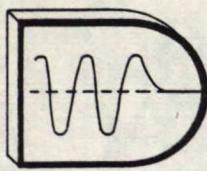


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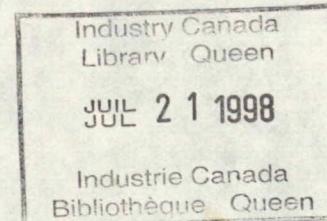


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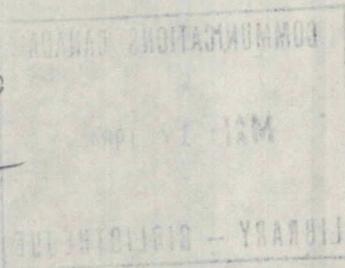


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TITLE: STRUCTURAL DYNAMICS MODEL FOR MSAT (*MARK I*)

AUTHOR(S):      P. C. Hughes  
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## SUMMARY

The aim of this report is to explain how to develop standard structural dynamics models for flexible satellites in a form suitable for computer simulation. The 'standard form' is first discussed, with an emphasis on how to distinguish 'rigid' degrees of freedom from 'elastic' ones. Modal coordinates are then introduced, being the most useful coordinate system in which to perform calculations. Input and output matrices are also formulated. The resulting equations are expressed both in state-space form and using transfer functions. Finally, several further simplifications are made in the special case of MSAT.

Four appendices contain the best data available at this time (*MARK I* model). System matrices and transfer function matrices are given for the design model (4 elastic modes) and the evaluation model (11 elastic modes).

## PREFACE

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The substructural data used to prepare the overall spacecraft models exhibited in the appendices were communicated by S. Sorocky of Spar/SASD.

Mrs. J. Hughes typed the report.

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### Units and Spelling

This report uses S.I. units and North American spelling.

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## 1. OBJECTIVES OF REPORT

The principal objective of this report is to construct the relationship between the structural dynamics model of MSAT, in the form

$$\underline{\underline{M}}\ddot{\underline{q}} + \underline{\underline{D}}\dot{\underline{q}} + \underline{\underline{G}}\dot{\underline{q}} + \underline{\underline{K}}\underline{q} = \underline{\underline{f}} \quad (1.1)$$

and the forms of the model most useful for computer simulation. Thus, this report begins (Section 2) by first explaining the form (1.1) in detail. The transformation to modal coordinates is then made in Section 3; this provides an opportunity for model order reduction, and is the best form for the structural dynamics kernel of the simulation. A structural dynamics model useful for control-design simulations must also have input and output modules. These inputs and outputs are the subject of Sections 4 and 5. The state-space form of the model is explained in Section 6, for those who wish to use this formulation, and Section 7 presents a derivation of input-output transfer functions. Certain additional specializations for MSAT are the subject of Section 8.

## 2. SPACECRAFT STRUCTURAL MODEL

Our point of departure is the structural model expressed in the form

$$\underline{\underline{M}}\ddot{\underline{q}} + \underline{\underline{D}}\dot{\underline{q}} + \underline{\underline{G}}\dot{\underline{q}} + \underline{\underline{K}}\underline{q} = \underline{\underline{f}} \quad (2.1)$$

where

$$\begin{aligned}\underline{\underline{M}}^T &= \underline{\underline{M}} > 0 \\ \underline{\underline{D}}^T &= \underline{\underline{D}} \geq 0 \\ \underline{\underline{G}}^T &= -\underline{\underline{G}} \\ \underline{\underline{K}}^T &= \underline{\underline{K}} \geq 0\end{aligned} \quad (2.2)$$

are, respectively, the mass matrix, damping matrix, gyricity matrix and stiffness matrix. Note that, for flexible space vehicles, unlike models of land-based structures,  $\underline{\underline{D}}$  and  $\underline{\underline{K}}$  are only semidefinite.

An important issue is the nature of the coordinates,  $\underline{q}(t)$ . It can be shown (although we shall not do so) that if  $\underline{K}$  is semidefinite (as here) a set of coordinates can always be found such that  $\underline{K}$  has the special form

$$\underline{K} = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{K}_e \end{bmatrix} \quad (2.3)$$

with

$$K_e > 0 \quad (2.4)$$

In other words, the first coordinates in  $\underline{q}$  are associated with the so-called 'rigid-body' degrees of freedom, and the remainder of the entries in  $\underline{q}$  are associated with 'elastic' degrees of freedom.

To be specific, let  $\underline{q}$  be  $N \times 1$ , comprising  $N_r$  rigid-body coordinates and  $N_e$  elastic coordinates:

$$N = N_r + N_e \quad (2.5)$$

Thus

$$\underline{q} = \begin{bmatrix} \underline{q}_r \\ \underline{q}_e \end{bmatrix} \quad (2.6)$$

where  $\underline{q}_r$  is  $N_r \times 1$  and  $\underline{q}_e$  is  $N_e \times 1$ . This partitioning agrees with (2.3); thus  $\underline{K}_e$  is  $N_e \times N_e$ . For MSAT it appears that  $N_r = 6$ , although if hinges or other types of articulation are employed,  $N_r > 6$ . For now we retain  $N_r$  arbitrary.

Next, we partition  $\underline{M}$ ,  $\underline{D}$ ,  $\underline{G}$  and  $\underline{A}$  to match  $\underline{K}$  and  $\underline{q}$ :

$$\underline{M} = \begin{bmatrix} M_r & M_{re} \\ M_{re}^T & M_e \end{bmatrix}; \quad \underline{\mathcal{D}} = \begin{bmatrix} 0 & 0 \\ 0 & D_e \end{bmatrix} \quad (2.7)$$

$$\underline{G} = \begin{bmatrix} G_r & G_{re} \\ -G_{re}^T & G_e \end{bmatrix}; \quad \underline{\mathbf{f}} = \begin{bmatrix} f_r \\ f_e \end{bmatrix}$$

Note that damping has been associated only with structural flexibility, not with the rigid-body degrees of freedom. It follows from (2.2) that

$$M_r^T = M_r > 0; \quad M_e^T = M_e > 0$$

$$D_e^T = D_e \quad (2.8)$$

$$G_r^T = -G_r \quad ; \quad G_e^T = -G_e$$

and since the structural damping will physically be complete, we must have

$$D_e > 0 \quad (2.9)$$

as an additional condition on  $D_e$ .

Before closing this section, the structural equations of motion, as represented thus far, are summarized for convenience:

$$\begin{aligned} M_r \ddot{q}_r + M_{re} \ddot{q}_e + G_r \dot{q}_r + G_{re} \dot{q}_e &= f_r \\ M_{re}^T \ddot{q}_r + M_e \ddot{q}_e - G_{re}^T \dot{q}_r + G_e \dot{q}_e + D_e \dot{q}_e + K_e q_e &= f_e \end{aligned} \quad (2.10)$$

More will be said presently about the nature of the coordinates  $q_r$  and  $q_e$ , and about their associated coefficient matrices.

### 3. MODAL COORDINATES

It is well known that there are many benefits to be derived from expressing the solution to the motion equations (2.1) in terms of 'characteristic motions' or 'eigenmotions.' For the applications in mind here, the damping is small (structural damping only) and the gyroscopic effects are also small (reaction wheels only). Therefore we adapt as our characteristic motions the undamped, nongyroscopic modes of the system, that is, the eigensolutions to

$$\ddot{\underline{M}\underline{q}} + \underline{K}\underline{q} = \underline{0} \quad (3.1)$$

In general, let the eigenvectors be  $\underline{e}_i$  and the associated eigenvalues (frequencies squared) be  $\omega_i^2$ . Then

$$\begin{aligned} \underline{e}_i^T \underline{M} \underline{e}_j &= 0 & (i \neq j) \\ \underline{e}_i^T \underline{K} \underline{e}_j &= 0 & (i \neq j) \end{aligned} \quad (3.2)$$

for  $i, j = 1, \dots, N$ . For our normalization condition, the 'natural' condition is chosen, viz.,

$$\underline{e}_i^T \underline{M} \underline{e}_i = 1 \quad (i = 1, \dots, N) \quad (3.3)$$

It follows that

$$\underline{e}_i^T \underline{K} \underline{e}_i = \omega_i^2 \quad (i = 1, \dots, N) \quad (3.4)$$

where  $\omega_i^2 = 0$  for  $i = 1, \dots, N_r$ . A succinct summary of this orthonormality information can be given in terms of the eigenmatrix  $\underline{E}$ ,

$$\underline{E} \triangleq [\underline{e}_1 \dots \underline{e}_N], \quad (3.5)$$

and the matrix of natural frequencies,

$$\underline{\Omega} \triangleq \text{diag}\{\omega_1, \dots, \omega_N\} \quad (3.6)$$

Using  $\underline{E}$  and  $\underline{\Omega}$ , (3.2) - (3.4) are written

$$\underline{E}^T \underline{M} \underline{E} = \underline{1} \quad (3.7)$$

$$\underline{E}^T \underline{K} \underline{E} = \underline{\Omega}^2 \quad (3.8)$$

How are these relationships specialized when the special rigid-elastic partitioning used in the last section is employed? To answer this question, recall that  $\underline{M}$  and  $\underline{K}$  are partitioned according to (2.3) and (2.7). We further assume (as is usual) that the first  $N_r$  natural frequencies are 0's, and the next (nonzero)  $N_e$  natural frequencies are listed in ascending magnitude. As a *change in notation*, it will also prove convenient to let  $\omega_1$  denote the first nonzero (elastic) natural frequency. Then with

$$\underline{\Omega}_e = \text{diag}\{\omega_1, \dots, \omega_{N_e}\} \quad (3.9)$$

we let

$$\underline{E} = \begin{bmatrix} \underline{E}_r & \underline{E}_{re} \\ \underline{0} & \underline{E}_e \end{bmatrix} \quad (3.10)$$

and

$$\underline{\Omega}^2 = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{\Omega}_e^2 \end{bmatrix} \quad (3.11)$$

However since the first  $N_r$  columns of  $\underline{E}$  are supposed to be rigid modes, the elastic coordinates are not involved in these modes. Therefore we have set

$$\underline{E}_{er} = \underline{0} \quad (3.12)$$

(These are also other ways to infer that  $\underline{E}_{er} = \underline{0}$ .)

One can in fact study the partitioned forms of the orthonormality conditions (3.7) - (3.8) to learn that

$$\underline{E}_r^T \underline{M}_r \underline{E}_r = 1 \quad (3.13)$$

$$\underline{M}_r \underline{E}_{re} + \underline{M}_{re} \underline{E}_e = 0 \quad (3.14)$$

$$\underline{E}_{e-e}^T \underline{M}_e \underline{E}_e - \underline{E}_{re-r}^T \underline{M}_r \underline{E}_{re} = 1 \quad (3.15)$$

$$\underline{E}_{e-e}^T \underline{K}_e \underline{E}_e = \underline{\omega}_e^2 \quad (3.16)$$

However this is not of primary importance to the present discussion.  
What is important is that the coordinate transformation

$$\underline{q} = \underline{E}\underline{n}; \quad \underline{n} = \text{col}\{\underline{n}_r, \underline{n}_e\} \quad (3.17)$$

applied to the full structural model (2.1) produces the following system model, in modal coordinates:

$$\begin{aligned} \ddot{\underline{n}}_r + \hat{G}_r \dot{\underline{n}}_r + \hat{G}_{re} \dot{\underline{n}}_e &= \underline{\gamma}_r \\ \ddot{\underline{n}}_e - \hat{G}_{re}^T \dot{\underline{n}}_r + \hat{G}_e \dot{\underline{n}}_e + \hat{D}_e \dot{\underline{n}}_e + \underline{\omega}_e^2 \underline{n}_e &= \underline{\gamma}_e \end{aligned} \quad (3.18)$$

where

$$\hat{D}_e \triangleq \underline{E}_{e-e}^T \underline{D}_e \underline{E}_e \quad (3.19)$$

$$\hat{G}_r \triangleq \underline{E}_r^T \underline{G}_r \underline{E}_r \quad (3.20)$$

$$\hat{G}_{re} \triangleq \underline{E}_r^T (\underline{G}_r \underline{E}_{re} + \underline{G}_{re} \underline{E}_e) \quad (3.21)$$

$$\begin{aligned} \hat{G}_e \triangleq & \underline{E}_{re-r}^T \underline{G}_r \underline{E}_{re} + \underline{E}_{e-e}^T \underline{G}_e \underline{E}_e \\ & + (\underline{E}_{re-r}^T \underline{G}_{re} \underline{E}_e) - (\underline{E}_{re-r}^T \underline{G}_{re} \underline{E}_e)^T \end{aligned} \quad (3.22)$$

However, as with (3.13 - 3.16), these latter four equations need not directly concern the person doing a simulation, provided he or she is given the transformed quantities as input data.

To complete the definition of new symbols in (3.18), we have

$$\begin{aligned}\underline{\gamma}_r &\triangleq \underline{E}_{r/r}^T \\ \underline{\gamma}_e &\triangleq \underline{E}_{re/e}^T + \underline{E}_{e/e}^T\end{aligned}\quad (3.23)$$

Further comments will be now made on the source of the forces  $\underline{f}_r$  and  $\underline{f}_e$  (and hence on  $\underline{\gamma}_r$  and  $\underline{\gamma}_e$ ).

#### 4. CLASSIFICATION OF FORCE INPUTS

For the purposes of this report, the inputs that affect the dynamics will be classified into three categories:

- gyric disturbances
- control inputs
- other disturbances

To comment on the first category, these disturbances are simply the gyric terms in (3.18). They are small; they couple otherwise uncoupled equations; and so they are best thought of as 'disturbances,' and accordingly placed on the right side of the motion equations. For MSAT, the gyricity is traceable to the reaction wheels. To avoid recalculating the whole matrix  $\underline{G}$  every time one of these wheels changes its spin rate, we set

$$\underline{G} = \sum_{i=1}^{N_w} G_i h_{si} \quad (4.1)$$

where  $h_{si}$  is the stored angular momentum in Wheel  $i$ , and  $N_w$  is the number of wheels. To the extent that the  $h_{si}$  are weak functions of time,  $\underline{G}$  is also a weak function of time: for this reason also, perhaps, it is best to look on the gyric terms as disturbances. Of course, after transformation to modal coordinates, we have

$$\hat{\underline{G}} = \sum_{i=1}^{N_w} \hat{G}_i h_{si} \quad (4.2)$$

where  $\hat{G}_i \triangleq \underline{E}^T \underline{G}_i \underline{E}$ , a relation that can be partitioned into rigid and elastic parts if desired.

