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CODING FOR FAST FREQUENCY HOPPED SPREAD-SPECTRUM COMMUNICATIONS TO ENSURE ANTI-JAM PROTECTION SECURED COMMUNICATIONS

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ABSTRACT

A main objective in the present study is to develop adaptive techniques for communications over a jammed fading channel. An adaptive rate control policy has been introduced in section 2 to compensate for channel fading. An algorithm to implement the adaptive rate control policy is described in section 3. It is shown that the adaptive rate controlled FH/MFSK signals exhibit similar cutoff rate performance as transmissions through an unfaded channel. In an adaptive rate controlled environment, coding offers superior performance to multichannel diversity transmission. An adaptive quantizing approach, which can potentially compensate for signal fading, is also proposed.

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1. Introduction

Communication in a highly stressed environment can be viewed as a game played by the communicator and the jammer. The success of either can be measured in terms of the bit error probability. While the jammer tries to maximize this bit error probability, the communicator seeks to minimize it in the presence of jamming and additive noise.

An effective way that the communicator can employ to evade the jammer is to hop its transmitted signal over a relatively wide bandwidth. This mode of communication is normally referred to as frequency-hopped spread spectrum communication. The basic modulation may be M-ary frequency shift keying (MFSK) or M-ary differential phase shift keying (MDPSK). From the detection point of view, noncoherent detection can be used to detect the MFSK signals and differentially coherent detection for the MDPSK signals. In either case, there is no need to maintain phase coherency at the receiver for demodulation purposes.

The jammer will devise strategies that will maximize the bit error probability. One way to accomplish this is to concentrate its available power to hit the communicator's signal as heavy as possible. That is, the jammer may choose to jam only a fraction ρ of the total spread spectrum bandwidth, W_{ss}. Thus, a fraction $(1-\rho)$ of the total spread spectrum bandwidth will be free of jamming. This mode of jammer operation is referred to as partial-band noise jamming. With probability ρ, the communicator's signal will be jammed; with probability $(1-\rho)$, it will be jamming free.

A second mode of jamming against frequency hopping is multitone or multiple CW tone interference. In this case, the jammer splits its available power into Q distinct, equal power, random phase CW tones and distribute these over the communicator's spread spectrum bandwidth in such a manner as to exert worst case jamming. Analogous to the parameter ρ in partial-band noise jamming, we may define a parameter μ as the Pr{any symbol in an M-ary band is jammed}. It is apparent that, in the above jamming models, the parameters ρ and μ are the variables that the jammer can change to maximize its effectiveness.

Initial work on the derivation of error probability expressions under both partial-band noise and multitone jamming models was done by Houston [1]. Presentation of similar error probability performance also appears in [2]. Houston [1] has focused attention on the maximization of the symbol or bit error probability from the jammer's point of view. In this study, we examine coding and diversity strategies that the communicator may employ to combat jamming. To set the stage for our investigation, we first define the relevant parameters and describe the bit error probability expressions given by Houston [1] for FH/MFSK and FH/MDPSK transmission plans.

The following notations describing the communicator and jammer parameters will be used throughout the report:

W_{ss} = *total spread spectrum bandwidth in Hz*

J = *total jammer power in watts*

$N_J = J/W_{ss}$ *is the jammer noise power spectral density*

S = *the signal power in watts*

E_b = *the bit energy*

R_b = *the bit rate*

$PG = W_{ss}/R_b$ *is the processing gain*

S/J = *the signal-to-jamming power ratio*

$E_b/N_J = (S/J)\cdot(W_{ss}/R_b)$

= *the bit energy-to-jammer power spectral density ratio*

R_c = *the chip rate*

R_s = *the M-ary symbol rate*

R_H = *the hop rate*

$$R_c = R_s = R_b/K \geq R_h$$

where K is the number of binary digits that comprises an M-ary symbol, i.e., $M = 2^K$. The received symbol energy is then $E_s = S/R_s = KE_b$, where $E_b = S/R_b$ is the bit energy.

1.1 FH/MFSK Signalling

With MFSK signalling in which the communicator's signal is hopped over a wide frequency band, it is impractical to attempt to maintain phase coherency. Noncoherent detection using a bank of energy detectors is normally employed. How the outputs of the energy detectors may be combined depends on the metric used. This is true for both partial-band noise and multitone jamming.

1.1.1 Partial-Band Noise Jamming

With M-ary signalling, the noncoherent receiver consists of a bank of M energy detectors. Let U_i, i=0,1,...,M−1 be the outputs of the energy detectors. For convenience, assume that symbol 0 is sent. The jammer may choose to jam a fraction ρ, $0<\rho\leq 1$, of the total spread spectrum bandwidth, W_{ss}. If ρ=1, then the jammer attempts to jam the entire spread spectrum band. In this case, the jamming interference may be viewed as having the effect of Gaussian noise. The output U_0 of the first energy detector will be the only one with a specular component so that it is a non-central chi-square random variable. If the background additive noise is negligible compared to the jammer power spectral density, the probability density function (pdf) of U_0 is given by

$$p_{U_0}(u_0) = \begin{cases} \exp(-u_0 - E_s/N_J) I_o(2\sqrt{u_0 E_s/N_J}); & u_0 \geq 0 \\ 0; & u_0 < 0 \end{cases}$$

where $I_o(\cdot)$ is the zeroth order modified Bessel function of the first kind. The other (M-1) energy detector outputs, u_i, $i=1,\ldots,(M-1)$, will not have a specular component. These are identically distributed central chi-square random variables with pdf given by

$$p_{U_0}(u_i) = \begin{cases} \exp(-u_i); & u_i \geq 0 \\ 0; & u_i < 0 \end{cases},$$

for $i = 1,\ldots,(M-1)$. On the basis that the M signals are orthogonal, the u_i's are all statistically independent. The symbol error probability is then given by

$$P_s = Pr\left\{ \bigcup_{i=1}^{M-1} (u_i \geq u_0) \right\}$$

$$= \frac{1}{M} \sum_{i=1}^{M-1} (-1)^{i+1} \binom{M}{i+1} e^{-(E_s/N_J)(1-\frac{1}{i+1})} .$$

The leading term on the right hand summation gives the union upperbound as follows:

$$P_s \leq (M-1) Pr\{u_1 \geq u_0\}$$

$$= \frac{M-1}{2} e^{-E_s/2N_J} .$$

Since $E_s = KE_b$, the bit error probability is given by

$$P_b = \frac{M}{2(M-1)} P_s$$

$$\leq \frac{M}{4} e^{-KE_b/2N_J} .$$

If the jammer chooses to jam only a fraction ρ of the total spread spectrum bandwidth, it can do so with power spectral density N_J/ρ. However, only with probability ρ the jammer hits the communicator's signal and the bit error probability becomes

$$P_b = \frac{\rho}{2(M-1)} \sum_{i=1}^{M-1} (-1)^{i+1} \binom{M}{i+1} e^{-(\rho K E_b/N_J)(1-\frac{1}{i+1})} . \tag{1.1}$$

The jammer can choose ρ to maximize P_b, i.e.,

$$P_b = \max_{0<\rho\leq 1} \left[\frac{\rho}{2(M-1)} \sum_{i=2}^{M} (-1)^i \binom{M}{i} e^{-(\rho K E_b/N_J)(\frac{i-1}{i})} \right.$$

Houston [1] has shown that

$$P_b = \begin{cases} \frac{1}{2(M-1)} \sum_{i=2}^{M} \binom{M}{i} e^{-(KE_b/N_J)(\frac{i-1}{i})}; & with\ \rho_o = 1;\ \frac{E_b}{N_J} \leq \beta \\ \frac{\alpha}{E_b/N_J}; & with\ \rho_o = \frac{\beta}{E_b/N_J};\ \frac{E_b}{N_J} \geq \beta \end{cases} \tag{1.2}$$

where ρ_o is the optimum fraction of the total spread spectrum band that the jammer can exert the worst damage to the communicator's signal, and α and β are parameters to be determined depending on the size of the M-ary alphabet, or m, the number of bits mapped onto the M-ary alphabet. The values of α and β corresponding to different values of m has been computed by Houston [1] as shown in Table 1.1.

1.1.2 Multitone Jamming

For MFSK, the frequency between the M-ary symbol keying tones is R_c Hz, which we shall refer to as a frequency slot. The hopping frequency bandwidth during any one use of the channel, which we shall refer to as the M-ary band, is then equal to $M \cdot R_c$ Hz. It is noted that the communicator places at most one symbol keying tone in one of the M slots within the M-ary band of $M \cdot R_c$ Hz. An intelligent

Table 1.1 Values of α and β for Worst Case Partial-Band Noise Jamming

m	α	β
1	0.3679	2.000
2	0.2329	1.170
3	0.1954	0.930
4	0.1803	0.872
5	0.1746	0.798

jammer is assumed to have perfect knowledge of the communicator's hopping bandwidth, but not the hopping sequence, i.e., the jammer knows neither the location of the hopping band within the total spread spectrum bandwidth W_{ss} nor the symbol keying tone within the hopping bandwidth. The total number of frequency slots is $N_t = W_{ss}/R_c$. If the jammer divides its available power into Q distinct tones, assuming that each jam tone is of a power level slightly exceeding the communicator's signal power S, and is placed exactly within one frequency slot, the fraction of frequency slots jammed will be Q/N_t. On the basis that the communicator's tone is said to have been jammed if the jam tone is placed in one or more of the $(M-1)$ unkeyed slots, the jammer has to choose one of two jamming strategies: to place one or two jam tones within an M-ary band. If the jammer chooses to place only one jam tone per M-ary band, there is a finite probability that the jam tone coincides with the communicator's keyed tone, in which case, the communicator's signal is declared unjammed. Placing two jam tones within one M-ary band can potentially increase its jamming effectiveness. However, the number of M-ary bands it can place two jam tones for a given value of Q is proportionately reduced, i.e., to $Q/2$.

Strategy 1: One Jam Tone per M-ary Band

The probability that the communicator's keyed symbol tone is jammed is

$$\mu_1 = (1-\frac{1}{M})\cdot\frac{Q}{W_{ss}/M\cdot R_c} . \tag{1.3}$$

The factor $(1-1/M)$ is the conditional probability that the jam tone falls in one of the $(M-1)$ unkeyed frequency slots within the hopping frequency bandwidth. The remaining fraction is the probability that Q of the total hopping frequency bands are jammed.

Strategy 2: Two Jam Tones per M-ary Band

With two jam tones per M-ary band, the conditional probability that the communicator's keyed symbol tone is jammed given that the jammer has placed its jam tones in the correct M-ary band is 1. Thus, in this case, the probability that the communicator's keyed symbol tone is jammed is simply given by the probability that the jammer places its jam tones in the correct M-ary band. Hence,

$$\mu_2 = \frac{Q/2}{W_{ss}/M\cdot R_c} . \tag{1.4}$$

In either case, Q is the parameter with which the jammer may vary to optimize its effectiveness. Define

$$P_H \equiv \frac{Q}{W_{ss}/M\cdot R_c} \tag{1.5}$$

as the probability that any of the M-ary band is jammed. Then, (1.3) and (1.4) can be written as

$$\mu_1 = (1-\frac{1}{M})\cdot P_H \tag{1.6}$$

and

$$\mu_2 = P_H/2 \ . \tag{1.7}$$

It is noted that $\mu_1 \geq \mu_2$ with equality if $M = 2$. That is, with BFSK the conditional probability that the communicator's keyed tone is jammed given that the jammer has placed its jam tone in the correct binary band is 1/2, which is the same fraction that Q has been reduced. In general it will not be beneficial for the jammer to place more than one jam tone per M-ary band. Thus, on the assumption that the jammer places jam tones in non-overlapping M-ary bands over the total spread spectrum band, placing only one jam tone per M-ary band is more effective.

1.2 Detection of FH/MFSK Signals

If the receiver is supplied with the jammer state information, one can introduce metrics for diversity combining and decoding in an optimal way. From the receiver's point of view, the objective is to reduce the bit error probabilities given by the expressions shown in section 1. Here, we will address the problem of how to devise methods or algorithms to estimate the jammer state information. The underlying assumption is that the jammer state is estimatable and is, therefore, a deterministic quantity. Since the received signal is a mixture of signal and interference, the jamming interference must possess certain identifiable feature or features.

For fast frequency hopping, the symbol to be transmitted is partitioned into L segments, generally referred as *chips*. The chips are ultimately frequency hopped over the spread spectrum bandwidth, W_{ss} Hz. If L represents a repeat of the transmitted symbol L times, as opposed to partitioning the symbol into L chips, then one symbol occupies L frequency hops. The L repeats provide an L-fold increase in bit (symbol) energy. In a severely fading channel, in may be necessary to increase the bit energy as a means to increase the signal-to-noise ratio. In either mode of operation, the L-fold transmission of the same symbol is referred to as diversity transmission. For FH/MFSK signalling, we are concerned with fast frequency hopping and the parameter L represents L partitions of a single symbol.

If the decoder input is comprised of a set of sufficient statistics, then a maximum likelihood type of detection can be used to affect optimal decoding. It is thus desirable that the coding channel appears memoryless to the decoder. Specifically, the L transmitted chips should be received as independent symbols. i.e., the interference corrupts the L chips in an independent manner. The type of interference channel dictates the complexity necessary to render the channel memoryless. For an unfaded channel, frequency hopping alone may provide sufficient randomization to make the channel appearing memoryless. If the channel exhibits fading, it may be necessary to use interleaving render the channel memoryless. The inclusion of an interleaver and a deinterleaver in the system undoubtedly increase the system complexity. With interleaving, each encoded symbol is partitioned into L chips first and then interleaved before passing them to the MFSK modulator. At the receiver, the reverse operations are performed. Without interleaving, the encoded symbols are first MFSK modulated and then partitioned into L chips. Thus, in the former case partitioning is done to the encoded symbol, whereas in the latter case, it is performed on the M-ary symbols. As discussed below, where partitioning takes place has a profound effect on the complexity of the diversity combining process.

In this report, it is assumed that the communicator's signal is jammed if and only if the jam signal or tone falls in the same M-ary band as the communicator's signal or keyed tone, whether the jammer employs a multitone or a partial-band noise jamming model.

With $L > 1$, L uses of the channel can be used to arrive at a decision of each information bit. Although it is possible that all of the L uses may experience jamming, the probability of such occurrence over a number of L uses of the channel is, hopefully, very small.

Consider the transmission of the i^{th} chip, $i = 1,...,L$. After frequency dehopping, the received signal is input to a bank of M non-coherent detectors. The j^{th} non-coherent detector produces an energy level U_{ij}. Suppose that the 0^{th} of the M-ary symbols is sent. If the i^{th} chip is jamming-free, we should have $U_{i0} > e_{ij}$, $j \neq 0$. If the i^{th} chip were jammed, then one or more of the $U_{ij}'s$, $j \neq 0$ would be as large or larger than U_{i0}. Under this situation, decisions based on the the observation of the $U_{ij}'s$ for any i^{th} chip can potentially introduce significant error. By postponing the decision making process until all L chips have been received should enable the making of a more intelligent decision. The question is how the L received corrupted chips should be processed. This is a diversity combining problem, which has a direct bearing on how the L chips are converted to M-ary symbols.

Consider the following scenario: The encoded binary symbols are mapped into M-ary FSK symbols, where $M = 2^K$. Each M-ary symbol is partitioned into L subsymbols or *chips* with energy $E_c = KE_b/L$. The individual chips are then frequency hopped over the spread spectrum bandwidth of W_{ss} Hz. A tacit assumption here is that the frequency hopping operation provides sufficient randomization of the L M-ary symbol chips for protection against partial-band noise or multitone jamming. In the absence of interference, consecutive blocks of L outputs, U_{i0}, $i = 1, ..., L$, from the 0^{th} energy detector, assuming that the 0^{th} M-ary symbol was sent, will be the same, and the outputs of the other (M-1) energy detectors will be zero. The presence of jamming and background noise will disrupt this idealized condition. The net effect is that, for a given i, more than one of the M energy detectors may have a high energy output. One detection strategy is to store the energy detector outputs for L time slots, i.e., store the U_{ij}, $i = 1,...,L$, $j = 0,1,...,M-1$, in an array. Discard all columns having more than one high energy levels. What remains is a reduced matrix; the row with the largest number of high energy levels is declared the estimate of the transmitted M-ary symbol. If all L chips are jammed, then all columns would have been deleted. Under this condition it would be necessary to introduce a suitable metric to extract the signal. Simon et al [2] have suggested a linear combining metric where the rows of the array are summed and the maximum of these sums is declared as the transmitted M-ary symbol. This detection procedure is a form of majority logic detection.

