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THE NUMERIC WEIGHTING OF ENVIRONMENTAL INTERACTIONS

BY

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FOREWORD

The difficulty in choosing between alternative courses of action is directly related to the amount of information pertaining to each alternative. This difficulty is especially acute in environmental problems, where facts and interrelationships usually have levels of importance that cannot be determined by the relatively simple economic rules generally employed by decision makers. The development of a body of techniques capable of establishing the importance of discrete environmental parameters is critical to the evolution of methodologies for the assessment of environmental alternatives.

This paper proposes a numeric method designed to aid decision makers in determining the importance of individual pieces of environmental information, and suggests a framework for the combination of these data into single quantitative measures of the environmental disruption of each of the alternatives examined. As such, the paper represents another step towards a theory of rational environmental use.

R.J. McCormack
Director General
Lands Directorate

AVANT-PROPOS

La difficulté à fixer son choix parmi toute une gamme de partis possibles est directement liée à la quantité d'information qui s'y rattache. Cette difficulté est particulièrement sérieuse dans le cas de l'environnement où les faits et les relations ont un degré d'importance qu'il est impossible de déterminer par les règles économiques relativement simples auxquelles les technocrates ont souvent recours. La mise au point d'un ensemble de techniques qui permettent d'établir l'importance de paramètres discrets est capitale pour l' évolution des méthodes d'évaluation des choix.

La présente étude propose une méthode numérique qui aidera les technocrates à déterminer l'importance d'éléments d'information isolés et suggère un cadre de travail pour la combinaison de ces données en mesures quantitatives simples des incidences environnementales de chacun des partis examinés. Cette étude représente ainsi une autre étape vers une théorie de l'utilisation rationnelle du milieu natural.

R.J. McCormack

Directeur général

Direction générale des terres

ABSTRACT

This paper sets forth a numeric methodology that uses the technique of non-metric multidimensional scaling to determine interval measures of the importance of environmental component interdependencies and then combines these to yield numeric indices of environmental impact.

The proposed methodology is applied to a real-world environmental impact assessment project. The results are found to be in very close agreement with those of a study group charged with performing the same task by conventional methods of impact assessment.

SOMMAIRE

La méthode numérique exposée dans cette étude fait appel à la technique de l'échelle non métrique multidimensionnelle pour déterminer les mesures par intervalle de l'importance de l'interaction des composants environnementaux; leur combinaison produit ensuite les indices numériques des incidences sur le milieu.

La méthode proposée est appliquée à un projet d'évaluation des incidences environnementales dans un milieu réel. Les résultats concordent étroitement avec ceux d'un groupe d'étude chargé d'effectuer les mêmes tâches d'évaluation au moyen de méthodes classiques.

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ACKNOWLEDGEMENTS

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PREAMBLE

Traditional approaches to the problem of assessing the environmental impact of alternative development options (see, for example, Leopold et. al., 1971) have not evolved rigorous methodologies for evaluating the myriad environmental parameters involved. The chief problems have been encountered in the identification of impacts beyond the first order, the derivation of importance weights for each impact and the design of a technique suitable for combining such weights to form single numeric indices of environmental impact. A previous paper (Ross, 1974 b) has dealt with the first of these problems. The purposes of this paper are to present a method for determining the importance of discrete environmental impacts, and to suggest a technique that may be used to combine these weights to form numeric impact indices.

In an earlier paper, the author has described a component dependency matrix, which identifies first order, or direct, interactions between discrete components of the environment (Ross, 1974 a, Appendix V, pp. 3-8). This matrix (Figure 1), in which a "1" in any cell denotes the dependency of the row component on the column component, is structured in such a way that second and higher orders of interaction may be extracted simply by powering the matrix. In the same paper, the author presents a set of environmental disruption matrices that record ordinal measures of the degree to which each of the environmental dependencies mentioned above would be disrupted by each of the proposed developments (Ross, 1974 a, Appendix V, pp. 9-11). In this latter matrix, (Figure 2) the measures of environmental disruption are expressed ordinally, that is, they cannot be added as interval measures could be. This is its chief weakness.

Figure 1

NANAIMO COMPONENT INTERACTION MATRIX

		CURRENTS 1	WIND 2	WATER TEMPERATURE 3	LIGHT 4	INTERTIDAL VEGETATION 5	UPLAND VEGETATION 6	BACTERIA 7	INSECTS 8	LARVAE 9	SHELL FISH 10	CRABS 11	OTHER CRUSTACEANS 12	PELAGIC FISH 13	BOTTOM FISH 14	WATERBIRDS 15	BIRDS OF PREY 16	SONG BIRDS 17	MARSH & SHORE BIRDS 18	UPLAND GAME BIRDS 19	AQUATIC & MARINE MAMMALS 20	UPLAND MAMMALS 21
CURRENTS	1		1																			
WIND	2						1															
WATER TEMPERATURE	3	1	1		1																	
LIGHT	4																					
INTERTIDAL VEGETATION	5	1		1	1																	
UPLAND VEGETATION	6				1			1	1									1	1	1		1
BACTERIA	7			1		1	1		1	1	1	1	1	1	1	1	1	1	1	1	1	1
INSECTS	8	1	1	1		1	1	1														
LARVAE	9	1		1		1		1			1	1	1	1	1							
SHELL FISH	10	1		1		1		1														
CRABS	11			1		1				1	1		1	1	1							
OTHER CRUSTACEANS	12			1		1		1	1	1	1	1		1	1							
PELAGIC FISH	13			1					1	1												
BOTTOM FISH	14			1		1			1	1	1	1	1	1								
WATERBIRDS	15					1			1	1	1	1	1	1	1							
BIRDS OF PREY	16						1		1			1	1	1	1	1		1	1	1		1
SONG BIRDS	17						1		1		1		1	1	1							
MARSH & SHORE BIRDS	18						1		1		1	1	1	1	1							
UPLAND GAME BIRDS	19						1		1													
AQUATIC & MARINE MAMMALS	20										1	1	1	1	1							
UPLAND MAMMALS	21						1				1	1	1					1		1		

Note: A(1) in any cell indicates that the row component is dependent on the column component.

Figure 2
NANAIMO DISRUPTION MATRICES

		CURRENTS 1	WIND 2	WATER TEMPERATURE 3	LIGHT 4	INTERTIDAL VEGETATION 5	UPLAND VEGETATION 6	BACTERIA 7	INSECTS 8	LARVAE 9	SHELL FISH 10	CRABS 11	OTHER CRUSTACEANS 12	PELAGIC FISH 13	BOTTOM FISH 14	WATERBIRDS 15	BIRDS OF PREY 16	SONG BIRDS 17	MARSH & SHORE BIRDS 18	UPLAND GAME BIRDS 19	AQUATIC & MARINE MAMMALS 20	UPLAND MAMMALS 21
CURRENTS	1		0 0 0 / 0 0																			
WIND	2						0 1 1 / 1 1															
WATER TEMPERATURE	3	1 1 1 / 1 1	2 1 1 / 1 1		3 1 1 / 2 1																	
LIGHT	4																					
INTERTIDAL VEGETATION	5	3 1 2 / 2 1		1 1 1 / 1 1	3 3 3 / 3 3																	
UPLAND VEGETATION	6				0 3 3 / 3 3			0 1 1 / 1 1	0 1 1 / 1 1									0 1 1 / 1 1	0 1 1 / 1 1	0 1 1 / 1 1		0 1 1 / 1 1
BACTERIA	7			1 1 1 / 1 1		2 2 2 / 2 1	0 2 2 / 2 2		1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	1 1 1 / 1 1	0 1 1 / 1 1	0 1 1 / 1 1	1 1 1 / 1 1	0 1 1 / 1 1	1 1 1 / 1 1	0 1 1 / 1 1
INSECTS	8	1 0 0 / 0 0	1 1 1 / 1 1	1 1 1 / 1 1		1 1 1 / 1 1	0 1 1 / 1 1	1 1 1 / 1 1														
LARVAE	9	2 2 2 / 2 2		2 2 2 / 2 2		3 3 3 / 3 3		2 2 2 / 2 2			2 2 2 / 2 2	2 1 2 / 2 0	2 2 2 / 2 2	2 2 2 / 2 2	2 2 2 / 2 2							
SHELL FISH	10	2 2 2 / 2 2		2 2 2 / 2 2		3 3 3 / 3 3		2 2 2 / 2 2														
CRABS	11			2 0 2 / 2 0		2 0 2 / 2 0				2 0 2 / 2 0	2 0 1 / 2 0		2 0 2 / 2 0	1 0 1 / 1 0	2 0 1 / 2 0							
OTHER CRUSTACEANS	12			2 2 2 / 2 2		2 2 2 / 2 2		1 1 1 / 1 1	1 1 1 / 1 1	2 1 1 / 2 1	2 2 2 / 2 2	1 0 1 / 1 0		1 1 1 / 1 1	1 1 1 / 1 1							
PELAGIC FISH	13			2 2 2 / 2 2					2 2 2 / 2 2	3 3 3 / 3 3												
BOTTOM FISH	14			2 2 2 / 2 2		2 2 2 / 2 2			2 2 2 / 2 2	3 3 3 / 3 3	1 0 1 / 1 0	1 0 1 / 1 0	3 3 3 / 3 3	1 2 2 / 2 2								
WATERBIRDS	15					3 1 1 / 2 1			1 0 0 / 1 0	2 0 1 / 2 0	3 1 1 / 2 0	3 0 1 / 2 0	3 1 1 / 2 0	2 1 1 / 2 1	2 1 1 / 2 0							
BIRDS OF PREY	16						0 3 3 / 3 3		0 1 1 / 1 1			1 0 0 / 1 0	1 0 1 / 1 0	1 1 1 / 1 1	1 0 1 / 1 0	2 1 1 / 2 1		0 2 2 / 2 2	0 0 1 / 1 0	0 1 1 / 1 1		0 2 2 / 2 2
SONG BIRDS	17						0 3 3 / 3 2		1 3 3 / 3 2		1 0 1 / 1 0		2 1 2 / 3 1	1 1 / 1 0	1 1 1 / 1 1							
MARSH & SHORE BIRDS	18						0 2 2 / 2 2		0 1 1 / 1 0		1 0 1 / 1 0	1 0 1 / 1 0	2 2 2 / 2 2	1 1 1 / 1 1	3 1 3 / 3 1							
UPLAND GAME BIRDS	19						0 3 3 / 3 3		0 1 1 / 1 1													
AQUATIC & MARINE MAMMALS	20										1 1 1 / 1 0	2 0 2 / 2 0	2 2 2 / 2 2	2 2 2 / 2 2	2 2 2 / 2 2							
UPLAND MAMMALS	21						0 3 3 / 3 3				0 2 3 / 2 2	0 0 1 / 1 0	0 3 3 / 3 3					0 1 1 / 1 1		0 1 1 / 1 1		