As for control inputs, one would generally have

$$\underline{A}(t) = \underline{B}\underline{u}(t) \quad (4.3)$$

where  $\underline{u}(t)$  are the control variables. Following the partitioning of (2.3), (2.6) and (2.7),  $\underline{B}$  can be shown to be of the form

$$\underline{B} = \begin{bmatrix} \underline{B}_r & \underline{B}_{re} \\ \underline{0} & \underline{B}_e \end{bmatrix} \quad (4.4)$$

This form presupposes that  $\underline{u}(t)$  has been partitioned

$$\underline{u} = \begin{bmatrix} \underline{u}_r \\ \underline{u}_e \end{bmatrix} \quad (4.5)$$

and, more subtly, that  $\underline{u}_r$  represents control forces (or torques) on the main bus while  $\underline{u}_e$  represents control forces (or torques) on the elastic appendages (solar arrays or antennas). Although both  $\underline{u}_r$  and  $\underline{u}_e$  can excite the rigid degrees of freedom, only inputs on the appendages *directly* cause elastic displacements. (Elastic displacements can, of course, be caused *indirectly* by  $\underline{u}_r$  via dynamic coupling with the rigid coordinates.)

Now, for MSAT, no actuators are planned for the flexible appendages. Therefore  $\underline{u}_e \equiv \underline{0}$ , and one can dispense with  $\underline{B}_{re}$  and  $\underline{B}_e$ . However, for the moment, we can retain the 'third-generation satellite' flavor with only a modest additional effort.

One must also be able to calculate the control distribution matrix,  $\underline{B}$ , for a modal coordinate representation - i.e., to calculate  $\hat{\underline{B}}$ . From

$$\hat{\underline{B}} = \underline{E}^T \underline{B} \quad (4.6)$$

and (3.10) and (4.4), we have

$$\hat{\underline{B}} = \begin{bmatrix} \hat{\underline{B}}_r & \hat{\underline{B}}_{re} \\ \hat{\underline{B}}_{er} & \hat{\underline{B}}_e \end{bmatrix} \quad (4.7)$$

where

$$\hat{\underline{B}}_r \triangleq \underline{E}_r^T \underline{B}_r \quad (4.8)$$

$$\hat{\underline{B}}_{re} \triangleq \underline{E}_r^T \underline{B}_{re} \quad (4.9)$$

$$\hat{\underline{B}}_{er} \triangleq \underline{E}_{re}^T \underline{B}_r \quad (4.10)$$

$$\hat{\underline{B}}_e \triangleq \underline{E}_{re}^T \underline{B}_{re} + \underline{E}_e^T \underline{B}_e \quad (4.11)$$

As with several of the previous, similar relationships, (4.8) - (4.11) need not directly concern the simulator if given the  $\hat{\underline{B}}$  elements directly.

Finally, to turn to the 'all other disturbances' category, we let

$$\underline{A}(t) = \underline{B}\underline{u}(t) + \underline{d}^*(t) \quad (4.12)$$

$$\underline{d}^*(t) = -\underline{G}\dot{\underline{q}} + \underline{d}(t) \quad (4.13)$$

Thus  $\underline{d}^*$  represents all non-control inputs and  $\underline{d}(t)$  denotes all disturbances other than gyric effects. (If gyric effects are deemed unimportant, the distinction between  $\underline{d}^*$  and  $\underline{d}$  can be dropped.) Typically,  $\underline{d}(t)$  for MSAT will be due to solar pressure, gravity-gradient torques, and other, lesser environmental interactions. It is not the purpose of this report to develop expressions for  $\underline{d}(t)$ . In modal coordinates, (4.12) and (4.13) become

$$\underline{x}(t) = \hat{\underline{B}}\underline{u}(t) + \hat{\underline{d}}^*(t) \quad (4.14)$$

$$\hat{\underline{d}}^*(t) = -\hat{\underline{G}}\dot{\underline{q}} + \hat{\underline{d}}(t) \quad (4.15)$$

where

$$\hat{\underline{d}}(t) = \underline{E}^T \underline{d}(t) \quad (4.16)$$

It is expected that the environmental inputs will be quasisteady for MSAT.

## 5. OUTPUT VARIABLES

Two types of outputs may be distinguished:

- output variables whose values are of special interest;
- output variables that are to be measured using sensing devices.

These two categories are not, of course, mutually exclusive. Moreover, it is also clear that any variables in the first category, unless measured (i.e., unless in the second category), must be calculated based on some type of simulation.

Let the variables of special interest be denoted  $\underline{y}(t)$  and let the measurements be  $\underline{z}(t)$ . Furthermore, it is assumed that there is a linear relationship between both  $\underline{y}(t)$  and  $\underline{z}(t)$  and the system coordinates,  $\underline{q}(t)$ :

$$\underline{y} = \underline{P}\underline{q} ; \quad \underline{z} = \underline{C}\underline{q} \quad (5.1)$$

After transformation to modal coordinates these straightforward algebraic definitions become

$$\underline{y} = \hat{\underline{P}}\underline{n} ; \quad \underline{z} = \hat{\underline{C}}\underline{n} \quad (5.2)$$

where

$$\hat{\underline{P}} \triangleq \underline{P}\underline{E} \quad (5.3)$$

$$\hat{\underline{C}} \triangleq \underline{C}\underline{E} \quad (5.4)$$

Rate measurements are not included in this formulation although it is relatively simple to do so. As a second remark, it is observed that both measurements and quantities of interest are fundamentally *physical* quantities not *modal* quantities. (The device that can directly measure a modal coordinate has yet to be invented!)

Of great importance in the theory of flexible spacecraft control is the idea of *sensor-actuator duality*. Mathematically, the condition for duality is simple:

$$\underline{P} = \underline{B}^T \quad (\text{for outputs of interest}) \quad (5.5)$$

$$\underline{C} = \underline{B}^T \quad (\text{for measurements}) \quad (5.6)$$

In flexible spacecraft control theory (which we do not go into here) the second of these is by far the most important, since only actual measurements can be fed back as control inputs. In order to satisfy (5.6) it is necessary (and sufficient) that two conditions be satisfied:

- a sensor-actuator pair must be *co-located*, i.e., located at the same physical point on the spacecraft (an approximation, obviously)
- the sensor-actuator pair must be *functionally dual*, which is to say that they must be members of the following table:

#### FUNCTIONALLY DUAL SENSOR-ACTUATOR PAIRS

Sensor Measures	Actuator Causes
Absolute Translational Displacement	External force
Absolute Rotational Displacement	External torque
Relative Translational Displacement	Internal force
Relative Rotational Displacement	Internal torque

Very often, persons who speak of 'co-located sensors and actuators' really mean 'dual sensors and actuators;' they thus emphasize co-location while not mentioning (or being ignorant of) the requirement of functional duality.

To ascertain the partitioned forms for (5.1) - (5.4), using rigid and elastic coordinates, one must set

$$\underline{y} = \begin{bmatrix} \underline{y}_r \\ \underline{y}_e \end{bmatrix} ; \quad \underline{z} = \begin{bmatrix} \underline{z}_r \\ \underline{z}_e \end{bmatrix} \quad (5.7)$$

where  $\underline{z}_r$  are measurements made on the bus and  $\underline{z}_e$  are measurements made on the elastic appendages (and similarly for  $\underline{y}_r$  and  $\underline{y}_e$ ). It can be argued that

$$\underline{c} = \begin{bmatrix} \underline{c}_r & \underline{0} \\ \underline{c}_{er} & \underline{c}_e \end{bmatrix} \quad (5.8)$$

That is, only rigid coordinates can be (directly) sensed on the main bus, while both rigid and elastic coordinates are involved when taking measurements on the appendages. Actually, for MSAT, it is not at present contemplated that measurements be made on any appendages. Thus  $\underline{c}_{er}$  and  $\underline{c}_e$  can be dropped for the present MSAT control configuration. If all sensors and actuators are in dual pairs, then the duality condition (5.6) becomes

$$\underline{c}_r = \underline{B}_r^T \quad (5.9)$$

$$\underline{c}_{er} = \underline{B}_{re}^T \quad (5.10)$$

$$\underline{c}_e = \underline{B}_e^T \quad (5.11)$$

Using modal coordinates, the appropriate output matrices are as given by (5.2). With partitions as given by (3.10) and (5.8), one concludes that

$$\hat{\underline{c}} = \begin{bmatrix} \hat{\underline{c}}_r & \hat{\underline{c}}_{re} \\ \hat{\underline{c}}_{er} & \hat{\underline{c}}_e \end{bmatrix} \quad (5.12)$$

where

$$\hat{\underline{c}}_r = \underline{c}_r \underline{E}_r \quad (5.13)$$

$$\hat{\underline{c}}_{er} = \underline{c}_{er} \underline{E}_r \quad (5.14)$$

$$\hat{\underline{c}}_{re} = \underline{c}_r \underline{E}_{re} \quad (5.15)$$

$$\hat{\underline{c}}_e = \underline{c}_{er} \underline{E}_{re} + \underline{c}_e \underline{E}_e \quad (5.16)$$

Once again, the relations (5.13) - (5.16) are not of direct interest to the simulator if the left sides are given as data. As to duality, it is easy to show that if (5.9) - (5.11) are satisfied, then

$$\begin{aligned}
 \hat{\underline{c}}_r^T &= \hat{\underline{B}}_r \\
 \hat{\underline{c}}_{er}^T &= \hat{\underline{B}}_{re} \\
 \hat{\underline{c}}_{re}^T &= \hat{\underline{B}}_{er} \\
 \hat{\underline{c}}_e^T &= \hat{\underline{B}}_e
 \end{aligned} \tag{5.17}$$

which is, of course, just the partitioned form of  $\hat{\underline{C}}^T = \hat{\underline{B}}$ .

## 6. STATE-SPACE FORM

Thus far, the motion equations have been written in second-order form. To integrate these equations or to apply many theories for automatic control it is often desirable to express them in first-order form.

If one does not distinguish between 'rigid' and 'elastic' coordinates, the system

$$\ddot{\underline{n}} + \hat{\underline{D}}\dot{\underline{n}} + \underline{\Omega}^2\underline{n} = \hat{\underline{B}}\underline{u} + \hat{\underline{d}}^* \tag{6.1}$$

$$\underline{y} = \hat{\underline{P}}\underline{n} \tag{6.2}$$

$$\underline{z} = \hat{\underline{C}}\underline{n} \tag{6.3}$$

can be written in first-order form by defining the state variables

$$\underline{x} \triangleq \begin{bmatrix} \underline{n} \\ \dot{\underline{n}} \end{bmatrix} \tag{6.4}$$

whereupon

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}\underline{u} + \underline{d}^* \tag{6.5}$$

$$\underline{y} = \underline{P}\underline{x} \tag{6.6}$$

$$\underline{z} = \underline{C}\underline{x} \tag{6.7}$$

where

$$\underline{A} \triangleq \begin{bmatrix} 0 & 1 \\ -\underline{\omega}^2 & -\hat{\underline{d}} \end{bmatrix}; \quad \underline{B} \triangleq \begin{bmatrix} 0 \\ \hat{\underline{B}} \end{bmatrix}; \quad \underline{d}^* \triangleq \begin{bmatrix} 0 \\ \hat{\underline{d}}^* \end{bmatrix} \quad (6.8)$$

$$\underline{P} \triangleq [\hat{\underline{P}} \quad \underline{Q}]; \quad \underline{C} \triangleq [\hat{\underline{C}} \quad \underline{Q}] \quad (6.9)$$

The matrices  $\underline{\omega}$ ,  $\hat{\underline{d}}$ ,  $\underline{B}$ ,  $\hat{\underline{d}}^*$ ,  $\hat{\underline{P}}$ , and  $\hat{\underline{C}}$  have already been partitioned into their 'rigid' and 'elastic' parts and so  $\underline{A}$ ,  $\underline{B}$ ,  $\underline{P}$ ,  $\underline{C}$  and  $\underline{d}^*$  can be further subdivided in this manner also.

## 7. TRANSFER FUNCTIONS

For the remainder of this report, we delete all sensors and actuators not on the main bus:

$$\underline{z}_e \equiv 0 \quad (7.1)$$

$$\underline{u}_e \equiv 0 \quad (7.2)$$

However, we continue to allow  $y_e \neq 0$ ; these may well be outputs of interest on the appendages (e.g., an array tip deflection). We also will assume a diagonally dominant damping matrix:

$$\begin{aligned} \hat{d}_{\alpha\beta} &= 0 & (\alpha \neq \beta) \\ \hat{d}_{\alpha\alpha} &= 2\zeta_\alpha \omega_\alpha \end{aligned} \quad (7.3)$$

Lastly, we neglect any direct disturbances to the elastic modes.

$$\underline{d}_e \equiv 0 \quad (7.4)$$

The elastic modes are therefore excited only by the controller. This assumption is related to the fact that the external disturbances are quasi-steady and do not really disturb the elastic modes. Under these assumptions, the system equations are

$$\ddot{\underline{n}}_r = \hat{\underline{B}}_r \underline{u}_r + \hat{\underline{d}}_r^*$$

$$\ddot{\underline{n}}_\alpha + 2\zeta_\alpha \omega_\alpha \dot{\underline{n}}_\alpha + \omega_\alpha^2 \underline{n}_\alpha = \hat{\underline{b}}_\alpha^T \underline{u}_r \quad (\alpha = 1, \dots, N_e)$$

$$\underline{y}_r = \hat{\underline{p}}_r \underline{n}_r + \sum_{\alpha=1}^{N_e} \hat{\underline{p}}_\alpha \underline{n}_\alpha \quad (7.5)$$

$$\underline{y}_e = \hat{\underline{p}}_{er} \underline{n}_r + \sum_{\alpha=1}^{N_e} \hat{\underline{p}}_{e\alpha} \underline{n}_\alpha$$

$$\underline{z}_r = \hat{\underline{c}}_r \underline{n}_r + \sum_{\alpha=1}^{N_e} \hat{\underline{c}}_\alpha \underline{n}_\alpha$$

where

$$\begin{bmatrix} \underline{n}_1 \\ \vdots \\ \vdots \\ \underline{n}_{N_e} \end{bmatrix} \triangleq \underline{n}_e ; \quad \begin{bmatrix} \hat{\underline{b}}_1^T \\ \vdots \\ \vdots \\ \hat{\underline{b}}_{N_e}^T \end{bmatrix} \triangleq \hat{\underline{B}}_{er} \quad (7.6)$$

$$[\hat{\underline{p}}_1 \dots \hat{\underline{p}}_{N_e}] \triangleq \hat{\underline{p}}_{re} \quad (7.7)$$

$$[\hat{\underline{p}}_{e1} \dots \hat{\underline{p}}_{eN_e}] \triangleq \hat{\underline{p}}_e \quad (7.8)$$

$$[\hat{\underline{c}}_1 \dots \hat{\underline{c}}_{N_e}] \triangleq \hat{\underline{c}}_{re} \quad (7.9)$$

We are interested in transfer functions between the input,  $\underline{u}_r$ , and the outputs  $\underline{y}_r$ ,  $\underline{y}_e$ ,  $\underline{z}_r$ .

