DESCRIPTION OF A COMPUTER PROGRAM
FOR OPTIMIZATION OF
SHAPED BEAM ANTENNAS

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2 DESCRIPTION OF A COMPUTER PROGRAM SHAPED BEAM ANTENNAS

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### 1.0 INTRODUCTION

In the initial phases of communication satellite antenna design, it is necessary to determine the feed horn locations and sizes that will provide adequate gain within a specified service area.

This task is normally accomplished by an antenna engineer who begins with an initial design, then refines the first guess by successive iterations until an acceptable solution is reached. This report is a description of a computer algorithm which calculates an initial feed horn configuration based on the desired service area and then accepts refinements to the first solution by means of a convenient interactive computer program.

The program then is designed for two levels of users, system planners and antenna designers. For system planners, the program automatically assigns feed locations. For the antenna designers the program gives the facility to change feed dimensions, locations, power, and phase.

The program is specifically designed for offset-fed parabolic reflectors with circular projected apertures. At the present time, rectangular or'square feed horns with either vertical or horizontal polarization are modelled.

This report first describes the analytical algorithms used for analysis as well as the geometrical parameters of the antenna. The next section gives a detailed description of each subroutine used, and a flow chart of the main program. Some recommendations for enhancement of the algorithms complete the body of the report. Four appendices are attached which contain: details of the analytical equations, a description of a test case, computer listings of the Fortran code, and copies of the main referenced papers.

### 2.0 THE ANALYTICAL MODEL

### 2.1 ANTENNA MODEL

This section describes the analytical basis of the approach used for the design of shaped-beam patterns using offset-fed parabolic antennas equipped with an array of horns placed in the focal plane of that reflector. The algorithm requires the user to specify the reflector geometry and a list of points located on the contour defining the required service area. The user can then adjust the individual feed phase, size, and positions to reduce or increase the gain over the beam coverage.

Estimates are also calculated for the following performance parameters:

- peak cross-polarization level due to feed horn geometry, reflector geometry, and the reflector surface roughness.
- loss of gain due to feed offset, and reflector surface roughness
- coma lobe level
- efficiency factor of polarization

These estimates allow the user to make adjustments to the antenna parameters by changing identified geometric parameters before running the relatively time-consuming far field pattern program. Descriptions of the required inputs and performance parameters are given in the following sections.
2.2 INPUTS

A list of user entered variables needed to run the program is as follows:

- contour points outlining the required service area.

These contour points may number up to a maximum of 15 and must be entered consecutively in a clockwise manner using positive values for degrees west in longitude and degrees north in latitude. Negative values may be entered to represent East of Greenwich and South of the Equator. Due to the foreshortening effect of the mapping of the service area from longitude and latitude into azimuth and elevation as seen from the spacecraft location, it is recommended to use specified contour points which are equally spaced around the contour, thereby avoiding long spans between points.

- geostationary orbital location of the satellite in longitude (degs W)
- frequency $f(\mathrm{MHz})$ or wavelength $\lambda(\mathrm{cm})$, of operation.

When one is entered, the other is calculated by the program.
antenna geometry

The geometry of an offset parabolic antenna having a circular projected aperture can be completely specified in terms of either of two sets of variables. The first set consists of the parent paraboloid focal length, $F$ expressed in meters; the offset distance from the plane of symmetry of the parent paraboloid to the center of the circular projected aperture, $X_{0}$ again expressed in meters; and the diameter of the circular projected aperture, D expressed in meters. These parameters are shown schematically in Figure 2.1.

The second set of variables consists of the half-angle subtended by the offset reflector as observed at the focal point of the parent parabola, $\theta_{*}$ expressed in degrees; the offset angle subtending the locations on the parent parabola


Figure 2.1 Reflector Geometry
of the projected center of the reflector and the center of the parent parabola, $\theta_{0}$ again expressed in degrees; and the diameter of the circular projected aperture, D expressed in meters.

These parameters are also shown in Figure 2.1 and are related to the first set of parameters through the equations presented in Appendix A, Section 1 . When one set is entered in the program, the second set is immediately calculated since various subsequent calculations are based on one or the other set. During the iterative design portion of the program (described later), the listing will recommend that certain of these paramters be increased or decreased. If a parameter other than $D$ is changed, then the remaining parameters in its set are left unchanged, and new values for the parameters in the other set are calculated. If D is changed then the remaining parameters in the set that was initially entered by the user are recalculated, and then new values for parameters in the other set are calculated.

- Horizontal or vertical polarization used by the satellite antenna.

Horizontal polarization is taken as referring the Electric Field Intensity to be perpendicular to the plane defined by the satellite location and the axis of rotation of the earth. Vertical polarization is taken as requiring the Electric Field Intensity to be in the above defined plane and also be perpendicular to the line connecting the satellite and the beam center of the earth coverage.

No capability is available at present for slanted polarizations, circular polarizations or eliptical polarizations.

- North, south, east or west antenna-satellite offsets.

The antenna-satellite offsets refer to the attitude of the spacecraft with respect to the earth. The choice of horn locations feeding the reflector as well as the number of horns required will depend on what attitude the spacecraft may have. North offset refers to the attitude where the feed structure is positioned north of the spacecraft antenna. South offset refers to the attitude where the feed structure is positioned south of the spacecraft antenna. These offsets are illustrated in Figure 2.2. Similar definitions apply for east and west offsets. No capability is available at present for slanted offsets.

Optional Inputs provide a selection of:

- type of feed horn used, either rectangular or square aperture. As is discussed in Section 2.3, the centers of the feed horns are located on a square grid on a plane passing through the focal point of the parent parabola. Consequently, the maximum physical dimensions of the feed are limited to this square grid size. However the optimum size for the horns as specified by optimum illumination of the reflector may be smaller than the grid for either rectangular or square feeds. This input allows the user to investigate a specific type of feed structure. If no information is provided, the program selects a rectangular feed horn geometry.

- RMS surface roughness of the reflector.

The RMS surface roughness is a source of cross-polarization radiation, a highly undesirable feature for satellites designed for frequency reuse in one or more service areas. Other significant sources of cross-polarization radiation are the offset reflector geometry and the feed horn geometry. A comparison of the relative levels of cross-polarization radiation due to these sources helps to define reasonable specifications for acceptable RMS surface roughness. Roughness also causes loss of gain for the antenna, and can be compared with other sources such as feed offset from the focus in order to estimate acceptable roughness. The program asks for roughness in cms, and defaults to zero surface roughness if no information is provided.

### 2.3 OPTIMIZED BEAM CENTRE CALCULATIONS

The optimized beam centre calculation is obtained using the following procedure.

Initially all reflector parameters and the wavelength (or frequency) are calculated using the equations presented in Appendix A.

The user-specified longitude and latitude service area contour points are converted to elevation and azimuth by coordinate transformation using the satellite position as $0^{\circ}$ latitude, the satellite longitude coordinate in degrees West, and an altitude of 35782.0 km .

The initial boresight of the antenna is established by finding the midpoint between the extremes of azimuth and elevation of the specified contour points. This initial boresight is shown in Figure 2.3 and minimizes the negative effects that occur when a beam is off axis by equalizing the offsets in opposite directions.


Figure 2.33 dB Service Area Viewed from Synchronous Height for - Translation into Azimuth and Elevation

The 6 dB beamwidth for the reflector aperture chosen is then calculated based on an interpolation from results given by Ruze (1). He presented an equation relating a parameter $W$ to the antenna beam width BW as follows:

$$
\frac{W}{2}=\frac{\pi D}{\lambda} \sin \left[\frac{B W}{2}\right]
$$

For a feed offset of zero, and an illumination taper of 10 dB , the following values of $W$ are found for the 3 dB and 10 dB beamwidths:

$$
\begin{array}{rl}
3 \mathrm{~dB} & W=3.6 \\
10 \mathrm{~dB} & \mathrm{~W}=6.1
\end{array}
$$

Further, Ruze shows that these factors are only weakly dependent on offset up to an offset distance of $\mathrm{X}=10$, which is a significant offset. For a 6 dB beamwidth then, $W$ is interpolated to be 4.8 and the expression for the 6 dB beamwidth becomes:
$B W(6 \mathrm{~dB})=2 \sin ^{-1}\left[\frac{2.4 \lambda}{\pi D}\right]$
as given in Appendix A, Section 3.

Off boresight beams are positioned so that the 6 dB beamwidths touch in the principal plane directions (azimuth and elevation) as shown in Figure 2.4. This requirement of having adjacent beams touch at the 6 dB level ensures that the total power density midway between beams is equal to the beam center power density, thereby minimizing the gain ripple over the composite beam. Only beams which are $65 \%$ within the closed polygon formed by connecting the user specified contour points are counted. It is apparent from Figure 2.4 that the initial boresight has not actually equalized the number of beam offsets in opposite directions due to this $65 \%$ criteria.


Figure 2.46 dB Beamwidths Filling Service Area


Figure 2.5 Count Routine (Grid Stepping)

To improve this situation and also to minimize the number of feeds required, the grid of 6 dB beamwidths is stepped through increments of 0.2 of the 6 dB beamwidth until a whole beamwidth has been stepped. This procedure is followed in both azimuth and elevation directions, and determines an offset which minimizes the number of elements inside the user specified contour (see the COUNT routine and Figure 2.5). The new boresight in the azimuth and elevation plane is obtained by the addition of the optimum offset to the original boresight, and new beam centre coodinates are calculated.

### 2.4 FEED HORN LOCATIONS AND DIMENSIONS

The feed horn locations and dimensions are found using the procedure described in this section.

The Beam Deviation Factor (BDF) is first calculated based on equation 6 of Lo (2) for zero feed offset. The assumption of zero feed offset for all feed horns is valid since Lo has shown in (2) that any offset has only a negligable second order effect on the Beam Deviation Factor. This factor relates the deviation from boresight (in azimuth or elevation) of a beam to an offset from the focal point of its illuminating feed horn. This is shown for the case of an offset in elevation in Figure 2.6. The calculated beam centre coordinates, reflector geometry, and beam deviation factor are used to locate the centres of the feed horns on the "optimum" feed plane passing through the focal point. The description of this optimum feed plane has been obtained by Mittra (3) and is shown in Figure 2.6. It should be mentioned that this optimum plane is not the optimum surface, and has aberrations somewhat above these for the optimum surface. However it is the plane which has the least aberrations of all possible planes passing through the focus. The equations are presented in Appendix A for the centres of feed horns for each case of north, south, east and west antenna-satellite offset. A sample derivation for the equations for the feed horn centres for a south antenna-satellite offset is outlined below.


Figure 2.6 Feed Horn Locations

Refering to Figure 2.7a, we have:

$$
\begin{equation*}
y^{\prime \prime}=P \sin \Theta_{1} \tag{1}
\end{equation*}
$$

Summing the angles in the large triangle gives:

$$
\theta_{1}+90+\theta_{0}+\eta=180
$$

or

$$
\begin{equation*}
\eta=90-\Theta_{1}-\Theta_{0} \tag{2}
\end{equation*}
$$

Also we have at the intersection of the $z$ axis and the vertical line located at the focal point

$$
n+\delta+\zeta=90
$$

hence

$$
\begin{equation*}
\delta=90-\zeta-\eta \tag{3}
\end{equation*}
$$

This allows the remaining angle $\varepsilon$ in the triangle bounded by $y^{\prime \prime}$ and $y^{\prime}$ to be expressed as:

$$
\begin{align*}
\varepsilon & =90-\delta \\
& =\zeta+\mathrm{n} \\
& =90+\zeta-\Theta_{1}-\Theta_{0} \tag{4}
\end{align*}
$$

Now

$$
\begin{aligned}
& y^{\prime}=\frac{y^{\prime \prime}}{\sin \varepsilon} \\
& =\frac{P \sin \theta_{1}}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}
\end{aligned}
$$

Finally:

$$
\begin{align*}
y & =-y^{\prime} \cos \zeta \\
& =-\frac{P \cos \zeta \sin \theta_{1}}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)} \tag{5}
\end{align*}
$$

as given in Appendix A Section 6.

Also using Figure 2.7a we get:

$$
\begin{align*}
z & =F-y^{\prime} \sin \zeta \\
& =F-\frac{P \sin \zeta \sin \theta_{1}}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)} \tag{6}
\end{align*}
$$

again given in Appendix A Section 6.

Referring to Figure 2.7 b we see that the triangle used to obtain the $x$ coordinate for the feed horn, has its side along the $z$ axis, $D$, foreshortened with respect to the focal length, F. The cause for this is the offset in the elevation plane of both the reflector and the optimum feed plane as seen in Figure 2.7a. Consequently:

$$
\begin{aligned}
D & =P \cos \theta_{0}-y^{\prime} \sin \zeta \\
& =P \cos \theta_{0}-\frac{P \sin \zeta \sin \theta_{1}}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}
\end{aligned}
$$

and hence:

$$
\begin{align*}
x & =D \tan \emptyset_{1} \\
& =\left[P \cos \theta_{0}-\frac{P \sin \zeta \sin \theta_{1}}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}\right] \tan \emptyset_{1} \tag{7}
\end{align*}
$$

The above expression is given in Appendix A Section 6.


Figure 2.7a South Offset Antenna Feed - Elevation Plane


The maximum feed horn dimensions are determined by the spacing required between the horn centres and are found from the above beam centre coordinates once the azimuth spacing and elevation spacing for 6 dB beamwidths are known. If these dimensions are greater than the optimum horn dimensions, developed by Truman and Balanis (4), and Herbison-Evans (5), then optimum feeds are used. These optimum feeds are designed based on the criteria of maximizing the gain of a reflector antenna by controlling the illumination of the reflector. For example if a feed horn used to illuminate a reflector is made too large, then the beam formed from the horn will not illuminate the reflector completely to its edges. Since the outer annular edge of the reflector is not being illuminated, the useful reflector size is less than its physical size, and the reflector gain will decrease. On the other hand if the feed horn is made too small, then the beam formed will spill over the edge of the reflector, resulting in a loss of power reflected towards boresight, and hence also causing a decrease in gain. The work described in (4) and (5) optimizes this trade-off for axially symmetric parabolic reflectors, not offset parabolic reflectors. However since the criteria is based on spillover power and since the offset parabola presents approximately a circular projection to the focal point, the results are equally useful for the two types of reflectors. The equations presented in the Appendix A Section 7 are curve fits to graphical data presented in Figure 2 of Reference (4) for rectangular feeds, and to graphical data presented in Figure 7 of Reference (5) for square feeds. The use of optimum feeds is shown schematically in Figure 2.8.


Figure 2.8 Feed Horn Dimensions

### 2.5 REFLECTOR PERFORMANCE ESTIMATES

The following estimates for various reflector performance characteristics are given with suggested changes in the antenna structure which allow the user a chance to improve the performance. Each estimate will only be calculated if it is within limits set by the equations. When all performance estimates are satisfactory the program commences execution of the antenna field pattern analysis.

Estimates for the cross-polarization level due to feed horn dimensions, reflector geometry, and reflector surface roughness are calculated based on work reported by Rudge and Adatia (6), Chu and Turrin (7), and Ghobrial (8). These equations are least squared error curve fits to data presented in the three reports. The effect due to feed horn dimensions is based on Figure 7 b of Reference 6. The effect due to reflector geometry is based on Figure 4 of Reference 7, while the effect due to reflector surface roughness is based on Figure 9 of Reference 8. The cross-polarization level indicates the relative power density of the unwanted orthogonal polarization compared to that of the desired polarization, either horizontal or vertical. This is shown schematically in Figure 2.9. It is important to have cross-polarization levels less than -28 $d B$ to reduce interference problems between adjacent coverage areas or even between signals in a single coverage area utilizing frequency reuse based on polarization isolation.

An estimate for loss of gain for the maximum, feed offset is obtained using the work of Ruze (1). A1so an estimate for the loss of gain due to reflector surface roughness is calculated based on another study by Ruze (9). Again these equations are least squared error curve fits to data presented in the reports. The effect due to feed offset is based on Figure 6 of Ruze(1) for a 10 dB tapered illumination. A complication arises here
in that the independent variable $x$ in Ruze's paper is a function of the number of three $d B$ beamwidths which the beam is off axis, M.

$$
X=\frac{M}{.02+\left[\frac{F}{D}\right]^{2}}
$$

However we are designing a shaped beam based on six dB beamwidths. Hence the loss of gain for a certain number of 6 dB beamwidths in which the beam is off axis, $N$ is found by relating $N$ to $M$. Using the parameter $W=4.8$ discussed previously for the six $d B$ bearwidths and reading $W=3.6$ for $X=0$ from Figure 3 of Ruze (1) we have:

$$
3.6 \mathrm{M}=4.8 \mathrm{~N}
$$

or

$$
\mathrm{M}=1.33 \mathrm{~N}
$$

and

$$
x=\frac{N}{.0150+.75\left[\frac{F}{D_{1}}\right]^{2}}
$$

as given in Appendix A Section 11. The maximum number of 6 dB beamwidths which any beam is off-axis is obtained from:

$$
\begin{aligned}
\mathrm{N} & =\frac{\text { Magnitude of Maximum Offset in Angle }}{6 \mathrm{~dB} \text { Beamwidth }} \\
& =\frac{M A X \sqrt{\mathrm{Az}^{2}+\mathrm{EL}^{2}}}{2 \sin ^{-1}\left[\frac{.76 \lambda}{\mathrm{D}}\right]}
\end{aligned}
$$

The loss of gain due to surface roughness is obtained using the least squared error curve fit to Figure 7 of Ruze (9). It should be noted that a conversion from RMS surface error values to peak surface error values has been made using a conversion factor of 3 as suggested by Ruze (9). The
loss of gain is a good indicator of the practical limitations imposed by defocusing the reflector, and should be kept less than 2 dB . The loss of gain is shown schematically in Figure 2.10 .

An estimate of the efficiency factor of polarization is calculated based on results reported by Dijk (10): This equation is again a least squared error curve fit to Figure 8 of Dijk (10). The decrease in the efficiency factor is caused by the loss of signal power density due to energy being radiated in the unwanted polarization. The efficiency factor should therefore be kept above $90 \%$.

The coma lobe level estimate is determined based on research by Rudge and Adatia (6) and Ruze (1). Once again the equation is based on a curve fit to the data presented in Figure 5 of Ruze (1). Since the independent variable $x$ is again based on three $d B$ beamwidths, the same considerations as discussed for the loss of gain with feed offset must be made. The coma lobe level is the relative power density of the largest co-polarized sidelobe compared to the main lobe. It is important to have the coma lobe level less than $-12 d B$ to reduce interference problems between separate earth stations. The coma lobe level for an off set beam is shown schematically in Figure 2.11.

The equations for all the above estimates are presented in Appendix A. Based on the parameters contributing to each of the performance estimates, the computer program prompts the user on which geometric parameters should be changed in order to improve performance. This iterative procedure is continued until all performance estimates are satisfactory to the user at which time the reflector geometry and feed horn locations, sizes and orientations are used as inputs to the surface currents and far field radiation pattern programs.


Figure 2.9 Cross-Polarization Level Estimate


Figure 2.10 Loss of Gain Estimate


Figure 2.11 Coma Lobe Level Estimate

### 3.0 SOFTWARE DOCUMENTATION

### 3.1 INTRODUCTION

This section contains a detailed description of the Satellite Antenna Modelling Program (SAMOD). The algorithms are described in sufficient detail so that the program may be modified by a Fortran programmer. This program uses a validated set of subroutines to calculate the far field pattern from a file of feed horn parameters. Input-process-output charts are provided for these programs.

Users interested in just the operator interface of the program, should refer to the User's Manual, which describes the inputs/outputs sufficiently for program use.

A Iist of input/output parameters is shown in Figure 3.2.

### 3.2 PROGRAM CONFIGURATION

Using Fortran based on the Sigma 9 system this program has been written in a structured format. The main program consists of subroutine calls, external program calls, data file creation, and the edit system. Each separate algorithm has its own subroutine, making documentation and understanding of the program much easier. Details of each subroutine are given in Section 3.3.

### 3.2.1 FLOW CHART

Figure 3.1 is a flow chart of the main program.

### 3.2.2 INPUT PARAMETERS

Figure 3.2 is a list of all the input parameters, their units and names of the variables within the program.

An initial set of values is required to configure the reflector geometry. In two places the user is given a choice of parameters. The parameters not chosen are then calculated. Specifically, if focal length and offset distance are chosen, offset angle and reflector half-angle will be calculated. All of the inputs are displayed on the screen and the user is given a chance to change any value. If the focal length or the offset distance is changed, the program will recalculate both the offset angle and the reflector half-angle, and vice-versa if an angle is changed. If the user chooses to change the projected diameter, recalculation will occur only on the parameters that were selected at the start of the program. In Figure 3.2, these inputs are listed in Part A. Figure 3.3, shows the location of each distance and angle dealing with the reflector geometry.

The next set of inputs is the latitude and longitude pairs that will define the service area. The program will handle 15 pairs. Section 3.3.1 describes restrictions on the choice of latitude/longitude points. The user is given a chance to change any or all points. The user is asked to specify the number of points he intends to use to define the service area. "This number cannot be changed throughout the rest of the program. After the list of points has been approved, the user is asked to input the longitude coordinate of the satellite (degs. West). It is stationary at $0^{\circ} \mathrm{N}$ 1atitude and at an altitude of 35782 km . See Part B of Figure 3.2 for a description of these inputs.


Figure 3.1a Flow Chart - SAMOD Main Program


Figure 3.1b


Figure 3.1c


Figure 3.1d

Contains: F, FL, X0, D,
XMIN, YMIN, XMAX, YMAX,TYPE,SOFF

| Subroutine COMAL Calculation of the coma lobes at the maximum feed offset | Changes may be made to the focal length jumps to FLNG. |
| :---: | :---: |
| 7 |  |
| Creation of the data file <br> 'INPUTS' | Contains: <br> F, FL, X0, D, XMIN, YMIN, XMAX, YMAX,TYPE, SOFF |



Contains; \# of"horns, centre coordinates, sizes, data mentioned in last three steps of the flow chart
Calculation of vector in E direction for each horn centre

Set phase of each horn to 0.0, make power ratios l-l
Calculation of horn centre normals



Figure 3.1f

## Figure 3.2 Input/Output Parameters

| Variable | Units | Definition |
| :---: | :---: | :---: |
| Part A |  |  |
| F | MHz | frequency |
| WL | cm | wavelength |
| FL | m | focal length |
| $\mathrm{x} \emptyset$ | m | offset distance |
| the $\emptyset$ | degs | offset angle |
| thest | degs | reflector half-angle |
| IFEED | Integer 1-2 | type of feed horn; rectangular, square |
| TYPE | Integer 1-2 | vertical or horizontal polarization |
| RIPLE | cm | RMS surface roughness of the reflector |
| SOFF | Integer 1-4 | satellite antenna feed offset: north, south, east, or west |
| Part B |  |  |
| NUMB | Integer 3-15 | number of LAT/LONG points that will specify the service area |
| $\mathrm{X}(\mathrm{I}), \mathrm{Y}(\mathrm{I})$ | $\begin{aligned} & \text { degs } N \text { (LAT) } \\ & \text { degs } W \text { (LONG) } \end{aligned}$ | lat/long pairs |
| SATLNG | degs W | longitude coordinate of the satellite |

### 3.2.3 DATA FILES

In the main program there are two data files created. One is called İNPUTS and consists of; frequency, focal length, offset distance, projected diameter, minimum/maximum values of the contour in azimuth and elevation, type of polarization (horizontal or vertical), and the satellite antenna feed offset. This file can be found on logical unit 500.

The second file is HRNDT. It contains the: number of horns necessary for optimum coverage, the location of each horn (w.r.t. the focal plane), normal vector of each horn (vector pointing from centre of horn towards reflector surface), the direction vector (E field) of each horn, and the power and phase of each horn. This file is stored on logical unit 501.

### 3.3 SUBROUTINES

The flowchart in Figure 3.1 details the workings of the main program. Described in this section are the algorithms of each separate subroutine that is referred to, without detail, in the flow chart. The documentation of the subroutines gives the calling sequence, definitions of the major parameters, and a brief description of the algorithm. More insight into the method of each subroutine may be gained by referring to the subroutine listing at the back of the manual.

### 3.3.0 INPUT SUBROUTINES

The following table lists the name of the input subroutine and the parameter(s) it passes into and out of the main program.

| FREO | (F,WL) | frequency, wavelength |
| :--- | :--- | :--- |
| FLNGT | $(F L)$ | focal length |
| DOFF | (X $\emptyset)$ | offset distance |
| DPROJ | (D) | projected diameter |
| AOFF | (The $\emptyset)$ | offset angle |


| HAREF | (thest) | reflector half-angle |
| :--- | :--- | :--- |
| FEED | (IFEED,HL) | type of feed horn, horn length |
| POLE | (IPOLE,TYPE) | vertical or horizontal, linear |
| SRFCE | (RIPLE) | polarization <br> RMS surface roughness of the <br> SOFFS |
|  | reflector |  |
|  | sOFF) |  |

Figure 3.3 shows the relation between the distances and angles used by the subroutines.


Figure 3.3 Reflector Geometry

### 3.3.1 SUBROUTINE INPUT

Calling Sequence:

$$
(X, Y, N, N U M B, F L A G, S A T L N G, F L A G 2)
$$

$X, Y$ - these arrays of dimension ' $N$ ' will contain the contour points, of the service area, input during this subroutine ( $N=15$ ).
$N O M B$ - number of contour points that the user is going to enter.

SATLNG - this is the longitude coordinate of the geostationary satellite.

At this point in the program, the contour points defining the service area will be asked for. The user inputs the number of points. Here it is suggested he input at least 10 points, for the reason that in latitude and longitude the connecting lines may be straight, but when they are converted to azimuth and elevation the lines become curved and the original shape of the coverage area may be lost. The format of the entered points must be as follows; input the points in clockwise order, with latitude (degs. north) first then longitude (degs. west).

There are a few restrictions on types of polygons which can be entered to define the service area. If an internal angle is greater than $180^{\circ}$ the program may not count the proper number of beamlet centres. This happens when a grid line (of an orthogonal grid) passes through the figure twice, the program is unable to determine which is the inside of the area and which is the outside. The diagrams in Figure 3.4 are intended to clarify this concept.


Figure 3.4 Service Area Definition Restrictions

Two software flags are used in conjunction with the input subroutine. 'FLAG', is set when the contour points have been entered. With 'FLAG' set the points will be recalled and displayed everytime the user reconfigures the satellite. The second flag 'FLAG2' is set in the subroutine LLTAE when the satellite longitude coordinate has been entered. It is also displayed with the points discussed above.

When the data is displayed the user may change any of the input data before going on.

### 3.3.2 SUBROUTINE LLTAE

Calling Sequence:

$$
\text { (X,Y,N,AZ , EL , NUMB , FLAG } 2, \text { SATLNG , BLAT , BLNG) }
$$

AZ, EL - these arrays will contain the LAT/LONG coordinate points, from the $X, Y$ arrays, after transformation into the $x, y$ coordinate system.

This subroutine converts the latitude and longitude points into $x$ and $y$ coordinates, with origin at the boresight. Figure 3.5 shows the setup of the coordinate systems used in this program.

Before calling the subroutine SVUE, the LLTAE subroutine sets up the following parameters:

| SALT | satellite altitude | 35782 km |
| :--- | :--- | :--- |
| SLAT | satellite latitude | $0.0^{\circ}$ North |
| SLNG | satellite longitude | user input (degs W) |
| RS | satellite altitude + earth radius | 42160.16 km |

RVS
vector position of satellite, with origin at the centre of the earth and the satellite lying in the $Z-Y$ axis.

$$
\begin{aligned}
& \overrightarrow{R V S}(1)=0.0 \\
& \operatorname{RVV}(2)=0.0 \\
& \text { RVS (3) }=-\operatorname{RS}
\end{aligned}
$$

BLAT, BLNG the boresight in LAT/LONG coordinates
AZIM, ELEV the boresight in AZ/EL coordinates

The boresight is calculated as the half-point between the extremes of latitude and longitude. The user is given a chance to change this position.

The first call to SVUE transforms the boresight, subsequent calls transform individual point.
$+Z$


### 3.3.3 SUBROUTINE SVUE

Calling Sequence:
(RVS , SLNG , SLAT , RS , A , B , CLT , CLG , PR , SAZ , SEL )

This subroutine does three sets of transformations. The first is to find the range vector from the centre of the earth to the latitude/longitude point (RVE). The next calculation finds the vector between the satellite and the latitude/longitude point ( $\vec{R}$ ). The third transformation is performed in a subroutine called TRANS (RT). This is a general subroutine which transforms a vector from one coordinate system to a rotated coordinate system using matrix multiplication.

The range vector is determined by the following equations using:

A - Iocation of boresight, azimuth
B - Iocation of boresight, elevation

CLT - Boresight latitude (degrees)

CLG - Boresight longitude (degrees)
$P R-6378.16$ radius of earth at equator (km)

SLNG - satellite Longitude coordinate

SLAT - satellite Latitude coordinate
$R E=P R x \sqrt{1-.0016 \sin ^{2}(C L T)}-r a d i u s$ of earth at given latitude.

- because of the oblatness of the earth, a correction factor of

$$
1-.0016 \sin ^{2}(\text { CLT }) \quad \text { has been included. }
$$

RVE (1) $=$ RE $\times \operatorname{SIN}(A)$
$R \vec{V} E(2)=-R E x \cos (A) x \sin (B-\operatorname{SLNG})$
$\operatorname{RVE}(3)=-R E x \cos (A) x \cos (B-S L N G)$

The next vector is found by:
$\overrightarrow{\mathrm{R}}(1)=\mathrm{R} \overrightarrow{\mathrm{V}} \mathrm{E}(1)-\mathrm{R} \vec{V} S(1)$
$\overrightarrow{\mathrm{R}}$ (2) $=$ R $\overrightarrow{\mathrm{V}} E$ (2) -RV S (2)
$\overrightarrow{\mathrm{R}}(3)=\mathrm{R} \vec{V} E(3)-\mathrm{RV} \mathrm{S}$ (3)

## RV̆S was found in subroutine LLTAE

See Figure 3.6 for the coordinate system that is being discussed here.

At this point in the program, the calculated coordinate is checked to see if it is actually visible from the satellite. If not visible, an error code is created and the user is notified but the program does not terminate. When the program asks if the number of horns needs changing, a yes response will return the user to the main program so that the change can be made.

The subroutine TRANS then transforms the satellite vector $\vec{R}$ into antenna coordinates, $\mathrm{R} \vec{T}$. The calling sequence for TRANS is (A, ANG, IAXIS, B) where:

$$
\vec{B}=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{array}\right] \vec{A}(\vec{A}=\vec{R})
$$

TRANS is a rotation by angle $\theta$ (ANG) about the IAXIS resulting in the new coordinate vector $\vec{B}$.

The points, now in the $x, y$ (earth coordinate) system are determined using the components of the new vector $\vec{R} T$ calculated in Subroutine TRANS as $\vec{B}$.

$$
\begin{aligned}
& \mathrm{SAZ}=\tan ^{-1}(\overrightarrow{\mathrm{R} T}(2) / \overrightarrow{\mathrm{R} T}(3)) \\
& \mathrm{SEL}=\tan ^{-1}\left[\overrightarrow{\operatorname{R} T}(1) / \sqrt{\vec{R} T(2)^{2}+\vec{R} T(3)^{2}}\right]
\end{aligned}
$$

These values are returned to the subroutine LLTAE where they become $A Z$ and EL.


Figure 3.6 Coordinate Systems for Transformation

### 3.3.4 SUBROUTINE BEMDF

Calling Sequence:

$$
\text { ( } \mathrm{D}, \mathrm{FL}, \mathrm{BDF} \text { ) }
$$

BDF - beam deviation factor.

In this subroutine the beam deviation factor is calculated.

### 3.3.5 SUBROUTINE BEAMW

Calling Sequence:


XMAX, YMAX - the eastern and northern limits of the service area.

XMIN, YMIN - the western and southern limits of the service area.
$B W \emptyset$ - is the width of the 6 dB beamilet diameter, in degrees.

This subroutine performs two tasks; first it determines the maximum and minimum values of the entered latitude/longitude contour points after they have been converted to azimuth/elevation, then it calculates $B W \emptyset$, the onaxis 6 dB width of a beamlet that will be used for the routines SHIFT and COUNT.

### 3.3.6 SUBROUTINE SHIFT

## Calling Sequence:

$$
(A Z, E L, N, N U M B, K, C F H X, C F H Y, M, B W \emptyset, P 1, P 2, X M I N, X M A X, Y M I N, Y M A X)
$$

K - number of chosen horns inside coverage area

CFHX, CFHY - position of centre of horns in AZ/EL, these are arrays of dimension $M(M=100)$

P1 - shift of horns (right) in degrees

P2 - shift of horns (up) in degrees

The main purpose of this routine is to choose a location for the boresight that minimizes the number of horns. In conjunction with the subroutine 'COUNT' it will shift the contour points around, simulating a shift of the horns on the reflector. After each shift the number of beamlet centres inside or 'just' outside the coverage area are counted and this number is recorded, thus recording the number of horns needed for illumination of the coverage area. The counting routine is documented in the next section.

The initial call to 'COUNT' uses the original azimuth/elevation coordinates. Then the azimuth.coordinates are transformed by adding. 2 of a beamwidth and the 'COUNT' routine is called again. This continues until a whole beamwidth has been covered in the shifts. Once this is completed the minimum count is found and its boresight position established.

The same process is done in the elevation direction from the position of minimun count found above.

The final call to the subroutine 'COUNT' uses the new optimum coordinate points.

The user is told how the horns were shifted, in angular degrees to the right and angular degrees upwards.

Each contour point has now been moved opposite to that of the horns. A display of the newly found coordinates is printed out and the limits of the service area are changed.

The parameters passed back to the main program include the number of beamlets needed tó cover the given area and the location, in azimuth and elevation, of the centre of each beamlet.

### 3.3.7 SUBROUTINE COUNT

Calling Sequence:
( $\mathrm{K}, \mathrm{CFHX}, \mathrm{CFHY}, \mathrm{M}, \mathrm{XX}, \mathrm{YY}, \mathrm{N}, \mathrm{NUMB}, \mathrm{BW}, \mathrm{XMIN}, \mathrm{YMIN}, \mathrm{XMAX}, \mathrm{YMAX}$ )

This routine is used to count the number of beamlets inside a given contour coverage area. The algorithm used is, quite simply, if the point is on the inside of every line, it is inside the coverage area and therefore it becomes the centre of one horn.

The program sets up a grid overlaying the coverage area and each intersection point of the $X, Y$ axis lines are considered centres of horns until eliminated by the checks described later. The grid is rectangular, using the minimum and maximum limits of the contour as borders. The size of each grid square is equal to the calculated size of a 6 dB beamlet.

The decision as to whether a beamlet centre is counted or not is made as follows:

- a distance called 'COUT' is calculated using, the sine and cosine of the angle made with the next contour point, with respect to the positive X axis, and the grid point in question. This actually transforms the line between the contour point and the grid point to the X axis giving a distance of how far it is outside the contour.
- a variable called 'CNST' is now calculated. CNST represents the amount of area of each beamlet that must be inside the service area before the beamlet will be counted. A point is counted if at least $35 \%$ of its area is inside the desired service area. This value was empirically set by evaluating a number of cases. It can be adjusted if the user finds too few horns are being included.
- This calculation is done with every contour point matching every grid point. The beamlet is only counted if it meets the requirements each time the point goes through the loop.

Figure 3.7 shows a few examples of a coverage area and the centres, located by the algorithm, necessary for coverage of the area.

If the contour plot, or the locations of the feed horns, shows that the routine (in the rough estimate) has missed points, the user can add (or delete) horns as desired.


Figure 3.7 Examples of COUNT Routine

### 3.3.8 SUBROUTINE LOFH

Calling Sequence:
( $\mathrm{BDF}, \mathrm{CFHX}, \mathrm{CFHY}, \mathrm{M}, \mathrm{BW} \emptyset, \mathrm{K}, \mathrm{FPX}, \mathrm{FPY}, \mathrm{FPZ}, \mathrm{FL}, \mathrm{THE} \emptyset, \mathrm{X} \emptyset, \mathrm{SOFF}$ )

FPX,FPY, FPZ - these arrays will hold the coordinates of the feed horns as determined by this subroutine. These are dimensioned to 100 .

This subroutine accepts the coordinates of the centres of each feed horn (found in the arrays CFHX,CFHY) and transforms them to the focal plane coordinate system. There are four cases of coordinate transformations depending on which satellite antenna feed offset (SOFF) was chosen at the beginning of the program. Each case is also dependent on the polarization type, vertical or horizontal. The formulas used can be found in Appendix A.

### 3.3.9 SUBROUTINE HFDIM

Calling Sequence:
(IFEED , FL , D, HL, WL , HHA , HWA , FPX , FPY , M, K, TYPE , BW $\emptyset$ )

HHA, HWA - are the $H$ and E plane dimensions of each individual horn. These are dimensioned to 100 .

Initially the subroutine finds the optimum size of a horn, dependant on the type of feed. For rectangular feed, vertical polarization dimensions are calculated, then if horizontal was picked, the dimensions are reversed.

The maximum actual size of each horn is then determined by comparing the distance between each centre to the optimum size of horn. By comparing the distance of a line between the two horn centres and the optimum widths (and heights), it can be ascertained if the horns overlap. If an overlap is
detected, the principle horn is shrunk (in height or width as necessary) from the optimum size. The program then loops back to see if the overlap has been cleared up. Each horn is looked at with respect to every other horn.

The subroutine passes back to the main program the height and width of each horn.

### 3.3.10 SUBROUTINES USED FOR ROUGH ESTIMATES

The following table lists the subroutines, with their parameters and a short definition of their name, used to give the user a rough estimate of many of the gain and coma lobe values. The formulas used are listed in Appendix A.

CPHF (WL, HW, IFEED, N)

- cross-polarization due to the feed horns

CPR (THEST,THE $\emptyset, N)$

- cross-polarization due to the offset reflector

CPST (RIPLE, WL, THEST, N)

- cross-polarization due to surface roughness of the reflector

GAINL (FL, D, X $\varnothing, N, A Z, E L, 15, A Z I M, E L E V, N U M B)$

- the estimated gain loss for maximum feed offset

GLST (FL, D,RIPLE,WL, $\dot{N}$ )

- gain loss due to surface roughness of the reflector

```
EFFF (THEØ,THEST,N)
    - estimated efficiency factor of polarization
```

COMAL (FL, D, BWØ, AZ, EL, $15, A Z I M, E L E V, N U M B, N, X \emptyset)$
- coma lobe level for maximum feed offset.

Each routine allows the user to improve the calculated value. Each variable that effects the calculation is indicated and the user may select one to change. The program then jumps back to the appropriate subroutine and asks the user to enter a new value. After the new value is entered the scenario is printed out. The program proceeds to print the service area points and then recalculates the beamwidth etc. based on the new value. The rough estimates are then re-displayed. An entry of ' NO ' will cause the program to proceed with the next estimate.

### 3.4 FAR-FIELD ANALYSIS PROGRAMS

This is a set of 4 programs (see Figure 3.8) in a validated software package used for analysis of parabolic reflector antennas. Each program has been designed to handle a specified aspect of a horn-fed reflector antenna configuration.

### 3.4.1 HORN CONFIGURATION PROGRAM (HORNCO)

The function of this program is shown in the $I-P-0$ diagram in Figure 3.9.

The program divides each horn into a number of elements and assumes uniform $\mathrm{E}-\mathrm{pl}$ ane and cosine $\mathrm{H}-\mathrm{pl}$ ane aperture distributions to estimate the radiation pattern of each horn. The program output is a data file called HORN1, which contains all the horn element locations, orientations, powers, and phases. The data file is stored on logical unit 502.

### 3.4.2 REFLECTOR CONFIGURATION PROGRAM (REFLCO)

The function of this program is shown in the I-P-O diagram in Figure 3.10. The output data is found in the two files DISH1 and DISH2, which are stored in logical units 503 and 504 , respectively.

### 3.4.3 REFLECTOR SURFACE CURRENTS PROGRAM (REFSC)

The function of this program is shown in the $\mathrm{I}-\mathrm{P}-\mathrm{O}$ diagram in Figure 3.11.

The program takes the previously prepared horn and reflector data files and computes the $X, Y$ and $Z$ components of current at each point on the reflector surface. This process could take anywhere from 5 to 10 minutes of computer time, depending on the number of horns. The surface current is corrected for inverse square law distance effects and path-length phase effects, as well as projected area effects.

The program outputs a data file of complex surface current values, each with $X, Y$ and $Z$ components.

### 3.4.4 FAR FIELD ANTENNA PATTERN PROGRAM (PATTERN)

The function of this program is shown in the $I-P-O$ diagram form in Figure 3.12.

The program takes the data file of reflector surface currents and computes the flux density of the radiation at any defined point (nominally in the far field of the antenna) by numerical integration of the contribution from the reflector elements. The integration is corrected for inverse square law, path length/phase, and projected areas effect for each individual reflector element. The electric field intensity at the far field point is computed in terms of components parallel to the antenna system $X$ and $Y$ axis and both have amplitude and phase.

The $X$ and $Y$ far field amplitudes are compared with the amplitude which would have been obtained by isotropic radiation of all the power fed into the feed horns. The ratio gives the gain relative to isotropic for each polarization.

The numerical integration is repeated over a range of azimuth and elevation values in the far field, thus providing a "matrix" of gain values for both polarizations. These values are stored on a file which may be used for a contour plotting program.
$\angle S$



FIGURE 3.4 CAL Antenna Analysis Program Package
58

HORN
POWERS AND
PHASE
P,PHS
FIGURE 3.9 I-P-O DIAGRAM FOR HORNCO
OUTPUT




### 3.5 DESCRIPTION OF FINAL OUTPUT FILES

The final output of the program is two arrays containing the gains that have been calculated using the information determined in the initial steps of the program. One array will contain the gain points of the crosspole pattern and the other will be the co-pole. Which is which depends on the user's choice of polarization (vertical or horizontal), the list below defines the 4 cases.

```
vertical }->\mathrm{ cross-pole }->\mathrm{ lst array
vertical }->\mathrm{ co-pole }->\mathrm{ 2nd·array
horizontal }->\mathrm{ cross-pole }->\mathrm{ 2nd array
horizontal }->\mathrm{ co-pole }->\mathrm{ 1st array
```

The diagram below shows the user how to interpret the printout of the gain matrix.

Example:


### 4.0 CONCLUSIONS AND RECOMMENDATIONS

### 4.1 CONCLUSIONS

A computer program has been written and installed on the main-frame SIGMA 9 computer which enables a user to specify the geometric properties of an offset focal point feed parabolic reflector and the contour of the required service area. The program will then calculate an initial estimate of the location and size of the feed horn array necessary to illuminate this service area. The program then returns with prompts to the user so that he may change any of the parameters of the feed array. In this way the patterns may be refined and further optimized with facility.

The analytic basis for the antenna feed configuration and performance estimates is presented as background to the description of the software. A test case and Fortran listing of the programs complete the report.

### 4.2 RECOMMENDATIONS

The following features would increase the capabilities and facilitate the use of the program:

- circular polarization

By running the program twice with orthogonal polarization and a $90^{\circ}$ phase shift between the two polarizations, far field patterns for circular polarization may be produced. The user would be given the choice of polarization at the start of the . program.

- service area definition

At the present time, convex polygons can be handled by the shift and count routine. For re-entrant polygons, the coverage could be broken into constituent convex figures.

- gain profile along arbitrary line

The contour plot of gain as a function of azimuth and elevation can be enhanced by a user-selected routine to choose a "cut line". The gain along this line would be calculated. This would be useful for adjacent beam interference analysis.

- horn location plotting

It would be very useful to have the program produce plots of the resulting horn configurations. The plot would be a plan view of the focal plane, upon which the outline of each horn aperture is drawn. A table of amplitude and phase for each of the horns would be given on the plot as well.

- circular horns

It would be an enhancement of the program to be able to choose circular aperture horns.
array processor for integration

The program presently runs on a mini-computer as well as the SIGMA 9. It is possible to use an array processor with the mini computer in order to speed the calculations. Improvements of factors of 3 to 10 have been achieved in other work using this approach. If a factor of ten improvement is achieved the contour plots would be produced in almost real-time. The iterative design of the feed assembly would then be implimented on a graphics terminal with light-pen or cursor inputs. The designer would quickly develop a heuristic feel for good or poor feed designs.

## REFERENCES

1) J. Ruze, "Lateral Feed Displacement in a Paraboloid", IEEE Trans A \& P, September, 1965
2) Y. Lo, "On the Beam Deviation Factor of a Parabolic Reflector", IEEE Trans A \& P, May, 1960
3) R. Mittra, " Efficient Computation of Radiation Patterns and Contour Beam Synthesis Using Reflector Antennas", University of Southern California, Continuing Engineering Education Technical Report, June, 1979
4) W. Truman, C. Balanis, "Optimum Design of Horn Feeds for Reflector Antennas", IEEE Trans A \& P, July, 1974
5) D. Herbison-Evans, "Optimum Paraboloid Aerial and Feed Design", Proc IEE, January, 1968
6) A. Rudge and N. Adatra, "Offset Parabolic Reflector Antennas: A Review", Proc IEEE, December, 1978
7) T.S. Chu, R.H. Turrin, "Deplorization Properties of Offset Reflector Antennas", IEEE Trans A \& P, May' 1973
8) S. Ghobrial, "Cross Polarization in Satellite and Earth Station Antennas", Proc IEEE, March, 1977
9) J. Ruze, "Antenna Tolerance Theory-A Review", Proc IEEE, April, 1966
10) J. Dijk; et al "The Polarization Losses of Offset Paraboloid Antennas", IEEE Trans A \& $\bar{P}$, July, 1974

## APPENDIX A - REFLECTOR ANTENNA EQUATIONS

## 1. GEOMETRIC PARAMETERS

The geometric parameters of an offset parabolic reflector are shown in Figure A.1 and are related by the following analytic equations:

$$
\begin{aligned}
& \theta_{*}(\operatorname{deg})=\cot ^{-1}\left[2\left(\frac{F}{D}\right)+\frac{\left(\frac{X_{0}}{D}\right)}{2\left(\frac{F}{X_{0}}\right)}-\frac{1}{8\left(\frac{F}{D}\right)}\right] \\
& \theta_{0}(\operatorname{deg})=\cot ^{-1}\left[\frac{F}{X_{0}}-\frac{1}{4\left(\frac{F}{X_{0}}\right)}+\frac{1}{16\left(\frac{F}{D}\right)\left(\frac{X_{0}}{D}\right)}\right] \\
& F=\left[\frac{\cos \theta_{*}+\cos \theta_{0}}{4 \sin \theta_{*}}\right] D \\
& X_{0}=\left[\frac{\sin \theta_{0}}{2 \sin \theta_{*}}\right] D \\
& P=\frac{2 F}{1+\cos \theta_{0}} .
\end{aligned}
$$

2. FREQUENCY - FAVELENGTH

$$
f(\mathrm{~Hz})=\frac{3 \times 10^{10}}{\lambda(\mathrm{~cm})}
$$

3. 6dB BEAMWIDTHS
$B W(\operatorname{deg})=2 \sin ^{-1}\left[\frac{.76 \lambda}{D}\right]$


Figure A. 1 Reflector Geometry

## 4. BEAM DEVIATION FACTOR

The bean deviation factor is given by the following equation, where the angles are shown in Figures A. 2 through A. 5 for North, South, East and West antenna-satellite offsets.

$$
\mathrm{BDF}=\frac{1+\left(\frac{1}{3}\right)\left(\frac{\mathrm{D}}{4 \mathrm{~F}}\right)^{2}}{1+\left(\frac{\mathrm{D}}{4 \mathrm{~F}}\right)^{2}}
$$

where $B D F=\frac{\sin (E L)}{\tan \left(\theta_{1}\right)}$, for an offset in the plane of symmetry (North or South Offset).
$\frac{\sin (A z)}{\tan \left(\emptyset_{1}\right)}$, for an offset in the plane of a symmetry (East or West Offset).

## 5. OPTIMUM FEED PLANE PASSING THROUGH THE FOCAL POINT

The angle $\zeta$ of the optimum feed plane with respect to the $y$ axis in Figure A. 2 is given by:

$$
\zeta=\tan ^{-1}\left(\frac{X}{F}\right)
$$

6. FEED HORN CENTRES ON THE OPTIMUM FEED PLANE

The $x$ y $z$ coordinates are dependent on which antenna-satellite offset is chosen. Again refering to Figures A. 2 through A. 5.

$\Gamma_{1}^{1} F+\frac{P \sin (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}$


Figure A. 2 Antenna Geometry for North Antenna-Satellite Offset

(a) Elevation Plane

(b) Azimuth Plane

Figure A. 3 Antenna Geometry for South Antenna-Satellite Offset

(a) Elevation Plane


Figure A. 4 Antenna Geometry for East Antenna-Satellite Offset

(a) Elevation Plane

(b) Azimuth Plane

Figure A. 5 Antenna Geometry for West Antenna-Satellite Offset

$$
\begin{aligned}
& x=-\left[P \cos \theta_{0}+\frac{P \sin (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}\right] \tan \left(\emptyset_{1}\right) \\
& y=-\frac{P \cos (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)} \\
& z=\quad F+\frac{P \sin (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}
\end{aligned}
$$

South

$$
\begin{aligned}
& x=-\left[P \cos \theta_{0}-\frac{P \sin (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}\right] \tan \left(\emptyset_{1}\right) \\
& y=-\frac{P \cos (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)} \\
& z=\quad F-\frac{P \sin (\zeta) \sin \left(\theta_{1}\right)}{\sin \left(90+\zeta-\theta_{1}-\theta_{0}\right)}
\end{aligned}
$$

East

$$
\begin{aligned}
& x=-\frac{P \cos (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\emptyset_{1}-\theta_{0}\right)} \\
& y=-P \cos \theta_{0}-\left[\frac{P \sin (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\emptyset_{1}-\theta_{0}\right)}\right] \tan \left(\theta_{1}\right) \\
& z=\quad F-\frac{P \sin (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\emptyset_{1}-\theta_{0}\right)}
\end{aligned}
$$

## West

$$
\begin{aligned}
& x=-\frac{P \cos (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\emptyset_{1}-\theta_{0}\right)} \\
& y=-\left[P \cos \theta_{0}+\frac{P \sin (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\phi_{1}-\theta_{0}\right)}\right] \tan \left(\theta_{1}\right) \\
& z=\quad F+\frac{P \sin (\zeta) \sin \left(\emptyset_{1}\right)}{\sin \left(90+\zeta-\emptyset_{1}-\theta_{0}\right)}
\end{aligned}
$$

## 7. OPTIMUM HORN DIMENSIONS

For a rectangular horn feed the optimum horn width and horn height are given by:
$H W(\mathrm{~cm})=-\left[\begin{array}{ll}\lambda\left[.34+2.08 \frac{F}{D}\right] & ; .25<\frac{F}{D} \leqslant .2+.2(\mathrm{HL}) \\ \lambda\left[.34+2.08 \frac{F}{D}\right]\left[1.06-.299 \frac{F}{D}+.0598 \mathrm{HL}\right] ; & .2+.2(\mathrm{HL})<\frac{F}{D} \leqslant 1.25\end{array}\right.$

where $H L$ is the horn length. If the horn length is not specified then the first equation for either horn width or horn height is used.

For a square horn feed, the optimum width is given by:

$$
\operatorname{HW}(\mathrm{cm})=\lambda\left[.08+1.80 \frac{\mathrm{~F}}{\mathrm{D}}\right] \quad ; .25<\frac{\mathrm{F}}{\mathrm{D}} \leqslant 1.5
$$

For a description of how the maximum horn dimensions are found see Section 3 for documentation on Subroutine HFDIM.
8. CROSS POLARIZATION LEVEL ESTIMATES DUE TO HORN FEED

For a square horn feed the cross polarization contribution is:
$\mathrm{CP}(\mathrm{dB})-64.1+61.06 \frac{\lambda}{\mathrm{HW}}-17.5\left(\frac{\lambda}{\mathrm{HW}}\right)^{2} ; 0.5<\frac{\lambda}{\mathrm{HW}}<1.5$
9. CROSS POLARIZATION LEVEL ESTIMATES DUE TO REFLECTOR

$$
\begin{array}{r}
C P(d B)=\left[5.587 \theta_{*}-70.88\right]^{\frac{1}{2}}+\left[7.092 \theta_{0}-72.97\right]^{\frac{1}{2}}-49.22 ; 15^{\circ}<\theta_{*}<45^{\circ} \\
155^{\circ}<\theta_{0}<90^{\circ}
\end{array}
$$

10. CROSS POLARIZATION LEVEL DUE TO SURFACE TOLERANCE

$$
C P(d B)=19.75+19.49 \log \left[\frac{\varepsilon}{\lambda}\right]-178.8\left(\theta_{*}\right)^{-1 / 2} ; 10^{-4}<\frac{\varepsilon}{\lambda} \leqslant 10^{-1}
$$

$$
10^{\circ}<\theta_{*} \leqslant 80^{\circ}
$$

where $\varepsilon$ is the RMS surface roughness.
11. LOSS OF GAIN WITH FEED OFFSET

Assuming a 10 dB taper for the reflector illumination function, the loss of gain is given as follows:

$$
\Delta G(d B)=-1.67 \times 10^{-2} \chi+2.67 \times 10^{-3} \chi^{2}
$$

where

$$
x=\frac{N}{.015+.75\left(\frac{\mathrm{~F}}{\mathrm{D}_{1}}\right)^{2}}
$$

$$
\mathrm{D}_{1}=2 \mathrm{x}_{0}+\mathrm{D}
$$

12. LOSS OF GAIN DUE TO SURFACE TOLERANCE

$$
\Delta G(\mathrm{~dB})=\left(\frac{\varepsilon}{\lambda}\right)^{2}\left[41.58+50.0 \frac{\mathrm{~F}}{\mathrm{D}}-19.28\left(\frac{\mathrm{~F}}{\mathrm{D}}\right)^{2}\right] ; .25<\frac{\mathrm{F}}{\mathrm{D}} \leqslant 1.25
$$

13. EFFICIENCY FACTOR DUE TO DEPOLARIZATION

$$
\begin{aligned}
& N=100-\left[3 \theta_{0}-80\right]\left[5.833 \times 10^{-5} \theta_{*}+10^{-5}\left(\theta_{*}\right)^{2}+4.167 \times 10^{-7}\left(\theta_{*}\right)^{3}\right] \\
& ; 30<\theta_{0} \leqslant 60 \\
& 0<\theta_{*} \leqslant 60
\end{aligned}
$$

14. COMA LOBE LEVEL WITH FEED OFFSET
$C L(d B)=[10.27 x+14.7]^{\frac{1}{2}}-25.83$
where

$$
x=\frac{N}{.015+.75\left(\frac{\mathrm{~F}}{\mathrm{D}_{1}}\right)^{2}}
$$

$$
D_{1}=2 x_{0}+D
$$

APPENDIX B

TEST RUNS

## APPENDIX B

## DEMONSTRATION RUNS OF PROGRAM

## B-1 EDITING FEATURES DEMONSTRATION

Test runs are described in this appendix to show the method of using the program. The first run demonstrates the use of the program during a typical session including editing of the horn data file to change the resulting coverage of the service area. This session was performed on the HP-1000 system at CAL so that the $x-y$ plotter could be used for graphic demonstration of the changes in far-field gain contours.

For this demonstration, a service area was selected which corresponds to the coverage of the Pacific time zone for a typical broadcast service. Figure B.l shows the results of the first estimate. No changes were made to the computed horn configuration. Following Figure B. 1 is a copy of the operator input part of the program.


Figure B. 1

## SAHOD



## 蜍复

## 综掌

## 

FOCAL LENGTH

OFFGE ANGLE
OFFSt D1S1ANCE OK
REFLECTUK HALE ANELE


PROLETES DIAMEIER

THE USER HAS THE CHOTE OF ENERING ETTHER GET OF vARIAGLES

ENTLR FOCAL LENGTH（M） 0.000
？ 3.658

| ENILR OFFSLT DISTANEE（H） | 0，000 |
| :---: | :---: |
| ENTLA PROTECTED DIMMETER 3.558. | 0.000 |

3.558

ENHR TYPE OF FEED 1
i hectangillak
2 SUUAPE
i
THPUI LEMGTH OF HORN（CA） 0,00
10
VERICAL（i）OR HORIZONTAL．（2）i
1
 .13
SAMELLITE OFFSLT：NORTH（1），SOUTH（2），EAST（3），WEST（4）？ 2
3
f hapuency
WAVLENGTH
2 HILAL LENGTH
12445,00
2． 140
3.658

3 UFSE DISTANCE
， 3.3
GHFET ANGLE
herlectur half angle
1.945

20．064
FWECTED DA能TER
3,658
peciangluat
LINEAK
－VEMILCAL
.030
9 kHE GHLACL VARLATION OF REFLECIOR
if EAST SATELLITE ANTENMA OHSET
DO YOU WAN 70 change ANY if THE ABOUE farameters？

i：

Nuhath of Lat:Long coopdnates to be uged
6
infoll in a caickuise qroer, coordinates of the 3dk contour polits



|  |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |

DO YOU MISH 10 CHANGE ANY DATA?

inglin longtrude coordinate of the geostationary gatelite 128

DO YOU HISH TI CHANGE THL LORESIGHI LOCATION?
N
ACMith/Elevation coordinates of uger entered Lat/long



THE NuMER (IF HONN NLEDED AKE: 10
 (Y) N)?

H
Tht b Siliated peak cross polarization level DUE 10 Subact fulemade I5:-51.1 dB
TO KEDUCE CNOSS POLAKIZATION CHODS OHE:
DECREASE SHIAASE VARIATION (SU) DECMLASE WLELECUH HALF ANGLE (KHA) NO CHANGE RLGURLD (NO)
H
THE ESTMAILD LOSS OF GAIN FOR MAXIMUA FLED OFFSET FROM THE FOCUS 151 . 602 dB
TO REDUCE LOSS OF GAIN CHOOSE OHLA
DECKLASE OFFSLT ANGLE (OA)
DECRGAEE RHLLELTOK HALF ANGLE (RHA) NO CHANGE REGU!KED (NO)
$N$
ThE LSGIMATED GAIN LOSE
DUE 10 SURHACL TGLERANCE 15: .06i03 dB
TO REDUE LOSS OF GAIN Choust ONt:
DECREASE SIRFALC VARIATIUN (EU)
INCHEAEE FLCAL LERGTH (FL)
WI CHANEE SLGU1RE) (NO)
N
IHI I Gilhated COMA loge level


INCHASE THE FOGAL LENGTH (HL)
mo Change koulked (no)

N


HORN DIM. (H,E) AND CENTEK $(X, Y, Z)=$

|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | . 0358 | . 137 |  |  |
|  | . 135 | . 1135 | . 001 |  |  |
|  | . 035 | . 1358 | . 081 | -. 005 |  |
|  | 59 | . 035 | . 034 | . 046 | 3,6568 |
|  |  | . 0 | 134 |  |  |
|  | . 0358 |  |  |  |  |
|  | . 0358 | . 1358 | 0702 |  |  |
|  | . 1358. | , 11 | $1 / 02$ |  |  |
|  | . 1334 | . 1354 | . 1060 | 066 | 36542 |

APERADEE NORMAL AL:EL (DCG)=

| 1859000 | 0.0001 |
| :---: | :---: |
| 181.9100 | 0. 10.10 |
| 192.9000 | 0.1000 |
| 181.9000 | 0.1000 |
| 181.9000 | 0.0000 |
| 281.9000 | 0.0000 |
| 181,700 | 0.8000 |
| 181.9000 | 0.0160 |
| 181.9100 | 4.0601 |
| 184,9000 | 0.01071 |
| E FIELD A | $\angle \mathrm{AND}$ EL |
| 0.0000 | 90.0009 |
| 0.0100 | 90.0000 |
| 0.0000 | 90.0000 |
| 0.0800 | 90.11000 |
| 0 ollit | 90.0010 |
| 0,0000 : | 70.1000 |
| 0.0050 | 910000 |
| 0.0000 | on, 060 |
| 0.0050 | 90.0000 |
| 0.0000 | 90.0000 |



| . 1000 | 0.0010 |
| :---: | :---: |
| -1004 | 0.0000 |
| . 1000 | 0.0000 |
| , 2000 | 0.0800 |
| . 1000 | 0.00010 |
| . 1000 | 0.0000 |
| . 1060 | 0.0000 |
| . 1980 | 9, (indil |
| 1041 | 0.0300 |
| 10.10 | 0,8000 |
| IDNH | ST0P mbni |



APERATURE F
/RSD : Siof 00003
II GLIUUTH AND ELEVATION COORDINATES DH THE EARTH


ENTEK AZIMUTH WIHING RANGE FOE FLOT (GTAKT, FINISH) $-1,5,2,5$
ENTER ELLUATION RANGE (START, FINISH)
$-1.5 .2 .5$

THE FROCRAM IS NOL GENERATING THE CONTOLE DATA HATRIX.


Yex Mill GATN $=42.30 \mathrm{~dB}$
APATB : STDF 000DA

WOLD YOU LIKE TO PLOT CROSS-PULE (1) OR CU-POLE (2)?
ᄃ
-1.6 6.6. 5.47.53.711.212.214.117.0.8.15.49.0.7.513.111.114.517.915.711.82.95.311.64.5

$-1.2 \quad-67,96,011,345,116,61 \%, 42,021,326,036,638,637,836,937,939,639,736,527,410,212,511,7$










 $1,21.55 .97 .3 .811 .914 .816 .415 .916 .117 .016 .011 .7 .76 .69 .36 .64 .64 .41 .1-93.2-3.5$ 1.4 $5.6-1,908.36 .64 .45 .810 .9 .26 .08 .87, \% 6.04 .54 .37 .79 .56 .5-7.7-1.6-17,4.9$






Figure B. 2 is the result of a few minor changes made to the horn configuration. The horn centre locations are numbered in the same way as the program counted them. In this part of the run, the relative power of a single horn was changed. Specifically, the power of number 7 horn was increased to a $3: 1$ ratio which altered the look of the copolar plot. Now the high gain contours are found surrounding the centre of the number 7 beamlet. Following Figure B. 2 is a copy of the output generated while making this change.


Figure B. 2
 $i$

| HOPN | COOPUINATS | Hopd | Poued | PHAGE |
| :---: | :---: | :---: | :---: | :---: |
| DIMERSIINS | CENTR |  | RAIII |  |
| H. E | $X \quad Y$ | 2 | \% |  |



DO YOU UTSH TO ADD OR DELETE A HDRR (Y OR N)?
H

IHPIT COLIM IETTER; ROX WUBEE TO EE CHAGED

ENTLR NEE PGUER RATIO FOR horN 7
IUUT COLIMN LETTER, ROU NUMER TO GE CHAMED.
 P.

GTiEK HEL PHASL FQR HORN $=1$ 50
inplif column letter, ron nuher to re changed
 LI

|  | HORN DTME NSTINS | CODRDNATES OF HORN CNHE | POUER <br> RATl0 | PHAGE |
| :---: | :---: | :---: | :---: | :---: |
|  | H E | $X \quad Y \quad Z$ | R | $p$ |
| 1. | .1359 .1358. | .137\% , 03066.3 .659 | 1.3000 | 60.800 |
| 2 | . 0359.0358 | . $07873-0.11533 .6593$ | 1.1000 | 1.0100 |
| 3. | .1059, 1046 | . 0015.05063 .6581 | 1:900 | 0.000 |
| 4 | .1035 .035 | . $0015-.00553 .6581$ | 1.0004 | 0.00015 |
| 5 | .1358. 1358 | -. 11343 , 0664. 3.8568 | 1.010 | 0.0000 |
| 4 | . 0558 .0558 | -, 1343 -1316 3, 6555 | 1.0100 | 0.0010 |
| 7 | , 035 . 1358 | -. $0343-.01533 .4568$ | 3.0000 | 6,060] |
| 8 | . 1358.156 | -.0702 . 06443.655 | 1.0000 | 0.0000 |
| 7 | . 0.558 . 0354 | -.11702 , 015063.655 | 1.0000 | 0.0600 |
| 10 | . 0392.0358 | -, 1060 , 1684 3.6542 | 1.0000 | [1. 1100 |
| 30 | Y\%0 WISH 70 HA | ke changes to lhe hik | CONH | Gubiolion |

Hew CUNFIGURATION
© of Hoxds = if

HORN DTM, (H,E) AND CEHTLR $(X, Y, Z)=$

|  | . 115 | , 4358 | 0373 | 0305 | 3.6593 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 0359 | . 035 | . $03 / 3$ | -. 10 |  |
|  | . 095 | . 1358 | . 0105 | . 0306 |  |
|  | .1159 | . 1358 | . 01015 | - 005 |  |
|  | . 0356 | . 035 | -. 0343 | 064 | 3.6568 |
|  | . 0358 | 1348 | -. 0343 | . 0310 | 3.6588 |
|  | . 0358 | . 055 | - | . 00 | 68 |
| 8 | . 1358 | . 0358 | . 1772 | . 066 | 5555 |
| \% | . 1358 | . 1358 | -1770 | . 0506 | 555 |
|  | 039 | .0559 | . 1060 | 11564 | 3,6542 |

ATERAIURE NORHAL AL, EL (DEB) $=$

| . 9000 | , |
| :---: | :---: |
| 181,9000 | 0.6000 |
| 189.9060 | 0. तापए |
| 181.9000 | 0.0000 |
| 181.9000 | 4, 0ino |
| 181.9070 | 0.0000 |
| 181.9000 | 0.8000 |
| 181,9000 | 0.0900 |
| 181,9000 | 1,0680 |
| 181.9001 | 0.1000 |

EFIELD AL AND EL (DEG)=

> 0.0900 90, 11000
> 0.100080 .1000
> 0.060090 .0060
> $0 ; 0100-90,0100$
> 0.1000090 .19600
> 0.000090 .0000
> 0,$000090 ; 0010$
> 0.010090 .6010
> 0.000090 .0100
> 0.1000 .90 .01000

HORN POLER (WATIS) AND PHBE (DEE $=$


ENTEK AZIMUTH UIEHING RANGE FOR FLOI (STAKT FINISH)
$-4.5,2.5$
ENTER ELEVATION RANGE (START,FINISH)
$-1.5 .2 .5$
THE PROGKAM is nol generating the contour mata matrix.


MiAXIUN GAIN $=45.57 \mathrm{dE}$
/PATB : STOP 00004

2

|  |  |
| :---: | :---: |
|  |  |
|  |  |
|  |  |
|  |  |
|  | . |
|  |  |
|  | $1,29,617,119,534,134,741,339,739,141,543,544,939,634,632,625,414,017,98,416,9-4$, |
|  |  |
|  | 4, 313, 356, 823,335 |
|  |  |
|  |  |
|  |  |
| . 0 | $3.65 .96 .312 .59,514.91 .511 .216 .413 .03 .62,19.49 .01 .711 .513,27.01 .13 .36 .38 .9$ |
| 1.2 | $4.27 .25 .76 .814 .113 .615 .113,16.319 .419 .714 .47 .47 .910 .68 .44 .69 .05 .15-188$ |
| 1.4 |  |
|  |  |
|  |  |
|  | 5.12 .26 .94 .6 , $7.57 .58 .211 .12010 .88 .4-62.86 .8-1-6.0-3.74 .2411 .71 .3$ |
|  |  |
|  |  |
|  | 4.35 .12 .23 .67 .17 .47 .67 .36 .3 3,52.0 1.51.78.08.3 |

Figure B． 3 was produced by making the following changes：

1）The power ratio on horn $⿰ ⿰ 三 丨 ⿰ 丨 三 一$ 7 was decreased to 1．5：1．
2）Horn $⿰ ⿰ 三 丨 ⿰ 丨 三 一$（ was moved up half a beamlet．

NOTE：The figure seen in the contour plot is mirror imaged in the focal plane．To ensure the correct movement，add distance if desired to move a horn to the left or downwards，subtract if desired to move a horn to．the right or upwards．Following figure $B .3$ is a copy of the output generated while making these changes．


Figure B. 3

TO YOU WISH THE HOW COWHGORATON FILE DGPLAVED?


DO YOU WISH TO ADD OR DELETE A HORN YY OR N?
N
IAFUT COLDTN LETER, RON HLHEER TO BE CHANED
 X, 4
ENTLR NEL ' $X$ ' COORDINATE OF HLON F 4
.0015
THPU COLUN LETTER, ROA NHBER TO RE CHANGED
 $Y .4$
ENTER NEL 'Y' COCRDIMATE OF HOREA 4
-. 10 159
IMPUT COLUN LETTER ROM MUMER TO BE CHAMGED
 I.I

| HOPN. DIFERENG |  |  | CORDDMATES © HORH CENIRE |  | $\begin{aligned} & \text { POUER } \\ & \text { RATIO } \\ & R \end{aligned}$ | PHASE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | F |  |
| 1 | .1359 | . 1358 |  |  | . 0373 . 03116 | 3.659 | . 01000 | 60.000 |
| 2 | . 0.65 | - 1358 | . $0373-.105$ | 3.6593 | 1. 1000 | 0.0100 |
| 3 | . 1359 | . 1156 | . 1015.0316 | 3.658i | $\underline{1} .00010$ | 0.0140 |
| 4 | . 035 | . 153 | . 0105 -.115 | 3.6591 | 1.0.00 | 0.0100 |
| 5 | . 10.45 | . 035 | -. 0343 . 0664 | 3.6568 | 1.0010 | (1.0u00 |
| 6 | . 135 | 11358 | -. 1343 -0306 | 3.6568 | 1. 1000 | 0.8000 |
| 7. | 055 | . 135 | -. $1343-11053$ | 3.658 | 3,0006: | 0.1060 |
| 3 | . 1358 | . 0350 | $-1770^{3}$. 1644 | 3.555 | L. 1600 | 1. 01000 |
| , | 11.558 | . 0358 | -. 1772 . 13163 | 3.6555 | 5, 0 (1ip | 0.0190 |
| 10 | . 1392 | . 1358 | - 2060.0664 | 3.6542 | 1. 1000 | 0.800 |
| D0 | प[4 415 | 10 hia | ine chances tu | IHE HOR | N CONH | culation |

OU YOU UTSH TO ADD UR DELETE A HORX (Y OR N) H

MFIM COLINN LETTER, RON NMMER TO HE CYANGED
 8.7

ENTE: NES FOUER RATIO FOR HORN * 7
1.5

INPIT COUMN LETTER, ROU NUMEER TO BE CHANEED

P.


Hotit configuration.

```
* OF H0R4S = 10
```

HORN DIN ( $\mathrm{H}, \mathrm{E}$ ) AND CENTER $(X, Y, Z)=$

|  |  | , 1355 |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | . 1358 |  | -. 00533.6593 |
|  | . 1135 | .1358 | 01015 |  |
|  | .05h? | , 1508 | . 0015 | . 01543.654 |
|  | . 0358 | . 135 | . 1343 | . 06643.6569 |
|  | . 135 | . 135 | 0.1543 | , 0106 |
|  | . 0.359 | . 135 | . 13.34 |  |
|  | . 1335 | . 0355 | . 0702 |  |
|  | . 0358 | . 0.558 | 02 |  |
|  | . 11392 | . 035 | 1060 | , 066 |

APERATLEE NOHMAL AL,EL (DLG)=

| 181.9000 | 1.0000 |
| :---: | :---: |
| 181, 9000 | 0.10 |
| 191.9000 | 0.00 |
| 381.7000 | If. 0100 |
| 101.9000 | 0.0000 |
| 181.9000 | 11,0700 |
| 191.9000 | 9.0000 |
| 181.5000 | 0.0000 |
| 192.9000 | 0.1000 |
| 181.9000 | (1. 0100 |
| E FICLD AZ AND EL |  |
| 10.0100 90,0000 |  |
|  |  |
| 10, 0400 . 0 , 0000. |  |
| 0.0000 | 90.0010 |
| 0.0100990 .5080 |  |
| 5. 300098.3000 |  |
| 0.010 | 90. 1000 |
| 0.010090 .1000 |  |
| 0.0010 | 90. 19 Alia |
| 0090 |  |



| , 1950 | 0.0000 |
| :---: | :---: |
| . 0950 | 0.0000 |
| . 0950 | 0.1000 |
| . 0950 | 0.1000 |
| . 0951 | 0, 1.10 |
| . 0950 | 0.11000 |
| . 1430 | 0.0000 |
| . 8950 | 0, (1010) |
| . 0750 | 0.0600 |
| , 185 | 0.9080 |
| TDHE | ctop 0 dod |


|  | $\frac{60102}{(01)}=-1.225$ |
| :---: | :---: |
| $X$ PUWL (DE) | $=-53,025$ |
| $Y$ Podete (DH) | $=-1020$ |
| 2 POUR (DH) | $=-23.359$ |
|  | $=-38.02586$ |
| -X Yodek(DE) | $=-34.22099$ |
| +Y POER (DG) | $=-39.79372$ |
| -Y Foluer (DB) | $=-31.08165$ |

APESAIURE PULR (DB) $=-15.7299$
RRSCB : 57 (1) 00003
IN GILHUTH AND ELEVATION COORDINATES ON THE CARTH

THE MWIMHE EL 15; -.98 THE MAXIMOM IS: . 46
EMTER AZIMUIH VIEHING RANGE FGS PLOI (START, FINISH)
$-1,5,2,5$
EATER ELLVAIIUN RANGE (START, FTNISH)
$-1.5,2.5$

ThE fROGRAM TS NOL GELERATING THE CQUTOR DATA MATRIX.


WhXIM GAIN $=42.78 \mathrm{~dB}$

PATB : STOP 00004

WOULD YUU LIKE TU PLOT CROSS-PULE (1) OR COPOLE (2)?
2
$-1.6 \quad 27.06 .57 .413 .910 .517 .817 .911 .25 .68 .510 .715,412.616 .017 .414 .812 .14 .45 .512 .45 .5$
$-1,4-5,92,88,02,818,114,810,47,62,328,830,328,455,76,629,350,126,412,610.42,011,9$

















 $2.43 .5,4-4.7 .68 .210 .68 .49 .415 .415 .210 .17 .93 .9-3.0-9.61 .33 .5-2.57 .46 .7-1149$


## APPENDIX B-2

COMPARISON OF RUNS :ON HP-1000 AND SIGMA 9 COMPUTERS

## B-2 COMPARISON OF SIGMA 9 and HP- 1000 GAIN MATRICES

This second example shows a comparis on of runs made on both the SIGMA 9 and the HP-1000 with the same input data. This example was run using an updated algorithm for the constituent beamlet size and therefore calculates 9 horns for the initial design instead of 10 as in the first test case. The purpose of showing this comparison case is to demonstrate that the program as transfered to the SIGMA 9 produces the same far-field gain matrices as the HP-1000 (minicomputer) version.

It may be noted that there are small differences in the horn positions and gain matrices between the two runs. This is because only single precision was used with the HP1000 minicomputer, and some inaccuracy was accepted to facilitate early development of the program on the in-house minicomputer.
!SAMODXEEXEC.

CHOOSE INPUT: FREQUENCY (F) OR WAUELENGTH ..... ( $\left.{ }^{( }\right)$
? ENTER FREQUENCY VALUE (MHz) ..... 000
FOCAL LENGTH OFFSET ANGLE OFFSET DISTANCE OR REFLECTOR HALF ANGLE PROJECTED DIAMETER: PROJECTED DIAMETER
THE USER HAS THE CHOICE OF ENTERING EITHER SET DF VARIAELES ..... i CHOOSE I OR 2? 1
ENTER FOCAL LENGTH(m) ..... 000
?3. 658
ENTER OFFSET DISTANCE(m) ..... 000
?. 132
ENTER PROJECTED DIAMETER000
?3. 658
ENTER TYPE OF FEED ..... 1
1 RECTANGULAR
square
?1
INPUT LENGTH OF HORN(cm) ..... 00
910
VERTICAL (1) OR HORIZONTAL (2) ..... 1
? 1
? 1
enter rits surface variation of reflector (cm) ..... 0000000
?. 03
SATELLITE OFFSET: NORTH(1),SOUTH(2),EAST(3), WEST(4)? ..... 2
? 3

12446.00
2.410
WAVELENGTH

3 OFFSET DISTANCE
OFFSET ANGLE
5 REFLECTOR HALF ANGLE
6 PROJECTED DIAMETER'
TYPE OF FEED TYPE OF POLARIZATION
9 RMS SURFACE UARIATION OF REFLECTOR 10 EAST SATELLITE ANTENNA OFFSET
DO YOU WANT TO CHANGE ANY OF THE ABOVE PARAMETERS?
PLEASE INPUT ACCORDING NUMBER OR "O" FOR NO
Number of lat,long coordinates to be used
96
INPUT TN A CLOCKHTSE ORDER COORDINATES OF THE 3dF CONTOUR POINTS
Latitude in deg north, Longitude in deg west. Lat cannot be > go degs.
COORDINATE POINT $\ddagger$ ..... 1
?75,120
COORDINATE POINT ..... 2
? 54,12

COORDINATE POINT $* 3$
? 49 , 114
COORDINATE POINT $\ddagger 4$ ?49, 125
COORDINATE POINT $\ddagger 5$ ? 60,141
COORDINATE POINT * 6 ?70,141

| 1 | 75.0000, | 120.0000 |
| :--- | :--- | :--- |
| 2 | 54.0000, | 120.0000 |
| 3 | 49.0000, | 114.0000 |
| 4 | 49.0000, | 125.0000 |
| 5 | 60.0000, | 141.0000 |
| 6 | 70.0000, | 141.0000 |

DO YOU WISH TO CHANGE ANY DATA?
INPUT $\ddagger$ OF LINE YOU WISH CHANGED OR "O" FOR NO ? 0 INPUT LONGITUDE COORDINATE OF THE GEOSTATIONARY SATELLITE ? 126

BORESIGHT LATITUDE $=62.000$ LONGITUDE $=127.500$
DO Y YOU WISH TO CHANGE THE BORESIGHT LOCATION? ?
aZImuTh/ELEVATION COORDINATES OF USER ENTERED LAT/LONG

| 1 | .28, | .46 |
| ---: | ---: | ---: |
| 2 | .73, | -.54 |
| 3 | 1.47 | -.98 |
| 4 | -.09 | -.96 |
| 5 | -1.00, | -13 |
| 6 | -.73, | .33 |

THE BEAMUIDTH OF A BEAMLET IS: . 59

THE HORNS WERE MOUED .353 TO THE RIGHT, AND .000 UPWARDS.

THE AZ/EL AFTER THE SHIFTING ROUTINE

$$
\begin{aligned}
& .6365, .4606 \\
& .0857,-.5373 \\
& .6249,-.9796 \\
& -.7558,-.1364 \\
& -.3784,3313 \\
& \text { THE NUMEER OF HORNS NEEDED ARE: } \\
& \text { TO DECREASE THE NUMEER CHANGE THE PROJECTED DIAMETER, } \\
& (Y \text { OR N)? } \\
& \text { ?N ESTIMATED PEAK CROSS POLARIZATION LEVEL } \\
& \text { THE ESTI } \\
& \text { DUE TO SURFACE TOLERANCE IS: }
\end{aligned}
$$

TO REDUCE CROSS POLARIZATION CHOOSE ONE;
DECREASE SURFACE VARIATION (SV)
DECREASE REFLECTOR HALF ANGLE (RHA) NO CHANGE REQUIRED (NO)

[^0]TO REDUCE LOSS OF GAIN CHOUSE ONE;
DECREASE OFFSET ANGLE (OA)
DECREASE REFLECTOR HALF ANGLE (RHA)
NO CHANGE REQUIRED (NO)
? N
THE ESTIMATED GAIN LOSS
DUE TO SURFACE TOLERANCE IS: . 01151 dB
to reduce loss of gain choose one;
decrease surface variation (su)
INCREASE FOCAL LENGTH (FL)
no change required (no)
? N
THE ESTIMATED COMA LOBE LEVEL
FOR MAXIMUM FEED OFFSET FROM FOCUS IS: -16.79 dB

TO DECREASE THE COMA LOBE LEUEL, CHOOSE ONE;
INCREASE THE FOCAL LENGTH (FL)
NO CHANGE REQUIRED (NO)
?


```
X POWER (DB) = -53.644
Y POWER(DB) = -.015
Z POWER(DB) = -24.624
+X POWER(DB) = -34.91147
-X POWER(DB) = -27.42418
+Y POWER(DB) = -4i.084i2
-Y POWER(DB) = -31.78119
APERATURE POWER(DB)= -15.6742i
*STOP* 3
    0024 - 1SET F:S00/INPUTSX:IN;SAUE
    0022 - SET F:503/DISH{X;IN;SAUE
    0023 - !SET F:505/RFIFIX;IN;SAUE
    0024 - ISET F:506/PATERNIX;GUT;SAVE
    0025 - !SET F:507/PATERNEX;OUT;SAVE
    0026 - ISET F:G01/Plotdata;jNGUT
    0027 - LLPATERNX
IN AZIMUTH AND ELEUATION COORDINATES ON THE EARTH
THE MINIMLM AZ IS; -.73 THE MAXIMUM IS; 1.82
ENTER AZIMUTH VIEWING RANGE FOR PLUT (START,FINISH)
?-1.5,2.5
?-4.5,2.5
```

THE PROGRAM IS NOW GENERATING THE CONTOUR DATA MATRIX. ABOUT 15 MINUTES COMPLETION TIME, 50 GO HAVE A COFFEE.
*STOP* 3
0021 - 1SET F:500/INPUTSX:IN SAUE
0022 - SET F:503/DISHiX; IN; SAUE
0023 - SET F:505/RFIFIX:INSAUE
0024 - !SET F:506/PATERNIX;OUT; SAUE
0025 - SET F:507/PATERN2X; QUT; SAUE
0026 - SET F: $601 / \mathrm{PLOTDATA}$; INOUT
0027 - ILPATERNX.
IN AZIMUTH AND ELEUATION COORDINATES ON THE EARTH $\begin{array}{lll}\text { THE MINIMUM AZ IS; } & -.73 & \text { THE MAXIHUM IS; } \\ \text { THE MINIMLM EL } \\ \text { IS } & -.88 \\ \text { THE MAXIMUM IS } & .46\end{array}$

ENTER AZIMUTH UIEWING RANGE FOR PLUT (START,FINISH)

?-1.5,2.5

THE PROGRAM IS NOW GENERATING THE CONTOUR DATA HATRIX. ABOUT 15 MINUTES COMPLETION TIME; SO GO HAVE A COFFEE.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 8.82 | 3.38 | 7.47 | 10.87 | 11.51 | 15.53 | 19 | 21.30 | 20.50 | 16.51 | 9.64 | 6.59 | 6.94 | -. 93 | 7.99 | 7.56 | . 72 | 3.34 | -8.87 | 2.09 |  | 1.64 |
| 7.09 | 10.78 | 11.91 | 10.47 | 5.46 | 6.93 | 13.99 | 17.19 | 17.95 | 16.69 | 13.36 | 9.23 | 10.22 | 9.69 | 1.13 | 7.42 | . 81 | -1.22 | -7.90 | 2.19 | . 96 | 93 |
| 14.13 | 6.56 | 4.88 | 11.69 | 16.44 | 19.38 | 22.91 | 25.13 | 24.23 | 20.20 | 16.56 | 14.88 | 7.13 | 8.03 | 9.01 | 2.24 | 5.52 | -2.52 | 3.67 | . 98 | 3.74 | 80 |
| 4.47 | 15.21 | 16.97 | 12.45 | 9.23 | 14.90 | 4.87 | 17.21 | 18.38 | 3.36 | 6.80 | 11.6 | 16.22 | 5.60 | 12.21 | . 48 | 5.24 | -3.65 | 7.26 | 6.47 | 3.11 | 4.07 |
| 12.27 | 17.99 | 6.91 | 15.44 | 24.99 | 33.28 |  | 35.20 | 34.66 | 4.78 | 32.65 | 24.08 | 12.05 | 10.23 | 9.82 | 4.55 | 10.00 | . 28 | 7.92 | . 60 | 4.6 | 1.38 |
| 11.52 | 8.68 | 22.31 | 24.28 | 26.92 | 37.45 | . 8. | 41.34 | 41.16 | , | 38.27 | 30.79 | -3.23 | 13.94 | 10.80 | 2.79 | 12.19 | 4.95 | 9.24 | 4.84 | 0.86 | -5.09 |
| -14.38 | 3.23 | 18.63 | 17.99 | 29.26 | 37.9 | 40.96 | 41.51 | 41.56 | 41.24 | 3858 | 30.19 | 16.20 | 15.97 | 15.28 | 11.35 | 11.64 | 4.07 | 11.20 | . 90 | 11.58 | 2.21 |
| . 45 | 24.59 | 31.14 | 34.36 | 36.86 | $38: 88$ | 39.67 | 39.75 | 40.28 | 40.38 | 37.5 | 27. | 21.54 | 18.43 | 16.26 | 13.88 | 2.6 | 8.19 | 1.44 | . 04 | 8.4 | 02 |
| 8.57 | 30.47 | 37.82 | 40.56 | 40.47 | 40.34 | 39.68 | 39.91 | 40.98 | 41.25 | 3896 | 29.99 | 17.31 | 17.62 | 14 | 2.18 | 13.53 | 0.9 | 9.7 | . 3 | 3.91 | 2.35 |
| 15.91 | 29.83 | 387 | 41.64 | 41.63 | 40.54 | 40.01 | 40.33 | 40.82 | 40.55 | 38.03 | 30.74 | -4.17 | 15.27 | 11.42 | 12.42 | 11.02 | 9.13 | 32 | 3 | 8.97 | 02 |
| 17.85 | 27.85 | 37.88 | 1. 22 | 40. | 40.21 | 40.01 | 39. | 37.15 | 34.71 | 31.56 | 24.70 | 5.59 | 14.77 | 4.65 | 12.32 | 2.91 | 10.44 | 6.79 | 7.74 | 6.31 | 39 |
| 12.15 | 28.71 | 37.89 | 40.58 | 40.41 | 40.08 | 40.01 | 37. | 29.39 | 13.23 | 16.73 | 12.74 | 6.65 | 13.21 | -6.60 | 9.92 | . 34 | 11.71 | 1.63 | 7.23 | 3.50 | 88 |
| 17.49 | 28.33 | 3.71 | 40.20 | 39.77 | 39.56 | 39.85 | 37.29 | 25.99 | 26.94 | 24.42 | 15.99 | 14.52 | 10.35 | 6.47 | 8.95 | -3.83 | 8.23 | $-4.27$ | 7.16 | 2.32 | 8.21 |
| 19.62 | 27.78 | $3{ }^{3} .96$ | 40.60 | 40.23 | 40.02 | 40.36 | 38.12 | 28.34 | 21.90 | 19.20 | 20.46 | 17.87 | 11.95 | 10.05 | 10.03 | 6.07 | 6.37 | $-1.45$ | 7.16 | 8.78 | 7.34 |
| 16.02 | 29.95 | 3 m 06 | 41.88 | 41.94 | 41.71 | 41.53 | 3915 | 30.87 | 17.47 | 20.50 | 17.88 | 10.30 | 15.38 | 9.36 | 11.24 | 4.25 | 8.29 | 30 | . 4 | 9.98 | 5.05 |
| 5.70 | 29.80 | 38.10 | . 06 | 41.31 | 41.08 | 40.68 | 76. 16 | 30.35 | 12.22 | 15.58 | 11.39 | 14.34 | 14.62 | 1.39 | 11.3 | 12.79 | 10.34 | 3.76 | . | 12 | 11 |
| 9.60 | 22.55 | 32.07 | 34:94 | - | ค | 34.49 | 32.15 | 23.63 | 15.67 | 13.12 | 11.89 | 5.50 | 5.96 | 7.23 | 8.98 | 4.88 | 8.43 | -2.20 | 5.84 | . 46 | 5.61 |
| 19 | 8.35 | 9.57 | 11.06 | 14.01 | 15.01 | 12.58 | 12.68 | 9.47 | 12.39 | 5.24 | 12.82 | 11.29 | 10.00 | 3.18 | 3.38 | 4.61 | . 64 | 6.43 | 4.87 | 4.64 | 5.42 |
| 3.84 | 7.63 | 13.79 | 17.78 | 20.91 | 20.97 | 17.68 | 13.63 | 11.15 | 5.87 | 9.57 | 8.97 | 6.17 | 4.20 | 4.69 | -1.98 | 4.08 | 2.42 | 3.86 | -3.60 | 1.50 | 9.21 |
| 8.24 | 12.16 | 15.02 | 17.02 | 18.30 | 18.85 | 17.81 | 14.34 | 10.01 | 8.92 | 7.85 | 6.84 | 9.41 | 9.58 | 7.65 | -9.61 | 6.67 | 2.17-1 | 2.61 | -5.07 | 2.79 | 65 |
| 3.20 | 6.15 | 11.16 | 15.56 | 17.92 | 17.67 | 14.66 | 8.66 | . 74 | -. 16 | 5.09 | 4.38 | -8.30 | 2.39 | 4.76 | 2.01 | $-5.90$ | 2.00 | 6.11 | 1.83 | 1.29 | 3.35 |
| 4.80 | 10.38 | 14.26 | 15.98 | 16.43 | 16.38 | 5.82 | 14.22 | 1.4 | 7.89 | 4.9 | 5.75 | 9.46 | 9.07 | 3.76 | -7.93 | $-2.01$ | 4.31 | 2. | -10.61 | $-9.02$ |  |

[^1]```
Ginhud
GHHMSA INPIST: FREGHINNCY (H) GK WAVELENGTH (W)
F
EMIEk FRELUJENLY VALUE (mHz) 0.000
```



```
THE USEK HAS THE CHOILE of ENIERING EITHER SET Of VARIABLES 1
ENTEK FUl:AL LENGTH(M) 0.000
3.658
0.000
LENER (IFtSLI DISIANLE(m)
thTEK PRUJECILD DIAMETLR
0.000
3. 6.5
1
WHLK TYPE OF FEED
1 kilitangllat
SHUfR1
1
JHPDI IENGIH GF HORN(cm
0.00
111
1
```



``` . 15
SATELLATE DFFSET: NURTH(1), SOUTH(2), EAST(3), WEST(4)? 2
3
```

1 FKL (JULENC:Y
WAVEIE NGITH
FAVEIRENGIH
. 5 brt SLI DISIANI:
4 Jrt SLL I ANGIIE
$\leftrightarrows$ REFLEECIUK liAIt ANGGE
6 FKUJKLTED MSAMF TEK
7 TYPE OF FEEED
3 TYPE CH POLALAKIZABIUN



id

InHH, IN A GIMERWISE ORDER, CODRDIHATES OF THE 3GE CONTOUR POINTS LATIIIJDL IN DEG NOKIH, LUNGTIUDE IN DEG WEST. LAT CANNGT EE $>$ gu DEG:
cuordinatt polni * 1
75,120
GOCRDINAIt HUNI $\ddagger 2$
54,120

4\%,114
:OURLINAIE PLINT * 4
60.141
:OORDINAIE PGINT * 5
70,141
CGOLRDINAIE RUINT $\# 6$
70,141
75.00.120.0
54.00,120.0

3 49.10, 154.0
4 60.00,141.0
$\begin{array}{ll}5 & 70.00,141.0 \\ 6 & 70.011,141.0\end{array}$
Di) Youl whst 10 CHANGE ANY DATA?

HHPG: UF LINE YHU WSGH CHANLIED GK JU, FOR NO
5
Gutk NtW ruini s 5
60, 141
7 75.00,120.0
$\begin{array}{ll}2 & 54.00,120.0 \\ 3 & 44.00,114.0\end{array}$
48.001214 .0
4
$40.00,141.0$

5 60.0II, 141.0
$6 \% \% .010141 .0$

DO YOU WISH TO CHANLE ANY DATA?

4
ENTEK NLW POHT 44
409,125
$\begin{aligned} & 1 \\ & 2\end{aligned} 75.00,1200.0$
54.06,120.0

3
4
4
$49.00,125.0$
5 40.00,125.0
s $70.011,141.0$

DU YOU WISH TO CHANG: ANY DAIA?
INPM \# OF LINE YILS WISH CHANGED UK 'O' FOR NO
 12 t

```
HORESIGHITIAITTUME = 62.000 LONGITUDE =127.500
HORESLGHIT IAITTHDE = 63.000 LONGITUDE =127.500
N
```

AZIMU'H/E LEVATION COORDINATES OF USER ENTERED LAT/LONG

| 1 | .28, | .46 |
| ---: | ---: | ---: |
| 2 | .73, | -.54 |
| 3 | 1.47, | -.98 |
| 4 | .29, | -.46 |
| 5 | -1.08, | -.13 |
| 6 | -.7 .5, | .35 |

THE MINIMUM ELAMWIDTH GF A BEAMLET IS

THE HURNS WERE MOULD .353 10 THE RIGHI, AND 0.00 UPWARDE.

IHE NIMEEK IF HURNS NEEDED ARE: 9
TG DEC:MEASE 1 HE NUMBLR CHANGE THE HKOIECTED DIAMETER, (Y OR N)?

N
TIIE LSJIMATED FLAK CROSS POLARIZATION LEVEL
DUE IU SLKkACt TULLKANCE $15:-51.1$ d

DECKLASL SURI ALCE VARIAIION (SU
DECREASE KELILLCIUK HALIF ANGLE (RHA)
no diance kt:ulired (NO)
$\stackrel{N}{\mathrm{~N}} \mathrm{H} \cdot \mathrm{HE}$
THE ESSIMATED I OSS OF GAIN FOR MAXIMUM FEED DFFSET FROM IHE FUEUS 15: .005 dE

TU ke mult IUSS LH GAIN CHOUSE CNAE;
ULDKEALSE UFFSET ANGIE (GA)
OLCNEASLL KEHEGGGK HALF ANGLE (RHA)
HO CHANGE HEGITRI:D (NO)

N
EGIMAILD GAIN Loss


IO regolict loss of GAIN CHOCSE ONE;

INCKLASL IUCAL LEN(IIH (FL)
No ClANCE: REMUIRED (NU)

N
FOR MAXIMUM fEED LHFSLET fKOM fulus 16: -16.6 db
TO De (:rease the cuma l.ubt level, choust onts
INCKLASS THE FOCAL LEN(ITH (FL)
NO CHANGE HEUUIRED (NG)
$N$


HORN CUNFIGURATIUN

- Dr HOKNS $=4$

HORN DIM, $(H, E)$ AND CEN)ER $(X, Y, Z)=$

| 1 | . 0392 | .0399 | .0332 | . 0265 | 3.6595 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | .1392 | . 0394 | .0332 | -.0135 | 3.6592 |
| 3 | . 1.359 | . 1359 | -. 11067 | 0654 | 3.65\%8 |
| 4. | . 13342 | . 0399 | -.0067 | . 0265 | 3.6578 |
| 5 | . 1389 | . 03998 | -. 01167 | -. 0135 | 3,65\% |
| 6 | . 10392 | . 0.399 | -. 0466 | . 0664 | 3.6563 |
| 7 | . 0.392 | . 0399 | -. 04466 | . 0266 | 3.6563 |
| 0 | . $11.34{ }^{2}$ | . 1399 | -. 01866 | . 0664 | 3.6549 |
| 9 | . 11398 | . 13399 | -. 0866 | . 0265 | 3.6549 |

AFERA(URE NORMAL AZ,EL. (DEEG)=

| 181.9010 | 0.0000 |
| :---: | :---: |
| 181.9000 | 0.0040 |
| 181.9100 | 0.0000 |
| 181.9000 | 0.01000 |
| 181.91000 | 0.0000 |
| 1191.9000 | 0.0000 |
| 161.91100 | 0.0000 |
| 184.9000 | 0.0000 |
| 161.9000 | 0.01110 |

[^2]