ALTERNATIVE 1 5 3 / 2 4

ALTERNATIVE 1 = INNER HARBOUR
ALTERNATIVE 2 = JACK POINT
ALTERNATIVE 3 = DUKE POINT (A)
ALTERNATIVE 4 = HARMAC SOUTH
ALTERNATIVE 5 = DUKE POINT (B)

DISRUPTION LEVEL

0 — NO NOTICEABLE DISRUPTION
1 — SLIGHT DISRUPTION
2 — APPRECIABLE DISRUPTION
3 — SEVERE DISRUPTION

The crux of the matter, obviously, is the development of a technique that would enable one to weight and summarize the information contained in the component interaction and environmental disruption matrices. Subsequent sections of this paper deal with a tentative methodology for weighting and summarizing environmental disruption, and present an application of the proposed techniques to a real-world problem of environmental assessment.

PART A -- METHODOLOGY

The body of theory from which the following techniques are taken can be mainly classified as psychometrics, a field that has long been concerned with the measurement of items generally held to be immeasurable. Although an attempt has been made to explain the necessary techniques in simple terms, some readers may experience considerable difficulty with a few of the underlying concepts. Those having problems are advised to refer to a general text on the topic (e.g. Moroney, 1968, Chapter 18) for further clarification.

The Evaluation of 'Intangibles'

Arranging objects or relationships in order of importance or quality, can be especially arduous if judgements are to be based on dimensions, or qualities, that are difficult to define and measure. One is often, for example, requested to priorize a number of alternative courses of action, each composed of many separate elements. In such a case, hard quantitative decisions are seldom possible because one finds it difficult to say anything more precise than, "I prefer alternative A over alternative B because...".

Clearly, when there are a large number of alternatives to be considered, the task becomes onerous and, as a result, often haphazard.

A logical structure, termed the technique of paired-comparisons, has been developed to aid in the solution of the above-mentioned problem. When using this technique, an individual who has been requested to assess a number (N) of objects is presented with every possible combination of these objects, and asked to make judgements as to which of each pair is favoured. His decisions are recorded in a paired-comparison matrix. An entry (C_{ij}) of "1" in this matrix denotes that the row object i (row stimulus) was judged as being better, or more desirable, than the column stimulus j. Once all possible pairs have been compared, and the decisions recorded in the matrix, the ranking of stimuli can be readily ascertained by summing the rows of the matrix. The stimuli are ranked in order of these row sums. An example comparison matrix is presented as Figure 3.

Figure 3 -- Example Comparison Matrix

Stimulus	1	2	3	4	5	6	Row Sum
1	-	1	1	1	0	0	3
2	0	-	0	1	0	0	1
3	0	1	-	1	0	0	2
4	0	0	0	-	0	0	0
5	1	1	1	1	-	1	5
6	1	1	1	1	0	-	4

The correct ordering of the stimuli in this case is 5, 6, 1, 3, 2, and 4. When the paired-comparison matrix is permuted according to the ranking derived, a characteristic pattern appears in which the upper right portion of the matrix is observed to be composed of 1's, and the lower left portion of 0's (see Figure 4).

Figure 4 -- Permuted Example Comparison Matrix

Stimulus	5	6	1	3	2	4	Row Sum
5	-	1	1	1	1	1	5
6	0	-	1	1	1	1	4
1	0	0	-	1	1	1	3
3	0	0	0	-	1	1	2
2	0	0	0	0	-	1	1
4	0	0	0	0	0	-	0

In the foregoing example, the individual making the comparisons, hereafter termed the judge, has been perfectly consistent in his judgements. It reveals that he has a clear idea of the stimuli, and that he has a good decision rule to follow while making the individual paired comparisons. Such is often not the case. Inconsistent judgements are revealed by the presence of 1's below the diagonal in the permuted matrix. In Figure 5, for example, the preference of the judge for 4 over 5, 5 over 1, and 1 over 4 (generally denoted 4>5, 5>1, 1>4, and termed an intransitive triad) is revealed. It is clearly a case of inconsistent judgement and may indicate an unclear understanding of the stimuli, or a confused or poor decision rule. It might also indicate that one attribute of stimulus 1 was so far superior to that of stimulus 4 that it became the sole determinant of the choice made in that particular comparison. The paired-comparison technique permits and identifies inconsistencies that would be lost in more traditional ranking approaches.

Figure 5 -- Permuted Example Comparison Matrix
(Inconsistent)

Stimulus	5	6	1	3	2	4	Row Sum
5	-	1	1	1	1	0	4
6	0	-	1	1	1	1	4
1	0	0	-	1	1	1	3
3	0	0	0	-	1	1	2
2	0	0	0	0	-	1	1
4	1	0	0	0	0	-	1

The degree to which the judge has been consistent can be determined by calculating the coefficient of consistency (K) defined by Kendall (1962):

$$K = 1 - \frac{24d}{n^3 - n}$$ if n is an odd number

or...

$$K = 1 - \frac{24d}{n^3 - 4n}$$ if n is an even number

where n = the number of stimuli and,

where d = the number of intransitive triads observed

The value of K equals 1.0 when d takes a value of 0.0 (i.e., when there are no intransitive triads), and declines to 0.0, as d approaches the maximum number possible. The number of intransitive triads (d) can be determined by inspecting each of the n(n-1) (n-2)/6 triads individually, but in general, it is best to compute its value from the equation:

$$d = \frac{T\ max - T}{2}$$

where T = the sum of the squared deviations of the row sums from their theoretically expected values, and,

$$T\ max = \text{the maximum possible } T = \frac{n^3-n}{12}$$

The expected number of 1's in each row of the matrix, assuming that the judgements are made randomly, is (n-1)/2. We can now extend Figure 5 as shown in Figure 6.

Figure 6 -- The Calculation of T

Stimulus	Row Sum	Expected	Deviation	Deviation^2
5	4	2.5	1.5	2.25
6	4	2.5	1.5	2.25
1	3	2.5	.5	.25
3	2	2.5	.5	.25
2	1	2.5	1.5	2.25
4	1	2.5	1.5	2.25
			T =	9.50

$$\text{Therefore } d = \frac{17.5 - 9.5}{2} = 4$$

$$K = 1 - \frac{96}{192} = .5$$

The interpretation of K is often difficult. Inspection of the permuted comparison matrix, Figure 3, reveals that only one "1" is out of place, yet the number of incorrect judgements this involves (4), is high enough to bring the coefficient of consistency well down. The statistical significance of K must be assessed by comparing d to its expected distribution. A table for this comparison developed for $n<8$ reveals that $d = 4$ is significant only at the .792 level (Kendall, 1962, p. 192).

The chi square distribution, to which the distribution of d tends as n increases (Kendall, 1962), is used to determine the probability of finding as few as d intransitive triads in a matrix where $n>7$. Chi square is found from the equation:

$$\chi^2 = 8/(n-4)\ \left|\tfrac{1}{4}\ ((n(n-1)\ (n-2)/6))-d+\tfrac{1}{2}\right|\pm v$$

where $$v = (n(n-1)\ (n-2))/(n-4)^2$$

In this case, the probability of χ^2 is the probability that an equal, or higher value of d would be found at random, rather than the inverse - as is usually the case.

The single-judge paired-comparison matrix may be extended to a multi-judge situation by summing the comparison matrices of a number of single judges to yield a group comparison matrix C (Figure 7). In such a case, one could hardly expect the level of unanimity to be as perfect as with a single judge matrix. However, this very fact later permits us to convert the ranking into a metric scale, in which the stimuli are given real values rather than ranks.