The transmission and reception scheme described in the preceding paragraph makes the detection process rather simple. However, the only protection against jamming is the frequency hopping mechanism. Specifically, interleaving/deinterleaving is not employed and diversity combining is done before MFSK demodulation.

If interleaving is to be used, it should be done to the baseband information symbols rather than the MFSK symbols. In this case, the encoded symbols should be partitioned into L chips, which are then interleaved before passing them to the MFSK modulator. Here, diversity combining must be performed after MFSK demodulation and deinterleaving. What this means is that the outputs from all energy detectors need to be MFSK demodulated and deinterleaved. The complexity thus increases many folds.

The added protection through interleaving/deinterleaving is sometimes needed when the channel exhibits fading. In a communications situation where fading is not a problem, frequency hopping provides sufficient randomization and the detection process is much simpler. The functional block diagrams of the two scenarios are shown in Figures 1.1 and 1.2.

Although the scheme of Figure 1.2 offers added protection by the interleaver/deinterleaver units, it is necessary to perform MFSK demodulation before diversity combining. This means that each of the M output energies U_{ij}, for *all* i and j will have to be MFSK demodulated and deinterleaved before diversity combining. In this way, all of the M energy detector outputs have to be passed through the MFSK demodulator and the deinterleaver before combining. The complexity of the detection process of Figure 1.2 is thus many folds of that of Figure 1.1.

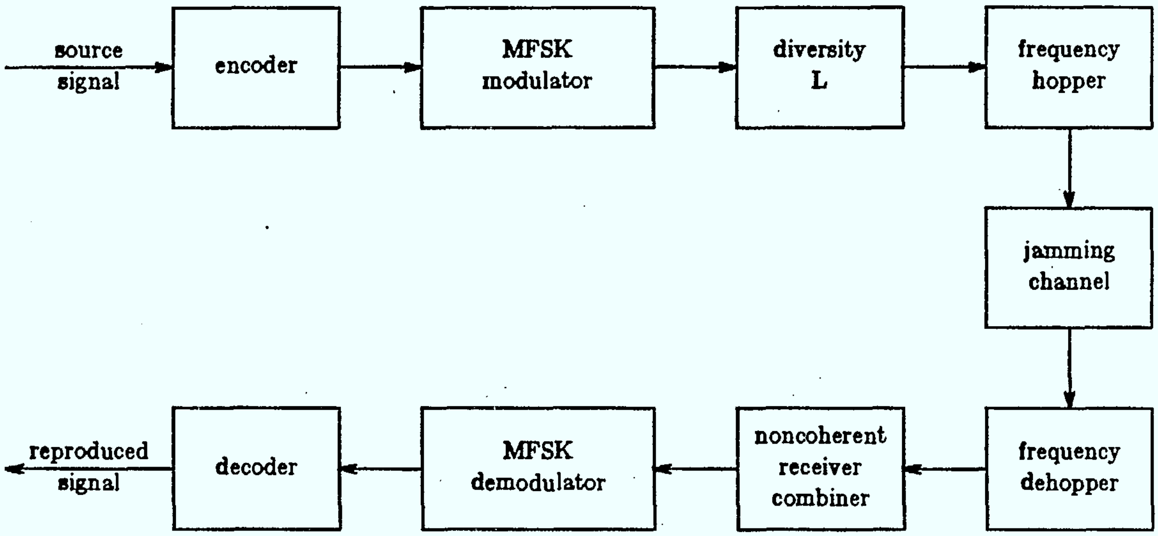

Figure 1.1 Diversity Signalling without Interleaving

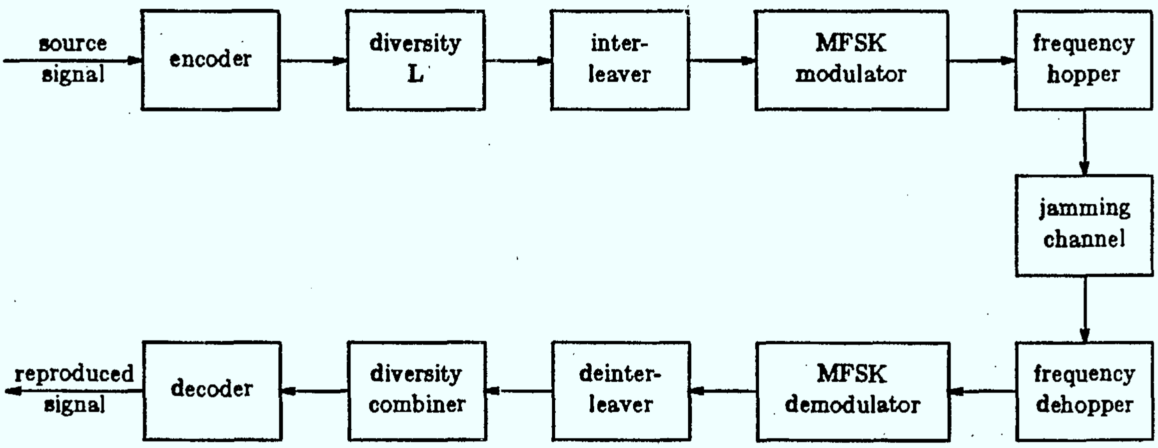

Figure 1.2 Diversity Signalling with Interleaving

1.3 Coding and Diversity Tradeoffs

The performance of a communications channel is governed by the channel capacity. Communication can be made reliable if the transmission rate is less than channel capacity. However, the capacity is a concave function with, possibly, a long tail. The cutoff rate R_o appears to be a more sensible parameter for the design of coding/modulation systems. With orthogonal signalling and soft decision detection, the cutoff rate R_o is given by the following parametric form:

$$R_o = 1 - \frac{1}{\log_2 M}\log_2(1 + (M-1)D) \tag{1.8}$$

where D represents the uncertainty in the detection process. Let $\mathbf{y} = (y_1, y_2, ..., y_M)$ be the coding channel output vector, x be a symbol of the correct sequence, and $\hat{x}$ be a symbol of the incorrect sequence. Let n be the number of times an incorrect symbol must be hit by the jammer so that the associated decision variable will be tie or exceed the keyed symbol decision variable. Then, the probability that a hop is jammed is

$$P_H = \frac{Mn}{r\gamma_b(n)\log_2 M} \tag{1.9}$$

where r is the code rate and $\gamma_b(n)$ is the bit energy-to-noise ratio. If maximum likelihood metrics are used in the diversity combining, we have

$$D = \left(\frac{M-1}{M}P_H(n)\right)^L \int_{\mathbf{y}} \left(p(\mathbf{y}\,|x)p(\mathbf{y}\,|\hat{x})\right)^{1/2} d\mathbf{y}\ . \tag{1.10}$$

Let $\mathbf{x} = (x_0, x_1, ..., x_{M-1})$ denote the number of times that each M-ary symbol has been hit by the jammer during the L transmissions of one symbol. Suppose the transmitted tone always occupies the 0^{th} subchannel (corresponding to x_0). Let

$$P\left(\max_{1\leq k\leq M-1} x_k = j\right)$$

be the probability that the maximum number of jamming tones hitting any symbol other than the keyed symbol equals j. On the assumption that $x_0 = 0$, j is in the range $\lceil L/M-1\rceil \leq j \leq L$. Assuming that ties are broken at random, the symbol error probability conditioned on that at least one of the L transmissions of the keyed symbol has not been jammed is given by

$$P_s(M,n) = P(j > n) + P(\text{error} \mid j{=}n)P(j{=}n) \ . \tag{1.11}$$

$P(j{=}n)$ is the probability of tie, which can occur only if the jammer tone power equals that of the communicator's keyed symbol power. Ties can be avoided if the jammer uses a slightly larger tone power.

The bit energy-to-noise ratio, $\gamma_b(n)$, with diversity L, can be expressed as:

$$\gamma_b(n) = \frac{(M-1)n}{r\log_2 M}\left(\frac{1-P_s(M,n))^{1/2} + ((M-1)P_s(M,n))^{1/2} - 1}{2^{(R_o-1)\log_2 M} - 1}\right)^{1/L} \ . \tag{1.12}$$

The jammer chooses n such that, at $r = R_o$,

$$n^* = \arg\max_n \gamma_b(n) \ .$$

The required bit energy-to-noise ratio for operation at $r = R_o$ is

$$\gamma_b = \gamma_b(n^*) \ . \tag{1.13}$$

It has been shown [9] that the worse case bit energy-to-noise ratio required for coded operation at the cutoff rate is expressible as

$$\gamma_b(l) = \frac{M(n^*-l)}{R_o^*(l)\log_2 M} , \quad 0 \leq l \leq n^* - \left\lceil \frac{L}{M-1} \right\rceil . \tag{1.14}$$

For binary signalling, the bit energy-to-noise ratio becomes

$$\gamma_b = \frac{L}{R_o(2^{1-R_o} - 1)^{1/L}} , \quad \gamma_b \geq 2L/R_o . \tag{1.15}$$

It is noted that $p(\mathbf{y}|x)$ in (1.9) is the conditional density when all L subbands are jammed and is given by

$$p(\mathbf{y}|x) = \delta(y_x - LE_s) \prod_{\substack{k=1 \\ k \neq x}}^{M} \delta(y_{\hat{x}} - x_k E_J) ,$$

where E_J is the jammer tone energy. Then, the expression for D becomes

$$D = \left[\frac{M-1}{M} P_H(n) \right]^L P_t(n) \tag{1.16}$$

where

$$P_t(n) = Pr(y_{\hat{x}} = LE_s | n)$$

is the probability that the decision variable corresponding to a particular incorrect symbol will tie the keyed symbol decision variable. This probability depends on n, the number of times an incorrect symbol must be jammed to cause a tie and is given by

$$P_t(n) = \frac{\binom{L}{n} (M-2)^{L-n}}{(M-1)^L} .$$

For an unfaded channel with an unfaded partial-band noise jammer, the cutoff rate R_o for BFSK signalling is given by

$$R_o = 1 - \log_2(1 - D) \tag{1.17}$$

where

$$D(\lambda) = \frac{\rho}{1-\lambda^2} e^{-\frac{\lambda}{1-\lambda}\rho\bar{\gamma}_b} \tag{1.18}$$

and

$$D = \min_{0 \le \lambda \le 1} D(\lambda)$$

1.4 Summary

This section presents the error performance of FH/MFSK signalling over a partial-band noise jamming channel and a multitone channel. The cutoff rate R_o is used as a communications parameter. Coding and diversity tradeoffs are culminated in a parametric equation relating R_o and D, which governs the error uncertainty due to errors in the detection process.

If interleaving is not used, diversity transmission repeats the M-ary symbol rather than the encoded symbol. In this manner, diversity combining can be performed prior to MFSK demodulation, so that the receiver complexity can be reduced. By examining the received signals corresponding to the L transmissions at a time, a form of majority capture procedure can be used to declare jamming-free conditions. Alternatively, suitable weights can be applied to the stored array elements and then combine the results using a suitable metric, e.g., a linear sum metric or a maximum likelihood metric.

Attention has been focussed on unfaded channel and unfaded jammer tone conditions. In the presence of channel fading, it will be necessary to estimate the fading channel and compensate for channel fading. Section 3 addresses the channel estimation problem and shows that, under an adaptive rate control policy, it is possible to render a time-varying channel to behave like a time-invariant channel. In this manner, the results of transmission through an unfaded channel can be applied.

2. Adaptive Transmission Over a Jammed Fading Channel

This section presents an analysis of the performance of an adaptive rate control technique for jammed fading channels. The performances of both uncoded and coded systems are examined in the presence of jamming. Adaptive signalling schemes are shown to offer negligible improvement for uncoded systems. However, for coded systems they provide substantial improvements both in the performance and the receiver implementation.

Adaptive rate control adjusts the duration of a signalling interval according to the channel fade state. Adjusting the signalling rate requires adjustment of the MFSK subband width, and in turn the spread spectrum bandwidth. It is assumed for the remainder of this section that the jammer has complete knowledge of the instantaneous spread spectrum bandwidth. This eliminates the improvement resulting from the increase in processing gain over a fixed rate system, caused by the bandwidth expansion required for the adaptive rate system. Only the improvement resulting from the rate adaptation is of interest. Furthermore, the limiting case of allowing an infinite instantaneous spread spectrum bandwidth is examined. This allows the determination of the maximum possible improvement of an adaptive rate system over a fixed rate system. The implications of using a finite bandwidth expansion are considered in section 3.

If the instantaneous spread spectrum bandwidth is $W(\gamma)$, then the instantaneous total jammer power is $J(\gamma) = N_N W(\gamma)$. The average total jammer power is $\overline{J} = N_J \overline{W}$, where $\overline{W} = NM\overline{R}$ with N being the number of subbands and $\overline{R}$ the average symbol rate.

2.1 Performance of Adaptive Rate Control in Noise Jamming

With partial band noise jamming, the MFSK symbol error probability conditioned on P_H, γ and the symbol rate $r = R_H$, is

$$P_{M \mid P_H, \gamma, r} = P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1} \binom{M-1}{n}}{n+1} e^{-\frac{P_H n S \gamma}{(n+1) N_J r}} \quad . \tag{2.1}$$

If the symbol rate is adjusted as a function of the channel fade state, then

$$P_{M \mid P_H, \gamma, r(\gamma)} = P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1} \binom{M-1}{n}}{n+1} e^{-\frac{P_H n S \gamma}{(n+1) N_J r(\gamma)}} \quad . \tag{2.2}$$

Both the receiver and transmitter must know the current symbol rate exactly. As in [6], $\gamma(t)$ is assumed to be an ergodic process, and therefore in a long time interval I, $\gamma(t)$ will be between γ and $\gamma + d\gamma$ for $I\, p(\gamma)\, d\gamma$ seconds. The average symbol rate is

$$\overline{R} = \frac{1}{I} \int_0^\infty r(\gamma)\, I\, p(\gamma)\, d\gamma = \int_0^\infty r(\gamma)\, p(\gamma)\, d\gamma \quad . \tag{2.3}$$

For a specified average symbol rate, the average symbol error probability is calculated by summing the error probability for each symbol in a very long string and dividing by the total length of the string. The contribution to the sum of the symbol error probabilities when the channel fade state is between γ and $\gamma + d\gamma$ is $I\, p(\gamma)\, r(\gamma)\, P_{M \mid P_H, \gamma, r(\gamma)}\, d\gamma$. This assumes that the feedback channel is delayless and the symbol rates are changed instantaneously. The symbol error probability averaged over the distribution of channel fade states is just

$$P_{M \mid P_H, r(\gamma)} = \frac{1}{\overline{R}} \int_0^\infty r(\gamma)\, P_{M \mid P_H, \gamma, r(\gamma)}\, p(\gamma)\, d\gamma \quad . \tag{2.4}$$

For convenience, define $\hat{r}(\gamma) := r(\gamma)/\overline{R}$ and $x := S\gamma / N_J \overline{R}$. If L^{th} order selective diversity is used, then

$$p(x) = \frac{1}{\bar{\lambda}_s} L \left(1 - e^{-\frac{x}{\bar{\lambda}_s}}\right)^{L-1} e^{-\frac{x}{\bar{\lambda}_s}} , \tag{2.5}$$

where

$$\bar{\lambda}_s := \frac{SE[\gamma]}{N_J \bar{R}} \tag{2.6}$$

is the average equivalent received symbol energy-to-noise ratio. The instantaneous symbol error probability from (2.2) is

$$P_{M \mid P_H, x, \hat{r}(x)} = P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1} \binom{M-1}{n}}{n+1} e^{-\frac{P_H n x}{(n+1)\hat{r}(x)}} . \tag{2.7}$$

The task is to find the normalized rate function $\hat{r}(x)$ which minimizes the average error probability

$$P_{M \mid P_H, \hat{r}(x)} = \int_0^\infty \hat{r}(x) \, P_{E \mid P_H, x, \hat{r}(x)} \, p(x) \, dx \tag{2.8}$$

subject to the constraints

$$E_x[\hat{r}(x)] = \int_0^\infty \hat{r}(x) \, p(x) \, dx = 1 \tag{2.9}$$

and

$$\hat{r}(x) \geq 0 \ . \tag{2.10}$$

This is a simple variational problem. To minimize $P_{M \mid P_H, \hat{r}(x)}$

$$H = P_{M \mid P_H, \hat{r}(x)} + \mu E_x[\hat{r}(x)]$$

$$= \int_0^\infty \left(\hat{r}(x) \, P_{M \mid P_H, x, \hat{r}(x)} + \mu \hat{r}(x)\right) p(x) dx \tag{2.11}$$

is minimized where μ is a Lagrange multiplier. Clearly, to minimize H $G(\hat{r}(x))$ is minimized for each normalized channel fade state x where

$$G(\hat{r}(x)) = \hat{r}(x)\, P_{M|P_H,\, x,\, \hat{r}(x)} + \mu\hat{r}(x)$$

$$= \hat{r}(x)\, P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} e^{-\frac{P_H n x}{(n+1)\hat{r}(x)}} + \mu\hat{r}(x) \quad (2.12)$$

Using differential calculus results in

$$\frac{dG(\hat{r}(x))}{d\hat{r}(x)} = P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} \left(1 + \frac{P_H x}{\hat{r}(x)} \frac{n}{n} + 1\right) e^{-\frac{P_H n x}{(n+1)\hat{r}(x)}} = -\mu \quad (2.13)$$

By noting that the right side of the above equation is only a function of $x/\hat{r}(x)$, the solution is of the form $\hat{r}(x) = kx$. By using the average value constraint (2.9), it can be verified that

$$k = \frac{1}{\overline{\lambda}_s \theta(L)} \quad (2.14)$$

and

$$\hat{r}(x) = kx = \frac{x}{\overline{\lambda}_s \theta(L)} \quad (2.15)$$

where

$$\theta(L) = \sum_{n=1}^{L} \frac{(-1)^{n+1}\binom{L}{n}}{n} \,. \quad (2.16)$$

The second derivative of $G(\hat{r}(x))$ is positive for all $\hat{r}(x)$, so that $\hat{r}(x) = kx$ is indeed the solution. An important property of adaptive rate control is that the optimal normalized rate function $\hat{r}(x)$ is independent of the distribution of modified channel fade states $p(x)$.

