CRL INTERNAL REPORT SERIES

No. CRL-63

Research on Self-Synchronization of Phase Codes

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April 1979

The report is the final report on DSS Contract 02SU.36100-8-9519, performed on behalf of the Department of Communications.

Abstract

The report describes the theoretical development of an algorithm to acquire and maintain synchronization for multi-h phase coded signals.

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CHAPTER 1

INTRODUCTION AND BACKGROUND

1.1 Introduction

Recent work in the area of bandwidth-efficient modulation [1-3] has seen the development of a new type of digital modulation known as multi-h phase coding. This modulation which may be regarded as a generalization of fast frequency-shift keying (FFSK) [4] has the property that it offers coding gains of between 2 and 4 dB at little or no expense of bandwidth.

However, as in the case of FFSK, it is a coherent modulation and requires that synchronization be established at the receiver in order that the coding gain may be realized. As with all coherent modulations, it is highly desirable that synchronization be acquired from the information carrying signal with no requirement that separate pilot or carrier signals be transmitted.

This report describes research performed during the period August 1978 to March 1979 to develop means of synchronizing multi-h phase coded signals. In particular we describe the underlying rationale and conceptual development of a particular synchronization algorithm. This algorithm which is analogous to one used in phase-shift keying demodulation and to one developed for use with FFSK [5] may be characterized as a brute force approach to synchronization. On the basis of the work described herein, it now appears that other, simpler and more elegant algorithms are possible and should be investigated.

In the remainder of this chapter we shall examine in part the demodulation of the multi-h phase codes in order to show exactly what coherent or synchronized reference signals are required. Then in Chapter 2, we will describe the rationale underlying the present synchronization algorithm and analyze the spectrum of the qth harmonic of the multi-h phase codes to demonstrate the existence of a spectral line structure which can be used for synchronization purposes. In Chapter 3 we will develop and show in block diagram form the proposed synchronization structure. In addition we will describe a possible procedure for resolving any possible phase ambiguities arising as a result of the synchronization process. Finally in Chapter 4, we will attempt to draw some conclusions and to indicate other promising avenues of research which may lead to other and possibly more effective synchronization algorithms.

1.2 On the Demodulation of Phase Codes

During the ith signalling interval $iT \leq t < (i+1)T$, a multi-h phase coded signal may be written in the form

$$s(t) = \sqrt{\frac{2E}{T}} \cos\left(\omega_c t + a_i\, \omega_i (t-iT) + \phi(iT) + \theta\right) \qquad (1.1)$$

$$iT \leq t < (i+1)T$$

where T is the duration of the signalling interval,

E is the signal energy,

ω_c is the nominal angular carrier frequency ($= 2\pi f_c$ where $f_c >> 1/T$)

and a_i is a binary information symbol having the equiprobable values ± 1. The quantity ω_i in eqn. (1.1) is the angular frequency deviation in the ith signalling interval. It may be written in the form

$$\omega_i = \frac{\pi h_i}{qT} \tag{1.2}$$

where the quantity h_i/q is known as the frequency deviation ratio or modulation index. Turning now to the two phase terms θ and $\phi(iT)$ in eqn. (1.1), the first is a random initial or carrier phase angle, assumed to be unknown and uniformly distributed on $(-\pi, \pi)$, and the second $\phi(iT)$ is known as the excess phase at time $t = iT$. It may be written in the form

$$\begin{aligned} \phi(iT) &= \sum_{k=0}^{i-1} a_k \omega_k T = \frac{\pi}{q} \sum_{k=0}^{i-1} a_k h_k \qquad (i \geq 1) \\ \phi(0) &\triangleq 0 \end{aligned} \tag{1.3}$$

It is clear that the presence of $\phi(iT)$ causes memory in the modulation, and it is this memory which permits coding gain to be achieved in the reception process.

In multi-h phase coding, the frequency deviation ω_i in each signalling interval is chosen from the ordered set $\{\omega_0, \omega_1, \ldots, \omega_{k-1}\}$ such that if ω_i is the deviation during the ith signalling period, $iT \leq t < (i+1)T$, then during the (i+K)th period $(i+K)T \leq t < (i+K+1)T$ we have

$$\omega_i = \omega_{i+K} = \omega_{i(\text{modulo}K)} \tag{1.4}$$

But this is exactly the same as defining the set of modulation indices $\{h_0/q, h_1/q, \ldots, h_{K-1}/q\}$, and during the ith signalling interval $iT < t < (i+1)T$ using the modulation index $h_{i'}/q$ where for convenience of notation, we have defined

$$h_{i'} = h_{i(\text{modulo}K)}.$$

In multi-h phase-coded signalling, the $\{h_i\}$ and q are all required to be integers. In addition we require $h_i < q$, so that the index in each interval is a rational fraction less than unity, implying that the modulated signal is a narrowband FM signal [6].

The information in the signal of eqn. (1.1) is entirely contained in the phase function

$$a_i \, \omega_i (t-iT) + \phi(iT)$$

$$= a_i \frac{\pi \, h_{i'}}{qT} (t-iT) + \frac{\pi}{q} \sum_{k=0}^{i-1} a_k \, h_{k'} \quad (iT \leq t < (i+1)T)$$

It has previously been shown [1-3] that when the indices h_i'/q are all rational fractions the phase function $\phi(t)$ traces a piecewise linear path through a periodic phase trellis. Furthermore from eqn. (1.3), it is clear that at the end of each signalling interval, there are q possible phases uniformly distributed on $(0, 2\pi)$. The period of the phase trellis T_s is defined by

$$T_s = KT \quad \text{for} \quad \Gamma = \sum_{i=0}^{K-1} h_i \quad \text{even}$$

and

$$T_s = 2\,KT \quad \text{for} \quad \Gamma = \sum_{i=0}^{K-1} h_i \quad \text{odd}$$

The number of distinct phase states in the trellis is related to the number of possible phase functions. Lereim [7] has shown that there are

$N = q$ phase positions for Γ even

and

$N = 2q$ phase positions for Γ odd.

The demodulation/decoding of signals of the type defined in eqn. (1.1) is accomplished by coherent demodulation and integrate and dump filtering. The outputs of the integrate and dump filters are then processed using the Viterbi Algorithm [8] to obtain estimates of the transmitted data symbols. This yields maximum likelihood decoding and is described in [1] for multi-h phase codes. A detailed exposition for conventional M-ary CPSK is given in [10], and shows many similarities to the present case.

