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ELASTIC STABILITY AND EQUILIBRIUM CONFIGURATION OF EARTH POINTING SATELLITES WITH LONG APPENDAGES

by

F.R. Vigneron and T. Garrett

CRC REPORT NO. 1206

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ELASTIC STABILITY AND EQUILIBRIUM CONFIGURATION OF EARTH POINTING SATELLITES WITH LONG APPENDAGES

by

F.R. Vigneron and T. Garrett

ABSTRACT

The elastic stability of the booms on a gravity gradient stabilized Alouette type satellite is investigated to determine the critical lengths above which the booms will take on large deflections under the influence of the gravitational and centrifugal force fields. If the booms are not structurally damaged by the large deflections, the satellite may take up one of several stable configurations. A criterion involving the boom lengths is derived for the stability of the gravity gradient orientation for zero boom deflections. The derivation includes the flexibility of the booms.

1. INTRODUCTION

The two most commonly used methods for passively stabilizing the attitude of a satellite in a circular, near-earth orbit are spin stabilization and gravity gradient stabilization. These methods are shown in Figure 1 for an Alouette type satellite fitted with four long and flexible sounder antennas (>30 ft.) along the principal axes x and y.

If spin stabilization is to be used, the satellite is given an angular velocity about the z-axis during the launch sequence. As a result of its angular momentum, the satellite will maintain a fixed attitude relative to inertial space during an orbit. Over a period of days the attitude will change in a slow and predictable manner due to the influence of gravity, magnetic, aerodynamic and solar forces. This is the type of stabilization that has been employed on the Canadian satellites, Alouettes I and II, and ISIS I*.

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* International Satellites for Ionospheric Studies, joint Canada and United States program.

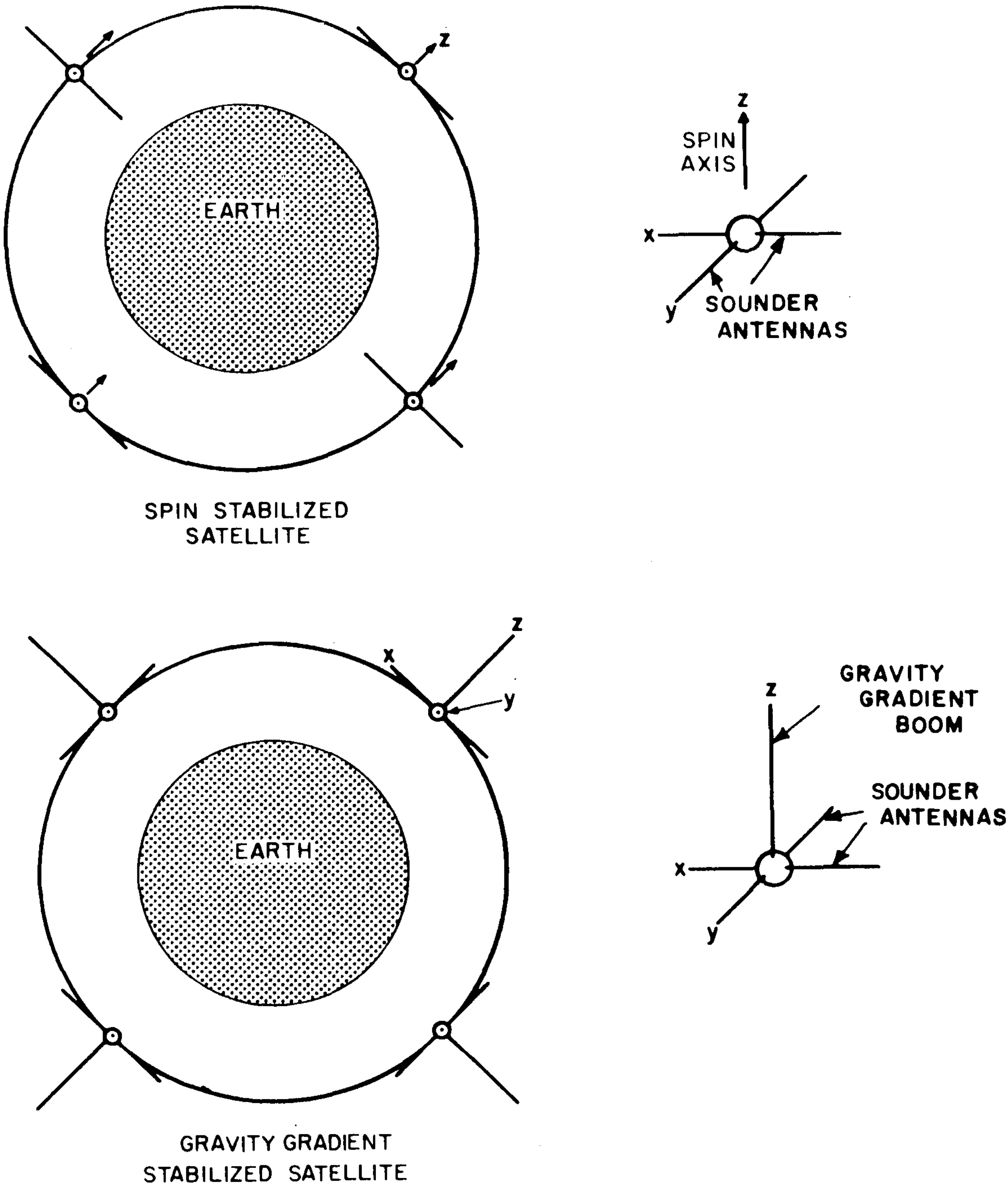

Fig. 1. Passive attitude stabilization methods for Alouette type satellites.

Gravity gradient stabilization could also be employed on these satellites to obtain a predictable attitude history, provided the orbit is circular. With this type of attitude stabilization, the satellite is not rotated about the z-axis, but another spar or gravity gradient boom is added along this axis. If the booms may be considered rigid, a fixed attitude relative to the local horizontal can be maintained by an appropriate choice of boom lengths. For an Alouette type satellite, the suggested attitude is for the gravity gradient boom to be coincident with the radius vector from the centre of the earth, the x-sounder antennas tangent with the orbit trajectory and the y-antennas perpendicular to the orbit plane as shown in Figure 1. This attitude of the satellite will be referred to as the rigid body equilibrium orientation.

However the booms are not rigid, but are flexible members which can be distorted by small forces and internal temperature gradients. Distortion of the booms may change the attitude of the satellite and as a result may negate any advantages inherent in this method of stabilization.

The following derives limits on the validity of assuming a gravity gradient stabilized Alouette type satellite to be a rigid body, when the only forces are due to gravity and centrifugal acceleration. Possible equilibrium configurations are suggested for the satellite when it can no longer be considered rigid. Consideration is also given to other factors which may affect the equilibrium shape of the satellite.

2. DISCUSSION OF THE SATELLITE EQUILIBRIUM

As shown in the Appendix, the following external forces act on each unit mass particle of a continuum in a circular orbit and in equilibrium under the influence of gravity and centrifugal forces.

$$\left.\begin{aligned} X &= 0 \\ Y &= -\rho\,\Omega^2\,y \\ Z &= 3\rho\,\Omega^2\,z \end{aligned}\right\} . \qquad \text{.....(1)}$$

Where X, Y and Z are, respectively, the forces in the x, y and z directions shown in Figure 2; ρ is the mass density of the continuum and Ω is the orbital angular velocity. At equilibrium these forces are balanced by equal and opposite internal reaction forces which are generated through distortion of the body. For an Alouette type satellite which can be assumed to be rigid, the equilibrium orientation will be such that the forces X, Y and Z on the booms will be zero (i.e., the booms will be aligned along the three axes x, y and z).