The average symbol error probability conditioned on the jamming fraction, obtained by substituting (2.7) and (2.15) into (2.8), is

$$P_{M|P_H} = P_H \sum_{n=1}^{M-1} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} e^{-\frac{P_H n \bar{\lambda}_s \theta(L)}{n} + 1} \quad . \tag{2.17}$$

The worst case jammer will choose the jamming fraction that maximizes P_M. This problem has bee well studied in the literature. For BFSK, it is easy to see from (2.17) that

$$P_H^* = \frac{2}{\bar{\lambda}_b \, \theta(L)} \tag{2.18}$$

resulting in

$$P_b = \frac{e^{-1}}{\bar{\lambda}_b \, \theta(L)} \quad . \tag{2.19}$$

It is clear from (2.17) that the improvement resulting from selective diversity is $\theta(L)$. The bit error probability using selective diversity signalling with a constant transmitted signal power is given by [17]

$$P_b = \max_a P_b(a) \;=\; \max_a \frac{a 2^L L!}{\bar{\lambda}_b \log_2 M} \sum_{n=1}^{M-1} \frac{(-1)^{n+1}\binom{M-1}{n}}{n+1} \prod_{i=0}^{L-1} \frac{1}{\frac{2na}{n+1} + 2(i+1)} \tag{2.20}$$

where $a = P_H \bar{\lambda}_b \log_2 M$. This expression can be used to determine the improvement resulting from adaptive rate control with selective diversity, over a nonadaptive system using selective diversity.

To summarize, Table 2.1 presents the bit error probability P_b, optimal fraction of bandwidth jammed P_H^*, and the improvement resulting from selective diversity $\theta(L)$, for BFSK. Also included is the improvement over selective diversity signaling with a constant signaling rate. For $L = 1$, the adaptive rate system results in a lower bit error probability than the fixed rate system.

As a final comment, note that the optimal rate function and optimal jammer fraction constitute a saddle point solution. This is evident from (2.13), where the optimal rate function does not depend on the jamming fraction.

Table 2.1 Performance of Adaptive Rate Control Against Partial Band Noise Jamming

L	Adaptive Rate Control $P_b\bar{\lambda}_b$	$P_H^*\bar{\lambda}_b$	$\theta(L)$ (dB)	Improvement over Selective Diversity (constant rate) (dB)
1	.36788	2.00000	0.00000	—
2	.24525	1.13578	1.76091	1.45685
3	.200066	.75976	2.63241	.97213
4	.17658	.62743	3.18759	.75278
5	.16112	.55777	3.58569	.64692

2.2 Performance of Adaptive Rate Control in Tone Jamming

With partial band tone jamming, the symbol error probability conditioned on P_H, γ and $r(\gamma) = R_H(\gamma)$ is

$$P_{M\,|\,P_H,\,\gamma,\,r(\gamma)} = \begin{cases} \dfrac{M-1}{M} P_H & , \quad \gamma S < \hat{S} \\ 0 & , \quad \gamma S > \hat{S} \end{cases} . \tag{2.21}$$

Defining

$$\lambda_s^t(\gamma) := \frac{E_H(\gamma)}{N_J} = \frac{S(\gamma)/R_H}{J/W_{ss}}$$

and

$$P_H = \frac{J/\hat{S}}{W_{ss}/MR_H}$$

where $S(\gamma)$ is the instantaneous transmitted signal power and $\hat{S}$ is the jammer tone power, and letting $R_H = r(\gamma)$, (2.21) becomes

$$P_{M\,|\,P_H,\,\gamma,\,r(\gamma)} = \begin{cases} \dfrac{M-1}{M} P_H & , \quad \gamma < \dfrac{MN_J}{(S/r(\gamma))P_H} \\ 0 & , \quad \gamma > \dfrac{MN_J}{(S/r(\gamma))P_H} \end{cases} \quad . \tag{2.22}$$

If $\hat{r}(\gamma) := r(\gamma)/\overline{R}$ and $x := S\gamma/N_J\overline{R}$, then the density of x is given by (2.5) and (2.6). With these definitions, the conditional error probability from (2.22) is

$$P_{M\,|\,P_H,\,x,\,\hat{r}(x)} = \frac{M-1}{M} P_H u\left(-x + \frac{M\hat{r}(x)}{P_H}\right) \quad . \tag{2.23}$$

Once again $P_{M\,|\,P_H,\,\hat{r}(x)}$ is minimized subject to the constraints (2.9) and (2.10). That is, H is minimized in (2.11) where $P_{M\,|\,P_H,\,x,\,\hat{r}(x)}$ is given in (2.23). To minimize H, $G(\hat{r}(x))$ is minimized for each modified channel state x where

$$\begin{aligned} G(\hat{r}(x)) &= \hat{r}(x)\, P_{M\,|\,P_H,\,x,\,\hat{r}(x)} + \mu\hat{r}(x) \\ &= \hat{r}(x)\frac{M-1}{M} P_H u\left(-x + \frac{M\hat{r}(x)}{P_H}\right) + \mu\hat{r}(x) \quad . \end{aligned} \tag{2.24}$$

It is clear that making $\hat{r}(x)$ as small as possible minimizes $G(\hat{r}(x))$. That is, $\hat{r}(x) = k$ and the average value constraint in (2.9) requires that $k = 1$. Once again this is a saddle point solution, because the optimal rate function does not depend on the jamming fraction. Using the density defined by (2.5) and (2.6) results in

$$P_{M\,|\,P_H} = \frac{M-1}{M} P_H \left(1 - e^{-\frac{M}{\overline{\lambda}_b P_H \log_2 M}}\right)^L \quad . \tag{2.25}$$

This is exactly the same error probability as L^{th} order selective diversity [17]. Therefore, adaptive rate control gives no additional improvement in performance against an unfaded tone jammer.

2.3 Coded Performance

The previous subsections have shown that the uncoded performance of adaptive feedback signalling techniques is very poor. However, it will be shown that significant improvements can be obtained by using coding in conjunction with these techniques. The main assumption made in evaluating the coded performance is that the coding channel is memoryless from hop to hop.

First consider a coded adaptive rate control system. Observe from section 2.1 and 2.2 that the optimal transmitted symbol rate depends on the type of jamming present (tone or noise). This can present some difficulties when implementing a practical system. To alleviate these difficulties, suppose that the transmitter uses the same normalized rate function for both tone and noise jamming as described by (2.15).

With binary signalling the cutoff rate is

$$R_o = 1 - \log_2(1 + D) \ . \tag{2.26}$$

For soft decision decoding using a weighted linear sum metric with jammer state information, the parameter D as a function of the normalized channel fade state is given by [2].

$$D(x) = \frac{P_H}{2}\, u\left(-x + \frac{2\hat{r}(x)}{P_H}\right) \tag{2.27}$$

when tone jamming is present. Using the rate function in (2.15) results in

$$D = E_x[D(x)] = \frac{P_H}{2}\, u\left(-1 + \frac{2}{P_H r \overline{\lambda}_b \theta(L)}\right) , \tag{2.28}$$

where r is the code rate. The worst case jamming fraction that minimizes R_o, or maximizes D, is

$$P_H^* = \frac{2}{r\overline{\lambda}_b\theta(L)} < 1 \ , \tag{2.29}$$

leading to $D = 1/r\overline{\lambda}_b\theta(L)$ and

$$R_o = 1 - \log_2\left(1 + \frac{1}{r\overline{\lambda}_b\theta(L)}\right) \ , \quad r\overline{\lambda}_b \geq \frac{2}{\theta(L)} \ . \tag{2.30}$$

When $r = R_o$, (2.30) can be rearranged in the form

$$\overline{\lambda}_b = \frac{1}{R_o(2^{1-R_o} - 1)\theta(L)} \ , \quad R_o\overline{\lambda}_b \geq \frac{2}{\theta(L)} \ . \tag{2.31}$$

If $R_o\overline{\lambda}_b < 2/\theta(L)$, then $P_H^* = 1$ and $R_o^* = 1 - \log_2(1 + 1/2)$. At this cutoff rate $\overline{\lambda}_b = 2/R_o^*\theta(L)$.

With partial band noise jamming,

$$D = \max_{P_H} \min_{\lambda} \int_0^\infty \frac{P_H}{1-\lambda^2} e^{-\frac{\lambda P_H}{1+\lambda} P_H \frac{x}{\hat{r}(x)}} p(x)\, dx \tag{2.32}$$

Using (2.32) and (2.26) along with (2.15), (2.5) and (2.6) yields

$$\overline{\lambda}_b = \frac{4e^{-1}}{R_o(2^{1-R_o} - 1)\theta(L)} \ , \quad \overline{\lambda}_b \geq \frac{3}{R_o\theta(L)} \ . \tag{2.33}$$

If the condition on $\overline{\lambda}_b$ in (2.33) is not satisfied, then $\overline{\lambda}_b$ is obtained by solving

$$R_o = 1 - \log_2\left(1 + \min_{\lambda} \frac{1}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda} R_o\overline{\lambda}_b\theta(L)}\right) \ . \tag{2.34}$$

The $\overline{\lambda}_b$ required for coded communication at $r = R_o$ using this adaptive feedback rate control technique is shown in Figures 2.1 and 2.2 for partial band noise and partial band tone jamming respectively. Included in these diagrams is the performance of a system using only selective diversity [17]. Note the improvement resulting from the rate adaptation. Also the performance is much worse for partial band noise jamming than for partial band tone jamming. Therefore for binary signaling, the performance of the proposed adaptive rate control system will be limited by noise jamming.

As a final comment, the rate control technique proposed here results in a coding channel where the received signal energy is a constant. This makes the selection of appropriate code structures and the generation of jammer state information relatively easy, because the coding channel is reduced to a time-invariant partial band jammed channel.

2.4 Summary

The adaptive rate control system described in the section is attractive in that it makes aa otherwise time-varying coding channel to appear time-invariant. This aspect simplifies the problem of generating jammer state information, as it allows us to use the techniques already available for estimating the jammer state for stationary partial band jammed channels. It also allows us to make use of coding option evaluations for these channels such as those in [2] and [9].

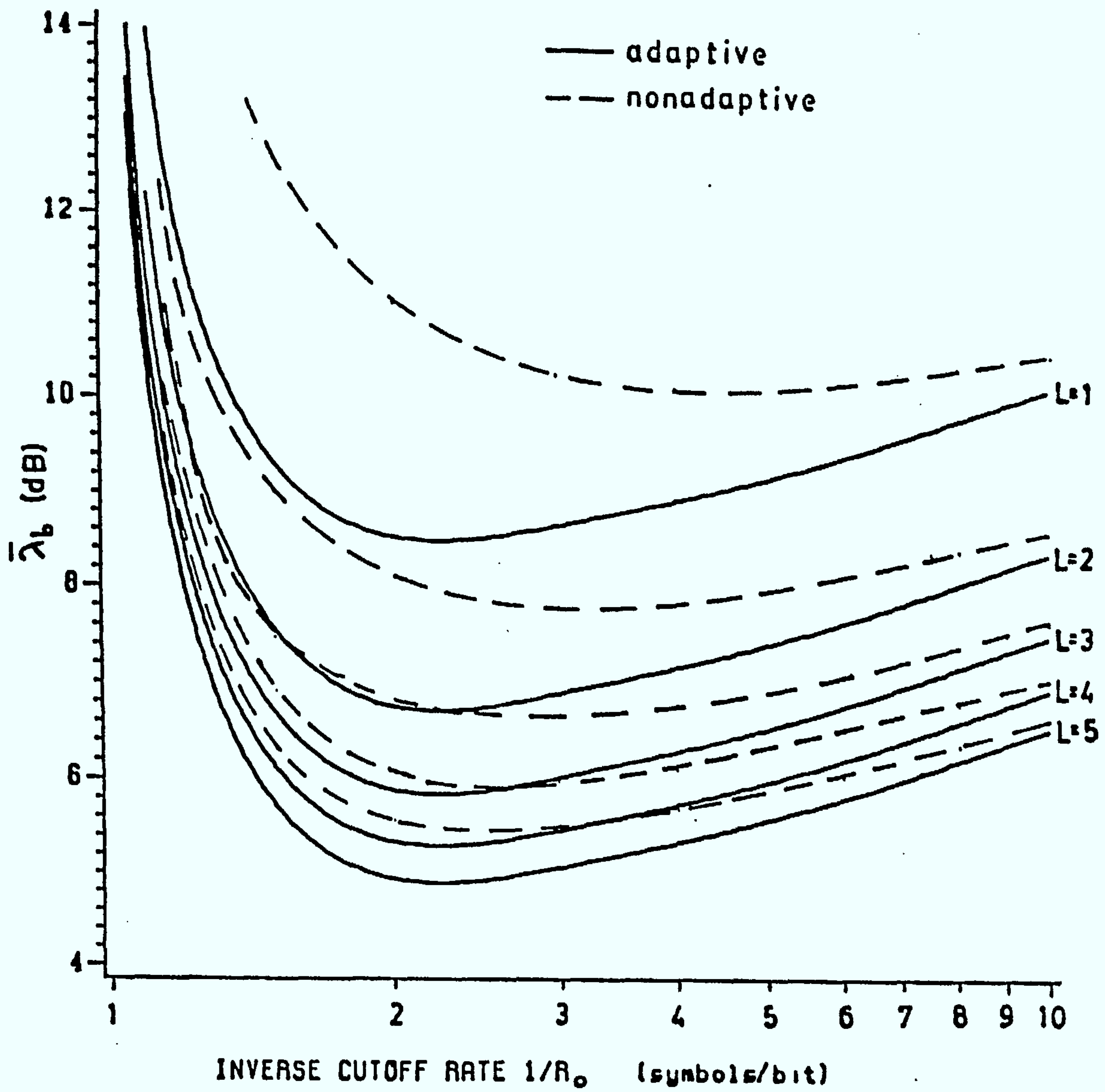

Figure 2.1 Performance of Coded FH/BFSK with Adaptive Rate Control and Selective Diversity for Soft Decision Decoding with Jammer State Information Against an Unfaded Partial Band Noise Jammer

