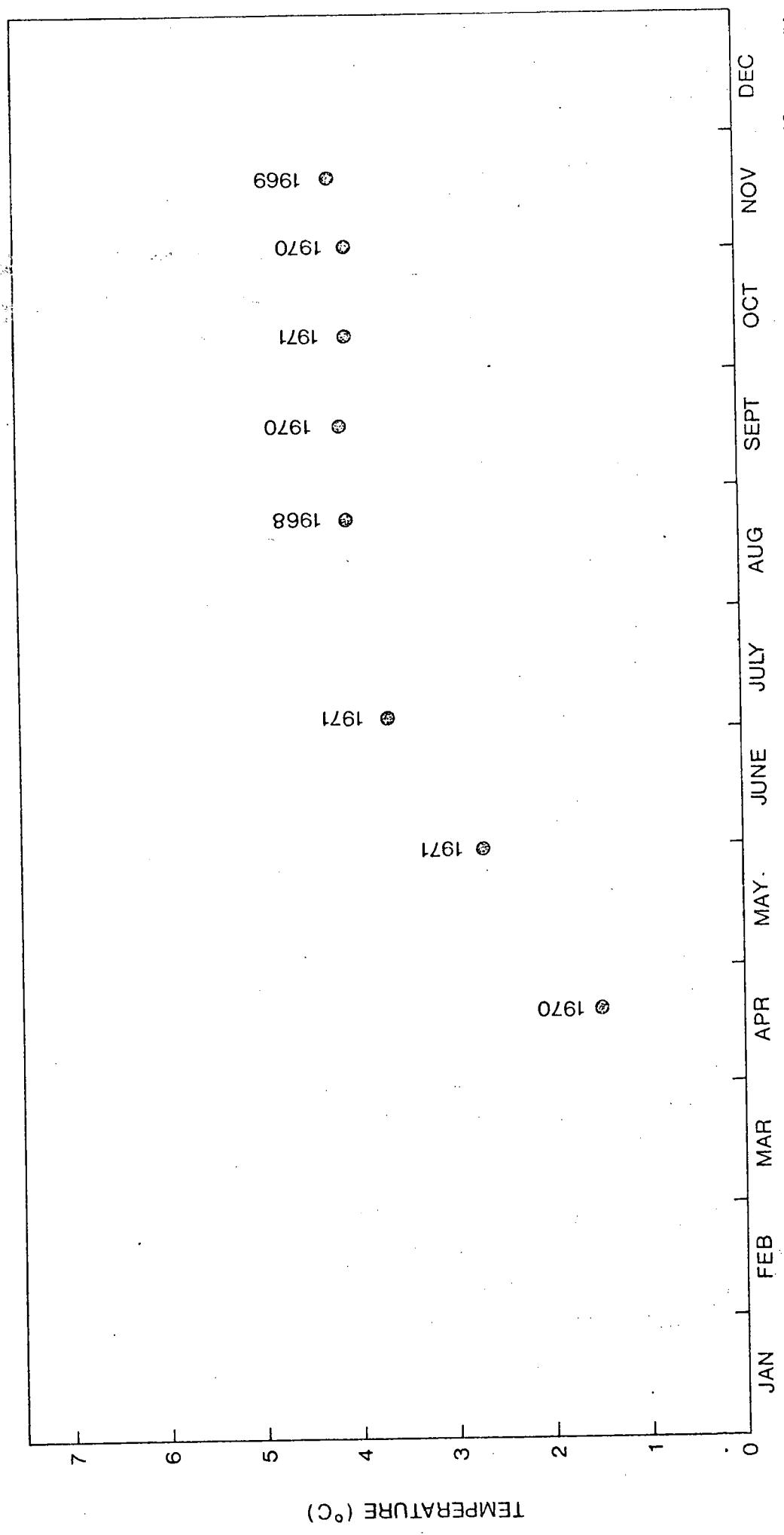
Slide 16. Oxygen % saturation in Lake Huron deep water

This slide has oxygen % saturation values for the deep cold water in Lake Huron. The data for different years are a little more scattered than for Lake Superior, but similar trends are evident. The oxygen depletion rate in late summer was twice as great in Lake Huron as it was in Lake Superior. The lowest mean oxygen % saturation value in Lake Huron bottom water was 94%.

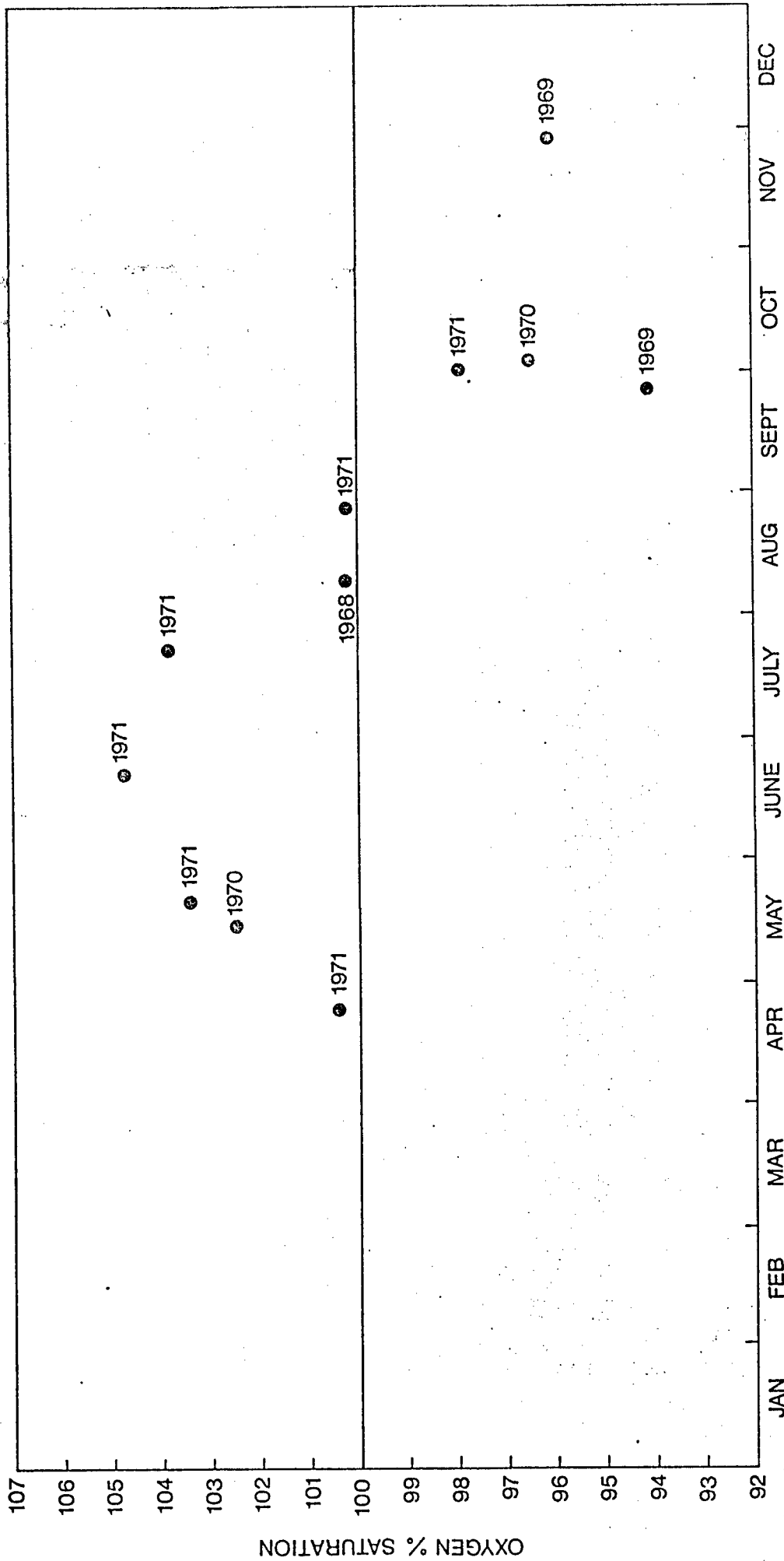
Slide 17. Central Erie, oxygen % saturation in surface waters and bottom waters



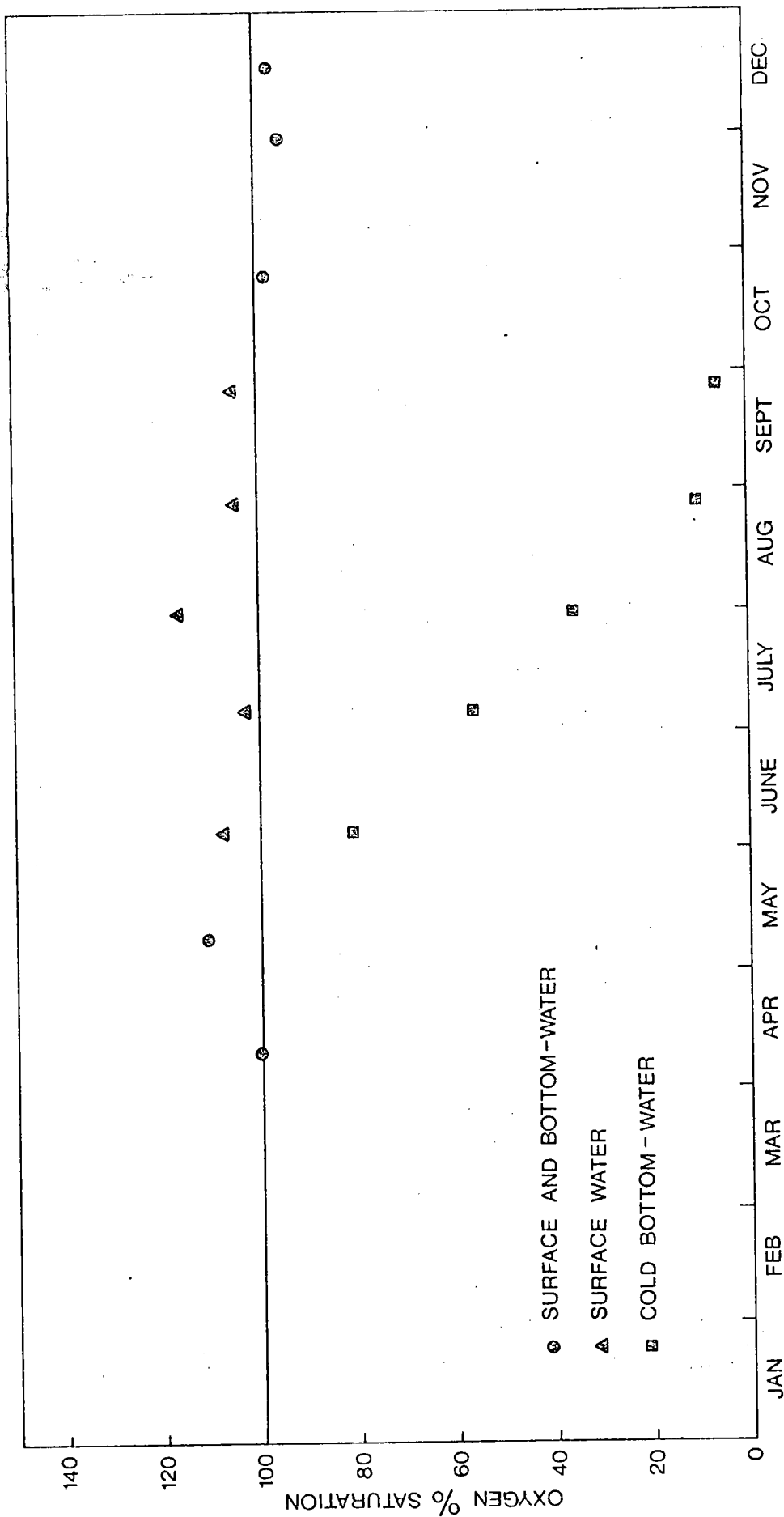
SLIDE 14. MEAN OXYGEN CONCENTRATIONS OF THE COLD WATER-MASS ($T < 5^{\circ}\text{C}.$) IN LAKE SUPERIOR, FOR CRUISES IN THE YEARS 1968 TO 1971. UNITS ARE MILLIGRAMS PER LITRE.



SLIDE 15. MEAN TEMPERATURES OF THE COLD WATER-MASS (T < 5°C.) IN LAKE SUPERIOR, FOR CRUISES IN THE YEARS 1968 TO 1971.



SLIDE 16. MEAN OXYGEN % SATURATION VALUES FOR THE COLD WATER-MASS ($T < 5^{\circ}\text{C}$) IN LAKE HURON, 1968 TO 1971.



SLIDE 17. CENTRAL LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES:
UNWEIGHTED MEAN OXYGEN % SATURATION VALUES AT A DEPTH OF 1 METRE,
AND ALSO IN THE COLD BOTTOM-WATER, FROM CRUISES OF THE MARTIN
KARLSEN DURING 1970.

In this graph and the following ones, the squares indicate cold bottom water or hypolimnion water, the triangles indicate surface water, and the circles are used when the lake is mixed completely.

In Central Erie bottom waters there is a steady decline in oxygen values during the period of thermal stratification, June to September, to values near zero. Surface waters remain near 100% saturation.

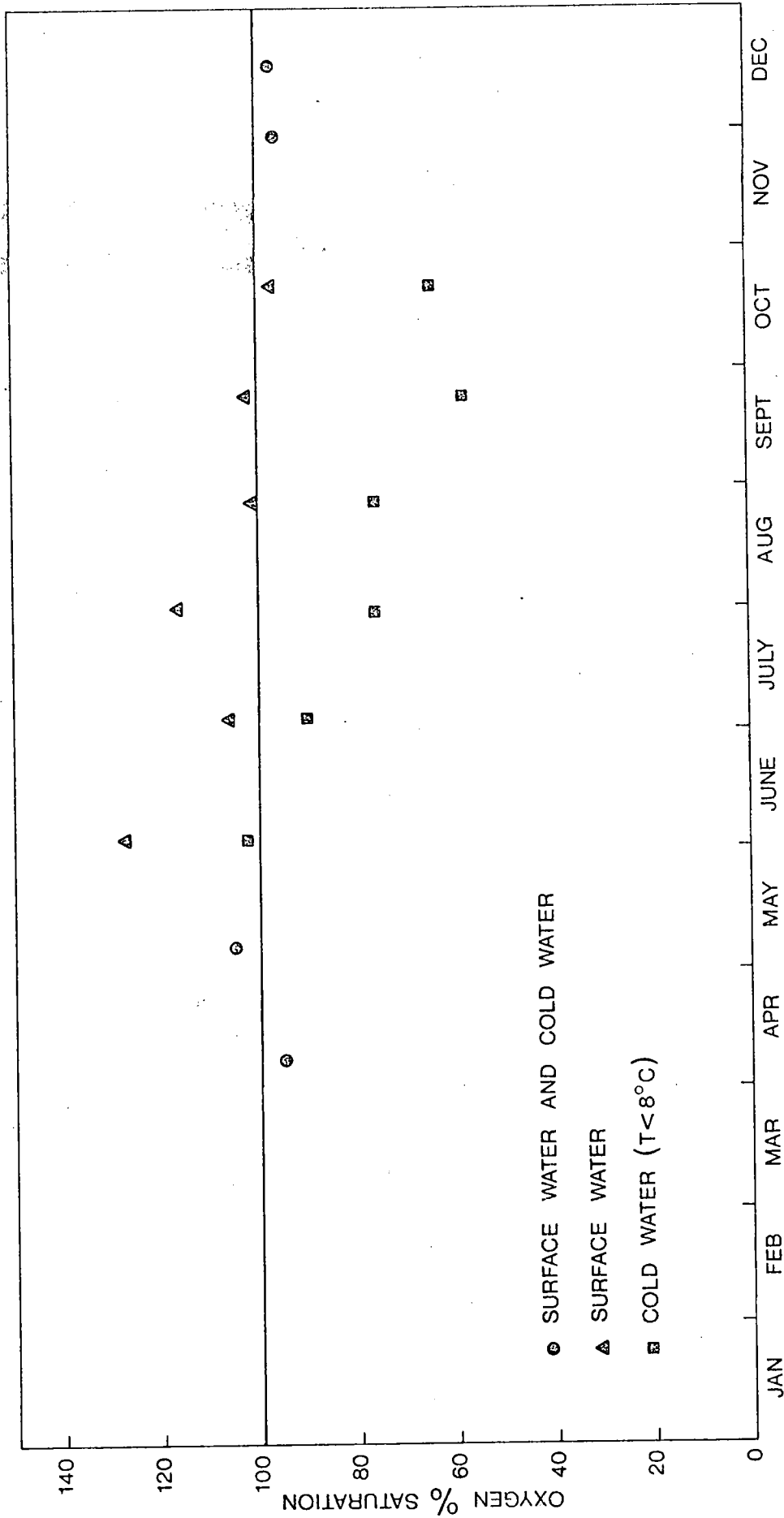
A worthwhile management goal for Central Lake Erie is to keep oxygen values in the bottom water above 50% saturation. In a moment I will discuss the necessary standing stock of plankton that would be required in summer, to preserve the dissolved oxygen.

Slide 18. Eastern Erie: oxygen % saturation in surface waters and bottom waters

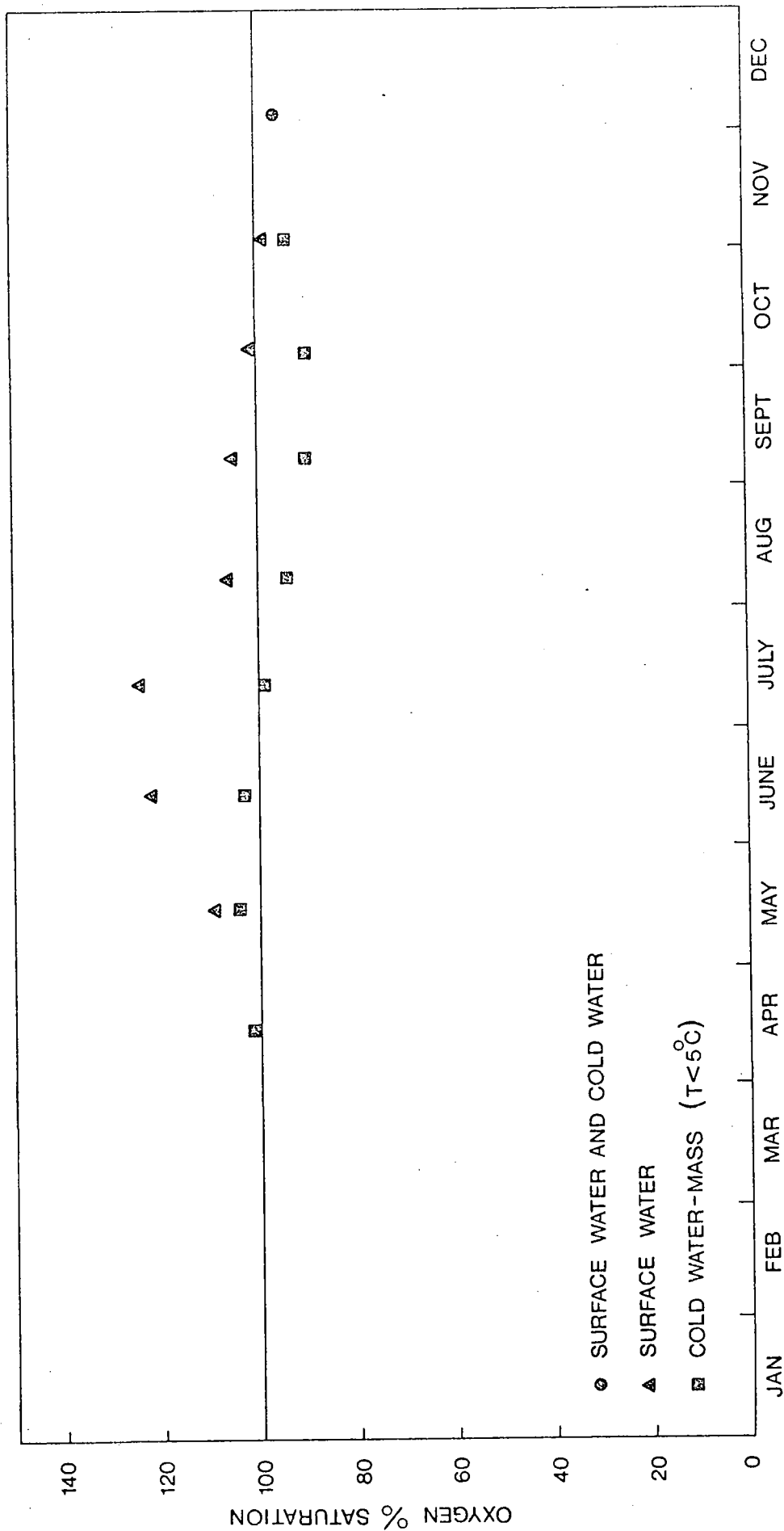
Oxygen in Eastern Lake Erie bottom water declines to about 60% saturation in late summer. If algae increase a bit in Eastern Lake Erie, its oxygen condition will become unsatisfactory.