```
        0.0000900.0000
        0.0000 90.0000
        0.0000 90.00000
        0.0000 90.0000
HURN PUWER(WATTS) AND PHASE (DE:G)=
\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|l|}{. 11100.0000} \\
\hline \multicolumn{3}{|l|}{.11100 .0000} \\
\hline . 1110 & \multicolumn{2}{|l|}{0.00000} \\
\hline , 1110 & \multicolumn{2}{|l|}{0.0000} \\
\hline .1110 & \multicolumn{2}{|l|}{0.01100} \\
\hline . 111.0 & \multicolumn{2}{|l|}{0.0000} \\
\hline .1114 & \multicolumn{2}{|l|}{0.0000} \\
\hline . 1110 & \multicolumn{2}{|l|}{0.0000} \\
\hline .1110 & \multicolumn{2}{|l|}{0.0000} \\
\hline /DNHM & STOP 00001 & 010001 \\
\hline \multicolumn{3}{|l|}{3} \\
\hline /REFG : & \multicolumn{2}{|l|}{STOP 0000e} \\
\hline gP fliluvek & \(\operatorname{Loss}(\mathrm{DH})=\) & \((\mathrm{DH})=-1.006\) \\
\hline \(X\) POWER(I) & & \(=-53.757\) \\
\hline Y Powtr \({ }^{\text {chem }}\) & (b) \(\quad=\) & \(=-.115\) \\
\hline 2 POWER (D) & & \(=-24.615\) \\
\hline + \(\times\) PUWER & DE) = & \(=-35.69421\) \\
\hline --x rowtr (did & DB) = & \(=-26.33 \% 88\) \\
\hline +Y POWERK( & DE) = & \(=-40.09579\) \\
\hline - Y Power (id & (18) \(=\) & \(=-31.66233\) \\
\hline
\end{tabular}
APE:KAIURE POOWER(DH)= -15.5%%24
KSCH: STGP 000003
IN nKIMIIH AND EL.EUATION CDORDINAIES IDN THE EARTH
THE MINIMIMM AZ 15; -1,0B THE. MAXIMIJM 15; 1.47
THE MINIMUM EL. 1S; -.98 THE MAXIMUM IS; .96
ENTEK AZIMUTH UIEWING KANGE FOR PlOI (START,FINISH)
-1.5,2.5
ENTER ELLUATIGN RANGE {SIAKT,FINISH)
-1.5,2.5
THE PROGRAM IG NDW GENLRATING THE CONTOUR DATA MATRIX.
ABOUI }15\mathrm{ MINIJES CUMPLEIION TIME,SO GO HAVE A CURHEE.
MAXIHUM GAIN= 41.74 #E
    /PATB : GTOP 00004
S=1(4 COMMAND) ?OF
GAMOD AHOKTED
    ABEND SAMGD AEOKTED
:
```

|  | 4.98 | 5.73 | 9.81 | 11.51 | 15.97 | 19.82 | 21.69 | 21.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 11.58 | 13.53 | 12 | 10 | 11.09 | 15.93 |  | 18 |  | 13.71 | 9. | 11 | 40 |  |  |  |  |  |  |  |  |
| 4. | 6.14 |  |  | 16.83 | 19 |  |  |  |  |  |  |  | 8.7 |  |  |  |  |  |  |  |  |
|  | 16. |  |  |  |  |  |  | 18 |  | 10 |  | 16 | 6. |  |  |  |  |  |  |  |  |
| 2.6 | 17. |  | 13 |  | 33 |  |  |  |  | 33 | 24. | 12. | 11 | 10 | 5.90 | 9.87 | -2 |  |  |  |  |
| 11.16 | 10 | 23 | 24.58 |  |  |  |  |  |  | 29.71 |  |  | 14. | 10. | 3. | 11. |  |  |  | 0.80 |  |
| -5.31 | 1.73 |  |  |  |  |  |  |  |  |  | 30. | 15. | 16 | 15. | 11.7 | 15.20 | 3.7 | 11.37 | -1.3 |  |  |
|  | 26 |  |  |  |  | 39 | 39.0 | 39 | 39 |  | 27 | 21. | 18. | 16. | 14. | 12.42 | 7.8 | 11.75 | 6.5 | 7.95 |  |
| 10.14 | 31 |  |  | 40.75 | 40.10 | 39 | 39 | 40 |  |  | 30 | 16 | 17. | 14. | 11. | 13.8 | 10.7 | 9.91 | 6. 2 | . 81 |  |
| 13.73 |  |  |  | 41.21 | 40.16 | 39 |  |  |  |  | 31 | -2 | 15. | 11. | 12. | 11.11 | 9.5 |  |  | 9.71 |  |
| 16 | 29 |  |  | 40.36 | 39.73 | 39.70 |  |  |  | 31 | 25 | 7.9 | 15 |  |  |  | 9. | . 9 |  | 7.75 |  |
|  |  |  |  |  |  |  |  |  | 15.5 |  |  | 8.7 | 13 | 7. | 10. | 3. | 11.76 | 2.6 | 6.90 | . 58 |  |
|  |  |  |  |  |  |  |  |  | 26.8 | 23.78 |  | 13 | 10.2 | 3 |  | -5. | 8.74 | -4. | 6.63 | . 57 |  |
|  | 29 | 3¢ |  | 39.5 |  |  |  | 27.5 | 21.7 | 17.60 | 20.89 |  |  |  | 10 |  | 5.82 | - 1.58 |  | 2.01 |  |
| 4 | 31 |  | 41.78 |  | 41.23 |  | 38. |  | 18.97 | 20 | 17 |  | 15 | B. | 11 |  |  |  |  |  |  |
| , | 31.65 | 39 | 32 |  |  | 40 | 2563 | 30.25 | 14. | 15.9 | 12.01 | 14 |  |  |  |  |  |  |  |  |  |
| 8.15 | 26.01 | 34 | 36.38 | 36 |  |  | 33 | 25.00 | 15.35 | 13.019 | 12.57 | 6.08 | 5. | 31 | 8.48 |  |  |  |  |  |  |
|  | $9 \%$ | 16.91 |  | 15.06 | 15.24 | 73 | 1\%.\%2 | 7.53 | 12.25 | 5.32 | 13.04 | 11.36 | 9.9 | 3.42 | 3.37 |  |  |  |  |  |  |
| 25 | 10.80 | 15.8\% | 19.72 | 22.58 | 22. 43 | 19.16 | 15.53 | 13.29 | 6.64 | 8.83 | 8.62 | 6.74 | 5.63 | 4.73 | -3.35 |  | 2.60 |  | 2 |  |  |
| 71 | 11.93 | 14.75 | 16.55 | 17.68 | 18.29 | 17. 25 | 13.64 | 9.55 | . | 8.95 | 7.26 | 10.1 | 10.14 | . 68 | 0.23 | . 16 | 2.80- | -19.78 | 4.18 | . 82 |  |
| 3.6 | 8.45 | 13.4 | 17.63 | 19.78 |  | 16.71 |  | 04 | 61 | 4.61 |  | -6. 8 | 1.86 | 4.58 | 1.55 | 4.5 | 15.96 | 5.84 | -2.33 | 80 |  |
| , | 0.9 | 14 | 15.80 | 16 | 15 | 15.22 | 13.64 | 11.13 | 8.07 | 5.80 | b. 1 |  | 9.1 | 3.98 | . 62 |  | 4.28 |  |  |  |  |

B-2 HP1000 DEMO RUN GAIN MATRIX

APPENDIX C

PROGRAM LISTITNGS


> DETLAATIOS
$\star \star \star \star \star \star \star \star * \star * *$

Tre fiest set of VABIGRLES ARE USER INPUTTED
RESL FM, WL

F - frecuency (ht)
nl - wavelengrh (cm)

## BLAL Fl, O, Xb

$F L$ - focal lengtn
D - proiected diameter
$x$. - offset distance

## GEI THEU,THEST

THEO - offset angle
ThEST - reflector half angle
IUTEGCA IFEED
$C$
$c$
$c$
$c$
$c$
$c$
$C$

IFEE - type of feed horn
T:TEGEA IPGE, TYPE
IPOLE - type of polarization
TYFE - vertical or norizontal polarization
if IFOLE is retargular
LSL MPLE
$C$
0
RIPLE - RWS Surface varistion of reflector




```
    TAE!=P)/2.-4TAE((FL/X0)-(1./(4.*FL/X0))+(1./(16.*(FL_**0/D**2))))
    TME|=THEN*FAD
    %1% 10 24
    14 CHi hOFF(THEO)
    0:1L UGOEF(YHEST)
    CAlu pfoma(D)
    FL=((COS(THEST/PGO)+COS(THEG/RAE))/(4.*STH(THEST/RAD))})*
    %=(SIH(THEO/KBD)/(Z.*SIM(THEST/RAD)))*D
    2L r:u, FEED (IFEED,HL)
    CH.L POLE (IPOLE,TYPE)
    C:LL SRFCE (RIPLE)
    COLL SOFFS (SOFF)
    22 :1,1%(5,B00,4) E,L
        {PITE(KBOD,5)FL,XO,THE0,THEST,D
        IF(TFEED.EG.1) PTTE(KEDO.15)
        IF(TFFED.E(.2) WGITE(KB00,16)
        \TE(кb!0.18)
        IF(IYPF.EQ.1) MRITE(KBDO,25)
    IF(TYPE.EQ.2) GRITE(KBOQ,23)
    HTTE(ABOO,9) RIPIF
    TF(SGFF.EQ.1) WHITE(KBDQ,1)
    IF(SOFF.RQ.2) N,GTE(KBOQ,21)
    TF(SOFF.EQ,3) प@TTE(KANO,932)
    IF(SOFF.EQ.4) wRTTE(KENO;933)
    12 MTE (4,00,11)
    f40(r,0|I,776) IA#S
    776 FOfmat(I)
            IF(IA#S.EQ.0) GO TO 100
            G0 T0 (10,20,30,40,50,60,70,80,90,91) IANS
c
C THFSG rOWABMS ARE FOR CHAMGTHG THE USERS INPUTS
C
    10 GNL FREQ (F.ML)
    20 call FligT(FL)
    THEST=PI/Z.-ATAN((Z.*FL/D) +((X0/D)/(2.*FL/X0))-(1./(B.*FL/D)))
    THST=THEST*RAD
```



```
    THF:=THEG*QAD
    30 COLL DOFF(X0)
    TH=ST=PT/2.-ATA4((2.*FL/0)+((X0/D)/(2,*FL/X0))-(1./(8.*FL/D)))
    THEST=THEST*PAD
```



```
    TH:G=THFO*RAD
```

```
        #!)}2
    40 C,AL \thereforeOFF(THEO)
    fL=((EOS(THEST/QAD)+COS(THEO/GAG))/(4.*SIM(THEST/RAD)))*O
    * = (SIM(THEO/GAO)/(2.*SIn(THEST/RAD)))*U
    B1]r.22
    50 &:口L. d&口RFF(THEST)
    FH=((CMSTIHEST/FAD)+COS(THEO/FAO)]/(4.*STH(THEST/RAD)))*D
    y:=(STH(THEO/HAD)/(2.*SIN(THEST/RAD)) **O
    i! r0 22
    60 [ML! iFROj(D)
    If(J.EQ.2),g0 TO 61.
    T:H丁=FI/2.-ATAL((2.*FL/D)+((X0/D)/(2.*FL/XO))-(1./(8.*FL/D)))
    TSST=THEST*RAD
```



```
    TH&G=THEO*PAD
    O| TG 22
    61 FL = ((COS(THEST/FSD) +COS(THEO/FAO))/(4.*SIN(THEST/RAD)))*D
    *u=(SIU(THEO/RAD)/(Z.*SIU(THEST/RAD))})*
    OT 10 22
    70 C!L_FFEO (IFEEO,HH,)
    6! T0 2%
    80 (.,LL POLE (IPOLE,TYPE)
    G! Tr 2%
    90 CrLL SRFCE (RIFLE)
    \0 T0 22
    91 (-1.SOFFS(SOFF)
    G1 10 22
```

```
C
```

C
C THF HEXT CALL STATENEUT IS FOR THE INPUT OF THE 3OB
C THF HEXT CALL STATENEUT IS FOR THE INPUT OF THE 3OB
Cum|GuR POTMTS.
Cum|GuR POTMTS.
100 CGLL IWPUT(X,Y,15,NUGG,FLAG,SATLRG,FLAGZ)
100 CGLL IWPUT(X,Y,15,NUGG,FLAG,SATLRG,FLAGZ)
c
c
C TYF COMTOLP FGTRTS WILL NON BE CONVERTED FRON LATITUDE AND

```
C TYF COMTOLP FGTRTS WILL NON BE CONVERTED FRON LATITUDE AND
```




```
C SuN DP TO GO THE COQROLUATE TRANSFOFMATIONS
```

C SuN DP TO GO THE COQROLUATE TRANSFOFMATIONS
C
C
OLL LITAE(X,Y,DF,AZ,EL,NUMB,FLAGZ,SATLNG,AZIM,ELEV)
OLL LITAE(X,Y,DF,AZ,EL,NUMB,FLAGZ,SATLNG,AZIM,ELEV)
C
C
C "bigF' biLL CALCULATE THE GEAM DEVIATION FACTOR.
C "bigF' biLL CALCULATE THE GEAM DEVIATION FACTOR.
CGL- HENOF(C,FL,OUF)
CGL- HENOF(C,FL,OUF)
C

```
C
```




```
C UGI:G THF BOFESJGHT REAMGIDTH A THE STGMDARD
```

CCI FEEAM(AZ,EL,15,HUPG,OL,D,THEST,THEO,

```

```

CSHIFT" STEPS THE COITOUR THRDUGH A SERIES A INCREMENTS HOPEING
Ti frm a BETTER COUNT OF BEAMLET COVERAGE THAN USINTG THE
ESTLHIEO DOFESIGGT, WhICHIS JUST. PICKED AS THE deviglis CENTRE,
the surimoitine count dOES all of the counting of cemtres of
meailets mistoe the comtular.
OML SHIFTOAZ,EL,15,ALMB,K,CFHK,CEHY,100,BWO,P1,PZ,
* % I:,YMHH,XHAX,YHAX)
-HIEC(G)(0.451)
451 FUHMAT"THE AZ/EL AFTER THE SHIFTIMG ROUTINE*,//)
a! 45? T=1, Numb
:RITE(KBOO,453) AZ(I),EL(I)
FDismat(F.4,*,O,F.4)
453 EG\&TINME
C
C IF THERE ARE NO PEANLETS COUNTEO INSIDE THE COVERAGE AREA IT
C THMT THE OH AXIS BEAMGIOTH HAS A DIAMETER THAT IS GREATER THAN
C THE CGVERAGE ARER. THE PROGRAM MILL THEN CHOSSE AIS ON AXIS
C BEPIGET TO COVER THE WROLE AREA.
IF(S.EG.0) GO TO 97
C RERAUSE IT IS THE COHTOUR THAT IS BEING MOVED AROUND THAT MEANS
C T:E RLKESIGHT IS REING SHIFTEO THE OPPOSITE WAY
C
*1:=A1年-P1
ELT:EEET-PZ
(i) 10 101
97-5%\&(1)=4%1%
CHCY(1)=ELEEV
=1
101 : 1TF(%600.102) :

```

```

    OT1E(%G04,105)
    ```

```

    *'(y C% (.):",1]
    GG(mR01,106) RES
    00 f, fot(A1)
    TF(nes.E(.1HY) GO TO b0
    C
TGF ShRMOUTINE LLESTIOM OF FEED HORTSS MJLL TAKE THE

```
```

C CE:THES OF THE FEED HOHMS, AMD FIND THE LOCATION OF THEH WITH
GEsprgto To the frcal plame
C.
C****************************************************************
c CHtMTE FLCTDATA* EILE:
C GTATM CEIqTGE OF EACH HORH IN AZIMUTH \& ELEVATION . ..
b:TTE(t,1,6001) K
6001 FOKM,隹(1^,IZ)
GTE(601,6002) (CFHX(I),CFHY(I),I=1,K)
6002 FOMmat(1x,F,3,1x,F,3)
C
C OHTAIM URER-OEFIMED SERYICE AREA FTS. AS MELL. ..
:17t(601,5001) N0,08

```
    :nTE (601,6002) (AZ(I),EL(I),I=1, H1 mb)

\(c\)
    CAL LOFH (EUF, CFHY, CFHY, \(100, B\) GO,K,FPX,FPY,FPZ,FL,THEO,XO, SOFF)
c
C THF HGRI: FEED DIUENSIORS APE CALCULATE FOR OPTIMUM SITE AND THEN
C THE A ARE SHRLNK ODH: IF IT IS DISCOVERED THAT THERE IS A OVERLAP
            CGLL HFUIM(IFEED,FL,D,HL, WL, HHA,HMA,FPX,FPY, 100,K, TYPE; BWO,HW)
C Tre fullouinc calls bill calculate the estimates of cross polar.
C IGATIOM, COS OF RAIT, EFFICIENCY LOSS, AND GOMA LOBE LEVFL

C SGGBETED PARAETERS. AMY SELECTIOL GILL RETURN THE USER TO THE
C begint:g of the frograr.
ChLL CPFF (6, HA, IFEER, 1)

    IF (InEO.40) GO Tn 4
    CILL CPR(THEST,THEOR
        IF (it.EO.40) GO TO 40
        IF (1, EFO. 50\()\) GO 7050

                IF (S.EN. 90) GO TO OO
            If (M.Ei.50) GOTO 50
            CALL GAIML (FL, N, N, AL,EL, 15, AZIM,ELFV,NUPB, XO, BWO)
        IF (M.EQ.40) GO T0 40
        iF( \(\because\) EQ. 50 ) GU 1050

        If \((1, E \pi, 90)\) Gi TO 90


```

C
211 01:=0.0
i!215 I=1,k
PHASE(I)=0.0
PGIDO(I)=1.
SON=SUW+FRATO(I)
215 rUSIINLE .--
C THE EOULOMTNG COMBAROS LTST THE COMPONENTS THAT NILL
C HF fOM,D If THE FILE *HRWDT.
C
444 :RITE(KBDO,302)

```

```

    *"Frase`./.
    ```


```

            #:407 I=1,K
            MGITE(KGCO,4\capट) I,HHA(I),HWA(I),FPX(I),FPY(I),FPZ(I),PRATO(I),
        * FHESE(f)
    402 FMRMAT(1X,I2.7(1x,F6.4))
    407 CUT1,NEE
C
C THE TOLIGIMG SECTEOG IS USEO ONLY IF THE USER HAS OPTED
C TH MEGETE g\& nOD A HDRO NHEH EDITTING THE DISPLAYED VALUES
C
455 R2ITE(KP00,382)
382 FUR:MAT(PALJO OR DELETE:")
RE40(K+30I,383) In00
383 Furmat(al)
IF(INOD.EG.1HD) FO TO 305
C
C THTS TLL ALLOH: THE USER TO AOD A HORN TO THE LIST
C ,rite(6B00,384)
384 FAKMAT(%,IGPUT X,Y,Z COORDTIATES OF HORN WRT FOCAL PLANE')
HGO(KgOL,777) FFX(K+1),FFY(K+1),FPZ(K+1)
777 FIRMAT.(3F)
\&TTE(RORO,385)
385 FOMAGTGINPUT A,E OIMENSIONS UF HORN (m)')
GEA0(KBOI,778) H4A(k+1),H%A(K+1)
778 FURW左(ZF)
VTTE KG100,380)
386 FORMA(GINPT PNYER RATIO,PHASE OF HORN')
HEMD(*BOI,779) PGATO(K+i), PHASE(k+1)

```

```

    0:3 10 370
    C THIS IS FOR DELETIOG A HUR:
C
395 FRITE(KbOO.387)
387 FOROAT(PNHICH HORN : ARE YDU DELETING?*)

```

```

    Figmat(I)
    0) 388 L=TU4,0-1
        FPX(L)=FPX(L+1)
        FPY(L)=FPY(L+1)
        FP:(L)=FPZ(L+1)
        Hi+A(L)=H:HA}(L+1
            H,N(L)=h:A(L+1)
            PRATO(L)=PRATO(L+1)
            SU|=GUiT-PRATO(GMm
            PHSSE(L)=PHSSE(L+1)
    388 OOnTIMME
k=n-1
391 covtinue
5:3 T0 370
C
460 UTTE(nb00.401)
401 FOR:IAT(DO YOU :ISH TO WAKE CHANGES TO THE HORN COHFIGURATIONP,
** FILE**)
ME40(KBOI,106) RES
YF(RES.EQ,1HAO GO TO 400
FLAS;=0
RITE(KBDO,380)
380 FOQWAT(/%OO YOU WTSH TJ ADOOR DELETE A HORN (Y QR W)??)
mequ(chot,381) RES!
381, FWi\mp@code{AT(4D)}
TF(PES1,EQ.1HY) FLAGI=1:GO TO 455
R(1F(KB100.3n4)

```

```

        *(ay ari imput of "0" the EDITTING WILL STOP,"-1" will list the",
        ** Table AgnI(i)")
    #E40(6R1,433) I,:ES
    433 O, %,\T(I,A, )
C
0.f The SIGMA ruIE THERE IS A FQRMAT STATEMENT THAT LETS
C vig WGE FORHAT(I,A!) \HIOH MEAHS.TF I IS 1 DIGIT OR a OR
C 3 IT AILL AEAD IT AS SUCH AUG MOT AOD AUYTHIUG
$\qquad$

|  |  |
| :---: | :---: |
|  |  |
|  |  |
|  | If (eFs.en. 1 HH) 60 T0 350 |
|  | If (RES.EV. 1 ite ) GO TO. 353 |
|  | Tf (RES.EX, 1 hx ) Go T0 355 |
|  | IF (nES.EO. 1 日Y) QU TO. 350 |
|  |  |
|  |  |
|  | IF (RES.EV.1nf 60 T0 363 |
| C |  |
| 35. | RITE(6000.351) |
| 351 |  <br>  |
| 352 | $\frac{\text { FGEGT(F.4) }}{60370}$ |
|  | 0010370 |
| 353 | 'ixTJE (Kセ00, 354) I. .-............. |
| 354 |  adu(abDI, 355) 40, (I) |
| 355 | FHRMT (F) |
|  | O: T0 370 |
| 356 |  |
| 357 |  |
| 358 | Fobgat (f) |
|  | 6070370. |
| 359 |  |
| 360 |  |
| 361 | VEatentiot, 369) FPY(T) |
| 36. | (i) T0 370 . |
| 362 | ARTE(KB00.3t3) I <br> FOBAT ("ERTER ME: yZ" COORDIMATE OF HORN $\#$ ". [3) |
| 363 |  |
|  |  |
| 364 |  |
|  | S0 10370 |
| 365 |  |
|  |  |
| 366 | FMKAT ( EMTEP NE': POKER RATIO FIR HORM \# *, I3) |
|  |  |
| 367 |  |
|  |  |
|  | $\begin{aligned} & \text { at } 370 \\ & \text { att } 6.600,360) 1 \end{aligned}$ |
| 368 |  |
| 369 |  |



```
    *',lGX,FPFOJECTED OIANETER',7K, 'FROTECTED OIANETER",/)
    3 F:G%ATU,"THE USEE HAS THE CHOICE OF ENTEKING EITHER?,
    *" SET OF VERIAFLES',6x,12,l,20x,'CHOOSE I OR 2.J
```




```
    * * S OFFSET DISTAMCE*,2OX,F7,3,/,
    * : 4 OFFSET ANGLEE,23X,F7.3,1,
    * - }5\mathrm{ REFlEETOR HALF 4mGLE:15x,F7.3.1.
    * G POO.jECTED DTQMETER-14X,F10.3)
    15 FGTMAT(7 TYPE OF FEED', E4X, *RECTANGULAR')
    15 FORMATP % TYPE OE FEED:.24%, SOUARE')
    da FOQmat(e s TYFE OF POLARIZATIO|',16X,ELINEAR:J
    23 FGm,TSN1x,=- NORIZOMTAL`S.
    Z5 HURMAT(41x,O- VERTICAL")
    9 FGRSTE: 7 RWS SURFACE VARIATIGL OF KEFLECTOR:,X,F7.3)
    11 FUFWAT(1, DO YOU YGHT TO CHANGE ANY OF THE ABQVE PARGNETERSP",/,
        * -GEASE IMPUT ACCORONMG HHAER GR "O" FOR NO!J.
        STr.m
        F,B
C
C SUAROUTINES TO HAUDLE INDIVIDUAL INPUTS
            SuRROUTINE FRE: (F,BL)
            COMOD DT,RAO,KBDI,KBDO
            ImTEGER VAR
            OITE(kBOD,1)
    I FGR~AT(/, 'CHOOSE INPIIT: FREQHENCY (F) OR WAVELENGTH (W)')
            REAN(rgot,2), VAR
    ? FGmi:AT(Al)
            IF(yAFE.EJ.1+F) GO 103
            IF(yAR.EO.1HIG) ARITE(KRDO,4) inc
    4 FOMMAT(PEOTER AVELENGTH (Cim)',6%,F10.3)
            #E4(%HDI,7) %L
    7 OM,GAT(F)
            F=3.Ea/i, L
            Gi T0 0
    3 NITE(NBDD,5) F
```



```
            #EA(:30)I,8) F
            frgamat(f)
            1=3.E4/F
            aETURH
            EO
C
```

```
    SGHROUTIAE FEED (IFEED,HL)
```



```
    AMTTE(KGOQ,1) TFEED
    1 FG&|G(/,EEITES TYFE OF FEED',GK,II%/,
    ** i &ECTAGGULARP,/.' 2 SOUARE*./J
        EESG(K日GT, 2) IFEED
    2 F(GWआप(I)
        If (IFEEG.IF.1) GG TO 5
        #HITE(k+DG,3) HL
    3 FORGMT(/, IMPUT LEUGTH OF HOFU(cm)',6x,F5,2)
        READ(KGCI,4) ML
        FUR|AT(F)
    ?&TUT%:4
    #0
C
    B:RPOUTIME POLE (TPOLE,TYPE)
    IWIEGEH IPOLE,TYPE
            GMOS PI,RAD, RPOI,KBDO
            IFOLE = 1
C T:AY IS TOSAY THAT THERE IS OMLY A CHOICE OF IINEAR
C POLaRI ATION ON THIS EDITIOG OF THE PROGRAM SAMOD
C
    GITTE(YAOU,1) TYPE
    l FGFDAT(/, VERTTCAL (1) OR HOPTZOHTAL (2)',6X,IZ)
    z FGMm&(I)
        QFT!m&
        E:口
C
        SHBNUUTIME SPFCE (RIPLE)
            CR*WOSPI,RAD,GBOL,KPOO
            YGTE(KBOO,1) FIPLE
            FGP:AT(/. FETER RES SURFACE VARIATIUN OF REFLECTOR(cm)",
        *与:,F1U.7)
            HES(NHOI,2) &TPLE
```



```
            GCTURM
            S%[
C
C
SURNGUTIHE FLHGTSFL）
```

```
    CHAMOH FI,FAD,KEDI,KEOT
    HETTE(FGOC,1) FL
    FGRIMT(/."ERTEF FOCAL LENGTH(;)",0X,F7.3)
        {EA0(ttol,2) FI
    C Functate
        ar!uk:
        -:0
    C
        SUERGUTINE DOFF(XO)
            CONOOOPI,FAD,KBOI,KPOD_.......
            Q&ITE(KBOO.1) X0
    1 FUFCAT(/, EGTEB OFFSET IISTAKCE(m):,6X,F7.3)
            ERAO(KBCjI,2) X0
            EGPGAT(F)
            TETUKH
            E,!
    C
        SUUROUTJME AOFF(THEO)
            CgamON FI,FAD,KBDT,KRDO
            MTTE(KGDO,1) THEO
    1 FOHMAT(/,FTTEF DFFSET ANGLE(GEZ)O,6X,F7.3)
            EEA0(KRBI,2) THE0
            FOR:|AT(F)
            \triangleETGRR
            G:0!
C
            S:HQGUTIUE HAREF(THEST)
                CuF#OH PI,RAD,KEOI,KRDO
            MGITE(KDDO&D) THEST
    1 COHAAT(/, EGTER PEFLFCTOR HALF ANGLE(deg)P,6X,F7.3)
            GEA!(KGOT,2) T:EST
            FuQ#AT(F)
            faTu&a
            F.ib
    C
    C
        SUFRGUTIUE GPROJ(D)
            CDM:MO: PI,RAO,"BOI,KAOO
            SITE(KHOO,1) D
    1 FOF!GA(/, FHTER PROJECTEO DIAEETER",6x,F7,3)
    FAD(KBNI,2)O
    2 FamgT(F)
```

```
OET|N:O
6:!
C
        SUEROUTINE SQFES(SOFE)
        CGmMDIPT,RMD,KEDI,KROO
        UMTEGEF SOEF
        NRITE(KDU0.1) SOFF
    1 FO##AT(%,SSATELLTTE OFFSET: NGRTH(1),SOUTH(2),EAST(3),NEST(4)?",
        * 5x,11:1)
            &AD(KHOT,Z) SOFF
            FORBET(I)
        SETUF
        F!(i)
C
        SugROUTIEL EEMDF(D,FL,BDF)
        REA! BOF
        AOF=(1+(1/3)*((0/(4*FL))**2))/(1+((D/(4*FL))**2))
        getura
        Em
C
            SHGROMTIWE FFOTA(IFEEO,FL,D,HL,HL,HHA,HMA,FPX,FPY,M,K,TYPE,BWO,HW)
        InTEGER TYPE,K
        HEAL FL,DL,HL,HH,HH,OL,FPX(M),FPY(M)
        REAL IMA(%),HOS(得)
        x=F1/O
        G0 in (1,2) IFEED
    C THESE aRE IHE OPTIUL: DIVESSION OF A HORN FEED
```



```
    C
    C
    FKTIGM POLABIzMTIOH
    1 IF(*.LT.(.2+.こ大值)) GO T0 A
        HG=((.34+2.08*X)*(1.06-. 299*X+5.98E-2*HL))*LL
        Mi4=((.325+1.30*x)*(1.047-.2345*x+4.69E-2*HL) ) ***1
```



```
            HM=(4.325+1.30*x)*M,
            IF(TYPE.E.1) GO TO 1u
    C
    BGMMOSTAL POLARTZATTOA
```





## $E 10$

```
C
    SUGROTINE LLTAE (X,Y,O,AZ,G,NU#AR,FLAGZ,SATLNG,AZIN,ELEV)
        COM,G&PI,RAD,KBOI,KQDO
        IGTLGER ANS,GMSZ, NUHQ,RES,FLAGZ
        HEML RUS(3),ALIM,ELEV,LAT,LOHG
        4EdL S4LT,GLAG,SLAT,PR,RS,BLAT,BLNG
        KEAL SAR,SEL,AZ(们),EL(N),X(G),Y(N)
C THE IMPMT LAT AMO LORG COORDIBATES WILL NOW BE TRARSFORMED
```



```
    A又I:=0.0
    P=578.16
    FLE:=0.0
    FS=u216i, 16
    SSLT=35782.0
    SLAT=0.ig
    IF(FLAGZ.EQ.1) GO TO II
    ~QJTE(KBOO,1)
    I FGHMGT INPUT LOMGITUDE COOPDIHATE OF THE GEOSIAIIONARYP.
        * " Satellite")
            kEAO(KROL,10) Satlhg
            F!P的自(F)
            FLbGr--
    11 SLMG=SATIFG/RAG
            H,S(1)=0.0
            F!S(2)=0.0
            R\cupS(3)=-RS
C
TMTS JECTAR DEFINES THE COORDIBATE SYSTEM. NITH THE DRIGIN
    AT TME CELTRE OF THE EARTH, FOSITIVE AXIS THROUGH THE NORTH
    OOLE, amD TME SATELLITE IN THE Z-X PLANE
    ND: FIND CENTRE OF CONTOVR IN LAT AMO LOMG COGRDINATES
    xim=x(1)
    x+:x=x(1)
    D0 z i=2,00wR
        LF}(X(I).LT,XHIN) XNTN=X(I
        IF(%(I).GT. XFAX) XGAX=X(I)
    Curtiwue
C
```





```
C T. GMEA TU FIMD THE AI:IMUM GUMGEF OF DEAMLETTES MEEDED
C fo cover tre coverage brea, this mibl be dONE by moving
    TrE COETOUR POINTS TO THE RIGHT OHF BEAMGIDTH AND THEN
    HP OE GEMMoTDTH. THSS MEAHS THAT THE HORHS ARE BEING SHIFTED
    The Sare afy.
        The ImitIAL COUOT is dome with the geamS IN THE ORIGINAL
        HGSITIO& USING AT AOD EL
```

c

comtrous
CA, COUMT $K, C F H X, C F H Y, 100, A Z, E L, 15, N U W B, B W O$,
*
$\operatorname{coj}(1)=\kappa$
c
C STARTIAG ITH SHIFTIMG TAE K -COMPONENTS AROUND
$\mathrm{J}=\mathrm{l}$
10 $15 \mathrm{~L}=1.4$
$j=\mathrm{j}+1$
$012 \mathrm{I}=1$, IUMA
$\therefore(I)=a(I)+.2 * B+1)$
$\begin{aligned} & \therefore(I)=1 \\ & \text { curtjente }\end{aligned}$
ט० $33 \quad I=1$, inne
33 comithue
CALL COU:STK, CFHX,CFHY, $100, A, E L, 15$, HUME, BWO,

contil!if
c.


C UTCH IS A WMLE PEANLET SHIFT
Arin = Th( $(\operatorname{cou}(1), \operatorname{cou}(2), \operatorname{cou}(3), \operatorname{cou}(4), \operatorname{cou}(5))$
$004 P=1.5$
IF (MIMEO.COU(P)) GO TG 5
coftinue
c
las count value is at cou(p)
Thit Imatum count value is at cou(p)
5 IF (OTHEFO.COO(1)) GO TO 11
$F_{1}=(P-1) *, 2 \times 2,0$

$02(1)=1.2(1)+p 1$



```
            C,NLL TRANS(ET,AZI:, !MX,R)
            CaLL TMACS(F,FLEV,IHY,RT)
            SAZ=4T4.(RT(2)/?T(3))
            SEL=ATAM(RT(1)/SGRT(RT(2)**2+RT(3)**2))
        AETURO
        F!:0
    C
C
```



```
            GEAL A(3),H(3),ANG
            INTEGER IAXIS
    C
    C Tr:% DOES A GELERAL TRASGORHATIOR ON ANY GIVEN VECTOR COORDINATE
        S=ST:(ANG)
        C=Cos(ASG)
        IF(T4XIS.EQ.1HY) GO TO 10
        JF(I4KIS.F(v.1HZ) G0 T0 20
        B(1)=A(1)
        B(2)=t(2)*C+4(3)*S
        H(3) = =A(2)*S+#(3)*C
    10 b(1)=n(1)*C-A(3)*S
            #(2)}=2(2
            H(3) 2A(1)*S+A(3)*C
            O T% 30
    2. <(1)=A(1)*C+A(2)*S
            (2) m-A(1)*S+A(2)*G
            h(3)=A(3)
        34 HETUR:4
        @!0
    C
c
```

        SURRUTIME HEAR * (AZ,EL, MUMQ, M, D,THEST,THEO,
    
Comand RI, fabarant, 600
UEAL G (C), ELC(G), L, D, THEST, THEO, BWO
InTriber nder
c
$x: I:=(1)$
$x+1 x=1)$



$i$
QUTIUE



```
            1=0
            F=0
            JF(OTPLE/RL).LE.1.F-3.OR.(PTPLE/NL).GE.1.) T=1
            IFGTHEST.LE.1G.OR,THEST.GT.80.) S=1
            IF(S.EW.1.OR.T.ER.1) GO TO 4
C
                            C=19.75+19.49*LOG10(RIPLE/FL)=(178.8/SQRT(THEST))
    C
    1 FORMAT('THE ESTIMOTEO PEAK CROSS POLARIZATION LEVEL',',
```



```
            HITE(FEDO,z)
    2 FOmfatroto merlue cross polarization choose one:",/,
    * 'Mecrease surface variation (SV)","pecrease reflector.
```



```
            HEAC(MBOL,3) RES
            fcgmat(A1)
            if(RES.E(v.1HS) P=90
            JF(EES.EO.1HR) P=50
        ETUR!
        E:1:
C
        SHG,CUTINE LOFH(BDF,CFHX,CFHY,N,BWO,K,FPX,FPY,FPZ,FL,THEO,XO,SOFF)
        Cum0: PT,RAD,KBOI,*日GO
            FEAL FPX(4),FPY(*),FPZ(*)
            REAL CFGX(M),CEHY(M)
            AEAL 3.O,SDF,FL,THEO,RHO
            JITEGER S,SIFF
C
C TH:SFER cEMTRES TO FOCAL PLSHE COORDIMATES
            SHO=OWAFL/(1+COS(THFO/RAD))
            1013 I=1,K
                    REFGj=aTAG(SI*(CFHY(I)/(BOF*RAO)))
                    REF& 己=ATAN(SIN(CFHX(I)/(BMF*RAD)))
            OPTA=ATAM(XG/FL)
            n,T TG (1,2,3,4) SOFF
C
C THES: THGEE EQUATIOIS AHE FOG THE TRANSFORVATION USIMG NORTHERN
C SAMLLLITE AMTEMA FFED OFFSET
C
    FPN(I)=-(FL*1On)+(PHO*SIF(OFTA)*SIN(PEFA1)/
    SI"(OG/RAD+UPTA-REFA1-THEG/RAO)))*TAN(REFAZ)
    FPY(I)=(-RHO*COS(OFTA)*SIN(REFA1))/SIN(90/RAO+
```



```
C
C
```



```
            Crgmoti PT,RAO,KEQI,GBOO
            INTEGFP F,RES,MHA
            NEALFL,AO,NO,D1,NAX,LGAIA,AZ(M),EL(N),AZO,ELO,D
            P=0
C
    CalmL DISN (AZ, FL, N, AZO, FLO, P,N(M, M, MAX)
C
    u1 = 2*x0, +u
    n=6m0/max
    y = N1/(.0143 + .716*((FL/D1)**2))
C -
GAI: = - 1.67Em2*X + 2.67Em3*X**
: FITE(H800,1) LGAIN
I TORMATC'THE ESTIMATED LOSS OF GAIN FOR MAXIMUM FEED OFFSET'
* ,/,*FROG THE FOCUS IS*',F10.3," da*,1/)
    WRITE(GBRO(O,z)
    z FOR*AT('TO KEDUCE LOSS OF GAIN CHOOSE ONE;':/I,
        * Vegrease gFfset anglf (oajo,/,
        * "deckease reflector hblf angle (rha)*,",
        * -OO (HATHE RERUIREO (NO)',1!/)
            RFAD(KEDI,3) RES
    3 FO&MAT(A1)
        TF(RES.EO.1HO) P=40
        1F(RES.FO.1HR) P=50
    4 ETURN
    Ha!
C
SgMFGUTAE GLST(FL,D,RIPLE,WL,P)
    Cu:aOM PI,R&D,REOI,K&iOO
    INTEGER RES,F
    MELL GI,FL,T,PIPLE,:OL
    F=0
    IF(IFL/U).LE..25.OR.(FL/D).GT.1.2)G0 TO 4
c
    GL=(RIPLE/:1)**?*(41.58+52,0*(F!/D)-19.28*((FL/O)**2))
1 NHTE(sGmo,1) GL
1
    FOQNT(TTHE ESTIMATEO GATN LOSS',1,ODE TO SURFACEP,
*
- ratramCE 1S;',F1%.5,' OB%,//)
```




1. TEGE GH

c
FEAL HM(99), HE(99)

| c | - hande uinenstots of I T |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

C Real a'ri(100), ELM(100)

GEGL $4 \%(100)$ efllH(100)
RE:L Pl: (1:0), PH(100)
RHL KH(100), YH(100), ZH(100)
MEAL Xf,YA,ZA
REPL OEX, OEY, UEZ, OHX, OHY, OHZ
REML Pris
fotebeg TM
THTEGER IA
$C$
$C$
$\therefore 80 \mathrm{i}=105$
nbou=10a
C $C$ ifle irncit was createg in the wain proaram sAMOD and
c i. :iseo to transfer tne number of lorns, location,
$i$ zasions, noral ard direction vectors, nower, and
Crise. The logical unit foir this file will be 501
10 READ(501~100) 12
10: Forririf)
C
a $10511=1,4$
READ (S01, 150) HH(I), HE (I), XH(I), YH(I), ZH(I)
FOQ4T(5F)

| 150 | FQQ4 |
| ---: | ---: |
| 056 | 0.11031 |

c
De $1000 \quad y=1, \mathrm{am}$

151 Fibnon (2F)
1060 GOTTT.HE



```
10:07 FG% 10,"32 DC/FEFLCOX.4290552
C*******&゙**************************************************************************
\begin{tabular}{|c|c|}
\hline c & \\
\hline ［ & PGRPUSE－ \\
\hline C & THIS PROGRAM TAKES A GIVE日 SET OF DIMEASIONS OF THE \\
\hline C & REFLECTOR AME BREAKS IT INTO ELEFENTAL AREAS（DEFINED）． \\
\hline C & IT THES CREATES TMD FIL．ES DISH1 AMO OISH？． \\
\hline C & OISH1－gTVES THE LOCATION OF THE CENTRE POIMTS OF \\
\hline C & ThF elemertal areas founo \\
\hline C & DTSHZ－GTVES ACTUAL AREA OF EACH ELENENT AS OPPOSED \\
\hline C & TO THE PROJEETEO AREA ANO，IT GIVES THE DIRECTION \\
\hline c & COSIUES OF EACH AfEG：－ \\
\hline C & calling sequence－Pu，reflolu \\
\hline
\end{tabular}
```



## FETL CL（90ウ），CN（GO），CH（900），AREAC（900）

CL，CM，CN－these are the oirection of the normal to each ooint on the reflectoc surface．．．．－．．．．．．．．．．
AREA－is the area of each reffector piece
－these can be foung in the data file DISHE＇
REAL XEGOOF）YR（900），RR（700）
Xerper－location of each ooint on the reflector surface
－these can ise found in the data file DISH：
$C$
$C$
$C$
$C$
$C$
$C$
FAL FL：$\times 6.0$
FL －focal lenath of the dish
$x \bar{u}$－offset distance of the reflector
（1）rroiectea oiameter

| $C$ | if GL RCX，RCY |
| :--- | :--- |
| $C$ | RCX，ROY－$X, Y$ dimensions of $t w e ~ d i s h ~$ |

c
GEM X X Y，YC，Z C
C





```
            "OTTE(5:3.100)]
C
            00 10ne I=1,j
                        irITE(503.106) XR(I),YP(I),ZR(I)
                        G1TE(5%4,107) CL(I),CM(I),C!C(I),AREAZ(I)
10n0 CC:TJMEF
```



```
100 FDF,AT(I)
106 FOG ST(3F.4)
107 FiFmet(4f.4)
    : TO O.90
```

996 :TTE(1):121)
121 FIG AT(1OA, AFPAIES XR(J) ETC OVERFLOMED, PROGRAM ABORTEDF)
10:07 F%% O.O2 OC/AEFSC%.42%555z
C
C REFLECTGU SHAFACF CURRENTS PROGRAR . V.A.STOTT 26 MAY 1970.
C CALIT G SEE. BG,RSEA,LU...
C
EEL IX(750),JY(750),JZ(750),JNX(750),NWY(750),JNZ(750)
C C jx.jy.jlZ - are the conmonents of the radiatino electric
field vectors (E direction)
N:X,JiY,JHZ - are the components of the radiatino
electric fiels vectors (H direction)
\#aL XA(750),Y:(75%),ZA(750)
C Ka,Ya,iA - tnese are the centres of each horn element
as read from the file HORN1?.
BaL P%S(750)
C
C
REML AR,YR,ZR,RL,PM,RO,AREA,XY,YV,ZV,RV
AR,YR,ZR - centres of each reflector element-
H,R,",R m direction cosines for each reflecton element
AfE\& - the actual area of each element (not proiected)
dy,Yyiy - the vector distance vetween the reflecton element
ang a horn elemerit
Ry - the maonitude of the arove vector
FPSL PX,PY,PT,PSP,PRXP,PRYP,PRX利,PRYM,PRAP
C C F,PY,B7- - v, oowers (dB)
c pso-snijlover loss (dB)
pixp,ppyp - bomer loss at the maximum x and y positjons (dB)

```

```

fimp - averture power (dB)

```
```

CJPMIEX HX,HY,HZ,CPX,RIX,RJY,RIZ.

```
CJPMIEX HX,HY,HZ,CPX,RIX,RJY,RIZ.
16at profr(5)
```

16at profr(5)

```


```

    ary Int 503
    ```
    ary Int 503
    - \(\operatorname{su}(5062200) F\)
    - \(\operatorname{su}(5062200) F\)
200
```

$\qquad$
$\because:=1$.

C

C : radiating elements the aperture E,H planes were divided
C ire.
$10500 \quad I=1, \mathrm{~mA}$


```
            i jix(I);{!Y(I),J#2(I),PHS(I)
C t: : ile contains: the centres of each horn element, the x,v,z
c co znents of the radiating electric field vectors (E,H), and
C t... .rase of each ingivigual norn
500 (%,TInut
C
    O reflector innut data files "fismy" and "DISH2"
    rar in number of reflector elements, NE
    EST(503.100)NE
C cinate reflector surface currents outout data file "RFIFI%
    O=5.14130265
    TOU=0.00:1*TP%*2AO.*PT/ME
    :
    y=0
    =?=1.
        1.299.703/F
        YM=-10.E.8
        OM:=10.E4
        Y \%=-1, E8
        #:=10,E%
        =1
    I~1
    &i, 4!% \=10.4E
C
C rint cut of tae file Clsht", location of reflector centres
```



```
C lua gut of ti:e file -DISHC", direction cosines of element,
C a: actual area of each element
```


$\qquad$

$\qquad$ foriector element kn

HITE(505,95)RJX,RJY,RJZ

$\sin 10 . * \operatorname{Loc} 10\left(F R /\left(T P A * 24 \pi P_{T}\right)\right)$
BAR=10. *LCO10(PGEF(1)/PR)


Cry $=10$ * * OG10(FOEE ( 4 )/PR)

$D=10 . * \operatorname{tg} 10(F \times / P E)$


-TE(10e, 1a3) propop,py,pz
-ITE (1) : , Ma) PK?
UIF (10k, 1,5)PRY:
SIE(1, R, lo6)PRYp

| 94 |  |  |
| :---: | :---: | :---: |
| 95 |  |  |
| 96 | Fform(4G) |  |
| 97 |  |  |
| 99 | Fifleat (F) |  |
| 100 | FOHAT (I) |  |
| 101 |  |  |
|  | 门3, "******************************** 1 , /1) |  |
| 103 |  |  |
|  |  |  |
| 104 |  |  |
| 105 |  |  |
| 106 |  |  |
| 107 |  |  |
| 108 |  |  |
|  | 60 T0 999 |  |
| 758 | - AITE (108,759) VAR |  |
| 759 | FEPMATCHT POSITIVE A VALUE, LOG10 WILL NOT WORK*.F9.3.1) |  |
|  | 60 90 Org |  |
| 999 | STot 3 |  |
|  | cid |  |
|  | -… . .... |  |

```
10:07 Fी.7 16x-12 0%/P4TE%4x,4290552
C
```



```
    revelored at Canacian Astronautics Limitec
    20 जAY 197? MIKE STOTT
    morified summer of al by
    Farnara Coll (CAl) + B. Zanjenkowsky (CRC)
    MFTGOAD NABE - PATERS
    T:j: ersion nuts the gain.matrix onto disc. This operation
    te reg about l5 minutes of the users time. The eno result
    #:t:e creatinn of tre file 'parrm* nhich is used in a cantour
    !tilma routine.
    FG4L XP(720),YR(720), 2R(720)
            XR,YF,XR - these are the coordinates of the centres
                of each reflector element
    -6, SVF(42),SFP(42)
    P:! Y*C(9),YxD(Q),INC
    CGPlEEX EX,EY,EZ,CPX,AX,HR,VT
    CGYLEx HX(720),HY(720),HZ(720)
C
C Hx,Hy,Hz - tnese are the total surface current for each reflector
                                    element
    I:TEGEF IGHF(1200),IGYT(1200),TGP(5)
    FMGIVALEICE (IGHR(1),HX(1)),(IGVT(1),HY(1))
C C G:ar irout data file 'DISHI', which lias created in the program 'REFLCO',
C G:-M inout data file 'DISHl', whict, bas createo in the program 'REF
C thi ism trat ere retermined also in the program PEFLCOP.
C
    O,O(5c3,98)4.4
    m 550 i=1, !H
    *ans location of centres of reflector elements from EnISHL file
    REnC(503,55) XR(I),YP(T),ZR(I)
    55: C0WTI:11F.
c
```

```
C lrgragd froalte file fifIFl" which was created in the proaram
    N\mp@code{Mb}
        &500 I=1.40
        EAO(505,96)
        HX(I),HY(I),HiZ(I)
    5%
C
C T.gmmout the prooram the total onwer of all horns has been set
    t" are, and the frecimency is foung in the file "INPUTS" which was
    Grarreg in the main Erogmam.samoj!. The maximum and minimum
    O. Etions in AZ/EL of the contour are also read in so they. can be
    *e to oive the user an idea of the.area that should be looked
    ar on the contour olnt.
C
    F:=1
    B!-1:5500
            TEBD(500,21) F
            FHPN4T(F)
            CALL. POSREC(500,3)
            &FAD(500,22) XITH,Y%TH,XHAX,Ymax
    2% FO&!AT(4F)
C
```



```
    7! FGRMAT(IN AZIMUTH AGD ELEVATIOY COORDINATES OH THE EARTHP,/,
    * THL HIGTMU! AZ IS: *FS.2,4X, THE MAXIMUM IS; *FS.2,%;
```



```
    * *:TER ATINUTH VIEGI*G RAEGE FOR PLOT (START,FINISH)*)
            {E:0 (1,5, 71) 4ZST, ADFI
            GTPGT RGTER.ELEVATING RANGE (START,FINISH).
            "F:H (19S, 71) ELST, ELFI
    71 FURMAT (2F)
C
    T:A butmoutine FXIGC xill set the incrementation of the scale
    a, aze sure that the livits chosem are divisable bv the
    incmeventation, tris may result in the scale beinc slightly
            írerent than whatwuras remestequg the user.
            CALFYIUC (EZST,AZFI,AZINC)
            G:IL FXIMC (ELST,E!NT,ELTYG)
C
            AITE(103.80)
    0) forgat(//: "the progran is don geveratirg the coutour data matrix.",
```



```
C
```

SERVICES D'ORDINATEUR
MINISTERE des COMMUNICATIONS




```
.155 +6n,4T (22F6.2)
    ron
C
    SHQROUTINE FXIUK(ST,FI,INC)
    HEBL THC,ST,FI,RIHT
    I:TEGEPI
C
C i increments chosen are based on the allowable memory of
C r:# :rourum.
    GI:T=(ADS(FI-ST))/IG
    TF(K\:T,GE..75) TMC=1,0
    TF(RTMT.GF.35.AN1, 2TIT.LT..75) TNC=.5
```



```
    TF(HIM.LT..1S) IUC=.1
C
C rimaie max and mins so thev wlll be divided evenly by the inc.
[:
    IF(amOCST.INC).EQ.O)GOTO 5
        i=1FTx(ST/INC)
        If(I.LT.!) I=I-1
        TF(I.GT.A) I=I+1
        ST=FLOMT(I)*ITC
    5 TF(ANOD(FI,INC).ED,O) GOTO 6
    I=TFl⿱艹\zh2(FT/INC)
        TF(I.LT.n) J=I-1
        TF(I.OT.0) I=I+1
        PI=Fi(iAT(I)*I*C
    G BETUNA
    Fr!
```

APPENDIX D

REFFRENCED USED

# Optimum paraboloid aerial and feed design. 

D. Herbison-Evans, M.A., D.Phil.

## Synopsis

A computer study was made of the spillover, crosspolarisation and illumination efficiency of paraboloids of various focal-length/paraboloid-diameter (F/D) ratios; fed by tectangular, square and circular waveguides in their lowest-order modes. The feed radiation patterns used were those derived by simple diffaction theory from the incident waveguide-mode field distribution. The approximations made in this derivation give unnaturally low spillovers for very small aperture feeds, and thus favour short $F / D$ systems, which would use such feeds. The feed dimensions were optimised to give a maximum figure of merit (gain/ total noise temperature) for each combination of feed, receiver noise temperature and F/D ratio. Some of the optimum horn dimensions and resulting illumination tapers, excess aerial noise temperatures; gain efficiencies and figures of merit produced by the computations are presented. The results show, among other things, that the figure of merit of a low-F/D-ratio system is about 0.8 dB less than that of a large$F / D$-ratio system. They also show that, if the system has a low receiver temperature, the feed dimensions should be larger than conventional design procedures indicate, to give up to 3 dB more edge-illumination taper.

## List of symbols

$a=$ Eplane halfwidth of rectangular feed aperture, radius of circular-feed aperture or halfwidth of square-feed aperture
$b=H$ plane halfwidth of rectangular-feed aperture
$D=$ diameter of paraboloid reflector
$E_{6}, E_{6}=$ components of electric field radiated by feed in spherical co-ordinates with origin at feed
$E_{\pi} E_{y}=$ components of electric field radiated by feed in rectangular co-ordinates with origin at feed
$F=$ focal length of paraboloid reflector
$G=$ gain of aerial relative to isotropic radiator
$\mathrm{J}_{1}(x)=$ Bessel function of first kind and order one with argument $x$
$k=2 \pi / \lambda$
$M=10 \log \left(\frac{G}{T}\right)-10 \log \left(\frac{\pi^{2} D^{2}}{\lambda^{2} T_{R}}\right)=$ figure of merit relative to dish with $100 \%$ efficiency and undergraded receiver temperature
$P_{00}=$ power radiated per unit solid angle by feed along its axis
$P_{1}=$ total power radiated by feed
$T=$ total equivalent noise temperature at aerial terminals
$T_{R}=$ equivalent noise temperature due to receiver and aerial main beam
$T_{s}=$ equivalent noise temperature of feed spillover past paraboloid
$\gamma=2 \pi /$ guide wavelength
$\Gamma=$ reflection coefficient at waveguide mouth
$\boldsymbol{\eta}_{\boldsymbol{I}}=$ aperture illumination taper efficiency
$\eta_{p}=$ efficiency factor due to crosspolarisation loss
$\eta_{\mathrm{s}}=$ efficiency factor due to spillover loss
$\theta=$ component of spherical co-ordinates about feed
$\lambda=$ free-space wavelength
$p=$ distance from point on paraboloid to focus
$\phi=$ component of spherical co-ordinates about feed
$\Phi=$ cone semiangle subiended by paraboloì at its focus

## 1 Introduction

One of the simplest methods of obtaining aerial gains in excess of 30 dB is the hom-fed paraboloid of revolution. The efficiency of this aerial depends on the focal-length/ aperture-diameter ( $F / D$ ) radio of the paraboloid and the noise temperature of the receiver, as well as the design and dimensions of its feed.

Paper 5428 E, first received $29 t h$ June and in revised form llh August 1967. Crown copyright

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England, and is now with the School of Physics, Universily of New South England, and is now with the
Wales, N.S.W., AustraliE.

Previous general design procedures for optimising paraboloids have assumed either a long focal length ${ }^{1.2}$ or approximations to the feed radiation pattern which lend themselves to integration. 3.4
The optimum design of the feed-paraboloid combination for a maximum signal-power/noise-power ratio depends on the particular application For radar, scatter and line-ofsight links and passive-satellite communication links, the received signal is proportional to the square of the aerial gain, so that the aerial may be designed for maximum $G^{2} /$ total-noise-power ratio, where $G$ is the gain above isotropic. For active-satellite communication ground systems, where the downlink is more critical than the uplink, as in the present generation of systems, the signal is proportional to the gain, and the aerial should give maximum G/total-noise-power ratio, i.e figure of merit.

The total noise power can be conveniently expressed as an equivalent noise temperature and split into the sum of components due to the receiver, aerial mainbeam, diffraction sidelobes, defects in the paraboloidal surface, feed-support blockage and spillover by the feed outside the cone subtended by the dish at the feed. This last component is important in this study, as it is the one which is traded' for gain by altering the feed dimensions. Assuming the noises due to each component are relatively incoherent, their powers can simply be summed In this study, all but the spillover will be lumped together and referred to as the 'receiver temperature'. The optimum feed-paraboloid configuration depends on this parameter.

## 2 Method

In this study, the effects of paraboloid F/D ratio and receiver noise temperature on the optimum design were investigated with more realistic approximations to the feed radiation patterns than, previous investigations ${ }^{3 ;} 4$ using digital computers. The study was directed to the design of ground-terminal aerials for an active-satellite communication system. Thus the figure of merit or gain/total-noise temperature ratio was maximised with the aerial presumed to be directed at low elevation ( $\sim 10^{\circ}$ ), when satellites tend to be at their maximum range and their signals weakest.
Three feeds which lend themselves to circular polarisation were investigated: rectangular, square and circular waveguides in their fundamental modes. The radiation pattems of the feeds were taken to be those derived by scalar diffraction theory from the transverse-field distribution of the modes?
For the rectangular and square feeds,

$$
\begin{align*}
& E_{\theta}=\left\{1+\left(\frac{\gamma}{k}\right)^{2} \cos \theta\right\}_{1}^{\cos (k a \sin \theta \cos \phi)} \frac{\left(\frac{2 k a \sin \theta \cos \phi}{2}\right)^{2}}{2 \pi} \\
& \frac{\sin (k b \sin \theta \sin \phi)}{k b \sin \theta \sin \phi} \sin \phi \tag{1}
\end{align*}
$$

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$E_{\phi}=\left\{\left(\frac{\gamma}{k}\right)^{2}+\cos \theta\right\}^{1-\left(\frac{2 k a \sin \theta \cos \phi}{\pi}\right)^{2}}$

$$
\begin{equation*}
\frac{\sin (k b \sin \theta \sin \phi)}{k b \sin \theta \sin \phi} \cos \phi \tag{2}
\end{equation*}
$$

and for the circular feed,

$$
\begin{align*}
& E_{\theta}=\left\{1+\left(\frac{\gamma}{k}\right)^{2} \cos \theta\right\} \frac{j_{1}(k a \sin \theta)}{k a \sin \theta} \sin \phi  \tag{3}\\
& E_{\phi}=\left\{\left(\frac{\gamma}{k}\right)^{2}+\cos \theta\right\} \frac{J_{1}(k a \sin \theta)}{1-\left(\frac{k a \sin \theta}{1 \cdot 841}\right)^{2} \cos \phi} \tag{4}
\end{align*}
$$

In both cases, the refiection coefficient appearing in the equations in Reference 5 has been approximated by that due to the change in refraciive index implied by the change in phase velocity. A more exact complex expression for the refiection coefficient is derived in Reference 5 , but the extra complexity of its use was not considered to be justified at this stage. Thus the reflection coefficient was taken as

$$
\begin{equation*}
\Gamma=\frac{k-\gamma}{k+\gamma} \tag{5}
\end{equation*}
$$

The calculation of the figure of merit of a particular combination of horn, paraboloid $F / D$ ratio and receiver noise temperature, fell into three parts: spillover, crosspolarisation and illumination efficiency.

The spillover is the proportion of power radiated by the ieed not intercepted by the paraboloid. It defines an efficiency factor $\eta_{s}$

$$
\begin{equation*}
\eta_{s}=\frac{\int_{0}^{\infty} \int_{0}^{2 \pi}\left(E_{\theta}^{2}+E_{\phi}^{2}\right) \sin \theta d \phi d \theta}{\int_{0}^{\pi} \int_{0}^{2 \pi}\left(E_{\theta}^{2}+E_{\phi}^{2}\right) \sin \theta d \phi d \theta} \tag{6}
\end{equation*}
$$

where $\Phi$ is the semiangular aperture of the paraboloid. The - denominator should be equal to the power leaving the waveguide aperture. So as 10 avoid the wiggly integral involved in its direct evaluation, the analytical formula for the gain was used to find it indirectly:

$$
\begin{equation*}
P_{\mathrm{r}}=\frac{4 \pi P_{00}}{G} \tag{7}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{1}=\int_{0}^{\pi} \int_{0}^{2-}\left(E_{6}^{2}+E_{\phi}^{2}\right) \sin \theta d \phi d \theta \tag{8}
\end{equation*}
$$

$$
\begin{equation*}
G=\text { axial gain of feed over isotropic radiator } \tag{9}
\end{equation*}
$$

and $\quad P_{00}=\left(E_{\theta}^{2}+E_{6}^{2}\right)$ at $\theta=\phi=0$
In our case ${ }^{6}$ for the rectangular and square feeds

$$
\begin{align*}
& P_{00}=\left\{1+\left(\frac{\gamma}{k}\right)^{2}\right\}^{2}  \tag{10}\\
& \left.G=k^{2} a b^{2}-8-1+\left(\frac{\gamma}{k}\right)^{2}\right\}^{2} \tag{11}
\end{align*}
$$

and for the circular feed

$$
\begin{align*}
& P_{0}=\frac{1}{4}\left\{+\left(\frac{\gamma}{k}\right)^{2}\right\}^{2}  \tag{12}\\
& G=\frac{1}{4}(k a)^{2} 0^{2}-8368\left\{1+\left(\frac{\gamma}{k}\right)^{2}\right\}^{2}\left(\frac{k}{\gamma}\right)^{2} \tag{13}
\end{align*}
$$

The crosspolarisation efficiency was found by integration across the focal plane of the wanted linear component;

$$
\begin{equation*}
\eta_{p} \int_{0}^{\int_{0} \int_{0}^{2 \pi}\left(E_{x}^{2}+E_{y}^{2}\right) \sin \theta d \phi d \theta}{ }_{0}^{2 \pi} E_{x}^{2} \sin \theta d \phi d \theta \mid \tag{14}
\end{equation*}
$$

where $E_{x}=E_{\theta} \sin \phi+E_{\phi} \cos \phi$

$$
\begin{equation*}
E_{y}=E_{\theta} \cos \phi-E_{\gamma} \sin \phi \tag{15}
\end{equation*}
$$

The aperture-illumination efficiency was found by using

$$
\begin{align*}
& \eta_{p}=\left.\int_{0}^{\infty} \int_{0}^{2 \pi} \frac{E_{x}}{p} p^{2} \sin \theta d \phi d \theta\right|^{2}  \tag{17}\\
& \int_{0}^{\infty} \int_{0}^{2 \pi} \rho^{2} \sin \theta d \phi d \theta \int_{0}^{\infty} \int_{0}^{2 \pi}\left(\frac{E_{x}}{\rho}\right)^{2} \rho^{2} \sin \theta d \phi d \theta
\end{align*}
$$

Where $\operatorname{req}^{2} \rho=\frac{2 F}{1+\cos \theta}$
and $F=$ focal length of paraboloid
The crosspolarisation was assumed to give rise to a final crosspolarisation pattern with most of its radiation near the axis. Hence it was assumed not to add to the noise temperature but only to diminish the gain. The total noise temperature was takenas

$$
T_{V} T_{R}+T_{S}\left(1-\eta_{S}\right)
$$

Where $T_{R}$ is the receiver temperature Assuming that one half of the spillover is absorbed by a black ground at $300^{\circ} \mathrm{K}$ and that the other half sees the sky at $0^{\circ} \mathrm{K}, T_{s}$ was taken as $150^{\circ} \mathrm{K}$ The results can be expressed simply for other dish orientations and sky and ground temperatures by linear scaling of $T_{R}$.

The figure of merit $M$ of the combination was approximated to by

$$
\begin{equation*}
M=10 \log \left\{\eta_{s} \eta_{p} \eta_{T} \frac{T_{R}}{T_{R}+150\left(1-\eta_{s}\right)}\right\} \tag{18}
\end{equation*}
$$

giving a value relative to aniformly illuminated aperture of the paraboloid diameter with no exess aerial noise No account was taken of feed blocking mismatching or other effects.

In the circular-fed case, the integrations were done analytically, and only single integrations were performed by the computer. For the rectangular and square feeds, the double integrations were done by computer. Integrations were done using an improved version of the adaptive Simpson procedure ${ }^{6}$ to a relative accuracy of better than 0.001 .

The approximations made in deriving the feed radiation patterns make them invalid in the range of feed dimensions used. This was brought out clearly by the spillover integration. This revealed that more energy was radiated into the cone subtended by the paraboloid than was transmitted through the waveguide aperture when the aperture transverse dimension dropped below about $0 \cdot 9 \lambda$. Thus the results are biased in favour of low $F \mid D$ ratios by having unnaturally low spillovers for small feed apertures.

The optimum horn dimensions were found by a goldensearch procedure for each $T_{R}$ and $F / D$. The golden search maximises a function of a variable over a limited range of that variable. This it does by examining the function at the golden points of the range; i.e at $\frac{1}{2}(\sqrt{ } 5-1)$ and $\frac{1}{2}(3-\sqrt{ } 5)$ of the range). Whichever is lower is taken as the boundary of a new range, and the procedure is recursively repeated it brackets the maximum of the function more and more closely, until a desired aecuracy is attained.

The results for a circular feed of the maximum figure of merit are shown in Fig, with the $H$ plane illuminations required at the edge of the paraboloid relative to that at the apex (including space taper due to the different distances from the focus to the edge and the apex of the paraboloid) in Fig. 2 , and the resulting aperture efficiencies and excers noise temperatures shown in Figs, 3 and 4 , Similar curves were obtained with rectangular and square feeds. Figs. 5 6 and 7 show the optimum feed dimensions for round, rectangular and squäre feeds. Fig. 8 shows figure of merit against $T_{R}$ for $F / D=0.4$ and 15 for all three feeds.

The scatter on many of the graphs is probably spurious and the result of the finite accuracy of the integration and search procedures.

The asymptotic values obtained for a roctangular feed as $F \mid D$ tends to infinity for $3000^{\circ} \mathrm{K}$ and $30^{\circ} \mathrm{K}$ were comparod with the results of Milne and Raab for a single-horn fcod, optimising gain alone and gain/excess temperature, respec: tively. The efficiencies and temperatures (allowing for the


Fig. 1
Figure of merit against $F / D$ for round feed


Fig. 2
Hplane edge illumination against F|D for round feed


Fig. 3
Gain efficiency agains! FID for round feed


Fig. 4
Excess temperature against FID for round feed


Fig. 5
Aperture diameter against $F / D$ for round feed.


Fig. 6
Aperture width against $F \mid D$ for rectangular feed O Hplape
$\times$ Eplane


Fig. 7
Aperiure width against FID for square feed


Fig. 1
Figures of meris compared
$-\nabla-$ quarc. $-0-$ - rectangularir. $\square \square$ round PROC. IEE, Vol 115 , NO. $1, J A N U X R Y J 968$
different environment models) agree closely, but the optimum horn dimensions derived here are slightly larger.

A point that arose in the calculations was that no optimum feed produced more than $2 \%$ of crosspolarised energy. This factor depends on the effective electric/magnetic dipole moment ratio at the feed aperture, and this is sensitive to the form taken for the refiection coefficient, since the mismatch at the foed aperture augments the electric field but diminishes the magnetic field there.?

## 3 Results

The following general conclusions can be made from the results:
(a) All three feeds give similarly shaped curves of figure of merit against $F \mid D$ or $T_{R}$. However, the curves for rectangular and circular feeds are identical to within 0.1 dB and show an advantage over a square feed of $0.2-0.4 \mathrm{~dB}$. For circular polarisation, the square and circular feeds can be used directly, but a rectangular feed has to be synthetised from a square one, with internal fins at the edges of the aperture to constrict it for each linear-polarisation component separately. However, the difference in performance between the optimum circular and rectangular feeds is probably less than the excess conduction losses that this fin loading would entail for a roctangular feed, and so a conical hom is the indicated feed for circular polarisation. ${ }^{8}$
(b) The usual feed-design procedures specify the size of the feed by the illumination occurring in the direction of the edge of the paraboloid relative to that in the direction of the apex. The present work shows that, for most values of $F / D$ and for all three types of feed, reducing the receiver temperature from 3000 to $300^{\circ} \mathrm{K}$ requires an increase in the optimum feed dimension to give an extra - 0.7 dB edge illumination in the Hplane. Reducing the receiver temperature from 3000 to $30^{c} \mathrm{~K}$ requires an extra -3 dB .
(c) Despite approximations leading to a bias in favour of low $F / D$ ratios, the results show an advantage in gain of about 0.8 dB for large over small $F / D$ ratios for all receiver temperatures. This advantage would also hold for Cassegrain or Gregorian systems with a short-focus main dish bul high magnification, as the spillover, crosspolarisation and aperture efficiency will be those apertaining to the long-focus equivalent paraboloid.? However, for these this advantage must be balance against extra losses due to blockage by the subreflector and diffraction effects due to its finite size.
(d) For all three feeds and over most $F \mid D$ ratios, there is litule change in the design for the optimum $G / T$ ratio between - receiver temperatures of 3000 and $300^{\circ} \mathrm{K}$. Small changes in aerial temperature are swamped by the large receiver temperatures, and the resulting designs are virtually for maximum $G$ alone. Thus the conclusions about optimum horn type and
dimensions, and paraboloid $F I D$ ratios are also true for applications requiring maximum $G^{2} / T$ for this range of receiver temperature. The similar excess aerial noise temperatures in this range for rectangular and circular feeds indicates a spillover of about $15 \%$ However, with a $30^{\circ} \mathrm{k}$ receiver temperature, the edge illumination should be tapered by a further 2-4dB The rectangular and circular feeds give excess aerial noise temperatures of 18 and $16^{\circ} \mathrm{K}$, respectively. These are nearly constant with respect to $F / D$ and indicate a spillover for the optimum designs of approximately 12 and $10.5 \%$ respectively.

## 4 Conclusions

The graphs of figure of merit against $F / D$ enable a system designer who is intent on using a hom-fed paraboloid to choose an FID ratio for the paraboloid he must use. He can trade the losses in the necessary waveguide run to a front feed, or the subreffector blocking and truncation losses of a Cassegrain feed, for the merit Poss as the F/D ratio of the paraboloid is reduced. The curves presented here guide the choice of feed and give the optimum feed-aperture dimensions. A small point is that the designer must ensure that there is room for the length of the flare of the feed from normal waveguide dimensions to the required feed-aperture dimensions, giving less than a $\lambda / 8$ phase error across the feed aperture.

He still has many other problems in the design of the aerial for the system. It is hoped that the results presented here can enable him to concentrate properly upon them.

## 5. Acknowledgments

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## 6 References

1. CROMPTON, J. w, On the optimum illumination taper for the objective of a microwave aefial, Proc, JEE. 1954, 101, Pt. III, pp. 371-382
2 - MILNE, K., and raAb, A. r.: Oplimmm illumination tapers for fourhorn and five-horn monopulse zerial systems", in "Design and construction of large stecrable aërials*:IEE Conf. Rep. Ser. 21.1966 pp. 12-16
2. SILYER, $s$. "Microwave antenna theory and designt (McGraw.Hill. 1949). p. 425

4 LIvingSTON. M. L.: 'The effect of anxerina characieristics on antenna' - noise temperature and system s.n.t. IRE Trans:1961.SET-7, p. 71

5 FRADIN, A. 2.: "Microwave antienmas' (Pergamon, 1961 ) ${ }^{5}$. 147
6 KUNCIR, G. F.: Alporithm 103, Commun. ACM, 1962, 5, p. 347
7 HANNAN, $P$. W, Microwave antenmas derived from the Cassegrain HANNAN
Ielescope,
IRE Trans., $1961 ;$ AP-9. pp. $140-153$
8 REINTJES, FA and COATE, GT: Principles of radar (McGrawHill. 1952. 3rd edn.), p. 955

Optimum Design of Horn Feeds for Reflector Antennas

## WILLIAM M. TRUMAN AND CONSTANTINE A. BALANIS

Abstract-A method of determining the optimum dimensions of a horn feed for a parabolic refiector using the power transferred to the feed as a criterion is described. To reduce the computation time, the focal plane and feed-horn aperture field distributions were expanded into finite power series whose coefficients were determined using collocation techniques. The paper extends previous work to include horns with flare angles greater than $10^{\circ}$ and contains useful desigi curves.

## INTRODUCTION

Although horns have been used as feeds for reflectors for many years, it was not until recently that a technique to determine the horn dimensions that produced maximum power transmission to the feed or to maxinize the aperture efficiency was reported. Rudge and Withers [1] developed the technique by utilizing a theorem reported by Aridgely [2]. The equation developed gives a measure of the power transferred to the feed as a function of the reflector focal plane field and the feed-horn aperture field. Rudge [3] found an integral equation for the focal plane field of the reflector for any $f / d$ ratio. However, his approximations leading to the integration of the focal plane field expression are not very accurate for the more practical arrangements of reflector systems ( $0.25<f / d<1.0$, or $90^{\circ}>\theta_{\text {ro }}>30^{\circ}$. Also, his assumed feed aperture field does not take into account the divergent phase front of the horn field. In this paper techniques are presented to eliminate the above shortcomings, reduce the computation time, and present useful design curves.

## THEORY

For two antennas with linearly polarized fields in the same direction, the power transmission is given [2] by

$$
\begin{equation*}
P_{r}=K \int_{S} E_{1} E_{2} d s \tag{1}
\end{equation*}
$$

where $E_{1} ; E_{:}$are the scalar fields crea ted by each antenna individually on a common surface $S$, and $K$ is a constant of proportionality. The expression is derived using the Lorentz reciprocity theorem [2]. In a reflector system arrangement, $E_{i}$ represents the reflector focal plane field, $E_{y}$ the feed-horn aperture field, and $S$ the horn aperture used as the common surface of integration. To find the optimurn dimensions of the horn for a given reflector, the power transmitted by the horn is kept constant while the dimensions are varied until $P_{r}$ is maximized.

Reierring to [4; fig. 2], Rudge [3] expressed the reffector focal plane field as

$$
\begin{equation*}
E_{1}(\psi)=G_{1} \int_{0}^{u} \frac{u}{\left(1-u^{2}\right)^{1 / 2}} J_{0}(\psi u) d u \tag{2}
\end{equation*}
$$

where; using the coordinate system of Fig. $1, u=\sin \theta, \hat{u}=\sin \theta_{\operatorname{sinx}}$, $\dot{\psi}=k l=k\left(x^{2}+y^{\dot{2}}\right)^{1 / 2}$, and $G_{1}$ is a constant.

In order to integrate $(2), J_{0}(\psi u)$ was expanded in a finite power series of the form

$$
\begin{align*}
& J_{0}(\psi u)=1+A_{1}(\psi u)^{2}+B_{1}(\psi u)^{4}+C_{1}(\psi u)^{6} \\
& \because \because+D_{1}(\psi u)^{8}+E_{1}(\psi u)^{10}+F_{1}(\psi u)^{12} \tag{3}
\end{align*}
$$

where

$$
\begin{align*}
& A_{1}=-0.2499275 \\
& B_{1}=0.1557649 \times 10^{-1} \\
& C_{1}=-0.4255879 \times 10^{-i} \\
& D_{1}=0.6194299 \times 10^{-8} \\
& E_{1}=-0.4867434 \times 10^{-i} \\
& F_{\mathrm{J}}=0.1635705 \times 10^{-8} \tag{4}
\end{align*}
$$

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Fig. 1.s Reflector and feed system coordinates.

The coefficients were determined using collocation and are valid in the range $0 \leq \psi i \leq 9$. With this expression for $J_{0}(\psi n), ~(2)$ can be integrated in a closed form, the accuracy of which will be discussed later.

In the horn aperture a $T E_{10}$ mode field is assumed of the form

$$
\begin{equation*}
E_{2}(X, Y)=G_{2} \exp \left(-j \frac{X^{2}}{4 \pi p_{z}}\right) \exp \left(-j \frac{Y}{4 \pi p_{y}}\right) \cos \left(\frac{\pi}{2 Y} Y\right) \tag{5}
\end{equation*}
$$

where $\hat{X}=k x, Y=k y, \hat{X}=k \hat{x}, \hat{Y}=k \hat{y}, \hat{x}$ is half the horn height, $\hat{j}$ is half the horn width, $\rho_{5}$ and $\rho_{y}$ are the hom lengths (in wave lengths) in the $E$ - and $H$-planes, respectively, and $G ;=G_{2}^{\prime} /(\hat{X} \hat{Y})^{1 / 2}$ is a normalization factor.
To perform the integration of (1) using (2) and (5), with reduced computation time, it was decided to expand the exponential and cosine terms of (5) into finite series and evaluate the coefficients as was done for (3). A detailed description of the expansions is given in [5].

## RESULTS AND CONCLUSIONS

When $\hat{u}=1$, (2) can be integrated into a closed form solution, and when $\hat{u}$ is small the approximate closed form solution used by Rudge and Withers [1] can be obtained as discussed in [5]. To check the validity of the collocation method used in the computations presented in this paper, we compared the series expansion technique against numerical iniegration and the special closed form solutions mentioned above.

For $\hat{u}=1$, the numerical integration and collocation methods agreed with the exact closed form solution to within one percent over the needed range of $\psi$. For small $\hat{\boldsymbol{u}}(\hat{u}=0.4)$, the collocation method agreed with the numerical integration to within one percent where the approximate closed form solution, used by Rudge and Withers [1] in their design, agreed only to within 15 percent with the results of numerical integration An expanded discussion of the accuracy: of the results is presented in [5]. The performed comparisons gave a degree of confidence in our series expansion method employed for our computations.

In Fig. 2 we plotted the optimum horn dimensions versus $\mathrm{f} / \mathrm{d}$ ratio for various horn lengths. The stars on the plots are the dimensions calculated using the method of Rudge sand Withers [1], which apply for horns of infinite length. With the curves shown in Fig. 2 , one can find the optimum dimensions of a horn for a reflector of any practical $j / d$ ratio and, by interpolation, for any practical horn length.
To find the optimum dimensions shown in Fig. 2 , the horn height and width were varied until $\cdot P_{r}$ of (1) was maximized. A topical variation of $P_{r}$ as a function of the horn dimensions is shown in Fig: 3 for $\hat{u}=0.25$ : This figure gives an indication of the sensitivity of $P_{r}$ to a change in the horn dimensions.

It should also be pointed out thist although the fechniques were applied to a pyramidal feed-horn with a TEio field distribution, the


Fig. 2. Optimum horn dimensions versus $f / d$ ratio for various horn lengths.


Fig. 3. Relative magnitude of power transfer as function of horn dimensions
methods can be used for any other feed with any desired field variation.

## REFERENCES

[1] A. W. Rudge and M. J. Withers, "Design of flared-horn primary feeds for parabolic refector antennas," Proc. IEE, 108B, pp. 1741 1749. 1961 .
12) D. Aidgely, "A theory of receiving aerials applied to the reradiation (3) or an electromannetic horn, " Proc. 1 EEEE, $108 \mathrm{~B}, \mathrm{pp}$. $645-650,1961$ [3] A. W. Rudge, "Focal-plane field distribution of parabolic refectors," Electron. Lett., vol. 5, pp, 510-512, 1969.
[4] P. A. Matthers and A. L. Cullen, A study of the fleld distribution at an axial focus of a square microrave lens," Proc. IEE, 103C, pp
(5) 4 . 456.1956
in . Truman "Optimization design methods of feeds for reflector antennas, MISEE thesis, Dep. Elec. Eng.. West Tirginia University, Morgantown, W. Va., Dec. 1973.

# Offset-Parabolic-Reflector Antennas: A Review: 

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#### Abstract

Although used for some decades, the offset-parabolicreflector antenna's electrical properties and performance were not accurately modeled and optimized until the 1960's. This paper reviews; in a tutorial fashion, the state of the art of this important antenna for readers who are not necessarily experts in antenna theory and tectnology. After a discussion of fundamentals, the performances of both singe- and double-reflector configurations are treated and compared, and practical primary feeds are described. Comments are given on the present status of analysis and design and on problems to be solved:


## I. Introduction

## A. Fundamental Advantages and Disadvantages

TTHE OFFSET-parabolic reflector has found applications as an antenna for many years and was certainly receiving some attention during the 1940 's. However, it is only in comparatively recent times that analytical and numerical models have been developed for this device which can provide reliable predictions of its electrical properties. Although the basic analytical techniques were available at the end of the World War II, the offset-reflector geometry did not readily lend itself to analysis without the aid of a digital computer. Hence, it was not until the 1960's that development in digitalcomputer technology provided a readily available and convenient means for accurate modeling and optimization of the offset antenna's electrical performance.
Since the offset-parabolic reflector is a somewhat more complicated strucrure to deal with both structurally and analytically, it will be as well to briefly review its principal advantages and disadvantages as an antenna. First and foremost, the offsetreflector antenna reduces aperture-blocking effects. This fact, which is illustrated in Fig. l, represents a very significant advantage for the offset configuration over comparable axisymmetric counterparts. Aperture blocking by a primary-feed or a subreflector, with their supporting struts, leads to scattered radiation which results in a loss of system gain on the one hand and a general degradation in the suppression of sidelobe and cross-polarized radiation on the other. These latter effects are becoming increasingly important as antenna spurious radiation specifications tighten and frequency reuse requirements demand higher levels of isolation between orthogonal hands of polarization.
A second major advantage of the offset configuration is that the reaction of the reflector upon the primary-feed can be reduced to a very low order. The excellent isolation between reflector and primary-feed which can be achieved implies that the primary-feed VSWR can be made to be essentially independent of the reflector. . When multiple-element or dual-

[^3]

Fig. 1. Basic single-offset-reflector configuration.
polarized primary-feeds are to be employed, the mutual coupling occurring between feed elements via the reflector can be reduced to an insignificant level.

Compared to an axisymmetric paraboloid, the offset configuration leads to the use of larger focal-length to diameter ratios ( $F / d$ ) while maintaining an acceptable structural rigidity: As a consequence, the offset-reflector primary-feeds employ relatively larger radiating apertures which, in the case of multipleelement primary-feeds, will resilt in lower direct mutual coupling between adjacent feed elements The use of larger aperture primary-feed elements in certain cases can also provide an opportunity for improved shaping of the primary-feed radiation pattern and better suppression of the cross-polarized radiation emanating from the feed itself
The offset-reflector configuration also has its disadvantages. When illuminated by a conventional linearly polarized primaryfeed, the offset reflector will generate a cross-polarized component in the antenna radiation field. When circular polarization is employed, the reflector does not depolarize the radiated field, but the antenna beam is squinted from the electrical boresight. For small offset reflectors this squinting effect has also been observed with linear polarization.
Structurally the asymmetry of the offset reflector might be considered as a major drawback, although there are many applications where its structural peculiarities can be used to good advantage. In the design of spacecraft antennas, for example, an offset configuration can often be accommodated more satisfactorily than an axisymmetric design, particularly when it is necessary to deploy the reflector after launching. Nevertheless, it is clear that the offset geometry is more difficult to deal with and generally more costly to implement. For these reasons its use in the past has tended to be restricted to applications where electrical performance specifications have been severe. The rapid growth in the use of offset antenna systems
in more recent times is an indication of the increasing demands being made upon antenna performance.

## B. Single- and Dual-Reflector Systems

Similar to its axisymmetric counterpart, the offset-parabolic reflector can be utilized as a single-reflector fed from the vicinity of its prime-focus or arranged in a dual-reflector system where the main offset reflector is illuminated by the combination of a primary-feed and subreflector. By this means offset Cassegrainian and offset Gregorian systems can be designed. A further dual-reflector system will be dealt with here in which the primary feed illuminates an offset section of a hyperboloid, and the combination feed an offset parabolic reflector. The geometry of this configuration can be adjusted such that no blocking of the optical path occurs either by the primary-feed or the subreflector. The primary-feed in this case is located below the main parabolic reflector. This arrangement contrasts with the open Cassegrainian configuration originated by Bell Laboratories in which the primary feed protrudes through the main reflector. To distinguish between these configurations, the no-blocking case will be termed a double-offset reflector antenna, while the general case will be referred to as dual-reflector offset antennas. Fig. 2 illustrates some of the configurations which are of particular interest here.

## C. Background

Much of the initial difficulty in dealing with the offsetparabolic reflector can be attributed to its asymmetric geometry. This geometry is the key to the analysis of the offset antenna and to ultimately understanding its electrical properties. One of the best analyses of the offset-reflector geometry can be found in a monograph issued by the Bell Telephone System [1]. This work by Cook et al., which was concerned with the analysis of a dual-reflector open Cassegrainian system, was published in 1965 [2]. Much of the subsequent analysis of the offset reflector either makes use of this geometry or, if performed independently, fellows a similar approach to that established by these authors.
The depolarization properties of asymmetric antennas have deservedly received considerable attention in the literature. While the polarization characteristics of the offset reflector were subject to the independent study of a number of authors, including the original work by Cook et al. [2], Chu and Turnin [3] first published detailed graphical data and provided a clear insight into the beam-squinting properties of the circularly polarized prime-focus-fed offset reflector. The radiation properties of offset-reflector antennas with off-axis feeds were studied by Rudge et al. [4]-[7], while Dijk et al. [8], [9] performed an in-depth analysis of the polarization losses of offset paraboloids. The low cross-polar radiation achievable with large $F / d$ ratio offset reflectors was confimed by Gans and Semplak [10].
The optimization of the geometry of dual-reflector offset antennas to reduce or eliminate cross-polarized radiation was first demonstrated by Graham [11] and confirmed theoretically by Adatia [12] in the U.K. Working independently, Tanaka and Mizusawa [13] established a simple geometric optics-based formula for this optimization process.
The reduction of offset reflector cross polarization by use of a field-matching primary-feed technique was proposed and demonstrated by Rudge and Adatia [14], [15]. Jacobsen [16] has made the point that, in principle, similar results could be


Fig. 2. Dual-offset-reflector configurations. (a) Double-offset system. (b) optimized double offset. (c) Open Cassègrainian system. Paraboloid vertex at 0 and feed phase center located at $0^{\prime}$.
achieved with an array of Huygen sources directed toward the vertex of the parent paraboloid and phased to direct the energy in to the cone of angles subtended by the offset portion of the reflector: A comparison of the radiation pattern and impedance properties of offset-Cassegrainian and offsetGregorian antennas with their symmetrical counterparts has been performed by Dragone and Hogg [17]. Trair results confirm the anticipated advantages of the offset structures with regard to both sidelobe radiation levels and VSWR. The use of offset-reflector antennas in applications where very low sidelobe radiation is mandatory is also receiving attention [18], [39].
The avoidance of aperture blockage implies that offset reflectors should offer good potential as multiple-beamantennas. This possibility has been investigated by a number of workers. Rudge et al. $[4]-[7],[19],[20]$ have studied the use of small clusters of feed elements, both linearly and circularly polarized, in conjunction with single offset reflectors. Ingerson [21] has also investigated the off-axis scan characteristics of offset reflectors, and Kaufmann and Croswell [22] have considered the effect of large axial displacements of the offset-reflector primary-feed. Ohm [23] has analyzed a proposed multiplebeam earth station based upon a dual-reflector offset system.

The use of offset-reflector systems to provide shaped or contoured beams for satellite communcations has also received attertion - Shaped beams have been achieved cither by deforming the offset-reflector surface, as described by Wood:
et al. [24], or by the use of a weighted array or primary-feed elements, as favored by Han [25] and his colleagues for the Intelsat V communications satellite.

Although their low sidelobe potential makes offset-reflector antennas attractive for many radar applications, difficulties were experienced in the past when a precision tracking capability was required. These difficulties, which arise as a result of the offset-reflector depolarization, are now well understood, and the means of compensating for these effects by use of improved monopulse primary feeds have been recently demon-: strated [26]

## D. Scope and Method

The purpose of this paper is to provide a tutorial review of the state of the art in offset-parabolic-reflector antennas. Although the information is intended to be of value to readers who are not necessarily familiar with this particular technical area, the authors have assumed a familiarity with basic antenna technology and a reasonable understanding of antenna theory.
The offset parabolic reflector antennas treated included both single- and dual-reflector configurations. However, space constraints do not permit attention to be given here to the various strictures with one-dimensional parabolic curvature, such as the parabolic cylinder and the parabolic torus, all of which can be designed in offset configurations. ${ }^{1}$

The paper commences by describing some basic techniques which can be applied to predict the vector radiation fields scattered from single-reflector structures with two-dimensional parabolic curvature and provides some indication of the validity and accuracy of those most commonly used. The geometry of the offset reflector is considered, and the authors' preferred definitions for the copolarized and cross-polarized radiation fields are stated. The formulation of a relatively simple analytical model for the offset-reflector antenna is then described, based upon the physical-optics approximation. Techniques for computing the two-dimensional integrals, which comprise the core of the mathematical model, are briefly discussed, and guidance is given on the choice of analytical models for the primary-feed radiation.

The electrical performance of the single-reflector offset antenna is then examined by comparing predicted and measured data.. The particular problems arising from the choice of polarization and the reflector dimensions are highlighted, and some applications involving multiple beams, shaped beams, contoured beams, and monopulse tracking are briefly reviewed,
Practical primary-feeds for offset-reflector antennas are discussed and the matched-feed concept for offset antennas is outlined. To clarify the matched-feed technique, the general characteristics of the focal-plane fields of the offset reflector are examined, and the matching of the electric fields in the primary-feed aperture to the reflector focal fields is illustrated. The improved performance potential of offset reflectors with matched feeds is then considered by examining their use in antennas for communication and for monopulse tracking.

The advantages and disadvantages of dual-reflector offset antennas are then examined, with particular emphasis upon the open Cassegrainian configuration devised by Bell Laboratories, and the optimized double-offset configuration

[^4]which offers, in principle, both freedom from blockage and low levels of cross-polarized radiation.

The paper concludes with some comments on the existing state of the art in offset-reflector analysis and design and the problems which remain to be solved in this area of antenna. technology.

## II. Single-OfFSET-REFLECTOR ANALYSIS

## A. Basic Techniques

A number of well-established techniques exist which can be applied to predict the vector radiation fields scattered from offset parabolic reflectors. The basic techniques are well described in texts by Silver [27], Collin and Zucker [28], and Rusch and Potter [29], and a wide range of general applications have been reported in the literature. The best known of these techniques are those based upon variations of the aperture-field and current-distribution methods. " In practical applications both methods make use of the physicaloptics approximation, which essentially demands that the reflector surface be large relative to the operating wavelength:

Referring to Fig. 3, the current-distribution method can be applied to determine the scattered field at a field point $P$ from a knowledge of the electric-current distribution on the reflector surface, when the reflector is illuminated by a primary radiator. Invoking the physical-optics approximation, the electric current on the illuminated surface of the reflector ( $\bar{J}_{s}$ ) is taken to be

$$
\begin{equation*}
\bar{J}_{s}=2\left(\bar{a}_{n} \times \bar{H}_{i}\right) \tag{1}
\end{equation*}
$$

where $\bar{H}_{i}$ is the incident magnetic field of the primary radiator at the reflector surface, and $\bar{a}_{n}$ is the surface unit normal.

Similarly, for the aperture-field method, the source of secondary radiation can be taken as the tangential electricfield distribution on an infinite plane immediately in front of the reflector and perpendicular to the axis of the parent paraboloid. This is illustrated in Fig. 3. The tangential elec-tric-field distribution within the aperture plane of the reflector is obtained approximately by assuming that the incident field of the primary radiator is reflected optically into this plane. The electric field reflected from the reflector is giyen by

$$
\begin{equation*}
\bar{E}_{r}=2\left(\bar{a}_{n} \cdot \bar{E}_{i}\right) \bar{a}_{n}-\bar{E}_{i} \tag{2}
\end{equation*}
$$

where $\bar{E}_{i}$ is taken as the radiation field of the primary-feed at the reflector surface.
In keeping with the physical-optics approximation, which is applied in both of these methods, it is assumed that in the shadowed regions of the reflector the fields and currents are assumed to be zero. In the current-distribution method this refers to the back of the reflector, while in the aperturefield case it implies a zero tangential electric field on the infinite plane outside of the reflector projected aperture To satisfy Maxwell's equations at the boundaries between the zero and nonzero field regions, a line distribution of electromagnetic charges can be introduced along the boundary curve in either of the above cases. The net effect of this boundary charge distribution is to ensure that the predicted radiation fields satisfy the radiation conditions and thus do not include a radiated field component in the direction of propagation.