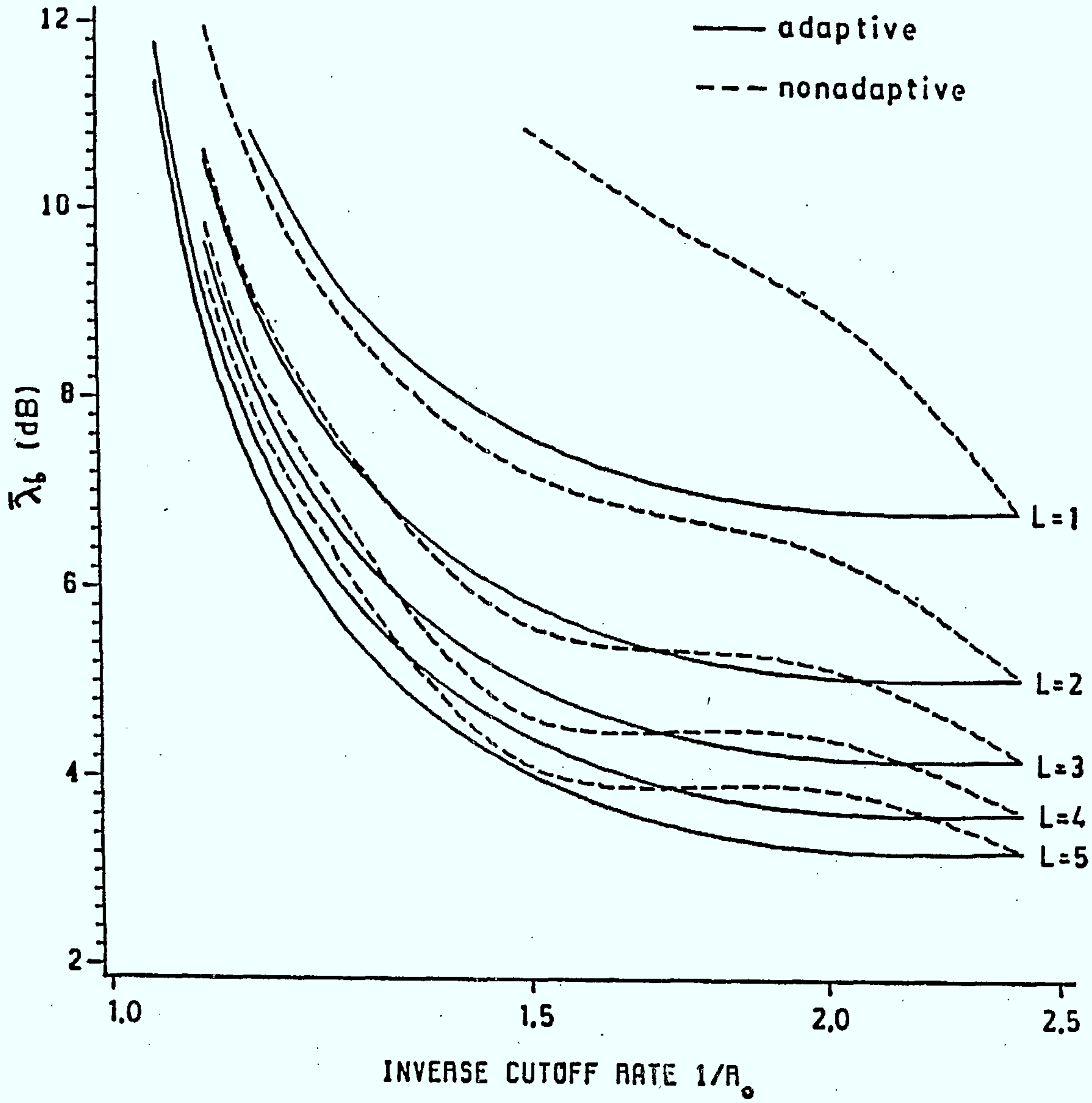

Figure 2.2 Performance of Coded FH/BFSK with Adaptive Rate Control and Selective Diversity for Soft Decision Decoding with Jammer State Information Against an Unfaded Partial-Band Tone Jammer

3. Channel State Estimation

3.1 Introduction

The objective of channel state estimation is to render the time-varying coding channel time-invariant and memoryless. This can be achieved by using the channel state information to control the coding rate which, in turn, greatly simplifies the generation of channel state information, since it allows the decoupling of the fade state and jammer state estimation aspects of the communication system.

The proposed system generates a linear predictive (LP) estimate of the channel fade state, and uses a thresholding technique to mitigate the effect of partial-band interference. Generating an LP estimate of the fade state is equivalent to obtaining a maximum entropy Doppler power spectral estimate. The receiver is assumed to have no prior Doppler power spectral information. If prior spectral information were available, then other techniques, such as minimum cross entropy spectral estimation, could be used to estimate the Doppler power spectrum. This would reduce the required order of the channel fade state predictor.

The proposed interference rejection technique uses errors and erasures decoding. Erasures are generated by using a linear threshold that generalizes the difference and ratio thresholds proposed by Berlekamp [11] and Viterbi [10] respectively.

Several factors will cause the performance of the proposed adaptive rate control system to deviate from the ideal performance with perfect jammer state information. These stem from the use of imperfect estimates of the channel fade state, which in turn are caused by the inherent randomness of the fading process that is being tracked, the finite predictor order, the estimation error resulting from partial band interference, the finite rate change period and the feedback channel delay. A further source of imperfection results from "noisy" estimates of the jammer state. As a result, the behaviour of the system is very complex and best evaluated by simulation studies.

Figure 3.1 illustrates the system under consideration. The receiver function will become clearer with the ensuing analysis. One aspect that should be clarified now is the waveform channel model. The spread spectrum bandwidth W_{ss} is divided into MFSK subbands of width $M\overline{R}B$, where $\overline{R}$ is the average transmitted code symbol rate and B is the bandwidth expansion defined by the ratio $B=R_{max}/\overline{R}$ with R_{max} being the maximum transmitted code symbol rate. Since orthogonal MFSK signalling is used, the separation of the subchannels in a subband must be adjusted along with the transmitted code symbol rate to maintain orthogonality.

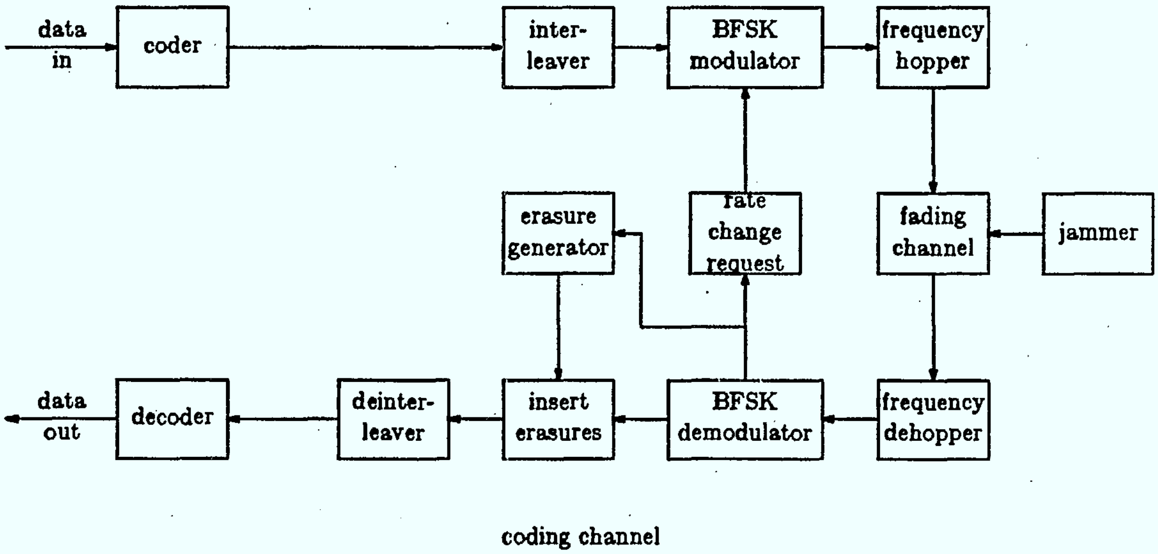

Figure 3.1 Coded FH/MFSK System with Adaptive Feedback Rate Control

During each signalling interval, the MFSK modulator uses only a fraction of a subband. A key assumption is that the exact portion of the subband used by the MFSK modulator is unknown to the jammer. In the analysis to follow, only partial-band noise jamming is considered. The jammer would probably prefer to use noise jamming over tone jamming because of the large degree of uncertainly the jammer would have in the frequency positions used by the transmitter resulting from the rate adaptation.

With our model the jammer always jams an entire subband. Consequently, the effectiveness of the jammer is expected to be reduced by a factor of B compared to a fixed rate system operating with rate $\overline{R}$. However, to allow a fair comparison, it is assumed that both systems are operating at the same average signalling rate, and therefore the fixed rate system can increase its bandwidth by a factor of B by increasing the number of subbands in the spread spectrum bandwidth. This also has the effect of reducing the effectiveness of the jammer by a factor of B.

Finally, the fading is assumed to be uniform and the partial-band interference is much stronger than the background noise. Therefore, ambient noise is neglected in the analysis.

3.2 Receiver Formulation

For convenience, we consider BFSK signalling, although the method described in this section can be extended to the MFSK case, for M > 2. The receiver formulation consists mainly of defining the memory contents required for the m^{th} order LP estimate. The structure of the proposed channel state estimation and BFSK demodulation is shown in Figure 3.2. A vector characterization is first obtained for the received signal $r(t)$ during each signalling interval by letting $\{\theta_j(t)\}_{j=0}^{\infty}$ be a complete complex orthonormal basis for $0 \leq t < T$. The received signal for the k^{th}

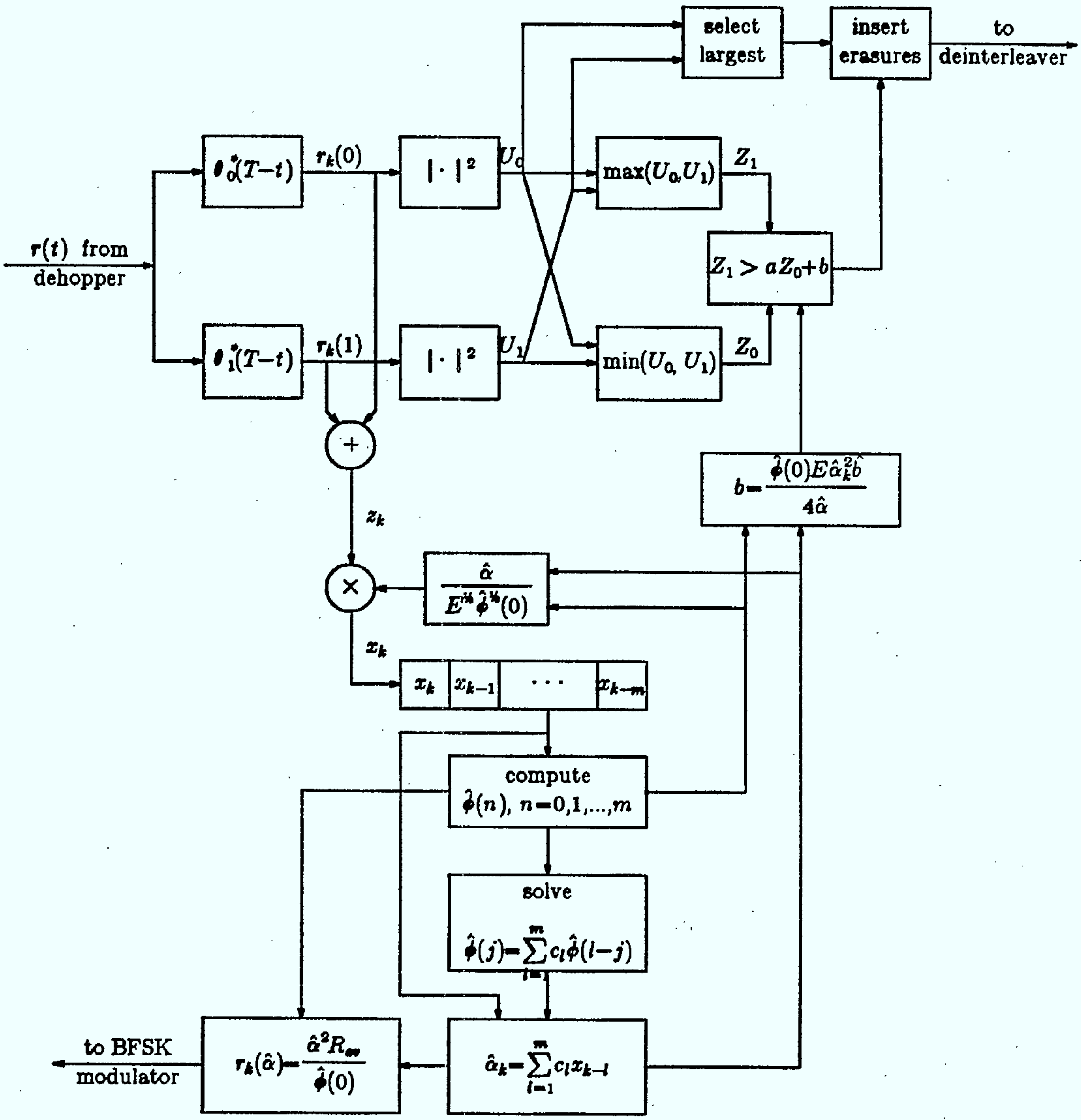

Figure 3.2 Channel State Estimation and BFSK Demodulation

interval is completely described by the vector $\mathbf{r}_k$ with complex components $r_k(j)$, $j = 0, 1, 2, \ldots$, defined by

$$r_k(j) = \int_{kT}^{(k+1)T} r(t)\theta_j^*(t)dt \quad . \tag{3.1}$$

Suppose that $\theta_0(t) = f_0(t)$ and $\theta_1(t) = f_1(t)$, so that on hypothesis $H(i)$, $i = 0,1$,

$$\begin{aligned} r_k(0) &= (1-i)E_k^{1/2}\alpha_k + n_k(0) \\ r_k(1) &= iE_k^{1/2}\alpha_k + n_k(1) \\ r_k(j) &= n_k(j)\ , \quad j = 2, 3,.. \end{aligned} \tag{3.2}$$

where $\{n_k(j)\}_{j=0}^{\infty}$ is a set of i.i.d. zero-mean complex Gaussian random variables having $E[|n_k(j)|^2] = 0$ with probability $1-\rho$, and $E[|n_k(j)|^2] = N_J/\rho$ with probability ρ. Note that at time $t = kT$, all the information concerning the fading gain α_k in the past reception $r(t)$, $(k-m)T \leq t < kT$, is summarized by the conditional density $p(\alpha_k \,|\, r_l(0), r_l(1)\ ,\ k-m \leq l < k-1)$ since α_k is independent of $n_k(j)$ for all k and j. Therefore the receiver only needs to retain the finite set of sufficient statistics $\{r_l(0), r_l(1)\}_{l=k-m}^{k-1}$.