Slide 19. Lake Ontario: oxygen % saturation in surface waters and bottom waters

This graph indicates the satisfactory oxygen conditions in Lake Ontario in the presence of quite large concentrations of algae - a characteristic stability of deep lakes.



SLIDE 18. EASTERN LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES; UNWEIGHTED MEAN OXYGEN % SATURATION VALUES AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD WATER - MASS (T < 8°C), FROM CRUISES OF THE MARTIN KARLSEN DURING 1970.



SLIDE 19. LAKE ONTARIO, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES:
UNWEIGHTED MEAN OXYGEN % SATURATION VALUES AT A DEPTH OF
1 METRE, AND ALSO IN THE COLD WATER-MASS ($T < 5^{\circ}\text{C}$), FROM
CRUISES OF THE LIMNOS AND MARTIN KARLSEN DURING 1969.

Oxygen supersaturation occurs in early summer in surface waters, due probably to the warming trend which lowers the solubility and increases the percent saturation. Perhaps there is also an influence by photosynthetic oxygen production.

In the bottom waters a slight decline in oxygen values occurs during summer. The lowest mean value in bottom waters in 1969 was 90% saturation.

Slide 20. Oxygen depletion rates in the Great Lakes

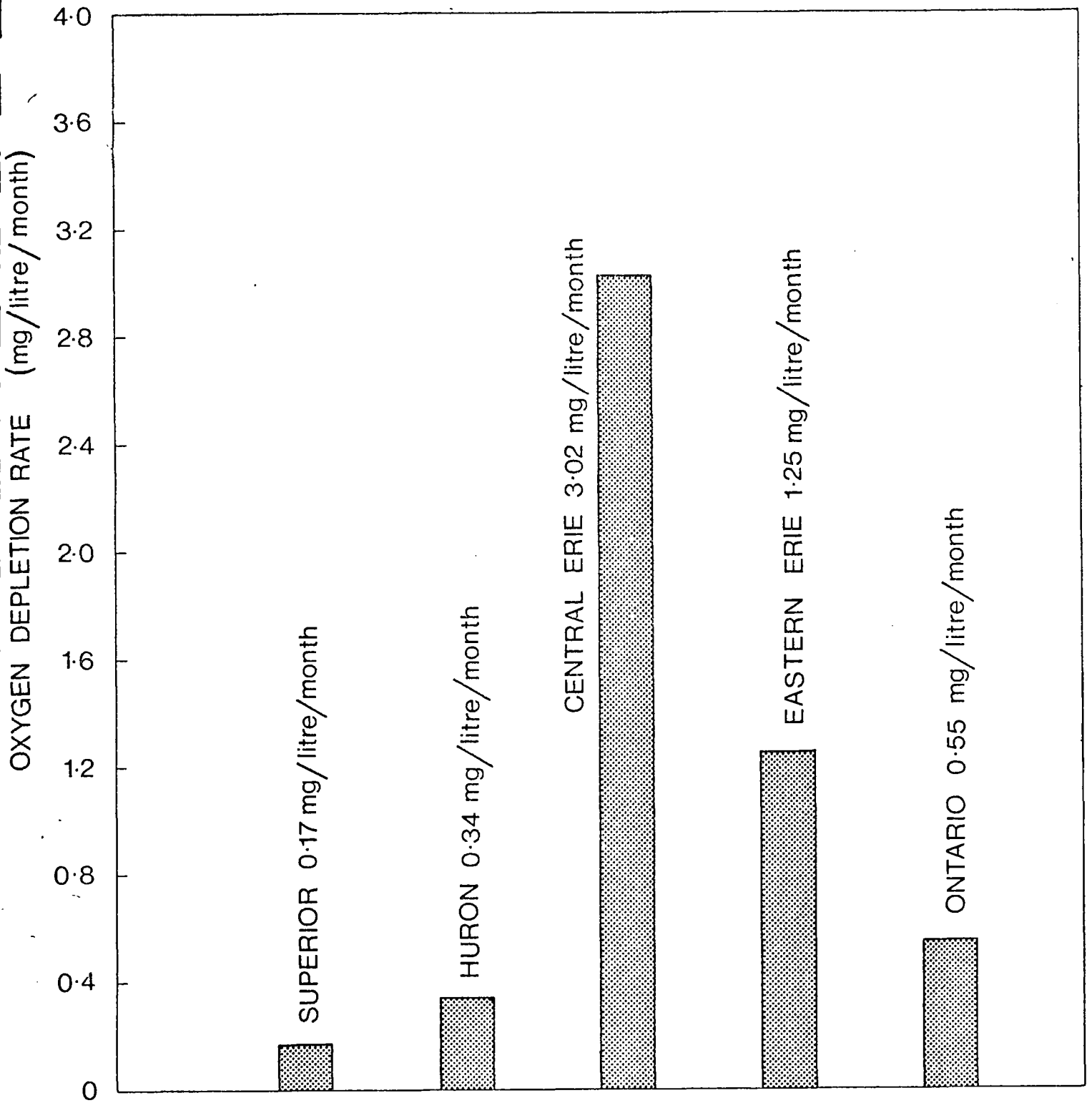
The rate of disappearance of oxygen in the bottom-waters during summer is a reflection of two things: the intensity of metabolism of surface waters, whose fallout of plankton affects the bottom layers, and also the thickness of the bottom-layer which gives it more or less resistance to change. Central Lake Erie has the largest oxygen depletion rate and Lake Superior the smallest.

I have shown in earlier work how the depletion rate in Central Erie bottom waters has increased since 1929.

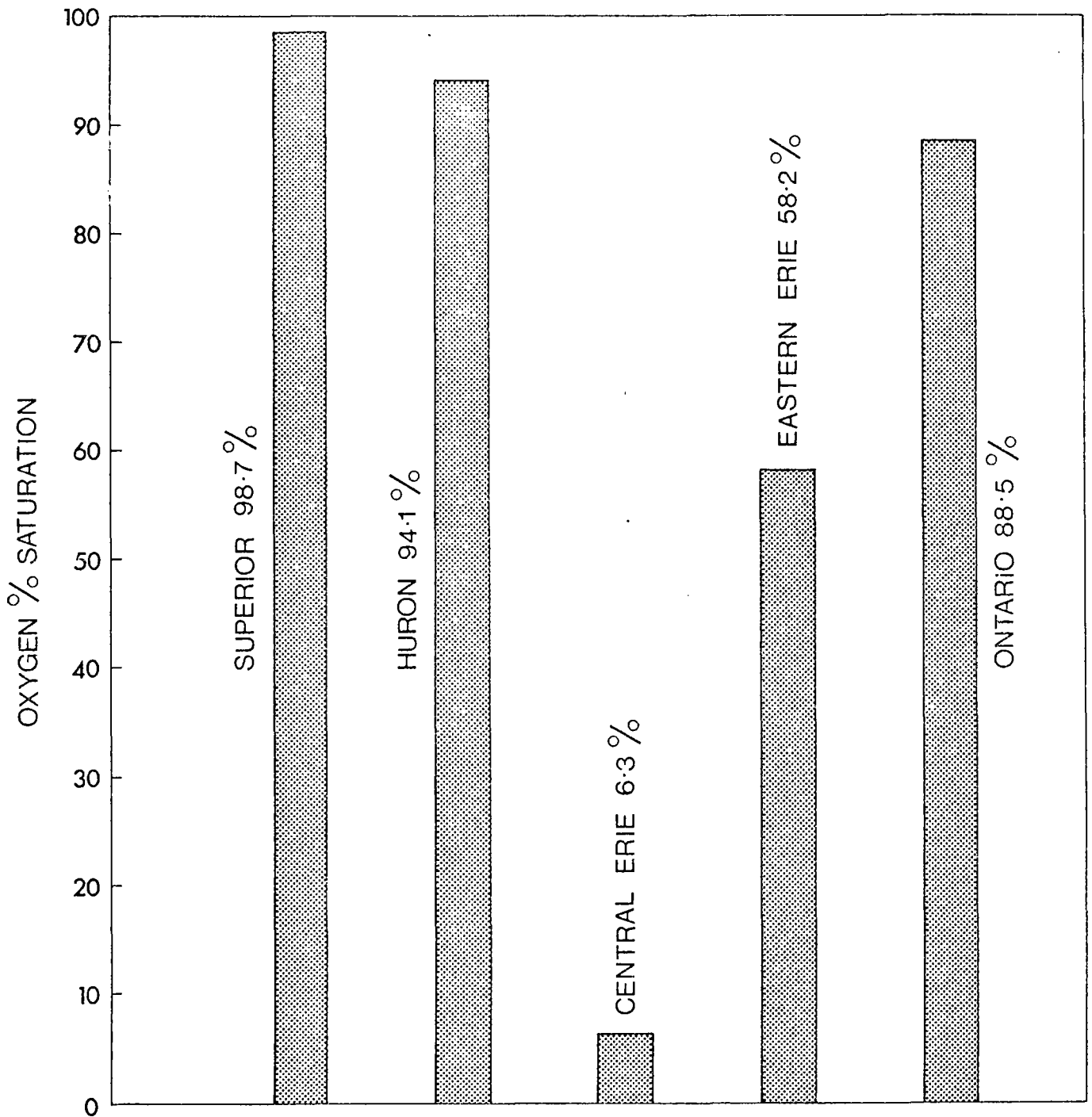
Slide 21. The lowest oxygen % saturation mean values in the hypolimnions of the Great Lakes

Here is a summary of oxygen conditions in the bottom-waters at the time of year having the lowest values.

Central Lake Erie has poor conditions near zero oxygen. Eastern Lake



S DE 20. OXYGEN DEPLETION RATES IN THE HYPOLIMNIONS OF THE GREAT LAKES DURING SUMMER 1970-1971.



SLIDE 21. THE LOWEST CRUISE - MEAN OXYGEN % SATURATION VALUES IN THE HYPOLIMNIONS OF THE GREAT LAKES DURING SUMMER STRATIFICATION, 1969 - 1970.

Erie is not much above 50% saturation so it too is approaching unsatisfactory conditions. The other deeper lakes are satisfactory. Lake Ontario is satisfactory even though it has as much algae in surface waters as there are in Central and Eastern Lake Erie.

Slide 22: oxygen : the relation between ordinary depletion rate and areal depletion rate

Here is the relation between ordinary oxygen depletion rate and the areal depletion rate which probably indicates better the trophic status of a lake.

It follows from this and from the relation between surface-water standing stock of particulate phosphorus and areal oxygen depletion rate in the lower Great Lakes that a 50% oxygen loss in summer is associated with a standing stock given by

$$\mu\text{g P/l} = 0.5 \times \text{hypolimnion thickness in metres. (See page 81.)}$$

In a deep lake the troublesome surface algae concentrations are reached before critical oxygen conditions, whereas central Lake Erie surface waters must be kept oligotrophic in their standing stock of algae in order to maintain adequate oxygen conditions in the hypolimnion. Eastern Lake Erie, being of intermediate depth, has marginal or troublesome algae conditions and oxygen conditions at the same time.

Slide 22

Dissolved oxygen: the relation between ordinary depletion rate
and areal depletion rate :-

$$\begin{aligned} &\text{Areal depletion rate (mg O}_2\text{/cm}^2\text{/month)} \\ &= \left[0.1 \times \text{hypolimnion thickness (metres)} \right. \\ &\quad \left. \times \text{ordinary depletion rate} \right] \\ &\quad \text{(mg O}_2\text{/litre/month)} \end{aligned}$$

Central Lake Erie needs a standing stock of particulate phosphorus of 2.8 $\mu\text{g P}/\ell$, in the oligotrophic range, in order to lose 50% of its bottom-water oxygen as it did in 1929. This was found by multiplying the present particulate phosphorus value by the ratio of the old and new oxygen depletion rates.

Slide 23. Historical data for nitrate in Lake Superior

Units for nitrogen fractions are micrograms nitrogen per litre.

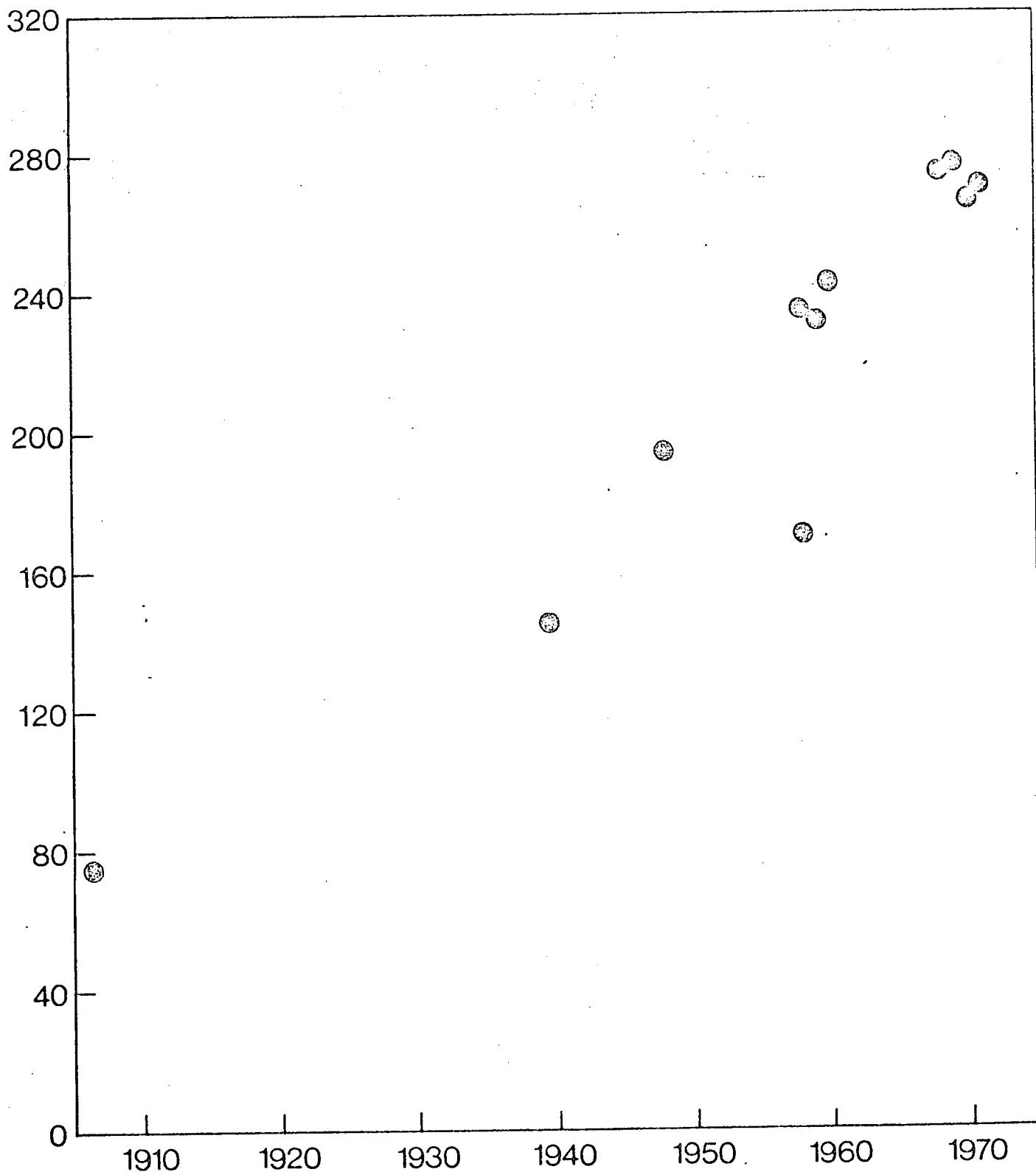
Historical data for nitrate in Lake Superior indicate a surprising increase over the years. Perhaps there has been a change in the amount of combined nitrogen in rainwater.

The amount of plankton in Lake Superior waters is limited by phosphorus. The increase in nitrate has probably had no biological effect.

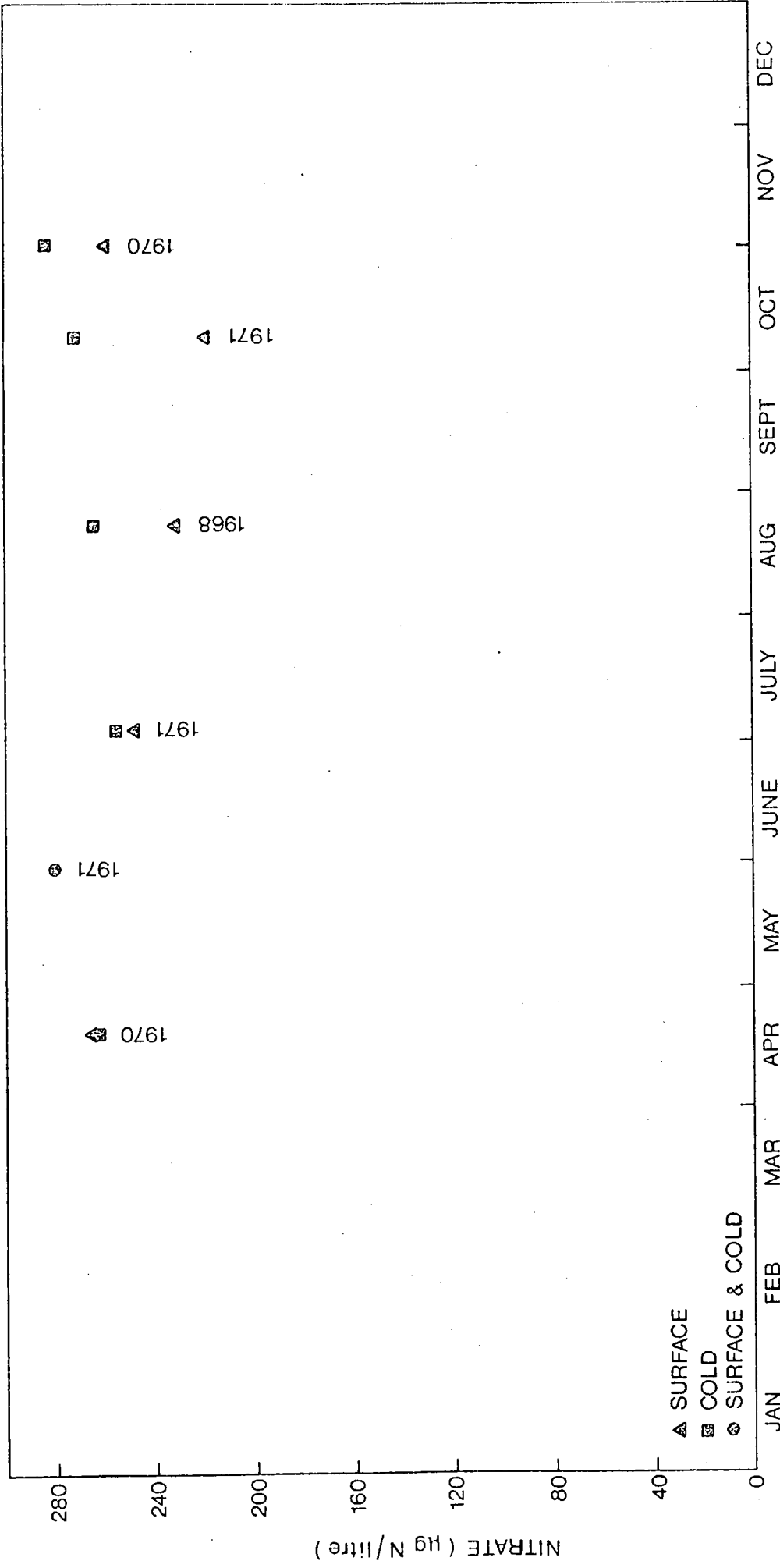
Slide 24. Nitrate in Lake Superior

This graph shows the nitrate values of surface waters and deep waters in Lake Superior during summer. Ammonia amounts to only about 8 $\mu\text{g N}/\ell$ in Lake Superior, so (nitrate + ammonia) would look similar.