Now consider the satellite in Figure 3 with the booms distorted from the principal axes. The forces of equations (1) are shown in magnitude and direction.

The forces on the gravity gradient boom tend to restore it to the z-axis. Consequently, this boom is in stable equilibrium when it is straight and aligned along the z-axis.

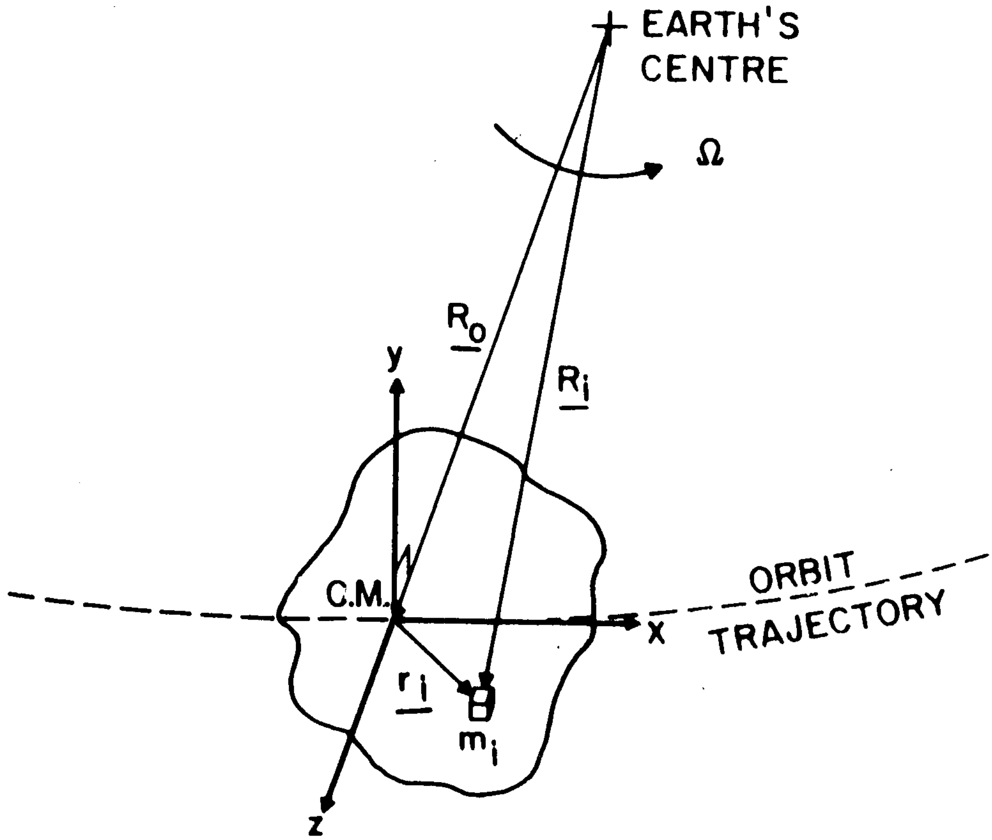

Fig. 2. Rotating, orbit referenced coordinate system.

The forces on the y set of booms are shown in Figure 3, and in Figure 4 (a) and (b). As a result of the direction of the force components and their dependence on the spatial coordinates y and z, these booms may be subjected to a condition of elastic instability. This situation has an analog in the problem of a long slender column as formulated by Euler[1]. For a given set of orbital and boom parameters a critical length, ℓ_{cr}, will exist. When the boom length, ℓ, is less than ℓ_{cr}, the internal resisting bending moments are sufficient to ensure that the boom remains straight. When $\ell > \ell_{cr}$ the straight boom is in a state of unstable equilibrium and will buckle elastically. With the onset of elastic buckling in the y - z plane, two stable equilibrium configurations are possible. These are shown schematically in Figure 4 (a) and (b).

The forces on the x set of booms are depicted in Figure 3, and in Figure 5 (a) and (b). Here again, the situation is analogous to that of the y set.

In the preceding qualitative discussion, it was assumed that the booms exhibit elastic stability behaviour when loaded. The Alouette satellite employs 'STEM'* antennas, which form tubes of open section when deployed as shown

* STEM--STORED TUBULAR EXTENDIBLE MEMBER, manufactured by Spar Aerospace Products Ltd., Toronto, Ontario.

in Figure 6. Buckling tests were performed to observe the behaviour of beryllium copper STEMs when subjected to column loads. The method of testing is shown schematically in Figure 7. The results confirm the underlying hypothesis that the booms do exhibit an elastic instability behaviour. The bending stiffness, EI, of the STEM tubes was found to be 11 lb-ft^2, which agrees reasonably well with the calculated value of 15.5 lb-ft^2 for a closed tube.

Since elastic buckling is characterized by a sudden large distortion of the booms, it is important to define the bounds of stability. In the following sections, the elastic stability of the booms is investigated analytically.

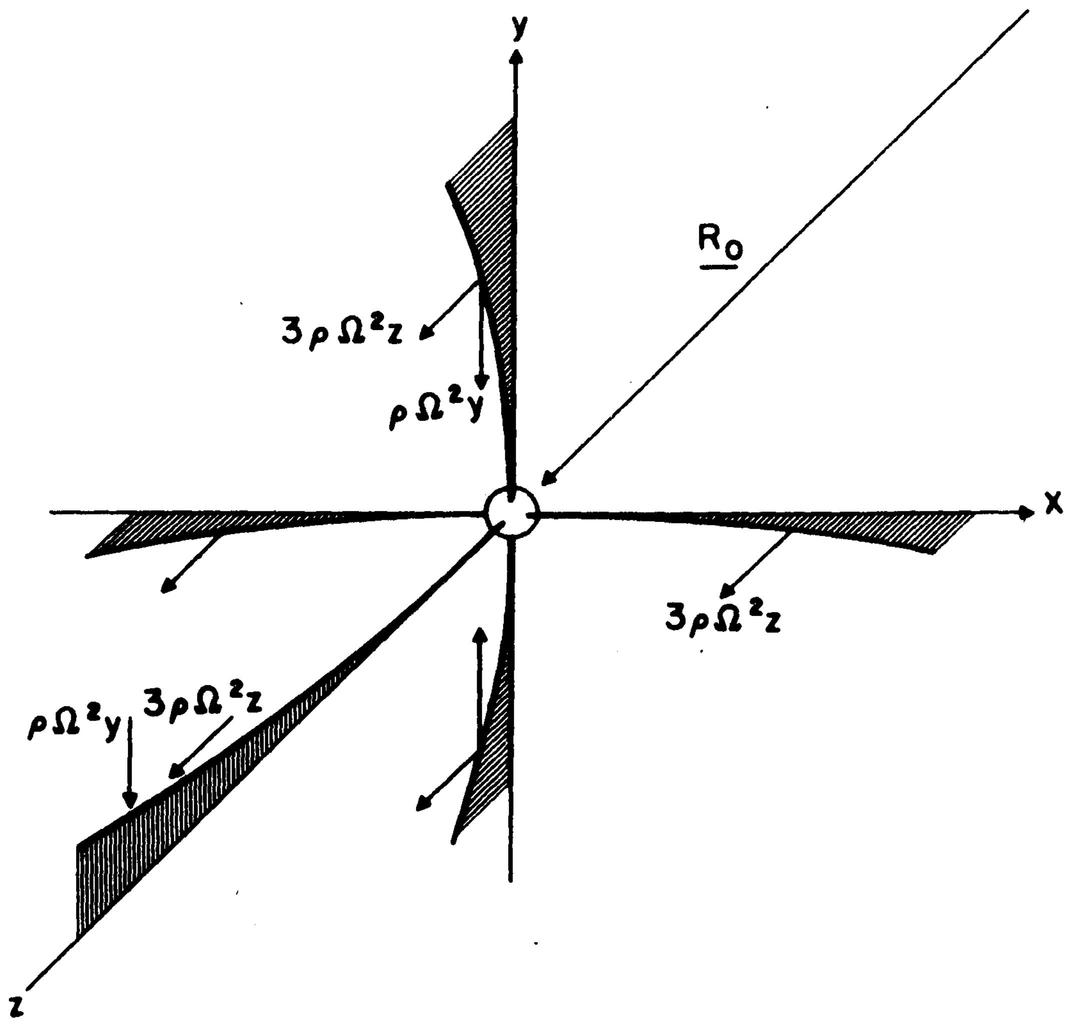

Fig. 3. Forces on deflected satellite booms.