The Viterbi Algorithm uses the coherently demodulated baseband components to calculate branch metrics. Coherent extraction of the baseband components requires the availability in the receiver of coherent frequencies or tones matched to those of eqn. (1.1). If, for the moment, we assume the availability of a coherent symbol timing clock of period T^{-1} and of a marker of period $(KT)^{-1}$, we then know which pair of signalling frequencies

$$\omega_c \pm \frac{\pi h_{i'}}{qT}$$

is being used in the ith interal $iT \leq t < (i+1)T$. We may then ideally compute branch metrics for this interval, where $r(t)$ represents the received signal, as

$$b(h_{i'},n) = \sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t) \cos\left(\omega_c t + \frac{\pi h_{i'}}{qT}(t-iT) + \frac{n\pi}{q} + \hat{\theta}\right)dt \qquad n = 0, 1, \ldots, (2q-1) \tag{1.5}$$

and

$$b(-h_{i'},n) = \sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t) \cos\left(\omega_c t - \frac{\pi h_{i'}}{qT}(t-iT) + \frac{n\pi}{q} + \hat{\theta}\right) dt \qquad n = 0, 1, \ldots, (2q-1) \tag{1.6}$$

where $n\pi/q$, $n = 0, 1, \ldots, 2q-1$ corresponds to the possible initial excess phase values and hence to the $2q$ trellis states, and $\hat{\theta}$ represents the locally estimated phase which ideally is equal to θ.

If we now define the ideal reference signals in the ith signalling interval as

$$\begin{aligned}
\theta_{i1}(t) &= \sqrt{\frac{2}{T}} \cos\left(\omega_c t + \frac{\pi h_{i'}}{qT}(t-iT) + \hat{\theta}\right) \\
\theta_{i2}(t) &= \sqrt{\frac{2}{T}} \sin\left(\omega_c t + \frac{\pi h_{i'}}{qT}(t-iT) + \hat{\theta}\right) \\
\theta_{i3}(t) &= \sqrt{\frac{2}{T}} \cos\left(\omega_c t - \frac{\pi h_{i'}}{qT}(t-iT) + \hat{\theta}\right) \\
\theta_{i4}(t) &= \sqrt{\frac{2}{T}} \sin\left(\omega_c t - \frac{\pi h_{i'}}{qT}(t-iT) + \hat{\theta}\right)
\end{aligned} \tag{1.7}$$

we may write the branch metrics of eqns. (1.5) and (1.6) as

$$b(h_{i'},n) = X(h_{i'}) \cos\frac{n\pi}{q} + Y(h_{i'}) \sin\frac{n\pi}{q} \tag{1.8}$$

$$n = 0, 1, \ldots, 2q-1$$

$$b(-h_{i'},n) = X(-h_{i'}) \cos\frac{n\pi}{q} + Y(-h_{i'}) \sin\frac{n\pi}{q} \tag{1.9}$$

where

$$\begin{aligned}
X(h_{i'}) &= (r,\theta_{i1}) = \sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t)\ \theta_{i1}(t)\ dt \\
Y(h_{i'}) &= -(r,\theta_{i2}) = -\sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t)\ \theta_{i2}(t)\ dt \\
X(-h_{i'}) &= (r,\theta_{i3}) = \sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t)\ \theta_{i3}(t)\ dt \\
Y(-h_{i'}) &= (r,\theta_{i4}) = \sqrt{\frac{2}{T}} \int_{iT}^{(i+1)T} r(t)\ \theta_{i4}(t)\ dt
\end{aligned} \tag{1.10}$$

Equations (1.8) to (1.10) show that the branch metrics required by the Viterbi Algorithm are easily calculated using in each signalling interval the outputs of the four clocked integrate and dump circuits represented by eqns. (1.10).

Also from eqns. (1.7) to (1.10), it is clear that three orders of synchronization are required, namely

a) phase synchronization of each of the signalling frequencies

$$\omega_c \pm \frac{\pi\ h_i}{qT} \quad (i = 0, 1, \ldots, K-1) \tag{1.11}$$

b) symbol timing or synchronization of the data clock at frequency 1/T

c) interval lock modulo K, so that the receiver knows in each signalling interval which pair of frequencies (cf. (1.11)) to use for coherent demodulation. this involves synchronization to a clock at frequency 1/KT.

In practice it is very difficult to generate exactly the signals of eqn. (1.7). It is difficult to guarantee a zero-crossing (for $\hat{\theta} = 0$) exactly at the beginning of each baud. In addition because of the nature of synchronization processes there will usually be a phase ambiguity of some form. The first of these problems turns out, as we shall now see, not to matter. The second is more difficult and will be dealt with in Chapter 3.

In practice, it turns out (cf. Chapter 3) to be fairly straightforward to regenerate the signals

$$\begin{aligned}
\hat{\theta}_{i1}(t) &= \sqrt{\frac{2}{T}} \cos\left(\omega_c t + \frac{\pi h_{i'}}{qT} + \frac{\alpha_i \pi}{q} + \hat{\theta}\right) \\
\hat{\theta}_{i2}(t) &= \sqrt{\frac{2}{T}} \sin\left(\omega_c t + \frac{\pi h_{i'}}{qT} + \frac{\alpha_i \pi}{q} + \hat{\theta}\right) \\
\hat{\theta}_{i3}(t) &= \sqrt{\frac{2}{T}} \cos\left(\omega_c t - \frac{\pi h_{i'}}{qT} + \frac{\alpha_i' \pi}{q} + \hat{\theta}\right) \\
\hat{\theta}_{i4}(t) &= \sqrt{\frac{2}{T}} \sin\left(\omega_c t - \frac{\pi h_{i'}}{qT} + \frac{\alpha_i' \pi}{q} + \hat{\theta}\right) \\
& \qquad i = 0, 1, \ldots, K-1
\end{aligned} \tag{1.12}$$

where α_i, α_i' represent 2q-fold phase ambiguities with α_i and α_i' being unknown integers in the set (0, 1, ..., 2q-1). If we then use these signals, instead of the ideal ones, in the coherent demodulation, integrate and dump operation specified by eqns. (1.10), we find that the branch metrics of eqns. (1.8) and (1.9) have the form

$$b(h_{i'},n) = \hat{X}(h_{i'}) \cos \frac{\pi}{q} (n-ih_{i'}-\alpha_i) + \hat{Y}(h_{i'}) \sin \frac{\pi}{q} (n-ih_{i'}-\alpha_i)$$

$$n = 0, 1, \ldots, 2q-1 \qquad (1.13)$$

and

$$b(-h_{i'},n)=\hat{X}(-h_{i'})\cos[\frac{\pi}{q} (n-ih_{i'}+\alpha_i')]+\hat{Y}(-h_{i'})\sin \frac{\pi}{q} (n-ih_{i'}+\alpha_i')$$

$$n = 0, 1, \ldots, 2q-1 \qquad (1.14)$$

It turns out that the effect of the $ih_{i'}$ terms is easy to compensate. It causes a rotation which, because each frequency pair is used in only every Kth signalling period, may except for an initial value be subtracted out. The initial value will be absorbed into the carrier phase estimate $\hat{\theta}$. However, the ambiguity terms, α_i and α_i', which follow no predictable pattern, cannot be simply tracked out. But, they do tend to remain constant over long periods of time, and by using the information within the Viterbi Algorithm can be compensated (cf. Chapter 3).