Using standard Laplace transform notation, the transform equivalent of (6.5) is

$$\underline{z}_r(s) = \underline{G}(s) \underline{u}_r(s) + \underline{G}_d(s) \underline{d}_r^*(s) \quad (7.10)$$

where the transfer-function matrices (or 'transfer matrices' for short)  $\underline{G}(s)$  and  $\underline{G}_d(s)$  are

$$\underline{G}(s) \triangleq \frac{1}{s^2} \left[ \hat{\underline{C}}_r \hat{\underline{B}}_r + \sum_{\alpha=1}^{N_e} \frac{s^2 K_\alpha}{s^2 + 2\zeta_\alpha \omega_\alpha s + \omega_\alpha^2} \right] \quad (7.11)$$

$$\underline{G}_d(s) = \frac{1}{s^2} \hat{\underline{C}}_r \quad (7.12)$$

and the 'modal gains' matrices  $\underline{K}_\alpha$  are

$$\underline{K}_\alpha \triangleq \hat{\underline{c}}_\alpha \hat{\underline{b}}_\alpha^T \quad (7.13)$$

If  $N_z$  and  $N_u$  are the number of sensors and actuators, respectively, then the  $\underline{K}_\alpha$  are  $N_z \times N_u$  matrices.

As for  $\underline{y}_r$  and  $\underline{y}_e$ , similar transfer functions can be derived in the above fashion if desired. Instead, we observe that they can be picked off from the simulation once  $\underline{n}_r$  and  $\underline{n}_\alpha$  are known, as indicated in (7.5c,d).

Before closing this section, alternative expressions are given for the 'rigid' transfer function  $s^{-2} \hat{\underline{C}}_r \hat{\underline{B}}_r$  and the modal gains  $\underline{K}_\alpha$ . From (5.13) and (4.8),

$$s^{-2} \hat{\underline{C}}_r \hat{\underline{B}}_r = s^{-2} \underline{C}_r \underline{E}_r \underline{E}_r^T \underline{B}_r \quad (7.14)$$

However, from (3.13),

$$\underline{E}_r \underline{E}_r^T = \underline{M}_r^{-1} \quad (7.15)$$

so that the 'rigid' transfer function is

$$s^{-2} \underline{C}_r \underline{M}_r^{-1} \underline{B}_r \quad (7.16)$$

Similarly, turning to the modal gains, if one combines (7.6) with (4.10) to get

$$\hat{\underline{b}}_\alpha^T = \underline{e}_{r\alpha}^T \underline{B}_r \quad (7.17)$$

and (7.7) with (5.15) to get

$$\hat{c}_\alpha = \underline{c}_r \underline{e}_{r\alpha} \quad (7.18)$$

where

$$[\underline{e}_{r1} \dots \underline{e}_{rN_e}] \triangleq \underline{E}_{re} \quad (7.19)$$

then the alternate expression for  $K_\alpha$  is

$$K_\alpha = \underline{c}_r \underline{e}_{r0} \underline{e}_{r\alpha}^T \underline{B}_r \quad (7.20)$$

Note that  $\underline{e}_{r\alpha}$  represents the participation of the 'rigid' coordinates in elastic mode  $\alpha$ .

## 8. CURRENT CASE FOR MSAT

In this section, the previous results will be specialized even further to MSAT by assuming the following:

- The control variables are

$$\underline{u} \equiv \underline{u}_r = \begin{bmatrix} \underline{f}_c \\ \underline{g}_c \end{bmatrix} \quad (6 \times 1) \quad (8.1)$$

where  $\underline{f}_c$  and  $\underline{g}_c$  are respectively the total control force and torque on the vehicle, all impressed on the main bus.

- The measurements are

$$\underline{z} \equiv \underline{z}_r = \underline{\theta} \quad (8.2)$$

where  $\underline{\theta}$  represents three small attitude librations for the main bus (roll, pitch, yaw).

- The other outputs of interest, in addition to  $\underline{\theta}$ , are yet to be specified.
- Disturbance inputs will be suppressed for the remainder of this section but can be dealt with in exactly the same manner as control inputs.

Under these assumptions, a number of simplifications, and specializations can be made to the results of previous sections. Thus,

$$\underline{q}_r = \begin{bmatrix} \underline{w} \\ \underline{\theta} \end{bmatrix} \quad (8.3)$$

where  $\underline{w}$  is the absolute translational displacement of some reference point O on the bus. (Note: the torque  $\underline{g}$  must be calculated about this same point.) The corresponding special form for  $\underline{M}_r$  is

$$\underline{M}_r = \begin{bmatrix} \underline{m}_1 & -\underline{c}^x \\ \underline{c}^x & \underline{J} \end{bmatrix} \quad (8.4)$$

where  $\underline{m}$  is the total mass, and  $\underline{c}$  and  $\underline{J}$  the first and second moment-of-inertia matrices of the entire vehicle about O.

Based on (6.1),  $\underline{B}_r$  is just the  $6 \times 6$  unit matrix, and based on (8.2),  $\underline{C}_r$  is just the following  $3 \times 6$  matrix:

$$\underline{C}_r = [0 \quad 1] \quad (8.5)$$

Then, from (7.13),

$$\hat{\underline{b}}_\alpha^T = [\underline{e}_{w\alpha}^T \quad \underline{e}_{\theta\alpha}^T] \quad (8.6)$$

and from (7.18),

$$\hat{\underline{c}}_\alpha = \underline{e}_{\theta\alpha} \quad (8.7)$$

where we have set out the  $\underline{w}$  and  $\underline{\theta}$  parts of  $\underline{e}_{r\alpha}$ :

$$\begin{bmatrix} \underline{e}_{w\alpha} \\ \underline{e}_{\theta\alpha} \end{bmatrix} \triangleq \underline{e}_{r\alpha} \quad (8.8)$$

These eigenvector pieces will give the parts of the modes that enter the measurements.

At this point, a literal inverse for  $\underline{M}_r$  is introduced.

Given (8.4) for  $\underline{M}_r$ , the inverse can be shown to be

$$\underline{M}_r^{-1} = \begin{bmatrix} m^{-1}\underline{I} - \underline{r}_c^x \underline{I}^{-1} \underline{r}_c^x & \underline{r}_c^x \underline{I}^{-1} \\ -\underline{I}^{-1} \underline{r}_c^x & \underline{I}^{-1} \end{bmatrix} \quad (8.9)$$

where

$$\underline{r}_c \triangleq \underline{c}/m \quad (8.10)$$

$$\underline{I} \triangleq \underline{J} + m\underline{r}_c^x \underline{r}_c^x \quad (8.11)$$

The physical interpretation is that  $\underline{r}_c$  is the location of the spacecraft mass center relative to O, and  $\underline{I}$  is the moment-of-inertia matrix of the spacecraft about its mass center. This means that the rigid transfer function given in (7.16) can be written

$$s^{-2} \underline{C}_r \underline{M}_r^{-1} \underline{B}_r = s^{-2} [-\underline{I}^{-1} \underline{r}_c^x \quad \underline{I}^{-1}] \quad (8.12)$$

Similarly, the modal gains  $\underline{K}_\alpha$  are

$$\underline{K}_\alpha = [\underline{K}_{f\alpha} \quad \underline{K}_{g\alpha}] \quad (8.13)$$

with

$$\begin{aligned} \underline{K}_{f\alpha} &\triangleq \underline{e}_{\theta\alpha} \underline{e}_{w\alpha}^T & (\alpha = 1, \dots, N_e) \\ \underline{K}_{g\alpha} &\triangleq \underline{e}_{\theta\alpha} \underline{e}_{\theta\alpha}^T \end{aligned} \quad (8.14)$$

All these gain matrices are now  $3 \times 3$ . There is no *a priori* reason to assume them to be diagonal, although the  $\underline{K}_{g\alpha}$  are evidently symmetric.

In summary, for MSAT, the transfer functions from control forces and torques on the main bus to attitude error are

$$\underline{\theta}(s) = \frac{1}{s^2} \left[ -\underline{I}^{-1} \underline{r}_c^X + \sum_{\alpha=1}^{N_e} \frac{s^2 \underline{K}_{f\alpha}}{s^2 + 2\xi_\alpha \omega_\alpha s + \omega_\alpha^2} \right] \underline{f}_c(s)$$

$$+ \frac{1}{s^2} \left[ \underline{I}^{-1} + \sum_{\alpha=1}^{N_e} \frac{s^2 \underline{K}_{g\alpha}}{s^2 + 2\xi_\alpha \omega_\alpha s + \omega_\alpha^2} \right] \underline{g}_c(s) \quad (8.15)$$

If only torques are applied, only the second transfer function is needed.

The time-domain equivalent of (8.15), expressed in second-order form is this:

$$\ddot{x}_w = \underline{f}_c(t) \quad (8.16)$$

$$\ddot{x}_\theta = \underline{g}_c(t) \quad (8.17)$$

$$\ddot{n}_\alpha + 2\xi_\alpha \omega_\alpha \dot{n}_\alpha + \omega_\alpha^2 n_\alpha = \underline{e}_{w\alpha}^T \underline{f}_c(t) + \underline{e}_{\theta\alpha}^T \underline{g}_c(t) \quad (\alpha = 1, \dots, N_e) \quad (8.18)$$

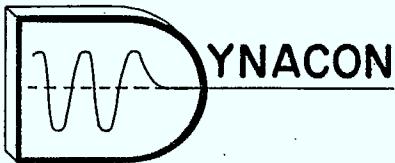
$$\underline{\theta}(t) = \underline{I}^{-1} [x_\theta(t) - \underline{r}_c^X x_w(t)] + \sum_{\alpha=1}^{N_e} \underline{e}_{\theta\alpha} n_\alpha(t) \quad (8.19)$$

The first three equations must be integrated and then  $\underline{\theta}(t)$  is then available from the last equation.

It should also be noted from (8.15) that the attitude dynamics is in general three-axis-coupled. That is, roll, pitch and yaw ( $\theta_1, \theta_2, \theta_3$ ) cannot in general be studied separately. Roll, pitch and yaw are uncoupled with respect to control torques only if both the following conditions are satisfied:

- $\underline{I}^{-1}$  is diagonal (i.e.,  $\underline{I}$  is diagonal)
- the  $\underline{K}_{g\alpha}$  are all diagonal matrices,  $\alpha = 1, \dots, N_e$

The second condition means that two of the three entries in each eigenvector-piece  $\underline{e}_{\theta\alpha}$  must be zero for all vehicle modes. This, in turn, requires certain inertial and elastic symmetries in the vehicle design. Such symmetry may sometimes be almost present (e.g., Hermes) and the 'uncoupled' approximation proves



YNACON

APPENDIX A

MSAT DYNAMICS MODEL

*MARK I*

(DESIGN MODEL:  $N_e = 4$ )

System Matrices

$$\omega_\alpha, \quad \alpha = 1, \dots, 4$$

$$\underline{E} = \begin{bmatrix} \underline{E}_r & \underline{E}_{re} \\ \underline{O} & \underline{E}_e \end{bmatrix}$$

$$\hat{\underline{d}}_e$$

$$\hat{\underline{G}}_1, \hat{\underline{G}}_2, \hat{\underline{G}}_3, \hat{\underline{G}}_4$$

$$\hat{\underline{B}} = \begin{bmatrix} \hat{\underline{B}}_r \\ \hat{\underline{B}}_{er} \end{bmatrix} \equiv \begin{bmatrix} \underline{E}_r^T \\ \underline{E}_{re}^T \end{bmatrix}$$

\*\*\* RETAINED FREQUENCIES \*\*\*

SELECTED MODES	(RAD/SEC)	FREQUENCY (HZ)
1	0. 000000000000000D+00	0. 000000000000000D+00
2	0. 000000000000000D+00	0. 000000000000000D+00
3	0. 000000000000000D+00	0. 000000000000000D+00
4	0. 000000000000000D+00	0. 000000000000000D+00
5	0. 000000000000000D+00	0. 000000000000000D+00
6	0. 000000000000000D+00	0. 000000000000000D+00
7	1. 1398259549566350D+00	1. 8140893499578850D-01
8	1. 1549710410779400D+00	1. 8381935031554660D-01
9	1. 5125592153215830D+00	2. 4073127583762840D-01
10	1. 5818256321765070D+00	2. 5175536847035330D-01

\*\*\* RETAINED EIGENVECTORS \*\*\*

ROW \ COL	7	8	9	10
1	-9.998D-06	-5.729D-03	1.518D-05	-1.099D-03
2	-3.310D-07	1.622D-04	-4.203D-04	-9.009D-03
3	6.147D-03	-4.600D-06	-1.012D-04	8.773D-06
4	8.401D-09	7.663D-05	-2.780D-04	-6.535D-03

ROW \ COL	7	8	9	10
5	5.094D-06	-2.842D-04	-1.519D-05	4.809D-04
6	6.791D-05	-1.358D-07	6.765D-03	-8.860D-05
7	-7.147D-01	-1.438D-01	-1.225D-01	6.812D-01
8	-1.273D-01	7.116D-01	8.508D-01	1.275D-01
9	1.032D-03	2.501D-03	2.820D-01	-5.170D-01
10	1.005D-03	4.045D-04	-5.571D-03	-1.114D-01
11	1.617D-04	1.853D-05	1.133D-03	5.823D-03
12	4.422D-06	-1.090D-04	-8.438D-04	8.370D-03
13	-3.810D-05	-4.264D-04	-5.460D-03	-1.497D-05
14	6.315D-04	5.634D-05	-1.305D-04	-7.545D-03
15	6.491D-04	2.271D-05	-3.768D-04	-5.295D-03
16	9.695D-05	-4.418D-05	-4.840D-04	2.686D-04
17	4.904D-05	1.313D-04	1.323D-03	-2.175D-04
18	-1.135D-04	3.596D-05	3.125D-04	4.735D-04
19	2.918D-05	-4.597D-05	-3.322D-04	-5.059D-04
20	-1.233D-05	-4.946D-05	-2.587D-04	1.393D-04
21	-8.784D-05	-1.165D-06	1.229D-05	3.639D-04
22	5.327D-05	7.940D-06	-5.416D-06	-2.250D-04
23	-3.708D-05	-3.285D-04	-1.092D-03	1.108D-04
24	2.398D-04	-2.804D-05	-1.757D-04	-1.068D-03
25	5.155D-05	1.173D-05	4.519D-05	-2.264D-04
26	-3.847D-07	-1.780D-05	-4.436D-05	2.877D-06
27	-1.397D-05	-5.180D-08	2.171D-06	3.724D-05
28	-1.306D-06	7.532D-07	4.953D-06	-7.796D-06
29	3.045D-07	1.219D-06	2.722D-06	-9.445D-07
30	-2.003D-08	-2.574D-07	-5.350D-07	-2.449D-07
31	-2.214D-07	-9.074D-07	-1.845D-06	-1.310D-07
32	-1.104D-07	-3.342D-07	-6.489D-07	-6.759D-08
33	-5.105D-07	1.633D-07	3.873D-07	1.298D-06
34	-7.147D-01	1.446D-01	-1.860D-01	-6.777D-01
35	-1.280D-01	-7.117D-01	8.379D-01	-1.475D-01
36	9.830D-04	-2.503D-03	2.897D-01	4.519D-01
37	1.005D-03	-4.061D-04	2.335D-03	1.114D-01
38	1.613D-04	-1.879D-05	7.174D-04	-5.850D-03
39	5.005D-06	1.091D-04	-1.470D-03	-8.335D-03
40	-3.664D-05	4.265D-04	-5.471D-03	1.767D-04
41	6.313D-04	-5.732D-05	4.459D-04	7.543D-03
42	6.494D-04	-2.370D-05	2.352D-05	5.303D-03
43	9.743D-05	4.403D-05	-5.101D-04	-2.536D-04
44	4.776D-05	-1.313D-04	1.355D-03	1.788D-04
45	-1.134D-04	-3.579D-05	2.751D-04	-4.826D-04
46	2.931D-05	4.593D-05	-2.943D-04	5.151D-04
47	-1.225D-05	4.949D-05	-2.700D-04	-1.317D-04
48	-8.777D-05	1.300D-06	-1.674D-05	-3.642D-04
49	5.343D-05	-8.022D-06	1.009D-05	2.252D-04
50	-3.694D-05	3.286D-04	-1.100D-03	-7.948D-05
51	2.400D-04	2.767D-05	-9.371D-05	1.073D-03
52	5.133D-05	-1.181D-05	6.549D-05	2.251D-04
53	-3.709D-07	1.780D-05	-4.440D-05	-1.605D-06
54	-1.397D-05	7.327D-08	-7.354D-07	-3.733D-05