To determine the scattered field at the field point it is necessary to integrate the effects of the elemental currents or fields


Fig. 3. Surfaces of integration for surface-current current technique ( $S_{2}$ ) and aperture-field method $\left(S_{2}\right)$.
over the relevant surfaces $S_{1}$ or $S_{2}$ as shown in Fig. 3. For offset reflectors with projected aperture diameters exceeding 20 wavelengths, the two techniques provide virtually identical predictions for the copolarized fieids over the main lobe and first four or five sidelobes. At wide angles from the boresight, the predictions differ and these differences tend to increase with increasing reflector curvature. For the cross-polarized radiation, the two methods differ significantly only when the peak levels of this radiation are very low (i.e., less than -50 dB relative to the peak value of the main beam copolarized field). The electric-current method is generally considered to be the more accurate of the two methods; but for most practical purposes the differences between the predictions tend to be insignificant. For small offset parabolic reflectors (i.e., projected aperture diameters of less than 20 wavelengths) the discrepancies between the predictions of the techniques become more discernable. Boswell and Ashton [30] have recently shown that a beam-squinting effect can occur with small linearly polarized offset reflectors. This effect is predictable using the surface-current technique but not with aperture fields. In their example a beam squint of 0.03 of a beamwidth occurred with a reflector of 6 -wavelengths diameter: This result is particularly interesting in that it is a comparatively rare example of experimental confirmation of the accuracy of the surface-current technique for small parabolic reflectors:
In dealing with the radiation from large offset-parabolicreflector antennas in a moderate cone of angles about the antenna boresight and over a dynamic range of the order of $50-60 \mathrm{~dB}$, there is, for most practical purposes, little significant variation between the predictions obtained by the two methods. Under these circumstances the technique which is more convenient analytically and computationally can be
employed. On this basis the aperture-field method; which involves an integration over a planar surface, results in generally more simple mathematical expressions and thus offers some saving in computational effort.

## B. Offset-Reflector Geometry

The geometry of the single-offset-parabolic reflector is shown in Fig. 4. The basic parameters of the reflector are shown as the focal length $F$ of the parent paraboloid, the offset angle $\theta_{0}$, and the half angle $\theta^{*}$ subtended at the focus by any point on the reflector rim. With $\theta^{*}$ maintained constant, a rotation about the inclined $z$ axis will generate a right circular cone with its apex at the reflector geometric focus. If the boundary of the parabolic surface is defined by its intersection with the cone, then the resultant reflector will have an elliptical contour, while the projection of this contour onto the $x$ 'y plane will produce a true circle.
To deal with this offset geometry it is desirable to obtain a coordinate transformation from the primary spherical coordinate system of the symmetrical parent paraboloid to an offset spherical coordinate system about the inclined $z$ axis. The reflector parameters which are readily expressed in terms of the symmetrical primary coordinates can then be transformed into the offset coordinate system. If the symmetrical coordinates are defined conventionally as $\rho^{\prime}, \theta^{\prime}, \phi^{\prime}$ with associated Cartesian coordinates $x^{\prime}, y^{\prime}, z^{\prime}$, then their relationships to the unprimed offset coordinates $\ddot{\rho}, \theta, \phi$ and $x ; y, z$ are obtained by simple geometry as:

$$
\begin{gather*}
\rho^{\prime}=\rho  \tag{3}\\
\cos \theta^{\prime}=\cos \theta \cos \theta_{0}-\sin \theta \sin \theta_{0} \cos \phi \tag{4}
\end{gather*}
$$

$$
\begin{equation*}
\sin \theta^{\prime} \sin \phi^{\prime}=\sin \theta \sin \phi \tag{5}
\end{equation*}
$$

$\sin \theta^{\prime} \cos \phi^{\prime}=\sin \theta \cos \theta_{0} \cos \phi+\cos \theta^{\sin } \theta_{0}$

$$
\begin{equation*}
\tan \phi^{\prime}=\sin \theta \sin \phi /\left(\cos \theta \sin \theta_{0}+\sin \theta \cos \theta_{0} \cos \phi\right) \tag{6}
\end{equation*}
$$

and these equations provide the basis for the transformations.
Some dimensions of particular relevance are given here and in Appendix A, while [1] provides an excellent source of further information. The distance ( $p$ ) from the reflector focus to a point on the parabolic surface is given by:

$$
\begin{align*}
& p=2 F /\left(1+\cos \theta^{\prime}\right) \\
& =2 F /\left(1+\cos \theta \cos \theta_{0}-\sin \theta \sin \theta_{0} \cos \phi\right) \tag{8}
\end{align*}
$$

The diameter of the projected aperture of the offset reflector $(d)$ is best obtained in the principal $x^{\prime} z^{\prime}$ plane. From the geometry we obtain

$$
\begin{equation*}
d=4 F \sin \theta^{*} /\left(\cos \theta_{0}+\cos \theta^{*}\right) \tag{9}
\end{equation*}
$$

and similarly the distance $\left(x_{0}^{\prime}\right)$ from the axis of the parent paraboloid to the centre of the projected aperture is obtained as

$$
\begin{equation*}
x_{0}^{\prime}=2 F \sin \theta_{0} /\left(\cos \theta_{0}+\cos \theta^{*}\right) \tag{10}
\end{equation*}
$$

## C. Copolar and Cross-Polar Definitions

The radiation fields from an antenna can be completely specified in terms of two vector components. The definition of the two components at a point in space and their identification in terms of a copolarized and cross-polarized compo-


Fig. 4. (a) Single-offset-reflector coordinate system. (b) Constant $\theta, \phi$ contours on projected-aperture plane.
nent is somewhat arbitrary. Ludwig [31] clarified and discussed some of the popular choices in his 1973 paper. For offset-reflector antennas which are operated in a predominantly. linearly polarized mode, the definition which is preferred by the authors is the third definition presented by Ludwig in the referenced paper. With this choice, the principally polarized or copolar field is that which would be measured by a conventional antenna-range technique with the polarization of the distant source initially aligned with that of the test antenna on boresight and remaining fixed while the test antenna is rotated in the normal way to produce the measured radiation patterns (e.g., see $[27$, ch. 15$]$ ). If the polarization of the distant source antenna is then rotated through $90^{\circ}$, and the radiationpattern measurements repeated, then the recorded data represents a measurement of the cross-polar field components.

In the primary spherical coordinate system $\rho, \theta, \phi$ shown in Fig. 4 the primary-feed spherical-coordinate fields will be defined as $E_{\theta}, E_{\dot{\phi}}$. If the primary-feed antenna has its principal electric vector along the $y$ axis then the feed copolar measuredfield component $\left(e_{p}\right)$ and the cross-polar component $\left(e_{q}\right)$ can be simply defined by [.7], [31]

$$
\left[\begin{array}{c}
e_{p}  \tag{11}\\
e_{q}
\end{array}\right]=\left[\begin{array}{cc}
\sin \phi & \cos \phi \\
\cos \phi & -\sin \phi
\end{array}\right]\left[\begin{array}{c}
E_{\theta} \\
E_{\phi}
\end{array}\right]
$$

Similarly, in terms of the secondary coordinate system, $r, \Psi$, $\Phi$; the overall radiation fields from the antenna will be defined as $E_{\Psi}, E_{\Phi}$, and the copolar $\left(E_{p}\right)$ and cross-polar ( $E_{Q}$ ) can be obtained from the right-hand side of (11) with $\Psi$, $\Phi$ replacing $\theta, \phi$, respectively.

It is worth noting that a zero cross-polarization primary-feed by the definition of (11) will produce a purely linearly polarized field in the projected aperture plane of an axisymmetric paraboloidal reflector. This condition will in turn result in a low level of cross-polarized radiation in the overall antenna far field, providing that the reflector aperture is large with respect to the wavelength and that the blockage effects are small. However, the field distribution in the mouth of the primary feed will not be purely linearly polarized but must exhibit some field-line curvature to establish this desired radiation condition [32].

## D. Projected-A perture Fields

The tangential electric-field distribution in the aperture plane of the offset reflector $\left(\bar{E}_{a}\right)$ can be determined approximately by assuming an optical reflection of the incident primary-feed fields and thus making use of (2). Since, after reflection, the wave travels as a plane wave, the tangential eleatric field in the projected-aperture plane is obtained by
the projection of (2) into the $x$ ' $y$ plane. Hence

$$
\begin{equation*}
\bar{E}_{a}=\bar{E}_{r} \exp j k(p-2 F) \tag{12}
\end{equation*}
$$

where $k=2 \pi / \lambda$ is the wavenumber. If the incident primary field at the reflector is taken as the radiation field of the primary-feed, then this can be expressed as

$$
\begin{equation*}
E_{i}=\frac{1}{p}\left(A_{\theta}(\theta, \phi) \bar{a}_{\theta}+A_{\phi}(\theta, \phi) \bar{a}_{\phi}\right) \exp (-j k p) \tag{13}
\end{equation*}
$$

where $A_{\theta}, A_{\phi}$ are the normalized spherical coordinate components of the primary feed radiation pattern.
Substituting in (12) from (2) and (13) and making use of the relationships derived in Section II-B above, the offset-reflector projected-aperture electric-field distribution can be expressed in Cartesian components ( $E_{a x}, E_{a y}$ ) as

$$
\left[\begin{array}{l}
E_{a y}(\theta, \phi)  \tag{14}\\
E_{a x}(\theta, \phi)
\end{array}\right]=K\left[\begin{array}{ll}
-S_{1} & C_{1} \\
C_{1} & S_{1}
\end{array}\right]\left[\begin{array}{l}
A_{\theta}(\theta, \phi) \\
A_{\phi}(\theta, \phi)
\end{array}\right]
$$

where

$$
\begin{align*}
S_{1} & =\left(\cos \theta_{0}+\cos \theta\right) \sin \phi  \tag{15}\\
C_{1} & =\sin \theta \sin \theta_{0}-\cos \phi\left(1+\cos \theta \cos \theta_{0}\right)  \tag{16}\\
K & =\exp (-2 k F) / 2 F . \tag{17}
\end{align*}
$$

Or for a circularly polarized primary feed

$$
\left[\begin{array}{l}
E_{a R}(\theta, \phi)  \tag{18}\\
E_{a L}(\theta, \phi)
\end{array}\right]=\frac{2 F K}{p}\left[\begin{array}{ll}
\exp (j-\Omega) & -j \exp (j-\Omega) \\
\exp (-j \Omega) & j \exp (-j \Omega)
\end{array}\right]\left[\begin{array}{l}
A_{\theta}(\theta, \phi) \\
A_{\phi}(\theta, \phi)
\end{array}\right]
$$

whère

$$
\begin{equation*}
\Omega(\theta, \phi)=\arctan S_{1} / C_{1} . \tag{19}
\end{equation*}
$$

If the circularly polarized primary-feed has a nomnalized radiation-pattern of the form

$$
\begin{equation*}
\bar{A}_{n}(\theta, \phi)=\left(A_{\theta}(\theta) \bar{a}_{\theta}-j A_{\phi}(\theta) \bar{a}_{\phi}\right) \exp (-j \phi) \tag{20}
\end{equation*}
$$

where the functions $A_{\theta}$ and $A_{\phi}$ are independent of $\phi$, then the offset-reflector aperture-plane fields have the form

$$
\begin{equation*}
E_{a M}=\frac{2 F K}{p}\left(A_{\theta}(\theta) \pm A_{\phi}(\theta)\right) \exp (-j \phi \pm \Omega) \tag{21}
\end{equation*}
$$

where $M$ is either $R$ or $L$ and $L$ takes the upper sign. Equation (21) can be satisfied by many practical types of circularly polarized conical horn, including fundamental mode, dual mode (Potter), and corrugated types [32], [50]. It is apparent that when $A_{\theta}(\theta)=A_{\phi}(\theta)$ which is the condition for zero crosspolarized radiation from the conical feed, then the reflector aperture plane will be purely copolarized with a beam-squinting phase distribution given by $\phi+\Omega$.
This can be contrasted with the linearly polarized case in which the introduction of a zero cross-polarized primary feed fails to reduce the projected-aperture field to a single linear polarization, unless the offset angle $\theta_{0}$ is put to zero.

## E. Mathematical Models for Offset-Reflector Antennas

Making use of the physical-optics approximation and wellestablished vector-potential methods, mathematical models can be established for offset-reflector antennas. For example, employing the tangential aperture-field approach, the nor-
malized copolar $\left(E_{p n}\right)$ and cross-polar $\left(E_{q n}\right)$ radiation patterns of a linearly polarized offset-reflector antenna can be expressed as

$$
\begin{align*}
& {\left[\begin{array}{l}
E_{p n}(\Psi, \Phi) \\
E_{q n}(\Psi, \Phi)
\end{array}\right]=\frac{1+\cos \Psi}{2}\left[\begin{array}{l}
1-t^{2} \cos 2 \Phi t^{2} \sin 2 \Phi \\
t^{2} \sin 2 \Phi+1+t^{2} \cos 2 \Phi
\end{array}\right]} \\
& {\left[\begin{array}{l}
F_{p}(\Psi, \Phi) \\
F_{q}(\Psi, \Phi)
\end{array}\right]} \tag{22}
\end{align*}
$$

where $F_{p}, F_{q}$ are the spatial Fourier transforms of the copolar and cross-polar tangential electric-field distribution in the projected-aperture plane of the offset reflector, and $t=$ tan $\Psi / 2$.

Similarly, for a circularly polarized antenna the far-field radiaison pattern can be expressed in terms of its normalized right- and left-hand components by [7]

$$
\begin{array}{r}
{\left[\begin{array}{l}
E_{R n}(\Psi, \Phi) \\
E_{L n}(\Psi, \Phi)
\end{array}\right]=\frac{1+\cos \Psi}{2}\left[1 \quad t^{2} \exp (j 2 \Phi)\right.} \\
t^{2} \exp (-j 2 \Phi)  \tag{23}\\
{\left[\begin{array}{l}
F_{R}(\Psi T \Phi) \\
F_{L}(\Psi y)
\end{array}\right]}
\end{array}
$$

where $F_{R}$ and $F_{L}$ are the spatial Fourier transformations of the right- and left-hand components of the projected-aperture tangential electric field.
The general form of the transform in terms of the geometrical parameters specified in (10) and Appendix $A$ is given by

$$
\begin{equation*}
\left.F=\int_{x, y} \int_{E_{a}}\left(x^{\prime}, y\right) \exp j k R^{\prime}\left(x^{\prime}, y, \Phi\right) d x^{\prime} d\right)^{\prime} \tag{24}
\end{equation*}
$$

where

$$
R^{\prime}\left(x^{\prime}, y\right)=\left(x_{0}^{\prime}-x^{t}\right) \sin \Psi \cos \Phi+y \sin \Psi \sin \Phi
$$

and $E_{a}$ is the tangential cmerture field as specified by either (14) or (18).

To predict the offset antenna radiation pattern it is necessary to set values for the enflector parameters $\left(F, \theta_{0}\right.$, and $\left.\theta^{*}\right)$, to specify the primary-feed radiations fields at the reflector ( $E_{\theta}, E_{\phi}$ ) and to then compute the two-dimensional transform integrals: Having calculated the transform functions, the computation of the radiating field by means of [22] or [23] is trivial. For accurate predictions the choice of primary-feed model is critical, while the evaluation of the two-dimensional integrals clearly represents the crux of the computational problem.

## F. Numerical Integration

There is a wide variety of numerical techniques which can be applied to the evaluation of two-dimensional integrals of the type given in (24) However any definition of an optimum technique will be dependent upon the particular type and range of applications which are envisaged. The critical factors of accuracy versus computational time and storage requirements will favor different techniques under different conditions and Weightings. No one technique appears to be superior in all cases.
In a study of computational techniques performed in 1973 [4] in connection with offset-reflector modeling, it vas concluded that high-order quadrature techniques are not well-
suited to this problem since they introduce excessive roundoff errors. Of the low-order quadrature methods, the repeated application of the one-dimensional Romberg method [33] was found to be the most efficient in that it required the least number of integrand evaluations to achieve a specified accuracy. The principal interest here was the computation of the main lobe and the first 4 or 5 sidelobes. Direct two dimensional integration techniques were found to be inferior to the preferred one-dimensional Romberg approach, both on the grounds of their efficiency and their general flexibility [34].
Lessow et al. [35] described a comparison made between a number of numerical integration techniques, including the Romberg method and a method devised by Ludwig [36]. Their conclusions wexe that for the main lobe and the near-in sidelobes of the radiation pattern, the repeated use of the onedimensional Romberg method was preferable. For computations out to the wide-angle sidelobes, they considered that the well-known Simpson method provided a preferable choice, particularly where extremely high accuracies are required. The Ludwig method was found advantageous for far-out sidelobe predictions where the phase factor in the integrand was varying very rapidly.

Ludwig's method is based upon the assumption that over a differential surface element with dimensions of the order of a square wavelength the amplitude and phase of the integrand can be expanded into linearized Taylor series form. This expansion allows the resulting expression to be integrated in closed form for each differential surface element. The total field contribution is then obtained by merely summing over the contributions from the constituent elements of the surface. It has been established that with incremental areas two thirds of a wavelength, for example, the absolute errors are more than 40 dB below the main field maxima.
The Romberg integration method employs the trapezoidal integration rule with successive interval halving and a mathematical correction technique known as Richardson's extrapolation. It offers the advantage that the number of iterations can be automatically adapted within the program to satisfy a specified accuracy. However, since it employs interval halving, it is not well-suited for use with a fixed integration grid.

## G. Primary-Feed Models

Techniques for theoretically modeling parabolic reflectors have received considerable attention in the literature in the past [1]-[7].. The need for accurate yet mathematically simple-models for the primary-feed radiation has tended to be somewhat overlooked. However, this aspect of reflectorantenna modeling is critical can raise more difficulties and produce more serious discrepancies than many of the differences observed between reflector-modeling techniques. The accuracy of primary-feed models becomes particularly critical when predictions of cross-polarized radiation are of concern.

For the prediction of radiation fields from microwave horns it has been found that analytical models based upon an assumed tangential electric-field distribution in the horn mouth can provide reasonable predictions of both the copolar and cross-polar radiation fields over the range of angles subtended by a typical reflector [4], [32], [37]. In its simplest form this approach, which has been termed the $E$-field model, essentially involyes an assumption that the hom mouth is sur-


Fig. 5. Radiation patterns in diagonal plane for $0.92 \lambda \times 1.11 \lambda$ pyramidal horn. Measured $\longrightarrow$. Predicted $\longrightarrow$


Fig. 6. Copolar and cross-polar radiation fields from a circularly polarized fundamental-mode conical horn with aperture diameter of $1.25 \lambda —$ Predicted ( $E$-field model). $\rightarrow \longrightarrow$ Predicted (Chu model) ----. Measured data.
rounded by an infinite flange. The expressions for the model in this form are identical to (22) or (23). The tangential aperture fields in this case can be taken as the wàveguide modes in a cross section of an infinite waveguide, and the model can be improved for larger horns by multiplying the aperture-field expression by a phase distribution function to account for the sphericity of the phase front in the hom mouth.

Studies have shown that for horns having aperture dimensions exceeding about 1 wavelength, good correiation can be


Fig. 7. (a) Peak cross-polar levels occurring in diagonal planes of funda-mental-mode conical horns of diameter $d_{h} .(A)$ : Predicted Chu model. (B): Predicted $E$-field model with superimposed measured data from a variety of conical horns. (b) Peak radiated cross-polar level in the diagonal plane of a fundamental-mode square-aperture waveguide horn with side dimensions $b$ wavelengths.
obtained between the $E$-field model and horns with finite flanges. Some typical results are shown in Figs. 5-7. It is noteworthy that the $E$-field model provides improved predictions over the method due to Chu [27] when applied to rectangular or conical horns of either the smooth-walled or corrugated varieties (32). The differences between the predictions obtained from the two methods can be very significant when cross polarization is of interest.

For horns having aperture dimensions of less than 1 wavelength the radiation pattern becomes increasingly flange dependent, and for small or nonexistent flanges the $E$-field model is not reliable. However, its validity can be extended into this region by using the $E$-field model in conjunction with the - geometric theory of diffraction (GTD). The GTD can be applied to predict the additional contribution of the finite flanges to the radiation field, and this hybrid model can provide significantly improved predictions for the vector radiation fields from small homs. A more rigorous solution for the vector radiation from small waveguide horns can be obtained by an application of the method of moments [38]. However, this technique is more appropriate for the analysis of electrically small waveguide homs alone, rather than in conjunc-


Fig. 8. Radiation fields from a $K$-band offset-reflector antenna. Copolar radiation in (a) plane of asymmetry $(\Phi=\pi / 2)$ and (b) plane of symmetry $(\Phi=0)$. Cross-polar radiation in (c) plane of asymmetry. Measured $\longrightarrow \quad$ Predicted - .
tion with an offset main reflector, when the combined computational effort may become excessive.

## III. Electrical Performance of the Single OFFSET-REFLECTOR ANTENNA

## A. Linear Polarization

The literature provides evidence of a number of analytical models, largely based upon physical optics techniques; which have been applied to study the radiation characteristics of the offset parabolic reflector Precise experimental data is somewhat more sparse, but sufficient material has been published to validate the main conclusions drawn from the analytical studies.
When fed by a purely linearly polarized primary feed (as defined by (11)) the offset-parabolic reflector exhibits a characteristic depolarizing effect which results in the generation of two principal cross-polarized lobes in the plane of asymmetry (i.e, the $y z$ plane or $\Phi=\pi / 2$ plane in the coordinate system of Fig. 4). Fig. 8 shows neasured data obtained with a precisely machined offset reflector fed by a fundamental-mode rectancular horn with a-12- c B illumination taper at the feflector rim [5], [7]. Predicted data obtained from a numerical


Fig. 9. Peak copolar sidelobe levels in planes of symmetry ( $S$ ) and asymmetry ( $A$ ) for offset reflectors with $\theta_{0}=\theta^{*}+5^{\circ}$, fed by uniformly illuminated circular aperture feeds producing $-10-\mathrm{dB}$ (subscript 1) and $-20-\mathrm{dB}$ (subscript 2) illumination tapers at $0=\theta$. Peak cross-polar levels (C) occurring in plane of asymmetry are also indicated.


Fig. 10. (a) Polarization loss-efficiency factor of ofset paraboloid reflector, offset angle $\theta_{0}$ being a parameter, jlluminated by - electric dīpole oriented along $x$, axis and Huygens source and - - - - electric dipole oriented along $y$ axis. (b) Polarization loss-efficiency factor of open Cassegrainian antenna, offset angle $\theta_{0}$ being a parameter, illuminated by - electric dipole oriented along $x^{\prime}$ and $y^{\prime}$ axis and --- - Huygens source.


Fig. 11. (a) Radiation fields from offset reflector ( $F=30.4 \lambda, \theta_{0}=35^{\circ}, \theta=30^{\circ}$ ) with hinearly polarized pyramidalhorn primary feed offset transwersely along $y$ axis by $0,0.83 \lambda$, and $2.5 \lambda$, respectively. (b) Copolar radiation field from circularly polarized offset reflector ( $F=22.7 \lambda$, $\theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ) with scalar-horn feed offset transversely by $0,1.4 \lambda$, and $2.8 \lambda$, respectively. __ Measured. $\rightarrow$ Predicted.
model has been superimposed, and, in general, an excellent correlation is observed. The cross-polar correlation is slightly inferior; and, in fact, better correlation was observed with later unpublished results. These radiation patterns are characteristic of the linearly polarized offset reflector, and similar results have been published by Cook et al. [1], [2], Chu and Turrin [3], and others. The cross-polar lobes which arise as a consequence of the reflector asymmetry have peak values in the vicinity of the $-6-\mathrm{dB}$ contour of the main copolarized beam, and the next subsidiary lobes are typically a further -20 dB down below the peaks. The copolar sidelobe radiation has the form of a well-defined diffraction pattern, with a comparatively rapid decay of sidelobe levels with increasing angle from boresight. Numerical models of the type discussed above reproduce these characteristics reliably and can be used profitably in performing parameter studies. For example, Fig. 9 illustrates some general sidelobe trends for fully offset anternas in which the reflector parameters are chosen such that the primary-feed hardware does not protrude into the projected aperture of the reflector.

It is evident from Fig. 9 that the peak value of the reflector cross polarization is dependent very largely upon the parameters $\theta_{0}, \theta^{*}$ and is relatively insensitive to the feed-imposed illumination taper It is also clear that large values of $\theta_{0}$ and $\theta^{*}$ result in higher peak levels of copolar and cross-polar radiation: However these results should not be assumed to indicate the low-sidelobe limitations of the offset system,
since at low levels this is strongly dependent upon the illumination taper introduced by the primary-feed anterina. In the example show, the primary-feed comprises a uniformly excited circular aperture which does not produce a good illumination characteristic when low sidelobes are a major concern.

The loss-of-aperture efficiency, arising from offset-reflectors' depolarization, tends to be small for values of $\theta_{0} ; \theta^{*}$; less than: $45^{\circ}$, but can become significant for larger angles. Some computed data due to Dijk et al [8], [9] are shown in Fig. 10.
Fig. 11 shows the predicted and measured radiation patterns of a linearly polarized offset reflector fed by an off-axis primary feed [4], [7]. These results provide further confidence in the quality of the analytical predictions and also serve to illustrate the comparative insensitivity of the peak cross-polar lobes to small transverse offsets in the primary-feed location. The formation of the copolarized coma lobe is the most evident source of pattern degradation. Studies have shown that the loss of gain suffered by an offset-reflector antenna with offset feed is essentially independent of the transverse plane in which the feed is offset However, the general radiation pattern deterioration appears to be more pronounced when feeds are offset in the plane of symmetry rather than the plane of asymmetry [19]-[21]. Some relevant computed data on gain and sidelobe performance from [19] is shown in Fig. 12. However no distinction as to the plane of offset is made for this data. The off-axis performance of an offset-reflector antenna is dependent upon the offset angle $\theta_{0}$ and is inferior to an


Fig. 12. Beam-scanning gain loss and coma-lobe levels. Illumination tapers of -6 dB (column I) and -10 dB (column II). (A): $\theta^{*}=30^{\circ}$, $\theta_{0_{0}}=0 .(B): \theta^{*}=45^{\circ}, \theta_{0}=0 .(C): \theta^{*}=30^{\circ}, \theta_{0}=45^{\circ} .(D): \theta^{*}=$ $45^{\circ}, \theta_{0}=45^{\circ}$.
ideal (unblocked) axisymmetric antenna with the same semiangle $\theta^{*}$. Approximate formulae for the beam deviation characteristics of offset reflectors have been derived which illustrate the role played by the offset angle $\theta_{0}$ [19], [20], [48].

The cross-polar characteristics of the offset-parabolic reflector are not unduly sensitive to small reflector profile errors. Small profile errors do not themselves generate a signinicant cross-polarized contribution to the radiated field but rather act as a phase-error distribution in the reflector aperture plane. As such, these errors will redistribute the existing copolar and cross-polar radiation in the far field of the antenna. However, like the main copolar lobe, the two main lobes of the cross-polarized field are relatively insensitive to the effects of small phase errors; and, although some increase in levels will occur in the subsidiary cross-polar lobes, the peak lobes will remain substantially unchanged [40].

The principal cross-polarized lobes radiated from a linearly polarized offset-reflector antenna are in phase quadrature with the main copolar beam and are essentially contained within the copolar beam envelope. The polarization over the main beam varies from linear at the beam center to an elliptical polarization in the region of the cross-polarized lobes. In some applications this will not constitute a serious drawback; but, if linear polarization is required over the main beam region, then the inherent limitations of an offset parabolic antenna with a conventional primary-feed are obvious. The peak levels of the cross-polarized lobes can be reduced to low levels (i.e., below -35 dB ), if the offset angle is made very small. This is evident from examination of Fig. 13, and Gans and Semplak [10] have described an antenna of this type with peak cross-polar levels of below -37 dB . However, if aperture blocking is to be avoided, then $\theta^{*}<\theta_{0}$; and this implies a long reflector structure which may be impractical in many cases. . It is worth noting, however, that a dual linearly polarized offset antenna can radiate two orthogonal elliptically polarized signals which are iresolvable into their orthogonal components by a suitably elliptically polarized receiving antenna [41]: Hence frequency reuse by polariza-


Fig. 13. Peak cross-polar levels radiated in the plane of asymmetry $(\Phi=\pi / 2)$ as a function of the offset-reflector parameters $\theta_{0}$ and $\theta^{*}$.
tion diversity is feasible even with a conventionally fed offset parabolic reflector. However, the pointing accuracy of the antenna must be maintained to a high order of accuracy if the use of a fast-operating adaptive polarizer is to be avoided.

## B. Circular Polarization

With a purely circularly polarized primary-feed illumination, the offset parabolic reflector does not depolarize the signal. Although each of the linearly polarized components of the incident wave effectively generates a cross-polarized component when the phase-quadrature relationship is introduced between the linear components, it is found that the combination of the two orthogonal copolarized vectors and the phaseasymmetric pair of cross-polarized vectors have the same direction of rotation. Hence the sum of these two signals results in a purely circularly polarized radiation, but the sum of the symmetric and asymmetric components fesults in a squinting of the radiation pattern from its boresight axis. The beam-squint effect acts to move the beam either forward or away from the axis of symmetry. The direction of the movement is dependent upon the hand of polarization.

Chu and Turrin [3] first published computed graphical data showing the effect of the reflector parameters $\theta_{0}, \theta^{*}$ upon the magnitude of the beam squint, and some key results are shown in Figs. 14, 15. Adatia and Rudge [42] have derived an approximate formula which gives the beam-squint angle $\left(\Psi_{s}\right)$ simply as

$$
\begin{equation*}
\Psi_{s}=\arcsin \frac{\left(\lambda \sin \theta_{0}\right)}{4 \pi F} \tag{25}
\end{equation*}
$$

This formula has been tested against computed and measured data and has been found, in all cases, to be accurate within 1.0 percent of the antenna half-power beamwidth.

## C. Practical Applications

The fully offset parabolic reflector is attractive for many applications, in that it offers the possibility of improved sidelobe performance and higher aperture efficiency : Some typical


Fig. 14. Beam displacement as a function of the offset-reflector parameters $\theta_{0}$ and $\theta^{*} . \Delta \beta$ is the angular squint of the beam in radians.


Fig. 15. Measured radiation patterns of an offsetreflector antenna operated with circular polarization at $18.5 \mathrm{GHz}\left(F=9.4 \lambda, \theta_{0}=45^{\circ}\right.$, $\theta^{*}=45^{\circ}$ )
applications are considered below. In these cases. unless stated otherwise, the primary-feed has been assumed to be of conventional design and with good polarization properties.


DUAL-GRIDDED OFFSET-FED REFLECTOR
Fig: 16. Illustrating a dual-polarized offset-reflector antenna employing polarized grids.

1) Point-to-Point Communications: With circular polarization the boresight-gain reduction arising from beam-squinting effects need not be a major problem. For systems employing a single hand of polarization the beam squint is simply compensated in the antenna alignment. For dual-polarized applications, where the direction of squint is reversed for oppositehands of polarization, the boresight-gain loss can be readily reduced to less than 0.03 dB per beam by suitable choice of the reflector parameters. This loss corresponds to a beamsquint angle of less than 10 percent of the antenna half-power beamwidth:

With linear polarization, a very exact alignment of the radiated beam is called for to remain within the boresight null of the cross-polarized radiation pattern. Practical dual-polarized operation, therefore, demands the use of a moderately lange reflector $F / d$ ratios [10]. Alternatively, good cross-polarization suppression can be achieved for a single linear polarization by use of a polarization-selective grid, either in the aperture plane or at the surface of the reflector [43], [66]. A dualpolarized system can then be formed by interleaving two orthogonal polarization-sensitive reflector surfaces with separate foci. This is illustrated in Fig. 16 [43], [49].
2) Shaped or Contoured Beams: For certain spacecraft applications, where circularly polarized area coverage is required, even a 5 -percent beam squint $c$ an produce significant changes in the gain at the edge of the coverage zone. This problem is simply overcome for a single hand of circular polarization, but for dual-polarized applications the $F / d$ of the reflector must be made as large as possible, consistent with the volumetric constraints. When the offset-reflector profile is deformed to provide a shaped or contoured beam, the beam squint compounds the design difficulty; numerical techniques are virtually essential to achieve a desired beamshape. Wood et al. [24] have described the design and evaluation of a dual circularly polarized offset antenna intended for the European Communication Satellite System. The reflector surface was deformed in one plane to provide an elliptical coverage pattern of $8.6^{\circ} \times 4.9^{\circ}$ A copolar/cross-polar ratio of better than 33 dB was predicted over the coverage zone. Limitations in the corrugated-horn primary-feed performance reduced this figure to 30 dB over 95 percent of the zone on the experimental model. The tolerances upon the reflector profile and the location of the primary feed are also likely to be more critical in these designs, and the beam shape and gain can be very sensitive to small feed misalignments [44]. For

Fig. 17. Variations in the peak gain of the offset antenna ( $F / d=0.52$ ) with primary-feed displacement by $\Delta_{t}$ wavelengths along the $y$ axis.
example, Fig. 17 which is taken from [44] shows the variations in the peak gain of an offset reflector with small primaryfeed displacements. This reflector has an $F / d$ ratio of 0.52 and is shaped in the plane of asymmetry to provide an elliptical beam with an aspect ratio of approximately $1.8: 1$.
For linear polarization, the effective phase error introduced by the departure of the shaped reflector from true parabolic results in a spatial redistribution of the cross-polar lobes generated by the reflector. Unless the reflector offset angle is made small, the antenna will exhibit very poor polarization purity in the coverage zone. Fasold [45] has described a linearly polarized offset-reflector system in which the perimeter of the reflector was contoured to provide an elliptical beam shape. However, the cross polarization from this antenna was predictably high with peak levels close to -20 dB . For a single linear polarization, good polarization purity can be restored by use of polarization-sensitive grids; for dual polarization, a larger $F / d$ ratio is necessary.

Contoured beam shaped can also be-achieved by the use of offset parabolic reflectors with multiple-element feed clusters. The feed elements are combined in amplitude and phase to generate a desired footprint over the coverage zone. Ultimately this technique offers considerably more flexibility in that the beam can be reconfigured to a variety of coverage patterns by changing thè relative excitation coefficients of the feed array. The implications on the offset-reflector design are as for the previous cases, and a relatively large $F / d$ ratio must be employed for dual-polarized applications. For applications demanding good polarization purity there are additional complications in that the feed array must itself exhibit good polarization properties. The combining network for the multiple elements will also be complex and is likely to be a critical feature of any practical design.
A multiple-feed offset antenna of this class is under development for the Intelsat V spacecraft. The circularly polatized system described by Han [25] employs an offset parabolic reflector with an $F / d$ ratio of approximately unity. The feed cluster has 78 square waveguide horns and produces 4 hemi-spheric-shaped beams, two having right-hand and two lefthand polarization. - The axial ratio of these beams within their respective coverage zones is less than 0.75 dB and, the isolation between beams is better than 27 dB . The power


Fig. 18. (a) Breadboard model of a multiple-element contoured-beam antenna for satellite communications. (b) Required coverages for Intelsat $V$ hemi/zone antenna.-
distribution networks for each beam employ low-loss air stripline with switches to provide some reconfiguration of the beams for different subsatellite locations. Fig. 18 shows a breadboard model of the antenna with a contour plot of the required coverage zones.
3) Multiple Spot Beams: The performance of the single offset reflector as a multiple-spot-beam antenna was studied in some detail by Rudge et al. [19],[201. In this investigation a three-beam circularly polarized configuration was designed and optimized using numerical techniques, and a detailed experimental evaluation was performed on a precisely made breadboard model. Since frequency meuse within each beam was not required, the antenna design made use of the beamsquint effect to reduce the beam spacing between the most closely spaced pair of beams. These meams were orthogonally


Fig. 19. Copolarized and cross-polarized radiation patterns of multiple-spot-beam antenina at 30 GHz . Measured $\qquad$ Predicted (a) Cut through beams 1 and $3: \Phi=90^{\circ}$. (b) Cut through beam 2: $\Phi=23.5^{\circ}$. (c) Photograph of breadboard spot-beam antenna:
polarized and had a minimum beam spacing of 1.1 HPBW with a beam efficiency of 60 percent. The feed elements were conical-horm elements with optimum dimensions for crossFolar suppression (see Fig. 7) and exhibited an isolation between beamports of better than 40 dB . Fig. 19(a) shows the
predicted and measured radiation patterns for the two beams. Fig. 19(b) shows the radiation characteristics of the third beam. A beam to beam isolation of better than 30 dB in the coverage zone was achieved for this antenna. which is shown in Fig. 19(c).


Fig. 20. Radiation pattern of a single-offset reflector fed by a 4-horn static-split monopulse feed (shown inset) in its tracking mode. Copolar $\quad$ Cross-polar - - - -.
4) Monopulse Tracking Radars: This application, in particular, is one in which the offset-reflector depolarization can be a serious handicap. The predicted copolar and cross-polar radiaion fields of an offset reflector fed by a 4 -horn staticsplit monopulse feed in its tracking mode are illustrated in Fig. 20. The cross-polar field can be seen to have a peak on the boresight axis. Randomly polarized signals from a radar target will result in an output from the tracking channel in which the boresight location appears to shift sporadically. Its precise location at any time will be dependent upon the relative magnitudes of the orthogonally polarized components in the returning signal. This boresight uncertainty, which is sometimes termed boresight-jitter, can impose serious limita: tions upon radar tracking accuracies.
Since the cause of boresight-jitter is the reflector depolarization, its cure (with conventional feeds) is to employ larger F/d ratio reflectors. Alternative approaches will be discussed in following sections of this paper.
5) Low-sidelobe antennas: One of the major attractions of the offset-reflector antenna is the possibility of lower sidelobe radiation. This feature has become increasingly important in recent years as a consequence of the pressure upon the available frequency spectrum and the need to avoid interference from both friendly and unfriendly sources.

Dragone and Hogg [17] have shown theoretically that the fully offset parabolic reflectors offer significant advantages over their (blocked) axisymmetric counterparts. Improvements of up to 10 dB in near-in sidelobe levels can be inferred. from their results given in Fig. 21. The data shown in Fig. 9 indicates that first sidelobe levels of the order of -30 dB can be realized with illumination tapers of the order of -20 dB . In fact more favorable tapers than those provided by the feed employed in Fig. 9 can reduce these levels considerably. Using corrugated horn feeds, pencil beams can be produced with first
sidelobes at the $-33-\mathrm{dB}$ level coupled with aperture efficiencies of 70 percent. An example of an antenna with this order of performance is shown in Fig. 22. Lower sidelobes are also feasible with a more sophisticated primary-feed design, although some reduction in aperture efficiency will be implied. Elliptical beams can be generated by offset antennas in which the first sidelobes in a specified critical plane can be suppressed below the $-40-\mathrm{dB}$ level [39].

The illumination asymmetry in the principal plane of the offset reflector does introduce an undesirable shoulder on the beam when larger offset angles are used. This effect can be detected on the data shown in Fig. 9 and cannot be completely obviated except by the use of larger $F / d$ ratios or more sophisticated primary-feeds.
Linearly polarized offset antennas will, of course, generate significant levels of cross-polar radiation unless the reflector has a large $F / d$ ratio. However, for most applications it is found that the reflector depolarization lays below the copolarized sidelobe envelope. Hence, the offset-reflector depolarization does not preclude the use of this antenna in a low-sidelobe role. For smaller offset reflectors, primary-feed spillover constitutes the main limitation on the overall sidelobe performance. These effects can be alleviated by good primaryfeed design and some use of shields or blinders about the antenna aperture.

## IV. Primary Feeds for Offset-Reflector Antennas

## A. Offset-Reflector Focal-Plane Fields

The offset parabolic reflect or has many obvious advantages, but the depolarizing beam-squinting phenomena clearly represent a major limitation in many applications. However, since analytical insight has provided us with a good understanding of the causal factors, it can also, indicate the necessary cure. To follow this vein, it will be useful to consider the form of the electromagrietic fields set up in the focal region of the offset reflector when a plane wave is normally incident upon the reflector aperture plane.
Bem [46] has performed an analysis of the focal-plane fields which, although limited to normally incident waves, has the particular merit that the transverse focal-plane fields $E_{x}, E_{y}$. can be expressed approximately in a simple closed form. Valentino and Toulios [47] confirmed Bem's results by extending them to include incident waves making small angles with the reflector boresight and comparing their results with measured data. Ingerson and Wong [48] have also employed a focal-region field analysis to determine beam-deviation factors for offset-reflector antennas. For offset-reflectors with long focal length and polatized in the plane of symmetry, we have from Bem's analysis:

$$
\begin{align*}
& E_{x}\left(u, \phi_{0}\right)=\frac{2 J_{1}(u)}{u}+\frac{j d \sin \theta_{0}}{F} \frac{J_{2}(u)}{{ }^{2}} \cos \phi_{0}  \tag{26a}\\
& E_{y}\left(u, \phi_{0}\right)=\frac{-j d \sin \theta_{0}}{F} \frac{J_{2}(u)}{u} \sin \phi_{0} \tag{26b}
\end{align*}
$$

where $r^{\prime}, \phi_{0}$ are polay coordinates in the reflector focal plane with origin at the geometric focus, $u\left(r^{\prime}\right)$ is a normalized parameter representing the distance $r^{\prime}$ to a point in the focal plane, and all multiplying constants have been suppressed. $J_{n}(u)$ is the Bessel function, of order $n_{\text {. }}$ The solutions for the wave polarized in the plane of asymmetry is achieved from (26) by interchanging $x$ and $y$ and replacing $\phi_{0}$ by $2 \pi-\phi_{0}$ -


Fig. 21. Effects of aperture blockage on the radiated sidelobe levels and the reflection coefficient at the primary-feed for symmetrical and of fise antennas. (a) Radiation pattern of blocked aperture ( $a_{2} / a_{1}=0.2$ ). (b) Radiation pattern for marginally blocked aperture ( $a_{2} / a_{1}=0.2$ ) in horizontal and vertical planes. (c) Reflection coefficients for illumination taper of 13 dB at subreflector edge.

Inspection of (26) and (27) reveals that the cross-polar component $\left(E_{x}\right)$ is an asymmetric function with a magnitude increasing with the offset angle $\theta_{0}$ and in phase quadrature with the principal axisymmetric copolar component. The axisymmetric copolar term is also modified by the presence of a quadrature component which is identical to the cross-polar term in all but its dependence upon $\phi_{0}$. Fig. 23 shows a
simplified contcur plot of the amplitude of the focal-plane iield in the vicinity of the geometric focus.

## B. The Offset-Reflector Matched-Feed Concept

If the primary-feed is to provide an optimum conjugate match to the incoming field, then its aperture fieids must exhibit similar polarization characteristics. Conventional high-

(a)

Fig. 22. Radiation patterns in principal planes for an offset reflector ( $F=22.7 \lambda, \theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ) fed by a circularly polarized corrugated horn with a $-18-\mathrm{dB}$ illumination taper at $\theta=\theta^{*}$. Frequency $=$ 30 GHz . Antenna efficiency 70 percent neglecting ohmic losses. (a) $\Phi=\pi / 2$ (b) $\Phi=0$. Measured $\qquad$


Fig. 23. Approximate contour plot of typical focal-plane field distribution of an offset parabolic reflector uniformly illuminated from a distant linearly polarized source.
performance axisymmetric feeds (such as the corrugated horn) provide a conjugate match to the copolar component only, which results in the apparently poor polarization properties of the offset reflector.
The focal-plane field distributions described by (26) and (27) can be matched very effectively by making use of higher order asymmetric waveguide modes. Recently, Rudge and Adatia [14], [15], [26] have proposed a class of new pri-mary-feed designs employing mode combinations in cylindrical, rectangular, and corrugated waveguides. To illustrate the general principal of this approach, Fig. 24(a) illustrates the nature of the symmetric and asymmetric components which make up the offset-reflector focal-plane fields for two linear polarizations.
To adequately match this characteristic in a smooth-walled cylindrical waveguide, the required asymmetric mode is the $\mathrm{TE}_{21}$. "Fig. 24(b) shows the field distribution in the conicalhorn mouth for the two orthogonal $\mathrm{TE}_{21}$ modes, which for
convenience are designated $\mathrm{TE}_{21}^{1}$ and $\mathrm{TE}_{21}^{2}$. The transverse field components of these modes ( $E_{x}^{1}, E_{y}^{1}, E_{x}^{2}, E_{y}^{2}$ ) can be readily derived from the solution of the vector wave equation in the circular pipe [27]. In terms of a normalized distance parameter $u^{\prime}$ and a polar angle $\phi_{0}$ and omitting multiplying constants, these components can be expressed as:

$$
\begin{align*}
& E_{x}^{1}\left(u^{\prime}, \phi_{0}\right)=K^{\prime}\left[J_{1}\left(u^{\prime}\right) \cos \phi_{0}+J_{3}\left(u^{\prime}\right) \cos 3 \phi_{0}\right]  \tag{27a}\\
& E_{y}^{1}\left(u^{\prime}, \phi_{0}\right)=-K^{\prime}\left[J_{1}\left(u^{\prime}\right) \sin \phi_{0}-J_{3}\left(u^{\prime}\right) \sin 3 \phi_{0}\right]  \tag{27b}\\
& E_{x}^{2}\left(u^{\prime}, \phi_{0}\right)=K^{\prime \prime}\left[J_{1}\left(u^{\prime}\right) \sin \phi_{0}+J_{3}\left(u^{\prime}\right) \sin 3 \phi_{0}\right]  \tag{28a}\\
& E_{y}^{2}\left(u^{\prime}, \phi_{0}\right)=K^{\prime \prime}\left[J_{1}\left(u^{\prime}\right) \cos \phi_{0}-J_{3}\left(u^{\prime}\right) \cos 3 \phi_{0}\right] \tag{28b}
\end{align*}
$$

where $K^{\prime}$ and $K^{\prime \prime}$ are constant factors proportional to the complex coefficients of the two $\mathrm{TE}_{21}$ modes.
In the principal planes ( $\phi_{0}=0$ and $\pi / 2$ ), (27) reduce to

$$
\begin{align*}
E_{x}^{\prime}\left(u^{\prime}, 0\right) & =4 K^{\prime} \frac{J_{2}\left(u^{\prime}\right)}{u^{\prime}}  \tag{29a}\\
E_{y}^{1}\left(u^{\prime}, 0\right) & =0  \tag{29b}\\
E_{x}^{1}\left(u^{\prime}, \pi / 2\right) & =0  \tag{29c}\\
E_{y}^{1}\left(u^{\prime}, \pi / 2\right) & =4 K^{\prime} \frac{J_{2}\left(u^{\prime}\right)}{u^{\prime}} \tag{29~d}
\end{align*}
$$

Comparing (29) with the asymmetric terms in (26), it can be seen that the coefficient of the $\mathrm{TE}_{21}^{1}$. mode can be selected to provide a perfect match to both the copolar and cross-polar asymmetric components when the reflector is polarized in the plane of symmetry ( $-x$ axis). For operation with the principal polarization along the $y$ axis (plane of asymmetry) from (28), $\mathrm{TE}_{21}^{2}$ mode gives in the principal planes:

$$
\begin{align*}
E_{x}^{2}\left(u^{\prime}, 0\right) & =0  \tag{30a}\\
E_{y}^{2}\left(u^{\prime}, 0\right) & =2 K^{\prime \prime} J_{2}^{\prime}\left(u^{\prime}\right)  \tag{30b}\\
E_{x}^{2}\left(u^{\prime}, \pi / 2\right) & =2 K^{\prime \prime} J_{2}^{\prime}\left(u^{\prime}\right)  \tag{30c}\\
E_{y}^{2}\left(u^{\prime}, \pi / 2\right) & =0 \tag{30d}
\end{align*}
$$

where

$$
J_{2}^{\prime}\left(u^{\prime}\right)=\frac{d}{d u^{\prime}} J_{2}\left(u^{\prime}\right)
$$

Equations (30) can be compared with the asymmetric components of the reflector focal-plane field for a wave polarized in the plane of asymmetry. : Making the necessary simple transformations in (26), the asymmetric components $\left(E^{a}\right)$ are given by:

$$
\begin{gather*}
\because E_{x}^{a}(u, 0)=0  \tag{31a}\\
E_{y}^{a}(u, 0)=\frac{j d \sin \theta_{0}}{2 F} \frac{J_{2}(u)}{u}  \tag{31b}\\
E_{x}^{a}(u, \pi / 2)=\frac{j d \sin \theta_{0}}{2 F} \frac{J_{2}(u)}{u}  \tag{31c}\\
E_{y}^{a}(u, \pi / 2)=0 \tag{31~d}
\end{gather*}
$$

Companison of (30) with (31) indicates a nonideal match. However, the Bessel differential functions $J_{2}^{\prime}(u)$ have very similar general characteristics to the $\left(f_{2}(u)\right) / u$ distribution; and, by judicious choice of the constant $K^{\prime \prime}$, the two functions can be closely matched over the aperture of the feed horn.

The differences in the principle-plane distributions of the


Fig. 24. (a) Field configurations of symmetric and asymmetric components of the offset-reflector focal-plane fields with the incident wave polarized in the plane of symmetry and in the plane of asymmetry. (b) Required field configuration in the aperture of a circular hom for focal-plane field matching in the orthogonal linear poiarizations.
orthogonal $T E_{21}$ modes is a consequence of the boundary conditions imposed by the smooth-walled cylindrical structure. The boundary conditions have similar implications with regard to the lack of axisymmetry in the copolarized radiation provided by the fundamental $\mathrm{TE}_{11}$ mode. To provide this axisymmetry it is necessary to add a component of the $\mathrm{TM}_{11}$ mode. This technique is well-established and forms the basis of the well-known dual mode or Potter horn [50].

For cylindrical corrugated structures in which the fields satisfy anisotropic boundary conditions, it can be shown that the corresponding ( $\mathrm{HE}_{21}$ ) hybrid modes have identical distributions in the principal planes. Combined with the fundamental $\mathrm{HE}_{11}$ mode, these structures can provide a close to ideal match in the two hands of principal polarization.

In Table I the modes required to provide focal-plane matching with three different feed structures are summarized. The cylindrical structures are also suitable for use with two hands of circular polarization. The corrugated rectangular case, although equally feasible, is not shown since the mode designations for this structure are not standardized.

Efficiency optimization techniques, such as those described in a recent review paper by. Clarricoats and Poulton [50] can be applied to optimize the performance of the offset antenna. If $\bar{E}_{1}, \bar{H}_{1}$ are the offset-reflector focal-plane fields and $\bar{E}_{2} ; \bar{H}_{2}$. the fields created at the aperture of the primary-feed when unit power is transmitted, then the efficiency can be obtained from an integration of these fields over the aperture plane of the feed (s).

$$
\begin{equation*}
\eta=\frac{1}{2} \int_{S}\left(\bar{E}_{1} \times \bar{H}_{2}+\bar{H}_{1} \times \bar{E}_{2}\right) \cdot d \bar{S} \tag{32}
\end{equation*}
$$

Optimization of this $\eta$ parameter will lead to the optimum values of the mods coefficients, but the optimum efficiency condition will not be the desired condition for all applications.

TABLE
Waveguide Modes for Offset-Reflector Focal-Plane:Matching

| Feed structure | Waveguide modes |  |
| :---: | :---: | :---: |
|  | Erincipal polarisation |  |
|  | plape of symattry ( $-x$ ). | plane or asymutry |
| Swooth-walled cylindrical* |  | $2 E_{y 1}^{2}+\mathrm{MH}_{11}^{2}+\mathrm{EE}_{21}^{2}$ |
| Corrugated cylindrical* | $\mathrm{HE}_{17}^{7}+\mathrm{HE}_{21}^{\mathrm{I}}$ | $\mathrm{HE}_{11}^{2}+\mathrm{HE}_{21}^{2}$ |
| Smooth-walled rectangular | $\mathrm{TE}_{10}+\mathrm{TE}_{11} / \mathrm{TE}_{\mathrm{ES}}$ | $\mathrm{TE}_{01}+\mathrm{TE}_{2}$ |

*Superscripts denote orthogonal pair of modes with otherwise identical mode designation.


Fig. 25. Protorype trimode offset-reflector matched-feed device.

Alternatively and more simply in this case, the antemna copolar performance can be optimized independently to whatever criteria is relevant, and the cross-polar performance simply optimized by use of the higher order mode coefficients. The interaction with the copolar ckaracteristic will be small and generally favorable in that the 西igher order modes act to compensate for the asymmetric space-attenuation factor introduced by the offset reflector.

The optimum values of the complex coefficients: of the modes can be determined by using the mathematical models described in the early sections, with the introduction of modified primary-feed models which include the higher order modes. By examining the cross-polar radiation pattern of the offset antenna, for a range of values of the mode coefficients; the optimum characteristics can be readily determined. The close similarity between the value of the first cross-polar lobe peaks and the required level of the higher order mode will be found to be an excellent guide in many cases.
Trimode devices based upon the matched-feed principle have been constructed in smooth-wall cylindrical guides, and dual-mode rectangular feeds have also been demonstrated. These prototype devices were constructed for operation with a. single linear polarization. A fully dual-polarized version of the trimode cylindrical structure has been recently developed [51]. The practical aspects of the matched-feed approach can best be realized by examining the design of the prototype trimode device introduced in 1975 [14]. The basic configuration is illustrated in Fig. 25.

## C. Prototype Trimode Matched Feed

The trimode primary feed is essentially a small flare-angle conical horn with two steps or discontinuities. The first step region ( $d_{3} / d_{2}$ ) is asymmetric and generates the $\mathrm{TE}_{21}$ mode. The diameter $d_{2}$ cuts off all higher modes. The second step ( $d_{2} / d_{1}$ ) is axisymmetric, and the guide dimensions cut off all modes above the $\mathrm{TM}_{11}$. The symmetry of this discontinuity avoids the further generation of the $\mathrm{TE}_{21}$ mode. The amplitudes of the modes are governed by the ratios $d_{3} / d_{2}$ and $d_{2} / d_{1}$, and the relative phases of the modes are adjusted by the constant-diameter phasing section which follows each discontinuity. The mode amplitudes required are a function of the offset angle $\theta_{0}$ and the semiangle $\theta^{*}$ of the offset reflector. Typically, the required mode amplitudes lay in the range $-20-30 \mathrm{~dB}$ below the fundamental. The diameter of the primary-feed aperture is selected in the usual way to satisfy the illumination requirements of the reflector. The overall length of the feed is between 0.25 and 1.0 wavelengths greater than a conventional axisymmetric dual-mode feed of the Potter type.

Predicted and measured radiation characteristics of this feed are shown in Fig. 26. The feed has an aperture diameter of 2.8 $\lambda$, and the measurements are made at 30 GHz . When used with a precision offset reflector with parameters $F=22.7 \lambda$, $\theta_{0}=44^{\circ}$, and $\theta^{*}=30^{\circ}$, the matched feed provides a significant improvement in cross-polar suppression over a conventional feed. Typically, feeds of this particular design can provide a minimum of 10 dB additional suppression of the reflector cross polarization (relative to a conventional primary-feed) over a 4-5-percent bandwidth. At midband the additional cross-polar suppression can approach 20 dB . Typical radiation pattern and bandwidth characteristics are shown in Figs. 27 and 28. These characteristics should not be interpreted as defining the fundamental bandwidth limitations of multimode matched-feed devices. A number of techniques can be applied to improve the bandwidth of the feeds, and bandwidihs of almost 7 percent have already been achieved [51].

## D. Matched Feed for Monopulse Radars

The undesirable effect of offset-reflector depolarization upon prècision tracking monopulse radars has been mentioned. Rudge and Adatia [15], [26] proposed that the matched-feed principle could be applied to this problem by incorporating the necessary higher order modes in a 4 hom static-split feed. The questionable features of this application were identified as: 1) the level of cross-polar suppression which could be achieved by an essentially off-axis rectangular feed element; and 2 ) whether the matched-feed condition could be maintained for both sum and difference excitations of the monopulse feed.
The proposal was investigated by mathematical modeling of the offset reflector with its multimoded feed elements and the construction and evaluation of one element of the 4 horn feed. Fig: 29 shows the computed cross-polar levels for a 4 horn matched feed and a conventional 4 horn device. The differ-ence-channel radiation pattern in the principal plane ( $\Phi=0$ ) has zero cross-polar radiation and is not shown in the figure. Fig. 30 shows the rectangular feed element, and Fig. 31 summarizes the measured radiation pattern and VSWR bandwidth performance of this $K$-band device. It was concluded that significant reductions in boresight jitter can be achieved by matched-feed techniques.


Fig. 26: Predicted and measured radiation characteristics for the $30-\mathrm{GHz}$ prototype matched feed. Predicted ( $H$ plane) $\rightarrow$. Measured ( $H$ plane) ——. (E plane) - - - -.


Fig. 27. Measured radiation patterns in plane of asymmetry for off-set-reflector antenna ( $F=22.7$. $\lambda, \theta_{i}=44^{\circ}, \theta^{*}:=30^{\circ}$ ). (a): Conventional Potter-horn feed. (b): Matched feed (initiai measuremerst) (c): Optimized matched feed. Copolar $\longrightarrow$. Cross-polar $\longrightarrow$


Fig. 28. Measured cross-polar suppression bandwidth of K-band trimode matched feed (I). (A): Nominal cross-polar level with conventional feed (reflector with $\theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ). (B): $1 \mathrm{D} 0-\mathrm{dB}$ improved suppression level, defining bandwidth $\Delta$.


Fig. 29. Predicted performance of offset reflector fed by 4-horn monopulse primary feed with matched-feed elements. (a) Sum channel $\Phi=0$. (b) Sum channel $\Phi=\pi / 2$. (c) Difference channel $\Phi=\pi / 2$. Copolar $\longrightarrow$. Cross-polar $\longrightarrow$. Dashed lines indicate crosspolar levels with conventional feed elements.


Fig. 30. Prototype rectangular-horn matched-feed configuration showing asymmetric mode aperture-plane-rield distributions ( $T E_{11}$ mode) in mouth of rectangular waveguide horn with fundamental mode polarization. Principal cross-polar regions are circled.

## V. Dual-Reflector Offset Antennas

## A. The Open Cassegrainian Antenna

For applications involving complex primary-feed structures, the use of a Cassegrainian feed system has some obvious advantages. In particular; the Cassegrainian configuration allows the feed elements and the associated circuitry to be located close to the main reflector surface, possibly avoiding long RF transmission paths and the need for extended feed support structure, while the for ward-pointing feed format can be a desirable attribute for applications requiring low-noise performance.

Of the variety of offset Cassegrainian systems proposed in the literature, perhaps the besi known is the open Cassegrain-


Fig. 31. (a) Measured radiation patterns of offset reflector fed by a rectangular matched-feed element. Lower trace is cross-polar level which has been suppressed from its nominal level of -23 dB . (b) Measured cross-polar suppression of prototype matched-feed element against frequency. Graph shows peak levels of lobes on either side. of boresight. (c) VSWR of prototype matched-feed with. 2-step transformer.
ian antenna introduced in 1965 by the Bell System Laboratories [1], [2]. The antenna, which is illustrated in Fig. 2, comprises an offset section of a paraboloid and an offset hyperboloid subreflector, fed by a primary feed which protrudes from an aperture in the main reflector surface.
With this configuration it is possible to design the antenna such that the subreflector does not block the aperture of the main reflector. However, as a direct consequence of the positions of the primary feed, some aperture blockage due to the feed system is unavoidable:

The analysis of the complete antenna can be performed by means of the physical-optics-based current distribution technique for the subreflector and the aperture-field integration method for the main reflector. Referring to Fig. 32, the current distribution on the subreflector ( $\bar{J}_{s}$ ) is given by (1), and the distant scattered fields arising from this current can be obtained from

$$
\begin{align*}
\bar{E}= & j \frac{k}{4 \pi R} \exp (-j k R) \int_{S} \bar{a}_{R} \times\left(\bar{a}_{R} \times \bar{J}_{S}\right) \\
& \cdot \exp \left(j k \bar{p} \cdot \bar{a}_{R}\right) d S \tag{33}
\end{align*}
$$

where $k$ is the wave number, and $S$ is the subreflector surface.
Expressed in spherical coordinate components ( $E_{\theta}, E_{\phi}$ ), the subreflector fields can be inserted into (14) or (18) to determine the tangential aperture fields of the main reflector, and, hence, via equations (22) or (24) to determine the far fields of the overall antenna. Thus the analysis essentially involves the evaluation of four two-dimensional diffraction integrals at each field point. : Under certain circumstances, use can be made of the axes of symmetry afforded by the subreflector geometry to eliminate the azimuthal dependent integrals, thereby alleviating the computational problem. Ierley and Zucker [52] have also described a technique for reducing the double integrals associated with the main reflector into a more convenient one-dimensional form. The technique, which is based upon an application of the stationary-phase approximation in the azimuthal part of the integral, allows more economical predictions of both the near-in and the far-out sidelobe performance of the open Cassegrainian antenna.
In general, the basic radiation characteristics of the open Cassegrainian antenna do not differ significantly from those of an equivalent single-offset-reflector antenna. To avoid aperture blockage the open Cassegrainian antenna must employ large offset angles and, when fed by conventional primary feeds, exhibits beam squinting and jepolarizing characteristics


Fig. 33. 6-m open Cassegrainian antenna located at the University of Birmingham, England.
which are similar to the single offset reflector However, for applications where these particular performance parameters are not of major concern, the open Cassegrainian configuration offers excellent potential for realizing high overall efficiency and low wide-angle sidelobe radiation. Fig. 33 shows an


Fig. 34. Radiation patterns of a $60-\mathrm{GHz}$ experimental model of the Bell open Cassegrainian antenna with parameters $F=152 \lambda \theta_{0}=47.5^{\circ}, \theta^{*}=30.5^{\circ}, \theta_{1}=7.5^{\circ}$, and a subreflector diameter of $40 \lambda$
example of an open Cassegrainian antenna of the Bell Laboratories. design located at the University of Birmingham. in England. Fig. 34 shows measured and predicted data made on a precision model of this design at the Bell Laboratories [1], [2]. This antenna had a computed efficiency of better than 65 percent"(including spillover and scattering losses, but not ohmic loss).

## B. The Double-Offset-Reflector Antenna

An alternative dual-offset-reflector configuration, which offers a number of attractive features, is the so-called doubleoffset antenna shown in Fig. 2. This antenna, which was first implemented by Graham [11], provides a convenient location for the primary-feed hardware by use of an offset section of a hyperboloidal subreflector in a Cassegrainian arrangement. Two variations of the double offset are illustrated in the figure. A Gregorian version, in which the subreflector comprises an offset portion of an ellipsoidal reflector, is also feasible and has been considered by Mizugutch et al. [53]. For either of the versions shown the overall antenna geometry can be designed to be completely free of aperture blockage.
Analyses performed by several workers [12], [13], [53], [54] has shown that the douile-offset antenna can be designed such that, when fed by a conventional linearly
polarized primary-feed, the depolarization arising from the two offset reflectors can be made to cancel, thus providing an overall low cross-polar characteristic. This performance is achieved by matching the scattered radiation fields from the subreflector to the main reflector. The principle is essentially similar to the matched-field approach previously described for single offset reflectors; and, in theory, the technique offers a greater potential for broad-band performance.

Approximate techniques based upon the use of geometric optics [13], [53], [54]. indicate that a perfect match can be achieved (i.e., giving zero cross-polar fields in the main reflector aperture) when the axis of the parent subreflector surface is depressed by an angle ( $\alpha$ ) from the axis of the parent paraboloid. This condition is illustrated in Fig. 2. A mathematical expression relating the depression angle $\alpha$ to the parameters of the subreflector has been derived by Mizugutch et al. [53]. In its simplified form this can be expressed as

$$
\begin{equation*}
\tan \frac{\alpha}{2}=\frac{1}{M} \tan \frac{\Psi_{0}}{2} \tag{34}
\end{equation*}
$$

where

$$
\begin{equation*}
K=\frac{1+e}{1-e} \tag{35}
\end{equation*}
$$



Fig. 35. The double-offset antenna geometry and parameters.
and $e$ is the eccentricity of the subreflector, $\Psi_{0}$ is the feed offset angle, and the geometry is illustrated in Fig. 35.
Structurally, the optimized geometry is not very compact, and this can be a drawback for applications where the volumetric constraints are severe. Earlier experimental results described by Graham [11] and the rigorous diffraction analysis performed by Adatia [12] confirmed the general validity of the optimization formula, although the level of cross-polar suppression is not independent of the subreflector dimensions and curvature. More recent results obtained by Adatia [55] have indicated that the major limiting factor in the realization of polarization purity with double-offset antennas is the diffraction effects introduced by the finite-sized subreflector. Diffraction analysis shows that the magnitude of the diffrac-tion-generated cross-polar field.component is primarily a function of the transverse dimensions of the subreflector in wavelengths, the eccentricity of the subreflector surface, and the primary-feed illumination taper. For good cross-polar suppression, subreflectors with large transverse dimensions and high values of eccentricity ( $e$ ) are favored.
In Fig. 36 the copolar and cross-polar components of the fields scattered from an offset subreflector with a diameter of $10 \lambda$ are shown for the plane of asymmetry. The associated phase characteristics are also indicated in the figure. The deviation from the ideal in-phase characteristics is a consequence of the nonoptical scattering by the finite subreflector. This phase deviation essentially limits the polarization purity of the overall antenna, since the cancellation of the depolarized field generated at the main reflector is impaired. This is apparent in Fig. 37, which also shows the residual cross-polar field distributions in the main offset-reflector projected aperture. The multilobe structure of this distribution is directly attributable to the nonuniform phase distributions of the subreflector fields. Fig. 38 shows the predicted secondary field characteristics in the principal planes for a double-offset antenna with the parameters given in Table II and a subreflector diameter of $10 \lambda$. The cross-polar fields are well above


Fig. 36. Scattered radiation fields from the offset subreflector in the plane of asymmetry. The broken curves:shows the cross-polar distribution in the aperture of the main reflection.


Fig. 37. Differential phase between the copolar and the cross-polar components of fields scattered from the offset subreflector (A) subreflector diameter $10 \lambda .(B)$ subreflectror diameter $20 \lambda$


Fig. 38. The radiation fields from the double-offset-reflector antenna. Copolar $\left(\Phi=90^{\circ}\right)$. $\longrightarrow$ Cross-polar $\left(\Phi=90^{\circ}\right)$. Copolar $\left(\Phi=0^{\circ}\right)$.

TABLE 11
Parameters of a Dual-Offset-Reflector Antenna with Geometry Optimized for Maximum Cross-Polar Cancellation

the levels indicated by geometric optics and can be reduced to below -40 dB only by use of a subreflector diameter of greater than $25 \lambda$
An alternative approach to the elimination of cross polarization from dual-offset-reflector systems has been followed by Albertsen [57], [58]. In this approach the two reflecting surfaces are shaped to provide the desired aperture-field conditions in the main reflector. The mathematical approach involves the solution of simultaneous partial differential equations with certain specified initial conditions. The solutions provide the profiles of the two reflector surfaces under constraints which provide for an in-phase cross-polar free-field distribution in the projected aperture of the main reflector. One of the principal attractions of this general approach is that the problem may be formulated for any prescribed position of the main and subreflector surfaces. Hence, at least in principle, highly compact dual-offset configurations can be
realized, although subreflector diffraction effects will still limit the cross-polar performance when small subreflectors are employed. An additional advantage is that, by appropriate choice of initial conditions, the method can provide solutions for elliptically contoured beams of any aspect ratio: [57], [58]. Hence, state-of-the-art axisymmetric low-cross-wolar primary-feeds, such as the cylindrical corrugated hom [50], could be employed to efficiently illuminate a dual-reffector system generating an elliptical main beam with good polanization purity. However, the full implications of this approach have yet to be investigated and confirmed experimentally.
$\AA$ further variation of the double-offset-reflector configuration was considered by Cha et al. [59]. In this work the offset subreflector was designed as a frequency selective filter. The double-offset antenna was thus fed from bot the prime focus of the main reflector and via the subreflector, which comprised an array of conformal printed-circuit crossed dipoles. The Cassegrainian geometry was utilized in the resonant band of the subreflector surface. The published paper lacks detail but significant gain reductions were encountered (i.e., of the order of $1 d B$ due to the insertion loss of the subreflector). An overall efficiency of 50 percent was estimated for the Cassegrainian system, but the sidelobe performance was poor.

## C. Applications

Dual-offset-reflector antennas can be designed to avoid (or minimize) aperture-blockage effects and thereby offer a good range of compromise between high efficiency and low sindelobes. The use of a subreflector provides a mechanincal advantage for some applications in that the antenna system can be folded to locate the primary feed either within or below the main reflector. Electrically the dual-reflector system offers the designer some additional degrees of freedicm, which can be employed to compensate for the depolarization effects introduced by the asymmetric reflectors. However, in this optimized configuration the dual-offset system is mot especially compact. In addition, the very-low-cross-polar performance (i.e., below -40 dB ), which is predicted by geometiric optics, can only be achieved if the subreflector dimensions are greater than about 20 wavelengths.
Hence, when good polarization purity is desired, dual-ofisetreflector systems are best suited for applications where their dimensions can be made large with respect to the operating frequency. Earth stations, radio telescopes, terrestrial communications, and ground-radar systems are likely examples.

The Bell System open Cassegrainian design [1], [2]: Has been implemented with a $6-\mathrm{m}$ main reflector and has been used in satellite communication experiments with the © $\mathbf{U} . \mathrm{K}$. Post Office and the University of Birmingham (U.K.) The design has operated successfully although it exhibits the predictable poor cross-polar performance. Magne and Buif-Hai [60] have described a 4-band dual-polarized double-ofifiset antenna for terrestrial microwave links. The antenna crosspolar performance is not given in their paper, but the decoupling between channels is specified as 35 dB . This figure presumably includes orthogonally polarized channels at the same frequency but gives no information on the cross-polar levels which may exist at angles off the antenna boresight. The Magne and Bui-Hai design is illustrated in Fig. 39. One example of the use of dual offset reflectors in a radar antenna is that manufactured by MEASL in the U.K. Performance data on this antenna is not available; but the system, wiaich is
 (Photograph reproduced with the permission of Thomson CSF.) (b) Schematic diagram of a four-band offset Cassegrainian antenna.
illustrated in Fig. 40, can be seen to make use of a fully offset configuration. Mizusawa et al. [61] have described the geo-metric-optics design and experimental modeling at 24 GHz of a dual-shaped-reflector configuration for a circular polarized


Fig. 40. Dual-offset radar reflector antenna. (Photograph reproduced by permission of Marconi Elliott Avionic Systems Ltd.)
shaped-beam antenna. The design was intended for a search radiar and provided a very narrow beam in the azimuth plane with a cosecant-squared beam in the elevation plane. This fully offset antenna incorporated an axisymmetric primaryfeed illumination which was converted by the shaped subreflector to feed the main elliptically contoured, shaped main reflector. The measured azimuthal-plane sidelobes of this antenna were only -18 dB down, and the axial ratio of the circular polarization was also poor. Both performance parameters were attributed to a nonoptimum primary-feed hom design.
More recently Semplak [56] has described measurements (at 100 GHz ) on a multiple-beam offset antenna which employs an interesting three-reflector configuration. Although the author indicates that the design is suitable for both satellites and Earth stations, the structure has a projected-aperture diameter of more than 200 wavelengths with an $F / d$ ratio of 1.9. Without deployment this would be difficult to accommodate on a satellite for frequencies lower than the 30 GHz possibility mentioned in the text. The three-reflector design exhibits good wide-angle beam performance, but the question of the minimum-achievable beam spacing, which is also critical in many multiple-beam satellite applications, is not discussed.

Much of the analysis of the dual-offset-reflector antenna has been performed in connection with either future Earth-station antennas [13], [53], [54] or spacecraft applications [55][58]. The spacecraft application tends to be limited by the volumetric constraints imposed by the launcher. Without deployment, the cross-polar performance of the antenna tends to be limited by diffraction effects arising from the relatively small subreflector. As an Earth-station antenna, however, the electrical performance of the optimized double-offset-reflector
antenna is very attractive. The increasing demands made upon the radiation performance of Earth-station antennas, and particularly the sidelobe and cross-polar specifications, suggest that optimized double-offset-reflector antennas may well be necessary to satisfy the electrical requirements of the next generation of large Earth stations.
The more general application of the dual-offset-reflector antenna may be hampered by the considerable computational effort which is involved in performing design and optimization with physical-optics techniques. Approximate techniques, based upon geometric optics and the GTD, are adequate for large reflectors, but subreflector diffraction must be accurately modeled for other cases. Analytical techniques of the type recently introduced by Galindo-Israel and Mittra [62]-[64] or the GTD methods under development by Pontoppidan [65] may prove to be useful here. As an interim step it is worth noting that much of the antenna optimization can be performed by examining the vector fields in the aperture plane of the main offset reflector and thus minimizing the need for the second (and costly) two-dimensional integration over the reflector surface.

## VI. Conclusions

This paper has presented a tutorial review of the state of the art in offset-parabolic-reflector antennas. The review has been concerned with offset antennas with both single and dualreflector configurations.
The emergence, since the mid-1960's, of computer-aided techniques for the analysis of the offset-parabolic reflectors and the modeling of the vector radiation of primary-feed antennas has led to significant advances in the design of offset antennas. The modern trends toward conservation of the spectrum, lower sidelobe radiation, frequency reuse, multiple beams, and contoured beams have also provided stimuli for further investigation of reflector systems which avoid aperture blockage. This paper has reviewed the fundamental advantages and disadvantages of offset-parabolic-reflector antennas, their analysis, and the resultant predicted and measured electrical performance.

In the past a major drawback to the use of offset-parabolic reflectors has been their inherently poor polarization'purity. This characteristic impinges upon the performance of the offset antenna in both communications and radar systems. Analysis of the offset reflector has led to a more complete understanding of this phenomena and has indicated several means of correcting the defect. The $F / d$ ratio of the structure can be made large, which is mechanically undesirable in some cases; an optimized dual-reflector geometry can be implemented, which is particularly effective for larger antennas; or, the reflector depolarization can be compensated for by use of an appropriately designed primary-feed. Employing these techniques, dual-polarized unblocked reflector antennas can be designed which offer significant advantages over their axisymmetric counterparts. In particular, a better compromise between high efficiency and low sidelobes can be realized with good polarization purity.
Further improvements in primary-feed design for both single- and dual-offset- reflector systems will be required if the full (dual-polarized) potential of these antennas is to be realized. Good suppression of unwanted cross-polarized radiation from the primpry-feed is becoming increasingly important for high-performance communication and ra dar systems. Side-
lobe and cross-polar radiation will be required to be suppressed to lower levels, and the condition maintained over wider bandwidths for many future systems. It is also worth noting that the full capabilities of dual-offset-reflector antennas have yet to be established, and much work remains to be done in this area.

Modern trends toward multiple beams, contoured beams, and generally more complex and higher performance antenna systems generally favor offset-antenna systems. With further development, it can be anticipated that the use of offset-parabolic-reflector antennas of all types will become increasingly popular in the future.

## Appendix A-Offset-Parabolic-Reflector Geometry

Distances from the reflector focus in the $z^{\prime}=0$ plane can be written

$$
\begin{align*}
x^{\prime} & =p \sin \theta^{\prime} \cos \phi^{\prime} \\
& =p\left(\cos \theta_{0} \sin \theta \cos \phi+\sin \theta_{0} \cos \theta\right)  \tag{A1}\\
y^{\prime} & =p \sin \theta^{\prime} \sin \phi^{\prime} \\
& =p \sin \theta \sin \phi . \tag{A.2}
\end{align*}
$$

The Jacobian for a surface element $d x x^{\prime} d y$ can be obtained from the equations given as

$$
\begin{align*}
d x^{\prime} d y & =p^{2} \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime} \\
\quad & =p^{2} \sin \theta d \theta d \phi . \tag{A3}
\end{align*}
$$

The unit normal to the reflector parabolic surface $\left(a_{n}\right)$ is given by

$$
\begin{equation*}
\bar{a}_{n}=-\left(\bar{a}_{\rho^{\prime}}+\bar{a}_{z^{\prime}}\right) /\left|\left(\bar{a}_{\rho^{\prime}}+\bar{a}_{z^{\prime}}\right)\right| \tag{A4}
\end{equation*}
$$

Since the unit vector along $\rho$ is equal to that along $\rho^{\prime}$ we have

$$
\begin{equation*}
\bar{a}_{\rho^{\prime}}=\sin \theta \cos \phi \bar{a}_{x}+\sin \theta \sin \phi \bar{a}_{y}+\cos \theta \bar{a}_{z} \tag{A5}
\end{equation*}
$$

and from the offset geometry

$$
\begin{equation*}
\bar{a}_{z^{\prime}}=\bar{a}_{z} \cos \theta_{0}-\bar{a}_{x} \sin \theta_{0} \tag{A6}
\end{equation*}
$$

Hence we obtain

$$
\begin{gathered}
\bar{a}_{n}=-\sqrt{(p / 4 \bar{F})}\left[\left(\sin \theta \cos \phi-\sin \theta_{0}\right) \bar{a}_{x}\right. \\
\left.\quad+\sin \theta \sin \phi \bar{a}_{y}+\left(\cos \theta+\cos \theta_{0}\right) \bar{a}_{z}\right] . \\
\quad \therefore \quad \text { REFERENCES }
\end{gathered}
$$

[1] J. S. Cook; E: M. Elam, et al., "The open Cassegrain antenna," Bell Telephone Syst. Tech Publ., Monograph 5051, 1965.
[2] J. S. Cook, E. M. Elam, and H. Zucker, "The open Cassegrain antenna-Part 1: Electromagnetic design and analysis," Bell Syst. Tech. J., vol. 44, pp. 1255-1300, 1965.
[3] T. S. Chu and R. H. Turrin, "Depolarization properties of offset reflector antennas," IEEE Trans. Antennas Propagat. vol. AP-21, pp. 339-345, May 1973.
[4] A. W. Rudge and M. Shirazi, "Multiple beam antennas: Offset reflectors with offset feeds," Univ. Birmingham, England, July 1973, Final Report ESA Contract $1725 / 72$ PP.
[5] A. W. Rudge, T. Pratt, and M. Shirazi, "Radiation-fields from offset reflector antennas," in Proc. European Microwave Conf. (Brussels, Belgium), 1973 , vol. C3.4, pp. 1-4.
[6] A. W. Rudge, "Multiple-beam offset reflector antennas for spacecraft," in Proc. Inst Elec. Eng. Int: Conf, on Antennas for Aircraft and Spacecraft (London, England), 1975, Conf. Publ. 128, pp. 136-141.
[7] Pp. "Multiple-beam antennas: Offset reflectors with offset feeds,' IEEE Trans. Antennas Propagat., vol. AP-23, pp. $317-$ 322 , May 1975.
[8] J. Dijk, C.T.W. van Diepenbeek, et al., "The polarisation losses of offset antennas," Eindhoven Univ. Tech., The Netherlands, June 1973, TH Rep. $73-\mathrm{E}-39$.
[9] -"The polarisation losses of offset parabuioid antennas,"

IEEE Trans Antennas Propagat., vol. AP-22, pp. 513-520, July 1974.
[10] M. J. Gans and R. A. Semplak, "Some far-field studies of an offset launcher," Bell Syst. Tech. J., vol. 54, pp. 1319-1 340, 1975.
[11] R. Graham, "The polarisation characteristics of offset Cassegrain aerials," in Proc. Inst Elec. Eng. Int Conf. on Radar Present and Future, 1973, Conf. Publ. 105, pp. 23-25.
[12] N. A. Adatia, "Cross-polarisation of reflector antennas," Ph.D. dissertation, Univ. Surrey, England, Dec. 1974.
[13] H. Tanaka and M. Mizusawa, "Elimination of cross-polarisation in offset dual reflector antennas," Elec. Commun. (Japan), vol. 58, pp. 71-78, 1975.
[14] A. W. Rudge and N. A. Adatia; "New class of primary-feed antennas for use with offset parabolic-reflector antennas," Inst. Elec. Eng. Electron. Lett., vol. 11, pp. 597-599, 1975.
[15] -" "Matched-feed for offset parabolic reflector antennas," in Proc. 6th European Microwave Conf. (Rome, Italy), Sept. 1976, pp. 143-147.
[16] J. Jacobsen, "On the cross polarisation of asymmetric reflector antennas for satellite applications," IEEE Trans. Antennas Propagat., vol: AP-25, pp. 276-283, Mar. 1977.
[17] C. Dragone and D. C. Hogg, "The radiation pattern and impedance for offset and symmetrical near-field Cassegrainian and Gregorian antennas," IEEE Trans Antennas Propagat., vol. AP-22, pp. 472-475, May 1974.
[18] H. P. Coleman et al., "Paraboloidal reflector offset feed with a corrugated conical horn," IEEE Trans. Antennas Propagat., vol. AP-23, pp. 817-819, Nov. 1975.
[19] A. W. Rudge, P. R. Foster, et al., "Study of the performance and limitations of muitiple-beam antennas," ERA (RF Technology Centre, Engiand), Sept. 1975 , Rep. ESA Con. 2277/74HP.
[20] A. W. Rudge and N. Williams, "Offset reflector spacecraft antennas: Design and evaluation at 30 GHz ," in Proc. Symp. on Advanced Satellite Communication Systems (Genoa, Italy), Dec. 1977, ESA Publ. SP-1 38, pp. 105-1 13.
[21] P. G. Ingerson, "Off-axis scan characteristics of offset fed parabolic reflectors," in IEEE Int. Symp. Digest AP-S (Urbana, IL), June 1975, pp. 382-383.
[22] J. F. Kaufmann and W. F. Croswell, "Off-focus characteristics of the offset fed parabola," in IEEE Int. Symp. Digest AP-S (Urbana, IL), June 1975, pp. 358-361.
[23] E. A. Ohm, "A proposed multiple-beam microwave antenna for earth stations and satellites," Bell Syst. Tech. J., vol. 53, pp. 1657-1665, Oct. 1974.
[24] P. Wood, A. G. P. Boswell et al., "Elliptical beam antenna for satellite applications," in Proc. Inst. Elec. Eng. Int Conf. on Antennas for Aircraft and Spacecraft (London, England), June 1975 , Conf. Publ. 128, pp. 83-94.
[25] C. C. Han, "A multifeed offset reflector antenna for the Intelsat V Communications Satellite," in Proc. 7th European Microwave Conf. (Copenhagen, Denmark), Sept. 1977, pp. 343-347.
[26] A. W. Rudge and N. A. Adatia, "Primary-feeds for boresightjitter compensation of offset-reflector radar antennas,' in Proc. Inst. Elec. Eng. Int. Conf. on Radar 77 (London, England), Oct. 1977, Conf. Publ. 155, pp. 409-413.
[27] S. Silver, Microwave Antenna Theory and Design. New York: McGraw-Hill, 1949.
[28] R. E. Collin and F. J. Zucker, Antenna Theory. Parts I and II. New York: McGraw Hill, 1969.
[29] W. V. T. Rusch and P. D. Potter, A nalysis of Reflector Antennas. New York: Academic, 1970.
[30] A. G. P. Boswell and R. W. Ashton, "Beam squint in a linearly polarised offset reflector antenna," Inst. Elec. Eng. Electron. Lett., vol. 12, pp. 596-597, Oct. 1976.
[31] A. C. Ludwig, "The definition of cross polarisation," IEEE Trans. Antennes Propagat., vol. AP-21, pp. 116-119, Jan. 1973.
[32] N. A. Adatia, A. W. Rudge, and C. Parini, "Mathematical modelling of the radiation fields from microwave primary-feed antennas,'" in Proc 7th European Microwave Conf. (Copenhagen, Denmark), Sèpt. 1977, pp. 329-333.
[33] A. Ralson, A First Course in Numerical Analysis. New York: McGraw-Hill, 1965.
[34] H. Schjaer-Jacobsen, "Computer"programs for one- and twodimensional Romberg integration of complex functions," Tech. Univ. of Denmark, Lab. of EM Theory, Rep. D187, 1973.
[35]. H. A. Lessow, W. V. T. Rusch, et al.,"On numerical evaluation of two-dimensional phase integrals," IEEE Trans. Antennas Prop. agat. vol. AP-23, pp. 714-717, Sept. 1975 :
[36] A. C. Ludwig, "Calculation of scattered patterns from asymmetrical reflectors." Jet Propulsion Lab., (Passadena, CA), 1970, Tech. Rep. 32-1430.
[37] A. W. Rudge, T. Pratt, and A. Fer, "Cross-polarised radiation from satellite reflector antennas," in Proc. AGARD Conf. on Antennas for A vionics (Munich, Germany), Nov. 1973, vol. 16, pp. 1-6.
[38] J. E. Hansen and L. Shafai, "Cross-polarised radiation from waveguides and narrow-angle horns," Inst. Elec. Eig. Electron. Lett., pp. 313-315, May 1977.
[39] P. R. Foster and A. W. Rudge, "Low sidelobe antenna study: Part 1: Literature survey and review,' ERA. RF Technology Centre, England; Oct. 1975, Rep. 190476/1..
[40] N. A. Adatia, P. R. Foster, and A. W. Rudge, "A study of the limitations in RF sensing signals due to distortions of large spacecraft antennas," ERA (RF Technology Centre, England), Sept. 1975, Rep. ESA Con. 2330/74 AK.
[41] D. F. DiFonzo, W. J. English, and J. L. Janken, بPolarisation characteristics of offset reflectors with multiple-element feeds," in IEEE Int. Symp. Digest PGAP (Boulder, Colorado), 1973, pp. 302-305.
[42] N. A. Adatia and A. W. Rudge, "Beam-squint in circularlypolarised offset reflector antennas,". Int. Elec. Eng. Electron. Lett., Pp. $513-515$, Oct. 1975.
[43] R. W. Gruner and W. J. English, "Antenna design studies for a" U.S. domestic satellite," Comsar Tech Rev., vol. 4, 2, pp. 413447, Fall 1974.
[44] N. A. Adatia and A. W. Rudge, "High performance offset-reflector spacecraft antenna development study." ERA RF Technology Centre, England, June 1976, Rep. ESA Con. $2654 / 76 /$ NLSW.
[45] D. Fasold, "Rechnergestützte Optimierung und Realisierung einer Offset-Reflektorantenne für Satelliten," NTG-Fachber, Germany, 1977, vol. 57, pp. 124-133.
[46] D. J. Bem, "Electric field distribution in the focal region of an offset paraboloid, Proc. Inst. Elec. Eng., vol. 116, pp. 579-684, 1974.
[47] A. R. Valentino and P. P. Toulios, "Fields in the focal region of offset parabolic reflector antennas," IEEE Trans Antennas and Propagat., vol. AP-24, pp. 859-865, Nov. 1976.
[48] P. G. Ingerson and W. C. Wong, "Focal region characteristics of offset fed reflectors" in IEEE Int. Symp. AP-S (Georgia), 1974 , pp. 121-1 23.
[49] K. C. Lang, M. K. Eick, and D. T. Nakatani, "A $6 / 4$ and $30 / 20$ dual foci offset paraboioidal reflector antenna," in IEEEE IntSymp. Digest AP-S (Urbana, IL), June 1975, pp. 391-395.
[50] P. J. B. Clarricoats and G. T. Poulton, "High efficiency microwave reflector antennas-A review," Proc. IEEE, vol. 65, pp. 1470-1504, Oct. 1977.
[51] B. K. Watson, A. W. Rudge, and N. Adatia, "Dual-polarised mode generator for cross-polar compensation in offset parabolic reflector antennas," presented at 8th European Microwave Conf., Paris, France, Sept. 1978.
[52] W. H. lerley and H. Zucker, "A stationary phase method for computation of the far-field of open Cassegrain antennas," Bell Syst. Tech. J., vol. 49, pp. 431-454, Mar. 1970.
[53] Y. Mizugutch, M. Akagawa, and H. Yokoi, "Offset dual reflector antenna," in IEEE Int. Symp. AP-S (Amherts, MA), Oct. 1976, pp. 2-5.
[54] M. Mizusawa and T. Katagi, "The equivalent parabola of a multireflector antenna and its application," Mitsubishi Elec. Eng., No. 49, pp. 25-29, Sept. 1976.
[55] N. A. Adatia, "Diffraction effects in dual offset Cassegrain antenna," to be presented at IEEE Int. Symp. AP-S; Washington, DC, May 1978.
[56] R. A. Semplak, " $100-\mathrm{GHz}$ measurement on a multiple-beam offset antenna," Bell Syst. Tech. J., vol. 56, pp. 385-398, Mar. 1977.
[57] N. C. Albertsen, "Shaped-beam antenna with low cross-polarisation" in Proc. 7th European Microwave Conf. (Copenhagen, Den mark), Sept. 1977, pp. 339-342.
[58] -"Dual offset reflector antenna shaped for low cross polatisation," TICRA (Copenhagen, Denmark), Mar. 1977, Rep. S-53-01.
[59] A. G. Cha, C. C. Chen, and D. T. Nakatani, "An offset Cassegrain reflector antenna system with frequency selective sub-reflector," in IEEE Int Symp. Digest AP-S (Urbana, IL), 1975, pp.
[60] P. Magne and N. Bui-Hai, "A modular offset Cassegrain antenna operating simultaneously in four frequency bands," in IEEE Int. Symp. Digest AP:S (Amherst Mass.), Oct. 19.76, pp. 10-19.
[61] M. Mizusawa, S . Betsudan, et al., "The dual doubly-curved reflectors for circularly polarised shaped-beam antennas," IEEE Int. Symp. Digest AP.S (Georgia), 1974, pp. 249-252.
[62] V. Galindo-Israel and R. Mittra, "A new series representation for the radiation integral with application to reflector antennas," IEEE Trans. Antennas Propagat. vol. AP-25, pp. 631-641, Sept. 1977.
[63] V. Galindo-israel and R. Mittra, "Synthesis of offset dual shaped reflectors with arbitrary control of phase and amplitude,' 'in Proc. IEEE Int. Symp. Digest A $-S$ (California), 1077.
[64] V. Galindo-Israel and R. Mittra, "New approaches to the analysis and synthesis of reflector antennas," in Proc. Symp. on Advanced Satellite Commun. Syst. (Genoa, Italy), Dec. 1977, ESA Publ. SP-138, pp. 75-79.
[65] K. Pontoppidan, "General analysis" of dual-offset reflector antennas," TICRA AP-S (Denmark), Oct. 1977, Final. Rep. S-66-02.
[66] A. R, Raab, "Cross-polarisation performance of the RCA Satcom frequency re-use antenna," IEEE Int. Symp. Digest AP.S (Am-
herst, M, A ), pp. $100-104$, Oct. 1976 .

# Depolarization Properties of Offset Reflector Antennas . 

TAsHING CHU aNd R. H. TURRIN

Abstract-The cross polarized radiation for lineatly polarized excitation and the beam displacement for circularly polarized excitation have been investigated for ofiset reflector antennas. Fumerical calculations are given to illustrate the dependence upon the angle $\theta_{0}$ between the feed axis and the refiector axis $a s$ well as upon the half-angle $\theta_{c}$ subtended at the focus by the reflector. In the case $\theta_{0}=\theta_{c}=45^{\circ}$, measured results hape been obtained for both lineariy and circularly polarized excitations with a dual mode feed illmminating an offset paraboloid.

The cross polarized radiation of horn refiector and open Cassegrainian antennas rises sharply to rather high values off the beam aris; however, in general, the manimum cross polarized radiation of offset reflector antennas can be made small by using a small angle between the feed and reffector axes. The cross polarization caused by offset is compared with that caused by an unbalanced feed pattern. The effect of the longitudinal current distribution and of departure of the surface from a paraboloid on cross polarization are also examined. The clarification of these cross polarization properties is found to be valuable in the design of reflector antennas.

## I. Introdoction

TN ORDER to increase the communication capacity of a transmission system by using orthogonal polarizations, it is essential to maintain the orthogonality, thereby

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preventing crosstalk. This requirement is easily fulfilled when the antenna is transmitting or receiving on the beam maximum. However poor pointing accuracy can rapidly increase depolarization in the vicinity of the beam maximum. In some satellite communication systems the ground stations are distributed over the beamwidth of the satellite antenna; in that case any polarization degradation within the satellite antenna beamwidth will give rise to cross polarization coupling.

The condition [1] that the directions of stationary polarization ${ }^{1}$ and maximum gain coincide is satisfied by all center-fed paraboloids where the aperture distribution of the polarization vectors are symmetrical with respect to the center of the aperture. Since it is difficult to predict the effect of aperture blocking on the sidelobe level and cross polarization properties, various versions of offset.. antennas such as the open Cassegrainian antenna [2] have been proposed. The purpose here is to discuss the depolarization properties of offset reflector antennss which can be conveniently characterized by two angles. Section II will calculate the cross polarization for linearly polarized excitation and the beam displacement fort,

[^5]ularily polarized excitation. Section III will describe measured results of an experimental example. Section IV will discuss the comparison of cross polarization in It wif-fed and offset paraboloids, the depolarization in ot Cassegrainian antennas; and other effects coneributing to depolarization.
Tpnes [3] showed that there will be no cross polarization he aperture field of a center-fed paraboloid if the feed plane wave source, i.e., a combination of electric and magnetic dipoles at right angles to each other with ratio al to the plane wave impedance. The deficiency in nlating this plane wave feed by a horn may be atEtributed to the questionable validity of the Kirchhoff roximation for a small horn. Recently, corrugated ns. [4], [5] and dual-mode horns [6], [7] with circularly symmetric radiation patterns have been recognized s apable of eliminating cross polarization in a center-fed aboloidal aperture. Theoretical prediction based on these feed patterns can be better realized in practice. We will use balanced feed radiation (defined by (5) in tion II) with circularly symmetric patterns to calculate cross polarization properties of offset reflectors, in $\therefore$ which case the cross polarization is only a consequence G the offset; it diminishes with a decrease in the angle ween the feed axis and reflector axis.

