Communications
Research
Centre

RECEPTION OF MULTIPLEXED SIGNALS OVER MULTIPLE CHANNEL AND DIVERSITY SYSTEMS

by

A.R. Kaye and D.A. George

CRC REPORT NO. 1211

DEPARTMENT OF COMMUNICATIONS
MINISTÈRE DES COMMUNICATIONS

CANADA

OTTAWA, MARCH 1970

COMMUNICATIONS RESEARCH CENTRE

DEPARTMENT OF COMMUNICATIONS
CANADA

RECEPTION OF MULTIPLEXED SIGNALS OVER MULTIPLE CHANNEL AND DIVERSITY SYSTEMS

by

A.R. Kaye and D.A. George

(National Communications Laboratory)

CRC REPORT NO. 1211

Published March 1970
OTTAWA

70-080

TABLE OF CONTENTS

ABSTRACT 1

1. INTRODUCTION 1

2. THE SYSTEM MODEL 4

3. GENERAL SOLUTION 6

4. SPECIAL CASES 7
   - 4.1 Single Channel Multiplex 7
   - 4.2 Diversity 9
   - 4.3 Multichannel Systems 10
   - 4.4 Interference in Multiple Terminal Systems 11

5. ADAPTIVE STRATEGIES 11

6. CONCLUSIONS 13

7. REFERENCES 14

APPENDIX A - Discussion of the General Solution 15

APPENDIX B - A Periodic Property of the Output Covariance Function 21

APPENDIX C - Results for Multipath Channels 23

APPENDIX D - Automatic Adjustment of Taps 30

RECEPTION OF MULTIPLEXED PAM SIGNALS OVER MULTIPLE CHANNEL AND DIVERSITY SYSTEMS

by

A.R. Kaye and D.A. George

ABSTRACT

It is well known that the optimum receiver for pulse-amplitude-modulated (PAM) signals transmitted over a single channel consists of a cascade of continuous and transversal filters. This report shows that an extended structure of this type is also optimum for the completely general case of M multiplexed PAM signals transmitted over random time and frequency dispersive, multi-channel systems with I inputs and N outputs, where each output is subjected to arbitrarily correlated additive noise. The random channels may also be arbitrarily correlated. In general, the receiver consists of MN continuous filters and M tapped delay lines with M^2 sets of taps. An adaptive version is also described. The results include as special cases the situations considered by Gonsalves and Tufts[8], Shnidman[5] and Kaye[3]. Also included as special cases are new results for diversity systems, multi-input systems such as multipair cables and multiple terminal systems such as multiple access repeaters in which co-channel interference occurs.

1. INTRODUCTION

It has been known for some time that the linear, least mean square receiving filter for pulse-amplitude-modulated (PAM) signals through a time-dispersive linear communication channel[1,2] is composed of a continuous filter, matched to a single received pulse, followed by a transversal filter, as shown in Figure 1. This allows minimization of the combined effects of additive noise and intersymbol interference and gives the least mean square error for each pulse separately. More recently Kaye[3,4] has shown that a continuous linear filter followed by a transversal filter is also the optimum format for the reception of PAM signals transmitted through linear channels which are both randomly time-variant and time-dispersive. In that case an estimate of the channel response was assumed to be available and the channel was assumed to have wide-sense stationary statistics. Gonsalves and Tufts[8] studied the particular case of a randomly selected channel. Their paper formulated the problem in the frequency domain, suggested a numerical method of solution for the frequency response of the receiver and carried out the solution for a class of random phase and delay problems, but did not discuss any general structures for the receiver.

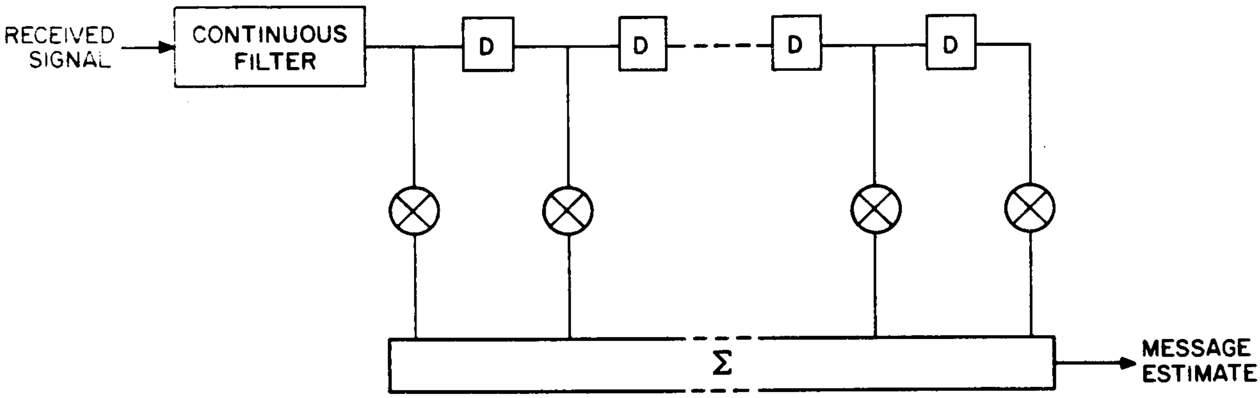

Fig. 1. Optimum receiver for PAM over a time-dispersive channel.

The work of Kaye[3,4] also included multiplex operation in which a linear combination of M PAM signals are transmitted through a single, linear, exactly known or randomly time-variant channel. The receiver consists of a bank of M continuous filters, each followed by a tapped delay line. Taps are taken from each delay line to each of M summers which are sampled for the estimates of each transmitted message signal. Thus M^2 sets of taps are required. Using a different optimization criterion, Shnidman[5] has obtained an identical form of receiver for the exactly known case, although the tap values are different. Shnidman minimized the squared error averaged over all the transmitted signals but added the additional constraint of zero intersymbol interference and crosstalk at the receiver output. Shnidman also first pointed out that intersymbol interference and crosstalk between multiplexed signals are essentially identical phenomena.

All the work referred to above is concerned with transmission over a single channel. The objective of this report is to present the general form of the receiver when both multiple channels, which may be randomly time variant, and multiplex transmissions are used. The previous work, summarized above, was limited to special cases of the results of this report. Diversity systems, in which there are more channel outputs than inputs, are a special case which is presented here for the first time. Another special case, for which the results given here are also new, is the problem of co-channel interference in multiple terminal systems in which each receiving terminal is required to detect only one of many transmitted signals.

The general form of the communications system treated is shown in Figure 2. M message sequences

$$\sum_k \theta_{km} \delta(t - kT), \; m = 1,\ldots,M$$

are transmitted and from these a linear pulse amplitude modulator forms a total of I inputs to the channel, which has N outputs all subject to arbitrarily correlated additive noise. All forms of multiplexed, diversity or multiple channel systems are included in this format. For instance a multipair cable, in which there may be linear crosstalk between channels in addition to time dispersion, is a case where $N = I > 1$. Another example is a diversity system in which $I = 1$ and $N > 1$, with an arbitrary number of multiplexed signals M. In general the receiver is required to estimate all M of the transmitted message sequences. However, in some multiple terminal systems, such as multiple access radio repeaters (including satellites) or where terminals are distributed along a multiple or single channel transmission line such as multipair or coaxial cable, each receiver is required to estimate only one, or some number $L < M$, of the sequences. The undesired sequences then cause co-channel interference. This is a special case of the general result and will be dealt with in Section 4; up to that point we shall discuss the receiver which estimates all M sequences. The receiver minimizes the mean square estimation error on all sequences simultaneously, subject to the constraint that the receiver be linear. That is we minimize

$$e_m = E\left\{(\theta_{km} - \hat{\theta}_{km})^2\right\}, \quad m = 1,\ldots,M. \qquad \ldots\ldots(1)$$

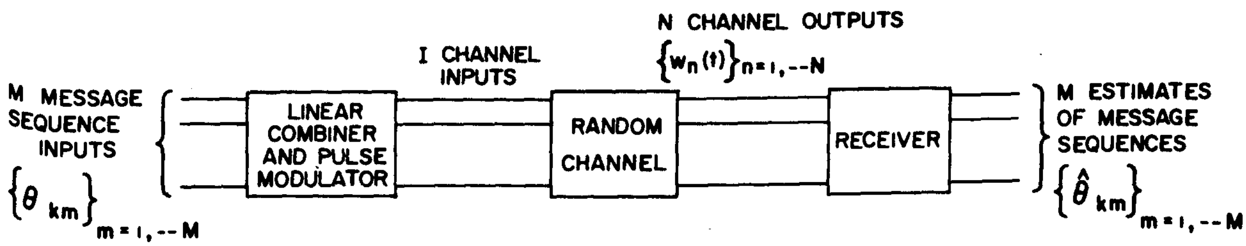

Fig. 2. General form of the communication system.