ROW \ COL	7	8	9	10
55	-1.310D-06	-7.512D-07	4.919D-06	7.648D-06
56	3.036D-07	-1.220D-06	2.775D-06	8.671D-07
57	-1.994D-08	2.575D-07	-5.310D-07	2.601D-07
58	-2.210D-07	9.077D-07	-1.866D-06	1.832D-07
59	-1.103D-07	3.344D-07	-6.588D-07	8.582D-08
60	-5.105D-07	-1.625D-07	2.960D-07	-1.310D-06
61	-2.035D-05	-5.247D-06	-3.564D-03	6.095D-04
62	-1.468D-04	-8.246D-05	-1.046D-06	1.800D-04
63	3.941D-07	1.493D-06	6.016D-05	-2.325D-04
64	2.185D-06	-5.100D-06	3.256D-08	-4.524D-06
65	-3.202D-08	-7.594D-08	-5.229D-06	1.001D-05
66	-6.578D-07	-8.741D-07	2.273D-08	-3.062D-07
67	-3.625D-09	-7.342D-09	-6.005D-07	9.146D-07
68	-5.410D-08	-6.914D-08	1.669D-09	-1.733D-08
69	-6.537D-08	5.956D-09	1.135D-09	2.903D-08
70	5.339D-10	1.070D-09	8.816D-08	-1.415D-07
71	4.057D-08	4.315D-08	-1.323D-09	1.392D-08
72	-5.816D-10	-1.569D-09	-9.392D-08	2.087D-07
73	-7.336D-10	-1.573D-09	-1.210D-07	1.996D-07
74	-1.098D-09	-2.433D-08	3.528D-10	-1.657D-08
75	4.037D-09	8.588D-09	-1.630D-10	3.374D-09
76	7.220D-09	-1.067D-08	-6.267D-12	-9.544D-09
77	1.598D-10	4.219D-10	2.590D-08	-5.488D-08
78	-1.220D-08	-9.034D-09	3.535D-10	-1.784D-09
79	-2.037D-05	5.330D-06	-3.611D-03	-5.066D-04
80	-1.488D-04	8.269D-05	9.644D-06	-1.808D-04
81	3.944D-07	-1.494D-06	7.827D-05	2.305D-04
82	2.232D-06	5.096D-06	-1.606D-07	4.536D-06
83	-3.217D-08	7.607D-08	-6.032D-06	-9.849D-06
84	-6.550D-07	8.751D-07	1.534D-08	3.026D-07
85	-3.644D-09	7.357D-09	-6.748D-07	-8.963D-07
86	-5.395D-08	6.922D-08	1.460D-09	1.703D-08
87	-6.568D-08	-5.855D-09	2.663D-09	-2.939D-08
88	5.360D-10	-1.072D-09	9.950D-08	1.388D-07
89	4.044D-08	-4.321D-08	-1.025D-09	-1.370D-08
90	-5.846D-10	1.571D-09	-1.107D-07	-2.057D-07
91	-7.374D-10	1.576D-09	-1.371D-07	-1.959D-07
92	-9.303D-10	2.433D-08	-2.941D-10	1.657D-08
93	4.005D-09	-8.594D-09	-7.011D-11	-3.351D-09
94	7.318D-09	1.066D-08	-4.151D-10	9.584D-09
95	1.607D-10	-4.225D-10	3.032D-08	5.407D-08
96	-1.219D-08	9.053D-09	3.532D-10	1.716D-09

\*\*\* RETAINED MODAL DAMPING MATRIX \*\*\*

ROW \ COL	1	2	3	4
1	1.170D-02	-8.171D-08	3.011D-05	1.953D-07
2	-8.171D-08	1.186D-02	-1.235D-06	-1.227D-05
3	3.011D-05	-1.235D-06	1.920D-02	8.620D-05
4	1.953D-07	-1.227D-05	8.620D-05	1.873D-02

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 1-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	0. 000D+00					
5	0. 000D+00	9. 014D-05				
6	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-9. 014D-05	0. 000D+00
7	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-7. 228D-07	4. 480D-08
8	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	1. 445D-09	-2. 407D-06
9	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-7. 200D-05	3. 712D-08
10	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	9. 430D-07	4. 071D-06
ROW \ COL	7	8	9	10		
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
4	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
5	-7. 228D-07	-1. 445D-09	7. 200D-05	-9. 430D-07		
6	-4. 480D-08	2. 407D-06	-3. 712D-08	-4. 071D-06		
7	0. 000D+00	1. 930D-08	3. 549D-08	-3. 311D-08		
8	-1. 930D-08	0. 000D+00	-1. 922D-06	2. 524D-08		
9	-3. 549D-08	1. 922D-06	0. 000D+00	-3. 252D-06		
10	3. 311D-08	-2. 524D-08	3. 252D-06	0. 000D+00		

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 2-AXIS \*\*\*

ROW \ COL	1.	2	3	4	5	6
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	0. 000D+00	-8. 198D-05				
5	0. 000D+00	7. 468D-08				
6	0. 000D+00	0. 000D+00	0. 000D+00	8. 198D-05	-7. 468D-08	0. 000D+00
7	0. 000D+00	0. 000D+00	0. 000D+00	6. 573D-07	-5. 988D-10	2. 986D-09
8	0. 000D+00	0. 000D+00	0. 000D+00	-1. 314D-09	1. 197D-12	-6. 490D-07
9	0. 000D+00	0. 000D+00	0. 000D+00	6. 548D-05	-5. 965D-08	2. 659D-06
10	0. 000D+00	0. 000D+00	0. 000D+00	-8. 576D-07	7. 813D-10	5. 534D-05
ROW \ COL	7	8	9	10		
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
4	-6. 573D-07	1. 314D-09	-6. 548D-05	8. 576D-07		
5	5. 988D-10	-1. 197D-12	5. 965D-08	-7. 813D-10		
6	-2. 986D-09	6. 490D-07	-2. 659D-06	-5. 534D-05		
7	0. 000D+00	5. 204D-09	-1. 894D-08	-4. 437D-07		
8	-5. 204D-09	0. 000D+00	-5. 184D-07	7. 677D-09		
9	1. 894D-08	5. 184D-07	0. 000D+00	-4. 423D-05		
10	4. 437D-07	-7. 677D-09	4. 423D-05	0. 000D+00		

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 3-AXIS \*\*\*

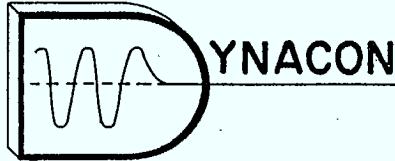
ROW \ COL	1	2	3	4	5	6
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	1. 030D-04	-2. 372D-07
5	0. 000D+00	0. 000D+00	0. 000D+00	-1. 030D-04	0. 000D+00	-4. 790D-07
6	0. 000D+00	0. 000D+00	0. 000D+00	2. 372D-07	4. 790D-07	0. 000D+00
7	0. 000D+00	0. 000D+00	0. 000D+00	-4. 930D-08	1. 343D-10	-2. 295D-10
8	0. 000D+00	0. 000D+00	0. 000D+00	2. 751D-06	8. 131D-07	1. 092D-08
9	0. 000D+00	0. 000D+00	0. 000D+00	1. 471D-07	-2. 959D-06	7. 499D-09
10	0. 000D+00	0. 000D+00	0. 000D+00	-4. 655D-06	-6. 955D-05	1. 385D-07
ROW \ COL	7	8	9	10		
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
4	4. 930D-08	-2. 751D-06	-1. 471D-07	4. 655D-06		
5	-1. 343D-10	-8. 131D-07	2. 959D-06	6. 955D-05		
6	2. 295D-10	-1. 092D-08	-7. 499D-09	-1. 385D-07		
7	0. 000D+00	-3. 927D-10	1. 416D-09	3. 329D-08		
8	3. 927D-10	0. 000D+00	-8. 018D-08	-1. 820D-06		
9	-1. 416D-09	8. 018D-08	0. 000D+00	-2. 330D-07		
10	-3. 329D-08	1. 820D-06	2. 330D-07	0. 000D+00		

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 4-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	5. 948D-05	-4. 747D-05
5	0. 000D+00	0. 000D+00	0. 000D+00	-5. 948D-05	0. 000D+00	5. 181D-05
6	0. 000D+00	0. 000D+00	0. 000D+00	4. 747D-05	-5. 181D-05	1. 323D-23
7	0. 000D+00	0. 000D+00	0. 000D+00	3. 510D-07	-4. 176D-07	2. 746D-08
8	0. 000D+00	0. 000D+00	0. 000D+00	1. 587D-06	4. 703D-07	-1. 758D-06
9	0. 000D+00	0. 000D+00	0. 000D+00	3. 789D-05	-4. 331D-05	1. 561D-06
10	0. 000D+00	0. 000D+00	0. 000D+00	-3. 183D-06	-3. 961D-05	3. 438D-05
ROW \ COL	7	8	9	10		
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00		
4	-3. 510D-07	-1. 587D-06	-3. 789D-05	3. 183D-06		
5	-4. 176D-07	-4. 703D-07	4. 331D-05	3. 961D-05		
6	-2. 746D-08	1. 758D-06	-1. 561D-06	-3. 438D-05		
7	-1. 292D-26	1. 392D-08	1. 037D-08	-2. 561D-07		
8	-1. 392D-08	3. 748D-25	-1. 455D-06	-1. 032D-06		
9	-1. 037D-08	1. 455D-06	0. 000D+00	-2. 755D-05		
10	2. 561D-07	1. 032D-06	2. 755D-05	2. 515D-22		

\*\*\* RETAINED MODAL CONTROL DISTRIBUTION MATRIX \*\*\*

ROW \ COL	1	2	3	4	5	6
1	2.641D-02	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
2	0.000D+00	2.641D-02	0.000D+00	0.000D+00	0.000D+00	0.000D+00
3	0.000D+00	0.000D+00	2.641D-02	0.000D+00	0.000D+00	0.000D+00
4	0.000D+00	1.340D-02	2.399D-20	9.680D-03	0.000D+00	0.000D+00
5	-1.474D-02	-1.221D-05	5.824D-05	-8.818D-06	1.064D-02	0.000D+00
6	3.394D-05	1.600D-05	-1.341D-07	4.502D-05	-2.451D-05	8.469D-03
7	-9.998D-06	-3.310D-07	6.147D-03	8.401D-09	5.094D-06	6.791D-05
8	-5.729D-03	1.622D-04	-4.600D-06	7.663D-05	-2.842D-04	-1.358D-07
9	1.518D-05	-4.203D-04	-1.012D-04	-2.780D-04	-1.519D-05	6.765D-03
10	-1.099D-03	-9.009D-03	8.773D-06	-6.535D-03	4.809D-04	-8.860D-05



SYNACON

APPENDIX B

MSAT DYNAMICS MODEL

MARK I

(DESIGN MODEL:  $N_e = 4$ )

Transfer Function Matrices

$\underline{I}$

$\underline{I}^{-1}$

$\underline{r}_c$

$-\underline{I}^{-1}\underline{r}_c^x$

$$\begin{bmatrix} \omega_\alpha \\ \zeta_\alpha \\ \underline{e}_{w\alpha}^T & \underline{e}_{\theta\alpha}^T \\ \underline{K}_{f\alpha} & \underline{K}_{g\alpha} \end{bmatrix}, \quad \alpha = 1, \dots, 4$$

INERTIA MATRIX I

1. 067D+04	8. 842D+00	-5. 671D+01
8. 842D+00	8. 827D+03	2. 550D+01
-5. 671D+01	2. 550D+01	1. 394D+04

INVERSE OF INERTIA MATRIX I

9. 370D-05	-9. 495D-08	3. 813D-07
-9. 495D-08	1. 133D-04	-2. 076D-07
3. 813D-07	-2. 076D-07	7. 172D-05

RC - POSITION OF MASS CENTER RELATIVE TO OB

5. 471D-03	0. 000D+00	1. 385D+00
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THE MATRIX PRODUCT -(I\*\*-1)(RC\*\*X)

1. 315D-07	1. 297D-04	-5. 195D-10
-1. 569D-04	-1. 303D-07	6. 198D-07
2. 874D-07	1. 355D-07	-1. 136D-09

FLEXIBLE MODE: ALPHA = .1

OMEGA = 1.140D+00 RAD/SEC

1

ZETA = 5.134D-03

1

T

E

W1

T

E

THETA1

-9.998D-06	-3.310D-07	6.147D-03	8.401D-09	5.094D-06	6.791D-05
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K

F1

K

G1

-8.399D-14	-2.781D-15	5.164D-11
-5.093D-11	-1.686D-12	3.131D-08
-6.789D-10	-2.248D-11	4.174D-07

7.057D-17	4.279D-14	5.705D-13
4.279D-14	2.594D-11	3.459D-10
5.705D-13	3.459D-10	4.611D-09

FLEXIBLE MODE: ALPHA = 2

OMEGA<sub>2</sub> = 1.155D+00 RAD/SEC

ZETA<sub>2</sub> = 5.134D-03

T E W2	
-5.729D-03	1.622D-04
	-4.600D-06

T E THETA2	
7.663D-05	-2.842D-04
	-1.358D-07

K F2	
-4.390D-07	1.243D-08
1.628D-06	-4.610D-08
7.779D-10	-2.203D-11
	-3.525D-10
	1.307D-09
	6.246D-13

K G2	
5.873D-09	-2.178D-08
-2.178D-08	8.076D-08
-1.041D-11	3.859D-11
	1.844D-14

FLEXIBLE MODE: ALPHA = 3

OMEGA = 1.513D+00 RAD/SEC

3

ZETA = 6.348D-03

3

T

E

W3

T

E

THETA3

1.518D-05	-4.203D-04	-1.012D-04	-2.780D-04	-1.519D-05	6.765D-03
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K

