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A STUDY OF THE PRODUCTIVE FACTOR AND FINANCIAL CHARACTERISTICS OF TELEPHONE CARRIERS

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The Canadian Department of Communications (DOC), contracted the Institute of Applied Economic Research (IAER), of Concordia University to carry out a study of the productive and financial characteristics for carriers operating in the telephone industry.

The work was done at the IAER during the latter part of 1976 and the first part of 1977, by the following team of researchers:

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We would like to thank the members of the DOC for their cooperation, and for the beneficial discussions with us while carrying out this study.

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EXECUTIVE SUMMARY

1. Purpose

This study purposes to evaluate the interaction of the productive factor and financial characteristics of telephone carriers. A model is developed, estimated, and simulated depicting the interaction of the corporate decision mechanisms with regards to pricing, output, factor and financial requirements. Conclusions are drawn as to the nature of the determinants of demand and hence revenues, the nature of the production processes, the ability of the carriers to affect the costs of their alternative financing instruments, the form of regulation, and finally the ability of the model to simulate the past behaviour of Bell Canada.

2. Structure

A general theoretical model is developed of a privately owned telephone carrier which maximizes profit. The profit maximizing utility is envisioned as being a monopolist in its product, and a competitor in it's labour markets. The carrier is financed by both debt and equity and has some degree of monopoly power in these financial capital markets. Indeed, this is one of the novel elements of the model, in the sense that the firm is able to influence the rate of return to debtholders and shareholders. Because of the imperfections in these markets, there is a difference between the average costs of the different methods of financing and the marginal costs of financing, with, of course, the marginal costs being the elements which are used in determining the portfolio composition. There is a tax on the net income of the firm, with interest payments on debt being tax exempt while dividend payments are not. This fiscal policy influences the relative marginal costs of debt and equity and so affects the leverage and capital budget decisions. Finally, in the general model the utility is restricted as to the maximum rate of return obtainable on the physical capital stock, which manifests itself in the constraint that the market factor price of capital must be less than the allowed price of capital. Having developed the general model one must proceed to estimate the various relationships which form the structure.

Demand functions are estimated for Bell Canada; British Columbia Telephone; the aggregation of Alberta Government Telephones, Edmonton Telephones, Saskatchewan Telecommunications, and Manitoba Telephone System, which are referred to as the public companies; and finally we aggregate Maritime Telegraph and Telephone, New Brunswick Telephone and Newfoundland Telephone, which we refer to as the private companies. For each set of carriers, we specify three types of demand equations, the linear, the double-log, and the Rotterdam models for different revenue categories, the most important being total, local, and toll revenues. The main determinants of demand are the price of a particular service divided by the price index for the region in which the carrier has the juridiction to operate, and income divided by the price, with the latter two variables being geographically specific to the operations.

The production module is the section where the relationships between output, inputs, and technological change are estimated for Bell Canada, British Columbia Telephone, the private companies and the public companies. We estimated Cobb-Douglas production functions and experimented with constant and variable returns to scale. In addition, two measures of technological change are defined, percentage of calls direct distance dialed, and the percentage of telephones in number five crossbar and electronic switching system.

The financial module encompasses the estimation of the determinants of the rates of return on debt, common equity, and preferred shares, for Bell Canada, B.C. Telephone and the private carriers. Obviously, due to the nature of ownership of the public companies rate of return equations, which summarize hypotheses concerning financial capital market structure, are rather less important, and meaningless with regards to equity financing. We experiment with the form of the function, the manner which debt and equity enter the equations, and we use variables representing alternative assets such as the long-term corporate bond rate, and the long-term government bond rate.

Finally, we integrate the general model with the estimated equations and parameters from the demand, production and financial modules. This integration is performed for Bell Canada, by far the most important telephone carrier, and simulation experiments are carried out for the period 1955-1975. These simulation experiments are in two parts; one part assumes that Bell Canada does not have any determining influence on the average costs of its different financing instruments; the second segment allows for monopoly power (which is clearly shown to exist in the financial module) on the part of Bell to influence these financial costs.

3. Conclusions

3.1 The Theoretical Model

1. The capital constraint (the market value of the balance sheet) is an important element in the integration of real and financial decisions.

2. Imperfections in the financial capital markets manifest themselves through the ability of the carrier to influence its average costs of financing and thus create a distinction between marginal and average costs.

3. The determination of the corporate output supply, labour, debt, and equity demands are simultaneously determined.

4. The integration of the real and financial aspects of the firm imply that the determination of the capital budget is equivalent to the determination of the value of physical capital.

3.2 The Demand Module

Bell Canada

The double-log model yielded a price elasticity of total demand equal to -1.3 and the income elasticity is .8. Thus Bell services can be considered a "normal necessity" from which marginal revenues are positive.

B.C. Telephone

The double-log model yielded a price elasticity of total demand equal to -1.1 and an income elasticity of 1.1. Thus B.C. Telephone services can be considered a "normal necessity", since the income elasticity is close to unity. Here also the marginal revenues are positive.

The Public Carriers

The linear model yielded an average price elasticity of total demand equal to -3.1 and an average income elasticity of 1.8 and so these services are considered a "normal luxury", from which marginal revenues are positive.

The Private Carriers

The double-log estimates of the price elasticity of total demand is -1.4 and the income elasticity is 1.3. So, once again, telephone services of these carriers are considered a "normal luxury" by their customers, who at the margin contribute positively to the carriers' revenues.

3.3 The Production Module

Bell Canada

The Cobb-Douglas function with capital, labour, and raw materials as inputs and direct distance dialing as the measure of technological change, characterize Bell as having constant returns to scale. The labour elasticity is .616, the capital elasticity is .305 and the materials elasticity is .079.

B.C. Telephone

The constant returns to scale Cobb-Douglas function with capital, and labour as inputs, and B.C.'s direct distance dialing as the measure of technological change, yield a capital elasticity of .625 and a labour elasticity of .375.

The Public Carriers

The carriers are characterized by decreasing returns to scale, and the two factors Cobb-Douglas production relation, gives a capital elasticity of .200 and a labour elasticity of output of .600.

The Private Carriers

These carriers exhibit constant returns to scale with a capital elasticity of .557 and a labour elasticity of .443.

3.4 The Financial Module

Bell Canada

The rate of return on debt equation was linear in the logs and both the composition and the sum of debt and equity influence this rate. In addition, although the equity rate of return equation was linear, both the composition and the sum of debt and equity influence the equity rate, and corporate bonds are viewed as being an alternative to holding Bell's shares.

B.C. Telephone

The rate of return equations show us that B.C. Telephone exhibits some degree of monopoly power in the market for its common shares, in which the composition of debt to common equity and the composition of preferred to common equity play the dominant role. On the other hand in the debt and preferred share markets the degree of monopoly power exerted by this carrier is insignificant.

The Private Carriers

These carriers do exhibit monopoly power in their common share market where the composition of debt to common equity, and preferred to common equity, along with the corporate bond rate, determine the rate of return on common equity. In a similiar fashion the private carriers exhibit some influence on the rate of return of their preferred shares, and on debt.

3.5 The Simulation Module

The results of the simulations are, indeed very encouraging. The difference between the simulated and actual values for the period 1955-1975, is generally around .3%. Thus, the accuracy by which are model reproduces the characteristics of Bell Canada establishes the fact that we have captured the essence of the behaviour of a privately owned regulated carrier.

4. Future Research

There are avenues in our model where, data permitting, disaggregations will be feasible. These disaggregations can occur in the supply of the product, instead of total services for example, one can use local, and toll. One can also deal with a greater refinement of the different types of financing instruments, along the lines of different classes of debt, and preferred shares.

There are important forecasting and simulation experiments that can be performed with this model. We envision, at least, four important areas in which simulation exercises are to be performed. The first pertains to the regulatory aspect. What is the impact on production, debt, equity, and the inputs when the firm faces a market rate of return on its physical capital rather than a regulated rate. Secondly, what are the effects of an exogeneous change in the production capabilities of the firm, for example a change in product mix or a change in factor intensity, such as the carrier becoming more labour-intensive. Thirdly, what is the impact if the firms are subject to maintaining a fixed debt/equity ratio, rather than one which is self-determined by the decision-making of the firm. Finally, what is the effect of an institutionally fixed level of investment while the debt/equity ratio is free to vary according to the optimal behaviour of the firm.

CHAPTER 1

INTRODUCTION

The purpose of this study is to evaluate the interaction of the product and factor requirements with the financial needs for important carriers in the telephone industry. A model is developed, estimated, and simulated, depicting the interaction of the corporate, real and financial decision mechanisms.

In recent years the telecommunications sector, in general, and the telephone industry, in particular, have been absorbing a significant proportion of Canada's resources. This phenomenon may create severe problems for telephone carriers' and policy-makers in the future development of the industry. Thus, a systematic analysis of financial and real needs will aid decision-makers in the assessment of corporate performance with respect to pricing policy, to labour, physical and financial capital needs.

The nature of the study necessitates that we must simultaneously investigate the determinants of revenues, production, and labour hirings, along with the financial considerations. Indeed, demand behaviour and technology are integral parts in influencing the size and composition of financial resources. Consequently, in Chapter 2, we develop a general model of regulated corporate activity with explicit recognition of the potentially important feedback mechanisms between real and financial considerations. In this model, we can isolate three fundamental aspects which determine financial needs: the nature of demand, the characteristics of production, and the determinants of the rates of return on the various financial instruments.

The study is then divided into three further Chapters, which are referred to as the Demand Module, the Production Module, and the Financial Module. In these Chapters, we isolate the three important parts of the general model, in order that we can econometrically test for the actual determinants of demand, production, and rate of return characteristics.

The general description of the sub-modules are:

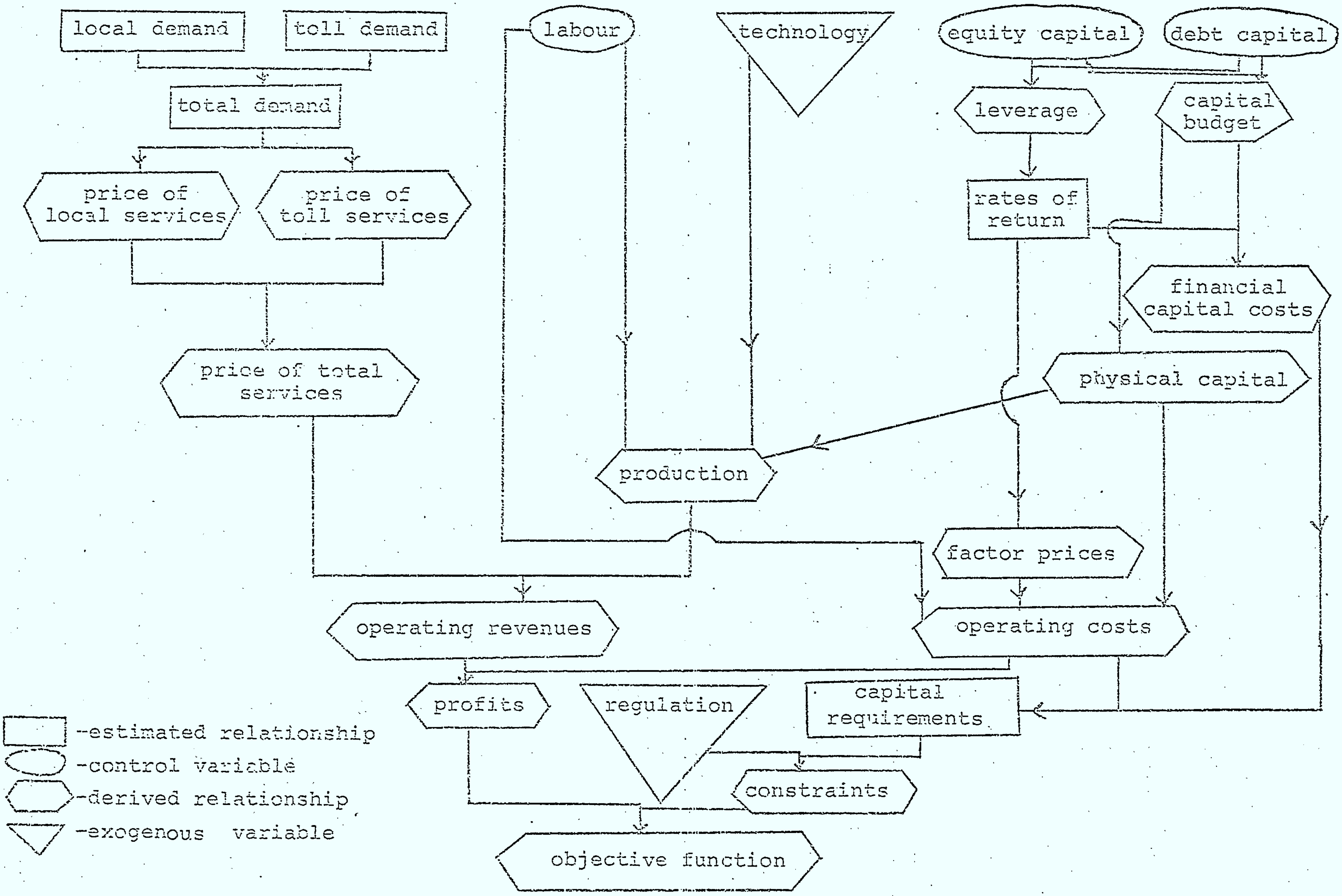
1) The determinants of the demand for telephone services, on a disaggregated (such as local, toll, etc.),as well as, aggregated levels.

2) The determinants of the production relations for telephone services, which will depend on the firm's demands for labour and capital services, in light of its technological capabilities.

3) The determinants of the rates of return on debt and equity capital, in the context of any monopoly power exhibited by the particular carriers. These rates of return will in general depend on the value of debt and equity, issued by any carrier along with variables which measure alternative portfolio endeavours for the investors.

The last Chapter of the study combines the empirical results of Chapters 3,4, and 5 with the general model, in order that we can simulate the corporate historical developments utilizing our model. The simulation Chapter will focus solely on Bell Canada, which by far is the most important telephone carrier in Canada. The simulation module consists of the estimated relationships from the demand, production, and financial modules, as well as, the optimality conditions, which are derived from the general model. This system of equations is then solved and the appropriate values of the endogenous variables are determined. A flow chart illustrates the economic feedback mechanisms that occur in our model.

General Structure of the Model



CHAPTER 2

THE GENERAL MODEL

1. Introduction

In the past, researchers have focused on the elements determining the demand and production characteristics of the telecommunications industry, in general, and the telephone industry, in particular. However, the financial and regulatory aspects have not been subjected to the same degree of intensive analysis. This state of affairs persists, although the roles of financing and regulation are now playing a crucial part in the present and (proposed) future development of the industry.

The main stumbling block to adequately understand the complete ramifications of the financial structure upon corporate growth, is the lack of a model integrating the financial decisions (capital budgeting and leverage) with the real decisions (output supply and factor demands). Thus, the first purpose of this project is to develop a model, which permits the integration of the financial and regulatory setting with the product demand (revenue) and production relations.

The model, itself, will centre around various fundamental behavioural equations, which are the demand and rates of return (on debt and equity) functions, a technological equation (the production function) and two constraints, a capital constraint (the market value of the balance sheet), and a regulatory constraint. These relations are then combined into a profit maximizing model of corporate behaviour.[1]

The derived equations from profit maximization will be a simultaneous system of equations. The endogenous variables of this system will be the demand for labour, supply of debt, supply of equity and a variable describing the impact of regulation. We will observe that the form of the rate of return equations will be the key elements in determining the interdependence and feedback mechanisms between the real and financial decisions. Consequently, in this theoretical part, we will pay particular attention to the rate of return specifications, in order to delimit the nature of the financial and real decision-making interdependence.

A noted feature of this model is, that we solve for debt and equity, and given certain exogenous variables (such as net money balances)[2], physical capital is then determined. In other words, once a firm has decided how to finance it s physical capital, given the price of the capital stock, it has simultaneously determined the quantity of physical capital. Moreover, since we can determine the debt and equity policy for any time period, we can then compute the change in the number of units of debt and equity, and so therefore, compute the real investment decision for the firm.

Finally, our model allows us to incorporate the regulatory environment. The impact of this environment is manifested by the regulatory constraint and the value of the "regulatory variable", which we simultaneously ascertain, along with the other aforementioned endogenous variables. Thus, we are able to determine the financial needs of any carrier, in light of regulation and the exigency to be consistent with the state of the product and factor markets.[3]

2. The Model

Let us begin our description of the general model by introducing the production function, which is defined by equation (1)[4],

$$y = F(K,L) \qquad (1)$$

where y is output, K is capital services, L is labour services, F represents the technology such that the marginal products of capital and labour are positive.[5] These marginal products for capital and labour respectively are, $\frac{\partial F}{\partial K} = F_k > 0$, $\frac{\partial F}{\partial L} = F_\ell > 0$

Demand behaviour may be summarized by the inverse function, represented by equation (2),

$$p = D(y), \qquad (2)$$

where p is the price of the product and D is the function with $\frac{dD(y)}{dy} = D' < 0$. So, we are assuming that the product is a normal commodity.

The pure profits for the firm are defined as,

$$\pi_g = py - w_\ell L - w_k K, \qquad (3)$$

where π_g are gross of taxes pure profits, w_ℓ is the factor price of labour and w_k is the factor price of capital. We define the factor price of capital to be related to the price of capital, the depreciation rate, the rate of return on physical capital, capital gains (or losses) and the corporate income tax rate, by the following formula,[6]

$$w_k = \left[\delta p_{kt} + rp_{kt-1} - (p_{kt} - p_{kt-1}\right] \frac{(1-ud)}{(1-u)}, \qquad (4)$$

where δ is the rate of depreciation, r is the nominal rate of return on physical capital (which is often loosely referred to as the cost of capital), p_{kt} is the price of the capital stock in period t, u is the corporate income tax rate and d is the discounted value of depreciation deductions on a unit value (dollar value) of real investment.

In regards to the factor markets, we assume that the firm is a price-taker in the labour market, but has some degree of monopoly power in the real capital market.[7] Consequently, the following equation, which combines (1), (2), (3) and (4) summarizes the product and factor market relations,

$$\pi_g = D(F(K,L))F(K,L) - w_\ell L - \left[\delta p_{kt} + rp_{kt-1} - (p_{kt} - p_{kt-1})\right]\frac{(1-ud)}{(1-u)}K. \qquad (5)$$

The financial structure of the firm may be represented by a set of relations, the first of which is the market value of the balance sheet, and we call this equation the capital constraint.

$$\overline{M} + p_k K = p_b B + p_c S_c + p_p S_p, \qquad (6)$$

where $\bar{M}$ is the exogenous net money balances, B is the number of bonds (long-term and short-term), S_c is the number of common shares, S_p is the number of preferred shares, p_b is the price of debt, p_c is the price of common shares, and p_p is the price of preferred shares.[8] The capital constraint reflects the fact that the market value of the corporation's assets must equal it's liabilities. In addition, we are using the capital stock in (6) and the flow of services from this stock in the production function. The dimension problem is overcome by noting that the stock-flow conversion parameter (which may be the rate of capacity utilization) is assumed to be unity.[9]

The nominal rate of return on physical capital in period t is the value of the capital stock in period t+1 (which was contracted in period t) minus the value of the stock in t divided by this value in period t. Similar definitions hold for common and preferred shares. The rate of return on debt must take into account the fact that interest payments are tax exempt. So, the nominal rate on debt is defined as the value in t+1 (contracted in t) minus the value, net of tax, in period t divided by this value in period t. Therefore, with these definitions and utilizing the capital constraint we get,

$$(1+r)p_kK = (1+r_b(1-u))\ p_bB + (1+r_c)p_cS_c + (1+r_p)p_pS_p - (1+r_m)\bar{M} \tag{7}$$

Dividing (7) by p_kK, subtracting 1 from both sides and using equation (6) yields,

$$r = (1-u)r_b\frac{p_bB}{p_kK} + r_c\frac{p_cS_c}{p_kK} + r_p\frac{p_pS_p}{p_kK} \qquad (8)$$

where all variables are defined in period t, and we are assuming the rate of return on nominal money balances is zero. It is important to realize, that the fact that the nominal rate of return on physical capital, is a weighted average of the rates of return on the different types of financial capital, arises, not from any ad-hoc definition of r, but rather from the correct procedure of explicitly incorporating the capital constraint. Indeed, to specify an equation like (8) and not utilize the capital constraint in other segments of the model, is to implicitly assume particular characteristics with respect to the rates of return on the financial commodities. These particularities, centre upon the rates being fixed, or that the behaviour of the returns are such that the liability side of the capital constraint is determined independently of the asset side. Manifestly, these assumptions are the antithesis for any meaningfully integrated financial and real decision-making model. It seems then that the capital constraint, along with the specification of the determinants of the rates of return equations, is fundamental to the nature of the integration.

The various rates of return for different commodities, need not be constant, since the rates are defined from the spot and forward prices. If the quantities of various commodities influence these spot or forward prices, then the rate of return may be a variable. This variable rate of return may be affected by the firm's own decisions, if the variables influencing the spot and forward prices fall under the firm's control. If this situation arises, then the firm possesses some degree of price-setting power or monopoly power in the capital markets. In our general model, we assume that the firm cannot influence spot prices, but only forward prices (for real and financial capital), and so there are imperfections in the financial capital markets. These imperfections are reflected in the following equations,[10]

$$r_b = B(p_b B, p_c S_c, p_p S_p) \tag{9.1}$$

$$r_c = C(p_b B, p_c S_c, p_p S_p) \tag{9.2}$$

$$r_p = P(p_b B, p_c S_c, p_p S_p) \tag{9.3}$$

where the rates of return depend on the values of debt, common equity and preferred equity. The rationale for including the values of the financial instruments in equation set (9) is quite obvious. The rate of return equations are inverse demand equations reflecting the outcome of the decisions by agents (individuals and firms) in their portfolio decisions, with regard to the equity and debt issued by the carrier. Obviously, these inverse financial demand equations will be influenced by the quantities of the other commodities that these agents are simultaneously demanding and supplying. The exact composition of these other commodities will depend on the preferences and endowments of the individual investors, the motivation (profit maximization, revenue maximization, etc.) and the ability of the corporate investors, and the process of aggregation. Nevertheless, the only variables that the carrier can control and thereby influence the nominal rates of return, are debt and equity: All other variables in the investor decision process are exogenous to the carrier. This means that, although equation set (9) is derived from a complex interaction of agents (in the same way that the inverse product demand is determined by a complicated mechanism), from a theoretical vantage point, because we want to focus on the monopoly power of the carrier, (9) includes all the relevant variables. However, for the empirical implementation, estimation and simulation, various forms of these exogenous variables must be accounted for in the rate of return equations.

Equations of similar, but less general, form can be found in the literature. It is often expressed, that the rates of return depend on the debt-equity ratio. Let us assume that common and preferred equity have an additive impact on the rates of return, so that only the total value of equity and the value of debt are in the domain of the rates. Next, assume that the rates of return are homogeneous of degree zero in debt, and equity, which means that proportional changes in the composition of the firm's portfolio do not affect the rates of return. We can then write the equations in (9) as only depending on the ratio of debt to equity.[1] While this proposition may be an interesting property to test empirically, for the theoretical formulation, we see no reason to impose such a restriction 'a priori' on the inverse demand functions for the different types of financial capital.

Thus, with equations (5), (6), (8), and defining $\pi_n = (1-u)\pi_g$, where π_n is the net of tax pure profits, we can summarize the real and financial characteristics by the equation set (9) and (10)

$$\pi_n = (1-u)\Big[\mathcal{D}(F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L))F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L) - w_\ell L\Big] - \Big[(r_b(1-u) + \delta-\Theta(1-\delta))p_bB + (r_c+\delta-\Theta(1-\delta))p_cS_c + (r_p+\delta-\Theta(1-\delta))p_pS_p - (\delta-\Theta(1-\delta))\bar{M}\Big] (1-ud)(1+\Theta)^{-1}, \qquad (10)$$

where $\Theta_t = \dfrac{p_{kt} - p_{kt-1}}{p_{kt-1}}$, and Θ_t is called the rate of price inflation of the physical capital stock.

The regulatory environment is typically characterized by the constraint,

$$\pi_n + rp_kK(1+\Theta)^{-1} + ur_bp_bB(1+\Theta)^{-1} \leq (1-u)(1+\Theta)^{-1}ip_kK \qquad (11)$$

where i is the before tax allowed nominal rate of return on capital net of depreciation. Using the capital constraint and equation (10), (11) becomes

$$(1-u)\Big[\mathcal{D}(F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L)) F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L)) - w_\ell L\Big]$$
$$-(\delta-\Theta(1-\delta))(1-ud)(1+\Theta)^{-1}(p_bB+p_cS_c + p_pS_p - \bar{M})$$
$$+ ud(1+\Theta)^{-1}\Big[r_b(1-u)p_bB + r_cp_cS_c + r_pp_pS_p\Big]$$
$$+ ur_b(1+\Theta)^{-1} p_bB \leq (1-u)(1+\Theta)^{-1}i(p_bB + p_cS_c + p_pS_p - \bar{M}). \qquad (12)$$

Before discussing the objective of the firm, it bears mentioning that, in our context, we view the factor prices of labour, and physical capital, the prices of debt and equity, and the depreciation rate, as random variables. Thus, since the prices of debt and the two types of equity are random, then the rates of return are also stochastic variables. Due to the presence of uncertainty, the objective function of the firm must incorporate the manner in which the firm maximizes the expected value of profit.[12] This implies that the firm is risk neutral, in that it's goal is to maximize the expected value of profit, irrespective of the variance of the distribution (or for that matter any other moments). Therefore, the firm maximizes the expected value of (10) subject to the expected value of (12) and equation set (9),

$$\begin{aligned}
L = E\Big[&(1-u)\Big[D(F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L)) \\
&F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L) \quad - w_\ell L\Big] - (1+\theta)^{-1} \\
&\Big[(B(p_bB, p_cS_c, p_pS_p)(1-u) + \delta - \theta(1-\delta))(1-ud)p_bB \\
&+ (C(p_bB, p_cS_c, p_pS_p) + \delta - \theta(1-\delta))(1-ud)p_cS_c \\
&+ (P(p_bB, p_cS_c, p_pS_p) + \delta - \theta(1-\delta))(1-ud)p_pS_p \\
&- (\delta - \theta(1-\delta))(1-ud)\bar{M}\Big] - \lambda\Big[(1-u)\Big(D(F((p_bB + p_cS_c \\
&+ p_pS_p - \bar{M})p_{kt}^{-1}, L))\ F((p_bB + p_cS_c + p_pS_p - \bar{M})p_{kt}^{-1}, L) \\
&- w_\ell L\Big) - (\delta - \theta(1-\delta))(1-ud)\ (1+\theta)^{-1}\ (p_bB + p_cS_c + p_pS_p - \bar{M}) \\
&- (1-u)(1+\theta)^{-1} i(p_bB + p_cS_c + p_pS_p - \bar{M}) + ud(1+\theta)^{-1} \\
&[r_b(1-u)p_bB + r_cp_cS_c + r_pp_pS_p] + ur_b(1+\theta)^{-1}p_bB\Big]\Big] \quad . \qquad (13)
\end{aligned}$$

where L is the Lagrangian function. Equation (13) is a function of five variables, debt (B), common equity (S_c), preferred equity (S_p), labour (L), and the regulatory variable (λ).[13]

The following optimality conditions, are derived by differentiating (13) with respect to each of the control variables and λ:

$$\frac{\partial L}{\partial L} = F_{\ell} \ (D'F + D) - E(w_{\ell})(1-\lambda)(1-u) = 0 \tag{14.1}$$

$$\begin{aligned}\frac{p_k}{p_b}\frac{\partial L}{\partial B} = {} & (1-\lambda)\left[(1-u)\left[F_k(D'F+D)\right] - E(\delta-\theta(1-\delta))(1-ud)(1+\theta)^{-1}p_{kt}\right] \\ & -E\left(\frac{\partial \mathcal{B}}{\partial B} p_b B(1-u) + \frac{\partial \mathcal{C}}{\partial B} p_c S_c + \frac{\partial \mathcal{P}}{\partial B} p_p S_p\right)(1-ud)(1+\theta)^{-1}p_{kt} \\ & - E(1-u)(1+\theta)^{-1}p_k(r_b(1-ud)-\lambda i) - E\ \lambda ud(1+\theta)^{-1}p_k\left[\frac{\partial \mathcal{B}}{\partial B} p_b B(1-u)\right. \\ & \left. + \frac{\partial \mathcal{C}}{\partial B} p_c S_c + \frac{\partial \mathcal{P}}{\partial B} p_p S_p\right] - E\ \lambda u(1+\theta)^{-1}\frac{\partial \mathcal{B}}{\partial B} p_b B p_k \\ & - E\ \lambda p_k(1+\theta)^{-1}p_k r_b u\left[1+d(1-u)\right] = 0.\end{aligned} \tag{14.2}$$

$$\begin{aligned}\frac{p_k}{p_c}\frac{\partial L}{\partial S_c} = {} & (1-\lambda)\left[(1-u)\left[F_k(D'F+D)\right] - E(\delta-\theta(1-\delta))(1-ud)(1+\theta)^{-1}p_{kt}\right] \\ & - E\left(\frac{\partial \mathcal{B}}{\partial S_c} p_b B(1-u) + \frac{\partial \mathcal{C}}{\partial S_c} p_c S_c + \frac{\partial \mathcal{P}}{\partial S_c} p_p S_p\right)(1-ud)(1+\theta)^{-1}p_{kt} \\ & - E(1-u)(1+\theta)^{-1}p_k\ (r_c(1-ud)(1-u)^{-1} - \lambda i) - E\ \lambda ud(1+\theta)^{-1}p_k \\ & \left[\frac{\partial \mathcal{B}}{\partial S_c} p_b B(1-u) + \frac{\partial \mathcal{C}}{\partial S_c} p_c S_c + \frac{\partial \mathcal{P}}{\partial S_c} p_p S_p\right] - E\ \lambda ud(1+\theta)^{-1}p_k r_c = 0.\end{aligned} \tag{14.}$$

$$\frac{p_k}{p_p}\frac{\partial L}{\partial S_p} = (1-\lambda)\left[(1-u)\left[F_k(\mathcal{D}'F+\mathcal{D})\right] - E(\delta-\theta(1-\delta))(1-ud)(1+\theta)^{-1}p_{kt}\right]$$

$$-\left(E\ \frac{\partial \mathcal{B}}{\partial S_p}\ p_b B(1-u) + \frac{\partial \mathcal{C}}{\partial S_p} p_c S_c + \frac{\partial \mathcal{P}}{\partial S_p} p_p S_p\right)\ (1-ud)(1+\theta)^{-1}p_{kt}$$

$$-\ E(1-u)(1+\theta)^{-1}p_k\ \ (r_p(1-ud)(1-u)^{-1} - \lambda i) - E\ \lambda ud(1+\theta)^{-1}p_k$$

$$\left[\frac{\partial \mathcal{B}}{\partial S_p} p_b B(1-u) + \frac{\partial \mathcal{C}}{\partial S_p} p_c S_c + \frac{\partial \mathcal{P}}{\partial S_p} p_p S_p\right] - E\ \lambda ud\ (1+\theta)^{-1}p_k r_p = 0. \quad (14.4$$

$$p_k\frac{\partial L}{\partial \lambda} = -E\left[(1-u)\mathcal{D}(F((p_b B + p_c S_c + p_p S_p - \overline{M})p_{kt}^{-1}, L))F((p_b B + p_c S_c + p_p S_p - \overline{M})\right.$$

$$p_{kt}^{-1}, L)) - w_\ell L - (\delta-\theta(1-\delta))(1-ud)(1+\theta)^{-1}(p_b B + p_c S_c + p_p S_p - \overline{M})$$

$$-\ (1-u)(1+\theta)^{-1}i(p_b B + p_c S_c + p_p S_p - \overline{M}) + ud(1+\theta)^{-1}$$

$$\left.\left[r_b(1-u)p_b B + r_c p_c S_c + r_p p_p S_p\right] + u(1+\theta)^{-1}r_b p_b B\right] = 0. \quad (14.5)$$

The equations in (14) tell us that the marginal revenue product of labour equals the expected value of the wage. Also, we can relate the first order conditions for debt and both types of equity to the same kind of economic meaning. For instance, the net of tax differential between the marginal revenue product of physical capital and the difference between the rate of return on debt and the allowed rate (everything adjusted for the presence of regulation) is equal to the expected marginal cost of financing capital due to an increase in debt. Finally, we can observe that the expected marginal costs of financing capital through debt, and equity (common or preferred) are equal. In our model, there is the simultaneous determination of real and financial decisions where both the optimal capital budget and various financial variable ratios are determined from the five fundamental equations.