## Il. Cross Polarization and Beam Shift

Let an ofiset reflector be illuminated by a feed as shown in Fig. 1 where $\theta_{0}$ is the angle between the feed s and the reflector axis, and $\theta_{c}$ is the half-angle subded by the reflector at the focus. In this section we will calculate the cross polarization properties of the qector as a function of these two angles. The far-field pression of the feed radiation can be written

$$
\begin{equation*}
\bar{E}_{y}=E_{\theta}^{\prime} \hat{\theta}^{\prime}+E_{\phi}^{\prime} \hat{\phi}^{\prime} \tag{1}
\end{equation*}
$$

reflected field from the paraboloid is

$$
\begin{equation*}
\bar{E}_{r_{n}}=-\bar{E}_{f}+2 \hat{n}\left(\bar{E}_{f} \cdot \hat{n}\right) \tag{2}
\end{equation*}
$$

- ere $\hat{n}$ is a unit vector normal to the reflector surface. Substituting (1) into (2) and using the transformations $\left(, \hat{\theta}^{\prime}, \phi^{\prime}\right) \rightarrow\left(\dot{x}^{\prime}, \hat{y}^{\prime}, \hat{z}^{\prime}\right) \rightarrow(\hat{x}, \hat{y}, \hat{z})$ (see Appendix) and the mal vector $\hat{n}=-(\hat{\rho}+\hat{z}) /(2 t)^{1 / 2}$ for a paraboloidal surface, one obtains the following form:

$$
\begin{align*}
= & \frac{\hat{x}}{t}\left\{\left[\sin \theta^{\prime} \sin \theta_{0}-\cos \phi^{\prime}\left(1+\cos \theta^{\prime} \cos \theta_{0}\right)\right] E_{\theta}^{\prime}\right. \\
& \left.+\sin \phi^{\prime}\left(\cos \theta^{\prime}+\cos \theta_{0}\right) E_{\phi}^{\prime}\right\}  \tag{3}\\
& +\frac{\hat{y}}{t}\left\{-\sin \phi^{\prime}\left(\cos \theta^{\prime}+\cos \theta_{0}\right) E_{\theta}^{\prime}\right. \\
& \left.+\left[\sin \theta^{\prime} \sin \theta_{0}-\cos \phi^{\prime}\left(1+\cos \theta^{\prime} \cos \theta_{0}\right)\right] E_{\phi}^{\prime}\right\}
\end{align*}
$$

$$
t=1+\cos \theta^{\prime} \cos \theta_{0}-\sin \theta^{\prime} \sin \theta_{0} \cos \phi^{\prime}
$$



Fig. 1. Geometry of offset refector.
The $\hat{z}$ component is absent in the aperture because of the unique focusing property of paraboloid [8]. However $z$ component currents flow on the reflector surface.

Let us consider a balanced feed radiation which can be written in the following form:

$$
\bar{E}_{f}=F\left(\theta^{\prime}, \phi^{\prime}\right)\left[\begin{array}{ccc}
\cos \phi^{\prime} & & \sin \phi^{\prime}  \tag{5}\\
\hat{\theta}^{\prime} \mp & \\
\sin \phi^{\prime} & \cdot \cos \phi^{\prime}
\end{array}\right] \frac{\exp \left(-j k_{\rho}\right)}{\rho}
$$

corresponding to two principal linear polarizations along the $x$ and $y$ axes, respectively. An important special case of (5) is a circularly symmetric radiation pattern where $F\left(\theta^{\prime}, \phi^{\prime}\right)$ is independent of $\phi^{\prime}$. The principal polarization component of the reflected field becomes

$$
\begin{align*}
M=\bar{E}_{r} \cdot \hat{y}= & \frac{F\left(\theta^{\prime}, \phi^{\prime}\right)}{t \rho}\left[\sin \theta^{\prime} \sin \theta_{0} \cos \phi^{\prime}-\sin ^{2} \phi^{\prime}\right. \\
& \left.\cdot\left(\cos \theta_{0}+\cos \theta^{\prime}\right)-\cos ^{2} \phi^{\prime}\left(1+\cos \theta_{0} \cos \theta^{\prime}\right)\right] \tag{6}
\end{align*}
$$

While the cross polarization component is

$$
\begin{align*}
N=\bar{E}_{\tau} \cdot \hat{y}= & \mp \frac{F\left(\theta^{\prime}, \phi^{\prime}\right)}{t_{\rho}}\left[\sin \theta^{\prime} \sin \theta_{0} \sin \phi^{\prime}\right. \\
& \left.-\sin \phi^{\prime} \cos \phi^{\prime}\left(1-\cos \theta^{\prime}\right)\left(1-\cos \theta_{0}\right)\right] \tag{7}
\end{align*}
$$

where $M^{2}+\lambda^{2}=F^{2} / \rho^{2}$ and $\lambda$ vanishes when $\theta_{0}=0$.
One notes that the rotation of the polarization vector due to offset in a paraboloidal aperture has the same magnitude and is in the same sense for any orientation of the incident linear polarization. It follows that the reflected field of a circularly polarized wave will remain circularly polarized but in opposite rotating sense, and with a phase shift $\tan ^{-1}(N / M)$. If the feed radiation is circularly polarized everywhere, no cross polarization will appear in the radiation from the reflector; however, a small beam displacement will occur because of variation in the phase shift across the reflector.

The projection of the intersection of a circular cone (with vertex at the feed) and the offset paraboloid onto
the $x y$ plane is a circle [2] with center

$$
\begin{equation*}
x_{c}=\frac{2 f \sin \theta_{0}}{\cos \theta_{c}+\cos \theta_{0}} \tag{8}
\end{equation*}
$$

and the diameter

$$
\begin{equation*}
d=\frac{4 f \sin \theta_{c}}{\cos \theta_{c}+\cos \theta_{0}} \tag{9}
\end{equation*}
$$

where $f$ is the focal length of the paraboloid. The relationships between points on the paraboloid and their projections onto the $x y$ plane are
$x=\rho\left(\cos \theta_{0} \sin \theta^{\prime} \cos \phi^{\prime}+\sin \theta_{0} \cos \theta^{\prime}\right)$
$y=\rho \sin \theta^{\prime} \sin \phi^{\prime}$
$\rho=\frac{2 f}{1+\cos \theta}=\frac{2 f}{1+\cos \theta^{\prime} \cos \theta_{0}-\sin \theta^{\prime} \sin \theta_{0} \cos \phi^{\prime}}$.

Then the far-field pattern of the antenna is given for small angles by
$A=\int_{-\pi}^{\tau} \int_{0}^{\delta_{c}} \frac{L F}{\rho} \exp \left\{j \frac{2 \pi v}{d}\left[\left(x-x_{c}\right) \cos \phi_{a}+y \sin \phi_{a}\right]\right\}$

$$
\begin{equation*}
\cdot \rho^{2} \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime} \tag{13}
\end{equation*}
$$

where $v=\sin \theta_{a} /(\lambda / d)$ and $L$ is a factor for the polarization component being calculated. The aperture surface element can be reduced to $\rho^{2} \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime}$ [2]. Assuming the circularly symmetric feed pattern to be the $H$-plane pattern of an open-end circular waveguide excited by $\mathrm{TE}_{11}$ mode

$$
\begin{equation*}
F\left(\theta^{\prime}\right)=\left\{\left[1-\left(\frac{u_{11}}{u}\right)^{2}\right]^{1 / 2}+\cos \theta^{\prime}\right\} \frac{J_{1}^{\prime}\left(u \sin \theta^{\prime}\right)}{1-\left(\frac{u \sin \theta^{\prime}}{u_{21}}\right)^{2}} \tag{14}
\end{equation*}
$$

Where $u$ is the circumference of the waveguide in wavelengths, and $u_{11}$ is the first root of $J_{1}^{\prime}(u)=0$. This is a good representation of a small aperture dual mode feed [6]. Substituting (6) for $L$ in (13) yields, for the principal polarization component,

$$
\begin{align*}
P=2 \int_{0}^{\pi} \int_{0}^{\epsilon_{0}} M & \exp \left[j \frac{2 \pi v}{d}\left(x-x_{c}\right) \cos \phi_{o}\right] \\
& \cdot \cos \left[\frac{2 \pi v}{d} y \sin \phi_{a}\right] \rho \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime} \tag{15}
\end{align*}
$$

For the cross polarization component, substituting (7) for $L$ in (13) gives

$$
\begin{align*}
C=2 j \int_{0}^{\pi} \int_{0}^{t_{c}} N & \exp \left[j \frac{2 \pi v}{d}\left(x-x_{e}\right) \cos \phi_{a}\right] \\
& \cdot \sin \left(\frac{2 \pi v}{d} y \sin \phi_{a}\right) \rho \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime} . \tag{16}
\end{align*}
$$



Fig. 2. Radiation patterns of offeet paraboloid ( $f / D-0.25$ ).
Putting $L=\exp \left[j \tan ^{-1}(N / M)\right]$, (13) will give the displaced radiation pattern for circular polarization as

$$
\begin{align*}
K= & 2 \int_{0}^{*} \int_{0}^{\theta_{c}} \exp \left[j \frac{2 \pi v}{d}\left(x-x_{c}\right) \cos \phi_{a}\right] \\
& \cdot \cos \left[\tan ^{-1} \frac{N^{\prime}}{M}+\frac{2 \pi v}{d} y \sin \phi_{a}\right] F^{\prime} \rho \sin \theta^{\prime} d \theta^{\prime} d \phi^{\prime} . \tag{17}
\end{align*}
$$

Oxing to cancellation by symmetry, the cross polarization will vanish when $\phi_{a}=0^{\circ}$. The numerical calculation will be made for $\phi_{c}=90^{\circ}$ where maximum cross polarization is expected. The parameter $u$ in (14) is selected to give a 10 dB taper for the feed pattern. For example, when $\theta_{c}=\theta_{0}=45^{\circ}$, numerical integration for (15)-(17) results in the plots in Fig. 2. The cross polarization has a maximum located beyond the -3 dB point of the main beam; however, at the -3 dB point it is only slightly below the maximum. In the far field of the main beam, the cross polarized and principally polarized components are in phase quadrature whereas they are in phase at the aperture of the reffector. The offset refiector is also characterized by a lack of polarization stationarity at the beam maximum, consequently the cross polarization rises sharply off axis.
The beam displacement $\Delta \beta$, as defined in Fig. 2, is the shift of the circularly polarized beam with respect. to the physical plane of symmetry. The direction of the shift is towards the right for left-handed circular polarization and towards the left for right-handed circular polarization. ${ }^{2}$ Since the half-power beamwidth $2 \beta_{1}$ of the linearly

[^6]3. Cross polarization and besm displacement versus $f / D$ ratio ( $\theta_{0}=\theta_{c}$ ).
polarized pattern equals that of the circularly polarized nattern: $\Delta \beta=\beta_{1}-\beta_{2}$ where $\beta_{2}$ is the angle between the is and the half-power direction of the circularly polarized ttern. The beam displacement for circularly polarized excitations results in some sacrifice of power if two posite circular polarizations are used simultaneousiy thin a specified angular region. The cross polarization oi linearly polarized excitation for an offiset reflector a be regarded as the consequence of the differential in of its two opposite circularly polarized components. This interpretation is analogous to the depolarization of arcularly polarized excitation for a pyramidal horn, consequence of the difierence between the patterns of two linearly polarized components.
Numerical data on the maximum cross polarization or linearly polarized excitation and the beam displacelent for circularly polarized excitations are summarized in three figures. Fig. 3 shows the variation with respect $f / D$ ratio (or the angle $\theta_{0}$ ) for the case of $\theta_{0}=\theta_{c}$. is seen that offset reflectors with large $f / D$ ratios of the order of unity will introduce little cross polarization. Insing $\theta_{0}$ as abscissa and $\theta_{c}$ as parameter, their effects on de maximum cross polarization and the beam displacement are presented, respectively, in Figs. 4 and 5. A feed pattern of 10 dB taper has been used in these alculations. Computation also shows that increasing de taper to 20 dB reduces the maximum cross polarization only about 1 dB .
The previously computed cross polarization of -23 dB Or an open Cassegrainian antenna [2] with $\theta_{0}=47.5^{\circ}$ and $\theta_{c}=30.5^{\circ}$ checks very well with our results. It is -lso interesting to compare these cross polarization data rith those of the pyramidal and conical horn reflectors [9], [10], where $\theta_{0}=90^{\circ}$. One recalls that cross polarizations are present for both linearly and circularly polarized xcitations of the horn reflector because this antenna


Fig. 4. Maximum cross polarization of linearly polarized excitation.


Fig. 5. Beam displacement of circularly polarized excitation (no circular cross polarization).
is not only offset but also has different aperture distributions for longitudinal and transverse polarizations. The aperture distribution of the horn reflector is a result of single-mode excitation of the horn.

## III. Measured Results

This section describes an 18.5 GHz experimental test of the cross polarization properties of an offset refiector antenna. The reflector consists of a paraboloidal section which was molded with Ultracal (a nonshrinking molding plaster) on a standard spun aluminum paraboloid with an $f / D$ ratio of 0.25 . This partial paraboloid corresponds to the case of $\theta_{0}=\theta_{c}=45^{\circ}$, and its projection onto the plane normal to the axis of the paraboloid is a circular aperture of 12 in diameter.


Fig. 6. Measured radiation pattern of ofiset fed paraboloid antenns linearly polarized at 18.5 GHz . Aperture diameter is 12 in , $f / D=0.25$.

A dual mode feed of circularly symmetric radiation provides a pattern taper of -10 dB at the periphery of the offset paraboloidal section. Because of the geometrical difference in path length from focal point to the paraboloidal section there exists an amplitude asymmetry of 6 dB in the aperture distribution for the case $f / D=0.25$. These conditions of illumination are the same as those of the theoretical calculations.

Radiation patterns were recorded for various polarization combinations in a plane transverse to the plane of symmetry of the offset paraboloid because this plane is expected to show maximum cross polarization for linearly polarized excitation and maximum beam displacement for circular polarization. Since the cross polarization patterns in the symmetry plane $\phi_{a}=0^{\circ}$ are theoretically null patterns, no attempt was made to measure them. Fig. 6 shows the in-line and cross polarized patterns with the principal linear polarization oriented in the plane of symmetry. The patterns were virtually identical with polarization either in the plane of symmetry or orthogonal to it.

Fig. 7 shows a composite of two measured patterns for the two senses of circular polarization, respectively. The angular displacement between the two patterns of opposite circular polarizations is twice the displacement of each circular polarization beam relative to the physical plane of symmetry with the direction as expected from the calculation in the preceding section. The maximum undesired circular polarization, which ideally should be zero: has been measured to be less than -38 dB relative


Fig. 7. Measured radiation patterns of offeet fed paraboloid antenna with circular polarization at 18.5 GHz . Aperture diameter is $12 \mathrm{in}, f / D=0.25$.
to the beam maximum. This residual depolarization can be attributed to the effect of the longitudinal current distribution (see Section IV-C), the experimental environment, and the imperfection of the feed.

To compare with the calculated results, the measured maximum cross polarization and beam displacement for the case of $\theta_{0}=\theta_{c}=45^{\circ}$ have been plotted in Fig. 3 where agreement is within about 5 percent. No elaborate efforts have been made to improve the imperfect symmetry of the cross polarization lobes in Fig. 6. The low sidelobe levels in both Figs. 6 and 7 can be explained by the nonorthogonality between the plane of the refiector edge and the plane of the measured patterns.

## IV. Discussion

## A. Comparison of Cross Polarization in Center-Fed and Offset Paraboloids

The cross polarization discussed in the preceding section originates entirely from the offset between the feed axis and the reflector axis. This kind of cross polarization vanishes when the angle of offset approaches zero. It is well known that cross polarization can also occur without offset when the feed radiation is unbalanced and does not satisfy (5). The characteristics of these two cross polarization mechanisms are enumerated in Table I.
Fig. 8 illustrates the cross polarization characteristics in the paraboloidal aperture for offset and unbalanced feed, respectively. In the offset cass, the two principal linear polarizations rotate in the same direction at a given

TABLE I
Comparison Between Two Cross Polbrization Mecennisms
Offset Paraboloid with Balanced Feed

Unbalanced Feed
[(5) invalid]

- Unstationary (cusp) polarization at beam maximum.

2. The polarization vectors rotate in the asme direction for two principal linear polarizations in the paraboloidal aperture [Fig. $8(\mathrm{a})$ ].
3. Cross polarized radiation in phase quadrature with principal polarization.
Cancellation of cross polarization in one principal plane only.
4. Maximum cross polarization in the other principal plane. No crose polarization but beam displacement for circularly polarized excitation.
5. Stationary (saddle point) polarization at beam maximum.
6. The polarization veciors rotate in opposite directions for two principal linear polarizations in the paraboloidal aperture [Fig. 8(b)].
7. Cross polarized radiation in phose ${ }^{2}$ with principal polariz ation.
8. Cancellation of cross polarization in both principal planes.
9. Maximum crobs polarization in the $45^{\circ}$ planes.
10. Equal cross polarization lobes in all planes for circularly. polarized excitation.
circularly symmetric feed pattern will. result in an elliptically shaped far-field radiation pattern with the same cross polarization properties as an offset circular aperture. However some sacrifice in gain will occur due to spillover.

## B. Depolarization in Offset Cassegrainian Antennas

A Cassegrainian antenna has an equivalent focal length much longer than that of its main reflector [11]. Thus an offset Cassegrainian antenna may have a large effective $f / D$ ratio and hence a small angle between the feed axis and the main reflector axis. The limiting case is that of two partial paraboloids, confocal and coaxial, which form a near field Cassegrainian or Gregorian antenna without aperture blocking. Bisected near-field Cassegrainian telescopes have been used in infrared propagation experiments [12]. In that case the feed axis is parallel to the main reflector axis, and there will be no cross polarization in the aperture of the main reflector provided no cross polarization illuminates the subreffector [13]. ${ }^{\text {a }}$ In other words, the combination of a plane wave feed and a confocal-paraboloid subreflector gives an effective balanced feed radiation which satisfies (5) in spite of the asymmetry of the geometrical configuration. This example illustrates that balanced feed radiation is not necessarily; circularly symmetric.
Some versions of offset Cassegrainian antennas are expected to have poor off-axis polarization discrimination (of the order of 20 dB ) because of the relatively large angle between the feed and refiector axes at certain orientations of the antenna. This poor polarization performance is the price paid for the mechanical advantage of the feed location in the open Cassegrainian antenna [2]. It is tempting to introduce additional subreflectors to bring the feed axis parallel to the reflector axis for open Cassegrainian antennas.

## C. Other Effects Contributing to Depolarization

1) Longitudinal Current Distribution: The cross polarization contributed by the longitudinal current distribution on a reflector antenna is expected to be of the order of the isotropic level. Therefore neglecting the longitudinal current distribution, as done in this paper, is only valid for a large reflector with high gain. In particular, for a center fed paraboloid uniformly illuminated by a feed radiation pattern which satisfies (5), the reflected field (obtained by geometrical optics ray tracing) contains no cross polarization in the paraboloidal aperture. However, it can be deduced from the radiation integral [14] of the current distribution on the reflector surface that the voltage ratio of the maximum cross polarization to the isotropic level is approximately $D / 8 f$.
2) Nonparaboloidal Surface Shape: In the design of a multibeam or scanning antenna, one often likes to use spherical or torus reflectors which are slight perturbations of true paraboloids. It is of interest to investigate the

[^7]contribution of such changes in the surface shape to the cross polarization:
The reflected field $\bar{E}_{\text {r }}$ from a perfectly conducting surface can be expressed in terms of the incident field $\tilde{E}_{i}$ and the unit vector normal $\hat{n}$ to the surface by geometrical optics as in (2) which is rewritten as
\[

$$
\begin{equation*}
\bar{E}_{\Gamma}=-\bar{E}_{i}+2 \hat{n}\left(\hat{n} \cdot \bar{E}_{i}\right) \tag{18}
\end{equation*}
$$

\]

If the surface deviates slightly from a paraboloid, i.e., $\hat{n}^{\prime}=\hat{n}+\bar{\Delta}$, then the reflected field becomes

$$
\begin{equation*}
\dot{E}_{r^{\prime}}=-\bar{E}_{i}+2 \hat{n}^{\prime}\left(\hat{n}^{\prime} \cdot \bar{E}_{i}\right) \tag{19}
\end{equation*}
$$

Subtracting (18) from (19) and taking its component along the direction of cross polarization, one obtains

$$
\begin{align*}
\left(\bar{E}_{r^{\prime}}-\bar{E}_{r}\right) \cdot \hat{X}=2(\hat{n} \cdot \hat{X})\left(\bar{\Delta} \cdot \bar{E}_{i}\right) & +2(\bar{\Delta} \cdot \hat{X})\left(\hat{n} \cdot \bar{E}_{i}\right) \\
& +2(\bar{\Delta} \cdot \hat{X})\left(\bar{\Delta} \cdot \dot{E}_{i}\right) . \tag{20}
\end{align*}
$$

It is obvious from (20) that the cross polarization will be reduced to a second-order effect if the $\hat{n}$ is nearly perpendicular to both $\hat{X}$ and $\bar{E}_{i}$. This latter condition is certainly satisfied by a reflector with large $f / D$ ratio. In the case of a multibeam reflector antenna a large $f / D$ ratio is often required in any case for reducing the phase error.
It is also evident from (20) that the cross polarization vanishes when both $\hat{n}$ and $\bar{\Delta}$ lie in, or parallel to, a constant plane of incidence. This observation is simply a rediscovery of the fact that no depolarization is produced by a cylindrical reflector.

## IV. Conclusion

The cross polarization of linearly polarized excitation and the beam displacement of circularly polarized excitation have been presented in Figs. 4 and 5 versus two characteristic angles for an offset reflector with a circularly symmetric feed pattern. An experimental example has verified the theoretical calculations. The nonstationary polarization at the beam maximum of an ofiset reflector implies stringent pointing tolerance. A larger efiective $f / D$ ratio always reduces the cross polarization. Some asymmetrical configurations of a multireflector antema such as the combination of two partial confocal paraboloids may keep the feed axis parallel to the reflector axis and hence avoid cross polarization due to offset.

## Appendix

With reference to Fig. 1, the transformations for the unit vectors $\left(\vec{p}^{\prime}, \hat{a}^{\prime}, \phi^{\prime}\right) \rightarrow\left(\hat{x}^{\prime}, \hat{y}^{\prime}, \hat{z}^{\prime}\right) \rightarrow(\hat{x}, \hat{y}, \hat{z})$ are the standard relations

$$
\begin{aligned}
& \hat{\rho}^{\prime}=\sin \theta^{\prime} \cos \phi^{\prime} \hat{x}^{\prime}+\sin \theta^{\prime} \sin \phi^{\prime} \hat{y}^{\prime}+\cos \theta^{\prime} \hat{z}^{\prime} \\
& \hat{\theta}^{\prime}=\cos \theta^{\prime} \cos \phi^{\prime} \hat{x}^{\prime}+\cos \theta^{\prime} \sin \phi^{\prime} \hat{y}^{\prime}-\sin \theta^{\prime} \hat{z}^{\prime} \\
& \hat{\phi}^{\prime}=-\sin \phi^{\prime} \hat{x}^{\prime}+\cos \phi^{\prime} \hat{y}^{\prime}
\end{aligned}
$$

and

$$
\begin{aligned}
& \hat{x}^{\prime}=\hat{x} \cos \theta_{0}-\hat{z} \sin \theta_{0} \\
& \hat{y}^{\prime}=\hat{y} \\
& \hat{z}^{\prime}=\hat{x} \sin \theta_{0}+\hat{z} \cos \theta_{0} .
\end{aligned}
$$

The position vectors in ( $x, y, z$ ) and ( $x^{\prime}, y^{\prime}, z^{\prime}$ ) are identical to each other; i.e., $p \equiv p^{\prime}$ and $\hat{p} \equiv \hat{p}^{\prime}$. Representing both sides of the last equation in terms of $(\hat{x}, \hat{y}, \hat{z})$ yields

$$
\begin{aligned}
\sin \theta \cos \phi & =\sin \theta^{\prime} \cos \phi^{\prime} \cos \theta_{0}+\sin \theta_{0} \cos \theta^{\prime} \\
\sin \theta \sin \phi & =\sin \theta^{\prime} \sin \phi^{\prime} \\
\cos \theta & =-\sin \theta^{\prime} \cos \phi^{\prime} \sin \theta_{0}+\cos \theta_{0} \cos \theta^{\prime} .
\end{aligned}
$$

Using the preceding transformations, one may obtain (4) by lengthy but straightforward algebra.

## Acirnowledgment

The authore wish to thank Ms. W. L. Mammel for assistance with the computation.

## References

[1] T. S. Chu and R. G. Kouyoumjian, "An analysis of polarization variation and its application to circularly-polarized radiators," IRE Trans. Antennas Propagat., vol. AP-10, pp. 188-192, Mar. 1962.
[2] J. S. Cook, E. M. Elam, and H. Zucker, "The open Cassegrain antenna: "Part 1 -Electromagnetic design and analysis," Bell Syst. Tech. J., vol. 41, pp. 1255-1300, Sept. 1965.
[3] E. M. T. Jones, "Paraboloid reflector and hyperboloid lens antennas," $1 R E$ Trans., Aniennas Propagat., vol. AP-2, pp.
119-127, July 1954 119-127, July 1954.
[4] F. C. Minnett and B. MacA. Thomas, "A method of synthesizing radiation patterns with axial symmetry," IRE Trans. Antennas Propagat., (Commun.), vol. AP-14, pp. 654-656, Sept. 1966.
(5) V. H. Rumeey, "Horn antennas with uniform power patterns - around their axes," IRE Trans. Antennas Propagat. (Commun.), vol. AP-14, pp. 656-658, Sept. 1966.
(6) R.H. Turrin, "Dual mode smail-aperture antennas," IRE Trans. Antennas Propaoat. (Commun.), vol. AP-15, pp. 307-308, Mar. 1967.
[7] P. D. Potter, "A new horn antenna with suppressed side lobes and equal beamwidths," Microwave J., vol. 6, pp. 71-78, June 1963.
[8] S. Silver, Microwave Antenna Theory and Design, (Radiation Lab. Series vol. 12). New York: Dover, 1965, p. 148.
[9] A. B. Crawford, D. C. Hogg, and L. E. Hunt, A A horn-reflector antenna for space communication," Bell Syst. Tech. J., vol. 40, pp. 1095-1116, July 1961.
[10] J. N. Hines, T. Li; and R. H. Turrin, "The electrical characteristics of conical horn-reflector antenna," Bell Syst. Tech. $J_{\text {., vol. }}$ 42, pp. 1187-1211, July 1963.
[11] P. W. Hannan, "Microwave antennas derived from the Cassegrain telescope," IRE Trans. Antennas Propagat., pp. 140-153, Mar. 1961.
[12] T. S. Chu and D. C. Hogg, "Effects of precipitation on propagation at $0.63,3.5$, and 10.6 microns," Bell Syst. Tech. J., vol.
47, pp. 723-759, May-June 1968. 47, pp. 723-759, May-June 1968.
[13] A. Saleh, private communication.
[14] M: S. Affi, "Scattered radiation from microwave antent
[14] M. S. Affi, "Scattered radiation from microwave antennas," and the design of a paraboloidal-plane refector antenna" Ph.D. dissertation, Tech. Univ., Delft, The Netherlands, June 1967.

The Radiation Pattern and Impedance of Offset and Symotrical Near-Field Cassegrainian and Gregorian Antennas

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Abstract-Most Cassegrainian and Gregorian antennas have axial symmetry, in which case the subreflector and associated supporting members partially block the aperture. Consequently, relatively high sidelobes appear in the radiation pattern, and a reflection is produced in the transmission line of the feed. These undesirable effects can be largely eliminated using aspmmetrical configurations. Here we compare axispmmetrical and offset near-feld Cassegrainians and Gregorians; expressions for the reflection coefficient and increase in sidelobe level are given. The offset designs are found to have superior performance in both respects.

To obtein low cross polarization. Chu [3] has pointed out that the aris of ine radiation pattern of the feed must be parallel to the axis of the main reflector.

## I. INTRODUCTION

Axisymmetric antennas of the near-field Cassegrain [1] (or Gregorian) type consist of two confocal paraboloids (Fig. I) and a feed which illuminates the subreflector through a centrally located aperture. The field produced by the feed, approximately a plane wave, is transformed by the subrefector into a spherical wave which, neglecting diffraction by the supports of the subreflector (not shown in Fig. 1), is transformed by the main reflector into a uniform phase front. However, a portion of the spherical wave from the subreflector is intercepted by the feed which partly absorbs and partly scatters the radiation; thus some of this energy represents impedance mismatch. Also, the interference forthcoming from high sidelobes generated by blockage imposes limitations in sharing of frequency bands by terrestrial and satellite communication systems [2].

Blockage can be minimized at the expense of symmetry, as in the offset antennas of Fig. 2. In Figs. 2(a) and (b), blockage is eliminated completely; in Fig. 2(c) there is some blockage, but the effect is small since the intensity over the blocked area is very low. In all of the designs, the field distribution over the aperture of the main reflector is essentially a replica of the field over the aperture of the feed.

The polarization properties of offset antennas are discussed in [3] where it is shom that antennas discussed here will have very low cross polarization, ${ }^{\text {a }}$ provided the feed has perfect polarization properties and a large effective focal length, which is characteristic of the near-field design.


Fig. 3. Conventional axially symmetrical antennas of Cassegrainian and Gregorian type.

## II. REFLECTION COEFFICIENT ${ }^{7}$

In Fig. 1 (a), the plane tangent to the vertex of the subreflector is illuminated by a linearly polarized field $E_{2}$. The field $E_{2}$ produced by the wave reflected from the subrefiector is polarized in the direction of $E_{2}$. At a point in $S_{2}$,

$$
\begin{equation*}
E:^{\prime}=E_{2} \exp (-j \psi) \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\psi=k\left(R-f_{2}\right) \tag{2}
\end{equation*}
$$

$R$ being the distance of the point from the focus and $f$ : the focal length of the subreflector. If the vertex of the subreflector is the origin of a soordinate system with the xy plane tangent at the vertex

$$
\begin{equation*}
\psi \simeq \frac{k}{f_{2}} \frac{x^{2}+y^{2}}{2} . \tag{3}
\end{equation*}
$$

The reflection coefficient on the feed transmission line (produced by the refiected wave) is given by

$$
\begin{align*}
|\Gamma| & =\left|\int_{S_{2}} E_{2} E_{2}^{\prime} d x d y\right| /\left|\int_{S_{z}}\right| E_{z}|=d x d y| \\
& =\left|\int_{S_{2}} E_{2}: \exp (-j \psi) d x d y\right| /\left.\left|\int_{S_{z}}\right| E_{2}\right|^{z} d x d y \mid \tag{4}
\end{align*}
$$

The principle of stationary pbase can be applied to obtain an estimat of $|\Gamma|$ for large $k$, the major contribution arising from the area in the vicinity of the vertex. If $E_{20}$ denotes the values of $E_{2}$ at the vertex, then
$-\int_{s:}\left|E_{2}\right|=\exp (-j \psi) d x d y$

$$
\begin{equation*}
\simeq\left|E_{20}\right|^{2} \int_{0}^{\infty} \int_{0}^{\infty} \exp \left(-j \frac{k x^{2}+y^{2}}{f_{2}}\right) d x d y \tag{5}
\end{equation*}
$$

convergence being assured if $k$ has a small imaginary component Then, for large $k$,

$$
\begin{equation*}
\int_{E_{z}}\left|E_{2}\right|^{2} \exp (-j \psi) d x d y \simeq-j \lambda y_{2}\left|E_{20}\right|^{2} \tag{6}
\end{equation*}
$$

2 The good match obtainable with a bisecied Cassegrainian (Fjg. 2(c)) was recopnized when used as an infrared telescope 14! feeding hikh-pata smplifiens.

(a) OFFEST CASSEGRAN

(b) BISECTED GREGORIAN

(c) Bisected cassegran

Fig. 2. (a) Cassegrainian confguration without blockage. (b) Gregorian conffguration without blockage. (c) Bisected Cassegrainian confguration with some blockage.
valid provided the rertex is pithin $S_{2}$. If the vertex is on the boundary of $S_{2}$, as in Fig. 2 (c),

$$
\begin{equation*}
\int_{S_{2}}\left|E_{2}\right|^{2} \exp (-j \psi) d x d y=\frac{3}{2}\left(-j \lambda f_{2}\left|E_{20}\right|^{2}\right) \tag{7}
\end{equation*}
$$

For Figs. 1 and 2 (c), (4) becomes

$$
|\Gamma|= \begin{cases}\frac{2 \beta}{\alpha}, & \text { (symmetrical) }  \tag{8}\\ \frac{\beta}{\alpha}, & \text { (ofiset) }\end{cases}
$$

Where $\alpha=k a_{2}^{2} / f_{2}, a_{2}$ being the subreflector radius. The quantity

$$
\begin{equation*}
\beta=\frac{\pi a_{2}^{2}\left|E_{20}\right|^{2}}{\int_{\delta z}\left|E_{2}\right|^{2} d x d y} \tag{10}
\end{equation*}
$$

is the ratio between the power density at the vertex of $S$ and the average power density over $S_{2}$.

Consider a circularly symmetric field distribution with quadratic dependence on the distance $p_{z}$ from the center of $S_{2}$, i.e.,

$$
\begin{equation*}
E_{2}=1-A\left(\frac{\rho_{2}}{a_{2}}\right)^{2} \tag{11}
\end{equation*}
$$

Let $A=0.7763$, which corresponds to a feed with a $-13-\mathrm{dB}$ illumination. Then in the cases of Figs. 1 and 2 (c), (10) results in $\beta=2.355$ and 0.1177, and, from (8) and (9),

$$
|\Gamma|= \begin{cases}\frac{4.71}{\alpha}, & \text { (symmetrical) } \\ \frac{0.1177}{a}, & \text { (ofiset). }\end{cases}
$$



Fig. 3. Reflection coefficients for cases of Figs, 1 and 2 (c), assuming an illumination taper of 13 dB at edge of subreflector.

Fig. 3 shows curves of $|\Gamma|$ versus a calculated from (3), (4), and (11), as well as curves given by the principle of stationary phase. In the ofiset case the reflection coefficient is $<-40 \mathrm{~dB}$, negligible for most practical purposes. The asymptotic relation (8) gives accurate results for $\alpha$ of practical interest. ${ }^{2}$ In an experiment [1] at. 6 GHz , on an axisymmetric $5-\mathrm{m}$ near-field Cassegrainian with a $1.8-\mathrm{m}$ focal length using a subreflector with $a_{2}=0.381 \mathrm{~m}$, $f:=0.28 t \mathrm{~m}$, and $\beta \approx 2.2$ (corresponding to an average illumination of -11 dB at the edge of the subrefiector), the reflection coefficient wias - 22 dB : the calculated value, from ( 8 ), is -21.5 dB .
Although ray tracing in Figs. 2 (b) and (c) shows that the latter involves some energy captured by the feed and the former does not, (9) formally applies to both cases. In the case of Fig. $2(8),|\Gamma|$ is near zero, since the difierence in phase between the two waves $E_{2}$ and $E$ : has no stationary point over the aperture of the subreflector.
It is interesting that the refiection coefincient accounts for only a small fraction of the total power intercepted by the feed aperture. Lising the aperture distribution of (11), one finds that the ratio between the power intercepted by the feed and the total power in the spherical wave reffected by the subrefiector, is, for configurations of Figs. 1 and $2(\mathrm{c}), ~ U=0.09,0.024,(-10.46 \mathrm{~dB},-16.2 \mathrm{~dB})$, respectively, using a -13 dB taper and $a_{2} / a_{1}=0.2$. The difierence between $U$ and $\left.|\Gamma|\right|^{\prime}$ is power not accepted by the feed which is radialed thereby degrading the radiation pattern; this depends on the particular scattering properties of the feed and will not be considered further here.

## III. RADIATION PATTERNS

The field distribution over the aperture of the main reflector in Figs. 1 and 2 (c) js assumed zero within the blocked region $S_{2}$. By superposition the overall antenna radiation pattern is approximated by the combination [5] of the fields from the unblocked and blocking apertures.
The envelope of the sidelobes of an unblocked aperture with a circularly symmetric field distribution typical of illuminations produced by dual mode [6] and hybridmode feed systems is shown in Figs. 4 and 5 by the dash-dot lines. By expressing the aperture field as a polynomial it can be shown that the largest increase in sidelobe level raused by the blockage is given by the simple expression

$$
\begin{equation*}
10 \log _{10}\left[\left(1+1.032 \epsilon^{1 / 2} \gamma\right)^{2} \eta\right] \tag{12}
\end{equation*}
$$

where $t=a_{:} / a_{1}$. is the ratio between the field amplitude at the

[^8]

Fig. 4. Radiation pattern of blocked aperture of Fig. $1\left(a_{z} / a_{3}=0.2\right)$.

(a)

(b)

Fjg. 5. Radiation pattern for marginally blocked aperture of Fig. 2(c) ( $a_{1} / a_{1}=0.2$ ). (a) Horizontal plane. (b) Vertical plane.
center of $S_{2}$ and that at the edge of $S_{1}$ (thus $\gamma=E(0) / E(1)$ for central blockage). $\eta$ is the ratio between the (power) gain of the aperture without and with blockage (for the cases of Figs. 1 and 2 (c), $10 \log _{10} \eta=0.68 \mathrm{~dB}$ and 0.25 dB , respectively). As seen in Fig. 4, both the exact computation and the approximate (12) result in "largest incresse" in sidelobe level of 10.7 dB .
Now consider the marginally blocked aperture of Fig. 2(c). According to (12) the increase in sidelobe level is much lower since $\gamma$ is now given by

$$
\begin{equation*}
\gamma=\frac{E(1-\epsilon)}{E(1)} \quad \epsilon=a_{2} / a_{1} . \tag{13}
\end{equation*}
$$

For the assumed illumination, (13) gives $\gamma=1.679$, which a ccording to expression (12) with $\epsilon=0.2$ and $10 \log _{10} \eta=0.25 \mathrm{~dB}$ corresponds to an increase in sidelobe level of 5.23 dB . Fig. 5 (a) shows the exact radiation pattern in the horizontal plane; the increase in level is 5.35 dB , close to the value given by (12). The pattern in the orthogonal plane is shown in Fig. 5 (b). The increase in level, for all sidelobes, is less than that given by (12), a consequence of the asymmetry.
The patterns of Figs 4 and 5 were calculated for $-13-\mathrm{dB}$ illumination at the edge of $S_{1}$; for a $-16-\mathrm{dB}$ taper the maximum increase in sidelobe level has also found to be given accurately by (12).
The offset Cassegrainian and bisected Gregorian (F:gs. 2 (a) and (b)) are unblocked antennas, thus the envelopes of the patierns are given by the dash-dot lines in Fig. 5.

## IV. DISCUSSION

Comparison of the properties of asymmetrical with axisymmetric antennas where an illumination taper of -13 dB and a blockage ratio $\epsilon=a_{2} / a_{1}=0.2$ are assumed, shows: the antennas' of Figs. 2 (a) and (b) have gain higher by about 0.7 dB , the level of near sidelobes is lower by sbout 10 dB , and their reffection coefficient is lower by 25 to 30 dB . The configuration of Fig. 2 (c) has similar advantages: the gain is higher by about 0.4 dB , sidelobes lower by 5 dB , and reflection coefficient down by 25 to 30 dB when compared with the axisymmetric case. An importent advantage of the various offset antennas is that they can be constructed without use of spars (or struts) which produce aperture blockage in the symmetrical case.

## ACKNOWLEDGMENT

We thank R. Kompfner for his interest in the bisected Gregorian design.

## REFERENCES

[1] D. C. Hogg and R. A. Semplak, "An experimental study of nearfield Cassegrainian antennas,' Bell Syst. Tech. J., vol. 43, pp. 26772704. Nov. 1964.
[2] L. C. Tillotson, "A model of a domestic satellite communications system, "Bell Syst. Tech, J. no. 10 , pp. $2111-213 \%$ Dec. 1968 .
[3] T. S. Chu and R. B. Turrin. "Depolarization properties of offset refector antennas." IEEE Trans. Antennas Propaoat., vol. AP-2h.
(4) T. S. Shu and D. C. Ho
14) T. S. Chu and D. C. Hogg, "Effects of precipitation on propagation at $0.63,3.5$ and 10.6 microns," Bell Syst. Tech, J., vol. 47, no. 5, pp. 723-759, Max-June 1968 .
15] C. C. Cutler, "Parabolic-antenna design for microwaves," Proc. IRE.
Vol. 35, pp. 1284-1294, Nov. 1947
[6] R. H. Turrin. "Dual mode small-aperture antennas," IEEE Trans Anlennos Propapot., vol. AP-15, pp. 307-308. Mar. 1967.

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## Abstract

The focal region characteristics of offset fed reflector antennas have been investigated both analytically and experimentally. In order for offset reflectors to have analogous focal region properties to those of front fed parabolic reflectors, an offset axis and an offser focal plane are definec. The definitions of the offset axis and offset focal plane are essential for satisfactory operarions of offset reflectors illuminated by a cluster of feeds. Numerical results using physicel optics approximation are presented to show the dependence of the beam deviation factor of offset reflectors upon the edge angle $\left(\epsilon_{E}\right)$ and the orientation of the feed axis ( $\Theta_{0}$ ).

## Introduction

The offset fed reflector antenna has become increasingly attractive for sacellite communications where low sidelobe levels are essential to achieve good isolation between adjacent high gain beams operated over the same frequency band. To meet these requirements, the use of offset reflectors is becoming desirable since the severe limiting efferts of aperture blockage on front fed parabolojd reflectors can be entirely removed. The focal region behavior of offset reflectors then becomes of major interest in dealing with scanned beams and multiple beau antennas (or a shaped beam antenna) involving a cluster of feeds.

The focal region properties of front fed paraboloicicl reflectors have been extensively invesrigated [1-4] in recent years. Although a few papers on offset reflectors have appeared in the literature $[5,6,7,8]$, the general focal region behavior has never been discussed. A few authors have presented data on laterally scanned beams, but the anklysis was restricted to the $\phi= \pm 90^{\circ}$ plane (see Figure 1). It will be shown that feed movements along the axis of the parent paraboloid (the paraboloid from which the offset section is described) causes the beam to scan vertically in the $\phi=0^{\circ}-180^{\circ}$. plane. Similarly, lateral feed motion in the focal plane of the parent paraboloid, except in the $\phi= \pm 90^{\circ}$ plane will not produce the familiar results:

- Beam scan proportional to feed dispiacement (i.e., a beam deviation factor).
- Equi-phase plane of secondary beam peaks.
- Maximum* gain plane for lateral scanned beams.

[^9]It has been found, however, that by redefining a new focal plane and axis, the analogous properties are restored.

## Offser Axis and Offser Focal Plane

For a front fed reflector, the axial defocusing curve is obtained by displacing the feed along the reflector axis. The secondary beam remains symmerrically disposed to the reflector axis, independent of the axial feed displacement. For an offser fed section of the parent reflector, however, the direction of the axis of the secondary beam is a function of axial defocusing as shown in Figure 2. This is because axial defocusing by $\Delta z$ in an offset reflector antenna produces in the $\mathrm{X}-2$ plane a non-uniform phase that changes monotonically from $\Delta Z \cos \Theta_{2}$ to $\Delta Z \cos \theta$, , where $\Theta_{y}$ and $\theta_{L}$ are the angles subtenced at the focal point by the edges of the reflector in $x-2$ plane. To insure that the direction of the axis of the secondary beam is always along the reflector axis, the feed must be moved along a direction © so that the phase variation across the aperture in the $\mathrm{X}-\mathrm{z}$ plane is symmetrical with respect to that direction. It can easily be shown that the path length, from the feed phase center to the focal plane, of a typical ray is $2 \mathrm{f}-\mathrm{LZ} \cos \left(\Theta_{-\infty}\right)$ where $f$ is the focal length of the reflector, $\Delta Z^{\prime}$ is the distance through which the feed is displaced along the direction $\Theta_{0}$, and $\Theta$ is the acure angle berween the typical ray and the reflector axis. Clearly, the rypical ray is an even funcrion of $\Theta$ about $\Theta_{0}$ for $\Theta_{\mathcal{L}} \leq \Theta \leq \Theta_{\mathrm{U}}$ if $\Theta_{0}$ is given by:

$$
\begin{equation*}
\Theta_{0}=\frac{1}{2}\left(\Theta_{U}+\Theta_{L}\right) \tag{I}
\end{equation*}
$$

Defining the offset axis as the direction $\Theta_{0}$, and $\Delta Z^{\prime}$ as the defocusing along the offset axis, a calculated offset axial defocusing curve along with some measured data are plotted in Figure 3. In both the measured and calculated results, the primary feed is oriented so that the feed axis coincides with $\oplus_{0}$. The close agreement between the measured and calculated results is evident.

If the offser axial defocusing is produced by feed displacement along the offset axis, then the equi-phase surface (offset focal plane), using the parallel ray approximation, is the plane containing the focal point and perpendicular to the offset axis.* Calculations show that this is indeed the case if the reference phase center is chosen as the projection on the parent focal plane of the intersection of the offset axis with the reflector surface.
*Dr. J. W. Duncan, TRW Systems Group, Private Communication.

## Beam Deviation Factor

Figure 4 shows the measured and calculated beam squints in the principal planes due to lateral feed displacements in the offset focal plane. Good agreement between the measured and calculated results is apparent. It is of interest to note that the beam squints in the principal planes corresponding to equal lateral displacements in the two planes are practically the same, indicating that the beam deviation factor (BDF) is independent of the azimuthal position of the feed. Figure 5 shows the calculated $B D F$ as a function of the edge angle $\Theta_{\mathrm{E}}$ (measured with respect to offset axis $\Theta_{0}$ ) for various $\Theta_{0}$. The primary feed is assumed to have a $\cos ^{\mathfrak{n}}$ type voltage pattern with $n$ adjusted to give a -10 dB edge taper (exclusive of space loss). We see that for a given $\Theta_{0}$ (or $\Theta_{\mathrm{E}}$ ), the BDF decreases with increasing $\Theta_{E}$ ( $O \Theta_{0} \Theta_{0}$ ). This is to be expected, as either increasing $\theta_{\mathrm{E}}$ for $a$ given $\theta_{0}$ or increasing $\theta_{0}$ for a given $\theta_{E}$ decreases the $f / D$, which is given by

$$
\begin{equation*}
E / D=\frac{\cos \theta_{E}+\cos \theta_{0}}{4 \sin \theta_{E}}, \tag{2}
\end{equation*}
$$

tinus decreasing the BDF.
It has been found that for a given set of $\theta_{E}$ and $\epsilon_{0}$ the ratio of the BDF (from Figure 5) to the $f / D$ (frow Equation 2) is constant for a given $\theta_{E}$. This means that the BDF of an offset reflector can be calculated from that of a front fed reflector with the same edge angle using the following equerion:

$$
\begin{aligned}
& (B D F)_{\text {offset }}(f / D)_{\text {front }} \text { fed } \\
= & (B D F)_{\text {front }} \text { fed }(f / D)_{\text {offset }}
\end{aligned}
$$

## Conclusion

An offset axis and an offset focal plane have been defined for offser reflectors so that the general focal region properties are analogous co those of front fed paraboloids. The theoretically predicted focal region behaviors of a particular offset reflector are also experimentally verified. The dependence of the BDF upon $\Theta_{0}$ and $\Theta_{E} i s$ presented for offset reflectors illuminated by a cos ${ }^{n}$ voltage illumination function with $n$ adjusted to give a -10 dB edge taper.

## References

[1] Y. T. Lo, "On the Beam Deviation Factor of a Parabolic Reflector," IRE Irans. Antennas and Propagation (Commun.), vol. AP-8, pp. 347-349, May 1960.
[2] J. Ruze, "Lateral-Feed Displacement in a Paraboloid," IEEE Trans. Antennas and Propagation, vol. AP-13, pp. 660-665, September 1965.
[3] P. G. Ingerson and W.V.T. Rusch, "Radiation from a Paraboloid with an Axially Defocused Feed,". IEEE Trans. Antennas and Propagation, vol. 21, No. 1, January 1973.
[4] W.V.T. Rusch and A. C. Ludwig, "Determination of the Kaximum Scan-Gain Contours of a BeamScanning Paraboloid and Their Relation to the Petzval Surface," IEEE Trans. Antennas and Propagation, vol. AP-21, No. 2, March 1973.
[5] D. F. DiFonzo, "Offset and Symmetrical Reflector Antennas," M.S. Thesis, San Fernando Valley State College, January 1972.
[6] T. S. Chu and R. H. Turrin, 'Depolarization Properties of Offset Reflector Antennas," IEEE Trans. Antennas and Propagation, vol. AP-21, May 1973.
[7] A. W. Rudge, "Offset-Reflector Antennas with Offset Feeds," Electronic Letters, November 17, 1973, pp. 611-613.
[8] J. A. Janken, W. J. English, and D. F. Difonzo, "Radiation from Multimode Reflector Antennas," 1973 G-AP Symposium Digest, pp. 306-309.



Figure 2. Beam squint characteristics.


Figure 3. Defocusing curve.


Figure 4. Beam deviation characteristics.


Figure 5. Beam deviation factor vs. the edge angle $\Theta_{E}$ for different offset angle © ${ }_{0}$.

# The Polarization Losses of Offset Paraboloid Antennas : 

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#### Abstract

In this paper the electric field in the aperture of offset front-fed paraboloid antennas and open Cassegrainian antennas, excited by an electric dipole or Huygens source in the focus, is compared with the fields of front-fed circularly spmmetrical paraboloid reflector antennas and classical Cassegrainian antennas. The aperture field forms the basis of expressions to calculate the polarization efficiency of all four types of antenna. Computed results are given, showing that offset antennas can compete with front-fed paraboloids if they are excited by an electric dipole; the classical Cassegrainian anterna, however, shows better results. If offset antennas are excited by a Huygens source, the result is very unfavorable compared with the symmetrical antennas which show no cross polarization.


## I. INTRODUCTION

IT. HAS BEEN known for several years that, if a paraboloid reflector antenna is fed by a linearly polarized electrical dipole, the antenna system will radiate energy not only in the main polarization, but, also a fair amount in an unwanted polarization called cross-polarization or depolarization.

Condon [1] was one of the first to give a detailed analysis of this phenomenon. It appears that cross-polarized lobes, also called Condon lobes, are formed having a maximum in planes at $4.5^{\circ}$ to the principal plane. Silver [ 2, p. 423] also mentioned this cross polarization, mainly as an abstract of Condon's work.

Cutler [3] gives a physical explanation as to the relation between aperture electric field lines and the polarization of the dipole feed, and explains the very unfavorable situation that occurs if the focus of the paraboloid falls between the aperture and apex of the paraboloid. This work has been continued by Jones [4], who investigated the radiation characteristics of paraboloid reflector antennas excited in their foci by a short electrical dipole feed, a short. magnetic dipole feed, and a plane wave source, being a combination of an electric and a magnetic dipole. If this dipole pair is represented by dipole fields of equal intensity, commonly known as a Huygens source, it has been proved that the cross-polarized component of the aperture illumination could be made to disappear [5].

Koffman [.5] has extended this work by considering other conical sections of revolutions as well as the paraboloid. The cross-polarized pattern of the reflector excited by any arbitrary feed system may be calculated, using the methods of Afifi [6], while Potter [7] has found an

[^10]analytical expression for the polarization loss or polarization efficiency. It is the latter expression that will also be reviewed in this paper. Potter [ $\delta$ ] has also found a similar expression for Cassegrainian antennas, which will be included in the present study.

Watson and Ghobrial [9] have investigated the crosspolarization isolation at off-axis incidence for classical Cassegrainian antennas and front-fed paraboloidal reflectors. It was shown that the Cassegrainian antenna was much superior to the equivalent front-fed antenna.

Not much is known so far about offset paraboloids and open Cassegrainian [10] antennas. Hanfing [11] has shown a stereographic mapping method that contains the aperture field lines of an offset antenna excited by several field sources, but without further details. Graham [12], [13] describes the polarization of offset antennas and states that an ofiset Cassegrainian antenna can be designed to have low cross-polarization losses, which was experimentally discovered by letting the axis of the main and subreflector differ only a few degrees. No calculations have been mentioned.

Since plans exist for frequency reuse, above 10 GHz , by polarization diversity, the interest in cross-polarization problems has recently increased considerably.

Ludwig [14] has published a paper on the definition of cross-polarization, and Kinber and Tischenko [1.5] calculated the current distribution of various reflector antennas with different illumination. Unfortunately no numerical results are given. Chu and Turrin [16] have discussed the beamshift of offset antennas with circular polarization and have calculated the level of cross-polarization sidelobes. The poor polarization performance of the open Cassegrainian antenna has been predicted.

It is the purpose of this paper to obtain a more detailed insight into the cross-polarization losses of offset antennas. For this purpose we shall compare the front-fed paraboloid, the true Cassegrainian antenna, the offset front-fed paraboloid, and the open Cassegrainian antenna. In all the cases we shall use a short electrical linearly polarized dipole and a Huygens source as a primary radiator. We will compare the aperture electric fields, define the polarization efficiency, and calculate this for different configurations. Finally, we will show a practical example.

## II. APERTURE FIELDS OF REFLECTOR ANTENNAS ILLUMINATED BY AN ELECTRIC DIPOLE

Let us consider a short electric dipole of length $l$ [ 2 , p. 92], lying along the $x$ axis of a Cartesian coordinate


Fig. 1. Electric dipole oriented along positive $x$ axis of Cartesian coordinate system.
system (Fig. 1), with a current $I$ flowing in the direction of the positive $x$ axis. Expressed in $\rho, \psi, \xi$ coordinates, the far zone components of the complex electric field are

$$
\tilde{E}=E_{\psi} \bar{a}_{\psi}+E_{k} \bar{a}_{\xi}
$$

or

$$
\begin{equation*}
\bar{E}=\frac{j \eta I l \exp \left(-j k_{\rho}\right)}{2 \lambda \rho}\left(-\bar{a}_{\psi} \cos \psi \cos \xi+\bar{a}_{\xi} \sin \xi\right) \tag{1}
\end{equation*}
$$

where $\eta=120 \pi \Omega, \bar{a}_{\psi}$ and $\bar{a}_{\xi}$ are unit vectors along the $\psi$ and $\xi$ axes, respectively, and $k$ is the wavenumber.
In $x, y, z$ coordinates (1) becomes
$\tilde{E}=-E_{0}\left[\bar{x}\left(\cos ^{2} \psi \cos ^{2} \xi+\sin ^{2} \xi\right)\right.$

$$
\begin{equation*}
\left.-\frac{1}{2} \bar{y} \sin ^{2} \psi \sin 2 \xi+\frac{1}{2} \bar{z} \sin \psi \cos \xi\right] \tag{2}
\end{equation*}
$$

where

$$
E_{0}=\frac{j \eta I l \exp \left(-j k_{\rho}\right)}{2 \lambda_{\rho}}
$$

If the dipole is oriented along the positive $y$ axis, it is readily seen that the electric field becomes

$$
\begin{equation*}
\bar{E}=\left(-\bar{a}_{\psi} \cos \psi \sin \xi-\bar{a}_{\xi} \cos \xi\right) \tag{3}
\end{equation*}
$$

$$
\begin{align*}
& \text { or } \\
& \begin{aligned}
& \tilde{E}=-E_{0}\left[-\frac{1}{2} \bar{x} \sin ^{2} \psi \sin 2 \xi+\bar{y}\left(\cos ^{2} \psi \sin ^{2} \xi+\cos ^{2} \xi\right)\right. \\
&\left.+\frac{1}{2} \bar{z} \sin 2 \psi \sin \xi\right] .
\end{aligned}
\end{align*}
$$

These fields will induce surface currents in any arbitrary reflector using geometrical optical techniques.

Using the method employed by Jones [4], the aperture field may now be found by calculating the surface-current density of the reflector $\bar{K}=2\left(\bar{n} \times \bar{H}_{i}\right), \bar{H}_{i}$ being the initial field and $\bar{n}$ the unit vector normal to the surface at the point of incidence and projecting $\bar{K}$ on the aperture.

A simpler way to find the aperture field may be followed by investigating what happens with the fields $E_{\downarrow} \bar{a}_{\nu}$ and $E_{\xi} \bar{a}_{\xi}$ at the point of incidence. From Fig. 2 it is readily seen that the vector $E_{\xi} \bar{a}_{\xi}$ is perpendicular to the plane comprising the $z$ axis, radius $\rho$ from focus to the surface of the refiector, the reflected ray; and the vector $n$ at the point of incidence (plane $F G H$ ). After reflection this vector


Fig. 2. Geometry of parabolic refiector with incident and reflected rays and vectors.
remains perpendicular to the surface, but its direction reverses. Therefore,

$$
\begin{equation*}
\bar{E}_{\xi^{r}}=-E_{\xi} \tilde{a}_{\xi} \tag{5}
\end{equation*}
$$

the index $r$ indicating reflection.
The vector $E_{\psi} \bar{a}_{\psi}$ lies in plane $F G H$ and is perpendicular to the radius. To find out what happens with $E_{\psi} \bar{a}_{\psi}$ we will use Fig. 2 and define the indices $n$ and $\tau$ as the directions normal and tangential to the paraboloid surface at the point of incidence. We now resolve $E_{\psi}$ in $E_{\psi, n}$ and $E_{\psi, \text { r }}$ resulting in

$$
\begin{align*}
& E_{\psi, n}=E_{\psi} \sin \frac{1}{2} \psi  \tag{6}\\
& E_{\psi, r}=E_{\psi} \cos \frac{1}{2} \psi .
\end{align*}
$$

After reflection, $E_{\psi, n}$ is continuous and $E_{\psi, r}$ retains its sign. Therefore,

$$
\begin{align*}
& E_{\psi, \pi^{r}}=E_{\psi} \sin \frac{1}{2} \psi \\
& E_{\psi, r^{r}}=-E_{\psi} \cos \frac{1}{2} \psi . \tag{7}
\end{align*}
$$

By means of the vectors $\bar{a}_{\perp}$ and $\bar{a}_{\| 1}$ (Fig. 2) and by resolving $E_{\psi, n}$ and $E_{\psi, r}$ along these vectors it is readily found, using (7) that

$$
\begin{align*}
& E_{1^{\top}}=-E_{\psi, n^{r}} \sin \frac{1}{2} \psi+E_{\psi, T^{\top}} \cos \frac{1}{2} \psi=-E_{\psi,}  \tag{8}\\
& E_{1 I^{r}}=E_{\psi, n^{r}} \cos \frac{1}{2} \psi+E_{\psi, T^{r}} \sin \frac{1}{2} \psi=0 . \tag{9}
\end{align*}
$$

The unit vector $\bar{a} \perp$ may be written

$$
\bar{a}_{\perp}=\bar{x} \cos \xi+\bar{y} \sin \xi .
$$

If we use an electric dipole oriented along the positive $x$ axis; the reflected field $\bar{E}_{\psi^{r}}$ follows from (1), (8), and (10) resulting in

$$
\begin{equation*}
\bar{E}_{\psi}{ }^{\tau}=E_{0} \cos \psi \cos \xi(\cos \xi, \sin \xi, 0) \tag{11}
\end{equation*}
$$

and (5) becomes $\tilde{E}_{\xi}^{r}=-E_{0} \sin \xi \bar{a}_{\xi}$.
By means of (1), (5), and (11) the aperture field $E_{A}$ yields

$$
\begin{aligned}
\bar{E}_{A}=E^{r}(x, y, z)=E_{0} \cos \psi \cos \xi & (\cos \xi, \sin \xi, 0) \\
& -E_{0} \sin \xi(-\sin \xi, \cos \xi, 0)
\end{aligned}
$$

or
$\bar{E}_{A}=E_{0}\left[\left\{1-\cos ^{2} \xi(1-\cos \psi)\right\} \bar{x}\right.$

$$
\begin{equation*}
\left.-\frac{1}{2} \sin 2 \xi(1-\cos \psi) \tilde{y}\right] \tag{12}
\end{equation*}
$$

where

$$
E_{0}=\frac{j \Pi I l \exp \left[-j k\left(F+z_{0}\right)\right]}{2 \lambda \rho}
$$

Tising the same technique we readily find the aperture field, if the dipole is oriented along the positive $y$ or $z$ axis [17].

The offset paraboloid is illustrated in Fig. 3. If the electric dipole is located in the focus of the paraboloid oriented along the positive $x^{\prime}$ axis of an $x^{\prime}, y^{\prime}, z^{\prime}$ coordinate system, the aperture field appears to be

$$
\begin{align*}
\bar{E}_{A}= & {\left[E_{0} \cos \Psi_{0}\left\{1-\cos ^{2} \xi(1-\cos \psi)\right\}\right.} \\
& \left.+E_{0} \sin \Psi_{0} \sin \psi \cos \xi\right] \bar{x} \\
& +\left[E_{0} \cos \Psi_{0}\left\{-\frac{1}{2} \sin 2 \xi(1-\cos \psi)\right\}\right. \\
& \left.+E_{0} \sin \Psi_{0} \sin \psi \sin \xi\right] \bar{y} . \tag{13}
\end{align*}
$$

The results for the case when the dipole is oriented along the $y^{\prime}$ or $z^{\prime}$ axis are found elsewhere [17].

The same technique used for the front-fed paraboloid may be employed to calculate the fields in the aperture of a Cassegrainian antenna. However, there are some fundamental differences because the dipole field is reflected twice before it arrives at the main reflector aperture. Therefore, the components $\bar{E}_{\xi}$ and $\bar{E}_{\psi}$ have to be known after this double reflection in order to calculate this aperture field. If the electric dipole is located in focus $F_{1}$ and oriented along the positive $x$ axis (Fig. 4), the aperture field is
$\bar{E}_{A}=-E_{0}^{\prime}\left[\left\{1-\cos ^{2} \xi\left(1-\cos \psi_{1}\right)\right\} \bar{x}\right.$

$$
\begin{equation*}
\left.-\frac{1}{2} \sin 2 \xi\left(1-\cos \psi_{1}\right) \tilde{y}\right] \tag{14}
\end{equation*}
$$

In (14) $E_{0}{ }^{\prime}=j \eta I l \exp \left(-j k r_{0}\right) / 2 \lambda \rho^{\prime}$, where $\rho^{\prime}$ is the distance between the primary focus and the surface of the main reflector. It is readily found that $r_{\mathrm{a}}=f / e+F+\mathrm{Z}_{0}$ and $\rho^{\prime}=2 F /(1+\cos \psi)+f / e$, where $f$ is the distance between the two hyperboloid foci, $e$ the hyperboloid cccen-


Fig. 4. Geometry of classical Cassegrainian antenna.


Fig. 5. Geometry of the open Cassegrainian antenns.
tricity, and $Z_{0}$ the depth of the paraboloid. If the dipole is oriented along the positive $y$ or $z$ axis similar equations may be found [17].
The calculation of the aperture field of an open Cassegrainian antenna is much more complicated than the previous ones. The geometry is presented in Fig. 5. In general, the planes $K G H$ (with the $z$ axis) and $F_{1} K G$ (with the $z^{\prime}$ axis) will not eoincide. Therefore, the ray from the primary focus $F_{1}$ to the subreflector and the ray reflected from the paraboloid ( $G H$ ) will generally not be located in the same plane. The calculation of the aperture field leads to long algebraic equations.. The reader is referred to a report recently issued [17], where a detailed description of these equations is given.

## III. THE APERTURE FIELDS OF REFLECTOR ANTENNAS ILLUMINATED BY A HUYGENS SOURCE

A combination of an electric dipole and a magnetic dipole of equal intensity and crossly oriented is often called a Huygens source [4]. If this source is located in the focus of a paraboloid antenna in such a way that the electric dipole orients along the positive $x$ axis and the magnetic dipole along the positive $y$.axis, the aperture fields are readily found by superposition of the aperture fields caused by illumination with electric and magnetic


Fig. 6. Polarization loss efficiency factor of parabolic reflector.
dipoles. The reader is referred to a report [17] recently issued for the aperture fields of reflector antennas fed by a magnetic dipole.

In accordance with Jones [4], we find

$$
\begin{equation*}
\bar{E}_{\hat{A}}=E_{0}(1+\cos \psi) \bar{x} . \tag{15}
\end{equation*}
$$

positive $y^{\prime}$. axis, or by an electric dipole oriented along the positive $y^{\prime}$ axis and the magnetic dipole along the negative $x^{\prime}$ axis. As will be noticed, the cross-polarization component does not disappear.

If we try to find the aperture fields of an open Cassegrainian antenna, it appears also that no simplification takes place. Therefore, it is of little value to rewrite the equations found before. As will be noticed, the crosspolarization component in the aperture does not disappear either.

## IV. THE POLARIZATION EFFICIENCY

In accordance with Potter [7], the polarization effciency of an antenna is defined by the ratio of antenna gain including the effects of cross polarization, to antenna gain if the cross-polarized energy were zero everywhere. Thus

$$
\eta_{p}=\left.\frac{\left|\int_{0}^{2 \pi} \int_{0}^{\psi} E_{m p}(\psi, \xi) \rho^{2} \sin \psi d \psi d \xi\right|^{2}}{\mid \int_{0}^{2 \pi} \int_{0}^{\psi}\left[E_{m p}{ }^{2}(\psi, \xi)+E_{\mathrm{cp}}{ }^{2}(\psi, \xi)\right]^{1 / 2} \rho^{2} \sin \psi d \psi d \xi}\right|^{2}
$$

where $E_{m p}(\psi, \xi)$ represents the electric field in the aperture with principal polarization and $E_{\mathrm{cp}}(\psi, 5)$ that of the crosspolarization. By means of (17) and the equations for the electric field in the aperture found in the previous paragraphs it is now possible to calculate the polarization efficiency.
In the case that a front-fed paraboloid is investigated, the distance $\rho$ between paraboloid and focus is $\rho=$ $2 F /(1+\cos \psi)$, and because all the fields involved are proportional to $\exp \left(-j k\left(F+z_{0}\right) / \rho\right.$, (17) may be replaced by

$$
\begin{equation*}
\cdots \eta_{p}=\frac{\left|\int_{0}^{2 \mathrm{r}} \int_{0}^{\Psi} \frac{E_{m p}(\psi, \xi)}{E_{0}} \tan \frac{1}{2} \psi d \psi d \xi\right|^{2}}{\left|\int_{0}^{2 \tau} \int_{0}^{\Psi}\left[\left(\frac{E_{m p}(\psi, \xi)}{E_{0}}\right)^{2}+\left(\frac{E_{\mathrm{cp}}(\psi, \xi)}{E_{0}}\right)^{2}\right]^{1 / 2} \tan \frac{1}{2} \psi d \psi d \xi\right|^{2}} . \tag{18}
\end{equation*}
$$

In the same way if the electric dipole is oriented along the $+y$ axis and the magnetic dipole along the $-x$ axis

$$
\begin{equation*}
\bar{E}_{A}=E_{0}(1+\cos \psi) \bar{y} . \tag{16}
\end{equation*}
$$

It appears that in both cases the cross-polarization component disappears. A classical Cassegrainian antenna shows. similar results and the cross-polarization component disappears as well.
The aperture field of an offiset antenna illuminated by a Huygens source may be found by combining the aperture fields originated by an electric dipole oriented along the positive $z^{\prime}$ axis and a magnetic dipole oriented along the

If the paraboloid is illuminated by an electric dipole oriented along the $+x$ axis, the aperture fields to be used are

$$
\begin{align*}
E_{m p} & =E_{0}\left[1-\cos ^{2} \xi(1-\cos \psi)\right]  \tag{19}\\
E_{\mathrm{op}} & =-\frac{1}{2} E_{0} \sin 2 \xi(1-\cos \psi) \tag{20}
\end{align*}
$$

where

$$
E_{0}=\frac{j_{\eta} I l \exp \left[-j k\left(F+z_{0}\right)\right]}{2 \lambda \rho} .
$$

It is possible to simplify (18) by substituting (19) and (20), but this does not increase the insight into the prob-
lem. An approximation of this equation as carried out by Potter [7], has the drawback that it gives only reliable results for very shallow paraboloid reflectors with subtending angles of less than $60^{\circ}$. The results of (18), computed without any approximation, applied to front-fed paraboloid reflector antennas are presented in Fig. 6.

In the case of a classical Cassegrainian antenna (Fig. 4), the integration is carried out over the angles $\xi$ and $\psi_{2}$. We can now replace (17) by
where the main and cross polarized fields for the offset paraboloid and open Cassegrainian antenna have been discussed in the previous sections for various illuminations.

In the case of an open Cassegrainian antenna the effciency factor becomes a little more complicated. It is readily shown that the factor $E_{0}$ in the aperture fields is equal to that of the classical Cassegrainian antenna and that the integration limits are the same as for the offset antenna. Fig. 8 shows the polarization efficiency of an

$$
\eta_{p}=\frac{\left|\int_{0}^{2 \pi} \int_{0}^{\psi} \frac{E_{m p}\left(\xi, \psi_{2}\right)}{E_{0}} \cdot \tan \left(\frac{1}{2} \psi_{2}\right) d \xi d \psi_{2}\right|^{2}}{\left|\int_{0}^{2 \pi} \int_{0}^{\psi}\left[\left(\frac{E_{m p}\left(\xi, \psi_{2}\right)}{E_{0}}\right)+\left(\frac{E_{\mathrm{cp}}\left(\xi, \psi_{2}\right)}{E_{0}}\right)\right]^{1 / 2} \tan \left(\frac{1}{2} \psi_{2}\right) d \xi d \psi_{2}\right|^{2}} .
$$

If the subreflector is illuminated by an electric dipole, oriented along the positive $x$ axis, the aperture fields to be used are

$$
\begin{align*}
& \frac{E_{m p}}{E_{0}^{\prime}}=-1+\cos ^{2} \xi {[1-\cos \xi} \\
&\left.\cdot 2 \arctan \left\{\frac{e-1}{e+1} \tan \left(\frac{1}{2} \psi_{2}\right)\right\}\right]  \tag{22}\\
& \frac{E_{\mathrm{cp}}}{E_{0}^{\prime}}=\frac{1}{2} \sin 2 \xi[1-\cos \xi \\
&\left.\cdot 2 \arctan \left\{\frac{e-1}{e+1} \tan \left(\frac{3}{2} \psi_{2}\right)\right\}\right] \tag{23}
\end{align*}
$$

Fig. 7 shows the computed results, where the polarization efficiency is given in relation to the subtended angle of the main reflector with the magnification ratio $M=$ $e+1 / e-1$ as a parameter.
When an offset paraboloid antenna is investigated (18) may still be used, however, the integration limits will differ. As explained before [17], $\psi$ will bave to be integrated between $\Psi_{0}-\Psi$ and $\Psi_{0}+\Psi$. The integration limits of $\xi_{,} \xi_{L}$, and $\xi_{R}$ are

$$
\begin{align*}
& \xi_{L}=-\arccos \left(\frac{\cos \Psi-\cos \Psi_{0} \cos \psi}{\sin \Psi_{0} \sin \psi}\right)  \tag{24}\\
& \xi_{K}=-\arccos \left(\frac{\cos \Psi-\cos \Psi_{0} \cos \psi}{\sin \Psi_{0} \sin \psi}\right) \tag{25}
\end{align*}
$$

where $\Psi_{0}$ is the offset angle and $\Psi$ the angular aperture of the mainrefiector. (In the open Cassegrainian antenna $\Psi$ is called $\Psi_{2}$ ). Equation 18 is then written as

$$
\eta_{p}=\frac{\left|\int_{\psi_{0}-\psi}^{\psi_{0+\psi}} \int_{\xi_{L}}^{\xi_{R}} \frac{E_{m p}(\xi, \psi)}{E_{0}} \tan \frac{1}{2} \psi d \psi d \xi\right|^{2}}{\left|\int_{\psi_{0}-\Psi}^{\psi \sigma+\psi} \int_{\xi L}^{l_{R}} \frac{\left(E_{m p^{2}}+E_{\mathrm{cp}}^{2}\right)^{1 / 2}}{E_{0}} \tan \frac{1}{2} \psi d \psi d \xi\right|^{2}}
$$

offset paraboloid illuminated by an electric dipole oriented along the positive $x^{\prime}$ axis and positive $y^{\prime}$ axis, respectively, as well as illuminated by a Huygens source. The results obtained with an open Cassegrainian antenna are given in Fig. 9. The eccentricity of the hyperboloid subrefiector was 1.5. For both offiset and open Cassegrainian antennas the offset angle served as a parameter.

## V. A PRACTICAL EXAMPLE

In the previous section a Huygens source was presented with equal intensities of a magnetic and an electric dipole. However, many feed patterns may be divided in electric and magnetic dipoles with unequal intensities. In this section we work out a practical example.
A popular feed system used to illuminate a refiector surface is the open waveguide excited with the $\mathrm{TE}_{10}$ mode described by Silver [2, p. 343] and Jones [4]. The field components of a rectangular waveguide excited in the $\mathrm{TE}_{10}$ mode and the electric field vector oriented along the $x$ axis is, in accordance with Silver, represented by:

$$
\begin{align*}
& \bar{E}_{\psi}(\psi, \xi)=C \frac{\cos \xi}{\rho}\left[1+\frac{\beta_{10}}{k} \cos \psi\right] F(\psi, \xi) \exp (-j k \rho) \bar{a}_{\psi} \\
& \bar{E}_{\xi}(\psi, \xi)=-C \frac{\sin \xi}{\rho}\left[\frac{\beta_{10}}{k}+\cos \psi\right] F(\psi, \xi) \exp (-j k \rho) \bar{a}_{\xi} \tag{27}
\end{align*}
$$

where

$$
\begin{array}{r}
F(\psi, \xi)=\frac{\cos [(\pi a / \lambda) \sin \psi \cos \xi]}{[(\pi a / \lambda) \sin \psi \cos \xi]^{2}-(\pi / 2)^{2}} \\
\quad \cdot \frac{\sin [(\pi b / \lambda) \sin \psi \sin \xi]}{(\pi b / \lambda) \sin \psi \sin \xi} .
\end{array}
$$

In this equation it is assumed that the reflection coefficient at the opening of the waveguide is zero. The symbols $a$ and $b$ are waveguide dimensions, and $C$ is a coefficient depending upon the wavelength and dimensions [2, p. 343]. Further, $\beta_{10}$ stands for the phase constant for the


Fig. 7. Polarization loss efficiency factor of classical Cassegrainian antenna.
$T E_{10}$ mode, and $k$ the propagation constant, equal to $2 \pi / \lambda$.

The polarization vector is

$$
\begin{equation*}
\bar{a}_{i}=\frac{\cos \xi}{\rho}\left(1+\frac{\beta_{10}}{k} \cos \psi\right) \bar{a}_{\psi}-\frac{\sin \xi}{\rho}\left(\cos \psi+\frac{\beta_{10}}{k}\right) \bar{a}_{\xi} \tag{28}
\end{equation*}
$$

If the dimensions of the waveguide are such that $\beta_{10} / k=1$ the polarization vector reduces to

$$
\begin{equation*}
\bar{a}_{i}=\cos \xi \bar{a}_{\psi}-\sin \xi \bar{a}_{\xi} \tag{29}
\end{equation*}
$$

which is equal to that of a Huygens source. However, in practice this cannot be realized as normally

$$
\begin{equation*}
\frac{\beta_{10}}{k}=\frac{\lambda}{\lambda_{g 10}} \tag{30}
\end{equation*}
$$

where $\lambda_{010}$ is the wavelength in the guide [2, p. 205]

$$
\begin{equation*}
\lambda_{0,0}=\frac{\lambda}{\left[1-(\lambda / 2 a)^{2}\right]^{1 / 2}} \tag{31}
\end{equation*}
$$

for the $\mathrm{TE}_{10}$ mode. Therefore, $\beta_{10}=k$ only for $\lambda \ll a$.
Nevertheless, this polarization vector is very popular and is used by several authors such as Affif [6], Carter


Fig. 8. Polarization loss efficiency factor of offiset paraboloid $\stackrel{\text { reflector, ofiset angle } \Psi_{0} \text { being a parameter, illuminated by -- }}{E}$ dipole oriented along $y^{\prime}$ axis.
[18], and Tartakovski [19], as it simplifies the complicated mathematical work considerably. If we want to study the cross-polarization properties of antennas illuminated by this feed, we must know the waveguide dimensions, frequency range, and cutoff frequency.

If we study a rectangular waveguide in the $X$ band ( $8200-12400 \mathrm{MHz}$ ) the dimensions $a$ and $b$ are $0.900 \times$ 0.400 in and the cutoff frequency is 6560 MHz . The proportions of the lowest and highest frequencies to the cutoff frequency are 1.25 and. 1.90 , a relationship that is also found for waveguides in other frequency bands. Let $\lambda_{1}$ (be the longer wavelength) $=3.66 \mathrm{~cm}$ and $\lambda_{2}$ (the shorter) $=$ 2.42 cm . The wavelength in the waveguide for $\lambda_{1}$ is then

$$
\lambda_{o 10}=\frac{3.66}{\left[1-(3.66 / 4.57)^{2}\right]^{1 / 2}}=6.11 \mathrm{~cm}
$$

and for $\lambda_{2}$

$$
\lambda_{010}=\frac{2.42}{\left[1-(2.42 / 4.57)^{2}\right]^{1 / 2}}=2.85 \quad \mathrm{~cm} .
$$

From (30) we then obtain for

$$
\beta_{10 / k}=\lambda_{1} / \lambda_{010}=3.66 / 6.11=0.60
$$



Fig. 9. Polarization loss efficiency factor of open Cassegrainian antenns, offset angle $\psi_{0}$ being a parameter, illuminated by $E$ dipole oriented along $x^{\prime}$ and $y^{\prime}$ axis and ---Huygens source.
and

$$
\beta_{10 / k}=\lambda_{2} / \lambda_{p 10}=2.42 / 2.85=0.85
$$

The polarization vector for $\lambda_{2}$ is now

$$
\begin{equation*}
\bar{a}_{i}=\frac{\cos \xi}{\rho}(1 \div 0.60 \cos \psi) \bar{a}_{\psi}-\frac{\sin \xi}{\rho}(0.60+\cos \psi) \bar{a}_{\xi} \tag{32}
\end{equation*}
$$

and for $\lambda_{2}$
$\bar{a}_{i}=\frac{\cos \xi}{\rho}(1+0.85 \cos \psi) \bar{a}_{\psi}-\frac{\sin \xi}{\rho}(0.85+\cos \psi) \bar{a}_{\xi}$.

The polarization properties apparently depend on the frequency. If such a feed is used to illuminate a front-fed paraboloid antenna, it is readily found by means of the theory developed in Section II that the aperture field
$\bar{E}_{A}=E_{0} \frac{\cos \xi}{1+\cos \psi}(1+m \cos \psi)(\cos \xi ; \sin \xi, 0) x$

$$
-E_{0} \frac{\sin \xi}{1+\cos \psi}(m+\cos \psi)(-\sin \xi, \cos \xi, 0)
$$



Fig. 10. Polarization loss efficiency factor of a circulerly symmetrical paraboloid antenna illuminated by open waveguide excited with the TE 10 mode.
or

$$
\begin{align*}
E_{A}= & \frac{E_{0}}{1+\cos \psi}\left[\left\{(1+m \cos \psi) \cos ^{2} \xi\right.\right. \\
& \left.+(m+\cos \psi) \sin ^{2} \xi\right\} \bar{x}+\{(1+m \cos \psi) \cos \xi \sin \xi \\
& -(m+\cos \psi) \cos \xi \sin \xi\} \bar{y}] \tag{34}
\end{align*}
$$

where $m$ is any value between 0.60 and $0.85 . E_{0}$ is the amplitude factor of the feed system and is in accordance with Silver [ 2, p. 343]

$$
\begin{align*}
& E_{0}=\frac{\cos [(\pi a / \lambda) \sin \psi \cos \xi]}{\left.[(\pi a / \lambda) \sin \psi \cos \xi)^{2}-1 / 2 \pi\right]} \\
& \quad \times \frac{\sin [(\pi b / \lambda) \sin \psi \sin \xi]}{(\pi b / \lambda) \sin \psi \sin \xi} . \tag{35}
\end{align*}
$$

The results for three different values of $m$ are given in Fig. 10.

## VI. CONCLUSIONS

It has been demonstrated that by calculating the aperture electric fields of antennas with a paraboloid (main) reflector, expressions may be derived for polarization effi-
ciency or polarization loss. These expressions are found not only for front-fed paraboloids, but also for classical Cassegrainian antennas, front-fed offset paraboloids, and open Cassegrainian antennas. Both electric dipole excitation and excitation by a Huygens source are investigated as they give a good insight into the problems and facilitate comparative studies. Moreover, there are a number of realistic feeds, such as a rectangular horn excited in the $T E_{01}$ mode, having polarization properties close to the Huygens source. An example of this kind has been worked out, showing that the polarization losses decrease considerably if the polarization vector approaches that of a Huygens source. If investigations are required for feeds with polarization properties different from those discussed here, the same techniques may be used.

After the electric aperture field has become known, an expression may be found for the polarization efficiency $\eta_{p}$. Carrying out the computation, it is readily seen that the front-fed paraboloid has very bad polarizatioli properties, becoming worse for decp paraboloids. In the case in which the focus falls within the aperture plane ( $\Psi_{2}=90^{\circ}$ ), the polarization efficiency falls to 89 percent (Fig. 6). On the other hand, the true Cassegrainian antenna has much better properties, which not only depend upon the subtended angle by the main reflector, but also on the magnification ratio $M=(\epsilon+1) /(e-1)$, which has been introduced as a parameter (Fig. 7). The result becomes worse for low $M$ values and deep main paraboloids; however, for $\lambda I=2$ and $\Psi_{2}=90^{\circ}$, the true Cassegrainian anterna still retains a polarization-efficiency of 99 percent; which is considerably more than in the case of front-fed paraboloids with equal $\Psi_{2}$. Offset paraboloid antennas show an increase in the losses at increasing subtended angle and increasing offset angle. If we compare the front-fed paraboloid with the offset paraboloid, it appears that the former shows better results for equal subtended angles than the offiset antenna with an electric dipole polarized along the $z^{\prime}$ axis; e.g., a front-fed paraboloid with a subtended angle of $60^{\circ}$ has a polarization efficiency of 98.5 percent, while an offset paraboloid nith subtended and ofiset angles of $60^{\circ}$ shows an efficiency of only 91 percent (Fig. 8). If the dipole is polarized along the $y^{\prime}$ axis, the efficiency even drops to 89 percent.

If we study the results obtained with an open Cassegrainian antenna illuminated by an electric dipole, it appears that not much difference is noticed if the dipole is oriented along the $x^{\prime \prime}$ axis or $y^{\prime \prime}$ axis. At offset angles and subtended angles of about 60 degrees it appears that the efficiency drops to 90 percent, which is of the same order as for offset front-fed paraboloids (Fig. 9). The results obtained by illumination by a Huygens source, for both offset antennas and open Cassegrainian antennas, are similar to those obtained by illumination by an electric dipole. The results cicarly depend on the ofiset and subtended angles rather more than on the polarization of the feed. At offset angles and main reflector subtended angers of about $60^{\circ}$, an efficiency of abrot 90 percent is noticed again.

We also investigated the losses of open Cassegrainian antennas in relation to the eccentricity of the hyperboloid subreflector. Using eccentricities of 2.0 and 2.5 , the results are very similar to those with eccentricities of 1.5 .

Compared with the symmetrical front-fed paraboloid antenna and the classical Cassegrainian antenna, offset antennas are very unfavorable when illuminated by a Huygens source. The Huygens source gives zero polarization losses for symmetrical paraboloid reflector antennas, but the losses of offset antennas are of the same order as those calculated for offset antennas illuminated by an electric dipole. This conclusion is supported by the fact that for eccentricities differing from $e=1.5$ similar results are obtained.

More study is required to find out whether feeds may be designed baving polarization properties that may improve the polarization losses of offset antennas. However, the present study makes the use of ofiset antennas for purposes where a polarization discrimination of more than 30 dB is required, very questionable.

## REFERENCES

[1] E. U. Condon, "Theory of radiation from paraboloid reflectors," Westinghouse Rep. no. 15, Sept. 24, 1941
[2] S. Silver, Microwave Antenna Theory and. Desion. New York: McGram-Hill, 1949.
[3] C. C. Cutler, "Parabolic antenna design for microwaves," Proc. IRE, vol. 3.5, pp. 1284-1294, Nov. 1947.
[4] E. M. T., Jones. "Paraboloid refiector and hyperboloid lens antennas," IRE Trans. Antennas Propagat., vol, 2, pp. 119-127, July 1954.
[5] I. Kofiman, "Feed polarization for parallel currents in refiectors generated by conic sections," IEEE Trans. Antennas Propagat, vol. AP-14, pp. 37-40, Jan. 1966.
[6] M. Affi, "Scattered radiation from microwave antennas and the design of a paraboloid plane refiector antenna," Ph.D dissertation, Delft Univ. of Technology, the Netberlands, 1967.
[7] P. D. Potter, "The aperture efficiency of large paraboloidal antennas as a function of their feed system radiation characteristics," Jet Propulsion Lab., Pasadena, Calif. Tech. Mep., no. 32-149, Sept. 25, 1961.
[8] P. D. Potter "Aperture illumination and gain of a Cassegrainian system:" IEEE Trans. Antennas Propaọl., vol. AP-11, pp. 373-375, May 1963.
(9) P. A. Watson and S. I. Ghobrial, "Off-axis polarization characteristics of Cassegrainian and front-fed paraboloidal antennas," IEEE Trans. Antennas Propagat., vol. AP-20, pp. 691-699, Nov. 1972.
[10] J. S. Cook, E. M. Elam, and H. Zucker, "The open Cassegrain antenna," Bell Syst. Tech. J., vol. 44, pp. 125j-1299, Sept. 1965.
[11] J. D. Hanfing, "Aperture felds of paraboloidal refectors by stereographic mapping of feed polarization," IEEE Trans. Aniennas Propagat., vol. AP-18, 110. 3, pp. 392-396, May 1970.
[12) R. Graham, "The polarization characteristics of ofiset Cassegrain aerials," in European Microwave Conf:, London Sept. $\delta-12,1969$, p. 352.
[13] - "The polarization characteristics of offset Cassegrain aerials," presented at the Int. Conf. Radar and Future, IEE, London, Oct. 23-25, 1973.
[14] A. C. Ludwig, "The dehnition of cross polarization," IEEE Trans. Antennas Propagat., vol. AP-21, pp. 116-119, Jan. 1973.
[15] B. E. Kinber and V. A. Tischenko, "Polarization of radiation of axisymmetric reflector antennas," Radio Eng. Electron. Phys., vol. 17, pp. 528-534, Apr. 1972, (published January 1973).
[16] T. S. Chu and R.'H. Turrin, "Depolarization properties of ofiset reflector aniennas," IEEE Trans. Antennas Propagal., vol. AP-21, pp. 339-345, May 1973.
[17] J. Dijk, C. T. W. van Djepenbeek, E. J. Maanders, and L. F. G. Thurlings, "The polarization losses of offset antennas," Eindhoven Uiniv. Tech., the Netherlands, TH Rep. 73-E-39, June 1873.
[18] $\overline{1}$. Carter, "Wide angle radiation in pencil beam antennas," J. Appl. Phys., vol. 26 , Mr. 6 pp. f44. 6 6.52, June 1925 .
[19] L. B. Tarakovski, "Side radiation from ideai paraboluid with circular aperture," Radio Eng. Electron. Phys., vol. 4, no. 4, no. 6, pp. 14-28, 1959.

# Multiple-Beam Antennas: Offset Reffectors with Offset Feeds, 

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#### Abstract

Two simple mathematical models are described for the prediction of the vector radiation fields from offset parabolic reflector antennas with offiset feeds. Experimental support for the predictions obtained from the models has been obtained by comparisons with measured data from antenna systems operating at 30 GHz . The principal radiation characteristics of the offset-reflector with offset-feed configuration are discussed.


## I. INTRODUCTION

IN THE DESIGN of antennas with a frequency reuse capability, an offset portion of a parabolic reflecting surface used in conjunction with either a single or multipleelement primary-feed offers a number of advantages. Compared to its full-paraboloidal counterpart, the offsetreffector avoids aperture-blocking effects, reduces the reflector reaction upon the primary-feed, and offers a reduction in astigmatism for off-axis feed locations [1], [2]. For practicable designs the offset structure leads to the use of larger effective focal-length to diameter ratios, with higher gain primary-feeds and a subsequent reduction in mutual coupling berween adjacent feed-elements. Although these advantages are significant, the asymmetry of the offset configuration raises certain questions regarding the vectorradiation characteristics of the overall antemna. In a multiple-beam application, which may include polarization diversity, the asymmetry is compounded by the off-axis location of the primary-feed elements. In such cases the beam cross-over levels and the levels of coma-lobes and cross-polarized sidelobes are of particular interest.

The vector-radiation fields from offset parabolic reflector antennas have been previously considered by Cook et al. [3] and more recently by Chu and Turrin [4] and Rudge et al. [5], [6]. The electric field distribution in the focal region of offset-reflectors under conditions of a normally incident plane-wave has been studied by Bem [7] and polarization losses for offisèt reflectors with dipole and Huygen-source primary-feeds have been calculated by Dijk et al. [8].

In the work described here two simple mathematical models are described that provide predictions of both the principally polarized (copolar) and cross-polarized (crosspolar) radiation from offset parabolic refiector antennas with either linearly or circularly polarized primary-feed elements. The theoretical approach adopted here differs

[^11]from that employed in [3] and [4] in that the predictions are not inherently limited to a small angular range about the antenna boresight. In addition, the theoretical models developed can accommodate small offsets in the location of the primary-feed relative to the reflector geometric focus and can thus be employed in the study and design of: multiple-beam antennas. The models have been developed with the prime objective of producing sufficiently accurate, yet comparatively simple, expressions that when programmed are suitable for repeated use in a design-optimization mode.
Experimental support for the mathematical models has been obtained by comparing their predictions with measured data obtained from antenna systems operating at a frequency of 30 GHz . The models have been applied to examine the radiation characteristics of the offset-reflector with offset-feed configuration and some of the principal features are reported.

## II. General Approach

The geometry of the offset reflector is shown in Fig. 1. The basic parameters of the reflector are shown as the focal length $F$ of the parent paraboloid, the offset angle $\theta_{0}$, and the half angle $\theta^{*}$ subtended at the focus by any point on the reflector rim. The physical contour of the reflector is elliptical but its projection into the $x^{\prime} y^{\prime}$ plane produces a true circle.
The far-field radiation arising from a known tangential electric field distribution in an infinite plane can be determined exactly, as shown for example in the text by Collin and Zucker [2]. In dealing with the offset reflector, the infinite surface is chosen as the $x^{\prime} y$ plane and the electric fields outside of the projected aperture region are assumed negligible. An electromagnetic field distribution is introduced around the boundary of the projected aperture to satisfy the continuity criterion, and thus the predicted radiation fields satisfy the radiation conditions in the forward hemisphere. The neglect of the electric fields outside of the projected-aperture region is acceptable providing that the dimensions of the aperture are large relative to the electrical wavelength (2). The tangential electric field distribution within the projected-aperture region is determined by use of the physical optics approximation. That is, the electric field $E_{r}$ reflected from the offset refiector is obtained from: [1], [2]

$$
\begin{equation*}
E_{r}=2\left(a_{n} \cdot E_{i}\right) a_{n}-E_{i} \tag{1}
\end{equation*}
$$

where $E_{i}$ is the incident electric field at the reffector and $a_{n}$ is the surface unit normal. The incident field $E_{i}$ is taken as the radiation field of the primary-feed.

The physical-optics aperture-field method was selected in preference to the surface-current method after comparisons between the two techniques indicated that there were no significant differences between the far-field predictions obtained from the methods over a cone of angles subtending a half-angle of at least $30^{\circ}$ about the antenna boresight. Since the aperture-field method leads to the more simple mathematical expressions, it offers a significant reduction in the required computational effort.

The distant radiation field from a linearly polarized antenna can be completely specified in terms of two spatially orthogonal vector components. The definition of these vectors in terms of a copolariz:d and cross-polarized component is, to some extent, an arbitrary one and at least three different definitions are commonly used in the literature. The definition employed here has the particular advantage that the predicted field components at any point in space, correspond directly to the components measured. using standard antenna-range techniques [1]. This definition has been discussed in a useful paper by Ludwig [9] and has found favor with a number of authors in recent publications [10], [11]. Employing this definition, with the antema having its principal electric vector along the $y$ axis, the copolar $\left(E_{p}\right)$ and cross-polar ( $E_{q}$ ) "measured-field components" can be related to the field components $E_{\psi}, E_{\phi}$ in a classical spherical coordinate system $r, \psi, \Phi$, by the matrix expression

$$
\left[\begin{array}{c}
E_{p}  \tag{2}\\
E_{q}
\end{array}\right]=\left[\begin{array}{rr}
\sin \Phi & \cos \Phi \\
\cos \Phi & -\sin \Phi
\end{array}\right]\left[\begin{array}{c}
E_{\psi} \\
E_{\Phi}
\end{array}\right] .
$$

## III. The Mathematical Model for Linearly-Polarized Antennas

The model employs a conventional spherical coordinate system ( $r: \psi, \Phi$ ) with origin at the center of the reflector projected-aperture, and makes use of the spatial Fourier transform formulation for the radiation from an infinite surface [2]. Using (2), the normalized radiation patterns of the offset antenna may be expressed directly in terms of a linear copolar field component ( $E_{p n}$ ) and a cross-polar component ( $E_{\text {gn }}$ ) as

$$
\begin{array}{r}
{\left[\begin{array}{c}
E_{p n} \\
E_{\text {on }}
\end{array}\right]=\frac{1+\cos \psi}{2 F_{p}(0,0)}\left[\begin{array}{cc}
1-t^{2} \cos 2 \Phi & t^{2} \sin 2 \Phi \\
t^{2} \sin 2 \Phi & 1+t^{2} \cos 2 \Phi
\end{array}\right]} \\
\cdot\left[\begin{array}{l}
F_{p}(\psi, \Phi) \\
F_{q}(\psi, \Phi)
\end{array}\right] \tag{3}
\end{array}
$$

where the functions $F_{p}$ and $F_{q}$ are the spatial Fourier transforms of the copolar and cross-polar components of the tangential electric field in the projected aperture plane and $t=\tan \psi / 2$.
The Fourier transforms are the transverse Cartesian components of the vector

$$
\begin{equation*}
F(\psi, \Phi)=\int_{x^{\prime}} \int_{y} \varepsilon\left(x^{\prime}, y\right) \exp \left[j k R^{\prime}\left(x^{\prime}, y, \psi, \Phi\right)\right] d x^{\prime} d y \tag{4}
\end{equation*}
$$

where $\varepsilon$ is the tangential electric fiefo distritution in the projected-aperture plane and $R^{\prime}$ is the distance from a


Fig. 1. Offset refiector geometry.
general point in the projected-aperture plane to a far-field point. The transform functions $F_{p}, F_{q}$ can be conveniently expressed in terms of an offset primary spherical coordinate system ( $\rho, \dot{\theta}, \phi$ ) with origin at the reffector focus, i.e., with $\theta$ measured to the inclined $z$ axis, and $\phi$ to the $x$ axis, of Fig. 1. Ignoring multiplying constants the transform integrals can be written
$F_{i}(\psi, \Phi)=\int_{0}^{2 \pi} \int_{0}^{\sigma^{*}} \varepsilon_{i} \exp [-j k R \sin \psi] p^{2} \sin \theta d \theta d \phi$
where $i$ is either $p$ or $q, k=2 \pi / \lambda$
$R=p\left[\left(\sin \theta \cos \theta_{0} \cos \phi+\sin \theta_{0} \cos \theta\right) \cos \Phi\right.$

$$
\begin{equation*}
-\sin \theta \sin \phi \sin \Phi] \tag{6}
\end{equation*}
$$

and the distance between a point on the refector surface and the reflector geometric focus ( $p$ ) is given by

$$
\begin{equation*}
p=\frac{2 F}{\left(1+\cos \theta \cos \theta_{0}-\sin \theta \sin \theta_{0} \cos \phi\right)} . \tag{7}
\end{equation*}
$$

The geometry involved in arriving at (5)-(7) from (4) is straightforward but somewhat lengthy. Relevant details of the geometry of offset parabolic surfaces can be found in [3]-[5] and will not be repeated here.