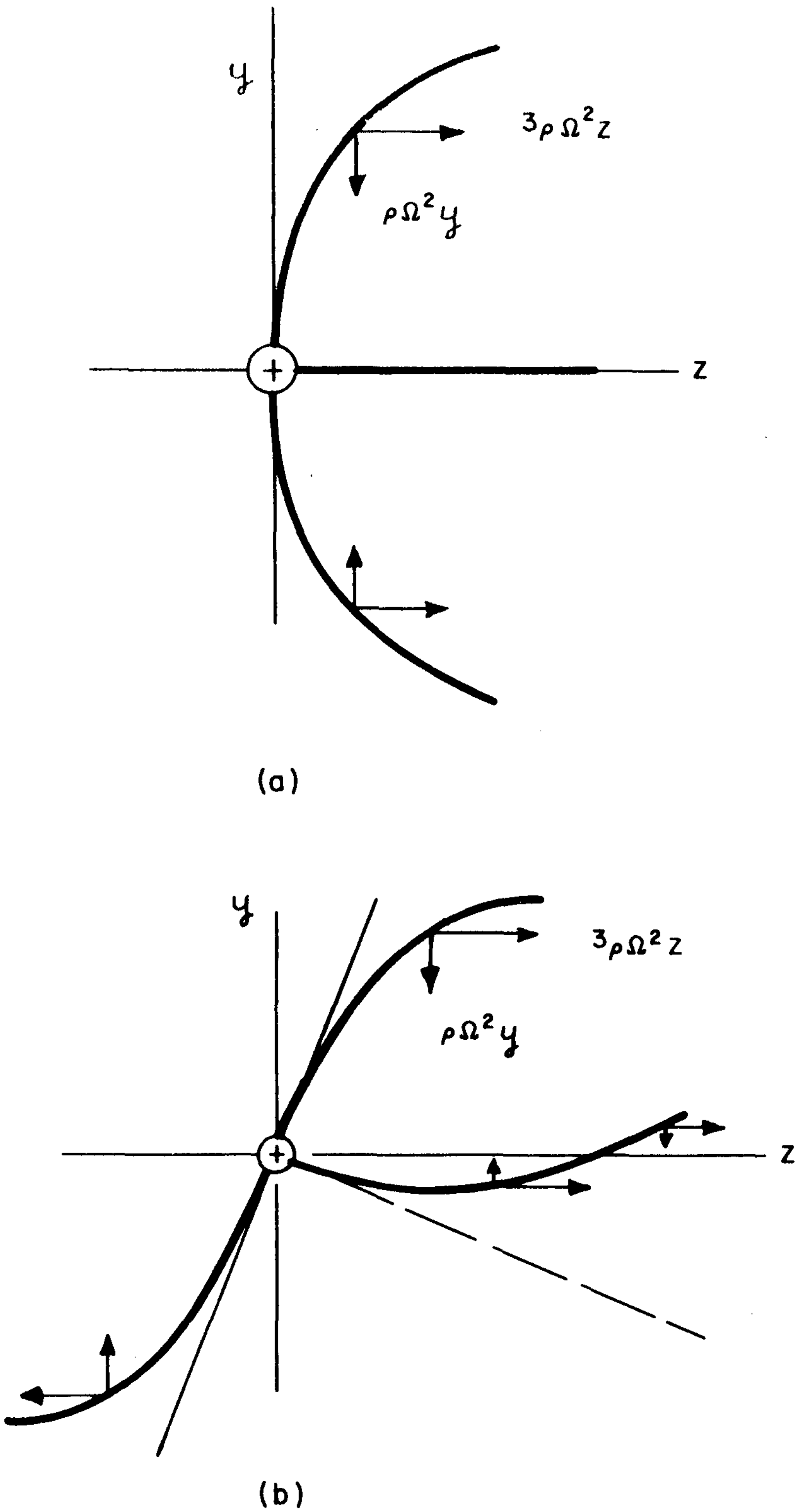

Fig. 4. Stable satellite orientations for large deflections of the y and z booms.

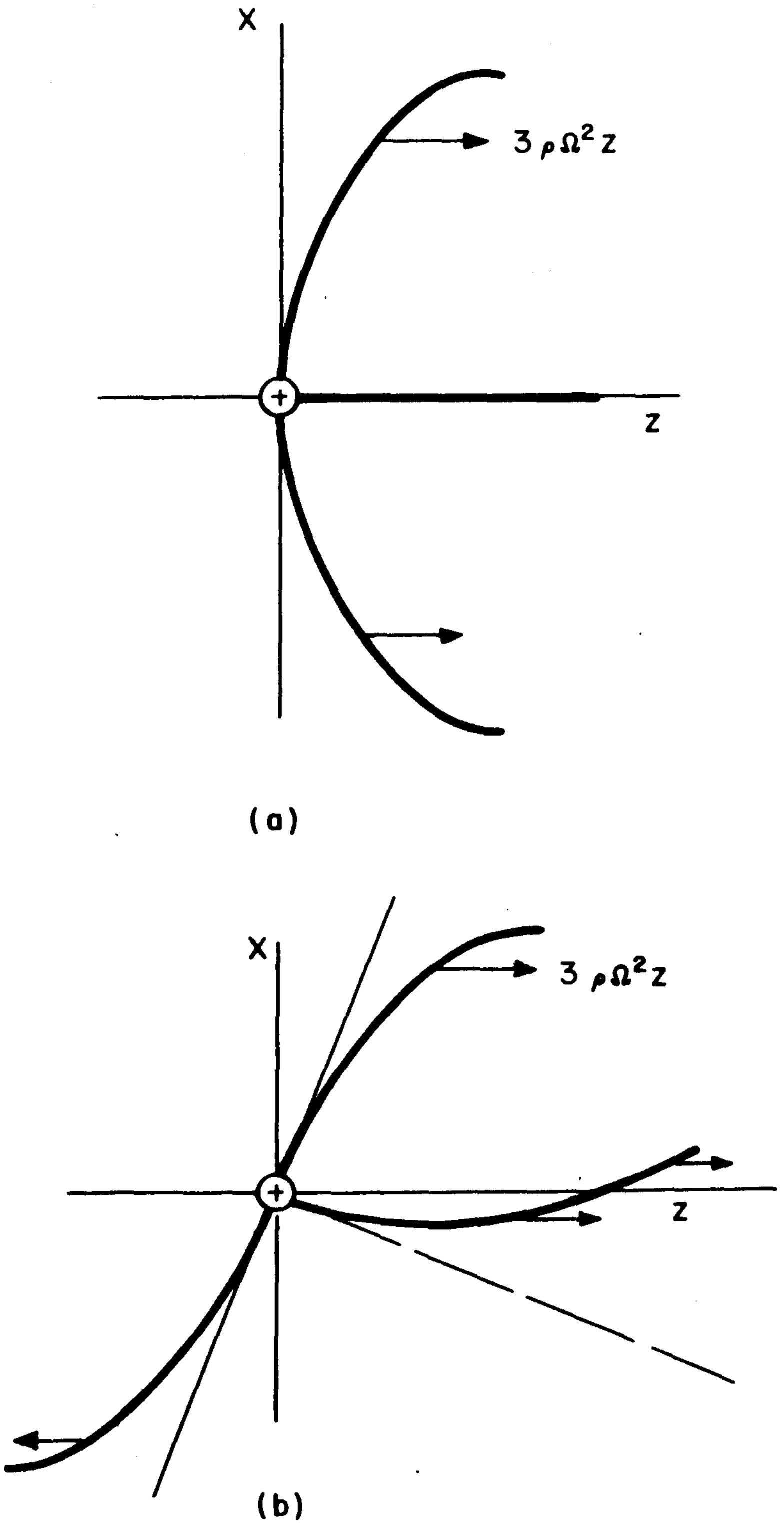

Fig. 5. Stable Satellite orientations for large deflections of the x and z booms.