It is quite clear, that the equations describing the different rates of return play a key role in determining the corporate equilibrium. This means that changes in these functional relationships due to changes in information or market power will affect our results. Nevertheless, our model is consistent with a varied array of equation forms of differing degrees of generality. Therefore, once we have estimated the different equations from each of the modules (demand, production, and financial) we will find the relevant functional forms and parameters to be substituted into equation set (14). This substitution will occur in the simulation module, when we solve (14) for Bell Canada.

Appendix 2.1 Derivation of the Relationship Between the Rates of Return

Suppose there is one type of equity, and the corporate tax rate is zero (or the interest on bonds is not tax exempt), then the relationship between the rates of return is denoted by,

$$r_{kt}p_{kt}K_t = r_{bt}p_{bt}B_t + r_{st}p_{st}S_t, \qquad (A.1)$$

where r_{kt} is the nominal rate of return on physical capital, r_{bt} is the nominal rate on bonds, and r_{st} is the nominal rate on equity, all defined in period t. Also, p_{kt} is the future price of physical capital, p_{bt}, and p_{st}, are the future prices of bonds and equity respectively; K_t is the quantity of physical capital, B_t the number of bonds, and S_t the number of shares; all defined in period t.

Now any real rate of return is defined as,

$$\rho_{it} = \frac{q_{it} - q_{it+1}}{q_{it+1}}, \; i = k,b,s, \qquad (A.2)$$

where ρ_{it} is the real rate of return on i in period t, and q_{it} is the forward price of i in period t. We must note that forward prices refer to contracts in the present for future delivery of a commodity, while future prices refer to contract formation and delivery both in the same future period.

The connection between forward and future prices may be established by the following equality,

$$\frac{p_{it}}{p_{nt}} = \frac{q_{it}}{q_{nt}} \quad i = k,\ b,\ s, \qquad (A.3)$$

which states that <u>relative</u> forward prices equal relative future prices; where n is the numeraire commodity. Since commodity n is the numeraire all of it's future prices are unity, i.e. $p_{nt} = 1$. This means that,

$$p_{it} = \frac{q_{it}}{q_{nt}} \quad i = k,b,s. \tag{A.4}$$

By the capital constraint, the market value of the balance sheet, we have,

$$p_{kt}K_t = p_{bt}B_t + p_{st}S_t, \tag{A.5}$$

we abstract from introducing money balances, which does not affect the procedures of the derivation irrespective of whether r_{mt} is zero or not. Therefore, with equations (A.5) and (A.3) (using the fact that $p_{nt} = 1$) we get (by multiplying out q_{nt}) the following,

$$q_{kt}K_t = q_{bt}B_t + q_{st}S_t \;, \tag{A.6}$$

which is the capital constraint measured in terms of forward prices. Then using (A.2), equation (A.6) is transformed to,

$$(1+\rho_{kt})q_{kt+1}K_t = (1+\rho_{bt})q_{bt+1}B_t + (1+\rho_{st})q_{st+1}S_t. \tag{A.7}$$

However, from (A.3), $p_{it+1}q_{nt+1} = q_{it+1}$ and so (A.7) becomes (by dividing out q_{nt+1})

$$(1+\rho_{kt})\,p_{it+1}K_t = (1+\rho_{bt})p_{bt+1}B_t + (1+\rho_{st})p_{st+1}S_t. \tag{A.8}$$

We can multiply and divide each term in (A.8) by the appropriate future price in period t and therefore,

$$(1+\rho_{kt}) \frac{p_{kt+1}}{p_{kt}} p_{kt}K_t = (1+\rho_{bt}) \frac{p_{bt+1}}{p_{bt}} p_{bt}B_t + (1+\rho_{st}) \frac{p_{st+1}p_{st}S_t}{p_{st}} .$$

(A.9)

The rate of inflation for any commodity i is defined as

$\frac{p_{it+1}}{p_{it}} = (1+\gamma_{it})$, where γ_{it} is the rate of inflation of i in period t and so (A.9) becomes,

$$(1+\rho_{kt})(1+\gamma_{kt})p_{kt}K_t = (1+\rho_{bt})(1+\gamma_{bt})p_{bt}B_t + (1+\rho_{st})(1+\gamma_{st})p_{st}S_t .$$

(A.10)

Last, the definition of one plus any nominal rate of return is equal to one plus the rate of inflation for that commodity times one plus the real rate of return of that commodity, i.e. $(1+r_{it}) = (1+\gamma_{it})(1+\rho_{it})$. Hence,

$$(1+r_{kt})p_{kt}K_t = (1+r_{bt})p_{bt}B_t + (1+r_{st})p_{st}S_t, \qquad (A.11)$$

and by (capital constraint) (A.5)

$$r_{kt}p_{kt}K_t = r_{bt}p_{bt}B_t + r_{st}p_{st}S_t,$$

which is the result we want to derive.

Therefore, we have established the nature of the relationship between the rates of return i.e. that the rate of return on physical capital is equal to the weighted average of the rates of return on the different types of financial capital, where these rates are nominal ones.

FOOTNOTES

[1]If we postulate revenue or sales maximization the essential structure of the model is not affected, but the derived demand and supply equations need to be slightly modified.

[2]By net money balances, we mean cash plus accounts receivable minus accounts payable and other residual balances.

[3]Our analysis builds on the works of Lermer and Carleton (7), Robichek and Myers (8), Turnovsky (9), and Vickers (10), (11).

[4]We do not include the time variable where it does not lead to confusion to omit it. But the time dependence of the variables is understood.

[5]F also has sufficient properties in order that it may be used in the profit maximization problem.

[6]The formula for w_k may be derived in an explicitly dynamic model. The fact that we include this formula means that we are indeed including the various dynamic elements. See Hall and Jorgenson (3).

[7]The exact nature of this monopoly power will be specified below when we discuss the components of the nominal rate of return on physical capital.

[8]For theoretical purposes there is no need to assume that M is exogenous However, for the empirical implementation of the model we want to focus on debt and equity, rather than net money balances.

[9]We can let the stock-flow parameter be some number different from 1, as long as it is an exogenous coefficient.

[10]These equations will also depend on other variables reflecting alternative assets. However, these other variables are exogenous to the firm's decision process, and therefore, we do not have to include them for the theoretical specification, but only for the empirical implementation.

[11]See, for example, the article by Turnovsky (9).

[12]We can alternatively assume, the firm maximizes the expected utility of profit. However, our ultimate purpose is to estimate the derived relationships, and since virtually nothing is known, concerning the empirical implementation of corporate utility functions, we have elected to assume that $U(\pi)=\pi$, where U is the utility function.

[13]According to the Averch-Johnson regulatory model, as described in Bailey (1), Baumol and Klevorick (2), and Johnson (4), $0<\lambda<1$.

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CHAPTER 3

THE DEMAND MODULE

1. Introduction

The nature of the telephone demand module is to describe the demand characteristics for the telephone services of the Trans-Canada Telephone System (TCTS) companies.[1] In describing the demand, and thereby the revenue, conditions for the system, we formulate a model which estimates the historical structure. This structural specification is then utilized to forecast the future trends of the carriers' revenues.

Consequently the purpose of the demand module is two-fold. Firstly, we estimate the demand aspects as a separate entity in the overall industry model. These estimated coefficients are then combined with the relevant segments of the production and financial modules so that the integrated model may be implemented and the appropriate forecasting experiments carried out. Therefore, one must view the results of this section not only in isolation, but also within the context of the complete model.

The study of demand behavior for telephone services is an important undertaking, because of its role in determining company revenues. Indeed, demand systems already exist depicting the Canadian telephone industry, in general; for example R. Dobell et. al. [3] and L. Waverman [9]. Moreover, other important works have focused on particular demand aspects, as in, V. Corbo [2], and I.I.Q.E. [5]. Our immediate interest is in the general structural form of the telephone demand relations.

Before proceeding to formulate the module, we must determine the appropriate aggregations across economic agents (in this case carriers) and commodities (in this case telephone services). The demand module disaggregates carriers into four categories. We treat Bell Canada and British Columbia Telephone separately; we aggregate Alberta Government Telephones, Edmonton Telephones, Saskatchewan Telecommunications and the Manitoba Telephone System into one category called public companies; we aggregate Maritime Telegraph and Telephone, New Brunswick Telephone and Newfoundland Telephone into one category called private companies. The rationale for this aggregation is based on the following reasons. Firstly, Bell Canada is the leader, in terms of market share, of the industry and so is dealt with individually. Secondly, the public companies, as their name suggests, are government owned while the three companies operating in the Maritimes are privately controlled. Finally, locational considerations suggest that the western carriers be separated from the eastern area , thus B.C. Telephone is dealt with separately from the Maritime private companies. Hence, our transactor disaggregations are derived from the market share, legal and spatial characteristics of the industry.

The disaggregation for the telephone services proceeds along the lines of local, toll and total revenues. However, in some cases, notably Bell Canada, where there exists a larger databank on revenue and price series, the services were further decomposed into local plus toll and local plus toll plus directory advertising.

The explication of the demand module is divided into four further sections. In section 2 we describe the various theoretical specifications and their rationale, in section 3 we describe the data and their limitations, in section 4 we present the empirical results and their evaluation.

2. The Theoretical Models

The theoretical basis for the demand model which is utilized in the econometric investigations is discussed in this section. The economic theory that we draw upon is largely the analysis of the individual household and also from the firm.

In developing the model, the first question to be answered is who are the demanders of telephone services. Manifestly, both households and firms are the demanders, since the telephone is a consumption product to households and a factor of production (part of intermediate inputs) to firms. Ideally, then, we would desire to construct demand equations disaggregated, not only along supplier and service categories, but, in addition, along demander groups. However, because of data limitations, we follow the usual route and aggregate the household and firms' demand for each revenue category into a single aggregate. We therefore assume that although the motivations and constraints of consumers and producers are different the ultimate elements affecting their telephone service demand are the same.

Individual demand behavior, according to economic theory, suggests that given the objectives of the demanders (preferences for consumers and generally profits for firms), that the quantity demanded of the i^{th} service by the k^{th} household in period t (X_{it}^{k}) depends on the nominal income of the k^{th} household in period t (Y_t^k), the price of the i^{th} service in period t (P_{it}), and the price of other commodities demanded and supplied by the household[2] (P_{jt}; $j=1,\dots,n$ and $j\neq i$). In a functional form we find that,

$$X^k_{it} = h^k_{it}\ (P_{1t},\dots,P_{nt},Y^k_t) \qquad (1)$$

where h^k_{it} is the demand function of the i^{th} service for the k^{th} household in period t.

To derive the aggregate household demand for any service i, in any period t, we must sum equation (1) over all households who are demanding the service.

$$\sum_{k=1}^{J} X^k_{it} = \sum_{k=1}^{J} h^k_{it}\ (P_{1t},\dots,P_{nt},Y^k_t) \qquad (2)$$

where J is the number of household demanders. So then,

$$X^H_{it} = h_{it}\ (P_{1t},\dots,P_{nt},Y^1_t,\dots,Y^J_t) \qquad (3)$$

where $X^H_{it} = \sum_{k=1}^{J} X^k_{it}$ and $h_{it}\ (P_{1t},\dots,P_{nt},Y^1_t,\dots,Y^J_t)$ $= \sum_{k=1}^{J} h^k_{it}\ (P_{1t},\dots,P_{nt},Y^k_t)$. Notice that in the aggregate demand function the income terms for each household enter separately and not as an aggregate. This fact takes into consideration that the distribution of income among households is not fixed. If we assume that the distribution of income among households in any period of time is fixed then we can write equation (3) as,

$$X^H_{it} = h_{it}\ (P_{1t},\dots,P_{nt},Y^H_t) \qquad (4)$$

where $Y^H_t = \sum_{k=1}^{J} Y^k_t$ is the aggregate income of the households.

Moreover, let us assume that the form of the demand function does not depend on the time period and so

$$X_{it}^{H} = h_i \ (P_{1t},\ldots,P_{nt},Y_{it}^{H}) \qquad (5)$$

For the producers, these demands for telephone services are derived, not from utility maximization procedures as in the case of households, but from cost minimization techniques. The quantity demanded of the i^{th} telephone service by the ℓ^{th} firm in period t (X_{it}^{ℓ}) depends on the nominal income (since output is given)[3] of the ℓ^{th} firm in period t (Y_t^{ℓ}), the price of the i^{th} service in period t (P_{it}), and the price of all other commodities demanded and supplied by the firm (P_{jt}; j=1,...,m and j≠i).[4] Hence we have

$$X_{it}^{\ell} = g_{it}^{\ell} \ (P_{1t},\ldots,P_{mt},Y_t^{\ell}) \qquad (6)$$

where g_{it}^{ℓ} is the ℓ^{th} firm's demand function for the i^{th} service in period t. Summing over all the firms yields,

$$X_{it}^{F} = g_{it} \ (P_{1t},\ldots,P_{mt},Y_t^{1},\ldots,Y_t^{I}) \qquad (7)$$

where I is the number of firms, $X_{it}^{F} = \sum_{\ell=1}^{I} X_{it}^{\ell}$ and

$$g_{it} \ (P_{1t},\ldots,P_{mt},Y_t^{1},\ldots,Y_t^{I}) = \sum_{\ell=1}^{I} g_{it}^{\ell} \ (P_{1t},\ldots,P_{mt},Y_t^{\ell}).$$

Again it is not aggregate output which affects the aggregate producer demand function for the i^{th} service in period t, but rather all the outputs separately which reflect the size and composition of output levels for firms demanding telephone services. By assuming that the output composition is fixed in every period and the demand functions do not change over time we get,

$$X_{it}^{F} = g_i \ (P_{1t}, \ldots, P_{mt}, Y_t^F) \qquad (8)$$

where $Y_t^F = \sum_{\ell=1}^{I} Y_t^{\ell}$.

To derive the consumer and producer demand for the i^{th} service in period t we must sum equations (5) and (8).

$$x_{it} = H_i \ (P_{1t}, \ldots, P_{rt}, Y_t^H, Y_t^F) \qquad (9)$$

where $x_{it} = X_{it}^H + X_{it}^F$, $H_i \ (P_{1t}, \ldots, P_{rt}, Y_t^H, Y_t^F) = h_i \ (P_{1t}, \ldots, P_{nt}, Y_t^H) + g_i \ (P_{1t}, \ldots, P_{mt}, Y_t^F)$ and so H_i is the aggregate (consumer and producer) demand function for the i^{th} telephone service.

Once again by assuming that the distribution of income between households and firms is fixed and by letting the prices of all commodities other than the i^{th} service be represented by a price index in period t (P_t), we can write equation (9) as,

$$x_{it} = H_i \ (P_{it}, P_t, Y_t) \qquad (10)$$

where $Y_t = Y_t^H + Y_t^F$.

Now that we have arrived at the aggregate demand function for any telephone service, we are able to impose the 'a priori' restrictions from economic theory. Economic theory does not predict the form of the demand function (H_i), but the theory does impose restrictions on the pattern of price and income effects in systems of demand behavior. Firstly, household and firm behavior is such that the demand function should be homogeneous of degree zero in the prices and income. In other words, if there is an equiproportionate change in all prices and income then the cost minimizing producer demand and utility maximizing consumer demand do not change. Consequently, the aggregate demand is not affected. This result implies that we can write equation (10) as,

$$x_{it} = H_i \ (p_{it}, y_t), \qquad (11)$$

where $p_{it} = P_{it}/P_t$, and $y_t = Y_t/P_t$. The variable p_{it} is the relative price of the i^{th} service in period t and y_t is the real income in period t.

The second proposition pertains to the nature of the effects of a change in the relative price and the real income on demand. Economic theory states that if the effect of a change in y_t is to increase the quantity demanded then it must be true that the effect of a change in p_{it} is to decrease the quantity demanded. Therefore the negativity condition is:

if $\frac{\partial x_{it}}{\partial y_t} > 0$ then it must be the case that $\frac{\partial x_{it}}{\partial p_{it}} < 0$.

The last restriction, in this context, is the so-called adding-up condition which states that the sum of the proportions of expenditure on all commodities out of income (or output) must equal unity. This means that if $p_{it}\ x_{it}$ is the expenditure on the

i^{th} service and there are r commodities then $\frac{\sum_{i=1}^{r} p_{it}x_{it}}{Y_t} = 1$.

This restriction, however, is not as important as the previous two because we are aggregating across households and firms. The reason is that, in general, this third condition holds for consumers, but does not do so for producers, unless their production functions exhibit constant returns to scale. Since the nature of the production functions for the producers who demand telephone services is outside the purview of our study, we shall develop demand models which do and do not incorporate this last condition. Moreover, whether this last condition is satisfied or not will not be a prerequisite for the acceptance or rejection of a particular functional form.

After the description of the relevant features of our specification which are derivable from the theory, for the empirical applications of equation (11), it is necessary to specialize the general form of the demand relation and to account for stochastic phenomena.[5]

2.1 The Linear Demand Model

The linear demand model assumes that the form of the aggregate demand function (H_i) is linear, so that,

$$x_{it} = \beta_0 + \beta_1 p_{it} + \beta_2 Y_t + e_t , \qquad (12)$$

where e_t represents the disturbance that can occur because H_t may not be strictly linear or there may exist measurement errors in the dependent variable and also other minor variables may have been omitted from the equation.

Here we must find that if $\beta_2 > 0$ then it should be the case that $\beta_1 < 0$. This means that if increases in income tend to increase demand then increases in the service's price tend to decrease demand. It bears mentionning that equation (12) satisfies the homogeneity and negativity conditions but does not satisfy the adding-up restriction. Nevertheless, in light of the caveat stated at the end of section 2, concerning the adding-up condition, the linear model should not be dismissed outright on these grounds.

2.2 The Double-Log Demand Model

In the double-log model we begin with the general demand equation, but instead of assuming that it is linear, we assume that it is multiplicative,

$$x_{it} = \alpha_0 \, p_{it}^{\beta_1} \, y_t^{\beta_2} \, u_t \, , \tag{13}$$

where u_t represents the error term and α_0 the constant. By taking logarithms of equation (13) we arrive at,

$$\log x_{it} = \beta_0 + \beta_1 \log p_{it} + \beta_2 \log y_t + e_t \, , \tag{14}$$

where $\log \alpha_0 = \beta_0$ and $\log u_t = e_t$.

The double-log formulation, as in the linear case, incorporates the homogeneity condition. Moreover, if $\beta_2 > 0$ then we should expect $\beta_1 < 0$. Notice that the magnitudes of β_0, β_1, and β_2 will be different from the linear model but the signs of the coefficients should be the same. The reason for this is that we are specifying an alternative hypothesis concerning the true structural form and in this case β_1 and β_2 are partial price and partial income elasticities rather than partial rates of change. Finally, the double-log equation does not incorporate the adding-up condition.

2.3 The Rotterdam Demand Model

The Rotterdam model, as applied to the demand for telephone services, imposes a more complicated set of restrictions upon the demand relations, (see H. Theil [8]).

Using the demand function, given by equation (11), take total differentials. This yields,

$$dx_{it} = \frac{\partial H_i}{\partial p_{it}} dp_{it} + \frac{\partial H_i}{\partial y_t} dy_t .$$

Using the equality $dz = z\, d \log z$, where z is any variable, we get,

$$x_{it}\, d \log x_{it} = \frac{\partial H_i}{\partial p_{it}} p_{it}\, d \log p_{it} + \frac{\partial H_i}{\partial y_t} y_t\, d \log y_t . \quad (15)$$

Next, multiplying both sides of equation (15) by $\frac{p_{it}}{y_t}$ we have,

$$\frac{p_{it}\, x_{it}}{y_t} d \log x_{it} = \frac{\partial H_i}{\partial p_{it}} \frac{p_{it}^2}{y_t} d \log p_{it} + \frac{\partial H_i}{\partial y_t} p_{it}\, d \log y_t . \quad (16)$$

However because equation (16) is a finite linear approximation there is implicitely a remainder term in $_{it}$. The approximation to the remainder is $1/2 \left[\left(\frac{p_{it}\, x_{it}}{y_t} - \frac{p_{it+1}\, x_{it+1}}{y_{t+1}} \right) d \log x_{it} + \alpha_0 \right]$.

Therefore including the remainder in equation (16) yields,

$$1/2\left[\frac{p_{it}\ x_{it}}{y_t} + \frac{p_{it+1}\ x_{it+1}}{y_t}\right] d \log x_{it} = \frac{\alpha_0}{2} + \frac{\partial H_i}{\partial p_{it}} \frac{p_{it}^2}{y_t} d \log p_{it} \tag{17}$$

$$+ \frac{\partial H_i}{\partial y_t} p_{it}\ d \log y_t .$$

Letting $a_{it} = \dfrac{p_{it}\ x_{it}}{y_t}$, $\alpha_{it} = 1/2\ (a_{it} + a_{it+1})$, $\beta_0 = \dfrac{\alpha_0}{2}$,

$\beta_1 = \dfrac{\partial H_i}{\partial p_{it}} \dfrac{p_{it}^2}{y_t}$ and $\beta_2 = \dfrac{\partial H_i}{\partial y_t} p_{it}$ then equation (17) becomes,

$$\alpha_{it} d \log x_{it} = \beta_0 + \beta_1\ d \log p_{it} + \beta_2\ d \log y_t . \tag{18}$$

Since $d \log z_t = \log z_t - \log z_{t-1}$ and allowing for stochastic phenomena then (18) can be written as,

$$\alpha_{it}(\log x_{it} - \log x_{it-1}) = \beta_0 + \beta_1\ (\log p_{it} - \log p_{it-1}) \tag{19}$$

$$+ \beta_2\ (\log y_t - \log y_{t-1}) + e_t .$$

Equation (19) represents a variant of the Rotterdam demand model. In this model, we not only have the homogeneity and negativity conditions, we also have the adding-up condition because of the presence of the weights in defining the dependent variable. Notice, also, that β_1 and β_2 do not mean the partial elasticities as they do in the double-log model. In this case to compute the partial elasticities we must divide β_1 and β_2 by α_{it}. Hence if we want to compute partial price and income elasticities we have to compare:

$\beta_1^L \dfrac{p_{it}}{x_{it}}$ to β_1^D to $\dfrac{\beta 1^R}{\alpha_{it}}$ for the partial price elasticities,

$$\beta_2^L \frac{Y_t}{x_{it}} \text{ to } \beta_2^D \text{ to } \frac{\beta_2^R}{\alpha_{it}} \text{ for the partial income elasticities,}$$

where the superscripts L, D, and R stand for linear, double-log and Rotterdam. This means that in the linear and Rotterdam models the partial (and therefore total) elasticities will be variable, while in the double-log model the partial elasticities (but not the total)[6] will be constant.

3. The Data

The data in the demand module consisted of published series which had to be collected from various sources.

3.1 The Quantity Demanded

In a study of the demand for telephone services, the quantity demanded should be measured in some homogeneous unit such as minutes of calls. Unfortunately, we do not have data at such a disaggregated level. Therefore, we used a variant of revenue deflated by it's price. We took the revenue for any service i (including uncollectibles, since they represent unpaid output) and substracted from it the proportion of revenue from service i out of total revenue times indirect taxes:[7]

$$SI_i = SR_i - \frac{SR_i}{SR} \times INT,$$

where SI_i is then defined for the i^{th} service, SR_i is the revenues from the i^{th} service, SR are total revenues, and INT are indirect taxes.

For the revenue figures, we utilized the income statements of the TCTS companies and Edmonton Telephones. Thus the level of disaggregation of revenues was limited to that which appears in the financial statements (which are local, toll and total). However for Bell Canada, the revenues were more disaggregated from R. Millen [6] and Bell Canada Rate Hearings Exhibits [1] , which included local, directory, intra-Bell toll, Trans-Canada and adjacent members toll, and U.S. and overseas toll revenues. Also because we dealt with the public and private carriers as seperate carriers we summed the revenues, as previously defined, for each category and used these figures for aggregate revenues.

To convert revenue figures into output figures we need the price index of each revenue category for each carrier. These price indexes were only available for Bell Canada in R. Millen [1] and the previously mentionned exhibits [6]. The procedure for the price index was to deflate the current dollar revenues for each service (as defined) by the constant dollar revenues (which were defined in the same manner as current). After, with this implicit price deflator for each category (for 1967 - 1.00) we obtained output,

$$SO_i = \frac{SI_i}{PI_i}$$

where SO_i is the demand for the i^{th} service and PI_i is the price index of the i^{th} service.

Having these price indexes for Bell Canada we assumed that the price indexes for any category and for any other carrier is a fixed proportion to Bell's over the sample period. If this assumption does not hold then the consequences of the error in the measurement of the price indexes are unknown, in terms of the bias and the inconsistency in the values of the parameters obtained in the demand equations. Nevertheless, our assumption is reasonable because Bell is the market leader in the industry. Thus, proceeding with this assumption we deflated the current appropriate revenues by the relevant price index and we obtained a measure of quantity demanded for any telephone service.