The optimum receiver is shown to consist of a bank of M continuous filters at each of the N outputs of the channel, followed by a sampled data filter containing M tapped delay lines and M summers with cross-connecting taps. Thus there are MN continuous filters, M delay lines and summers and M^2 sets of taps. The structure for $M = 2$, $N = 3$ is shown in Figure 3. From an extrapolation of the earlier results for single channels it might have been expected that MN delay lines, one for each continuous filter, and hence M^2N sets of taps, would be required. The result that only M delay lines, one for each message sequence, are necessary is therefore somewhat surprising and of considerable practical importance.

Section 2 of the report describes the system model and defines the various functions and parameters needed to express the results. In Section 3 the general solution of the problem is presented. A proof of the solution is included in Appendix A which also includes an analysis and synthesis of the receiver sampled data filter in Z-transform terminology. Appendix B develops a property of the covariance function of the channel output; this property is required in the proof in Appendix A. Section 4 shows how the general solution may be specialized to deal with specific system configurations, in particular: single channel multiplex, diversity systems and multichannel systems. Some numerical results for signalling, using a single sequence, over a random multipath channel, are described in Appendix C. An adaptive version of the general receiver structure is described in Section 5 and additional analysis of this is given in Appendix D.

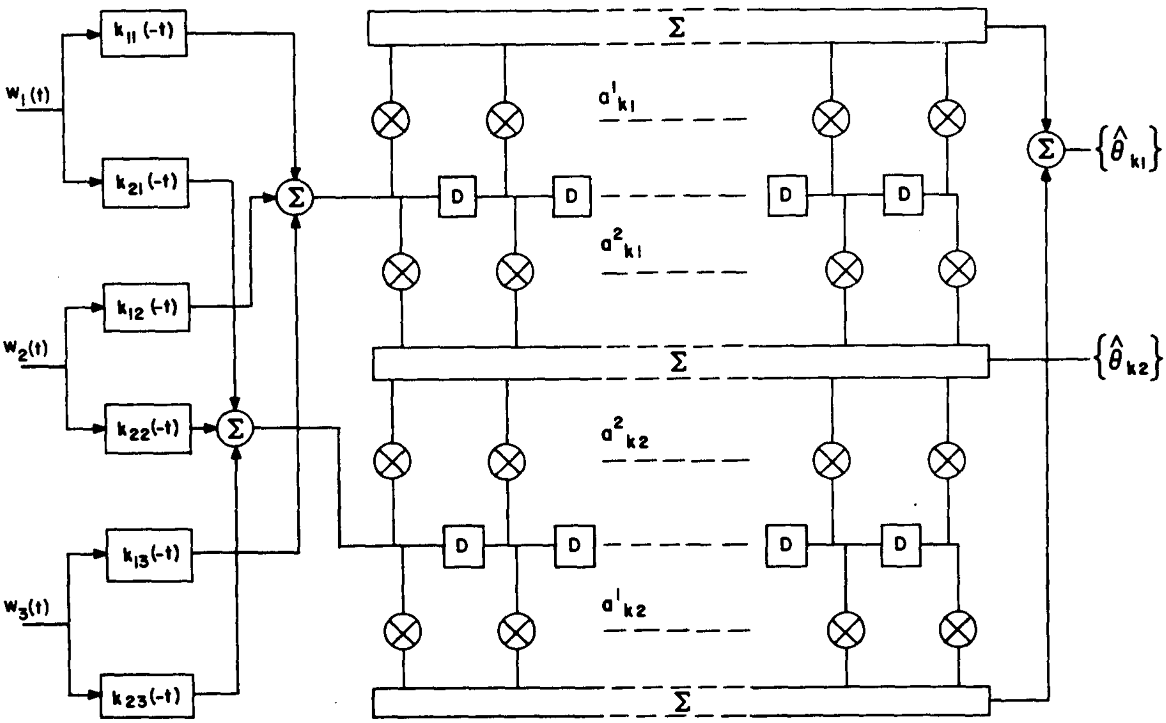

Fig. 3. Optimum receiver for two multiplexed signals and a third order diversity or three channel system.

2. THE SYSTEM MODEL

We are concerned with the transmission of M sequences

$$\sum_k \theta_{km}\delta(t - kT), \; m = 1,\ldots,M$$

by means of a set of PAM multiplex signals of the form

$$s_m(t) = \sum_k \theta_{km} p_m(t - kT), \; m = 1,\ldots,M \qquad \ldots\ldots(2)$$

over a random channel with I inputs and N outputs.

In stating the results it is only necessary to consider the response produced at each output of the channel by each input signal $s_m(t)$. Thus, at this stage, it is not necessary to know whether the M signals are added before transmission over a channel with a single input and single or multiple (diversity) outputs, or whether they each drive a separate input (M pair cable) or, indeed, any combination of these. Only in applying the results to a particular situation is it necessary to consider the specific system used in order to evaluate the functions defined below.

The noise-free response, at the n^{th} output of the channel, to the component $\theta_{km}p_m(t - kT)$ of the transmitted signal is denoted by $\theta_{km}z_{kmn}(t)$. It is assumed that $z_{kmn}(t)$ is the result of the interaction of the signal component with a linear channel, whose time varying weighting function is drawn from a wide-sense stationary ensemble with non-zero mean. This non-zero mean corresponds to the assumption that an estimate $\overline{z}_{mn}(t - kT)$ of the response $z_{kmn}(t)$ is available at the receiver. Thus

$$z_{kmn}(t) = \overline{z}_{mn}(t - kT) + \tilde{z}_{kmn}(t) \qquad .....(3a)$$

where $\tilde{z}_{kmn}(t)$ is of the form

$$\tilde{z}_{kmn}(t) = \int_{-\infty}^{\infty} \tilde{a}_{mn}(t,\alpha)\; p_m(t - kT - \alpha)d\alpha \qquad .....(3b)$$

and $\tilde{a}_{mn}(t,\alpha)$ is a random weighting function. The total random, signal dependant component of the n^{th} output signal is then

$$y_n(t) = \sum_k\sum_{m=1}^{M}\theta_{km}\; \tilde{z}_{kmn}(t). \qquad .....(4)$$

The covariance function of this component and the corresponding component at the ℓ^{th} output is denoted by

$$R_{n\ell}(t,s) = E\{y_n(t)\; y_\ell(s)\},\; n = 1,...,N, \quad \ell = 1,...,N. \qquad .....(5)$$

At each output there may also be an additive noise component $n_n(t)$, $n = 1,...,N$, and these are related by the covariance functions

$$\psi_{n\ell}(t,s) = E\{n_n(t)\; n_\ell(s)\},\; n = 1,...,N,\; \ell = 1,...,N. \qquad .....(6)$$

The total signal at the n^{th} channel output is therefore

$$w_n(t) = \sum_k\sum_{m=1}^{M}\theta_{km}\overline{z}_{mn}(t - kT) + y_n(t) + n_n(t). \qquad .....(7)$$