Surface waters in Lake Superior are depleted slightly of nitrate during the period of thermal stratification, but the lowest values are still quite high. We would not expect blue-green algae to replace diatoms in Lake Superior during summer, if nitrogen shortages cause blue-greens to dominate.



SLIDE 23. SUMMARY OF HISTORICAL DATA FOR NITRATE IN LAKE SUPERIOR. UNITS ARE MICROGRAMS NITROGEN PER LITRE.



SLIDE 24. NITRATE IN LAKE SUPERIOR DURING 1968, 1970 AND 1971: MEAN VALUES IN THE COLD WATER-MASS (T < 5°C.), AND MEAN VALUES AT A DEPTH OF ONE METRE, FOR EACH CRUISE OF THE "THERON" AND "MARTIN KARLSEN". UNITS ARE MICROGRAMS NITROGEN PER LITRE.

Slide 25. Nitrate in Lake Huron

Here are some nitrate data for Lake Huron. The record as shown is not long enough to indicate the return of the surface waters to higher values in winter.

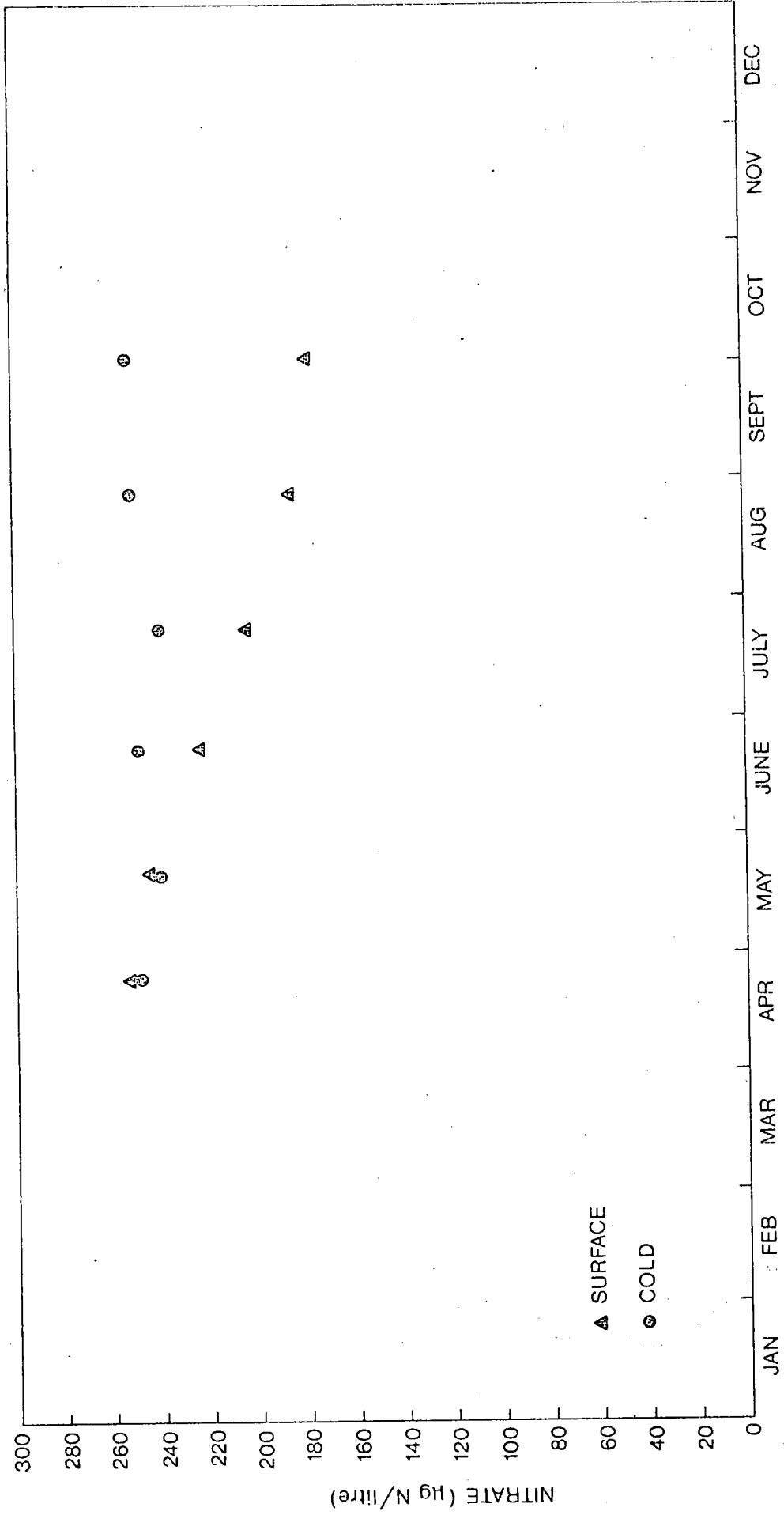
Depletion of nitrate and silicate in the surface waters of Lake Huron and Lake Superior proceeds only during thermal stratification. The stratification commences earlier near the shore, so that offshore surface waters have higher nitrate and silicate values in midsummer than do the nearshore waters. I have omitted the slides that prove this.

Slide 26. Western Erie: nitrate + ammonia in surface waters

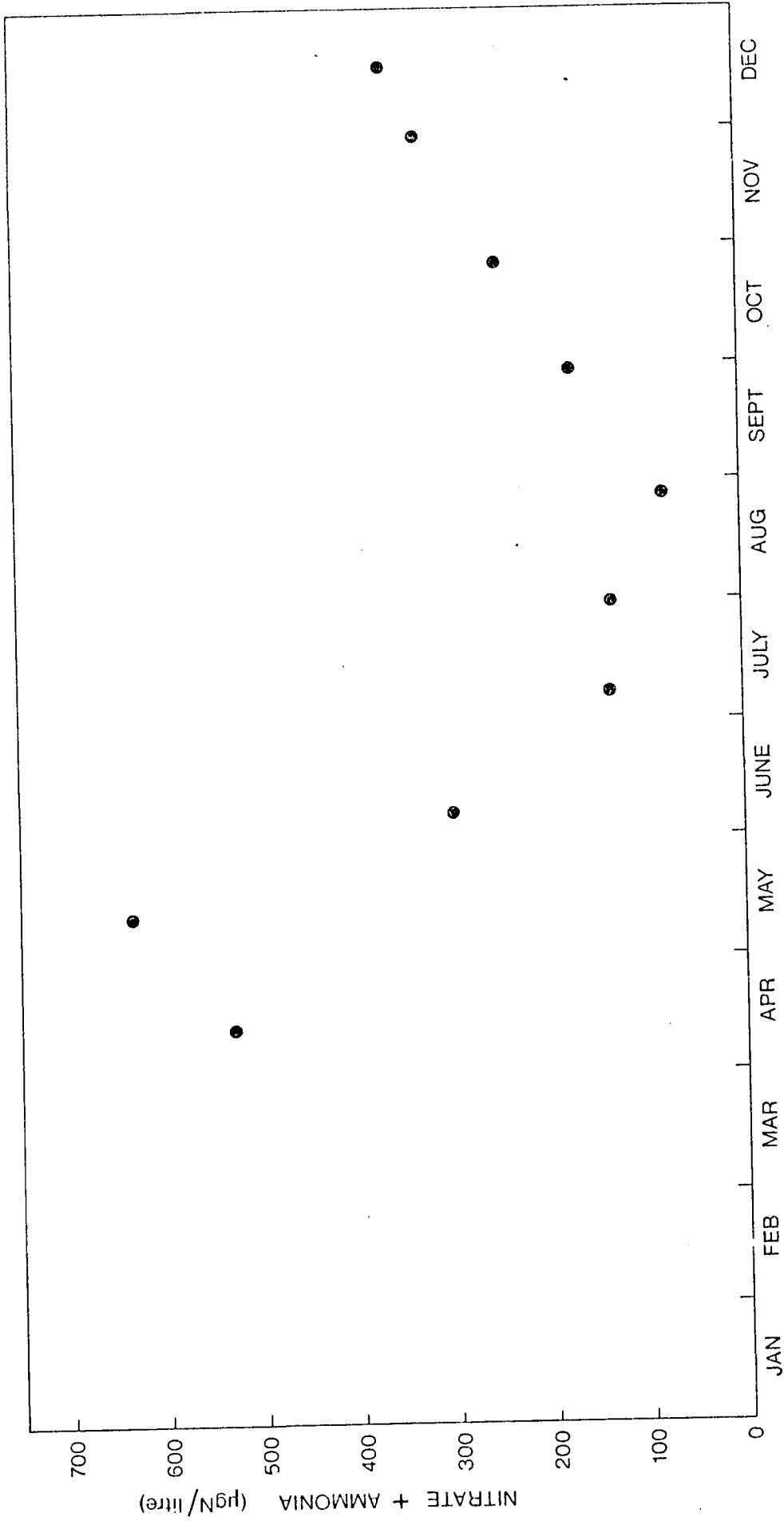
The springtime values of (nitrate + ammonia) in Western Lake Erie surface waters are about 600. $\mu\text{g N/litre}$, much higher than in Lake Huron. There must be local sources such as Detroit and the Maumee River basin.

There is depletion of these nitrogen fractions in summer, which is reasonable but hard to explain fully without measurements of other nitrogen fractions and measurements of seasonal variations of inflows and outflows.

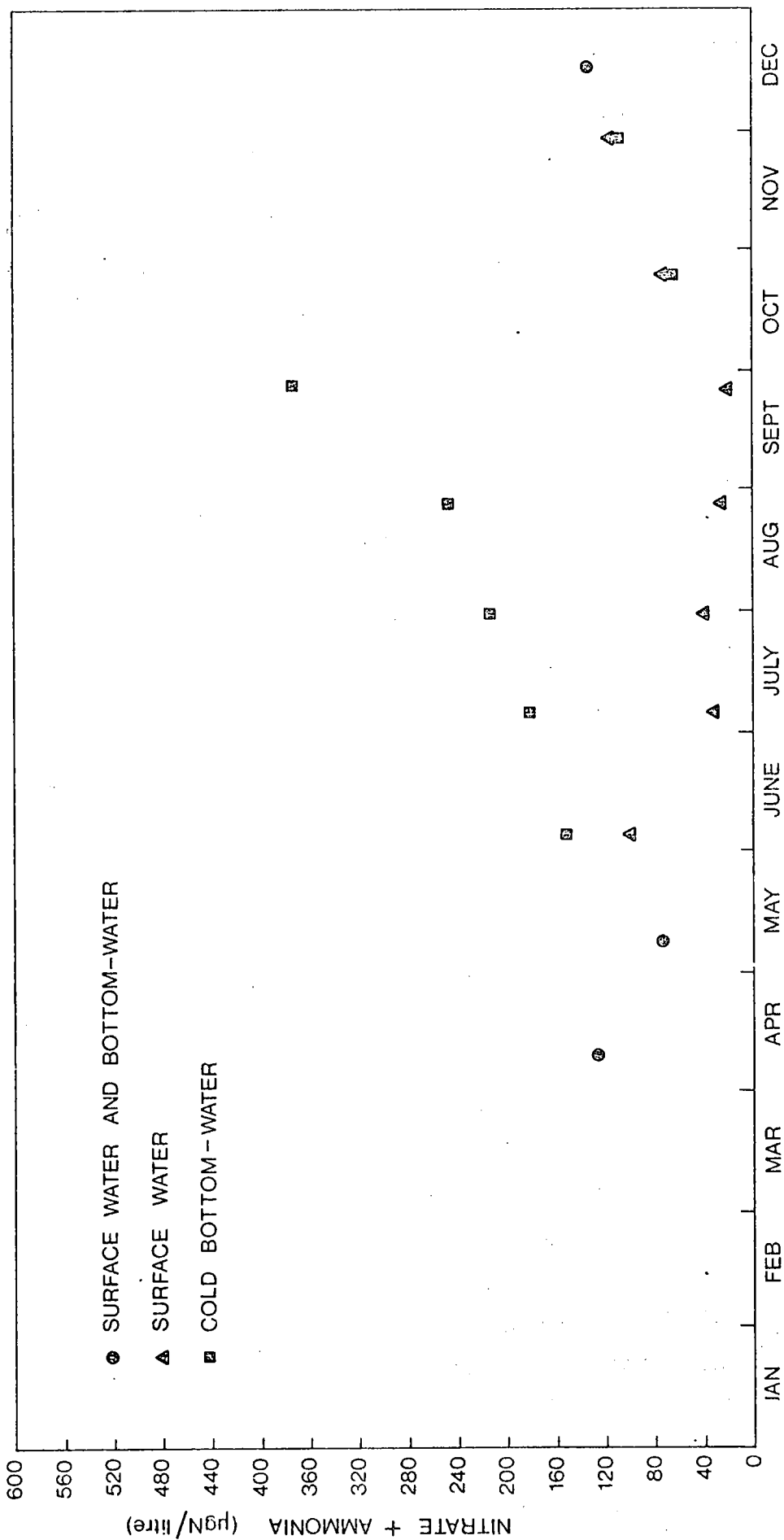
Slide 27. Central Lake Erie: nitrate + ammonia in surface waters and bottom waters



SLIDE 25. NITRATE IN LAKE HURON DURING 1971: MEAN VALUES IN THE COLD WATER-MASS ($T < 5^{\circ}\text{C}$), AND MEAN VALUES AT A DEPTH OF 1 METRE, FOR EACH CRUISE OF THE "MARTIN KARLSEN". UNITS ARE MICROGRAMS NITROGEN PER LITRE.



SLIDE 26. WESTERN LAKE ERIE: UNWEIGHTED MEAN (NITRATE + AMMONIA) CONCENTRATIONS AT A DEPTH OF 1 METRE, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS NITROGEN PER LITRE.



SLIDE 27. CENTRAL LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN (NITRATE + AMMONIA) CONCENTRATIONS AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD BOTTOM-WATER, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS NITROGEN PER LITRE.

(Nitrate + ammonia) are depleted from surface waters of Central Lake Erie in summer, with regeneration in bottom waters at the same time.

The low value of the mixture in October is a reflection of the relatively small volume of the hypolimnion the month before.

Slide 28. Eastern Lake Erie: nitrate + ammonia in surface waters and bottom waters

(Nitrate + ammonia) are depleted from the surface waters of eastern Lake Erie in summer, mostly during June. Surface values are nearly zero in September.

There is some regeneration in bottom-waters of Eastern Lake Erie during summer.

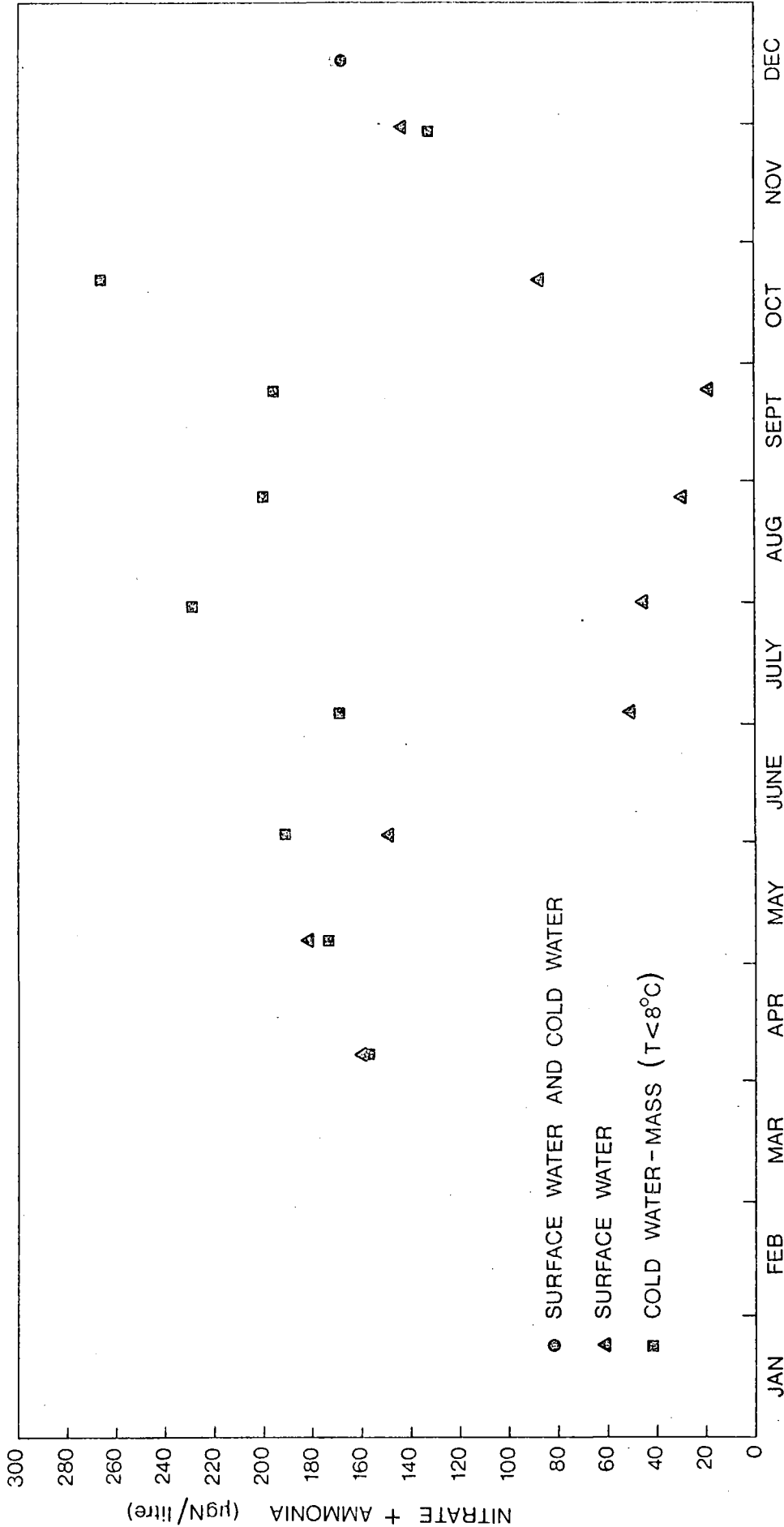
The rise in surface-water values of (nitrate + ammonia) during autumn is partly from the mixture with deeper waters as the thermocline descends.