It is assumed that the receiver does not employ decision feedback and therefore is unable to decide whether the signal term $E_k^{1/2}\alpha_k$ is present in $r_k(0)$ or $r_k(1)$. The use of decision feedback may not be successful for interference channels because of the relatively low signal-to-noise ratios. The receiver forms the variable

$$z_k := r_k(0) + r_k(1) \tag{3.3}$$

so that regardless of the hypothesis,

$$z_k = E_k^{1/2}\alpha_k + \hat{n}_k \quad , \tag{3.4}$$

where $\hat{n}_k$ has twice the variance of $n_k(j)$. At time $t = kT$, the information set at the receiver is defined as

$$z^k := \{z_l = r_l(0) + r_l(1)\ ,\ k - m \leq l < k - 1\}\ . \tag{3.5}$$

3.3 Estimation of the Channel Covariance Function

Suppose that $r(x_k)$ represents the signalling rate for the k^{th} signalling interval, where $x_k := S\alpha_k^2/N_J\overline{R}$ is the normalized channel fade state and S is the average transmitted code symbol power. The theoretical analysis of adaptive rate control [17] shows that the optimal normalized rate function $\hat{r}(x_k) = r(x_k)/\overline{R}$, assuming perfect knowledge of the channel fade state and a delayless feedback channel, is of the form

$$\hat{r}_k(x_k) = \begin{cases} \dfrac{x_k}{\overline{\gamma}_s} & ,\ x_k < x^* \\[2ex] \dfrac{x^*}{\overline{\gamma}_s} & ,\ x_k > x^* \end{cases} \tag{3.6}$$

where x^* depends upon the bandwidth expansion B, and $\overline{\gamma}_s$ is defined as the average equivalent received code symbol energy-to-noise ratio, obtained with nonadaptive transmission. However, $\overline{\gamma}_s = S\hat{E}[\alpha_k^2]/N_J\overline{R}$ where $\hat{E}[\alpha_k^2] = E[\alpha_k^2]/2$ is the average energy in the bandpass signal. Therefore the signalling rate for the k^{th} interval is

$$\begin{aligned} r_k(\alpha_k) &= \hat{r}_k(\alpha_k)\overline{R} \\ &= \frac{\alpha_k^2}{E[\alpha_k^2]}\overline{R} \end{aligned} \tag{3.7}$$

However, $E[\alpha_k^2] = \phi(0)$ where $\phi(n) = E[\alpha_k\alpha_{k+n}]$ is the channel covariance function. Without prior information concerning the channel covariance function $\phi(n)$, the receiver must generate an estimate of the mean square value denoted as $\hat{\phi}(0)$. In addition, the receiver must generate an estimate of α_k, denoted as $\hat{\alpha}_k$. Hence during the k^{th} interval, the signalling rate is

$$r_k(\hat{\alpha}_k) = \begin{cases} \dfrac{\hat{\alpha}_k^2}{\hat{\phi}(0)}\overline{R} & , \ \hat{\alpha}_k \le \alpha^* \\ \dfrac{\alpha^{*2}}{\hat{\phi}(0)}\overline{R} & , \ \hat{\alpha}_k > \alpha^* \end{cases} . \tag{3.8}$$

Since B is the bandwidth expansion, $\alpha^* = (B\hat{\phi}(0))^{1/2}$. Therefore if E_s is the average transmitted code symbol energy, then $E_k^{1/2} = (E_s\overline{R}/r_k(\hat{\alpha}_k))^{1/2}$ and the received statistic z_k in (3.4) is

$$z_k = \begin{cases} \dfrac{\hat{\phi}^{1/2}(0)E_s^{1/2}}{\hat{\alpha}_k}\alpha_k + \hat{n}_k & , \ \hat{\alpha}_k \le (B\hat{\phi}(0))^{1/2} \\ \dfrac{E_s^{1/2}}{B^{1/2}}\alpha_k + \hat{n}_k & , \ \hat{\alpha}_k > (B\hat{\phi}(0))^{1/2} \end{cases} . \tag{3.9}$$

To derive an estimate of the channel covariance function $\phi(n)$, the receiver first normalizes the z_k. To do so, the receiver forms the new variables

$$x_k = \begin{cases} \dfrac{\hat{\alpha}_k}{E_s^{1/2}\hat{\phi}^{1/2}(0)}z_k & , \ \hat{\alpha}_k \le (B\hat{\phi}(0))^{1/2} \\ \dfrac{B^{1/2}}{E_s^{1/2}}z_k & , \ \hat{\alpha}_k > (B\hat{\phi}(0))^{1/2} \end{cases} . \tag{3.10}$$

The normalized information set at the receiver at time $t = kT$ is

$$x^k := \{x_l \ , \ k-m \le l < k-1\} \ . \tag{3.11}$$

The receiver estimates the channel covariance function by forming the sum

$$\hat{\phi}(n) = \frac{1}{N}\sum_{i=1}^{N-n} x_i x_{i+n} \quad , \qquad n = 0, 1, ..., m$$

$$= \frac{1}{N}\sum_{i=1}^{N-n}\alpha_i\alpha_{i+n} + \frac{1}{NE_s^{1/2}\hat{\phi}^{1/2}(0)}\sum_{i=1}^{N-n}\alpha_i\hat{\alpha}_{i+n}\hat{n}_{i+n} \tag{3.12}$$

$$+ \frac{1}{NE_s^{1/2}\hat{\phi}^{1/2}(0)}\sum_{i=1}^{N-n}\alpha_{i+n}\hat{\alpha}_i\hat{n}_i + \frac{1}{NE_s\hat{\phi}(0)}\sum_{i=1}^{N-n}\hat{\alpha}_i\hat{\alpha}_{i+n}\hat{n}_i\hat{n}_{i+n} \; .$$

Observe that as $N \to \infty$

$$\hat{\phi}(n) = \phi(n) + \lim_{N\to\infty}\frac{1}{NE_s\hat{\phi}(0)}\sum_{i=1}^{N-n}\hat{\alpha}_i\hat{\alpha}_{i+n}\hat{n}_i\hat{n}_{i+n} \tag{3.13}$$

because α_k and $\hat{n}_i$ are independent for all k,i. Recall that the noise is uncorrelated, and therefore for $n \neq 0$, (2.13) results in the asymptotically unbiased estimate

$$\hat{\phi}(n) = \phi(n) \; . \tag{3.14}$$

Unfortunately (3.12) will result in a biased estimate of $\phi(0)$. To determine the value of $\hat{\phi}(0)$, it is noted that with the bandwidth expansion constraint B, the signalling rate is actually adjusted by using a clipped version of α_k, that is

$$\hat{\alpha}_k^2 = \begin{cases} \hat{\alpha}_k^2 & , \; \hat{\alpha}_k^2 < B\hat{\phi}(0) \\ B\hat{\phi}(0) & , \; \hat{\alpha}_k^2 > B\hat{\phi}(0) \end{cases} . \tag{3.15}$$

If the channel gain α_k is Rayleigh distributed, the approximation is made that $\hat{\alpha}_k$, before clipping, is also Rayleigh distributed. That is

$$p(\hat{\alpha}_k) = \frac{2\hat{\alpha}_k}{\hat{\phi}(0)} e^{-\hat{\alpha}_k^2/\hat{\phi}(0)} \; , \tag{3.16}$$

so that after clipping

$$E[\hat{\alpha}_k^2] = B\hat{\phi}(0)e^{-B} + \hat{\phi}(0)\Big(1 - e^{-B}(B+1)\Big)$$
$$= \hat{\phi}(0)(1 - e^{-B}) \ . \tag{3.17}$$

From (3.13)

$$\hat{\phi}(0) = \phi(0) + \lim_{N\to\infty} \frac{1}{NE_s\hat{\phi}(0)} \sum_{i=1}^{N} \hat{\alpha}_i^2 \hat{n}_i^2$$
$$= \phi(0) + \frac{1}{E_s\hat{\phi}(0)} E[\hat{\alpha}_k^2]E[\hat{n}_k^2] \ . \tag{3.18}$$

Using (3.17) along with $E[\hat{n}_k^2] = 2N_J$ results in

$$\hat{\phi}(0) = \phi(0) + \frac{2N_J E[\hat{\alpha}_k^2]}{\hat{\phi}(0)E_s}$$
$$= \phi(0) + \frac{2N_J}{E_s}(1 - e^{-B}) \ . \tag{3.19}$$

Therefore by using knowledge of the average transmitted signal energy E_s and the jammer noise spectral density N_J, an improved estimate of $\phi(0)$ can be obtained by subtracting the bias term or at least an estimate of the bias term from $\hat{\phi}(0)$.

3.4 Estimation of the Channel Fade State

The receiver generates an LP estimate of the channel fade state. Specifically, the fade level estimate for the k^{th} signalling interval is given by the linear combination

$$\hat{\alpha}_k = \sum_{l=1}^{m} c_l x_{k-l} \tag{3.20}$$

where the c_l are the predictor coefficients and the x_k are given by (3.10). To determine the predictor coefficients c_l, the mean square prediction error ξ_r is minimized where

$$\xi_r = E[\tilde{\alpha}]$$
$$= E[(\alpha_k - \hat{\alpha}_k)^2] \tag{3.21}$$

and $\hat{\alpha}_k$ is given by (3.20). Using simple differential calculus leads to the following set of linear equations for the predictor coefficients

$$\phi(j) - \sum_{l=1}^{m} c_l \phi(l-j) - \sum_{l=1}^{m} c_l \frac{E[\hat{\alpha}_{k-l}\hat{\alpha}_{k-j}]E[\hat{n}_{k-l}\hat{n}_{k-j}]}{E_s\hat{\phi}(0)} = 0 \ , \tag{3.22}$$

$$j = 1, 2, , \ldots, m \ .$$

However, instead of using the actual values of $\phi(l-j)$, the estimates $\hat{\phi}(l-j)$ are used. Therefore, if (3.13) is used to substitute for $\phi(l-j)$ in (3.22), the following set of linear equations is solved to obtain the predictor coefficients c_l:

$$\hat{\phi}(j) = \sum_{l=1}^{m} c_l \hat{\phi}(l-j) \ , \quad j = 1, 2, ..., m \ . \tag{3.23}$$

An estimate of the fade level for the $(k+j)^{th}$ interval can be calculated recursively by using

$$\hat{\alpha}_{k+j} = \begin{cases} \sum_{l=1}^{j} c_l \hat{\alpha}_{k+j-l} + \sum_{l=j+1}^{m} \xi_l x_{k+j-l} & , \ j+1 \le m \\ \sum_{l=1}^{j} c_l \hat{\alpha}_{k+j-l} & , \ j+1 > m \end{cases} . \tag{3.24}$$

3.5 Generation of Side Information

The performance of coded antijam communication systems is tremendously improved by supplying the decoder with side information. In this section, the proposed method of generating side information is examined. Basically, the receiver uses errors and erasures decoding to mitigate the effects of partial band interference. The task is to determine an effective method of generating erasures that will improve the coded performance in the presence of partial band interference. A linear thresholding technique is proposed for generating erasures. The receiver tests the following condition:

$$Z_1 \geq aZ_0 + b \tag{3.25}$$

where $Z_1 = \max(U_0, U_1)$, $Z_0 = \min(U_0, U_1)$, and the decision variables, $U_i = |r_k(i)|^2$, $i = 0, 1$, are at the output of the noncoherent detector. If the condition in (3.25) is not satisfied, then an erasure is generated. Otherwise a hard decision is made on the received code symbol.

The linear threshold is a generalization of Viterbi's ratio threshold [10] where $b = 0$, and Berlekamp's difference threshold [11] where $a = 1$. We now proceed to evaluate the performance of the linear threshold for a stationary partial band noise jammed channel. The rationale for this procedure is that adaptive rate control tends to neutralize the effects of fading.

The decision variables U_0 and U_1 at the detector output for hypothesis $H(0)$ are

$$U_0 = |\alpha_k E_k^{1/2} + n_0|^2 \tag{3.26}$$

$$U_1 = |n_1|^2$$

The densities of the decision variables are

$$p(u_0) = \frac{\rho}{2N_J} e^{-\frac{u_0 + \alpha_k^2 E_k}{2N_J/\rho}} I_o\left(\frac{\sqrt{u_0 \rho}\,\alpha_k E_k^{1/2}}{N_J}\right)$$

$$p(u_1) = \frac{\rho}{2N_J} e^{-u_1\rho/2N_J} \; . \tag{3.27}$$

The probability of a code symbol being in error is

$$P_e = \rho P(U_1 > aU_0 + b) \tag{3.28}$$

$$= \rho \int_R P(U_1 > aU_0 + b \mid U_0 = u_0) p(u_0) du_0 \ .$$

It can be shown that

$$P(U_1 > aU_0 + b \mid U_0 = u_0) = \begin{cases} 1 & , \ b \leq -au_0 \\ e^{-\rho(au_0 + b)/2N_J} & , \ b > -au_0 \end{cases} \ . \tag{3.29}$$

Therefore,

$$P_e = \rho \int_0^{-b/a} p(u_0) du_0 + \rho \int_{-b/a}^{\infty} e^{-\rho(au_0 + b)/2N_J} p(u_0) du_0 \ . \tag{3.30}$$

Substituting the density $p(u_0)$ into (3.30) and simplifying results in the expression

$$P_e = \rho \int_0^{-\hat{b}/a} f(u, \epsilon_1) du + \frac{\rho}{1 + a} e^{-\rho\gamma_s\left(\frac{\hat{b}}{4} + \frac{a}{1+a}\right)} \int_{-\frac{(1+a)\hat{b}}{a}}^{\infty} f(u, \epsilon_2) du \ , \tag{3.31}$$

where

$$f(u,\epsilon) = \frac{\rho\gamma_s}{4} e^{-\rho\gamma_s \frac{u + 4\epsilon}{4}} I_o(\rho\gamma_s\sqrt{\epsilon u}) \ , \tag{3.32}$$

$\epsilon_1 = 1$, $\epsilon_2 = 1/(1 + a)$, $\hat{b} = 4b/\alpha_k^2 E_k^2$, and $\gamma_s = \alpha_k^2 \hat{E}_k / N_J$ is the code symbol energy-to-noise ratio with $\hat{E}_k = E_k/2$.

The probability of a code symbol being erased is

$$P_{er} = \rho\left(1 - P\left(U_1 < \frac{U_0 - b}{a}\right)\right) - P_e \tag{3.33}$$

$$= \rho\left(1 - \int_R P\left(U_1 < \frac{U_0 - b}{a} \mid U_0 = u_0\right) p(u_0) du_0\right) - P_e \ .$$

But,

$$P\left(U_1 < \frac{U_0 - b}{a} \mid U_0 = u_0\right) = \begin{cases} 0 & , \ u_0 \le b \\ 1 - e^{-\frac{P_H}{2N_J}\left(\frac{u_0 - b}{a}\right)} & , \ u_0 > b \end{cases} . \tag{3.34}$$

Therefore

$$P_{er} = \rho - \rho \int_b^{\infty} p(u_0)du_0 + \rho \int_b^{\infty} e^{-\frac{\rho}{2N_J}\left(\frac{u_0 - b}{a}\right)} p(u_0)du_0 - P_e \ . \tag{3.35}$$

Using the density $p(u_0)$ and the expression for P_e in (3.30) results in

$$P_{er} = \rho - \rho \int_{\hat{b}}^{\infty} f(u, \epsilon_1)du \tag{3.36}$$

$$+ \frac{\rho a}{1 + a} e^{-\frac{\rho \gamma_s}{4}\left(\frac{-\hat{b}}{a} + \frac{4}{1 + a}\right)} \int_{\frac{(1+a)\hat{b}}{a}}^{\infty} f(u, \epsilon_3)du - P_e \ ,$$

where $\epsilon_1 = 1$, $\epsilon_3 = a/(1 + a)$, and $f(u, \epsilon)$ is given by (3.32). The cutoff rate of a binary erasure channel is

$$R_o = 1 - \log_2\left(1 + P_{er} + 2\left(P_e(1 - P_e - P_{er})\right)^{½}\right) \ . \tag{3.37}$$

If (2.37) is rewritten as

$$P_{er} + 2\left(P_e(1 - P_e - P_{er})\right)^{½} = 2^{1 - R_o} - 1 \ , \tag{3.38}$$

and both sides of (3.38) are multiplied by $\gamma_s = R_o \gamma_b$ where γ_b is the bit energy-to-noise ratio, then (3.38) can be rearranged in the form

$$\gamma_b(\mu, a, \hat{b}) = \frac{2\hat{P}_e + \hat{P}_{er}\chi - 2\left(\hat{P}_e\left(\hat{P}_e + \chi\left(\hat{P}_{er} - \chi(\hat{P}_e + \hat{P}_{er})\right)\right)\right)^{½}}{\chi^2} \ , \tag{3.39}$$

where

$$\chi = 2^{1-R_o} - 1$$

$$\hat{P}_e = \mu \int_0^{-\hat{b}/a} f(u, \epsilon_1)du + \frac{\mu}{1+a} e^{-\frac{\mu}{4}\left(-\hat{b} + \frac{4a}{1+a}\right)} \int_{-\frac{(1+a)\hat{b}}{a}}^{\infty} f(u, \epsilon_2)du \quad , \tag{3.40}$$

$$\hat{P}_{er} = \mu - \mu \int_{\hat{b}}^{\infty} f(u, \epsilon_1)du \tag{3.41}$$

$$+ \frac{\mu a}{1+a} e^{-\frac{\mu}{4}\left(\frac{-\hat{b}}{a} + \frac{4}{1+a}\right)} \int_{\frac{(1+a)\hat{b}}{a}}^{\infty} f(u, \epsilon_3)du - \hat{P}_e \ ,$$

with

$$f(u, \epsilon) = \frac{\mu}{4} e^{-\mu\frac{u+4\epsilon}{4}} I_o(\mu\sqrt{\epsilon u}) \tag{3.42}$$

and $\rho = \mu / r\gamma_b$. With worst case jamming and the best threshold parameters, the bit energy-to-noise ratio required for coded operation at $r = R_o$ is

$$\gamma_b = \min_{a,\hat{b}} \max_{0 \le \mu \le R_o \gamma_b} \gamma_b(\mu, a, \hat{b}) \Big|_{r = R_o} \ . \tag{3.43}$$

Optimal threshold parameters have been computed for the linear and ratio threshold mitigation techniques. These are tabulated in Table 3.1 along with the required bit energy-to-noise ratios and worst case jammer parameters. Figure 3.3 is a plot of γ_b against R_o^{-1} for these techniques, where the linear threshold apparently performs the best. Observe from Table 3.1 that the best performance is achieved at $R_o^{-1} \approx 3.00$. Also, at $R_o^{-1} \approx 3.00$, the minimax solution for the ratio threshold mitigation technique occurs at $a = 2.52$ and $\rho = 0.73$, requiring 9.576 dB. This will be important when the adaptive rate control algorithm is considered.