F3

K

G3

-4.219D-09	1.169D-07	2.813D-08
-2.306D-10	6.386D-09	1.537D-09
1.027D-07	-2.843D-06	-6.843D-07

7.730D-08	4.224D-09	-1.881D-06
4.224D-09	2.309D-10	-1.028D-07
-1.881D-06	-1.028D-07	4.576D-05

FLEXIBLE MODE: ALPHA = 4

OMEGA = 1.582D+00 RAD/SEC  
4

ZETA = 5.921D-03  
4

T  
E  
W4

-1.099D-03 -9.009D-03 8.773D-06

T  
E  
THETA4

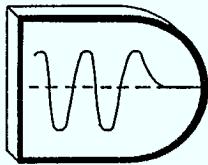
-6.535D-03 4.809D-04 -8.860D-05

K  
F4

7.180D-06 5.887D-05 -5.733D-08  
-5.285D-07 -4.333D-06 4.219D-09  
9.736D-08 7.982D-07 -7.773D-10

K  
G4

4.270D-05 -3.143D-06 5.790D-07  
-3.143D-06 2.313D-07 -4.261D-08  
5.790D-07 -4.261D-08 7.850D-09



**YNACON**

APPENDIX C

MSAT DYNAMICS MODEL

*MARK I*

(EVALUATION MODEL:  $N_e = 11$ )

System Matrices

$$\omega_\alpha, \quad \alpha = 1, \dots, 11$$

$$\underline{E} = \begin{bmatrix} E_r & E_{re} \\ 0 & E_e \end{bmatrix}$$

$$\hat{\underline{\rho}}_e$$

$$\hat{\underline{G}}_1, \hat{\underline{G}}_2, \hat{\underline{G}}_3, \hat{\underline{G}}_4$$

$$\hat{\underline{B}} = \begin{bmatrix} \hat{\underline{B}}_r \\ \hat{\underline{B}}_{er} \end{bmatrix} \equiv \begin{bmatrix} \underline{E}_r^T \\ \underline{E}_{re}^T \end{bmatrix}$$

\*\*\* RETAINED FREQUENCIES \*\*\*

SELECTED MODES	FREQUENCY (RAD/SEC)	FREQUENCY (HZ)
1	0. 000000000000000D+00	0. 000000000000000D+00
2	0. 000000000000000D+00	0. 000000000000000D+00
3	0. 000000000000000D+00	0. 000000000000000D+00
4	0. 000000000000000D+00	0. 000000000000000D+00
5	0. 000000000000000D+00	0. 000000000000000D+00
6	0. 000000000000000D+00	0. 000000000000000D+00
7	1. 1398259549566350D+00	1. 8140893499578850D-01
8	1. 1549710410779400D+00	1. 8381935031554660D-01
9	1. 5125592153215830D+00	2. 4073127583762840D-01
10	1. 5818256321765070D+00	2. 5175536847035330D-01
11	1. 6077254388481950D+00	2. 5587745072727690D-01
12	1. 6139546382229340D+00	2. 5686885859927150D-01
13	2. 1868930860301580D+00	3. 4805484465518910D-01
14	2. 3182132897469090D+00	3. 6895510420454470D-01
15	2. 3204058952163090D+00	3. 6930406820325000D-01
16	2. 3659197534602500D+00	3. 7654782372195710D-01
17	2. 5868329848692610D+00	4. 1170725649510500D-01

\*\*\* RETAINED EIGENVECTORS \*\*\*

ROW \ COL	7	8	9	10	11	12
1	-9.998D-06	-5.729D-03	1.518D-05	-1.099D-03	1.057D-03	1.391D-04
2	-3.310D-07	1.622D-04	-4.203D-04	-9.009D-03	-8.724D-03	-1.971D-03
3	6.147D-03	-4.600D-06	-1.012D-04	8.773D-06	1.432D-05	-8.362D-05
4	8.401D-09	7.663D-05	-2.780D-04	-6.535D-03	-6.316D-03	-1.443D-03

ROW \ COL	7	8	9	10	11	12
5	5.094D-06	-2.842D-04	-1.519D-05	4.809D-04	-4.809D-04	-5.953D-05
6	6.791D-05	-1.358D-07	6.765D-03	-8.860D-05	5.472D-04	-3.874D-03
7	-7.147D-01	-1.438D-01	-1.225D-01	6.812D-01	6.268D-01	2.237D-01
8	-1.273D-01	7.116D-01	8.508D-01	1.275D-01	1.821D-01	-3.919D-01
9	1.032D-03	2.501D-03	2.820D-01	-5.170D-01	3.880D-01	7.126D-01
10	1.005D-03	4.045D-04	-5.571D-03	-1.114D-01	-1.153D-01	-2.565D-02
11	1.617D-04	1.853D-05	1.133D-03	5.823D-03	5.978D-03	6.876D-04
12	4.422D-06	-1.090D-04	-8.438D-04	8.370D-03	8.542D-03	2.807D-03
13	-3.810D-05	-4.264D-04	-5.460D-03	-1.497D-05	-2.504D-04	3.787D-03
14	6.315D-04	5.634D-05	-1.305D-04	-7.545D-03	-7.619D-03	-1.900D-03
15	6.491D-04	2.271D-05	-3.768D-04	-5.295D-03	-5.283D-03	-1.135D-03
16	9.695D-05	-4.418D-05	-4.840D-04	2.686D-04	3.202D-04	4.062D-04
17	4.904D-05	1.313D-04	1.323D-03	-2.175D-04	-3.445D-04	-9.875D-04
18	-1.135D-04	3.596D-05	3.125D-04	4.735D-04	5.105D-04	-7.843D-05
19	2.918D-05	-4.597D-05	-3.322D-04	-5.059D-04	-5.098D-04	9.266D-05
20	-1.233D-05	-4.946D-05	-2.587D-04	1.393D-04	1.309D-04	2.092D-04
21	-8.784D-05	-1.165D-06	1.229D-05	3.639D-04	3.756D-04	9.166D-05
22	5.327D-05	7.940D-06	-5.416D-06	-2.250D-04	-1.954D-04	-5.078D-05
23	-3.708D-05	-3.285D-04	-1.092D-03	1.108D-04	4.691D-05	7.499D-04
24	2.398D-04	-2.804D-05	-1.757D-04	-1.068D-03	-1.055D-03	-1.649D-04
25	5.155D-05	1.173D-05	4.519D-05	-2.264D-04	-2.636D-04	-9.818D-05
26	-3.847D-07	-1.780D-05	-4.436D-05	2.877D-06	1.944D-06	3.026D-05
27	-1.397D-05	-5.180D-08	2.171D-06	3.724D-05	3.740D-05	8.790D-06
28	-1.306D-06	7.532D-07	4.953D-06	-7.796D-06	-7.261D-06	-5.143D-06
29	3.045D-07	1.219D-06	2.722D-06	-9.445D-07	-8.917D-07	-2.067D-06
30	-2.003D-08	-2.574D-07	-5.350D-07	-2.449D-07	-2.447D-07	2.962D-07
31	-2.214D-07	-9.074D-07	-1.845D-06	-1.310D-07	-1.393D-07	1.210D-06
32	-1.104D-07	-3.342D-07	-6.489D-07	-6.759D-08	-6.710D-08	4.222D-07
33	-5.105D-07	1.633D-07	3.873D-07	1.298D-06	1.297D-06	9.765D-08
34	-7.147D-01	1.446D-01	-1.860D-01	-6.777D-01	-6.493D-01	-6.577D-02
35	-1.280D-01	-7.117D-01	8.379D-01	-1.475D-01	-6.263D-02	-4.478D-01
36	9.830D-04	-2.503D-03	2.897D-01	4.519D-01	-6.354D-01	5.596D-01
37	1.005D-03	-4.061D-04	2.335D-03	1.114D-01	1.150D-01	2.791D-02
38	1.613D-04	-1.879D-05	7.174D-04	-5.850D-03	-5.787D-03	-2.053D-03
39	5.005D-06	1.091D-04	-1.470D-03	-8.335D-03	-8.768D-03	-1.183D-03
40	-3.664D-05	4.265D-04	-5.471D-03	1.767D-04	-7.953D-04	3.698D-03
41	6.313D-04	-5.732D-05	4.459D-04	7.543D-03	7.659D-03	1.621D-03
42	6.494D-04	-2.370D-05	2.352D-05	5.303D-03	5.261D-03	1.304D-03
43	9.743D-05	4.403D-05	-5.101D-04	-2.536D-04	-4.115D-04	2.481D-04
44	4.776D-05	-1.313D-04	1.355D-03	1.788D-04	5.946D-04	-7.990D-04
45	-1.134D-04	-3.579D-05	2.751D-04	-4.826D-04	-4.576D-04	-3.010D-04
46	2.931D-05	4.593D-05	-2.943D-04	5.151D-04	4.524D-04	3.173D-04
47	-1.225D-05	4.949D-05	-2.700D-04	-1.317D-04	-1.798D-04	1.394D-04
48	-8.777D-05	1.300D-06	-1.674D-05	-3.642D-04	-3.773D-04	-8.043D-05
49	5.343D-05	-8.022D-06	1.009D-05	2.252D-04	1.967D-04	4.267D-05
50	-3.694D-05	3.286D-04	-1.100D-03	-7.948D-05	-2.481D-04	6.856D-04
51	2.400D-04	2.767D-05	-9.371D-05	1.073D-03	1.034D-03	3.185D-04
52	5.133D-05	-1.181D-05	6.549D-05	2.251D-04	2.745D-04	2.117D-05
53	-3.709D-07	1.780D-05	-4.440D-05	-1.605D-06	-1.006D-05	2.764D-05
54	-1.397D-05	7.327D-08	-7.354D-07	-3.733D-05	-3.747D-05	-8.434D-06

ROW \ COL	7	8	9	10	11	12
55	-1.310D-06	-7.512D-07	4.919D-06	7.648D-06	8.144D-06	-1.171D-06
56	3.036D-07	-1.220D-06	2.775D-06	8.671D-07	1.398D-06	-1.544D-06
57	-1.994D-08	2.575D-07	-5.310D-07	2.601D-07	1.471D-07	4.005D-07
58	-2.210D-07	9.077D-07	-1.866D-06	1.832D-07	-2.029D-07	1.229D-06
59	-1.103D-07	3.344D-07	-6.588D-07	8.582D-08	-5.396D-08	4.403D-07
60	-5.105D-07	-1.625D-07	2.960D-07	-1.310D-06	-1.242D-06	-4.959D-07
61	-2.035D-05	-5.247D-06	-3.564D-03	6.095D-04	2.301D-04	2.470D-03
62	-1.468D-04	-8.246D-05	-1.046D-06	1.800D-04	-1.874D-04	-1.918D-05
63	3.941D-07	1.493D-06	6.016D-05	-2.325D-04	-2.249D-04	-9.834D-05
64	2.185D-06	-5.100D-06	3.256D-08	-4.524D-06	4.607D-06	5.296D-07
65	-3.202D-08	-7.594D-08	-5.229D-06	1.001D-05	9.408D-06	5.945D-06
66	-6.578D-07	-8.741D-07	2.273D-08	-3.062D-07	2.991D-07	5.803D-08
67	-3.625D-09	-7.342D-09	-6.005D-07	9.146D-07	8.468D-07	6.229D-07
68	-5.410D-08	-6.914D-08	1.669D-09	-1.733D-08	1.656D-08	3.759D-09
69	-6.537D-08	5.956D-09	1.135D-09	2.903D-08	-3.029D-08	-1.991D-09
70	5.339D-10	1.070D-09	8.816D-08	-1.415D-07	-1.315D-07	-9.328D-08
71	4.057D-08	4.315D-08	-1.323D-09	1.392D-08	-1.348D-08	-2.929D-09
72	-5.816D-10	-1.569D-09	-9.392D-08	2.087D-07	1.977D-07	1.142D-07
73	-7.336D-10	-1.573D-09	-1.210D-07	1.996D-07	1.858D-07	1.294D-07
74	-1.098D-09	-2.433D-08	3.528D-10	-1.657D-08	1.674D-08	2.191D-09
75	4.037D-09	8.588D-09	-1.630D-10	3.374D-09	-3.333D-09	-5.521D-10
76	7.220D-09	-1.067D-08	-6.267D-12	-9.544D-09	9.740D-09	1.047D-09
77	1.598D-10	4.219D-10	2.590D-08	-5.488D-08	-5.187D-08	-3.080D-08
78	-1.220D-08	-9.034D-09	3.535D-10	-1.784D-09	1.630D-09	5.672D-10
79	-2.037D-05	5.330D-06	-3.611D-03	-5.066D-04	-8.267D-04	2.215D-03
80	-1.488D-04	8.269D-05	9.644D-06	-1.808D-04	1.861D-04	2.728D-05
81	3.944D-07	-1.494D-06	7.827D-05	2.305D-04	2.375D-04	8.059D-06
82	2.232D-06	5.096D-06	-1.606D-07	4.536D-06	-4.587D-06	-6.501D-07
83	-3.217D-08	7.607D-08	-6.032D-06	-9.849D-06	-1.044D-05	1.397D-06
84	-6.550D-07	8.751D-07	1.534D-08	3.026D-07	-3.052D-07	-2.223D-08
85	-3.644D-09	7.357D-09	-6.748D-07	-8.963D-07	-9.634D-07	2.087D-07
86	-5.395D-08	6.922D-08	1.460D-09	1.703D-08	-1.706D-08	-8.107D-10
87	-6.568D-08	-5.855D-09	2.663D-09	-2.939D-08	2.968D-08	5.565D-09
88	5.360D-10	-1.072D-09	9.950D-08	1.388D-07	1.486D-07	-2.908D-08
89	4.044D-08	-4.321D-08	-1.025D-09	-1.370D-08	1.385D-08	7.198D-10
90	-5.846D-10	1.571D-09	-1.107D-07	-2.057D-07	-2.164D-07	1.923D-08
91	-7.374D-10	1.576D-09	-1.371D-07	-1.959D-07	-2.094D-07	3.888D-08
92	-9.303D-10	2.433D-08	-2.941D-10	1.657D-08	-1.675D-08	-2.136D-09
93	4.005D-09	-8.594D-09	-7.011D-11	-3.351D-09	3.370D-09	3.327D-10
94	7.318D-09	1.066D-08	-4.151D-10	9.584D-09	-9.673D-09	-1.443D-09
95	1.607D-10	-4.225D-10	3.032D-08	5.407D-08	5.701D-08	-5.856D-09
96	-1.219D-08	9.053D-09	3.532D-10	1.716D-09	-1.743D-09	9.805D-11