Slide 29. Lake Ontario: nitrate + ammonia in surface waters and bottom waters

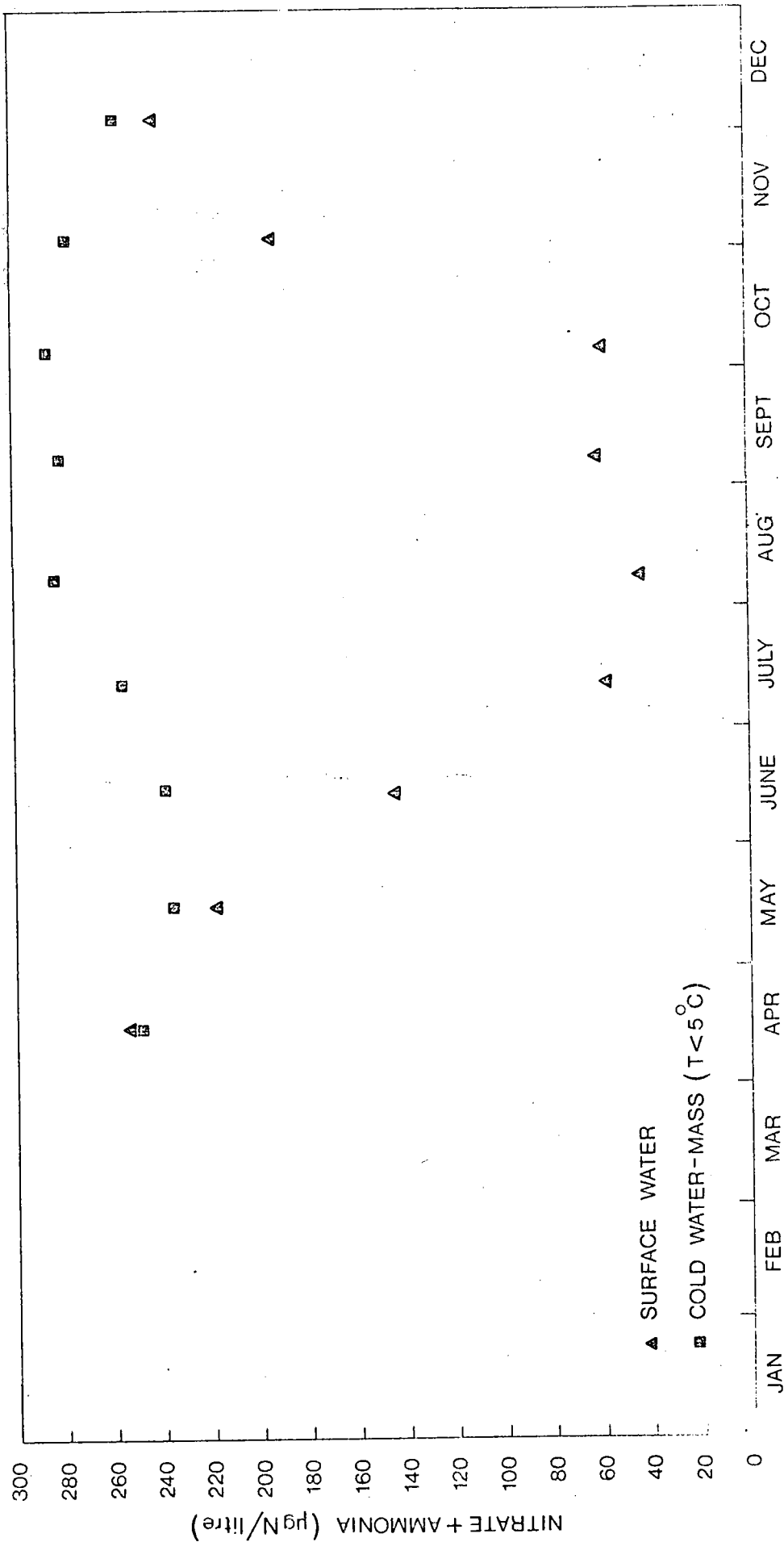
(Nitrate + ammonia) are depleted from surface waters of Lake Ontario during June and July while the summer standing stock of plankton is building up.

In bottom waters there is little regeneration during summer. The mixture in autumn and winter has high concentrations due to the relatively large volume of the hypolimnion in this deep lake.

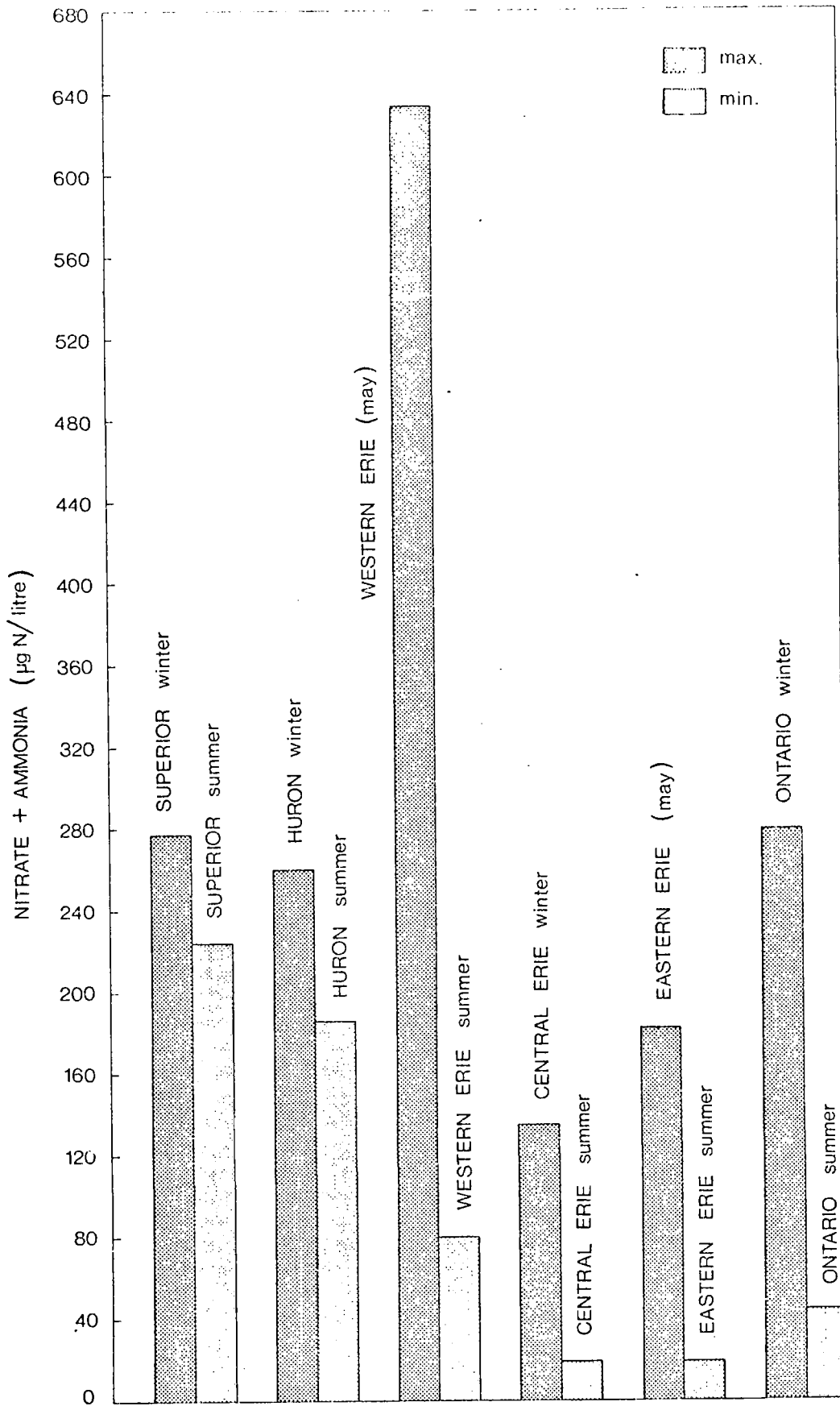
Slide 30. Nitrate + ammonia in Great Lakes surface waters



SLIDE 28. EASTERN LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN (NITRATE + AMMONIA) CONCENTRATIONS AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD WATER - MASS (T < 8°C), FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS NITROGEN PER LITRE.



SLIDE 29. LAKE ONTARIO, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES:
UNWEIGHTED MEAN (NITRATE + AMMONIA) CONCENTRATIONS AT A DEPTH
OF 1 METRE, AND ALSO IN THE COLD WATER-MASS (T < 5°C), FROM
CRUISES OF THE LIMNOS AND MARTIN KARLSEN DURING 1969.
UNITS ARE MICROGRAMS NITROGEN PER LITRE.



SLIDE 30. SUMMARY OF (NITRATE + AMMONIA) CONCENTRATIONS IN SURFACE WATERS OF THE GREAT LAKES, 1969-1971 (FROM CRUISE - MEAN VALUES AT A DEPTH OF 1 METRE IN EACH BASIN.)

Here is a summary of (nitrate + ammonia) concentrations in surface waters of the four lakes.

(Downstream is on the right hand side of the graph. For each lake the winter maximum is on the left and the summer minimum is on the right).

In winter the upper lakes have high concentrations of these inorganic nutrients, similar to the values in Lake Ontario. Western Erie is higher and Central Erie lower than the others in winter. In summertime in the upper lakes (nitrate + ammonia) are not much depleted whereas in the lower lakes they are depleted to near zero.

Slide 31. Standing-stock potential indicated by winter-time (nitrate + ammonia) concentrations

In this table I have indicated the amount of plankton that the winter (nitrate + ammonia) values would allow, in units of particulate phosphorus, $\mu\text{g P/l}$, by dividing the nitrogen value by 13, the approximate ratio of N to P in plankton.

Also I have listed the actual maximum standing stock observed for particulate phosphorus.

The nitrogen fractions are unused in Lake Superior and Lake Huron, where (nitrate + ammonia) are present in concentrations that would allow quite eutrophic conditions if only phosphorus were added.

Slide 32. Silicate in Lake Superior

Units for reactive or dissolved silicate are micrograms SiO_2 per litre.

Slide 31

Standing stock potential indicated by winter-time

(nitrate + ammonia) concentrations.

	nitrate + ammonia in winter ($\mu\text{g N}/\ell$)	trophic potential ($\mu\text{g P}/\ell$)	observed maximum particulate phosphorus concentration ($\mu\text{g P}/\ell$)
--	--	--	--

Lake Superior

277.

21.

1.4

Lake Huron

260.

20.

2.4

Western Erie

634.

49.

56.5

Central Erie

135.

10.

18.4

Eastern Erie

182.

14.

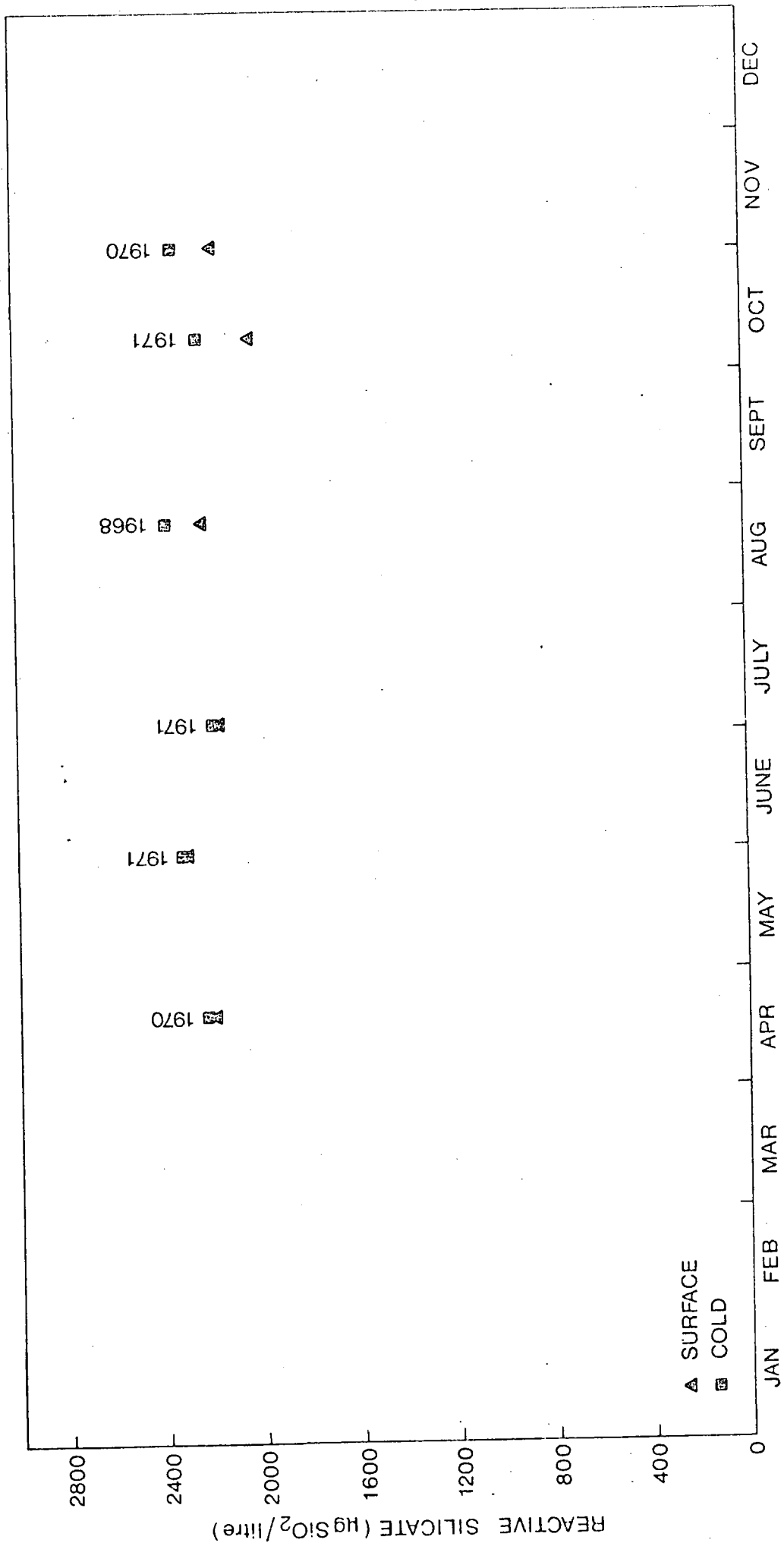
12.6

Lake Ontario

279.

21.

13.5



SLIDE 32. REACTIVE SILICATE IN LAKE SUPERIOR DURING 1968, 1970 AND 1971: MEAN VALUES IN THE COLD WATER-MASS (T<5°C.), AND MEAN VALUES AT A DEPTH OF ONE METRE, FOR EACH CRUISE OF THE "THERON" AND "MARTIN KARLSEN". UNITS ARE MICROGRAMS SiO₂ PER LITRE.

The concentrations of reactive silicate in Lake Superior are quite high, with a slight depletion of surface waters in summer.

Slide 33. Silicate in Lake Huron

There is evidence for a long-term trend towards lower silicate values in Lake Huron, in line with Schelske and Stoermer's ideas about Lake Michigan.

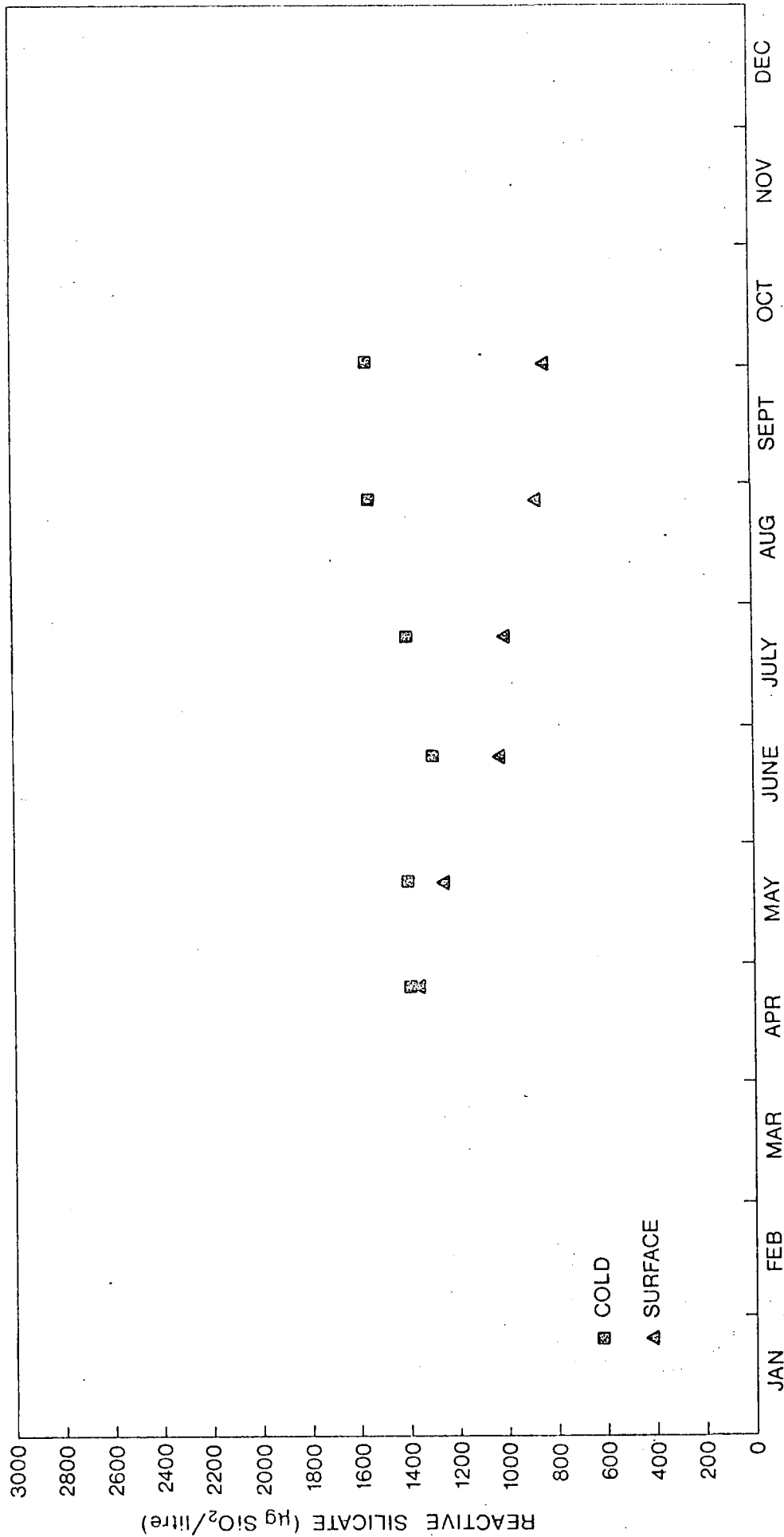
Deep-water values in Lake Huron were about 2600. $\mu\text{g SiO}_2/\ell$ in 1954 and 1400 $\mu\text{g SiO}_2/\ell$ in 1971, a decrease to about half of the earlier value.

This trend in silicate is a reasonable consequence of increased sedimentation rates for particulate silicate in diatoms. There has probably been an increase in diatom production due to increasing fertilization with phosphorus.