When the phase center of the primary-feed is located in the general vicinity of the reflector geometric focus, the incident fields at the reflector can be expressed in the form,

$$
\begin{equation*}
E_{i}=\frac{\left[A_{\theta}(\theta, \phi) a_{\theta}+A_{\phi}(\theta, \phi) a_{\phi}\right] \exp \left[j k\left(R_{1}-p\right)\right]}{p} \tag{8}
\end{equation*}
$$

where $A_{\theta}$ and $A_{\dot{\psi}}$ are normalized functions describing the radiation pattern characteristics of the primary-feed radiation, $a_{\theta}$ and $a_{\phi}$ are the associated unit vectors, and the function $R_{1}$ is a phase-compensation term that accounts for small offsets in the primary-feed location, from the geometric focus. In the approach adopted here, the effect on the overall antenna radiation pattern of the amplitude variations in the field at the reflector, due to a small feed offset, have been assumed to be negligible. Thus if $\Delta_{t}$ is a small transverse offst, $\Delta$ a a small axial offset, and the angle $\phi_{0}$ (measured to the $x$ axis in the $x y$ plane) denotes ine plane of the oifset, then providing $\Delta_{t}$ and $\Delta_{2}$ are much less than $F$, it can be
shown that [5], [12]

$$
\begin{equation*}
R_{1} \approx \Delta_{t} \sin \theta \cos \left(\phi-\phi_{0}\right)+\Delta_{z} \cos \theta . \tag{9}
\end{equation*}
$$

The unit normal to the reffector surface ( $a_{n}$ ) can be expressed in terms of the offset primary coordinate system (with unit vectors $a_{x}, a_{y}, a_{z}$ ) as

$$
\begin{array}{r}
a_{n}=-\left\{( \frac { p } { 4 F } ) ^ { 1 / 2 } \left[\left(\sin \theta \cos \phi-\sin \theta_{0}\right) a_{x}+\sin \theta \sin \phi a_{y}\right.\right. \\
\left.\left.+\left(\cos \theta+\cos \theta_{0}\right) a_{z}\right]\right\} \tag{10}
\end{array}
$$

Hence making use of (1), (8), and (10), and assuming that the primary-feed has its principal electric vector in the $a_{y}$ direction, then the Cartesian components of the tangential electric field in the projected-aperture plane can be resolved. The $a_{y}$ directed, or copolar component of the aperture field ( $\varepsilon_{p}$ ) and the $a_{x}$ directed, or cross-polar component ( $\varepsilon_{q}$ ) can be related directly to the radiation characteristics of the primary-feed by the matrix expression

$$
\left[\begin{array}{l}
\varepsilon_{p}  \tag{11a}\\
\varepsilon_{q}
\end{array}\right]=K\left[\begin{array}{cc}
s_{1} & -c_{1} \\
c_{1} & s_{1}
\end{array}\right]\left[\begin{array}{c}
A_{\theta} \\
A_{\phi}
\end{array}\right]
$$

where

$$
\begin{align*}
& K=\frac{-\exp \cdot\left[j k\left(R_{1}-2 F\right)\right]}{2 F}  \tag{11b}\\
& s_{1}=\left(\cos \theta_{0}+\cos \theta\right) \sin \phi  \tag{1ic}\\
& c_{1}=\sin \theta \sin \theta_{0}-\cos \phi\left(1+\cos \theta \cos \theta_{0}\right) \tag{11d}
\end{align*}
$$

## IV. Computation of the Model

The mathematical model for the offset reflector with a linearly polarized feed is effectively given by (3), (5), (9), and (11). To predict the overall antenna radiation it is necessary to specify the primary-feed directivity characteristics ( $A_{\theta}, A_{\phi}$ ) and to compute the two-dimensional integrals given by (5). Expressions for a wide variety of primary-feed types are available in the literature [1], [2], [4], [5], although the cross polarization characteristics of the feed models are not always adequate and must be examined carefully [13]. Having calculated $F_{p}$ and $F_{q}$, the computation of the radiation fields by means of (3) is trivial. It is apparent that the evaluation of the twodimensional integrals represents the crux of the computational problem.
The computational problem can be alleviated by imposing a minor constraint upon the primary-feed functions $A_{\theta}, A_{\phi}$. Providing $A_{\theta}$ is an odd function and $A_{\phi}$ an even function, with respect to the variable phi ( $\phi$ ), which is certainly true for many practical primary-feed types, then (5) can be readily reduced to a half-range form with respect to phi with a subsequent saving in computation time. The twodimensional integral can be accomplished numerically employing a variety of techniques including fast Fourier transforms. A Romberg integration technique was adopted for the models described here [5], [14]. This metbod involves the use of trapezoidal-rule integration with successive interval-halving and an extrapolation technique to remove increasing orders of error.

## V. Circularly Polarized Antennas

The right- and left-hand components of the far-field radiation from a circularly polarized antenna can be simply: defined in terms of the linearly polarized components as.

$$
\left[\begin{array}{c}
E_{R}  \tag{12}\\
E_{L}
\end{array}\right]=\frac{1}{2}\left[\begin{array}{rr}
1 & j \\
1 & -j
\end{array}\right]\left[\begin{array}{l}
E_{q} \\
E_{p}
\end{array}\right] .
$$

However, it will be both informative and useful with regard to minimizing computational effort, to express the normalized radiation patterns in the form

$$
\begin{array}{r}
{\left[\begin{array}{l}
E_{R_{n}} \\
E_{L_{n}}
\end{array}\right]=\frac{1+\cos \psi}{2 F_{0}}\left[\begin{array}{cc}
1 & t^{2} \exp [j 2 \Phi] \\
t^{2} \exp [-j 2 \Phi] & 1
\end{array}\right]}  \tag{13}\\
\ddots\left[\begin{array}{l}
F_{R} \\
F_{L}
\end{array}\right]
\end{array}
$$

where the subscript $n$ denotes the mormalization, $F_{R}, F_{L}$ are the Fourier transformations of the right- and left-hand components of the reflector projected-aperture tangential electric field ( $\varepsilon_{R}, \varepsilon_{L}$ ), and $F_{0}$ is the normalizing constant. In this form the circularly polarized aperture-field components can be related directly to the radiation characteristics of the primary-feed $\left(A_{\theta}, A_{\phi}\right)$ by

$$
\begin{array}{r}
{\left[\begin{array}{l}
\varepsilon_{R} \\
\varepsilon_{L}
\end{array}\right]=\frac{2 F K}{p}(1+\cos \theta)\left[\begin{array}{ll}
\exp [j \Omega] & -j \exp [j \Omega] \\
\exp [-j \Omega] & j \exp [-j \Omega]
\end{array}\right]} \\
\cdot\left[\begin{array}{l}
A_{\theta} \\
A_{\phi}
\end{array}\right] \tag{14}
\end{array}
$$

where

$$
\begin{equation*}
\Omega(\theta, \phi)=\arctan \left(s_{1} / c_{1}\right) \tag{15}
\end{equation*}
$$

The numerical integration time of the Fourier transform functions can again be reduced by imposing a constraint upon the primary-feed radiation. For example, (5) can again be reduced to a half-range integration with respect to phi if the normalized circularly polarized primary-feed radiation $\left(E_{n}\right)$ takes the form

$$
\begin{equation*}
E_{n}=\left(A_{1}(\theta) a_{\theta}-j A_{2}(\theta) a_{\phi}\right) \exp [-j \phi] \tag{16}
\end{equation*}
$$

where the functions $A_{1}, A_{2}$ are independent of the phi variable. This expression is satisfied by the radiation fields from many practical types of circularly polarized feed including fundamental-mode conical-horns, dual-mode horns [15], and corrugated horns [16]. In these cases the reflector aperture-plane fields may be obtained from (14) as

$$
\begin{equation*}
\varepsilon_{M}=\frac{2 F K}{p}(1+\cos \theta)\left(A_{1}(\theta) \pm A_{2}(\theta)\right) \exp -j[\phi \pm \Omega] \tag{17}
\end{equation*}
$$

where $M$ is either $R$ or $L$, and $L$ takes the upper sign: It is apparent that when $A_{1}(\theta)=A_{2}(\theta)$, which, for 2 . circularly symmetric feed, is the condition for zero cross: polarized primary-feed radiation [9], then the reflector-aperture-plane field will be purely copolarized with a "beam-squinting" phase distribution given by $\Omega+\phi$.

## VI. Experimental Verification

The measurements reported here were made over a period of many months at the University of Birmingham: The antenna range was investigated using feld-probing


Fig. 2. Copolar radiation fields from ofiset reflector antenna ( $F=$ $22.72, \theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ) fed by linearly polarized rectangular horn with aperture dimensions $1.571 \times 2.141$, _measured $\rightarrow$ predicted. (a) Plane of asymmetry, $\Phi=90^{\circ}$. (b) Plane of symmetry, $\Phi=0^{\circ}$.
techniques and the perturbation level for linearly polarized waves determined as -40 dB . The isolation between copolarized and cross-polarized signals incident at the test antenna was measured as better than 37 dB employing a pyramidal-horn probe.
In Fig. 2 the copolar radiation patterns are shown for an offset refiector fed by a linearly polarized $T E_{10}$ mode rectangular horn with aperture dimensions of $1.571 . \times$ 2.14 , which produces a $-12-\mathrm{dB}$ illumination taper at $\pm 30^{\circ}$. The reflector surface was precision machined to a tolerance of better than 0.1 mm and has parameters $F=$ $22.7 \lambda, 0_{0}=44^{\circ}, \theta^{*}=30^{\circ}$. The predicted radiation patterns in the principal-planes are shown superimposed upon experimental data obtained with a $30-\mathrm{GHz}$ antenna system. The correlation can be seen to be excellent although the cross-polarized results shown in Fig. 3 are slightly inferior. The cross-polarized data is shown for the plane of asymmetry ( $y z^{\prime}$ plane), which contains the peak levels of the reflector generated cross-polarized radiation. In the plane of symmetry ( $x^{\prime} z^{\prime}$ plane) the reflector generated cross polarized radiation is zero.

In Fig. 4 the $30-\mathrm{GHz}$ copolar and cross-polar radiation patterns are shown for a second offset reflector with $F=$ $30.4 \lambda, \theta_{0}=35^{\circ}$, and $\theta^{*}=30^{\circ}$. The refiector was again fed with the linearly polarized rectangular horn and the cases shown are for transverse primary-feed offsets of $0,0.832$., and $2.5 \%$, respectively. The feed offsets are along the $y$ axis and correspond to an E-plane array of 4 beams with crossover levels of -5 to -6 dB , a maximum coma-lobe level (on the outer beams) of approximately -21 dB and a peak cross-polar level of -26 dB . Reducing the primary-feed spacing (and hence the feed apenure dimensions) to provide a -3 dB beam cross-over level, results in an outerbeam coma-lobe level of approximately -18 dB with a


Fig. 3. Cross-polar radiation field in plane of asymmetry ( $\Phi=90^{\circ}$ ) for ofiset antenna of Fig. 2; measured; $\rightarrow$ predicted.
-25 dB peak cross-polar level in the plane of asymmetry.
Fig. 5 shows the copolar radiation field of the first reflector fed by a circularly polarized corrugated scalarhorn feed with a diameter of 2.82 . Employing circular polarization the offset reffector does not generate a cross polarized component and, since the scalar-feed produces a. very low level of cross-polarized radiation, only the copolar result is of concern in this case. The scalar horn produces a -17 dB illumination taper at $\pm 30^{\circ}$ and thus the sidelobe levels are significantly lower than previously. In Fig. 6 the radiation pattern is shown in the plane of asymmetry with the horn moved transversely along the $y$ axis by distances of $0,1.4 \%$, and $2.8 \%$, respectively. For this case the coma-lobe levels have been reduced to better than -27 dB but the larger primary-feed horn implies a minimum beam crossover level of approximately -19 dB .

## Vil. Features of the Offset Configuration

Having gained some confidence in the quality of the predictions obtained, the theoretical models were employed to examine the principal features of the offset-reflector configuration. A few of the many points of interest that emerged during the course of this work will be briefly mentioned.
For focused primary'feed locations the cross-polarized radiation generated by the refector with linearly polarized illumination is primarily a function of the parameters $\theta_{0}$ and $\theta^{*}$. The peak level of the cross polarized radiation increases with increasing values of $\theta_{0}$ and $\theta^{*}$ and is comparatively insensitive to the amplitude taper of the primaryfeed illumination. The peak values of the refector-generated cross polarization occur in the plane of asymmetry ( $y z^{\prime}$ plane) at the angles corresponding approximately to the $-6-\mathrm{dB}$ levels of the main copolar beam. In the plane of symmetry ( $x^{\prime} z^{\prime}$ plane) the refiector-generated cross polarization is zero. Using refiector parameters of the order of $\theta_{0}=35^{\circ}$ and $\theta^{*}=30^{\circ}$ the peak values of the copolar and cross-polar sidelobes can be reduced below -25 dB using simple primary-feed types, which provide an illumination taper of at least -13 dB . To reduce the peak cross polarized levels to below - 30 dB , the offset angle must be reduced to $30^{\circ}$ or less. When a circularly polarized primaryfeed illumination is employed, offset refiectors of this type


Fig．4．Radiation fieids from offiset refiector（ $F=30.4 \%, \theta_{0}=35^{\circ}$ $\partial^{*}=30^{\circ}$ ）with primary－feed ofset transversely by $0,0.83 \lambda$ ，and 2．5；，respectively＇；－measured；－predicted．


Fig．5．Copolar radiation field from an offset reflector antenna （ $F=22.7 \%, \theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ）fed by circularly polarized scalar horm uith aperture－diameter 2．82；（a）Plane of asymmetry＇，$\Phi=90^{\circ}$ ． （b）Plane of symmerry，$\Phi=0^{\circ}$ ．


Fig．6．Copolar radiation field from circularly polarized offset refiector antenna（ $F=22.7 \lambda, \theta_{0}=44^{\circ}, \theta^{*}=30^{\circ}$ ）with scalar horn feed offset transversely by $0,1.4 \lambda$ ，and $2.8 \lambda$ ，sespectively； －measured；－predicted．
do not generate a cross－polarized component and the only significant cross－polarized radiation that occurs is due directly to the primary－feed．However，the beam formed by the antenna exhibits a small squint，the angle and direc－ tion of which is dependent upon the refiector offiset－angle and the band of polarization．When a duplex dual－polarized configuration is utilized，some loss of gain will occur in the boresight direction due to the squint－angle difierence of the two beams．The results obtained for this focused configura－ tion are in general agreement with those obtained by Chu and Turrin［4］，which are valid for small angles about the antema boresight．

For a linearly polarized multiple－beam application，a small transverse offset of the primary－feed results in the expected spatial shift of both the copolar and cross－polar distributions．However，it is significant that the peak levels of the cross－polarized lobes remain essentially unchanged and that these lobes are comparatively insensitive to phase－ errors of the type introduced by small offsets in the primary－ feed location．For offset feed locations，the formation of the copolar coma－lobe remains the most evident source of pattern deterioration．The results described earlier provide an indication of the tradeoff between beam cross－over levels and copolar and cross－polar sidelobes，which is available using an $E$－plane array of rectangular feed elements．

With circularly polarized multiple－beam antennas，the lack of a reflector－generated cross－polarized component is a useful attribute，although the beam－squint problem may be significant if polarization diversity is required．For the circularly polarized reflector described here，the squint of each beam from boresight is of the order of 0.07 of the antenna -3 dB beamwidth and the resultant loss of gain
in the boresight direction is approximately 0.03 dB per beam. These figures could be improved to 0.05 and 0.02 dB , respectively, by reducing the offset angle from $44^{\circ}$ to $35^{\circ}$. Although the reflector does not contribute to the cross polarized radiation the contribution from the circularly polarized primary-feed may not be negligible, particularly' for multiple-beam applications. Fundamental mode conical horns, for example, which are a possible choice of feed-array element, radiate a very significant cross-polar component.

## ViII. Conclusions

Two theoretical models have been described for the prediction of the far-field radiation from offset reflector antennas. The models can accommodate small offsets in the primary-feed location with respect to the reflector geometric focus and can thus be usefully employed in the study or design of multiple-beam antennas. The models are based upon the physical-optics approximation and employ' a known aperture-field approach, the validity of which is not inherently restricted to angles close to the antenna boresight. Experimental verification of the models, and comparisons with the surface-current technique, have indicated that reliable predictions of the significant copolarized and cross-polarized radiation can be obtained by use of these methods over a moderate range of angles about the antenna boresight.
The models have been employed to examine the radiation characteristics of the offset-reflector with offset-feed configuration and some of the principal features have been briefy discussed. A particularly interesting result is that the maximum levels of the refiector-generated cross polarized radiation are comparatively insensitive to small ofisets in the primary-feed location from the geometric focus.

## Acknowledanent

The author wishes to thank Dr. M. Shirazi for the computation of the mathematical nodels.

## Referencas

[1] S. Silver, Microwave Antenna Thery and Design. New York: McGraw-Hill, 1949.
[2] R. E. Collin and F. J. Zucker, Akerna Theory, Parts I and II. New York: McGraw-Hill, 1969.
[3] J. S. Cook, E. M. Elam, and H. Zukker, "The open Cassegrain antenna: Part I-Electromagnetic design and analysis," Bell Syst. Téch. J., vol. 44, pp. 1255-130, Sept. 1965.
[4] T. S. Chu and R. H. Turrin, "Depharization properties of offset reflector antennas," IEEE Trans. Arıennas Propagat., vol. AP-21, pp. 339-345, May 1973.
[5] A. W. Rudge and M. Shirazi, "Multiple-beam antennas: Offset reflectors with offset feeds." Univerity of Birmingham, U.K., Final Rep. ESRO/ESTEC Contraci 1725/72PP, July 1973.
[6] A. W. Rudge, "Offset refectors with offiset feeds," IEE Electron. Lett., vol. 9, pp. 611-613, Dec. 197s.
[7] D. J. Bem, 'Electric-field distribution in the focal region of an offset paraboloid," Proc. Inst. Elc. Eng., vol. 116, no. 5, pp. 579-684, 1969.
[8] J. Dijk et al., "The polarization losses of offset paraboloid antennas," IEEE Trans. Antennas Propagat., vol. AP-22, pp. 513-520, July 1974.
[9] A. C. inudwig, "The definition of enss polarization," IEEE Trans. Antennas Propagat. (Commun.), vol. AP-21, pp. 116-119, Jan. 1973.
[10] P. J. Wood, "Depolarization with Cassegrain and front-fed refiectors," IEE Electran. Lett., vol. 9, pp. 181-183, May 1973.
[11] P. A. W'atson and S. I. Ghobrial, "Crosspolarization in Cassegrain and front-fed antennas," IEE Electron. Lett., vol. 9, pp. 297-298, June 1973.
[12] J. Ruze, "Lateral-feed displacement in a paraboloid," IEEE Trans. Antennas Propagat., AP-13, np. 660-665, Sept. 1965.
[13] A. W. Rudge, T. Pratt, and A. Frr. "Cross-polarised radiation from satellite refiecior antennas," in Proc. AGARD Conf. on Anternas for Avionics, Munich, Nor: 1973, pp. 16.1-16.8.
[14] W. V. T. Rusch and P. D. Potter, Analysis of Reflector Antennas. New York: Academic, 1970.
[15] P. D. Potter, "A new' horn antenna with suppressed sioe Jobes and equal beamwidths," Microwart J., vol. 6, pp. 71-78, June 1963.
[16] P. J. B. Clarricoats and P. K. Saha, "propagation and radiation behavior of corrugated feeds," Proc. Inst. Elec. Eno., vol. 118, pp. 1177-1186, Sept. 1971.

## NEW CLASS OF PRIMARY-FEED ANTENNAS FOR USE WITH OFFSET PARABOLIC-REFLECTOR ANTENNAS

# Indexing Jerms: Antenno feeders, Refiector ansennas 

In many praclical applications the performance of an offise parabolic-refiector antenna is limited by the ofisel refiectors depolarising and beam-squinting properties. The lether describes a neu class of primary-iced antennas which overcome these limitanions. The new primary-ieed types ofier a signt ficant improvement in the crosspolar and beam-squinting properties of the ofiset refiector amenna, withoul adoding stpnificanlly to the complexity or mass of the primary-feed system.

In many antenna systems where high gain and very low sidelobe radiation is required, the offsel parabolic reflector offers a number of significant advantages over its axisymmetric counterparts. In particular, since the primary feed and its supporting structure need not protrude into the optical path of the incident or reflected wavefronts, spurious scattering, and the associated gain loss due to blockage effects, can be avoided. The avoidance of blockage is a major factor in achieving high overall efficiencies and low sidelobe performance. ${ }^{1,2}$

For many practical applications the offset reflector configuration has a serious disadvantage, however, in that the refiectior has undesirable depolarising properties when illuminated efficiently by a perfectly linearly polarised primaryfoed radiation. The overall radiation field from such an antenna typically exhibits a pair of crosspolarised lobes, with peaks close to the -6 dB contour of the antenna main beam, in one of the principal planes of the reffector. In the other principal plane the crosspolarised field is zero. ${ }^{3,4}$ The plane containing the peak crosspolarised lobes will be referred to here as the plane of asymmetry. When similarly illuminated by a purely circularly polarised primary feed, the offset refiector does not introduce a crosspolarised field, but the main beam of the antenna is squinted from the antenna boresight. ${ }^{3-5}$ These depolarising and squinting phenomena are often undesirable in high-performance antennas, especially when frequency reuse is required. In many cases these effects constitute a major limitation on the application of the offset reflector, regardless of its other desirable properties.

To overcome these limitations in offset reflector antennas, without adding excessively to the complexity or mass of the


Fig. 1
a Coniour plot of rypieal focai-plane field characleristics for an offsel parabolie reflector. Crosspolarised field peak tevel aypically -20 to -30 dB below peak
copolarised held
b Aperture-field distribution of overmoded waveguide horn
feed radiator, a set of novel primary-feed devices have been devised and are under development at the RF Technology Centre. ${ }^{6}$ This letter briefly describes the principle behind these feeds and shows predicled and preliminary experimental data for a protolype of one of the new feed types.

The general concept behind the improved feeds can be understood in a simple fashion by a consideration of the nature of the focal-plane eleciric-field distribution of an offset reflector operating in its reception mode. Fig. $1 a$ shows an approximate contour map of the electric-field distribution with a distant source located on the antenna boresight. To provide a conjugate match to this incident field distribution, the primary-feed aperture-plane fields must exhibit similar polarisation properties. In Fig. I $b$ this general aperture-field characteristic is approximated by cylindrical waveguide modes in which a particular higher-order mode provides the necessary crosspolarisation characteristics. This waveguide mode is the


Fig. 2 Experimental trimode offset reflector conical-horn feed Asymmetric diseontinuity introduced by iwo small posis al right angles
$\mathrm{TE}_{21}$ in a cylindrical smooth-walled guide or the $\mathrm{HE}_{21}$ hybrid mode in a cylindrical corrugated guide. A similar effect can be achieved in a rectangular waveguide structure, and, in this case, the crosspolarisation characteristics are matched by the addition of the $T E_{11}$ mode to the fundamental $\mathrm{TE}_{10}$ mode. For the corrugated cylindrical structures and the rectangular structures, the use of the fundamental mode plus the one higher-order mode is sufficient. For the smooth-walled cylindrical structure; a third mode, the $\mathrm{TM}_{12}$, can be employed to improve the axisymmetry of the feed copolar radiation pattern, and to remove crosspolarised fields which otherwise radiate into the diagonal planes of the feed far-field pattern. This technique is well known in axisymmetric reflector design ${ }^{7}$. Although all three feed configurations are of interest, in view of space constraints, only the trimode cylindrical configuration will be discussed further here. The general structure for this feed is illustrated in Fig. 2.


Fig. 3 Predicted and measured copolar and crosspolar charac. teristics for $30 \mathrm{GH} \geq$ trimode feed
Predicied ( $H$-plane): -
Measured ( $H$-plane):-
(E-plane):---


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Briefly, the trimode primary feed is essentially a small-flare-angle conical horn with two discontinuities or steps. The first step $\left(d_{3} / d_{2}\right)$ is asymmetric and generates the $T E_{21}$ mode. The diameter $d_{2}$ is chosen to cut off all higher modes above the $\mathrm{TE}_{21}$, which is the first propagating mode above the fundametal. The second step $\left(d_{2} / d_{1}\right)$ is axisymmetric and the guide dimensions cut off all modes above the $\mathrm{TM}_{1}$, mode. The symmetry of this discontinuity avoids the further generation of the $\mathrm{TE}_{21}$ mode. The amplitudes of the modes generated are governed by the ratios $d_{3} / d_{2}$ and $d_{2} / d_{1}$, and the relative phases of the modes are adjusted by the constant-diameter phasing section which follows each discontinuity. The mode amplitudes required are a function of the parameters of the offset reflector; in particular, the ofiset angle $\theta_{0}$ and the semiangle $\theta^{*}$ subtended by the reflector rim at the geometric focus. Typically, the mode amplitudes are of the order of -20 dB below the fundamental mode. The diameter of the primaryfeed aperture is selected in the usual way to satisfy the illumination requirements of the reflector. The overall length of the feed is between 0.25 and 1.0 wavelengths greater than a conventional axisymmetric dual-mode feed of the Potter-horn variety. ${ }^{7}$ The new dual-mode corrugated and rectangular feed types referred to above can be even more compact.

The predicted radiation pattern of a trimode feed with $D=2.8 \%$ is shown in Fig. 3. The $T E_{21}$ mode amplitude for this case is -20 dB below the fundamental $T E_{11}$ mode. The TM ${ }_{11}$ mode amplitude is as for a standard Potter dual-mode hom. This feed would be well suited to feed an offset reflector with $\theta_{0}=35^{\circ}$ and $\theta^{*}=30^{\circ}$. The required pair of crosspolarised lobes are generated in one principal plane, and the crosspolarised field is zero in the other principal plane. In Fig. 3, measured data are superimposed for a prototype trimode feed operating at 30 GHz . Some spurious crosspolarised radiation is evident around the -30 dB level in the $E$-plane of the horn, but the correlation is, in general, very satisfactory.

In Fig. $4 a$ the 30 GHz overall radiation characteristics are shown. in the plane of asymmetry, for.a precision offset refiector fed by a conventional low-crosspolar feed with diameter $2.8 \%$. The ofiset reflector has parameters $F=22.72$, $\theta_{0}=44^{\circ} ; \theta^{2}=30^{\circ}$, and the large crosspolar peaks are predictable for a refiector with this relatively large ofiset angle. In Fig. $4 b$ the radiation characteristics in this plane are shown when the feed is replaced by the trimode feed. The reduction in the crosspolarised radiation (to the curve $b_{1}$ ) is very significant, if not entirely complete. The imperfect cancellation is predictable, and arises from the fact that the prototype feed is not optimised for the parameters of this particular ofiset refiector. By artificially modifying the amplitude of the $T E_{21}$ mode, the measured performance is improved to that shown in Fig. 4b, curve $b_{2}$. This result demonstrates the level of performance which can be expecied when the mode
amplitude is correctly chosen to match the reflector $\theta_{0}$. $\theta^{*}$ parameters.

The smooth-walled and corrugated structures referred to here can be designed for use in circularly polarised offset reflector systems. In such cases, the feed can be employed either to remove or to enhance the beam-squinting effects normally incurred in these configurations. Further development of all three feed configurations is proceeding. ${ }^{6}$

The principal advantages of the new class of primary-feed antennas described here can be briefly summarised as follows:
(a) The feeds are essentially optimised for operation with offset parabolic-reflector antenias. Used in conjunction with such reflectors, highly efficient antennas with low copolarised and crosspolarised sidelobe radiation can be constructed.
(b) The feeds provide a very significant improvement in the crosspolar performance of ofiset reffector antennas by cancelling the depolarising properties of the offset reflector.
(c) The corrugated dual-mode and conical trimode feeds will also remove beam-squinting effects when used for circularly polarised applications.
(d) The feeds are relatively simple to construct and do not involve any significant increase in complexity or mass over existing feeds designed for axisymmetric systems.

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## References

I DRAGONE, C.. and hOGG. D. C.: 'The radiation panerr and impedance of offsel and symmelrical near-ñeld Cassegrainian and Gregorian amiennas'. /EEE Trans.. 1974. AP-22, pp 472-475
2 RUDGE, A. W'. FOSTER. P. R., eI al.: 'Siudy of the performance and limitations of multiple-beam antennas'. ES'TEC Conitaet 2277/74 HP.ERA-11TR1 RF Technology Certic. 1975
3 CHU, T. S. and TURRIN. R. H.: 'Depolarisation properties of ofisel refiector antennas', /EEE Trans., 1973, AP-21, pp. 339-345
4 Runge. A. W.: 'Multiple-beam antennas: ofiset refiectors with ofiset feeds'. ibió., 1975 A P-23, pr. 317-322
5 ADATIA, N. A., and RUDGE, A. W.: 'Beam-squint in circularly polarised adatia, N. A., and rubge, A. W.: 'Beam-squint in circularly pola
ofiset-reficcior aniennas'. Eleciron. LeII.. $1975,11, \mathrm{pp} .513-515$
6 RUDGE, a. W.. and ADATIA. N.: llmproved primaryefed aniennas for RUDGE, A. W'a and ADATIA. N.: 'Improved primary-iced antennas for
ofisel parabolic-refiector aniennas'. British Patent Application $44505 /$ ofisel parabolic
75 , Oci. 1975
7 POTTER, P. D.: 'A new horn antenna with suppressed sidelobes and equal beamwidihs", Microwave J., 1963, 6, pp. 71-78

On the Cross Polarization of Asymmetric Reflector Antennas for Satellite Applications.

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Abstroct-It is well known that focussed, axial symmetrical reflector antennas collimate the co- and cross-polar components of the primary field separately, i.e., the reflector does not create a contribution to the cross polarization of the far-field. By a simple extension of a classical physical argument it is demonstrated that this separability does not depend on the symmetry of the antenna, and that it, therefore, holds even for off-set fed reflectors. A new mathematical formulation of the collimation is derived in which this is shown. Yet the separability does depend on how the co- and cross-polar fields are defined, and the cross polarization of feeds for asymmetric reflectors is discussed in detail in the light of this. It is further suggested how to design low cross polari-

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zation feeds fer off-set fed antennas. As a consequence of the separate collimation such feeds will lead tolow cross-polarization of the secondary fields. Two simple examples are treated. The only limitations of the results are those due to the application of the aperture field version of the physical optics approximation.

## I. INTRODUCTION

In order to save frequency bandwidth it is desirable to reuse the frequency by: operating in two orthogonal polarizations either within the same beam or as a means of providing isolation between neighboring beams operating in the same frequency band. In such systems the isolation between channels depends on the suppression of cross polarization. In a typical system such suppression is required to be in the order of 30 dB within the complete coverage zone. This translates into an isolation due to the antenna hardware of 40 dB as atmospheric depolarization and other sources outside the onboard antenna will also contribute to the interchannel coupling.

Off-set reflector antenna configurations offer several advantages compared to center-fed systems. The blockage by the feed horn (or subreflector in Cassegranian systems) and the supporting struts which decrease the gain in front-fed systems is avoided; this is particularly important in multibeam antennas, because the feed system is relatively large. The field scattered by the struts also has a large content of depolarized signal and, therefore, causes a limitation to which polarization purity can be achieved in front-fed systems [1]; this limitation is particularly severe in relatively small reflectors such as is typical for on-board $X$-band antennas. Furthermore, in a multibeam antenna, the coupling between the feeds via the reflector is considerably lower in an off-set system since there is no direct reflection back into the feed. Finally, there are mechanical advantages in that more freedom is allowed in the choice of supporting structures and in the means of thermal control.

Extensive literature has appeared in recent years dealing with analysis of cross polarization and other characteristics of off-set reflector antennas, and references have been given by several authors [2]-[6]. It has been observed that such antennas usually have a high level of cross polarization at least when operated in inear polarization, whereas for circular polarization the two (mutually orthogonal) linear cross-polar components may add up to a co-polar circular component causing a beam squint as shown by Chu and Turrin [3].

The purpose of the present paper is to demonstrate that the cross polarization is not a basic and unsurmountable characteristic of off-set reflectors. By using a definition of cross polarization, in which the reference is a Huygens source oriented orthogonal to the axis of the reflector system, it is shown that the reflector does not contribute to the cross polarization in the far-field as is sometimes stated, neither is it a necessary consequence of the asymmetry of the configuration. The cross polarization of the far-field is shown to be merely the cross polarization of the primary field after collimation by the reflector. It is suggested, therefore, that the cross polarization can be significantly decreased by improvements in the feed, and there seems to be no physical limitation to the extent to which this can be acheived independently of the type of symmetry involved in the antenna system. Two simple feeds for off-set illumination of elliptically contoured reflectors with low cross polarization are treated as examples of how the techniques work.

## II. ON THE DEFINITION OF CROSS POLARIZATION

Co- and cross polarization is usually defined by comparing the source under consideration with a reference source [7]. The co-polar field of the given source is then taken to be the component of the field which is parallel to the field of the reference source and the cross-polar field is the orthogonal component. This means that the co-polar field of a given source is defined by

$$
\begin{equation*}
\bar{E}_{\mathrm{co}}=\frac{\bar{E} \cdot \bar{E}_{\mathrm{ref}}}{\bar{E}_{\mathrm{ref}}{ }^{2}} \bar{E}_{\mathrm{ref}}, \tag{1}
\end{equation*}
$$

where $\bar{E}=E(\xi, \psi)$ is the electric vector field of the given source, $\bar{E}_{\text {ref }}=\bar{E}_{\text {ref }}(\xi, \psi)$ is the electric vector field of the reference source ((1) is to be properly mended in directions
where $\bar{E}_{\text {ref }}=0$ ), and the cross-polar field is obtained by (1) by replacing $\bar{E}_{\text {ref }}$ with the orthogonal field. It should be clear that the definition of cross polarization then depends not only on which source has been chosen as the reference, but also on how it is oriented.

Various definitions have been discussed by Ludwig [8] who named them 1 st, 2nd, and 3rd definition according to the reference field being 1) a plane wave, 2) the radiated $E$-field from a short electric dipole, and 3) the $E$-field radiated by a Huygens source, and recommended the 3rd definition be used in connection with feed systems because of the following essential merits.
a) Interchanging the co- and cross-polar fields as measured in any direction corresponds to a $90^{\circ}$ rotation of the reference source.
b) The definition is logically associated with normally used primary feed measurement setup as described by Silver [9] and Hollis et al., [10]. It should be noted that the reference Huygens source is understood to be oriented perpendicular to the $\xi=0$ axis of the measurement coordinate system.
c) When using this definition rigorously the collimation performed by a focused reflector system treats the co- and crosspolar components of the primary field separately. Thus no coupling is introduced by the reflector. This last charateristic will be dealt with in detail in the following sections.

The recommendation of Definition 3 has been widely accepted in recent publications; yet, as will be pointed out in the following, the statement in item c) above holds only if the definition is used in a strict sense. Fig. 1, which corresponas to the relevant part of Fig. 1 of Ludwig's paper [8], shows the electrical field lines on a sphere surrounding a Huygens source which is orthogonal to the $z$ axis and which has the electric dipole oriented towards the $y$ and $x$ axis, respectively. For convenience this will be referred to as. $p$ - and $q$-polarization throughout this paper; $\hat{p}$ and $\hat{q}$ are supposed to be unit vector fields with directions defined by the $\bar{E}$-fields from the two Huygens sources, so that the two polarization components, according to (1) are

$$
\begin{align*}
E_{p} & =\bar{E} \cdot \hat{p} \\
E_{q} & =\bar{E} \cdot \hat{q} \tag{2}
\end{align*}
$$

where $\hat{p}$ and $\hat{q}$ are related to $(\xi, \psi)$ coordinates by

$$
\left\{\begin{array}{l}
\hat{p}  \tag{3}\\
\hat{q}
\end{array}\right\}=\left\{\begin{array}{c}
\sin \psi \cos \psi \\
\cos \psi-\sin \psi
\end{array}\right\}\left\{\begin{array}{c}
\hat{\xi} \\
\hat{\psi}
\end{array}\right\} .
$$

## III. RADIATION FROM FOCUSED REFLECTORS

## A. Currents on a Focused Paraboloidal Reflector Surface

Let a parabolic reflector be illuminated by a Huygens source which is located in its focal point and oriented with the electric dipole in the $x$ direction and the magnetic dipole in the $y$ direction (i.e., the source is $q$-polarized). The associated coordinate system is defined in Fig. 2. On the basis of the paraboloidal geometry and by the use of the physical optics approximation by which the surface current density $\bar{J}$ on the reflector is given by $\bar{j}=2(\hat{n} \times \bar{H})$, where $\bar{H}$ is the incident magnetic field and $\hat{n}$ is the outgoing unit normal to the surface, it has then been shown by Jones [11] and Koffmann


Fig. 1. $\hat{p}$ and $\hat{q}$ vector fields on unit sphere. Fields are defined by $\bar{E}$ field from Huygens source placed at origin and oriented orthogonal to $z$ axis with electric dipole paralled $t o$ (a) $y$ axis and (b) $x$-axis, respectively. This figure, and some of following, are drawn for stereoscopic viewing.


Fig. 2. Parabolic reflector illuminated by $q$-oriented linearly polarized source. Currents on reflector and $q$ field lines on sphere surrounding feed are indicated.
[12] that the currents satisfy the condition $J_{y}=0$ at all points on the reflector. Briefly speaking, this means that the far-field radiated by the currents on the reflector will be linearly polarised in the $x$ direction, and the Huygens source is; therefore, an "ideal source" in this particular sense. Obviously, any other $q$-polarized feed will excite currents in the same direction even if the amplitude distribution is different (e.g., asymmetric), and the same conclusion for the farfield therefore holds. In Fig. 2 the currents on the reflector are shown, and a unit sphere with the $q$-field lines are shown also in order to illustrate the polarization performance of the feed.
it is worthy of note that the condition $J_{y}=0$ is satisfied all over the surface. This means that the polarization purity of the far-field does not depend on a concellation of cross polarization contributions from various parts of the reflector. The secondary field therefore remains linearly polarized if some part of the reflector is removed or new parts are added. It may thus be deduced in general, that the polarization of the secondary field is independent of how the contour of the re-


Fig. 3. Off-set parabolic reflector illuminated by $q$-oriented linearly polarized source. Currents on reflector and $q$ field lines on sphere surrounding feed are indicated.


Fig. 4. Coordinate system for focused paraboloid and primary ano secondary field.
flector is shaped. As an example an off-set-fed reflector antenna is shown in Fig. 3. Again, the vertical currents and the unit sphere showing the feed polarization are shown. The conclusion drawn here is in contrast to the frequently appearing statement that cross polarization is created by the reflector or by the asymmetry of the antenna configuration; a statement which seems to have impeded straight forward design of offset reflector antennas for polarization diversity. A more rigorous treatment of the problem is given in the following.

## B. Mathematical Formulation of the Reflector Transformation

In order to obtain the mathematical relationship between the primary and the secondary field for a focal-fed paraboloidal reflector the aperture technique will be used because of its simplicity. This technique is well established, and is used here as described by Collin and Zucker [13]. It is worthwhile to note that the technique has been shown by Rudge [14] to work well for multibeam antennas as well.

The coordinate system for the paraboloid and the feed is shown in Fig. 4. The feed radiation is described by.

$$
\begin{equation*}
\bar{E}^{i}(\rho, \xi, \psi)=\bar{E}^{i}(\xi, \psi) \cdot \frac{e^{i k \rho}}{\rho} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{E}^{i}(\xi, \psi)=E_{p}{ }^{\mathbf{i}}(\xi, \psi) \hat{p}+E_{Q}^{i}(\xi, \psi) \hat{q} . \tag{5}
\end{equation*}
$$

Here $\hat{p}$ and $\hat{q}$ are the polarization vectors defined in Section II, and in the following $E_{p}{ }^{i}$ will be referred to as the "co-polar" component essentially oriented towards the $y$ axis, and $E_{q}{ }^{i}$ as
the "cross-polar" component. Throughout this paper the time dependence factor $\exp (-i \omega t)$ is understood to multiply all field quantities, and $k=\omega \sqrt{\mu \epsilon}$ to be the free space wavenumber. The geometry of the reflector is given by

$$
\begin{equation*}
\rho=\frac{2 f^{\circ}}{1+\cos \xi}=\frac{f}{\cos ^{2}(\xi / 2)} \tag{6}
\end{equation*}
$$

where $f$ is the focal length, and its outgoing normal unit vector by

$$
\begin{equation*}
\hat{n}=-\cos \left(\frac{\xi}{2}\right) \hat{\rho}+\sin \left(\frac{\xi}{2}\right) \hat{\xi} \tag{7}
\end{equation*}
$$

A.ssuming a physical optics reflection and neglecting a constant phase term, the aperture field is expressed by

$$
\begin{equation*}
\bar{E}^{a}=2\left(\hat{n} \cdot \bar{E}^{i}\right) \hat{n}-\bar{E}^{i} \tag{8}
\end{equation*}
$$

The aperture field will now be evaluated in terms of the copolar and cross-polar components of the primary field. First, $\bar{E}^{i}$ as given by (5) is expressed in spherical vectors, by using (3); carrying out the multiplication in (8) in spherical coordinates then gives

$$
\begin{align*}
\bar{E}^{a}= & -\sin \xi\left(\sin \psi E_{D}^{i}+\cos \psi E_{Q}^{i}\right) \hat{\rho} \\
& -\cos \xi\left(\sin \psi E_{D}^{i}+\cos \psi E_{Q}^{i}\right) \hat{\xi} \\
& -\left(\cos \psi E_{D}^{i}-\sin \psi E_{Q}^{i}\right) \hat{\psi} . \tag{9}
\end{align*}
$$

By further substituting the following coordinate transformation

$$
\begin{align*}
& \hat{\rho}=\sin \xi \cos \psi \dot{x}+\sin \xi \sin \psi \hat{y}+\cos \xi \hat{z} \\
& \hat{\xi}=\cos \xi \cos \psi \dot{x}+\cos \xi \sin \psi \hat{y}-\sin \xi \hat{z} \\
& \hat{\psi}=-\sin \psi \dot{x}+\cos \psi \hat{y} \tag{10}
\end{align*}
$$

in (9). Taking the $\rho$-dependent term of (4) into account again, the following expression for $\bar{E}^{o}$ in Cartesian coordinates is then found:

$$
\begin{equation*}
\bar{E}^{a}=\frac{-1}{\rho}\left\{E_{Q}^{i}(\xi, \psi) \hat{x}+E_{p}^{i}(\xi, \psi) \hat{y}\right\} \tag{11}
\end{equation*}
$$

neglecting a constant phase term. Here $\rho$ is given by (6).
The aperture field is assumed (for physical reasons) to be a TEM wave and, hence, the radiation from the aperture is given in the coordinate system of Fig. 4 by [13].

$$
\begin{align*}
& E_{\theta}=\frac{-i k e^{i k r}}{2 \pi r}\left(f_{x} \cos \phi+f_{y} \sin \phi\right) \\
& E_{\phi}=\frac{-i k \cos \theta e^{i k r}}{2 \pi}\left(f_{y} \cos \phi-f_{x} \sin \phi\right) \tag{12}
\end{align*}
$$

where

$$
\begin{align*}
& \quad \bar{f}\left(k_{x}, k_{y}\right)=f_{x} \dot{x}_{1}+f_{y} \hat{y}_{1} \\
& \quad=\iint_{\text {aperture }} \bar{E}^{a^{a}}\left(x_{1}, y_{1}\right) e^{-i \bar{k} \cdot \bar{r}} d x_{1} d y_{1}  \tag{13}\\
& \bar{k}=k_{x} \dot{x}+k_{y} \hat{y} \\
& k_{y}=k \sin \theta \cos \phi \\
& k_{y}=k \sin \theta \sin \phi
\end{align*}
$$

and

$$
r=x_{1} \hat{x}_{1}+y_{1} \hat{y}_{1}
$$

In the aperture the coordinates $x_{1}$ and $y_{1}$ are related to the primary feed coordinates by

$$
\begin{align*}
x_{1} & =\bar{\rho} \cdot \hat{x}=\rho \hat{\rho} \cdot \hat{x} \\
& =\frac{2 f}{1+\cos \xi} \sin \xi \cos \psi  \tag{14a}\\
y_{1} & =-\bar{\rho} \cdot \hat{y}=-\rho \hat{\rho} \cdot \hat{y} \\
& =\frac{-2 f}{1+\cos \xi} \sin \xi \sin \psi \tag{14b}
\end{align*}
$$

Substituting the expression (11) for the aperture field in (13) gives

$$
\begin{align*}
\bar{f}\left(k_{x}, k_{y}\right)= & \iint_{\text {aperture }} \frac{1}{\rho}\left\{-E_{Q}^{i}(\xi, \psi) \hat{x}_{1}\right. \\
& \left.+E_{p_{1}}{ }^{i}(\xi, \psi) \dot{y}_{1}\right\} e^{-i \bar{k} \cdot \bar{r}} d x_{1} d y_{j} \tag{15a}
\end{align*}
$$

where the change of sign is caused by the change of coordinate system. By change of parameters (which is a quite lengthy but straightforward operation) this expression may be rewritten as

$$
\begin{equation*}
\bar{f}\left(k_{x}, k_{y}\right)=\iint r\left\{-E_{q}^{i}(\xi, \psi) \hat{x}_{1}+E_{p}^{i}(\xi, \psi) \hat{y}_{y}\right\} e^{-i \bar{k} \cdot \bar{r}} d \xi d \psi \tag{15b}
\end{equation*}
$$

where the integration is carried out over the range of ( $\xi, \underline{\psi}$ ) values subtended by the aperture. The expression of $f\left(k_{x}, k_{y}\right)$, in (15b) is now used in (12) in order to find the far-field. Using also the transformation between $(\hat{p}, \hat{q})$ and $(\hat{\theta}, \hat{\phi})$ equivalent to (3), the far-field in terms of co- and cross-polar components turns out to be

$$
\begin{align*}
E(\theta, \phi) & =E_{p}\left(k_{x}, k_{y}\right) \hat{p}+E_{Q}\left(k_{x}, k_{y}\right) \hat{q} \\
& =\frac{-i k e^{i k r}}{2 \pi r}\left(f_{y} \hat{p}+f_{x} \hat{q}\right), \tag{16}
\end{align*}
$$

which gives

$$
\begin{gather*}
E_{p}\left(k_{x}, k_{y}\right)=\frac{-i k e^{i k r}}{2 \pi r} \iint r E_{p} i(\xi, \psi) e^{-i \bar{k} \cdot \bar{r}} d \xi d \psi  \tag{17a}\\
E_{q}\left(k_{x}, k_{y}\right)=\frac{i k e^{i k r}}{2 \pi r} \iint r E_{q}(\xi, \psi) e^{-i \bar{k} \cdot \bar{r}} d \xi d \psi \tag{i7b}
\end{gather*}
$$

This is the "reflector transformation". which expresses the coand cross-polar components of the radiated field. It shows that when Ludwig's 3rd definition of polarization is used (in the present strict sense) then the co- and cross-polar components are transformed separately for symmetric as well as unsymmetric antennas. This is the mathematical proof of the more heuristic conclusion of the previous section.

For simplicity, the present analysis has been limited to parabolic antennas, but the conclusion will yet hold even for compound focused reflector systems. It may be shown [15] that the curnents which flow on any. reflector surface generated by a closed conic section (i.e., ellipsoid or hyper-
boloid) when it is illuminated by a Huygens source type of feed placed in one focal point will have the same direction at each point as the current produced by the same type of feed placed in the opposite focal point. Therefore, the secondary field from such a reflector will have the same polarization characteristics as the primary feed, and, thus when it is acting as a primary field for a new reflection, will lead to the same conclusion, although the actual transformation is not simple to carry out. Again, this conclusion is valid within the limits of the physical optics approximation. It is anticipated that the analysis can be extended to cover the case where the feed is moved off the focus in order to steer the beam off axis [2] or in order to improve the polarization characteristics of a Cassegranian antenna [16].

## IV. CROSS POLARIZATION IN FEED SYSTEMS FOR ASYMMETRIC REFLECTOR ANTENNAS

## A. Definitions

While the previous section covered focused reflector antennas irrespective of whether the configurations and feed patterns were symmetric or not, it is worthwhile to look at the feeds for off-set antennas in more detail, because for these it is less obvious how to deal with the definition of cross polarization than in the symmetric case. Consider, again, a linearly polarized primary source with the $q$-polarization as shown in Fis. l(b). If this source is tilted, around the $y$ axis, the polarization of course remains polarization clean assuming that the reference Huygens source is tilted together with the horn. However, such a tilted polarization reference makes less sense in relation to the reflector system because the condition for separate transformation of co- and cross-polarized fields is that the Huygens sources defining the $p$ - and $q$-vector fields are oriented orthogonal to the axis of the reflector system. Therefore, it is useful to find the field components of the tilted feed horn in che $p$-, $q$-system defined in Fig. 1. In Fig. 5, the electric field lines of the tilted horn are shown together with the $q$-vector field from Fig. 1(b), and it is observed that the field lines cross. This means that the tilted feed does have a cross polarized component in the reflector coordinate system although it was polarization clean in the coordinate system oriented towards its own axis.

A simple example will demonstrate the effect of a reorientation of a feed horn. Consider an ideaiized circular horn situated in a set of coordinate systems as shown in Fig. 5. The associated $(\xi, \psi, n)$ and ( $\xi^{\prime}, \psi^{\prime}, n^{\prime}$ ) coordinates are defined in the usual manner. Let the radiation pattern of the horn by given by

$$
\begin{aligned}
& E_{\xi^{\prime}}=\sin \psi^{\prime} \frac{J_{1}\left(k a \sin \xi^{\prime}\right)}{k a \sin \xi^{\prime}} \\
& E_{\psi^{\prime}}=\cos \psi^{\prime} \frac{J_{1}\left(k a \sin \xi^{\prime}\right)}{k a \sin \xi^{\prime}}
\end{aligned}
$$

where $a$ is the radius of the horn aperture, and $J_{1}$ is the Bessel function of first order and first kind. The co- and crosspolarized field components in the same coordinate system are then
$E_{p^{\prime}}=\frac{\dot{J}_{1}\left(k a \sin \xi^{\prime}\right)}{k a \sin \xi^{\prime}}$

$$
E_{Q},=0,
$$

where $p^{\prime}$ and $q^{\prime}$ refer to Huygens sources with the electric di-


Fig. 5. E-field lines for tilted Huygens source compared to $q$-field lines.


Fig. 6. Patterns for $p$ - and $q$-polarized fields radiated by ideal, circulat feed, which is tilted compared to reference coordinate system.
pole along the $y^{\prime}$ and the $x^{\prime}$ axis, respectively. The co- and cross-polarised field components, $E_{p}$ and $E_{Q}$, in the ( $x, y . z$ ) coordinates may then be found by carrying out the appropriate coordinate transformations. The mathematical details of this are straightforward, and the resulting $E_{p}$ and $E_{Q}$ patterns are shown in Fig. 6. With the parameters chosen (radius of the horn: $a=1.15 \lambda$, tiltangle: $\theta_{0}=45^{\circ}$ ) the co-polar $\left(E_{p}\right)$ radiation pattern is almost unchanged in the new coordinates, whereas, there is a significant cross-polar component ( $E_{Q}$ ); the pattern of this is antisymmetric around the $\phi=0$. axis and has a maximum value of approximately 23 dB relative to the beam peak of $E_{p}$.

If the tilted horn is used as feed in an off-set reflector system, the cross polarized field component found above transforms into a cross polarized component of the secondary field. Such a cross polarized far-field component in off-set reflector antennas has been discussed by several authors based on studies of specific examples [2]-[4], [6], and the conclusion has been drawn, logically, but not entirely correct, that a long focal length is advantageous because the tilt angle $\theta_{0}$ becomes smaller, and the cross polarization thus decreases. Some authors [6] have even extended the conclusion to state that the use of off-set antennas for purposes where a polarization discrimination of more than 30 dB is required is very questionable. In view of the separability of the collimation of coand cross-polar fields it is now clear that this conclusion is not correct:

Bearing in mind that the cross-polarized far-field is merely the near-field cross polarization transformed according to (17b), the author [17] suggested two other obvious ways to suppress it. One is a mode matching technique similar to that
used by Potter [18]. In the so-called Potter horn, the sidelobes and cross polarization radiated by the $\mathrm{TE}_{11}$ mode of a circular aperture are partially cancelled by adding in opposite phase a small portion of the $\mathrm{TM}_{11}$ mode which has (almost) identical sidelobe and cross-polar patterns. Obvjously this technique can also be used to cancel the cross polarized radiation pattern in Fig. 6 by adding a suitable mode. This method will be referred to as the mode-matching technique, and an example in which a rectangular horn is treated will be given in Section $V$.

The second method is based on the observation that a necessary and sufficient condition for a plane'aperture field to radiate a far-field with zero cross polarization is that the tangential $E$ - and $H$-field components in the aperture satisfy the plane wave impedance relation. This can be seen from the expressions for the far-field radiated by an aperture as given by Collin and Zucker [13]. The requirement that $E_{\xi}$ and $E_{\psi}$ are to satisfy the relation for zero cross polarization, $E_{\xi} \cos \psi=$ $E_{\psi} \sin \psi$, is easily seen to be equivalent to the requirement that $f_{y}=-\xi_{0} g_{x}$ where $\xi_{0}$ is the free space impedance for plane waves. This implies that the equivalent electric and magnetic currents should be related as in a plane wave; hence, the equivalent surface source radiating a far-field with zero cross polarization is a Huygens layer. An open ended circular waveguide operated near cutoff radiates a field with very low cross polarization; and a Huygens source layer may, therefore, be approximately realized as an array of open ended circular waveguides. This method will be called the Huygens source technique, and a very simple example with an asymmetric beam will be given also in Section V .

## B. The "Wedge Setup"

The equjvalence between the Huygens source definition and the standard antenna test setup discussed in Section II exists only under the assumption that the Huygens reference source is oriented orthogonal to the $\xi=0$ axis. This means that if one wished to use measured results for a primary field directly in the reflector transformation (17) in order to achieve the secondary field, then the primary field must be measured in a test-setup in which it is tilted the same amount relative to the $\xi=0$ axis as it will eventually be tilted relative to the axis of the reflector system in which it is to be used as the primary source. This is illustrated in Fig. 7 where a wedge is shown to provide the appropriate alignment.

## - $\because$ V. TWO EXAMPLES

## A. The Mode Matching Technique

To demonstrate this technique a rectangular horn with a 2.5 by $2.5 \lambda$ aperture has been chosen. The aperture is situated in the ( $x^{\prime}, y^{\prime}, z^{\prime}$ ) coordinate system shown in Fig. 5 with its center in the origin and its sides parallel to the $x^{\prime}$ and $y^{\prime}$ axis and radiates in the direction of the positive $z^{\prime}$ axis. The aperture is excited by the $\mathrm{TE}_{01}$-mode where the first index refers to $y^{\prime}$ axis and the second to the $x^{\prime}$ axis; so, the horn is essentially $p$-polarized. The radiated field is found from a model in which the electrical aperture field is assumed to be the $T E_{01}$ mode-field and the magnetic field is assumed to be zero. This model is similar to that used by Silver [9], and it is known to give a realistic prediction of the cross-polar as well as the copolar field [14] whereas a straight use of Silver's model leads to too optimistic values for the cross-polar field.

Because of the relatively large size of the aperture, a yery


Fig. 7. "Wedge setup" for measurement of feed systems for off-set reflector antennas.
low cross polarization (below 45 dB ) is found when the pattern is calculated in the ( $x^{\prime}, y^{\prime}, z^{\prime}$ ) coordinate system, but when the $p$ and $q$ components are evaluated a peak level for the $q$ component of about 25 dB is found, as in the example treated in Section IV, whereas the $p$ component is almost the same as the original co-polar component. The $p$ and $q$ components are shown in Figs. 8(a) and 8(b). The $p$ and $q$ component of the radiation from a $\mathrm{TE}_{20}$ mode excited in the same aperture are shown in Figs. 8(c) and (d). (The same theoretical model has been used.)

A striking similarity between the $q$-polar components of the $T E_{01}$ and the $T E_{20}$-modes is evident, which means that a proper combination of the two modes can lead to a (partial) cancellation of the $q$ component. This is shown in Figs. 8(e)and ( $f$ ). In the $\psi^{\prime}=90^{\circ}$ plane, the $q$ component is below -41 dB inside the -10 dB . points of the $p$ pattern, i.e., a $15-\mathrm{dB}$ improvement compared to the pattern of the $\mathrm{TE}_{01}$-mode has been achieved in this plane. The cancellation is less perfect but still good in other planes, and a ratio of about -40 dB can be anticipated for a reflector antenna illuminated by the present feed.

It is observed from Fig. 8 that the $q$ pattern of the $T E_{20^{-}}$ mode is considerably broader than the $q$ pattern of the $T E_{01}-$ mode. A better cancellation of the cross polarization could, therefore, be achieved, by equalizing the two patterns. This could be done, for instance, by adding a finned section to each side of the horn in the $E$-plane of the $T E_{01}$-mode with the fins in the $y^{\prime}$ direction. This would not effect the aperture distribution of the $T E_{01}$-mode but would give a broader aperture for the $T E_{20}$-mode and, therefore, serve to namow the beam and to match it to the $\mathrm{TE}_{01}$-mode.


Fig. 8. Radiation patterns for rectangular horn excited by (a) and (b) $\mathrm{TE}_{01}$ mode; (c) and (d) $\mathrm{TE}_{20}$ mode; and (e) and (f) combined $T E_{01}$ and $T E_{20}$ mode.

## B. The Huygens Source Technique

An open ended circular waveguide operated near cutoff is known to radiate a field with very low cross polarization thus approximating a Huygens source. A feed with two open ended circular waveguides has been discribed by Gruner and English [19]. The waveguides were fed in phase in order to produce an elliptical beam for illumination of an elliptical reflector. As the coupling between small circular apertures is very low [14], no reason is seen to abstain from phasing the two waveguides relative to each other in order to squint the beam so that it can illuminate an off-set, elliptical reflector.

As an example of this, the calculated $p$ and $a$ patterns for an array consisting of two open ended waveguides are shown in Figs. 9(a) and (b). The following parameters have been used: radius of the wayeguide $a=0.321 \lambda$, distance between the centres $d=0.70 \lambda$, and phase difference $d p=-2.2 \mathrm{rad}$. The beam maximum occurs at about $\xi=25^{\circ}, \psi=0^{\circ}$, and because the axis of the waveguides remains oriented towards the $z$ axis (i.e., $\xi=0^{\circ}$ ) the cross-polar component remains low. The element pattern has been obtained by using Chu's model as described by Silver [9] for the open ended waveguide. This model is known'from experiments to give a realistic prediction of the co-polar pattern. For the cross-polar pattern, experiments have shown that the predicted levels are far too pessimistic when the waveguides are operated near cutoff [14], and in the present work 6 dB has been subtracted from the crosspolar values predicted by the Chu model. This is a rather arbitrary value, but is is considered to be conservative.

The results show a cross-polar peak level better than -32 dB relative to the co-polar beam peak inside the -6 dB contour of the co-polar beam. Thus secondary patterns can be expected to give cross-polar peaks in the range of $35-40 \mathrm{~dB}$ below the co-polar peak. By adding more elements to the feed array a more refined beam shape could be modelled.

The two examples shown here do not pretend to be optimized feeds. Rather, they have been chosen because of their simplicity in order to demonstrate the priniciples outlined in the previous sections. Recently, an elegant feed design based on mode matching technique has been presented by Rudge


Fig. 9. Radiation patterns for Huygens source array of two elements.
[20] who used a Potter horn as basic configuration and added the $\mathrm{TE}_{21}$-mode to cancel the cross polarization.

## VI. CONCLUSIONS

It is shown that co- and cross-polar components of the radiation from a feed system are collimated separately by focused paraboloidal reflectors, even for offset and for elliptical beam antennas. However, the separability depends on a rigorous use of the Huygens source definition (Ludwig's 3rd definition) of cross polarization by which it is assumed that the Huygens source with which the feed system under consideration is compared shall be properly aligned with the axis of the reflector system in which the feed is supposed to be used. A test setup which conforms with this definition is shown.

The consequence of the above separability is that cross polarization of a reflector antenna system may be removed by
using a clean feed system. Two different methods for design of clean feed systems have been pointed out, one based on mode matching technique, one based on array technique. A theoretical example on each of these techniques have been studied and a considerable improvement of cross polarization performance has been observer. An experimental verification of the designs has been beyond the scope of the present investigation, but it is believed that the very simple principles behind the designs will prove useful in many practical antenna systems.

The proof of the separation of co- and cross-polar fields has only been carried out in detail for the parabolic reflector, but it has been pointed out that it holds also for compound, focused systems. The physical optics approximation has been used in its aperture field version. This is a well established technique, and the limitations its use impose on the results are mostly well known. However, it should be mentioned that edge effects, not treated by the theory, may have an unknown depolarization effect, in particular for nonsymmetric configurations.

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## REFERENCES

[1] W. V. T. Rusch and $O$. Sorensen, "Aperrure blocking of a focused paraboloid," Techn. Univ. of Denmark, Electromagnetics Inscitute, R. 126, July 1974.
[2] A. W. Rudge, "Multiple-beam antennas: Offset reflectors with offset feeds," JEEE Trans. Antennas Propagat., vol. AP-23, pp. 317-322, May 1975.
il) T. S. Chu and R. H. Turrin, "Depolarisation properties of offset teflector antennas," IEEE Trans. Antennas Propagat., vol. AP-21, pp. 339-345, May 1973.
[1] M. J. Pagorines, "Gain factor of an offset-fed paraboloidal reflector," IEEE Trans. Antennas Propagat., vol. AP-16, pp. 536-541, Sept. 1968.
[2] A. W. Rudge, "Offset reflector antennas with offset feeds," Electron. Lett., vol. 9, pp. 611-613, Dec. 27, 1973.
[3] M. J. Gans and R. A. Semplak, "Some far field studies of an offset launcher," Bell Syst. Tech. J., vol. 54, pp. 1319-1340, Sept. 1975.
[4] H. Tanaka and M. Mizusawa, "Elimination of cross polarization in offset dual reflector antennas," Electron and Commun. in Japan, vol. 58-B, pp. 71-78, 1975.
[5] J. F. Kauffman and W. F. Croswell, "Off focus characteristics of the offset fed parabola," in IEEE AP.S Symp. Dig., June 1975, pp. 358-361.
[6] N. A. Adatia and A. W. Rudge, "Beam squint in cir-
[4] M. J. Gans and R. A. Semplak, "Some far-fjeld studies of an offset launcher," Bell Sysiem Tech. Jour., vol. 54, pp. 13191340, Sept. 1975.
[5] P. J. Wood, "Depolarisation with cassegrain and front-fed reflectors," IEE Electron. Lett., vol. 9, pp. 181-183, May 1973.
[6] J. Dijk, C. T. W. v. Diepenbeek, E. J. Maanders, and L. F. G. Thurlings, "The polarization losses of offset paraboloid antennas," IEEE Trans. Antennas Propagat., vol. AP-22, pp. 513-520, July 1974.
[7] "IEEE Standard Definitions of Terms for Antennas," IEEE Trans Antennas Propagat., vol. AP-17, pp. 262-269, May 1969.
18] A. C. Ludwig, "The definition of cross polarisation," IEEE Trans. Antennas Propagat., vol. AP-21, pp. 116-119, Jan. 1973.
[9] S. Silver, Microwave Antenna Theory and Design. New York: McGraw-Hill, 1949.
[10] J. S. Hollis, T. J. Lyon, and L. Clayton, "Microwave antenna measurements," Scientific-Atlanta, Inc. Atlanta, Georgia, USA, July 1970.
[11] E. M. T. Jones, "Paraboloid reflector and hyperboloid lens antennas," IRE Trans. Antennas Propagat., vol. AP-2, pp. 119127, July 1954.
[12] I. Koffman, "Feed polarization for parallel currents in reflectors generated by conic sections," JEEE Trans. Antennas Propagat., vol. AP-14, pp. 37-40, Jan. 1966.
$[13]$ R. E. Collin and F. J. Zucker, Antenna Theory. New York: McGraw-Hill, 1969.
[14] A. W. Rudge, P. R. Foster, N. Williams, and N. Adatia, "Stung of the performance and limitations of multiple-beam antennas, final report on ESTEC contract no.: 2277/74 HP,"ERA, RF Technology Centre, England, Sept. 1975.
[15] G. T. Poulton, private communication.
[16] R. Graham, "The polarization characterisitcs of off-set cassegrain aerials," International Conference on Radar and Future, IEE, London, Oct. 23-25, 1973.
[17] J. Jacobsen, "Study of limitations of RF sensing systems due to distortions of large spacecraft antennas," Technical Specifications to ESTEC Contract No.: 2330/74 AK, Aug. 1974.
[18] P. D. Potter, "A new horn antenna with suppressed sidelobes and equal beamwidths," Microwave J., vol. 6, pp. 71-78, June 1963.
$[19]$ R. W. Gruner and W. J. English, "Antenna design studies for a U.S. domestic satellite," COMSAT Techn. Rev., vol. 4, pp. 413447, Fall 1974.
[20] A. W. Rudge and N. A. Adatia, "A new primary feed for offset reflector antennas," IEE Electron. Lett., vol. 11, pp. $597-$ 599, Nov. 1975.

## Bibliography for Part VI

cularly polarized offset reflector antennas," Electron. Lett., vol. 11, pp. 513-515, Oct. 16, 1975.
[7] H. P. Coleman, R. M. Brown, and B. D. Wright, "Paraboloidal reflector offset fed with a corrugated conical horn," IEEE Trans. Antennas Propagat., vol. AP-23, pp. 817-819, Nov. 1975.
[8] A. G. P. Boswell and R. W. Ashton, "Beam squint in a linearly polarised offset reflector antenna,". Electron. Lett., vol. 12, pp. 596-597, Oct. 28, 1976.
[9] A. R. Valentino and P. P. Toulios, "Fields in the focal region of offset parabolic antennas," IEEE Trans. Antennas Propagat., vol. AP-24, pp. 859-865, Nov. 1976.
[10] T. S. Chu, "Cancellation of polarization rotation in an offset paraboloid by a polarization grid," Bell Syst Tech. J., vol. 56، pp. 977-986, July-Aug. 1977.

# Lateral Feed Displacement, Scanning, and Multiple Beam Formation 

When the feed in a reflector antenna is moved away from the focus in a direction transverse to the axis, the beam is displaced in the opposite direction and is said to be squinted, or scanned. Because such a displacement of the feed produces higher odd order, as well as linear phase terms in the aperture, the angle through which the beam is squinted is less than the angle (mea. sured at the paraboloid vertex) through which the feed is displaced. The ratio of the beam angle to the feed angle is called the beam deviation factor, abbreviated BDF. The calculation of this factor is carried out in the first paper of this part, by Lo, in which analytical and experimental results are given as a function of $f / D$ ratio.
Accompanying the beam squint due to feed displacement are beam broadening, loss in gain, and the incidence of coma lobes. These phenomena are analyzed by Ruze, in the second paper, and the range of validity is given for the approximations that he finds necessary to use. Figs. 3-8 in this paper are useful graphical presentations of the analytical results that enable the designer to estimate the magnitudes of the various effects as a function of feed displacement or of the number of beamwidths scanned. In general, scanning is limited to a very few beamwidths before the beam degradation becomes intolerable. In the thiro paper, Rudge and Withers present a method which has experimentally been shown to permit scanning through $\pm 15$ beamwidths with little pattern degradation and minimal gain loss. Their technique is based on the use of a number of feeds arrayed in the plane of scan and arranged to move on an appropriate locus in that plane. Basically, the feed array carries out a spatial Fourier transform of the distorted focal region fields which result when an off-axis plane wave is incident on the reflector.

The correct locus on which a feed should be moved and the orientation of the feed for optimum scanning of the beam are the subjects of investigation in paper four, by Rusch and Ludwig. The analysis is based on physical optics and results are given in relation to what is called the Petzval surface in optics. One interesting result that is not evident a priori is that a higher scan gain is obtained when the feed axis remains parallel to the reflector axis rather than being pointed to the vertex. This is true unless the $f / D$ ratio is very large, and it holds up until the point where spillover loss begins to dominate.

Imbriale, Ingerson, and Wong have used a vector formulation that is more accurate than the scalar approximation used by Ruze to investigate the effects of large lateral displacement of the feed in a paraboloid. Their results are given in the fifth
paper, and quite reasonable agreement with experimental results is shown for displacements of up to 16 wavelengths, corresponding to a beam scan of about 29 half power beamwidths. At this point, the pattern degradation is severe and the loss in gain amounts to about 14 dB . One significant conclusion is that the approximate scalar analysis does succeed in predicting the scan angle quite accurately, even for the largest displacements.
Although all of the above papers, and at least two in Part V1 (those by Ingerson and Wong, and by Rudge), have great relevance for multiple beam formation in paraboloidal reflectors, they are incomplete in the sense that they are concerned only with the effects of a single feed when it is displaced from the axis. The formation of multiple, simultaneous beams requires the use of multiple, contiguous feeds in the focal plane, only one of which can be on axis and at the geometric focus. Thus, a new set of problems is introduced and new questions are raised. Is it physically. possible to stack feed horns side by side to give adjacent beams with reasonable crossover levels; how severe will be the cross talk due to the sidelobes in adjacent beams; what is the nature of the matrix that is used to access the beams? Some of these questions are very briefly considered in the provocative short paper by Shelton. Many, however, remain unanswered at the present time, although there is a great deal of effort being expended in these areas because of the undoubted future importance of multiple beam antennas in satellite communications systems.

The seventh and final paper, by Ohm, represents a good, practical approach toward solving some of the problems peculiar to multiple beam forming antenna design. An offset paraboloid is suggested for the practical reason that aperture blocking by the feed cluster can thereby be avoided. These notions have been successfully put to practical test in an offset Cassegrainian system at 100 GHz . The results are reported in a paper by Semplak which appeared after this volume was in preparation [1].

Provided that spherical aberration can be either corrected or minimized, then spherical reflectors have great potential for the formation of multiple beams in space. These kinds of reflectors comprise the subject matter for Part IX in this volume and will not be further mentioned here. If multiple beams are required to be generated only in one plane, then toroidal reflectors have advantages, but again, the spherical aberration in that plane must be reckoned with. This approach has been adopted for an unattended earth terminal antenna and is described in detail in a paper by Hyde, Kreutel, and Smith [2] .

# On the Beam Deviation Factor of a Parabolic Reflector* 

Y. T. LO $\dagger$

IN a parabolic reflector antenna, the scanning can be achieved for a range by displacing the feed in the focal plane without resorting to a costly steerable mechanism for the whole antenna system. It is wellknown that the range of scan by this method is limited by the increasing coma and astigmatism. However, it is still widely used in some specific applications. It is not the intention of this paper to investigate the aberrations of such a system, which may be referred to elsewhere, but rather to derive a formula for another characteristic of interest, namely the beam deviation factor. Such a formula does not seem to appear in literature, although there are available some experimental results for specific cases.
The beam deviation factor has been defined as the ratio of the beam deflection angle $\Theta_{b}$ to the angular displacement of the feed $\Theta_{f}$, both measured from the axis of the reflector with the vertex as origin. Let the feed be at $F^{\prime}$ at a distance $d$ from the focus $F$ and let $d$ be much smaller than the focal length $f$; then with a certain plausible approximation, the field at $P(R, \Theta \Phi)$, in Fig. 1 is given by

$$
\begin{align*}
& I=K \iint_{A} \frac{f(\phi, \theta)}{r} \cos \left[\beta_{F} \sin \Theta \sin \dot{\Phi} \sin \phi\right] \\
& \quad \times \cos [\beta \rho(\sin \Theta \cos \Phi-d / r) \cos \phi] \rho d \phi d \rho, \tag{1}
\end{align*}
$$

where $(\rho, \phi, z)$ or ( $r, \theta, \phi$ ) are the coordinates of a typical point $q$ of integration.
$K$ is a proportional constant, including the inverse $R$ factor and the phase delay function due to $r$ and $R$; $A$ is the aperture of the refiector, i.e., for $\rho=0$ to $D / 2$ ( $D=$ diameter), and $\phi=0$ to $2 \pi$,

$$
r=f \sec ^{2} \theta / 2, \text { and }
$$

$f(\phi, \theta)$ is the primary pattern of the feed.
If the plane containing $F^{\prime}$ is defined as one for which $\Phi=0$ and $\pi$, the maximum of $I$ must appear in the same plane, since in practice $f(\theta, \phi)$ is real and positive for $q$ in $A$. Thus if the maximum of $I, I_{m}$, occurs at $\Theta_{m}$,
$I_{m}=K \iint_{\Delta} \frac{f(\phi, \theta)}{r} \cos \left[\beta \rho\left(\sin \Theta_{m}-d / r\right) \cos \phi\right] \rho d \phi d \rho$,

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$\dagger$ University of Illinois, Urbana, Illinois.


Fig. 1-Geometry of the refiector.
and $\Theta_{m}$ satisfies the following equation:

$$
\begin{align*}
\iint_{A} \frac{f(\phi, \theta)}{r} \sin \left[\beta \rho\left(\sin \Theta_{m}-d / \tau\right)\right. & \cos \phi] \beta \rho^{2} \\
& \cdot \cos \phi d \phi d \rho=0 \tag{3}
\end{align*}
$$

Since for $(\rho, \phi)$ in $A, f(\phi, \theta)>0$, and

$$
\sin \left[\beta \rho\left(\sin \Theta_{m}-d / \tau\right) \cos \phi\right] \cos \phi
$$

is a continuous function of $p$ and $\phi$. This equation will be satisfied

$$
\sin \left[\beta_{\rho}\left(\sin \Theta_{m}-d / \tau\right) \cos \phi\right] \equiv 0
$$

for some ( $\rho, \phi$ ) in $A$, by invoking mean value theorem for integrals. As $d=0$, the first condition can be met with $\Theta_{m}=0$, as expected. Were $r$ a constant the condition could still be satisfied with $\Theta_{m}=\sin ^{-1} d / r$. Now $r$ is a variable with a range from $f$ to $f+Z_{0}$ where $Z_{0}=$ the depth of the dish. Therefore, it will be expected that in general there are many solutions of $\Theta_{m}$ to satisfy this second condition. For the maximum of the main beam, as shown in Fig. 2, the following inequality must be satisfied:

$$
\left(1+Z_{0} / f\right)^{-1} d / f<\sin \Theta_{b}<d / f
$$

It is easily seen that for a shallow dish $\Theta_{b}$ can be determined accurately without going to the complicated


Fig. 2-The range of $\sin \Theta_{b}$ which is a portion of the hyperbola $d / r$.
integration in (3). Now we can write

$$
\begin{align*}
\sin \theta_{b} & =\frac{d}{f}\left(\frac{1}{1+Z_{0} / f}+k \frac{Z_{0} / f}{1+Z_{0} / f}\right) \\
& =\frac{d}{f} \frac{1+k Z_{0} / f}{1+Z_{0} / f}=\frac{d}{f} \frac{1+k(D / 4 f)^{2}}{1+(D / 4 f)^{2}}, \tag{4}
\end{align*}
$$

where $k<1$. Or

$$
\begin{equation*}
\frac{\sin \Theta_{b}}{\tan \Theta_{j}}=\frac{1+k(D / 4 f)^{2}}{1+(D / 4 f)^{2}} . \tag{5}
\end{equation*}
$$

If the usual definition of the beam deviation factor ( BDF ) is used,
$\begin{aligned} \mathrm{BDF} & =\frac{\Theta_{\mathrm{b}}}{\tan ^{-1} d / f}=\frac{\sin ^{-1}\left[\frac{d}{f} \frac{1+k(D / 4 f)^{2}}{1+(D / 4 f)^{2}}\right]}{\tan ^{-1} d / f} \\ & \approx(\mathrm{BDF})_{0}\left\{1+\frac{1}{3}(d / f)^{2}\left[\frac{1}{2}(\mathrm{BDF})_{0^{2}}+1\right]+O(d / f)^{4}\right\},\end{aligned}$ where

$$
\begin{equation*}
(\mathrm{BDF})_{0}=\frac{1+k(D / 4 f)^{2}}{1+(D / 4 f)^{2}} \tag{6}
\end{equation*}
$$

It is seen that by the usual definition the BDF , up to the approximations assumed so far, depends also upon (d/f). ${ }^{1}$

Although $k$ is a function of $f, D, f(\theta, \phi)$, its value is not critical, especially for large $f / D$ as seen from (5) and (6). From one experimental point (or computation) it should be possible to predict the rest as a function of $f / D$ with good accuracy. In Fig. 3, there shows the


Fig. 3-Beam deviation factor as a function of f/D. Experimental data: Silver and Pao: © Keller and Coleman; $\times 20-$ do taper, $\Delta 10-\mathrm{db}$ taper; computed curve: —.
computed curves and the experimental results by Silver and $\mathrm{Pao},{ }^{2,8}$ and by Kelleher and Coleman. ${ }^{1}$ For the former, there is no information available for $f(\theta, \phi)$ function, and $k=0.59$ is established by its point where $f / D=0.5$. For the latter, one case has a $10-\mathrm{db}$ tapering, the other 20 db with $k=\frac{1}{5}$ and 0.48 respectively. The extraction of the experimental data from Kelleher and Coleman's report is rather difficult because they measured for various values of $\Theta_{b}$, those indicated here being for $\Theta_{b}=2 \times$ beamwidth. It seems that one can conclude that the values of $k$ are from about 0.3 to 0.7 , and that $k$ becomes larger with higher tapering, as expected.

A second method to determine $\Theta_{b}$ is by assuming that the system is circularly symmetrical and $f(\theta, \phi)=\cos ^{n} \cdot \theta$.
${ }^{1}$ K. S. Kelleher and H. P. Coleman, "Off-Axis Characteristics of the Paraboloidal Reflector, ${ }^{n}$ NRL Rept. No. 4088, Washington, D. C.; December, 1952.
${ }^{2}$ S. Silver and C. S. Pao, "Paraboloid Antenna Characteristics as a Function of Feed Tilt," Rad. Lab. Rept. No. 479, Cambridge, Mass.; 1944.
${ }^{3}$ S.'. Silver, "Microwave Antenna Theory and Design," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 12, p. 487; 1949.

In this case (3) becomes

$$
\begin{array}{r}
\int_{0}^{D / 2} \frac{\cos ^{n} \theta}{1+(\rho / 2 f)^{2}} J_{1}\left[\beta \rho\left(\sin \Theta_{b}-\frac{d / f}{1+(\rho / 2 f)^{2}}\right)\right] \\
\cdot \rho^{2} d \rho=0 \tag{7}
\end{array}
$$

Since

$$
\begin{aligned}
\rho^{\prime} & \equiv \rho / 2 f=\tan \theta / 2, \\
\cos ^{n} \theta & =\left(1-\rho^{\prime 2}\right)^{n}\left(1+\rho^{\prime 2}\right)^{-n}
\end{aligned}
$$

It does not seem that (7) can be evaluated in a closed form. However, by using the series-expansion for the Bessel Function it can be integrated term by term since the integrand involves algebraic functions only. Suppose that $D$ is so small that only the first term is sig-
nificant; then

$$
\begin{align*}
\Theta_{b} & =\sin ^{-1} \frac{d}{f}\left[1-\frac{2}{3} \frac{Z_{0}}{f}+k^{\prime}\left(\frac{Z_{0}}{f}\right)^{2}\right] \\
& =\sin ^{-1} \frac{d}{f}\left[1-\frac{2}{3}\left(\frac{D}{4 f}\right)^{2}+k^{\prime}\left(\frac{D}{4 f}\right)^{4}\right], \tag{8}
\end{align*}
$$

where $k^{\prime}=1 / 2,13 / 18,15 / 18$ for $n=0,2,3$, respectively. For small $d / f$, the first term corresponds to the reflection by a flat sheet, and the remaining terms appear as a result of the curvature of the reflector. If the result in (4) is expanded in $Z_{0} / f$ and compared with (8), it will be found that $k=\frac{1}{3}$, a value we obtained before. It may also be mentioned that the same method can be carried out if the illumination function $f(\theta)$ be any polynomial in $\rho$, such as $\left(1-\rho^{2}\right)^{n}$, type.

# Lateral-Feed Displacement in a Paraboloid. 

JOHN RUZE, fellow, IeEE

Aostract-The beam shift and degradation of a paraboloidal reflector with an offset feed is analyzed by the scalar plane prave theory. Higher order coma terms are included with the feed at its opimum axial position. The beam characteristics for a tapered circularly symmetric illumination are presented. The range of validity of the approximate analysis is indicated.

## I. Introduction

THE BEAM degradation due to a lateral feed displacement in a paraboloidal mirror is part of the classical study of aberrations of optical systems. These have been extensively investigated by Nijboer and Nienhuis [1], Kingslake [2], and others. The text by: Born and Wolf [3] provides an excellent presentation of the geometric and wave theory of optical aberrations.

However, these optical papers are not immediately applicable to antenna technology. The reasons for this may be summarized as:

1) In many optical instruments, we are primarily interested in the spatial distortion (ray aberration) of an image point from its desired or Gaussian focus;

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in an antenna, however, angular beam distortions, and factors such as gain, beam width, and side lobe level are of primary interest.
2) Optical systems usually have a higher " $f$ " number ( $f / D$ ratio) than antenna reflectors, where values as low as 0.25 are not uncommon. This lower ratio increases the importance of the higher order aberrations.
3) The optical analysis is generally concerned with uniform illumination of the exit pupil, whereas antenna apertures are invariably tapered or apodized.
4) Optical systems further are many orders of magnitude greater in aperture to wavelength ratio. They follow much more closely geometric optics' behavior than antenna systems, where diffraction theory is mandatory.
5) Some optical systems are limited by other factors than the phase aberrations, so that much larger wavelength distortions may be tolerated.
6) The expansion of the aberration function, in terms of Zernike polynomials [1], is useful when dealing with a single aberration, but becomes unmanageable when higher order effects and apodization are included. With modern computing machines, it is preferable to deal with the original diffraction integral than with its general evaluation in series form [4].

I'nfortunately, the antenna literature is not very extensive. We have the original experimental work of Silver and Paio [5]; the theoretical derivation, limited to primary coma, and experimental work of Kelleher and Coleman [6]; a paper on the beam deviation factor by Lo [7]; and a more extensive analysis by Sandler [s].
It is the purpose of this paper to present the properties of the offset fed paraboloid in the form of graphs of the significant characteristics. These are derived by means of the scalar plane wave theory; and series expansion of the phase aberration function. The results have the same small-angle limitation as the on-axis patterns normally used. In addition, the range of validity of the expansion is examined.

## II. ANimisis

Let us consider a paraboloid of diameter $D=2 a$, with a spherical coordinate system centered at the focus (Fig. 1). The field at a far-field point with a foral feed is

$$
\begin{equation*}
E(\theta, \phi)=\int_{0}^{2 x} \int_{0}^{c} f\left(r, \phi^{\prime}\right) e^{j k\left(r-\bar{p} \cdot \bar{R}_{0}\right)} r d r d \phi^{\prime}, \tag{1}
\end{equation*}
$$

where $f\left(r, \phi^{\prime}\right)$ is an effective aperture distribution, and where we have suppressed constant factors. The bars represent vector quantities, and the subscript " 0 " their unit values.
With feed displacement to the lateral point " $\epsilon$ ", we have

$$
\begin{equation*}
I:(\theta, \phi)=\int_{0}^{2 x} \int_{0}^{0} f\left(r, \phi^{\prime}\right) e^{j\left(n^{\prime}-\overline{n^{\prime}} \cdot \bar{R} \vec{R}^{\prime}\right)} r d r d \phi^{\prime}, \tag{2}
\end{equation*}
$$

where we have assumed that the magnitude of the effective aperture distribution has remained unchanged.'

We have the geometric relations:

$$
\begin{align*}
p & =\frac{2 f}{1-\cos \theta^{\prime}}  \tag{3a}\\
\bar{p} & =p\left[\cos \phi^{\prime} \sin \theta^{\prime} \bar{x}+\sin \phi^{\prime} \sin \theta^{\prime} \bar{y}+\cos \theta^{\prime} \bar{z}\right]  \tag{3b}\\
\bar{R}_{0} & =\left[\cos \phi \sin \theta_{\bar{x}}+\sin \phi \sin \theta \bar{y}+\cos \theta \overline{\bar{z}}\right] \tag{3c}
\end{align*}
$$

1 This implies that the vertex ray is the principal ras, and that the optical stop is at the mirror (11]. The feed should, therefore, be pointed at the vertex.

$$
\begin{align*}
p^{\prime} & =\bar{\gamma}+\epsilon_{x} \bar{x}+\epsilon_{z} \bar{z}  \tag{3d}\\
p^{\prime} & =p\left\{1+\frac{2 \epsilon_{z}}{p} \cos \phi^{\prime} \sin \theta^{\prime}+\frac{2 \epsilon_{z}}{p} \cos \theta^{\prime}+\frac{\epsilon^{2}}{p^{2}}\right\}^{1 / 2} \tag{3e}
\end{align*}
$$

For feed displacements small compared to the focal length (small-field angle),

$$
\begin{equation*}
\frac{\epsilon_{x}}{p}<\frac{\epsilon_{x}}{f} \ll 1 \tag{4}
\end{equation*}
$$

we can write the phase factor (2), neglecting terms higher than the square of this parameter, as

$$
\begin{align*}
p^{\prime}-\bar{p}^{\prime} \cdot \widetilde{R}_{0}= & 2 f-\epsilon_{\mathrm{I}} \cos \phi \sin \theta-\epsilon_{\mathrm{z}} \cos \theta \\
& -p \sin \theta^{\prime} \sin \theta \cos \left(\phi^{\prime}-\phi\right)+\epsilon_{\mathrm{z}} \cos \phi^{\prime} \sin \theta^{\prime} \\
& +\epsilon_{z} \cos \theta^{\prime}+\frac{\epsilon_{z}^{2}}{2 p}+p \cos \theta^{\prime}(1-\cos \theta) \\
& -\frac{\epsilon_{z}^{2}}{2 p} \cos ^{2} \phi^{\prime} \sin ^{2} \theta^{\prime} . \tag{5}
\end{align*}
$$

The first three terms are independent of the integration coordinates and may be taken out of the integral; they represent a phase pattern of the far field. Recalling that $t=p \sin \cdot \theta^{\prime}$, the next term is the normal phase factor due to an in-phase aperture. The fifth term represents the beam shift and comatic aberrations. The next three terms are field curvature (terms proportional to $r^{2}$ ), and higher terms of even power. The last term is the astigmatism.

The field curvature may be eliminated by axially refocusing the feed. This condition gives the Petzval surface in optics [9] which is defined as that locus which contains a sharp image when the other aberrations are absent. It may be obtained from (5), by setting all the field curvature terms to zero (including those hidden in the last two terms), as

$$
\begin{equation*}
\epsilon_{z}=\frac{\epsilon_{z}^{2}}{2 f} . \tag{6}
\end{equation*}
$$

For small aberrations, (6) defines the feed locus for sharpest nulls, as another paraboloid of focal length " $f / 2$ " tangent to the focal plane. The vertex feed distance must, therefore, be slightly increased as we scan off axis.


Fig. 1. Coordinate system.

For this optimum feed position, the magnitude of the far field can be written, from (2), as

$$
\begin{align*}
|E(\theta, \phi)|= & \int_{0}^{2 \pi} \int_{0}^{a} f\left(r, \phi^{\prime}\right) \\
& \cdot e^{\left.-j k \mid r \sin \theta \cos \left(\phi^{\prime}-\phi\right)-t \pi \operatorname{tin} \theta^{\prime} \cos \phi^{\prime}\right] r d r d \phi^{\prime}}, \tag{2a}
\end{align*}
$$

where, for the present, we have neglected the astigmatism.

The second term of the exponential causes the beam shift and the beam degradation as it represents the phase departure from the nonscanned in-phase aperture.

Since

$$
\begin{align*}
\sin \theta^{\prime} & =\frac{r / f}{1+(r / 2 f)^{2}} \\
& =\frac{r}{f}\left[1-\left(\frac{r}{2 f}\right)^{2}+\left(\frac{r}{2 f}\right)^{4}-\cdots\right] \tag{7a}
\end{align*}
$$

this phase departure may be written

$$
\begin{equation*}
\dot{\delta}=\frac{2 \pi}{\lambda} u_{2} r \cos \phi^{\prime}\left[1-\left(\frac{r}{2 f}\right)^{2}+\left(\frac{\dot{r}}{2 f}\right)^{4}-\cdots\right], \tag{7b}
\end{equation*}
$$

where

$$
u_{s}=\frac{\epsilon_{x}}{f}=\tan \theta_{s}
$$

is a measure of the feed squint.
The first term of (7b) is a phase shift linear with " $x$ " across the aperture and causes an undistorted beam shift, equal to the feed squint. The second term, proportional to " $u, r^{3} \cos \phi^{\prime \prime}$, is what is known as primary coma, and creates beam degradation and a beam shift in the opposite direction. The remaining terms are higher order coma terms normally neglected in high " $f / D$ " systems but retained here.

It is of interest to examine the ratio of the total coma aberration to the neglected astignatism. We have for the ratio of the relative edge errors

$$
\begin{equation*}
\frac{\text { ASTIGMATISM }}{\text { Total Coma }}=\frac{2 u_{s}(f / D)}{\left[1+(D / 4 f)^{2}\right]^{2}} . \tag{8}
\end{equation*}
$$

We see that for normal parabolic antennas, the astigmatism is a small quantity. However, for astronomical mirrors and Cassegrainian systems of high magnification, the astigmatism becomes the limiting factor on the field of view. Dimitroff and Baker [10] give the comparative image errors for an $f / 3$ and $f / 10$ parabolic telescope. When the astigmatism becomes significant, the Petzval surface loses its utility and the feed focus will be poorly defined.

With the notation

$$
u=\sin \theta ; \quad M(r)=1+(r / 2 f)^{2},
$$

the exponential in the integral of (2a) may be written

$$
\begin{equation*}
\ddot{k r}\left[u \cos \left(\phi^{\prime}-\phi\right)-\frac{u_{1} \cos \phi^{\prime}}{M(r)}\right]=A k r \cos \left(\phi^{\prime}-\alpha\right), \tag{9}
\end{equation*}
$$

where " $A$ " and " $\alpha$ " may be determined as

$$
\begin{align*}
A^{2} & =u^{2}-\frac{2 u u_{i}}{M(r)} \cos \phi+\frac{u_{s}^{2}}{M^{2}(r)}  \tag{9a}\\
\tan \alpha & =\frac{u \sin \phi}{u \cos \phi-u_{s} / M(r)} \tag{9b}
\end{align*}
$$

The magnitude of the far field is:

$$
\begin{equation*}
|E(\theta, \phi)|=\int_{0}^{2 \tau} \int_{0}^{a} f\left(r, \phi^{\prime}\right) e^{j \pi r \Delta \cos \left(\phi^{\prime}-a\right)} r d r d \phi^{\prime} . \tag{2b}
\end{equation*}
$$

For circularly symmetric illumination functions, the " $\phi^{\prime \prime}$ integration can be performed with the result

$$
\begin{equation*}
E(\theta, \phi)=2 \pi \int_{0}^{a} f(r) J_{0}(k r A) r d r . \tag{2c}
\end{equation*}
$$

By means of computing machines, this integral can be evaluated for a specified illumination. We note that the pattern is symmetric about the plane of scan ( $\phi=0$ ); for zero feed squint ( $A=u$ ), it reduces to the normal circular diffraction pattern; and with off-axis feeds, it contains all the comatic aberrations (first order in $\left.\epsilon_{=} / f\right)$.