For implementation purposes, it is desirable to keep the threshold parameters constant. In the sequel, the receiver is assumed to use a linear threshold with parameters $a = 1.45$ and $\hat{b} = 1.72$ corresponding to $R_o^{-1} = 3.00$. To study the

Table 3.1 Parameters for Threshold Interference Mitigation Techniques Versus a Partial-Band Noise Jammer

R_o^{-1}	Linear				Ratio		
	a	$\hat{b}$	γ_b(dB)	ρ	a	γ_b(dB)	ρ
1.25	2.00	3.07	13.398	0.18	---	---	---
1.50	1.81	2.69	11.173	0.32	---	---	---
1.75	1.70	2.43	10.223	0.44	---	---	---
2.00	1.62	2.23	9.739	0.54	3.51	10.238	0.46
2.50	1.52	1.93	9.343	0.70	2.86	9.723	0.61
3.00	1.45	1.72	9.272	0.83	2.52	9.576	0.73
3.50	1.40	1.56	9.333	0.93	2.28	9.586	0.84
4.00	1.38	1.48	9.454	1.00	2.12	9.671	0.92
5.00	1.35	1.73	9.711	1.00	2.01	9.926	1.00
6.00	1.34	1.97	9.943	1.00	2.01	10.169	1.00
8.00	1.31	2.40	10.345	1.00	2.01	10.586	1.00
10.00	1.29	2.79	10.680	1.00	2.01	10.937	1.00

effect of using these parameters at other cutoff rates, the worst case jamming fractions and signal-to-noise ratios have been computed and are presented in Table 3.2. Comparison of Tables 3.1 and 3.2 shows that the performance is insensitive to the use of suboptimal threshold parameters, except at higher cutoff rates.

3.6 Rate Control Algorithm

Several assumptions are made concerning the implementation of the adaptive rate control system. First, the receiver generates rate change requests every N_r channel uses. When a rate change is requested, it requires N_r code symbols to take effect. This is intended to model the delay characteristics of the feedback link, although in an actual system the rate change period and channel delay will probably be distinct. The receiver computes a short time autocorrelation every $N_a = KN_r$ channel uses. Finally, it is noted that the receiver only needs to retain the information set x^k in (2.11), because the autocorrelation can be computed as the code symbols are received. The algorithm basically proceeds as follows:

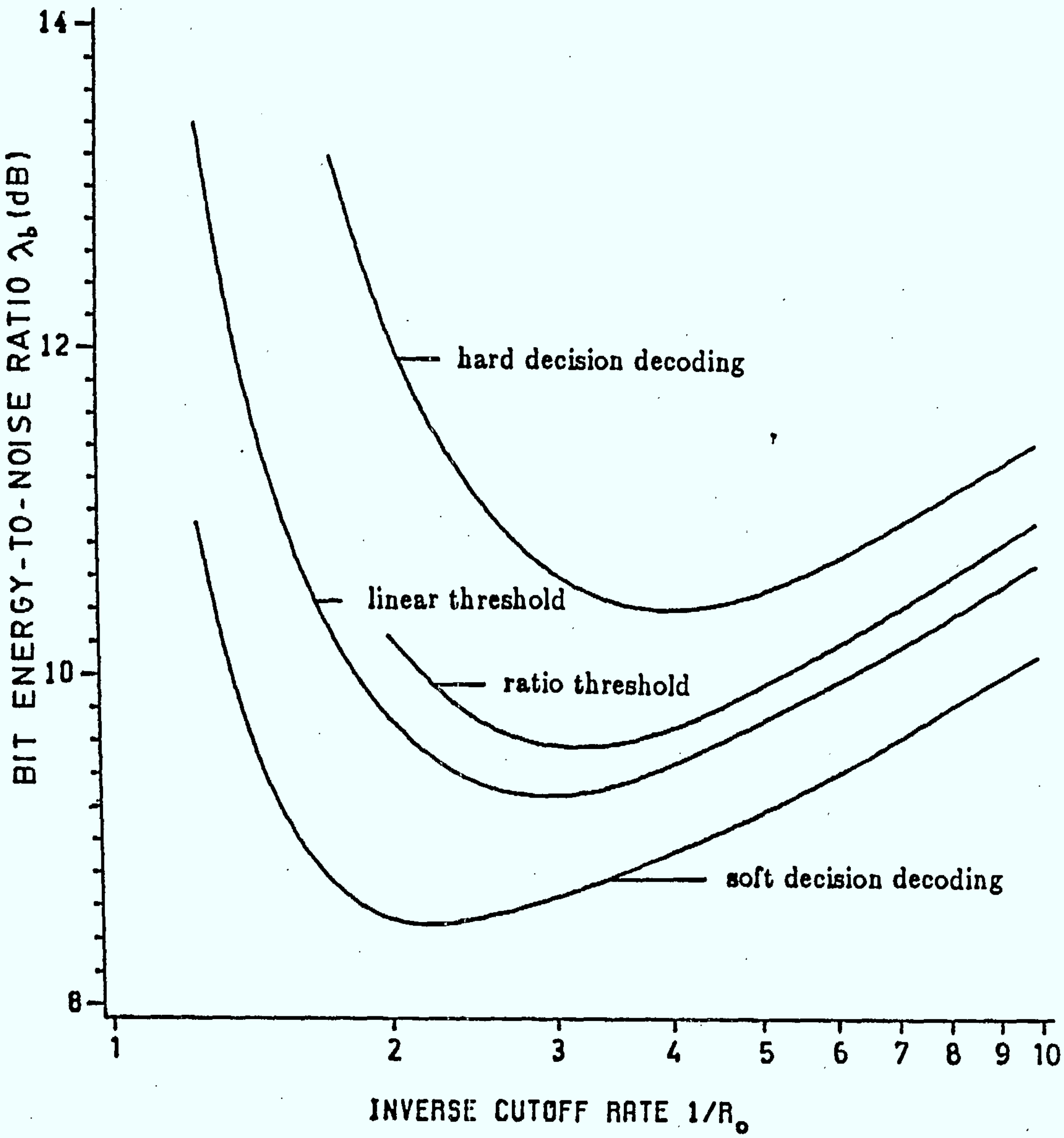

Figure 3.3 Performance of Threshold Interference Mitigation Techniques

Table 3.2 Performance of Linear Threshold Interference Mitigation Techniques with Suboptimal Parameters

R_o^{-1}	ρ	γ_b(dB)
1.50	0.122	12.256
1.75	0.227	10.750
2.00	0.340	9.993
2.50	0.582	9.384
3.00	0.838	9.272
3.50	1.000	9.355
4.00	1.000	9.472
5.00	1.000	9.715
6.00	1.000	9.947
8.00	1.000	10.360
10.00	1.000	10.713

1. The receiver generates an estimate of the channel covariance function $\phi(n)$, $n = 0, 1, 2, ..., m$. During this time the receiver has no initial estimate of the channel fade state and consequently the value of b is unknown. It therefore uses the ratio threshold to generate erasures with the parameter $a = 2.52$.

Since neither the receiver nor the transmitter has an initial estimate of α_k, the initial value $\hat{\alpha}^2 = E[\alpha_k^2] = \hat{\phi}(0)$ is used. This implies that the transmitter initially signals with the average rate $\overline{R}$ and average lowpass energy $E_s = S/\overline{R}$. After the reception of N_a code symbols, the receiver computes the initial channel covariance estimate

$$\hat{\phi}(n) = \begin{cases} \dfrac{1}{N_a E_s} \displaystyle\sum_{i=1}^{N_s - n} z_i z_{i+n} & , \ n=1, 2, ..., m \\ \dfrac{1}{N_a E_s} \displaystyle\sum_{i=1}^{N_s} z_i^2 - \dfrac{N_J}{E_s}(1 - e^{-B}) & , \ n=0 \end{cases} . \tag{3.44}$$

Note that the receiver must know the average transmitted signal energy E_s, and be supplied with an estimate of the jammer noise spectral density N_J. It is assumed here that the receiver knows these values exactly.

2. After estimating the channel covariance function, the current channel fade state is estimated by solving the system of linear equations in (3.23). This can be done very efficiently by using Durbin's recursive procedure [16, pp. 411-12]. The fade level estimate $\hat{\alpha}$ is computed using (3.20). An improved fade level estimate for the $(k + N_r - 1)^{th}$ interval can be computed recursively by using (3.24).

3. The receiver then generates a rate change request

$$r_{k+N_r-1} = \frac{\hat{\alpha}^2}{\hat{\phi}(0)}\overline{R} \ , \tag{3.45}$$

where the rate change request is assumed to take effect N_r channel symbols later.

4. Since the receiver can now estimate the channel fade state, it uses the linear threshold to generate erasures with the values of a and $\hat{b}$ at the optimal operating point $R_o^{-1} \approx 3.00$. From the previous section $b = \hat{\alpha}_k^2 E_k \hat{b} / 4$. However $E_k = \hat{\phi}(0)E_s / \hat{\alpha}^2$, and therefore the receiver tests the condition

$$Z_1 > aZ_0 + \frac{\hat{\phi}(0)E_s}{4}\frac{\hat{\alpha}_k^2}{\hat{\alpha}}\hat{b} \ . \tag{3.46}$$

5. The receiver continues to request rate changes every N_r code symbols until a new channel covariance estimate is to be computed.

6. When estimating the channel covariance function, the receiver averages the last P covariance function estimates. Therefore,

$$\hat{\phi}(n) = \frac{\hat{\phi}^l(n) + \sum_{i=1}^{P-1} \hat{\phi}^{l-i}(n)}{P} \ , \tag{3.47}$$

where $\hat{\phi}^i(n)$ is the i^{th} covariance estimate. This enables the receiver to track slow variations in the Doppler spectrum.

3.7 Simulation Study

For the simulation study the assumed fading model is

$$\phi(n) = \sigma^2 \varsigma^{|n|}, \quad 0 < \varsigma \leq 1 \quad . \tag{3.48}$$

This model implies that the piecewise constant fading process $\{\alpha_k\}$, is a Markov process with the state equation

$$(a_{k+1}, b_{k+1}) = (a_k, b_k) + (w_{1k}, w_{2k}) \quad . \tag{3.49}$$

In (3.49), w_{1k} and w_{2k} are i.i.d. zero-mean stationary white Gaussian sequences with $E[w_{ik}w_{il}] = \sigma^2(1-\varsigma^2)\delta_{kl}$, $i = 1, 2$. The model in (3.48) can also be rewritten in the familiar form

$$\phi(n) = \frac{\beta}{2} e^{-\nu |n|} , \tag{3.50}$$

where $\nu = -\ln(\varsigma)$ is the fading bandwidth and $\beta = 2\sigma^2$.

Throughout the simulation study, it was arbitrarily assumed that $\sigma^2 = 0.25$, $\varsigma = 0.95$, $m = 10$, $N_a = 500$, and $P = 10$. The results of the simulation study are summarized in Figure 3.4. Figure 3.4 shows the performance of a nonadaptive system with hard decision decoding and no side information, the performance of a nonadaptive system using the ratio threshold, and the steady state performance of the adaptive system with the linear threshold. For the adaptive system, the steady state performance is plotted for several rate change periods. In all cases $\rho = 1$, or broadband jamming, resulted in the worst performance.

Observe from Figure 3.4 that the performance is improved by about 3 to 4 dB by using the adaptive rate control algorithm, if the channel coherence time is large. The effectiveness of adaptive rate control is largely determined by the $\nu\tau$ and νN_r products, where τ is the channel propagation delay in units of code symbols. As ς increases or ν decreases, the performance will improve. When $N_r = 1$, $\nu\tau = \nu N_r = 0.51$, and some additional improvement is expected with a smaller fading bandwidth.

In Figure 3.5 we plot the performance of FH/BFSK signalling for a Rayleigh fading channel with varying degrees of fading. In all cases noncoherent detection is used, and decoding is performed using a weighted linear sum metric with perfect knowledge of the jammer state.

When the code symbol rate is changed very frequently to match the channel conditions, the performance is best, as expected. If the receiver has a perfect estimate of the channel fade state and performs soft decision decoding with a weighted linear sum metric, then the cutoff rate is

$$R_o = 1 - \log_2(1 + D) \tag{3.51}$$

where

$$D = \max_{\rho} \min_{\lambda} \int_0^{\infty} \frac{\rho}{1 - \lambda^2} e^{-\frac{\lambda\rho}{1+\lambda}\frac{x}{\hat{r}(x)}} p(x)dx \quad , \tag{3.52}$$

$$p(x) = \frac{1}{R_o\bar{\gamma}_b} e^{-\frac{x}{R_o\bar{\gamma}_b}} \quad , \tag{3.53}$$

and

$$\hat{r}(x) = \frac{x}{R_o\bar{\gamma}_b} \quad . \tag{3.54}$$

By using (3.52) and (3.53) in (3.51) the value of D simplifies to

$$D = \frac{\rho}{1 - \lambda^2} e^{-\frac{\lambda}{1+\lambda}\rho R_o\bar{\gamma}_b} \quad , \quad 0 \le \lambda \le 1 \, . \tag{3.55}$$

This is exactly the same as an unfaded channel with an unfaded partial-band noise jammer prescribed by (1.17) and (1.18).