ROW \ COL	13	14	15	16	17
1	8.272D-07	-3.155D-05	-1.468D-08	9.489D-05	-4.485D-07
2	-3.806D-07	1.730D-03	-2.888D-07	9.009D-03	3.884D-07
3	-4.659D-04	-1.176D-07	8.494D-05	-2.626D-07	7.187D-06
4	-1.096D-08	1.259D-03	-3.082D-08	6.566D-03	-9.309D-09
5	-6.073D-07	9.118D-06	-2.745D-08	-3.779D-05	2.688D-07
6	6.850D-05	1.262D-05	4.689D-05	6.535D-05	-7.560D-05
7	2.764D-03	-8.653D-02	-1.301D-03	-4.458D-01	9.320D-04
8	5.915D-03	-1.563D-02	3.290D-03	-8.144D-02	-5.205D-03

ROW \ COL 13	14	15	16	17	
9	-5. 493D-04	-3. 212D-03	-4. 177D-04	-1. 536D-02	5. 478D-04
10	7. 073D-01	-1. 769D-01	-2. 239D-04	-6. 981D-01	-7. 761D-05
11	-1. 973D-04	-6. 838D-01	-7. 084D-01	1. 753D-01	6. 891D-05
12	-6. 407D-05	-1. 112D-02	-6. 089D-05	-7. 258D-02	-7. 061D-01
13	-1. 792D-04	-1. 296D-04	-1. 677D-04	-5. 636D-04	5. 254D-04
14	-2. 533D-04	4. 445D-03	6. 176D-05	2. 494D-02	-2. 951D-06
15	-2. 597D-04	2. 968D-03	5. 282D-05	1. 661D-02	1. 550D-05
16	-4. 997D-05	-1. 729D-04	-2. 497D-06	-9. 306D-04	2. 537D-05
17	1. 890D-05	1. 844D-04	3. 041D-05	9. 434D-04	-6. 049D-05
18	4. 769D-05	-2. 411D-04	-2. 795D-06	-1. 334D-03	-1. 413D-05
19	-1. 697D-05	2. 250D-04	-3. 654D-06	1. 242D-03	1. 223D-05
20	-2. 672D-06	-6. 724D-05	-5. 389D-06	-3. 607D-04	9. 694D-06
21	2. 613D-05	-1. 625D-04	-5. 527D-06	-8. 840D-04	-4. 250D-07
22	-1. 575D-05	9. 153D-05	3. 364D-06	5. 052D-04	1. 925D-07
23	-1. 488D-05	-6. 042D-05	-2. 015D-05	-3. 024D-04	3. 689D-05
24	-7. 135D-05	4. 444D-04	1. 200D-05	2. 424D-03	5. 774D-06
25	-1. 330D-05	1. 048D-04	3. 912D-06	5. 605D-04	-1. 547D-06
26	-9. 099D-07	-2. 200D-06	-7. 356D-07	-1. 045D-05	1. 459D-06
27	3. 917D-06	-1. 551D-05	-7. 975D-07	-8. 424D-05	-9. 464D-08
28	4. 838D-07	3. 387D-06	8. 280D-09	1. 857D-05	-1. 786D-07
29	-2. 182D-08	4. 583D-07	6. 171D-08	2. 393D-06	-8. 874D-08
30	-6. 430D-09	9. 076D-08	-9. 573D-09	5. 210D-07	1. 710D-08
31	1. 960D-08	1. 309D-08	-4. 225D-08	1. 601D-07	5. 912D-08
32	1. 597D-08	1. 360D-08	-1. 672D-08	1. 080D-07	2. 059D-08
33	1. 498D-07	-5. 317D-07	-2. 407D-08	-2. 906D-06	-1. 361D-08
34	2. 763D-03	8. 619D-02	-1. 305D-03	4. 440D-01	9. 307D-04
35	5. 912D-03	1. 747D-02	3. 288D-03	9. 086D-02	-5. 205D-03
36	-5. 604D-04	2. 996D-03	-4. 182D-04	1. 428D-02	5. 516D-04
37	7. 071D-01	1. 770D-01	-2. 321D-04	6. 983D-01	-7. 913D-05
38	-1. 964D-04	6. 863D-01	-7. 058D-01	-1. 759D-01	6. 936D-05
39	-6. 478D-05	1. 108D-02	-6. 144D-05	7. 236D-02	-7. 081D-01
40	-1. 800D-04	4. 045D-05	-1. 677D-04	5. 062D-05	5. 265D-04
41	-2. 531D-04	-4. 442D-03	6. 197D-05	-2. 493D-02	-2. 925D-06
42	-2. 598D-04	-2. 970D-03	5. 295D-05	-1. 662D-02	1. 572D-05
43	-5. 022D-05	1. 672D-04	-2. 513D-06	8. 992D-04	2. 557D-05
44	1. 953D-05	-1. 698D-04	3. 044D-05	-8. 629D-04	-6. 097D-05
45	4. 765D-05	2. 443D-04	-2. 812D-06	1. 351D-03	-1. 411D-05
46	-1. 701D-05	-2. 280D-04	-3. 643D-06	-1. 259D-03	1. 227D-05
47	-2. 690D-06	6. 472D-05	-5. 390D-06	3. 470D-04	9. 712D-06
48	2. 610D-05	1. 625D-04	-5. 537D-06	8. 838D-04	-4. 082D-07
49	-1. 582D-05	-9. 151D-05	3. 365D-06	-5. 050D-04	2. 406D-07
50	-1. 480D-05	5. 058D-05	-2. 013D-05	2. 492D-04	3. 690D-05
51	-7. 138D-05	-4. 456D-04	1. 203D-05	-2. 430D-03	5. 807D-06
52	-1. 321D-05	-1. 043D-04	3. 921D-06	-5. 578D-04	-1. 607D-06
53	-9. 035D-07	1. 809D-06	-7. 311D-07	8. 340D-06	1. 453D-06
54	3. 916D-06	1. 552D-05	-7. 981D-07	8. 426D-05	-9. 490D-08
55	4. 705D-07	-3. 344D-06	-1. 593D-09	-1. 834D-05	-1. 584D-07
56	-2. 242D-08	-4. 342D-07	6. 131D-08	-2. 262D-06	-8. 812D-08
57	-6. 701D-09	-9. 544D-08	-9. 780D-09	-5. 462D-07	1. 759D-08
58	1. 912D-08	-2. 938D-08	-4. 264D-08	-2. 480D-07	6. 010D-08

ROW \ COL 13	14	15	16	17	
59	1. 571D-08	-1. 934D-08	-1. 693D-08	-1. 390D-07	2. 108D-08
60	1. 500D-07	5. 347D-07	-2. 396D-08	2. 921D-06	-1. 393D-08
61	-7. 673D-05	-2. 490D-04	-5. 929D-05	-1. 352D-03	1. 195D-04
62	4. 126D-05	7. 248D-06	-8. 601D-06	-3. 186D-05	-6. 289D-07
63	1. 466D-06	9. 628D-05	1. 128D-06	5. 230D-04	-2. 269D-06
64	-6. 075D-07	-2. 289D-07	1. 268D-07	8. 329D-07	7. 383D-09
65	-1. 188D-07	-4. 129D-06	-9. 151D-08	-2. 241D-05	1. 836D-07
66	1. 836D-07	-1. 891D-08	-3. 761D-08	5. 913D-08	-4. 283D-09
67	-1. 346D-08	-3. 768D-07	-1. 036D-08	-2. 045D-06	2. 079D-08
68	1. 509D-08	-1. 201D-09	-3. 096D-09	3. 447D-09	-3. 413D-10
69	1. 822D-08	1. 297D-09	-3. 756D-09	-5. 208D-09	-3. 529D-10
70	1. 981D-09	5. 834D-08	1. 526D-07	3. 167D-07	-3. 061D-09
71	-1. 132D-08	8. 808D-10	2. 320D-09	-2. 702D-09	2. 584D-10
72	-2. 158D-09	-8. 609D-08	-1. 662D-09	-4. 673D-07	3. 335D-09
73	-2. 723D-09	-8. 225D-08	-2. 097D-09	-4. 464D-07	4. 206D-09
74	3. 136D-10	-8. 748D-10	-5. 749D-11	3. 079D-09	-2. 703D-11
75	-1. 127D-09	2. 019D-10	2. 304D-10	-6. 460D-10	2. 799D-11
76	-2. 009D-09	-4. 826D-10	4. 168D-10	1. 757D-09	3. 126D-11
77	5. 930D-10	2. 263D-08	4. 566D-10	1. 229D-07	-9. 162D-10
78	3. 404D-09	-1. 352D-10	-6. 986D-10	3. 638D-10	-7. 480D-11
79	-7. 682D-05	2. 172D-04	-5. 937D-05	1. 180D-03	1. 196D-04
80	4. 215D-05	-7. 225D-06	-8. 554D-06	3. 192D-05	-1. 183D-06
81	1. 469D-06	-9. 568D-05	1. 135D-06	-5. 197D-04	-2. 270D-06
82	-6. 253D-07	2. 285D-07	1. 263D-07	-8. 337D-07	1. 923D-08
83	-1. 195D-07	4. 079D-06	-9. 221D-08	2. 215D-05	1. 845D-07
84	1. 827D-07	1. 901D-08	-3. 759D-08	-5. 889D-08	-3. 625D-09
85	-1. 354D-08	3. 712D-07	-1. 044D-08	2. 015D-06	2. 090D-08
86	1. 506D-08	1. 209D-09	-3. 093D-09	-3. 427D-09	-3. 094D-10
87	1. 835D-08	-1. 287D-09	-3. 750D-09	5. 232D-09	-4. 363D-10
88	1. 991D-09	-5. 751D-08	1. 535D-09	-3. 122D-07	-3. 074D-09
89	-1. 128D-08	-8. 872D-10	2. 319D-09	2. 688D-09	2. 295D-10
90	-2. 172D-09	8. 519D-08	-1. 676D-09	4. 625D-07	3. 353D-09
91	-2. 740D-09	8. 112D-08	-2. 113D-09	4. 403D-07	4. 229D-09
92	2. 522D-10	8. 749D-10	-5. 866D-11	-3. 079D-09	1. 482D-11
93	-1. 117D-09	-2. 026D-10	2. 302D-10	6. 445D-10	2. 045D-11
94	-2. 047D-09	4. 814D-10	4. 158D-10	-1. 759D-09	5. 629D-11
95	5. 971D-10	-2. 239D-08	4. 605D-10	-1. 215D-07	-9. 216D-10
96	3. 401D-09	1. 372D-10	-6. 981D-10	-3. 593D-10	-7. 207D-11

\*\*\* RETAINED MODAL DAMPING MATRIX \*\*\*

ROW \ COL	1	2	3	4	5	6
1	1. 170D-02	-8. 171D-08	3. 011D-05	1. 953D-07	3. 938D-06	-2. 579D-05
2	-8. 171D-08	1. 186D-02	-1. 235D-06	-1. 227D-05	-4. 331D-05	-8. 481D-06
3	3. 011D-05	-1. 235D-06	1. 920D-02	8. 620D-05	4. 743D-04	-2. 364D-03
4	1. 953D-07	-1. 227D-05	8. 620D-05	1. 873D-02	2. 796D-03	6. 623D-04
5	3. 938D-06	-4. 331D-05	4. 743D-04	2. 796D-03	1. 887D-02	4. 201D-04
6	-2. 579D-05	-8. 481D-06	-2. 364D-03	6. 623D-04	4. 201D-04	1. 769D-02
7	-2. 978D-05	2. 023D-08	4. 795D-05	-4. 473D-07	4. 092D-06	-2. 743D-05
8	3. 312D-08	7. 393D-06	-2. 211D-05	-6. 376D-04	-6. 216D-04	-1. 450D-04
9	5. 896D-06	-3. 668D-09	3. 293D-05	-2. 519D-07	2. 943D-06	-1. 959D-05
10	1. 904D-07	3. 093D-05	-1. 164D-04	-3. 350D-03	-3. 258D-03	-7. 598D-04
11	2. 154D-08	7. 619D-09	-5. 508D-05	4. 709D-07	-4. 861D-06	3. 254D-05
ROW \ COL	7	8	9	10	11	
1	-2. 978D-05	3. 312D-08	5. 896D-06	1. 904D-07	2. 154D-08	
2	2. 023D-08	7. 393D-06	-3. 668D-09	3. 093D-05	7. 619D-09	
3	4. 795D-05	-2. 211D-05	3. 293D-05	-1. 164D-04	-5. 508D-05	
4	-4. 473D-07	-6. 376D-04	-2. 519D-07	-3. 350D-03	4. 709D-07	
5	4. 092D-06	-6. 216D-04	2. 943D-06	-3. 258D-03	-4. 861D-06	
6	-2. 743D-05	-1. 450D-04	-1. 959D-05	-7. 598D-04	3. 254D-05	
7	2. 187D-02	5. 123D-08	-2. 260D-07	2. 621D-07	-7. 219D-07	
8	5. 123D-08	2. 333D-02	3. 233D-08	7. 602D-04	-6. 090D-08	
9	-2. 260D-07	3. 233D-08	2. 320D-02	1. 669D-07	-4. 574D-07	
10	2. 621D-07	7. 602D-04	1. 669D-07	2. 765D-02	-3. 155D-07	
11	-7. 219D-07	-6. 090D-08	-4. 574D-07	-3. 155D-07	2. 587D-02	

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 1-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0. 000D+00	0. 000D+00				
2	0. 000D+00	0. 000D+00				
3	0. 000D+00	0. 000D+00				
4	0. 000D+00	0. 000D+00				
5	0. 000D+00	9. 014D-05				
6	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-9. 014D-05	0. 000D+00
7	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-7. 228D-07	4. 480D-08
8	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	1. 445D-09	-2. 407D-06
9	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-7. 200D-05	3. 712D-08
10	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	9. 430D-07	4. 071D-06
11	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-5. 824D-06	-4. 059D-06
12	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	4. 123D-05	-5. 991D-07
13	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-7. 290D-07	-3. 464D-09
14	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-1. 343D-07	7. 753D-08
15	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-4. 991D-07	9. 167D-10
16	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-6. 956D-07	-3. 184D-07
17	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	8. 046D-07	4. 239D-10

ROW \ COL	7	8	9	10	11	12
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	0. 000D+00					
5	7. 228D-07	-1. 445D-09	7. 200D-05	-9. 430D-07	5. 824D-06	-4. 123D-05
6	-4. 480D-08	2. 407D-06	-3. 712D-08	-4. 071D-06	4. 059D-06	5. 991D-07
7	0. 000D+00	1. 930D-08	3. 549D-08	-3. 311D-08	3. 544D-08	-1. 569D-08
8	-1. 930D-08	0. 000D+00	-1. 922D-06	2. 524D-08	-1. 556D-07	1. 101D-06
9	-3. 549D-08	1. 922D-06	0. 000D+00	-3. 252D-06	3. 245D-06	4. 615D-07
10	3. 311D-08	-2. 524D-08	3. 252D-06	0. 000D+00	2. 206D-07	-1. 868D-06
11	-3. 544D-08	1. 556D-07	-3. 245D-06	-2. 206D-07	0. 000D+00	1. 896D-06
12	1. 569D-08	-1. 101D-06	-4. 615D-07	1. 868D-06	-1. 896D-06	0. 000D+00
13	-3. 901D-10	1. 947D-08	-3. 067D-09	-3. 289D-08	3. 261D-08	6. 430D-09
14	5. 549D-10	3. 585D-09	6. 187D-08	-6. 878D-09	1. 106D-08	-3. 457D-08
15	-2. 407D-10	1. 332D-08	5. 267D-10	-2. 255D-08	2. 253D-08	2. 897D-09
16	-2. 899D-09	1. 858D-08	-2. 546D-07	-2. 808D-08	1. 075D-08	1. 503D-07
17	4. 033D-10	-2. 148D-08	6. 700D-10	3. 633D-08	-3. 621D-08	-5. 542D-09