The message sequences may have arbitrary correlation:

$$E\{\theta_{jq}\theta_{km}\} = \phi_{k-j}^{mq}. \qquad .....(8)$$

A special case occurs when the sequences are white and uncorrelated with each other; in this case

$$\phi_{k-j}^{mq} = \delta_{m-q}\delta_{k-j}. \qquad .....(9)$$

Many systems operating over time-variant channels transmit a test or reference signal together with the information bearing signal. Such a signal might be used to form the signal estimates used here. This use is not considered directly, but the presence of a reference signal results in greater random disturbances at the receiver due to its interaction with the random channel and the system model described can be modified very simply to take account of this. If an estimate of the response of the channel to a signal component, such as $p_m(t)$, is available then the response to a reference signal can also be estimated. Thus at the n^{th} output there is a reference signal $r_n(t)$ which can be expressed as the sum of an estimate $\overline{r}_n(t)$ and a random part $\tilde{r}_n(t)$:

$$r_n(t) = \overline{r}_n(t) + \tilde{r}_n(t), \quad n = 1,\ldots,N.$$

The portion $\overline{r}_n(t)$ can be subtracted at the receiver leaving only the random component to interfere with message estimation. Thus, when a reference signal is used, it is only necessary to redefine the set of covariance functions (5) as

$$R_{n\ell}(t,s) = E\left\{\left[y_n(t) + \tilde{r}_n(t)\right]\left[y_\ell(s) + \tilde{r}_\ell(s)\right]\right\}.$$

3. GENERAL SOLUTION

It is shown in Appendix A that the estimate of θ_{ji}, with the least mean square criterion defined in (1), can be written in the form

$$\hat{\theta}_{ji} = \sum_k \sum_{m=1}^{M} a^i_{km} \sum_{n=1}^{N} \int_{-\infty}^{\infty} k_{mn}\left[t - (k + j)T\right] w_n(t)dt. \qquad \ldots\ldots(10)$$

Thus M filters, with impulse responses $k_{mn}(-t)$, $m = 1,\ldots,M$, at each output operate on the received signals $w_n(t)$, $n = 1,\ldots,N$. The outputs of the m^{th} filters at each of the N channel outputs are summed and used as the input to a delay line tapped at multiples of T seconds delay. To estimate the i^{th} message sequence $\{\theta_{ji}\}$, taps are taken from each delay line leading to the i^{th} summer. This summer is sampled at intervals of T seconds and the outputs at these times are the estimates of the i^{th} message sequence. The receiver structure for two multiplexed signals with third order diversity is shown in Figure 3.

The weighting functions $k_{mn}(-t)$, $m = 1,\ldots,M$, $n = 1,\ldots,N$, of the continuous filters, where $k_{mn}(-t)$ is the m^{th} filter at the n^{th} output, are defined by M sets of N simultaneous Fredholm integral equations:

$$\overline{z}_{mn}(t) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t,s) + \psi_{n\ell}(t,s)\right] k_{m\ell}(s)\, ds \qquad \ldots\ldots(11)$$

$n = 1,\dots,N$, $m = 1,\dots,M$. Thus the solution of the m^{th} set of N equations determines the m^{th} filters at each of the N outputs*.

The tap gains in the sampled data filter are determined by M sets of simultaneous algebraic equations. The set of all taps leading to the i^{th} summer is determined by

$$\phi_k^{mi} = a_{km}^i + \sum_j \sum_{p=1}^M C_{k-j}^{mp} \, a_{jp}^i, \qquad \text{.....(12)}$$

$m = 1,\dots,M$, $k = 0, \pm 1, \pm 2,\dots$

where

$$C_{k-j}^{mp} = \sum_{n=1}^N \sum_v \sum_{q=1}^M \phi_{k-v}^{mq} \int_{-\infty}^{\infty} k_{pn}(s - jT)\, \overline{z}_{qn}(s - vT)\,ds \qquad \text{.....(13)}$$

$m = 1,\dots,M$, $p = 1,\dots,M$. This is an infinite set of equations, reflecting the theoretically infinite length of the delay lines. A solution of these equations, using Z-transforms, is given in Appendix A. Constraining the set to a suitable finite number of taps on each delay line produces a good approximation. In the simpler case of white, uncorrelated sequences substitution of (9) in (12) and (13) gives:

$$\delta_{m-i}\, \delta_k = a_{km}^i + \sum_j \sum_{p=1}^M C_{k-j}^{mp} \, a_{jp}^i, \qquad \text{.....(14)}$$

$m = 1,\dots,M$, $k = 0, \pm 1, \pm 2\dots$

where

$$C_{k-j}^{mp} = \sum_{n=1}^N \int_{-\infty}^{\infty} k_{pn}(s - jT)\, \overline{z}_{mn}(s - kT)\, ds. \qquad \text{.....(15)}$$

The minimized estimation error on the i^{th} sequence is shown in Appendix A to be

$$e_i = a_{0i}^i$$

and, since the sequences have unit variance, this is the error per unit variance.

4. SPECIAL CASES

4.1 SINGLE CHANNEL MULTIPLEX

In a single channel multiplex system we have, referring to Figure 2, $I = N = 1$, $M > 1$. Since $N = 1$ we can write (5) and (6) as

$$R_{n\ell}(t,s) = R(t,s) \qquad \text{.....(16)}$$

$$\psi_{n\ell}(t,s) = \psi(t,s). \qquad \text{.....(17)}$$

* In general Fredholm equations can be solved only by numerical means and this is frequently accomplished by converting the kernel to a matrix by means of an integration formula[9]. Using a similar method a set of simultaneous equations results in a partitioned matrix.

The receiver consists of a bank of M filters $k_m(t)$ related to the M estimated pulse shapes $\bar{z}_m(t)$ by a single set of Fredholm equations:

$$\bar{z}_m(t) = \int_{-\infty}^{\infty} \left[R(t,s) + \psi(t,s) \right] k_m(s)\, ds \qquad \ldots\ldots(18)$$

$m = 1,\ldots,m$. There are M delay lines and M^2 sets of taps, defined by (12) with (13) modified to be:

$$C^{mp}_{k-j} = \sum_v \sum_{q=1}^{M} \phi^{mq}_{k-v} \int_{-\infty}^{\infty} k_p(s - jT)\, \bar{z}_q(s - vT)\, ds. \qquad \ldots\ldots(19)$$

This is the result previously stated in a less specific notation by Kaye[3]. The receiver structure for M = 2 is shown in Figure 4.