Figure 7 -- Permuted Example Group Comparison Matrix (n = 100)

Stimulus	1	2	3	4	5	6	Row Sum
1	-	80	95	97	100	100	472
2	20	-	75	85	95	100	375
3	5	25	-	60	70	90	250
4	3	15	40	-	70	80	208
5	0	5	30	30	-	75	140
6	0	0	10	20	25	-	55

As in the single-judge case, we are concerned with the consistency of the paired comparisons in the matrix, and can use the same methodology to determine both the ranking of the stimuli and the coefficient of consistency. The fact that a number of judges have assessed the stimuli and have not been unanimous in their judgements leads us to calculate the probability of one stimulus being judged better than any other stimulus. This task is performed by calculating the P matrix (Figure 8) in which each entry P_{ij} is defined as:

$$P_{ij} = \frac{C_{ij}}{C_{ij} + C_{ji}}$$

Figure 8 -- Example Proportions Matrix

Stimulus	1	2	3	4	5	6
1	-	.80	.95	.97	1.00	1.00
2	.20	-	.75	.85	.95	1.00
3	.05	.25	-	.60	.70	.90
4	.03	.15	.40	-	.70	.80
5	0.00	.05	.30	.30	-	.75
6	0.00	0.00	.10	.20	.25	-

In this case, each entry P_{ij} designates the proportion of the judgements between row stimulus i and column stimulus j, that favoured i. This matrix, in turn, is transformed into a proximity matrix which indicates the spacing of the stimuli in an M dimensional preference space. In this paper, discussion is restricted to uni-dimensional space, but, the writer wishes to acknowledge the possible existence of preference spaces of many dimensions (hyperspaces).

Each entry in the proximity matrix P^1 (Figure 9) is defined as:

$$P^1_{ij} = |.5 - P_{ij}|$$

In this matrix each entry P^1_{ij} is interpreted as being a measure of the closeness of stimulus i and stimulus j. The smaller P^1_{ij} becomes, the closer the stimuli lie in preference space. In Figure 9, for example, stimuli 1 and 2 are .30 units apart while stimuli 2 and 3 are separated by .25 units. It is legitimate, therefore, to conclude that 2 and 3 are closer together than 1 and 2. Whether it is also valid to conclude that, for example, the distance 2, 1 is thrice as great as distance 4, 3 is debatable, and furthermore, the conclusion is unnecessary to the analysis, as Abler et. al., pp. 97-98 so succinctly demonstrate.

Figure 9 -- Example Proximity Matrix

Stimulus	1	2	3	4	5	6
1	-	.30	.45	.47	.50	.50
2	.30	-	.25	.35	.45	.50
3	.45	.25	-	.10	.20	.40
4	.47	.35	.10	-	.20	.30
5	.50	.45	.20	.20	-	.25
6	.50	.50	.40	.30	.25	-

If we consider the P^1 entries to be ordinal in nature (i.e., we treat .30 merely as being greater than .10) and examine each in turn, a great deal of metric information about the positions of the stimuli in preference space may be deduced by applying a technique known as non-metric Multidimensional Scaling. Although this technique is not described in detail here, a short discussion of the underlying theory is pertinent. Readers wishing to become familiar with the field of measurement by scaling, are referred to the text Mathematical Psychology by Coombs, Dawes and Tversky, 1970.

Let us first consider a hypothetical case in which four stimuli A-D have been compared. From these comparisons, the proximity matrix presented as Figure 10 has been prepared.

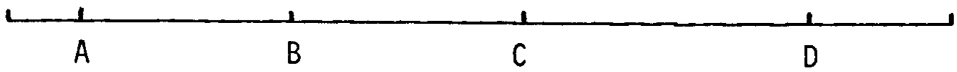
Figure 10 -- Proximity Matrix (Simple Case)

Stimulus	A	B	C	D
A	.00	.11	.23	.34
B	.11	.00	.15	.27
C	.23	.15	.00	.19
D	.34	.27	.10	.00

These proximities are then arranged in rank order: AB (smallest), BC, CD, AC, BD, AD (largest). There are many ways in which the four stimuli could be located in linear space in such a way that all these proximal relationships are satisfied (Figure 11a). It is clear that each of the stimuli may be moved without violating any proximity relationship, but it is also evident that the range of any such perturbation is quite limited (Figure 11b), and that the correlation between possible solutions will be high.

Figure 11 -- The Constraints of Proximal Rankings

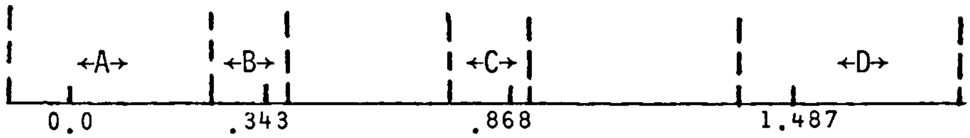
(a) Rank order to stimuli as deduced from the ranking of proximities (spacing consistent with the proximal constraints)



Proximal Constraints

AB<BC<CD<AC<BD<AD

(b) Ranges of stimuli consistent with these constraints



Logic:

A cannot move further to the left without violating AB<BC
" " " " " " right " " CD<AC

B cannot move further to the left without violating BC<CD
" " " " " " right " " AB<BC

C cannot move further to the left without violating AB<BC
" " " " " " right " " BC<CD

D cannot move further to the left without violating BC<CD
" " " " " " right " " CD<AC

Stimulus values as derived by non metric-multidimensional scaling (TORSCA-9):

A = 0.000

B = 0.343

C = 0.868

D = 1.487

In the four-stimuli example presented, the amount by which the stimuli can be perturbed is relatively large, but as the number of stimuli, and consequently the number of proximal relationships, increases,

> the points are forced to satisfy more and more....(constraints, and) the spacing tightens up until any but very small perturbations of the points will usually violate one or more of the inequalities. (Shepard, 1966, pp. 288)

Thus it is shown that it is not necessary to assume that the individual entries in the proximity matrix (the P^1_{ij}) are metric in nature.

It is sometimes impossible to order a set of points in one dimension in such a way as to satisfy all the interpoint distance constraints. In such cases, one would have to introduce additional dimensions in order to satisfy all constraints or, if only a one-dimensional scale is desired to accept a solution that in some way minimizes the number of constraints broken.

The extent to which a matrix of proximities can be fitted to a metric scale determined by these methods is indicated by a 'stress coefficient', which expresses the degree to which the ordering of the interpoint proximities of the derived configuration agrees with those of the original proximity matrix.

In order to test the ability of the paired-comparison/multidimensional scaling technique to determine an underlying preference scale, the following simulation experiment was conducted.

Twenty objects were hypothesized, ranked, and each assigned a true preference score according to its rank. If people's perceptions of these objects were accurate, they would experience no difficulty in performing paired comparisons and all choices would be unanimous since the highest valued stimulus-object would always be favoured in the comparison.* If, however, people had only a rough idea of the value of a stimulus, some would perceive it as being higher than it really is, and some lower, and thus their choices would not be unanimous. In this experiment, the true values of the stimuli are altered by varying amounts in such a way that the average perturbed value of a stimulus is equal to its true value, but that the distribution of perturbed values about the true value is normal and can be described by a set standard deviation. This experiment results in comparisons between similarly valued stimuli (e.g., 15 and 16) sometimes being judged correctly and sometimes incorrectly. This situation closely models the real world where people are required to choose between pairs of alternatives, the true values of which are rather poorly defined.

In the simultation experiment, the number of hypothetical subjects performing the comparisons between the twenty stimuli was varied. One would expect that the reliability of the individual P_{ij} entries in the proportion matrix, - and consequently the proximities and the final scale derived - would increase as the number of subjects increased, the standard deviation of the distribution of the perceived values being held constant.

* This unanimity would permit us to rank the stimuli, but because all entries of the proximity matrix would have the same value (.50), we would not be able to order the proximities, and would be unable to determine a metric scale.

This was indeed the case; the results, presented in Figure 12, reveal that the recovery of the original scale of stimuli values was relatively good in spite of the high stress values. The recovered scale, it is obvious, becomes more accurate as sample size increases, and as the standard deviation, analogous to the degree of uncertainty, decreases.

The application of the paired-comparison/multidimensional scaling methodology to the component interactions identified by Ross (1974b, p. 19) resulted in the identification of ratio scale weights for each of the interactions he noted.

Figure 12 -- Scale Deduction, Varying Sample Size

Sample Size	Standard Deviation	Stress	Correlation of Recovered Scale with Original Scale
10	2.0	.422	.8875
20	2.0	.345	.9710
50	2.0	.295	.9963
100	2.0	.255	.9964
200	2.0	.252	.9959
500	2.0	.250	.9965
2	20.0	.480	.1241
5	20.0	.500	.2968
10	20.0	.461	.5873
20	20.0	.477	.6678
50	20.0	.355	.9898
100	20.0	.316	.9944
200	20.0	.218	.9943
500	20.0	.166	.9976

To accomplish this task, a paired-comparison matrix is constructed. The stimuli are the interdependencies. One next sets forth the criterion on which the judgements must be made, and directs a number of judges to complete the paired-comparison matrix. In the case of an environmental impact study, they are likely to be the study team although a sample of the well-informed public would be suitable. Figure 13 presents a hypothetical comparison matrix based on the interdependencies identified in Figure 1. The application of the foregoing methodology to this data produces weights for each of the interdependencies, and brings us one step closer to our goal.