3.2 The Relative Price of Telephone Services

There is no information on price data relating to a homogeneous unit such as minutes of calls. We used the price indexes for the revenue categories of Bell (as described in section 3.1). Moreover, we used this series for all carriers.

To define relative prices we divided the price index of any service by the consumer price index of a large metropolitan area, within the region in which the carrier has juridiction to operate.[8] For Bell Canada, we considered the weighted arithmetic mean of the consumer price indexes for Toronto and Montreal; for B.C. Telephone we used the consumer price index of Vancouver; for the public carriers we used the consumer price index of Winnipeg, and for the private carriers we used the weighted arithmetic mean of the consumer price indexes of St. John's, St. John and Halifax.

3.3 The Real Income

The demand equations pertain to households and firms, so then the income variable must include more than consumption expenditures. Indeed, for the income variable we used the gross provincial product.[9] For Bell Canada we considered the sum of the Gross Provincial Products of Quebec and Ontario; for B.C.

Telephone we used the Gross Provincial Product of B.C.; for the public companies we use the sum of the Gross Provincial Products of Alberta, Saskatchewan and Manitoba; for the private companies we used the sum of the Gross Provincial Products of New Brunswick, Nova Scotia and Newfoundland.

Finally to deflate these nominal income variables we utilized the appropriate consumer price indexes from each of the jurisdictions, as explained in section 3.2.

4. The Empirical Results

4.1 Bell Canada

4.1.1 The Linear Demand Model

The linear model, in the context of Bell Canada, may be represented by the following set of equations (the sample period for Bell is 1950-1975);[10]

$$BLTSO_t = \beta_0 + \beta_1 \; BLPDTS_t + \beta_2 \; BLGPD_t$$

$$BLLSO_t = \beta_0 + \beta_1 \; BLPDLS_t + \beta_2 \; BLGPD_t$$

$$BLTTO_t = \beta_0 + \beta_1 \; BLPDTT_t + \beta_2 \; BLGPD_t$$

$$BLTOO_t = \beta_0 + \beta_1 \; BLPDTO_t + \beta_2 \; BLGPD_t$$

$$BLDAO_t = \beta_0 + \beta_1 \; BLPDDA_t + \beta_2 \; BLGPD_t$$

$$BLMSO_t = \beta_0 + \beta_1 \; BLPDMS_t + \beta_2 \; BLGPD_t$$

$$BLLTO_t = \beta_0 + \beta_1 \; BLPDLT_t + \beta_2 \; BLGPD_t$$

$$BLBMO_t = \beta_0 + \beta_1 \; BLPDBM_t + \beta_2 \; BLGPD_t$$

$$BLBDO_t = \beta_0 + \beta_1 \; BLPDBD_t + \beta_2 \; BLGPD_t \; .$$

The results for the ordinary least squares regression, which are found in table 4.1.1, show us that $\beta_1 < 0$, and $\beta_2 > 0$, except for the directory and miscellaneous categories but the Durbin-Watson statistic points out that positive autocorrelation is present.

Upon correcting for autocorrelation, we find that, although the results improve, the Durbin-Watson is still quite low. When

Table 4.1.3

Linear Demand Model: N.L.R.

(t-values in parentheses)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Toll	-17.485 (-1.270)	-134.088 (-1.305)	-.005 (-2.222)	1.062 (4.192)	.024 (.090)	1.946	.999
Local	-3.892 (-1.046)	-35.247 (-.864)	-.0003 (-.298)	1.523 (6.552)	-.473 (-1.919)	2.010	.999
Toll	-29.322 (-2.058)	-110.274 (-2.227)	-.001 (-.791)	.907 (3.821)	.249 (.954)	2.178	.998
Toll-Misc. Toll	-22.017 (-2.058)	-83.874 (-2.159)	-.001 (-1.064)	.924 (3.867)	.218 (.838)	2.163	.998
Directory	3.188 (1.429)	-10.237 (-.568)	-.001 (-1.572)	1.348 (5.503)	-.377 (-1.631)	1.634	.896
Misc.	3.503 (1.284)	-6.098 (-.211)	-.001 (-1.783)	1.397 (5.745)	-.416 (-1.774)	1.625	.886
Local+Toll-Misc. Toll	-17.111 (-1.337)	-108.323 (-1.243)	-.003 (-1.439)	1.083 (4.391)	.009 (.032)	2.035	.999
Local+Toll+Directory	-13.151 (-1.110)	-106.695 (-1.126)	-.004 (-2.015)	1.135 (4.450)	-.056 (-.207)	1.929	.999
Local+Toll	-23.507 (-1.536)	-138.817 (-1.488)	-.003 (-1.485)	1.000 (4.118)	.103 (.395)	2.059	.999

Table 4.1.1.

Linear Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	1651.598 (4.223)	-1379.052 (-5.068)	.011 (3.889)	.401	.978
Local	643.238 (3.424)	-558.150 (-4.188)	.008 (5.733)	.362	.981
Toll	546.127 (1.598)	-456.734 (-2.165)	.005 (1.753)	.220	.935
Toll-Misc. Toll	311.036 (1.244)	-275.093 (-1.783)	.005 (2.333)	.242	.934
Directory	-30.123 (-4.235)	37.008 (5.376)	.0005 (9.566)	1.035	.821
Misc.	-38.932 (-3.860)	48.339 (4.902)	.0005 (9.221)	1.053	.806
Local+Toll-Misc. Toll	1446.241 (3.545)	-1173.877 (-4.230)	.009 (2.993)	.352	.973
Local+Toll+Directory	1399.301 (3.855)	-1159.831 (-4.609)	.010 (3.935)	.388	.977
Local+Toll	1738.657 (3.813)	-1410.870 (-4.557)	.009 (2.671)	.360	.973

Table 4.1.2

Linear Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	2055.058 (4.454)	-1043.557 (-4.777)	.006 (1.267)	.971 (20.206)	.827	.992
Local	955.353 (4.770)	-444.680 (-4.512)	.004 (1.823)	.968 (19.209)	.713	.994
Toll	104.957 (1.362)	- 68.863 (-1.299)	-.003 (-3.377)	1.097 (18.200)	2.261	.998
Toll-Misc. Toll	152.973 (2.847)	- 86.921 (-2.381)	-.0004 (-.636)	1.134 (19.201)	2.083	.998
Directory	80.327 (3.137)	-11.548 (-.878)	-.0006 (-1.904)	.932 (12.899)	1.198	.893
Misc.	101.224 (3.038)	-12.809 (-.689)	-.0008 (-1.971)	.938 (13.492)	1.161	.878
Local+Toll-Misc. Toll	1741.904 (4.027)	-834.902 (-4.229)	.006 (1.323)	.973 (21.002)	.711	.992
Local+Toll+Directory	1901.555 (4.486)	-869.403 (-4.490)	.005 (1.136)	.972 (20.700)	.800	.992
Local+Toll	2063.695 (3.851)	-964.082 (-4.280)	.006 (1.226)	.976 (22.459)	.720	.992

Table 4.1.3

Linear Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Toll	-17.485 (-1.270)	-134.088 (-1.305)	-.005 (-2.222)	1.062 (4.192)	.024 (.090)	1.946	.999
Local	-3.892 (-1.046)	-35.247 (-.864)	-.0003 (-.298)	1.523 (6.552)	-.473 (-1.919)	2.010	.999
Toll	-29.322 (-2.058)	-110.274 (-2.227)	-.001 (-.791)	.907 (3.821)	.249 (.954)	2.178	.998
Toll-Misc. Toll	-22.017 (-2.058)	-83.874 (-2.159)	-.001 (-1.064)	.924 (3.867)	.218 (.838)	2.163	.998
Directory	3.188 (1.429)	-10.237 (-.568)	-.001 (-1.572)	1.348 (5.503)	-.377 (-1.631)	1.634	.896
Misc.	3.503 (1.284)	-6.098 (-.211)	-.001 (-1.783)	1.397 (5.745)	-.416 (-1.774)	1.625	.886
Local+Toll-Misc. Toll	-17.111 (-1.337)	-108.323 (-1.243)	-.003 (-1.439)	1.083 (4.391)	.009 (.032)	2.035	.999
Local+Toll+Directory	-13.151 (-1.110)	-106.695 (-1.126)	-.004 (-2.015)	1.135 (4.450)	-.056 (-.207)	1.929	.999
Local+Toll	-23.507 (-1.536)	-138.817 (-1.488)	-.003 (-1.485)	1.000 (4.118)	.103 (.395)	2.059	.999

we then adjust once more, we can observe from table 4.1.3 that β_2 consistently has the wrong sign. Thus the linear model does not perform well for Bell Canada.

4.1.2 The Double-Log Demand Model

For this model we estimated the equations in the form given by the following subset of the nine regressions:

$$\log BLTSO_t = \beta_0 + \beta_1 \log BLPDTS_t + \beta_2 \log BLGPD_t$$

$$\log BLLSO_t = \beta_0 + \beta_1 \log BLPDLS_t + \beta_2 \log BLGPD_t$$

$$\log BLTTO_t = \beta_0 + \beta_1 \log BLPDTT_t + \beta_2 \log BLGPD_t$$

$$\log BLMSO_t = \beta_0 + \beta_1 \log BLPDMS_t + \beta_2 \log BLGPD_t$$

$$\log BLBDO_t = \beta_0 + \beta_1 \log BLPDBD_t + \beta_2 \log BLGPD_t \ .$$

The results for the O.L.S. regressions are presented in table 4.1.4. In most cases $\beta_1 < 0$, and $\beta_2 > 0$, however the presence of autocorrelation discounts these positive findings. Correcting once for autocorrelation significantly improves the results, except for the directory and miscellaneous categories. Moreover, the statistical tests show us that, from table 4.1.6, local and the last three categories perform best (in this context) when we adjust twice for autocorrelation.

Table 4.1.4

Double-Log Demand Model: O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	-10.503 (-9.750)	-.196 (-.692)	1.595 (15.694)	.507	.985
Local	-10.333 (-9.520)	.054 (.184)	1.528 (14.879)	.467	.981
Toll	-12.914 (-9.861)	-.743 (-2.678)	1.728 (14.098)	.547	.992
Toll-Misc. Toll	-11.331 (-8.186)	-.691 (-2.316)	1.562 (12.079)	.512	.989
Directory	-6.468 (-7.802)	1.562 (5.424)	.937 (11.466)	.858	.870
Misc.	-5.402 (-7.052)	1.552 (4.961)	.836 (11.385)	1.005	.864
Local+Toll-Misc. Toll	-10.100 (-8.549)	-.225 (-.756)	1.546 (13.893)	.469	.985
Local+Toll+Directory	-10.124 (-8.860)	-.128 (-.427)	1.552 (14.406)	.478	.983
Local+Toll	-10.668 (-9.260)	-.283 (-.985)	1.605 (14.793)	.477	.987

Table 4.1.5

Double-Log Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	-2.156 (-1.997)	-1.325 (-8.318)	.816 (8.044)	.812 (6.954)	1.228	.998
Local	-1.825 (-1.742)	-1.061 (-7.021)	.734 (7.454)	.821 (7.191)	1.012	.998
Toll	-7.311 (-5.711)	-1.566 (-7.631)	1.205 (10.031)	.692 (4.795)	1.600	.998
Toll-Misc. Toll	-6.280 (-4.720)	-1.455 (-6.586)	1.092 (8.753)	.673 (4.555)	1.448	.997
Directory	11.817 (2.598)	-.400 (-.979)	-.770 (-1.832)	.894 (9.954)	1.191	.947
Misc.	13.286 (2.800)	-.454 (-.941)	-.887 (-2.028)	.899 (10.289)	1.162	.929
Local+Toll-Misc. Toll	-2.787 (-2.646)	-1.291 (-7.719)	.863 (8.702)	.776 (6.149)	1.178	.998
Local+Toll+Directory	-1.963 (-1.850)	-1.268 (-7.879)	.790 (7.923)	.802 (6.712)	1.209	.998
Local+Toll	-3.190 (-2.998)	-1.343 (-8.176)	.907 (9.061)	.785 (6.337)	1.214	.998

Table 4.1.6

Double-Log Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	-.068 (1.447)	-.420 (-2.538)	.166 (1.376)	1.334 (10.337)	-.339 (-2.750)	2.386	.999
Local	-.053 (-1.407)	-.276 (-2.248)	.149 (1.553)	1.423 (13.623)	-.427 (-4.305)	2.032	.999
Toll	.067 (2.624)	-.666 (-2.773)	.439 (2.105)	1.078 (5.859)	-.082 (-.458)	2.356	.999
Toll-Misc. Toll	.055 (2.024)	-.578 (-2.411)	.364 (1.599)	1.153 (6.304)	-.151 (-.847)	2.382	.998
Directory	.784 (1.691)	-.654 (-1.130)	-.777 (-1.078)	1.318 (5.862)	-.385 (-1.873)	1.508	.942
Misc.	.836 (1.715)	-.578 (-.744)	-1.133 (-1.488)	1.326 (5.900)	-.378 (-1.797)	1.560	.926
Local+Toll-Misc. Toll	.046 (1.044)	-.381 (-2.146)	.232 (1.768)	1.366 (10.247)	-.367 (-2.885)	2.398	.999
Local+Toll+Directory	.063 (1.209)	-.400 (-2.269)	.176 (1.343)	1.359 (9.988)	-.364 (-2.815)	2.276	.999
Local+Toll	.053 (1.356)	-.431 (-2.473)	.261 (2.057)	1.331 (9.985)	-.333 (-2.615)	2.432	.999

4.1.3 The Rotterdam Demand Model

The Rotterdam model for Bell Canada can be represented by the following subset of equations:[11]

$$\alpha_{it}\left[\log BLTSO_t - \log BLTSO_{t-1}\right] = \beta_0 + \beta_1\left[\log BLPDTS_t - \log BLPDTS_{t-1}\right] + \beta_2\left[\log BLGPD_t - \log BLGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BLLSO_t - \log BLLSD_{t-1}\right] = \beta_0 + \beta_1\left[\log BLPDLS_t - \log BLPDLS_{t-1}\right] + \beta_2\left[\log BLGPD_t - \log BLGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BLTTO_t - \log BLTTO_{t-1}\right] = \beta_0 + \beta_1\left[\log BLPDTT_t - \log BLPDTT_{t-1}\right] + \beta_2\left[\log BLGPD_t - \log BLGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BLMSO_t - \log BLMSO_{t-1}\right] = \beta_0 + \beta_1\left[\log BLPDMS_t - \log BLPDMS_{t-1}\right] + \beta_2\left[\log BLGPD_t - \log BLGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BLBDO_t - \log BLBDO_{t-1}\right] = \beta_0 + \beta_1\left[\log BLPDBD_t - \log BLPDBD_{t-1}\right] + \beta_2\left[\log BLGPD_t - \log BLGPD_{t-1}\right].$$

The results for the ordinary least squares estimates are found in table 4.1.7. We can observe that the signs of β_1 and β_2 are generally correct. However, price appears to affect local services in a positive fashion and this can also account for the positive β_1 in the local plus toll - miscellaneous toll category. After adjusting for the positive autocorrelation, we see that the estimates are generally insignificant and so we must reject the Rotterdam model, even though in all cases β_1 has the right sign and only for miscellaneous and directory categories does β_2 have the wrong sign.

Table 4.1.7

Rotterdam Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	.001 (7.839)	-.004 (-1.338)	.001 (.452)	1.135	.079
Local	.0007 (7.153)	.0004 (.278)	.0003 (.221)	.707	.009
Toll	.0004 (4.651)	-.004 (-2.989)	.002 (1.084)	1.644	.302
Toll-Misc. Toll	.0004 (4.613)	-.003 (-2.309)	.001 (.828)	1.629	.207
Directory	.0001 (3.244)	.0004 (1.602)	-.001 (-1.719)	1.245	.169
Misc.	.001 (9.969)	-.005 (-2.781)	.002 (1.075)	2.441	.320
Local+Toll-Misc. Toll	.0001 (3.843)	.0007 (1.922)	-.001 (-2.449)	1.174	.251
Local+Toll+Directory	.001 (6.737)	-.003 (-1.008)	.002 (.694)	1.041	.052
Local+Toll	.001 (6.949)	-.002 (-.795)	.001 (.402)	1.008	.030

Table 4.1.8

Rotterdam Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	.001 (9.970)	-.005 (-2.781)	.002 (1.075)	.158 (.765)	2.441	.320
Local	.001 (10.580)	-.002 (-1.705)	.001 (1.281)	.394 (2.057)	2.135	.432
Toll	.0004 (4.265)	-.004 (-2.968)	.002 (1.221)	.124 (.600)	1.915	.351
Toll-Misc. Toll	.0003 (4.417)	-.003 (-2.607)	.001 (.994)	.084 (.407)	2.022	.277
Directory	.0001 (1.006)	-.0004 (-1.732)	-.0004 (-1.318)	.746 (5.372)	1.570	.467
Misc.	.0001 (2.074)	-.0002 (-.466)	-.0008 (-2.381)	.636 (3.951)	1.617	.463
Local+Toll-Misc. Toll	.0009 (8.096)	-.004 (-2.264)	.002 (1.485)	.222 (1.094)	2.352	.284
Local+Toll+Directory	.0001 (8.470)	-.004 (-2.099)	.002 (1.083)	.238 (1.176)	2.338	.260
Local+Toll	.001 (8.683)	-.006 (-2.966)	.003 (1.714)	.150 (.727)	2.322	.353

4.1.4 Partial Price Elasticities of Demand

In this section we report the partial price elasticities of demand for Bell Canada. For the linear model a subset of the formulae are:

$\beta_1 \frac{BLPDTS_t}{BLTS0_t}$ - partial price elasticity of total demand in period t.

$\beta_1 \frac{BLPDLS_t}{BLLS0_t}$ - partial price elasticity of local demand in period t.

$\beta_1 \frac{BLPDTT_t}{BLTT0_t}$ - partial price elasticity of toll demand in period t.

$\beta_1 \frac{BLPDMS_t}{BLMS0_t}$ - partial price elasticity of miscellaneous demand in period t.

$\beta_1 \frac{BLPDBD_t}{BLBD0_t}$ - partial price elasticity of local plus toll demand in period t.

In the linear model the elasticities are variable over time, because β_1 is just the rate of change. Indeed, we can see by the formulae that the trend of the elasticities, in the linear model, is defined by the trend of relative prices to demand.

The elasticities for the O.L.S. estimates ranged from -12.724 in 1950 to -.750 in 1975 for the total demand, for local demand -7.897 in 1950 to -.565 in 1975, for toll -16.899 to -.573, for toll minus miscellaneous toll -10.600 to -.452, for directory 4.233 to .881, for miscellaneous 4.092 to 1.014, for local plus toll minus miscellaneous toll -12.185 to -.733, for local plus toll plus directory -11:155 to -.703, for local plus toll -14.461 to -.789. Therefore we find that for all the cases where $\beta_1 < 0$ the elasticities increase over the sample, and for the cases where $\beta_1 > 0$ the elasticities decrease over the period. Indeed, for total demand a 1% increase in the price of total services led to 12.7% decrease in demand in 1950 while only to 8% decrease in 1976.

For the Cochrane-Orcutt estimates of the linear model the ranges of the elasticities are, for total demand -9.629 in 1950 to -.568 in 1975, for local demand -6.292 to -.450, for toll demand -2.548 to -.086, for toll minus miscellaneous toll -3.349 to -.143, for directory -1.321 to -.275, for miscellaneous -1.084 to -.269, for local plus toll minus miscellaneous toll -8.666 to -.521, for local plus toll plus directory -8.362 to -.527, and for local plus toll -9.882 to -.539. In this set of elasticities the signs are all correct because $\beta_1 < 0$ for all the services under the C-O.L.S. estimates. Moreover for all the elasticities the values are monotonically increasing.

The nonlinear regression estimates for the linear model exhibit the same trend as in the previous cases. For total demand the range is -1.237 to -.073, for local demand -.499 to -.036, for toll -4.080 to -.138, for toll minus miscellaneous toll demand -3.232 to -.138, for directory -1.171 to -.244, for miscellaneous

-.516 to -.128, for local plus toll minus miscellaneous toll -1.124 to -.068, for local plus toll plus directory -1.026 to -.065, for local plus toll -1.423 to -.078.

The double-log demand model incorporates the assumption of constant price (and income) elasticities over the sample period. Thus for each category (given the estimation technique) we get a single elasticity. For the O.L.S. estimates, the elasticities are, for total -.196, for local .054, for toll -.743, for toll minus miscellaneous toll -.691, for directory 1.562, for miscellaneous 1.552, for local plus toll minus miscellaneous toll -.225, for local plus toll plus directory -.128, for local plus toll -.283.

The Cochrane-Orcutt estimates are, for total -1.3, for local -1.1, for toll -1.6, for toll minus miscellaneous toll -1.5, for directory -.4, for miscellaneous -.5, and for the last three categories the elasticities are all -1.3.

The nonlinear regression estimates for each category are -.420, -.276, -.566, -.578, -.654, -.578, -.381, -.400, -.431.

The last group of elasticities pertains to the Rotterdam model and it's method of estimation. The elasticities, in this context, are computed by dividing the β_1 estimates in tables 4.1.7 and 4.1.8 by α_{it}. Since α_{it} (as defined in section 2 and footnote 11) is a variable, the partial price elasticity of demand is variable.

For ordinary least squares the elasticities tend to fluctuate over the sample period for each type of service. The range for total demand is -.210 to -.410, for local demand :047 to .102, for toll -.561 to -1.425, for toll minus miscellaneous toll -.444 to -.931, for directory 2.454 to .494, for miscellaneous 3.384 to .730, for local plus toll minus miscellaneous toll -.179 to -.350, local plus toll plus directory -.143 to -.280, for local plus toll -.265 to -.516.

The C-O.L.S. estimates of the Rotterdam model yield the following range of values for the elasticities in the nine demand categories, -.311 to -.606, -.154 to -.330, -.575 to -1.460, -.484 to -1.014, -.490 to -2.430, -.183 to -.848, -.292 to -.570, -.271 to -.528, -.353 to -.689.

We previously stated that for Bell Canada the equations which generally yielded the best results were the double-log C-O.L.S. estimates. Moreover, because we ultimately will integrate the demand module into a more complex framework relating such variables as revenues, costs and rates of return, we are interested in the value of the price elasticity of demand. From economic theory we are aware that a monopolist (given a particular geographical location) must always have a price elasticity smaller or equal to -1.000. We can see that the price elasticity for total demand given from table 4.1.5 is -1.3, and this figure is consistent with economic analysis.

4.2 B.C. Telephone

4.2.1 The Linear Demand Model

The first equation we estimated for British Columbia Telephone was the linear model given by equation (12). Table 4.2.1 presents the results for the linear case when we used ordinary least squares and estimated the equations for the period 1961-1975.

$$BCTSO_t = \beta_0 + \beta_1\ BCPDTS_t + \beta_2\ BCGPD_t$$
$$BCLSO_t = \beta_0 + \beta_1\ BCPDLS_t + \beta_2\ BCGPD_t$$
$$BCTTO_t = \beta_0 + \beta_1\ BCPDTT_t + \beta_2\ BCGPD_t$$
$$BCMSO_t = \beta_0 + \beta_1\ BCPDMS_t + \beta_2\ BCGPD_t$$
$$BCBDO_t = \beta_0 + \beta_1\ BCPDBD_t + \beta_2\ BCGPD_t$$

The results for the O.L.S. regressions are such that, although in all cases $\beta_2 > 0$ and so we should and do find $\beta_1 < 0$, there is serial correlation, which is reflected by the low D.W. statistic.

Correcting for autocorrelation we find that the results significantly improve. Indeed, for the toll category there is a radical change in the coefficients and their importance. We also find that the estimate for ρ_1 in each case is significant but the serial correlation still persists, as found in table 4.2.2.

This time we estimated the linear model using the nonlinear approach because we are twice correcting for autocorrelation. These results appear in table 4.2.3. We can observe from table 4.2.3, that

Table 4.2.1

Linear Demand Model: O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	352.602 (2.096)	-318.667 (-2.631)	.014 (2.130)	.429	.979
Local	166.888 (3.488)	-134.175 (-3.863)	.005 (2.482)	1.208	.989
Toll	3.166 (.033)	-47.920 (-.723)	.015 (3.787)	.487	.965
Miscellaneous	23.838 (7.137)	-22.712 (-3.692)	.0005 (4.014)	.958	.985
Local + Toll	282.308 (1.746)	-262.804 (-2.272)	.015 (2.402)	.471	.978

Table 4.2.2

Linear Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	386.142 (3.624)	-386.848 (-5.183	.016 (3.352)	.764 (4.437)	1.332	.994
Local	176.676 (3.582)	-144.058 (-4.061)	.005 (2.284)	.344 (1.371)	1.805	.990
Toll	111.444 (1.818)	-159.695 (-3.630)	.014 (4.815)	.749 (4.234)	1.198	.991
Miscellaneous	15.372 (2.668)	-17.132 (-4.510)	.0008 (3.037)	.700 (3.666)	1.578	.990
Local + Toll	365.383 (3.471)	-364.283 (-4.895)	.016 (3.294)	.753 (4.281)	1.380	.993

Table 4.2.3

Linear Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	-30.728 (-1.701)	-116.986 (-2.219)	-.0007 (-.116)	1.016 (2.248)	.164 (.322)	1.837	.998
Local	-4.728 (-.511)	-21.542 (-.585)	(-.005 (-1.050)	.488 (.960)	.653 (1.167)	2.079	.994
Toll	-18.345 (-1.892)	-64.925 (-2.643)	.004 (1.075)	1.243 (3.830)	-.004 (-.010)	2.120	.998
Miscellaneous	4.402 (1.159)	-14.356 (-3.268)	.001 (3.289)	.892 (3.013)	-.286 (-1.042)	2.034	.993
Local + Toll	-26.279 (-1.507)	-108.132 (-2.033)	-.001 (-.252)	1.058 (2.276)	.106 (.202)	1.787	.998

using the non-linear regression in order to compute the estimates from the double correction specification, that for all categories ρ_2 is insignificant and so the second adjustment is inappropriate. This outcome could be due to the fact that we are using Bell's data as an approximation for the output prices.

It appears then that the C-O.L.S. approach to the linear model for B.C. Telephone yields the best results.

4.2.2 The Double-Log Demand Model

In this section we estimated the double-log equations, which for B.C. Telephone demand categories are:

$$\log BCTSO_t = \beta_0 + \beta_1 \log BCPDTS_t + \beta_2 \log BCGPD_t$$

$$\log BCLSO_t = \beta_0 \quad \beta_1 \log BCPDLS_t + \beta_2 \log BCGPD_t$$

$$\log BCTTO_t = \beta_0 + \beta_1 \log BCPDTT_t + \beta_2 \log BCGPD_t$$

$$\log BCMSO_t = \beta_0 + \beta_1 \log BCPDMS_t + \beta_2 \log BCGPD_t$$

$$\log BCBDO_t = \beta_0 + \beta_1 \log BCPDBD_t + \beta_2 \log BCGPD_t$$

The results for the O.L.S. estimates are presented in table 4.2.4. We find that for all cases $\beta_2 > 0$ and $\beta_1 < 0$. In addition, the coefficients for each category are all significant while there does seem to be a minimal degree of autocorrelation; except for miscellaneous demand, because of the residual nature of the component, the autocorrelation is quite severe.

For the Cochrane-Orcutt adjustment the results are found in table 4.2.5. Again we find that the price and income effects

Table 4.2.4

Double-Log Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	-5.296 (-11.072)	-1.077 (-8.634)	1.138 (21.057)	1.543	.999
Local	-3.183 (-5.255)	-.929 (-5.924)	.826 (12.037)	1.878	.997
Toll	-10.5766 (-15.496)	-.873 (-5.191)	1.641 (21.384)	1.334	.999
Miscellaneous	-7.000 (-3.616)	-2.409 (-4.168)	.950 (4.324)	.809	.944
Local + Toll	-5.486 (-12.197)	-1.016 (-8.779)	1.156 (22.743)	1.932	.999

Table 4.2.5

Double-Log Demand Model: C-O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	-5.417 (-9.609)	-1.069 (-7.860)	1.152 (18.111)	.180 (.686)	1.956	.999
Local	-3.319 (-4.741)	-.907 (-5.288)	.841 (10.628)	.030 (.113)	1.996	.996
Toll	-10.209 (-13.042)	-.936 (-5.194)	1.600 (18.195)	.336 (1.333)	1.644	.999
Miscellaneous	-12.086 (-3.048)	-1.517 (-2.023)	1.516 (3.412)	.557 (2.510)	2.136	.969
Local + Toll	-5.586 (-11.593)	-1.000 (-8.317)	1.167 (21.462)	-.024 (-.089)	1.998	.999

Table 4.2.6

Double-Log Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	-7.156 (-2.647)	-1.015 (-6.004)	1.184 (16.347)	.128 (.350)	-3.829 (-1.175)	2.134	.999
Local	-4.568 (-2.189)	-.958 (-3.731)	.832 (7.646)	.083 (.212)	-.493 (-1.022)	2.173	.996
Toll	-9.706 (-1.832)	-1.030 (-3.041)	1.564 (9.850)	.572 (1.734)	-.554 (-1.593)	2.202	.999
Miscellaneous	-12.314 (-6.411)	-.493 (-1.095)	2.156 (7.207)	.266 (1.571)	.042 (.296)	1.358	.992
Local + Toll	-8.958 (-2.372)	-.962 (-5.706)	1.188 (16.897)	-.040 (-.098)	-.512 (-1.347)	2.254	.999

have the appropriate sign, however ρ_1 is insignificant in all cases except the miscellaneous category. This result is what we expected, given our conclusion concerning the O.L.S. estimates.