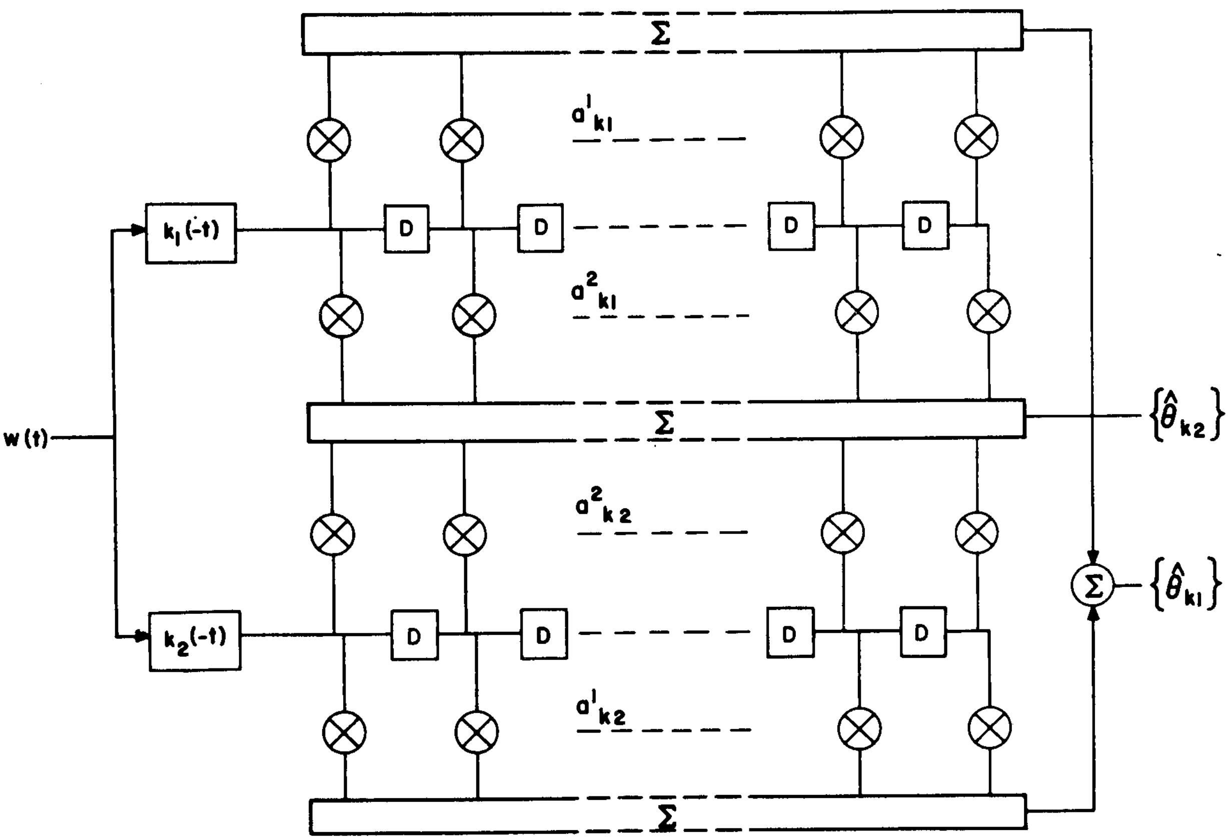

Fig. 4. Optimum receiver for two multiplexed signals over a single output channel.

A subcase is the situation when the channel is exactly known, so that $R(t,s) = 0$, and if the additive noise is white, with spectral density N_0, then (18) becomes

$$k_m(t) = \frac{\bar{z}_m(t)}{N_0}. \qquad \ldots\ldots(20)$$

Thus we have a bank of filters matched to the M exactly known received pulse shapes. This corresponds to the case considered by Shnidman[5] although, as was pointed out in the introduction, Shnidman used a different optimization criterion. The difference between the two results lies in the details of the sampled data filter.

A further subcase is the situation when only a single message sequence ($M = 1$) is transmitted. Clearly the receiver consists of a single filter and a single delay line. Some numerical examples of this problem are solved in Appendix C for a random multipath channel.

4.2 DIVERSITY

For simplicity we shall consider the case of a single message sequence $M = 1$. The method of transmission (value of I) is unimportant; only the number of outputs, N, affects the receiver structure. We have a set of N estimated pulse shapes $\overline{z}_n(t)$, one at each output, and a set of covariance functions as defined in (5) and (6). The receiver has one continuous filter at each channel output, defined by

$$\overline{z}_n(t) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t,s) + \psi_{n\ell}(t,s)\right] k_\ell(s)\, ds \qquad \text{.....(21)}$$

$n = 1,\ldots,N$. The outputs of these filters are summed and sampled, as shown in Figure 5, and used to drive a single tapped delay line. Since there is only a single message sequence $\{\theta_k\}$, (8) becomes:

$$E\left\{\theta_j \theta_k\right\} = \phi_{k-j}. \qquad \text{.....(22)}$$

A single set of taps, leading to a single summer, has gains determined by

$$\phi_k = a_k + \sum_j C_{k-j} a_j \qquad \text{.....(23)}$$

where

$$C_{k-j} = \sum_{n=1}^{N} \sum_v \phi_{k-v} \int_{-\infty}^{\infty} k_n(s - jT)\, \overline{z}_n(s - vT)\, ds. \qquad \text{.....(24)}$$

If the random disturbances in the paths from the transmitter to each diversity output are uncorrelated then (21) becomes a set of independent Fredholm equations. The relationship of this result with more conventional diversity combining can be seen by assuming each path to be exactly known with additive white noise of spectral density N_n at the n^{th} output. In this case we see from (21) that

$$k_n(t) = \frac{\overline{z}_n(t)}{N_n} \qquad \text{.....(25)}$$

so that we have a matched filter at each output which scales each diversity output before summing according to the ratio $\sqrt{\{E_n/N_n\}}$, where

$$E_n = \int_{-\infty}^{\infty} \overline{z}_n^2(t)\, dt.$$

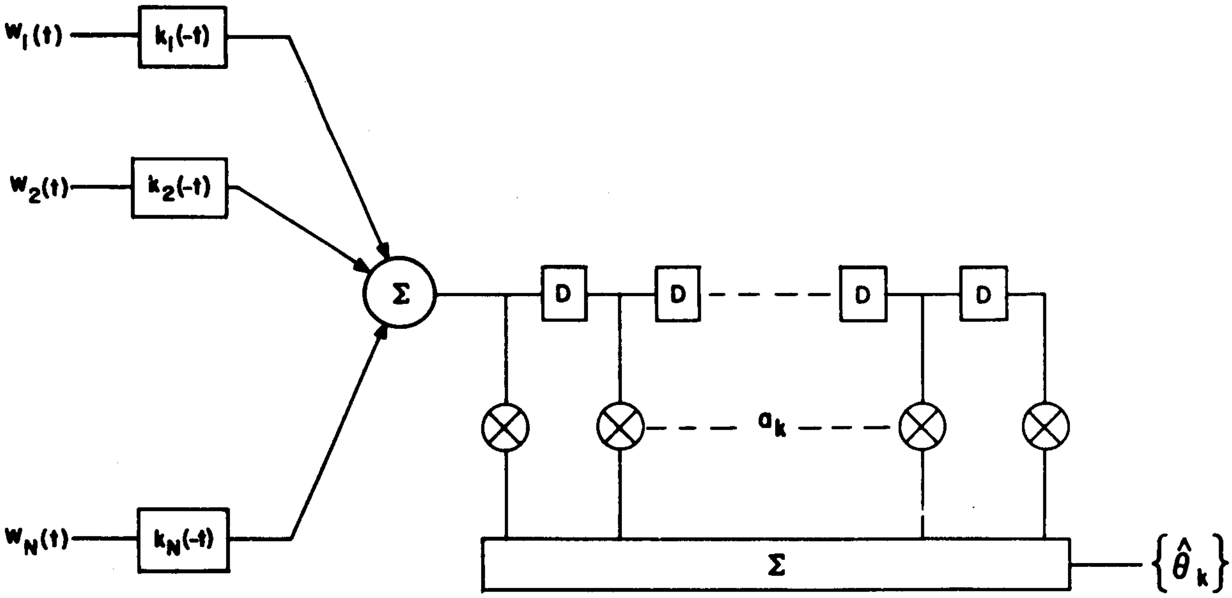

Fig. 5. Optimum receiver for a single message signal and n^{th} order diversity.

4.3 MULTICHANNEL SYSTEMS

In this section we consider the multichannel situation typified by a multipair cable with crosstalk between pairs. Such a cable would not be time varying, except possibly at a very low rate, but, if the transfer and crosstalk functions are not exactly known, might still be random in the sense of being randomly selected.

It has already been pointed out, in Section 2, that the number of inputs and the way in which the various message signals are divided between them is of no consequence in determining the general form of the receiver. Thus a multipair cable with crosstalk can be regarded as a special diversity system in which the number of channel outputs, that is the number of wire pairs, equals the number of message signals, i.e., N = M. Because of crosstalk, each output will carry information about each transmitted signal and this can be used rather than being regarded simply as a nuisance. The equations defining the various filters and tap gains are those given in Section 3. The receiver structure, for the case of a three-pair cable, is as shown in Figure 3.