Figure 13 -- Hypothetical Comparison Matrix Based on Figure 3

Stimulus	1,2	2,6	3,1	3,2	3,4
1,2	0	5	10	15	20
2,6	15	0	15	12	18
3,1	10	5	0	7	14
3,2	5	8	13	0	4
3,4	0	2	6	16	0

The foregoing discussion sets forth a methodology capable of determining the values of stimuli from the quantitatively much weaker information obtained by presenting paired comparisons to a number of subjects. It describes a simulation experiment in which this methodology was used to recover values from a set of simulated paired-comparison data; and it assesses the degree of relationship between the recovered values and the original values on which the comparisons are based.

The following sections of this paper present a method of combining the stimuli values with the ranked disruption measures discussed previously, a method of summarizing these data in order to permit alternatives to be selected, and a practical application of this methodology to the field of environmental impact prediction.

The Combination of Interdependency Weights and Disruption Rankings

Ross (1974b) formed a three-dimensional matrix D (Figure 2 above) in which two dimensions were the stimulii and the third was that of alternative courses of action. The entries in this matrix were ordinal estimates of the damage that would accrue to each interaction as a result of the choice of each alternative. In order to make use of the information contained in this matrix D, it was necessary to combine the individual D_{ijk} with the corresponding W_{ij} which express the importance of each interaction.

If the interdependency weights derived from the paired-comparison matrix are entered in their appropriate cells in a weighting matrix W, i.e., the weight derived for dependency ij is assigned to W_{ij}, it is evident that the same cells will be occupied as in each 'layer' of the three-dimensional disruption matrix D. If one now considers these data, with the objective of being able to summarize them, a decision can be made about the impact of each of the alternatives. It becomes evident that one may summarize by calculating the total disruption that would be caused by each alternative. This summarization is accomplished by combining for each alternative the rank of each disruption ij with the weight of the corresponding dependency ij. The question, of course, is how to proceed with this combination. It would not be valid to calculate these summary measures by summing the product of the rankings and the corresponding weights (i.e. using the equation:

$$S_k = \sum_{i=j}^{NV} \sum_{j=i}^{NV} D_{ijk} * W_{ij}$$

where S_k = the summary measure of kth alternative,
NV = the number of variables in the interdependency Matrix A,
D = the disruption matrix, and
W = the weighting matrix)

because the rank data contained in D cannot be multiplied, since they denote order, not interval spacing.

One may, however, utilize this information in a paired-comparison way. Another comparison matrix (X) is constructed, this time between alternatives rather than variables,(Figure 14).

Figure 14 -- Hypothetical Alternative Comparison Matrix (X)

Alternative	1	2	3
1	0.0	X_{12}	X_{13}
2	X_{21}	0.0	X_{23}
3	X_{31}	X_{32}	0.0

One then inspects all the cells ij in the disruption matrix D and increment X_{kl} each time that alternative k is observed to be more disruptive than alternative l. The amount X_{kl} is incremented is, in each case, equal to W_{ij}. Each entry in X is therefore defined as -

$$X_{kl} = \sum_{i=1}^{NV} \sum_{i=1}^{NV} W_{ij} d \text{ where } d = 1 \text{ if } D_{ijk} > D_{ijl}, \text{ 0 otherwise}$$

It should be noted that in cases where all the alternatives have been judged to be equally disruptive d is always zero, and no contribution to X is made. In this case, it would be unnecessary to calculate a weight for the interdependency, and it is best omitted from the paired-comparison matrix. Such omissions, it is evident, will greatly reduce the number of comparisons that must be made by the subjects because the number of these increases as N(N-1)/2 (where N is the number of stimuli - in this case interdependencies).

This comparison matrix is then converted to the proportions matrix Y (Figure 15) by the formula:

$$Y_{ij} = \frac{X_{ij}}{X_{ij} + X_{ji}}$$

Figure 15 -- Hypothetical Alternative Proportions Matrix (Y)

Alternative	1	2	3	Z
1	0.00	Y_{12}	Y_{13}	Z_1
2	Y_{21}	0.00	Y_{23}	Z_2
3	Y_{31}	Y_{32}	0.00	Z_3

where $Z_i = \sum_{j=i}^{NA} Y_{ij}/(NA)$

Interpretation of these Y_{ij} is difficult because of the introduction of the dependency weights. However, each Y_{ij} may be considered to be a measure of the probability that Alternative i causes more disruption than Alternative j. From this matrix, we may calculate a single value for each alternative by finding the average of each row in the matrix, denoting it as Z. The value of Z_i, then represents a metric measure of the disruption which would result from the choice of Alternative i.

The foregoing sections of this paper have outlined a methodology for determining a numeric measure of environmental disruption. The following section presents a real-world application of the methodology.

PART B -- THE NANAIMO CASE

The foregoing sections of this paper have outlined a tentative methodology designed to estimate the magnitude of the environmental disruptions that may accrue to an environmental system as a result of the construction of alternative development projects. This section of the paper will present the application of the proposed methodology to the problem of selecting a location for a lumber transshipment facility on the east coast of Vancouver Island (Ross, 1974a).*

Interdependency Weighting

The sixty-nine first order interdependencies identified as having different disruption levels (Figure 1) were arranged in pairwise comparison form and presented individually to each member of the study team for judgement. For each ij comparison, they were directed to formulate the question: "In terms of its contribution to the production of biomass, is interdependency i more important than interdependency j?". If the answer was in the affirmative, a "1" was entered in the ijth cell. To reduce fatigue in completing the P/C matrix, only the upper triangular portion was completed. The lower part was filled in during the analysis by entering the complement of each response in the appropriate cell below the diagonal.

---

*Ross was faced with the task of assessing the environmental disruption that would be caused by the construction of a major facility at one of five alternative sites. These are not discussed here, but are listed in Appendix I, along with the definitions of environmental components identified.

FIGURE 16 -- COMPARISON MATRIX - SUBJECT 1

SORTED MATRIX

```
      016554611565630325444542253363043364610331002065622301221611221044545
      590121432710237926789867656837321090516540424889685217915746308953443
  5   011111111111111111111111111111111111111111111111111111111111111111111
 19   001111111111111111111111111111111111101111111111111111111111111111111
 60   000001111111111111111111111111111111111111111111111111111111111111111
 51   001010011111110111111111111111111111111111111111111111111111111111111
 52   001000001111111111111111111111111111111111111111111111111111111111111
 41   000110100111110111111111111111111111111111111111111111111111111111111
 64   000110000111110111111111111111111111111111111111111111111111111111111
 13   000011100111110111111111111111111111111011111111111111111111111111111
 12   000001110111110111111111111111111111111011111011111111111111111111111
 57   000000000001111101111111111111111111111111111111111111111111111111111
 61   000000000101001001111111111111011111111111111111111111111111111111111
 50   000000000000011101111111111111111111111111111111011111111111111111111
 62   000000000011010100111111111111111101011111111111111111111111111111111
 33   000000000010001011111111110111011111111111111111111111111111111111111
  7   000101111000100010010010011111111111111111111111111111111111111111111
 39   000000000010011010111111101110110111111111111111111111111111111111111
 22   000000000111100001111100111111111101011111111111111111111111111111111
 56   000000000000101100001011111111111111111111111111111111111111111111111
 47   000000000000001001011111110111111111111111111111111111111111111111111
 48   000000000000000001000111111111111111111111111111111111111111111111111
 49   000000000000001000010111101011111110101111111111011111111111111111111
 58   000000000000001001000000111101111110101111111110111111111111111111111
 46   000000000000000000000001011010111110111111111111111111111111111111111
 27   000000000000000101000000011011110111101111111111111111111111111111111
 26   000000000000000101000000000111111111011111111111111111111111111111111
 55   000000000000000100001010101101101001111111111111111111111110111111111
 36   000000000000100000000000110000110111111111111111111111111111111111111
 38   000000000000000000000010100001011100111011111111111111111111111111111
 63   000000000000000000000010010001100010010001111111111111111111111111111
 37   000000000000000001000000000000010100111111111101111111111111111111111
  3   000000000010010000000000000000000010010111011111111111111111111111111
 42   000000000000000000000000101111100110110011100110011101111111111111111
 31   000000000000000100000000000011100011001110111011111111111111011111111
 30   000000000000000000000000100100001010110110001111011111111111111111111
 69   000000000000100000001100010010010001110110001111101110011111111111111
 40   000000000000000100000000000000101000000111111111101101111011111111111
 65   000000000000100000001101100110001100000110000001110101111111111111111
 11   010000011000000010000000000000000110001001001010001111111111111111111
  6   000000000000000000000000000000010010110011010111110111111111111111111
 35   000000000000000000000000000000001001000010111111100111011111111111111
 34   000000000000000000000000000000000010001000111110111111111111111111111
 10   000000000000000000000000000000000011101100000010101111111111111111111
  4   000000000000000000000000000000000100101010010001111101111111111111111
  2   000000000000000000000000000000000010010110001100111001111111111111111
 24   000000000000000000000000000000000000101000110101111111111111111111111
  8   000000001000000000000000000000000010001100010000111111111101111111111
 68   000000000000000000000010000000000010101110110001001101110110100111111
 59   000000000000000000000000000000001010000001010100000001001110111111111
 66   000000000001000000001000000000000000000000001000001011111111111111111
 28   000000000000000000000000000000001100000001001100001111111111111111111
 25   000000000000000000000000000000000000101000100001110111111101111111011
 32   000000000000000000000000000000000010000000000000100101110111111111111
  1   000000000000000000000000000001000010000000000000000001001111111111111
 17   000000000000000000000000000000010000000000000001000010010111100111111
 29   000000000000000000000000000000011000000000000001000010010100110111111
 21   000000000000000000000000000000000000000000000100000011001011011111111
 15   000000000000000000000000000000000000000000000100000000100111111111111
 67   000000000000000000000000010000000100000000000000001100010000111011111
 14   000000000000000000000000000000000000000000000100000000100100111111111
 16   000000000000000000000000000000000000000000000100000000100110001111111
 23   000000000000000000000000000000000000000000000000000001010001011110111
 20   000000000000000000000000000000000000000000000000000001000001001101111
 18   000000000000000000000000000000000000000000000000000000010000000011111
  9   000000000000000000000000000000000000000000000000000000001000000011111
 45   000000000000000000000000000000000000000000000000000000000000010001111
 43   000000000000000000000000000000000000000000000000000000000000100000111
 54   000000000000000000000000000000000000000000000000001000000000000000001
 44   000000000000000000000000000000000000000000000000000000000000000000101
 53   000000000000000000000000000000000000000000000000000000000000000000000
```