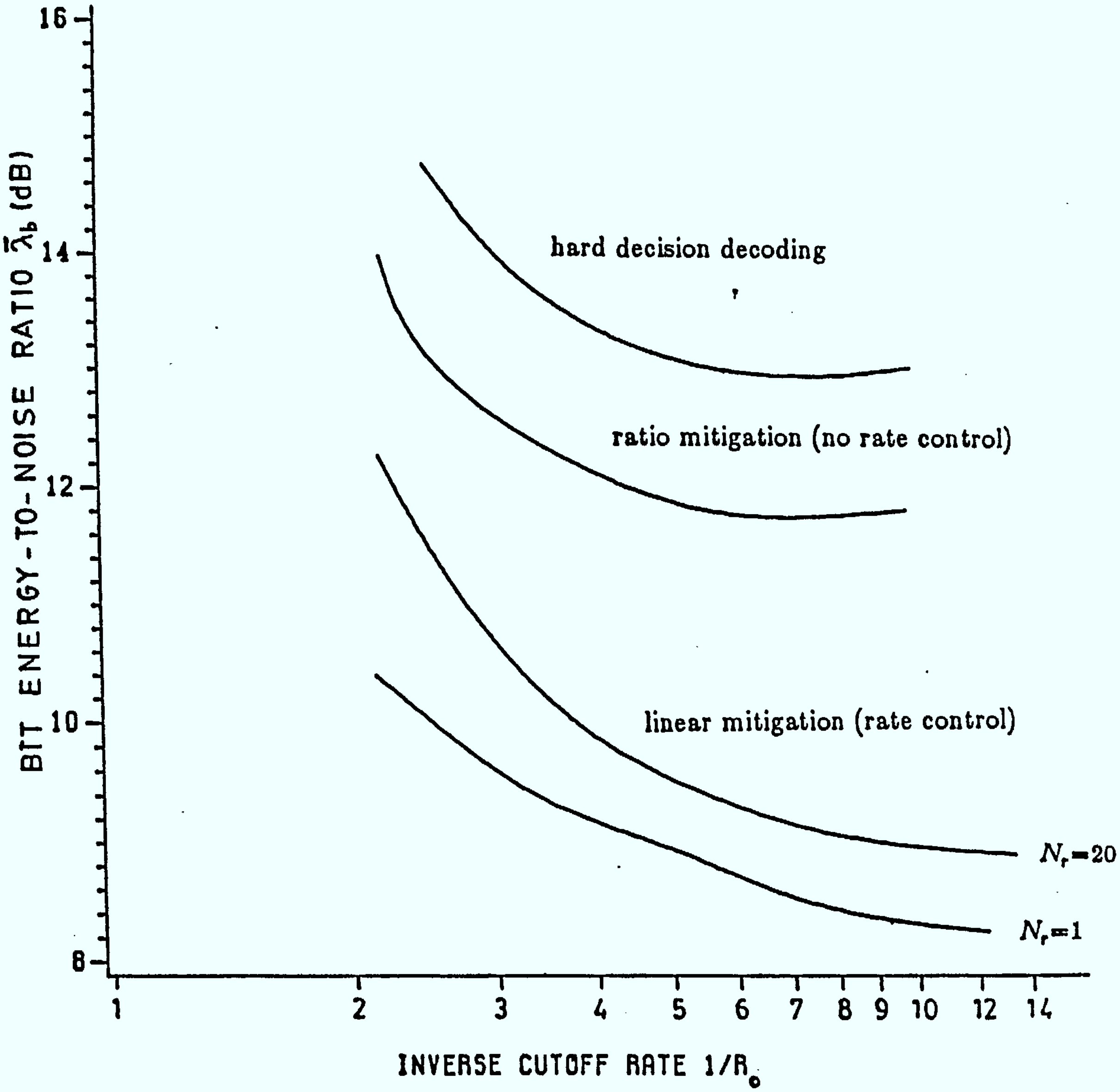

Figure 3.4 Performance of the Adaptive Rate Control Technique

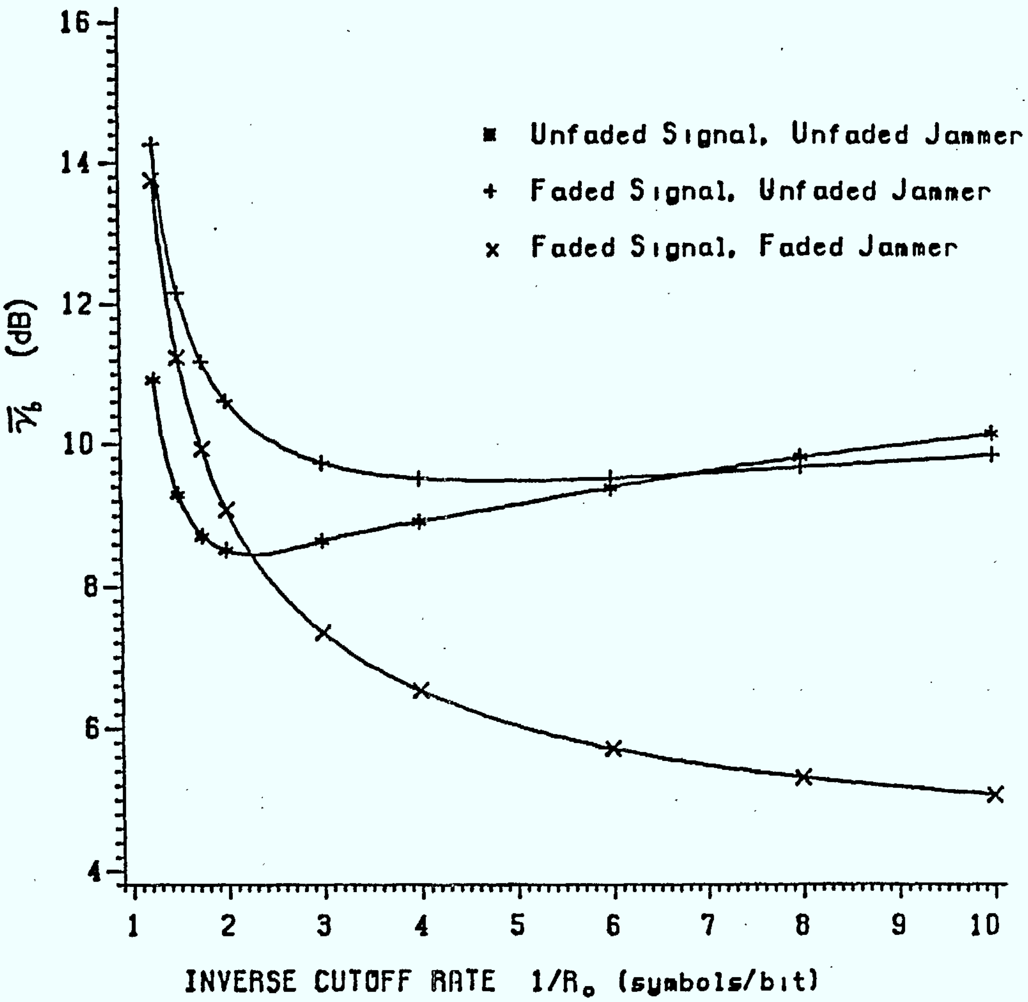

Figure 3.5 Minimum $\overline{\gamma}_b$ Required for Coded Operation at the Cutoff Rate with Soft Decision Decoding and Jammer State Information Using a Weighted Linear Sum Metric

However at low rates the performance digresses from ideal. This is because the receiver is generating a noisy estimate of the channel fade state. The estimation error is due to the finite order predictor, and the partial band interference. Furthermore as the rate change period becomes large, it is expected that the actual and estimated channel states will be independent. When this happens, the value of D becomes

$$D = \max_{\rho} \min_{\lambda} \int_0^{\infty}\int_0^{\infty} \frac{\rho}{1-\lambda^2} e^{-\frac{\lambda}{1+\lambda}\rho \frac{x}{y} R_s \bar{\lambda}_b} e^{-x} e^{-y} \, dx \, dy \quad . \tag{3.56}$$

where x and y are i.i.d. with the density in (3.53). This is identical to the expression describing the performance of constant rate signalling over a Rayleigh faded channel with a Rayleigh faded noise jammer, assuming that soft decision decoding is used with a weighted linear sum metric and perfect jammer state information. The performance curves for this channel and decoder in Figure 3.5 are similar to the ones that characterize the system at hand.

3.8 Summary

In this section an adaptive rate control algorithm was presented for jammed fading channels. The algorithm tracks variations in the received signal strength, and by doing so dynamically changes the transmitted code symbol rate. It also generates side information by using a linear thresholding technique. The adaptive rate control algorithm was shown to offer significant improvement in the range of 3 to 4 dB over a nonadaptive system. Even when the actual and estimated channel fade states are (nearly) independent, the performance is improved by about 2 dB.

No attempt has been made to determine the effects of rate quantization, background noise, tone jamming, faded jamming, and the problem of generating an estimate of N_J. These problems are for future research.

4. FH/MDPSK Signalling

4.1 Generation of the FH/MDPSK Signal

In DPSK signalling, the phase information is carried by the differential phase between the phases of the adjacent signal symbols. It follows that the determination of the information carrying phase is obtained by estimating the difference in phase between adjacent received signal symbols, and the need to acquire the absolute phase for each transmitted symbol is circumvented. This mode of detection is referred to as differentially coherent detection. Since detection requires two adjacent received symbols, there will be an uncertainty in the detection of the first symbol. Further, error in the detection of the first symbol may propagate, the result of which may be viewed as a transient behaviour.

Since the information is carried in the phase, the envelope of a DPSK signal in maintained constant. The MDPSK signal can be represented by

$$s_k(t) = A\,\cos(\omega_o t+\theta_{ac}^{(k)}) \quad (k-1)T_s \le t \le kT_s \tag{4.1}$$

where T_s is the symbol duration and $\theta_{ac}^{(k)}$ is the accumulated phase up to the transmission of the k^{th} signal symbol. We have

$$\theta_{ac}^{(k)} = \theta_{ac}^{(k-1)} + \theta^{(k)} \tag{4.2}$$

where $\theta_{ac}^{(k-1)}$ is the accumulated phase up to the $(k-1)^{st}$ transmission interval and $\theta^{(k)}$ is the differential phase that takes on one of the M possible phases from the set

$$\left\{\phi_m = \frac{m2\pi}{M}\,, \quad m = 0, 1, ..., M-1\right\} \tag{4.3a}$$

which can also be written as

$$\left\{\phi_m = \frac{m2\pi}{M}, \quad m = 0 \pm 1, ..., \frac{M-2}{2}, \frac{M}{2}\right\}. \tag{4.3b}$$

Let R_s and R_h be respectively the symbol rate and the hopping rate. Define

$$L = \frac{R_h}{R_s}.$$

Slow frequency hopping implies that $L{<}1$ (for fast frequency hopping $L{>}1$). The parameter $\Gamma = 1/L$ represents the number of symbols per frequency hop. There will be Γ data symbols contained in any one frequency hop. That is, Γ MDPSK signal symbol intervals are mapped onto one hop interval. Let T_h be the hop interval and $(\overline{\nu-1}T_h, \nu T_h)$, where $\overline{\nu-1} := (\nu-1)$, be the ν^{th} hopping interval. We then have

$$\nu T_h = (\nu-1)T_h + \sum_{k=1}^{\Gamma} kT_s .$$

Define Ω to be the set of Γ symbol intervals. Let $s_{h,\nu}(t)$ denote the transmitted signal during the ν^{th} hop, i.e., during the interval $[(\nu-1)T_h, \nu T_h]$. Then the signal set in the ν^{th} hop is given by

$$s_{h,\nu}(t) = \sum_{k \in \Omega} s_k(t) \quad (\nu-1)T_h \le t \le \nu T_h . \tag{4.4}$$

4.2 Differentially Coherent Detection

The reason that the differentially encoded and difference detected system works well is due to the tacit assumption that the received carrier phase during any one hop is constant. As mentioned earlier, the first data symbol in each hop is either lost or contains a phase uncertainty, since a phase reference has not yet been established for its detection. Because of this uncertainty, the information rate of an MDPSK system is reduced. It is necessary to make Γ large to minimize this loss in information rate. On the other hand, a larger value of Γ also means that more data symbols are affected by jamming during any one hop interval.

Let $\hat{\theta}_{ac}^{(k)}$ and $\hat{\theta}_{ac}^{(k-1)}$ be respectively the estimates of $\theta_{ac}^{(k)}$ and $\theta_{ac}^{(k-1)}$. Let $\hat{\theta}_{u}^{(k)}$ and ϕ_a be respectively the unambiguous estimate and the ambiguous phase (which may include the carrier phase). Then

$$\begin{aligned}\hat{\theta}^{(k)} &= \hat{\theta}_{ac}^{(k)} - \hat{\theta}_{ac}^{(k-1)} \\ &= (\hat{\theta}_{u}^{(k)} + \phi_a) - (\hat{\theta}_{u}^{(k-1)} + \phi_a) \\ &= \hat{\theta}_{u}^{(k)} - \hat{\theta}_{u}^{(k-1)} . \end{aligned} \tag{4.5}$$

Thus, within any one hop interval only the errors in the unambiguous estimates of the phases in successive symbol intervals give rise to an event error when a given hop is jammed. Here, it is assumed that when a given hop is jammed, all Γ symbols in that hop are affected. A simplified MDPSK demodulator is shown in Figure 4.1.

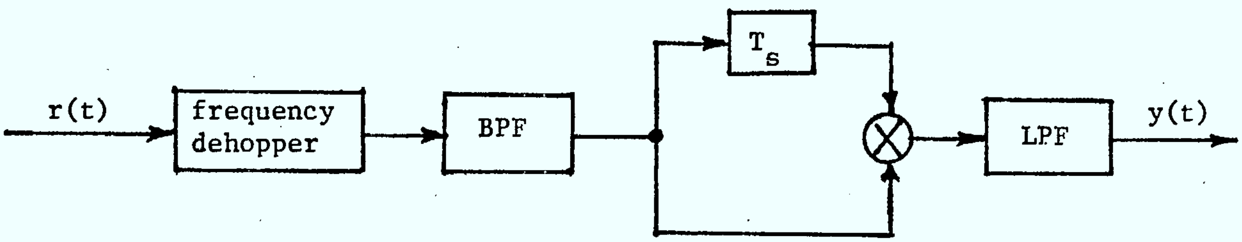

Figure 4.1 Differentially Coherent Demodulator

Each decision region in the differentially coherent detection of an MDPSK signal is of size π/M. An error event occurs if a given hop is jammed. Then, in terms of phase error estimates, an error event occurs if

$$|\theta^{(k)} - \hat{\theta}^{(k)}| > \frac{\pi}{M}, \quad k \in \Omega . \tag{4.6}$$

Suppose that the actual value of $\theta^{(k)}$, $k \in \Omega$, is ϕ_n, where the value of ϕ_n is specified by (4.3b). Then, the error event can be written as

$$|\phi_n - \hat{\theta}^{(k)}| > \frac{\pi}{M} \quad n = 0, \pm 1, \pm 2, \ldots, \pm\left(\frac{M-2}{2}\right), \frac{M}{2}, \quad k \in \Omega . \tag{4.7}$$

Let $Z^{(k)}$, $k \in \Omega$ denote the signals received corresponding to the ν^{th} hop. Assuming that the jammer signal in any one hop interval is constant, both in amplitude and phase, the jamming signal can be represented, in complex notation, by

$$J^{(\nu)} = Ie^{j\theta_J^{(\nu)}}$$

where θ_J is a random phase uniformly distributed in the interval $(0, 2\pi)$ and the superscript (ν) denotes jamming occurrence in the ν^{th} hop. Then, successive received signals corresponding to the symbols transmitted in the ν^{th} hop are given, in complex notation, by

$$Z^{(k-1)} = Ae^{j\theta_{ac}^{(k-1)}} + Ie^{j\theta_J^{(\nu)}} \tag{4.8a}$$

and

$$Z^{(k)} = Ae^{j\theta_{ac}^{(k-1)} + \theta^{(k)}} + Ie^{j\theta_J^{(\nu)}} , \tag{4.8b}$$

where $k \in \Omega$. It is noted that, if $\theta^{(k)}$ equals 0, i.e., 0 is the transmitted phase, then the received signals corresponding to consecutive MDPSK symbols in which the differential phase is zero will be identical. In this event, there can be no phase error. Define the phase estimate $\hat{\theta}^{(k)}$ by

$$\hat{\theta}^{(k)} := arg(Z^{(k)} - Z^{(k-1)}) , \quad k \in \Omega . \tag{4.9}$$

Following the notation of [2], let

$$Q_{2\pi n/M} = Pr\left\{ |\phi_n - \hat{\theta}^{(k)}| \right\} \tag{4.10}$$

for $n = 0, \pm 1, \pm 2, \ldots, \frac{M-2}{2}, \frac{M}{2}$ be the probability of error. By symmetry, we have

$$Q_{2\pi n/M} = Q_{-2\pi n/M}\,; \quad n = 1, 2, \ldots, \frac{M-2}{2}\,. \tag{4.11}$$

It is noted that the above formulation of error events and error event probabilities is equally applicable to partial-band noise or multitone jamming.

4.3 Multitone Jamming

In the presence of multitone jamming, the symbol error probability is given by [2]

$$P_s(M) = \frac{\rho}{M}\left[Q_0 + Q_\pi + 2\sum_{n=1}^{(M-2)/2} Q_{2\pi n/M}\right]. \tag{4.12}$$

If $\phi_0 = 0$ is a transmitted phase, then the received signals corresponding to the $(k-1)^{th}$ and k^{th} symbols that have been transmitted during the ν^{th} hop are equal. That is, if $\theta^{(k)} = 0$, then $\hat{\theta}^{(k)}$ also equals 0 and $Q_0 = 0$. The symbol error probability then becomes

$$P_s(M) = \frac{\rho}{M}\left[Q_\pi + 2\sum_{n=1}^{(M-2)/2} Q_{2\pi n/M}\right]. \tag{4.13}$$

It follows that the bit error probability is approximately given by

$$\begin{aligned} P_b(M) &= \frac{M}{2(M-1)} P_s(M) \\ &= \frac{\rho}{2(M-1)}\left[Q_\pi + 2\sum_{n=1}^{(M-2)/2} Q_{2\pi n/M}\right]. \end{aligned} \tag{4.14}$$

Let Q be the number of distinct jamming tones. Then the jamming power per tone is $J_o = J/Q$. Let β^2 denote the jamming power per tone to signal power ratio, i.e.,

$$\beta^2 = \frac{J/Q}{S}$$

$$= I/A \ .$$

The probabilities $Q_{2\pi n/M}$ need to be evaluated using a geometric consideration [1], [2]. For the 2 and 4-ary FH/DPSK signals, the worst case bit error probabilities are given by

$$P_b(2) = \begin{cases} \dfrac{1}{2E_b/N_J} & , \ E_b/N_J > 1 \\ \dfrac{1}{2} & , \ E_b/N_J \leq 1 \end{cases} \tag{4.15}$$

and

$$P_b(4) = \begin{cases} \dfrac{0.2592}{E_b/N_J} & , \ \dfrac{E_b}{N_J} > 1.85 \\ \dfrac{1}{3\pi}\left[\cos^{-1}\left(\dfrac{2E_b/N_J-1}{2\sqrt{2E_b/N_J}}\right) + \cos^{-1}\left(\dfrac{2E_b/N_J+1}{4\sqrt{E_b/N_J}}\right)\right] & , \ \dfrac{1}{2} < \dfrac{E_b}{N_J} < 1.85 \\ 0.5 & , \ \dfrac{E_b}{N_J} < \dfrac{1}{2} \end{cases} \tag{4.16}$$