Let us now consider the pattern in the plane of scan ( $\phi=0$ ) where the major pattern distortions occur, and where

$$
\begin{equation*}
A=u-\frac{u_{s}}{M(r)} ; \quad \alpha=0 \tag{9c}
\end{equation*}
$$

For small feed displacements, the position of the beam maximum, " $u_{m}$," may be found as that value of " $u$ " that minimizes the illumination-weighted squared phase error, or

$$
\frac{\partial}{\partial u} \int_{0}^{2 \pi} \int_{0}^{0} f(r)\left[k r A \cos \phi^{\prime}\right]^{2} r d r d \phi^{\prime}=0 .
$$

Performing this operation, we have for the beam deviation factor.

$$
\begin{equation*}
B D F=\frac{u_{m}}{u_{z}}=\frac{\int_{0}^{a} \frac{f(r) r^{3}}{M(r)} d r}{\int_{0}^{a} f(r) r^{3} d r} \tag{10}
\end{equation*}
$$

This may be evaluated in closed form for various illumination functions, as has been done by Lo [7]. The field reduction, at the beam peak $\left(u_{m}\right)$, can also be obtained from (2c).

## III. Computed Results

For computational purposes, (2c) was put in a normalized form and evaluated on an IBM 7090.
$E(w, 0)$

$$
\begin{equation*}
=2(p+1) \int_{0}^{1}\left(1-r^{2}\right)^{n} J_{0}\left[\left(w-\frac{w_{z}}{M(r)}\right) r\right] r d r, \tag{2d}
\end{equation*}
$$

where

$$
w_{z}=\frac{2 \pi a}{\lambda} \tan \theta_{s} ; \quad w=\frac{2 \pi a}{\lambda} \sin \theta .
$$

Computations were made for the illumination function $\left(1-r^{2}\right)^{p}$, and for the 10 dB tapered illumination $f(r)=0.3+0.7\left(1-r^{2}\right)$, with $f / D=0.25,0.33,0.4,1.0$, and 2.0 .

Figure 2 shows typical scan plane patterns. We note that the gain drops with scan, the beam broadens, the beam scan is less than the feed squint, the sidelobe on the axis side (coma lobe) increases; whereas, the first sidelobe on the other side decreases, changes sign, and merges with the main beam and second sidelobe causing additional beam broadening. Complete patterns or image plane isophots may be found in Born and Wolf [3]. Figs. 9.6 to 9.8. These are for primary coma only, and represent parabolic reflectors of large " $f / D$ " ratio, with no astigmatism. They are expressed in maximumedge coma error which, in our notation, is

$$
\begin{equation*}
\delta_{c}(a)=\frac{w_{t}}{M(a)}\left(\frac{D}{4 f}\right)^{2} \tag{12}
\end{equation*}
$$

The problem remained of expressing the large amount of computed data in terms of useful curves. It was determined that if the pattern characteristics were plotted against the quantity

$$
\begin{equation*}
X=\frac{\frac{w_{m}}{2 w_{0}}(D / f)^{2}}{1+0.02(D / f)^{2}}, \tag{13}
\end{equation*}
$$

then over the region of interest the data was essentially independent of the $f / D$ ratio. The factor " $w_{m} / 2 w_{0}{ }^{n}$ " is the number of half-power beamwidths scanned, a convenient variable. Figures 3 to 6 show these results.

The curves permit expressing the scanning limit in terms of a simple formula. If we choose 1 dB loss of gain (the Rayleigh limit) as our criteria, then for the 10 dB taper, $X=22$, and the number of beamwidths scanned is

$$
\begin{equation*}
\frac{w_{m}}{2 w_{0}}=0.44+22(f / D)^{2} \tag{14}
\end{equation*}
$$

This criteria may not be adequate for many applications as the coma lobe has increased to -10.5 dB at the scan limit.

Figure 7 shows the beam deviation factor. To a first approximation (10), the beam deviation factor is independent of the number of beamwidths scanned. The machine computation shows its dependence on $X$. We have indicated in the figure the small increase, by means of vertical bars, as our parameter $X$ approaches 50 .

The question naturally arises as to the range of validity of the approximation made in going from the phase function of (2) to the truncated series expansion of (2b). As the two expressions are known, their maximum difference, or the phase error made by this analysis, can be calculated. This difference depends on the number of beamuidths scanned, the diameter in wavelengths, and the $f / D$ ratio.

In Fig. 8, we show the loci of quarter wayelength phase errors. Also the 1 dB scanning limit from (14) is shown. The use of this figure requires an example. Consider an $f / D$ of 0.4 ; from (14) or Fig. 6 or 8 we can scan 4 beamwidths for a 1 dB loss of gain. From Fig. 8 we have, that for a half-power beamwidth of 1.9 degrees or less, our approximate analysis is within a maximum phase error of a quarter wavelength. For greater beamwidths, larger errors will be incurred, and more precise and laborious methods, based on (2), may be necessary [4].


Fig. 2. Typical scanned patterns.


Fig. 3. Half-power beamwidth.


Fig. 4. Tenth-power beamwidth.


Fig. 5. Coma lobe


Fig. 6. Loss of gain.


Fig. 7. Beam deviation factor.


Fig. 8. Region of validity.

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## References

[1] B. R. A. Nijboer, "The diffraction theory of aberrations," pts. 1 and. 2, Physica X, pp. 679-692, October 1943; Physica XIII, pp. 605-620, December 1947; with K. Nienhuis, pt. 3, Physica ${ }_{X I} V_{1}$ pp. 590-608, January 1949.
[2] R. Kingslake, "The diffraction structure of the elementary coma
image, ${ }^{n}$ Proc. Phys. Soc. (London), vol. 6, pp. 147-158, 1948
[3] M. Born and E. Wolf, Principles of Oplics. New York: Pergamon, chs. 5 and 9, 1959.
[4] C. C. Allen, "Numerical integration methods for antenna pattern calculations," IRE Trans. on Antennas and Propagation (Special Supplement), vol. AP-7, pp. S387-S401, December 1959.
[5] S. Silver and C.S. Pao, "Paraboloidal antemna characteristics as a function of feed tilt," Radiation Lab., Mass. Inst. Tech., Cambridge, Rept. 479, February 1944.
[6] K. S. Kelleher and H. P. Coleman, "Off-axis characteristics of paraboloidal reflectors," Naval Research Lab,, Washington, D. C., Rept. 4088, December 1952.
[7] Y. T.' Lo, "On the beam deviation factor of a parabolic reflector," IRE Trans. on Antennas and Propagation, vol. AP-8, pp. 347349, May 1960.
[8] S. S. Sandler, "Paraboloidal reflector patterns for off-axis feed," ibid., pp. 368-379, July 1960.
[9] L. C. Martin, Technical Optics, vol. 2. London: Sir Isaac Pitman \& Sons, p. 74, 1954.
[10] G. Z. Dimitroff and J. G. Baker, Telescopes and Accessories. London: Churchill, p. 86, 1946.
[11] A. S. Filler, "Primary aberrations of mirrors," Amer. J. Phys., vol. 29, pp. 687-694, October 1961.

# New technique for beam steering with fixed parabolic reflectors 

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Indexing term: Reflector antennas


#### Abstract

A technique is described which offers the potential of achieving wide-angle beam steering with fixed parabolic reflectors. The technique inyolves a primary-feed device with an aperture field distribution which can be adapted to match the distorted field distributions resulting from a parabolic reflector when an off-axis plane wave is incident. To provide an adaptation without deterioration of the system signal/noise ratio, which requires only a movement of the primary feed along a given locus and an adjustment of noninteracting phase shifters, the primary feed carries out a spatial Fourier transformation of the intercepted fields. The technique has been implemented in an experimental Xband antenna and beam steering of $\pm 15$ beamwidths achieved with negligible distortion of the directional pattern and less than 0.5 dB loss in gain.


## List of principal symbols

$f=$ focal length of parabolic refiector
$d=$ diameter of refiector
$\theta=$ halfangle subtended at the focus by a point on the refiector
$g^{*}=$ maximum value of $\theta$
$u=\sin \theta$
$\dot{u}=\sin \theta^{*}$
$\phi=$ rotational angle in refiector-aperture plane
$t, \phi^{\prime}=$ focal-plane polar co-ordinates
$x, y=$ focal-plane rectangular co-ordinates
$p=\sin 6 \cos \phi$
$\theta=\sin \theta \sin \phi$
$\dot{p}=\sin \theta^{*} \cos \phi$
$\dot{o}=\sin \theta^{*} \sin \phi$
$\psi=$ beam-steering angle, from antenna boresight
$\beta_{r}=$ component of phase error referred to reflectoraperture plane
$k=2 \pi / \lambda$
$\lambda=$ wavelength

## 1 Introduction

In the majority of present-day applications of large parabolic-refiector antennas, it is required that the antennas' directional patterns should be steerable over large angles. This is normally accomplished by mechanically steering the refiector structure. In a large antenna, the refiector and associated backing structure may have a deadweight of the order of hundreds of tons, and thus the steering requirement constitutes a major part of the antenna-design problem, particularly with respect to the economics of the construction. The steering problem becomes further aggravated when the patuern-steering requirements demand rapid movement coupled with highly accurate positioning and maintenance of pointing under adverse weather conditions.

To achieve a relaxation in the specification pertaining to the mechanical steering requirements, it would be advantageous if a moderate degree of beam steering could be effected by means of a suitable primary-feed design. While this may involve movement of a complex feed assembly, the problem compares favourably with that involved in moving the massive reflector structure.

Parabolic-antenna radiation-pattern steering can be achieved over a small range of angles by displacing a conventional primary feed radially about the reflector vertex. ${ }^{1,2}$ However, unless the reffector-aperture illumination is severely tapered, with a resultant reduction in aperture efficiency, the introduc-

[^12]tion of phase-error effects restricts the range of scanning to within a few beamwidths of the boresight before the radiationpattern deterioration becomes excessive.
A number of studies have been reported describing methods leading to the reduction of distortion of the antenna radiation pattern at small to moderate scan angles. For example, Takesima ${ }^{3}$ has described a defocusing technique to achieve balancing of two or more aberrations, a compensatory phaseerror technique involving tilting of the subrefiector has been developed by Hannan ${ }^{4}$ for Cassegrainian systems, and both Loux and Martins and Assaly and Ricardi6 have described focal-plane-array techniques which carry out weighting, phasing and summing of the intercepted energy.

The technique described here appears similar to the focal-plane-array techniques mentioned above. However, here the location of the feed is not restricted to the focal plane, and the signal processing employed is that of a spatial Fourier transformation of the intercepted electric fields. The advantages of this approach lie in the fact that, with a movement of the feed array along a defined locus, only an adjustment of phase shifters is required to achieve aberration-free scanning. The technique described here ${ }^{7,8}$ is equally applicable to either transmission or receplion, it provides a greater angle of scan than comparative systems, and requires no amplitude weighting of the intercepted energy to maintain optimum signal/noise performance of the antenna system.

## 2 Reflector electric-field distributions

### 2.1. Normally incident waves

It has been shown that the principal component of the electric-field distribution $E$ in the focal region of a circular parabolic reflector can be related to the electric field $F$ in the reflector-aperture plane by a scalar equation of the form ${ }^{\text {g }}$, 10

$$
E\left(t, \phi^{\prime}\right)=j \frac{k}{2 \pi} \int_{0}^{2 \pi} \int_{0}^{\dot{u}} \frac{F(u, \phi)}{\sqrt{\left(1-u^{2}\right)}} \exp \left\{j k t u \cos \left(\phi-\phi^{\prime}\right)\right\} u d u d \phi
$$

where the geomerry is given in Fig. 1, $\theta^{*}$ is the maximum halfangle subtended by the reflector from the geometric focus, $u=\sin \theta, \hat{u}=\sin \theta^{*}$ and $k=2 \pi / \lambda$, where $\lambda$ is the operating wavelength.
While this equation is an approximation, it has been found to provide reasonable solutions for the principal components of the focal-plane electric-field distribution for paraboloids of any focal-length/diameter ( $f / \sigma^{\prime}$ ) ratio. ${ }^{10}$
For parabolic refiectors with a rectangular aperture; which will be pertinent to the following discussion, eqn. 1 may be expressed in rectangular co-ordinates as ${ }^{9}$

$$
\begin{equation*}
E(x, y)=j k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(p, q) \exp \{j k(x p+y q)\} d p d q \tag{2}
\end{equation*}
$$

where $p=\mu \cos \phi$
$q=u \sin \phi$
$x=1 \cos \phi^{\prime}$
$y=r \sin \phi^{\prime}$

For reflectors having $f / d$ ratios greater than about 0.5 , the transform relationship can be considered to exist directly between the reflector-aperture-plane field $F$ and that of the focal plane.
When a linearly polarised plane wave is normally incident

and the function $G$ is defined as

$$
G=\left\{\begin{array}{ll}
\left(1-p^{2}-q^{2}\right)^{-1 / 2} F(p, q) & |p| \leqslant \dot{p}  \tag{3}\\
0 & |q| \leqslant \dot{q} \\
0 & |p|>\dot{p}
\end{array}|q|>\dot{q}\right\}
$$

where $\dot{p}=\dot{u} \cos \phi$ and $\dot{q}=\hat{u} \sin \phi$
The function $G$ has been termed the 'modified' aperture-field distribution since it can be considered as an amplitudeweighted version of the true aperture distribution. ${ }^{9}$

The existence of a spatial 2-dimensional Fourier-transform relatjonship beiween the modified-aperture-plane field distribution $G$ and the focal-plane field $E$ is indicated by eqn. 2.


Predicted focal-plane electric-field distributions from eqns. 4
B $\mathrm{fld}=0.25$
B $/ \mathrm{d}=0$
6 fld $=0.5$
$c / 1 d=1.0$
on a rectangular parabolic-reflector aperture, the aperture illumination observed from the reflector focal point has a uniform amplitude and phase distribution. The electric-field distribution along the principal axes of the focal plane may then be obtained from eqn. 2, putting the complex function $F$ equal to a constant. The solutions take the form

$$
E(x) \alpha\left\{\begin{array}{ll}
\frac{2 \hat{p} \sin k x \dot{p}}{k x \dot{p}} & \dot{p}<0.5  \tag{4}\\
\pi J_{0}(k x) & . \dot{p}=1.0
\end{array}\right\}
$$

and similarly for $E(y)$, replacing $\dot{p}$ with $\hat{q}$ by $x$ and $y$. These functions are illustrated in Fig. 2.

### 2.2 Inclined incident waves

Eqns. 1-3 are not restricied to the case of a normally incident plane wave at the refiector aperture and may be usefully applied to incident waves inclined to the antenna boresight. However, for inclined incident waves, at a given angle of incjdence, eqns. 1 and 2 express a relationship between the reffector-aperture plane and one of a set of transform planes which are normally inclined to the centreline of the angular cone subtended by the perimeler of the refiector aperture (i.e. the angle subtended at the new 'focal point ${ }^{2}$ must remain constant and equal to $2 \theta^{*}$ ). This relationship is valid providing the original assumptions hold; ${ }^{9,10}$ in particular, that the spread of the energy in the transform plane is small compared with the distance separating this plane from the reflector-aperture plane, and that most of the energy is concentrated within the region about the new focal point. The locus of the iransform planes is shown in Fig. 3.

The mathematical description of the function $G$, for a given angle of arrival of the incident wave, has still to be determined. We will first discuss the general effect on the refiector fields of an inclined incident wave.

Observed from the refiector geometric focus, the inclined incident wave effectively produces a reflector-aperture electricfield distribution which is nonuniform in both phase and amplitude. The aperture phase distribution $\beta$. can be. expressed as a series of the form ${ }^{11}$

$$
\begin{equation*}
\beta(a)=\beta_{1} a+\beta_{2} a^{2}+\beta_{3} a^{3}+\beta_{4} a^{4} \tag{5}
\end{equation*}
$$

where $a=u / \hat{u}$ and $\beta_{n}$ is the value of the respective phase term at the edge of the aperture. $\beta_{n}$ will be a function of the angle of inclination of the incident wave and the reflector fld ratio. The first four terms of the infinite series predominate and are commonly termed linear, focus, coma and spherical aberrations. Detailed analyses of these aberrations appear in PROC. IEE, Vol. 1J8, No. 7, JULY 1971
the literature, ${ }^{11-13}$ and it will be sufficient for our purpose to merely outline the effects of these phase deviations.
A linear phase distribution is not, in effect, an aberration, since it does not distort the focal-plane field distribution, but merely shifts it along the transverse axis. At points along this axis, however, the system geometry introduces both focus and


Fig. 3
Locus of transform planes with associated (1ypical) electric-field distribuions
Circle diameter $=f(1 \div(d / 2 f): \sec \theta$.
coma aberrations. The focus error tends to defocus the field distribution in a symmetrical fashion, while the coma error introduces an asymmetrical distortion, which results in a rapid increase in the level of the minor lobes of the focal field distribution on one side of the main lobe.
To achieve even a small degree or beam steering in a highgain fixed-refiector system, it is necessary to move the phase centre of the primary feed from its boresight location to obtain the required linear tilt in the phase distribution at the reflector" aperture. However, once the primary feed is so moved, the effects described above are incurred, and, in addition to the desired linear term in the reflector-aperture phase distribution, there appear additional terms owing to focus, coma and spherical aberrations.
The focus and spherical terms are symmetrical errors and can be minimised by reducing the spacing between the reflector and the primary feed, thereby introducing compensatory phase errors by defocusing the aperture illumination. In addition, a primary-feed location can be determined which provides a relatively uniform aperture-amplitude distribution. Consequently, it is the coma phase error which initially constitutes the major limitation to achieving wide-angle beam steering.
To illustrate the effect of the coma phase error, consider a parabolic reflector curved in one dimension, with an $f / d$ ratio not less than $0 \cdot 5$. We will consider a wave incident on the refiector aperture at an angle $\psi_{1}$ to the boresight and assume a transform plane with a phase centre which compensates for all but the coma aberration. Along the transverse PROC. IEE, Yol. 118, No. 7, JULY 1971
axis of the transform plane $x^{\prime}$, the electric-field distribution will be given by ${ }^{9}, 10$

$$
\begin{equation*}
E(x)=K \int_{-\infty}^{\infty} G^{\prime}(p) \exp \left(k x^{\prime} p\right) d p \tag{6}
\end{equation*}
$$


$|E|$

|E|


Fig. 4


Predicted electric-field distributions in transform planes, from eqn. 6
2. $=3.2 \mathrm{~cm}, \mathrm{fld}=0.5$
$0 G_{1}=0$
$b$
$G_{1}=0$
$\begin{gathered}0 \\ S_{3}=0.64 \\ 0\end{gathered}=1.28$

where $K$ is a constant, and $G^{\prime}$ comprises a uniform amplitude distribution with a coma phase-error distribution of $\beta_{3}\left(\psi_{1}\right)$ at the edge of the aperture

$$
G^{\prime}(p)=\left\{\begin{array}{ll}
\exp \left\{j \dot{\beta}_{3}(p / \tilde{p})^{3}\right\} & |p| \leqslant \dot{p}  \tag{7}\\
0 & |p|>\hat{p}
\end{array}\right\}
$$

Eqn. 6 has the form of an Airy function ${ }^{14}$ which has been plotted elsewhere for another application. ${ }^{15}$ The function is shown in Fig. 4 with $\lambda=3.2 \mathrm{~cm}$ and $\dot{p}=0.8$ for several values of $\beta_{3}\left(\psi_{1}\right)$.

## 3 Beam-steering technique

To design an efficient primary feed for a parabolicreflector antenna, it is necessary to achieve, a best-match condition between the electric-field distribution across the aperture of the primary feed on transmission and that field produced at the same surface by the reflector when it is illuminated by a distant source. ${ }^{16}$ Applying this condition to the beam-steering problem, it is evident that, to achieve wide-angle beam steering, it is first necessary to move the primary feed to a region where it intercepts the energy from the reflector, and then effectively to adapt the primary-feed aperiure field distribution, so that the required matching characteristics are obtained for the given angle of beam steering.
From Fig. 4, it can be seen that the variation in both the amplitude and phase distribution of the transform plane field at different $\beta_{3}$ (corresponding to different steered angles) is severe. Direct replacement of a conventional primary feed with a more exterisive multielement array will thus demand an adaptive system with the capability of achieving combination of nonuniform signals. To avoid signal/noise-ratio degradation in the combining process, it is necessary to weight adaptively the gain of each array element with its own signal/noise ratio. ${ }^{17}$

To avoid these complications, an alternative approach has been proposed. The method employs the existence of the 2-dimensional spatial Fourier-transform relationship berween the electric-field distributions in the modified aperture plane $G$ and a set of transform planes. The technique consists of carrying out a second spatial Fourier transformation on the electric-field distribution in the relevant transform plane, so that, at the output of the transforming device, the amplitude of the field distribution always has the spatial form $|G|$ ( $a$ constant for a given refiector), while the phase distribution in this plane is an image of that in the refiector-aperture plane.

If the transforming device is followed by a suitable phaseshifting network and a combining matrix, compensation for aperture-plane phase errors can be made by means of phaseshifter adjustments at the primary feed, without adjustments to either the transforming-device or the combining-matrix components. ${ }^{9}$ Beam steering can thus be achieved by a combination of movement of the primary feed along a defined locus with switching in preset values of phase in the phaseshifter network. The basic primary feed can be constructed as a passive device and can be used for either transmission or reception:

Employing an ideal Fourier-transforming device, i.e, one having infinite dimensions, the spatial distribution at the device output would be that given by eqn. 3, where $\rho$ and $q$ now' correspond to linear distances across the output, Use of a finite device tends to impose an amplitude weighting on the output distribution, which is directly related to the reflectoraperture illumination taper produced by any finite primary feed. Although this weighting represents a decrease in reflect oraperture efficiency, it has practicai applications in providing a reduction of sidelobe levels in the overall antenna radiation pattern. The effect of the amplitude weighting provides an additional bonus, in that the spatial output distribution of the finite transformer tends to be uniform and is thus in convenient form for combining under optimum signal/noiseratio conditions.

A good approximation to the ideal Fourier transformation can be achieved provided that the collecting aperture of the finite device intercepts most of the energy in the transform plane. Examination of both Figs. 2 and 4 indicates that this
can be achieved for both normal and inclined incident waves in the reflector aperture, with a primary-feed collecting aperture having dimensions of the order of wavelengths.

## 4 Operation of primary feed

For simplicity, the operation of the primary feed will be described in terms of a reflector curved in one dimension and having an fld ratio of the order of $0 \cdot 5$. Consider Fig. $3 a$. The circle diagram illustrates a parabolic reflector with a normally incident plane wave. The array primary feed situated at the focus of the refiector 'sees' a uniform phase and amplitude distribution over the angle $2 \theta^{*}$. The output of the primary-feed transforming device thus comprises a uniform phase and amplitude distribution, which may be summed directly to maximise the power output.

Now consider a plane wave incident at an angle to the reflector boresight. Viewed from the geometric focus, the reffector aperture has a nonuniform amplitude and phase distribution. The amplitude distribution is a consequence of the fact that, after refiection, the incident wavefront is no longer convergent in the immediate vicinity of the reflector geometric focus. If the primary feed is moved from the geometric focus along a locus which maintains the angle $2 \theta^{*}$ constant, a point on this locus can be reached where the linear and focus phase errors and the nonuniform amplitude distribution introduced by the incident wave are largely compensated. Fig. $3 b$ illusirates the configuration. However, while the observed aperture/amplitude distribution is now uniform over the angle $2 \theta^{*}$, the phase distribution will still contain uncompensated phase-error terms resulting from the asymmetry of the configuration.

The electric-field distribution across the aperture of the primary feed will now be of the form shown in Fig. 4, while, at the output of the primary-feed transforming device, the field distribution will be that given by egn. 7; i.e. a uniform amplitude distribution with a phase distribution containing a dominant coma term, which can be compensated by a phaseshifting network prior to summation. In this fashion, the array primary feed is capable of achieving beam steering over a range of angles, limited only by the constraint that the spread of energy' in the relevan transform plane should not exceed the collecting aperture of the primary feed.

## 5 Experimental system

An experimental primary-feed system was constructed completely in Xband waveguide for operation at 10 GHz in conjunction with a 1 -dimensionally curved parabolic refiector having a diameter of 1.8 m and an f/d ratio of 0.5 . The feed system, which has been described in an earlier pubication, ${ }^{9}$ is shown in block-schematic form in Fig. 5. The system comprises a linear array of eight waveguide feeds with an interelement spacing of $\lambda / 2 \sin \theta^{*}$, followed by an 8 -port Butler matrix, to provide a sampled spatial Fourier transformation. ${ }^{9}$ Each of the matrix output ports is taken via an adjustable phase shifter to the combining matrix. The Butler matrix and the combining matrix are constructed individually as 1 -piece units by a dip-brazing technique. ${ }^{18}$ Overall, the experimental feed system introduces an insertion loss of the order of dB , this being largely atributable to the imperfect construction of the r.f. components forming the matrices.

The primary-feed subassembly was mounted on a movable carriage, to permit the aperture plane of the array to be moved around the locus shown in Fig. 3, and the combination of reflector and primary-feed assembly was rotatable, to allow measurements of the overall antenna radiation pattern.

For measurements of the electric-field distributions in the transform planes, the linear array was replaced by a single waveguide feed which was positionally coupled to an xyrecorder. The single feed was then mechanically scanned across the relevant transform plane, and the measured electric-field spatial distribution recorded.

## 6 Measurements

The proposed beam-steering technique is based on the existence of the spatial Fourie-transform relationship between the electric-field distribution in the reflector modified-aperture
plane and that of a defined transtorm pianc. in addition to measurements of the system performance, it was considered desirable to oblain experimental verification of the theory on this point. This verification was achieved by means of additional field measurements in the transform planes. The
system output was calibrated by comparison with the output from a reference horn antenna.

With the array feed removed, the field distribution in the relocated transform plane was recorded on the $x y$ plotter by traversing the travelling feed in the plane defined by the


Fig. 5
Adaptive-primary-feed block schematic
measured field distributions in the transform planes were compared with the fields predicted in Fig. 4 and with the radiation-patterns of the overall antenna. An explanation of the raciation-patiern comparison follows.

It is a well established fact that the far-field radiation patiern of a parabolic-reflector antenna (in $\sin \psi$ measure) is related to the renecior-aperture field distribution by a spalial Fourier transformation. ${ }^{1}$ Thus the antenna radiation patiern measured uith a single-element primary feed illuminating the reflector from a point on the $2 \theta^{\circ}$ locus (with the feed aperture plane aligned with the relevant transform place at that point) will constitute the Fourier transformation of the reflector-aperure field distribution including the coma phase errors arising from the geometrical asymmeiry. The main beam of the antenna will appear at an angle $\psi_{1}$ to the boresight. If now, with the antenna aligned so that the target transmitter is situated at the same angle $\psi_{1}$ to the reflector boresight, a travelling feed is employed to measure the feld distribution in the relevant transform plane (about the point at which the single feed had been localed), for refiectors having $f / d$ ratios of not less than about 0.5 , this distribution should be an identity with the radiation pattern previously measured, the two distributions being related by a double Fourier transformation.

Measurements of the system performance comprised anienna-radiation-pattern measurements and power-outputlevel measurements using the array primary feed and steering the radiation pattern over angles of up to $\pm 15$ beamwidths. The reflector system was positioned so that the target transmitter made an angle of $\psi$ to the reflector boresight. The primary-array feed was moved along the $20^{*}$ locus and locked initially at the point giving the best output power level. The array-feed phase shifters were then adjusted to maximise the outpui power. No significant interaction between the phase shifters was observed, as would be expected in view of the orthogonality of the beams formed by the matrix. The radiation pattern of the antenna was then recorded by rotating the complete antenna assembly. The array feed was then moved incrementally on the $2 \theta^{*}$ locus, and the procedure repeated until the optimum radiation pattern with respect to gain and sidelobe levels was obtained. The power level at the PROC. IEE, Yol. II8, No, 7, JULY 197J
aperture of the array. Finally, with the travelling feed at the centre of the relevant transform plane, the antenna radiation pattern was again recorded by rotating the assembly. Measurements were made at offset angles $\psi$ of $0,5,10$ and $\pm 15^{\circ}$.

Some experimental results are shown in Fig. 6. The critical results for $\psi=0$ and $15^{\circ}$ are given, as these illustrate the range of the experiments. The results are shown for each $\psi$ in the form
(a) antenna radiation pattern with a single-element primary feed
(b) relevant transform-plane field distribution
(c) antenna radiation pattern employing array primary feed.

## 7 Experimental results

The measure transform-plane field distributions were in good general agreement with these predicted by eqn. 6. In addition, the similatity of the relevant antenna radiation patterns employing the single-element feed and the associated transform-plane field disiributions, for all $\psi$ employed, provided further experimental support for the predicted double-Fourier-transform relationship (see Figs, 2, 4 and 6). For refiectors of very small $\mathrm{f} / \mathrm{d}$ ratio, the antenna radiation pattern and the transform-plane field distribution will be related by the 'modified'-a perture distribution. In such cases, the transform-plane field will be modified and will no longer be directly similar to the antenna radiation pattern. However, for the $f / d$ ratio of 0.5 employed in the experimental system, the curvature effect will be small, and, consequently, considerable similarity can be expected between the two distributions,

The results obtained from the beam-steering measurements were very encouraging. The antenna radiation pattern was steered over a range of $\pm 15^{\circ}$ (or $\pm 15$ beamwidths) with less than 0.5 dB reduction in the system gain. Fig. 6 shows a redistribution of power in the sidelobe distribution at $15^{\circ}$, but the peak level of the sidelobe distribution bas not changed significantly. The slightly higher sidelobe-levels in the antenna radiation patterns in all cases involving use of the array feed are a result of the aperture-shadowing effect
of the array hardware. This would be reduced to a negligibie level in a practical configuration involving a large refector, since the dimensions of the primary-feed aperture are governed by the fld ratio, rather than the diameter of the reflector. Hence, to achieve $\pm 15$ beamwidths of scanning at 10 GHz with a refiector having $f / d=0.5$ demands a primary-feed aperture of the order of 20 cm diameter, regardless of the diameser of the main reflector.

$|E|$

$|E|$


Fig. 6
Experimental results
c Sicered antensa radiation patterns with conventional single-eiement primary c Sicer
(i) Fred on axis
(ii) Fred oflse:

Trassform-plane field distributions
(i) Fogi plane with Lerget tuansmitter along antenna boresigit

Steredocaled transiom p)ane with target transmituer offiset $15^{\circ}$

| $c$ Sterred-zntenna rad |
| :--- |
| $\lambda=3.2 \mathrm{~m} . ~$ |
| $1 a^{\circ}=0.5$ |

(i) Primary feed in focal plane
(ii) Primary feed offiel

## Conclusions

The beam-steering technique developed shows considerable promise with respect to achieving wide-angle beam steering of parabolic refiectors. While, owing to mechanical limitations, the experimental system did not exceed $\div 15$ beamwidths of beam stecring, this does not constitute a fundamental limitation on the technique, nor does it necessarily represent the maximum range obtainable with the 8 -element array feed. Further work is planned to determine the fundamental limitations and to determine an analytical relationship between the dimensions of the collecting aperture of the array feed and the maximum obtainable range of beam steering. From a superficial examination based on the geometry of Fig. 3, the overall limits of the beam-steering technique appear to be given by

$$
\begin{equation*}
\psi^{*}=90-\theta^{*} \tag{8}
\end{equation*}
$$

where $\psi^{*}$ is the maximum steered angle from the antenna boresite. Verification of this equation, however, awaits the outcome of further studies.
Any fixed-aperture antenna system will suffer from a reduction of gain proportion to $\cos ^{2} \psi$, when the antenna radiation pattern is steered to an angle $\psi_{1}$ from the boresight. At 15 beamwidths, with the experimental system, this would result in a 0.3 dB reduction in gain, which would, within the experimental tolerance, account for virtualiy all the 0.5 dB gain loss measured.
In many applications of large-refiector antennas (e.g. radar, radioastronomy and satellite communications), it is of en desirable to have the facility of rapidly changing the direction of the antenna radiation pattern at electronic switching speeds. Early results of experiments aimed at the evaluation of purely electronic beam steering (i.e. by electronically controlling the adjustment of the phase shifters only) indicate that the range of such a technique is of the order of a few beamwidths. However, this would be sufficient to achieve conical scarning of the beam.
In addition to conventional beam-steering applications, the techniques described would be particularly suitable where an ofisel primary-feed configuration for a parabolic refiector is desired. Such a confguration would permit more than one independent radiation pattern to be formed by employing twe or more primary-array feeds on the one reflector.
In the interest of clarity, the description of the beam-steering technique has been largely confined to reflectors curved in one dimension. Nevertheless, the basic technique may be applied similarly $t 0$ the case of 2 -dimensionally curved surfaces. Application of the spatial-transform technique to a 2 -dimensionally curved refectior demands a 2 -dimensional Fourier transformer. In this case, the use of alternative methods of achieving the spatial transformation; and, in particular, the use of a second refiector or a microwave lens, appears attractive in offering a reduction of the complexities involved in the construction of a large planar matrix.

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## 10 References

silver, s.: "Microwave antenna theory and design' (McGraw-Hill, 1949) JASIK, H.: 'Antenna engineering handbook' (McGraw-Hill, 1961)

TAKESHIMA, T. : 'Beam scanning of parabolic antenna by defocusing'. Electronic Engng.• 1969, pp. 70-72
HANNAN, $P$. W. "Microwave antennas derived from Cassegrain telescopes', IRË Trans., 1961, AP-9, pp. 140-153
5 Loux, P. C., and martin, R. W.: 'Efficient abertation correction with a iransverse focal plane array technique', JEEE Iniernat. Convention Record, 1964, 12, p. 125

PROC. IEE, Vol. 118, No, 7, JULY 1971

6 asuly, R. N., and RICARDI, i. ..; 'A theoretical siudy of a multietement scanning system for a parabolic cylinder', IEEE Trans. 1966. AP-14, p. 601

7 Rudige, A. W.: British Provisional Patent 23145/68, May 1968
8 RUDGE: A. W', and WIHEERS, M. J.: 'Beam-scanning primary feed for parabolic refiectors', Electron. Lett.: 1969, 5, pp. 39-41
RUDGE, $A$. W., and DAVIES, D. E. N.: 'Electronically controllable primary feed for profile-error compensation of large parabolic refiectors', Proc. IEE, 1970, 117, (2), pp. 351-358
10 RLDGE, A. W.: "Focal-plane field distribution of parabolic reflectors' Electron. Lell., 1969, 5, pp. S10-512
11 welpord. w. T.: 'Geometrical optics, optical instrumentation' (North Holland, 1962)
12 born', M., and wolf, e:: 'Principles of optics' (Pergamon Press 1964)

13 LUM, Y. F., and PAVLASEK, T. 3. F.: 'The innuence of aberrations and aperture inclinations, on the phase and intensity structure in 14 ABRAMowitz $M$ a ABRAMOWITZ, M., and STEGUN, 1.: Handbook of mathematical KRAMER, S. A.: 'Doppler and acceleration volerances of high-gain, wideband linear f.m. correlation sonars', Proc. Inst. Elect. Electron. wideband $19 n e a r$.m. correiatio
Engrs., 1967,55, pp. 627-636
16 RUDGE, $A$. W., and wITHERS, M. J.: 'Design of flared-hom primary feeds for parabolic sefiecior antennas', Proc. IEE, 1970,117 . (9), pp. 1741-1749

17 GRENNAN, D. G.: On the maximum signal-to-noise ratio realizable for several noisy channels', Proc. Insl. Radio Engrs., 1955, 43, p. 1530 . 'An X-band Butler matrix aray' Proceedings of the 18 Thraves, J.: "An X-band Butler matrix array"

# Determination of the Maximum Scan-Gain Contours of a Beam-Scanning Paraboloid and Their Relation to the Petzval Surface 

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#### Abstract

The scan-plane fields in the focal region of a beamscanoing paraboloid are determined from physical optics. Amplitude and phase contours are presented, and comparisons are made with the geometrical-optics results. Contours for maximum scsn-gain are determined as a function of $F / D$ and illumination taper and compared with the Petzval surface. Doless the $F / D$ is very large or spillover is excessive, a bigher scan gain is achieved when the aris of a directional feed is parallel to the axis of the reflector than Fhen the feed is directed toward the vertex The contour of maxirom scan-gain is a function of both illumiation taper and $F / D$. In general, larger $F / D$ values tend to have a maximum-gain contowr close to the focal plane, while the smaller $F / D$ values tend to bave a maximum-gain contour closer to the Petzral surface. Increasing the illumination taper raoves the maximum-gain contour closer to the Petzval surface. Normalized maximum-gain contours are presented as a function of beampidths of scan. The frequency dependence of these results is discussed.


## I. Introduchion

PARABOLOIDAL antennas in radar, radio-astronomy, and microwave communication systems frequently employ lateral displacement of the feed to achieve a beam-scanning capability. Features of beam scanning achieved in this manner have been dealt with in [1]-[7]. However, the simple but important problem of determining the proper position and orientation of the feed to achieve maximum scan gain still remains relatively unresolved. This paper describes a numerical study of that problem.

In the receive mode, a linearly polarized plane wave, incident from some direction of axis, includes currents on the reflector which then give rise to a particular field distribution in the focal region. In the transmit mode, a linearly polarized directional point-source feed is conceptually placed at some point in the focal region in order to scan the far-zone transmitted beam off axis. These two situations are related by the reciprocity principle [8], [9], which is particularly simple when the trans-

[^13]mitting feed is an infinitesimal dipole. For actual feeds the relationship is more complex, but it can be shown that the efficiency of an antenas is given by the correlation (over any closed surface) between the focal-region fields of the reflector receiving an incident plane wave and the fields of the transmitting feed [10], [11]. Thus the focal-region fields correspond to the optimum feed aperture distribution. This fact has been used with considerable success in the development of corrugated feeds, for example [10], [12]. Rudge has also applied this approach to the design of an array feed for a beam-scanning paraboloid, where the feed aperture fields are syathesized to match the focal region fields on a given surface [13]. The primary objective of this paper is to determine the optimum orientation and location of displaced feeds. This may be interpreted in terms of correlation as moving the feed aperture until the feed fields in the aperture most. effectively match the focal region fields over that same surface.

## II. Analysis by Geometrical Optics

Geometrical ray tracing has been used by several authors to determine surfaces of sharpest focus for scanned paraboloidsl reflectors [14]-[19]. Fig. 1 indicates the reflected rays in the plane of scan ${ }^{1}$ of a paraboloid receiving a plane wave from $16^{\circ}$ off axis. If only single reflections are considered, two of these reflected rays will pass through a given point in the plane of scan. The envelope of these rays ( $A O B$ in Fig. 1) forms a caustic curve in the plane of scan [20]. This caustic will also define a forbidden region within which no rays will pass. Consequently, the geometrical-optics (GO) field is zero within this region.

A second caustic, COD (sometimes referred to 39 a "ridge line"), also forms a boundary beyond which no reflected rays from out of the plane of scan will cross the plane of scan. As a result, four rays will pass through every point in the plane of scan below COD in Fig. 1, two rays from points of reflection in the plane of scan, and two rays from points of reflection out of the plane

[^14]

Fig. 1. Reflected rays in plane-of-scan of paraboloid receiving plane Fave from $16^{\circ}$ off axis.


Fig. 2. Comparison of geometrical-optics and physical-optics fields.
f scan, one from above and one from symmetrically below. However, truncation of the reflector may limit his number to as few as one.
Superposition of the contributions of each ray passing through a point will give the total GO field at that point. The feld intensity calculated from $G O$ and physical pptics ( PO ) - is plotted in Fig. 2 for the line LMNO of Fig. 1. In general, the $G O$ field is significantly larger than the PO field. From $L$ to $M$, only one geometric ray rosses the path and the field shows a slow, monotonic increase. From $M$ to $N$ two rays cross the path and two jnterference fringes are observed in both the GO and PO ntensities. From $N$ to the caustic COD four rays cross he path: the GO intensity becomes unrealistically large as the caustic is approached, while the PO field exhibits th primary maximum. Beyond $O$ the GO field ceases to Wist, while the PO field drops to smaller but nonzero values. Such results may be easily obtained using GO for a njde renge of parameters which are useful in locat-
ing the general position of the most intense focal-region fields and for grossly characterizing these fields. However, the classical GO analysis is incapable of yielding useful intensity information on the focal-region fields.

## III. Analysis by Phystcal Optics

The analytical PO technique has been used to produce the numerical results presented in this paper. PO assumes the surface currents on the reflector to be approximated by the GO currents; the free-space dyadic Green's function is then integrated over these currents to obtain the fields. Analytic justification for the use of this technique may be found in [21]. Only paraboloidal reflectors are considered, and such parasitic effects as aperture blocking, surface irregularities, etc., are not included in the analysis.

It has been pointed out in the literature that, except under special circumstances, the PO approximation fails to satisfy the reciprocity principle [22]. One set of special conditions for which the PO fields do satisfy the reciprocity principle is the focused paraboloid and a distant observation point on the reflector axis [23]. However, the fields considered primarily in this paper arise from defocused conditions. The reciprocal properties of these defocused PO fields were examined numerically in the following manner:

1) An infinitesimal unit-amplitude electric dipole was conceptually placed at point $(x, y, z)$ in the focal region of the paraboloid. $E_{i}(x, y, z, \theta, \phi)$, the resulting PO fieldexclusive of the $(\exp (-j / \tau) / r)$ factor-radiated to a distant field point in the direction $(\theta, \phi)$, was numerically computed as described elsewhere [24].
2) An incoming unit-amplitude vector plane wave from the direction $(\theta, \dot{\varphi})$ then illuminated the reflector and the resulting field at $(x, y, z), E_{r}(x, y ; z, \theta, \phi)$ was numerically computed.

The reciprocity principle requires that ${ }^{2}$

$$
\begin{equation*}
E_{i}(x, y, z, \theta, \phi)=E_{r}(x, y, z, \theta, \phi) \tag{1}
\end{equation*}
$$

For all sets of parameters of the 34 wavelength paraboloid considered in this paper, the two fields agreed to mithin 0.4 percent or better, usually to within 0.2 percent. How much of this small difference is due to different numerical integration algorithms is unknown. It is felt that, on the basis of this evidence, the PO solution and the computer programs used were sufficient for the accuracies required for this study.

## A. Scan-Plane Fields in Focal Region of Receiving Paraboloid

Phase and amplitude contours in the focal region plane of scan of a paraboloid receirring an incident plane wave are plotted for 3 different angles of incidence in Fig. 3. The electric vector is perpendicular to the plane of scan.

[^15]

Fig. 3. Phase and amplitude contours in focal region plane-of-scan of receiving paraboloid.

The origin is the reflector focus. The coordinate axes are the reflector axis marked in wavelengths toward the reflector (the vertex of which is 14.7 wavelengths from the origin) and a line at right angles to the reffector axis marked in wavelengths to the left of the focus. When the incident wave travels along the refiector axis, the set of contours surrounding the focus is generated. These contours are similar to results for the normal-incidence case [12], [25]. However, the phase is retained in its exact form [26], [2i]. Consequently; the contours are not symmetrical about the fucal plane, and closely agree with the physically measured values [28]. The maximum received field intensity occurs at the focus. The equiphase contours are separated by 180 electrical degrees. It is evident from the figure that, in terms of an equivalent wave travelling along the refiector axis, the effective wavelength is greater than the actual wavelength.

When the incident field arrives from a direction to the right of the reflector axis, the phase and amplitude contours shift to the left as shown. The maximum intensity points for $8^{\circ}$ and $16^{\circ}$ incidence are, respectively, 1.32 and 4.28 dB below the original maximum intensity for axial incidence. The amplitude and phase contours distort and disperse. At the maximum intensity point for each angle of incidence, the constant phase contour is approximately parallel to the aperture plane, i.e., perpendicular to the reflector axis.

Reciprocity permits the contours in Fig. 3 to be interpreted in terms of an infinitesimal electric dipole feed polarized perpendicular to the plane of scan, transmitting to a distant field point. For example, if the distant field point is eight degrees on the right side of the reflector axis, the set of amplitude and phase contours for eight degrees will yield the phase and magnitude of the field observed at that distant point. Placing the dipole at the maximum intensity point will transmit the maximum possible field to the distant point; this field, however, will be 1.32 dB below the field radiated to a distant axial


Fig. 4. Comparison of maximum field locus and Petzval surface for receiving paraboloid.
field point when the dipole transmitting feed is at the focus.

The Petzval surface, a term from classical optics, is the surface of best focus in an optical system in the absence of astigmatism [29]-[31]. For a single mirror the radius of curvature of the Petzval surface is one half the radius of curvature of the mirror [31]. In the primary microwave reference to the subject, the Petzval surface of a paraboloidal mirror is derived by Ruze [7] to be another paraboloid of half the focal length, tangent to the focal plane at the focus: and described by the equation

$$
\begin{equation*}
\left(\frac{X}{\lambda}\right)^{2}=2 \frac{F}{\lambda} \frac{Z}{\lambda} \tag{2}
\end{equation*}
$$

where $F$ is the focal length, $\lambda$ is the wavelength, and $X$ and $Z$ are defined in Fig. 1. The Petzval surface is superimposed on Fig. 3. It comes close to, but does not pass through, the scan maxima.

Plots of the scan maxima, without the corresponding amplitude and phase contours, are shown in Fig. 4 for $F / D=0.433$ and $F / D=0.604$. The remaining parameters are the same as for Fig. 3. The appropriate Petzval surfaces for the two focal lengths are also plotted. As before, these curves can be interpreted in either the transmit or receive mode. The plot for $F / D=0.433$ contains the same information as Fig. 3, except that additional scan maxima for $4^{\circ}, 10^{\circ}, 12^{\circ}$, and $14^{\circ}$ are included. For $F / D=0.604$ the maximum-field locus remains relatively far from the Petzval surface for scan angles as large as $16^{\circ}$, while the contour for $F / D=0.433$ droops toward the Petzval surface more rapidiy.

For axial incidence the maximum field intensity for $F / D=0.604$ does not occur at the focus but a small fraction of a wavelength toward the reffector. This effect can best be understood in terms of an infinitesimal electric dipole transmitting feed illuminating the reflector. By moving the feed away from the focus, perfect phase synchronism is lost for a distant point on axis. However,
br moring the feed closer to the reflector, more power is intercepted by the reflector and becomes available for secondary radiation. Thus the phase loss is initially overcome bra a decrease in spillover, and the maximum occurs closer to the reflector. This effect, which is exclusively geometrical, becomes less pronounced for the deeper reflector ( $F / D=0.433$ ). This example, however, illustrates the fact that the microwave problem is generally more comples than the scalar-optical problem. In addition to phase eñects, the vector nature of the fields, the directionalitr of the sources, spillover, etc., must be taken into account.

For a paraboloidal mirror, the $S$ (sagittal) focal surface from third-order Seidel aberration theory [31] is the focal plane, and the $T$ (tangential) focal surface lies between the focal plane and the mirror. The $T$ surface is three times as far from the Petzval surface as the $S$ surface [31], [33]. The best compromise focus of an optical srstem is expected to lie between the $S$ and $T$ suriaces. In Fig. 4 the maximum field locus for $F / D=0.433$ out to $12^{\circ}$ and the entire locus for $F / D=0.604$ satisfies this criterion. It will be shown later that at low illumination tapers and small scan angles the maximum field locus Hips betreen the $S$ and $T$ suriaces. However, as the taper andior the scan angle increases, the maximum gain locus crasses the focal plane and droops toward the Petzval suriace defined in (2). This difference between the thirdorder optical and microwave vector diffraction theory results is atributed to: 1) the breakdown of third-order aberration theory at wide angles; 2) optical systems iawally have a much larger $f$ value ( $F / D$ ratio) than anienia refiectors, for which values may be as small as 0.25 ; and 3) the unique differences associated with the compiete vecior result, not included in scalar optics, e.g., as illustrated in the previous paragraph.

## B. Orieniation oj Directional Transmit Feed to ficiove Maximum Gain

The remainder of the paper is concerned with the problem of positioning a directional transmit feed to achieve maximum scan gain. The class of feed patterns to be consicered bas a spherical phase front and a linearly polarized: axially symmetric voltage pattern of the idealized form $\cos ^{n} \theta$, where $\theta$ is the polar angle measured from the axis of the pattern maximum.

From a purely geometrical point of view, optimum gain for a given position of the feed (when scanned) is achieved by orienting the feed pattern to minimize spillover berond the edge of the dish. Generally speaking, this is achiered by directing the feed maximum toward the renector vertex. However, analysis has consistently revealed that unless spillover is excessively large (of the arder of dB 's), a higher gain is achieved for moderate $F / D$ whices riken the feed axis is parallel to the axis of the refistor then when the feed is pointed toward the vertex. For example, if $F i D=0.433, D / \lambda=34.0$, and a $\cos ^{2} \theta$ feed (11.5 dE taper at the reflector edge), a maximum scanned gán $\varepsilon^{\ddagger}$ a scen angle of $20^{\circ}$ (approximately 10 beam-


Fig. 5. Comparison of induced current density distributions for two orientations of feed.
widths of scan) is achieved if the feed is positioned 5.8 wavelengths laterally from the refiector axis ( $1 / 3$ of the distance to the edge of the aperture). If the feed is directed parallel to the reflector axis, i.e., "straight ahead," the spillover loss is 0.19 dB greater than when the feed is directed toward the reffector vertex. However, the scan gain for the former orientation is 0.85 dB higher than for the latter orientation. For smaller scan angles the scan gain is also larger for the straight-ahead orientation, although the magnitude of the effect is proportionately reduced. Kelleher and Coleman [3] have poinied out that, based on geometrical ray tracing; the straight-ahead orientation yields an aperture field centered symmetrically in the aperture.

An explanation of the straight-ahead orientation yielding maximum scan gain is based on the constant-phase contours in Fig. 3. The contours through the maximum scan intensity points are nearly perpendicular to the reflector axis. Consequently; by the reciprocity/correlation principle, the scan gain will be maximized when the phase front of the feed is parallel to this. A more quantitative derivation is contained in the Appendix.

It is also instructive to examine the components of reflector current for the two orientations of the feed. The principally polarized components of the induced current density in the scan plane are plotted in Fig. 5 for an $H$-plane scan with the two different orientations of the feed. The straight-ahead orientation yields a maximum current density that is $1.00 \cdot \mathrm{~dB}$ higher than the current density produced by the vertex-look orientation. (For an $E$-plane scan the difference is $0,95 \mathrm{~dB}$ ). The two scan-plane stationary points (i.e., points of geometrical reflection) for that feed position-observation angle combination are indicated by $S_{1}$ and $S_{2}$, neither being particularly close to either current maximum. Since the relative phases of the contributions to the total field by various parts of the current distribution is a complicated function of position, the interpretation of these relative maxima is not completely clear. However, they probably account for the fact that the straight-ahead orientation produces 0.85 dB higher gain in spite of having 0.19 dB more spillover, e.g., compare $1.00-0.19=0.51 \mathrm{~dB}$ versus

H-FLAF SCAN

## D = 3f wavenctics



Fig. 6. Comparison of meximum transmit-gain contours and Petzral surface; $F / D=0.433$.
0.85 dB . Consequently, all maximum gain contours computed in this paper are for the feed axis directed parallel to the axis of the reflector. ${ }^{3}$

## C. Location of Transmit Feed to Achieve Maximum Gain

Fig. 6 shows the maximum scan-gain contours for $F / D=0.433$ and the four feed functions $n=1,2,3$ and infinitesimal dipole. Fine structure of the order of 0.1 dB peak-to-peak has not been included. The dipole curve is identical to the maximum receive-field contour in Fig. 4. The Petzval surface is also plotted in the figure. It is evident that the maximum-gain contours are slightly on the focal-plane side of the Petzval surface, although the difierence is a relatively small fraction of a wavelength. As the scan angle is increased, or the edge taper is increased, the maximum-gain contours approach the Petzval suriace.

Fig. 7 presents the scan losses along the maximumgain contours. The abscissa is half-power beamwidths (HPBWs) scanned for the 34 wavelength aperture. The scan-loss curves do include spillover loss, since spillover occurs as the feed is scanned laterally while pointing straight ahead. Also plotted in the figure, however, is the component of loss due exclusively to the decreasing fraction of the total power intercepted by the reflector as the feed is scanned. This spillover loss amounts to a fraction of a $d B$, while the remaining scan loss is due to phase defocusing and other effects.

Fig. 8 presents a similar set of maximum-gain contours for the same class of feeds but an $F / D$ value of 0.604 . Essentially, the same phenomena take place as for the deeper dish of Fig. 6, but there is a larger and more significant separation between the maximum-gain contours and the Petzval surface. For this value of $F / D$ the contours lie relatively close to the focal plane before drooping toward the Petzval surface at larger scan angles. These results are consistent with the limited experimental data of [3]. Fig. 9 is a plot of scan loss versus beamwidths of scan for $F / D=0.604$ and the 3 feeds under consideration. For a given edge taper this shallower reflector is subject

[^16]

Fig. 7. Beam-scan losses; $F / D=0.433$.


Fig. 8. Comparison of maximum transmit-gain contours and Petzval surface; $F / D=0.604$.


Fig. 9. Beam-scan losses; $F / D=0.604$.


Fig. 10. Comparison of maximum-gain and Petzval felds.
to larger spillover loss but less total scan loss with scan sngle.

Fig. 10 presents the $d B$ difference between the fields of the maximum-gain contour and the fields of the Petzval surface. For $F / D=0.433$ the difference is only a tenth of a dB or so. However, for $F / D=0.604$ the difference is a significant fraction of a dB . Since the refiectors with higher $F / D$ values are generally used for beam scanning applications because of reduced scan loss, this effect is of practical interest in wide-angle scanning systems.

## IV. Somalary of Results and Conclosions

1) Unless the $F / D$ is very large or spillover is excessive, a higher scan gain is achieved when the axis of a directional feed is parallel to the axis of the reflector than when the feed is directed toward the reflector vertex.
2) The contour of maximum scan-gain is a function of illumination taper and reflector $F / D$. In general, larger $F / D$ values (greater than 0.5 ) tend to have a maximumgain contour close to the focal plane, while the smaller $F / D$ values tend to have a maximum-gain contour closer to the Petzral surface. Increasing the illumination taper moves the maximum-gain contour closer to the Petzval surface.
3) The maximum-gain contours in Figs. 6 and 8 are blotted on coordinate axes expressed in terms of waveengths for two different $F / D$ values. Similar contours are plotted in Fig. 11 for 10, 15, and 20 dB illumination baige taper for the same class of feed functions $\cos ^{n} \theta$. However, the axial and lateral components of feed displarement are divided by the $F / D$ value. Three solid naximum-gain curves corresponding to $F / D$ values of 1.433 ( $60^{\circ}$ edge angle), 0.604 ( $45^{\circ}$ edge angle), and 0.687 ( $40^{\circ}$ edge angle) are plotted for each edge taper. ${ }^{4}$ Superimposed on each figure are dashed curves to indicate the can angle in HPBWs. Normalizing the coordinate axes Dy the $F / D$ values makes the maximum-gain contours considerably less sensitive to $F / D$ value than the results xhibited in Figs. 6 and 8. In a sense, then, these curves re "universal" and can be used to select the feed position for maximum scan gain over a wide range of scan angle Ind reflector shapë by simple interpolation.
4) Because of limitations imposed by available computer time and the requirements of the program study, all results to this point were computed for an aperture iameter of 34.0 wavelengths. Data for two additional maximum-gain contours were also generated for twice the frequency ( $D=68.0$ wavelengths). This limited study felded the following results:
a) For'a given illumination taper and $F / D$, the lateral component of scan (in wavelengths) for a particular Pan angle (in HPBWs) was the ssme at both 34 and 68 avelengths diameter. If this result can be extrapolated

[^17]

Fig. 11. Maximum transmit-gain contours versus beamwidths of scan; 10,15 , and 20 dB edge taper.
to other frequencies, it implies that the lateral component of the curves in Fig. 11 are also "universal."
b) For a given illumination taper and $F / D$, the axial component of scan for a particular scan angle was the same at the two frequencies for small scan angles. However, for large scan angles, the axial component of scan was smaller at 68 wavelengths than at 34 wave lengths.
c) For a given illumination taper $F / D$ and scan angle (in HPBWs), the 68-wavelength gain underwent considerably less scan loss than the 34-wavelength gain.

## Appendin

The correlation between the focal region field $\dot{E}_{1}$ and the feed aperture field $\bar{E}_{2}$ is approximately ${ }^{5}$.

$$
\begin{equation*}
\eta \cong\left|\int_{S} \bar{E}_{1} \cdot \bar{E}_{2} d S\right|^{2} \tag{A-1}
\end{equation*}
$$

where $S$ is the feed aperture. Considering only the principle polarization,

$$
\begin{align*}
& E_{1} \cong 1-\left(\frac{r}{\alpha}\right)^{2}  \tag{A-2}\\
& E_{2} \cong\left[1-\left(\frac{r}{\alpha}\right)^{2}\right] \exp \left(j k_{\mathrm{eff} r} \cos \phi \sin \delta\right) \tag{A-3}
\end{align*}
$$

where

$$
\begin{array}{ll}
r, \phi & \text { circular coordinates of the feed aperture } \\
\alpha & \text { the feed aperture radius, wavelengths } \\
\delta & \text { the tilt of the feed aperture relative to the focal- } \\
& \text { region field phase front } \\
k_{\mathrm{e} f 1} \quad=2 \pi / \lambda_{\mathrm{eff}} .
\end{array}
$$

[^18]Then
$\eta \cong\left|\int_{0}^{\pi} \int_{0}^{\alpha}\left[1-\left(\frac{\tau}{\alpha}\right)^{2}\right]^{2} \exp \left(j k_{\text {eff }} \cos \phi \sin \delta\right) r d \tau d \phi\right|^{2}$.

$$
\begin{equation*}
=\left[1_{\pi} \pi \alpha^{2} \frac{J_{3}\left(k_{\mathrm{ef}} \alpha \sin \delta\right)}{\left(k_{\mathrm{eff} \mathcal{} \alpha} \sin \delta\right)^{3}}\right] . \tag{A-4}
\end{equation*}
$$

For the example considered in Section III-B, $\lambda_{\text {eff }}=1.23 \lambda$ (cf., Fig. 3). If $\alpha=0.53 \lambda$, the field of a circular aperture closely approximates the assumed $\cos ^{2} \theta$ feed pattern. If $\alpha=0.71 \lambda$ the received field contours are closely matched. For the purpose of evaluating (A-4), an average value of $\alpha=0.62 \lambda$ was used. If the hypothetical feed is pointed toward the vertex of the paraboloid, it would be rotated by $\delta=21.5^{\circ}$ relative to the focal-region phase front, and equation (A-4) gives a predicted loss (relative to $\delta=0^{\circ}$ ) of 0.74 dB . This value is in reasonable agreement with the computed value of 0.85 dB .

## References

[1] S. Siver and C. S. Pao, "Paraboloid antenna characteristics as a function of feed tilt," Radiation Lab., MIT, Cambridge, M18s5., Rep. 479: 1944.
[2] F. B. Hildebrand, "The alternation in the radiated field of a parsboloid due to a shift in the position of the dipole feed." Rsidistion Lab. MIT, Cambridge, Mass., Rep. 1078, Feb. 20, 1946.
[3] K. S. Kelleher and H. P. Coleman, "Off-axis characteristics of the paraboloidal reflector," Naral Research Laboratory, Frashingion, D. C., NRL Rep. 4088, Dec. 31, 1952.
[4] Y. T. Lo, ton the beam deviation factor of a parabolic refiector," ${ }^{\prime \prime}$ IRE Trans. Artennas Propagat. (Commun.), vol. AP- $\mathcal{L}, \mathrm{pp}$. $347-349$, May; 1960.
i5: S. S. Sandler, "Paraboloidel reflector patterns for off-axis fed." IRE Trans. Antennas Propapat., vol. AP-8, pp. 368-379, Julr 1960.
[6] R. C. Hansen, Microwave Scanning Antennas. New York: Academic Press, 1964, vol. I, ch. 2.
[7] J. Puze, "Lateral-feed displacement in a paraboloid," IEEE Trans. Andennas Propagat., vol. AP-13, pp. 660-665, Sept. 1965.
[8] J. Ven Bladel, ElectromagneticFields. New York: McGraw-Hill, 1964, p. 205.
19] P. E. Collin and F. J. Zucker, Antenna Theory. New York: M1cGram-Hill, 1969, pt. 1, pp. 93-98.
[10] B. MacA. Thomas, "Matebing focal-region fields with bybrid modes," IEEE Trans Antennas Propagat. (Commun.), vol. AP-18, pp. 404-405, May 1970.
[11] A. W. Rudge and M. J. Withers, "Design of flared-horn primary
feeds for parabolic reflector antennas," Proc. Inst. Elec. Eng. vol. 117, no. 9, pp. 1741-1744, Sept. 1970
[12] H. C. Minnett and B. MacA. Thomas, "Fields in the image space of symmetrical focusing reflectors," Proc. Inst. Elec. Eng., vol. 115, no. 10, pp. 1419-1430, Oct. 1968.
[13] A. W. Rudge and M. J. Withers, "New technique for beam steering with fixed paraboloid reflectors," Proc. Inst. Elec. Eng., vol. 118, no. 7, pp. 857-863, July 1971.
[14] "Calculation of the caustic (focal) surface when the reflecting surface is a paraboloid of revolution and the incoming rays are parallel,"" Parke Math. Lab., Concord, Mass., Study 3, Contract AF 19(122)-484 for AFCRL, May 1952.
[15] "Calculations of the caustic surface of a paraboloid of revolution for an incoming plane wave of twenty degrees incidence," Parke Math. Lab., Concord, Mass., Rep. 1, Contract AF19(604)263 for AFCRL, May 1952.
[16] I. W. Kay and M. Goldberg, "Investigation of electromagnetic fields in the focal regions of a paraboloid receiving off-axis," Conductron, Ann Arbor, Mich., Rep.D5220-448-P410, Contract AF19(628)-5812, June 24, 1966.
[17] F. S. Holt, "Application of geometrical optics to the design and analysis of microwave antennas," AFCRL, Bedford, Mass., AFCRL-67-0501, Sept. 1967.
[18] R. E. Collin, and F. J. Zucker, Antenna Theory. New York: McGraw-Hill, 1969, pt. 2, pp. 16-103.
[19] M. S. Affif, "Aberration and dispersion off the focus of a parabola," in 1971 G-AP Symp. Dig., p. 219.
[20] J. B. Scarborough, "The caustic curve of an off-axis parabola," Appl. Opt., vol. 3 , no. 12, pp. 1445-1446, Dec. 1964.
[21] W. H. Watson, "The field distribution in the focal plane of a paraboloidal reflector," IEEE Trans. Antennas Propagat., vol. AP-12, pp. $561-569$, Sept. 1964.
[22] R. G. Kouyoumjian, "Asymptotic high frequency methods," Proc. IEEE, vol. 53, pp. 861876 , Aug. 1965.
[23] W. V. T. Rusch and P. D. Potter, Analysis of Reflecior Antennas. New York, Academic Press. 1970, 141-142.
[24] A. C. Iudwig, "Computation of Radiation patterns involving numerical double integration," IEEE Trans. Aniennas Propagat. (Commun.), vol. AP-16, pp. 767-769, Nov. 1968.
[25] E. M. Kennaugh and R. H. Ott, "Fields in the focal region of a parabolic receiving antenna," Antenna Lab., the Ohio State Uniy. Research Foundation, Columbus, Ohio, Rep. 1223-15, Contract AF33(616)-8039, Aug. 31, 1963.
[26] H . Gniss and $G$. Ries, "Feldbild um den Brennpunkt., von Parabolreflektoren mit kleinem f/D-Verhaeltnis," Arch., Elek. Uberiragung, vol. 23. no. 10, pp. 481-488, Oct. 1969.
[27] P. G. Ingerson and W. V. Tusch, "Studies of an axially defocused paraboloid." in 1969 G-AP Symp. Dig., pp. $62-68$.
[28] M. Landry and $Y$. Chasse, "Measurement of electromagnetic field intensity in focal region of wride-angle paraboloid refiector," IEEE Trans. Antennas Propagal., vol. AP-19, pp. 530-543, July 1971 .
[29] A. E. Conrady, Applied Optics and Optical Design. New York: Dover 1960, p. 290.
[30] F. A. Jenkins and H. E. White, Fundamentals of Optics, 3rd ed. New York: McGraw-Hill, p. 150.
[31] H. P. Brueggemann, Conic Mirrors: London, England: Focal Press, p. 30.
[32] K. G. Habell and A. Cox, Enoineering Optics. London, England: Pitman \& Sons, pp. 62-93.
[33] L. C. Martin, Technical Optics, volume II. Nen York: Pitman Publ., p. 75.

# Large Lateral Feed Displacements in a Parabolic Reflector o 

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#### Abstract

The radiation patterms of a parabolic reflector with large lateral-feed displacements are computed utilizing both the vector carrent method and scalar aperture theory, and compared to erperimental results. The theory is general enough to include asymmetric primary pattem illumination. The scalar and vector solutions are derived from the same initial equation so that the approximations used in obtaining the scalar solution are clearly displaped. Results from the vector and scalar theories are compared and the range of validity of the approximate analysis is indicated.


## I. INTRODUCTION

$A^{\mathrm{s}}$THE TREND toward multiple beams in spacecraft antennas continues, and in particular when multiple beam systems are obtained utilizing multiple feeds in a parabolic reffector, the evaluation of the performance of refiector systems Then there is large lateral feed displacement becomes increasingly important. Previous analyses have treated the case of a few wavelengths of lateral displacement but there is little discussion of the limits of the range of scan, nor whether the concept of a beam deviation factor is valid for large ( 10 to 20 wavelengths) amounts of scan [1]-[5]. The purpose of this paper is to examine reflector performance under the condition of large feed displacements. In particular, the complete solution taking into account all of the phase errors resulting from lateral feed displacement is compared to the approximate formulation of Ruze [1]. The complete and approximate solutions are derived from the same initial equation so that the approximations used in obtaining the Ruze type of solution are clearly displayed. The approximate formulation is generalized to include asymmetric primary illumination. The calculated results are compared to experimental measurements obtained using a $9-\mathrm{ft}$ reflector.

## $" \rightarrow \quad$ II. ANALYSIS

The far-field secondary pattern for the reflector system of Fig. I is given by

$$
\begin{array}{r}
\dot{E}(P)=\frac{-j \omega \mu}{4 \pi} \frac{\exp \cdot(-j k R)}{R} \int_{s u r f a c e}\left[\bar{J}_{s}-\left(\bar{J}_{s} \cdot \bar{a}_{R}\right) \bar{a}_{R}\right] \\
\cdot \exp \left(j k \bar{p} \cdot \bar{a}_{R}\right) d S \tag{1}
\end{array}
$$

where $\bar{J}^{\prime}$ is the reflector surface current, $k=2 \pi / \lambda, \bar{a}_{R}$ is the unit position vector of the far-field point, $d S$ is the incremental surface area, and the quantities $R, \rho$, and $P$ are as defined in Fig. 1.

[^19]

Fig. 1. Reflector coordinate system.
The solution will be determined in the following manner. 1) The incident field is formulated in the double primed coordinate system, hence the induced current $\bar{J}$ : obtained using the physical optics approximation is expressed in the same coordinate system. 2) A change of variables converts the $\bar{J}_{\text {s }}$ to the reflector (prime) coordinate system. 3) For the complete solution a two-dimensional numerical integration of (1) is required and for the scalar solution, approxiniations can be made to reduce (1) to a one-dimensional integral. Before exploring the solution further, it will prove convenient to rerrite (1) in a slightly: different form.
Letting

$$
\begin{equation*}
\left[\bar{J}_{s}-\left(\bar{J}_{s} \cdot \bar{a}_{R}\right) \bar{a}_{R}\right]=\bar{G}\left(\theta^{\prime}, \phi^{\prime}, \theta ; \phi\right) \frac{\exp \left[-j k p^{\prime \prime}\left(\theta^{\prime} ; \dot{\phi}^{\prime}\right)\right]}{. \rho^{\prime \prime}\left(\theta^{\prime} ; \phi^{\prime}\right)} \tag{2}
\end{equation*}
$$

and further utilizing the definitions,

$$
r=\frac{\rho}{a} \sin \theta^{\prime} \quad u=\frac{\pi D}{h} \sin \theta \quad D=2 a
$$

the tro-dimensional vector integral for the far zone electric field given in (I) can be cast in the form

$$
\begin{align*}
\dot{E}(P)= & -j \frac{\exp (-j k R)}{R} \frac{k \eta o a^{2}}{2 f} \int_{0}^{1}\left[\int_{0}^{i \pi} \bar{G}\left(\theta^{\prime}, \phi^{\prime}, \theta ; \phi\right) \csc \frac{\theta^{\prime}}{2}\right. \\
& \cdot \exp \left(-j k\left(\rho^{\prime \prime}-\rho \cos \theta \cos \theta^{\prime}\right)\right)\left(\frac{\rho}{\rho^{\prime \prime}}\right) \\
& \left.\cdot \exp \left(j u r \cos \left(\phi-\phi^{\prime}\right)\right) d \phi^{\prime}\right]\left(1-\cos \theta^{\prime}\right) r d r \tag{3}
\end{align*}
$$

The only remaining step for the vector solution is to specify the vector quantity $\bar{G}$. To this end we cast the incident field in its most general form in the double primed coordinate system as follows:

$$
\begin{equation*}
\bar{E}_{\mathrm{inc}}=\frac{\exp -j k \rho^{\prime \prime}}{\rho^{\prime \prime}}\left(F_{1}\left(\theta^{\prime \prime}, \phi^{\prime \prime}\right) \hat{a}_{t^{\prime \prime}}+F_{2}\left(\theta^{\prime \prime}, \phi^{\prime \prime}\right) \hat{a}_{\phi^{\prime \prime}}\right) \tag{f}
\end{equation*}
$$

Recalling that the physical optics approximation inplies that $\bar{J}_{s}=2 \hat{n} \times \bar{H}_{\text {inc, }}$ where $\bar{n}$ is the outward normal from the surface and that

$$
\bar{H}_{\text {inc }}=\frac{\hat{a}_{0}{ }^{\prime \prime} \times \bar{E}_{\text {ine }}}{\eta_{0}}
$$

we can calculate the current $\bar{J}_{z}$ induced on the reflector surface by the incident $E$-field from (4). A simple trigonometric manipulation is then required to transform the double primed angles ( $\theta^{\prime \prime}, \phi^{\prime \prime}$ ) into the ( $\theta^{\prime}, \phi^{\prime}$ ) coordinate system. This transformation is necessary so that the incident field given by (4) is stationary with respect to the feed. Fnowing $\bar{J}_{s}$, the vector $\bar{G}$ given by (2) can easily be obtained by vector manipulation. Equation (3), along with the quantities $F_{1}$ and $F_{2}$ from the incident field allow one to calculate the efiects of lateral scan from the complete vector description of the problem.

The scalar aperture result will now be developed from (3). The incident field is approximated in the following manner:
$\bar{E}_{\text {inc }}=\frac{\exp \left(-j k \rho^{\prime \prime}\right)}{\rho^{\prime \prime}}\left(E_{P}^{\prime}\left(\theta^{\prime}\right) \sin \phi^{\prime} \hat{u}_{\mathrm{e}^{\prime}}-H_{P}\left(\theta^{\prime}\right)\right.$

$$
\begin{equation*}
\left.\cdot \cos \phi^{\prime} \hat{a}_{e^{\prime}}\right) \tag{5}
\end{equation*}
$$

The suriace current is again obtained using the physical oprics approximation where $E_{P}\left(\theta^{\prime}\right)$ is the $E$-plane radiation pattern and $H_{P}\left(\theta^{\prime}\right)$ is the $H$-plane radiation pattern.
There will in general be $x, y$, and $z$ directed current. components. For the scalar case, which assumes fields near the boresight, we ignore the $z$ and $x$ directed current components and utilize only the $y$ directed currents so that the secondary radiation field has a principal polarization vector component ( $\hat{\mathrm{a}}_{\mathrm{f}}$ directed) in the ( $\theta_{2}, \phi_{2}$ ) coordinate system ö Fig. 1. The vector $\bar{G}$ is therefore approximated by

$$
\begin{align*}
\bar{G}=-\left(1-\sin ^{2} \theta \sin ^{2} \phi\right)^{1 / 2} \frac{2}{\eta_{0}} & \left(-\sin \frac{\theta^{\prime}}{2}\left[\sin ^{2} \phi^{\prime} E_{P}\left(\theta^{\prime}\right)\right.\right. \\
& \left.\left.+\cos ^{2} \phi^{\prime} H_{P}\left(\theta^{\prime}\right)\right]\right) \delta_{\theta_{\theta}} \tag{6}
\end{align*}
$$

In the vector formulation when the feed is laterally defocused, the illumination function is assumed stationary with respect to the feed and the distance function $\rho^{\prime \prime}$ is obtained exactly using the law of cosines. In the scalar formulation, the illumination function is assumed stationary with respect to the reflector so that the trans. formation from double primed to single primed crordinates is not necessary; and the lateral feed displacement is
accounted for by the distance function $\rho^{\prime \prime}$ in the phase only, $\rho^{\prime \prime}$ being approximated by the parallel ray approximation, as follows. Let $\bar{\rho}^{\prime \prime}=\bar{\rho}-\bar{d}$, where the feed displacement $\bar{d}=\epsilon_{\tau} \bar{a}_{z}+\epsilon_{y} \bar{a}_{\nu}+\epsilon_{\pi} \bar{a}_{z}$ whence, if we assume that ( $d / \rho$ ) < 1 , we obtain
$\rho^{\prime \prime} \approx \rho-\left(\epsilon_{x} \sin \theta^{\prime} \cos \phi^{\prime}+\epsilon_{\mu} \sin \theta^{\prime} \sin \phi^{\prime}+\epsilon_{\mathrm{x}} \cos \theta^{\prime}\right)$.
Further, we make the assumption that the observation point is near the boresight so that $\cos \theta \approx 1$ and $\sin \theta \ll 1$, which implies that $\rho\left(1-\cos \theta \cos \theta^{\prime}\right) \sim 2 f$, and the square root of (6) can be replaced by 1. With these approximations, (3) can be shown to reduce to

$$
\begin{align*}
F(P)= & -j \exp (-j 2 f) \frac{\exp (-j k R)}{4 \pi R} k\left(\frac{a^{2}}{f}\right) \\
& \cdot \int_{0}^{1} \exp \left(j h \epsilon_{2} \cos \theta^{\prime}\right) I(r)\left(1-\cos \theta^{\prime}\right) r d r \tag{8}
\end{align*}
$$

where

$$
\begin{align*}
I(\tau)= & \int_{0}^{2 \tau}\left[\sin ^{2} \phi^{\prime} E_{P}^{\prime}\left(\theta^{\prime}\right)+\cos ^{2} \phi^{\prime} H_{P}\left(\theta^{\prime}\right)\right] \\
& \cdot \exp \left[j k\left(\epsilon_{x} \sin \theta^{\prime} \cos \phi^{\prime}+\epsilon_{y} \sin \theta^{\prime} \sin \phi^{\prime}\right)\right] \\
& \cdot \exp \left[j u r \cos \left(\phi-\phi^{\prime}\right)\right] d \phi^{\prime} . \tag{9}
\end{align*}
$$

Note that this would be identical to the formulation given by Ruze if we used the assumed illumination function

$$
\begin{equation*}
f\left(r, \phi^{\prime}\right)=\sin ^{2} \phi^{\prime} E_{P}\left(\theta^{\prime}\right)+\cos ^{2} \phi^{\prime} H_{P}\left(\theta^{\prime}\right) \tag{10}
\end{equation*}
$$

Now; making use of some trigonometric manipulations similar to those employed by Ruze, incorporating the plane wave to cylindrical wave transformation, and using the following definitions

$$
\begin{aligned}
& \omega=\left[(u r)^{2}+2 k \epsilon_{R} \sin \theta^{\prime} u r \cos (\phi-\xi)+\left(k \epsilon_{R} \sin \theta^{\prime}\right)^{2}\right]^{1 / 2} \\
& \alpha=\tan ^{-1} \frac{\left(u r \sin \phi+k \epsilon_{R} \sin \theta^{\prime} \sin \xi\right)}{\left(u r \cos \phi+k \epsilon_{R} \sin \theta^{\prime} \cos \xi\right)} \\
& \epsilon_{R}=\left(\epsilon_{x}{ }^{2}+\epsilon_{\nu}^{2}\right)^{21 / 2}
\end{aligned}
$$

$$
\begin{equation*}
\xi=\tan ^{-1} \frac{\epsilon_{y}}{\epsilon_{x}} \tag{11}
\end{equation*}
$$

we obtain the final result

$$
\begin{align*}
E(P)= & -j \exp (-2 j f) \frac{\exp (-j k R)}{R}\left(\frac{\pi D}{\lambda}\right)\left(\frac{D}{4 f}\right) \\
& \cdot \int_{0}^{1}\left(1-\cos \theta^{\prime}\right) \exp \left(j k \epsilon_{z} \cos \theta^{\prime}\right) \\
& \cdot\left\{J_{0}(\omega) \frac{E_{P}+H_{P}}{2}+\frac{E_{P}-H_{P}}{2} J_{2}(\omega) \cos 2 \alpha\right\} r d r \tag{12}
\end{align*}
$$

Note that (12) reduces to Ruze's result for $\epsilon_{z}=0$ and $E_{P}=H_{P}$.

Equation (12) can now be interpreted as the principal


Fig. 2. Measured secondary patterns as function of lateral primary feed displacement.


Fig. 3. Secondary beam squint and beam scan loss as function of latersl displacement of primary feed in focal plane.
polarization component in the ( $\theta_{2}, \phi_{2}$ ) coordinate system utilizing approximations for the incident field $\rho^{\prime \prime}$ and some small angle assumptions.

## III. DESCRIPTION OF EXPERINIENTAL MEASUREMENTS

Heasurements were performed on a $1750-\mathrm{ft}$ antenna test range. A $9-\mathrm{ft}$ diameter high precision experimental reflector, with an $f / D=0.4$ and a rms surface error of 0.004 in, was used with a circular 1 -in inside diameter open ended waveguide primary feed. At the testing frequency of 7.9 GHz the primary feed bad a $3-\mathrm{dB}$ beamwidth of $64^{\circ}$ in the $E$-plane and $70^{\circ}$ in the $H$-plane, which gives an edge taper of approximately 9.5 dB in the $E$-plane and 8.7 dB in the $H$-plane. A standard Scientific Atlanta positioner was used to support the reflector but a special


Fig. 4. Comparison of theoretical and measured gains as function of axisl defocusing.
track was added to the base arm of the positioner. A carriage was mounted on this track, and a thin support mast was placed vertically up from the carriage and supported the feed in front of the reflector. This experimental arrangement allows minimum blockage and accurate movement of the feed axially and laterally. Further details of the experimental setup can be found in [6].

The primary feed's radiation patterns were recorded everywhere over the far-field sphere. In particular, the functions $F_{2}\left(\theta^{\prime \prime}, \phi^{\prime \prime}\right)$ and $F_{2}\left(\theta^{\prime \prime}, \phi^{\prime \prime}\right)$ of (4) and the functions $E_{P}\left(\theta^{\prime}\right)$ and $H_{P}\left(\theta^{\prime}\right)$ of (5) were extracted from the measured data, digitized, and used as input for the computer programs implementing (3) and (12).

A sample of the measured secondary patterns as a function of lateral primary feed displacement in the focal plane with the feed aligned parallel to the $z$ axis is shown in Fig. 2. The axis of the feed was maintained parallel to the $z$ axis because a higher scan gain was achieved than by directing the feed towards the apex [7]. The measured data is summarized and compared to the calculated data in the following figures. Fig. 3 shows the pattern peak gain and angular position as a function of lateral defocusing in the focal plane. The half-power beamwidth for the focused feed is approximateiy $1^{c}$, bence the beam has been scanned 29 half-power beamwidths at the extreme end. "Vector" refers to the results using the vector formulation of (3), and "scalar"' refers to the results using (12). It is noted that for beam squint the agreement is very good for both calculations and, in fact, the slight discrepancy at the extreme angles was traced to the primary fixture sag at this extreme range. Observe that the scalar analysis yields higher gain than both the vector formulation and the measured data.

Fig. 4 explores the scan-plane fields for axial feed movements with no lateral displacement and lateral displaced 9.7 wavelengths in the $H$-plane (approximately 18 halfpower beamwidths of scan). Several pertinent results are illustrated. The scalar formulation indicates that the highest gain occurs in the focal plane whereas the measured


Fig. 5. Coms-lobe gain level relative to beam peak as function of lsieral displacement of primary feed in focal plane.


Fig. 6. Comparison of scalar and vector gain versus dish diameter.
data peshs of the focal plane. The difference in the measured anci the scaled peak gain is small. Since the data in Fig. 3 was measured in the focal plane, it is to be expected that the scalar gain, when it differs from the vector gain, rould yield ralues too high. The vector calculation correlates well with the measured data. Even though the antenna range on which the data was measured was approximately $2 \frac{1}{2} D^{2} / \lambda$, there is a very slight near-field shift in the measured on-axis defocusing curve, but this shift introduces negligible error into the data in Figs. 2, 3: and 5 .

Fig. 5 presents the coma-lobe gain level relative to the beam peak as a function of lateral displacement. Again, the approximate analysis can be misleading for scans greater than 10 half-power beamwidths.

For the measured data the reflector diameter was 72 Wavelengths. A plot showing the difference between the
peak gain in the focal plane of the scalar and the vector calculation for various scan angles and dish diameters is shown in Fig. 6. For a $10^{\circ}$ scan there is very little peak gain error for any dish diameter from 10 to 200 wavelengths even though at 10 wavelengths the scan is 1.5 half-power beannwidths and at 200 wavelengths the scan is about 29 half-power beamwidths. This plot indicates that the peak gain difference is more dependent upon the actual angular scan as contrasted to the scan measured in half-power beamwidths. For the various cases calculated the scalar and vector formulations compared favorably for the angular position of the peak.

## IV. CONCLUSIONS

Vector and scalar analyses were used to examine reflector performance under the condition of large feed displacement. Results from the two formulations were compared to experimental data obtained using a precision reffector with minimal blockage. Several pertinent observations can be made. The beam peak angle position is accurately predicted by both the vector and scalar theories. The peak gain is accurately predicted by the vector theory, but the scalar theory is several $d B$ in error for large scan angles. The scalar theory can be greatly in error for predicting. the coma-lobe peak. A comparison of the peak gain difference between the two theories indicates that the accuracy of the scalar theory is more dependent upon the actual amount of angular scan rather than upon the scan in terms of half-power beamwidths. The scalar analysis indicates that the highest gain level for a given lateral displacement occurs with the feed in the focal plane, whereas the vector formulation correctly predicts scanplane fields.

## REFERENCES

[1] J. Ruze, "Lateral-feed displacement in a paraboloid," IEEE Trans. Antennas Propagat., vol. AP-13, pp. 660-665, Sept. 1965.
[2] Y.T.Lo, "On the beam deviation factor of a parabolic Refiector," IRE Trans. Antennas Propagat. (Commun.), vol. AP-8, pp. 347-349, May 1960.
[3] K. S. Kelleher and H. P. Coleman; "Off-axis characteristics of the paraboloidal refiector," National Research Laborstories, Washington, D.C., Rep. 4088, Dec. 1952.
[4] S. S. Sandler, "Paraboloidal reflector patterns for of-axis feed"" IRE Trans. Antennas Propagat., vol. A.P-8, pp. 368-379, July 1960.
[5] S. Siver, Microwave Andenna Theory and Design. New York: McGraw-Hill 1949.
[6] W. A. Imbriale, P. G. Ingerson, and W. C. Wong, "Experimental verification of the anslysis of unbrella parabolic reflectors," IEEE Trans. Antennas Propagat., vol. AP-21, pp. 705-708, Sept. 1973.
[7] W. V.T. Rusch and A. C. Ludrig; "Determination of the maxi-mum scan-gain contours of a beam-scanning paraboloid and their relation to the Pitzval surface," IEEE Trans. Antennas Propagat.;: vol. AP-21, pp. 141-147, Mar. 1973.

## Multiple-Feed Systems 0 for Objectives

Before the advent of large high-performance array-type antennas, the microwave optical focusing objective, such as the lens or reflector, was the basic tool. Multiple beams have occasionally been formed, as in stacked beam height finders, by arraying feeds in the neighborhood of the focus.

The problem of multiple feeds for a focusing objective is a long-standing one in the field of microwave antennas. The dilemma is that simple independent feed radiators cannot be placed close enough together to obtain good beam crossover level if reasonable aperture efficiency is specified.

Since, in one dimension, the feed system is just a linear array, one might expect that a soiution can be obtained in terms of array factors and independent of element patterns, thereby eliminating the element partern and spacing problem.

Based on the results for a linear-array muluiple-beam system, one concludes that the secondary patterns, for maximum efficiency and minimum beam spacing, should be of the general form, $\sin \theta / \theta$. Further, the aperture illumination at the objective should be of uniform amplitude. Thus, the patien of an individual feed should be as nearly rectangular as possible, and finally, the amplitude distribution at the feed radiator should be of the form $\sin x / x$. The preceding chain of conclusions is based on the approximation that an aperture distribution and its far-field radiation patterns are Fourier transforms of each other.

Perhaps the best place to start on the problem, based on the previously mentioned ground rules, is to determine the accuracy with which the rectangular feed pattern can be realized. The maximum slope that can be achieved at the edge of the rectangular pattern is proportional to the aperture of the feed system. If the multiple-element feed system is considered as an array, the approximation of the required primary beam can be described by Fig. 1. The entire feed array is used to generate a set of pencil beams covering the objective, and the directivity of these beams, and, therefore, the slope at the edge of the composite pattern is proportional to the total aperture of the array.

At this point, since it starts to look like the solution to the problem may be an array fed by a hybrid matrix, one might ask why the objective is used at all; why not just use an array as the primary radiator and save all of the intervening steps? The answer is related to the original choice between the arrey and the geometrical focusing objective. The important factors are gain and coverage. If one needs a versatile antenna of low or medium gain, the array is usually the best answer. Further, if both high gain and large-solid-angle coverage, such as five or ten thousand square degrees, are required, the array is probably still called for. For the case of a high-gain antenna with a relatively small number of simultaneous beams, such
as ten or twenty, the array would represent a great waste of complexity, and a focusing objective is the best answer. Therefore, although the solution may seem complex, it must be remembered that once a multiplefeed system is designed, it can be used with any focusing objective of any size.

Referring again to Fig. 1, it is seen that the relative phase of the narrow beams determines the effective phase center of the fan beam that they comprise. If the narrow beams are fed in phase, the phase center of the fan beam is in the center of the array. If they are fed in progressive phase, the phase center is displaced from the center of the feed array. Thus, the ports of the hybrid matrix generating the pencil beams illuminating the objective can be fed by a second, smaller matrix to realize the optimally spaced phase centers corresponding to the desired aperture distributions.

Since no study or design program related
to this technigue has been carried out, a detailed theoretical example is presented. The radiating elements and feed matrices are shown schematically in Fig. 2. A 16-element array is fed by a 16 -port matrix. The initial design choice is the number of beams to use, which is determined by the angle subtended by the objective. In the case shown, the outer two beams on each side are discarded, and the angular coverage in $\psi(=2 \pi d \sin \theta) / \lambda$ is between $\pm(3 \pi / 4)$ and $\pm(13 \pi / 16)$, corresponding to the hall-power points and first nulls of the outer beams. If the radiators are half-wave spaced, the actual range of angular coverage is $\theta= \pm 48.6^{\circ}$ to $\theta= \pm 54.4^{\circ}$. The maximum allowable element spacing in this case is approximately $19 \lambda / 32$, for which the real angle coverage falls in the range $\theta= \pm 39.3^{\circ}$ to $\theta= \pm 44.3^{\circ}$.

Once the number of output beams has been selected, the size of the second matrix is determined- 12 ports in the example.


Fig. 1. Illumination of focusing objective with multiple beams from feed array.

16-element arroy of focol plone


10 useful feed positions
Fig. 2. Illustrative multifeed system for focusing objective.


Fig. 3. Representative patterns from multifed system.


Fig. 4. Aperture distributions and phase centers for multifed system.

There are 12 effective feed positions located along the 16 -element array. The performance of the system for each feed position is measured by the quality of the radiation pattern and by its effective phase center. These parameters have been calculated for the first and fifth off-axis positions. The patterns are shown in Fig. 3, and the aperture discributions and phase centers are illustrated in Fig. 4. It is found that the cheracteristics degrade slightly for the fifth position off axis. The radiation pattern b:comes more distorted and a poorer approximation to the desired rectangular shape. The phase center deviates further from the desired position. The degradation is not excessive, bowever. The inner feed position has an efiective phase center that is displaced from the ideal by only 0.027 element spacing. The outer feed position is displaced by 0.219
element spacing. When it is considered that the effective feed positions are separated by 1.333 element spacings, these errors are within reason.

The outermost feed positions are discarded because the performance will be sharply degraded by the splitting of the "main lobe" of the aperture distribution. One of the strongly excited elements slips off one end of the array and reappears on the other end with resultant pattern and phasecenter distortion. Thus, ten feed positions are obtained.

The relationship between secondarypattern beamwidth and beam spacing is a function of the focal length and aperture of the objective and the primary pattern beamwidth. It is interesting to note, however, that optimum secondary-pattern characteristics are obtained if the Abbe sine condition
is satisfed; that is, the aperture is given by $2 J \sin \theta_{0}$, where $f$ is the focal length and $\theta_{0}$ the half angle subtended by the objective. The sine condition is the criterion for optimum off-axis focus, and examples of objectives which satisfy it are the Luneberg lens and the spherical zoned-plate refiector.

This design approach is not perfect, of course, because some energy is inevitably spilled over the edge of the focusing objective, and the fan-shaped feed patterns also tend to deteriorate as the phase center is displaced farther from the center of the feed system. However, the important point is that the type of system shown in Fig. 2 does approach perfection in the limit of very many elements in the feed array.

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# A Proposed Multiple-Beam Microwave Antenna for Earth Stations and Satellites . 

By E. A. OHM<br>(Manuscript received October 3, 1973)


#### Abstract

An offset Cassegrainian anienna with essentially zero aperture blockage is expected to support closely spaced well-isolated beams suitable for earth stations and satellites. Each beam is fed with a separate small-fare-angle corrugated horn and has good area efficiency over a 1.75:1 bandwidth. Each beam also has good cross-polarization properties. The antenna is compact, and the design appears practical for a 4 and 6-GHz earth station, a 20- and SO-GHz earth station, and a 20- and $50-\mathrm{GHz}$ satellite.


## l. INTAODUCTION

Satellite communication systems with large capacities can be achieved if the satellites and earth stations are provided with multiple-narrow-beam antennas. ${ }^{1}$ The capacity is proportional to the number of satellites, and thus it is important to use as many as practical in the limited orbital space. A moderate number of the resulting closely spaced satellites can be served by a single antenna at each earthstation site if the antenna is patterned after the offset Cassegrainian antenna shown in Fig. 1. This design allows an orderly expansion in communication capacity by the addition of feed horns. Since only one antenna is needed at each site, the design also permits a large saving in earth-station costs. Good multiple-beam performance can be achieved across all up/down pairs of satellite frequency bands, including those well below 10 GHz . At 20 and 30 GHz , a large earth-station antenna with acceptable thermal and wind distortion is hard to achieve. However, with the design outlined here, these problems can be largely overcome because the main reffector and subreflector can be fixed in position, thus allowing a stiffer structure. The steering of each beam is achieved by moving one of the feed horns, resulting in a steerable angle sufficient for tracking near-synchronous satellites.

[^20]

Fig. 1-Geometry of antenna and feed system. The feed horns are scaled for a 3 m 20 and $30-\mathrm{GH} z$ satellite antenna. For a 30 m 4 and 6 -GHz earth-station antenna, $L$ and $2 a$ are half as large as shown.

The offset Cassegrain is also appropriate for use aboard a satellite because all beams, including those moderately far off-axis, have high area efficiencies and low side-lobe levels. However, good results on a satellite are restricted to bands well above 10 GHz because the antenna size is limited by the launch vehicle.

It has been previously shown that a multiple-beam antenna can be achieved in a variety of ways, ${ }^{2-5}$ where each approach has emphasized one feature desired in a practical antenna. By combining several of these with a corrugated feed horn ${ }^{6}$ and an enlarged subreflector, it is possible to achieve a compact antenna with exceptionally good multiple-beam characteristics. In particular, in the offset Cassegrainian antenna shown in Fig. 1:
(i) An offset design essentially eliminates beam blockage, thus allowing a significant reduction in side-lobe level.' This, in turn, results in higher isolation between beams and a lower antenna noise temperature.
(ii) The Cassegrainian feed system is compact and has a large focal-length-to-diameter ( $F / D$ ) ratio. ${ }^{8}$ The large $F / D$ ratio reduces aberrations to an acceptable level, even when a beam is moderately far off-axis.
(iii) A corrugated feed horn is essentially a Gaussian-beam launcher ${ }^{9}$ and, as such, it can be used to achieve beams with low side-lobe levels. The corresponding feed-horn aperture ${ }^{6}$ is small enough to allow the beams to be closely spaced.
(iv) An enlarged subreflector, as indicated by the dashed line in Fig. 1, allow's the main refiector to be properly illuminated, even when a beam is moderately far off-axis.

These features can be achieved over a wide range of antenna parameters. Using the results developed here, the sample calculations summarized in Table I show that ( $i$ ) the off-axis beam angles are practical, (ii) the coma aberration is small, (iii) the feed-horn dimensions are reasonable, and (iv) the isolations between beams are large.