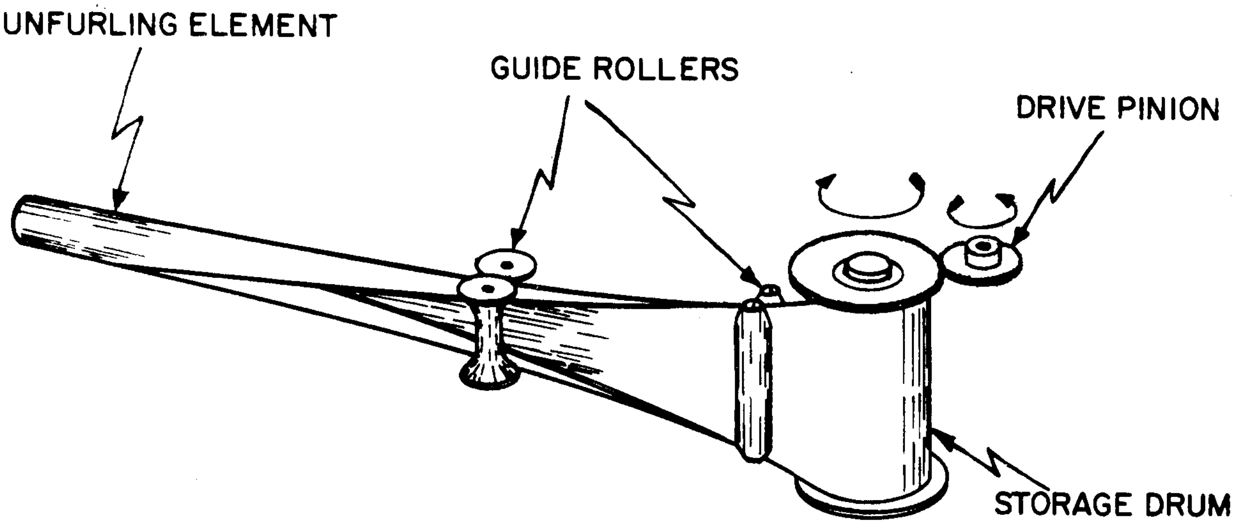

Fig. 6. STEM boom.

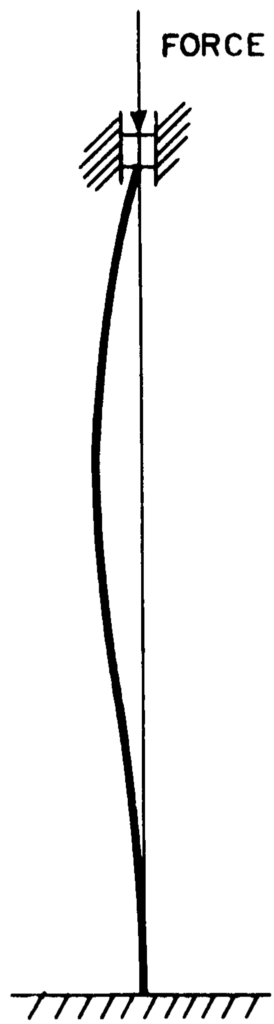

Fig. 7. Buckling test of a STEM boom.

3. ELASTIC STABILITY OF THE BOOMS

From small deflection theory the differential equations for the shape of a boom are[2]

$$\frac{dT}{dy} + Y = 0 \qquad .....(2)$$

$$EI\,\frac{d^4z}{dy^4} - \frac{d}{dy}\left(T\,\frac{dz}{dy}\right) - Z - EI\,\frac{d^2K_T}{dy^2} = 0 \qquad .....(3)$$

where y is the coordinate along the undeflected axis of the boom, z is the deflection, Y and Z are the body forces in the y and z directions, respectively, EI is the section modulus, T is the internal tension and K_T is the thermal curvature of the boom. Also, the constitutive equation, which relates the geometric and thermal curvatures and the internal moments in the boom, is

$$\frac{d^2z}{dy^2} = \frac{M}{EI} + K_T(y) \qquad .....(4)$$

where M is the internal moment.

In determining the elastic stability of the booms, the thermal and inherent boom curvatures will be neglected. A qualitative discussion of the effect of these curvatures will be given later.

3.1 ELASTIC STABILITY OF THE Y BOOMS

Consider the y set of booms as depicted in Figure 4. Here, $Y = -\rho\Omega^2 y$, $Z = 3\rho\Omega^2 z$. Equation (2) may be integrated immediately to yield

$$T = \frac{-\rho\Omega^2}{2}\,(\ell^2 - y^2) \qquad .....(5)$$

for the tension in a boom of length ℓ. Assume that the thermal curvature is zero along the length of the boom. Equation (3) then becomes,

$$EIz'''' + \left[\rho\,\frac{\Omega^2}{2}\,(\ell^2 - y^2)z'\right]' - 3\rho\Omega^2 z = 0 \qquad .....(6)$$

where the superscript primes denote differentiation with respect to the coordinate y.

Introducing the non-dimensional variables $\varepsilon = y/\ell$ and $N = z/\ell$ into equation (6) yields

$$N'''' + b\left\{\frac{1}{2}\,\left[(1 - \varepsilon^2)\,N'\right]' - 3N\right\} = 0 \qquad .....(7)$$

where $$b = \rho\Omega^2\ell^4/EI. \qquad .....(8)$$

In the absence of thermal curvature (i.e., $k_T = 0$) the geometric and static boundary conditions for the booms as shown in Figure 3 are, respectively,

$$\left.\begin{aligned} N(0) &= N'(0) = 0 \\ N''(1) &= N'''(1) = 0 \end{aligned}\right\} . \qquad \text{.....(9)}$$

Equation (7) with boundary conditions (9) defines an eigenvalue problem. The first eigenvalue, which will be denoted as b_{cr}, will supply the critical stability condition, in accordance with the theory of buckling. Equation (7) is not readily solvable in terms of known functions, but an approximate solution may be obtained by transforming the equation into a variational problem and then employing the Ritz method[3,4]. The transformation to a variational problem is accomplished by multiplying the equation by N and integrating over the length of the boom to obtain

$$\int_0^1 N\left[N'''' + b\left\{\left[\frac{1}{2}(1-\varepsilon^2)N'\right]' - 3N\right\}\right]d\varepsilon = 0. \qquad \text{.....(10)}$$

If the term in N'''' is integrated by parts twice and the term in N' once, the result is

$$\psi = \int_0^1 \left\{(N'')^2 - b\left[\frac{1-\varepsilon^2}{2}(N')^2 + 3N^2\right]\right\} d\varepsilon = 0. \qquad \text{.....(11)}$$

In physical meaning equation (11) represents the principle of 'conservation of potential energy' during buckling. The first term is the strain energy of the boom and the second the potential energy or work of the conservative force field of equation (1).