4.4 Partial-Band Noise Jamming

The derivation of the symbol error probability involves a geometric consideration. Pawula et al [23] have derived a concise expression for the average symbol error probability performance in an additive white Gaussian noise channel:

$$P_s(M) = \frac{\sin\frac{\pi}{M}}{\pi} \int_0^{\pi/2} \frac{\exp\left[-(log_2 M)\frac{E_b}{N_o}\left(1 - \cos\frac{\pi}{M}\cos\alpha\right)\right]}{1 - \cos\frac{\pi}{M}\cos\alpha} d\alpha \, . \tag{4.17}$$

The average symbol error probability performance of FH/MDPSK in partial-band noise jamming is obtained by replacing E_b/N_o in the above equation by $\rho E_b/N_J$

and multiplying the result by ρ, where ρ is the fraction of the spread spectrum bandwidth the jammer chooses to concentrate its jamming power. The bit error probability performance of FH/MDPSK in partial-band noise jamming is then given by

$$P_b(M) = \frac{M}{2(M-1)} P_s(M, \rho)$$

$$= \frac{M}{2(M-1)} \left(\frac{E_b}{N_J}\right)^{-1}$$

$$\times \left[\gamma_b \left(\frac{\sin\frac{\pi}{M}}{\pi} \right) \int_0^{\pi/2} \frac{\exp\left[-(log_2 M)\gamma_b(1 - \cos\frac{\pi}{M}\cos\alpha)\right]}{1 - \cos\frac{\pi}{M}\cos\alpha} d\alpha \right] \tag{4.18}$$

where

$$\gamma_b := \frac{\rho E_b}{N_J} \tag{4.19}$$

is the bit energy-to-jammer noise power spectral density ratio. In (4.18) the parameter ρ has been absorbed into the effective bit energy-to-jammer power spectral density ratio, γ_b. For a fixed value of E_b/N_J, the bit error probability is maximized when γ_b attains its maximum value, and the optimal value of ρ is given by

$$\rho_o = \frac{\gamma_{b_{max}}}{E_b/N_J} .$$

The maximum bit error probability is then given by [2]

$$P_{b_{\max}} = \begin{cases} \dfrac{\dfrac{M}{2(M-1)}\dfrac{\sin\frac{\pi}{M}}{\pi}\displaystyle\int_0^{\pi/2}\exp\left[-\log_2 M\frac{E_b}{N_J}(1-\cos\frac{\pi}{M}\cos\alpha)\right]}{1-\cos\frac{\pi}{M}\cos\alpha}d\alpha & , \dfrac{E_b}{N_J} < \gamma_{b_{\max}} \\ \left(\dfrac{E_b}{N_J}\right)^{-1}\left[\dfrac{MP_{\max}}{2(M-1)}\right] & , \dfrac{E_b}{N_J} \geq \gamma_{b_{\max}} \end{cases} \tag{4.20}$$

where

$$P_{\max} := \gamma_{b_{\max}} P_s(M) .$$

For binary FH/DPSK transmission in partial-band noise jamming, the bit error probability is given by

$$P_b(2) = P_s(2) = \frac{\rho}{2}\exp\left(-\frac{\rho E_b}{N_J}\right) . \tag{4.21}$$

The worst case bit error probability performance is obtained by differentiating the right-hand side of (4.20) and equating the results to zero. The resultant optimal jamming fraction, ρ_o, is given by

$$\rho_o = \frac{1}{E_b/N_J}$$

and the maximum bit error probability is given by

$$P_{b_{\max}}(2) = \begin{cases} \dfrac{1}{2}\exp\left(-\dfrac{E_b}{N_J}\right) & , \quad \dfrac{E_b}{N_J} < 1 \\ \dfrac{1}{2e(E_b/N_J)} & , \quad \dfrac{E_b}{N_J} \geq 1 \end{cases} \tag{4.22}$$

4.5 Adaptive Quantization

While the adaptive rate control policy discussed in section 2 is from the MFSK signalling point of view, a similar approach can be used with MDPSK signalling to render a fading channel seemingly stationary. In this subsection, we consider an adaptive quantization strategy as a means of coping with channel fading.

It was pointed out in an earlier subsection that, to reduce the complexity of Since a group of Γ M-ary symbols will experience similar distortion during any one use of the channel, the channel will appear to the encoder/decoder pair as having memory. To render the channel memoryless, it will be necessary to use interleaving at the transmitter and deinterleaving at the receiver. With a sufficient depth of interleaving, the information symbols sent via a single use of the channel would be randomized and the coding channel will appear memoryless to the encoder/decoder pair.

Interleaving/deinterleaving increases system complexity. To reduce the complexity of the deinterleaver, it is necessary to quantize the demodulated signal to at least 8 levels (for soft quantization).

If the channel exhibits fading, the demodulated signal will fluctuate. The quantizing threshold should follow the signal fluctuation. This implies a necessity to adaptively adjust the quantizing thresholds. Let there be $2N$ quantizing levels and let $y(t)$ be the input to the quantizer. The quantizer output is given by

$$x(t) = \begin{cases} N & \text{if } T_{N-1}(t) \leq r(t) \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & \text{if } 0 \leq r(t) < T_1(t) \\ -1 & \text{if } -T_1(t) < r(t) < 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ -N & r(t) \leq -T_{N-1}(t) \end{cases}$$

where $T_i(t)$, $i = 1, 2, ..., N-1$, are the dynamic threshold values. In practice, we choose a set of fixed threshold values and amplitude scale the input $y(t)$ based on an estimate of the fade level of the channel. A crude approach is to use a moving average of the demodulator output as the scaling factor. That is, the moving average $\bar{y}(t)$ is given by

$$\bar{y}(t) = \int_0^{T_a} y(t)dt \ .$$

Let $y^*_i(t)$ be the sample value of the i^{th} chip. The moving average $\overline{y^*}(t)$ is then given by

$$\overline{y^*}(t) = \sum_{i=1}^{I} y_i(t)$$

where I is the moving average window size and $y_i(t) := y(t-iT_c)$, with T_c being the encoded symbol interval. The quantizer is as shown in Figure 4.2. Here, the estimation of the fade level and the quantization operation are independent. A more efficient approach is to use feedback to generate an estimate of the scaling factor.

We propose to compute the quantizer range factor α_n (the quantizer range factor is a factor used to normalize the quantizer input so that the actual quantizer is a fixed device) using an iterative formula,

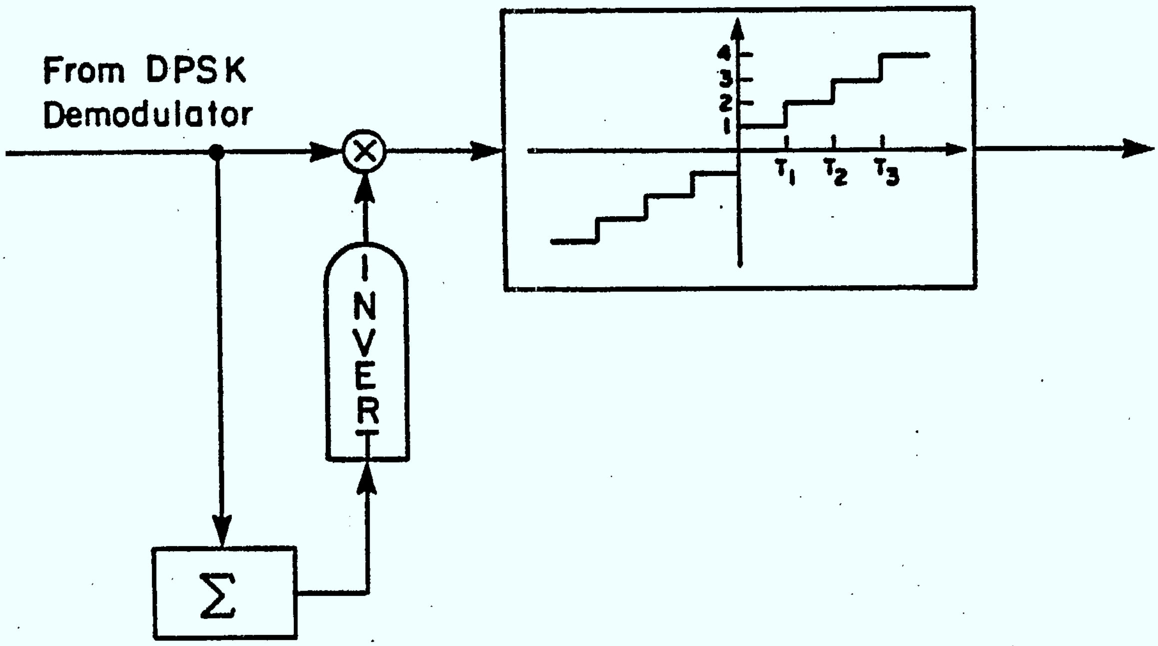

Figure 4.2 Quantization of Normalized Input using Moving Average

$$\alpha_{n+1} = M_i \alpha_n \ , \tag{4.23}$$

where $\{M_i\}$ is a set of multiplicative constants that satisfy the condition

$$1 > M_1 \leq M_2 \leq \cdots \leq M_N > 1 \ .$$

Intuitively, if α_n is too small, the quantizer input amplitude will be too large and the output will have a magnitude larger than the correct value. The M_i selected to compute α_{n+1} should be larger than unity, so that $\alpha_{n+1} > \alpha_n$. On the other hand, if α_n is too large, the quantizer input would be small compared to the correct value. Then an $M_i < 1$ should be selected to yield an $\alpha_{n+1} < \alpha_n$. Equation (4.23) is thus a random walk process. The trick is to execute the random walk in an efficient manner. This amounts to making a suitable assignment of the set $\{M_i\}$ and selecting the M_i's in a correct way.

With a correct normalization factor (quantizer range factor), we expect the magnitude of the quantizer input to be unity. With a $2N$-level midriser symmetric quantizer, we only need to consider the positive excursion half. Let us consider the positive half, i.e., a positive amplitude pulse has been transmitted. If the input is within a quantum perturbation, i.e., $(\pm k\delta)$, of unity, where $k \geq 1$, the output will be the level $N/2$, where k and δ are chosen to making the random walk process of (4.23) converge. The quantum size referred to above in thus $2k\delta$. Thus, the input in the range $[1-k\delta,\ 1+k\delta)$ is mapped onto the output level $N/2$. δ may be determined as a function of k as follows:

$$\frac{(1-k\delta)}{N/2} = 2k\delta$$

or

$$\delta = \frac{2}{k(N+2)\delta} \ . \tag{4.24}$$

For an 8-level quantizer, $N = 4$. Choosing $k = 2$ in (4.24), we have $\delta = \frac{1}{6}$ and the threshold settings are 0, 2/3, 4/3, 6/3, with the mappings

$$[0,\ 2/3) \longrightarrow 1$$

$$[2/3,\ 4/3) \longrightarrow 2$$

$$[4/3,\ 2) \longrightarrow 3$$

and

$$[2,\ \infty) \longrightarrow 4 \ .$$

The M_i's are chosen such that

$$M_{N/2} = 1.0,\ M_{N/2\pm i} = 1 \pm i\delta,\ \ i = 1, 2, \cdots, N/2-1 \ .$$

For the $N = 4$ case, the multiplicative constants are $M_1 = 1-1/6$, $M_2 = 1$, $M_3 = 1+1/6$, and $M_4 = 1+1/3$. The output, y_n = *level* i, is used to select M_i. The adaptive 8-level quantizer is shown in Figure 4.3, where the initial value, α_0, is to be obtained by averaging the first few received chips.

It is noted that choosing a value of k in (4.24) greater than unity has the effect of decreasing δ, and hence the values of the M_i's will be closer to unity. The net effect is that the random walk process takes a smaller step size during each iteration. For a slowly fading environment, slow quantizer adaptation would be desirable. A computer simulation study of the adaptive quantizer described in this section has been discussed in [24] for a similar application. Adaptive quantization can potentially compensate for signal fading.

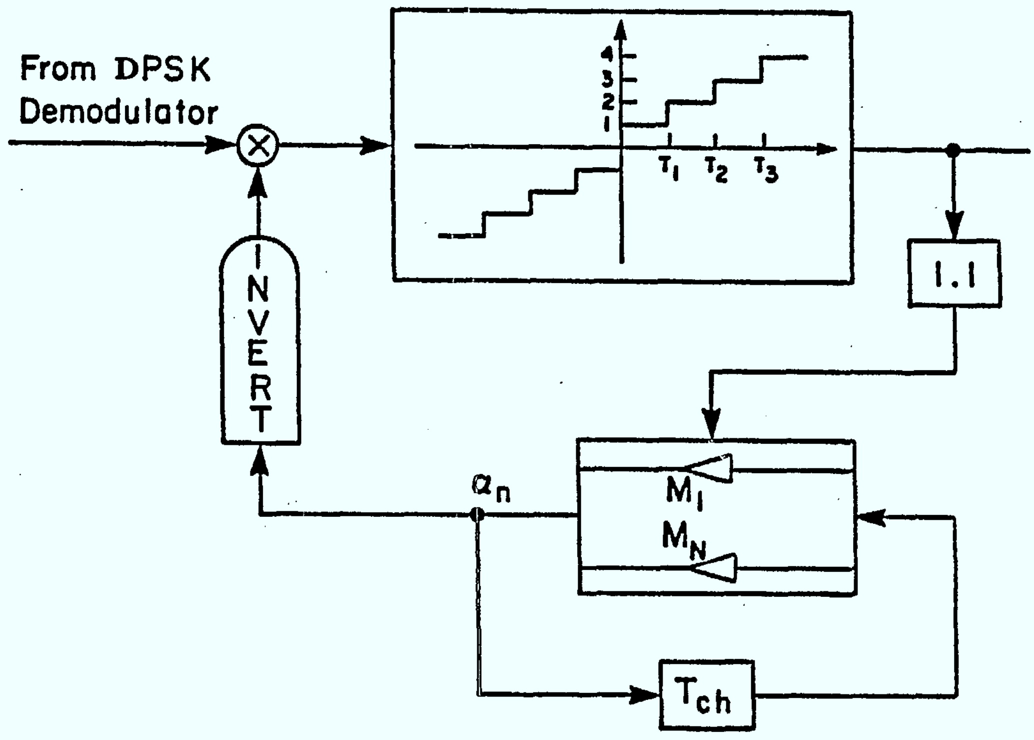

Figure 4.3 An 8-Level Adaptive Quantizer

4.6 Summary

This section has examined the error probability performance of FH/MDPSK signal transmission in the presence of multitone and partial-band noise jamming. Since differentially coherent demodulation requires two adjacent received M-ary symbols to generate one information symbol estimate, at least two M-ary symbols per frequency hop are needed. This is referred to as slow frequency hopping, which has applications in, for example, the Milsatcom program [25].

The derivation of the bit error probability expressions presented in this section under the influence of multitone and partial-band noise jamming assumes an unfaded channel. Practical channels normally exhibit some form of fading. Adaptive rate control algorithms of the form proposed in section 2 can be used here to render a fading channel seemingly stationary. An adaptive quantization approach, which is less complex and can potentially compensate for signal fading, is proposed for the reception of MDPSK signals over a fading channel. The effectiveness of adaptive quantization as a means of coping with rain attenuation has been studied in [28]. Extensive computer simulation study for the present situation will be necessary to ascertain the effectiveness of adaptive quantization for MDPSK signalling over a fading channel with jamming.

Except for the change in the bit error probability performance, the code performance studied in [9] for FH/MFSK signalling is also applicable to the FH/MDPSK system examined in this section.

5. Conclusions

In this report we have examined adaptive processing techniques for communication over a jammed fading channel. Section 2 introduces an adaptive rate control method for FH/MFSK signalling over a jammed fading channel. An adaptive rate control algorithm that tracks the variations in the received signal by dynamically changing the transmitted code symbol rate is described and discussed in section 3. The algorithm also generates side information for decoding by using a linear thresholding technique. The adaptive rate control algorithm offers a significant improvement, in the range of 2 to 4 dB, over a nonadaptive system. Of particular importance is that the adaptive rate control algorithm renders the faded coding channel seemingly stationary. In this way, the code ensemble performance for a stationary coding channel discussed in a previous report [9] is also valid for an adaptive rate contolled communications system in a jammed fading environment.

In certain applications, such as the Milsatcom program, slow frequency hopping with MDPSK signalling is preferred over MFSK transmission. The bit error probability performance is studied in section 4. An adaptive quantizing approach that can potentially compensate for signal fading is also discussed in section 4.

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