In order to complete the demodulation/decoding process we make use of the Viterbi Algorithm. Assuming a set of path metrics P_n, $n = 0, 1, \ldots, 2q-1$, initially cleared to zero, then using Schonhoff's notation [10], we may express the Viterbi Algorithm in the ith signalling interval $iT \leq t < (i+1)T$ as

i) update the path metrics: $\text{new } P_n = \max_{n'} [\text{old } P_{n'} + b(m,n')]$

$m = \pm h_{i'}$

$n' = 0,1,\ldots,2q-1$

ii) update path histories: new H_n = {old $H_{n'}$, data bit corresponding to branch from n' to n}

iii) output the oldest bit of the most likely path.

Because the values of $m = \pm h_{i'}$ to be used in each signalling interval are known, and because each pair of values is used only in every Kth interval, it is a straight forward matter to track out the $ih_{i'}$ term in eqns. (1.8) and (1.9) by in each interval offsetting the values of n by $Kh_{i'}$.

In this section we have shown the types of synchronizing information required and the coherent signals which must be regenerated in the receiver structure. In the remainder of the report we will be concerned with the generation of these signals.

CHAPTER 2

RATIONALE FOR THE SYNCHRONIZATION ALGORITHM

2.1 Introduction

In any system used to acquire and maintain synchronization to a signal using information in the signal itself, it must be established that there is non-zero average power at the frequency to which it is desired to synchronize. If such is not the case then some nonlinear operation must be performed to create this non-zero power or spectral line condition at the frequency of interest or at some harmonic of it. For example, in previous work with the fast frequency-shift-keying (FFSK) modulation [5], spectral lines suitable for use in synchronization were created at double the signalling frequencies by squaring the received FFSK signal. This nonlinear operation is necessary because the FFSK signal contains no lines in its spectrum, and therefore no coherent frequency component or spectral line.

Turning now to the multi-h phase coded signals, a careful spectral study [7] has shown that they contain no spectral lines, provided that the modulation characteristic function defined as

$$C(1;KT) = \prod_{i=0}^{K-1} \cos \frac{\pi h_i}{q} \qquad (2.1)$$

is always strictly less than 1 in magnitude. For the codes of interest, $h_i/q < 1$ for all i, and this condition is clearly satisfied.

In order to generate a line structure, we must, therefore, perform a nonlinear operation on the received signal. From eqn. (1.1), we may write the received signal during the ith signalling interval as

$$s(t) = \sqrt{\frac{2E}{T}} \cos \left[\omega_c t + a_i \frac{\pi h_i}{qT} (t-iT) + \phi(iT) + \theta\right] \tag{2.2}$$

where

$$\phi(iT) = \frac{\pi}{q} \sum_{k=0}^{i-1} a_k h_k, \tag{2.3}$$

By passing $s(t)$ through a qth order power-law nonlinearity and filtering the output to obtain the qth harmonic we obtain

$$s_q(t) = A \cos \left[q\omega_c t + a_i \frac{\pi h_i}{T} (t-iT) + q\phi(iT) + q\theta\right] \tag{2.4}$$

$$iT \leq t < (i+1)T$$

where A is a suitable amplitude factor. It is clear that eqn. (2.4) is a multi-h phase coded signal having the set of integer modulation indices $h_0, h_1, \ldots, h_{K-1}$, and for which the modulation characteristic function is

$$C_q(1;KT) = \prod_{k=0}^{K-1} \cos \pi h_k \tag{2.5}$$

Since the h_k are all integers, it is clear that

$$|C_q(1;KT)| = 1$$

so that $S_q(t)$ has a line structure in its power spectrum. In the next section we will determine the characteristics of this line structure.

2.2 Spectral Analysis of the qth Harmonic

Because s(t) as defined in eqn. (1.1) is actually a digital FM signal, it may be written as

$$s(t) = \sqrt{\frac{2E}{T}} \cos \left[\omega_c t + \int_0^t \psi(t')\, dt'\right]$$

where

$$\psi(t) = \frac{\pi}{qT} \sum_{j=0}^{\infty} a_j h_{j'} g(t-jT)$$

and g(t) is a unit-amplitude rectangular pulse on [0,T]. It is then clear that the qth harmonic of s(t) is given by

$$s_q(t) = A \cos \left[q\omega_c t + \int_0^t \theta(t')\, dt'\right] \tag{2.6}$$

where

$$\theta(t) = q\, \psi(t) = \frac{\pi}{T} \sum_{j=0}^{\infty} a_j h_{j'} g(t-jT) \tag{2.7}$$

This phase function may readily be written in the form

$$\theta(t) = \sum_{r=0}^{\infty} u_r (t - r T_b) \tag{2.8}$$

where

$$u_r(t-rT_b) = \frac{\pi}{T} \sum_{m=0}^{K-1} a_{m+rK} h_{m'} g(t - mT - rT_b) \tag{2.9}$$

and $T_b = KT$ is loosely referred to as the superbaud duration.