Therefore one would have to say that, for the double-log equation, the simple ordinary least squares estimates are preferable (except for the miscellaneous category) and it is of little value to analyse the nonlinear regression, but for completeness we present the results in table 4.2.6.

4.2.3 The Rotterdam Demand Model

In this section we estimate equation (19) which in this context gives the following system of equations:

$$\alpha_{it}\left[\log BCTSO_t - \log BCTSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log BCPDTS_t - \log BCPDTS_{t-1}\right] + \beta_2 \left[\log BCGPD_t - \log BCGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BCLSO_t - \log BCLSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log BCPDLS_t - \log BCPDLS_{t-1}\right] + \beta_2 \left[\log BCGPD_t - \log BCGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BCTTO_t - \log BCTTO_{t-1}\right] = \beta_0 + \beta_1 \left[\log BCPDTT_t - \log BCPDTT_{t-1}\right] + \beta_2 \left[\log BCGPD_t - \log BCGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BCMSO_t - \log BCMSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log BCPDMS_t - \log BCPDMS_{t-1}\right] + \beta_2 \left[\log BCGPD_t - \log BCGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log BCBDO_t - \log BCBDO_{t-1}\right] = \beta_0 + \beta_1 \left[\log BCPDBD_t - \log BCPDBD_{t-1}\right] + \beta_2 \left[\log BCGPD_t - \log BCGPD_{t-1}\right].$$

The results for the ordinary least squares estimates are found in table 4.2.7. We can observe that the income (β_2) and

Table 4.2.7

Rotterdam Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	.0008 (2.125)	-.019 (-2.743)	.010 (2.295)	1.940	.496
Local	.0008 (2.856)	-.00005 (-.002)	-.008 (-.264)	1.786	.009
Toll	.004 (1.465)	-.006 (-2.321)	.008 (2.613)	1.393	.542
Miscellaneous	-.0005 (-.698)	-.002 (-3.269)	.001 (1.225)	1.527	.519
Local + Toll	.009 (2.964)	-.009 (-2.630)	.009 (2.539)	2.030	.506

Table 4.2.8

Rotterdam Demand Model: C-O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	.0009 (2.127)	-.012 (-2.622)	.010 (2.117)	.017 (.057)	1.671	.495
Local	.0008 (2.757)	.0004 (.113)	-.001 (-.400)	.090 (.313)	1.837	.018
Toll	.0004 (1.807)	-.008 (-2.961)	.008 (2.593)	.225 (.801)	1.715	.620
Miscellaneous	-.0002 (-2.020)	-.003 (-3.634)	.002 (2.840)	.494 (1.970)	1.317	.604
Local + Toll	.0009 (2.899)	-.009 (-2.601)	.009 (2.380)	-.044 (-.154)	1.846	.510

price (β_1) effects have the correct sign for all categories. In addition, for the total and local plus toll regressions, not only are all the variables significant, but there is virtually no auto-correlation. The toll category cannot be analysed due to the autocorrelation, while the local results show us that the adding-up restrictions do not approximate the true structural characteristics. Therefore because of the generally good results for the Rotterdam model using ordinary least squares we adjusted for autocorrelation.

The results for the model corrected for autocorrelation are presented in table 4.2.8. We find that for total and local plus toll where the results were favourable the correction was not a factor and where the results were unfavourable as in the local category there was basically no improvement. One must conclude then that on the whole the Rotterdam model, in particular the imposition of the adding-up constraint, does not perform well for B.C. Telephone.

4.2.4 Partial Price Elasticities of Demand

In this section we report the partial price elasticities of demand for B.C. Telephone. For the linear model the formulae are:

$\beta_1 \frac{BCPDTS_t}{BCTSO_t}$ - partial price elasticity of total demand in period t.

$\beta_1 \frac{BCPDLS_t}{BCLSO_t}$ - partial price elasticity of local demand in period t.

$\beta_1 \frac{BCPDTT_t}{BCTTO_t}$ - partial price elasticity of toll demand in period t.

$\beta_1 \frac{BCPDMS_t}{BCMS0_t}$ - partial price elasticity of miscellaneous demand in period t.

$\beta_1 \frac{BCPDBD_t}{BCBD0_t}$ - partial price elasticity of local plus toll demand in period t.

We must notice that in the linear model the elasticities are variable over time because the estimated coefficient gives the rate of change and not the percentage rate of change.

First we discuss the elasticities which were computed for the linear model using O.L.S. estimates. The range for the total demand was from -5.835 in 1961 to -.758 in 1975, with monotonic increasing values for the figures. The local demand numbers were also monotonically increasing with a range from -3.901 in 1961 to -.735 in 1975. The toll demand ranged from -2.865 in 1961 to a continual increase to -.222. In addition the miscellaneous and local plus toll elasticities also increased from a low of -7.451 to a high of -1.226 for miscellaneous, and -5.125 to -.655 for local plus toll. Hence the linear model restrictions yield there to be a gradual decrease in the responsiveness of demand to a change in the price of the telephone service; for example a 1% increase in the price of total telephone services caused a 5.8% decrease in the total demand in 1961, while a 1% price increase for services only caused a .8% decrease in 1975.

When we estimated the equations using Cochrane-Orcutt least squares technique we found the elasticities to monotonically increase over the sample period. For total demand the range was from -6.753 to -.878; for local demand the range was from -4.188 to -.789; for toll demand the range was from -9.549 to -.740; for miscellaneous demand the range was from -5.621 to -.925; and finally for local plus toll demand the range was from -7.104 to -.907. Again there is a gradual diminution of the responsiveness of demand for telephone services to the prices of these services.

As in the previous two cases, for the linear model which was twice corrected for serial correlation the elasticities exhibited a decreasing demand responsiveness to price. For total demand the elasticity ranged from -2.142 to -.278, for local demand the range was -.626 to -.118, for toll demand -3.882 to -.301, for miscellaneous -4.710 to -.775, and for local plus toll -2.109 to -.269.

In the double-log model we constrain the partial price elasticities to be constants. Indeed they are the coefficients called β_1 in tables 4.2.4, .5, .6. From the ordinary least squares we have, -1.077 for total demand, -.929 for local demand, -.873 for toll demand, -2.409 for miscellaneous demand, and -1.016 for local plus toll demand. From the Cochrane-Orcutt method the elasticities are -1.069 for total, -.907 for local, -.936 for toll, -1.517 for miscellaneous, and -1.000 for local plus toll. Finally the twice corrected estimates yield the elasticities to be -1.015 for total, -.958 for local, -1.030 for toll, -.493 for miscellaneous, and -.963 for local plus toll.

The third model estimated was the Rotterdam and once more we had variable partial price elasticities of demand. In each case the Rotterdam model elasticities are the β_1 coefficients from tables 4.2.7 and 4.2.8 divided by α_{it}. Since α_{it}, which is the two-year average share of the expenditure on any telephone service out of the income in the carrier's jurisdiction, is variable, then of course the elasticity is variable.

For ordinary least squares the elasticities tend to fluctuate over the sample period. This is due to the imposition of the adding-up restriction, which was described in section 2. The elasticities for total demand ranged from -.641 to -1.254, for local demand from -.0005 to -.001, for toll demand from -.580 to -1.133, for miscellaneous demand from -2.500 to -5.005, and for local plus toll demand from -.520 to -1.009.

For C-O.L.S. estimates we have, a range of -.639 to -1.250 for total, .035 to .086 for local, -.785 to -1.534 for toll, -3.242 to -6.493 for miscellaneous, -.517 to -1.010 for local plus toll.

Recalling that for B.C. Telephone the equations which yielded the best results, on average, were the double-log estimated by ordinary least squares. Since demand conditions determine revenues, and we are interested (for the integrated model) in total revenues, and therefore total demand, the partial price elasticity of total demand will play a crucial role in our simultaneous model.

Moreover from economic theory we know that a firm operating as a monopoly in its own jurisdiction must always have an elasticity of total demand smaller than -1.000, then the price elasticity from the double-log O.L.S. specification of -1.1 is consistent with this theoretical result.

4.3 Public Carriers

In this section we estimate the demand characteristics for the aggregation of Alberta Government Telephones, Edmonton Telephones, Saskatchewan Telecommunications, and Manitoba Telephone System.

4.3.1 The Linear Demand Model

The linear equations for the public companies are

$$TGTS0_t = \beta_0 + \beta_1 \; TGPDTS_t + \beta_2 \; TGGPD_t$$
$$TGLS0_t = \beta_0 + \beta_1 \; TGPDLS_t + \beta_2 \; TGGPD_t$$
$$TGTT0_t = \beta_0 + \beta_1 \; TGPDTT_t + \beta_2 \; TGGPD_t$$
$$TGMS0_t = \beta_0 + \beta_1 \; TGPDMS_t + \beta_2 \; TGGPD_t$$
$$TGBD0_t = \beta_0 + \beta_1 \; TGPDBD_t + \beta_2 \; TGGPD_t$$

The results for the O.L.S. estimates are found in table 4.3.1. In all cases $\beta_2 > 0$ and $\beta_1 < 0$ (in other words all telephone services are normal commodities). The results are generally good except for the presence of autocorrelation, suggested by the low D.W. statistic, although the miscellaneous demand equation does not show an important price coefficient. This occurs because of the residual nature of the miscellaneous category. We must notice that the strong price effects from local and toll swamp the weak response from miscellaneous demand, as depicted by the highly significant price term in the demand equation for total telephone services. We should be careful in interpreting the t - values, because of the presence of autocorrelation.

Table 4.3.1

Linear Demand Model: O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	291.147 (2.575)	-315.392 (-4.079)	.019 (5.877)	1.137	.990
Local	161.665 (6.074)	-136.732 (-7.451)	.005 (5.891)	.652	.995
Toll	22.489 (.291)	-96.948 (-1.913)	.016 (6.840)	1.379	.9816
Miscellaneous	-1.260 (-.197)	-4.043 (-.851)	.001 (7.870)	1.753	.981
Local + Toll	246.137 (2.225)	-278.272 (-3.703)	.019 (5.914)	1.103	.988

Table 4.3.2

Linear Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	524.163 (4.744)	-482.819 (-6.338)	.014 (4.295)	.555 (2.493)	1.541	.994
Local	181.542 (7.880)	-152.215 (-9.354)	.004 (5.954)	.587 (2.715)	1.257	.997
Toll	231.815 (2.629)	-253.830 (-4.144)	.011 (4.417)	.469 (2.000)	1.753	.981
Miscellaneous	20.311 (2.946)	-18.214 (-3.794)	.0006 (3.165)	.499 (2.152)	2.071	.988
Local + Toll	500.809 (4.800)	-473.233 (-6.428)	.013 (4.374)	.567 (2.573)	1.531	.994

Table 4.3.3

Linear Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	-77.447 (-3.896)	-260.275 (-8.440)	.002 (1.387)	.797 (3.014)	.428 (1.386)	2.385	.999
Local	-12.010 (-1.823)	-97.265 (-6.567)	.002 (3.013)	1.106 (3.444)	-.009 (-.025)	2.109	.999
Toll	-44.032 (-3.166)	-150.209 (-6.009)	.0002 (.113)	1.106 (5.246)	.109 (.426)	2.226	.998
Miscellaneous	-.064 (-.084)	1.484 (.291)	-.0009 (-.439)	1.069 (3.076)	.070 (.182)	2.220	.993
Local + Toll	-72.061 (-3.689)	-251.989 (-8.141)	.002 (1.354)	.926 (3.721)	.289 (.987)	2.484	.999

Correcting for autocorrelation we find that from table 4.3.2 the results improve over the O.L.S. estimates. In all cases $\beta_2 > 0$ and $\beta_1 < 0$, and the t-values show that the variables are more significant, once we adjust for serial correlation. However, in two cases, local and local plus toll the D.W. statistic suggests that we should correct for autocorrelation.

The results for the nonlinear regression are found in table 4.3.3. We see that ρ_2 is insignificant for all the demand categories, and the income effect (β_2) is not significant for all services except local demand. Thus for the linear model the C-O.L.S. estimates appear to be the best ones.

4.3.2 The Double-Log Demand Model

The double-log model, in this context, is represented by the following equations:

$$\log TGTSO_t = \beta_0 + \beta_1 \log TGPDTS_t + \beta_2 \log TGGPD_t$$

$$\log TGLSO_t = \beta_0 + \beta_1 \log TGPDLS_t + \beta_2 \log TGGPD_t$$

$$\log TGTTO_t = \beta_0 + \beta_1 \log TGPDTT_t + \beta_2 \log TGGPD_t$$

$$\log TGMSO_t = \beta_0 + \beta_1 \log TGPDMS_t + \beta_2 \log TGGPD_t$$

$$\log TGBDO_t = \beta_0 + \beta_1 \log TGPDBD_t + \beta_2 \log TGGPD_t$$

The results for the ordinary least squares estimates are found in table 4.3.4. For all the categories, except miscellaneous, the price effect is negative and for all the services β_2, the income effect, is positive. However, we cannot

Table 4.3.4

Double-Log Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	-5.845 (-2.162)	-1.411 (-2.581)	1.181 (4.057)	.454	.982
Local	-3.455 (-1.565)	-1.284 (-2.896)	.829 (3.482)	.371	.980
Toll	-7.600 (-2.476)	-1.754 (-2.997)	1.303 (3.958)	.594	.983
Miscellaneous	-17.829 (-9.440)	.842 (1.885)	2.147 (10.520)	1.790	.970
Local + Toll	-5.228 (-1.963)	-1.544 (-2.907)	1.109 (3.868)	.468	.983

Table 4.3.5

Double-Log Demand Model: C-O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	1.800 (1.146)	-1.103 (-5.276)	.423 (2.653)	.918 (8.675)	1.588	.998
Local	1.401 (1.031)	-.921 (-5.223)	.352 (2.555)	.918 (8.633)	.995	.997
Toll	.445 (.185)	-1.019 (-3.396)	.524 (2.165)	.923 (9.004)	1.510	.996
Miscellaneous	-3.536 (-1.765)	-1.161 (-3.787)	.622 (2.902)	.630 (3.036)	1.447	.991
Local + Toll	1.741 (1.017)	-1.098 (-4.790)	.428 (2.481)	.925 (9.099)	1.418	.997

Table 4.3.6

Double-Log Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	.203 (1.935)	-.940 (-4.118)	.314 (1.979)	.850 (2.793)	.091 (.306)	2.328	.998
Local	.064 (.782)	-.894 (-3.284)	.383 (2.827)	1.381 (4.397)	-.444 (-1.420)	2.375	.998
Toll	.146 (.994)	-1.118 (-3.682)	.370 (1.649)	.990 (3.654)	-.058 (-.220)	2.452	.997
Miscellaneous	.109 (1.144)	-.382 (-.828)	.050 (.199)	1.139 (5.480)	-.161 (-.865)	2.864	.994
Local + Toll	.175 (1.742)	-.982 (-3.906)	.332 (2.027)	.966 (3.340)	-.023 (-.080)	2.443	.998

take much stock in these results because of the very high auto-correlation which of course biases the values of the parameters and their t-values.

Once we correct for autocorrelation, we see from table 4.3.5, an enormous improvement in the D.W. statistics, except for the local services. Moreover, $\beta_2 > 0$ and $\beta_1 < 0$ for all of the demand categories, and the values of these coefficients are significant.

Because of the inconclusiveness of the D.W. statistic, in local category, it appears interesting to run the model correcting twice for serial correlation. These results are presented in table 4.3.6. In all cases we find that ρ_2 is insignificant and therefore we can immediately dismiss these estimates. Consequently, for the double-log model the Cochrane-Orcutt estimates appear to be the best ones.

4.3.3 The Rotterdam Demand Model

The equations of the Rotterdam model are:

$$\alpha_{it}\left[\log TGTSO_t - \log TGTSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log TGPDTS_t - \log TGPDTS_{t-1}\right] + \beta_2 \left[\log TGGPD_t - \log TGGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log TGLSO_t - \log TGLSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log TGPDLS_t - \log TGPDLS_{t-1}\right] + \beta_2 \left[\log TGGPD_t - \log TGGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log TGTTO_t - \log TGTTO_{t-1}\right] = \beta_0 + \beta_1 \left[\log TGPDTT_t - \log TGPDTT_{t-1}\right] + \beta_2 \left[\log TGGPD_t - \log TGGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log TGMSO_t - \log TGMSO_{t-1}\right] = \beta_0 + \beta_1 \left[\log TGPDMS_t - \log TGPDMS_{t-1}\right] + \beta_2 \left[\log TGGPD_t - \log TGGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log TGBDO_t - \log TGBDO_{t-1}\right] = \beta_0 + \beta_1 \left[\log TGPDBD_t - \log TGPDBD_{t-1}\right] + \beta_2 \left[\log TGGPD_t - \log TGGPD_{t-1}\right].$$

Table 4.3.7

Rotterdam Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	.001 (4.805)	-.014 (-2.927)	.004 (1.329)	2.045	.540
Local	.0004 (3.859)	-.005 (-2.832)	.0008 (.715)	1.572	.465
Toll	.0007 (3.758)	-.007 (-2.007)	.003 (1.300)	1.768	.423
Miscellaneous	.0001 (3.388)	.0004 (.847)	.0001 (.368)	1.680	.072
Local + Toll	.001 (4.280)	-.014 (-2.815)	.004 (1.282)	1.782	.530

Table 4.3.8

Rotterdam Demand Model: C-O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	.001 (5.788)	-.015 (-3.452)	.003 (1.277)	-.151 (-.530)	2.164	.621
Local	.0003 (2.230)	-.005 (-2.483)	.002 (2.050)	.465 (1.820)	1.994	.502
Toll	.0007 (4.642)	-.010 (-3.326)	.003 (1.366)	.008 (.027)	2.440	.644
Miscellaneous	.0001 (6.617)	.0005 (2.918)	.0003 (2.030)	-.354 (-1.311)	2.975	.194
Local + Toll	.001 (5.156)	-.015 (-3.430)	.004 (1.395)	-.044 (-.153)	2.244	.630

With respect to the Rotterdam model, the results are presented in table 4.3.7. For the total demand, there is virtually no autocorrelation and $\beta_1 < 0$, $\beta_2 > 0$, but the income effect is a marginal variable. The toll and local plus toll are acceptable equations but again the income effect is insignificant. The miscellaneous category, due to the random noise from the data, is a very poor fit. When we adjust for autocorrelation there is only a marginal improvement in the demand for local services. This result is expected because the D.W. statistic in Table 4.3.7 indicates an absence of autocorrelation.

One can then say that in general, except for the local services, the Rotterdam model does not perform well for the public carriers.

Indeed, we find that for the total, toll, miscellaneous and local plus toll the linear model using the Cochrane-Orcutt least squares yields the best results; for the total services the Rotterdam model using ordinary least squares also does quite well, while for local services the Rotterdam model utilizing the C-O.L.S. estimates gives the best results.

4.3.4 Partial Price Elasticities of Demand

In the linear model the price elasticities are computed from:

$\beta_1 \dfrac{TGPDTS_t}{TGTS0_t}$ - partial price elasticity of total demand in period t

$\beta_1 \dfrac{TGPDLS_t}{TGLS0_t}$ - partial price elasticity of local demand in period t

$\beta_1 \dfrac{TGPDTT_t}{TGTT0_t}$ - partial price elasticity of toll demand in period t

$\beta_1 \dfrac{TGPDMS_t}{TGMS0_t}$ - partial price elasticity of miscellaneous demand in period t

$\beta_1 \dfrac{TGPDBD_t}{TGBD0_t}$ - partial price elasticity of local plus toll demand in period t

With the ordinary least squares estimates all the elasticities showed a monotonic decrease in demand responsiveness to price, because of the fact that given a constant β_1 the ratio of relative prices to output declined over time. The range for total demand is from -4.305 to -.563, for local -3.670 to -.700, for toll from -3.195 to -.293, for miscellaneous from -1.195 to -.155, and for local plus toll from -4.015 to -.521.

The Cochrane-Orcutt estimates yield the ranges of the elasticities to be, for total -6.727 to -.879, for local -4.086 to -.776, for toll -8.363 to -.767, for miscellaneous -5.384 to -.699, for local plus toll -6.828 to -.886.

The nonlinear estimates are, for total -3.553 to -.464, for local -2.611 to -.496, for toll -4.950 to -.454, for miscellaneous .439 to .057, for local plus toll -3.636 to -.472.

The double-log elasticities are: for ordinary least squares, total -1.411, local -1.284, toll -1.754, miscellaneous .842, local plus toll -1.544; for C-O. least squares, total -1.103, local -.920, toll -1.019, miscellaneous -1.161, local plus toll -1.098; for N.L.R. estimates, total -.940, local -.894, toll -1.118, miscellaneous -.382, local plus toll -.982.

The Rotterdam elasticities vary over the sample period because they are defined by β_1 divided by α_{it}.

The ordinary least squares results are: for total a high of -.828 and a low of -1.077, for local -.717 (high) and -1.700 (low), for toll -.661 (high) and -1.291 (low), for miscellaneous .932 (high) and .409 (low), for local plus toll -.856 (high) and -1.691 (low).

The C-O least squares estimates are: for total -.801 (high) and -1.691 (low), local -.692 (high) and -1.642 (low), toll -.950 (high) and -1.869 (low), miscellaneous 1.359 (high) and .596 (low), local plus toll -.849 (high) and -1.785 (low).

For the total demand services, it appears that the linear model corrected for autocorrelation yielded the best results and if we calculated the mean of the range of the elasticities, we find that the average partial price elasticity of total demand for the public companies is -3.1.

4.4 Private Carriers

This section deals with the estimation of the demand structure for the aggregation of the private companies, Newfoundland Telephone, New Brunswick Telephone, and Maritime Telegraph and Telephone.

4.4.1 The Linear Demand Model

The linear equations which were estimated for the private companies are,

$$OPTSO_t = \beta_0 + \beta_1 \; OPPDTS_t + \beta_2 \; OPGPD_t$$
$$OPLSO_t = \beta_0 + \beta_1 \; OPPDLS_t + \beta_2 \; OPGPD_t$$
$$OPTTO_t = \beta_0 + \beta_1 \; OPPDTT_t + \beta_2 \; OPGPD_t$$
$$OPMSO_t = \beta_0 + \beta_1 \; OPPDMS_t + \beta_2 \; OPGPD_t$$
$$OPBDO_t = \beta_0 + \beta_1 \; OPPDBD_t + \beta_2 \; OPGPD_t$$

The results for the ordinary least squares estimates are found in table 4.4.1. We can observe from this table that, although, the price and income effects have the correct sign, the problem of autocorrelation is quite severe.

After adjusting for the positive autocorrelation, we see from table 4.4.2 that the signs of the coefficients are correct and ρ_1 is significant, which implies that we were correct in carrying out the adjustment. However, if we adjust once more then ρ_2 is not significant (table 4.4.3) and so we have to conclude that, in general, the linear model does not adequately describe the private companies demand structure.

Table 4.4.1

Linear Demand Model: O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	235.569 (1.882)	-223.778 (-2.514)	.016 (1.767)	.436	.968
Local	91.779 (2.259)	-84.054 (-2.887)	.006 (2.328)	.639	.980
Toll	23.544 (.326)	-50.060 (-1.017)	.016 (2.924)	.439	.948
Miscellaneous	12.639 (5.184)	-12.438 (-6.555)	.0005 (3.140)	1.002	.969
Local + Toll	186.232 (1.518)	-185.127 (-2.137)	.018 (2.016)	.429	.967

Table 4.4.2

Linear Demand Model: C-O.L.S.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	198.438 (2.330)	-235.941 (-4.148)	.025 (3.244)	.760 (4.379)	1.325	.990
Local	79.351 (2.377)	-81.766 (-3.571)	.009 (3.230)	.624 (2.992)	1.583	.990
Toll	54.779 (1.242)	-108.043 (-3.569)	.020 (4.596)	.778 (4.628)	1.230	.987
Miscellaneous	15.270 (4.064)	-14.188 (-5.304)	.0003 (1.039)	.414 (1.700)	1.463	.976
Local + Toll	178.611 (2.258)	-220.790 (-4.157)	.025 (3.510)	.766 (4.459)	1.356	.991

Table 4.4.3

Linear Demand Model: N.L.R.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	59.147 (.632)	-249.072 (-2.759)	.019 (1.057)	1.109 (2.508)	-.352 (-.765)	1.261	.990
Local	-.206 (-.050)	-3.770 (-.115)	-.010 (-1.464)	1.045 (1.731)	.039 (.060)	2.058	.996
Toll	-1.900 (-.216)	-22.837 (-2.377)	.013 (7.386)	1.887 (5.975)	-.317 (-.807)	2.389	.999
Miscellaneous	-.388 (-.924)	.942 (.317)	-.001 (-1.506)	1.097 (2.728)	.018 (.166)	2.076	.992
Local + Toll	-9.169 (-1.245)	-42.558 (-1.113)	-.010 (-.811)	1.099 (2.309)	.031 (.060)	2.030	.998

4.4.2 The Double-Log Demand Model

The equations for the double-log model, in this section, are:

$$\log OPTSO_t = \beta_0 + \beta_1 \log OPPDTS_t + \beta_2 \log OPGPD_t$$
$$\log OPLSO_t = \beta_0 + \beta_1 \log OPPDLS_t + \beta_2 \log OPGPD_t$$
$$\log OPTTO_t = \beta_0 + \beta_1 \log OPPDTT_t + \beta_2 \log OPGPD_t$$
$$\log OPMSO_t = \beta_0 + \beta_1 \log OPPDMS_t + \beta_2 \log OPGPD_t$$
$$\log OPBDO_t = \beta_0 + \beta_1 \log OPPDBD_t + \beta_2 \log OPGPD_t .$$

The results for the double-log model, estimated by ordinary least squares are presented in table 4.4.4. Once more, the problem of autocorrelation is severe, even though the coefficients (β_1 and β_2) have the right sign. Table 4.4.5 shows us the adjusted results and there is an improvement. The problem of autocorrelation has been overcome, especially in the total, local, and local plus toll categories. In addition the estimates have the right sign and are all significant for these three categories.

The second correction for autocorrelation (table 4.4.6) brings about a significant improvement in the results for each demand category. Indeed, it seems that for the linear and double-log models the best estimates are the double-log nonlinear regression estimates.

4.4.3 The Rotterdam Demand Model

The equations for the Rotterdam model are:

Table 4.4.4

Dougle-Log Demand Model: O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	-6.777 (-9.396)	-1.293 (-6.798)	1.327 (15.215)	1.381	.998
Local	-5.433 (-5.108)	-1.171 (-4.210)	1.077 (8.359)	.914	.992
Toll	-10.101 (-12.589)	-1.239 (-6.215)	1.639 (16.989)	1.529	.998
Miscellaneous	-9.560 (-9.675)	-2.018 (-6.723)	1.244 (10.377)	.858	.984
Local + Toll	-6.910 (-9.136)	-1.251 (-6.363)	1.339 (14.649)	1.374	.998

Table 4.4.5

Double-Log Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	-7.514 (-10.439)	-1.182 (-6.809)	1.415 (16.294)	.178 (.675)	1.911	.998
Local	-6.938 (-5.309)	-.941 (-3.412)	1.257 (7.983)	.440 (1.831)	1.712	.995
Toll	-10.333 (-12.617)	-1.238 (-6.291)	1.667 (16.932)	.150 (.566)	1.240	.998
Miscellaneous	-9.177 (-3.662)	-1.987 (-4.384)	1.200 (4.000)	.569 (2.588)	1.100	.998
Local + Toll	-7.626 (-10.616)	-1.149 (-6.654)	1.424 (16.440)	.167 (.637)	1.906	.998

Table 4.4.6

Double-Log Demand Model: N.L.R.
(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	ρ_2	D.W.	R^2
Total	-8.566 (-3.266)	-1.347 (-5.744)	1.329 (12.239)	.325 (1.003)	-.585 (-2.316)	1.994	.999
Local	-5.087 (-2.280)	-1.233 (-2.700)	1.104 (4.951)	.683 (1.945)	-.582 (-1.754)	1.880	.996
Toll	-9.880 (-3.551)	-1.457 (-6.831)	1.530 (14.667)	.461 (1.997)	-.536 (-2.471)	2.311	.999
Miscellaneous	-6.833 (-3.656)	-2.558 (-7.279)	1.087 (7.679)	.989 (4.825)	-.814 (-3.827)	2.012	.995
Local + Toll	-8.746 (-3.272)	-1.301 (-5.616)	1.343 (12.351)	.312 (.970)	-.571 (-2.354)	2.083	.999

$$\alpha_{it}\left[\log OPTSO_t - \log OPTSO_{t-1}\right] = \beta_0 + \beta_1\left[\log OPPDTS_t - \log OPPDTS_{t-1}\right] + \beta_2\left[\log OPGPD_t - \log OPGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log OPLSO_t - \log OPLSO_{t-1}\right] = \beta_0 + \beta_1\left[\log OPPDLS_t - \log OPPDLS_{t-1}\right] + \beta_2\left[\log OPGPD_t - \log OPGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log OPTTO_t - \log OPTTO_{t-1}\right] = \beta_0 + \beta_1\left[\log OPPDTT_t - \log OPPDTT_{t-1}\right] + \beta_2\left[\log OPGPD_t - \log OPGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log OPMSO_t - \log OPMSO_{t-1}\right] = \beta_0 + \beta_1\left[\log OPPDMS_t - \log OPPDMS_{t-1}\right] + \beta_2\left[\log OPGPD_t - \log OPGPD_{t-1}\right]$$

$$\alpha_{it}\left[\log OPBDO_t - \log OPBDO_{t-1}\right] = \beta_0 + \beta_1\left[\log OPPDBD_t - \log OPPDBD_{t-1}\right] + \beta_2\left[\log OPGPD_t - \log OPGPD_{t-1}\right].$$

The results for the ordinary least squares estimates for the Rotterdam model (table 4.4.7) are not very promising; for all the services the income effect has the wrong sign and for the local demand the price effect is also signed incorrectly. In addition the Cochrane-Orcutt results, for the correction of the problem of autocorrelation, do not improve the explanatory power of the model.