In the present problem a trial function

$$N = a_1\varepsilon^2 + a_2\varepsilon^3 \qquad \text{.....(12)}$$

shall be selected in accordance with the procedure of the Ritz method. Constructing the minimization of ψ leads to

$$\frac{\partial\psi}{\partial a_m} = \int_0^1 \left\{2N''\frac{\partial N''}{\partial a_m} - b\left[(1-\varepsilon^2)N'\frac{\partial N'}{\partial a_m} + 6N\frac{\partial N}{\partial a_m}\right]\right\}d\varepsilon = 0 \qquad \text{.....(13)}$$

where $m = 1,2$.

Performing the integrations and algebraic manipulations yields

$$\left.\begin{aligned} (4 - .867b)a_1 + (6 - .75b)\,a_2 &= 0 \\ (6 - .75b)a_1 + (12 - .686b)a_2 &= 0 \end{aligned}\right\} . \qquad \text{.....(14)}$$

Hence, for a non-trivial solution, the determinant of equations (14) must equal zero. Expanding the determinant and solving for the two roots for b gives

$$b_1 = 2.967 \text{ and } b_2 = 127.3.$$

Then, $$b_{cr} \simeq b_1 = 2.967. \qquad \text{.....(15)}$$

Alternately, the critical condition may be written (for the y set of booms) as

$$\ell_{cr_y} = 1.314 \left(\frac{EI}{\rho\Omega^2}\right)^{1/4}. \qquad .....(16)$$

In general the Ritz method gives a first eigenvalue which is one to two per cent high.

3.2 ELASTIC STABILITY OF THE X BOOMS

For the x set of booms which are subjected to the forces X = 0 and Z = $3\rho\Omega^2 z$, equations (2) and (3) reduce to

$$N'''' - 3bN = 0 \qquad .....(17)$$

in terms of the non-dimensional variables $\varepsilon = x/\ell$ and $N = z/\ell$. The boundary conditions are

$$N(0) = N'(0) = N''(1) = N'''(1) = 0. \qquad .....(18)$$

Equations (17) and (18) are again the formulation of an eigenvalue problem. In this case the exact value of b_{cr} may be obtained as

$$b_{cr} = 4.12^*. \qquad .....(19)^3$$

The critical length for the x set of booms is then

$$\ell_{cr_x} = 1.425 \left(\frac{EI}{\rho\Omega^2}\right)^{1/4}. \qquad .....(20)$$

4. THE STABILITY OF THE RIGID BODY EQUILIBRIUM ORIENTATION

The stability of the rigid body equilibrium orientation shown in Figure 1 is dependent on the relative lengths of the booms. To determine the stability conditions, the satellite is given a small angular displacement β about an axis as shown in Figure 8. With this satellite orientation the forces on the y and z booms produce moments M_y and M_z, respectively, on the satellite. The moment M_y tends to increase the angular displacement and the moment M_z tends to decrease the angular displacement. The equilibrium orientation of the satellite in the undeflected shape will be stable for small angular displacements about the x-axes if $M_z > M_y$. In terms of the boom curvatures the stability condition is

$$|N_z''(0)| > 2\,|N_y''(0)|. \qquad .....(21)$$

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* The corresponding Ritz technique gives a value of 4.16, when a trial function $N = a_1\varepsilon^2 + a_2\varepsilon^3$ is assumed.

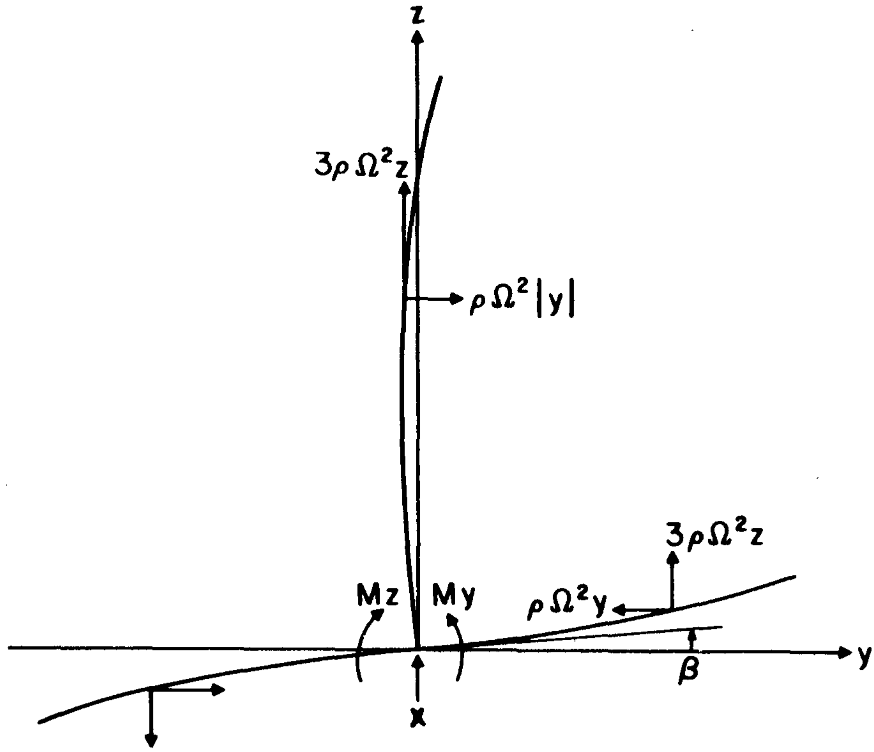

Fig. 8. Moments on the satellite body resulting from a small rotation about the x-axis.

The curvatures $N_y''(0)$ and $N_z''(0)$ may be calculated as follows. The equation of the y boom, as given previously is

$$N'''' + b\left\{\left[\frac{1}{2}(1-\varepsilon^2)N'\right]' - 3N\right\} = 0. \qquad .....(7)$$

The boundary conditions, as shown in Figure 8, are

$$N(0) = N''(1) = N'''(1) = 0 \;;\; N'(0) = \beta. \qquad .....(22)$$

Equation (7) cannot be solved readily in terms of known functions, but for $b < 1$ a perturbation technique may be employed. Assume a solution of the form

$$N = N_0 + bN_1 + b^2N_2 + ... \qquad .....(23)$$

The expansion may be expected to converge rapidly for small values of b. Substitution of equation (23) into equation (7), yields

$$\begin{aligned} N_0'''' &+ b\left[N_1'''' + \frac{1}{2}(1-\varepsilon^2)N_0'' - \varepsilon N_0 - 3N_0\right] \\ &+ b^2\left[N_2'''' + \frac{1}{2}(1-\varepsilon^2)N_1'' - \varepsilon N_1 - 3N_1\right] \\ &+ b^3 \;...... \qquad\qquad = 0. \qquad .....(24) \end{aligned}$$

For arbitrary values of b, each coefficient of b must vanish independently. This requirement provides the following set of equations which enable $N_0, N_1, \ldots$ ...to be calculated successively:

$$N_0'''' = 0 \qquad \text{.....(25a)}$$

$$N_1'''' + \frac{1}{2}(1 - \varepsilon^2)\, N_0'' - \varepsilon N_0' - 3N_0 = 0. \qquad \text{.....(25b)}$$

The solution of equation (25a) chosen to satisfy the boundary conditions is

$$N_0 = \beta\varepsilon. \qquad \text{.....(26)}$$

Substitution of equation (26) into (25b), yields a differential equation for N_1. The boundary conditions of the problem are satisfied by N_0; therefore, N_1 must satisfy zero boundary conditions, i.e.,

$$N_1(0) = N_1'(0) = N_1''(1) = N_1'''(1) = 0.$$

The solution of N_1 with these boundary conditions is

$$N_1 = (\beta/30)(\varepsilon^5 - 10\varepsilon^3 + 20\varepsilon^2). \qquad \text{.....(27)}$$