Rather than work directly with $s_q(t)$, let us consider the complex signal

$$x(t) = A \exp i \left(q\omega_c t + \int_0^t \theta(t')\, dt'\right) \tag{2.10}$$

where $s_q(t) = \mathrm{Re}\{x(t)\}$. Then following Lucky [6], we may write the power spectral density of $x(t)$ for positive frequency as

$$G(f) = \lim_{\lambda\to\infty} 2 \frac{G_\lambda(f)}{\lambda} \qquad f > 0 \tag{2.11}$$

where

$$G_\lambda(f) = E\left\{\left|\int_0^\lambda x(t)\, e^{-2\pi i f t}\, dt\right|^2\right\} \tag{2.12}$$

Then letting $\lambda = NT_b$, it is a straightforward matter to expand eqn. (2.12) to the form

$$G_N(f) = E\left\{\sum_{k=0}^{N-1}\sum_{s=0}^{N-1} \int_{kT_b}^{(k+1)T_b} \int_{sT_b}^{(s+1)T_b} x(t_1)\, x^*(t_2)\right.$$

$$\left. e^{-2\pi i f t_1}\, e^{2\pi i f t_2}\, dt_1\, dt_2\right\} \tag{2.13}$$

Next consider the quantity

$$Q_k(f) = \int_{kT_b}^{(k+1)T_b} x(t)\, e^{-2\pi i f t}\, dt\ .$$

Substituting for $x(t)$ and making use of eqn. (2.8) in this we obtain

$$Q_k(f) = A \int_{kT_b}^{(k+1)T_b} e^{-2\pi i(f-qf_c)t} \exp(i \int_0^t \sum_{r=0}^{\infty} u_r(t'-rT_b)dt')dt$$

where $\omega_c = 2\pi f_c$. Replacing t with $t-kT_b$ in this and letting $\beta = 2\pi(f-qf_c)$, we may write

$$Q_k(\beta) = A \int_0^{T_b} e^{-i\beta kT_b} e^{-i\beta t} \exp(i \sum_{r=0}^{\infty} \int_0^{t-(r-k)T_b} u_r(\tau)dt$$

where we note that $u_r(\tau) = 0$ for $\tau < 0$. Finally making use of the fact that $u_r(\tau) = 0$ for $\tau < 0$ and $\tau > T_b$, we may with a little algebra arrive at

$$Q_k(\beta) = A\ e^{-i\beta kT_b} \prod_{r=0}^{k-1} \exp\ (ib_r)\ F_k(\beta) \qquad (2.15)$$

where

$$b_r = \pi \sum_{m=0}^{K-1} a_{m+rK}\ h_m, \qquad (2.16)$$

$$F_k(\beta) = \int_0^{T_b} e^{-i\beta t} \exp\ [ib_k(t)]\ dt \qquad (2.17)$$

$$b_k(t) = \sum_{m=0}^{K-1} a_{m+rK}\ h_m, \frac{\pi}{T} \int_0^t g(\tau-mT)\ d\tau \quad 0 \leq t < T \qquad (2.18)$$

Substituting into eqn. (2.13) and regrouping the terms, we then obtain

$$G_N(\beta) = A^2\, E\, \{ \sum_{k=0}^{N-1} |F_k(\beta)|^2$$

$$+ 2\, \mathrm{Re}\, [\sum_{s=0}^{N-3} \sum_{k=s+2}^{N-1} e^{-i\beta(k-s)T_b} F_k(\beta) [F_s^*(\beta)\, e^{ib_s}] \exp[i \sum_{r=s+1}^{k-1} b_r]$$

$$+ \sum_{s=0}^{N-2} e^{-i\beta sT_b} F_{s+1}(\beta)\, F_s^*(\beta)\, e^{ib_s}]\} \qquad (2.19)$$

We now want to compute the expected value of this expression. Because the data sequence $\{a_i\}$ consists of independent, identically distributed binary digits with values ± 1, we may compute the expected values of the various terms separately. First we define

$$C_q(\alpha;T_b) = C_q(\alpha;KT) \triangleq E\, \{\exp\, i\alpha b_r\} \qquad (2.20)$$

Making use of eqn. (2.16) in this we readily obtain

$$C_q(\alpha;KT) = \prod_{m=0}^{K-1} \cos\, (\alpha\pi h_m) \qquad (2.21)$$

Setting $\alpha = 1$ in eqn. (2.21), we obtain the modulation characteristic function of eqn. (2.5) as

$$C_q(1;KT) = \prod_{m=0}^{K-1} \cos\, \pi h_m, = \prod_{m=0}^{K-1} (-1)^{h_m}, = e^{i\Gamma\pi} \qquad (2.22)$$

where

$$\Gamma \triangleq \sum_{m=0}^{K-1} h_m.$$

Next we compute from eqn. (2.17)

$$F(\beta) = E\{F_k(\beta)\} = E\, \{ \int_0^{KT} e^{-i\beta t} \exp\, i\, b_k(t)\, dt\}$$

After some manipulation this may be reduced to the form

$$F(\beta) = \sum_{p=0}^{K-1} e^{-i\beta pT} \prod_{m=0}^{p-1} \cos(\pi h_m) \int_0^T e^{-i\beta t} \cos \frac{\pi h_p t}{T} dt \tag{2.23}$$

The integral in eqn. (2.23) is readily evaluated, and we finally obtain

$$F(\beta) = 2\beta \sum_{p=0}^{K-1} e^{-i\beta pT} \exp[\frac{-i}{2} (\beta T - \pi h_p)] [\prod_{m=0}^{p-1} (-1)^{h_m}] \frac{\sin 1/2 \ (\beta T - \pi h_p)}{\beta^2 - (\pi h_p/T)^2} \tag{2.24}$$

We now want to evaluate the quantity

$$F_b^*(\beta) = E \ \{F_s^*(\beta) \ e^{ib_s}\}.$$

Following a similar line of reasoning to that used in deriving eqn. (2.24), we find that for the present case of integer modulation indices

$$F_b^*(\beta) = C_q(1;KT) \ F^*(\beta) \tag{2.25}$$

We note that the results in both eqns. (2.24) and (2.25) are independent of the indices (k or s) of summation.

Substituting eqns. (2.24) and (2.25) into eqn. (2.19), it is a straightforward matter to obtain

$$G_N(\beta) = A^2 \sum_{k=0}^{N-1} |F_k(\beta)|^2 + 2 A^2 \ \mathrm{Re} \ \{F(\beta) \ F_b^*(\beta)$$

$$[(N-1) \ e^{-i\beta KT} + \sum_{s=0}^{N-3} \sum_{k=s+2}^{N-1} e^{-i\beta(k-s)KT} \ C^{k-s-1}(1;KT)]\}$$

Substituting this into eqn. (2.12), and taking the limit as $N\to\infty$, we obtain the desired power spectral density in the form

$$G(\beta) = \frac{2A^2}{KT} \{P(\beta) + 2 \text{ Re } [F(\beta) F_b^*(\beta) e^{-i\beta KT} [1 + \Lambda e^{i\beta KT}]\} \tag{2.26}$$

where

$$P(\beta) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} E \{|F_k(\beta)|^2\}$$

and

$$\Lambda = \lim_{N\to\infty} \frac{1}{N} \sum_{s=0}^{N-3} \sum_{k=s+2}^{N-1} e^{-i\beta(k-s)KT} C^{k-s-1}(1;KT) \tag{2.27}$$

The $P(\beta)$ term results in a continuous component of the power spectral density, and has no line structure. It is therefore of no interest for synchronization purposes and will not be considered further in this report.