As an additional illustration of the nature of the receiver it is worth considering the case of exactly known channels. For simplicity we shall also assume white, uncorrelated sequences, and white additive noise with spectral density N_n at the n^{th} output. If the channels are exactly known the filter responses are given by

$$k_{mn}(t) = \frac{\overline{z}_{mn}(t)}{N_n} \quad \text{.....(26)}$$

Thus the filters are matched to the exactly known pulse shapes, arriving at each output from each input. The coefficients in the tap gain equations (12) are:

$$C_{k-j}^{mp} = \sum_{n=1}^{N} \frac{1}{N_n} \int_{-\infty}^{\infty} \bar{z}_{pn}(s - jT)\, \bar{z}_{mn}(s - kT)\, ds. \qquad .....(27)$$

Bearing in mind that the m^{th} filters at all N outputs are summed, we see that C_{k-j}^{mp} is the total covariance of the responses to the m^{th} and p^{th} message sequences, summed over all channel outputs.

4.4 INTERFERENCE IN MULTIPLE TERMINAL SYSTEMS

Multiple terminal systems, in which each receiver is required to estimate only L (< M) of the M transmitted sequences, were introduced in Section 1. Although the general theory developed in Section 2 assumes M synchronous transmissions, it is useful, particularly in multiple terminal systems, to consider the asynchronous case as well and these two situations are dealt with in this section.

If the various transmissions are synchronized, then synchronous, but uncorrelated, crosstalk from undesired signals occurs at each receiving terminal. This situation is identical to the single channel multiplex problem discussed in Section 1 except that each receiver estimates only L < M message sequences. The optimum receiver then consists of M filters and M delay lines but only L summers and the tap gains are determined by (19) and (12). Such a structure is shown in Figure 6, for L = 1. In practice, filters and delay lines would be provided only for those signals which interfere significantly with the desired signals. The structure applies, of course, only when estimates of all of the received pulse shapes exist, or can be obtained, at the receiver. When no estimates of pulse shapes, other than those of the desired signals, are available, the synchronous interference can be treated only as non-stationary additive noise. The optimum receiver then has only L filters and delay lines and the filters are determined by (18) for m = 1,...,L where the desired sequences are $\{\theta_{km}\}$, m = 1,...,L and where R(t,s) includes the autocovariance function of the synchronous interference.

In non-synchronous systems the optimum receiver again has L filters and delay lines, with the filter determined by (18), but the crosstalk now causes stationary additive interference which is included in the function $\psi(t,s)$ instead of in R(t,s) as in the synchronous case.

5. ADAPTIVE STRATEGIES

So far in this report we have been concerned with presenting optimum receiver structures based on the existence of an estimate of the received pulse shapes. A common feature of all these structures is that they contain tapped delay lines which are used to combat intersymbol interference and crosstalk. For the special case of the transmission of a single message over a single, randomly selected or slowly time varying channel, substantial literature

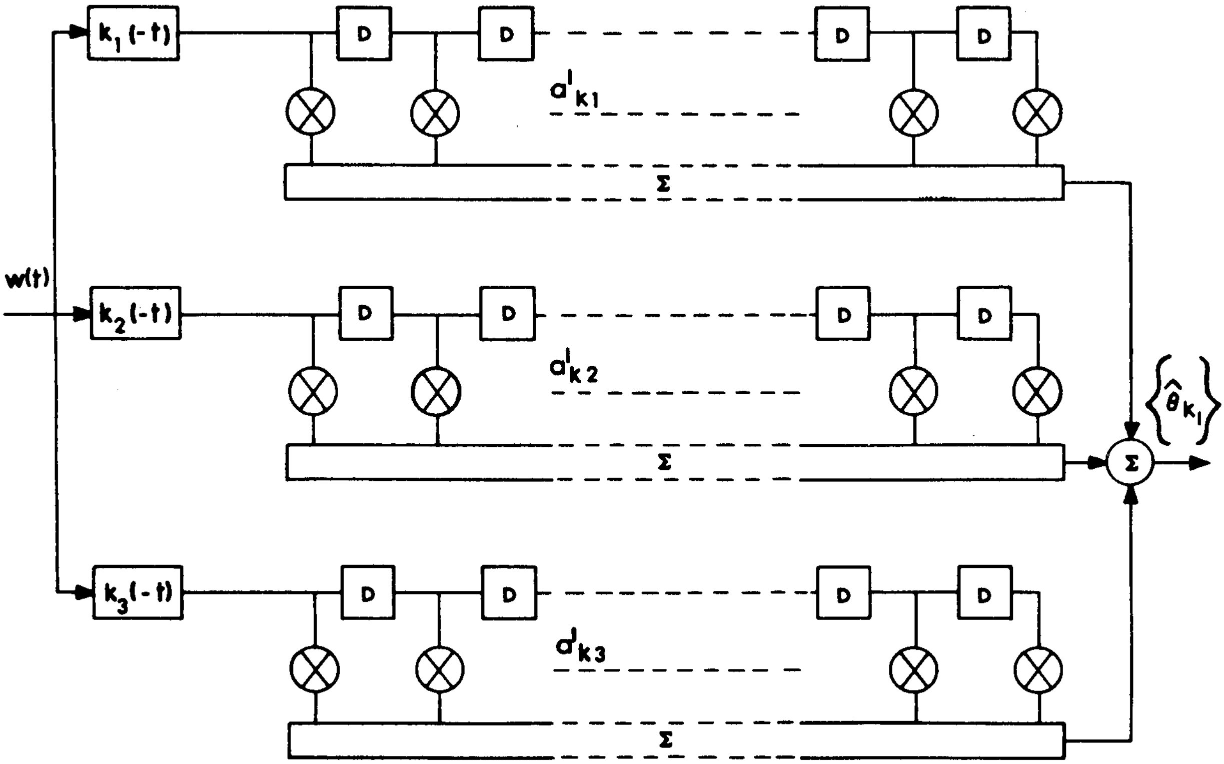

Fig. 6. Optimum receiver for a single message with two synchronous interfering signals.

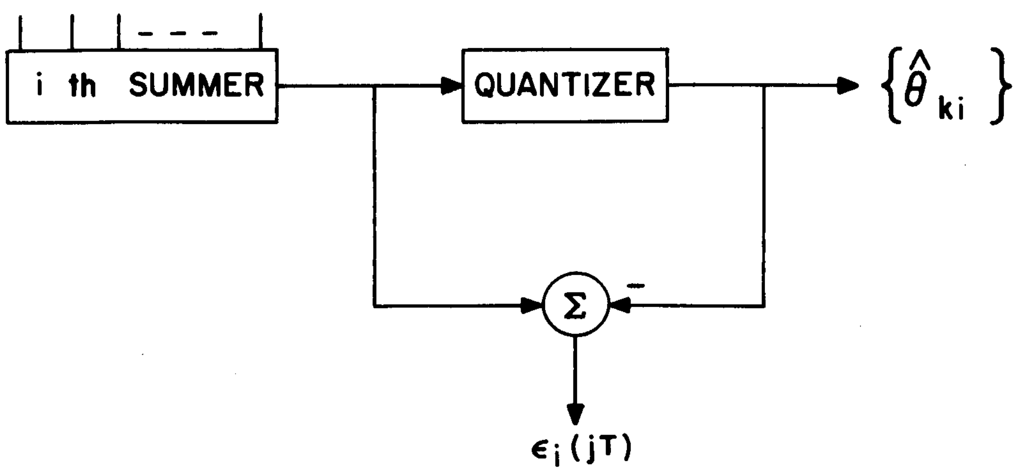

Fig. 7. Formation of the error signal in an adaptive receiver.

exists on methods of making the tapped delay line adaptive. The techniques involved have been summarized and discussed in the tutorial paper by Proakis and Miller[6] who also provide an extensive bibliography. When the transmitted signal is quantized rather than continuous PAM, the most successful techniques depend on the fact that the derivative of mean square error with respect to a tap gain is proportional to the correlation between the signal at the tap and an error signal derived from the output of the receiver. This concept is just as applicable to the multi-delay line structures for crosstalk described here. To be specific, if we form an error signal $\varepsilon_i(t)$ from the i^{th} receiver output as shown in Figure 7, we have

$$\frac{\partial e_i}{\partial a^i_{km}} \propto E\{\varepsilon_i(jT)\ u_{km}(jT)\}$$

where $u_{km}(jT)$ is the signal at the k^{th} tap of the m^{th} delay line at $t = jT$ and the expectation is independent of j because of the stationariness of the system. The right hand side of this equation can be determined by taking a time average of $[\varepsilon_i(jT)\ u_{km}(jT)]$ and the adaptive procedure is started by setting up the receiver with the centre tap on each delay line (a^i_{0i}, $i = 1,\ldots,M$) set to unity and all others to zero. It is shown in Appendix D that a steepest descent convergence to minimum error is then obtained by incrementing each tap in accordance with the sign of the derivative of the error with respect to the tap gain. Thus a^i_{km} is incremented according to the sign of $\partial e_i/\partial a^i_{km}$. A simulation of such a system for three pairs of a multipair cable has recently been carried out successfully by Harrison[7].