## II. OFF-AXIS DESIGN CRITERIA

Consider a parabolic reflector that is circularly symmetric and illuminated with a feed at its prime focus. If the aperture is large in wavelengths and the prime focal-length-to-diameter, $F^{\prime} / D$, ratio is 2 or more, it is well-known that a beam can be scanned over tens of beamwidths by lateral displacement of the feed. ${ }^{2}$ A Cassegrainian antenna normally has a secondary focal length $F$ larger than $F^{\prime}$, and thus a larger $F / D$ ratio. ${ }^{8}$ Consequently, a scanned beam can also be obtained by displacing a feed at the secondary focus. ${ }^{5}$ For the small ofi-axis angle reported in Ref. 5 ( 4 beamwidths $\equiv 0.9^{\circ}$ ), the on-axis and off-axis beam characteristics are nearly identical, and the residual differences can be readily explained in terms of an equivalent parabola. ${ }^{5,8}$ The equivalent parabola, in turn, has characteristics identical to those of a prime-focus parabola. Consequently, the prime-focus theory: can be used to predict the off-axis equivalent-parabola results, and thus the Cassegrainian results. This chain of reasoning assumes that the equiv-alent-parabola concept is valid for the antenna parameters ( $F^{\prime} / D$ and $F / D$ ratios and off-axis angles) considered here. In support of this assumption, it is of interest to note that the chief off-axis beam parameter of a prime-focus parabola, namely, ${ }^{2}$

$$
\begin{equation*}
X^{\prime}=\frac{N\left(\frac{D}{F^{\prime}}\right)^{2}}{1+0.02\left(\frac{D}{F^{\prime}}\right)^{2}} \tag{1}
\end{equation*}
$$

MULTIPLE-BEAM MICROWAVE ANTENNA
where $N$ is the off-axis angle in half-power beamwidths, has a value in Ref. 5 of about 30 . Thus, the equivalent-parabola concept is valid for $X^{\prime}$ values at least through 30 . Furthermore, the known results indicate that the region of validity can be extrapolated to $X^{\prime}$ values well beyond 30 . In particular, Ref. 5 shows that the coma lobe, which is the first side lobe aimed toward the on-axis direction, increases very slowly as a function of off-axis beam angle. From Ref. 2, it is also known that an increase in coma-lobe level is a sensitive leading indicator of serious aberration problems, and that $X^{\prime}$ increases rapidly with coma-lobe level. It follows that $X^{\prime}$ in Ref. 5 can be much larger than 30 before a larger increase in coma-lobe level signals the onset of serious aberrations. The upper limit of $X^{\prime}$ should and can be calculated but, in the meantime, some of the results in Table I include an engineering judgment that the equivalent-parabola concept is valid for $X^{\prime}$ values through 45 . Even if the upper limit turns out to be somewhat less, the offset Cassegrain can still support a respectable number of multiple beams, i.e., for $X^{\prime}=30$, the number of $1^{\circ}$-spaced beams from the earth-station antenna of Table $I$ is 7 rather than 11.

An important parameter of an off-axis beam is the third-order phase error across the beam at the antenna aperture. This error, $\Delta \phi$, increases the level of the coma lobe. ${ }^{2}$ For a symmetrical parabola illuminated with a feed displaced laterally from the prime focus, the peak value of $\Delta \phi$ at the edge of the aperture can be calculated from eq. (12) of Ref. 2. Similarly, when an offset parabola (as in Fig. 1) is illuminated with a feed displaced laterally from the prime focus (in the $x$ direction in Fig. 1), the maximum third-order phase error, $\Delta \phi^{\prime}$, which occurs at the side edge of the aperture, can be calculated from ${ }^{10}$

$$
\begin{equation*}
\Delta \phi^{\prime}=\frac{2 \pi}{32} \frac{F^{\prime}}{\lambda} \frac{\sin \theta}{\left(F^{\prime} / D\right)^{5}} \frac{1}{1+\left(Y_{2} / 2 F^{\prime}\right)^{2}}, \tag{2}
\end{equation*}
$$

Where $F^{\prime}$ is the prime focal length; $\theta$ is the off-axis angle of the beam, $D$ is the diameter of the offset aperture, and $Y_{2}$ (see Fig. 1) is the ofiset beight of the aperture. Equation (2) assumes that the feed is also displaced slightly in the longitudinal direction (the $-z$ direction in. Fig. 1) to cancel field curvature.

Comparison of eqs. (12) and (13) of Ref. 2 shows that $\Delta \phi^{\prime}$ is proportional to $X^{\prime}$. Noting that $\Delta \phi^{\prime}$ in (2) is defined in terms of the aperture diameter, $D$, independently of whether the aperture is centered or off set, it follows that $D$ in eq. (1) should be interpreted in the same way, i.e., it is the diameter of the offset aperture, $D$, and not the diameter of the aperture of the full parabola ( $8 / 3 D$ in Fig. 1).

If the prime-focus feed illuminating the offset parabola is replaced with a Cassegrainian feed system, as in Fig. 1, and the equivalentparabola concept is valid, $F^{\prime}$ in (1) and (2) can be replaced with the Cassegrainian focal length $F$. In Fig. $1, F$ is the distance $Z$ times the ratio of centerline-ray heights where they intercept the main and subreflector heights, i.e., $F=Z\left(Y_{2} / Y_{1}\right)$. For the antenna parameters listed in Table I, the values of $\Delta \phi$ calculated from (2) are substantially less than $90^{\circ}$. For these values, the first side lobe, or coma lobe, is increased in amplitude, but the side lobes which are positioned further out, i.e., those that determine the minimum spacing of well-isolated beams, are virtually unchanged. Accordingly, in the remainder of this paper, it is assumed that $\Delta \phi$ is zero. The corresponding values of $X$, which are calculated from (1) after replacing $F^{\prime}$ by $F$, are found to be 4.5 or less. From the plots given in Ref. 2, the off-axis and on-axis beam characteristics are essentially identical for these values of $X$.

## III. BEAM SPACING

Suppose the amplitude distribution across an unblocked aperture is that of a dominant-mode Gaussian beam, that the amplitude at the edge is truncated at the $-15-\mathrm{dB}$ point, and that the phase front is uniform. The envelope of the resulting radiation pattern is shown in Fig. 2. For the offset Cassegrain shown in Fig. 1, the above amplitude and phase distribution can be achieved by placing a corrugated feed horn ${ }^{6}$ at the secondary focal point, $f$. Comparison of Dragone's results ${ }^{9}$ with the standard Gaussian-beam equations ${ }^{11}$ shows that the radius of the beam, $\omega$, at the $-8.686-\mathrm{dB}$ (or $1 / e$ amplitude) point, is related to the feed-aperture radius, $a$, by

$$
\begin{equation*}
\omega=0.647 a . \tag{3}
\end{equation*}
$$

The comparison also shows that the phase-front radius is equal to the slant length of the feed-horn, $L$. Using Gaussian-beam equations, ${ }^{11}$ the beam parameters in any other region in the feed system can be calculated. One result is that the required feed-born length, $L$, can be found from the half-angle, $\gamma$, subtended at the focus $f$ by the subreflector, and the illumination taper, $T$, in dB , at the edge of the subreflector.

$$
\begin{equation*}
L=0.076 \frac{\lambda}{\gamma^{2}} T_{\mathrm{dR}} . \tag{4}
\end{equation*}
$$

Equation (4) includes the feed-horn design criterion ${ }^{6}$

$$
\begin{equation*}
a^{2} / \lambda L=1 \tag{5}
\end{equation*}
$$



Fig. 2-Estimate of the side-lobe envelope resulting from a Gaussian illumination taper truncated at the -15 dB point, courtesy of T. S. Chu.
where, for a $1.75: 1$ bandwidth, $\lambda$ is specified at the low end of the frequency range. Equation (4) is strictly valid only when $\gamma \gg \lambda / D_{\text {sub }}$, where $D_{\text {sub }}$ is the diameter of the subreflector. For an equivalent parabola with focal length $F,{ }^{8}$ it can be shown that the $\gamma$ criterion is automatically satisfied when the $F / D$ ratio is less than 5 .

The corresponding feed-horn aperture radius, $a$, is found by solving $a^{2} / \lambda L=1$ for $a$, and substituting $L$ from (4):

$$
\begin{equation*}
a=0.275 \frac{\lambda}{\gamma} \sqrt{T_{\mathrm{iB}}} . \tag{6}
\end{equation*}
$$

Suppose the antenna shown in Fig. I has a diameter-to-wavelength ratio, $D / \lambda$, in the hundreds, an equivalent focal length, $F$, and an $F / D$ ratio larger than 2 . Then if a second feed-horn is placed adjacent to the on-axis feed, the second beam will be aimed in an off-axis direction,
$\theta_{1}=2 a / F$. Inserting $a$ from Eq. (6) and noting ${ }^{8}$ that $\gamma=D / 2 F$;

$$
\begin{equation*}
\theta_{1}=1.1 \frac{\lambda}{D} \sqrt{T_{\mathrm{dB}}} \tag{7}
\end{equation*}
$$

Inserting (7) into the parameter on the abscissa of Fig. 2, the value of $u$ for contiguous corrugated feed horns is

$$
\begin{equation*}
u_{1}=3.46 \sqrt{T_{\mathrm{dB}}} \tag{8}
\end{equation*}
$$

For $T=15 \mathrm{~dB}, u_{1}=13.4$. From Fig. 2, the $-3-\mathrm{dB}$ beamwidth is 3.62 ; thus, $u_{1}$ corresponds to $13.4 / 3.62=3.7$ beamwidths. For $u_{1}=13.4$, Fig. 2 shows that the side-lobe envelope level is -37 dB ; this is approximately equal to the isolation of two beams spaced $\theta_{1}$ degrees apart. The isolations for typical beam spacings are included in Table I. In the earth-station example, the minimum beam spacing is $0.6^{\circ}$, but the corresponding isolations, 37 and 43 dB at 4 and 6 GHz , respectively, are too small for allowable adjacent-satellite interference. ${ }^{12}$ These isolations can be increased to 45 and 49 dB , respectively, by increasing the beam (and satellite) spacing to $1^{\circ}$. The increased beam spacing also allows room between feed horns, so they can be moved individually to track small errors in satellite positioning.

## IN. AREA EFFICIENCY

Suppose an off-axis plane wave is incident on the main-reflector aperture shown in Fig. 1. The rays intercepted and reflected by the main reflector are displaced laterally with respect to those from an on-axis beam. But if the subreflector surface is sufficiently broadened, each of these rays will be intercepted and focused to a new point that is displaced laterally with respect to focal point $f$. To accommodate off-axis beams in the horizontal plane, the subreflector width is increased; similarly, for beams in the vertical plane, the beight is increased, as indicated by the dashed line in Fig. 1.

The lateral displacement of the focus, corresponding to an off-axis beam at an angle $\theta$, is equal to $\theta$ times the equivalent focal length, $F$. It is assumed that a separate corrugated feed horn is optimally positioned about the focus of each off-axis beam, i.e., each feed is pointed such that the original on-axis amplitude distribution is maintained across the main-reflector aperture, and each feed is longitudinally positioned to minimize aberrations.
The phase center of a corrugated horn can be calculated as a function of frequency.' This in turn allows the longitudinal position of the feed to be optimized for broadband performance.

Assuming the foregoing precautions are observed, each beam of the antenna in Fig. 1 has a computed gain about 1 dB less than that obtainable from an aperture with a uniform amplitude distribution. The underlying reasons for the good area efficiency, 80 percent, are (i) the main reflector does not have to be enlarged to accommodate off-axis beams, and (ii) the $F / D$ ratio of a Cassegrainian antenna is fairly large.

## V. POLARIZATION CROSS-COUPLING

T. S. Chu and R. H. Turrin have shown that the cross-coupling of an offset reflector is a function of (i) the angle between the feed axis and the reflector axis and (ii) the half-angle subtended at the focus by the reflector. ${ }^{13}$ In an offset Cassegrainian antenna with a moderate $F / D$ ratio, these angles are fairly small; thus, the cross-coupling is very small. In particular, in Fig. 1, $\psi=14^{\circ}$ and $\gamma=8.5^{\circ}$. For linearly polarized excitation, the cross-polarized lobes have a peak value of -45 dB . It is anticipated that, in beams with small off-axis angles, as in Table I, the cross-coupling will be about the same.

## VI. MULTIPLE-BEAM ANTENNA PARAMETERS

The off-axis beam parameters and corresponding feed-horn dimensions of an offset Cassegrainian anterina fed with corrugated horns can be calcuiated once the main-aperture diameter and operating

Table 1-Multiple-beam antenna parameters

|  | Earth Station at $4 / 6 \mathrm{GHz}$ | Satellite at $20 / 30 \mathrm{GHz}$ |
| :---: | :---: | :---: |
| Aperture dismeter, $D$ | 30 meters | 3 meters |
| Wavelength, $\lambda$ | $7.5 \mathrm{~cm} / 5 \mathrm{~cm}$ | $1.5 \mathrm{~cm} / 1 \mathrm{~cm}$ |
| Beamuidth, $\beta$ | $0.165^{\circ} / 0.11^{\circ}$ | $0.33^{\circ} / 0.22^{\circ}$ |
| Primary focal length, $F^{\prime}$ | 30 meters | 3 meters |
| OF-exis beam angle, $\theta=0 / B$ | $5^{\text {c }}$ $30 / 45$ | $\stackrel{4}{4} 12$ |
| Of-axis parameter, $X^{\prime}$ | 30/45 | 12/18 |
| Msin-refiector ofiset, $\mathrm{Y}_{z}$ | 25 meters | 2.5 meters |
| Subrefiector ofiset, $Y_{1}$ | 5 meters | 0.5 meter |
| Equivalent focal length, $F$ | 100 meters | 10 meters |
| $F / D$ ratio | 3.33 | 3.33 |
| Coma aberration, $\Delta \phi$ | $18^{\circ} / 27^{\circ}$ | $14^{\circ} / 21^{\circ}$ |
| Off-axis parameter, $X$ | 3.0/4.5 | 1.2/1.8 |
| Feed-horn length, $L$ | 3.8 meters | 76 cm |
| Feed-horn diameter, $2 a$ | 1.03 meters | 20.5 cm |
| Beam spacing $\theta_{1}$ | $0.6{ }^{\circ}$ | $1.2^{\circ}$ |
| Isolation at $\theta_{1}$ spacing | $37 \mathrm{~dB} / 43 \mathrm{~dB}$ | $37 \mathrm{~dB} / 43 \mathrm{~dB}$ |
| Isolation at. $1^{\circ}$ spacing | $45 \mathrm{~dB} / 49 \mathrm{~dB}$ |  |
| No. of available beams | 16 (in a row) | 18 (within U.S.) |

wavelengths are specified. Typical results for an earth-station antenna at 4 and 6 GHz and a satellite antenna at 20 and 30 GHz are givenin Table I. Similar results for other diameters and wavelengths can be found by following the text and performing the calculations in the order listed in Table I.

## VII. CONCLUSIONS

An offset Cassegrainian antenna fed with corrugated horns is expected to have well-isolated multiple beams that are broadband and dual-polarized. The antenna has good area efficiency and is relatively compact. This combination of properties makes the antenna wellsuited for earth stations and satellites.

## VIII. ACKNOWLEDGMENT

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## REFERENCES

1. L. C. Tillotson, "A Model of a Domestic Satellite Communication System," B.S.T.J., 47, No. 10 (December 1968), pp. 2111-2137.
2. John Ruze, "Lateral Feed Displacement in a Paraboloid," IEEE Trans. on Antennas and Propagation, September 1965, pp. 660-665'.
3. T. S. Chu, "A Multibeam Spherical Reflector Antenna," IEEE Antennas and Propagation Int. Symp., Program and Digest, December 9, 1969, pp. 94-101.
4. Henry Zucker, "Ofiset Parabolic Refiector Antenne," U. S. Pateni 3,696,435, filed Nov. 24, 1972.
5. William C. Wong, "On the Equivalent Parabola Technique to Predict the Performance Characteristics of a Cassegrain System with an Offset Feed," IEEE Trans. on Antennas and Propagation, AP-21, No. 3 (May 1973).
6. S. K. Buchmeyer, "Corrugations Lock Horns with Poor Beamshapes," Microwaves, January 1973, pp. 44-49.
7. C. Dragone and D. C. Hogg. "The Radiation Pattern and Impedance of Ofiset and Symmetrical Near-Field Cassegrainian and Gregorian Antennas," IEEE Trans. on Antennes and Propagation, AP-22, No. 3 (May 1974), pp. 472-475.
8. Peier W. Hannan, "Microwave Antennas Derived from the Cassegrain Telescope," IRE Trans. on Antennas and Propagation, March 1961, pp. 140-153.
9. C. Dragone, unpublished work, June 1972.
10. H. Zucker, unpublished work, December 1969.
11. H. Kogelnik and Tingye Li, "Laser Beams and Resonators," Appl. Opt., 5, No. 10 (October 1966); pp. 1550-1567.
12. AT\&T Application for a Domestic Communications Satellite System, before the FCC, March 3, 1971, Table IX.
13. Ta-Shing Chu and R. H. Turrin, "Depolarization Properties of Ofset Refector Antennas," IEEE Trans. on Antennas and Propagation, AP-21, No. 3 (May 1973), pp. 339-345.

## Bibliography for Part VII

[1] R. A. Semplak, " 100 GHz .measurements on a multiple beam offset antenna," Bell Sjrst. Tech. J., vol. 56, pp. 385-398, Mar. 1977.
[2] G. Hyde, R. W. Kreutel and L. V. Smith, "The unattended earth terminal multiple beam antenna," COMSAT Tech. Rev., vol. 4, pp. 231-262, Fall 1974.
[3] S. S. Sandler, "Paraboloidal reflector patterns for off-axis feed," IRE Trans. Antennas Propagat., vol. AP-8, pp. 368379, July 1960.
[4] J. B. Scarborough, 'The caustic curve of an off-axis parabola," Appl. Opt., vol. 3, pp. 1445-1446, Dec. 1964.
[5] T. Takeshima, "Beam scanning of parabolic antenna by defocusing،" Electron. Eng., vol. 41, pp. 70-72, Jan.
1969.
[6] A. W. Rudge and M. J. Withers, "Beam-scanning primary feed for parabolic reflectors," Electron. Lett., vol. 5, pp. 39-41, Feb. 6, 1969.
[7] M. S. Afiffi, "Aberration and dispersion off the focus of a parabola," in IEEE G-AP Symp. Dig., 1971, p. 219.
[8] O. Sorensen and W. V. T. Rusch, "Application of the geometrical theory of diffraction to Cassegrain subreflectors with laterally defocused feeds," IEEE Trans. Antennas Propagat., vol. AP-23, pp. 698-702, Sept. 1975.
[9] B. H. C. Liesenkötter, "Raising the crossover level of dual beam parabolic antennas," Electron. Lett., vol. 12, pp. 559-560, Oct. 14, 1976.

The first paper in this section, by Cheng, presents a simple nalysis whereby the maximum loss in gain can be estimated when phase errors are present in the aperture of an antenna. The exact nature of the phase and amplitude distributions need hot be known. To estimate the maximum change in beamwidth, it turns out to be necessary to know either the amplitude distribution or else the slope of the secondary pattern near the half-power point.
The next two papers are concerned with the validity of the simulation technique that makes use of axial defocusing of the feed in a reflector antenna so that Fraunhofer patterns may be freasured in the Fresnel region. An inconsistency in Cheng's treatment (second paper) is resolved by Chu in the third paper. Perhaps the earliest statistical treatment of the effect of fandom phase errors in the aperture of an antenna was given by Ruze in 1952. He published this work in an Italian journal [1] and about ten years later produced the well-known review baper which appears here, as the fourth in this part. It ranks igh among the most oft-quoted antenna papers of all time. It is followed by two papers in similar vein. The one by $V_{u}$ is concerned only with the effect of phase errors on the boresight gain. In the sixth paper, Zarghamee notes that the assumption of a uniform error distribution almost invariably produces a pessinistic estimate of the gain, although the effect is not serious until the surface deviations that cause the phase errors become an appreciable fraction of a wavelength. The same assumption, he finds, may significantly influence the scatter pantern even when the "roughness" is small.
The seventh and eighth papers are of importance for large ground-based reflector antennas in which astigmatism induced by gravitational deformation is likely to occur. Cogdell and Davis present a brief but useful discussion of astigmatism in
their paper, and describe a procedure for focusing an antenna and for diagnosing the existence of astigmatism. Von Hoerner and Wong introduce the important concept of homologous deformation whereby a large antenna is deliberately designed to deform under tilting, but in such a way as to remain a paraboloid of revolution with a different focal length. It is clear, then, that the effects of distortion can be rendered innocuous by commanding the feed to follow the focal point as it changes with tilt angle.

The determination of errors in the surface profile of large reflector antennas can be a difficult and time-consuming task, even with the use of sophisticated optical surveying techniques. Bennett et al. describe a holographic technique, in the ninth paper, that can be applied to reflectors at microwave frequencies. The method is simple, in principle, and requires only a fixed reference antenna, the antenna under test (with its scanning pedestal); and a source, which in the case of large antennas may be a radio star. Not only can surface profile errors be determined with this powerful technique, but other vital information is also obtained, for example, the $E$ and $H$ plane phase centers of the feed. Accurate prediction of both near and far field radiation patterns may also be made from the holographic data.

In the tenth and final paper of this part, Ingerson and Pusch return to a consideration of deterministic (as opposed to random) phase errors of the kind caused by axial displacement of the feed from the focal point. They show that defocusing effects are not symmetrical about the focus and that, when defocusing is deliberately used to achieve beam broadening, the feed should produce a well-tapered aperture distribution. For less tapered illumination, the main beam becomes bifurcated before appreciable broadening occurs.

# Effect of Arbitrary Phase Errors on the Gain and Beamwidth Characteristics of Radiation Pattern* 

D. K. CHENG $\dagger$


#### Abstract

Summary-Simple erpressions hape been obtained for predicting the meximum loss in antenas gain when the peak value of the apertare phase devistion is known. It is not necessary to know the exat smplitude or phase distribution function as long as the phese errors are relstively small; and the same expressions may be used for both rectangular and circular aperture cases. Relations have also been established such that the marimum change in the main-lobe beamwiath can be predicted from the knowledge of the amplitude distribntion function and the peak phase deviation.


## Introduction

IT IS KNOWN that for a given a mplitude illumination function, a uniform phase distribution over the aperture plane of a microwave antenna reflector gives a maximum gain. A uniform phase distribution requires an exact parabolic surface in addition to a correct primary feed. Any deviation from the exact parabolic surface will introduce phase errors, which in turn will cause a reduction in gain. Unfortunately phase errors are of ten quite arbitrary and it is in general not possible to insert them under integral signs, weight them properly with the amplitude function, and perform integrations. Ruze ${ }^{1}$ has investigated the effect of random phase errors on the radiation pattern as a statistical problem and obtained approximate formulas for the reduction in gain. However, as a statistical problem, only the average behavior of a large number or an ensemble of seemingly identical antennas and the probability distribution of the members of the ensemble about an average radiation patiern can be discussed; the individual patterns will dififer from the system-average pattern. By a leastsquare analysis, Spencer ${ }^{2}$ obtained an approximate expression for the fractional loss in gain due to small phase errors. In order to estimate the loss quantitatively it would be necessary to determine the plane least-square solution of the wavefront from a complete knowledge of the amplitude illumination function and the phase-error function over the aperture. The integration process involved is in general very difficult to carry out.

In practice, it is desirable to be able to predict the maximrum effect on the gain, main-lobe beamwidth, etc., if the peak phase error is given for an individual antenna, even when the exact phase distribution is not known or too complicated for analysis. This paper presents a simplified approach with which the maximum

[^21]loss in gain and the maximum change in beamwidth due to small arbitrary phase errors can be estimated.

## Phase-Error Effect on Antenna Gan

Consider the case of a rectangular aperture with separable, symmetrical field distribution. The maximum value of gain function, or simply gain, can be written as

$$
\begin{equation*}
G=\frac{2 \pi A}{\lambda^{2}} \frac{\left|\int_{-1}^{1} f(x) d x\right|^{2}}{\int_{-1}^{1}|f(x)|^{2} d x} \tag{1}
\end{equation*}
$$

in which all notations are conventional. The aperturefield distribution function $f(x)$ is in general

$$
\begin{equation*}
f(x)=F\left(x: \epsilon^{j \phi(x)},\right. \tag{2}
\end{equation*}
$$

where $F(x)$ is the amplitude illumination function and $\phi(x)$ represents the phase function: It is implied in (1) that the phase error is small and that maximum radiation occurs along the axis of the reflector. $\phi(x)$ may vary in an unknown manner across the aperture but it is assumed that the maximum deviation from an average value is known or can be estimated:

$$
\begin{equation*}
|\Delta \phi(x)|=|\phi(x)-\overline{\phi(x)}| \leqq m, \quad-1 \leqq x \leqq 1 \tag{3}
\end{equation*}
$$

In (3), $\overline{\phi(x)}$ is the average value; it will have no effect on the gain since the term $\epsilon^{\sqrt{\rho(z)}}$ can be taken out from under the integral sign. Only the phase deviation $\Delta \phi(x)$ from this average value is of importance. Substituting (2) in (1), one has

$$
\begin{equation*}
G=\frac{2 \pi A}{\lambda^{2}} \frac{\left|\int_{-1}^{1} F(x) \epsilon^{j \partial \phi(x)} d x\right|^{2}}{\int_{-1}^{1}|F(x)|^{2} d x} \tag{4}
\end{equation*}
$$

When there is no phase error, the maximum gain is

$$
\begin{equation*}
G_{0}=\frac{2 \pi A}{\lambda^{2}} \frac{\left|\int_{-1}^{1} F(x) d x\right|^{2}}{\int_{-1}^{1}|F(x)|^{2} d x} \tag{5}
\end{equation*}
$$

Hence, for $F(x) \geqq 0$,
$\frac{G}{G_{0}}=\frac{\text { gain with phase error }}{\text { gain without phase error }}=\frac{\left|\int_{-1}^{1} F(x) \epsilon^{j \Delta \phi(x)} d x\right|^{2}}{\left|\int_{-1}^{1} F(x) d x\right|^{2}}$

Examining the numerator of (6) for small $\Delta \phi(x)$ values, i.e., when

$$
\begin{equation*}
\epsilon^{j \Delta \phi(x)} \cong 1-\frac{1}{2}[\Delta \phi(x)]^{2}+j \Delta \phi(x), \tag{7}
\end{equation*}
$$

which holds for $(\Delta \phi)^{3} / 3!\ll \Delta \phi$, or $\Delta \phi \ll \sqrt{6}=2.45$ $=\pi / 1.28$,

$$
\begin{align*}
\left|\int_{-1}^{1} F(x) \epsilon^{i \Delta \phi(t)} d x\right|^{2} \cong & \left\{\int_{-1}^{1} F(x)\left[1-\frac{(\Delta \phi)^{2}}{2}\right] d x\right\}^{2} \\
& +\left\{\int_{-1}^{1} F(x) \Delta \phi d x\right\}^{2} \tag{8}
\end{align*}
$$

The first term on the right-hand side of (8) is

$$
\begin{align*}
& \left\{\int_{-1}^{1} F(x)\left[1-(\Delta \phi)^{2} / 2\right] d x\right\}^{2} \\
& \quad=\left\{\int_{-1}^{1} F(x) d x-(1 / 2)[\Delta \phi(\xi)]^{2} \int_{-1}^{1} F(x) d x\right\}^{2} \\
& \quad \geqq\left(1-\frac{m^{2}}{2}\right)^{2}\left|\int_{-2}^{1} F(x) d x\right|^{2} \tag{9}
\end{align*}
$$

Eq. (9) is the result of the application of the mean-value theorem $[\Delta \dot{\rho}(x)$ is continuous and $F(x)$ is positive within the range of integration; $-1 \leqq \xi \leqq+1$ ] and relation (3). The second term on the right-hand side of (8) is

$$
\begin{equation*}
\left\{\int_{-1}^{1} F(x) \Delta \phi d x\right\}^{2} \geqq 0 \tag{10}
\end{equation*}
$$

Substituting (9) and (10) in (8), and then back into (6), one obtains

$$
\begin{equation*}
\frac{G}{G_{0}} \geqq\left(1-\frac{m^{2}}{2}\right)^{2} . \tag{11}
\end{equation*}
$$

Eq. (11) is a useful relation because it sets the lower bound for the gain when the maximum phase deviation $m$ is known; it is independent of the amplitude illumination function $F(x)$ and the exact variation of $\Delta \phi(x)$. From (11), one also readily obtains the maximum fractional reduction in gain

$$
\begin{equation*}
\frac{\Delta G}{G_{0}}=1-\frac{G}{G_{0}} \leqq m^{2}\left(1-\frac{m^{2}}{4}\right) . \tag{12}
\end{equation*}
$$

It can be shown that for reflectors with circular aperture and symmetrical field distribution, (6) will be changed to the following:

$$
\begin{equation*}
\frac{G}{G_{0}}=\left.\frac{\left|\int_{0}^{1} F(\rho) \epsilon^{i \Delta d(\rho)} \rho d \rho\right|^{2}}{\mid \int_{0}^{1} F(\rho) \rho d \rho}\right|^{2} \tag{13}
\end{equation*}
$$

Provided (7) is satisfied for small phase deviations, the derivation procedure is entirely similar, and one also obtains (11) and (12) as the result.

As an example, if $m=\pi / 16=0.1964$ (corresponding to $\lambda / 32$ in terms of wavelength $\lambda$ ), the minimum $G / G_{0}$ is 0.9618 which corresponds to a maximum reduction in gain of 0.169 db . The maximum fractional reduction in gain from (12) is 0.0382 or 3.82 per cent.

When the phase distribution function is such that the maximum radiation does not occur along the axis of the aperture, for instance, if $\phi(x)$ is not an even function of $x$ in the rectangular aperture case, then the gain formula as given by (6) does not give the maximum gain ratio, but the lower bound as given by (11) still holds and is on the safe side.


Fig. 1-Normalized radiation patterns. $u=(\pi D / \lambda) \sin \theta$

## Phase-Error Effect on Beamwidth

When the phase error is small, an estimate on the maximum change in the main-lobe beamwidth of the radiation pattern can also be obtained. Consider this time the case of a circular aperture. It is necessary to assume here that both the amplitude and the phase distribution functions in the aperture plane are circularly symmetrical. In Fig. 1 are plotted two normalized radiation patterns, one without phase error $|g(0) / g(u)|$ and one with phase error $\left|g^{\prime}(0) / g^{\prime}(u)\right|$, where

$$
\begin{align*}
g(u) & =\int_{0}^{1} F(\rho) J_{0}\left(u_{\rho}\right) \rho d_{\rho}  \tag{14}\\
g^{\prime}(u) & =\int_{0}^{1} F(\rho) J_{0}(u \rho) \rho \epsilon^{j \Delta_{\rho}(\rho)} d \rho . \tag{15}
\end{align*}
$$

The condition for equal normalized radiation level is

$$
\begin{equation*}
\left|\frac{g^{\prime}(0)}{g^{\prime}\left(u_{1}\right)}\right|=\left|\frac{g(0)}{g\left(u_{0}\right)}\right| \tag{16}
\end{equation*}
$$

which would be equal to $10^{3 / 20}$ at 3 db down. It is noted that for $u \leqq 2.405=\pi / 1.3$, the integrand in (14) is always greater than or equal to zero and the following inequality holds:

$$
\begin{equation*}
\left|g^{\prime}(u)\right| \leqq g(u) . \tag{17}
\end{equation*}
$$

When the small phase error condition as indicated in (7) is satisfied (with $x$ changed to $\rho$ in this case), one may go through the same reasoning that led to (11) and get the relationship.

$$
\begin{equation*}
\left|g^{\prime}(u)\right| \geqq\left(1-\frac{m^{2}}{2}\right) g(u) . \tag{18}
\end{equation*}
$$

Combining (16), (17) and (18), one obtains

$$
\begin{equation*}
g\left(u_{1}\right) \geqq\left|g^{\prime}\left(u_{1}\right)\right| \geqq\left(1-\frac{m^{2}}{2}\right) g\left(u_{0}\right) . \tag{19}
\end{equation*}
$$

For small phase errors, the change in beamwidth is small and it is convenient to write

$$
\begin{equation*}
u_{2}=u_{0}+\Delta u, \quad \Delta u \ll 1 . \tag{20}
\end{equation*}
$$

Substitution of (14), (15) and (20) in (19) yields the following result:

$$
\begin{equation*}
\Delta u \leqq \frac{m^{2}}{2} \frac{\int_{0}^{1} F(\rho) J_{0}\left(u_{0} \rho\right) \rho d \rho}{\int_{0}^{2} F(\rho) J_{1}\left(u_{0} \rho\right) \rho^{2} d \rho}=\frac{m^{2}}{2}\left[\frac{g(u)}{-\frac{d}{d u} g(u)}\right]_{u_{0}} . \tag{21}
\end{equation*}
$$

Ec. (21) furnishes an upper bound in the change in halfbeamwidth for a given maximum phase error; it is dependent upon the amplitude illumination function $F(\rho)$.

For the case of a rectangular aperture, the following expression is obtained by a similar procedure (for $u \leqq \pi / 2$ ):
$\Delta u \leqq \frac{m^{2}}{2} \frac{\int_{0}^{1} F(x) \cos \left(u_{0} x\right) d x}{\int_{0}^{1} F(x) x \sin \left(u_{0} x\right) d x}=\frac{m^{2}}{2}\left[\frac{g(u)}{-\frac{d}{d u} g(u)}\right]_{u_{0}}$.
$\|$ is seen that (22) is entirely similar to (21). Results
listed in Table I are for the simplest case of uniform amplitude illumination function. The maximum change in beamwidth for other typical amplitude illumination functions can similarly be computed; there is no need to know the exact phase deviation curve. Eqs. (21) and (22) do not give useful information for very small $u_{0}$ values but are very helpful in estimating changes in $3-\mathrm{db}$ beamwidth. Moreover, it can be proved that when $u_{0}$ is small compared with unity, $\Delta u$ is always smaller than $u_{0}$.

TABLE I
Maximum 3-da Beamwidth Changes for Uniform Amplitude Function

|  |  | Rectangular aperture | Circular aperture |
| :---: | :---: | :---: | :---: |
| Amplitude function |  | $F(x)=1$ | $F(p)=1$ |
| Half 3-db beamwidth (no phase error), $u_{0}$ |  | 0.45\% | 0.51\% |
| $m=0.1$ | $\triangle u$ | $\leqq 0.00918$ | $\leqq 0.011$ |
|  | $\Delta u / u_{0}$ | $\leqq 0.65$ per cent/ | $\leqq 0.68$ per cent |
| $m=0.2$ | $\Delta u$ | $\leqq 0.0367$ | §0.044 |
|  | $\Delta u / u_{0}$ | $\leqq 2.6$ per cent | $\leqq 2.8$ per cent |

## Conclusion

It has been shown that for small phase errors, simple expressions (11) or (12) can be used to compute the maximum possible loss in antenna gain when the peak values of the phase deviation is known. It is not necessary to know the exact amplitude or phase distribution function in the aperture; and the same expressions apply to both rectangular and circular aperture cases. Similarly, (21) and (22) can be used to compute the maximum possible change in half $3-\mathrm{db}$ beamwidth, which is dependent upon the amplitude illumination and is different for the rectangular and circular aperture cases.

## Acknowiedgment

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# On the Simulation of Fraunhofer Radiation Patterns in the Fresnel Region* 

DAVID K. CHENG $\dagger$

Summary-Physical limitations on the size of obstacle-free test sites give rise to the need of making radiation-partern measurements on higi-gain antemas at a reduced distance. The general practice is to defocus the primary source along the principal axis of the antenna refiector by a small distance so that Fraunhofer patterns may be simulated in the Fresnel region. This note summarizes and compares tiree difierent approaches with which the proper amount of defocus may be determined.

THE PERFORMANCE of an antenna is usually specified in terms of the characteristics of its radiation pattern in the Fraunhofer region. This is because of the fact that Fraunhofer radiation patterns do not change with the distance from the antenna as long as the far-zone approximations are satisfied. Radiation patterns.in the Fresnel region tend to be very complex, and they change considerably: with distance.

While there is no clear-cut boundary between the Fraunholer and the Fresnel regions, a common and acceptable criterion is that $2 D^{2} / \lambda$ represents a safe farzone distance, where $D$ is the maximum dimension of

[^22]the antenna aperture and $\lambda$ is the operating wavelength. At a distance of $2 D^{2} / \lambda$ the maximum path-length difference between the contribution from the edge of the aperture and that from the center corresponds to $\lambda / 16$ or $\pi / 8$ radians. In practice, an unobstructed, open space with a dimension of $2 D^{2} \lambda$ is of ten not available for testing high-resolution antennas. For example, the $2 D^{2} / \lambda$ distance for a 20 -foot antenna at 3 cm would be about 1.53 miles. Higher gain requirements would demand even larger test sites. Unfortunately, calculation of Fraunhofer radiation patterns from measurements in the Fresnel region is not of practical value at the present time both because of the difficult and laborious process of extrapolation and because of the inherent difficulties in making accurate amplitude and phase measurements. ${ }^{1}$ The need for the technique of testing microwave anternas at reduced ranges is, therefore, both real and urgent.

A commonly used practice is to displace the primary source of the antenna assembly slightly from the focal position in a direction away from the reflector. The

[^23]present note summarizes three different methods with which the required amount of defocus may be determined. ${ }^{\text {T The results are plotted and compared. }}$

## The Geometrical Approach

It is a well-known fact that for an effective point source of excitation, the best radiation pattern will be obtained from a paraboloidal reflector at a field point in the far zone when the source is located at the focal point of the reflector. Geometrically, this may be explained by equal path length from the source to all points in an aperture plane by virtue of the inherent property of a focused paraboloid. If the field point is far enough away from the reflector, the path lengths from the aperture points to the field point will be again approximately equal, resulting in an optimum additive effect. When the field point lies in the quasi-near zone (Fresnel region), the path-length differences from the points in an aperture plane to the field point must be compensated in some way in order that the measured radiation pattern may approach the true far-zone (Fraunhofer) pattern. This is done by slightly defocusing the source along the reflector axis in the direction away from the reflector. Since the amount of on-axis defocus is the only adjustable variable here, one cannot expect to achieve equal path length for all points in the aperture plane. For simplicity, the conventional approach is to make the path length from the source to the field point by way of the apex of the paraboloid equal to that by way of the points on the edge of the reflector.

Reíer to Fig. 1, which represents a cross section of a sxmmetrical paraboloidal reflector with focal length $\overline{O F}=f$ and defocused point source at $F^{\prime}$, the above requirement is equivalent to making

$$
\begin{equation*}
\overline{F^{\prime} O}+\overline{O O^{\prime}}=\overline{F^{\prime} A}+\overline{A B} \tag{1}
\end{equation*}
$$

Call $\overline{Z A^{\prime}}=D$ (aperture diameter), $\bar{O}^{\prime} \bar{P}=\overline{B P}=R$ (distance of measurement), $\overline{F F}^{\prime}=\epsilon$ (defocus distance). Eq. (1) can be reduced to give

$$
\begin{equation*}
\epsilon=\frac{f^{2}}{R}\left[\left(\frac{R}{R-f}\right)+\left(\frac{D}{4 f}\right)^{2}\right] . \tag{2}
\end{equation*}
$$

When $(f / R)^{2} \ll 1$, it is accurate enough to write (2) as

$$
\begin{equation*}
\epsilon=\frac{f^{2}}{R}\left[1+\frac{f}{R}+\left(\frac{D}{4 f}\right)^{2}\right] . \tag{3}
\end{equation*}
$$

Normalizing all quantities with respect to the focal length and introducing new notations $\epsilon^{\prime}=\epsilon / f, R^{\prime}=R / f$, and $D^{\prime}=D / f$, one can rewrite (3) as

$$
\begin{equation*}
\epsilon^{\prime}=\frac{1}{R^{\prime}}\left[1+\frac{1}{R^{\prime}}+\left(\frac{D^{\prime}}{4}\right)^{2}\right] . \tag{4}
\end{equation*}
$$

The normalized amount of defocus needed is seen to increase when $R^{\prime}$ decreases and when $D^{\prime}$ increases. As $R^{\prime}$ approaches infinity, $\epsilon^{\prime}$ correctly goes to zero.

[^24]

Fig. 1-Geometry of on-axis defocusing arrangement for paraboloidal refiector.

## The Aperture-Phase Approach

The defocusing problem can also be approached from a consideration of the phase distribution in an aperture plane of the reflector together with the diffraction integral for the field at a point in space. When the point under consideration is in the quasi-near zone of a paraboloidal reflector, the normalized diffraction integral which gives the field pattern in a horizontal plane can be approximated as ${ }^{\text {a }}$

$$
\begin{equation*}
I(u)=\int_{0}^{1} F(r)^{-j k D^{2} r^{2} / 8 R_{r} J_{0}(u r) d r .} \tag{5}
\end{equation*}
$$

In (5), $r$ is the radial dimension of the aperture plane normalized with respect to $D / 2 ; u=(\pi D / \lambda) \sin \theta, \theta$ being the azimuth angle; $k=2 \pi / \lambda$; and $F(r)$ is the circularly symmetrical amplitude illumination function over the aperture. The explicit exponential term is the Fres-nel-region contribution; terms above the second order are neglected. When $R$ is very large, (5) reduces to the far-zone pattern function

$$
\begin{equation*}
I_{0}(u)=\int_{0}^{1} F(r) \tau J_{0}(u \tau) d \tau . \tag{6}
\end{equation*}
$$

When the primary source is displaced from the focus of a paraboloidal reflector along the reflector axis in the direction away from the reflector with a view to simulating far-zone patterns in the quasi-near zone, there will be a relative phase variation over the aperture. It has been found that this phase variation referred to the center point can be approximated satisfactorily by

$$
\begin{equation*}
\delta \cong-2 \epsilon\left[1-\frac{r^{2}}{\left(\frac{4 f}{D}\right)^{2}+1}\right] \tag{7}
\end{equation*}
$$

Eq. (7) is exact for $r=0$ (center) and $\tau=1$ (edge of aperture). For other values of $r$, the $|\delta|$ given by (7) is slightly too large; the error decreases when the ( $f / D$ )

[^25]ratio of the reflector increases. The diffraction integral now becomes
\[

$$
\begin{equation*}
I(u)=\int_{0}^{1} F(r) e^{i k\left[r-D^{2} r^{2} / 8 R\right]} J_{0}(u r) d r \tag{8}
\end{equation*}
$$

\]

In order to simulate Fraunhofer radiation patterns in the Fresnel region, the exponent under the integral sign in (8) should be made to vanish. This yields

$$
\frac{\epsilon}{f}=\frac{f}{R}\left[1+\left(\frac{D}{4 f}\right)^{2}\right]
$$

or

$$
\begin{equation*}
\epsilon^{\prime}=\frac{1}{R^{\prime}}\left[1+\left(\frac{D^{\prime}}{4}\right)^{2}\right] \tag{9}
\end{equation*}
$$

which checks with (4) when $R^{\prime}=R / D>1$. If it is desirable to write

$$
\begin{equation*}
R^{\prime}=n D^{\prime 2} / \lambda^{\prime} \tag{10}
\end{equation*}
$$

with $\lambda^{\prime}=\lambda / f, n$ a numeric, then (9) reduces to

$$
\begin{equation*}
\frac{\epsilon^{\prime}}{\lambda^{\prime}}=\frac{\epsilon}{\lambda}=\frac{1}{n}\left[\left(\frac{1}{D^{\prime}}\right)^{2}+\left(\frac{1}{4}\right)^{2}\right] . \tag{11}
\end{equation*}
$$

Eq. (11) shows that for a given value of $D^{\prime},(\epsilon / \lambda)$ plotted vs $n$ gives a hyperbola in linear scales, and a straight line in $\log -\log$ scales. ${ }^{4}$ It is noted that for $n=2$ ( $R=2 D^{2} / \lambda$ ), appreciable defocus is still necessary.

## The Ellipsoidal-Reflector Approach

The purpose of defocusing the primary source in the case of a paraboloidal reflector is to simulate far-zone radiation patterns at points in the quasi-near zone. In terms of geometrical optics, it is quite easy to see that this could be achieved by means of an ellipsoidal reGector. If the primary source is placed at one of the two foci of an ellipsoidal reflector, the reflected rays will converge at the other.

The equation in the $x z$ plane of a cross section of an ellipsoidal reflector with focal lengths $f_{1}$ and $f_{2}$ is

$$
\begin{equation*}
z=\frac{f_{1}+f_{2}}{2}\left[1-\sqrt{1-\frac{x^{2}}{f_{1} f_{2}}}\right] . \tag{12}
\end{equation*}
$$

Subject to the condition

$$
\begin{equation*}
\sqrt{1-\frac{x^{2}}{f_{1} f_{2}}} \cong 1-\frac{x^{2}}{2 f_{1} f_{2}} \tag{13}
\end{equation*}
$$

(13) can be approximated as

$$
\begin{equation*}
z=\frac{f_{1}+f_{2}}{4 f_{1} f_{2}} x^{2} \tag{14}
\end{equation*}
$$

which is the equation for a parabola of focal length

$$
\begin{equation*}
f=\frac{f_{1} f_{2}}{f_{1}+f_{2}} \tag{15}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{1}{f}=\frac{1}{f_{1}}+\frac{1}{f_{2}} . \tag{16}
\end{equation*}
$$

Hence, for reflected rays to converge at $R=f_{2}$, the primary source should be placed at $z=f_{1}$, and

$$
\begin{equation*}
\epsilon=f_{1}-f=\frac{f^{2}}{R-f} \tag{17}
\end{equation*}
$$

or, in normalized form,

$$
\begin{equation*}
\epsilon^{\prime}=\frac{1}{R^{\prime}-1}=\frac{1}{R^{\prime}}\left[1+\frac{1}{R^{\prime}}+\frac{1}{\left(R^{\prime}\right)^{2}}+\cdots\right] \tag{18}
\end{equation*}
$$

Eq. (18) should be compared with both (4) and (9). $\epsilon^{\prime}$ can also be expressed in terms of the focal lengths as

$$
\begin{equation*}
\epsilon^{\prime}=\frac{f_{1}}{f_{2}} \tag{19}
\end{equation*}
$$

which is extremely simple.
The basis of the ellipsoidal-refector approach lies in the fact that an ellipsoidal reflector of focal lengths $f_{1}$ and $f_{2}$ approximates a paraboloidal reffector of focal length $f$ as given by (15) or (16). An examination of (13) shows that it implies the condition $(1 / 8)\left(x^{2} / f_{2} f_{2}\right)^{2} \ll 1$. Now the maximum value of $x$ is $D / 2 \leq 2 f_{1}$. This reduces the condition to

$$
\begin{equation*}
\frac{f_{2}}{f_{1}} \gg \sqrt{2} \tag{20}
\end{equation*}
$$

which is undoubtedly true in practice. An ellipsoidal reflector with focal lengths $f_{1}$ and $f_{2}$ has its semimajor and semiminor axes equal to ( $f_{1}+f_{2}$ )/2 (arithmetical mean) and $\sqrt{f_{1} f_{2}}$ (geometrical mean), respectively; it approaches very closely a paraboloidal reflector when (20) is satisfied. As an example, with $f_{2}=R=50 f_{1}$ the maximum error is less than 0.09 per cent.

## Comparison of Defocusing Methods

Curves plotting $\epsilon^{\prime}$ vs $R^{\prime}$ based upon (4), (9), and (19) from the three different approach es discussed above are shown in Fig. 2. It is seen that except for small values of $R^{\prime}$, the required $\epsilon^{\prime}$ from the geometrical approach is nearly the same as that from the aperture-phase approach, both of which increase with increasing $D^{\prime}$. The required $\epsilon^{\prime}$ from the ellipsoidal-reflector approach is the smallest of the three methods and is independent of $D^{\prime}$.

A review of the geometrical approach reveals that there is really no plausible justification in requiring equal path length from the source to the field point by way of the apex and by way of the points on the edge of the paraboloidal reflector only; the path lengths by way of the intermediate points on the reflector would then all be longer. Besides, there is no guarantee that the rays emanating from $F^{\prime}$ will be reflected to pass through the point $P$ except the ray along the principal axis. The approximation ( 7 ) used in the aperture-phast approach is


Fig. 2-Comparison of defocusing methods.
exact for $r=0$ and $r=1$ only; for $0<r<1, \delta$, given by (7), is numerically too large resulting in an $\epsilon^{\prime}$ which is also too large. It can be shown that the maximum error in $\delta$ introduced by (7) is

$$
\begin{equation*}
1-\frac{1}{\sqrt{1+q}}\left(2-\frac{1}{\sqrt{1+q}}\right) \tag{21}
\end{equation*}
$$

where $q=(D / 4 f)^{2}$. For a reflector with $q=0.35$ or $D^{\prime}$ $=2.36$, the maximum error is about 5 per cent.
Although the geometrical approach and the aperturephase approach yield approximately the same results, the aperture-phase approach makes it clear that this method would not be useful when $R$ is too small because it would then be necessary to include terms higher than the second order in the exponent that appears in (5); the geometrical approach gives no indication of this resctriction. It is believed that $\epsilon^{\prime}$ in (18) derived from the ellip-soidal-reflector approach gives the most nearly correct results because the approximation implied by (13) is very good; it does not restrict its correctness only to the edge of the reflector.

It should be noted that in all three methods the required amount of defocus is not a function of the operating wavelength and that diffraction phenomena are neglected.

## A Note on Simulating Fraunhofer Radiation Patterns in the Fresnel Region

Abstract-This communication resolves the inconsistency among the earlier results of Cheng [1]. The region of validity for this simulation technique is clarifed by calculating the residual phase deriation.

A test range for the measurement of antenna radiation patterns should be free of scattering obstacles and satisfy the far zone ( $2 D^{2} / \lambda$ ) criterion. The practical difficulty of providing such test sites for large antennas stimulates the interest in simulating Fraunhofer radiation patterns by measurements in the Fresnel region. The required defocusing for the feed of a paraboloidal reflector in order to achieve this simulation was discussed by Cheng [1]. However, be obteined inconsistent answers from different approaches. It is the purpose of this communication to resolve this ambiguity by calculating the residual phase deviation.
The focal plane phase front of a defocused paraboloid deviates from that of a focused paraboloid by $\delta=\epsilon(1+\cos \theta)$ as shown in Fig. I(a). Making use of the relation $\tan \theta / 2=r D / 4 f$ yields

$$
\begin{align*}
\delta & =\frac{-2 \epsilon}{1+(D / 4 f)^{2} r^{2}} \\
& =-2 \epsilon\left[1-\frac{r^{2}}{(4 f / D)^{2}+r^{2}}\right] \tag{1}
\end{align*}
$$

Fhere $\tau$ is the normalized radius of the refiector aperture. Cheng proposed the following approximation for (1)

$$
\begin{equation*}
\delta=-2 \epsilon\left[1-\frac{r^{2}}{(4 f / D)^{2}+1}\right] \tag{2}
\end{equation*}
$$

Within the preceding approximation, a spherical wavefront of quadratic approximation [ $\exp \left(-j k D^{r^{2}} / 8 R\right)$ ] will be obtained from the defocused parsboloid if the defocus distance is

$$
\begin{equation*}
\varepsilon=\frac{f^{2}}{R}\left[1+\left(\frac{D}{4 i}\right)^{2}\right] \tag{3}
\end{equation*}
$$

Equations (1) and (2) coincide at $\tau=0$ and $\tau=1$. The difference between (1) and (2) at intermediate points $(0<r<1)$ is

$$
\begin{equation*}
\frac{\Delta}{\lambda}=-\frac{1}{2} \frac{r^{2}\left(1-r^{2}\right)(D / 4 f)}{1+(D / 4 J) r^{2}} \frac{(D / 2)^{2}}{\lambda R} \tag{4}
\end{equation*}
$$

Cbeng also compared an ellipsoidal reflector with a defocused paraboloid. The equation of an ellipsoidal reflector with focal lengths $f_{1}$ and $R$ is

$$
\begin{equation*}
z=\frac{f_{1}+R}{2}\left[1-\left(1-\frac{\rho^{2}}{f_{1} R}\right)^{1 / 2}\right] \tag{5}
\end{equation*}
$$

Taking the first wioterms of the binomial expansion for the square root, ( 5 ) becomes a paraboloid $z=\rho^{2} / 4 f$, where the focal length of the paraboloid is $f=f_{1} R /\left(f_{1}+R\right)$. The required defocus from this approach is

$$
\begin{equation*}
\epsilon=f_{1}-f=\frac{f}{R[1-f / R]} \approx \frac{f}{R} \tag{6}
\end{equation*}
$$

where $f / R \ll I$. One notes that $f / R$ is also small compared with $(D / 4 f)^{2}$ in the case of a large microwave paraboloidal antenna. The deviation betreen the wavefronts reflected from an ellipsoid and a defocused paraboloid is $\Delta z(1+\cos \theta)$ as shown in Fig. 1 (b). Estimating $\Delta z$ by the third term in the square root expansion of (5), we have the phase deviation

$$
\begin{equation*}
\frac{1}{\lambda}=\frac{1}{2} \frac{r^{4}(D / 4 f)^{2}}{1+(D / 4 f)^{2} r^{2}} \frac{(D / 2)^{2}}{\lambda R} . \tag{7}
\end{equation*}
$$

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(a)

(b)

Fig. 1. (a) Defocusing of paraboloid. (b) Ellipsoldal-reflector approach.


Fig. 2. Residual phase errors of defocused paraboloids with unity FTesnel number. ( $D / 2$ ):/ $\lambda R=1$.

Numerical comparison between (4) and (7) in the rese of wity Fresmel member ( $D / 2)^{2} / \lambda R$ has been plotited in Fig. 2 for various $f / D$ ratios. It is seen that the maximum total phase deviation with the defocus of (6) is more than three times that with the defocus of (3). Cheng [1] suggested (6) as the most nearly correct answer perhaps becanse he overlooked the effect of the large factor out side the bracket in (5). The ellipsoidal reflector approach will also give an answer identical to (3) if three terms of the square root expansion of (i) are taken and if coincidences at the center and the edge are imposed.

The phase deviation in (4) is similar to that of an optimally defocused spherical reflector for obtaining approximate plane phase front, while (7) is similar to that of feeding a spherical reflector. at the half-radius point [2].

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## References

[1] D. K. Cheng, 'OOn the simulation of Fraunhofer radiation patterns in the Fresnel region, IRE Trans. Antennas Propagat., vol. AP-5, Oct. 1957, pp. 399-402.
[2] A. S. Dunbar, "Optics of microwave directive systems for wideangle scanning." Naval Res. Lab., Rep, R-3312. Sept. $7,1048$.

## Antenna Tolerance Theory-A Review

JOHN RUZE, fellow, reee

Abstract-The theoretical basis of anterna tolerance theory is reviewed. Formulas are presented for the arial loss of gain and the pattern degradation as a function of the reflector surface rms error and the surface spatial correlation.

Methods of determining these quantities by astronomical or ground-based electrical measurements are described. Correlation betreen the theoretical predictions and the performance of actual large antenna structures is presented.

## I. Introduction

TYHE REQUIREMENT of precise optics for good image quality is well known in optical technology, and methods of testing and contour shaping have been developed to obtain precisions in excess of one part in $10^{7}$. Optical systems of very large $D / \lambda$ (diameter to wavelength), ratio are therefore common. Large antennas, such as required for radio astronomy or interplanetary probes, are engineering structures subject to gravity, wind, and thermal strains. Contour measurement and adjustment to the accuracy desired is also extremely difficult. Normal civil engineering structures have a precision of about one part in a thousand. Significant progress has been made in recent large parabolic antennas both in precision of construction (one part in 30000 ) and in the computer prediction of deformation under various loads. Nevertheless, the tolerance of the structure sets a limit on the highest frequency of operation and thereby on the $D / \lambda$ ratio. It is desirable to review the theory of aperture errors and their effect on the antenna radiation pattern.

We begin with a simple approach and attempt to develop a tolerance theory in an heuristic manner. The axial gain of a circular aperture with an arbitrary phase error or aberration $\delta(r, \phi)$ may be written as

$$
\begin{equation*}
G(0)=\frac{4 \pi}{\lambda^{2}} \frac{\left|\int_{0}^{2 \pi} \int_{0}^{a} f(r, \phi) \epsilon^{\bar{x}(r, \phi)} d d r d \phi\right|^{2}}{\int_{0}^{2 \pi} \int_{0}^{a} f^{2}(r, \phi) r d r d \phi} \tag{1}
\end{equation*}
$$

where $f(r, \phi)$ is the in-phase illumination function in terms of the aperture coordinates $r, \phi$,

For small phase errors, the exponential may be expanded in a power series with the result that the ratio

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of the gain to the no-error gain $G_{0}$ is

$$
\begin{equation*}
\frac{G}{G_{0}} \approx 1-\overline{\delta^{2}}+\bar{\delta}^{2} \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
\overline{\delta^{2}} & =\frac{\int_{0}^{2 \pi} \int_{0}^{a} f(r, \phi) \delta^{2}(r, \phi) r d r d \phi}{\int_{0}^{2 \pi} \int_{0}^{a} f(r, \phi) r d r d \phi} \\
\bar{\delta} & =\frac{\int_{0}^{2 r} \int_{0}^{a} f(r, \phi) \delta(r, \phi) r d r d \phi}{\int_{0}^{2 \pi} \int_{0}^{a} f(r, \phi) r d r d \phi}
\end{aligned}
$$

In general, the phase reference plane can be chosen so that $\bar{\delta}$, the illumination weighted mean phase error, is zero. The loss of gain is then simply

$$
\begin{equation*}
\frac{G}{G_{0}}=1-\overline{\delta_{0}^{2}}, \tag{3}
\end{equation*}
$$

where $\overline{\delta_{0}{ }^{2}}$ is calculated from the mean phase plane.
This simple relation (3), that the fractional loss of gain is equal to the weighted mean-square phase error was probably first pointed out by Marechal [1] and, in antenna technology, by Spencer [2]. It is valid for any illumination and reflector deformation, provided the latter is small in wavelength measure. It indicates that for a one dB loss of gain the rms phase variation about the mean phase plane must be less than $\lambda / 14$ or, for shallow reflectors, the surface error must be less than $\lambda / 28$.

We next seek a more exhaustive analysis valid for large phase errors and one that would give information on the radiation pattern: The problem is common to a class of problems, illustrated in Fig. 1, where a plane wave is distorted into an error phase front. Alternately, we can say that to a narrow or "diffracted limited" direction of transmission is added a wider angular spectrum of scattered energy.

If we have detail knowledge of the phase front error, the radiation pattern or angular spectrum can be obtained by machine computation of the standard Kirchhoff integral [3]. Unfortunately, such detail knowledge is not available, and we must fall back on various statistical estimates of the character of the surface distortion and obtain a probable radiation pattern.

We can begin the analysis by subdividing the aperture into $N$ subregions, each with a phase error and with no relation or correlation with contiguous regions. This crude model is shown in Fig. 2, where the aperture phase front is represented by a number of hatboxes of random heights. The axial field is the sum of these individual vector contributions. With no phase error the power sum of $N$ unit vectors is $N^{2}$ (see Fig. 3). If we now assume that the phase of each vector is randomly in error by an amount taken from a Gaussian population of standard deviation $\delta$, in radians, then the expected or average power sum is

$$
\begin{equation*}
\bar{P}=N^{2} e^{-\overline{\delta^{2}}}+N\left(1-\epsilon^{-\overline{b^{2}}}\right) . \tag{4}
\end{equation*}
$$

The first term may be considered as the coherent power and the second as the incoherent. For small or large errors, we get the limiting forms for the coherent or incoherent addition of waves. The distribution of the sum is also of interest [4]. For small phase errors, the distribution is Gaussian in voltage with a standard deviation $\sqrt{\bar{\lambda}} \delta$, so that the distribution becomes relatively more peaked with a larger number of vectors and smaller phase errors. For large phase errors, we have the wellknown Rayleigh distribution characterized by the mean power $N$.

The expected or average radiation function of the crude model shown in Fig. 2 can be derived. The procedure is briefly as follows [5]: the field at a general farfield point is expressed as a Kirchhoff surface integral. The power pattern is obtained by multiplying by the conjugate integral yielding a double surface integral of the two running surface vector variables. A correlation function is defined as a function of the vector difference of these variables. The average or expected value is then obtained. To perform the integration, assumptions must be made on the spatial nature of the correlation and on the frequency distribution of the phase errors. For the model chosen, these assumptions are that the phase values are completely correlated in a diameter " $2 c$ " and completely uncorrelated for larger distances. In addition, the various phases come from a Gaussian population of rms error " $\delta$." As in all statistical problems, the number of components must be large so that

$$
\begin{equation*}
N^{\top} \approx\left(\frac{D}{2 c}\right)^{2} \gg 1 . \tag{5}
\end{equation*}
$$

The result of this process is

$$
\begin{equation*}
G(\theta, \phi)=G_{v}(\theta, \phi) e^{-\overline{s^{2}}}+\left(\frac{2 \pi c}{\lambda}\right)^{2}\left(1-e^{-\delta^{2}}\right) \Lambda_{1}\left(\frac{2 \pi c u}{\lambda}\right) \tag{6}
\end{equation*}
$$

where
$G_{0}(\theta, \phi)$ is the no-error radiation diagram whose axial value is $\eta(\pi D / \lambda)^{2}$
$\eta$ is the aperture efficiency
$u$ is $\sin \theta$
$\Lambda_{1}()$ is the Lambda function.


Fig. 1. A class of problems. (a) Refection from a rough surface. (b) Transmission through a random medium. (c) Diffraction from an imperfect paraboloid.


Fig. 2. Aperture subdivided into a number of hatboxes.


Fig. 3. Addition of vectors. (a) No phase error.
(b) rms phase error " $\delta$."

Although the model chosen is a crude one, (6) illustrates the changes in the radiation pattern and its similarity to (4) should be noted. We see that the noerror radiation diagram has been reduced by an exponential tolerance factor. A broad scattered field has been added whose "beamwidth" is inversely proportional to the size of the correlated region in wavelengths, so that smooth reflectors (large $c$ ) scatter more directively and rough reflectors (small $c$ ) more diffusely. For small phase errors, the relative magnitude of the axial scattered field is

$$
\begin{equation*}
\frac{1}{\eta}\left(\frac{2 c}{D}\right)^{2} \overline{\delta^{2}} \tag{7}
\end{equation*}
$$

The model chosen can be considerably improved by replacing the hatboxes with hats as shown in Fig. 4. If


Fig. 4. Aperture subdivided into a number of hats.


Fig. 5. Special model constructed to test theory.
the phase front distortions are assumed to be of Gaussian shape, the required integrations can again be performed [5] with the following result:

$$
\begin{align*}
G(\theta, \phi)= & G_{0}(\theta, \phi) e^{-\overline{\delta^{2}}} \\
& +\left(\frac{2 \pi c}{\lambda}\right)^{2} e^{-\bar{b}^{\delta^{3}}} \sum_{n=1}^{\infty} \frac{\overline{\delta^{2}}}{n \cdot n!} e^{-(\tau \cos / \lambda)^{2} / n} . \tag{8}
\end{align*}
$$

Although (8) is more complex, the general effects are similar to those discussed previously.
We have considered a two-dimensional distribution of errors. It is of interest to present the one-dimensional case derived by Bramley [6] in our notation

$$
\begin{equation*}
G(\theta)=G_{0}(\theta) e^{-\overline{\delta^{2}}}+\frac{\sqrt{\pi} c}{\lambda} e^{-\overline{b^{2}}} \sum_{n=1}^{\infty} \frac{\overline{\delta^{2} n}}{\sqrt{n} \cdot n!} e^{-(\tau \cos / \lambda)^{2} / n} . \tag{9}
\end{equation*}
$$

The gain reduction and pattern degradation predicted by (8) was checked in the original reference [5] by the construction of a special model, Fig. 5, which fuifilied the statistical assumptions necessary for the theoretical development.

## II. Discussion

From (8), we can write the reduction of axial gain as

$$
\begin{equation*}
\frac{G}{G_{0}}=e^{-\overline{b^{2}}}+\frac{1}{\eta}\left(\frac{2 c}{D}\right)^{2} e^{-\overline{\delta^{2}}} \sum_{n=1}^{\infty} \frac{\overline{\delta^{2^{n}}}}{n \cdot n!} . \tag{10}
\end{equation*}
$$

In the region of interest, i.e., reasonable tolerance losses, and for correlation regions that are small compared to the antenna diameter, the second term may be neglected and we have for the gain

$$
\begin{equation*}
G=G_{0} e^{-b^{2}}=\eta\left(\frac{\pi D}{\lambda}\right)^{2} e^{-(\Delta+\epsilon / \lambda)^{2}}, \tag{11}
\end{equation*}
$$

where we define " $\epsilon$ " as the effective reflector tolerance in the same units as $\lambda$; i.e., that rms surface error on a shallow reflector (large $f / D$ ), which will produce the phase front variance $\overline{\delta^{2}}$. In Fig. 6 we plot the loss of gain (11) as a function of the rms error and the peak surface error. The ratio used, $3: 1$, is one found experimentally for large structures and results, in part, from the truncation used in the manufacturing process (i.e., large errors are corrected).

It should be noted that for small errors (11) is identical with (3), with the exception that the former is independent of the illumination function and the latter is not. For the statistical analysis, it was necessary to assume a uniform distribution of errors, for which case the illumination dependence factors out in (3) and becomes identical to (11).

For deep (nonshallow) refiectors, the surface tolerance is not exactly equal to the effective tolerance " $\epsilon$." In addition, structural people at times measure the reflector deformations normal to the surface and at times in the axial direction. The relation between these quantities is

$$
\begin{align*}
& \epsilon=\frac{\Delta z}{1+(r / 2 f)^{2}}  \tag{12a}\\
& \epsilon=\frac{\Delta n}{\sqrt{1+(r / 2 f)^{2}}} . \tag{12b}
\end{align*}
$$

The result is that the tolerance gain loss in dB , as computed from the reflector axial or normal mean square error, is too high by a factor $A$. This factor is given in Fig. 7. For shallow reflectors, this correction factor approaches unity.

Equation (11) indicates that if a given reflector is operated at increasing frequency, the gain, at first, increases as the square of the frequency until the tolerance effect take over and then a rapid gain deterioration occurs. Maximum gain is realized at the wavelength of

$$
\begin{equation*}
\lambda_{m}=4 \pi \epsilon, \tag{13}
\end{equation*}
$$

where a tolerance loss of 4.3 dB is incurred. This maximum gain is


Fig. 6. Gain loss due to refiector tolerance.
A


Fig. 7. Correction factor due to reflector curvature.

$$
\begin{equation*}
G_{\max } \approx \frac{\eta}{43}\left(\frac{D}{\epsilon}\right)^{2} \tag{14}
\end{equation*}
$$

and is proportional to the square of the precision of manufacture ( $D / \epsilon$ ).

This behavior is illustrated in Fig. 8, where we show some of the world's large antennas. The frequency region where the smaller and more precise structure is superior
to the larger ard coarser antenna, and the converse, is evident.

Next we consider the effect of surface errors on the radiation diagram. In Fig. 9, we show the pattern of a $12-\mathrm{dB}$ tapered circular aperture with random phase errors and with $D=20 c$. Wie plot from (8) the expected power diffraction and scatter patterns for mean-square phase errors of $0.2,0.5,1.0,2.0$, and 4.0 in radian squared measure. These correspond to tolerance gain losses of $0.87,2.24 .3,8.6$, and 16.6 dB , respectively. The complete radiation diagram is the power sum of the diffraction and scatter patterns. It should be noted that the diffraction pattern is reduced by the exponential tolerance factor and that the energy lost appears in the scattered pattern, which broadens as the surface error increases.


Fig. 8. Gain of large paraboloids (based on published estimates).


Fig. 9. Radiation patterns of phase distorted circular aperture, 12 dB illumination taper, $D=20 c$.

With further increase in loss, the diffraction pattern is submerged in the scattered energy and disappears. Scheffler [7], ${ }^{1}$ in a similar analysis, has pointed out that for large phase errors the scattered pattern approaches

$$
\begin{equation*}
G_{\Delta}(\theta)=\left(\frac{2 \pi c}{\lambda}\right)^{2} \frac{\left[1-e^{-\overline{\delta^{2}}}\right]}{\overline{\delta^{2}}} e^{-(x c u / \lambda)^{2} / \overline{\delta^{2}}}, \tag{15}
\end{equation*}
$$

so that for extremely large phase errors the radiated energy is scattered over an angular region with the intensity equal to

$$
\begin{equation*}
G_{z}(\theta)=\left(\frac{c}{2 \epsilon}\right)^{2} \epsilon^{-(c u / 4 \epsilon)^{2}} \tag{16}
\end{equation*}
$$

[^26]We note that under these extreme conditions the beamwidth is defined by the average surface slopes and is wavelength independent, a result we would have expected from geometric optics.
Before leaving the theoretical discussion, it should be recalled that the distribution in the focal plane has the same shape as the radiated angular spectrum. Therefore, the same relation (8) can be used to determine the spot size due to surface imperfections or small scale atmospheric inhomogeneities.

## III. Application to Antenna Structures

The experimental check of the theory afforded by the specially constructed model (Fig. 5) merely verifies the mathematical development. We turn now to practical structures and list those factors which deviate from the theoretical assumptions.

1) The surface errors are not random, but to a large part are due to calculable gravity, wind, and thermal strains. However, analysis of actual antenna photogrammetric measurements indicates that the reflector deviations, if not strictly random and Gaussian, are distributed in a bell-shaped curve [ 8 ], [ 9 ].
2) The actual reflector errors are not uniformly distributed over the aperture. Again, photogrammetric measurements and deformation calculations after structural compensation indicate that this condition is not grossly violated [8], [9].
3) The theory assumes a fixed, circular correlation region. As the contour adjustment points are normally spaced in a uniform grid, there is a tendency for this condition; however, various structural factors such as pie-panel segments would yjeld elliptical correlated regions of varying size.
4) The theory also requires that the number of uncorrelated regions in the aperture be large, that is $D \gg 2 c$. It has been found that for compensated structures the number of regions is related to the panel size or spacing of the target points.
5) It was also assumed that the spatial phase correlation function had a particular shape, namely Gaussian. Another smooth deformation surface would have yielded slightly different functional forms in the shape of the scattered power.
6) Finally, we have developed a statistical theory and obtained the average power pattern of the ensembie of such antennas. We apply the theory to one sample.
Therefore, a check of the performance of actual antenna structures with the above theory is necessary. Correlation has been obtained between frequency-gain measurements and optical photogrammetric measurements [9]. We present here other confirmation.

In Fig. 10, we show a horn reflector antenna [10]. The


Fig. 10. Horn refiector antenna.
gain of this antenna was precisely measured over $6: 1$ range of frequencies [11]. Equation (11) can be written as

$$
\begin{equation*}
10 \log G \lambda^{2}=10 \log \eta(\pi D)^{2}-\left(\frac{4 \pi \epsilon}{\lambda}\right)^{2} 10 \log e, \tag{17}
\end{equation*}
$$

which is the straight line

$$
y=c-b x
$$

when $G \lambda^{2}$ in $d B$ is plotted against reciprocal wavelength squared. The vertical intercept is a measure of the aperture efficiency and the reffector tolerance can be obtained from the slope.

The experimental data is shown in Fig. 11, where outside of a gain droop at low frequencies, due to diffraction effects, the data follow a straight line with a mean deviation of 0.166 dB . The indicated aperture efficiency also lies between the calculated efficiencies of 78.34 and 76.13 percent for the two polarizations used. The predicted surface tolerance is an effective value of 33 mils or 48 mils normal to the parabolic surface. The agreement of the measured data with the predicted straight line relationship is a confirmation of antenna tolerance theory. In addition, this procedure, combined with a linear regression analysis of the experimental data, to establish confidence limits, is probably the most convenient and accurate method of determining the surface precision [12].

We next consider the determination of the size of the correlation region by means of electrical measurements. The temperature measured on an extended astronomical source is equal to the product of the fractional enclosed power and the source brightness temperature. With no surface errors, practically all the radiated power is enclosed by the source if it is at least several beamwidths in extent. With reflector errors, some of the scattered energy is outside of the source and the measured temperature is decreased. This reduction depends on both


Fig. 11. Gain vs. frequency-horn reflector antenna.

The rms surface error and the size of the correlation egion.
The fractional enclosed power in a cone angle $y_{0}=\sin \theta_{0}$ (several beamwidths) can be obtained by ntegration of (8) witn the result:

$$
\begin{equation*}
E P\left(\theta_{0}\right)=[1-S]\left[1-e^{-\overline{t^{2}}} \sum_{n=1}^{\infty} \frac{\overline{\hat{\sigma}^{2^{n}}}}{n!} e^{-\left(\pi\left(\omega_{0} / \lambda\right)\right)^{2} / n}\right] \tag{18}
\end{equation*}
$$

where $S$ is the fractional energy very widely scattered by aperture blockage (feed supports, etc.). It, should be loted that. either with no surface error or with large one angle, the enclosed power is a constant.
Fig. 12 shows the reduction of enclosed power as a unction of the rms error and the enclosed cone angle. If temperature measurements are now made of the same source at two different frequencies, the enclosed power is
ifferent. If we enter the temperature ratio (after orrection for atmospheric effects and spectral index) as an ordinate into Fig. 12 and the frequency ratio as an abscissa, then we can obtain a set of values of tolerance rror and correlation intervals which satisfy this condition (with a known source cone angle). If the reflector tolerance is known from point source gain measurehents, the required correlation radius is determined.
This type of measurement was applied to the HAY'STACK radio telescope ( 120 -foot diameter in a metal Pace frame radome) at the frequencies of 7750 and $5500 \mathrm{Mc} / \mathrm{s}$. The moon was used as an extended source and the planet Jupiter as a point source. By means of the procedure outlined, it was concluded that the rms urface error $\epsilon$ was 0.053 inch and that the correlation radius $c$ was 4.4 feet.

A check of antenna tolerance theory is obtained by omparison of the predicted antenna pattern based on We astronomically determined values of $(\epsilon, c)$ and the experimentally measured pattern with a ground-based Fansmitter. Figure 13 shows this comparison at the freuency where the tolerance effect is significant. The


Fig. 12. Reduction in enclosed power.


Fig. 13. Comparison of measured and predicted patterns, HAl'STACK ( $15.745 \mathrm{Gc} / \mathrm{s}$ ).
agreement of the predicted pattern and the actual measured characteristic is excellent considering the statistical nature of the problem and that the sidelobe peaks should be 3 dB higher than the average intensity. A corresponding pattern, taken with a similar feed, at $7750 \mathrm{Mc} / \mathrm{s}$ where the tolerance effects are not significant showed sidelobe levels of about 25 dB down.

From the above measurements and those cited in the references, it may be inferred that the present status of antenria tolerance theory is such that the behavior of large antennas may be determined by the specification of two quantities: the rms surface error and the correlation interval. These quantities may be determined from electrical ground-based or astronomical data. Detail correlation of these electrically determined values and actual mechanical measurements is lacking. However, available photogrammetric or other structural estimates are not in variance with the theoretical predictions.

## Appendix

## A. Derization of (4)

Consider the power sum of $N$ unit vectors whose phases " $\delta$ " come from a normal distribution of zero mean
and variance $\overline{\delta^{2}}$

$$
\begin{equation*}
P=\sum_{i}^{N} \sum_{j}^{N} e^{j\left(\delta_{i}-i_{j}\right)}=\sum_{i}^{N} \sum_{i}^{N} e^{j y \bar{i}} . \tag{19}
\end{equation*}
$$

$y$ is another statistic, defined as the difference of two samples taken from the original distribution. It can be readily shown that it is normally distributed with zero mean and variance $2 \overline{\delta^{2}}$ or

$$
\begin{equation*}
W(y)=\frac{1}{\sqrt{4 \pi \delta^{2}}} e^{-v^{2} / 4 \delta^{2}}, \tag{20}
\end{equation*}
$$

now

$$
\bar{P}=\sum_{i}^{N} \sum_{j}^{N} \overline{\cos y}+i \overline{\sin y^{\prime}}
$$

for $i \neq j$

$$
\begin{align*}
& \overline{\cos y}=\int_{-\infty}^{\infty} \cos y^{\prime W}(y) d y=e^{-\delta^{2}}  \tag{21}\\
& \overline{\sin y}=\int_{-\infty}^{\infty} \sin y^{\prime W}(y) d y=0 . \tag{22}
\end{align*}
$$

Therefore

$$
\begin{equation*}
\bar{P}=N^{72} e^{-\overline{\varepsilon^{2}}}+N\left(1-e^{-\overline{\delta^{2}}}\right) . \tag{23}
\end{equation*}
$$

## B. Derivation of (8)

The gain function of an aperture is written:

$$
\begin{equation*}
G\left(\theta_{1} \phi\right)=\frac{4 \pi}{\lambda^{2}} \frac{\left|\int f(\bar{F}) e^{\overline{j r} \cdot \overline{F^{j}} e^{j(\tau, \phi)} d S}\right|^{2}}{\int f^{2}(\bar{F}) d S} \tag{24}
\end{equation*}
$$

where
$\bar{i}=\bar{F}$ is an aperture vector position variable, $\bar{t}=(2 \pi / \lambda) \overline{\hat{p}_{0}}$ is a vector in the direction of observation, $\dot{\delta}(r, \phi)$ is the aperture phase perturbation function,
$d S$ is an elemental aperture area,
$f(\bar{f})$ is the aperture illumination function.
The numerator may be written as:
denoting $\bar{\tau}=\bar{f}_{1}-\bar{f}_{2}$ as the vector difference between the two aperture running variables and $y(\bar{j})=\delta_{1}-\delta_{2}$ as the phase difference of the two points. We have

$$
\iint f\left(\bar{\tau}_{1}\right) f\left(\overline{\bar{r}_{1}}+\bar{\tau}\right) e^{\overline{\bar{k}} \cdot \overline{F_{T}}} e^{j v \overline{\bar{r}})} d S_{1} a S_{r}
$$

defining $\phi(\xi)$ as the illumination correlation function

$$
\begin{equation*}
\phi(\bar{F})=\frac{\int f\left(\bar{r}_{1}\right) f\left(\bar{r}_{1}+\bar{\tau}\right) d S_{1}}{\int f^{2}\left(\bar{r}_{1}\right) d S_{1}} \tag{25}
\end{equation*}
$$

We now rewrite (24) as:

$$
\left.G\left(\theta_{1} \phi\right)=\frac{4 \pi}{\lambda^{2}} \int \phi(\bar{\tau}) e^{\bar{j} \cdot \bar{F}} e^{j \nu(\bar{\tau}}\right) d S_{T}
$$

and the average or expected value

$$
\begin{equation*}
\overline{G\left(\theta_{1} \phi\right)}=\frac{4 \pi}{\lambda^{2}} \int \phi(\bar{\tau}) e^{\bar{j} \cdot \bar{\tau}}[\cos y(\tau)+i \overline{\sin y(\tau)}] d S_{\tau} \tag{26}
\end{equation*}
$$

for $\tau$ large compared to " $c$," the phase correlation distance where the two phase samples are uncorrelated, $y(\tau)$ is normally distributed with zero mean and variance $2 \bar{\delta}^{2}$. When $\tau$ approaches zero; $y(\tau)$ approaches zëro with zero variance. Some convenient form must be assumed for the variance function. Taking

$$
\begin{equation*}
\overline{y^{2}(\tau)}=2 \overline{\sigma^{2}}\left[1-e^{-\tau^{2} / c^{2}}\right] \tag{27}
\end{equation*}
$$

from (21) and (22), we have

$$
\begin{aligned}
& \overline{\cos y(\tau)}=e^{-\overline{\delta^{2}}\left\{1-\tau-\tau^{2} / \epsilon^{2}\right\}} \\
& \overline{\sin y(\tau)}=0 .
\end{aligned}
$$

Equation (26) may be rewritten as

$$
\begin{aligned}
\overline{G(\theta, \phi)} & =\frac{4 \pi}{\lambda^{2}} e^{-\overline{\delta^{2}}} \int \phi(\bar{\tau}) e^{\bar{k} \cdot \overline{F^{*}} e^{\overline{\bar{z}^{2}} c \tau^{2} / c^{2}} d S_{\tau}} \\
& =\frac{4 \pi}{\lambda^{2}} e^{-\overline{\sigma^{2}}} \sum_{n=0}^{\infty} \int \phi(\bar{\tau}) e^{j \bar{k} \cdot \bar{F}} \frac{\overline{\bar{\sigma}^{2}}}{n!} e^{-n r^{2} / c^{2}} d S_{r} .
\end{aligned}
$$

The first term is the unperturbed pattern $G_{0}\left(\theta_{1} \phi\right)$

$$
\begin{aligned}
\overline{G\left(\theta_{1} \phi\right)}= & G_{0}\left(\theta_{1} \phi\right) e^{-\overline{\delta^{2}}} \\
& +\frac{4 \pi}{\lambda^{2}} e^{-\bar{\delta}} \sum_{n=1}^{\infty} \int \phi(\bar{\tau}) e^{j \bar{k} \cdot \bar{\tau}} \frac{\overline{\delta^{2}}}{n!} e^{-n r^{2} / c^{2}} d S_{\tau:}
\end{aligned}
$$

Due to the exponential factor, the remaining terms have their principal contribution for $\tau<c$. As we have assumed that $c$ is small compared to the aperture dimensions, the illumination correlation function (25) may be assumed as unity in evaluating these terms. The angular integration can be immediately performed with the result:

$$
\begin{aligned}
G\left(\theta_{1} \phi\right)= & G_{0}\left(\theta_{1} \phi\right) e^{-\bar{\delta}^{2}} \\
& +\frac{8 \pi^{2}}{\lambda^{2}} e^{-\bar{\sigma}^{2}} \sum_{n=1}^{\infty} \frac{\bar{\delta}^{2^{n}}}{n!} \int J_{0}\left(\frac{2 \pi}{\lambda} u \tau\right) e^{-n r^{2} / c^{2}} \tau d \tau .
\end{aligned}
$$

The integral can be evaluated by extending the limits and recalling that

$$
\int_{0}^{\infty} J_{0}\left(\frac{2 \pi}{\lambda} u \tau\right) e^{-n r^{2} / c^{2} \tau d \tau}=\frac{c^{2}}{2 n} e^{-(x c u / \lambda)^{2} / n}
$$

with the final result

$$
\begin{align*}
& G\left(\theta_{1} \phi\right)=G_{0}\left(\theta_{1} \phi\right) e^{-\overline{\delta^{2}}} \\
&  \tag{28}\\
& \quad+\left(\frac{2 \pi c}{\lambda}\right)^{2} e^{-\delta^{2}} \sum_{n=1}^{\infty} \frac{\overline{\delta^{2} n}}{n \cdot n!} e^{-(x c u / \lambda)^{2} / n}
\end{align*}
$$

In this derivation, we have assumed that we are dealing with highly directive antennas. The obliquity factor has, therefore, been suppressed and we have used the small angle formulation of the Kirchhoff integral.

## C. The Function

$$
S(m, x:)=e^{-z} \sum_{n=1}^{\infty} \frac{x^{n}}{n \cdot n!} e^{-m^{2} / n}
$$

|  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

D. Daia for Fig. 12

Ratio Enclosed Poifer to Angle uo dB
Total Power

|  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

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## References

[1] A. Marechal, "The diffraction theory of aberrations," Rep. Progr. in Phys. (GB), vol. XIV; p. 106, 1951; for English summary see E . Wolf.
[2] R. C. Spencer, "A least square analysis of the effect of phase errors on antenna gain," Air Force Cambridge Research Center, Bedford, Mass., AFCRC Rept. E5025, January 1949.
[3] A. R. Dion, "Investigation of effects of surface deviations on HAYSTACK antenna radiation patterns, ${ }^{n}$ M.I.T. Lincoln Lab., Lexington, Mass., Rept. 324, July 1963.
[4] P. Beckman, "The probability distribution of the vector sum of $N$ unit vectors with arbitrary phase distributions," ACTA Tech. (Czechoslovakio), vol. 4, no. 4, pp. 323-334, 1959.
[5] J. Ruze, "The effect of aperture errors on the antenna radiation pattern" Suppl. al Nuovo Cimento, vol. 9, no. 3, pp. 364-380, 1952. This work used for the theoretical basis of antenna tolerance, was first prepared by the author as a Ph.D. dissertation under the direction of Prof. L. J. Chu at M.I.T. in 1952.
16] E. N. Bramley, "Some aspects of the rapid directional fluctuations of short radio waves reflected from the ionosphere, ${ }^{n}$ Proc. IEE (London), vol. 102B, pp. 533-540, 1955.
[7] H. Scheffer, "Uber die Genauigkeitsforderungen bei der Herstellung optischer Flachen fur astronomische Teleskope," 2. Astrophys. (Germany), vol. 55, pp. 1-20, 1962.
[8] J. W. Findlay, "Operating experience at the National Radio Astronomy Observatory, ${ }^{n}$ Ann. N. Y. Acad. Sci., vol. 116, pp. 25-40, June 1964.
[9] P. G. Mezger, "An experimental check of antenna tolerance theory using the NRAO 85 -foot and 300 -foot telescopes," 1964 Internol'l Symp. on Antennas and Propagation, pp. 181-185.
[10] R. W. Friis and A. S. May, "A new broadband microwave antenna system;" Trans. AIEE (Communicotion ond Electronics), vol. 77, pp. $97-100$, March 1958.
[11] A. Sotiropoulos and J. Ruze, "HAYSTACK calibration antenna," M.1.T. Lincoln Lab., Lexington, Mass., Tech. Rept. 367. December 1964.
[12] J. Ruze, "Reflector tolerance determination by gain measurement," 1964 A'EREM Conv: Rec., pp. 166-16\%.
[13] R. H. T. Bates, "Random errors in aperture distributions," IRE Trans. on Antennas and Propagation, vol. AP-7, pp. 369372, October 1959.
[14] H. G. Booker, J. A. Ratcliffe, and D. H. Shinn, "Diffraction from an irregular screen with application to ionospheric problems," Plitl. Trans. (GB), vol. 212 , ser. A pp. 579-609, 1950.
[15] R. N. Bracewell, "Tolerance theory of large antennas," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 49-58, January 1961.
[16] B. V. Brande, N. A. Esepkina, N. L. Kaidanovskii, and S. E. Khaikin, "The effects of random errors on the electrical characteristics of high-directional antennae with variable-profile reflectors," Radiotcknika i Elektranika (USSR), vol. 5, no. 4, pp. 7592, 1960.
[17] D. K. Cheng, "Effect of arbitrary phase errors on the gain and beam width characteristics of radiation pattern," IRE Trans. on Antennas and Propagation, vol. AP-3, pp. 145-147, July 1955.
[18] A. Consortini, L. Rouchi, A. M. Scheggi, and G. Toroldo DiFrancia, "Gain limit and tolerances of big reflector antennas," Alta Frequensa (Ilaly), vol. 30, pp. 232-276, March 1961.
[19] C. Dragone and D. C. Hogg, "Wide angle radiation due to rough phase fronts," Bell Sys. Tech. J., vol. 42, pp. 2285-2296, September 1963.
[20].]. Robieux, "Influence of the precision of manufacture on the performance of aerials," Am. Radio Elect., vol. 11, pp. 29-56, January 1956.
[21] Ya. S. Shifrin, "The statistics of the field of a linear antenna," Radio Engrg. Electronic Phys., vol. 8, pp. 351-358, March 1963.
[22] R. A. Shore, "Partially coherent diffraction by a circular aperture," in Electromagnetic Theory and Antennas, E. C. Jordan, Ed. New York: Pergamon, 1963, pp. 787-795.

## The Effect of Phase Errors on the Forward Gain

The efiect of random phase errors, which are caused by refector surface irregularities, on the radiation characteristics of a refiector antenna had been treated by a number of authors in the past. By treating the case as a statistical problem, Ruze [1] had obtained an approximate expression for the average loss in gain in the iorward direction in terms of the standard deviation of the error. Cheng [2], on the other hand, proposed a way to estimate the upper bound in the loss as a function of the peak phase error. Cheng's results are, however, not very usef ul in practice because estimates based on his formulas tend to be too conservative; in other words, they rend to underestimate the actual capability of the antenna system. Ruze's idea of finding the "average" performance of the antenna is basically sound, but to know the "average" radiation characteristics is not enough because the actual radiation pattern of a given antenna at any particular time is likely to be difierent from "average" pattern. One is therefore interested in finding the actual loss in the forward gain if the rms error at any particular instant is known. The problem becomes even more important when it bad been learned that, with the Haystack antenna, the effects of snow and wind loads are completely eliminated. It is therefore possible to obtain the error distribution at any antenna pointing direction, and to optimize the feed position simply by feeding the bearing data of the antenna to an appropriate computer.

Since we are only interested in the gain in the forward direction, the antenna can be replaced by a circular aperture (Fig. 1). If the illumination is uniform across the aperture, the field strength in the forward

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Fig. 1.
direction in the Fraunhofer region is given by

$$
\begin{equation*}
E_{0}=\text { constant } \int_{0}^{1} \int_{0}^{2 r} \exp [j \phi]_{\rho d \rho d \theta} \tag{1}
\end{equation*}
$$

where $\exp [j \phi]$ represents the error term. The normalized intensity is therefore equal to

$$
\begin{aligned}
P_{\lambda} & =\frac{1}{\pi^{2}}\left|\int_{0}^{1} \int_{0}^{2 \pi} \exp [j \phi] \rho d \rho d \theta\right|^{2} \\
& =\left|\frac{\int_{0}^{1} \int_{n}^{2 r} \exp [j \phi] \rho d \rho d \theta}{\int_{0}^{1} \int_{0}^{2 \pi} \rho d \rho d \theta}\right|^{2}
\end{aligned}
$$

For small value of $\phi$, we have

$$
\begin{aligned}
& \exp [j \phi] \doteqdot\left(1-\frac{\phi^{2}}{2}\right)+j \phi \\
& P_{\lambda}=\left|\frac{\int_{0}^{1} \int_{0}^{2}\left[\left(1-\frac{\phi^{2}}{2}\right)+j \phi\right] \rho d \rho d \theta}{\int_{0}^{1} \int_{0}^{2} \rho d \rho d \theta}\right|^{2} \\
&=\left|1-\frac{\overline{\phi^{2}}}{2}+j \bar{\phi}\right|^{2}
\end{aligned}
$$

$$
\doteqdot 1-\left[\bar{\phi}^{2}-(\bar{\phi})^{2}\right]
$$

where $\overline{\phi^{2}}$ and $\phi$ are the mean value of $\phi^{2}$ and. $\phi$, respectively. But $\bar{\delta}^{2}$, the mean square phase deviation is given by

$$
\begin{align*}
\overline{\delta^{2}} & =(\overline{\phi-\phi})^{2}=\frac{\int_{0}^{1} \int_{0}^{2 r}(\phi-\bar{\phi})^{2} \rho d \rho d \theta}{\int_{0}^{1} \int_{0}^{2 r} \rho d \rho d \theta} \\
& =\overline{\phi^{2}}+(\bar{\phi})^{2}-2 \bar{\phi} \cdot \bar{\phi} \\
& =\overline{\phi^{2}}-(\bar{\phi})^{2} \\
\therefore P_{N} & =1-\overline{\delta^{2}} . \tag{2}
\end{align*}
$$

This result has been derived directly from the optical case. In practice, however, it is very unlikely to find that the antenna is uniformly illuminated, and (2) does not apply. One must therefore find a different way to solve a more general problem with the errors not necessarily small and the amplitude illumination function of any form.

Referring to Fig. 2, it can be shown [3] that, when errors are present, the field strength in the forward direction is given by


Fig. 2.

$$
\begin{equation*}
E=C \int_{0}^{2 r} \int_{0}^{\gamma 0}[G(\xi, \gamma)]^{1 / 2} e^{j \beta} \tan \frac{\gamma}{2} d \gamma d \xi \tag{3}
\end{equation*}
$$

where $C=$ constant, $F(\xi, \gamma)$ represents the amplitude illumination function, and exp [ $j \phi]$ again represents the error term.

Imagine that we rotate the reflector about the axis of the antenna while keeping the feed horn fixed. We shall find that the field strength at any point in the far field region will change because the phase errors caused by surface irregularities of the reflector upset the symmetry of the radiation pattern. There is one exception however; that is, the field strength in the forward direction is not disturbed in any way by this rotation as it is in the direction of the axis of rotation. This is true irrespective of the shape of the radiation pattern. Now, since we only perform the rotation in our mind, the error pattern is not changed by gravitational or any other effects. On the other hana the phase error corresponding to any other point ( $x, y, s$ ) which is fixed relative to a fixed system of reference axes will change with the rotation. In other words, if we change the magnitude of the angle of rotation in a random manner, we therefore achieved a random variation of phase errors. The problem can therefore be treated as a statistical problem. It is important to note, tha: while the actual Geld strength in any direction is different from its "average" value, the average field strength in the forward direction is exactly equal to its actual value. The actual field strength in the forward direction in the presence of the error is
therefore given by

$$
\begin{align*}
& E_{a t t}=E=C \int_{0}^{2 \pi} \int_{0}^{\pi 0}[\ddot{G}(\xi, \gamma)]^{1 / 2} \\
& \cdot(\overline{\cos \phi+j \sin \phi}) \tan \frac{\gamma}{2} d \gamma d \xi . \tag{4}
\end{align*}
$$

Now

$$
\begin{equation*}
\phi=\frac{2 \pi}{\lambda} \epsilon \cdot 2 \cos \frac{\gamma}{2} \tag{5}
\end{equation*}
$$

where $|\epsilon|$ is the magnitude of the surface irregularity, and $\lambda$ is the wavelength of the operating frequency. Since it is quite justifiable to assure that $\epsilon$ is normally distributed with zero mean, the same thing can be said of $\phi$, the standard deviation of which is given by

$$
\begin{equation*}
\phi_{\mathrm{s}, \mathrm{~d} .}=\frac{4 \pi}{\lambda} \sigma \cos \frac{\gamma}{2} \tag{6}
\end{equation*}
$$

where $\sigma$ is the standard deviation of $\epsilon$. We also have

$$
\begin{aligned}
\overline{\sin \phi}= & \frac{1}{\sqrt{ } 2 \phi_{\text {s.d. }}{ }^{2}} \int_{-\infty}^{+\infty} \sin \phi \\
& \cdot \exp \left[-\frac{\phi^{2}}{2 \phi_{\text {s.d. }}{ }^{2}}\right] d \phi=0 \\
\overline{\cos \phi}= & \frac{1}{\sqrt{ } 2 \pi \phi_{\text {s.e. }}{ }^{2}} \int_{-\infty}^{+\infty} \cos \phi \\
& \cdot \exp \left[-\frac{\phi^{2}}{2 \phi_{\text {s.d. }}{ }^{2}}\right] d \phi \\
= & \exp \left[-\frac{\phi_{\text {s.d. }}{ }^{2}}{2}\right]
\end{aligned}
$$

or

$$
\begin{equation*}
\overline{\cos \phi}=\exp \left[-\frac{8 r^{2}}{\lambda^{2}} \sigma^{2} \cos ^{2} \frac{\gamma}{2}\right] \tag{7}
\end{equation*}
$$

Equation (4) can be rewritten as

$$
\begin{aligned}
E_{\mathrm{act}}= & \bar{E}=C \int_{0}^{2 x} \int_{0}^{\gamma 0}[G(\xi, \phi)]^{1 / 2} \\
& \cdot \exp \left[-\frac{8 \pi^{2} \sigma^{2}}{\lambda^{2}} \cos ^{2} \frac{\gamma}{2}\right] \tan \frac{\gamma}{2} d \gamma d \xi
\end{aligned}
$$

The actual value of the field strength in the forward direction can, therefore, be estimated for any error pattern irrespective of the amplitude illumination function.