Consequently, the nonlinear regression estimates for the double-log model, in general, are the best results.

4.4.4 The Partial Price Elasticities of Demand

In the linear model the price elasticities are computed from:

$\beta_1 \dfrac{OPPDTS_t}{OPTSO_t}$ - partial price elasticity of total demand in period t

$\beta_1 \dfrac{OPPDLS_t}{OPLSO_t}$ - partial price elasticity of local demand in period t

Table 4.4.7

Rotterdam Demand Model: O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	D.W.	R^2
Total	.003 (2.411)	-.007 (-.970)	-.021 (-1.122)	1.430	.299
Local	.002 (3.439)	.001 (.416)	-.023 (-2.419)	1.996	.423
Toll	.001 (1.534)	-.005 (-1.442)	-.002 (-.173)	.878	.182
Miscellaneous	.0001 (.596)	-.0001 (-1.895)	-.0002 (-.095)	1.118	.350
Local + Toll	.003 (2.515)	-.006 (-.896)	-.021 (-1.159)	1.416	.283

Table 4.4.8

Rotterdam Demand Model: C-O.L.S.

(t-values in parenthesis)

Demand Category	β_0	β_1	β_2	ρ_1	D.W.	R^2
Total	.002 (1.793)	-.010 (-1.326)	-.008 (-.426)	.216 (.765)	1.770	.332
Local	.002 (3.213)	.002 (.433)	-.022 (-2.200)	-.035 (-.122)	2.004	.385
Toll	.0005 (.790)	-.006 (-2.105)	.011 (1.144)	.523 (2.125)	1.390	.547
Miscellaneous	.0001 (.744)	-.0009 (-2.129)	-.0004 (-.263)	.411 (1.561)	1.540	.494
Local + Toll	.002 (1.857)	-.009 (-1.318)	-.008 (-.413)	.215 (.762)	1.790	.323

$\beta_1 \dfrac{OPPDTT_t}{OPTTO_t}$ - partial price elasticity of toll demand in period t

$\beta_1 \dfrac{OPPDMS_t}{OPMSO_t}$ - partial price elasticity of miscellaneous demand in period t

$\beta_1 \dfrac{OPPDBD_t}{OPBDO_t}$ - partial price elasticity of local plus toll demand in period t

The price elasticities for the O.L.S. estimates in the linear case monotonically increase since the ratio of the relative price to output for each service declines. The ranges are: for total -7.324 to -.882, for local -4.766 to -.786, for toll -4.546 to -.373, for miscellaneous -10.075 to -1.217, for local plus toll -6.375 to -.761. The C-O.L.S. estimates are: for total -7.722 to -.930, for local -4.636 to -.765, for toll -9.811 to -.805, for miscellaneous -11.494 to -1.389, for local plus toll -7.604 to -.908. The N.L.R. estimates are: for total -8.152 to -.982, for local -.214 to -.035, for toll -2.074 to -.170, for miscellaneous .092 to .763, and for local plus toll -1.466 to -.175.

The double-log elasticities are: For O.L.S., total -1.293, local -1.171, toll -1.239, miscellaneous -2.018, local plus toll -1.251; C-O.L.S., total -1.182, local -.942, toll -1.238, miscellaneous -1.987, local plus toll -1.149; N.L.R., total -1.347, local -1.233, toll -1.457, miscellaneous -2.558, local plus toll -1.301.

The Rotterdam O.L.S. estimates vary over the sample, for total -.347 (high) to -.672 (low), for local .320 (high) to .159 (low), for toll -.470 (high) to -.905 (low), for miscellaneous -1.083 (high) to -2.025 (low), and for local plus toll -.311 (high) to -.603 (low). The C-O.L.S. estimates also vary and they are: for total -.468 (high) to -.905 (low), for local .345 (high) to .171 (low), for toll -.550 (high) to -1.059 (low), for miscellaneous -1.175 (high) to -2.198 (low), and for local plus toll -.446 (high) to -.864 (low).

It has been stated that the best results for the total demand services are the double-log N.L.R. estimates. In this case the partial price elasticity is -1.4 which is indeed lower than -1.0 and therefore consistent with economic theory.

Appendix 3.1 Demand Module Symbols

1. Provinces and Country

QU	Quebec
ON	Ontario
NS	Nova Scotia
PE	Prince Edward Island
NF	Newfoundland
NB	New Brunswick
MN	Manitoba
SK	Saskatchewan
AL	Alberta
BC	British Columbia
CA	Canada

2. Cities

ML	Montreal
TR	Toronto
HL	Halifax
JF	St. John's
JB	St. John
CL	Charlottetown
WN	Winnipeg
ST	Saskatoon
RG	Regina
ED	Edmonton
VC	Vancouver

3. Companies

BL	Bell Canada
BC	British Columbia Telephone Company
MT	Maritime Telegraph & Telephone Company
NF	Newfoundland Telephone Company
NB	New Brunswick Telephone Company
AG	Alberta Government Telephones
MN	Manitoba Telephone System
SK	Saskatchewan Telecommunications
ED	Edmonton Telephones
OP	Private telephone companies other than BL and BC (MT + NF + NB)
TG	Total public telephone companies (AG + MN + SK + ED)
ID	Total telephone industry

4. Revenues

TSR	Total Revenue
LSR	Local Revenue
TTR	Total Toll Revenue (including miscell. Toll Rev.)
TOR	Toll Revenue (excluding miscell. Toll Revenue)
DAR	Directory Advertising Revenue
MSR	Miscellaneous Revenue (TSR - LSR - TTR)
LTR	LSR + TOR
BMR	LSR + TOR + DAR (All revenue but miscell.)
BDR	LSR + TTR (All revenue but directory advertisement and miscell.)

TSI	TSR minus Indirect Taxes
LSI	LSR " " "
TTI	TTR " " "
TOI	TOR " " "
DAI	DAR " " "
MSI	MSR " " "
LTI	LTR " " "
BMI	BMR " " "
BDI	BDR " " "

TSO	TSI deflated (output of the first revenue category)
LSO	LSI " " " second " "
TTO	TTI " " " third " "
TOO	TOI " " " fourth " "
DAO	DAI " " " fifth " "
MSO	MSI " " " sixth " "
LTO	LTI " " " seventh " "
BMO	BMI " " " eighth " "
BDO	BDI " " " nineth " "

5. <u>Price Indices</u>

PITS	Price Index (of the first revenue category)	TSI/TSO
PILS	" " (" second " ")	LSI/LSO
PITT	" " (" third " ")	TTI/TTO
PITO	" " (" fourth " ")	TOI/TOO
PIDA	" " (" fifth " ")	DAI/DAO
PIMS	" " (" sixth " ")	MSI/MSO
PILT	" " (" seventh " ")	LTI/LTO
PIBM	" " (" eighth " ")	BMI/BMO
PIBD	" " (" nineth " ")	BDI/BDO

PDTS	PITS deflated by the consumer price index
PDLS	PILS " " " " "
PDTT	PITT " " " " "
PDTO	PITO " " " " "
PDDA	PIDA " " " " "
PDMS	PIMS " " " " "

PDLT	PILT deflated by the consumer price index
PDBM	PIBM " " " " "
PDBD	PIBD " " " " "

CPI	Consumer Price index
GPI	Gross Provincial product price index

6. Gross Provincial Product

GPP	Gross Provincial Product (current prices)
GPD	" " " (deflated)

Footnotes

1. Along with the T.C.T.S. carriers, we include Edmonton Telephones because of its large share of the local revenues from the telephone services in the province of Alberta.

2. There are an arbitrary number (n) of commodities in each household's decision process.

3. Since we are only dealing with the telephone industry and not the Canadian economy we are using partial equilibrium analysis. Hence, we are assuming the outputs (and revenue) of all non-telephone producing firms are given.

4. There is no reason for the type and number (m) of commodities, in any one firm's decision, should be the same as the type and number (n) in the household's choice problem.

5. Of course we have tried other variables in the aggregate demand function such as population, and the percentage of direct distance calls, as a measure of technological change, but they created problems of multicollinearity, or had the wrong sign, or were insignificant.

6. The partial price elasticity of demand is $\frac{\partial \log x_{it}}{\partial \log p_{it}}$, whereas the total price elasticity is $\frac{d \log x_{it}}{d \log p_{it}}$, and for the income elasticities replace p_{it} by y_t.

7. The need for this procedure arises because we do not have information on indirect taxes by the type of revenue services.

8. The data source for the consumer price indexes is Cansim.

9. The data source for the gross provincial products is Cansim.

10. The list of mnemonics is given in Appendix 1 of the demand module. The first two letters pertain to the company and the remaining letters refer to the particular type of variable.

11. α_{it} refers to the average value (between two periods) of the share of expenditure on the i^{th} service out of total expenditure in period t. Thus α_{it} changes for any service over time and it changes across different services.

REFERENCES

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CHAPTER 4

THE PRODUCTION MODULE

1. Introduction

The purpose of the production module is to investigate the structural characteristics of both the technology and the factors of production of telephone services for the TCTS companies and Edmonton Telephones.

In this module we estimate functional forms which describe production relationships such as the marginal products and the degree of returns to scale. These relevant estimated parameters are then integrated with the demand and financial modules, so that the overall model can be solved and the various forecasting experiments carried out.

The analysis of the production configurations for telephone services has generally been studied at two levels of carrier aggregation. One level is at the industry, where production functions are estimated for all the important carriers combined; as is found in the study by R. Dobell et. al. [7].

The other level is at the firm level, but for a particular firm, which is Bell Canada; as is found in the studies by the I.A.E.R. [10], J. Carr [5] and R. Millen [13] . In this study, we estimate production functions not only for Bell Canada, but also for British Columbia Telephone Company; the aggregation of Maritime Telegraph and Telephone Company, New Brunswick Telephone and Newfoundland Telephone Company. Finally, we estimate functions for the aggregation of Alberta Government

Telephones, Edmonton Telephones, Manitoba Telephone System and Saskatchewan Telecommunications.

In addition to the types of carriers usually studied, there is generally only an aggregate production relationship specified. This arises because of the detailed data which is needed in order to estimate sets of disaggregated (by services) functions. In particular, one would need the contribution of capital, labour, and any other factors to each service category. Because we do not have such "micro" data, we follow the route of previous authors, and utilize an aggregate production function for each set of carriers under consideration.

The production module is divided into four sections, where each section pertains to a different set of carriers in the industry.

2. Bell Canada

The production function for Bell Canada, as for all the other carriers, is a variant of the Cobb-Douglas specification. Basically, we have

$$y_t = AK_t^{\alpha_1}L_t^{\alpha_2}e^{\alpha_3 I_t}u_t \qquad (1)$$

where y_t is output, A is a constant (representing the transformation of inputs into the output), K_t is capital, L_t is labour, I_t is technological change, u is the random error. By taking natural logarithms of (1) we get,

$$\ell n y_t = \alpha_0 + \alpha_1 \ell n K_t + \alpha_2 \ell n L + \alpha_3 I_t + v_t \qquad (2)$$

where $\ell n A = \alpha_0$, and $\ell n u_t = v_t$. We can observe from equation (2) that α_1 is the elasticity of output with respect to capital, α_2 is the elasticity of output with respect to labour, and α_3 is the average effect of a change in output with respect to a change in the technology.

An immediate question arises as to the appropriateness of the Cobb-Douglas function as a representation of the technology for Bell Canada, or for that matter, any other carrier in the telephone industry. Indeed, this question has been tested by hypothesizing a more general production function, called the trans-logarithmic production function. This so-called translog function can be considered as a second order approximation to any production function around a point in which the logarithms of each of the inputs are made equal to zero.[1]

We write the translog function as,

$$\ell n y_t = \alpha_0 + \alpha_1 \ell n K_t + \alpha_2 \ell n L_t + \tfrac{1}{2}\gamma_{11}(\ell n L_t)^2$$
$$+ \tfrac{1}{2}\gamma_{22}(\ell n K_t)^2 + \gamma_{12}(\ell n K_t \ell n L_t) +$$
$$\alpha_3 I_t + v_t. \qquad (3)$$

The essential ingredient of the above relationship is that it allows for a non-unitary elasticity of substitution between capital and labour, besides non-constant returns to scale. This function was estimated for Bell Canada in [10] and it was found that we could not reject the hypothesis that the technology was Cobb-Douglas. This essentially means that $\gamma_{11} = \gamma_{22} = \gamma_{12} = 0$, and with this restriction we see that equations (2) and (3) are identical. We should remark that we still have not imposed constant returns to scale, we have only claimed that the production function exhibits a particular form of separability.

2.1. The Data

The data series that we used to estimate equation (2) came from two sources, the Bell Rate Hearings [3] and R. Millen [13]. The output variable we used was defined as total telephone service revenues minus indirect taxes (but including uncollectibles) deflated by the price index for total services, as in the Bell Rate Hearings [3].

The labour input in production was defined in terms of manhours worked (excluding hours spent in construction) rather than by the number of employees. In addition, we adjusted the manhours for differences in the quality of work among different types of labour. A complete description of the method of adjustment for differences in skills can be found in Bell Rate Hearings [3] . It does bear mentioning that differences in nominal wages reflected the differences in skills and these weights were computed for 1967 and assumed to be constant over the sample period. Moreover, the definition of the payment to labour to compute the weights included not only wages but also other forms of remuneration, such as fringe benefits.

The physical capital input is defined to be the net capital stock as defined in R. Millen [13] . The capital stock is comprised of plant and equipment, including plant under construction. The rate of depreciation is economic depreciation computed from the life expectancy curves for the different vintages of capital.

For a measure of technological change, we used two different variables; the percentage of calls direct distance dialed, and the percentage of telephones in number five crossbar and electronic switching system.[2]

2.2 The Empirical Results

We have three different specifications for Bell Canada. Two equations include technological change and one does not. We also estimated the equations using ordinary least squares and the

Cochrane-Orcutt adjustment for autocorrelation. The equations that we estimated for 1950-1975 were the following:[3]

$$\ell nBLTSO_t = \alpha_0 + \alpha_1 \ell nBLK_t + \alpha_2 \ell nBLL_t$$

$$\ell nBLTSO_t = \alpha_0 + \alpha_1 \ell nBLK_t + \alpha_2 \ell nBLL_t + \alpha_3 BLDD_t$$

$$\ell nBLTSO_t = \alpha_0 + \alpha_1 \ell nBLK_t + \alpha_2 \ell nBLL_t + \alpha_3 BLX_t .$$

The O.L.S. estimates are found in Table 2.1 and the C-O.L.S estimates are presented in Table 2.2.

We can observe from Table 2.1 that the equation without technological change shows that α_2 (the labour coefficient) is not significantly different from zero and that α_1 is not significantly different from one. Indeed, when we introduce technological change, whether through a direct distance dialed variable or number five crossbar, the labour coefficient becomes significant. The problem is, not only autocorrelation as the Durbin-Watson statistics point out, but also we must restrict α_2 to be equal to $1-\alpha_1$. In other words, we must impose constant returns to scale. These results are presented in Tables 2.3 and 2.4. If we test for constant returns to scale for the no technical change O.L.S equation, we find that the computed F statistic is 7.59 and the tabulated $F_{1,23}$ at the 1% level of significance is 7.88 and so, we accept the existence of constant returns to scale for this equation.[4] The computed values of F for the other

TABLE 2.1

Cobb-Douglas Production Function
Variable Returns to Scale: O.L.S.
(t-values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	D.W.	R^2
None	-1.519 (-.783)	.956 (8.621)	.134 (1.205)		1.553	.929
Direct Distance Dialed (Bell)	-1.752 (-4.434)	.404 (7.881)	1.143 (7.214)	.957 (13.446)	.830	.997
Number Five Crossbar (Bell)	-1.040 (-2.517)	.530 (11.694)	.743 (4.915)	1.648 (12.772)	.749	.997

TABLE 2.2

Cobb-Douglas Production Function
Variable Returns to Scale: C-O.L.S.
(t-values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	ρ	D.W.	R^2
None	-1.298 (-.471)	.968 (7.131)	.057 (.064)		.189 (.964)	1.935	.919
Direct Distance Dialed (Bell)	.253 (.679)	.328 (8.335)	.773 (6.792)	1.118 (19.404)	.494 (2.842)	1.483	.999
Number Five Crossbar (Bell)	1.513 (4.850)	.436 (16.453)	.266 (3.144)	2.045 (25.187)	.532 (3.145)	1.619	.999

TABLE 2.3

Cobb-Douglas Production Function
Constant Returns to Scale: O.L.S
(t-values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	D.W.	R^2
None	-1.210 (-5.430)	.970 (15.078)		1.549	.905
Direct Distance Dialed (Bell)	-.040 (-.255)	.552 (10.254)	.849 (9.315)	.484	.993
Number Five Crossbar (Bell)	-.142 (-1.203)	.587 (14.461)	1.597 (11.586)	.623	.995

TABLE 2.4

Cobb-Douglas Production Function
Constant Returns to Scale: C-O.L.S.
(t-values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	ρ	D.W.	R^2
None	-1.210 (-4.044)	.971 (11.401)		.190 (.966)	1.935	.896
Direct Distance Dialed (Bell)	.691 (5.627)	.316 (7.678)	1.144 (18.626)	.540 (3.205)	1.523	.999
Number Five Crossbar (Bell)	.257 (3.427)	.460 (18.189)	1.903 (24.457)	.409 (2.243)	1.146	.999

two equations found in Tables 2.2 and 2.4 also tell us that we should accept constant returns to scale. Since it appears that autocorrelation is particularly strong we need only carry out our tests using Tables 2.2 and 2.4. From these Tables we see that by single equation criteria, the direct distance dialed measure of technological change yields the best results, so we accept contant returns to scale for Bell Canada. We compute that $\hat{\alpha}_1=.32$ and $\hat{\alpha}_2=.68$ so that $\hat{\alpha}_1+\hat{\alpha}_2=1$. Given that we are interested in a complete integrated model the final acceptance or rejection of a particular equation will be determined by the equation's performance in tracking the actual data in the sample period, when we simultaneously solve the modules.

The production functions which we have estimated contain equipment and labour as inputs, and revenues in constant dollars as the output. Consequently, because we have excluded materials as an input, we are assuming that materials affects output through a fixed coefficient technological process. This process may be represented by

$$y = \min. \left(\frac{F(K,L)}{\mu} \; , \; \frac{R}{\nu} \right);$$

where R represents materials and μ and ν are positive constants. The preceeding equation implies that

$$y = \frac{F(K,L)}{\mu} = \frac{R}{\nu} \, .$$

Since we have already estimated the relationship $y = \frac{F(K,L)}{\mu}$ we now need to find the estimated value of ν. We ran regressions of the form $R = \nu y$ using the ordinary least squares and Cochrane - Orcutt adjustment and found that,

$$R = \underset{(13.163)}{.14} (y_t - .956 y_{t-1}) + \underset{(16.226)}{.956} \; R_{t-1}$$

$$D.W. = 2.421 \qquad R^2 = .989$$

where $\hat{\nu}$= .14 which is the estimated share (or average product) of materials to output.

There also exists the possibility that the fixed coefficient assumption is not valid and that the actual technological process of Bell is

$$y = F(K, L, R)$$

where R, materials, is included, with equipment and labour, in the group of factors which are potentially subtitutable.

It should be mentioned here that the figures of materials series also include services rent and supplies and it has been taken directly from the Memorandum on Productivity, Exhibit No. B-73-62, by Bell Canada.

The results from the estimation, when we included materials in the Cobb-Douglas Production function, are presented in tables 2.5 - 2.8. For the variable returns to scale model we utilized the ordinary and Cochrane-Orcutt least squares methods, while for the constant returns to scale formulation we used the restricted least squares estimation technique. In the following tables α_4 represents the materials elasticity of output.

It is clear from the variable and constant returns to scale results that technological change must be included in the production relationship. Moreover, we find when we perform our F-test

TABLE 2.5

Cobb-Douglas Production Function
Variable Returns to Scale: O.L.S.
(t- values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	α_4	D.W.	R^2
None	-1.982 (-2.260)	.335 (2.926)	-.689 (-2.659)		.998 (6.419)	.655	.991
Direct Distance Dialed (Bell)	-.973 (-1.570)	.345 (5.581)	.830 (3.339)	.818 (7.386)	.216 (1.596)	.919	.998
Number Five Crossbar (Bell)	-.236 (-.410)	.437 (6.736)	.442 (2.072)	1.368 (7.150)	.256 (1.904)	.810	.998

TABLE 2.6

Cobb-Douglas Production Function
Variable Returns to Scale: C-O.L.S.
(t- values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	α_4	ρ	D.W.	R^2
None	7.316 (1.647)	-.121 (-.258)	.016 (.057)		.277 (2.000)	.952 (15.512)	.873	.998
Direct Distance Dialed (Bell)	.329 (.790)	.317 (7.560)	.688 (4.613)	1.043 (13.574)	.082 (1.100)	.457 (2.570)	1.619	.999
Number Five Crossbar (Bell)	1.600 (5.510)	.406 (14.035)	.172 (1.939)	1.860 (2.500)	.123 (2.000)	.501 (2.891)	1.827	.999

(for constant returns), and indeed solely by adding $\alpha_1, \alpha_2\ \alpha_4$ in tables 2.5 and 2.6, that Bell exhibits constant returns to scale. This means that we estimate α_1, and α_2 but α_4 is defined from $1-\alpha_1-\alpha_2$. The constant returns results are presented in tables 2.7 and 2.8. From these tables we can observe that the direct distance dialed measure of technological change in table 2.8 seems to perform best, with a capital elasticity of output equal to .305, a labour elasticity of output equal to .616, and a materials elasticity of output equal to 1-.616-.305 = .079.

TABLE 2.7

Cobb-Douglas Production Function
Variable Returns to Scale: R.L.S.
(t- values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	D.W.	R^2
None	.565 (1.620)	.343 (2.835)	-.270 (-5.489)		.471	.988
Direct Distance Dialed (Bell)	.640 (6.270)	.324 (9.185)	.463 (9.112)	.906 (15.049)	1.084	.999
Number Five Crossbar (Bell)	.300 (3.653)	.442 (15.500)	.454 (11.545)	1.725 (19.250)	1.004	.999

TABLE 2.8

Cobb-Douglas Production Function
Variable Returns to Scale: A.R.L.S.
(t- values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	ρ	D.W.	R^2
None	-.326 (-1.533)	.533 (34.375)	.469 (3.143)		1.061 (34.375)	1.244	.999
Direct Distance Dialed (Bell)	.716 (3.618)	.305 (4.818)	.616 (5.366)	1.071 (8.130)	.519 (3.225)	1.673	.999
Number Five Crossbar (Bell)	.338 (3.306)	.430 (12.361)	.480 (9.006)	1.782 (14.537)	.362 (2.348)	1.132	.999

3. British Columbia Telephone

We estimated variants of the Cobb-Douglas production function for B.C. Telephone under both variable and constant returns to scale.

3.1 The Data

The output measure was defined as total revenue minus indirect taxes plus uncollectibles (these were obtained from the income statements) deflated by the price index of Bell Canada for total services. This variable was defined and described in the demand module of the study.[5]

The labour input in the production process was defined in terms of weighted manhours. We had data in the number of employees (obtained from the Annual Reports of B.C. Telephone [4]. We then converted employee data to manhours by assuming seven hours of work per day, 5 work-days per week, and 50 work-weeks. Thus 1750 hours were worked per year per employee times the number of employees gives total manhours. Moreover, we utilized the weights of Bell Canada to convert manhours to weighted manhours in order to account for quality differences of workers. Therefore, we assumed the weights were not only constant in a temporal sense, but also constant across different carriers.

The capital input was computed from the book values of the net capital stock reported in the balance sheets. We computed the ratio of net capital stock (for Bell) in constant 1967 prices obtained from R. Millen [13] to it's net book value in current prices. We then multiplied this number by the net book value in current prices of B.C. Telephone. This computed value is net capital stock for B.C. Telephone in 1967 prices. We are assuming that the proportion of the market value of net capital to book value is the same for all companies and that the price of capital for Bell Canada is identical to all other carriers. There are two advantages to this approach. Firstly, the longer a unit of capital (plant, equipment, etc.) remains in the production process, the more obsolete becomes it's book value. Hence, our method uses a measure of the market value of capital. Secondly, our method shows the manner that one can move from book values to market values for capital or from market to book values, depending on one's immediate interest.[6]

The technological change variable that we used was the percentage of calls direct distance dialed for B.C. Telephone.[7]

3.2 The Empirical Results

We have two different specifications for the Cobb-Douglas production functions, and two methods of estimation. The equations we estimated for 1961-1975 were:

$$\ell n BCTSO_t = \alpha_0 + \alpha_1 \ell n BCK_t + \alpha_2 \ell n BCL_t$$

$$\ell n BCTSO_t = \alpha_0 + \alpha_1 \ell n BCK_t + \alpha_2 \ell n BCL_t + \alpha_3 BCDD_t$$

The results for the variable returns to scale equations (i.e. $\alpha_1 + \alpha_2 \gtreqless 1$) are presented in tables 3.1 and 3.2. The constant returns to scale equations (i.e. $\alpha_1 + \alpha_2 = 1$) are found in tables 3.3. and 3.4. After performing our tests, we found that we could not accept constant returns to scale for British Columbia Telephone. However, it seems quite implausible that B.C. Telephone should exhibit increasing returns to scale solely from capital, as shown in tables 3.1 and 3.2. This is true, especially in light of the relatively old machinery and little technological innovation by the carrier. This leads us to the conclusion that for an appropriate specification of the production relations one needs an exact measure of physical capital for B.C. Telephone itself. In any event the results which seem most credible are found in the second equation of table 3.4. In this case we estimate $\hat{\alpha}_1 = .63$ and $\hat{\alpha}_2 = .37$.

Table 3.1

Cobb-Douglas Production Function
Variable Returns to Scale: O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	D.W.	R^2
None	4.613 (9.711)	1.131 (7.220)	.382 (2.752)		1.082	.995
Direct Distance Dialed (B.C.)	4.279 (6.993)	1.007 (4.738)	.397 (2.814)	.258 (.878)	1.021	.995

Table 3.2

Cobb-Douglas Production Function
Variable Returns to Scale: C-O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	ρ	D.W.	R^2
None	4.402 (7.972)	1.049 (5.352)	.444 (2.771)		.394 (1.606)	1.400	.995
Direct Distance Dialed B.C.	4.321 (7.431)	1.027 (4.965)	.402 (2.400)	.217 (.647)	.501 (2.165)	1.534	.995

Table 3.3

Cobb-Douglas Production Function
Constant Returns to Scale: O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	D.W.	R^2
None	1.557 (.895)	-.193 (-.372)		.365	.011
Direct Distance Dialed (B.C.)	2.954 (5.104)	.501 (2.776)	1.136 (10.574)	1.261	.904

Table 3.4

Cobb-Douglas Production Function
Constant Returns to Scale: C-O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	ρ	D.W.	R^2
None	5.267 (12.674)	.721 (6.064)		.940 (10.290)	1.488	.935
Direct Distance Dialed (B.C.)	3.300 (4.536)	.625 (2.563)	1.221 (5.920)	.265 (1.026)	1.314	.860

4. The Public Carriers

In this section we estimated Cobb-Douglas production functions for the aggregation of Alberta Government Telephones Edmonton Telephones, Saskatchewan Telecommunications and Manitoba Telephone System.