6. CONCLUSIONS

It has been shown that an expanded version of the well known continuous filter-tapped delay line structure applies to a completely general PAM system employing multiplex, diversity and multichannel operation simultaneously. With M message signals and N total channel outputs the receiver contains MN continuous filters, M delay lines, L summers and ML sets of taps where L of the M sequences are to be estimated. This is independent of the number of channel inputs so that a multichannel system, such as a multipair cable, may be regarded as a type of diversity system.

Adaptive strategies similar to those used for single-message, single-channel systems may be used.

7. REFERENCES

1. George, D.A. *Matched filters for interfering symbols.* Trans. IEEE, Vol. IT-11, pp 153-154, January 1965.

2. Tufts, D.W. *Nyquist's problem - the joint optimization of transmitter & receiver in pulse-amplitude modulation.* Proc. IEEE, Vol. 53, pp 248-259, March, 1965.

3. Kaye, A.R. *Reception of pulse-amplitude modulated symbols transmitted over a dispersive fading channel with nonstationary statistics.* Proc. IEEE, Vol. 56, pp 1087-1088, June 1968.

4. Kaye, A.R. *Reception of digital signals over randomly time variant dispersive channels.* Ph.D. thesis, Carleton University, Ottawa, Canada, April 1968.

5. Shnidman, D.A. *A generalized Nyquists criterion and an optimum linear receiver for a pulse modulation system.* Bell Systems Technical Journal, Vol. 46 pp 2163-2177, November 1967.

6. Proakis, J.G., and J.H. Miller. *An adaptive receiver for digital signalling through channels with intersymbol interference.* IEEE Trans. on Information Theory, Vol. IT-15, pp 484-497, July 1969.

7. Harrison, D.J. *Adaptive equalization for channels with crosstalk.* M.Eng. thesis, Carleton University, Ottawa, Canada, December 1969.

8. Gonsalves, R.A., and D.W. Tufts. *Data transmission through a random noisy channel by PAM.* Trans. IEEE, Vol. COM-16, pp 375-379, June 1968.

9. Kantorovich, L.V., and V.I. Krylov. *Approximate methods of higher analysis.* Interscience Publishers, New York, 1958.

APPENDIX A

DISCUSSION OF THE GENERAL SOLUTION

Since the message sequences and the channels are wide-sense stationary we can concentrate on the estimation of one element, say the 0^{th} element, of a sequence. Thus we wish to estimate θ_{0i}, $i = 1,\ldots,M$ by a linear estimator of the form

$$\theta_{0i} = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} h_\ell^i(s)\, w_\ell(s)\, ds. \qquad \ldots\ldots(A.1)$$

By the principle of orthogonality* this must satisfy the equation

$$E\left\{(\theta_{0i} - \hat{\theta}_{0i})\, w_n(t)\right\} = 0,\ n = 1,\ldots,N. \qquad \ldots\ldots(A.2)$$

Substituting (A.1) and (7) in (A.2) and using (8) gives

$$\sum_k \sum_{m=1}^{M} \phi_k^{mi}\, \overline{z}_{mn}(t - kT) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} R_{n\ell}^{w}(t,s)\, h_\ell^i(s)\, ds \qquad \ldots\ldots(A.3)$$

where

$$\begin{aligned} R_{n\ell}^{w}(t,s) &= E\left\{w_n(t)\, w_\ell(s)\right\} \\ &= \sum_k \sum_{m=1}^{M} \sum_v \sum_{q=1}^{m} \phi_{k-v}^{mq}\, \overline{z}_{q\ell}(s - vT)\, \overline{z}_{mn}(t - kT) \\ &\qquad + R_{n\ell}(t,s) + \psi_{n\ell}(t,s) \qquad \ldots\ldots(A.4) \end{aligned}$$

from (7), (8), (5) and (6).

We shall now make the assumption that the solution of (A.3) for the various $h_\ell^i(s)$ can be written in the form

$$h_\ell^i(s) = \sum_j \sum_{p=1}^{M} a_{jp}^i k_{p\ell}(s - jT) \qquad \ldots\ldots(A.5)$$

where the functions $k_{p\ell}(s)$, $p = 1,\ldots,M$, $\ell = 1,\ldots,N$, are defined by the following set of simultaneous Fredholm integral equations

$$z_{pn}(t) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t,s) + \psi_{n\ell}(t,s)\right] k_{p\ell}(s)\, ds. \qquad \ldots\ldots(A.6)$$

This assumption may be justified by substituting (A.5), (A.6) and (A.4) in (A.3) and showing that it leads to a consistent set of equations for the

---

* Papoulis, A. 'Probability, Random Variables and Stochastic Processes', McGraw-Hill Book Co., New York, 1965.

factors a^i_{jp}, $p = 1,\ldots,M$, $i = 1,\ldots,M$, $j = 0, \pm 1, \pm 2,\ldots$. The results of Appendix B are also required. Carrying out this substitution leads to

$$\sum_k \sum_{m=1}^M \phi_k^{mi} \, \overline{z}_{mn}(t - kT) = \sum_j \sum_{p=1}^M \sum_k \sum_{m=1}^M C_{k-j}^{mp} \, a^i_{jp} \, \overline{z}_{mn}(t - kT)$$

$$+ \sum_j \sum_{p=1}^M a^i_{jp} \, \overline{z}_{pn}(t - jT), \quad n = 1,\ldots,N. \qquad \ldots\ldots(A.7)$$

where

$$C_{k-j}^{mp} = \sum_{\ell=1}^N \sum_{q=1}^M \sum_v \phi_{k-v}^{mq} \int_{-\infty}^{\infty} k_{p\ell}(s - jT) \, \overline{z}_{q\ell}(s - vT) \, ds. \qquad \ldots\ldots(A.8)^*$$

Now by changing the dummy indexes in the last sum of (A.7) from j to k and from p to m it becomes possible to equate coefficients of $\overline{z}_{mn}(t - kT)$ for $m = 1,\ldots,M$, $n = 1,\ldots,N$, $k = 0, \pm 1, \pm 2,\ldots$. We note first that the result is independent of n, thus the factors a^i_{km} are defined as the solutions of the following set of algebraic equations:

$$\phi_k^{mi} = a^i_{km} + \sum_j \sum_{p=1}^M C_{k-j}^{mp} \, a^i_{jp}, \quad m = 1,\ldots M, \; k = 0, \pm 1, \pm 2\ldots \; . \qquad \ldots\ldots(A.9)$$

Although this is an infinite set of equations it is clear that there is exactly one equation per unknown. An alternative expression of the set of equations, using Z-transform notation, is given later.

To obtain the full expression for $\hat{\theta}_{0i}$ we substitute (A.5) in (A.1), changing the dummy index j to k as in (A.9), to give

$$\hat{\theta}_{0i} = \sum_k \sum_{p=1}^M a^i_{kp} \sum_{\ell=1}^M \int_{-\infty}^{\infty} k_{p\ell}(s - kT) \, w_\ell(s) \, ds. \qquad \ldots\ldots(A.10)$$

Now, because of the stationariness of the message sequence, the estimate θ_{ji} will be obtained from the same filter at time $t = jT$ compared with $t = 0$ for $\hat{\theta}_{0i}$. Thus

$$\hat{\theta}_{ji} = \sum_k \sum_{p=1}^M a^i_{kp} \sum_{\ell=1}^N \int_{-\infty}^{\infty} k_{p\ell}\left[(s - (k + j)T\right] w_\ell(s) \, ds. \qquad \ldots\ldots(A.11)$$

This is the desired result (10) except for changes in the dummy variables which have been made in (10) in order, for the sake of clarity, to use the same variables as in Figure 2.