The solution for N_y using the first two terms of the series is

$$N_y = \beta\left[\varepsilon + (b/30)(\varepsilon^5 - 10\varepsilon^3 + 20\varepsilon^2)\right]. \qquad \text{.....(28)}$$

This equation is valid for $b \leq 0.1$. The boom curvature, $N_y''(0)$, is obtained by differentiation of equation (28) as

$$N_y''(0) = \frac{4}{3}\beta b_y. \qquad \text{.....(29)}$$

The equation of position of the z booms is derived from equations (2) and (3) with

$$Y = -\rho\Omega^2 y \text{ and } Z = 3\rho\Omega^2 z$$

$$EI\frac{d^4y}{dz^4} - \frac{3}{2}\rho\Omega^2(\ell^2 - y^2)\frac{d^2y}{dz^2} + 3\rho\Omega^2\frac{dy}{dz} + \rho\Omega^2 y = 0. \qquad \text{.....(30)}$$

In the non-dimensional variables $N = y/\ell$ and $\varepsilon = z/\ell$ the above equation becomes

$$N'''' - \frac{3}{2}b(1 - \varepsilon^2)N'' + 3b\varepsilon N' + bN = 0. \qquad \text{.....(31)}$$

The boundary conditions are identical to equation (22). The solution of equation (31) may be obtained in a manner similar to the above solution of equation (7). The deflection N_z becomes

$$N_z = \beta\left[\varepsilon - \frac{b}{30}(\varepsilon^5 - 10\varepsilon^3 + 20\varepsilon^2)\right] \qquad \text{.....(32)}$$

which gives the curvature $N_z''(0)$ as

$$N_z''(0) = -\frac{4}{3}\beta b_z. \qquad \text{.....(33)}$$

From equations (21) (29) and (33), it is found that the equilibrium orientation of the satellite in the undeflected shape will be stable for

$$b_z > 2b_y \qquad \text{.....(34)}$$

or in terms of the boom lengths

$$\ell_z > 1.19\ \ell_y. \qquad \text{.....(35)}$$

In a similar manner the stability criteria for small rotations about the y and x axes are found to be

$$\ell_z > 1.19\ \ell_x \qquad \text{.....(36)}$$

and

$$\ell_x > \ell_y \qquad \text{.....(37)}$$

respectively. The combined stability criterion is then

$$\ell_z > 1.19\ \ell_x > 1.19\ \ell_y. \qquad \text{.....(38)}$$

This result does not differ significantly from the following stability criterion for a completely rigid satellite

$$\ell_z > 1.26\ \ell_x > 1.26\ \ell_y \qquad \text{.....(39)}$$

(i.e., the satellite will then be stable in accordance with the results of Ref. 5).

5. OTHER FACTORS INFLUENCING BOOM SHAPE

In addition to the gravity and centrifugal forces other factors may influence the shape of the booms. In this section some of these will be analyzed for the y set of booms for boom lengths below ℓ_{cr}.

5.1 THE EFFECT OF THERMAL OR INHERENT CURVATURE

Consider the y set of booms. If they have an inherent curvature, or are subject to solar heating, they will be deflected. Assume that the induced curvature is a constant (K_τ) along the length of the boom[2]. Equation (4) then gives the boundary condition at $y = \ell$.

$$\frac{d^2 z(\ell)}{dy^2} = K_\tau \qquad \text{.....(40)}$$

since, at $y = \ell$ the internal moment M is zero. Equation (40) in non-dimensional form is

$$N''(1) = K_\tau. \qquad \text{.....(41)}$$

The additional boundary conditions in non-dimensional form are

$$N(0) = N'(0) = N'''(1) = 0. \qquad \text{.....(42)}$$

The equation of position of the boom is deduced from equations (2) and (3) with $d^2K_\tau/dy^2 = 0$, and is therefore identical to equation (7)

$$N'''' + \frac{1}{2} b \ (1 - \varepsilon^2) \ N'' - b\varepsilon N' - 3bN = 0. \qquad \text{.....(43)}$$

A perturbation solution of equation (43) with the boundary conditions (41) and (42) gives

$$N = K_\tau \ell \left[\frac{\varepsilon^2}{2} + \frac{b}{240} \ (2\varepsilon^6 - 5\varepsilon^4 - 20\varepsilon^3 + 60\varepsilon^2)\right]. \qquad \text{.....(44)}$$

The deflection at the tip of the boom is then

$$N(1) = K_\tau \ell \left(\frac{1}{2} + \frac{37}{120} b\right). \qquad \text{.....(45)}$$

5.2 THE EFFECT OF A SMALL SLOPE

As a characteristic of the mechanical design, the booms may extend from the satellite with a small slope β. The boundary conditions are then

$$N(0) = N''(1) = N'''(1) = 0; \ N'(0) = \beta. \qquad \text{.....(46)}$$

A perturbation solution of equation (43) with these boundary conditions gives the boom shape as

$$N = \beta \left[\varepsilon + \frac{6}{30} \ (\varepsilon^5 - 10\varepsilon^3 + 20\varepsilon^2)\right] \qquad \text{.....(47)}$$

and the tip deflection as

$$N(1) = \beta\left(1 + \frac{11}{30} b\right). \qquad \text{.....(48)}$$

5.3 THE EFFECT OF A SMALL DISPLACEMENT

If the axis of the antenna set is displaced from the mass centre of the satellite, the boundary conditions are then

$$N'(0) = N''(1) = N'''(1) = 0; \ N(0) = S. \qquad \text{.....(49)}$$

The solution of equation (43) with these boundary conditions is

$$N = S \left[1 + \frac{b}{8} \ (\varepsilon^4 - 4\varepsilon^3 + 6\varepsilon^2)\right] \qquad \text{.....(50)}$$

and the tip deflection is

$$N(1) = S \left(1 + \frac{3}{8} b\right). \qquad \text{.....(51)}$$

Since equation (7) is linear the various solutions of the boom shape given in equations (28), (44), (47), and (50) for small values of b may be superimposed.

The preceding analyses apply to the y set of booms. Similar results may be easily obtained for the x set of booms. The tip deflections for the x booms will be smaller, as is evident by comparing the force systems of Figures 4 and 5.

If $\ell > \ell_{cr}$ (elastic instability) the booms will undergo large deflections. In this instance the use of large deflection theory is required in order to predict the boom shape[3,6].

6. DISCUSSION

In the preceding analyses, it has been assumed that the booms are closed tubes whereas the STEMs are actually open overlapped tubes. Tubes of overlap construction have a lower 'effective' bending stiffness than closed tubes of equal diameter, because of the possibility of bending-torsion interactions. Also the idealized boundary conditions utilized in the analyses are not likely to be found in practice. Hence, the critical lengths predicted herein are subject to some error, and are most likely optimistic values.

By combining equations (16) and (A-11), the formula for the critical lengths may be written in the form,

$$\ell_{cr} \sim \left(\frac{EI}{\rho GM}\right)^{\frac{1}{4}} R_0^{3/4}. \qquad \text{.....(52)}$$

This equation indicates that the calculated critical lengths will reflect uncertainties in the above discussed bending stiffness, only in proportion to the one-fourth power of EI. Hence the theory developed in this report is expected to give meaningful numerical results.

It is evident also, from equation (52) that the lengths at which the distortion due to gravity is significant decrease in proportion to the three-quarters power of the orbit radius R_0.

7. APPLICATION OF THE RESULTS TO EXISTING SATELLITE BOOMS

Properties of STEMs of 1/2 inch diameter beryllium copper (BeCu) and 1 inch diameter steel material (presently utilized on Alouette and ISIS satellites) are listed in Table 1.