COEF OF CONSISTENCY= 0.89331

FIGURE 17 -- COMPARISON MATRIX - SUBJECT 2

SORTED MATRIX

```
    001325253324225333234354263602120310510565665454666654204644111110140
    342247290900858386756158134269865732397476452702819011389654756411039
 3  011111111111111111111111111111111111111111111111111111111111111111111
 4  001111111111111111111111111111111111111111111111111111111111111111111
12  000111111111111111111111111111111110111111111111111111111111111111111
32  000000101011111111111111111111111110111111111111111111111111111111111
24  000101001101111111111111111111111110111111111111111011111111111111111
57  000100000101111111111111111111111110111111111111111111111111111111111
22  000011010000111111111111111111111110111111111111111111111111111111111
59  000111001101111010101111110111111110111111111111111111111111111111111
30  000001100100101111111111111111111110111111111111111111111111111111111
39  000100100011011101111101111111111110111111111111111111111111111111111
20  000011111000111111110111011111011110101111111111111111111111111111111
40  000000101010011001111101111111111110111111111111111111111111111111111
28  000000000101001111001111111111111110111111111111111111111111111111111
25  000000001000100011111111111111111110111111111111111111111111111111111
58  000000000000010111111100111111111111111111111111111111111111110111
33  000000010001010010011111111011111110111111111111111111111111111111111
38  000000000101000001111101111111111110111110111111111111111111111111111
36  000000010000000010010111111011111110111110111111111111111111111111111
27  000000000000100100011010111111111110111111111111111111111111111111111
35  000000010000010000100111100111111110111110111111111111111111111111111
46  000000000010000000000100111110111110111111111111111111111111111111111
31  000000000000000000010001010101111110111111111111111111111111111111111
55  000000000101001010001001010101000110110111111111111111111111111111111
48  000000000000001000101100110010111000111111111111111111111111111111111
21  000000000010000000010010001011111110111111111111111111011111111111111
63  000000000000000001010100100101110111011111111110111111111111111111111
34  000000010000000000000011010101001111111111111111111111101111111111111
62  000000000000000100000101100001110111011111110111111111111111111111111
 6  000000000000000000000010011100110100111111111111111111011111111111111
29  000000000000000000001001000010101111011010111111111111111111111111111
18  000000000010000000000010001000001101111111111111111111011111111111111
26  000000000000000000000010001001101100011011111111111111111111111111111
 5  000000000000000000000010010110000010111111111111111111011111111111111
37  000000000000000000000000100000001011101110111111111111111111111111111
13  000000000000000000000001000010110001011011111111111111011111111111111
 2  001111111111110111111111111000100110001011000000000000000101000000011111
53  000000000000000000000000010101010010010101101111111111111111111111111
19  000000000010000000000000000000000101000111111111111111011111111111111
 7  000000000000000000000010000000000000110111111111111111011111111111111
54  000000000000000000000000000001010010000010111111111111111111111111111
67  000000000000000000000000000000000001100000101111111111111111111111111
56  000000000000000011010000000001000101000100101010101111111011111111111
64  000000000000000000000000000000000001000010011001111111111111111111111
65  000000000000000000000000000000000001100001000111011111110111111111111
52  000000000000000000000000100000000001000001010101101111111011111111111
47  000000000000000000000000000000000001000001100001101110111111111111111
50  000000000000000000000000000000000001000000101101101110101011111111111
42  000000000000000000000000001000000001000001000000111111100111111111111
68  000000000000000000000000000000000001000000110000011101110111111111111
61  000000000000000000000000000000000001000001000110100101110011111111111
69  000000000000000000000000000000000001000000000000010110111011111111111
60  000010000000000000000000000000000001000000100000000011100111111111111
51  000000000000000000000000000000000001000000000000101001110111111111111
41  000000000000000000000000000000000001000000000001101010101100111111101
23  000000000000000000000000100010101011011000000000000000100000111111101
 8  000000000000000000000000001000000000000000000001100000100111111111111
49  000000000000000000000000000000000001000000000000101010010001111111111
66  000000000000000000000000000000000000000001011010111101010000001000011
45  000000000000000000000000000000000001000000000000000011001101111110111
44  000000000000000000000000000000000001000000000000000001100100111110111
17  000000000000000000000000000000000001000000000000000000000100011111111
15  000000000000000000000000000000000001000000000000000000000100001111111
16  000000000000000000000000000000000001000000000000000000000100000111111
14  000000000000000000000000000000000001000000000000000000000000000011111
11  000000000000000000000000000000000000000000000000000000000100000001111
 1  000000000000001000000000000000000000000000000000000000000111000000010
10  000000000000000000000000000000000000000000000000000000001000000001011
43  000000000000000000000000000000000000000000000000000110000000000000001
 9  000000000000000000000000000000000000000000000000000000000000000001000
```

COEF OF CONSISTENCY= 0.85269

FIGURE 18 -- COMPARISON MATRIX - SUBJECT 3

SORTED MATRIX

```
     026553522526062363516554433033051010603501110121514134642243244462664
     580109253944722815793860136847453322560490411775369817629682036491785
  5  011111111111111111111111111111111111111111111111111111111111111111111
 28  001111110111111111111111111111111111111111111111111111111111111111111
 60  000011110111111111101111111111111111111111111111111111111111111111111
 51  001001011111111110111111111111111111111111111111111111111111111111111
 50  000100111011111111111111111111111111111111111111111111111111111111111
 39  000010110111111111111111111111111111111111111111111111111111111111111
 52  000100010111111111011111111111111111111111111111111111111111111111111
 25  000000000111111111111111111111111111111111111111111111111111111111111
 23  011001110011110011111111111111111111111111111101110111111111111101111
 59  000010001000001111111111111111111111111111111111111111111111111111111
 24  000000000101110111111111111111011111111111111111111111111111111111111
 64  000000000100011111111111011111111111111111111111111111111111111111111
  7  000000000101011111111101111111101111111111111111111111111111111111111
 62  000000000100001001101111111111111111111111111111111111111111111111111
 22  000000001010000111101110101111111111111111111111111111111111111111111
 38  000000001000010001101111011111111111111111111111111111111111111111111
 61  000000000000010101001111011111111111111111111111111111111111111111111
 35  000100000000000000111110111111111111111111111111111111111111111111111
 57  000000100000000010000111111111111111111111111111111111111111111111111
 19  001000000000011110101110100001110111111111111111111111111111111111111
 63  000000000000000000100011011111111111111111111111111111111111111111111
 58  000000000000000000001011111111111111111111111111110111111111111111111
 56  000000000000100000000001011111111110111111111111111111111111111111111
 40  000000000000001001010000011111101111011111111111111111111111111111111
 41  000000000001000110001011001001011110110111111111111111111111111111111
 33  000000000000001000010000100111111111111101111111111111111111111111111
 36  000000000000000000010000010111101111111111111111111111111111111111111
  8  000000000000000000010000100010111011111111111111111111111111111111111
 34  000000000000000000010000100000111111110011111111111111111111111111111
 37  000000000000000000000000000110101111011111111101111111111111111111111
  4  000000000000000000000000100000001011111111111111111111111111111111111
 55  000000000000100000000001001001100001010111111111111110111111111111111
 13  000000000010000000010000000000010001111011111111111111111111111111111
  3  000000000000000000000010000100111010011111111011111111111111011111111
 12  000000000000000000000000000000011001111001111111111111111111111111111
  2  000000000000000000000000100000000100011111111111111111111111111111111
 65  000000000000000000000001000001010101010001010111111101011111111111111
  6  000000000000000000000000000000000000001111111111111111111111111111111
 30  000000000000000000000000110010010000100101011010101111111111111111111
 54  000000000000000000000000000010001010101000101001110111111111011111111
  9  000000000000000000000000000000000010101000100111111111111111111111111
 10  000000000000000000000000000000000000000110110111111111011111111111111
 14  000000000000000000000000000000000000101000000111111111111111111111111
 11  000000000000000000000000000000000000000110100111111111100111111111111
  1  000000000000000000000000000000000100000111110101010110111101101111111
 17  000000000000000000000000000000000000001000000011011111011111111111111
 27  000000001000000000000000000000000000000000001000101011111111111111111
 15  000000000000000000000000000000000000001000000010011111011111111111111
 53  000000000000000000000000000001000000000100001101010101110010111111111
 16  000000000000000000000000000000000000001000000100011110111111111111111
 49  000000001000000000000100000000000000100000001000100001001111111111111
 18  000000000000000000000000000000000000000000000010001011111111111111111
 31  000000000000000000000000000000000000100000000010100111101111111111111
 47  000000000000000000000000000000000000000000001000001000101111111111111
 66  000000000000000000000000000000010000000001000101010000000001011111111
 42  000000000000000000000000000000000000000000010000001001100010111111111
 29  000000000000000000000000000000000000000000010000100000110010111111111
 26  000000000000000000000000000000000000000000000000100010111001011111111
 48  000000000000000000000000000000000000000000001000000000100101111111111
 32  000000000000000000000000000000000000000000000000100000011000011111111
 20  000000000000000000000000000000000100000100000000000000100101000011111
 43  000000000000000000000000000000000000000000001000000000000000101111111
 46  000000000000000000000000000000000000000000000000000000000000100111111
 44  000000000000000000000000000000000000000000000000000000000000100011011
 69  000000001000000000000000000000000000000000000000000000000000000000111
 21  000000000000000000000000000000000000000000000000000000000000000010111
 67  000000000000000000000000000000000000000000000000000000000000000100011
 68  000000000000000000000000000000000000000000000000000000000000000000001
 45  000000000000000000000000000000000000000000000000000000000000000000000
```