For integer modulation indices the limit in eqn. (2.27) is a distribution sum which may be evaluated following Barnard [9] so that we finally obtain the desired power spectral density in the form

$$G(\beta) = \frac{2A^2}{KT} \{P(\beta)+|F(\beta)|^2 [2\pi \sum_{n=-\infty}^{\infty} \delta(\beta KT-\arg C_q(1;KT)-2\pi n)-1]\} \tag{2.28}$$

where

$$\arg Cq(1;KT) = \begin{cases} 0 & \text{if} \quad \Gamma = \sum_{i=0}^{K-1} h_i \text{ is even} \\ \pi & \text{if} \quad \Gamma = \sum_{i=0}^{K-1} h_i \text{ is odd} \end{cases} \tag{2.29}$$

The second term in (2.28) contains an infinite spectral line structure where the power in each line is proportional to the value of $|F(\beta)|^2$ at the frequency of the line where

$$|F(\beta)|^2 = 4\beta^2 \sum_{p=0}^{K-1} \sum_{q=0}^{K-1} \cos\left[\beta(p-q)T + \frac{\pi}{2}(h_p - h_q)\right]$$

$$\cdot \left[\prod_{m=0}^{p-1} \cos\pi h_m\right] \left[\prod_{m=0}^{q-1} \cos\pi h_m\right] \frac{\sin 1/2\,(\beta T - \pi h_p)}{\beta^2 - (\pi h_p/T)^2} \cdot \frac{\sin 1/2\,(\beta T - \pi h_q)}{\beta^2 - (\pi h_q/T)^2} \qquad (2.30)$$

The positions of the spectral lines are defined from (2.28) by the equation

$$\beta = \frac{\arg Cq(1;KT)}{KT} + \frac{2\pi n}{KT} \qquad (-\infty < n < \infty) \qquad (2.31)$$

This may be rewritten in terms of actual frequencies as

$$f = \begin{cases} qf_c + \frac{n}{KT} & r \text{ even} \\ qf_c + \frac{(2n+1)}{2KT} & r \text{ odd} \end{cases} \qquad -\infty < n < \infty \qquad (2.32)$$

In the next chapter, we will deal with how to use this line structure to obtain synchronization for the multi-h phase-coded signals. However, before doing so, it is of interest to consider the FFSK system which after squaring ($q = 2$) yields a digital FM signal with $K = 1$; $h_0 = 1$ and $r = 1$. This can readily be seen to yield spectral lines at

$$f = 2f_c \pm \frac{1}{2T},$$

and an examination of eqn. (2.30) shows that all other spectral lines fall at the nulls of $|F(\beta)|^2$. The two surviving lines have previously been used [5] to synchronize the FFSK system.

CHAPTER 3

SYNCHRONIZATION STRUCTURES

3.1 Structures

From eqn. (2.32), we see that the spectrum of $s_q(t)$ contains an infinite structure of spectral lines spaced by 1/KT Hz where K is the constraint length of the particular phase code being considered. The first problem to be considered is which of these lines should be used for synchronization.

In principle, any non-zero line in the spectrum can be used for synchronization. In practice we want to use those lines which contain the most energy, and these will be the lines which fall at or near a maximum of $|F(\beta)|^2$ as defined in eqn. (2.30).

In general, it has been found [7] that for practical phase codes the modulation indices

$$\frac{h_0}{q}, \frac{h_1}{q}, \ldots, \frac{h_{K-1}}{q}$$

tend to be closely spaced rational fractions, and that the numerators the h_0, h_1, ..., h_{K-1} tend to be a set of consecutive or near-consecutive integers with at most one interior member of the set missing. The consequence of this is that $|F(\beta)|^2$ has a zone where it is maximum or close to maximum in the neighborhood of the normalized frequencies

$$\beta T = \pi h_p \quad p = 0, 1, \ldots, K-1$$

In fact in all cases of interest, we find this maximum occurring at or near

$$\beta T = \pi \bar{h}$$

where $\bar{h}$ is the mean index

$$\bar{h} \triangleq \frac{1}{K} \sum_{k=0}^{K-1} h_k .$$

Based on this it appears that we want to use those lines which occur in the vicinity of the normalized qth harmonics $\pm h_0/T$, $\pm h_1/T$, ..., $\pm h_{K-1}/T$ of the signalling frequencies. In most cases, the line structure will not coincide with the locations of the qth harmonics of the signalling frequencies (the possible exceptions are the K=1 and K=2 cases), but as it turns out, the line structure in the spectrum of $s_q(t)$ can be used to generate the required synchronization information.

To begin with, let us consider the recovery of interval lock modulo K. For this purpose, we consider two adjacent lines in the vicinity of the maximum of $|F(\beta)|^2$. In all cases, there are at least three strong lines in the vicinity of the maximum of $|F(\beta)|^2$. Furthermore, there appears to be one line which for any phase code always occurs at the maximum value of $|F(\beta)|^2$ with at least one strong line to either side.

Let m be the value of n in eqn. (2.32) at which the maximum value of $|F(\beta)|^2$ occurs, and consider also the lines at $n = m-1$ and $n = m+1$. If we now employ two high-gain phase-locked loops (PLL) having their voltage controlled oscillators (VCO) centered at the free-running frequencies

$$f_c = qf_c + \frac{m-1}{KT} \quad \text{and} \quad f = qf_c + \frac{m}{KT} \qquad (r \text{ even})$$

or

$$f = qf_c + \frac{2(m-1)+1}{2KT} \quad \text{and} \quad f = qf_c + \frac{2m+1}{2KT} \qquad (r \text{ odd})$$

we may mix the VCO outputs and pass the mixer output through a low-pass filter to obtain a phase-coherent signal at the frequency $1/KT$. This is the signal required for interval lock modulo K.

Two points about this signal are of interest. First, the recovered signal at frequency $1/KT$ is ambiguity-free since no frequency-division process is involved in its generation. Second it is a very straightforward matter to generate from this signal the symbol timing clock at frequency $1/T$ Hz by multiplying the frequency $1/KT$ by K where for practical phase codes $K = 2$, 3 or 4. We have, therefore, solved in a very straightforward manner, two of the three synchronization problems. It remains to solve the problem of coherently recovering the signalling frequencies $f_c \pm h_i/2qT$.