The critical lengths of BeCu booms projected in the x and y directions, respectively, are as obtained from equations (20) and (16),

$$\ell_{cr_x} = 625 \text{ ft} \quad \ell_{cr_y} = 576 \text{ ft}.$$

These values apply to a circular orbit at 1000 Km. The corresponding lengths of steel antennas are,

$$\ell_{cr_x} = 915 \text{ ft} \quad \ell_{cr_y} = 845 \text{ ft}.$$

These results are included in Figure 9, which shows the calculated values of b versus boom length.

TABLE 1

Boom Parameters

	BeCu	Steel
Mass, slugs/ft	.000449	.00212
Tape width, inches	2.0	4.0
Thickness, inches	.0020	.005
Diameter, inches	.500	1.0
E, lb/in^2	18.5×10^6	30×10^6
EI average, $lb\text{-}ft^2$	15.5	351
K_τ ft^{-1}	.002	.0064

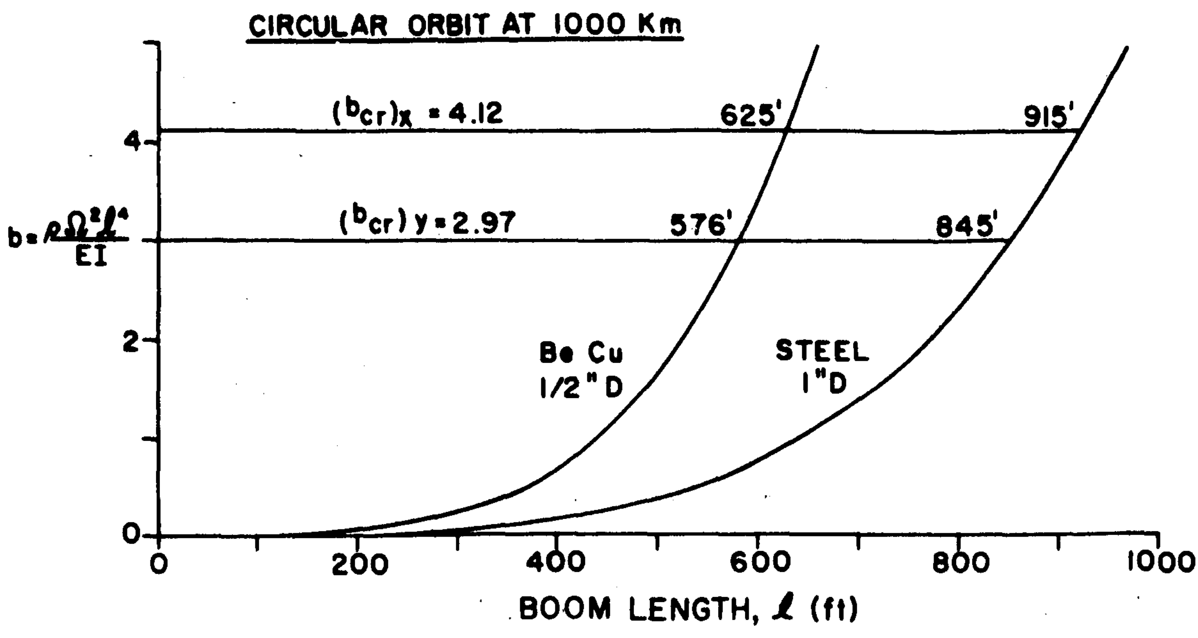

Fig. 9. Parameter b versus boom length.

STEMs on the present design of ionospheric sounding satellites are of the order of 100 ft in length, for which $b < .01$. Thus these booms are not likely to be subjected to buckling instabilities, or appreciable deformation effects as described in Section 5.

The influence of the orbit radius on the critical length for the y booms is shown in Figure 10.

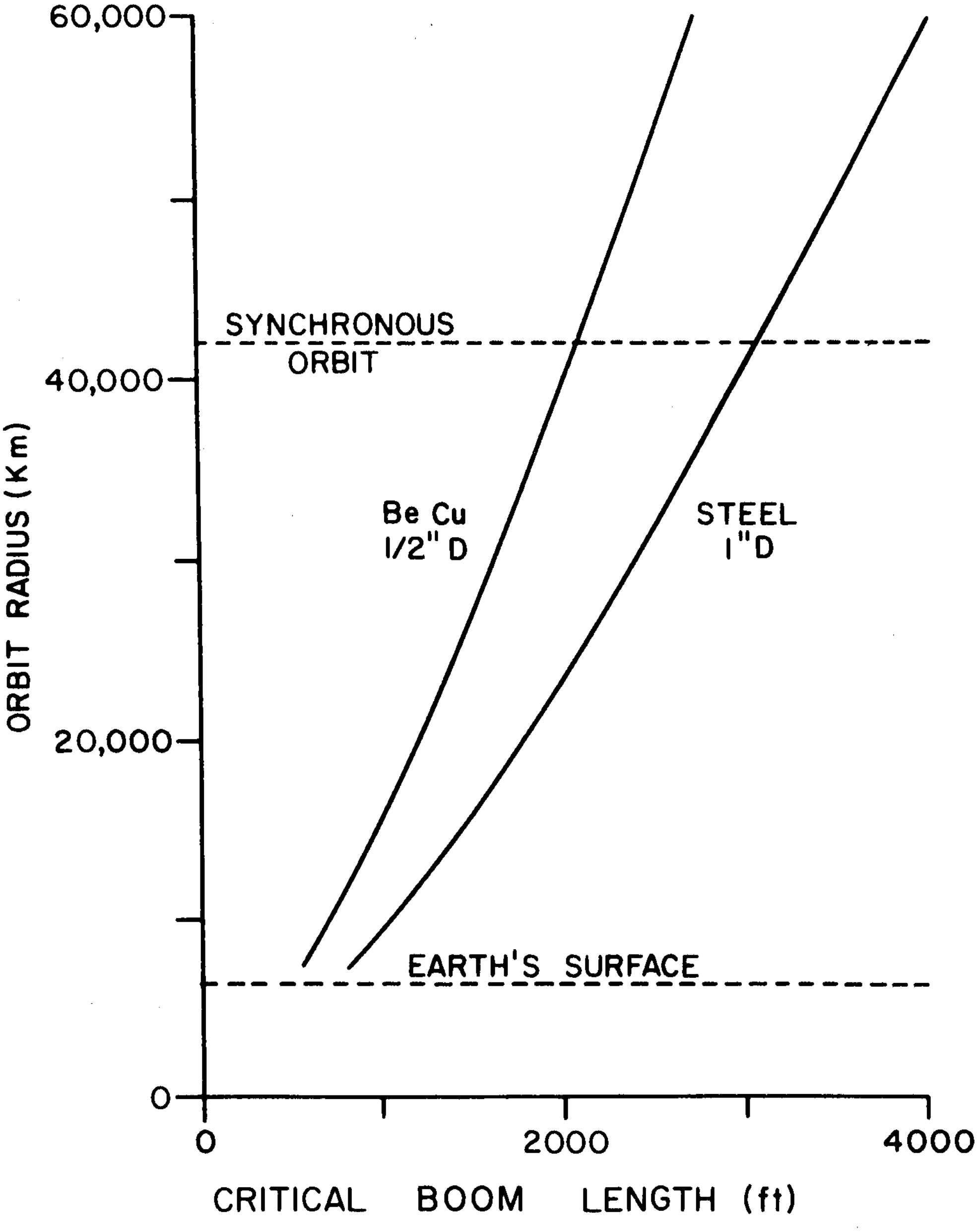

Fig. 10. Orbit radius versus critical length of the y booms.

8. CONCLUSIONS

The sounder antennas of a gravity-stabilized Alouette type satellite can exhibit elastic instability in the form of boom buckling under the influence of gravity and centrifugal forces.