COEF OF CONSISTENCY= 0.92539

FIGURE 19 -- COMPARISON MATRIX - SUBJECT 4

SORTED MATRIX

```
     654660665565252542103624360654346434344334233325252521102101111001100
     411205182736280998273942456707606896543705582164339578341987645210139
 64  011111101111111111111011111111111111111111111111111111111111111111111
 51  001101110110111111111111111111111111111111111111111111111111111111111
 41  000100111111111111111111111111111111111111111111111111111111111111111
 62  000011101111111101111011111111111111111111111111111111111111111111111
 60  011000101101111111111011111111111111111111111111111111111111111111111
  5  001010010011110111111111111111111111111111111111111111111111111111111
 61  000001001101111111111011111111111111111111111111111111111111111111111
 68  100110101111010111001101110111111111111111111111111111111111111111111
 52  010001000100111111111011111111111111111111111111111111111111111111111
 57  000001000001011111111111111111111111111111111111111111111111111111111
 63  000010101101011101101011110111111111111111111111111111111111111111111
 56  010000001000001011111101111111111111111111111111111111111111111111110
 22  000000010111010110110001111111111111111111111111111111111111111111111
 58  000000000001001111001011110111111111111111111111111111111111111111111
 20  000001010000100010110110001111111111111111111101111111011111111111111
 59  000000000001001011001011100011110111111111111111111111111111111111111
 49  000100000100000000110011101110111111111111111111111111111111111111111
 28  000000000000101010110110011111101101111111110111111111111111111111111
 12  000000010000010110000110011111111111111111111101111111011111011111111
  7  000000010010010100100110010101111111111111111101111111011111111111111
 33  000000000000101001110110011110101000011111111111111111111111111111111
 69  100110101010010110000001000011010111011011011101111111111111111011111
 24  000000010001000010000100010111101101111101111111111111111111111111111
 42  000000000000101001111010101001110111111111111111010101110111111101111
 34  000000000000101001111110011100101000100110111111111111111111111111111
 65  000000000000101100000101000011000011110110111111111111111011111111111
  6  000000010010010110010110010111111111111111011100110111100111100001111
 67  000000000000000010000010101001100111011011111111111111111011111111111
 50  000000000000000000001001100000000101111111111111111111111111111111111
 47  000000000000000000000010001000100111111111111111111111111111111111111
 36  000000000000000001000100010110010111010111111101111111111111111111111
 40  000000000000000000100101011010000101010011011111111111111111111111111
 66  000000000000000010000010100001100111011011111111111111111011111111111
 48  000000000000000000000010001000000000111111111111111010101111111111111
 39  000000000000000000100101010000000100001001101111111111111111111111111
 46  000000000000000000000010001000000000101111111111111010101111111111111
 35  000000000000000000000011000101000010000000110011111111111111111111111
 44  000000000000000000000000010000000100101011111111111010101111111111111
 43  000000000000000000000000010000000100101001111111111010101111111111111
 37  000000000000000000000100010100001000000000101011111111111111111111111
 30  000000000000000100011001000100000000000000101111111111111111111111111
 45  000000000000000000000000010000000100101001101111111010101111111101111
 25  000000000000000000000000010000000000100000000111111111111111111111111
 38  000000000000000000000000000000000100000001101011101010101111111111111
 32  000000000000000000010000000001000000000000000011111111111111111111111
 31  000000000000000000000000000100000000000000010011111111111111111111111
 26  000000000000000000000000010000000000000000000110111101111111111111111
 54  000000000000000000000001000000010101011001010000100100101111111111111
 23  000000000000000100011000000100000000000000000001010001111111111111111
 53  000000000000000000000001000000010101011001010000000010010111111111111
 29  000000000000000000000000000000000000000000000001110111111111111111111
 55  000000000000000000000001000000010101011001010011110010000101101000101
 27  000000000000000000000000000000000000000000000000100001111111111111111
 18  000000000000000000000000010000000000000000000001010100110011111111111
 13  000000000000000000000000010000000000000000000001010100000111111111111
  4  000000000000000000010000101010001000000000000000000100101001111011101
 21  000000000000000000000000000000000000000000000001010101100101111110111
 19  000000000000000000000000000000000000000000000000000001010011111111111
  8  000000000000000000000000000000000000000000000000000100011001101011111
 17  000000000000000000000000010000000000000000000000000000000011111111111
 16  000000000000000000000000010000000000000000000000000000000001111111111
 14  000000000000000000000000010000000000000000000000000100000010000111111
 15  000000000000000000000000010000000000000000000000000000000001010111111
  2  000000000000000000000000000000000000000000000000000100010010000011111
  1  000000000000000000000001010000000000000001000000000100000000000000011
 10  000000000000000000000000000000000000000000000000000100001000000010011
 11  000000000000000000000000000000000000000000000000000000000000000011011
  3  000000000000000000000000000000000000000000000000000100010000000000001
  9  000000000001000000000000000000000000000000000000000000000000000000000
```

COEF OF CONSISTENCY= 0.83507

FIGURE 20 -- COMPARISON MATRIX - SUBJECT 5

SORTED MATRIX

```
    055066334503225322524364233566211240050643635445464652121146301111100
    512440861933354784020277195553723067638181208956482979569736421064891
 5  011111111111111111111111111111111111111111111111111111111111111111111
51  001111111111111111111111111111111111111111111111111111111111111111111
52  000111111111111111111111111111111111111111111111111111111111111111111
 4  000011111111111111111111111111111111111111111111111111111111111111111
64  000001111111111111111011111111111111111111111111111111111111111111111
60  000000101101111111111111111111111111111111111111111111111111111111111
38  000000011111111101111111111111111111111111111111111111111111111111111
36  000001000101111111111111111111111111111111111111111111111111111111111
41  000000010101111111011110111111111111111111111111111111111111111111111
59  000000000011011111110111111111111111111111111111111111111111111111111
 3  000001011000111111111101111011111111111111110111111111111111111111111
33  000000000010011111100111111111111111111111111111111111111111111111111
23  000000000101001101011110111111111111111111111111111111111111111111111
25  000000000000101111010110111111111111111111111111111111111111111111111
54  000000000000000010101111111111111111111111111111111111111111111111111
37  000000000000001010010111111111111111111111111111111111111111111111111
28  000000100000100000111101111101111111111111111111111111111111111111111
24  000000000000001110101111011011111111111111111111111111111111111111111
50  000000001000110100011101111101100111111111111111111111111111111111111
22  000000000001001001001010111111111111111111111111111111111111111111111
40  000000000101010100000110011011111111111111111111111111111111111111111
32  000010000000000000010011111111111111111111111111111111111111111111111
67  000000000010000010100001001111111111111111111111111111111111111111111
47  000000001000110000011000111000111111101111111111111111111111111111111
21  000000000000000001001010010111111111111111111111111111111111111111111
39  000000000000000000000010000111111111111111111111111111111111111111111
35  000000000000000000000000110111111101111111111111111111111111111111111
55  000000000010000001001001000011011111111110101111111111111111111111111
65  000000000000000010100001000000111111101111010111111111111111111111111
63  000000000000000000000001000010100111111111111111111111111111111111111
27  000000000000000000000000000100011111111111111111111111111111111111111
12  000000000000000000100000000001001110111111111111111111111111111111111
13  000000000000000000100000000001000110111111111111111111111111111111111
20  000000000000000000000000000000000011111111111111111111111111111111111
46  000000000000000000000000001000000001101111111111111111111111111111111
 7  000000000000000000000000000000011000011101111111111111110111111111111
 6  000000000000000000000000000000000001011101111111111111111111111111111
53  000000000000000000000001000010000100001101011111111111111111111111111
 8  000000000000000000000000000000000001011111111111111110111111111111111
61  000000000000000000000000000000000000000010111111111111111111111111111
48  000000000000000000000000000000000001100000111011111111111111111111111
31  000000000000000000000001000000000101101011111111110110111111111111111
62  000000000000000000000000000010000000000000011011111111111111111111111
30  000000000000000000000001000000000100010010101111111111111111111111111
58  000000000010000000000000000010000000000000000101010111111111111111111
49  000000000000000000000000000000000000000010110010100011111111111111111
45  000000000000000000000000000000000000000000001001101111111111111111111
56  000000000000000000000000000000000000000000010100011101111111101111111
44  000000000000000000000000000000000000000000001001001111011011111111111
68  000000000000000000000000000000000000000000000110100110111111001111111
42  000000000000000000000000000000000000000000001100010111011001101111111
69  000000000000000000000000000000000000000000000100000010111111001111111
57  000000000000000000000000000000000000000000000001000000111110101111111
29  000000000000000000000000000000000000000000000000010110001001111111111
15  000000000000000000000000000000000000000001000000101001000111110011111
26  000000000000000000000000000000000000000000000000000001101100111111111
19  000000000000000000000000000000000001001000000000000000100100111111111
17  000000000000000000000000000000000000000001000000101001000011110011111
43  000000000000000000000000000000000000000000000000001001011000101111111
66  000000000000000000000000000000000000000000000000000010011010001111111
34  000000000000000000000000000000000000000000000000010100000001001111111
 2  000000000000000000000000000000000000000000000001011110000011100000101
11  000000000000000000000000000000000000000000000000000000100100010111111
10  000000000000000000000000000000000000000000000000000000100100010011111
16  000000000000000000000000000000000000000000000000000000000000010001111
14  000000000000000000000000000000000000000000000000000000000000010000111
18  000000000000000000000000000000000000000000000000000000000000000000011
 9  000000000000000000000000000000000000000000000000000000000000010000001
 1  000000000000000000000000000000000000000000000000000000000000000000000
```