ROW \ COL	13	14	15	16	17
1	0. 000D+00				
2	0. 000D+00				
3	0. 000D+00				
4	0. 000D+00				
5	7. 290D-07	1. 343D-07	4. 991D-07	6. 956D-07	-8. 046D-07
6	3. 464D-09	-7. 753D-08	-9. 167D-10	3. 184D-07	-4. 239D-10
7	3. 901D-10	-5. 549D-10	2. 407D-10	2. 899D-09	-4. 033D-10
8	-1. 947D-08	-3. 585D-09	-1. 332D-08	-1. 858D-08	2. 148D-08
9	3. 067D-09	-6. 187D-08	-5. 267D-10	2. 546D-07	-6. 700D-10
10	3. 289D-08	6. 878D-09	2. 255D-08	2. 808D-08	-3. 633D-08
11	-3. 261D-08	-1. 106D-08	-2. 253D-08	-1. 075D-08	3. 621D-08
12	-6. 430D-09	3. 457D-08	-2. 897D-09	-1. 503D-07	5. 542D-09
13	0. 000D+00	-6. 322D-10	-2. 659D-11	2. 549D-09	2. 749D-11
14	6. 322D-10	0. 000D+00	4. 279D-10	1. 073D-09	-6. 927D-10
15	2. 659D-11	-4. 279D-10	0. 000D+00	1. 770D-09	-1. 053D-11
16	-2. 549D-09	-1. 073D-09	-1. 770D-09	0. 000D+00	2. 839D-09

ROW \ COL 13

14

15

16

17

17 -2.749D-11 6.927D-10 1.053D-11 -2.839D-09 0.000D+00

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 2-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
4	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	-8. 198D-05
5	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	7. 468D-08
6	0. 000D+00	0. 000D+00	0. 000D+00	8. 198D-05	-7. 468D-08	0. 000D+00
7	0. 000D+00	0. 000D+00	0. 000D+00	6. 573D-07	-5. 988D-10	2. 986D-09
8	0. 000D+00	0. 000D+00	0. 000D+00	-1. 314D-09	1. 197D-12	-6. 490D-07
9	0. 000D+00	0. 000D+00	0. 000D+00	6. 548D-05	-5. 965D-08	2. 659D-06
10	0. 000D+00	0. 000D+00	0. 000D+00	-8. 576D-07	7. 813D-10	5. 534D-05
11	0. 000D+00	0. 000D+00	0. 000D+00	5. 297D-06	-4. 825D-09	5. 352D-05
12	0. 000D+00	0. 000D+00	0. 000D+00	-3. 750D-05	3. 416D-08	1. 205D-05
13	0. 000D+00	0. 000D+00	0. 000D+00	6. 630D-07	-6. 040D-10	3. 177D-09
14	0. 000D+00	0. 000D+00	0. 000D+00	1. 222D-07	-1. 113D-10	-1. 066D-05
15	0. 000D+00	0. 000D+00	0. 000D+00	4. 539D-07	-4. 134D-10	2. 372D-09
16	0. 000D+00	0. 000D+00	0. 000D+00	6. 326D-07	-5. 762D-10	-5. 560D-05
17	0. 000D+00	0. 000D+00	0. 000D+00	-7. 318D-07	6. 666D-10	-3. 325D-09

ROW \ COL	7	8	9	10	11	12
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	-6. 573D-07	1. 314D-09	-6. 548D-05	8. 576D-07	-5. 297D-06	3. 750D-05
5	5. 988D-10	-1. 197D-12	5. 965D-08	-7. 813D-10	4. 825D-09	-3. 416D-08
6	-2. 986D-09	6. 490D-07	-2. 659D-06	-5. 534D-05	-5. 352D-05	-1. 205D-05
7	0. 000D+00	5. 204D-09	-1. 894D-08	-4. 437D-07	-4. 289D-07	-9. 797D-08
8	-5. 204D-09	0. 000D+00	-5. 184D-07	7. 677D-09	-4. 107D-08	2. 971D-07
9	1. 894D-08	5. 184D-07	0. 000D+00	-4. 423D-05	-4. 257D-05	-1. 084D-05
10	4. 437D-07	-7. 677D-09	4. 423D-05	0. 000D+00	4. 135D-06	-2. 519D-05
11	4. 289D-07	4. 107D-08	4. 257D-05	-4. 135D-06	0. 000D+00	-2. 526D-05
12	9. 797D-08	-2. 971D-07	1. 084D-05	2. 519D-05	2. 526D-05	0. 000D+00
13	1. 320D-12	5. 249D-09	-1. 897D-08	-4. 476D-07	-4. 326D-07	-9. 890D-08
14	-8. 550D-08	1. 138D-09	-8. 521D-06	2. 909D-08	-7. 687D-07	4. 860D-06
15	2. 487D-12	3. 593D-09	-1. 283D-08	-3. 064D-07	-2. 961D-07	-6. 779D-08
16	-4. 459D-07	5. 900D-09	-4. 443D-05	1. 547D-07	-4. 006D-06	2. 534D-05
17	-2. 973D-15	-5. 793D-09	2. 108D-08	4. 940D-07	4. 775D-07	1. 091D-07

ROW \ COL	13	14	15	16	17
1	0. 000D+00				
2	0. 000D+00				
3	0. 000D+00				
4	-6. 630D-07	-1. 222D-07	-4. 539D-07	-6. 326D-07	7. 318D-07
5	6. 040D-10	1. 113D-10	4. 134D-10	5. 762D-10	-6. 666D-10
6	-3. 177D-09	1. 066D-05	-2. 372D-09	5. 560D-05	3. 325D-09
7	-1. 320D-12	8. 550D-08	-2. 487D-12	4. 459D-07	2. 973D-15
8	-5. 249D-09	-1. 138D-09	-3. 593D-09	-5. 900D-09	5. 793D-09
9	1. 897D-08	8. 521D-06	1. 283D-08	4. 443D-05	-2. 108D-08
10	4. 476D-07	-2. 909D-08	3. 064D-07	-1. 547D-07	-4. 940D-07
11	4. 326D-07	7. 687D-07	2. 961D-07	4. 006D-06	-4. 775D-07
12	9. 890D-08	-4. 860D-06	6. 779D-08	-2. 534D-05	-1. 091D-07
13	0. 000D+00	8. 625D-08	-1. 597D-12	4. 498D-07	-1. 466D-12
14	-8. 625D-08	0. 000D+00	-5. 904D-08	5. 800D-10	9. 519D-08
15	1. 597D-12	5. 904D-08	0. 000D+00	3. 079D-07	-2. 766D-12
16	-4. 498D-07	-5. 800D-10	-3. 079D-07	0. 000D+00	4. 964D-07

ROW \ COL 13	14	15	16	17	
17	1.466D-12	-9.519D-08	2.766D-12	-4.964D-07	0.000D+00

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 3-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
2	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
3	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
4	0.000D+00	0.000D+00	0.000D+00	0.000D+00	1.030D-04	-2.372D-07
5	0.000D+00	0.000D+00	0.000D+00	-1.030D-04	0.000D+00	-4.790D-07
6	0.000D+00	0.000D+00	0.000D+00	2.372D-07	4.790D-07	0.000D+00
7	0.000D+00	0.000D+00	0.000D+00	-4.930D-08	1.343D-10	-2.295D-10
8	0.000D+00	0.000D+00	0.000D+00	2.751D-06	8.131D-07	1.092D-08
9	0.000D+00	0.000D+00	0.000D+00	1.471D-07	-2.959D-06	7.499D-09
10	0.000D+00	0.000D+00	0.000D+00	-4.655D-06	-6.955D-05	1.385D-07
11	0.000D+00	0.000D+00	0.000D+00	4.655D-06	-6.723D-05	1.765D-07
12	0.000D+00	0.000D+00	0.000D+00	5.762D-07	-1.536D-05	3.805D-08
13	0.000D+00	0.000D+00	0.000D+00	5.878D-09	-1.220D-10	2.761D-11
14	0.000D+00	0.000D+00	0.000D+00	-8.826D-08	1.340D-05	-3.127D-08
15	0.000D+00	0.000D+00	0.000D+00	2.657D-10	-3.283D-10	1.991D-12
16	0.000D+00	0.000D+00	0.000D+00	3.658D-07	6.988D-05	-1.592D-07
17	0.000D+00	0.000D+00	0.000D+00	-2.602D-09	-9.671D-11	-1.188D-11
ROW \ COL	7	8	9	10	11	12
1	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
2	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
3	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00
4	4.930D-08	-2.751D-06	-1.471D-07	4.655D-06	-4.655D-06	-5.762D-07
5	-1.343D-10	-8.131D-07	2.959D-06	6.955D-05	6.723D-05	1.536D-05
6	2.295D-10	-1.092D-08	-7.499D-09	-1.385D-07	-1.765D-07	-3.805D-08
7	0.000D+00	-3.927D-10	1.416D-09	3.329D-08	3.217D-08	7.350D-09
8	3.927D-10	0.000D+00	-8.018D-08	-1.820D-06	-1.832D-06	-4.147D-07
9	-1.416D-09	8.018D-08	0.000D+00	-2.330D-07	3.774D-08	-5.377D-09
10	-3.329D-08	1.820D-06	2.330D-07	0.000D+00	6.180D-06	1.083D-06
11	-3.217D-08	1.832D-06	-3.774D-08	-6.180D-06	0.000D+00	-3.180D-07
12	-7.350D-09	4.147D-07	5.377D-09	-1.083D-06	3.180D-07	0.000D+00
13	-5.074D-14	4.965D-11	-1.687D-10	-3.973D-09	-3.830D-09	-8.757D-10
14	6.413D-09	-3.585D-07	-1.660D-08	6.652D-07	-5.479D-07	-6.179D-08
15	-1.568D-13	1.086D-11	-7.164D-12	-1.942D-10	-1.586D-10	-3.778D-11
16	3.344D-08	-1.863D-06	-1.103D-07	2.911D-06	-3.396D-06	-4.454D-07
17	-4.967D-14	-1.796D-11	7.489D-11	1.752D-09	1.703D-09	3.885D-10
ROW \ COL	13	14	15	16	17	
1	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	
2	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	
3	0.000D+00	0.000D+00	0.000D+00	0.000D+00	0.000D+00	
4	-5.878D-09	8.826D-08	-2.657D-10	-3.658D-07	2.602D-09	
5	1.220D-10	-1.340D-05	3.283D-10	-6.988D-05	9.671D-11	
6	-2.761D-11	3.127D-08	-1.991D-12	1.592D-07	1.188D-11	
7	5.074D-14	-6.413D-09	1.568D-13	-3.344D-08	4.967D-14	
8	-4.965D-11	3.585D-07	-1.086D-11	1.863D-06	1.796D-11	
9	1.687D-10	1.660D-08	7.164D-12	1.103D-07	-7.489D-11	
10	3.973D-09	-6.652D-07	1.942D-10	-2.911D-06	-1.752D-09	
11	3.830D-09	5.479D-07	1.586D-10	3.396D-06	-1.703D-09	
12	8.757D-10	6.179D-08	3.778D-11	4.454D-07	-3.885D-10	
13	0.000D+00	7.645D-10	-1.841D-14	3.988D-09	-8.600D-15	
14	-7.645D-10	0.000D+00	-3.428D-11	-1.075D-07	3.386D-10	
15	1.841D-14	3.428D-11	0.000D+00	1.814D-10	-8.541D-15	
16	-3.988D-09	1.075D-07	-1.814D-10	0.000D+00	1.765D-09	

ROW \ COL 13	14	15	16	17
17	8.600D-15 -3.386D-10	8.541D-15 -1.765D-09	0.000D+00	

\*\*\* RETAINED MODAL ANGULAR MOMENTUM MATRIX FOR 4-AXIS \*\*\*

ROW \ COL	1	2	3	4	5	6
1	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
2	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
3	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
4	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00	5. 948D-05	-4. 747D-05
5	0. 000D+00	0. 000D+00	0. 000D+00	-5. 948D-05	0. 000D+00	5. 181D-05
6	0. 000D+00	0. 000D+00	0. 000D+00	4. 747D-05	-5. 181D-05	1. 323D-23
7	0. 000D+00	0. 000D+00	0. 000D+00	3. 510D-07	-4. 176D-07	2. 746D-08
8	0. 000D+00	0. 000D+00	0. 000D+00	1. 587D-06	4. 703D-07	-1. 758D-06
9	0. 000D+00	0. 000D+00	0. 000D+00	3. 789D-05	-4. 331D-05	1. 561D-06
10	0. 000D+00	0. 000D+00	0. 000D+00	-3. 183D-06	-3. 961D-05	3. 438D-05
11	0. 000D+00	0. 000D+00	0. 000D+00	5. 745D-06	-4. 218D-05	2. 866D-05
12	0. 000D+00	0. 000D+00	0. 000D+00	-2. 132D-05	1. 496D-05	-6. 632D-06
13	0. 000D+00	0. 000D+00	0. 000D+00	3. 862D-07	-4. 213D-07	-1. 499D-10
14	0. 000D+00	0. 000D+00	0. 000D+00	1. 957D-08	7. 660D-06	-6. 130D-06
15	0. 000D+00	0. 000D+00	0. 000D+00	2. 622D-07	-2. 886D-07	1. 900D-09
16	0. 000D+00	0. 000D+00	0. 000D+00	5. 764D-07	3. 995D-05	-3. 238D-05
17	0. 000D+00	0. 000D+00	0. 000D+00	-4. 240D-07	4. 649D-07	-1. 682D-09