The critical length above which a sounder boom will exhibit elastic instability is proportional to

$$\left(\frac{EI}{\rho GM}\right)^{\frac{1}{4}} R_0^{\;3/4}$$

i.e., the boom properties, the gravitational constant and the orbit radius.

For BeCu STEMs employed on present designs of Alouette satellites, an approximate estimate of the critical length is found to be 575 ft for a circular orbit at 1000 Km.

For an Alouette type satellite which is gravity gradient stabilized and which has booms with lengths below the critical, the rigid body equilibrium orientation is stable if $\ell_z > 1.19\ell_x > 1.19\ell_y$.

For lengths above the critical, large boom deflections will occur. If the booms are not structurally damaged by the large deflections, the satellite may take up one of several stable configurations.

When small deflection theory applies the boom equilibrium shape may be readily determined, provided its curvature and non-zero boundary conditions at the root are known.

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APPENDIX

FORCES OF DEFORMATION

This appendix evaluates the forces of deformation due to gravity and centrifugal acceleration that act on a mass particle m_i of a flexible continuum of total mass M in a circular orbit. The continuum is in the gravity gradient equilibrium orientation which is obtained when the body maintains a fixed orientation relative to the rotating, orbit referenced coordinate system shown in Figure 2.

First some results will be developed for the flexible continuum when it is not in the gravity gradient equilibrium orientation. Newton's Second Law states that:

$$\underline{F}_i = m_i \frac{d^2 \underline{R}_i}{d\ t^2} \qquad \text{.....(A-1)}$$

where $\underline{F}_i$ is the force external to the particle, t is time and $\underline{R}_i$ is the position vector measured from an inertial frame. For this problem a non-rotating frame with origin at the centre of the earth is sufficient. $\underline{F}_i$ may be divided into the two components $m_i\underline{g}$, the gravity force and $\underline{f}_i$, the internal force exerted by the surrounding continuum. Further, if m_i is referenced to the x y z coordinate system rotating with speed Ω equation (A-1) may be written as

$$m_i\ddot{\underline{R}}_i = \underline{f}_i + m_i\underline{g} - m_i\left[2\underline{\Omega} \times \dot{\underline{R}}_i + \dot{\underline{\Omega}} \times \underline{R}_i + \underline{\Omega} \times (\underline{\Omega} \times \underline{R}_i)\right] \qquad \text{.....(A-2)}$$

where the superscript dots denote partial differentiation with respect to time.

Consider the gravitational force per unit mass

$$\underline{g} = -\ GM\underline{R}_i/R_i^3 \qquad \text{.....(A-3)}$$

where GM is the gravitational constant for the earth. Denoting $\underline{R}_0$ as the vector to the centre of mass of the continuum and $\underline{r}_i$ as a vector from the centre of mass to a mass particle, then

$$\underline{R}_i = \underline{R}_0 + \underline{r}_i . \qquad \text{.....(A-4)}$$

Substitution in equation (A-3) gives

$$\underline{g} = -\frac{GM\ (\underline{R}_0 + \underline{r}_i)}{|\underline{R}_0 + \underline{r}_i|^3} . \qquad \text{.....(A-5)}$$

Let $$\underline{r}_i = x\hat{i} + y\hat{j} + z\hat{k}$$

and $$\underline{R}_0 = R_0\hat{k}$$

then
$$\underline{g} = -\frac{GM(\underline{R}_0 + \underline{r}_i)}{[x^2 + y^2 + (R + z)^2]^{3/2}} = -\frac{GM(\underline{R}_0 + \underline{r}_i)}{R_0^3\left(1 + \frac{2z}{R_0} + \frac{r_i}{R_0^2}\right)^{3/2}} \qquad \text{.....(A-6)}$$

As $r_i << R_0$, equation (A-6) may be approximated by

$$\underline{g} = -\frac{GM}{R_0{}^3}\left(\underline{R}_0 - \frac{3z}{R_0}\underline{R}_0 + \underline{r}_i\right) \qquad \text{.....(A-7)}$$

in which terms of order $r_i{}^2/R_0$ and higher have been neglected.

Substitution of equation (A-7) in equation (A-2) gives

$$m_i(\ddot{\underline{R}}_0 + \ddot{\underline{r}}_i) = \underline{f}_i - m_i\left[\frac{GM}{R_0^3}\left(\underline{R}_0 - \frac{3z}{R_0}\underline{R}_0 + \underline{r}_i\right)\right.$$
$$+ 2\underline{\Omega} \times \dot{\underline{R}}_0 + \dot{\underline{\Omega}} \times \underline{R}_0 + \underline{\Omega} \times (\underline{\Omega} \times \underline{R}_0) + 2\underline{\Omega} \times \underline{r}_i$$
$$\left. + \dot{\underline{\Omega}} \times \underline{r}_i + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_i)\right]. \qquad \text{.....(A-8)}$$

Assuming that $m_i \neq m_i(t)$, then summing over the total mass of the body yields

$$\ddot{\underline{R}}_0 = -\left[\frac{GM}{R_0^3}\underline{R}_0 + 2\underline{\Omega} \times \dot{\underline{R}}_0 + \dot{\underline{\Omega}} \times \underline{R}_0 + \underline{\Omega} \times (\underline{\Omega} \times \underline{R}_0)\right] \qquad \text{.....(A-9)}$$

since

$$\sum_i \underline{f}_i = \sum_i m_i z = \sum_i m_i\underline{r}_i = \sum_i m_i\dot{\underline{r}}_i = \sum_i m_i\ddot{\underline{r}}_i = 0.$$

Equation (A-9) describes the well known Keplerian elliptical orbit. For investigation of gravity gradient satellites, the circular orbit solution,

$$\underline{R} = R_0\hat{k} \qquad \text{.....(A-10)}$$
$$\underline{\Omega} = \Omega\hat{j} = \left(\frac{GM}{R_0^3}\right)^{\frac{1}{2}}\hat{j} \qquad \text{.....(A-11)}$$

where R_0 and Ω are constants, will be utilized.

Equations (A-9), (A-10) and (A-11) are well known results for an orbiting mass point. The preceding development shows that they also apply approximately for a flexible satellite, provided the satellite's dimensions are small compared to the orbit radius. It should also be noted that the effects of the oblateness of the earth are ignored in this presentation.

The above results will now be used to determine the gravitational and centrifugal forces of deformation acting on the continuum, when it is in the gravity gradient equilibrium orientation. Substitution of equations (A-10) and (A-11) into equation (A-8) and applying the condition that the satellite achieves the gravity gradient equilibrium orientation when $\dot{\underline{r}}_i = \ddot{\underline{r}}_i = 0$ yields

$$\underline{f}_i = m_i \left[\Omega^2\left(-\frac{3z\underline{R}_0}{R_0} + \underline{r}_i\right) + \underline{\Omega} \times (\underline{\Omega} \times \underline{r}_i)\right]. \qquad \text{.....(A-12)}$$

Expanding in component form leads to

$$\underline{f}_i = m_i\Omega^2 \ (y\hat{j} - 3z\hat{k}) \qquad \text{.....(A-13)}$$

which gives the components of the force exerted by the surrounding continuum on the mass particle m_i. The negative of $\underline{f}_i$ is the resultant force of deformation due to gravity and centrifugal acceleration acting on m_i and may be interpreted as a force field referenced to the orbit fixed coordinate system[7]. The force field components x, y, z are then

$$\left.\begin{aligned} X &= 0 \\ Y &= -\rho\Omega^2 y \\ Z &= 3\rho\Omega^2 z \end{aligned}\right\} \qquad \text{.....(1)}$$

where ρ is the density of the continuum.

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