COEF OF CONSISTENCY= 0.94563

Analysis of the resultant individual comparison matrices (permuted examples of which are presented as Figures 16-20) revealed that each of the five subjects was extremely consistent in his judgements, as evidenced by the coefficients of consistency K presented in Figure 21. It is of interest to note that although all individuals revealed a remarkably high degree of consistency, indicating that each had a very firm idea of the "true" ordering of the sixty-nine stimuli, by no stretch of the imagination could the agreement among individuals be considered high. The rank correlations between the individuals' rankings are presented as Figure 22.

Figure 21 -- Nanaimo Individual Consistency Measures

Subject	Consistency (K)	Significance
1	.89331	>.001
2	.85269	.001
3	.92539	.001
4	.83507	.001
5	.94563	.001

Figure 22 -- Individual Ranking Correlation Matrix

Subject	1	2	3	4	5	Consensus
1	1.000	- .050	.337*	.104	- .023	- .034
2	- .050	1.000	- .071	.031	.456*	.054
3	.337*	- .071	1.000	- .203	- .126	.015
4	.104	.031	- .203	1.000	.170	.148
5	- .023	.456*	- .126	.170	1.000	.004
Concensus	- .034	.054	.015	.148	.004	1.000

*Statistically significant at the .95 level

Inspection of the sixth column of Figure 22 reveals that the concensus ranking derived from the five individual paired-comparison matrices was only weakly related to any of the individual scales.

The permuted group-comparison matrix (Figure 23) reveals the composite ranking of the interdependencies. The matrix shows a reasonable degree of unanimity, and has a consistency of .252 which is over thirty-three standard deviations from the mean and therefore very significantly non-random.

The proximity matrix (Figure 24) was scaled using the TORSCA-9 Multidimensional Scaling Program, which deduced the weights to be assigned to each component interdependency. The configuration reached during the analysis had a stress of .414; 78% of the 52,394 distance constraints were satisfied.

FIGURE 23 -- NANAIMO PERMUTED GROUP COMPARISON MATRIX

	056526334152256335640526030531534102464323243564206206526111414141100
	514220931298472680307851453673507967856432091472688129396810475534619
5	0444454454444344445545444444545545554454444354555455445545455555555555
51	102244443443444443443445434344444344444454454444445445444445555555555
64	130343343343344443442435344443444344454443454455454454444545555555555
52	132042343443444433444444434344534445444444544454534544444555555555555
22	111101222332553443523534353544544354545443355554455455555455555555555
60	012340445443543344434444443434444442444444444444444534534444555555555
39	112231042222323432434444343334343445444534444444554444554555454545555
33	111131102323311133324243333354534454454434445554554545555555555555555
41	022230330333334334423342334434342444443444544444354454454554545455555
12	112121322032333334331433132335444354545324455344535445555545555555554
59	111121332203332234332333243454444443443434544445454435335445555555555
28	122232322320223344344323232334434443434342444443444545554544555555555
24	112100222223033333333333342443334444454335344553454444555544555555555
57	211201342223202433243432333543434344444453434444445434344445555454555
62	111122242232230323332423333443344333443433334444554445335555555555555
36	111112141232212003122132323154344444545444443455535545455555555555555
38	111111222221223502442232344254243344334454344443454544445455545454555
50	022221322111222230344342343342444334424454344432544445444245555555555
63	011202121222232412022323333433334533334543343444345445345555555555555
40	111131231221212311302242353244142444343534434453453443545555454545555
7	023121113431223331330323333234333223433132344443553455454545555555555
58	111101132122211432232031334543333334243533334444543453455555454555545
25	112121112223223221313203342334432444343344334433443534554544454545555
61	100111121222232333232420332532344233433433343334444353344455555555555
4	112122223433222222222220212243332433532344533455334534444544454545555
35	121101122212122311202212303234442443342432334443453444455554545454555
3	111122223233222122221334203233332433333343334343444344444444444444435
56	021201221212101432133020332042342233233433353434444345354455454555554
37	111111201202111001212122323104243233423343335533544554454554545454555
13	012211112011222113211213112310333452433212343234444254434444545454555
55	011101201111212231242212212232003332435232342424434235433545555554545
30	111111122112221111112221212112502333223333325433433434554554444444454
47	011011211111111121032231232322230223414343343223444444244344545454454
19	022223113211122122212213313331223022232212332333243253223445353535555
6	011101101021112112213212111220223302232312242444445335545455545454455
27	111111011112112011213112222223322330213342333233444434444444444444454
48	111101111011111121222321222321131333024333213233534434234444545454454
65	010011101122011023111112012212234224303233433224432344444444555554544
46	1111011110111121112222222232232021332120432324233444443423344454544444
34	111111012222201111102021312123322322231033322342223344445455343435555
23	101111221311222100224212222224221441222202412422242541422544353424545
32	112121111103012111121221213121332233322323032233323352355243343444444
20	211121111111222222113222122222222232312212033432434224434333444343434
49	0000011100111112111222221222021131212423343202233422341343344545454454
31	111101101011111011111111221222230233222133323032322443455443343434444
54	0111011012010110121111121111031132133332121320343332233332334444444434
67	010101101111011013101112201220232321223213222320323332334233335353433
42	010011111102210220221121021121122212212332322120223332313434444343444
26	111111001011100011112011212121111311011333113233033334353444343434444
8	000000002211011001000111201101221111221312233223203222334355454544555
68	011202110021121111120221121111121201131232131222220223223343435343344
21	111101101100111001112102011213311321121200321332233023422432344434444
2	100010111121120110011020212100221022212113312323233303323344222242254
69	011102100020112011220212111001211201111142141222132220212343334444434
53	011101001000012011100102101211003301313110120214223123023233444443443
29	111101001010010001011001101012101311121213021023202333430334332323344
66	010101100011010013000011001101211201112033121221212323220323333354434
18	111101001000000001001001101111202111111101221322122122322022323232234
11	0000100001011000000000010111001001101111101221222110121121330323233333345
10	000000000001100000000000100010001110011110122122211023122223202323333345
44	000000101000010010010110111111010201000221101101211232122233022233323
17	000000000000000000000000010000110111011021121221021321323223024445534
45	000000101000010010010110111111101020100022110110121013112223333033313
15	000000000000000000000000000100001101110111121212210213111323223120444
43	000000101000000010010010111011010201001231101111211211120222212102233
14	000000000000000000000000000100011101111101121111211121312213222121301
16	000000000000000000000000000100001101110100111112110213112132220213403
1	000000000000000000000010000200010000001101120122310110211221132422220
9	000000000100000000000000000100011001111001111112110111121110021212112

TABLE 24 -- NANAIMO PROXIMITY MATRIX

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             111111111122222222223333333333444444444455555555556666666666
    123456789012345678901234567890123456789012345678901234567890123456789