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## References

[1) J. Ruze, *The efiect of aperture errors on tine antenna radiation pattera," Supplemento del $N_{\text {uoro }}$ Cimerto, vol. 9. no. 3. pp. 364-380, 1952.
[2] D. K. Cheng, Efiect of arbitrary phase errors on the gein and beamuidth characteristics of radiathe gain and beamundth charactenstics of radiation pattern. $1 . R, E$, Trans. on Aniennas and
Propagation. vol. AP-3, pp. 145-147. July 1955.
[3] S. Silver, Microniave A rilenna Theory ard Desigr. S. Silver, Micronuave A rienna Theory n
New York: McGraw-Hill, 1949, ch. 12.

# On Antenna Tolerance Theory 

MEHDI S. ZARGHAMEE

Abstract-To predict the loss of gain of antennas due to surface deviauions which are not distributed uniformly over the aperture, an extension of Ruze's theory is presented. It is found that the assumpion of uniform error distribution, in general, underestimates the axial gain of an antenna whose surface deriations have regional variations over the aperture. This effect becomes significant only when the surface deviations cannot be considered small as compared to the wavelength. Furthermore, it is found that the assumption of a uniform distribution of error may have a significant effect on the predicted scatter even when surface deviations are not large. Assuming that the deviations from uniform distribution are also random, a correction term to the theory is also presented.
notation
$A=$ area of aperture
$c=$ correlation radius
$f=$ aperture illumination function
$G=$ antenna gain
$G_{0}=$ no error gain, a function of direction of observation
$\bar{k}=2 \pi \bar{p} / \lambda$, where $\bar{p}$ is a unit vector in the direction of observation
$\bar{r}=$ aperture position vector (Fig. 1)
$u=\sin \theta$
$\bar{\delta}=$ phase error, a function of position

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$\epsilon=$ effective surface deviation, a function of position
$\epsilon_{0}=$ rms of effective surface deviations [see (9)]
$\eta_{0}=$ fourth root of the second variance of surface deviations [see (10)]
$(\theta, \phi)=$ angles defining the direction of observation (Fig. 1)
$\lambda=$ wavelength
$\sigma=$ standard deviation of phase-error distribution function, a function of position
$\sigma_{0}=$ averaged standard deviation of phase error over the aperture [see (7)].

## Introduction

THE DEVIATIONS of an antenna reflector from its ideal shape cause, in general, loss of gain and pattern degradation. These deviations may result from manufacturing and rigging tolerances and from gravity, wind, and thermal effects. The effects on surface deviations of manufacturing and rigging tolerances (including nondeterministic errors in the measuring instruments employed) are usually random in nature and can be estimated through stochastic analyses. ${ }^{[1]}$ Recently developed automated computation techniques permit structural engineers to predict accurately the deformations of the reflector surface caused by known wind, gravity, and temperature changes. ${ }^{[2]}$

The effects of surface deviations on the radiation pattern and gain may be predicted from the actual distribution of surface deviations over the aperture. ${ }^{[1] .[3]}$ A simpler, approximate method for computing these effects is presented by


Fig. 1. Aperture position and direction of observation coordinate systems.
uze, ${ }^{15}$. 51 in which only two quantities, namely the root mean square of the effective surface deviations and the cor--plation radius, are employed as the measure of the deviation f the refiector from its ideal shape. (Effective surface deviation is defined as one half the change in the RF path length. As shown in the Apppendix, it is equal to the axial compoent of the normal deviation from the ideal surface.) In the Luze formulation, the assumption was made that the effective surface deviation at any point is a random sample from single Gaussian distribution with zero mean and a stanard deviation equal to the rms of the effective surface deviation of the refiector. An additional assumption was that the surface deviations are correlated in small regions. he gain was then expressed as follows:

$$
\begin{align*}
& G(\theta, \phi)=G_{0}(\theta, \phi) e^{-(4 \pi \in \theta / \lambda)^{2}} \\
& +\left(\frac{2 \pi c}{\lambda}\right)^{2} e^{-\left(4 \pi \in(\lambda)^{2}\right.} \sum_{n=1}^{\infty} \frac{1}{n n!}\left(\frac{4 \pi \epsilon_{0}}{\lambda}\right)^{2 \pi} e^{-(\pi c u / \lambda)^{2} / n} . \tag{1}
\end{align*}
$$

The actual effectrve surface deviations of a reflector may ave a distribution which differs significantly from the aforementioned assumption of uniform error distribution over the perture. For example, the random errors of many parabobidal reflectors frequently increase with radial distance from the axis; the magnitude of the surface deformations also may xhibit nonrandom regional variations over the aperture.
It is of interest to determine the range of validity of (1) in predicting both the loss of axial gain and the half-power beamwidth for a reflector with nonuniform distribution of its urface deviations.
In this paper an equation which is a generalization of (1) to the cases where the error has nonuniform distribution ver the aperture is presented. Also, for cases where the eviations from a uniform distribution can be considered random over the aperture: a correction term to the usual plerance theory is proposed.

## Theoretical Development

The gain of an antenna may be expressed by the following equation:

$$
\begin{equation*}
G(\theta, \phi)=\frac{4 \pi}{\lambda^{2}} \frac{\left|\int_{A} f(\bar{r}) e^{j \bar{k} \cdot \bar{r}} e^{j \delta(\bar{r})} d S\right|^{2}}{\int_{A} f^{2}(\bar{r}) d S} \tag{2}
\end{equation*}
$$

Consider the phase-error function $\delta$ at point $\dot{z}$ as a random sample from a Gaussian distribution with zero mean and a standard deviation $\sigma(\bar{r})$, a function of position within the aperture. Let us further assume that the phase errors are so correlated that for the difference in the phase errors at points $\bar{r}_{1}$ and $\bar{r}_{2}$; we can write

$$
\begin{equation*}
\sigma^{2}\left(\bar{r}_{1}-\bar{r}_{2}\right)=\left[\sigma^{2}\left(\bar{r}_{1}\right)+\sigma^{2}\left(\tilde{r}_{2}\right)\right]\left(1-e^{-\tau^{2} / \epsilon^{2}}\right) \tag{3}
\end{equation*}
$$

where $\tau$ is the distance between $\bar{r}_{1}$ and $\bar{r}_{2}$ and $c$ is referred to as the radius of the correlation region. This assumption on the shape of the spatial phase-correlation function is not believed to affect the shape of the scattered power significantly if it is replaced with a similar smooth function.

For $c$ sufficiently small as compared to the dimensions of the aperture, the expected gain of the antenna may be expressed as follows:

$$
\begin{align*}
& G(e, \phi)=\frac{4 \pi}{\lambda^{2}} \frac{\mid \int_{A} f(\bar{r}) e^{-\sigma^{2} /\left.2 e^{j \bar{k} \cdot \bar{T}} d S\right|^{2}}}{\int_{A} f^{2}(\bar{r}) d S} \\
& +\left(\frac{2 \pi c}{\lambda}\right)=\sum_{n=1}^{\infty} \frac{1}{n n!} e^{-(\pi \epsilon u / \lambda)^{2} / n} \frac{\int_{A} f^{2}(\bar{r}) e^{-\sigma^{2}\left(\sigma^{2}\right)^{n} d S}}{\int_{A} f^{2}(\bar{r}) d S} \tag{4}
\end{align*}
$$

where $u=\sin \theta$. Equation (4) reduces to (1) if $\sigma(\bar{r})$ is assumed to be constant over the aperture.
. The complexity of (4) reduces its suitability for use in approximate design; therefore, certain simplifications will be made in this equation for this purpose. However, these simplifications limit the applicability of the theory to the prediction of the loss of gain alone. To predict the scatter, (4) must still be employed. If the corresponding simplifications are performed for (1), it reduces to the following well-accepted simpler form:

$$
\begin{equation*}
G=G_{0} e^{-\left(4 \pi \pi_{0} / \lambda\right)^{2}} \tag{5}
\end{equation*}
$$

Considering only the first term of (4), the axial gain of the antenna can be written in the following form:

$$
\begin{equation*}
G=\frac{4 \pi}{\lambda^{2}} e^{-\sigma_{0}^{*}} \frac{\left|\int_{A} f(\bar{r}) e^{-\xi / 2} d S\right|^{2}}{\int_{A} f^{2}(\bar{r}) d S} \tag{6}
\end{equation*}
$$

where $\xi=\sigma^{2}(\hat{r})-\sigma_{1}{ }^{2}$, and $\sigma_{0}{ }^{2}$ is the averaged variance of the
phase error defined as

$$
\begin{equation*}
\sigma_{0}{ }^{2}=\frac{\int_{A} \sigma^{2}(\bar{r}) f(\bar{r}) d S}{\int_{S} f(\bar{r}) d S} \tag{7}
\end{equation*}
$$

Note that the average $\xi$ over the surface is zero. Then, if we assume that $\xi$ at each point on the aperture is a random sample from a single Gaussian distribution with zero mean and standard deviation $\sigma_{\xi}$, the expected value of gain can be written as follows:

$$
\begin{equation*}
G=G_{0} e^{-\sigma 0^{2}} e^{\sigma} \xi^{1 / 4} . \tag{8}
\end{equation*}
$$

To express (8) in terms of surface deviations, let us introduce the rms of the effective surface deviations, defined as follows:

$$
\begin{equation*}
\epsilon_{0}^{2}=\mathrm{rms}^{2}=\frac{\int_{A} \epsilon^{2} f(\bar{r}) d S}{\int_{A} f(\bar{r}) d S} \tag{9}
\end{equation*}
$$

where the function $\epsilon$ is the effective surface deviation from the best-fit paraboloid for deterministic errors and it is the standard deviation of the effective surface deviations for random errors that have Gaussian distributions with zero means. Let us also define a quantity called the second variance of surface deviations as follows:

$$
\eta_{0}{ }^{4}=\text { second variance of surface deviations }
$$

$$
\begin{equation*}
=\frac{\int_{A}\left(\epsilon^{2}-\epsilon_{0}^{2}\right)^{2} f(\bar{r}) d S}{\int_{A} f(\bar{r}) d S} \tag{10}
\end{equation*}
$$

Then (8) may be written as follows:

$$
\begin{equation*}
G=G_{0} e^{-(4 \times 00 \lambda)^{2}} e^{\{(4-70 / \lambda) \alpha} . \tag{11}
\end{equation*}
$$

## Discussion

To show the accuracy of the usual tolerance theory [(1) and (5)] in predicting the loss of gain of antennas due to surface deviations, the axial loss of a uniformly-illuminated paraboloidal reflector with various radially-linear distributions of surface deviations is calculated and the results are compared with the corresponding values obtained assuming uniform error distribution. For the purposes of this comparison, (5) and (6) are employed instead of the more general (1) and (4). The effect of neglecting the second parts of these equations, which become important for large tolerance lesses, is not expected to alter the conclusions reached from the results obtained herein.
The function $\sigma$ is assumed to be varying linearly with radius, that is

$$
\begin{equation*}
\sigma(\bar{r})=\sigma_{0}+\nu\left(r-r_{0}\right) \tag{12}
\end{equation*}
$$

where the constant $\sigma_{0}$ is the averaged standard deviation of the phase error in (7), and $\nu$ and $r_{0}$ are arbitrary constants. Let us consider a number of possible values for these constants and compare the values for the tolerance loss computed from (6) to the corresponding values obtained from (5). Four cases are considered for this purpose, as follows.

Case I: The function $\sigma(f)$ vanishes at the outer edge of the reflector.

Case II: The function $\sigma(j)$ vanishes at the center of the reflector.

Case III: An intermediate condition in which the rate of change of $\sigma$ with radius is one half that for Case I.

Case IV: An intermediate condition in which the rate of change of $\dot{\sigma}$ with radius is one half that for Case II.

The results are shown in Fig. 2 and indicate that the assumption of uniform distribution of surface deviations over the aperture involves errors that become siguificant for hightolerance losses.

For a $120-\mathrm{ft}$ ( 36.58 m ) uniformly-illuminated paraboloidal refiector with surface deviations that are distributed as in Cases I and II and an $\mathrm{mms} \epsilon_{0}=0.1$ in ( 2.54 mm ), the axial gain was computed from (5), (6), and (11), and the results are compared in Fig. 3. The agreement between the results of (6) and those obtained using the simpler theory (11) is quite good in most ranges of interest.

For the cases examined, a point of interest is that the usual antenna tolerance theory always underestimates the gain. The difference between the actual axial gain and that predicted by assuming uniform distribution of error is negligible for wavelengths which are large compared with surface errors and becomes increasingly more pronounced as the frequency increases. For $\sigma_{0}$ equal to unity, which corresponds to $\lambda / \epsilon \approx 12.5$ and to a loss of axial gain of 4.34 dB as computed by (5), the usual tolerance theory underestimates the axial gain by 8.6 percent for surface deviations assumed in Case II with a uniform illumination. This erro increases with an increase in the taper of the illumination function until it reaches 12.5 percent for the case where the illumination function vanishes at the edge. (In these calculations the illumination was assumed to be radially parabolic.) If we employ (11) instead, the error in predicting the axial gain reduces by a factor of at least four.

In the general case of arbitrary distribution of error over the aperture, the tendency of (5) to underestimate the gain can also be shown by examining (8). This equation has been derived assuming that the variable $\xi$, the deviation from mean variance of phase errors, has a Gaussian distribution. with zero mean. If the distribution is not Gaussian, then the expected value of $\exp (-\xi / 2$ is $)$ is given by

$$
\begin{equation*}
E\left(e^{-\frac{1}{2}}\right)=\int_{-\infty}^{\infty} f_{t}(t) e^{-t / 2} d t \tag{12}
\end{equation*}
$$

where $f_{z}$ is the frequency distribution function for $\xi$. If $f_{k}$ is symmetric about its zero mean then, from the mean-value theorem,

Fig. 2. Tolerance loss for various distributions of surface deviations.


Fig. 3. Comparison of calculated gain of a $120-\mathrm{ft}(36.58 \mathrm{~m})$ antenna
 with $\epsilon_{0}=0.1$ in ( 2.54 mm ) and with various distributions of surface deviations:-

$$
\begin{equation*}
E\left(e^{-\xi / 2}\right)=\cosh \frac{\bar{i}}{2} \int_{-\infty}^{\infty} f_{\xi}(t) d t=\cosh \frac{\bar{i}}{2} \tag{13}
\end{equation*}
$$

for some real $\bar{i}$. Obviously, $E\left(e^{-\xi / 2}\right)$ is always greater than or equal to unity, which shows that the usual tolerance theory hlways underestimates the gain. Only for $f_{\xi}$ having extremely large skew can the expected value of $\exp (-\xi / 2)$ be less than

The increase in scatter with an increase in the rms error may be predicted by employing (1). For this purpose a size must be assumed for the correlation radius $c$. For the two imiting cases of radially-linear variation of $\sigma$, namely for Cases I and II, the half-power beamwidth was also calculated as a function of the rms error of the reflector, employ-


Fig. 4. HPBW of a $120-\mathrm{ft}(36.58 \mathrm{~m})$ uniformly illuminated reflector with various distributions of surface deviations; $\lambda=1$ in ( 25.4 mm ); $c=5.6 \mathrm{ft}(1.7 \mathrm{~m})$.
ing (4). The computation was performed for a uniformlyilluminated circular aperture of $120-\mathrm{ft}(36.58 \mathrm{~m})$ radius and $c=5.6 \mathrm{ft}(1.71 \mathrm{~m})$. The results are shown in Fig. 4.

It can be observed that the half-power beamwidth is significantly affected by the distribution of the variance of phase error. For antennas having less error at the center the half-power beamwidth is greater than that predicted by (1). When an antenna bas its greatest surface deviations at the center, the half-power beamwidth decreases with increasing rms. This effect of the error distribution on the half-power beamwidth may be explained by noting that when the antenna has its larger errors at its edge the effective taper of the illumination function is increased, which corresponds to an increase in the half-power beamwidth. Similarly, for an antenna with its larger errors at its center, we expect a reduction of the effective taper and thus a decrease in the halfpower beamwidth; for large center errors, we have effectively an annular ring.

## CONCLUSIONS

A generalization of Ruze's eq. (1) is derived including the effect of variation of phase error over the aperture and the accuracy of his equation is examined. It is found that the assumption of uniform error distribution, in general, underestimates the gain. This error is small for effective surface deviations less than about a twentieth of the wavelength, but becomes rapidly larger as the surface deviations increase. In radio astronomy applications, deviations in excess of a twentieth wavelength are not uncommon.

If the rms of the surface deviations is calculated ${ }^{[5]}$ by substituting into (5) measurements of the gain at two different frequencies, the resulting rms value, in general, will be too small, since the error in (5) increases with frequency.

The distribution of the surface deviations is found to have an effect on beamwidth. In the absence of large feed-support surface deformations, the centrally supported antennas have their larger surface deviations near their edges. As is shown here the beamwidths of this class of antennas are larger than the values predicted using (1). This phenomenon has in fact been observed for many antennas.

## ZARGHAMEE: ANTENNA TOLERANCE THEORY



Fig. 5. Change in RF path length:

## APPENDK

The total change in the RF path length at a point on a reflector surface at which the normal surface deviation from the theoretical surface is equal to $u_{n}$ (Fig. 5) is $d_{1}+d_{2}$. On the other hand, the axial component of the normal deviation may be expressed as follows:

$$
\begin{aligned}
\epsilon & =u_{n} \cos \alpha=d_{1} \cos ^{2} \alpha \\
& =\frac{1}{2}\left(d_{1}+d_{1} \cos 2 \alpha\right) \\
& =\frac{1}{2}\left(d_{1}+d_{2}\right) .
\end{aligned}
$$

It is thus show'n that the effective surface deviation at a point is equal to the axial component of the normal deviation of the surface at that point.

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## References

mj M. S. Zarghamee and H. Simpson, "Feasibility study for rerigging the Haystack amenna," M.I.T. Lincoln Lab., Lexington, Mass., Tech. Rept. ESD-TR-67-235, February 1967.
${ }^{12} \mathrm{H}$. Simpson and J. Antebi, "Space frame analysis and applications to other types of structures," Proc, SHARE Design Automation Workshop (Atlantic City, N. J., June 1964), pp. 24-26.
is A. R. Dion, "Investigation of effect of surface deviations on haystack antenna radiation patterns," M.I.T. Lincoln Lab., Tech. Rept. 324, July 1963.

14 J . Ruze, "The effect of aperture errors on the antenna radiation pattern:: Suppl. Nuove Cimento, vol. 9, pp. 364-380, 1952.
[5) J. Ruze, "Antenna tolerance theory-A review," Proc. IEEE, vol. 54, pp. 633-640, April 1966.

## Astigmatism in Reflector Antennas

## JOHN R. COGDELL and JOHN H. DAVIS

Abstract-The characteristics of the astigmatic phase error in large parabolic refiector antennas are described. A procedure for focusing an antenna and diagnosing the presence and degree of estigmetism is given.

Astignstism is a term used to describe one of several common eberrations (imperfections) in optical reflectors [1]. Astigmatism is common in microwave reflector antennss and is often the dominant error in degrading antenna performance. The astigmatic problem may be csused by feed displacement [2] or feed phase center problems [ $3, \mathrm{pp}$. 157-160, and other references cited therein]. but the more serious problems are due to the shape of the reflector. While good discussions of common aberrations, including astigmatism, in microwave antennas exist. [3, p. 139], workers with lsege reffector antennas seemingly fail to realize the importance of scigmstism or recognize its effects on antenna properties. The purpose of this communication is to describe the nature of the satigmstic phase error and its effects on the antenna properties and to outine a simple procedure for detecting astigmatism in refiectior antennes. This discussion is placed within the context of the practical matter of locating the optimum focus of the reflector.
No refector can be manufactured without errors. The errors in a refiecior may be traced io various physical causes but naturally fill into ino classes. Errors which decorrelate over a region small with respect to the antenns have been treated by Ruze [4] and otbers with probabilistic models. Errors which affect the entire sienns stucure lead to the various aberrations, of which astigmatism is often the dominant form.
Let us investigate the nature of large scale errors in reflectors. A peffect parcbola will transform a spherical wave originating from its tue focus inw a plane wave perpendicular to its axis. If the refector is imperfect, the surface of constant phase will deviate from the sperture plane by an amount $z_{r}(x, y)$ which may be expressed $\varepsilon \leq$ a power series

$$
z_{r}(x, y)=\sum_{n=0}^{\infty} \sum_{n=0}^{\infty} a_{n m} \frac{x^{n} y^{m}}{(D / 2)^{n+m}}
$$

where $x$ and $y$ are Cartesian coordinates in the aperture plane and $D$ is the antenna diameter. The error coefficients $a_{n m}$ are normalized to be tise maximum deviation of the $(n, m)$ th term in the series st the eage of the antenna.

Anotker source of phase error is feed displacement from the true focus. If the spherical waves originate at a point displaced from the focus by $x^{\prime}, y^{\prime}$, and $z^{\prime}$, then an additional error $z_{f}(x, y)$ will result, given by
$z_{f}(x, y)=F+\frac{1}{4 F}\left(x^{2}+y^{2}\right)$

$$
-\left[\left(x-x^{\prime}\right)^{2}+\left(y-y^{\prime}\right)^{2}+\left(F+z^{\prime}-\frac{x^{2}+y^{2}}{4 F}\right)^{2}\right]^{2 / 2}
$$

Where $F$ is the focal length of the reflector. Adding the two results and convering to phase error in the aperture plane, we may expand

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The alihors are rith the Electrical Enginearing Research Laboratory, Depariment of Electical Engineering, Unirersity of Texas, Austin, TEX, 8 8i2.
the two expressions as

$$
\begin{aligned}
\varphi(x, y)=\frac{2 \pi}{\lambda}\left(z_{r}+z_{j}\right)= & \text { constant term (absolute phase) } \\
& + \text { first-order terms (beam steering) } \\
& + \text { second-order terms (defocusing and } \\
& \text { astigmatism) } \\
& + \text { third- and higher order odd terms } \\
& \text { (coma and others) } \\
& + \text { fourth-and higher order even terms } \\
& \text { (spherical aberration and others). }
\end{aligned}
$$

The constant and first order terms are without physical significance. The third- and higher order odd terms lead to various effects, notably a large principal sidelobe (coma). Normally one would equalize the principal sidelobes through lateral adjustment of the equivalent feed position during the focusing procedure, thus reducing coma.

The second-order term takes the form:

$$
\text { second-order term }=\left[\frac{\alpha\left(x^{2}-y^{2}\right)+\gamma\left(x^{2}+y^{2}\right)}{(D / 2)^{2}}\right] \frac{2 \pi}{\lambda}
$$

where

$$
\begin{aligned}
& \gamma=\left(\frac{z^{\prime}}{8}\left(\frac{D}{F}\right)^{?}+\frac{a_{20}+a_{00}}{2}\right) \\
& \alpha=\frac{a_{20}-a_{02}}{2}, \text { the astigmatism parameter. }
\end{aligned}
$$

For the second-order term to take the simple form given in the preceding kith no $x y$ term, the $x, y$ axes must be aligned with the direction of maximum astigmatism. We see two second-order terms: a. radially symmetric error which is parabolic due to both reflector errors ( $a_{20}$ and $a_{02}$ ) and axial feed position ( $z^{\prime}$ ). Note that this term can be reduced to zero by setting the axial position of the feed properly. This feed position will maximize the peak gain of the antenna. With the feed in the maximum gain position, the remaining second-order phase error is the astigmatism term, proporijonal to $\alpha$. Thus we reach the following conclusion: The proper feed location in the lateral plane is that which eliminates coma (equalizes sidelobes) and in the axial dimension is that which maximizes gain. With this optimum feed position, phase errors up to fourth order are eliminated except, astigmatism, which is second order. It is precisely this which makes astigmatism so important to recognize, as it cannot be eliminated through focusing.

Physically; astigmatism can be thought of as an effect of squeezing the reflector at opposite edges. Opposite sides would move near the focus while the other two quadrants would move further away as the reflector acts. like a shell. Thus the phase in the aperture plane leads in opposite quadrants and lags in the orthogonal quadrants. Gravitational sag tends to produce a deformation of this type, as will thermal expansion in certain cases. We would expect to see the effects of astigmatism in nearly all large movable antennas, which is indeed the case. In the following, we discuss the effects. that astigmatism produces on the measurable characteristics of the antenna.

It is important in the evaluation of a large antenna to be aware of the effects of astigmatism on the gain and pattern of the entenna for several reasons. For one, moving the feed position cannot eliminate astigmatism and hence proper focusing of the antenna amounts to reducing the phase error to the astigmatic form. In other words, one is through focusing when one obtains the characteristics of pure astigmatism. Another benefit of recognizing the effects of astigmatism is that the engineer can weight the benefits of reworking
(in some way) the antenns suriace or structure to reduce severe astigmatism. Finally, exploring the effects of astigmatism leads us to a simple procedure for detecting the presence and degree of astigmatism in an antenna.

The effects of astimgmatism are most easily seen in the patterns of the antenna. Let us consider the effects on the patterns with astigmatism present as the feed is moved axially. Recall the phase error is

$$
\varphi(x, y)=\left[\frac{\alpha\left(x^{2}-y^{2}\right)+\gamma\left(x^{2}+y^{2}\right)}{(D / 2)^{2}}\right] \frac{2 \pi}{\lambda} .
$$

As $\gamma$ is varied by moving the feed axially, the amount of parabolic phase errors in each principal direction is varied. This will be evidenced in the patierns by broadened beamwidths, increased sidelobes, and an absence of nulls, with patterns in $x$ and $y$ behaving differently due to the astigmatism. For example, if $\gamma=-\alpha$, there is no phsee error in the $x$ direction but there is a $2 \alpha$ parabolic error in the $y$ direction. This condition will produce a well formed pattern in the $x$ direction, with a narrow beamwidth, small sidelobes, and deep nulls. The pattern in the $y$ direction will be defocused and will exhibit a broadened main lobe, high principal sidelobes, and poor nulls. On the other hand, with $\gamma=+\alpha$, the opposite effects will be seen: the $y$ pattern will be narrow and well formed and the $z$ pattern will be broad and badly formed. If the fned is placed in the compromise position, both pstterns will be equally defocused but the gsin will be maximum.

Oi ine several effects of astigmatism which we have described above, the effect which gives the best guantitative evaluation of the astigastism of $s$ refiector is that of beam broadening. Half-power beamridths can be conveniently and precisely measured on a paitern range or through astronomical measurements of point sources [5], or the solar limb [6]. The $3-\mathrm{dB}$ beamwidths are generally a good diegnostic for the sharpness of the focus of an antenna. For example, if one varies the axial feed position for a perfect refiector and examines the beamwidths, one finds both beampidths minimumst the best focus, i.e., the focus for maximum gain.

In the previous section we have described how the astigmatic satenna focuses differently in orthogonal planes. Thus we would expect the two beamridths to minimize at two separate feed positions. This is shown in Fig. 1, which gives calculated beammidths versus sicis feed position for several values of the astigmatism parsmeter. These calculations are for $F / D=0.5$ and a 10 dB illuminstion iaper. Note that the beamwidths do minimize at diferent sxial foci, and that the distance between the axial foci increases njth the asigmatism parameter. Note that the beamwidths are equed at the compromise focus but are broadened with increasing astigmstism.

Comparison of experimental beamwidth dats with theoretical curve like those in Fig. 1 allows one to estimate the estignatism. The dsta ploted on Fig. 1 were taken on the University of Texas 16-ft antenne at 94.0 GHz in 1969 . From the data we judged the astigmatism parameter to be 1.3 , indicating a displacement of the reflector of 0.66 mm at the edge from a true parabola. This astigmatism has since been corrected by a reshimming of the antenns bachup striucture.

We might summarize the focusing and diagnostic procedures as follows:

1) Locste a priori focus. Mechanical or optical messurements can lecete the focus approximately. Fine adjustments must be based on the patterns.


Fig. 1. Relative beam fidths rersus axial defocusing for several values of astigmarism parameter $\alpha^{\prime}=2 \pi \alpha / \lambda$, in radians of phase error st edpe of reffector. This chart allows one to estimate degree of astigmatism from measured beamwidths. (Data ploted here are from matism firn reity of Texas 16-it antenna in 1869.)
2) Remove coma. Lateral movements are made to remove coma. To correct coms, move the feed awsy from the prominent (coma) sidelobe until symmetry is obtsined in both directions. Coms should be checked in more than two orthogonal planes in the pattern.
3) Locate the direction of maximum astigmatism. With the feed axially defocused from the maximum gain position, make a contour of the beam at the 3 or 10 dB level. If the beam is elliptical, astigmatism is possibly present. If defocusing on the opposite side of the maximum rotates the ellipse by $90^{\circ}$, then astigmatism is present. If not, unequal illumination tapering is indicated.
4) Measure degree of astigmatism. Measure beamwidths in major and minor axis directions versus axial feed position. Dats can then be compared with theoretical calculations as in Fig. 1 to estimate astipmatism.

The preceding procedure is premised on using a pattern range. All of the preceding procedures, or equivalent, can be accomplished through radio astronomical metbods in the centimeter- and millimeter-wavelength range, although not as conveniently as on a pattern range. In the event that the antenna astignatism depends on some independent variable such as arabient temperature or antenns elevation angle, the evaluation procedure is complicated though not confused. Modifications frould need to be developed to fit the specific case.

## References

[1] M. Born and E. Wrolf. Principles of Optics, 4th ed. Oxford, England: Pergamon Press, Ch. IX 1870.
[2] Pergamen "Lateral-feed displacement in a paraboloid," IEEE Trans. Antennus Propagat. vol. AP-1a, pp. B60-665. Sopt. 1965 .
[3] R. C. Hansen, Ed, Microwave Scanning Antennas Now York: Academic Press, 1964, vol. 1, pp. 139, 157-180 snd other reference cited therein.
[4] J. Ruzerin. Antenna tolerance theory-a review." Proc. IEEE, vol.
(5] $54, \mathrm{py}$. $633-640$, Apr. 1960 .
 IEEE Trans. Antennas Propagat., vol. AF-18, pp. 515-529, July
[6] $\underset{\text { E }}{ }$. Jocobs and H . King, "'2.8-minute beampldths, millimeter Fape antoms measurements and evaluation," in 1965 IEEE Int. Cond. Rec.. pt. 5. pp. 92-100.
tenna system for satellite or earth station applications," presented at 1975 Int. IEEE/AP-S Symp., J une 2-4, 1975.
[16] D. F. DiFonzo, W. J. English, and J. A. Janken, "Polarization characteristics of offset reflectors with multiple element feeds," presented at 1973 G-A.P Int. Symp., Aug. 22-24, 1973.
[17] T. S. Chu and R. H. Turrin, "Depolarization properties of offset reflectors," IEEE Trans. Antennas Propagat., vol. AP-21, pp. 339-345, May 1973.
[18] F. Taormina et. al, "INTELSAT IV-A communications antennasfrequen cy reuse through spatial isolation," presented at IEEE Int. Conf. Communications, June 14-16, 1976.
[19] R. W. Gruner and W. J. English, "Antenna design studies for a U.S. domestic satellite," COMSAT Tech. Rev., vol. 4, no. 2, pp. '413-448, Fall 1974.
[20] S. I. Ghobrial, "Cross-polarization in satellite and earth station antennas," this issue, pp. 378-387.
\{21] D. J. Sommers, L. I. Parad, and J. G. DiTullio, "Beam waveguide feed with frequency reuse diplexer for satellite communication earth station," Microwave J., vol. 18, no. 11, pp. 51-59, Nov. 1975.
[22] G. Hyde, R. W. Kreutel, and L. Smith, "The unattended earth terminal multiple-beam torus antenna," COMSAT Tech Rev, vol. 4, no. 2, pp. 231-262, Fall 1974.
$[23] \mathrm{S}$, Soma et. $a L$, "A high performance mode transducer/polarizer assembly in the 4 and 6 GHz bands," presented at 1974 Int. IEEE/AP.S Symp., June 10-12, 1974.
[24] W. J. English and.R. W. Gruner, "Earth station antenna system depolarization measurements with boresight ranges," in 1974

Int. IEEE/AP.S Symp. Dig., pp. 383-386, 1974.
[25] Telespazio S.p.A. per le Communicazioni Spaziaje (Italy), "Earth station antenna depolarization measurement study," Rep. for the INTELSAT, Sept. 15, 1974.
[26] D. F. DiFonzo, W. J. English, and W. S. Trachtman, "Antenna depolarization measurements using satellite signals," presented at 1976 Int. IEEE/AP.S Symp., Oct. 11-14, 1976.
[27] W. J. English, D. F. DiFonzo, and W. S. Trachtman, "Polarization diagrams for CP antenna analysis," presented at 1976 Int. AP-S Symp., Oct. 11-14, 1976.
[28] T. Oguchi, "Attentuation and phase rotation of radio waves due to rain: Calculated at 19.3 and 34.8 GHz ," Radio Sci, yol 8, no. 1, pp. 31-38, Jan. 1973.
[29] R.Taur, "Rain depolarization: Theory and experiment," COMSAT Tech. Rev., vol. 4, no. 1, pp. 187-190, Spring 1974.
[30] D. DiFonzo, A. E. Williams and W. S. Trachtman, "Adaptive polarization control for satellite frequency reuse systems," COMSAT Tech. Rev., vol. 6, no. 2, Fall 1976.
[31] T. S. Chu, "Restoring the orthogonality of two polarizations in radio communications systems, I," Bell Syst. Tech. J., vol. 50, no. 9, pp. 3063-3069, Nov. 1971.
[32] "Restoring the orthogonality of two polarizations in radio communications systems, II," Bell Syst. Tech. J., vol. 52, no. 3, pp. 319-327, Mar. 1973.
[33] W. S. Trachtman, "Adaptive orthogonalization of polarized signals with applications to satellite $\infty$ mmunications," M.Sc. thesis, Massachusetts Inst. Technol., Cambridge, June 1975.
[34] A. Williams, Private Commun.

# Cross-Polarization in Satellite and Earth-Station Antennas 

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#### Abstract

Cross-polarization in axially symmetric reflector antennas can be reduced, theoretically, to zero by use of special feeds like the Huygens' source. Alternatively, paraboloidal reflectors with large $f / D$ ratio do not deteriorate further the cross polarization level relative to the value due to the feed itself. The Cassegrainian optics is equivalent to a large $f / D$ paraboloid. The reflector of linearly polarized off set fed antennas contribute more cross-polarization than symmetrical reflectors fed by the same feed. With symmetrical reflectors the crosspolarized component generated by the reflector vanishes in the principal planes and is confined to four main lobes that have peak values in planes at $45^{\circ}$ to the principal planes. In the case of offset fed reflectors cross-polarization vanishes in the plane of symmetry and has its peak in the plane of asymmetry. The reflector generated crosspolarization with offset fed antennas may be reduced by use of small offset angles and large $f / D$ ratios. Feed offsetting has but little effect on the peak level of cross-polarization. This is usually accompanied with an asymmetry in the cross-polarization radiation pattern. Feed offsetting also results in spatial tilt in the copolarized and crosspolarized lobes with the cross-polar minimum always coinciding with the main beam peak. The effect of surface emors on the antenna crosspolarization is to partly fill the cross-polar along boresight. The peak cross-polarization, however, changes but slightly.


> -.

## I. Introduction

$-\pi D$ECENTLY, INCREASING interest has been shown in employing orthogonally polarized channels over the same microwave link for satellite communications [1][12]. While in theory orthogonal channels can be received in-

[^27]dependently, in practice limitations are set by the crosspolarization introduced by the antenna system and in the propagating medium. Most reflector antennas (symmetrical or offset front fed, Cassegrainian fed) has the property of having high cross-polarization discrimination along the main beam, i.e., on-axis. However, cross-polarized radiation rapidly increases and reaches a peak near the $3-\mathrm{dB}$ point of the copolarized beam maximum. This is of particular importance to satellite antennas since earth stations are usually distributed within the $3-4-\mathrm{dB}$ beamwidth of the satellite antenna. In this case, stations not in line with the satellite antenna axis will receive a signal with a certain amount of cross talk due to crosspolarization. It is evident therefore that every attempt should be made to reduce antenna cross-polarization not only along boresight but also off-axis.
Over the past few years considerable effort was put by different workers who sought means of cross-polarization reduction. The techniques that were suggested may be divided into two types: i) use of special primary feeds, ii) use of reflectors with large $f / D$ ratio.
As will be shown later the Huygens' source, if used to illuminate a symmetrical reflector, yields a unipolarized secondary pattern. On the other hand, with linearly polarized offset fed reflectors the Huygens' source offer but little improvement over the short dipole. When a long focal-length optical system (paraboloid, spherical reflector, lens) is used the contribution of the optical system is negligible compared to the feed. This results in an acceptable system, if the feed itself is reasonably
cross-polarization clean. However, practical consideration may prohibit the use of front-fed reflectors with large f/D ratio. For these cases the Cassegrainian geometry presents a solution since the effective f/D ratio of the Cassegrainian can be much greater than that of its main reflector. This depends on the eccentricity of the hyperboloidal subreflector.
In Section II, the definition of cross-polarization as employed in the present work will be introduced. Section III addresses itself to the analysis of front-fed and Cassegrainian antennas. The use of the equivalent parabola technique for crosspolarization is introduced. In Section IV, the important features of offset reflectors are discussed. This leads to the problem of feed off setting and a comparison between linear and circular polarization which is discussed in Section V. The problem of surface errors and the associated on beam crosspolarization will be discussed in Section VI.

## II. Cross-Polarization Reduction

## A. The Definition of Cross-Polarization

Ludwig [1] discussed three definitions for cross-polarization that are based on the direction of unit vectors which are function of position. Two of these definitions are significant: i) The IEEE standard [2] definition for cross-polarization in which a unit vector along one of the axes of the Cartesian is taken as the reference polarization (copolar). The direction of cross-polarization is taken along an orthogonal axis; ii) reference and cross-polarization are defined to be what one measures when antenna patterns are taken in the usual manner. Now interference between orthogonal channels in a frequency reuse system is a function of the antenna (transmitting and receiving) characteristics and not on the definition of crosspolarization one adopts. One should therefore adopt a definition which can predict the worst possible interference that may taken place in a frequency reuse link due to the antennas system. In this work instead of defining cross-polarization in terms of a reference unit vector the scalar voltage generated across a pick-up probe placed at the focus of the antenna will be used. Thus we will speak of the copolar and cross-polar voltages which we define as follows: With the transmitter, the satellite in Fig. 1, polarized along some reference axis, the $x$-axis say, the receiving antenna is adjusted to 'look' at the transmitter and the pick-up probe is oriented to be polarized along the $x$-axis. Copolar azimuth pattern ( $H$-plane in this case) is taken by rotating the antenna in the $x=0$ plane. The voltage given by a voltmeter connected across the pick-up probe will be referred to as the copolar voltage. The crosspolar voltage is measured in the same manner except that the pick up probe is initially adjusted to be polarized along the $y$ axis. It is evident that cross-polarization as defined by the above measurement procedure makes it possible to predict the level of cross-polarization that can taken place in a practical link employing orthogonal channels when an off-axis beam is received; e.g., due to refraction or reflection.

## B. Voltage at Terminals of Primary Feed

The voltage induced at the terminals of a primary feed placed at the focus of a reflecting system (this may comprise more than one reflector) when the reflector is excited by a plane wave is given by [3]:

$$
\begin{equation*}
V=C \iint_{S} \bar{E}_{r} \cdot \bar{E}_{f} \exp (-j \delta) \bar{r} \cdot d \bar{S} \tag{1}
\end{equation*}
$$



Fig. 1. On the definition of cross polarization.

(a)

(b)

Fig. 2. (a) Geometry of the front-fed system. (b) The Cassegrainian geometry.
where $\bar{E}_{r}$ is a unit vector describing the polarization of the reflected field; $\bar{E}_{f}$ is a unit vector describing the polarization of the field radiated from the primary feed when transmitting; $\delta$ is the phase angle associated with $\bar{E}_{r} ; \bar{r}$ is a unit vector along the ray associated with $\bar{E}_{r} ; d \bar{S}$ is a vector normal to the surface at the point of reflection with magnitude equal to the differential element of area of the reflector; and $C$ is a constant. The integration in (1) is to be carried over the surface of the reflector.

## C. The Copolar Voltage

Equation (1) gives the copolar voltage if the incident wave has the same polarization as the primary feed when the reflector is 'looking' along boresight. If the incident wave is polarized along the $x$-axis and is traveling along the axis of the reflector ( $z$-axis), Fig. 2, then the incident field is denoted by $\bar{E}_{i}^{x}$, the superscript designates the axis of polarization. The reflected field then becomes $\bar{E}_{r}^{x}$ and the field due to the feed
when transmitting is denoted by $\widetilde{E}_{f}^{x}$, here the superscript refers to the direction of the feed radiated field in the direction of maximum power flow.
Thus (1) is now written for the copolar voltage as

$$
\begin{equation*}
V_{\mathrm{co}}=C \iint_{s} \bar{E}_{r}^{x} \cdot \bar{E}_{f}^{x} \exp (-j \delta) \bar{r} \cdot d \bar{S} \tag{2}
\end{equation*}
$$

If the wave is incident along the axis of the reflector then the phase angle $\delta$ will be constant for all rays and by properly choosing the reference plane the exponential term may be set equal to unity. Under these conditions the copolar voltage is maximum, thus

$$
\begin{equation*}
\left.V_{\mathrm{co}}\right|_{\max }=C \iint_{S} \bar{E}_{r}^{x} \cdot \bar{E}_{f}^{x} \bar{r} \cdot d \bar{S} \tag{3}
\end{equation*}
$$

## D. The Cross-Polar Voltage

In this case the polarization of the incident field and that of the primary feed should be orthogonal when the incident wave is traveling along the axis of the reflector. This may be achieved by either: i) rotating the primary feed through $90^{\circ}$ keeping the incident polarization unchanged, or ii) rotating the incident polarization through $90^{\circ}$ keeping the primary feed unchanged. The latter method leads to the following equation for the cross-polar voltage

$$
\begin{equation*}
V_{\mathrm{cross}}=C \iint_{s} \bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x} \exp (-j \delta) \bar{r} \cdot d \bar{S} \tag{4}
\end{equation*}
$$

The superscript ' $y$ ' denotes the polarization of the wave when travelling along the axis of the reflector.
From the above expression for cross-polar voltage it is readily seen that the condition for zero cross-polarization is

$$
\begin{equation*}
\bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x}=0 \tag{5}
\end{equation*}
$$

This requires that the field due to the primary feed when transmitting and that due to the reflector when receiving should be orthogonal everywhere. An example for a feed that satisfies (5) when used with a paraboloid is offered by the Huygens' source. The credit for first pointing this goes to Jones [6]. Koffman [7] extended the work of Jones to other reflectors generated by conic sections.
It is interesting to note that (4) is symmetrical in $\bar{E}_{r}^{y}$ and $\bar{E}_{f}^{x}$. This suggests that the roles of reflector and feed on the cross-polarization of the system are equal. According to the third definition of Ludwig for the cross-polarization vector $\widetilde{E}_{r}^{y}$ is the reference polarization and hence any crosspolarization is attributed to the feed. Furthermore, according to (5) for any given reflector there exists a feed, at least theoretically, that satisfies the zero cross-polarization condition. Equation (5) also suggests that if a feed-reflector combination satisfies the zero cross-polarization requirements, then if the system is offset; zero cross-polarization is no longer maintained except if the feed has a corresponding asymmetry. This is indeed the case as was noted by a number of workers [8], [9]. Another possibility for elimination of cross-polarization is suggested by (4). This requires that the whole integral should vanish for all $\delta$ instead of the integrand vanishing. Thus even if $\bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x}$ is not zero, yet if the integral

$$
\iint \bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x} \exp (-j \delta) \bar{r} \cdot d \bar{S}=0, \quad \text { for all } \delta
$$

then no cross-polarization is introduced by the antenna system. The physical interpretation of this is that the field at the aperture of the hom feed should be identical to that formed at the focal plane of the reflector when excited by a plane wave. This follows directly from the transform property of the farfield and exit-pupil field. Using this, Ghobrial suggested a low cross-polarization feed that comprises five dipoles [10]. A main dipole polarized to match the required polarization and four auxiliary dipoles that are cross-polarized to simulate the cross-polarized field that appears in the reflector's focal plane diffraction image [11]. Measurements [12] with such a feed indicated that $9-\mathrm{dB}$ improvement in cross-polarization can be attained when the auxiliary elements are introduced.
The above discussed methods for cross-polarization reduction are not always possible to realize in practice. For example although small conical homs were reported by DiFonzo and Kreutel [13] to have polarization characteristics that come close to those of the Huygens' source, yet.these are only useful for symmetrical reflectors. With offset reflectors the Huygens' source does not eliminate cross-polarization. We now further investigate (4) for means of cross-polarization reduction. Expanding the dot product in (4), we have

$$
\begin{equation*}
\bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x}=E_{r x}^{y} E_{f x}^{x}+E_{r y}^{y} E_{f y}^{x}+E_{r z}^{y} E_{f z}^{x} . \tag{6}
\end{equation*}
$$

Cross-polarization reduction may be achieved by minimizing the absolute value of each term on the right-hand side (RHS) (6), thus,

$$
\begin{equation*}
\left|\bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x}\right|_{\min } \leqslant\left|E_{r x}^{y} E_{f x}^{x}\right|_{\min }+\left|E_{r y}^{y} E_{f y}^{x}\right|_{\min }+\left|E_{r z}^{y} E_{f z}^{x}\right|_{\min } \tag{7}
\end{equation*}
$$

To minimize the RHS, we need $E_{f f^{x}}^{x}, E_{r x}^{y}, E_{f z}^{x}$, and $E_{r z}^{y}$ all be minimum. The components $E_{r x}^{y}, E_{r z}^{y}$ are the $x$ and $z$ components of the field reflected by the reflector. Both of these components can be reduced by use of a long focal-length system (small angular semi-aperture or large $f / D$ ratio). That small offset angles minimize cross polarization in offset reflectors is also evident. The large $f / D$ ratio condition also results in reduction of the $y$ and $z$ components of the feed radiated field ( $E_{f y}^{x}, E_{f z}^{x}$ ). This follows since illuminating a reflector that subtends a small angle at the focus requires use of large aperture feed to reduce spill over. For most primary feeds the above components increase with off-axis angle. Hence one concludes that using reflectors with large f/D ratio can be considered as an alternative to using special primary feeds for cross-polarization reduction.

To achieve large $f / D$ ratios in practice using a single reflector presents difficulties. A double reflector system (e.g., the Cassegrainian) with a rather deep main reflector may have a large $f / D$ ratio and is more convenient from the point of view of practical considerations. The mechanism by which crosspolarization is reduced in the Cassegrainian system can also be explained in terms of the double reflection process that takes place. Since the main reflector and the subreflector have opposite curvatures the unwanted components of the field generated by reflection from the main reflector are restored on the second reflection by the subreflector. The extent to which this is accomplished is a function of the curvature of the hyperboloid and therefore a function of its eccentricity. In fact if the hyperboloidal subreflector is replaced by a paraboloidal subreflector, then the curvatures of the two reflectors become equal in magnitude and opposite in sense with the result that no depolarization takes place. However, under


Fig. 3. Cassegrainian employing two paraboloids.
these conditions the reflected rays from the subreflector do not converge to any focal point but rather form a plane wave; i.e., the focal length of the system becomes infinite, Fig. 3.

This last result may be readily proved by considering the expression for the $f / D$ ratio of the Cassegrainian in terms of the $f / D$ ratio of its main reflector. If $R_{c}$ is the $f / D$ ratio of the Cassegrain and $R$ is the $f / D$ ratio of the main reflector, then we have

$$
\begin{equation*}
R_{c}=\frac{e+1}{e-1} R \tag{8}
\end{equation*}
$$

where $e$ is the eccentricity of the hyperboloid. Now as $e$ approaches unity the hyperboloid transforms into a paraboloid and the RHS of (8) becomes infinite.

## II. Analysis of Front-Fed and Cassegrainian Systems

## A. The Reflecting Matrix

In (1); (2), and (3), $\bar{E}_{r}$ is a unit vector that describes the polarization of the reflected field. This may be determined in terms of the incident field polarization by use of the reflecting matrix (or using Ludwig's nomenclature the polarization matrix.). For any reflector the matrix is

$$
\left[\begin{array}{lll}
2 n_{x}^{2}-1 & 2 n_{x} n_{y} & 2 n_{x} n_{z} \\
2 n_{y} n_{x} & 2 n_{y}^{2}-1 & 2 n_{y} n_{z} \\
2 n_{z} n_{x} & 2 n_{z} n_{y} & 2 n_{z}^{2}-1
\end{array}\right]
$$

where $n_{x}, n_{y}$, and $n_{z}$ are the components of the unit vector normal to the surface at the point of reflection.

For the paraboloidal reflector and in terms of the coordinate system shown in Fig. 2, $n_{x}, n_{y}$, and $n_{z}$ are given by

$$
\begin{align*}
& n_{x}=\sin \theta \sin \phi / \sqrt{2(1+\cos \theta)} \\
& n_{y}=\sin \theta \cos \phi / \sqrt{2(1+\cos \theta)} \\
& n_{z}=-\sqrt{(1+\cos \theta) / 2} \tag{9}
\end{align*}
$$

Now if the incident electric vector has components $E_{i x}, E_{i y}$ and $E_{i z}$, then the components of $\bar{E}_{r}$ are obtained as

$$
\left[\begin{array}{l}
E_{r x}  \tag{10}\\
E_{r y} \\
E_{r z}
\end{array}\right]=\left[\begin{array}{lll}
2 n_{x}^{2}-1 & 2 n_{x} n_{y} & 2 n_{x} n_{z} \\
2 n_{y} n_{x} & 2 n_{y}^{2}-1 . & 2 n_{y} n_{z} \\
2 n_{z} n_{x} & 2 n_{z} n_{y} & 2 n_{z}^{2}-1
\end{array}\right]\left[\begin{array}{l}
E_{i x} \\
E_{i y} \\
E_{i z}
\end{array}\right]
$$

If the reflecting system consists of more than one reflector, e.g., the Cassegrainian, then the components of $\bar{E}_{r}$ are obtained by multiplying the column matrix of $\bar{E}_{i}$ by two matrices one for each reflector. Let $u_{x}, u_{y}$, and $u_{z}$ be the components of a $\therefore$ unit vector normal to the hyperboloidal subreflector in the Cassegrainian, then $\bar{E}_{r}$ is obtained as follows

$$
\begin{align*}
& {\left[\begin{array}{l}
E_{r x} \\
E_{r y} \\
E_{r z}
\end{array}\right]=\left[\begin{array}{lll}
2 u_{x}^{2}-1 & 2 u_{x} u_{y} & 2 u_{x} u_{z} \\
2 u_{y} u_{x} & 2 u_{y}^{2}-1 & 2 u_{y} u_{z} \\
2 u_{z} u_{x} & 2 u_{z} u_{y} & 2 u_{z}^{2}-1
\end{array}\right]} \\
&  \tag{11}\\
& \cdot\left[\begin{array}{lll}
2 n_{x}^{2}-1 & 2 n_{x} n_{y} & 2 n_{x} n_{z} \\
2 n_{y} n_{x} & 2 n_{y}^{2}-1 & 2 n_{y} n_{z} \\
2 n_{z} n_{x} & 2 n_{z} n_{y} & 2 n_{z}^{2}-1
\end{array}\right]\left[\begin{array}{l}
E_{i x} \\
E_{i y} \\
E_{i z}
\end{array}\right]
\end{align*}
$$

$u_{x}, u_{y}$, and $u_{z}$ are given in terms of the coordinate system of Fig. 2 by

$$
\begin{align*}
& u_{x}=-\sin \theta \sin \phi q^{-1} \\
& u_{y}=-\sin \theta \cos \phi q^{-1} \\
& u_{z}=(e+\cos \theta) q^{-1} \tag{12}
\end{align*}
$$

where $q^{2}=1+2 e \cos \theta+e^{2}$
That a Cassegrainian consisting of two paraboloids maintains the incident polarization may be deduced from (11). This is done by setting $u_{z}=n_{x}, u_{y}=n_{y}$, and $u_{z}=n_{z}$ in the subreflector matrix. On multiplication and using $n_{x}^{2}+n_{y}^{2}+n_{z}^{2}=1$ the required result is obtained immediately.

## B. The Front-Fed Paraboloid

Assuming a general primary feed with radiated field

$$
\begin{equation*}
\bar{E}_{f}=\bar{i} E_{f x}+\bar{j} E_{f y}+\bar{k} E_{f z} \tag{13}
\end{equation*}
$$

The copolar voltage measured across a pick-up probe at the focus of the parabola when excited in the receive mode is obtained using (2) and (10) as

$$
\begin{align*}
V_{\mathrm{co}}(\zeta, \eta)= & C \int_{0}^{2 \pi} \int_{0}^{\theta^{\prime}}\left\{E_{f x}\left(\sin ^{2} \phi(1-\cos \theta)-1\right)\right. \\
& \left.+E_{f y} \cos \phi \sin \phi(1-\cos \theta)-E_{f z} \sin \theta \sin \phi\right\} \\
& : \exp (-j \delta(\theta, \phi, \zeta, \eta))\left(\frac{\sin \theta}{1+\cos \theta}\right) d \theta d \phi \tag{14}
\end{align*}
$$

where
$\delta(\theta, \phi, \zeta, \eta)=\frac{4 f \pi}{\lambda(1+\cos \theta)}[\cos \theta(\cos \zeta \cos \eta-1)$
$-\sin \theta \sin \phi \sin \zeta-\sin \theta \cos \phi \cos \zeta \sin \eta]$
$\zeta$ and $\eta$ are the off-axis angles in elevation and azimuth, respectively [14], $\lambda$ is the wavelength. In deriving (14) it was assumed that the incident wave is polarized along the $x$-axis; thus the column matrix for $\bar{E}_{i}$ is.

$$
\left[\bar{E}_{i}\right]=\left[\begin{array}{l}
1  \tag{15}\\
0 \\
0
\end{array}\right]
$$

where the magnitude of the field was normalized to unity. It must be emphasized, however, that the matrix of (15) is true if and only if the incident wave is propagating along the axis of the reflector. If incidence if off-axis, i.e., the plane wave is tilted in elevation and azimuth by $\eta$ and $\zeta$ respectively, then the components of $\overline{E_{i}}$ will assume the form [14]

$$
\begin{align*}
& E_{i x}=\cos \zeta \\
& E_{i y}=-\tan \zeta \sin \eta \\
& E_{i z}=\sqrt{-\left(1-\sin ^{2} \zeta-\sin ^{2} \eta\right)} \tan \zeta . \tag{16}
\end{align*}
$$

The error introduced by using (15) rather than the more accurate (16) on the computations of the main characteristics of the copolar and cross-polar radiation pattems is negligible if the diameter of the reflector is large compared with the wavelength. 'This is true for earth-station antennas. For small reflectors such as those used in satellites using (15) introduces a small error in the second sidelobes of the main and cross-polar beams.

The cross-polar yoltage equation can be readily obtained by rotating the incident wave polarization through $90^{\circ}$, thus

$$
\left[\bar{E}_{i}\right]=\left[\begin{array}{l}
0 \\
1 \\
0
\end{array}\right]
$$

and hence

$$
\begin{align*}
V_{\text {cross }} & =C \int_{0}^{2 \pi} \int_{0}^{\theta^{\prime}}\left\{E_{f x} \sin \phi \cos \phi(1-\cos \theta)\right. \\
& \left.+E_{f y}\left(\cos ^{2} \phi(1-\cos \theta)-1\right)-E_{f z} \sin \theta \cos \phi\right\} \\
& \cdot \exp (-j \delta)\left(\frac{\sin \theta}{1+\cos \theta}\right) d \theta d \phi \tag{17}
\end{align*}
$$

Equations (14) and (17) are general equations and apply to any symmetrical paraboloid front-fed with any feed. We now discuss (17) in the light of the methods of cross polarization reduction discussed previously. For (17) to vanish for all $\zeta$ and $\eta$; i.e., for any off-axis angle the integrand must vanish, or

$$
\begin{array}{r}
E_{f x} \sin \phi \cos \phi(1-\cos \theta)+E_{f y}\left(\cos ^{2} \phi(1-\cos \theta)-1\right) \\
-E_{f z} \sin \theta \cos \phi=0 \tag{18}
\end{array}
$$

Thus one looks for a feed the $x, y$, and $z$ components of its radiated field satisfy the above condition. One feed that satisfies (18) is the Huygens' source. For the Huygens' source $\bar{E}_{f}^{x}$ assumes the form

$$
\begin{array}{r}
\bar{E}_{f}^{x}=\bar{i}\left(\sin ^{2} \phi(1-\cos \theta)-1\right)+\bar{j} \sin \phi \cos \phi(1-\cos \theta) \\
\\
-\bar{k} \sin \phi \sin \theta .
\end{array}
$$

The Huygens' source is only a conceptual source. Nonetheless, a number of workers investigated the possibility of constructing practical feeds that offer close approximation to the Huygens' source. Of these mention should be made to DiFonzo and Kreutek[13] who first observed that conical homs with - small apertures have polarization characteristics which come close to those of the Huygens' source. Cowan [15] reported that corrugated homs have optimum polarization characteristics if the aperture diameter is $0.8 \lambda$. This is particularly suitable for satellite antennas.
For most of the commonly used primary feeds the left-hand side (LHS) of (18) reduces to

$$
\begin{equation*}
F(\theta) \sin \phi \cos \phi \tag{19}
\end{equation*}
$$

where $F(\theta)$ is a function of $\theta$ as well as the geometry of the feed and is independent of $\phi$. From (19) one deduces that cross-polarization in symmetrical reflectors vanishes in the principal planes and therefore along boresight. It is also seen that maximum cross-polarization occurs in planes inclined at $45^{\circ}$ to the principal planes.

## C. The Cassegrainian System

For the Cassegrainian system, $\bar{E}_{r}^{x}$ is obtained as follows:

$$
\begin{align*}
& E_{r x}^{x}=A\left(1-(1-e)^{2}(1-\cos \psi) \sin ^{2} \phi q^{-2}\right) \\
& E_{r y}^{x}=-A \sin \phi \cos \phi(1-\cos \psi)(1-e)^{2} q^{-2} \\
& E_{r z}^{x}=-A \sin \phi \sin \psi\left(1-e^{2}\right) q^{-2}
\end{align*}
$$

when the incident wave is polarized along the $x$-axis. With th wave polarized along the $y$-axis we have

$$
\begin{align*}
& E_{r x}^{y}=-A \sin \phi \cos \phi(1-\cos \psi)(1-e)^{2} q^{-2} \\
& E_{r y}^{y}=A\left(1-(1-e)^{2}(1-\cos \psi) \cos ^{2} \phi q^{-2}\right) \\
& E_{r z}^{y}=-A \cos \phi \sin \psi\left(1-e^{2}\right) q^{-2} \tag{2}
\end{align*}
$$

where $A=B(1+e \cos \psi) /(1+\cos \psi), \quad B=2 e f / F\left(e^{2}-1\right)$, and $F$ are the focal lengths of the main reflector and the Cass grainian system, respectively. The factor $A$ is introduced $t$ take care of the increased field intensity at the surface of th subreflector due to collimation by the main reflector.

Now assuming the feed to be polarized along the $x$-axis th copolar and cross polar voltages are readily written as

$$
\begin{aligned}
V_{c o}(\zeta, \eta)= & C^{\prime} \int_{0}^{2 \pi} \int_{0}^{\psi^{\prime}}\left[E _ { f x } ^ { x } \left\{1-(1-e)^{2}(1-\cos \psi)\right.\right. \\
& \left.\cdot \sin ^{2} \phi q^{-2}\right\}-E_{r y}(1-e)^{2}(1-\cos \psi) \sin \phi \\
& \left.\cdot \cos \phi q^{-2}+E_{f z}^{x}\left(1-e^{2}\right) \sin \phi \sin \psi q^{-2}\right\} \\
& \cdot \frac{\sin \psi}{1+\cos \psi} q^{-2} \exp (-j \delta) d \psi d \phi \\
V_{\text {cross }}= & C^{\prime} \int_{0}^{2 \pi} \int_{0}^{\psi^{\prime}}\left[-E_{f x}^{x}(1-e)^{2}(1-\cos \psi) \sin \phi\right. \\
& \cdot \cos \phi q^{-2}+E_{f y}^{x}\left\{1-(1-e)^{2}(1-\cos \psi)\right. \\
& \left.\left.\cdot \cos ^{2} \phi q^{-2}\right\}+E_{f z}^{x}\left(1-e^{2}\right) \sin \psi \cos \phi q^{-2}\right] \\
& \cdot\left(\frac{\sin \psi}{1+\cos \psi}\right) \exp (-j \delta) d \psi d \phi
\end{aligned}
$$

## D. Equivalence of Front-Fed and Cassegrainian Cross-Polarization

We define a new variable $\alpha$ related to $\theta$ by

$$
\begin{aligned}
\cos \theta & =\left(\cos \alpha+e^{2} \cos \alpha+2 e\right) /\left(1+e^{2}+2 e \cos \alpha\right) \\
\sin \theta & =\left(e^{2}-1\right) \sin \alpha /\left(1+e^{2}+2 e \cos \alpha\right)
\end{aligned}
$$

If we substitute for $\sin \theta$ and $\cos \theta$ in (14) and (17) using ( ${ }^{2}$ we get

$$
\begin{aligned}
V_{c o}(\zeta, \eta)= & C^{\prime \prime} \int_{0}^{2 \pi} \int_{0}^{\alpha^{\prime}}\left[E _ { f x } ^ { x } \left\{(1-e)^{2}(1-\cos \alpha)\right.\right. \\
& \left.\cdot \sin ^{2} \phi p^{-2}-1\right\}+E_{f y}^{x}(1-e)^{2}(1-\cos \alpha) \sin \phi \\
& \left.\cdot \cos \phi p^{-2}-E_{f z}^{x}\left(1-e^{2}\right) \sin \alpha \sin \phi p^{-2}\right] \\
& \cdot \frac{\sin \alpha}{1+\cos \alpha} p^{-2} \exp (-j \delta) d \alpha d \phi
\end{aligned}
$$

$$
\begin{align*}
V_{\text {cross }}= & C^{\prime \prime} \int_{0}^{2 \pi} \int_{0}^{\alpha^{\prime}}\left[E_{f x}^{x}(1-e)^{2}(1-\cos \alpha) \sin \phi\right. \\
& \cdot \cos \phi p^{-2}+E_{f y}^{x}\left\{(1-e)^{2}(1-\cos \alpha) \cos ^{2} \phi-1\right\} \\
& \left.-E_{f z}^{x}\left(1-e^{2}\right) \sin \alpha \cos \phi p^{-2}\right] \\
& \cdots \frac{\sin \alpha}{1+\cos \alpha} p^{-2} \exp (-j \delta) d \alpha d \phi \tag{26}
\end{align*}
$$

where $p^{2}=1+e^{2}+2 e \cos \alpha, \alpha^{\prime}$ is related to $\theta^{\prime}$ by

$$
\begin{equation*}
\theta^{\prime}=\cos ^{-1}\left(\left(\cos \alpha^{\prime}+e^{2} \cos \alpha^{\prime}+2 e\right) /\left(1+e^{2}+2 e \cos \alpha^{\prime}\right)\right) \tag{27}
\end{equation*}
$$

Now the angular semi-aperture $\theta^{\prime}$, is related to the $f / D$ ratio of the front-fed system by

$$
\begin{equation*}
\cos \theta^{\prime}=\frac{16 R^{2}-1}{16 R^{2}+1} \tag{28}
\end{equation*}
$$

where $R=f / D$.
On the other hand, the $f / D$ ratio of the Cassegrainian is given in terms of the angular semi-aperture $\psi^{\prime}$ by

$$
\begin{equation*}
16 R_{c}^{2}=M^{2}\left(1+\cos \psi^{\prime}\right) /\left(1-\cos \psi^{\prime}\right) \tag{29}
\end{equation*}
$$

where $R_{c}$ is the $f / D$ ratio of the Cassegrainian;

$$
M=\left(\frac{e+1}{e-1}\right)
$$

Now if the $f / D$ ratios of the Cassegrainian and that of the front-fed system are equal, then using (21) and (29) one obtains the following relation $\theta^{\prime}$ and $\psi^{\prime}$.

$$
\begin{equation*}
\theta^{\prime}=\cos ^{-1}\left(\left(\cos \psi^{\prime}+e^{2} \cos \psi^{\prime}+2 e\right) /\left(1+e^{2}+2 e \cos \psi^{\prime}\right)\right) \tag{30}
\end{equation*}
$$

Comparing (30) with (27) it is immediately seen that $\alpha^{\prime}=\psi^{\prime}$ : Also comparing (25) and (26) (which were obtained for the front-fed antenna by introducing transformations (24) with (22) and (23), one finds that the Cassegrainian and front-fed antennas have the same polarization characteristics if their $f / D$ ratios are equal.
It is therefore concluded that the Cassegrainian polarization properties are deduceable from those of the equivalent parabola. An effect that is negligible but worth noting is that of


Fig. 5. The offset geometry.
blockage by the subreflector on cross-polarization in the case of the Cassegrainian. The effect of blockage on the copolar voltage is to reduce the gain of the antenna. Thus in (22) and (23) the lower limit with respect to $\psi$ should be taken as $\psi_{0}$ instead of zero; where $\psi_{0}$ is the half-angle subtended by the geometrical shadow of the subreflector at the paraboloid focus, Fig. 4. The effect of blockage on the cross-polar voltage is in general much less on the copolar voltage. This is the case since the shadow of the subreflector falls on the almost plane part of the paraboloid, which region introduces but little crosspolarization. One may therefore ignore the effect of blockage on cross-polarization.

## IV. Cross-Polarization in Offset Reflector Antennas

The problem of aperture blockage by the subreflector in Cassegrainian antennas may be overcome by use of the so called open Cassegrainian or offset Cassegrainian. Beside eliminating blockage the offset geometry has other advantages. These include reduction of reflector reaction on primary feed as well as reduction of astigmatism for off-axis feed locations. A number of workers studied the cross-polarization properties of offset reflectors. Of these mention is made to Cook et al. [16], Ghobrial and Watson [17] Chu and Turrin [8] and Rudge [9]. Cook et al. and Ghobrial and Watson studied the offset Cassegrainian cross-polarization when linear-polarization is employed. The conclusion that was drawn is that the peak of the cross-polar lobes lies in the plane of asymmetry rather than in planes inclined at $45^{\circ}$ to the principal planes. Chu and Turrin [8] and more recently Rudge [9] studied offset frontfed reflectors. The following are the major conclusions that emerged from the work of these authors.
Assuming a Huygens' source primary feed cross-polarization does not vanish in offset reflectors, cross-polarization originates entirely from the offset angle between the feed axis and the reflector's axis. The peak cross-polarization generated under these conditions is primarily a function of the offset angle $\theta_{0}$ and the half angle $\theta^{\prime}$; Fig. 5. The peak cross-polarization level increases with increasing values of $\theta_{0}$ and $\theta^{\prime}$ and is comparatively insensitive to amplitude taper of the primary feed illumination. Increasing the edge taper from 10 dB to 20 dB was reported by Chu and Turrin to reduce the peak cross-polarization by 1 dB . Fig. 6 which is due to Chu and Turrin, shows the maximum cross-polarization of linearly polarized excitation as


Fig. 6. Peak cross-polarization of off-set reflectors with linear excitation (after Chu and Turrin).


Fig. 7. Beam displacement as a function of the offset angle and the half-subtended angle for offset antennas with circularly polarized excitation (after Chu and Turrin).
a function of $\theta_{0}$ and $\theta^{\prime}$. The nonvanishing cross-polarization in offset reflectors excited by a Huygens' source can be explained in terms of the dot product of (4); thus while for symmetrical reflectors $\bar{E}_{f}^{x} \cdot \bar{E}_{r}^{y}=0$, indicating that these vectors are orthogonal, for offset reflectors the feed is rotated through the offset angle and $\bar{E}_{f}^{x}$ is no longer orthogonal to $\bar{E}_{r}^{y}$. Obviously as the offset angle is increased the deviation from orthogonality be-
tween $\bar{E}_{f}^{x}$ and $\bar{E}_{f}^{y}$ increases which explains the monotonically increasing behavior of the peak cross polarization with $\theta_{0}$.. It is worth noting that Dijk et al. [18] in their study of the polarization efficiency of offset reflectors noticed that the polarization efficiencies of offset paraboloids fed by a Huygens' source are very similar to those fed by an electric dipole. These workers also noticed that the polarization losses increase at increasing subtended angle $\theta^{\prime}$ and offset angle $\theta_{0}$.
While the polarization properties of linearly polarized offset reflectors are poor with circular-polarization no crosspolarization is generated. However, a beam displacement is produced. This is defined as the shift of the circularly polarized beam with respect to the physical plane of symmetry. The direction of shift is towards the right-for left-handed circular polarization and towards the left for right-handed circular polarization. Thus the beam displacement $\Delta \beta$ for circular polarization excitation results in some sacrifice of power if two opposite circular polarizations are used simultaneously within a specified angular region. Fig. 7 shows the beam displacement of circularly polarized excitation as a function of the offset angle $\theta_{0}$ and the half subtended angle $\theta^{\prime}$. This Fig. is due to Chu and Turrin.

## V. Feed Offsetting and Circular Polarization

## A. Symmetrical and Offset Reflectors with Offset Feeds

Studies of cross-polarization in symmetrical reflectors with offset feed were conducted by Kelleher and Coleman [19], Sandler [20] and more recently by Ghobrial [21]. The following conclusions were arrived at: i) the effect of feed offsetting on the peak cross-polarization is relatively not appreciable. Strictly speaking transverse displacement of feed is accompanied with a small change in peaks of cross-polar lobes and cross polar pattern asymmetry that increase with increasing feed offsetting. ii) Small transverse displacements result in a spatial shift in the cross-polar and copolar patterns, with the null of the cross-polar pattern always coinciding with the copolar peak. This point is of importance since for multibeam applications feed offsetting is a necessity. Fig. 8 shows theoretical copolar and cross-polar patterns for a feed displacement of 0.34 wavelength. The feed assumed in these computations was short dipole.
Rudge studied offsetting reflectors with offset feeds. The conclusions he arrived at are similar to those stated above for symmetrical reflectors.

## B. Circular-Polarization

With circular-polarization some of the cross-polarization characteristics of symmetrical and offset reflectors are modified. For example while with linear-polarization crosspolarization is confined to four main lobes, with circularpolarization and for a circularly symmetric feed crosspolarization is symmetric about the axis of the reflector. The off-axis angle at which peak cross-polarization occurs is approximately the same as for linear-polarization. The crosspolarization peak value is again the same as for linearpolarization provided that the same type of feed is used. For example if two orthogonal dipoles in phase quadrature are used as a circularly polarized primary feed to illuminate a symmetrical reflector, then the peak cross-polarization of the antenna will be the same as for a linearly polarized antenna with dipole primary feed. The off axis angle at which peak cross polarization occurs will also be the same. However, with circularpolarization the cross-polar pattern will be the same in any


Fig. 8. Copolar and cross-polar radiation patterns for a symmetrical reflector with offset feed. Feed displacement $=0.34$ wavelength, $f / D=0.25$.
plane while with linear-polarization peaks occur in the $45^{\circ}$ planes.

The analysis of circularly polarized antennas is similar to that of linearly polarized ones. Assuming a plane circularly polarized wave impinges on the surface of the reflector, then we can write the incident wave matrix as

$$
\left[\begin{array}{l}
1 \\
e^{ \pm j \pi / 2} \\
0
\end{array}\right]
$$

The $\pm$ sign indicate the hand of polarization. The reflected electric vector $\bar{E}_{r}^{M}$ is now obtained from the reflecting matrix as before, thus

$$
\left[\begin{array}{l}
E_{r x}^{M}  \tag{31}\\
E_{r y}^{M} \\
E_{r z}^{M}
\end{array}\right]=\left[\begin{array}{l}
\text { Reflecting } \\
\text { matrix }
\end{array}\right]\left[\begin{array}{l}
1 \\
e^{ \pm j \pi / 2} \\
0
\end{array}\right]
$$

where the superscript $M$ is $L$ or $R$ denoting left- or right-hand polarization. To compute the copolar and cross-polar voltages one uses the expressions:

$$
\begin{gather*}
V_{\mathrm{co}}=C \iint_{S} \bar{E}_{r}^{L} \cdot \bar{E}_{f}^{L} \exp (-j \delta) \bar{r} \cdot d \bar{S}  \tag{32}\\
V_{\mathrm{cross}}=C \iint_{S} \bar{E}_{r}^{L} \cdot \bar{E}_{\dot{J}}^{R} \exp (j \delta) \bar{r} \cdot d \bar{S} \tag{33}
\end{gather*}
$$

where the vectors $\bar{E}_{f}^{L}$ and $\bar{E}_{f}^{R}$ are unit vectors describing the polarization of the feed when left- and right-hand polarized, respectively.

## VI. Surface Errórs

While cross-polarization in both symmetric and offset reflectors vanishes along boresight, this is not true if the reflector's surface is not ideal; i.e., not true paraboloidal. If there are surface errors, then cross-polarization may not vanish along


Fig. 9. Boresight isolation as a function of peak surface error for reflectors with different angular semi-apertures. The dotted curve give the corresponding loss in antenna gain.
boresight. This problem is particularly relevent to earthstation antennas where the size of the reflector sets a limit on the accuracy to which the surface may be constructed [22]. Thus one requires a criterion for surface tolerance. The criterion that is usually employed is based on the loss in gain [22]. Although this criterion is useful for antennas using one polarization, for orthogonal-polarization applications the effect of surface errors on both antenna gain and crosspolarization should be investigated. A criterion based on the more serious effect should then be adopted.

Since cross-polarization has the property of being in antiphase in adjacent quadrants (considering linearly polarized symmetric reflectors) it follows that if surface errors are periodic with two periods about the axis of the reflector, then for a given peak surface error cross-polarization along boresight will be maximum. This is true if the pick-up probe is not a Huygens' source. If a Huygens' source is used then cross polarization will vanish independent of surface errors. Assuming the worst case, i.e., phase errors penodic with two periods in $\phi$, we then have

$$
\begin{equation*}
V_{\text {cross }}=C \iint_{s} \bar{E}_{r}^{y} \cdot \bar{E}_{f}^{x} \exp (-j m \sin \phi) \bar{r} \cdot d \bar{S} \tag{34}
\end{equation*}
$$

where $m$ is the peak phase error.
Now while for the Huygens' source the dot product vanishes, this is not necessarily the case for other types of primary feeds. For the short electric dipole and for pyramidal horns the cross polar voltage as obtained from (34) can be as high as -20 dB relative to the copolar peak; being dependent on the $f / D$ ratio of the reflector, and the peak surface error.
The expression for the boresight isolation which is true for paraboloids fed by a short dipole and small pyramidal horns is [23]

$$
\begin{equation*}
\text { isolation }(\mathrm{dB})=20 \log _{10}\left|J_{0}(m) F\left(\theta^{\prime}\right) / J_{1}(m)\right| \tag{35}
\end{equation*}
$$

where $J_{0}(m)$ and $J_{1}(m)$ are Bessel functions of the first kind, $m$ is the peak phase error $=(4 \pi / \lambda)$ peak surface error, $F\left(\theta^{\prime}\right)=$ $\left(1-\cos \theta^{\prime}\right) /\left(0.386-2 \log _{e}\left(1+\cos \theta^{\prime}\right)+\cos \theta^{\prime}\right), \theta^{\prime}$ is the angular semi-aperture as defined in Fig. 2.

Close investigation of (35) reveals that for a given $m$, increasing $\theta^{\prime}$ results in decreasing isolation. Fig. 9 shows a family of curves of isolation as a function of the peak surface error.


Fig 10. The effect of surface errors on the cross-polar radiation pattern. Shown patterns are for a reflector with flD $=0.46$, peak surface error $=0.01$ wavelength.

With the Cassegrainian antenna it can be easily demonstrated that surface errors have a much lesser effect on crosspolarization. This follows from the equivalence of Cassegrain and front-fed antennas with equal $f / D$ ratios. To determine the boresight isolation for a Cassegrainian with surface errors (35) may be employed using $\theta^{\prime}$. of the equivalent paraboloid as given by (30).
The effect of surface errors on the off-axis polarization performance of the antenna is to introduce a negligible change in the peak of the cross-polar lobe. Fig. 10 gives the cross-polar radiation patterns for a reflector with and without surface errors. The peak surface error is taken as 0.01 wavelength. It is seen that the effect of errors on cross polarization is essentially to fill the boresight null.
Finally, perhaps it should be emphasized that (35) and Fig. 10 are based on the assumption that reflector surface errors are periodic with two periods in $\phi$. Although this distribution is not impossible [24] and, in many cases, it was noticed that errors show periodicity about the reflector's axis, yet the above results represent the worst case and are valid for conventional feeds. With special feeds, e.g., corrugated homs, a considerable improvement in the boresight isolation is expected.

## VII. Conclusions

The polarization characteristics of reflector antennas were discussed and methods of cross-polarization reduction were reviewed. Two methods were discussed: i) use of special primary feeds, ii) use of long focal-length reflecting systems, e.g., the Cassegrainian system.
A method for the analysis of the polarization properties of reflector antennas based on the scalar copolar and cross polar voltages was outlined. This was further employed to demonstrate the equivalence of Cassegrainian and front-fed antennas with equal $f / D$ ratios. The polarization characteristics of offset reflectors were reviewed. Offset reflectors with linear exci-
tation have poor cross-polarization performance which improves with decreasing offset and subtended angles. With circular polarization offset reflectors generate no cross polarization, however beam shift is produced.

Feed offsetting in symmetrical and offset reflectors have the effect of tilting the copolar and cross-polar beams such that the null of the latter always coincides with the peak of the former. The peaks of the cross polar lobes are affected by feed offsetting but slightly. Circular polarization was also discussed. While with linear polarization cross polarization is confined to four main lobes, with circular polarization circular symmetry is maintained.

The boresight cross polarization due to antenna surface errors was discussed for both front-fed and Cassegrainian antennas. Maximum boresight cross polarization occurs if surface errors are periodic with two periods about the axis of the reflector. This is true if the primary feed is not a Huygens' source. With the Huygens' source cross polarization vanishes completely.

## References

[1] A. C. Ludwig, "The definition of cross polarization," IEEE Trans. Antennas Propagar., vol. AP-21, pp. 116-119, Jan. 1975.
[2] "IEEE Standard Definitions of Terms for Antennas," IEEE Trans. Antennas Propagat., vol. AP-17, pp. 262-269, May 1969.
[3] S. I. Ghobrial, "The effect of the polarization characteristics of the receiving feed on cross polarization of receiving reflector antennas,". Radio and Electron. Eng., vol. 45, no. 7, pp. 346-350, July 1975.
[4] P. A. Watson and S. I. Ghobrial, "Cross polarization in Cassegrain and front-fed antennas,". Electron. Lett., vol. 9, no. 14, pp. 297-298; July 12, 1973.
[s] P. J. Wood, Depolarization with Cassegrainian and front-fed reflectors," Electron. Letr., vol. 9, no. 8/9, May 3, 1973.
[6] E. M. T. Jones, "Paraboloid reflector and hyperboloid lens antennas," IRE Trans. Antennos Propagat., vol. AP-2, pp. 119-127, July 1954.
[7] 1. Koffman, "Feed polarization for parallel currents in reflectors generated by conic sections," IEEE Trans. Antennas Propagat., vol. AP-14, no. 1, pp. 37-40, Jan. 1966.
[8] T. S. Chu and R. H. Turrin, "Depolarization properties of offset reflector antennas," IEEE Trans. Antennas Propagat., vol. AP-21, Pp. 339-345, May 1975.
[9] A. W. Rudge, "Multi-beam antennas: Off-set reflectors with offset feeds," IEEE Trans. Antennas Propagat., vol. AP-23, No. 3, pp. 317-322, May 1975.
[10] S. 1. Ghobrial, "Some data for the design of low cross polarization feeds," Electron. Letr., vol. 9, no. 20،.pp. 465-466, Oct. 4, 1973.
[11]. "Co-polar and cross polar diffraction images in the focal plane of paraboloidal reflectors: A comparison between linear and circular polarization," IEEE Trans. Antennas Propagat., July 1976.
[12] S. l. Ghobrial and M. M. Futuh, "Cross polarization measurements using a composite feed with parasitic elements," Electron. Lett., vol. 11, Oct. 1975.
[13] D. F. DiFonzo and R. W. Kreutel, "Communication satelite antennas for frequency reuse," in Proc. 1971 G-AR. Int. Symp. (Univ. California, Los Angeles), 1971.
[14] P. A. Watson and. S. I. Ghobrial, "Off-axis polarization characteristics of Cassegrainian and front-fed antennas," IEEE Trans. Antennas Propagat., vol. AP-20, pp. 691-698, Nov. 1972.
[15] J. H. Cowan, "Dual band reflector feed element for frequency reuse applications," Electron. Lett., vol. 9, no. 25, pp. 596-597, Dec. 13, 1973.
[16] J. S. Cook, E. M. Elam, and H. Zucker, "The open Cassegrain antenna: Part 1: Electromagnetic design and analysis," Bell Syst. Tech. J., vol. 44, pp. 1255-1300, Sept. 1965.
[17] S. I, Ghobrial and P. A. Watson, "Polarization characteristics of off-set Cassegrain antennas," Postgraduate School of Electrical and Electronic Engineering, Univ. Bradford, Bradford, England, Tech. Rep. 110, May 1972.
[18] J. Dijk et al., "The polarization losses of off-set paraboloid antennas," IEEE Trans. Antennas Propagat., vol. AP-22, pp. 513520, July 1974.
[19] K. S. Kelleher and H. P. Coleman, "Off axis characteristics of the paraboloidal reflector," NRL Rep. 4083, Dec. 31, 1952.
[20] S. S. Sandler, "Paraboloidal reflector patterns for off-axis feed," IRE Trans. Antennas Propagat., vol. AP-8, pp. 368-379, July
1960.
[21] S. I. Ghobrial, "Cross polarization in reflector antennas," Ph.D. dissertation, Univ. Bradford, Bradford, Engiand, Dec. 1972.
[22] J. Ruze, "Antenna tolerance theory-A Review," Proc. IEEE, vol. S4, pp. 633-639, 1966.
[23] S. I. Ghobrial, "Cross polarization effect of paraboloidal reflector
antennas surface errors," presented at IEE Conf. Satellite Com munication Systems Technology, London, England, Apr. 7-10, 1975 ; IEE Conf. Publ. 126, pp. 246-252, 1975.
[24] Schneider and W. Schonbach: "Design study for 80-meter radio telescope," in Rec. IEE Conf. on Design and Construction of Large Steerable Aerials, pp. 237-241.

# Traveling Wave Tubes for Communication Satellites 

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#### Abstract

Traveling wave tubes (TWT's) have contributed markedly to the development of communications satellites. As the prime-power consuming and transmitting device, the major transponder gain element, and the largest contributor to transmission nonlinearities, the TWT has been the focal point for continuous but carefully measured evolutionary improvements. Efficiency improvements continue to be made without compromising desired communications characteristics or tube lifetimes. These improvements have been made primarily in the RF circuit through loss reduction and phase-velocity tapering techniques, and in the spent-beam region through better multielement collector designs. Traveling wave tubes developed for satellites at 4 and 12 GHz are used as examples.

Since TWT's are life-limited devices, emphasis has been placed on techniques ensuring long life in satellite applications. Both oxide- and dispenser-type cathodes are discussed and data on life characteristics are presented. During the past decade, while generally demonstrating excellent space lifetimes, operating TWT's continue to approach their potential cathode wear-out life, which is theoretically of the order of $10^{5} \mathrm{~h}$.


## I. Introduction

SINCE the inception of active communications satellites, traveling wave tubes (TWT's) have played a fundamental role in their development. Typically, the TWT constitutes the major satellite power-consuming element as well as the sole telecommunications microwave power transmitter: It is also the transponder's major amplification element, and it is the largest contributor to transmission nonlinearities. Along with radiation-degradable solar cells, batteries, and station keeping fuel, TWT's remain among the satellite's basic lifelimiting items. These features, in addition to the TWT's presently untaxed broad band capability, will continue to focus attention on this device.

The paper, after a brief review, discusses the most important design aspects of current space-type TWT's. First is the maximization of dc-toRF tube conversion efficiency at a given level

[^28]of RF output power. The output-power operating point relative to saturation is drive selected, and is based on an acceptable level of intermodulation "noise" contribution to the total noise budget in the transmission path. Therefore, improvements in the tube's linearity which also preserve efficiency are of interest. The second major issue is the TWT lifetime and the techniques used to obtain acceptable life performance. The output power required (typically in the range of 4 to 20 W) determines, in part, the selection of the TWT's long-lived design. To date all C-band satellite TWT's have, for example, used oxide-coated cathodes with electron emission densities ranging from 100 to $300 \mathrm{~mA} / \mathrm{cm}^{2}$.
Although other TWT characteristics, such as gain (typically 40 to 70 dB ), gain flatness, group delay; VSWR match, noise figure, overdrive response, dissipation, and weight, are important, each in its own way, they do not constitute sources of significant potential improvement at the present high level of device maturity. The major competitive incursion to TWT's comes from the longer lifetime potential and reduced weight of solid-state amplifiers. An indication of this trend is the use of solid-state driver amplifiers ( 3.7 to $4.2 \mathrm{GHz}, 40-\mathrm{dB}$ gain, $\sim 1$-W saturation power) to "back up" or replace driver TWT"s. The recently launched SATCOM and COMSTAR satellites, for example, carried bipolar, transistor driver amplifiers. Future satellites are gradually expected to have ever greater complements of solid-state microwave devices as advances in power handling, combining, and efficiency occur.

## II. Background

In the early 1960's, when the first experimental active communications satellites (TELSTAR, RELAY, and SYNCOM) were launched, TWT's provided the RF output amplification. By that time, the TWT (invented 20 years earlier by Kompfner [1]) was already an established device. It had been mathematically characterized by Pierce in 1950 [2], and by the mid1950's "line-of-sight" radio relay stations began using TWT's as transmitter output amplifiers [3], [4]. These early TWT's typically housed the electron gun, the helix slow-wave structure, and the beam collector in a glass envelope surrounded by a beam-focusing solenoid. This format, although usable in maintainable ground applications, was not attractive for weight-constrained space applications.




[^0]:    ?N
    THE ESTIHATED LOSS OF GAIN FOR MAXIMUM FEED OFFSET
    FROM THE FOCUS IS: -.006 dB

[^1]:    $=.0232 \mathrm{CON}=00: 03: 00$ INT $=5 \mathrm{CHG}=\$ .1$

[^2]:    11.000090 .0000
    0.000089 .0000
    0.000090 .0000
    0.000090 .0000

[^3]:    Manuscript recejved April 5, 1978 ; revised May 31, 1978:
    The authors are with the RF Technology Centre at ERA, Leatherhead, Surrey; England.

[^4]:    $\because$ Axisymmetric parabolic antennas have recently been subject to a detailed review by Clarricoats and Poulton [50].

[^5]:    ${ }^{1}$ Stationary polarization here means that the slope of the cross polarized radiation pattern is zero in the direction of the main beam maximum.

[^6]:    2 Circular polarization sense by IEEE definition: Wave receding from observer having clockwise rotation of the electric field is right circularly polarized.

[^7]:    : The cross polarizations caused by the subreflector and the main refector cancel each other.

[^8]:    S For Fig. 2 (c), convergence | $\Gamma$ | to the asymptotic re)ation ( 9 ) is much slower: this is because the stationary point is not a point of maximum illumination, as in the symmetrical case,

    - Some of this Imum is amin radjated toward the subreflector. -Gultine in multiple bounces between feed and subreflecior: this gives rise io a poor iransmission charanteristic for broad-band signals..

[^9]:    *For small angles of scan, Reference [4],

[^10]:    Manuscript received October 3, 1972; revised January 23, 1974. J. Dijk, E. J. Maanders, and L. F. G. Thuriings are with the Einchoven University of Technolugy, Eindhoven, the Netherlands.
    C. T. W. van Diepenbeek is with the Max Planck-Institut für Radioastronomie, Bonn, Germany.

[^11]:    Manuscript received July 24, 1974; revised October 30, 1974. This work was largely carried out under contracts with the European Space Research Organization and the Royal Radar Establishment, Malvern, England.
    The author was with the Department of Electronic and Ejectrical Engineering, University of Birmingham, Birmingham, England. He is now with the ERA-IITRI RF Technology Centre, Leatherhead, Surrey KT22 7SA, England.

[^12]:    Paper 6444 E. first received 3rd Seplember 1970 and in revised form 19th April 1971
    Dr. Rudge and Prof. Withers were formerly with the Department of Eleerical Engineering; University of Birmingham, Birmingham, England. Dr. Rudge is now with IIT Research Institule, 10 Wesi 35 Sireet, Chicago, I11. 60616, USA, and Prof. Wihers is a visiling prolessor of lelecommunicalions at the Instilulo Technologico de Acronáulica, Sáo José dos Campos. Sào Paulo, Brazil

[^13]:    Manuscript received February 22, 1972; revised August 14, 1972. This work was supported by NASA under Contract NAS7-100. W. V. T. Rusch is with the Department of Electrical Engineering, University of Southern Californis, Los Angeles, Calif. He is also a Consultent for the Jet Propulsion Laborstory, Californis Institute of Technology, Passdens, Calif. 91103.
    A. C. Ludwig is with the Jet Prapulsion Laborstory, Californis Institute of Technology, Pasadena, Calif. 91103.

[^14]:    ${ }^{1}$ The plane of scan contains the incident wave normal and the reflector axis.

[^15]:    2 A complete statement of reciprocity includes the currents on the reflect or. Howevei, the tangenibl $E$ field is zero on the corivetir, and (1) results.

[^16]:    ${ }^{2}$ Nio attempt was made to find other orientations of the feed which would optimize the scan gain.

[^17]:    4 In the process of generating these curves, small variations of order of 0.1 mavelength have been suppressed. However, these iations were not present for the $20-\mathrm{dB}$ edge taper, indicating perhaps, the existence of "edge-diffracted" rays in the focal region in: the less tapered illumination.

[^18]:    ${ }^{5}$ The true correlarion involves both $E$ and $\bar{H}$ fields [10].

[^19]:    Manuscript received November 13, 1972: revised May 8, 1974.
    The authors are with TRW Systems Group, Redondo Beach, Calif. 90278 .

[^20]:    Reprinted with permission from Bell Syst Tech. J., val. 53, pp. 1657-1665, Oct. 1974. Copyright © 1974 by the American Telephone and Telegraph Company.

[^21]:    * Original manuscript received by the PGAP, February 18, 1955; revised manuscript received, April 14, 1955.
    $\dagger$ Electrical Engrg. Dept., Syracuse Liniversity, Syracuse, N. Y.
    ${ }^{2}$ J. Ruze, "Efiect of Aperture Distribution Errors on the Radiation Patten," Antenna Lab. Memo., AF Cambridge Res. Center; lanuar: 22. 1952.
    $=$ R. C. Spencer, "A Least Square Analysis of the Effect of Phase Errors on Antenna Gain," Rep. No. F.5025, AF Cambridge Res. Center: lanuary; 1949.

[^22]:    * Manuscript received by the PGAP, February 27, 1956; revised manuscript received February 4, 1957 . This work was supported by the kome Air Development Center, USAF, under Contract No. AF $30(60) \geq 1360$.
    $\dagger$ Elec. Eng. Dept., Syracuse Liniv., Sıtacuse, N. Y'.

[^23]:    ${ }^{1}$ A. F. Kay, "Far Field Data at Close Distances. ${ }^{n}$ Final Rep. f.rr Contract No. AF 19(604)-1126, Tech. Res. Group; New York, N. Y.; October, 1954.

[^24]:    :D. K Cheng, "Microwave aerial testing at reduced ranges," Witeless Ertg. vol. 33, np. 234-237; October, 1956.

[^25]:    : S. Silver, "Microwave Antenna Theory and Design," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 12, ch. 6; 1949.
    ©D. K. Cheng and S. T. Moseley, "On-axis defocus characteristics of the paraboloidal reflector, ${ }^{n}$ IRE TRANS., vol. AP-3, pp. 214-216; October, 1955.

[^26]:    ${ }^{1}$ This reference was first brought to my attention by Dr. P. Mezger of the National Radio Astronomy Observators:

[^27]:    Manuscript received April 16, 1976; revised September 15, 1976.
    The author is with the Department of Electrical Engineering, Faculty of Engineering and Architecture, University of Khartoum, Khartoum, Sudan.

[^28]:    Manuscript received July 20, 1976; revised October 14, 1976. This paper is based upon work done in COMSAT Laboratories, Clarksburg, MD; AEG-Telefunken, Ulm, Federal Republic of Germany; and Thom-son-CSE, Velizy, France. It was supported in part by COMSAT Corporation and (INTELSAT) Organization. Views expressed in this paper are not necessarily those of INTELSAT or COMSAT.
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