4.1 The Data

The output measure, as for Bell Canada and B.C. Telephone was the total service demand variable which was defined and described in the demand module. The labour and capital factors of production were derived in the same fashion as those for B.C. Telephone. For technological change, we tried three different measures, the percentage of calls direct distance dialed for Bell, for B.C., and the number five crossbar variable for Bell. These measures were used as proxies for the public companies. Nevertheless, it is a reasonable assumption to use Bell's data given that it is the industrial leader in innovation and the public companies would tend, with a lag, to adopt the new technology of the largest carrier.

4.2 The Empirical Results

The equations which we estimated for 1961-1975 were:

$$\ell nTGTSO_t = \alpha_0 + \alpha_1 \ell nTGK_t + \alpha_2 \ell nTGL_t$$

$$\ell nTGTSO_t = \alpha_0 + \alpha_1 \ell nTGK_t + \alpha_2 \ell nTGL_t + \alpha_3 BLDD_t$$

$$\ell nTGTSO_t = \alpha_0 + \alpha_1 \ell nTGK_t + \alpha_2 \ell nTGL_t + \alpha_3 BCDD_t$$

$$\ell nTGTSO_t = \alpha_0 + \alpha_1 \ell nTGK_t + \alpha_2 \ell nTGL_t + \alpha_3 BLX_t$$

The results for the variable returns to scale equations are presented in tables 4.1 and 4.2. The constant returns equations are presented in tables 4.3 and 4.4. Clearly, from table 4.1 there is no significant autocorrelation. Next when we performed the tests for the acceptance or rejection of constant returns we concluded that we must reject the hypothesis of constant returns. For the equation which performs relatively better than any of the others for the public carriers, we accept decreasing returns to scale. We compute that $\hat{\alpha}_1 = .2$, $\hat{\alpha}_2 = .6$, and so $\hat{\alpha}_1 + \hat{\alpha}_2 = .8 < 1$. This equation is the one in table 4.1 with number five crossbar as the measure of technological change. Here .2 is the capital elasticity of output and .6 is the labour elasticity of output.

Table 4.1

Cobb-Douglas Production Function
Variable Returns to Scale : O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	D.W.	R^2
None	-.137 (-.129)	.372 (1.809)	1.739 (5.599)		1.442	.989
Direct Distance Dialed (Bell)	2.275 (3.018)	.163 (1.343)	.726 (2.762)	.013 (5.094)	2.044	.997
Direct Distance Dialed (B.C.)	-.415 (-.388)	.199 (.794)	1.649 (5.235)	.603 (1.173)	1.647	.991
Number Five Crossbar (Bell)	2.763 (3.775)	.180 (1.656)	.582 (2.293)	.025 (5.784)	2.101	.998

Table 4.2

Cobb-Douglas Production Function
Variable Returns to Scale : C - O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	ρ	D.W.	R^2
None	.086 (.072)	.438 (1.832)	1.674 (4.799)		.232 (.861)	1.888	.98
Direct Distance Dialed (Bell)	2.097 (2.877)	.179 (1.538)	.796 (3.120)	.012 (4.941)	-.121 (-.438)	2.161	.99
Direct Distance Dialed (B.C.)	-.145 (-.119)	.306 (1.021)	1.628 (4.523)	.399 (.673)	.187 (.686)	1.902	.98
Number Five Crossbar (Bell)	2.540 (3.590)	.190 (1.839)	.667 (2.707)	.023 (5.600)	-.164 (-.600)	2.152	.99

Table 4.3.

Cobb-Douglas Production Function
Constant Returns to Scale : O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	D.W.	R^2
None	8.785 (7.132)	1.894 (5.438)		.404	.711
Direct Distance Dialed (Bell)	2.006 (4.557)	.169 (1.465)	.011 (18.109)	2.139	.991
Direct Distance Dialed (B.C.)	.366 (.253)	.015 (.044)	2.120 (6.383)	1.490	.939
Number Five Crossbar (Bell)	2.163 (5.304)	.191 (1.770)	.021 (19.231)	2.308	.992

Table 4.4

Cobb-Douglas Production Function
Constant Returns to Scale : C-O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	ρ	D.W.	R^2
None	2.721 (2.918)	-.102 (-.361)		.942 (10.079)	2.366	.976
Direct Distance Dialed (Bell)	2.037 (4.964)	.180 (1.667)	.012 (19.466)	-.133 (-.484)	2.183	.989
Direct Distance Dialed (B.C.)	2.442 (2.164	-.118 (-.381)	.002 (.004)	.927 (8.925)	2.201	.975
Number Five Crossbar (Bell)	2.174 (6.117)	.196 (2.078)	.021 (22.062)	-.233 (-.862)	2.302	.991

5. The Private Carriers

In this section we estimated Cobb-Douglas production functions for the aggregation of Newfoundland Telephone, New Brunswick Telephone, and Maritime Telegraph and Telephone companies.

5.1 The Data

The data, with respect to output, labour, capital, and technological change are defined in the identical manner as for the public carriers.

5.2 The Empirical Results

The regression estimates are found in tables 5.1 through 5.4 for the private companies. The equations we estimated were;

$$\ell nOPTSO_t = \alpha_0 + \alpha_1 \ \ell nOPK_t + \alpha_2 \ell nOPL_t$$

$$\ell nOPTSO_t = \alpha_0 + \alpha_1 \ \ell nOPK_t + \alpha_2 \ell nOPL_t + \alpha_3 BLDD_t$$

$$\ell nOPTSO_t = \alpha_0 + \alpha_1 \ \ell nOPK_t + \alpha_2 \ell nOPL_t + \alpha_3 BCDD_t$$

$$\ell nOPTSO_t = \alpha_0 + \alpha_1 \ \ell nOPK_t + \alpha_2 \ell nOPL_t + \alpha_3 BLX_t$$

We immediately observe from table 5.1 the severe problem of serial correlation. This problem is subsequently alleviated by the Cochrane-Orcutt adjustment under variable returns to scale. In addition for constant returns to scale we must also deal with the C - O.L.S. estimates from table 5.4. When we perform the tests whether to accept or reject constant returns, between the equations found in tables 5.2 and 5.4, we find that we can accept constant returns. With the hypothesis of constant returns to scale, we see that the equation in table 5.4 with the number five crossbar yields the best results. In this case we have a capital elasticity of output of .55 and a labour elasticity of output of .45.

6. Conclusion

Therefore we accept constant returns to scale for Bell Canada, B.C. Telephone, and the private carriers, and we accept decreasing returns to scale for the public carriers. Comparing the companies by capital elasticities of output we find, writing the highest to the lowest carrier gives, a ranking of B.C., Bell, private, public. Comparing the companies by labour elasticities gives a ranking of highest to lowest carrier which is public, private, Bell and B.C. The fact that each ranking is the converse of the other is not surprising, given that we accepted constant returns to scale for three out of four sets of carriers.

Table 5.1

Cobb-Douglas Production Function
Variable Returns to Scale : O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	D.W.	R^2
None	3.553 (3.302)	1.013 (4.368)	.915 (2.622)		.612	.984
Direct Distance Dialed (Bell)	3.159 (7.415)	.168 (1.217)	.118 (.700)	.020 (8.156)	1.163	.998
Direct Distance Dialed (B.C.)	1.443 (1.037)	.434 (1.254)	1.136 (3.481)	1.071 (2.080)	1.021	.988
Number Five Crossbar (Bell)	3.376 (11.639)	.119 (1.253)	.015 (.123)	.039 (12.430)	1.258	.999

Table 5.2

Cobb-Douglas Production Function
Variable Returns to Scale : C - O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_2	α_3	ρ	D.W.	R^2
None	3.954 (3.408)	1.126 (3.932)	.791 (2.156)		.602 (2.818)	1.865	.99[illegible]
Direct Distance Dialed (Bell)	3.497 (6.778)	.366 (2.144)	.138 (.745)	.017 (6.644)	.436 (1.814)	1.771	.998
Direct Distance Dialed (B.C.)	2.359 (2.062)	.708 (2.534)	.894 (3.040)	1.020 (2.556)	.319 (1.260)	1.977	.99[illegible]
Number Five Crossbar (Bell)	3.563 (10.694)	.249 (2.157)	.041 (.333)	.036 (10.756)	.354 (1.418)	1.812	.999

Table 5.3

Cobb-Douglas Production Function
Constant Returns to Scale : O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	D.W.	R^2
None	9.820 (8.790)	2.194 (6.933)		.415	.787
Direct Distance Dialed (Bell)	2.990 (5.134)	.449 (2.934)	.012 (13.505)	.672	.987
Direct Distance Dialed (B.C.)	.506 (.304)	.065 (.166)	2.203 (5.980)	1.529	.947
Number Five Crossbar (Bell)	3.091 (5.597)	.455 (3.112)	.021 (14.096)	.611	.988

Table 5.4

Cobb-Douglas Production Function
Constant Returns to Scale : C- O.L.S.
(t - values in parenthesis)

Technological Change Variable	α_0	α_1	α_3	ρ	D.W.	R^2
None	4.642 (7.351)	.434 (2.168)		.951 (11.452)	1.166	.986
Direct Distance Dialed (Bell)	3.437 (5.580)	.583 (3.504)	.012 (12.367)	.560 (2.530)	1.887	.993
Direct Distance Dialed (B.C.)	.762 (.612)	.176 (.597)	2.352 (8.456)	-.029 (-.109)	1.746	.966
Number Five Crossbar (Bell)	3.415 (5.975)	.557 3.578)	.022 (13.004)	.588 (2.718)	1.792	.994

Now that we have estimated the production functions we can determine the capital - labour ratios for any given ratio of factor prices. From optimality conditions of corporate bahaviour we know that the ratio of the marginal products of the two factors equals the factor price ratio. This means that,

$$\frac{\alpha_2 \frac{TSO}{L}}{\alpha_1 \frac{TSO}{K}} = \frac{W_\ell}{W_k} \quad ,$$

where W_ℓ is the factor price of labour, W_k the factor price of capital, α_1 is the capital elasticity of output, α_2 the labour elasticity of output. Therefore

$$\frac{K}{L} = \frac{\alpha_1 W_\ell}{\alpha_2 W_k}$$

and so given the factor price ratio for each carrier, we can determine the capital intensity. This implies that for the same factor price ratio we can rank the capital intensity of firms by the ratio of $\frac{\alpha_1}{\alpha_2}$, which we have estimated. For Bell Canada the ratio is .50, for B.C. Telephone it is 1.70, for the public companies it is .33 and for the private carriers the ratio is 1.22. These ratios tell us the number of units of capital per unit of labour for any level of output, when the ratio of factor prices is unity.

Appendix 4.1 Production Module Symbols

1. Provinces and Country

QU	Quebec
ON	Ontario
NS	Nova Scotia
PE	Prince Edward Island
NF	Newfoundland
NB	New Brunswick
MN	Manitoba
SK	Saskatchewan
AL	Alberta
BC	British Columbia
CA	Canada

2. Factors of Production and Output

K	Physical Capital
L	Labour
DD	Percentage of Calls Direct Distance Dialed
X	Percentage of Telephones in Number Five Crossbar and Electronic Switching System
R	Materials
TSO	Output of Total Services

Footnotes

1. See for example Christensen, Jorgenson, and Lau [6].

2. See the thesis by R. Millen [13] .

3. See appendix 1 for the definition of the symbols.

4. The test is
$$F = \frac{\dfrac{SSR_{vn} - SSR_{cn}}{1}}{\dfrac{SSR_{cn}}{23}} .$$

   Where F is the computed F - statistic for the O.L.S. equation without technological change, SSR_{vn} is the regression sum of squares for the variable returns to scale with no technological change equation, SSRcn is the regression sum of squares for constant returns and no technological change. An equivalent formula is
$$F = \frac{(R^2_{vn} - R^2_{cn})\ 23}{(1 - R^2_{vn})} .$$

5. As in the demand module, we utilized Bell's price index for total services, because as long as the price indexes for the other carriers are in a fixed proportion to Bell's there will not be any bias in the estimated values of the parameters.

6. We do not have access to Bell's computation methods for the market values of net capital and their capital price index. We are, consequently, assuming that the vintages of capital are identical for Bell, and B.C. (and also identical to the other carriers under consideration).

7. The only measures of technological change available to us are Bell and B.C.'s direct distance dialed and Bell's number five crossbar.

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CHAPTER 5

THE FINANCIAL MODULE

1. Introduction

The financial module describes the relationships which determine the rates of return on the various financing instruments for the different classes of telephone carriers. The formulation of such a system, which estimates the past structure, enables us to determine the impact of a firm's financing and factor hiring decisions on the rates of return. In fact, if these impacts exist, then we have found empirical evidence which shows that the carrier has monopsony power in its capital markets. This monopsony power is manifested by the significant coefficients which arise out of our estimation.

An immediate question concerns the types of financing instruments. Although there are many instruments, we can define three broad classes, debt (both long and short term), common equity and preferred equity. These different types of financial commodities hold relatively different position of importance in the portfolios of the various companies.[1]

Due to our aggregation over financial commodities we must compute the rates of return on debt, common, and preferred equity. These rates of return play an important role in the integrated model, of which this model is one segment. The rates are part of the computation of the marginal costs associated with a particular financial commodity. Consequently, differences in both the return and the functions which determine them, will cause differences in the marginal financial costs, and thereby, affect the attractiveness of the alternative instruments.

The financial module is divided into three broad sections. Each section describes the equations and results for a different carrier; the first is Bell, the second is B.C. Telephone, and the third is the private carriers. Because the nature of the financial characteristics depends, not only on the market structure confronting any carrier, but also on its internal ownership structure, we feel that it is inappropriate to develop the financial module for the publicly-owned companies. This justification stems from the fact that it is completely meaningless to analyse equity and in most cases bond debt for crown corporations. Whereas, with regards to demand characteristics, which depend on product market structures, and production relations, which depend on technology, the nature of the ownership of a particular carrier is completely irrelevant and so demand and production functions were estimated for the crown corporations.

2. Bell Canada

2.1 Introduction

In the case of Bell Canada the three types of financial instruments, common equity, preferred equity, and debt, play distinct roles. Debt and common equity are by far the most important, since the company just began to issue preferred shares in 1970. We discuss the data which was used for the Bell Canada financial module.

2.2 The Data

Firstly we needed to calculate the rates of return.[2] The rate of return on debt for any time period is defined as the interest payments and other fixed charges, on debt during the period, divided by the value of outstanding debt. This definition is sufficiently general in that it is sensitive to changes in the maturity composition of debt. The data on interest payments and outstanding debt were obtained from the financial statements of Bell.

The rate of return on common equity was derived in terms of a more complicated formula. Investors who purchase common shares must expect some minimum level or return to induce them to invest. This compensation, which may be in the form of income (dividends) or capital gains (or both), when related to the market price of common shares is the rate of return. With this conception of the rate of return we can apply the discounted cash flow formula to find the rate. This formula is,

$$r_{ct} = \frac{D_{ct+1}}{P_{ct}} + g_t ,$$

where r_{ct} is the rate of return in period t, D_{ct+1} is the value of dividends per common share at the end of period t (or the beginning of period t+1), p_{ct} is the market price in period t, and g_t is the rate of growth of dividends per share in period t. Obviously D_{ct+1} and p_{ct} can be obtained from the balance sheets. In addition, we adjusted the issue price for any premiums or discounts. The rate of growth, however, reflects investor's expectations and so cannot be directly measured. To arrive at a plausible estimate of g_t for every time period, the mean of the log-linear least squares growth rates of dividends per share was computed for the past ten years. The growth rate (i.e. the regression coefficient) was significant for the years 1957, 1958, 1964, and 1971 - 1975. Thus we used g_t in the r_{ct} formula for these years and for the other years we took g_t to be equal to zero.[3]

The rate of return on preferred shares was calculated according to the same type of formula we used for common shares. However, because Bell has only issued preferred shares since 1970, we took g_t to be a constant and therefore it does not enter into the method of measuring the return. The preferred issues for Bell have various distinct classes. Therefore to determine the rate of return on preferred shares we weighted the returns on the different series by the proportion of each series out of the total outstanding value of preferred shares for each year according to the data described in Bell's balance sheets.

2.3. The Empirical Results

We estimated relationships for the rates of return, in other words inverse investor demand functions, which depended on the value of debt, equity, income, long-term government bond yield, and a long-term corporate bond yield. The rationale for this selection of regressors rested on the grounds that the values of debt and equity influence the rates of return. In other words we want to test for the existence of monopsony power on the part of Bell. In addition, the government and corporate bond yield represent the alternative forms of investment available to the bondholders and shareholders, while income stands for the aggregate measure of economic activity which facilitates investors attempts to increase their investible funds.[4] In general, then, we can write the functional specification of the rates of return as depending on the financial commodities of Bell, the returns on alternative assets and the general level of economic activity.

Before we proceed to the estimation of the rate of return equations, let us recall the fact that Bell has only begun to issue preferred shares in 1970. Therefore we believe that any results obtained from the rate of return on preferred shares equation will not be sufficiently robust. Therefore, after computing the rates of return on common and preferred shares we formed a weighted average return on equity such that

$$r_{st} = r_{ct} \frac{(\text{value of common equity}_t)}{\text{value of equity}_t} + r_{pt} \frac{(\text{value of preferred equity}_t)}{\text{value of equity}_t}$$

where r_{st} is the rate of return on equity, r_{ct} is the return on common shares and r_{pt} is the return on preferred, all defined in period t.

The equations that we estimated were

$$r_{bt} = \mathcal{B}(p_{bt}B_t, p_{st}S_t, r_{at}, Y_t) \qquad (2.1)$$

$$r_{st} = \mathcal{S}(p_{bt}B_t, p_{st}S_t, r_{at}, Y_t) \qquad (2.2)$$

where r_{bt} is the rate of return on debt, $\mathcal{B}$ and $\mathcal{S}$ are the functions for debt and equity respectively, p_{bt} is the price of bonds, p_{st} is the price of shares, r_{at} is the return on the alternative asset, and Y_t is income, with all the variables defined in period t. We estimated equations (2.1) and (2.2) in linear and log-linear forms. The linear form can be represented by

$$r_{bt} = \gamma_0+\gamma_1 p_{bt}B_t+\gamma_2 p_{st}S_t+\gamma_3 r_{at}+\gamma_4 Y_t \qquad (2.3)$$

$$r_{st} = \gamma_0+\gamma_1 p_{bt}B_t+\gamma_2 p_{st}S_t+\gamma_3 r_{at}+\gamma_4 Y_t, \qquad (2.4)$$

while the log-linear or double-log equations can be expressed as

$$\log r_{bt} = \gamma_0+\gamma_1 \log p_{bt}B_t+\gamma_2 \log p_{st}S_t+\gamma_3 \log r_{at}+\gamma_4 \log Y_t \qquad (2.5)$$

$$\log r_{st} = \gamma_0+\gamma_1 \log p_{bt}B_t+\gamma_2 \log p_{st}S_t+\gamma_3 \log r_{at}+\gamma_4 \log Y_t. \qquad (2.6)$$

We also estimated equations using the ratio of the value of debt to the value of equity, but these results were not quite as robust as the ones we accepted when debt and equity were not constrained to be in ratio form.

For r_{st}, we can observe that when we assume that the alternative asset's rate of return does not affect the return on equity then the equations suffer from a high degree of positive autocorrelation. Moreover, once we adjust for autocorrelation, we find that the income variable is insignificant and that the equations still do not perform well. We feel, then, that the rate of return on long-term corporate bonds, should be included as representatives of substitutable or complementary choices for the investor. From Tables 2.1 and 2.2 we find that the corporate bond rate is a somewhat better explanatory variable than the government bond rate. Once again, income is not a significant variable, and by the Durbin-Watson statistic, there is no autocorrelation for the last two regressions in Table 2.1. It does bear mentioning, that the values of the estimated coefficients are remarkably stable; that is, for example, for all the regressions in Table 2.1 and 2.2, the estimates of γ_1 are .0001. A word should be said concerning the values of R^2. We can observe that from the following tables the R^2's are remarkably high for rate of return equations. Giving us one more indication that we are on the right track towards an adequate representation of the determinants for the rate of return on equity.

TABLE 2.1

Rate of Return on Equity
Linear Model: O.L.S.

(t - value in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	.057 (11.541)	.0001 (2.902)	-.0001 (-2.333)			1.070	.783
None	.048 (6.283)	.0001 (1.379)	-.0001 (-2.469)		.000001 (1.581)	1.44	.805
Government Bond	.073 (7.516)	.0001 (3.246)	-.00005 (-2.373)	-.478 (-1.853)		1.305	.812
Government Bond	.065 (3.764)	.0001 (1.742)	-.00005 (-2.380)	-.358 (-1.083)	.000001 (.600)	1.242	.815
Corporate Bond	.080 (7.845)	.0001 (3.755)	-.00006 (-2.853)	-.548 (-2.479)		1.467	.830
Corporate Bond	.073 (4.816)	.0001 (2.197)	-.0001 (-2.802)	-.477 (-1.887)	.00001 (.616)	1.400	.833

TABLE 2.2

Rate of Return on Equity

Linear Model: C-O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	$\bar{R}^2$
None	.049 (6.811)	.0001 (1.485)	-.00002 (-.882)			.546 (3.260)	1.686	.843
None	.047 (5.414)	.0001 (1.082)	-.00003 (-1.184)		.000001 (.660)	.482 (2.749)	1.687	.846
Government Bond	.059 (4.668)	.0001 (1.649)	-.00002 (-.924)	-.267 (-.850)		.486 (2.782)	1.674	.848
Government Bond	.056 (3.573)	.0001 (1.211)	-.00003 (-1.074)	-.235 (-.707)	.000001 (.467)	.455 (2.558)	1.674	.849
Corporate Bond	.066 (5.250)	.0001 (1.878)	-.00002 (-1.064)	-.383 (-1.433)		.457 (2.569)	1.704	.856
Corporate Bond	.063 (4.198)	.0001 (1.416)	-.00003 (-1.233)	-.362 (-1.306)	-.000001 (.480)	.423 (2.332)	1.698	.857

Proceeding to the double-log model, we find that from Table 2.3 the corporate bond rate performs quite well as the explanatory variable representing the alternative asset. Although, in this case income is significant, it becomes so at the expense of the t-values of γ_1 and γ_3, which are the coefficients of the rates of return on debt and the corporate bond rate. In Tables 2.3 and 2.4 we also observe that adjusting for autocorrelation does not adequately improve the results, and indeed, for the last equation in Table 2.3, which seems the best for the double-log model, that the D.W. statistic is in the acceptable region and so the Cochrane-Orcutt adjustment is not really necessary.

Again, one should notice that the values of R^2 for the double-log are not as high as for the linear model. Suggesting, among other criteria, that the linear specification performs better than the log-linear.

TABLE 2.3

Rate of Return on Equity

Double-Log Model: O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-2.421 (-2.874)	1.175 (2.455)	-1.094 (-2.036)			.991	.560
None	-8.221 (-4.416)	.498 (1.117)	-1.134 (-2.538)		1.007 (3.362)	1.226	.709
Government Bond	-2.750 (-1.367)	1.230 (2.137)	-1.127 (-1.947)	-.072 (-.181)		1.028	.560
Government Bond	-8.710 (-3.540)	.575 (1.111)	-1.182 (-2.456)	-.104 (-.313)	1.010 (3.300)	1.269	.710
Corporate Bond	-2.711 (-1.588)	1.230 (2.176)	-1.130 (-1.950)	-.070 (-.092)		1.015	.560
Corporate Bond	-10.814 (-4.180)	.740 (1.580)	-1.359 (-2.922)	-.430 (-1.411)	1.146 (3.708)	1.446	.734

TABLE 2.4

Rate of Return on Equity

Double-Log Model: C-O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	-4.622 (-4.562)	.298 (.824)	-.009 (-.022)			.664 (4.438)	1.736	.742
None	-8.346 (-3.346)	.194 (.479)	-.690 (-1.384)		.900 (1.830)	.456 (2.559)	1.635	.761
Government Bond	-4.876 (-1.884)	.298 (.806)	.009 (.020)	-.040 (-.101)		.664 (4.442)	1.737	.742
Government Bond	-8.490 (-2.585)	.199 (.473)	-.686 (-1.323)	-.026 (-.068)	.900 (1.787)	.455 (2.556)	1.636	.761
Corporate Bond	-6.848 (-2.830)	.306 (.853)	.145 (.334)	-.367 (-.986)		.686 (4.711)	1.761	.754
Corporate Bond	-10.780 (-3.210)	.250 (.614)	-.689 (-1.396)	-.370 (-1.043)	.997 (2.050)	.442 (2.464)	1.658	.773

TABLE 2.5

Rate of Return on Debt

Linear Model: O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	.030 (25.261)	-.000003 (-.308)	.00001 (2.040)			1.497	.969
None	.031 (16.205)	.000002 (.150)	.00001 (2.036)		-.000001 (-.629)	1.634	.969
Government Bond	.027 (11.344)	-.000004 (-.447)	.00001 (2.007)	.076 (1.192)		1.503	.970
Government Bond	.027 (6.286)	-.00001 (-.372)	.00001 (1.946)	.081 (.986)	.000003 (.102)	1.481	.970
Corporate Bond	.029 (10.491)	-.000004 (-.423)	.00001 (2.050)	.029 (.491)		1.470	.969
Corporate Bond	.030 (7.338)	.000003 (.017)	.00001 (2.005)	.015 (.224)	-.000001 (-.443)	1.589	.969

TABLE 2.6

Rate of Return on Debt

Linear Model: C-O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	.028 (18.846)	-.00002 (-1.746)	.00002 (3.413)			.345 (1.835)	1.668	.974
None	.028 (14.124)	-.00002 (-1.297)	.00002 (3.185)		-.000007 (-.252)	.323 (1.705)	1.708	.974
Government Bond	.025 (8.829)	-.00002 (-1.893)	.00002 (3.387)	.090 (1.267)		.345 (1.839)	1.737	.976
Government Bond	.024 (6.202)	-.00002 (-1.754)	.00002 (2.918)	.101 (1.270)	.00001 (.333)	.357 (1.911)	1.702	.976
Corporate Bond	.026 (8.497)	-.00002 (-1.847)	.00002 (3.414)	.044 (.674)		.355 (1.897)	1.684	.975
Corporate Bond	.026 (6.729)	-.00002 (-1.510)	.00002 (3.098)	.043 (.627)	-.00001 (-.002)	.354 (1.894)	1.684	.975

TABLE 2.7

Rate of Return on Debt

Double-Log Model: O.L.S.

(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-4.805 (-14.666)	.250 (1.344)	.011 (.054)			1.184	.921
None	-7.131 (-10.013)	-.021 (-.125)	-.005 (-.027)		.404 (3.526)	1.260	.950
Government Bond	-4.010 (-5.272)	.117 (.539)	.092 (.418)	.174 (1.155)		.945	.926
Government Bond	-6.369 (-7.026)	-.142 (.743)	.070 (.395)	.161 (1.324)	.400 (3.546)	1.087	.954
Corporate Bond	-4.142 (6.298)	.126 (.590)	.093 (.424)	.159 (1.160)		.994	.926
Corporate Bond	-6.915 (-6.992)	-.042 (-.222)	.014 (.076)	.036 (.294)	.392 (3.178)	1.210	.950

TABLE 2.8

Rate of Return on Debt
Double-Log Model: C-O.L.S.
(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	-6.403 (-20.432)	-.300 (-2.913)	.720 (6.082)			.696 (.4851)	1.691	.974
None	-6.783 (-7.014)	-.317 (-2.737)	.634 (3.563)		.108 (.519)	.659 (4.384)	1.608	.974
Government Bond	-6.090 (-8.061)	-.302 (-2.880)	.700 (5.455)	.052 (.461)		.698 (4.878)	1.717	.974
Government Bond	-6.479 (-5.432)	-.321 (-2.708)	.614 (3.277)	.052 (.452)	.109 (.516)	.660 (4.388)	1.631	.974
Corporate Bond	-6.472 (-9.025)	-.299 (-2.831)	.723 (5.657)	-.013 (-.122)		.695 (4.832)	1.689	.974
Corporate Bond	-6.895 (-5.571)	-.317 (-2.669)	.638 (3.464)	-.017 (-.150)	.111 (.518)	.659 (4.384)	1.608	.974

The rate of return on debt equations are presented in Tables 2.5 to 2.8. Table 2.5 and 2.6 refer to the linear model. These results show us that from the investor's point of view the government bond rate is not much more adequate as an alternative rate of return compared to the corporate bond rate, although we must observe that both rates do not perform exceptionally well. Moreover, from economic theory we should expect that an increase in the value of debt tends to increase the rate of return on debt. In other words, we expect to find that $\gamma_1>0$, and $\gamma_2<0$. On these grounds we can reject all the regressions in Table 2.6 and most in 2.5. For the other equations in Table 2.5 the value of γ_1, although positive, is insignificant and so the linear model as it appears in these Tables, is inadequate.