 1  551555551335511111513535535535335555555551113355555531355355555553113 1
 2  551131313333351111151133311131111333353333131113135551113151555555 31111
 3  115333135333113333313111111113113111111311115333111113333311131111113133
 4  513533115533355553135111111311311111113111113333311111133333111111111155533
 5  533355555553555553351333333333333333333355555535355533555551335335335555
 6  5133551355555333333115333313311151331333333555111313353111311131111315355
 7  53115155555335555531111311113313113151555555133333113551113333353335513355
 8  51315355555133555131153553133111515555555131113333355513535555555553 3111
 9  1355555555553533333533555533353335555555555311133335551135555555555531133
10  33355555555155111111511533333333511553555555531113333555115555555555531111
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13  551355335535555553311331331311113133333313333113313131111133311111113133
14  113553533115553331513535533513335555555553111333355513355555555533113
15  1135535531155353315135355335131353555555551311133335553355555555555551113
16  11355355311553355153355553351333555555555531113333555335555555555553113
17  11355355311553355153355553351311535555555513113333555335555555555551111
18  1133333131153111155313555533515335355535555311113131333111355553355331111
19  551151135531355553511133311311111313311331111111111111111111131111111111
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21  3135311113111313333311555351351335513553533311133331333311133531333311111
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23  33113333533133355533535131331111111315113111111313553331111133111131111 3
24  5311331553311555553135151331513511313111111555333313355131115111115553 3
25  51113313533313555553151315331513331311113311333111133357351111311111133113
26  3111331133353333331313133533531151351353311113353333311335533353331113
27  51113113333313333311333333513111311513533133311113333311133331111333133
28  5311333353313555553351311315513111111313115553333311533111111111113335
29  313333131351111111115155535555555355355331111131333311353533355331113
30  51113111335313333511331113115551113333333113331111333535311333333313113
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32  31333111311131333131151153111511531133333313331111333511131531111111 11
33  531135355551355555335111353151535313513113555333313355513113131355535
34  53113111555115353311131111111311135331355111113111333311355133333153 13
35  5311331555313555553135133311533113511335113331111131353311133111335313
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40  5311331555513555553331113333333315513315313331111333535313113111335511
41  53115311555115555313313113313133111331135333333335111333311153131335355
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43  133355531115313131131515313513335135333333351135555553355555555555333
44  113355531115311111131515313513335135333333315155555553353335555555515 31
45  31335553111531111113351531351333513533333331155555555335333555555551553
46  3111311333351333331135131313113133131331313555331333115131331113111133
47  53115113333313333311333313133111131131111131555353333351111131333353331 33
48  51113111333533333311351315131111131131131315553353333311313133331313133
49  5311531133353333311115331313311131131331515551335355113531333115111113
50  5511513355531555531131513333333313313113155553333511313111331111111133
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52  5533333555531555531333333331333333333131555535351153331133133115331 3
53  31335551111531333113353551351555535533553333311113335151335311333131 1
54  113353313111133333113153531131311513553333333311111133153333531333 11111
55  311151115353135551111511331335115131111333555513333353513133111311131
56  5311511335511555531115131311531113133111335331115111113155533533313135
57  511113135551155555113511153131333513311131353333331313335531311113333 13
58  353131135553155555113511153151311513313113533111113333153513331335311
59  5111311355513555553351311331333511315111155553333333553311531111313311
60  551153355553355553133335333133333331333535553333333133333335311133311
61  5511311355511555531113111311331111311111313355513331531111513135115131 31
62  55113113555115555511311115115331331131111555513311133131311115133533 1
63  5511511555511555551335111311533113131331315551511131331311111153353 31
64  55313315555115555313333113313331333333131555533351113333333153355553 3
65  3315513133331355531313153331311115135313333555131113531113313133553113
66  1135555331153313111131515313313315555533553511133113311133533335553511 1
67  11155331111311111111151511131111533553353135511111333111333313351151 1
68  3133555131353111111131513113311313113333151335333133311133111133331115 1
69  1133555131353333111131533333353331533553315131333333331115311111113311 15
```

Figure 25 -- Nanaimo Alternative P/C Analysis -- Weighted

Comparison Matrix

Alternative	1	2	3	4	5
1	-	3.186	6.703	12.593	12.517
2	9.829	-	6.159	14.250	12.686
3	9.614	0.308	-	11.861	8.723
4	8.747	0.000	0.000	-	.493
5	9.135	0.000	0.000	3.930	-

Proportions Matrix

Alternative	1	2	3	4	5	Z
1	-	.245	.411	.589	.578	.455
2	.755	-	.952	1.000	1.000	.927
3	.589	.048	-	1.000	1.000	.659
4	.411	.000	.000	-	.110	.106
5	.422	.000	.000	.890	-	.328

Combination of Weights and Disruptions

The interdependency weights and disruption matrix (Figure 2) were combined by the methods outlined above. The results of this combination are presented in Figure 25. From the two scales presented therein, one can see that the sites are ranked in the order 2, 3, 1, 5, 4. This ranking indicates that the Harmac South alternative would cause the least environmental disruption. The ordering of the alternatives according to their environmental disruption is, therefore:

(a) Jack Point

(b) Duke Point (a) (with barge channel)

(c) Inner Harbour

(d) Duke Point (b) (without barge channel), and

(e) Harmac South.

In order to test the procedure by which the Z scale was calculated, a proximity matrix formed from the data of Figure 25 was submitted for multidimensional scaling. The resulting scales are presented in Figure 26. The product moment correlation between the two scales is .884, indicating a very close agreement between them.

In order to test the sensitivity of the analysis, the alternative P/C analysis was performed again. This time the interdependencies were assigned equal weights. The results of the check are shown in Figure 27, which indicates that the solution reached is quite stable. The product moment correlation between the weighted and unweighted Z scales is .955.

Figure 26 -- Scale Comparison (Weighted Scales)

Alternative	Z Scale	MDS Scale
1	.455	.766
2	.927	1.339
3	.659	1.339
4	.106	.280
5	.328	.106

Figure 27 -- Nanaimo Alternative P/C Analysis -- Equally Weighted

Comparison Matrix

Alternative	1	2	3	4	5
1	-	9	17	36	35
2	34	-	13	36	34
3	32	1	-	31	25
4	32	0	0	-	0
5	31	0	9	0	-

Proportions Matrix

Alternative	1	2	3	4	5	Z
1	-	.209	.347	.529	.530	.404
2	.791	-	.929	1.000	1.000	.930
3	.653	.071	-	1.000	1.000	.681
4	.471	.000	.000	-	.000	.118
5	.470	.000	.000	1.000	-	.368

Discussion

The close agreement between the ordering of alternatives derived by the foregoing quantitative analysis and that reached via non-analytical processes (Ross, 1974a, p. 58 ff) led to a number of pointed questions. The study group, in ranking the alternatives 1, 2, 3, 5, 4, was not restricted to the information contained in this numerical analysis. The group was free to draw - and probably could not have been prevented from drawing - on a wide variety of additional information which may or may not have had a direct bearing on the problem, but which undoubtedly influenced their "subjective" decisions - whether or not they were aware of it.

The most obvious of these bits of outside information is the question of uniqueness. This analysis has not considered the uniqueness of the particular environmental interactions per se, although, it might be expected that these would be taken care of in the paired-comparison weighting portion of the analysis. Ross' group, when confronted with this suggestion, replied that the lack of information concerning the magnitude of the interactions, as contrasted with their importance, militated against the reduction of the problem to a quantitative exercise. They were unanimous in stating that environmental uniqueness parameters had not been included, and were adamant in their view that uniqueness considerations had played no part in their paired-comparison decisions relating to the environmental interactions. When presented with the quantitative replication of their decision, they expressed agreement with the ranking on purely biological grounds, but stated that the study area's relative scarcity of the estuarine sub-environment and its relative plenitude of rocky beach and wooded upland environments caused the former to have a much greater value.

The choice of the original environmental components, from which Ross' interaction and, subsequently, disruption matrices were prepared, is obviously critical to the whole process of investigating environmental disruption in this manner. It is evident that a careful selection of these components would allow an unscrupulous researcher to bias a study at will, especially if the analysis did not weight the interactions as a base for quantitative analysis.

Although, in this analysis, the judgements of all judges were given equal weight, it is obvious that this need not necessarily be so. One could easily make a case for the differential weighting of the judges' responses according to their experience with the environment under investigation. The problem would occur in assigning the weights for each. Any scheme adopted would be arbitrary, and therefore, difficult to defend.

Another problem in the application of the technique discussed above, lies in the sheer amount of work that the judges must perform. In this study, each of the five judges was required to make 2,346 individual paired comparisons! This was even more onerous than it seems, because of the complex nature of the stimuli being compared.

Finally, the question of testing the final scale derived from the analysis must be discussed. In most modelling attempts, the analyst has a well-defined answer to use as an objective. In this experiment, there was only a rank ordering of the alternatives, and this ordering was necessarily assumed to be correct. Under this assumption, the model performed quite well, for both the equally and unequally weighted trials. Whether it would perform as well in a different situation is another question.

PART C -- SUMMARY

The foregoing sections of this paper have presented a proposed methodology for determining the importance weights of discrete environmental impacts and combining these weights into numeric impact indices, and have described an application of this methodology to a real-world environmental assessment problem. The task of this section is to summarize the technique and the application, and to suggest how it may be employed in future environmental impact assessment problems.

The technique outlined above proceeds from an environmental interaction matrix which identifies dependencies among components of the environmental system being studied. An environmental disruption matrix, prepared from the former, identifies the dependencies for which weights must be determined. These, in turn, are submitted in a paired-comparison format to a panel of judges. The resulting data are reduced to interval weights through the use of multidimensional scaling, and are then combined with the entries of the disruption matrix, again in a paired-comparison mode. The resulting proportions matrix is scaled to derive interval indices of disruption for each alternative course of action.

The foregoing techniques were applied to a problem concerning port development. The results were in close agreement with those of a study group charged with performing the same task by conventional methods of impact assessment.

The future uses of the methodology outlined above depend on the ability of the scientific community to refine methods for the measurement of still more subjective concepts such as uniqueness, and to establish firm quantitative relationships between environmental components. The chief drawback of the outlined methodology is the often large amount of work that is required to judge the importance of the environmental dependencies.

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