---

* To see that the right hand side of (A.8) is in fact a function of (k - j) observe that the integral is a function of (j - v) and the product of this with ϕ_{k-v}^{mq} is summed over the infinite range of v.

The mean square estimation error for the i^{th} sequence is

$$e_i = E\left\{(\theta_{0i} - \hat{\theta}_{0i})\right\}$$

$$= E\left\{(\theta_{0i} - \hat{\theta}_{0i})\theta_{0i}\right\} \qquad \text{from.....(1)}$$

$$= 1 - E\left\{\hat{\theta}_{0i}\ \theta_{0i}\right\}. \qquad .....(A.12)$$

Now substituting from (A.10) for $\hat{\theta}_{0i}$ and using (7) and (8), gives

$$e_i = 1 - \sum_k \sum_{p=1}^{M} a^i_{kp} \sum_{\ell=1}^{N} \sum_{m=1}^{M} \sum_j \phi^{mi}_j \int_{-\infty}^{\infty} k_{p\ell}(s - kT)\ \overline{z}_{m\ell}(s - jT)\ ds.$$

But by the general property of cross-correlation functions

$$\phi^{mi}_j = \phi^{im}_{-j}$$

and, by the definition (A.8),

$$\sum_{\ell=1}^{N} \sum_{m=1}^{M} \sum_j \phi^{im}_{-j} \int_{-\infty}^{\infty} k_{p\ell}(s - kT)\ \overline{z}_{m\ell}(s - jT)\ ds = C^{ip}_{-k}$$

so that

$$e_i = 1 - \sum_k \sum_{p=1}^{M} a^i_{kp}\ C^{ip}_{-k}. \qquad .....(A.13)$$

Now the member of the set of equations (A.9) for which m = i and k = 0 is:

$$\phi^{ii}_0 = 1 = a^i_{0i} + \sum_j \sum_{p=1}^{M} C^{ip}_{-j}\ a^i_{jp}. \qquad .....(A.14)$$

Comparing (A.13) with (A.14) shows that

$$e_i = a^i_{0i}. \qquad .....(A.15)$$

Z-Transform Representation

Useful insight into the meaning of (A.9) can be obtained by using Z-transform techniques*.

We first express (A.9) in slightly different form:

$$\phi^{mi}_k = \sum_j \sum_{p=1}^{M} \left(\delta_{p-m}\ \delta_{j-k} + C^{mp}_{k-j}\right) a^i_{jp}. \qquad .....(A.16)$$

$$m = 1,...M$$

$$k = 0, \pm 1, \pm 2... .$$

* See, for instance, 'Analysis of Linear, Time Invariant Systems', W.M. Brown, McGraw-Hill Book Co. Inc., New York, 1963.

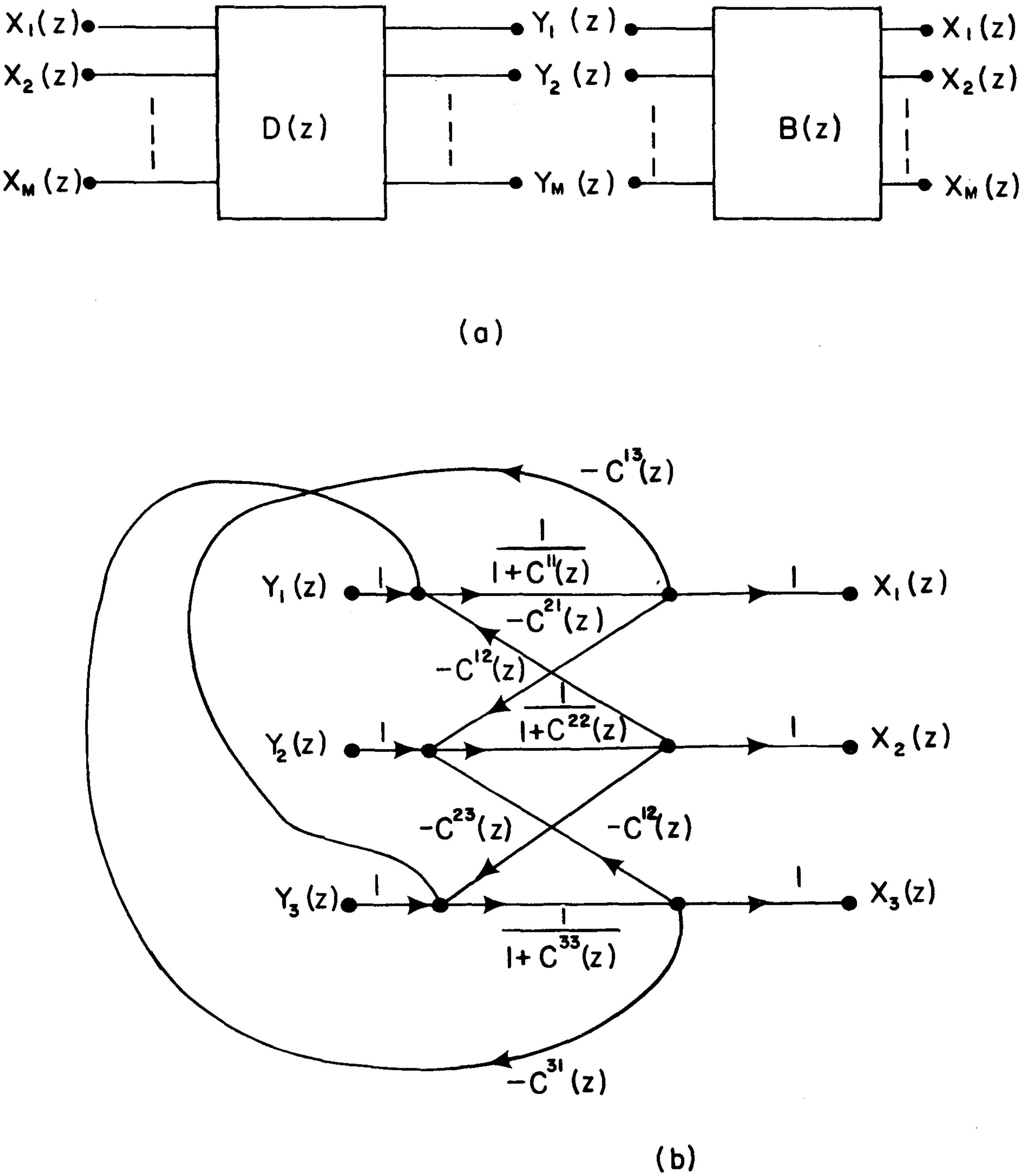

Fig. A1. Synthesis of the discrete filter.

(a) Discrete filter equivalent of equation (A.13)
(b) Flow graph representation of the filter B(z).