First let us consider the recovery of the nominal carrier frequency f_c. This is readily accomplished, albeit with a phase ambiguity, from the line structure in the spectrum of $s_q(t)$. An examination of $G(\beta)$ in eqn. (2.28) reveals that it is symmetric about $\beta = 0$ and therefore about $f = qf_c$. Furthermore, the spectral lines are symmetrically located about $f = qf_c$ and $|F(\beta)|^2$ is symmetric about $\beta = 0$. Therefore to recovery the carrier at frequency f_c Hz, we again use two phase-locked loops, one with a free-running frequency of

$$f = \begin{cases} qf_c + \frac{m}{KT} & r \text{ even} \\ qf_c + \frac{2m+1}{2KT} & r \text{ odd} \end{cases}$$

and the other with a free-running frequency of

$$f = \begin{cases} qf_c - \frac{m}{KT} & r \text{ even} \\ qf_c - \frac{2m+1}{2KT} & r \text{ odd} \end{cases}$$

THe VCO outputs from these two loops are then mixed and the mixer output is high-pass filtered to produce the frequency $2qf_c$ Hz. This is then divided by 2q, possibly using a further phase-locked loop to produce the nominal carrier frequency in the form

$$A \cos \left(2\pi f_c t + \frac{\pi\alpha}{q} + \hat{\theta}\right)$$

where $\hat{\theta}$ is the estimate of the received carrier phase and $\pi\alpha/q$ represents a 2q-fold phase ambiguity with α being an integer in the range 0, 1, ..., 2q-1.

In order now to regenerate the actual signalling frequencies

$$f_c \pm \frac{h_i}{2qT} \qquad i = 0, 1, \ldots K-1 \qquad (3.1)$$

we first divide the unambiguous clock signal at frequency 1/T Hz by 2q to obtain a signal at frequency 1/2T Hz. As a result of the division, the resulting signal will have a 2q-fold phase ambiguity. This portion of the synchronization system, which generates the interval clock at 1/KT Hz, the symbol clock at 1/T Hz, the nominal carrier frequency of f_c Hz and the signal at frquency 1/2qT Hz, is shown in block diagram form in Figure 3.1.

The recovered carrier at f_c Hz and the signal at frequency 1/2qT are now used to drive a signal generation structure to regenerate the actual signalling frequencies specified in (3.1). The structure of this generator is shown in Figure 3.2, and except for the number of frequencies to be generated is common to all phase codes. It consists essentially of an open loop chain of multipliers, $\pi/2$ phase shifters and filters. We note that very high Q's may be required in the filters which may, therefore, have to be implemented as phase-locked loop structures. The signalling frequency components appearing at the output of this structure will have the ambiguous form given by eqns. (1.12). These equations indicate a 2q-fold phase ambiguity which in general will be different at each signalling frequency. For successful demodulation/decoding the ambiguity must be resolved. This is the subject of the next section.

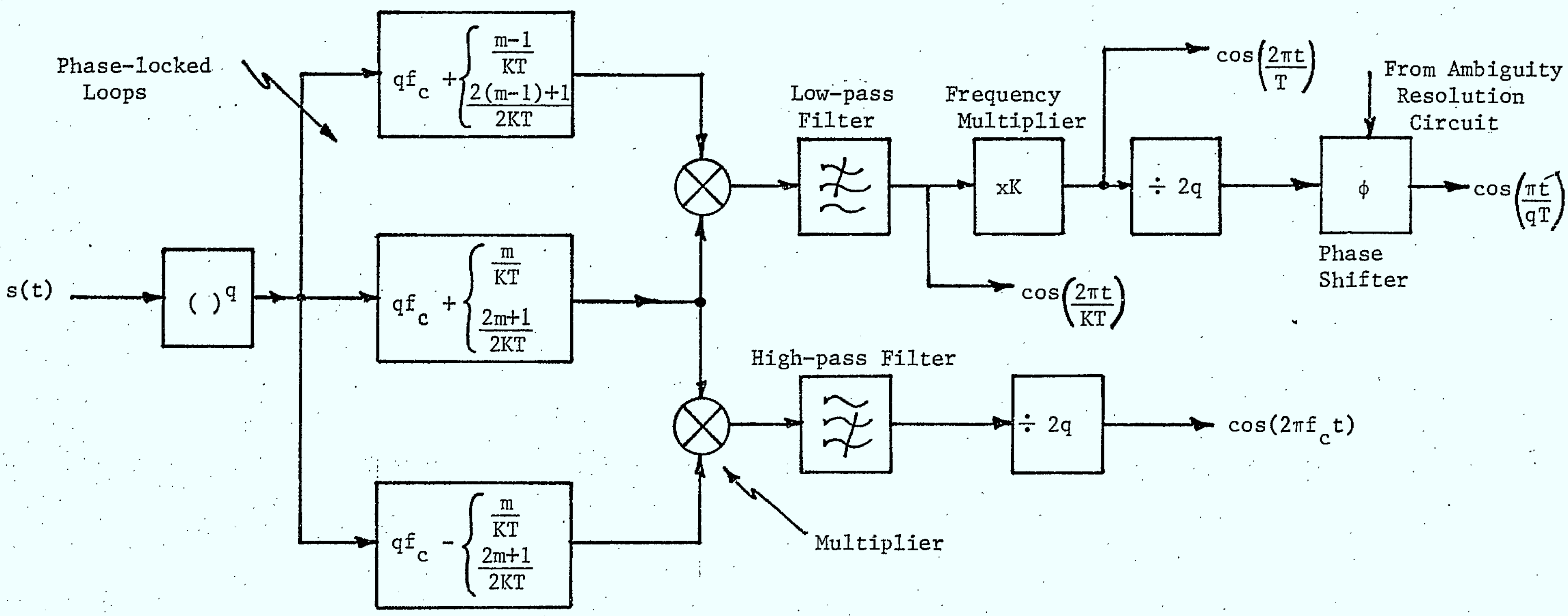

Figure 3.1: Block diagram of subsystem for carrier and clock recovery.