In Tables 2.7 and 2.8, we find the double-log model for r_{bt}. We reach the same conclusions as in the linear model, that is, the alternative rates of return do not perform well. One must also say that in general, in Tables 2.7 and 2.8, that γ_1 has the wrong sign, and when γ_1 is positive, it is insignificant. Thus, all the equations in Tables 2.5 to 2.8 have limitations. The problem may be that the dependent variables do not properly show the investors' preferences towards the different types of capital utilized by Bell Canada. We are saying that, not only should the prices of debt and equity be included, also the price of physical capital should be one of the regressors.

Moreover, we also tested for an alternative definition of the rate of return on equity. Because equity includes, not only common and preferred shares but also capital surpluses, retained earnings etc., we can define the rate of return as net income divided by equity for any time period. We then ran regressions including the price of physical capital in the returns on debt and equity equations, along with the new definition of r_{st}. The best results are

$$\log r_{bt} = \underset{(-9.806)}{-4.521} + \underset{(2.700)}{.618} \quad \log \frac{p_{bt}B_t}{p_{kt}} - \underset{(-1.370)}{.358} \quad \log \frac{p_{st}S_t}{p_{kt}}$$

D.W. = 1.200 $\qquad R^2 = .853$

where $\hat{\gamma}_1 = .618 > 0$ and $\hat{\gamma}_2 = -.358 < 0$ (the t- values are in parenthesis). the rate of return on equity equation is,

$$\begin{aligned} r_{st} &= .054\ (1+.051) + .00005\ (p_{bt}B_t \\ &\quad + .051\ (\frac{p_{bt-1}B_{t-1}}{p_{kt-1}} - .00002\ (\frac{p_{st}S_t}{p_{kt}} \\ &\quad + .051\ \frac{p_{st-1}S_{t-1}}{p_{kt-1}}) - .290\ (r_{at} + .051 r_{at-1}) \\ &\quad - .051 r_{st-1} \end{aligned}$$

D.W. = 1.650 $\qquad R^2 = .500$

where $\hat{\gamma}_1 = .00005 > 0$ and $\hat{\gamma}_2 = -.0002 < 0$, with r_{at} defined as the corporate bond rate of return.

3. British Columbia Telephone

3.1 Introduction

In this section we discuss the results for the rate of return equation which are applicable to B.C. Telephone.

3.2 The Data

The data for the dividends per share (common and preferred) were obtained from the companies financial statements, as were interest payments and the value of debt. Moreover, from the accompanying financial data, we were able to obtain market prices of the different classes of shares. With this data, we computed the rates of return on common equity, preferred shares, and debt in the same fashion as for Bell Canada.

3.3 The Empirical Results

The equations that we estimated tested for the form of the function i.e. linear, double-log, semi-log etc., and the manner in which the regressors entered the function i.e. in ratio form, additively etc. The results which we present in Tables 3.1 to 3.8 arise from the equations of the form

$$r_{it} = f_i\left(\frac{p_{bt}B_t}{p_{st}S_{ct}}, \frac{p_{pt}S_{pt}}{p_{st}S_{ct}}, r_{at}, Y_t\right) \quad i = b,c,p \qquad (3.1)$$

where the subscript $i = b$ refers to debt, $i = c$ refers to common equity, and $i = p$ refers to preferred shares. The equations which we present are for common equity, and debt in linear and double-log form. These Tables illustrate B.C. Tel.'s position in financial markets.

Upon inspection of the rate of return on common equity equations (Tables 3.1 to 3.4), that the linear equation with $\gamma_3 = \gamma_4 = 0$ in Tables 3.2 is the best one in terms of the values of the t - statistics, D.W., and R^2 which is 98.5%. Indeed, we see that $\gamma_1 > 0$ and $\gamma_2 > 0$ which should be the case, for as the debt to common equity and preferred shares to common equity ratios increase, the holders of common shares require a higher rate of return. We see then, that B.C. Tel. exhibits monopoly power in the common share market.

Turning to Tables 3.5 to 3.8, the rate of return on debt equations, we observe by the values of the t - statistics, that in the cases where there is no autocorrelation the degree of monopoly power in the debt market is minimal. We see that the motivating variable determining the investors' required return on debt, is not the variables which B.C. Tel. have direct control over, namely debt and equity, but rather income.

Finally, the preferred rate of return equations (which are not presented here) showed that the carrier has no rate determining power in the preferred shares market. In other words, the more senior the security or asset (from the investor's viewpoint) the smaller the degree of monopoly power exhibited by B.C. Tel. in that particular market.

TABLE 3.1

Rate of Return on Common Equity
Linear Model: O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	.018 (.280)	.178 (2.126)	-.375 (-3.147)			.716	.459
None	-.143 (-7.802)	.020 (.896)	.263 (5.172)		.00001 (14.846)	2.400	.974
Government Bond	-.045 (-.226)	.176 (2.014)	-.296 (-1.128)	.628 (.339)		.664	.464
Government Bond	-.104 (-2.392)	.019 (.900)	.218 (3.248)	-.415 (-1.008)	.00001 (14.811)	2.212	.977
Corporate Bond	.039 (.198)	.179 (2.035)	-.405 (-1.390)	-.179 (-.113)		.740	.460
Corporate Bond	-.095 (-2.193)	.022 (1.010)	.197 (2.666)	-.041 (-1.207)	.00001 (15.191)	2.132	.978

TABLE 3.2

Rate of Return on Common Equity
Linear Model: C - O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	.240 (2.634)	.026 (.725)	.126 (1.089)			.964 (13.480)	.822	.900
None	-.177 (-13.617)	.046 (2.617)	.287 (8.891)		.00001 (23.873)	-.551 (-2.473)	2.099	.985
Government Bond	.270 (2.027)	.026 (.690)	.126 (1.039)	-.271 (-.295)		.964 (13.522)	.866	.897
Government Bond	-.200 (-5.302)	.048 (2.634)	.311 (6.262)	.200 (.700)	.00001 (23.211)	-.576 (-2.634)	2.324	.986
Corporate Bond	.279 (2.137)	.029 (.755)	.116 (.947)	-.442 (-.530)		.962 (13.157)	.897	.898
Corporate Bond	-.203 (-5.095)	.048 (2.656)	.321 (5.514)	.184 (.700)	.00001 (23.600)	-.584 (-2.690)	2.352	.986

TABLE 3.3

Rate of Return on Common Equity
Double-Log Model: O.L.S.
(t-values in parenthsis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-4.608 (-9.939)	2.151 (2.501)	-1.625 (-3.918)			.850	.569
None	-14.131 (-9.715)	1.233 (2.898)	1.034 (2.319)		1.398 (6.621)	1.575	.914
Government Bond	-1.694 (-.349)	2.184 (2.466)	-1.102 (-1.143)	.829 (.604)		.773	.583
Government Bond	-15.629 (-4.913)	1.196 (2.685)	.872 (1.580)	-.360 (-.550)	1.432 (6.295)	1.453	.916
Corporate Bond	-4.205 (-.903)	2.153 (2.396)	-1.539 (-1.430)	.114 (.087)		.833	.570
Corporate Bond	-16.425 (-5.908)	1.198 (2.798)	.668 (1.145)	-.578 (-1.000)	1.434 (6.671)	1.392	.921

TABLE 3.4

Rate of Return on Common Equity
Double-Log Model: C - O.L.S.
(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	.290 (.225)	.422 (1.175)	.432 (1.044)			.963 (13.424)	.850	.921
None	-18.335 (-6.427)	.139 (.505)	.373 (1.282)		1.719 (5.728)	.824 (5.441)	1.604	.968
Government Bond	.861 (.411)	.410 (1.106)	.438 (1.027)	-.007 (-.011)		.969 (14.669)	.870	.923
Government Bond	-18.393 (-5.089)	.141 (.500)	.369 (1.225)	-.062 (.200)	1.739 (5.253)	.832 (5.603)	1.580	.968
Corporate Bond	.111 (.053)	.428 (1.140)	.414 (1.000)	-.181 (-.255)		.965 (13.847)	.893	.922
Corporate Bond	-18.485 (-5.377)	.145 (.495)	.361 (1.143)	-.107 (-.222)	1.707 (5.478)	.816 (5.273)	1.665	.968

TABLE 3.5

Rate of Return on Debt
Linear Model: O.L.S.
(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	.067 (4.025)	.029 (1.360)	-.111 (-3.651)			1.119	.535
None	.030 (2.850)	-.007 (-.587)	.035 (1.205)		.000003 (5.956)	2.429	.900
Government Bond	.061 (1.220)	.029 (1.285)	-.104 (-1.553)	.050 (.106)		1.1221	.536
Government Bond	.048 (1.911)	-.007 (-.590)	.014 (.369)	-.190 (-.799)	.000003 (5.908)	2.128	.897
Corporate Bond	.079 (1.554)	.029 (1.319)	-.128 (-1.724)	-.102 (-.255)		1.137	.538
Corporate Bond	.048 (1.861)	-.006 (-.510)	.010 (.300)	-.154 (-.744)	.000003 (5.875)	2.175	.896

TABLE 3.6

Rate of Return on Debt
Linear Model: C - O.L.S.
(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	.110	-.018	-.002			.955	2.067	.731
	(3.396)	(-1.163)	(-.045)			(11.980)		
None	.015	.007	.037		.000003	-.562	2.246	.907
	(1.817)	(.600)	(1.776)		(8.439)	(-2.542)		
Government Bond	.165	-.018	-.003	-.550		.958	1.667	.784
	(3.530)	(-1.256)	(-.070)	(-1.559)		(12.544)		
Government Bond	-.002	.009	.059	.169	.000003	-.648	2.651	.914
	(-.182)	(.804)	(1.900)	(.877)	(8.595)	(-3.180)		
Corporate Bond	.160	-.016	-.013	-.456		.956	1.789	.774
	(3.356)	(-1.070)	(-.268)	(-1.385)		(12.240)		
Corporate Bond	-.006	.008	.066	.151	.000003	-.651	2.635	.914
	(-.262)	(.760)	(1.806)	(.914)	(8.805)	(-3.208)		

TABLE 3.7

Rate of Return on Debt
Double-Log Model: O.L.S.
(t - values in parenthesis)

Alternative Asset	Y_0	Y_1	Y_2	Y_3	Y_4	D.W.	R^2
None	-3.512 (-21.162)	.364 (1.181)	-.579 (-3.899)			1.215	.569
None	-6.388 (-8.366)	.087 (.388)	.224 (.957)		.422 (3.809)	2.248	.814
Government Bond	-3.299 (-1.870)	.366 (1.137)	-.541 (-1.541)	.061 (.122)		1.228	.569
Government Bond	-7.696 (-4.730)	.054 (.239)	-.083 (.292)	-.314 (-.913)	.452 (3.884)	1.903	.828
Corporate Bond	-3.943 (-2.373)	.361 (1.127)	-.670 (-1.745)	-.121 (-.260)		1.212	.571
Corporate Bond	-7.719 (-5.345)	.066 (.298)	.011 (.037)	-.335 (-1.082)	.443 (3.968)	1.920	.834

TABLE 3.8

Rate of Return on Debt
Double-Log Model: C - O.L.S.
(t - values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	-2.656 (-4.424)	-.266 (-1.061)	-.090 (-.311)			.929 (9.371)	2.165	.681
None	-6.619 (-10.361)	.362 (1.630)	.277 (1.429)		.459 (4.920)	-.447 (-1.870)	1.982	.822
Government Bond	-4.019 (-3.307)	-.281 (-1.213)	-.075 (-.281)	-.682 (-1.579)		.953 (11.780)	1.802	.746
Government Bond	-9.561 (-4.612)	-.377 (-1.371)	-.079 (-.309)	-.656 (-1.794)	.525 (3.436)	.494 (2.127)	1.923	.838
Corporate Bond	-3.996 (-3.170)	-.235 (-.986)	-.151 (-.546)	-.648 (-1.442)		.951 (11.491)	1.905	.737
Corporate Bond	-9.290 (-4.776)	-.325 (-1.159)	-.157 (-.560)	-.622 (-1.790)	.504 (3.498)	.438 (1.821)	1.967	.838

4. The Private Carriers

4.1 Introduction

In this section of the financial module we estimate the rate of return equations for the aggregation of Newfoundland Telephone, New Brunswick Telephone, and Maritime Telegraph and Telephone.

4.2 The Data

We calculated the rates of return in the same fashion as for Bell Canada. However, in this case, we are dealing with three distinct carriers. Consequently, after computing the rates of return on debt, common equity, and preferred shares for each individual company, we then formed the various rates of return for the private carrier by taking the weighted average of the appropriate rates. In other words for example, with regards to common equity, we computed the rate of return for each carrier by finding dividends per share divided by the price per share and adding this latter ratio to the expected growth rate of dividends per share. Where the growth rate was computed in the same fashion as that for Bell Canada, and these rates were set equal to zero for the years in which their values were statistically insignificant. We then computed the ratio of each carrier's common equity to the total common equity of the three companies and used these values as the weights for each carrier's rate and then summed these weighted rates of return. This sum we refer to as the rate of return on common equity for the private companies. The same procedure was utilized in computing the rates of return on debt and preferred shares for the private carriers.

4.3 The Empirical Results

We find, upon observing the balance sheets for the companies comprising the private carriers that preferred shares played an important role in their financial picture. This is contrary to the role of preferred shares in the case of Bell Canada. Indeed, we observed from the balance sheets that the proportion of preferred shares to total equity is roughly 30% for Newfoundland Telephone for 1961, 1962, and 1963 and 10% for 1972 to 1975.

The data for the computation of the rates of return were obtained from the financial statements of the carriers in question and the methods of computing the rates were identical to those for Bell Canada.

We tested many functional forms for the rates of return equation and we found that the equations which are homogeneous of degree zero in common equity, preferred shares and debt, performed best. Thus, we have,

$$r_{bt} = \mathcal{B}\left(\frac{p_{bt}B_t}{p_{ct}S_{ct}}, \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}}, r_{at}, Y_t\right) \qquad (4.1)$$

$$r_{ct} = \mathcal{C}\left(\frac{p_{bt}B_t}{p_{ct}S_{ct}}, \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}}, r_{at}, Y_t\right) \qquad (4.2)$$

$$r_{pt} = \mathcal{P}\left(\frac{p_{bt}B_t}{p_{ct}S_{ct}}, \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}}, r_{at}, Y_t\right) \qquad (4.3)$$

Specializing these functions to the double-log form we get,

$$\log r_{bt} = \gamma_0 + \gamma_1 \log \frac{p_{bt}B_t}{p_{ct}S_{ct}} + \gamma_2 \log \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}} + \gamma_3 \log r_{at} + \gamma_4 \log Y_t. \tag{4.4}$$

$$\log r_{ct} = \gamma_0 + \gamma_1 \log \frac{p_{bt}B_t}{p_{ct}S_{ct}} + \gamma_2 \log \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}} + \gamma_3 \log r_{at} + \gamma_4 \log Y_t. \tag{4.5}$$

$$\log r_{pt} = \gamma_0 + \gamma_1 \log \frac{p_{bt}B_t}{p_{ct}S_{ct}} + \gamma_2 \log \frac{p_{pt}S_{pt}}{p_{ct}S_{ct}} + \gamma_3 \log r_{at} + \gamma_4 \log Y_t. \tag{4.6}$$

The results for the estimation of equations (4.4) to (4.6) are presented in Tables 4.1 to 4.6.

We would expect 'a priori' that $\gamma_1 > 0$, and $\gamma_2 > 0$ for the common equity rate of return equation. Indeed, as debt to common equity increases, and preferred shares to common equity also increases the shareholders would desire a larger rate of return on common equity because of the fact that preferred shares and debt are more senior financial commodities, in terms of the payment obligations by the suppliers. We find that in Tables 4.1 and 4.2 that income does not affect the rate of return when γ_1 and γ_2 have the right sign and when there is no autocorrelation. Also, the corporate bond rate is a more appropriate variable to represent alternative sources of investment to the common shareholders. Therefore we feel that the equation, with the corporate bond rate being r_{at} and $\gamma_4 = 0$, which was estimated using ordinary least squares provides the best description for r_{ct}, explaining 94% of the variance.

Table 4.1

Rate of Return on Common Equity
Double-Log Model: O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-1.166 (-3.695)	2.684 (4.820)	-.072 (-1.185)			.977	.682
None	-9.568 (-4.369)	-.043 (-.053)	.016 (.346)		.828 (3.855)	.539	.865
Government Bond	2.869 (4.757)	.741 (1.956)	.053 (1.604)	1.741 (6.886)		1.383	.940
Government Bond	1.173 (.353)	.522 (.910)	.053 (1.571)	1.563 (3.633)	.127 (.521)	1.279	.942
Corporate Bond	2.122 (4.336)	.654 (1.715)	.068 (2.036)	1.535 (7.015)		1.305	.942
Corporate Bond	.619 (.199)	.459 (.818)	.068 (1.941)	1.390 (3.720)	.117 (.488)	1.221	.943

Table 4.2

Rate of Return on Common Equity
Double-Log Model: C-O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
None	-1.330 (-1.988)	.185 (.405)	-.002 (-.065)			.958 (12.409)	1.063	.928
None	-14.759 (-5.298)	-.556 (-1.209)	-.057 (-1.641)		1.353 (4.681)	.677 (3.441)	1.106	.961
Government Bond	2.668 (3.391)	.856 (2.347)	.025 (.740)	1.682 (5.362)		.401 (1.636)	1.682	.947
Government Bond	-10.137 (-3.199)	-.376 (-1.034)	-.052 (-1.867)	.925 (2.911)	1.117 (3.880)	.743 (4.151)	1.019	.980
Corporate Bond	1.969 (2.795)	.704 (1.908)	.041 (1.212)	1.506 (5.117)		.466 (1.972)	1.658	.949
Corporate Bond	-10.738 (-3.248)	-.467 (-1.304)	-.044 (-1.545)	.927 (3.028)	1.166 (3.751)	.775 (4.581)	.933	.980

The results for the return on preferred shares are presented in Tables 4.3 and 4.4. Again, we believe that γ_1 must be greater than zero but γ_2 may be negative. Because as the preferred shares to common equity ratio increases, all other things being equal, the riskiness of the carrier has not increased, but rather decreased and so preferred shareholders may be satisfied with a lower rate of return. Once again, income is not a meaningful determinant of r_{pt}, while the corporate bond rate performs marginally better than the government bond rate. Consequently, the second to last equation in Table 4.3, statistically and in an economic sense is the best equation.

Finally, for the rate of return on debt, we refer to Tables 4.5 and 4.6. Obviously an increase in the debt to common equity ratio must lead to an increase in the rate of return to the demanders of this debt. On the other hand, an increase in the ratio of preferred shares to common equity should, in general, lead to an increase in r_{bt}, if the debtholders are acutely aware of other competing senior financial instruments in the corporate portfolio. Hence, because of the lack of autocorrelation and by the signs of γ_1, γ_2, and the values of the t-statistics for the regressors, the equation with $\gamma_4 = 0$ and the corporate rate as r_{at} in Table 4.5 is the best equation in explaining 84% of the variance.

Table 4.3

Rate of Return on Preferred Shares
Double-Log Model: O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-2.416 (-31.984)	.793 (5.944)	-.061 (-4.201)			1.054	.755
None	-2.032 (-2.554)	.917 (3.145)	-.065 (-3.790)		-.038 (-.485)	1.160	.760
Government Bond	-2.846 (-9.335)	1.000 (5.224)	-.074 (-4.465)	-.186 (-1.452)		1.481	.794
Government Bond	-4.494 (-2.782)	.788 (2.819)	-.073 (-4.433)	-.358 (-1.711)	.123 (1.039)	1.443	.814
Corporate Bond	-2.793 (-11.298)	1.025 (5.320)	-.077 (-4.532)	-.176 (-1.590)		1.509	.801
Corporate Bond	-4.601 (-3.100)	.791 (2.956)	-.078 (-4.689)	-.350 (-1.969)	.141 (1.235)	1.432	.827

Table 4.4

Rate of Return on Preferred Shares
Double-Log Model: C-O.L.S.
(t-values in parenthesis)

lternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
one	-2.383 (-22.840)	.801 (4.650)	-.056 (-3.132)			.741 (2.015)	1.668	.763
one	-2.316 (-1.937)	.812 (2.889)	-.056 (-2.859)		-.007 (-.056)	.463 (1.956)	1.666	.763
overnment Bond	-2.760 (-7.052)	.890 (4.632)	-.059 (-3.321)	-.152 (-.968)		.344 (1.370)	1.676	.779
overnment Bond	-4.453 (-2.051)	.715 (2.400)	-.065 (-3.350)	-.316 (-1.218)	.131 (.792)	.331 (1.313)	1.645	.794
orporate Bond	-2.731 (-8.468)	.915 (4.659)	-.062 (-3.420)	-.151 (-1.100)		.338 (1.344)	1.688	.784
orporate Bond	-4.955 (-2.334)	.695 (2.433)	-.072 (-3.547)	-.353 (-1.507)	.178 (1.059)	.329 (1.305)	1.669	.808

Table 4.5

Rate of Return on Debt
Double-Log Model: O.L.S.
(t-values in parenthesis)

Alternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	D.W.	R^2
None	-2.220 (-13.782)	1.223 (4.302)	-.023 (-.745)			1.487	.643
None	-7.157 (-7.711)	-.333 (-1.000)	.027 (1.391)		.473 (5.297)	1.227	.900
Government Bond	-.573 (-1.163)	.430 (1.391)	.028 (1.036)	.711 (3.445)		1.820	.829
Government Bond	-6.144 (-2.958)	-.288 (-.800)	.030 (1.421)	.127 (.471)	.416 (2.731)	1.413	.902
Corporate Bond	-.853 (-2.157)	.379 (1.230)	.035 (1.298)	.638 (3.609)		1.772	.837
Corporate Bond	-5.934 (-3.026)	-.280 (-.792)	.033 (-.792)	.148 (1.491)	.397 (2.626)	1.450	.903

Table 4.6

Rate of Return on Debt
Double-Log Model: C-O.L.S.
(t-values in parenthesis)

lternative Asset	γ_0	γ_1	γ_2	γ_3	γ_4	ρ	D.W.	R^2
one	-2.073 (-4.660)	-.364 (-1.227)	.014 (.561)			.959 (12.579)	1.319	.865
one	-11.524 (-5.378)	-.802 (-2.703)	-.018 (-.799)		.937 (4.177)	.749 (4.235)	1.704	.930
overnment Bond	-.621 (-1.179)	.403 (1.242)	.032 (1.107)	.692 (3.152)		.042 (.158)	1.842	.818
overnment Bond	-13.325 (-5.491)	-.864 (-3.019)	-.020 (-.918)	-.380 (-1.514)	1.023 (4.742)	.715 (3.828)	1.411	.944
orporate Bond	-.890 (-2.101)	.337 (1.044)	.041 (1.407)	.624 (3.322)		.061 (.228)	1.820	.829
orporate Bond	-12.803 (-5.121)	-.819 (-2.791)	-.022 (-.955)	-.319 (-1.272)	.990 (4.467)	.693 (3.598)	1.493	.940

FOOTNOTES

1. One might want to distinguish between short term and long term debt but for our purposes we were interested in the relative benefits and costs of debt versus equity financing.

2. In this module we are discussing rates of return which are unadjusted for the existence of taxes.

3. We adopted this procedure because, even though g_t was not equal to zero in many years it was also not statistically different from zero.

4. Of course, we used other variables such as population and the price index of services, but none of them performed as well as the variables we mentioned in the text.

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CHAPTER 6

SIMULATION MODULE

1. The Structure

The simulation module integrates the general theoretical model with the three estimation modules. In this part of the study we bring forth the demand for total services, the production function, the rates of return equations, that we accepted for Bell Canada, and combine them with the first order conditions, regulatory and capital constraints.

The first set of equations we estimated were the demand equations. In the demand module we stated that the double-log demand equation estimated by the Cochrane-Orcutt method performed quite well. This equation is:

$$y_t = \frac{.116^{\,.188}\; p_t^{-1.325}\; Y_t^{.816}\; Y_{t-1}^{.812}\; P_{t-1}^{-.413}}{P_t^{-.519}\; p_{t-1}^{-1.076}\; y_{t-1}^{.663}} \qquad (1)$$

where y_t is output (which is the same as demand in equilibrium), p_t is the price index of output, P_t is the consumer price index, Y_t is the provincial product. From equation (1) we get that the price elasticity of demand is -1.325 and we can solve for p_t,

$$p_t = \frac{P_t^{.384}\; Y_t^{.616}\; Y_{t-1}^{.613}\; p_{t-1}^{.812}}{1.358\; Y_{t-1}^{.500}\; y_t^{.755}\; P_{t-1}^{.312}} \qquad (2)$$

this equation determines the price index of total telephone services.[1]

The production function we selected for Bell was the constant returns to scale Cobb-Doublas one with direct distance dialing as the measure of technological change, and materials explicitely included as an input. The estimated equation is,

$$Y_t = \frac{1.411K_t^{.305} \; L_t^{.616} \; R_t^{.079} \; e^{1.0711DD_t} \; Y_{t-1}^{.519}}{K_t^{.158} \; L_{t-1}^{.320} \; R_{t-1}^{.041} \; e^{.556DD_{t-1}}} \qquad (3)$$

where K is capital, L is labour, DD is direct distance dialing, and R is materials. The production function yields marginal products of capital and labour which are $\frac{\partial y}{\partial K} = .305 \frac{y}{K}$, $\frac{\partial y}{\partial L} = .616 \frac{y}{L}$.

We see that the marginal product of capital is .305 times the average product of capital, and the marginal product of labour is .616 times its average product.

The rate of return on debt equation is

$$r_{bt} = \frac{.011 \; D_t^{.618}}{p_{kt}^{.260} \; E_t^{.358}}, \qquad (4)$$

where p_{kt} is the price index of physical capital, $D_t = p_{bt}B_t$ is the value of debt, and $E_t = p_{st}S_t$ is the value of equity. With equation (4), we can compute that,

$$\frac{\partial r_{bt}}{\partial D_t} = .618 \frac{r_{bt}}{D_t}, \quad \frac{\partial r_{bt}}{\partial E_t} = -.358 \frac{r_{bt}}{E_t}.$$

The rate of return on equity equation is,

$$r_{st} = .057 + .00005 \left(\frac{D_t}{p_{kt}} + .051 \frac{D_{t-1}}{p_{kt-1}}\right) - .00002 \left(\frac{E_t}{p_{kt}}\right.$$

$$\left. + .051 \frac{E_{t-1}}{p_{kt-1}}\right) - .290 \left(r_{at} + .051 \, r_{at-1}\right) - .051 \, r_{st-1}, \qquad (5)$$

where r_{at} is the rate of return on long-term corporate bonds.

Equation (5) implies that

$$\frac{\partial r_{st}}{\partial D_t} = \frac{.00005}{p_{kt}}, \quad \frac{\partial r_{st}}{\partial E_t} = \frac{-.00002}{p_{kt}} .$$

These are then the equations and the important derivatives, which are obtained from the three estimation modules. The next step is to integrate the preceeding equations with the first order conditions in the general model. Before doing so, we must modify these conditions, in view of our aggregation of common and preferred equity, and the fact that we determine the value of debt and equity.[2] In addition we tested for the actual regulatory constraint in the context of Bell Canada and found it to be

$$(1 - u)(py - w_\ell L - w_r R) - \delta(1 - ud)p_k K$$
$$+ udr\, p_k K + r_b u(D - M) \leqq i(1 - u)p_k K , \qquad (6)$$

where w_r is the price index of materials and r_b is now also the rate of return on the exogeneous variable M. We must note that changes in the price of equipment are not considered by either the tax or the regulatory authorities and therefore for the simulation we omit θ_t from the regulatory constraint.

The test for the constraint was carried out by noting that from the theory of regulation we must have

$$\frac{py - w_\ell L - w_r R}{K} = S , \qquad (7)$$

where S is called the allowed factor price of capital. Also, in the context of our model the first order conditions imply an equation for S (the allowed factor price).