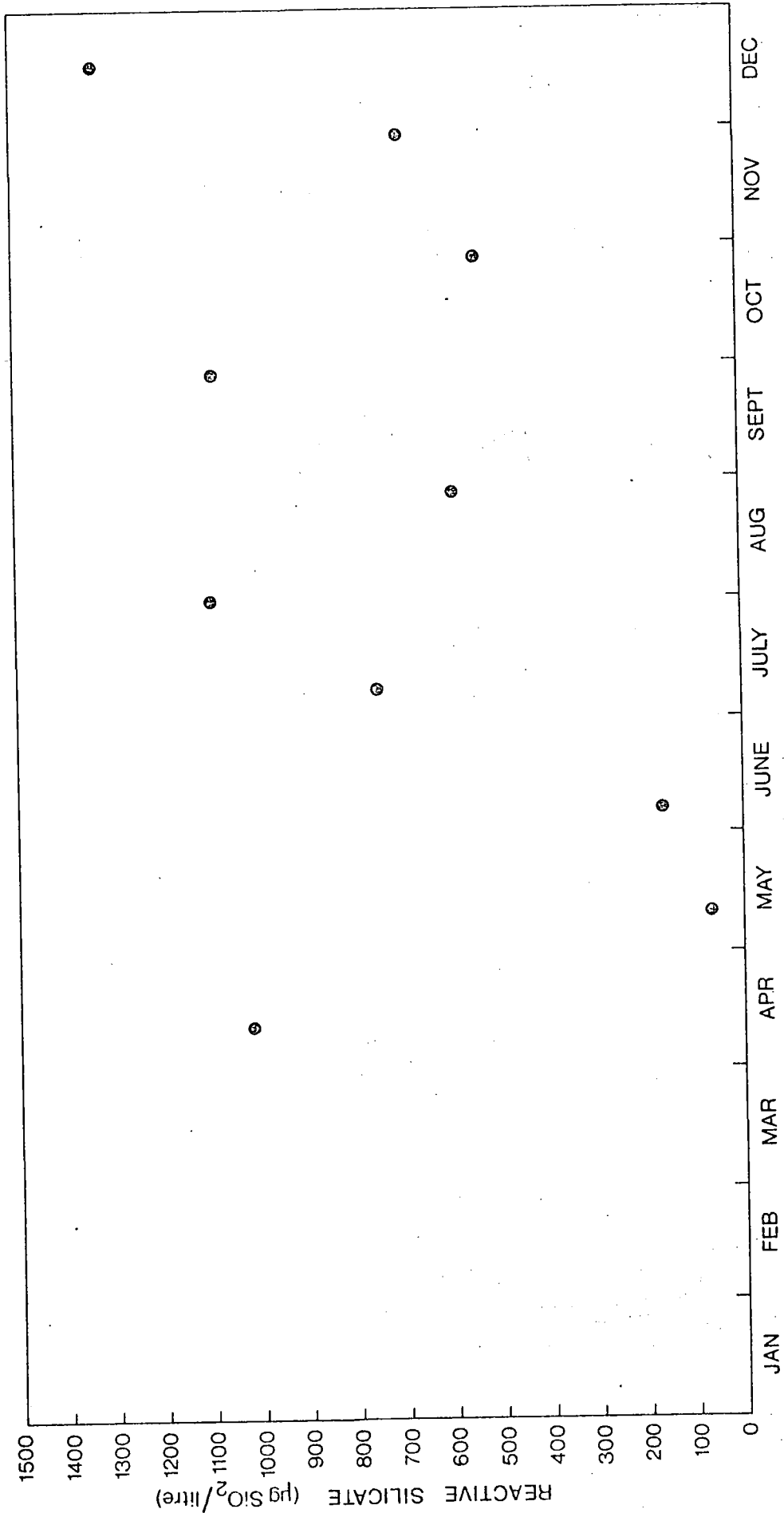
In this slide we have part of the seasonal cycle of dissolved silicate in Lake Huron. Surface waters become partially depleted during summer, and they return to higher values during winter when thermal stratification disappears.

Slide 34. Silicate in Western Lake Erie

Western Lake Erie is usually unstratified, so I have not shown any bottom-water values. Most of these silicate values for western Lake Erie are equal to or somewhat lower than the values in the source water in Lake Huron.



SLIDE 33. REACTIVE SILICATE IN LAKE HURON DURING 1971: MEAN VALUES IN THE COLD WATER-MASS ($T < 5^{\circ}\text{C}$), AND MEAN VALUES AT A DEPTH OF 1 METRE, FOR EACH CRUISE OF THE "MARTIN KARLSEN". UNITS ARE MICROGRAMS SiO_2 PER LITRE.



SLIDE 34. WESTERN LAKE ERIE: UNWEIGHTED MEAN REACTIVE-SILICATE CONCENTRATIONS AT A DEPTH OF 1 METRE FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS SiO₂ PER LITRE.

However, in May and June the values are much lower than those in Lake Huron, due probably to a diatom bloom in Western Lake Erie.

Slide 35. Silicate in Central Lake Erie

Dissolved silicate is extremely low in Central Lake Erie during springtime, from extraction by the diatoms.

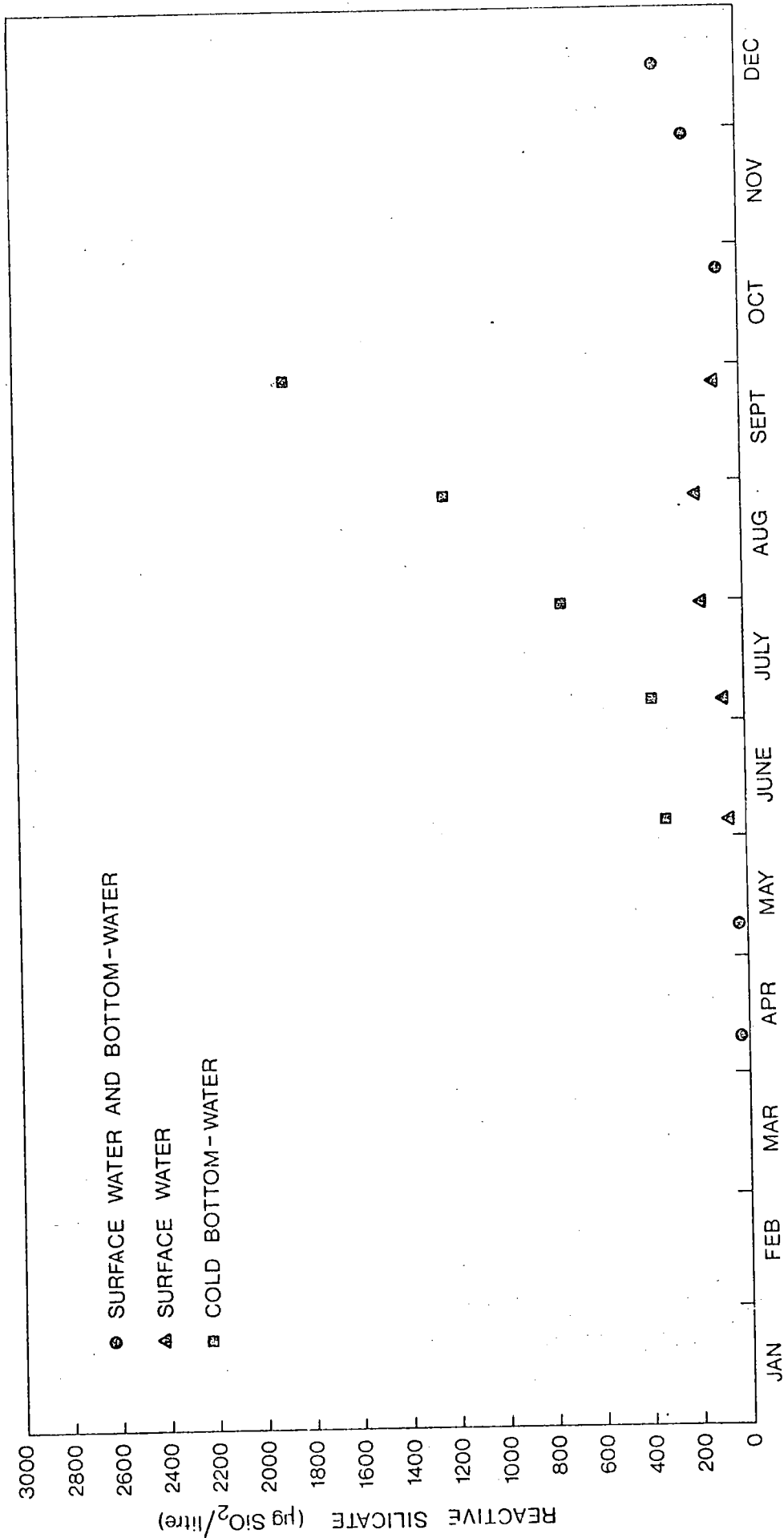
Regeneration of dissolved silicate in the bottom-water is evident during the summer period having thermal stratification, but the mixed water in the autumn has low values because the volume of the hypolimnion is small relative to the epilimnion in Central Lake Erie.

Slide 36. Silicate in Eastern Lake Erie

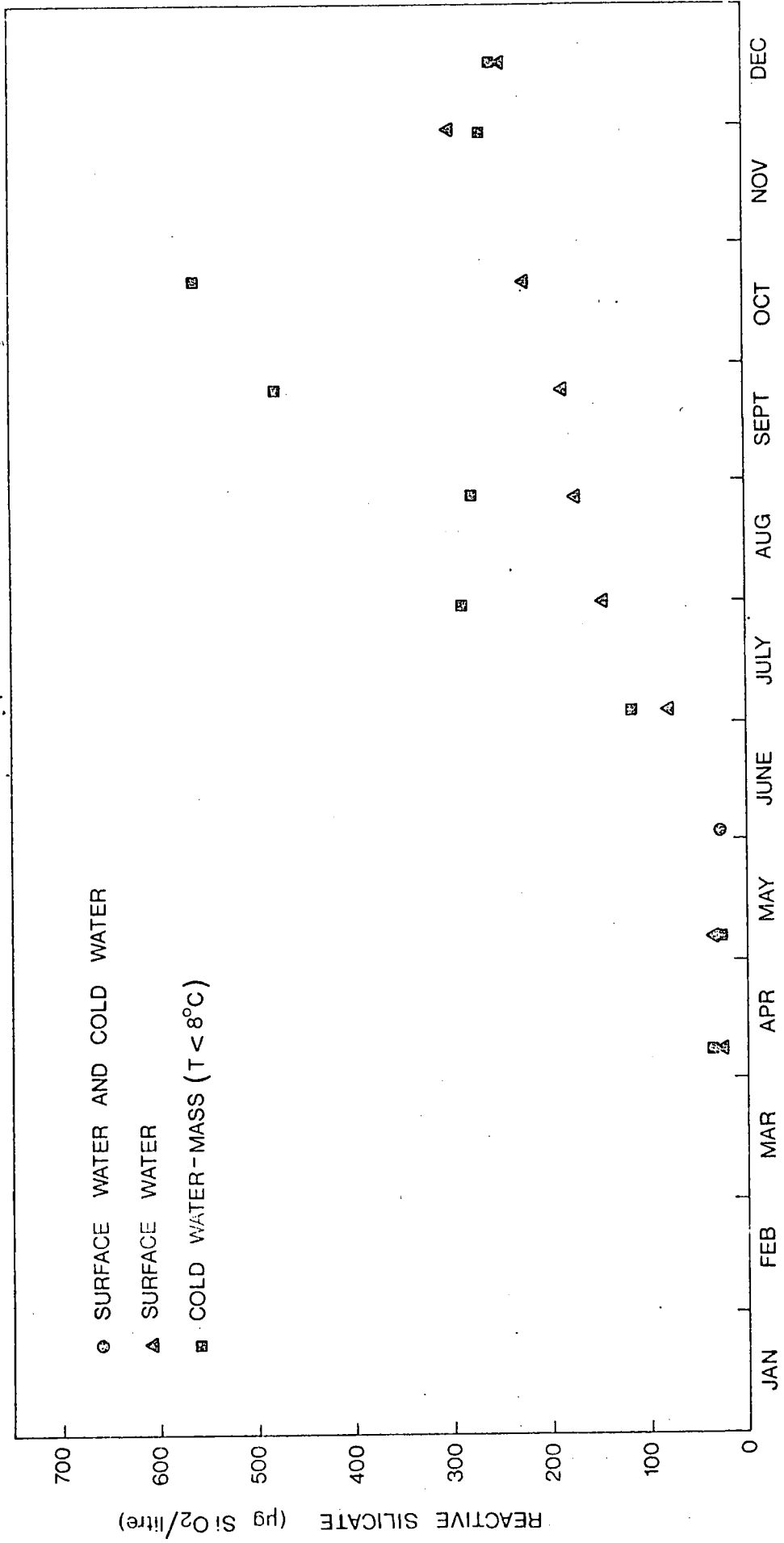
Silicate values are very low in springtime in Eastern Lake Erie, probably because of spring diatom growth.

In summer and autumn there are increasing silicate values in surface waters. Regeneration in the surface waters themselves probably causes this in early summer. In autumn it may be the effect of mixture with bottom-water as the thermocline descends.

Regeneration of silicate in bottom-waters is evident during the period of thermal stratification.



SLIDE 35. CENTRAL LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN REACTIVE-SILICATE CONCENTRATIONS AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD BOTTOM-WATER, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS SiO₂ PER LITRE.



SLIDE 36. EASTERN LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN REACTIVE-SILICATE CONCENTRATIONS AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD WATER-MASS ($T < 8^\circ\text{C}$), FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS SiO_2 PER LITRE.

Slide 37. Silicate in Lake Ontario

In Lake Ontario there is depletion of silicate from surface waters and bottom waters during springtime. In summer there is regeneration in bottom waters.

The winter values reflect the large volume of the hypolimnion in summer.

These silicate values are much lower than those of Lake Huron and Lake Superior, but not as low as in Central and Eastern Lake Erie in springtime.

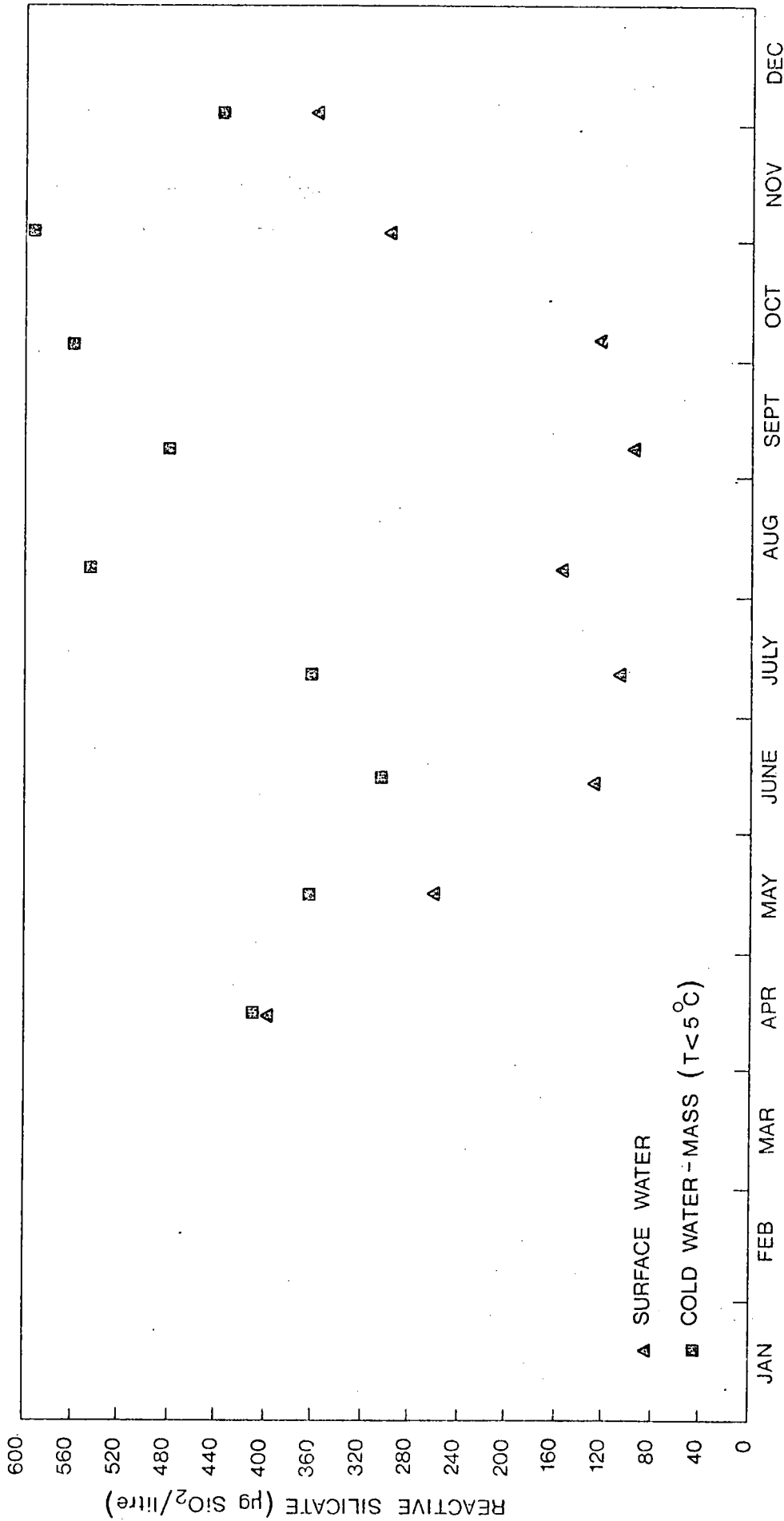
Slide 38. Silicate in Great Lakes surface waters

This bar-graph summarizes dissolved silicate concentrations in the four lakes. Superior is on the left side of the graph and Lake Ontario is on the right. For each lake the left bar shows the winter maximum and the right bar shows the summer or spring minimum.

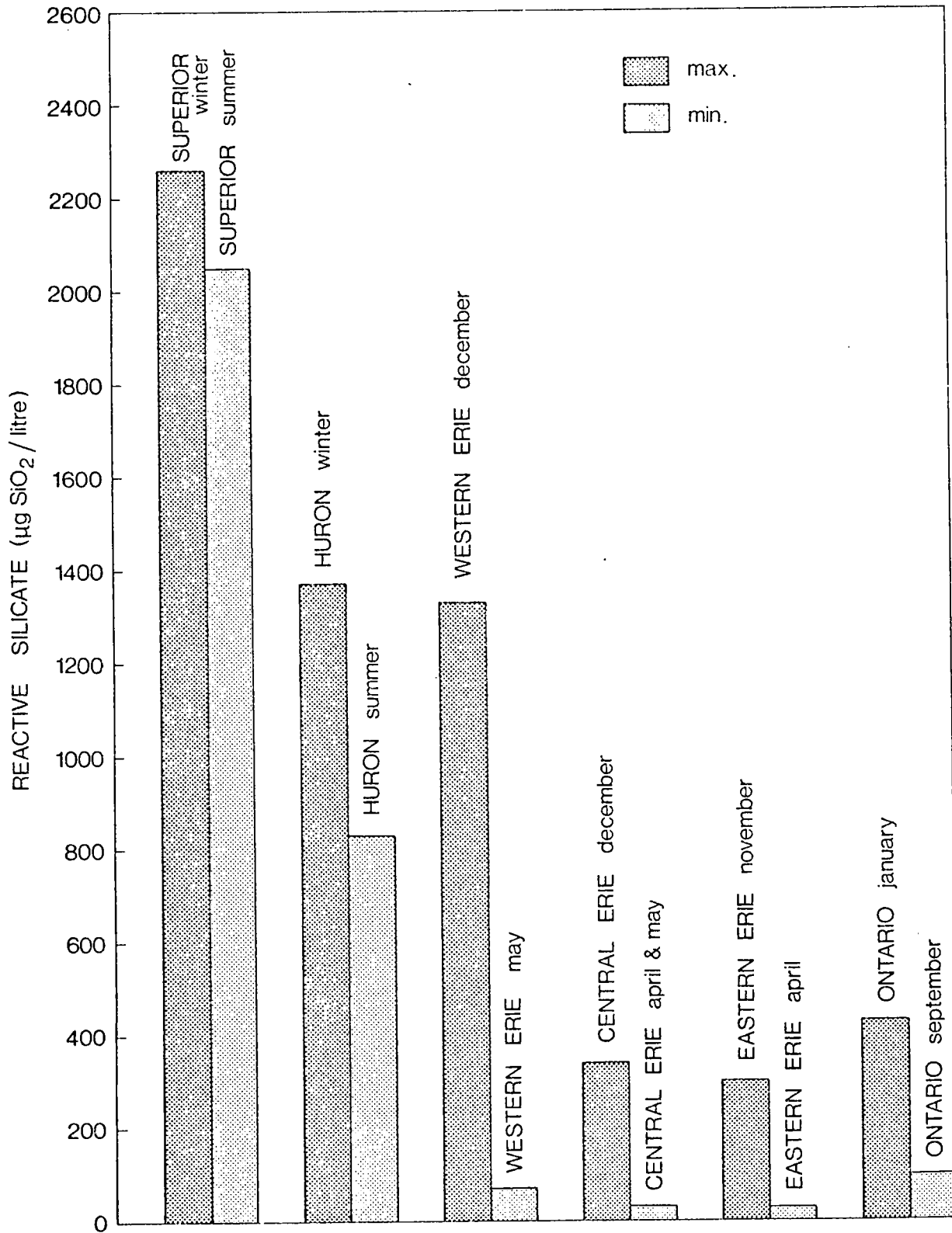
The upper lakes have high silicate concentrations. Lake Michigan may have lower values than Lake Huron but I have not studied Lake Michigan yet.