ROW \ COL	7	8	9	10	11	12
1	0. 000D+00					
2	0. 000D+00					
3	0. 000D+00					
4	-3. 510D-07	-1. 587D-06	-3. 789D-05	3. 183D-06	-5. 745D-06	2. 132D-05
5	4. 176D-07	-4. 703D-07	4. 331D-05	3. 961D-05	4. 218D-05	-1. 496D-05
6	-2. 746D-08	1. 758D-06	-1. 561D-06	-3. 438D-05	-2. 866D-05	-6. 632D-06
7	-1. 292D-26	1. 392D-08	1. 037D-08	-2. 561D-07	-2. 086D-07	-6. 138D-08
8	-1. 392D-08	3. 748D-25	-1. 455D-06	-1. 032D-06	-1. 171D-06	5. 677D-07
9	-1. 037D-08	1. 455D-06	0. 000D+00	-2. 755D-05	-2. 269D-05	-5. 995D-06
10	2. 561D-07	1. 032D-06	2. 755D-05	2. 515D-22	6. 083D-06	-1. 500D-05
11	2. 086D-07	1. 171D-06	2. 269D-05	-6. 083D-06	0. 000D+00	-1. 367D-05
12	6. 138D-08	-5. 677D-07	5. 995D-06	1. 500D-05	1. 367D-05	2. 118D-22
13	-2. 245D-10	1. 430D-08	-1. 282D-08	-2. 797D-07	-2. 332D-07	-5. 389D-08
14	-4. 534D-08	-2. 043D-07	-4. 894D-06	3. 969D-07	-7. 538D-07	2. 750D-06
15	-1. 376D-10	9. 774D-09	-7. 106D-09	-1. 900D-07	-1. 581D-07	-3. 749D-08
16	-2. 398D-07	-1. 061D-06	-2. 587D-05	1. 754D-06	-4. 267D-06	1. 446D-05
17	2. 328D-10	-1. 576D-08	1. 260D-08	3. 072D-07	2. 558D-07	5. 999D-08

ROW \ COL	13	14	15	16	17
1	0. 000D+00				
2	0. 000D+00				
3	0. 000D+00				
4	-3. 862D-07	-1. 957D-08	-2. 622D-07	-5. 764D-07	4. 240D-07
5	4. 213D-07	-7. 660D-06	2. 886D-07	-3. 995D-05	-4. 649D-07
6	1. 499D-10	6. 130D-06	-1. 900D-09	3. 238D-05	1. 682D-09
7	2. 245D-10	4. 534D-08	1. 376D-10	2. 398D-07	-2. 328D-10
8	-1. 430D-08	2. 043D-07	-9. 774D-09	1. 061D-06	1. 576D-08
9	1. 282D-08	4. 894D-06	7. 106D-09	2. 587D-05	-1. 260D-08
10	2. 797D-07	-3. 969D-07	1. 900D-07	-1. 754D-06	-3. 072D-07
11	2. 332D-07	7. 538D-07	1. 581D-07	4. 267D-06	-2. 558D-07
12	5. 389D-08	-2. 750D-06	3. 749D-08	-1. 446D-05	-5. 999D-08
13	3. 231D-27	4. 987D-08	-1. 629D-11	2. 634D-07	1. 502D-11
14	-4. 987D-08	0. 000D+00	-3. 386D-08	-6. 108D-08	5. 475D-08
15	1. 629D-11	3. 386D-08	1. 010D-28	1. 789D-07	-7. 682D-12
16	-2. 634D-07	6. 108D-08	-1. 789D-07	1. 323D-23	2. 892D-07

ROW \ COL 13

14

15

16

17

17 -1.502D-11 -5.475D-08 7.682D-12 -2.892D-07 0.000D+00

\*\*\* RETAINED MODAL CONTROL DISTRIBUTION MATRIX \*\*\*

ROW \ COL	1	2	3	4	5	6
1	2. 641D-02	0. 000D+00				
2	0. 000D+00	2. 641D-02	0. 000D+00	0. 000D+00	0. 000D+00	0. 000D+00
3	0. 000D+00	0. 000D+00	2. 641D-02	0. 000D+00	0. 000D+00	0. 000D+00
4	0. 000D+00	1. 340D-02	2. 399D-20	9. 680D-03	0. 000D+00	0. 000D+00
5	-1. 474D-02	-1. 221D-05	5. 824D-05	-8. 818D-06	1. 064D-02	0. 000D+00
6	3. 394D-05	1. 600D-05	-1. 341D-07	4. 502D-05	-2. 451D-05	8. 469D-03
7	-9. 998D-06	-3. 310D-07	6. 147D-03	8. 401D-09	5. 094D-06	6. 791D-05
8	-5. 729D-03	1. 622D-04	-4. 600D-06	7. 663D-05	-2. 842D-04	-1. 358D-07
9	1. 518D-05	-4. 203D-04	-1. 012D-04	-2. 780D-04	-1. 519D-05	6. 765D-03
10	-1. 099D-03	-9. 009D-03	8. 773D-06	-6. 535D-03	4. 809D-04	-8. 860D-05
11	1. 057D-03	-8. 724D-03	1. 432D-05	-6. 316D-03	-4. 809D-04	5. 472D-04
12	1. 391D-04	-1. 971D-03	-8. 362D-05	-1. 443D-03	-5. 953D-05	-3. 874D-03
13	8. 272D-07	-3. 806D-07	-4. 659D-04	-1. 096D-08	-6. 073D-07	6. 850D-05
14	-3. 155D-05	1. 730D-03	-1. 176D-07	1. 259D-03	9. 118D-06	1. 262D-05
15	-1. 468D-08	-2. 888D-07	8. 494D-05	-3. 082D-08	-2. 745D-08	4. 689D-05
16	9. 489D-05	9. 009D-03	-2. 626D-07	6. 566D-03	-3. 779D-05	6. 535D-05
17	-4. 485D-07	3. 884D-07	7. 187D-06	-9. 309D-09	2. 688D-07	-7. 560D-05

useful for control system design in such cases. For accurate simulations, however, all couplings should be included.

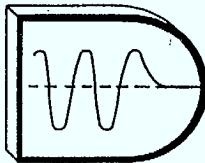
## 9. NUMERICAL DATA

A set of numerical data has been prepared for MSAT in the form described in the preceding sections. It is not the purpose of this report, however, to explain how this data was obtained — only to present it clearly so that simulation can proceed. This model will be called the *MARK I* model because it is anticipated that future improvements in it will be made, especially as better substructural models become available.

The *MARK I* data is presented in four appendices, as follows:

- Appendix A: Design Model — System Matrices
- Appendix B: Design Model — Transfer Function Matrices
- Appendix C: Evaluation Model — System Matrices
- Appendix D: Evaluation Model — Transfer Function Matrices

The idea is that a relatively low-order model (4 flexible modes) will do for control system design, but that a higher-order model (11 flexible modes) is required for control system evaluation.



YNACON

APPENDIX D

MSAT DYNAMICS MODEL

*MARK I*

(EVALUATION MODEL:  $N_e = 11$ )

Transfer Function Matrices

$$\begin{array}{l} \underline{I} \\ \underline{I}^{-1} \\ \underline{r}_c \\ -\underline{I}^{-1}\underline{r}_c^x \end{array}$$

$$\begin{bmatrix} \omega_\alpha & \\ & \zeta_\alpha \\ \underline{e}_{w\alpha}^T & \underline{e}_{\theta\alpha}^T \\ \underline{K}_{f\alpha} & \underline{K}_{g\alpha} \end{bmatrix}, \quad \alpha = 1, \dots, 11$$

INERTIA MATRIX I

1. 067D+04	8. 842D+00	-5. 671D+01
8. 842D+00	8. 827D+03	2. 550D+01
-5. 671D+01	2. 550D+01	1. 394D+04

INVERSE OF INERTIA MATRIX I

9. 370D-05	-9. 495D-08	3. 813D-07
-9. 495D-08	1. 133D-04	-2. 076D-07
3. 813D-07	-2. 076D-07	7. 172D-05

RC - POSITION OF MASS CENTER RELATIVE TO OB

5. 471D-03	0. 000D+00	1. 385D+00
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THE MATRIX PRODUCT -(I\*\*-1)(RC\*\*X)

1. 315D-07	1. 297D-04	-5. 195D-10
-1. 569D-04	-1. 303D-07	6. 198D-07
2. 874D-07	1. 355D-07	-1. 136D-09

FLEXIBLE MODE: ALPHA = 1

OMEGA = 1.140D+00 RAD/SEC

1

ZETA = 5.134D-03

1

T  
E  
W1

-9.998D-06 -3.310D-07 6.147D-03

T  
E  
THETA1

8.401D-09 5.094D-06 6.791D-05

K  
F1

-8.399D-14 -2.781D-15 5.164D-11  
-5.093D-11 -1.686D-12 3.131D-08  
-6.789D-10 -2.248D-11 4.174D-07

K  
G1

7.057D-17 4.279D-14 5.705D-13  
4.279D-14 2.594D-11 3.459D-10  
5.705D-13 3.459D-10 4.611D-09

FLEXIBLE MODE: ALPHA = 2

OMEGA = 1.155D+00 RAD/SEC  
2

ZETA = 5.134D-03  
2

T  
E  
W2

-5.729D-03 1.622D-04 -4.600D-06

7.663D-05 -2.842D-04 -1.358D-07

T  
E  
THETA2

K  
F2

-4.390D-07 1.243D-08 -3.525D-10  
1.628D-06 -4.610D-08 1.307D-09  
7.779D-10 -2.203D-11 6.246D-13

K  
G2

5.873D-09 -2.178D-08 -1.041D-11  
-2.178D-08 8.076D-08 3.859D-11  
-1.041D-11 3.859D-11 1.844D-14

FLEXIBLE MODE: ALPHA = 3

OMEGA = 1.513D+00 RAD/SEC  
3

ZETA = 6.348D-03  
3

T

E

W3

1.518D-05 -4.203D-04 -1.012D-04

-2.780D-04 -1.519D-05 6.765D-03

T

E

THETA3

K

F3

K

G3

-4.219D-09 1.169D-07 2.813D-08  
-2.306D-10 6.386D-09 1.537D-09  
1.027D-07 -2.843D-06 -6.843D-07

7.730D-08 4.224D-09 -1.881D-06  
4.224D-09 2.309D-10 -1.028D-07  
-1.881D-06 -1.028D-07 4.576D-05

FLEXIBLE MODE: ALPHA = 4

OMEGA = 1.582D+00 RAD/SEC  
4

ZETA = 5.921D-03  
4

T  
E  
W4

-1.099D-03 -9.009D-03 8.773D-06

T  
E  
THETA4

-6.535D-03 4.809D-04 -8.860D-05

K  
F4

7.180D-06 5.887D-05 -5.733D-08  
-5.285D-07 -4.333D-06 4.219D-09  
9.736D-08 7.982D-07 -7.773D-10

K  
G4

4.270D-05 -3.143D-06 5.790D-07  
-3.143D-06 2.313D-07 -4.261D-08  
5.790D-07 -4.261D-08 7.850D-09

FLEXIBLE MODE: ALPHA = 5

OMEGA = 1.608D+00 RAD/SEC  
5

ZETA = 5.868D-03  
5

T  
E  
W5

1. 057D-03 -8.724D-03 1. 432D-05

T  
E  
THETA5

-6.316D-03 -4.809D-04 5.472D-04

K

F5

-6.678D-06 5.510D-05 -9.042D-08  
-5.084D-07 4.195D-06 -6.884D-09  
5.785D-07 -4.774D-06 7.834D-09

K

G5

3.989D-05 3.037D-06 -3.456D-06  
3.037D-06 2.313D-07 -2.631D-07  
-3.456D-06 -2.631D-07 2.994D-07

FLEXIBLE MODE: ALPHA = 6

OMEGA = 1.614D+00 RAD/SEC  
6

ZETA = 5.480D-03  
6

T  
E  
W6

1.391D-04 -1.971D-03 -8.362D-05

-1.443D-03 -5.953D-05 -3.874D-03

T  
E  
THETA6

K  
F6

K  
G6

-2.007D-07	2.845D-06	1.207D-07
-8.279D-09	1.173D-07	4.977D-09
-5.388D-07	7.636D-06	3.239D-07

2.083D-06	8.590D-08	5.591D-06
8.590D-08	3.543D-09	2.306D-07
5.591D-06	2.306D-07	1.501D-05

FLEXIBLE MODE: ALPHA = 7

OMEGA = 2.187D+00 RAD/SEC

7

ZETA = 5.001D-03

7

T

E

W7

T

E

THETA7

8.272D-07 -3.806D-07 -4.659D-04 -1.096D-08 -6.073D-07 6.850D-05

K

F7

K

G7

-9.068D-15 4.173D-15 5.108D-12  
-5.023D-13 2.311D-13 2.829D-10  
5.666D-11 -2.607D-11 -3.191D-08

1.202D-16 6.657D-15 -7.509D-13  
6.657D-15 3.688D-13 -4.160D-11  
-7.509D-13 -4.160D-11 4.692D-09

FLEXIBLE MODE: ALPHA = 8

OMEGA = 2.318D+00 RAD/SEC  
8

ZETA = 5.031D-03  
8

T  
E  
WB

T  
E  
THETAB

-3.155D-05	1.730D-03	-1.176D-07	1.259D-03	9.118D-06	1.262D-05
------------	-----------	------------	-----------	-----------	-----------

K  
F8

K  
G8

-3.972D-08	2.178D-06	-1.481D-10
-2.876D-10	1.577D-08	-1.073D-12
-3.981D-10	2.183D-08	-1.485D-12

1.585D-06	1.148D-08	1.589D-08
1.148D-08	8.314D-11	1.151D-10
1.589D-08	1.151D-10	1.593D-10

FLEXIBLE MODE: ALPHA = 9

OMEGA = 2.320D+00 RAD/SEC  
9

ZETA = 5.000D-03  
9

T	E	W9	T	E	THETA9
-1.468D-08	-2.888D-07	8.494D-05	-3.082D-08	-2.745D-08	4.689D-05
K			K		
4.523D-16	8.902D-15	-2.618D-12	9.499D-16	8.460D-16	-1.445D-12
4.029D-16	7.928D-15	-2.332D-12	8.460D-16	7.535D-16	-1.287D-12
-6.882D-13	-1.354D-11	3.983D-09	-1.445D-12	-1.287D-12	2.199D-09

FLEXIBLE MODE: ALPHA = 10

OMEGA = 2.366D+00 RAD/SEC  
10

ZETA = 5.844D-03  
10

T  
E  
W10

9.489D-05 9.009D-03 -2.626D-07

T  
E  
THETA10

6.566D-03 -3.779D-05 6.535D-05

K  
F10

6.230D-07 5.915D-05 -1.725D-09  
-3.586D-09 -3.404D-07 9.925D-12  
6.201D-09 5.887D-07 -1.716D-11

K  
G10

4.311D-05 -2.481D-07 4.291D-07  
-2.481D-07 1.428D-09 -2.470D-09  
4.291D-07 -2.470D-09 4.271D-09

FLEXIBLE MODE: ALPHA = 11

OMEGA = 2.587D+00 RAD/SEC  
11

ZETA = 5.000D-03  
11

T	E	W11	T	E	THETA11
K	F11		K	G11	
-4.485D-07	3.884D-07	7.187D-06	-9.309D-09	2.688D-07	-7.560D-05
4.175D-15	-3.616D-15	-6.690D-14	8.665D-17	-2.503D-15	7.037D-13
-1.206D-13	1.044D-13	1.932D-12	-2.503D-15	7.228D-14	-2.032D-11
3.391D-11	-2.937D-11	-5.433D-10	7.037D-13	-2.032D-11	5.715D-09

CACC / CCAC



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HUGHES, P. C.  
--Structural dynamics model for  
MSAT (Mark 1)

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1983

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