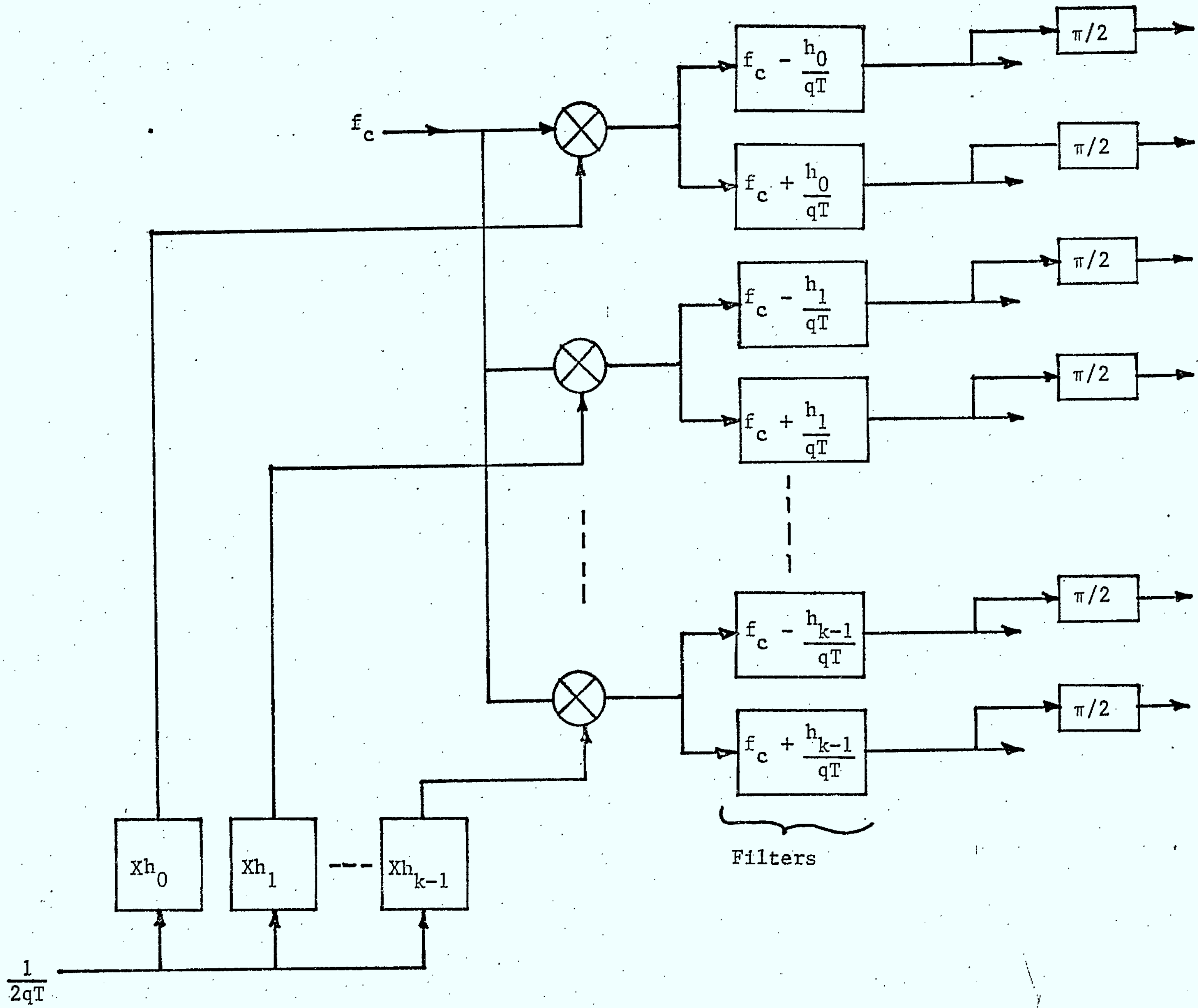

Figure 3.2: Basic form of the frequency generator used to derive signalling frequencies from the outputs of the phase and timing recovery circuits.

3.2 Phase Ambiguity Resolution

Recall the expressions (1.13) for the branch metrics at the end of the ith signalling interval $iT \leq t < (i+1)T$. These may be written as

$$b(h_{i},n) = \hat{X}(h_{i}) \cos \frac{\pi}{q} (n-ih_{i}-\alpha_i) + \hat{Y}(h_{i}) \sin \frac{\pi}{q} (n-ih_{i}-\alpha_i)$$

$$n = 0, 1, \ldots, 2q-1$$

$$\alpha_i \in \{0, 1, \ldots, 2q-1\}$$

and

$$b(-h_{i},n) = \hat{X}(-h_{i}) \cos \frac{\pi}{q} (n-ih_{i}+\alpha_i') + \hat{Y}(-h_{i}) \sin \frac{\pi}{q} (n-ih_{i}+\alpha_i')$$

$$n = 0, 1, \ldots, 2q-1$$

$$\alpha_i' \in \{0, 1, \ldots, 2q-1\}$$

It is clear from these that the correct branch metrics are contained in this set but with an unknown offset in the value of n. In other words the ordering of the metrics with respect to n has been "scrambled". The effect of this scrambling will be to cause errors in the receiver output data, and hence the phase ambiguity must be resolved or at least reduced to a value which is common to all signalling frequencies.

If we examine in detail the phase trellis structure as used by the Viterbi Algorithm for decoding the phase codes, we find that the effect of the unknown phase ambiguities is to cause discontinuous jumps in the most likely path through the trellis. In other words, if either the phase ambiguity is zero or is a common value at all signalling frequencies then the most likely path through the phase trellis will be (piece-wise) continuous.

These statements suggest a possible structure to generate ambiguity resolving signals, and a block diagram is shown in Figure 3.3. The block labelled ambiguity resolution circuit monitors the continuity of the most likely path through the trellis by comparing the present n_i, the identity of the terminal node in the ith signalling interval corresponding to the most likely path, with the two possible terminating node identities as calculated from the previous n_{i-1}. A match indicates continuity and no match indicates discontinuity. When discontinuity is detected, the phase of the cos $(\pi t/qT)$ signal (cf. Fig. 3.1) used to drive the signal generator of Figure 3.2 is step-changed by π/q radians. This procedure is repeated in each signalling interval until continuity is detected by the ambiguity resolution circuit over a succession of signalling intervals.

In order to verify that this algorithm actually works to resolve phase ambiguity, a computer simulation was used. This simulation actually simulates a receiver for multi-h phase codes under the assumption that the phase coherent, but ambiguous signals of eqns. (1.12) have been regenerated.