Lake Erie and Lake Ontario have values near zero suggesting that silicate may sometimes limit diatom growth in the lower lakes.

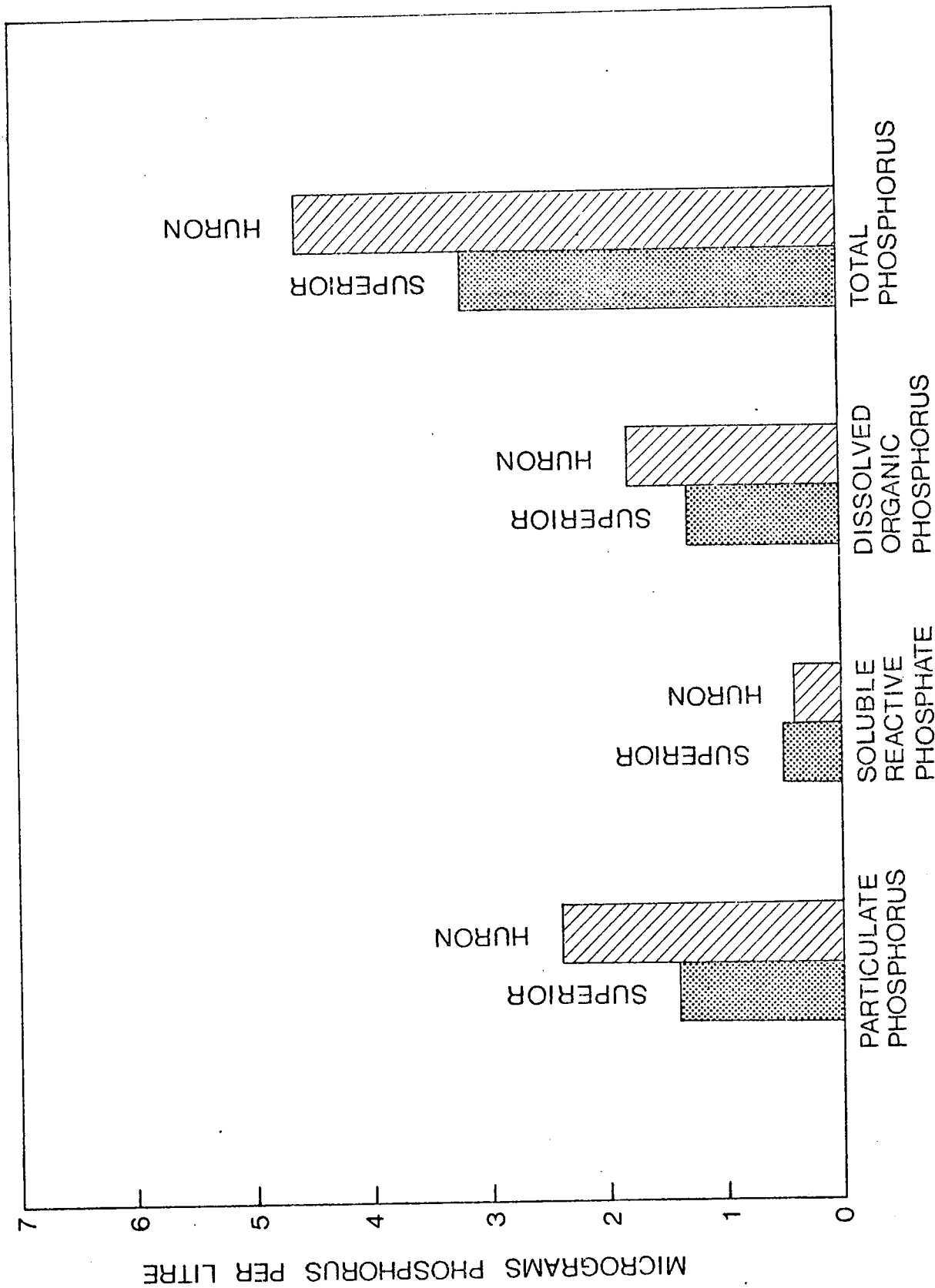
Slide 39. Phosphorus in Lake Superior and Lake Huron



SLIDE 37. LAKE ONTARIO, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES:
UNWEIGHTED MEAN REACTIVE SILICATE CONCENTRATIONS AT A DEPTH OF
1 METRE, AND ALSO IN THE COLD WATER-MASS (T < 5°C), FROM
CRUISES OF THE LIMNOS AND MARTIN KARLSEN DURING 1969.
UNITS ARE MICROGRAMS SiO₂ PER LITRE.



SLIDE 38. SUMMARY OF REACTIVE SILICATE CONCENTRATIONS IN SURFACE WATERS OF THE GREAT LAKES, 1969 - 1971 (FROM CRUISE-MEAN VALUES AT A DEPTH OF 1 METRE IN EACH BASIN).



SLIDE 39. COMPARISON OF THE PHOSPHORUS CONCENTRATIONS IN LAKE SUPERIOR (1970 and 1971) AND LAKE HURON (1971).

NOTE: A PARTICULATE PHOSPHORUS VALUE OF 10 µg P/LITRE CORRESPONDS TO THE LOWER LEVEL FOR EUTROPHY.

Here is a summary of mean values for the various fractions of phosphorus in Lake Superior and Lake Huron. Particulate phosphorus is in the oligotrophic range, less than 5 $\mu\text{g P}/\ell$, with Lake Superior having lower values than Lake Huron. There is almost no dissolved inorganic phosphorus even in deep water or in winter, suggesting that phosphorus is the effective limiting factor in the upper Great Lakes.

Slide 40. Western Lake Erie: Three forms of phosphorus in surface waters

This graph shows for western Lake Erie surface waters the 3 phosphorus fractions in a cumulative way, with particulate phosphorus along the baseline, then dissolved inorganic phosphorus above, and then on top dissolved organic phosphorus.

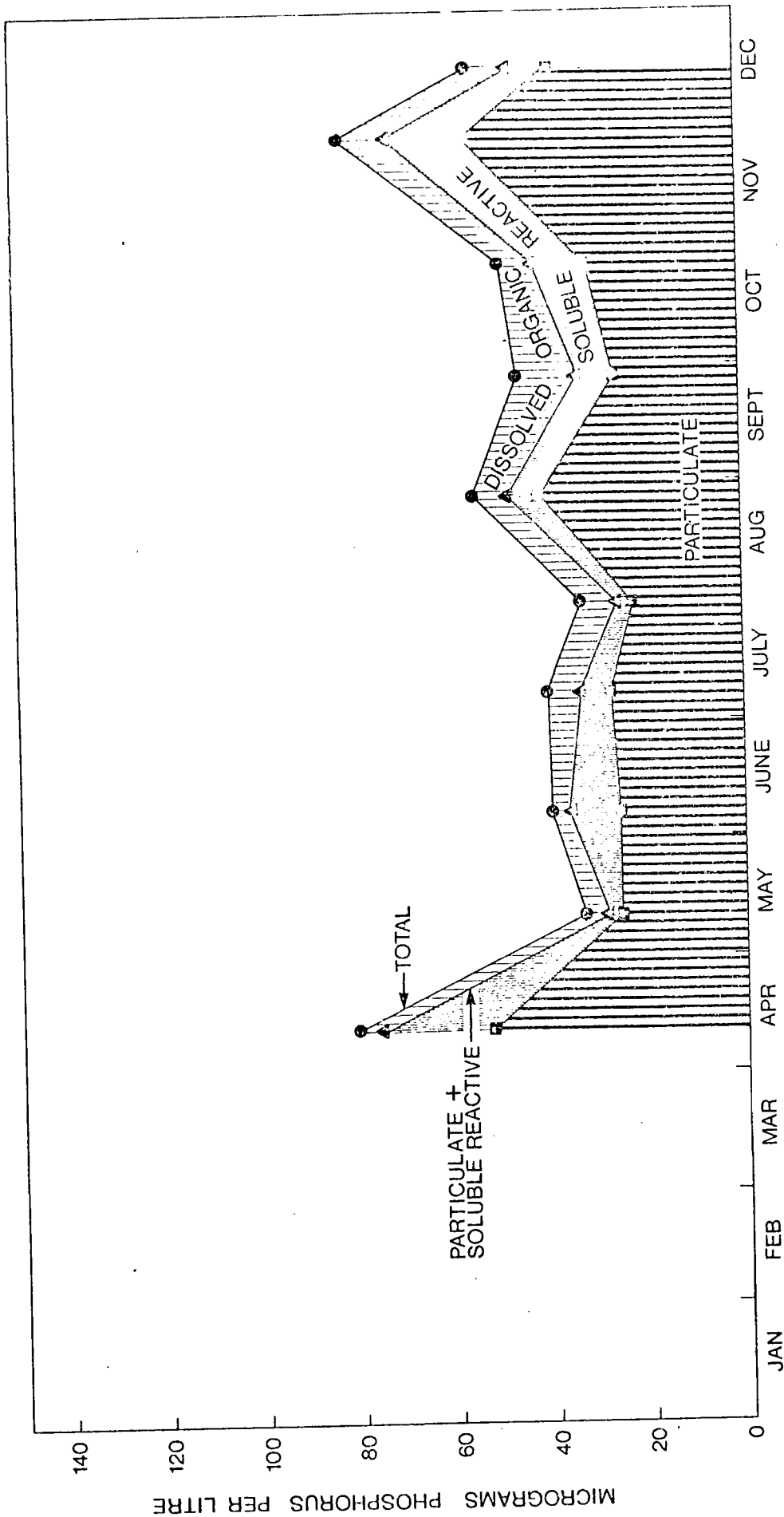
The maximum values for total phosphorus are about 80. $\mu\text{g}/\ell$ and the particulate fraction contains the major portion, being in the range 25 to 55 $\mu\text{g P}/\ell$, which are extremely eutrophic values.

The higher values for particulate phosphorus are perhaps at the times of resuspension of dead plankton from the bottom.

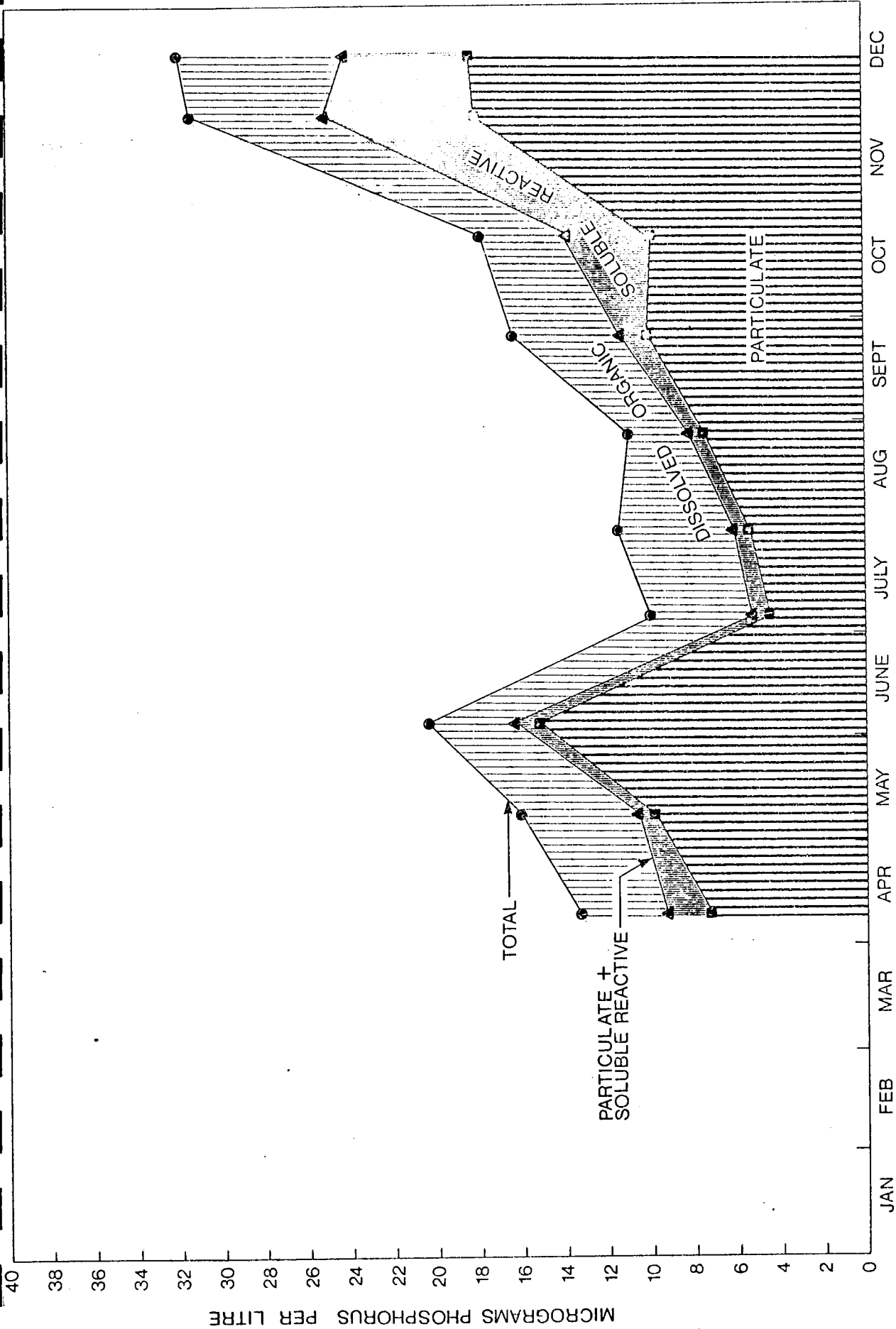
Slide 41. Central Lake Erie: Three forms of phosphorus in surface waters

A similar graph for Central Lake Erie surface waters indicates lower values of particulate phosphorus and dissolved inorganic phosphorus than in Western Lake Erie.

Particulate phosphorus is in the range 4.5 to 18.4 $\mu\text{g P}/\ell$ in Central



SLIDE 40. WESTERN LAKE ERIE: UNWEIGHTED MEAN VALUES FOR THE VARIOUS PHOSPHORUS FRACTIONS AT A DEPTH OF 1 METRE, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.



SLIDE 41. CENTRAL LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN VALUES FOR THE VARIOUS PHOSPHORUS FRACTIONS AT A DEPTH OF 1 METRE, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.

Lake Erie, with the lowest values occurring in July. Central Lake Erie is sometimes mesotrophic and sometimes eutrophic.

In early winter there may be resuspension of detritus, suggested by the high values for particulate phosphorus.

Slide 42. Central Lake Erie: Soluble reactive phosphate in surface waters and bottom waters

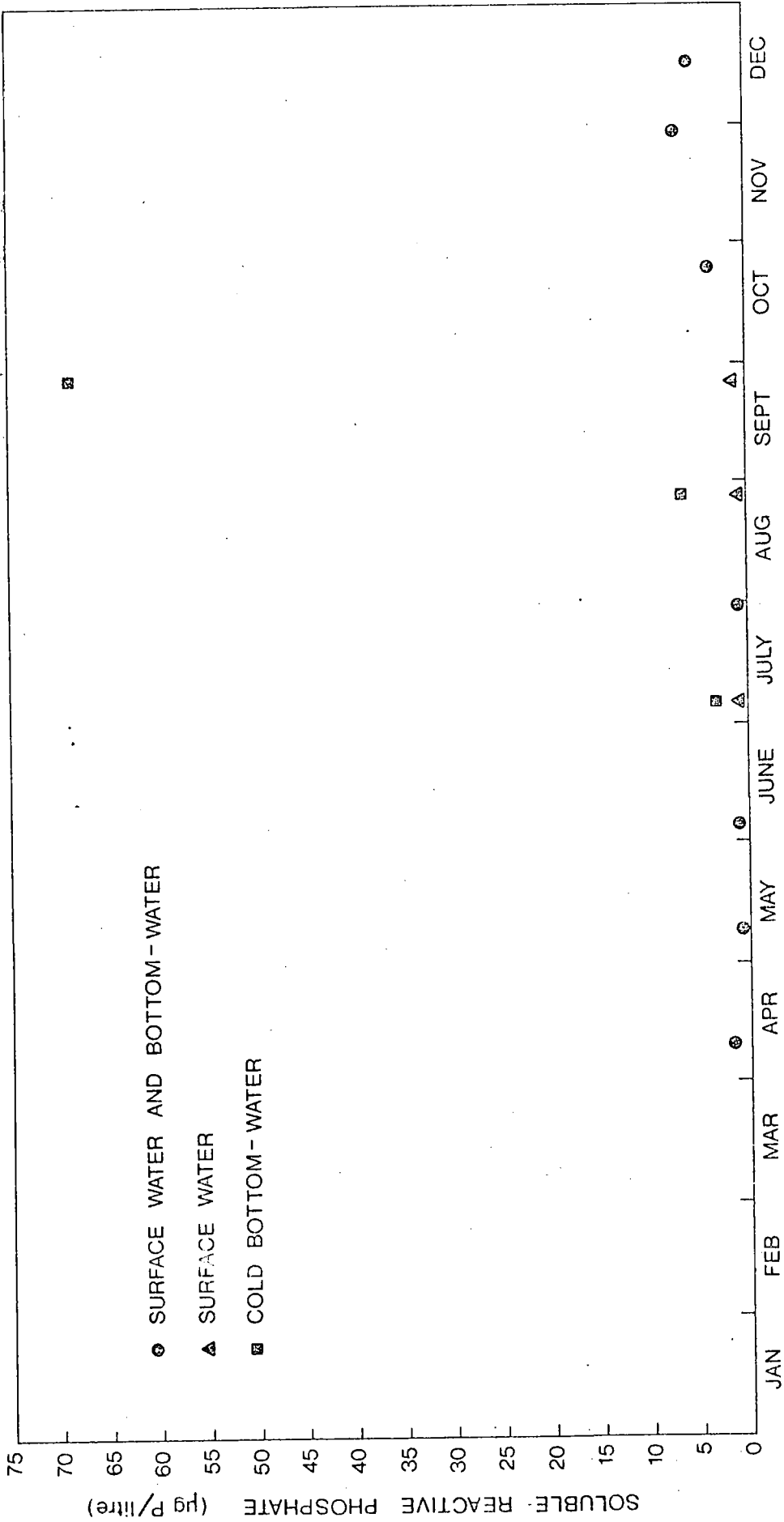
Here is a picture of dissolved inorganic phosphorus in Central Lake Erie surface waters and bottom waters.