This derived equation for S must be consistent with (7) and with equation (6), and we computed S to be

$$S = \frac{p_k}{(1-u)} \left[\delta(1-ud) + i(1-u) - udr - ur_b \left(\frac{D-M}{p_k K}\right)\right] . \quad (8)$$

Indeed the values of S implied by equations (7) and (8) are identical. Consequently, with the manner in which the authorities treat changes in the price of equipment, and aggregating the values of common and preferred equity we can rewrite the net profit equations as,[3]

$$\Pi_n = (py - w_\ell L - w_r R)(1-u) + \frac{\Theta}{(1+\Theta)} p_k K - \delta(1-ud) p_k K - r p_k K(1-ud)$$

and recalling the capital constraint, the regulatory constraint, and noting that R and its price are exogeneous, the first order conditions now become

$$\frac{\partial \mathcal{L}}{\partial L} = F_\ell(\mathcal{J}'F+\mathcal{J}) - w_\ell = 0 \quad (9.1)$$

$$\frac{\partial \mathcal{L}}{\partial D} = F_k(\mathcal{J}'F+\mathcal{J}) - (1-\lambda)\delta(1-ud)p_k + \frac{\Theta}{(1+\Theta)} p_k$$

$$- p_k(1-ud(1-\lambda)\left[\frac{\partial r_b}{\partial D}(D-M)(1-u)\right.$$

$$\left. + \frac{\partial r_s}{\partial E}E\right] - \lambda u \frac{\partial r_b}{\partial D} p_k (D-M) - p_k r_b (1-u) \left[1-ud(1-\lambda)\right]$$

$$- \lambda p_k u r_b + \lambda i(1-u) p_k = 0 \quad (9.2)$$

$$\frac{\partial \mathcal{L}}{\partial E} = F_k(\mathcal{J}'F+\mathcal{J}) - (1-\lambda)\delta(1-ud)p_k$$

$$+ \frac{\Theta}{1+\Theta} p_k - p_k (1-ud(1-\lambda)) \left[\frac{\partial r_b}{\partial E}(D-M)(1-u)\right.$$

$$+ \frac{\partial r_s}{\partial E} E] - \lambda u \frac{\partial r_b}{\partial E} p_k \ (D-M) - p_k r_s \ (1-u) \ [1-ud(1-\lambda)]$$

$$+ \lambda i(1-u)p_k = 0 \qquad (9.3)$$

To implement equation set (9) for simulation we must substitute the values of the various estimated terms. We can rewrite equation (9.1) as, $F_\ell (\mathcal{E})' F^{-1} + 1)$ which is $F_\ell p (\mathcal{E})' \frac{Y}{p} + 1)$. Given that the price elasticity of demand is -1.3, we can write,

$$F_\ell p (\mathcal{E})' \frac{Y}{p} + 1) = F_\ell p (-\frac{1}{1.3} + 1) = .23 F_\ell p \ .$$

Also, from our computations on the marginal product of labour, we have

$$F_\ell p(.23) = (.616)(.23)\frac{pY}{L} = .142 \ \frac{pY}{L}$$

Thus equation (9.1) becomes

$$.142 \ p_t Y_t - w_\ell L^*_t = 0 \ , \qquad (10)$$

where L* is the equilibrium quantity of labour. Because the carrier may not be in equilibrium in the labour market, we must incorporate an equation which depicts the labour input dynamic adjustment process. This process is denoted by the following difference equation,

$$L_t - L_{t-1} = \mu(L^*_t - L_{t-1}) \ . \qquad (11)$$

Thus with the values of L^*_t we can estimate μ and then we have an equation for the computed value of labour. Upon estimating equation (11) we found that,

$$L_t = 1.703L_{t-1} - .015L^*_t - .698L_{t-2} + .010L^*_{t-1} \,, \qquad (12)$$

where (12) will be used to determine the simulated quantity of labour.

Next equation (9.2) becomes,

$$(1-\lambda)(1-u)(.070)\frac{yp_t}{K_t} - (1-\lambda)\,\delta(1-ud)p_k$$

$$+\frac{\theta}{(1+\theta)}p_k - (1-ud)(1-\lambda))\Big[.618\frac{p_k}{D^*_t}r_{bt}(D^*_t-M)(1-u)$$

$$+ .00005E^*_t\Big] - \lambda u(.618)\frac{p_k r_{bt}}{D^*_t}(D^*_t-M)$$

$$- p_k r_{bt}(1-u)\big[1-ud(1-\lambda)\big] - \lambda p_k u r_{bt} + \lambda i(1-u)p_k = 0 \qquad (13)$$

and equation (9.3) becomes

$$(1-\lambda)(1-u)(.070)\frac{Y_t p_t}{K_t} - (1-\lambda)\,\delta(1-ud)p_k$$

$$+\frac{\theta}{(1+\theta)}p_k - (1-ud(1-\lambda))\Big[-.358\frac{p_k r_{bt}}{E^*_t}(D^*_t-M)(1-u)$$

$$- .00002E^*_t\Big] + \lambda u\,(.358)\frac{p_k r_{bt}}{E^*_t}(D^*_t-M)$$

$$- p_k r_{st}\big[1-ud(1-\lambda)\big] + \lambda i\,(1-u)\,p_k = 0 \,, \qquad (14)$$

where i was computed by making (6) an equality. In the same fashion that we provided a dynamic adjustment equation for labour we also specify one for debt and one for equity. The equations are

$$D_t - D_{t-1} = \phi\,(D^*_t-D_{t-1})$$

$$E_t - E_{t-1} = \Psi\,(E^*_t-E_{t-1}).$$

and with the values of D* and E* which are found by simultaneously solving equations (13) and (14) for these variables, we get;

$$D_t = 1.113\ D_{t-1} - .113D^*_t \ , \tag{15}$$

and

$$E_t = 1.431E_{t-1} - .343E_{t-2} - .131E^*_t + .040E^*_{t-1} \ . \tag{16}$$

To close the system we include the rate of return on physical capital equation, the factor price equation, and the capital constraint,

$$r_t = r_{bt} \frac{(D_t - M)}{p_k K_t} (1-u) + r_{st} \frac{E_t}{p_k K_t} \tag{17}$$

$$w_{kt} = p_k\ (\delta + r_t) \frac{(1-ud)}{(1-u)} - \frac{\theta}{(1+\theta)} \frac{p_k}{(1-u)} \tag{18}$$

$$K_t = \frac{D_t}{p_k} + \frac{E_t}{p_k} - \frac{M}{p_k} \ . \tag{19}$$

Therefore the set of equations; (2), (3), (4), (5), (6), (10), (12), (13), (14), (15), (16), (17), (18), (19), gives us fourteen equations in fourteen unknowns, p, y, r_b, r_s, λ, L^*, L, D^*, E^*, D, E, r, w_k, K . In addition the exogenous variables are; the consumer price index (P), the provincial product (Y), technological change (DD), the ware rate (w_ℓ), the price of physical capital (p_k), the corporate income tax rate (u), the depreciation rate (δ), the before tax nominal allowed rate of return (i), net non-capital assets (M), the corporate bond rate (r_a), and also the constant (d = .54079), which is the discounted value of depreciation deductions on a dollar value of investment in physical capital.

This constant was computed from

$$d = \left[\frac{1}{(1-u)\rho T}\right]\left[1-e^{-(1-u)\rho T}\right]$$

for the continuous case and as a discrete approximation,

$$d = \left[\frac{1}{(1-u)\rho T}\right]\left[1 - \left(\frac{1}{(1+(1-u)\rho}\right)^{T}\right],$$

where ρ is the before tax rate of return and T is the lifetime of the asset. The average value of T for Bell Canada was obtained by dividing the average capital by the depreciation rate for tax purposes which is 5.3% so T = 18.868 years. For u (the corporate tax rate) we used the average, and for ρ we used 15% which was close to the average before tax allowed rate of return.[4]

2. Simulation Within the Sample Period

The tests of the adequacy of multiple equation simulation models are performed in two stages. The first stage pertains to the selection of the appropriate equations from the estimation process. This selection is based on two criteria; the first being consistency with economic propositions and the second being statistical analysis. Indeed, we have completed the first stage in the previous parts of the study dealing with the demand, production and financial modules. The second stage encompasses the simulation of the equation system within the estimation time period. This equation system was explicitly derived in section 1 of this module. We are now ready to specify the form in which the equations have been placed into the computer and to observe the values of the endogenous variables, which have been calculated using our model in order to compare them with the actual data.

The simulation was performed in two parts. The first part pertained to a model which assumed that Bell Canada did not possess any monopoly power in its financing ability, while the second part allowed for monopoly power. As it turns out, the model without monopoly power is a subset of the model with monopoly power, and so we are able to compare both sets of results and in that way test for the ability of our model to integrate the real and financial characteristics of Bell.

2.1 Simulation Without Monopoly Power

The assumed absence of monopoly power for Bell in the capital markets implies that the carrier cannot affect the rates of return on its financing instruments. Thus the equations determining r_b and r_s are dropped and these variables as well as r and also w_k become exogenous. This means that the share of debt to physical capital, and equity to physical capital is fixed but the capital budget (debt plus equity) is still an endogenous variable. Hence, because these later shares are fixed the first order conditions for debt and equity collapse into one equation for physical capital. The equation is,

$$(.070)py(1-\lambda)-(w_k-\lambda s)K^* = 0 \qquad (20)$$

Then with the value of K* obtained from (20) we estimated the dynamic adjustment equation for physical capital and obtained,

$$K_t = 1.105K_{t-1} - .105K^*_t . \qquad (21)$$

Finally, instead of solving the Lagrangian multiplier λ from the system of equations by including the constraint as a separate equation, we can estimate λ from equation (20). By letting $K = K^*$ (which is the case in steady-state equilibrium) and then solving for $M_k = \frac{w_k K}{py}$ we get,

$$M_k = .070(1-\lambda) + \lambda\frac{sK}{py} .$$

Due to the nature of technological innovations and other structural changes we split the sample period into three sections of eight years, nine years, and nine years. The estimated value of λ for the first sample period was .339, for the second .672, and for the third .707. We can observe then that, as λ increases, the impact of regulatory policies on the output, input, and financing requirements has tended to become much more intensified. One can also say that, because of the movement of λ over the sample period, by taking a single estimate of the multiplier the results would tend to be quite distorted, for purposes other than looking at average behavioural characteristics of Bell. In addition, by observing the ratio $\frac{w_k}{s}$, which is the <u>market price</u> of physical capital services over the <u>allowed price</u>, (λ must be less than this ratio for regulation to be effective) we can detect evidence of regulatory lag; especially in 1951, 1952, 1953, 1958, 1965, 1970, 1974. Therefore the values of λ, found in appendix 1 of this chapter, have been adjusted for nine and twelve month regulatory lags. We also took the price of telephone services as exogenous for this part of the simulation and so the system is comprised of equations (3), (10), (12), (20), (21) and equation (19). This means we can solve for y, L*, L, K*, K, (D+E). Equation (19), in this case, determines the capital budget (D+E), and not physical capital. The simulated values of these endogenous variables are compared with the actual values in appendix 2. We can observe that this model performs extremely well, in that the simulated values are very close to the actuals. Indeed we can conclude that our model reproduces the behaviour of Bell Canada over the sample period, quite accurately. The remaining test is whether the inclusion of the financial segment will reproduce these fine results.[5]

2.2 Simulation With Monopoly Power

For the simulation in this part we endogenized the rates of return on capital and so also the factor price of physical capital. We retained the values of the Lagrangian multiplier which we calculated in section 2.1. Thus our system of equations is (2), (3), (4), (5), (10), (12), (13), (14), (15), (16), (17), (18), (19), in the thirteen unknowns (excluding λ).

The values of the endogenous variables are compared with the actual values in appendix 3 of this chapter. We can observe once more that the results are very good. Starting with the price for total services, we compute that the average proportional difference between the simulated and actual values is .003. The average proportional difference for the output variable is -.002, for labour -.001, for physical capital -.003, for debt -.0004, for equity .004, for the rate of return on debt -.008, for the rate of return on equity .008, for the rate of return on physical capital .0002,for the factor price of physical capital -.006.

Therefore we can conclude that our model successfully integrates the real and financial characteristics of Bell Canada and this simulation model, as a whole, can be utilized to forecast the important aggregates of output, labour, physical capital, debt and equity requirements (as well as other variables) for this regulated carrier.

Appendix 6.1 <u>Exogenous Variables for the Monopoly System</u>

CPI	Consumer Price Index of Montreal and Toronto
Y	Gross Provincial Product of Quebec and Ontario in Current Dollars
BLR	Raw Materials for Bell
BLDD	Percentage of Calls Direct Distance Dialed for Bell
BLWL	Wage Rate for Bell
PI	Price Index of Investment Products
BLPK	Price Index of Physical Capital for Bell
BLM	Net Non-Capital Assets in Current Dollars for Bell
RBUS	Long-Term Corporate Bond Rate of Return
BLAR	Allowed Rate of Return of Capital for Bell
BLDEL	Rate of Depreciation for Bell
BLLAM	Regulatory Constraint Multiplier for Bell
THETA	Rate of Price Inflation for Investment Products
U	Corporate Tax Rate

YEAR	CPI	Y	BLR	BLOD	BLWL
1955.00	.789545	18171.0	53.3500	0.	1.96100
1956.00	.800772	20431.0	62.5100	.600000E-02	2.00700
1957.00	.828745	22082.0	62.9400	.130000E-01	2.09500
1958.00	.851868	22576.0	69.2800	.530000E-01	2.21000
1959.00	.856521	23927.0	72.8300	.910000E-01	2.31700
1960.00	.865386	24903.0	76.1300	.159000	2.46300
1961.00	.872320	26105.0	79.3700	.224000	2.60600
1962.00	.881129	27946.0	84.9800	.263000	2.72200
1963.00	.896067	30327.0	89.6600	.311000	2.81200
1964.00	.910725	33245.0	89.8100	.373000	2.88500
1965.00	.931549	36681.0	97.9500	.433000	2.97500
1966.00	.967554	41065.0	101.880	.471000	3.15300
1967.00	1.00000	44113.0	98.7100	.507000	3.40400
1968.00	1.03648	47692.0	103.960	.568000	3.69100
1969.00	1.07519	52326.0	123.780	.523000	3.99800
1970.00	1.10057	56691.0	123.180	.678000	4.42200
1971.00	1.11974	61735.0	138.500	.721000	4.85300
1972.00	1.16488	68171.0	153.000	.766000	5.32600
1973.00	1.24410	74546.0	168.520	.825000	5.83700
1974.00	1.37699	81448.0	182.030	.885000	6.41600
1975.00	1.52603	89175.0	200.700	.946000	7.04300
1	2	3	4	5	6

YEAR	PI	BLPK	BLM	RBUS	BLAR
1955.00	.742230	.838000	235.235	.370000E-01	.800288E-01
1956.00	.783672	.851000	259.660	.438000E-01	.736504E-01
1957.00	.802321	.856000	290.403	.528000E-01	.662212E-01
1958.00	.806879	.861000	275.226	.492000E-01	.644540E-01
1959.00	.818483	.862000	313.287	.570000E-01	.830149E-01
1960.00	.827600	.866000	320.209	.576000E-01	.858427E-01
1961.00	.828844	.863000	365.177	.552000E-01	.888427E-01
1962.00	.844177	.872000	435.890	.552000E-01	.914036E-01
1963.00	.867385	.881000	439.621	.546000E-01	.873062E-01
1964.00	.897223	.879000	511.001	.554000E-01	.926279E-01
1965.00	.938666	.894000	521.574	.566000E-01	.953728E-01
1966.00	.982594	.937000	703.374	.640000E-01	.856937E-01
1967.00	1.00000	1.00000	653.602	.692000E-01	.867467E-01
1968.00	1.00456	1.04900	634.384	.776000E-01	.632785E-01
1969.00	1.04683	1.09900	549.687	.864000E-01	.767424E-01
1970.00	1.09863	1.17800	544.422	.922000E-01	.789549E-01
1971.00	1.15126	1.24100	568.116	.828000E-01	.714870E-01
1972.00	1.20887	1.31200	422.238	.828000E-01	.653195E-01
1973.00	1.27352	1.38750	301.308	.842000E-01	.649593E-01
1974.00	1.33751	1.47080	245.686	.836000E-01	.647269E-01
1975.00	1.40489	1.55470	266.091	.816000E-01	.762728E-01
1	2	3	4	5	6

YEAR	BLDEL	BLLAM	THETA	U
1955.00	.529760E-01	.338650	.374667E-01	.434968
1956.00	.527800E-01	.338650	.385185E-01	.432968
1957.00	.581600E-01	.338650	.346577E-01	.436111
1958.00	.578420E-01	.503720	.274128E-01	.428099
1959.00	.597270E-01	.671620	.201416E-01	.469802
1960.00	.590840E-01	.671620	.172191E-01	.473053
1961.00	.589900E-01	.671620	.136012E-01	.486333
1962.00	.596060E-01	.671620	.158549E-01	.484833
1963.00	.614020E-01	.671620	.179413E-01	.481151
1964.00	.620860E-01	.671620	.203689E-01	.483993
1965.00	.634920E-01	.503720	.264358E-01	.482855
1966.00	.649110E-01	.503720	.321663E-01	.478934
1967.00	.653440E-01	.503720	.294102E-01	.466347
1968.00	.665120E-01	.707190	.241746E-01	.472282
1969.00	.587200E-01	.707190	.293915E-01	.477334
1970.00	.690090E-01	.530390	.346596E-01	.487045
1971.00	.692690E-01	.530390	.381321E-01	.453299
1972.00	.736610E-01	.707190	.411433E-01	.434876
1973.00	.755650E-01	.707190	.439045E-01	.425997
1974.00	.773800E-01	.530390	.453113E-01	.414050
1975.00	.796270E-01	.707190	.476423E-01	.397579
1	2	3	4	5

Appendix 6.2 Endogenous Variables for the Competitive System

BLTSO	Total Services for Bell
BLL	Labour for Bell
BLK	Physical Capital for Bell
BLDE	Debt Plus Equity Capital for Bell

The simulated values have the same symbols but with an S at the end.

YEAR	BLTSO	BLTSOS	BLL	BLLS
1955.00	246.010	235.409	51.8890	49.8131
1956.00	274.850	264.257	55.6610	54.6163
1957.00	303.390	291.428	57.7980	58.4560
1958.00	327.210	326.829	57.5960	59.4522
1959.00	354.180	346.213	56.5290	57.6009
1960.00	377.900	383.020	54.5970	55.9293
1961.00	404.530	409.532	52.4420	53.3837
1962.00	446.520	426.818	52.2790	51.0711
1963.00	475.290	475.197	53.5180	52.2816
1964.00	513.080	520.540	54.4270	54.4899
1965.00	561.820	573.287	55.7990	55.1602
1966.00	615.310	612.474	57.4700	56.8778
1967.00	669.880	670.315	56.5780	58.7677
1968.00	725.710	724.092	55.4880	56.0854
1969.00	802.553	773.651	56.5980	54.8556
1970.00	876.350	872.093	57.8350	57.4959
1971.00	923.111	948.717	58.1250	58.8229
1972.00	1032.88	987.143	58.9980	58.4629
1973.00	1103.19	1088.21	59.9070	59.7395
1974.00	1220.47	1221.86	60.7580	60.6656
1975.00	1375.19	1352.49	61.5050	61.4752
1	2	3	4	5

YEAR	BLK	BLKS	BLDE	BLDES
1955.00	856.650	790.153	953.108	897.384
1956.00	973.810	392.716	1088.37	1019.36
1957.00	1119.16	1027.89	1239.84	1170.27
1958.00	1259.93	1169.34	1360.07	1282.03
1959.00	1405.65	1326.37	1524.96	1456.61
1960.00	1554.70	1493.61	1666.58	1613.67
1961.00	1693.41	1666.94	1826.59	1903.74
1962.00	1827.35	1806.85	2029.34	2011.46
1963.00	1971.52	1939.29	2176.53	2148.13
1964.00	2110.69	2037.76	2366.30	2302.20
1965.00	2241.63	2239.92	2525.59	2524.06
1966.00	2384.18	2320.11	2937.35	2877.32
1967.00	2538.86	2549.60	3192.46	3203.21
1968.00	2684.91	2687.43	3450.86	3453.50
1969.00	2836.99	2725.34	3667.54	3544.83
1970.00	2994.12	3035.17	4059.72	4119.86
1971.00	3147.16	3182.57	4473.74	4517.68
1972.00	3312.24	3173.91	4767.90	4586.40
1973.00	3490.75	3304.35	5144.74	4886.09
1974.00	3676.50	3723.89	5653.08	5722.78
1975.00	3873.37	3939.21	6288.02	6399.37
1	2	3	4	5

Appendix 6.3 Endogenous Variables for the Monopoly System

BLPITS	Price Index of total Services for Bell
BLTSO	Total Services for Bell
BLL	Labour for Bell
BLK	Physical Capital for Bell
BLD	Debt in Current Dollars for Bell
BLE	Equity in Current Dollars for Bell
BLRD	Rate of Return on Debt for Bell
BLRE	Rate of Return on Equity for Bell
BLRK	Rate of Return on Physical Capital for Bell
BLWK	Factor Price of Physical Capital for Bell

The simulated values have the same symbols but with an S at the end.

YEAR	BLPITS	BLPITSS	BLTSO	BLTSOS
1955.00	.961587	1.00479	246.010	235.409
1956.00	.963289	1.01684	274.850	264.257
1957.00	.962919	1.01749	303.390	291.428
1958.00	.968797	.951476	327.210	326.829
1959.00	1.02595	.989854	354.180	346.213
1960.00	1.03125	.998331	377.900	383.020
1961.00	1.02919	1.01574	404.530	409.532
1962.00	1.01393	1.04409	446.520	426.818
1963.00	1.01769	1.01911	475.290	475.197
1964.00	1.01715	1.01167	513.080	520.540
1965.00	1.01543	1.00324	561.820	573.287
1966.00	1.00484	1.05837	615.310	612.474
1967.00	1.00000	1.00734	669.080	670.315
1968.00	.996404	1.01146	725.710	724.092
1969.00	.998717	1.01496	802.550	773.651
1970.00	1.02393	1.01501	876.350	872.093
1971.00	1.05217	1.03141	923.111	948.717
1972.00	1.07243	1.06008	1002.88	987.143
1973.00	1.09233	1.05962	1103.19	1088.21
1974.00	1.11707	1.11601	1220.47	1221.86
1975.00	1.14165	1.12641	1375.19	1352.49
1	2	3	4	5

YEAR	BLL	BLLS	BLK	BLKS
1955.00	51.8890	49.8131	856.650	790.153
1956.00	55.6610	54.6163	973.810	892.716
1957.00	57.7980	58.4560	1109.16	1027.89
1958.00	57.5960	59.4522	1259.98	1169.34
1959.00	56.5290	57.6009	1405.65	1326.37
1960.00	54.5970	55.9293	1554.70	1493.61
1961.00	52.4420	53.3837	1693.41	1666.94
1962.00	52.2790	51.0711	1827.35	1806.85
1963.00	53.5180	52.2816	1971.52	1939.29
1964.00	54.4270	54.4899	2110.69	2037.76
1965.00	55.7990	55.1602	2241.63	2239.92
1966.00	57.4700	56.8778	2384.18	2320.11
1967.00	56.5780	58.7677	2538.86	2549.60
1968.00	55.4880	56.0854	2684.91	2687.43
1969.00	56.5980	54.8556	2836.99	2725.34
1970.00	57.8350	57.4959	2984.12	3035.17
1971.00	58.1250	58.8229	3147.16	3182.57
1972.00	58.9980	58.4629	3312.24	3173.91
1973.00	59.9070	59.7395	3490.76	3304.35
1974.00	60.7580	60.6656	3676.50	3723.89
1975.00	61.5050	61.4752	3873.37	3939.21
1	2	3	4	5

YEAR	BLD	BLDS	BLE	BLES
1955.00	266.402	285.606	686.706	620.982
1956.00	305.019	279.044	783.353	762.984
1957.00	343.407	328.051	896.437	840.858
1958.00	423.000	370.477	937.069	979.037
1959.00	453.000	451.632	1071.96	1011.60
1960.00	545.000	485.734	1121.58	1182.25
1961.00	570.000	591.715	1256.59	1177.95
1962.00	630.000	614.357	1399.34	1368.22
1963.00	710.000	681.477	1466.53	1518.49
1964.00	735.000	765.908	1631.30	1562.40
1965.00	794.353	796.982	1731.24	1758.25
1966.00	944.803	851.521	1992.55	1865.65
1967.00	1070.23	1027.07	2122.23	2168.66
1968.00	1193.07	1160.78	2257.78	2303.67
1969.00	1262.50	1299.99	2405.04	2441.48
1970.00	1336.50	1384.04	2673.21	2556.32
1971.00	1541.50	1521.15	2932.24	2902.49
1972.00	1652.24	1685.28	3115.66	3266.76
1973.00	1772.24	1906.88	3372.50	3638.26
1974.00	1963.45	1962.69	3669.63	3602.06
1975.00	2160.93	2283.77	4127.09	4290.34
1	2	3	4	5

YEAR	BLRD	BLRDS	BLRE	BLRES
1955.00	.364450E-01	.376084E-01	.465672E-01	.476396E-01
1956.00	.366670E-01	.343013E-01	.446146E-01	.424320E-01
1957.00	.370410E-01	.365597E-01	.402003E-01	.412126E-01
1958.00	.348350E-01	.372667E-01	.415114E-01	.421458E-01
1959.00	.395280E-01	.416211E-01	.469086E-01	.442050E-01
1960.00	.407630E-01	.411234E-01	.477113E-01	.422674E-01
1961.00	.453560E-01	.465673E-01	.459108E-01	.494628E-01
1962.00	.457510E-01	.450501E-01	.466542E-01	.470768E-01
1963.00	.443540E-01	.461513E-01	.455684E-01	.481588E-01
1964.00	.464480E-01	.491348E-01	.476547E-01	.521599E-01
1965.00	.458850E-01	.480591E-01	.499787E-01	.495169E-01
1966.00	.452930E-01	.484205E-01	.461535E-01	.477730E-01
1967.00	.486590E-01	.506532E-01	.511428E-01	.495236E-01
1968.00	.505900E-01	.528032E-01	.506377E-01	.504925E-01
1969.00	.544110E-01	.548023E-01	.472742E-01	.514345E-01
1970.00	.536160E-01	.550343E-01	.498509E-01	.502502E-01
1971.00	.559290E-01	.549975E-01	.502313E-01	.525163E-01
1972.00	.591220E-01	.553585E-01	.528903E-01	.531753E-01
1973.00	.633590E-01	.566650E-01	.536415E-01	.550084E-01
1974.00	.616540E-01	.570166E-01	.504983E-01	.559957E-01
1975.00	.682540E-01	.579745E-01	.740601E-01	.584962E-01
1	2	3	4	5

YEAR	BLRK	BLRKS	BLWK	BLWKS
1955.00	.454395E-01	.456596E-01	.641874E-01	.644374E-01
1956.00	.433107E-01	.418626E-01	.591855E-01	.575213E-01
1957.00	.391221E-01	.403299E-01	.651879E-01	.665893E-01
1958.00	.385705E-01	.402986E-01	.739017E-01	.759008E-01
1959.00	.439163E-01	.415417E-01	.952139E-01	.923346E-01
1960.00	.433317E-01	.397377E-01	.986690E-01	.942730E-01
1961.00	.427415E-01	.453428E-01	.104313	.107534
1962.00	.438420E-01	.443227E-01	.103616	.104216
1963.00	.429016E-01	.448321E-01	.101558	.103983
1964.00	.447948E-01	.483999E-01	.997036E-01	.104237
1965.00	.464057E-01	.461769E-01	.936261E-01	.933332E-01
1966.00	.437162E-01	.461146E-01	.859776E-01	.891735E-01
1967.00	.470115E-01	.462192E-01	.103906	.102796
1968.00	.458888E-01	.462838E-01	.121433	.122018
1969.00	.429679E-01	.460769E-01	.117036	.121885
1970.00	.444974E-01	.448060E-01	.120264	.120787
1971.00	.453327E-01	.469672E-01	.119022	.121823
1972.00	.473769E-01	.470715E-01	.130385	.129843
1973.00	.483957E-01	.481246E-01	.137300	.136795
1974.00	.458795E-01	.487046E-01	.141172	.146675
1975.00	.636946E-01	.509568E-01	.184299	.158493
1	2	3	4	5

FOOTNOTES

[1]In this chapter we have rounded the numbers to the third and fourth decimal place. In the actual simulation we did not round the numbers because of the errors this would have caused.

[2]Given the fact that we have aggregated the financial instruments of Bell Canada into two categories, it is simpler, from the viewpoint of forming aggregate price indices of debt and equity, to treat the values of debt and equity as the carrier's decision variables. Given our assumption that the prices of bonds and equity are exogenous variables, the two treatments are essentially identical.

[3]We drop the expectations operator for ease of notation.

[4]The values of θ were obtained from an eight year moving average of the Canadian price index of investment products. This also means that the price by which we multiply θ is not p_{I_t}, but rather the former price index. The justification for this procedure stems from the fact that θ is the expected rate of price inflation of physical capital and these expectations are captured by the Canadian index.

[5]It should be recalled that most of our data series for Bell Canada were from 1952-1972 and we extrapolated to 1950 and to 1975. Therefore one should focus on the simulation results for the period 1955-1972.

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