The simulation was run for several phase codes at different values of input signal-to-noise ratio (SNR) ranging from 3.0 dB to 7.0 dB. The reason for choosing such low SNR values was that if the ambiguity resolution algorithm will work effectively in this range it will work

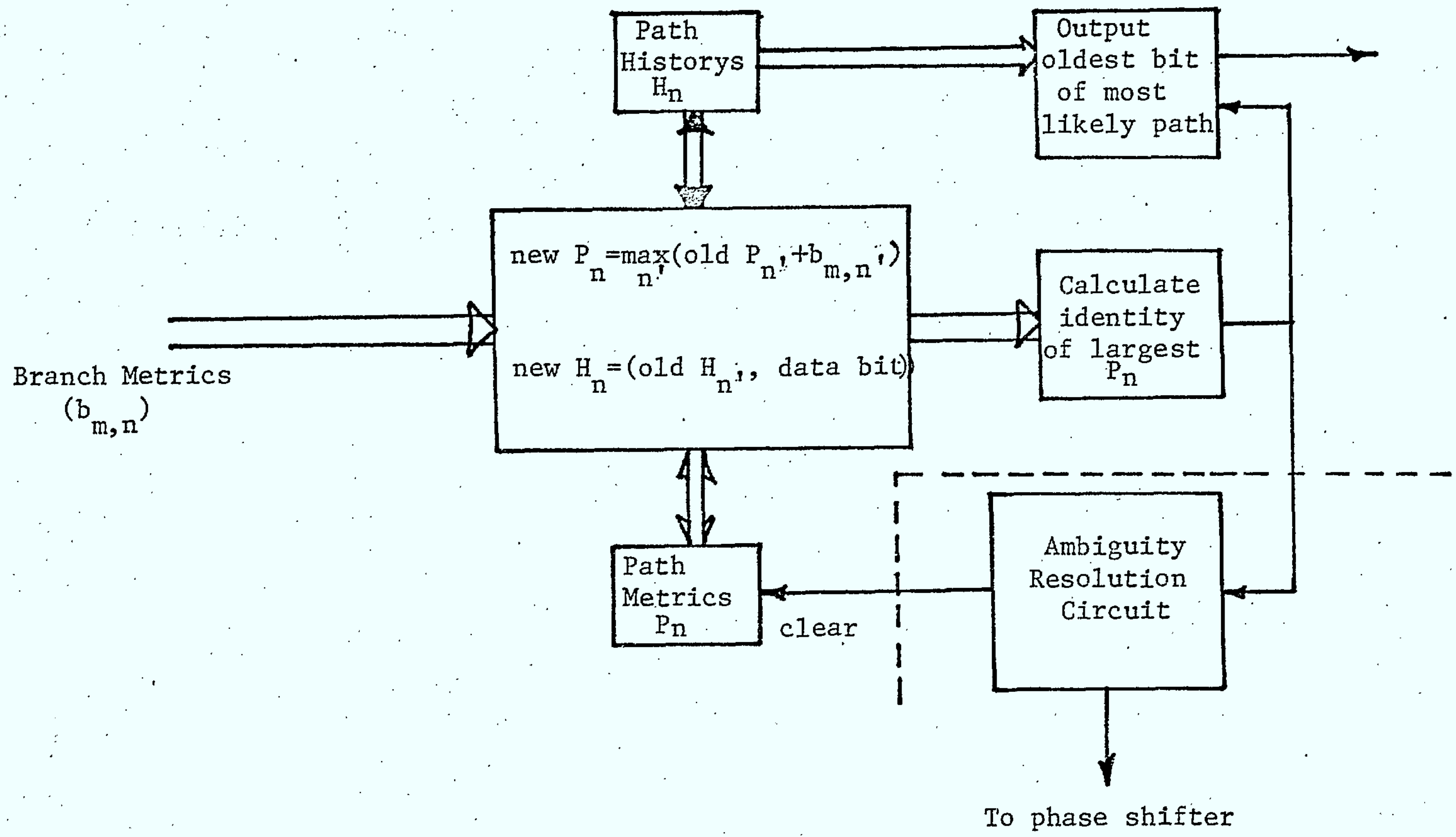

Figure 3.3: Illustration of algorithm for ambiguity resolution.

even more effectively at high SNR values. We found as a result that provided the number of bits of quantization Q_T (see below) is sufficient, then for SNR values of 6 dB and above phase ambiguities were virtually always resolved.

The only problem with this algorithm is that we found that it typically requires 100 to 300 signalling intervals at the beginning of a transmission in order to ensure successful resolution of ambiguity. Provided the system is operating in a continuous rather than a burst mode, this represents a fairly small penalty. This is particularly true in view of the fact that once the phase ambiguity is resolved it tends to stay that way or if it does drop out it tends to shift by $\pm\ \pi/q$ which is easily corrected by the algorithm.

As a side result of the simulation, we have been able to determine the number of bits of quantization required in the Viterbi Algorithm. We found that error-rate performance within a few tenths of a dB of the theoretical results in [1] were obtained for any phase code provided that the number of bits of quantization Q_T is such that

$$Q_T > \log_2 (2q) = 1 + \log_2 q$$

Hence for an eight-phase ($q = 8$) phase code we should use a minimum of 5 bits of quantization. In practice we have found that using more than $2 + \log_2 q$ bits of quantization results in very small improvements, and hence we conclude that for all practical purposes $2 + \log_2 q$ bits is sufficient.

CHAPTER 4

CONCLUSIONS AND SUGGESTED FURTHER WORK

In this study we have developed a self-synchronization algorithm for phase and timing recovery from multi-h phase coded signals. The algorithm presented in this report is based on the use of the spectral line structure generated by raising the received signal to the qth power and filtering to obtain the qth harmonic zone. In this respect, it is analogous to the procedure often used for M-ary phase-shift-keyed systems. The algorithm so developed will work for any multi-h phase code with its complexity governed only by the constraint length of the code and hence by the number of signalling frequencies which must be regenerated.

One problem with this algorithm is that in general each regenerated signalling frequency will contain a distinct 2q-fold phase ambiguity. Hence an algorithm to resolve this based on monitoring the continuity of the most likely path through the phase trellis was developed. This algorithm was found to work for any phase code provided the number of bits of quantization Q_T used in the Viterbi Algorithm was sufficient for the code being used and provided the received SNR was 6 dB or higher.

Unfortunately, as can be seen from Figures 3.1 to 3.3, the implementation of this self-synchronization algorithm tends to be rather complex. However, based both on the work in the present report and on the realization that multi-h phase coded signals are actually digital FM signals and may therefore be incoherently demodulated using a discriminater, it now appears that simpler algorithms based on remodulation or data-aided techniques [11] are possible. It also appears that such algorithms may be very close to the optimum estimator of phase and timing as predicted by estimation theory [11]. It also appears that such an algorithm could lead to a much simpler resolution of any phase ambiguity since the output of the frequency discriminator is inherently ambiguity-free. This approach to the synchronization study is, however, beyond the scope of the present study.

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