A large increase in the concentration in the bottom water occurred in September, when the bottom water had zero oxygen levels. The mean value for dissolved inorganic phosphorus in the bottom water was then about 69 $\mu\text{g P/l}$, which causes all the other points on this graph to appear nearly zero.

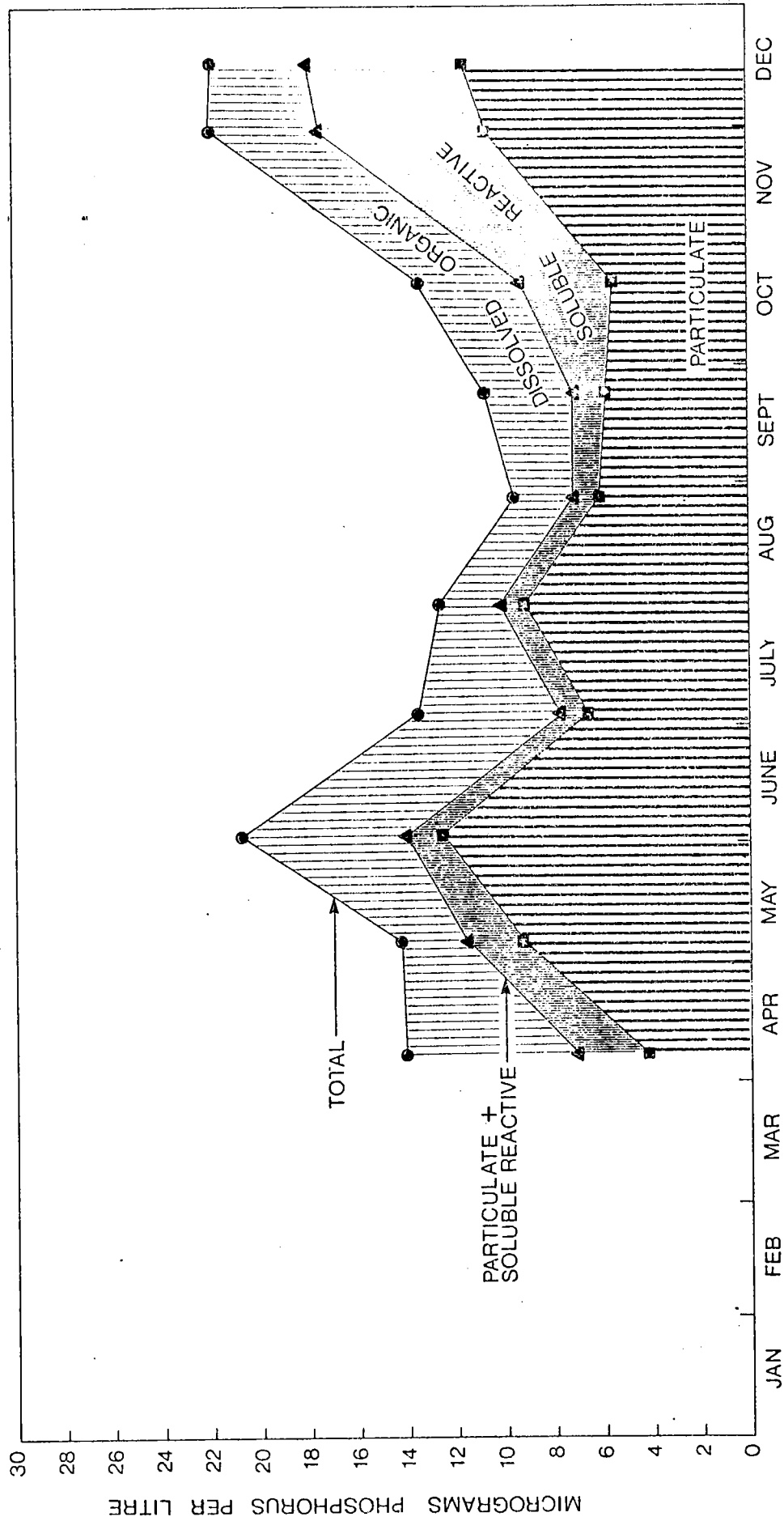
In Central Lake Erie the dissolved inorganic phosphate values after the end of summertime stratification were quite low. The volume of the cold water layer with high phosphate values in September is quite small, and therefore regeneration of phosphate in the bottom water has no great influence on surface waters after mixing.

Slide 43. Eastern Lake Erie: Three forms of phosphorus in surface waters

In Eastern Lake Erie surface waters, particulate phosphorus varies from 4 to 13



SLIDE 42. CENTRAL LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS > 18 METRES:
UNWEIGHTED MEAN SOLUBLE REACTIVE PHOSPHATE CONCENTRATIONS AT A
DEPTH OF 1 METRE, AND ALSO IN THE COLD BOTTOM-WATER, FROM CRUISES
OF THE MARTIN KARLSEN DURING 1970.
UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.



SLIDE 43. EASTERN LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN VALUES FOR THE VARIOUS PHOSPHORUS FRACTIONS AT A DEPTH OF 1 METRE, FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.

$\mu\text{g P/l}$, mostly in the mesotrophic range. Summer values are about 6 or 7 $\mu\text{g P/l}$. About half of the total phosphorus is in particulate form.

Slide 44. Eastern Lake Erie: Soluble reactive phosphate in surface waters and bottom waters

In Eastern Lake Erie there is depletion of dissolved inorganic phosphorus at all depths during springtime before thermal stratification begins.

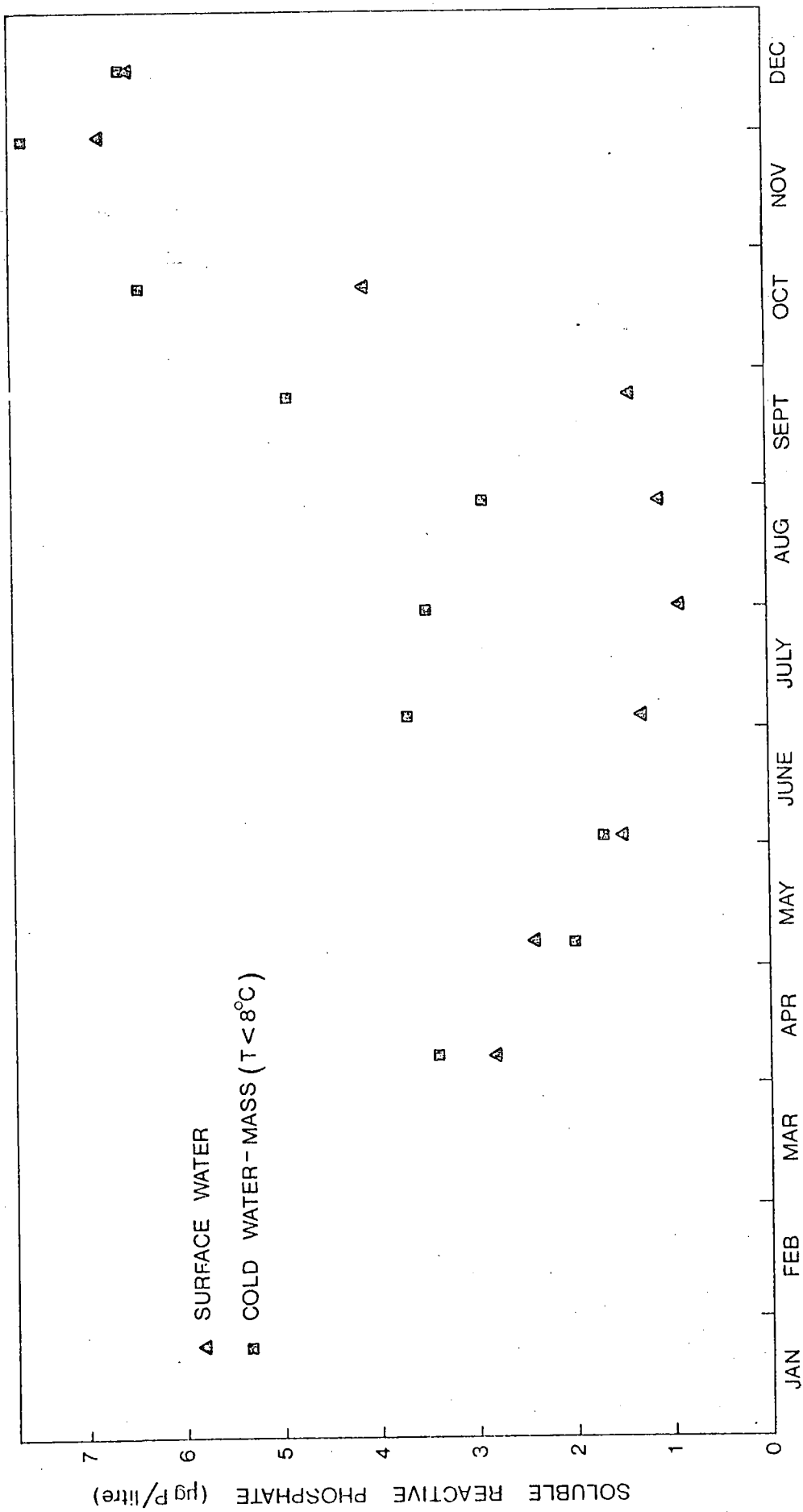
In summer there is regeneration in bottom waters lasting into November.

In autumn the increase in the inorganic phosphate values of surface waters is at least partly the result of mixing with deeper water as the thermocline gradually descends.

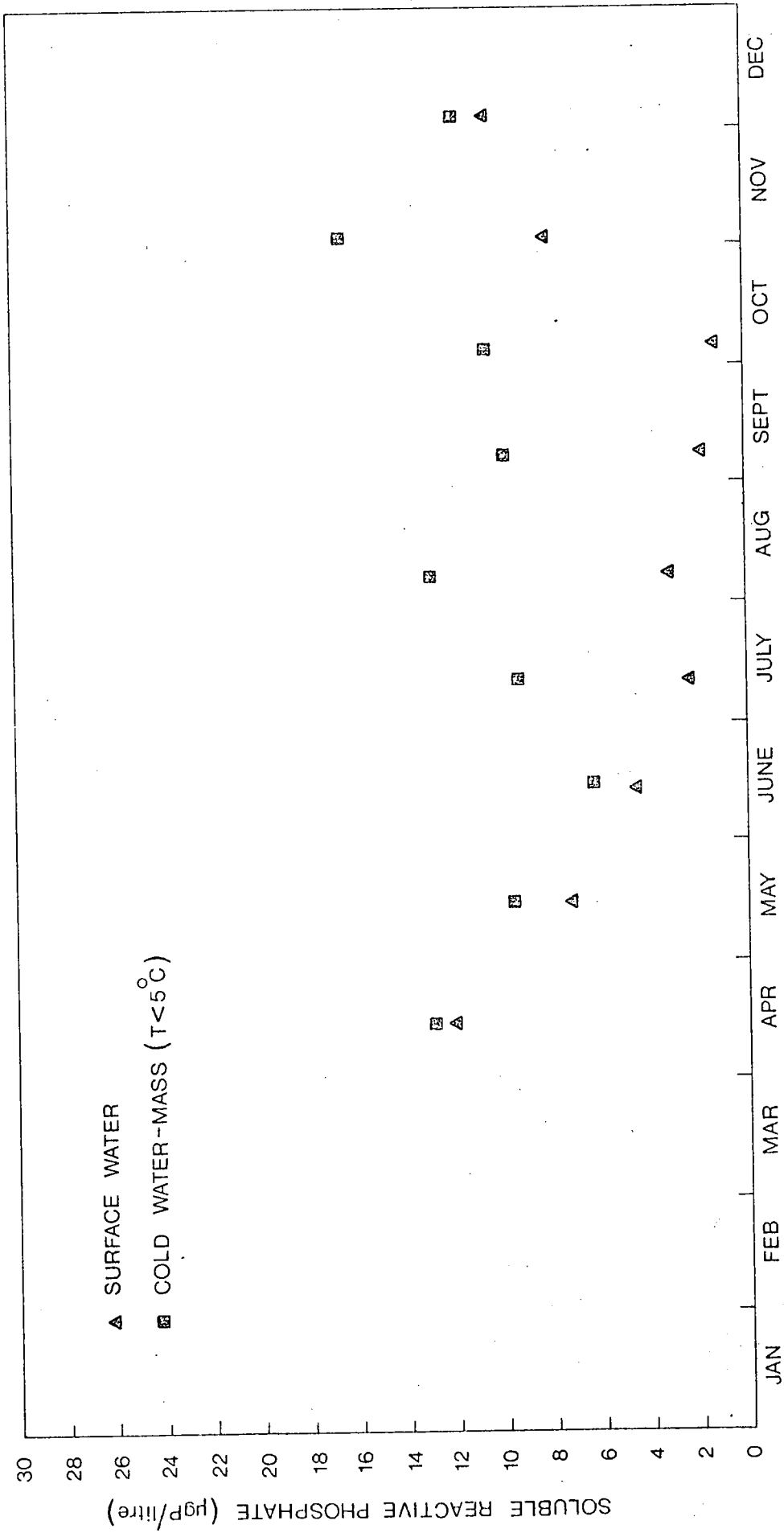
Slide 45. Lake Ontario: Soluble reactive phosphate in surface waters and bottom waters

For Lake Ontario the only phosphorus fraction that has been measured throughout one year is dissolved inorganic phosphorus, so I cannot show a cumulative graph as shown for Lake Erie.

In Lake Ontario, inorganic phosphate is depleted in surface waters and in bottom waters during April, May and June when much of the lake is still unstratified. This seems surprising to me, that algae growing at the surface could influence this deep lake from top to bottom. In summer there is more



SLIDE 44. EASTERN LAKE ERIE, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES: UNWEIGHTED MEAN SOLUBLE REACTIVE PHOSPHATE CONCENTRATIONS AT A DEPTH OF 1 METRE, AND ALSO IN THE COLD WATER-MASS (T < 8°C), FROM CRUISES OF THE MARTIN KARLSEN DURING 1970. UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.



SLIDE 45. LAKE ONTARIO, OFFSHORE PART WHERE THE SOUNDING IS >18 METRES:
UNWEIGHTED MEAN SOLUBLE REACTIVE PHOSPHATE CONCENTRATIONS AT A
DEPTH OF 1 METRE, AND ALSO IN THE COLD WATER-MASS (T < 5°C),
FROM CRUISES OF THE LIMNOS AND MARTIN KARLSEN DURING 1969.
UNITS ARE MICROGRAMS PHOSPHORUS PER LITRE.

gradual depletion of surface waters and some regeneration in bottom waters.

In autumn the mixing of surface waters with bottom waters occurs gradually. The mixture in December has inorganic phosphate values near 11. $\mu\text{g P}/\ell$.

Slide 46. Particulate phosphorus in Great Lakes surface waters.

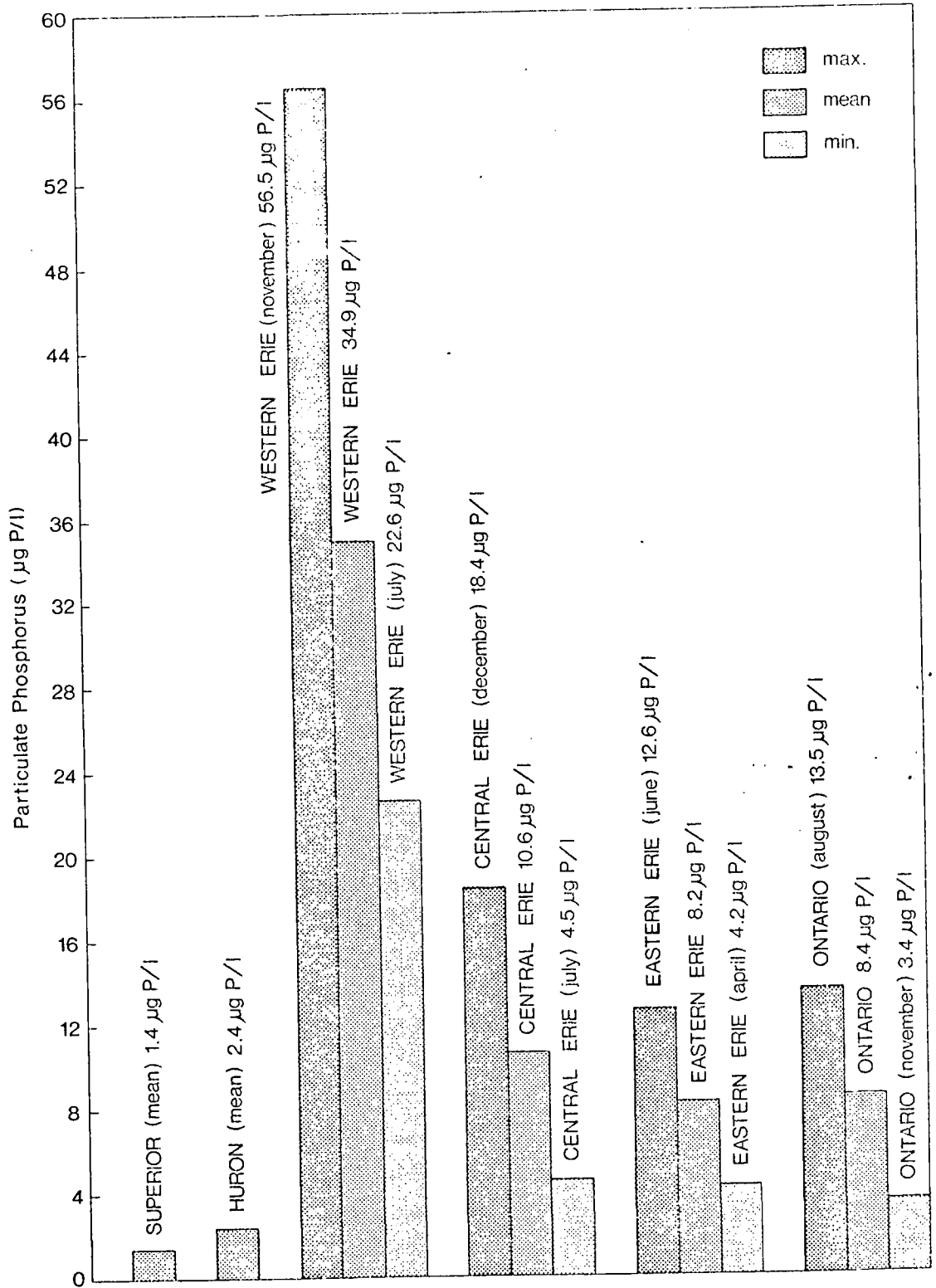
This is a summary of particulate phosphorus values in surface waters of the four lakes, based on basin-wide mean values on each cruise. The bars indicate maximum values, annual mean values, and minimum values.

The mesotrophic range is 5 to 10 $\mu\text{g P}/\ell$. Lake Superior and Lake Huron are oligotrophic or poorly fertilized. Western Lake Erie is extremely eutrophic or over fertilized. Central Erie and Eastern Erie and Lake Ontario are sometimes of the year mesotrophic, sometimes slightly oligotrophic and sometimes eutrophic.

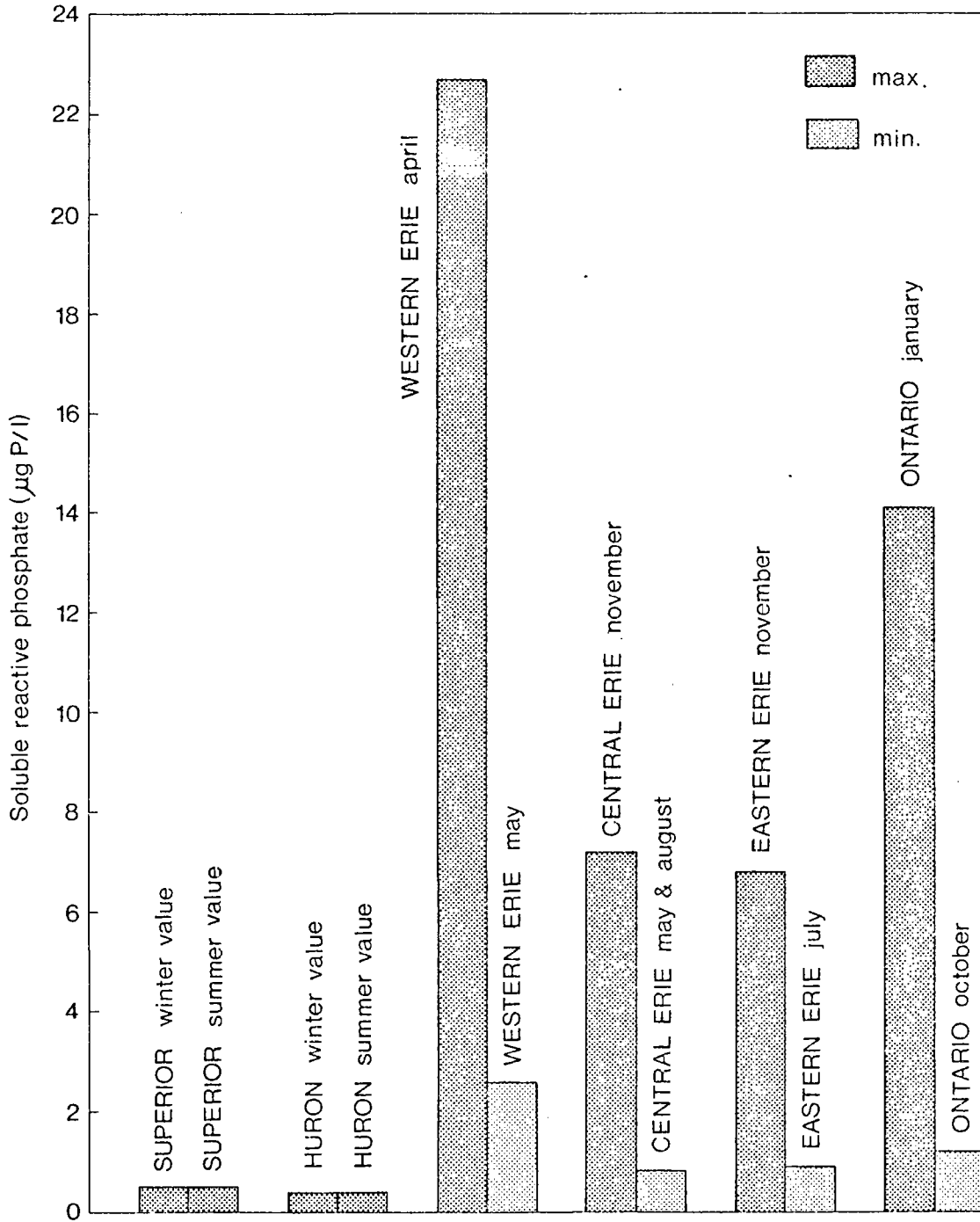
Slide 47. Soluble reactive phosphate in Great Lakes surface waters

This bar-graph shows the dissolved inorganic phosphorus values of surface waters in the four lakes. For each basin the left bar indicates the winter maximum and the right bar indicates the summer minimum.

Winter values and summer values are very low in Lake Superior and Lake Huron. Summer values are low in Lake Erie and Lake Ontario.



SLIDE 46. MEAN AND EXTREME PARTICULATE PHOSPHORUS CONCENTRATIONS IN SURFACE WATERS OF THE GREAT LAKES, 1970-1971, BASED ON CRUISE-MEAN VALUES AT A DEPTH OF 1 METRE IN EACH BASIN.



SLIDE 47. SUMMARY OF SOLUBLE REACTIVE PHOSPHATE CONCENTRATIONS IN SURFACE WATERS OF THE GREAT LAKES, 1969-1971 (FROM CRUISE-MEAN VALUES AT A DEPTH OF 1 METRE IN EACH BASIN).

Western Lake Erie and Lake Ontario have the highest winter values.

Inorganic phosphate is the only major nutrient having low values year-round in Lake Huron and Lake Superior, where it is probably the main limiting factor for plankton.

In Lake Erie and Lake Ontario, inorganic phosphate is near zero in summer which suggests that it is a limiting factor although perhaps not the only one. Reductions of the phosphorus loading to Lake Erie and Lake Ontario will reduce the concentrations of plankton.

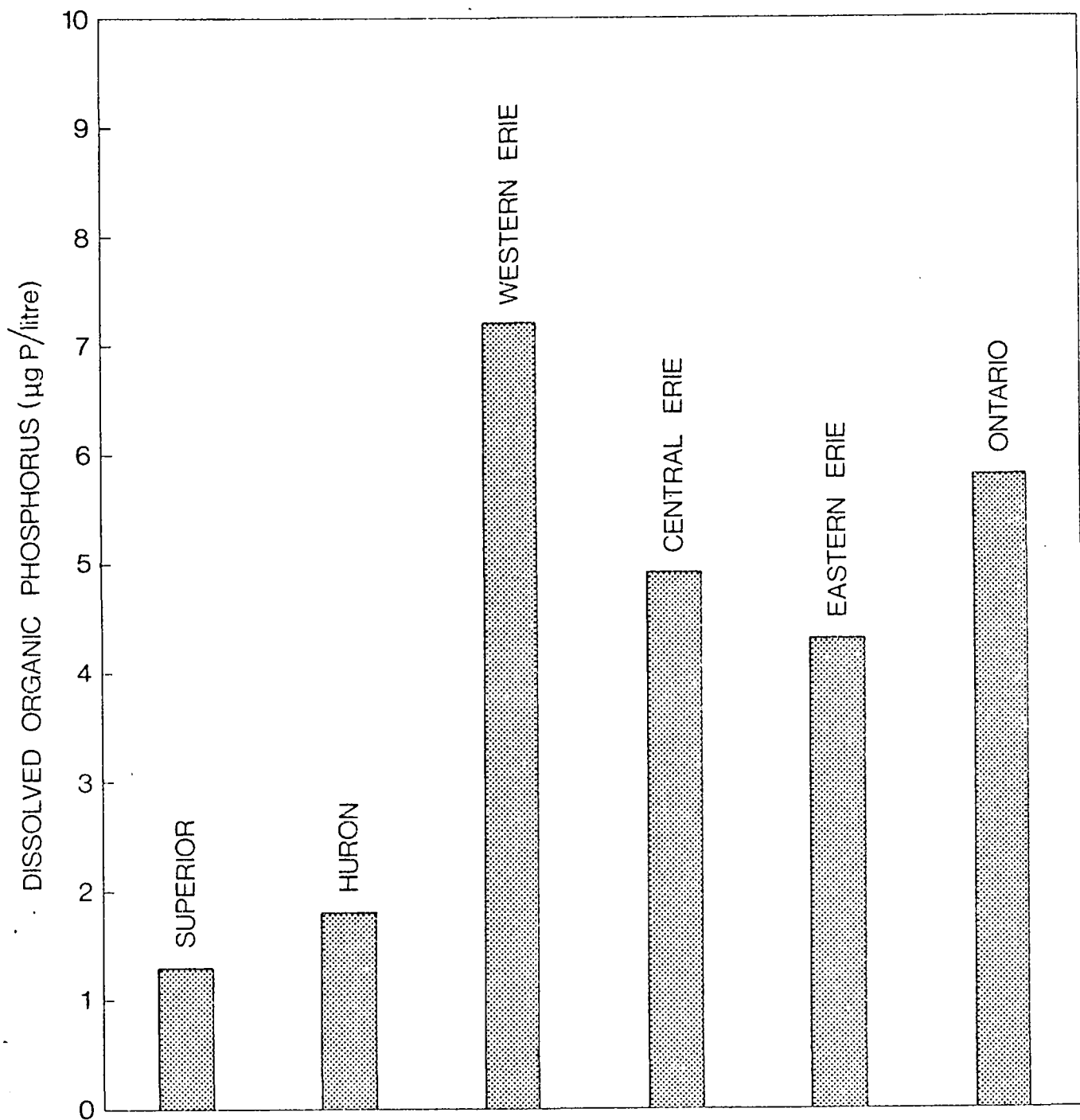
Slide 48. Dissolved organic phosphorus in Great Lakes surface waters

Dissolved organic phosphorus does not show as great a range among the four lakes as particulate phosphorus or dissolved inorganic phosphorus, although dissolved organic phosphorus is lowest in Lake Superior and Lake Huron.

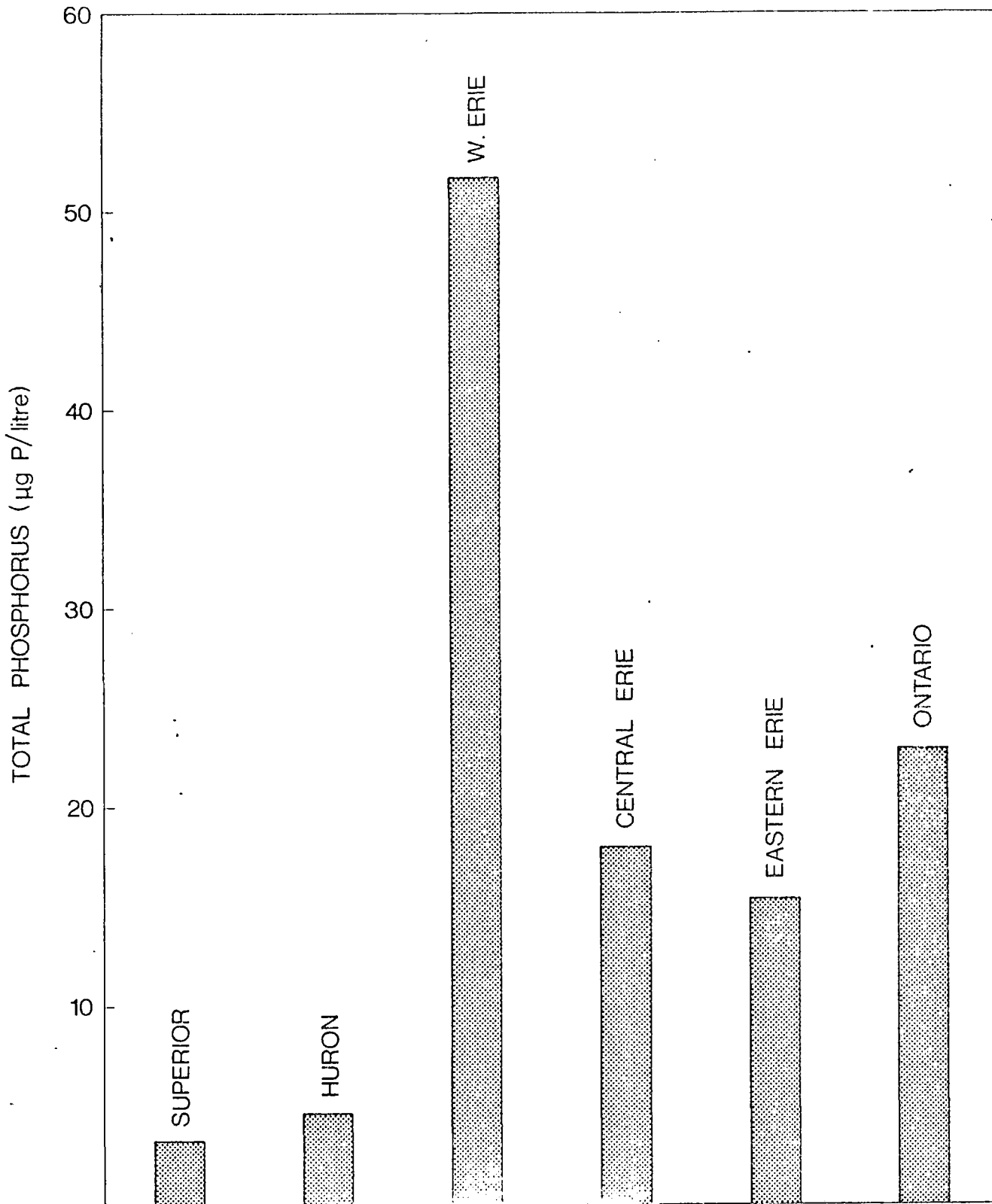
Perhaps dissolved organic phosphorus is partly end-products of plankton growth, which bacterial decay doesn't degrade easily. Perhaps the concentration of these end products is a reflection of the standing stock of plankton and the residence time of each lake.

Slide 49. Total phosphorus in Great Lakes surface waters.

Here is a bar-graph comparing total phosphorus concentrations in the four lakes. This is a quantity related to the phosphorus loading, water



SLIDE 48. ANNUAL MEAN DISSOLVED ORGANIC PHOSPHORUS CONCENTRATIONS IN SURFACE WATERS OF THE GREAT LAKES, 1970-1971.



SLIDE 49. ANNUAL MEAN TOTAL PHOSPHORUS CONCENTRATIONS (EXCLUDING PHOSPHORUS IN FISH) IN SURFACE WATERS OF THE GREAT LAKES, 1969-1971.

outflow, and net sedimentation within each lake.

Now we will look at the magnitude of these various fluxes for Lake Erie and Lake Ontario.

Slide 50. Net sedimentation of phosphorus in Lake Erie and Lake Ontario

Net sedimentation rate for phosphorus is found here by solving the steady state equation, given external loading, outflow of water, and concentration of phosphorus in the outflow.

There is 90% retention of phosphorus in the sediments of Lake Erie, and 61% retention in Lake Ontario. Direct estimates of phosphorus sedimentation, by Dr. Kemp and Dr. Lewis, gave somewhat lower but still quite similar sedimentation rates.

I have no slides illustrating carbon, but a brief statement can be given. The bicarbonate concentration of Lake Superior is about 50. milligrams HCO_3 per litre, equals about 10. milligrams carbon per litre, which would allow a potential standing stock of particulate phosphorus of about

$$\frac{10,000}{130} = 77 \text{ } \mu\text{g P/l}$$

Slide 50

Phosphorus in Lake Erie and Lake Ontario,
1967: Solving for the net sedimentation term
in the equation:

$$\text{concentration in outflow} = \frac{(\text{outside loading minus net sedimentation})}{\text{outflow of water}}$$

	Lake Erie	Lake Ontario
Outside loading (Kg P/yr)	27.3×10^6	12.4×10^6
Outflow of phosphorus (Kg P/yr)	2.6×10^6	4.8×10^6
Net sedimentation (Kg P/yr)	24.7×10^6	7.6×10^6
% retention	90%	61%

or 8 times the lower limit for eutrophy, whereas in fact Lake Superior is oligotrophic with particulate phosphorus at $1.4 \mu\text{g P/l}$. Carbon cannot be a growth limiting factor in the Great Lakes.

Now I will conclude with a brief general statement. Depletion of dissolved nutrients in surface waters is related to thermal stratification which isolates the surface waters from bottom waters during summer. Due to the sinking of plankton, the process of nutrient regeneration tends to occur mostly deep in the lake, below the thermocline where it cannot immediately affect the surface waters. Assimilation of nutrients, rather than regeneration, dominates in the surface waters in summer, and therefore nutrient depletion of surface waters occurs. As winter, in contrast, the water column is well-mixed vertically and the regeneration of nutrients at any depth can affect the surface waters by turbulent mixing.

Another possibly-influential factor is the light regime. Surface light is greatest in June. Also, the amount of light in surface waters is influenced by the mixed surface layer thickness, which is least in early summer and gradually increases as the thermocline deepens. A thermocline increases the light in the surface layer because the mean depth in the lake of each algal cell is reduced.

The thermocline tends to isolate the assimilation of nutrients from their regeneration, and light is increased in summer: both of these effects favor depletion of inorganic nutrients from surface waters during summer.

At other times of year, perhaps light is an influential factor limiting production of algae. Light becomes more important with large concentrations of algae which become self-shading.

An increase in the phosphorus loading to a lake, and an increase in the winter and spring concentrations of inorganic phosphorus, result in more plant production in early summer. Whether this continues through the summer depends on retention of phosphorus in the surface layers. It may instead be lost to the bottom waters by sedimentation of the plankton.

Plant production causes depletion of silicate and nitrate in surface waters in summer, and with enough permanent sedimentation of these constituents, the winter and spring values for silicate and nitrate may decrease over the years.

The effect of silicate and nitrate depletion in summer is the partial replacement of diatoms and green algae by the blue-green algae which can obtain their nitrogen from dissolved nitrogen gas. Fertilization with phosphorus causes a proportional increase in summer-time standing stock of plankton and a change in the type of algae occurring during summer. Perhaps only phosphorus is the effective limiting factor for the standing-stock of blue-green algae, until they become self-shading.

Phosphorus is apparently limiting the standing stock of plankton in Lake Superior and Lake Huron.

In the lower Great Lakes, Lake Erie and Lake Ontario, the inorganic nutrients phosphate, nitrate and silicate are all depleted to near zero during summer, and therefore shortages of all three are significant, but blue-green algae perhaps do not need silicate or nitrate so then phosphorus remains the likely key element in the lower Great Lakes as well. Phosphorus control is a wise choice for managing the quantity of plankton in the Great Lakes.

Appendix.

A note on the allowable standing stock to maintain oxygen in the hypolimnion of a lake above 50% saturation at the end of the stratification period.

Relationships:

$$\begin{aligned} & \text{Areal depletion rate (mg O}_2\text{/cm}^2\text{/month)} \\ &= 0.1 \times \text{hypolimnion thickness (metres)} \\ & \quad \times \text{ordinary depletion rate (mg O}_2\text{/l/mo.)} \end{aligned}$$

$$\begin{aligned} & \text{Particulate P (}\mu\text{g P/l)} \times 0.23 \\ &= \text{Areal depletion rate (mg O}_2\text{/cm}^2\text{/mo.)} \\ & \text{[observed in lower Great Lakes]} \end{aligned}$$

Then

$$\begin{aligned} & \text{Particulate P (}\mu\text{g P/l)} \\ &= 0.435 \times \text{hypolimnion thickness (metres)} \\ & \quad \times \text{ordinary depletion rate (mg O}_2\text{/cm}^2\text{/month)}. \end{aligned}$$

Now the allowable oxygen loss is, say,

$$\begin{aligned} & 5.75 \text{ mg } O_2/\ell/5 \text{ months} \\ & = 1.15 \text{ mg}/\ell/\text{month} \end{aligned}$$

Then

$$\begin{aligned} \text{Particulate P } (\mu\text{g P}/\ell) & \\ & = 0.435 \times 1.15 \times \text{hypolimnion thickness (metres)} \\ & = 0.5 \times \text{hypolimnion thickness (metres)} \end{aligned}$$

Some results:

hypolimnion thickness (m)	allowable standing stock ($\mu\text{g P}/\ell$)
1	0.5
4 (Central Erie)	2.0
10	5.0
20 (Eastern Erie)	10.0
40	20
60 (Ontario)	30

] mesotrophy