Defining the following Z-transforms*:

$$\Phi^{mi}(z) = Z\left(\phi_k^{mi}\right)$$

$$A_m^i(z) = Z\left(a_{mk}^i\right)$$

$$C^{mp}(z) = Z\left(C_k^{mp}\right)$$

allows (A.16) to be expressed as

$$\Phi^{mi}(z) = \sum_{p=1}^{M} \left[\delta_{p-m} + C^{mp}(z)\right] A_p^i(z), \quad \begin{matrix} m = 1,\ldots M \\ i = 1,\ldots M \end{matrix}$$

$$= \sum_{p=1}^{M} D^{mp}(z)\, A_p^i(z), \quad \begin{matrix} m = 1,\ldots M, \\ i = 1,\ldots M \end{matrix} \qquad \ldots\ldots(A.17)$$

where

$$D^{mp}(z) = \delta_{p-m} + C^{mp}(z), \quad \begin{matrix} m = 1,\ldots M \\ p = 1,\ldots M. \end{matrix} \qquad \ldots\ldots(A.18)$$

At this point is is convenient to consider an equation closely related to (A.17):

$$\sum_{p=1}^{M} D^{mp}(z)\, B^{pq}(z) = \delta_{m-q} \quad \begin{matrix} m = 1,\ldots M \\ q = 1,\ldots M. \end{matrix} \qquad \ldots\ldots(A.19)$$

(The relation between the two equations will be discussed later.) We shall rely, in our development, on an interpretation of (A.19) in terms of the linear, discrete system shown in Figure A.1(a). The discrete filter D has, as its inputs, a set of sequences $\{x_{km}\}$ with Z-transforms $X_m(z)$, $m = 1,\ldots M$. Its outputs are the sequences $\{y_{kp}\}$ with transforms $Y_p(z)$, $p = 1,\ldots M$, related to the inputs by:

$$Y_p(z) = \sum_{\ell=1}^{M} D^{\ell p}(z)\, X_\ell(z), \quad p = 1,\ldots M. \qquad \ldots\ldots(A.20)$$

A second filter, B, is cascaded with D and is determined, according to (A.19) so that the overall system reproduces the inputs exactly: thus

$$X_q(z) = \sum_{p=1}^{M} B^{pq}(z)\, Y_p(z). \qquad \ldots\ldots(A.21)$$

Thus (A.19) requires that B be the inverse of D, where D is determined by the parameters of our channel as in (A.12). A signal flow-graph of a synthesis of B is shown in Figure A.1(b) for M = 3. It is not immediately obvious that this is, in fact, a form which satisfies (A.19) but this can be seen by expanding the identity:

$$X_p(z) = \frac{1}{D^{pp}(z)} \left[D^{pp}(z)\, X_p(z)\right].$$

---

* The Z-transform of a sequence a_k is defined as

$$A(z) = \sum_k a_k z^k.$$

Thus

$$X_p(z) = \frac{1}{D^{pp}(z)} \left[\sum_{\ell=1}^{M} D^{\ell p}(z)\, X_\ell(z) - \sum_{\ell \neq p} D^{\ell p}(z)\, X_\ell(z) \right].$$

But, from (A.20) this is

$$X_p(z) = \frac{1}{D^{pp}(z)} \left[Y_p(z) - \sum_{\ell \neq p} D^{\ell p}(z)\, X_\ell(z) \right]$$

and, substituting for $D^{\ell p}(z)$ from (A.12)

$$X_p(z) = \frac{1}{1 + C^{pp}(z)} \left[Y_p(z) - \sum_{\ell \neq p} C^{\ell p}(z)\, X_\ell(z) \right]. \qquad \text{.....(A.22)}$$

But this is precisely the equation which specifies the action of the filter shown in Figure A.1(b) so that we see that this filter does, in fact, reproduce the system inputs $X_p(z)$ given the filter inputs $Y_p(z)$.

Having solved (A.19) by means of a synthesis of the filter B, it is now necessary to show that this solution is relevent to the main problem, represented by (A.17). First we see that if the data sequences $\{\theta_{km}\}$ are white and uncorrelated, $\Phi^{mi}(z) = \delta_{m-i}$, then (A.17) and (A.19) are identical. This in this case the filter B is the optimum sampled data filter for the receiver and

$$A_p^i(z) = B^{pi}(z). \qquad \text{.....(A.23)}$$

In the general case we note that multiplying (A.19) by $\Phi^{qi}(z)$, $q = 1,\ldots M$, $i = 1,\ldots M$, and summing over q yields (A.17). Thus

$$\sum_{p=1}^{M} D^{mp}(z) \sum_{q=1}^{M} B^{pq}(z)\, \Phi^{qi}(z) = \sum_{q=1}^{M} \Phi^{qi}(z)\, \delta_{m-q}$$

$$= \Phi^{mi}(z)$$

and setting

$$A_p^i(z) = \sum_{q=1}^{M} B^{pq}(z)\, \Phi^{qi}(z) \qquad \text{.....(A.24)}$$

gives (A.17). Now the right hand side of (A.24) corresponds to a cascade of two filters, B and Φ, where Φ is a filter whose response between the q^{th} input and the i^{th} output is $\Phi^{qi}(z)$. Φ can be synthesized in exactly the same way as B.

The Z-transform analysis shows that the sampled data filter in the receiver can be regarded as a cascade of two filters, one determined solely by the channel and one determined solely by the correlation of the data sequences. A synthesis of both filters has been described which does not require inversion of any of the functions describing the channel or sequence correlation. Unfortunately both filters, when synthesized this way, involve non-causal feedback and so are unrealizable. They do, however, offer insight into the nature of the receiver.

APPENDIX B

A PERIODIC PROPERTY OF THE OUTPUT COVARIANCE FUNCTION

In this Appendix we prove a theorem on the periodicity of the covariance functions $R_{n\ell}(t,s)$ and a corollary required for the derivation in Appendix A.

Theorem

$$R_{n\ell}(t - kT,\ s - kT) = R_{n\ell}(t,s), \text{ all } k \qquad .....(B.1)$$

$$\psi_{n\ell}(t - kT,\ s - kT) = \psi_{n\ell}(t,s), \text{ all } k. \qquad .....(B.2)$$

Proof

From (4) and (3b) we have

$$y_n(t) = \sum_j \sum_{m=1}^{M} \theta_{jm}\ \tilde{z}_{jmn}(t)$$

$$= \sum_j \sum_{m=1}^{M} \theta_{jm} \int_{-\infty}^{\infty} \tilde{a}_{mn}(t,\alpha)\ p_m(t - jT - \alpha)\ d\alpha. \qquad .....(B.3)$$

Hence, substituting from (B.3) and (8) in (5),

$$R_{n\ell}(t,s) = \sum_j \sum_{m=1}^{M} \sum_v \sum_{q=1}^{M} \phi_{j-v}^{mq} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E\left\{\tilde{a}_{mn}(t,\alpha)\ \tilde{a}_{q\ell}(s,\sigma)\right\} p_m(t - jT - \alpha) \cdot p_q(s - vT - \sigma)\ d\alpha\ d\sigma. \qquad .....(B.4)$$

But, since the channel is taken to be wide-sense stationary, the covariance function in the integrand of (B.4) is periodic:

$$E\left\{\tilde{a}_{mn}(t - kT,\alpha)\ a_{q\ell}(s - kT,\sigma)\right\} = E\left\{\tilde{a}_{mn}(t,\alpha)\ \tilde{a}_{q\ell}(s,\sigma)\right\}, \text{ all } k. \qquad .....(B.5)$$

Thus replacing t and s by $(t - kT)$ and $(s - kT)$ in the right hand side of (B.4) does not affect its value since the sums are over the infinite range $(-\infty,\infty)$ for the indexes j and v. This proves the result (B.1).

The proof of (B.2) is trivial since the additive noise is taken to be stationary.

Corollary

If, as in (14), $k_{m\ell}(t)$ is defined by

$$\bar{z}_{mn}(t) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t,s) + \psi_{n\ell}(t,s)\right] k_{m\ell}(s)\, ds \qquad \ldots\ldots(B.6)$$

then

$$\bar{z}_{mn}(t - kT) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t,s) + \psi_{n\ell}(t,s)\right] k_{m\ell}(s - kT)\, ds. \qquad \ldots\ldots(B.7)$$

Proof from (B.6)

$$\bar{z}_{mn}(t - kT) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t - kT,s) + \psi_{n\ell}(t - kT,s)\right] k_{m\ell}(s)\, ds. \qquad \ldots\ldots(B.8)$$

By a change in the dummy variable of integration we then have

$$\bar{z}_{mn}(t - kT) = \sum_{\ell=1}^{N} \int_{-\infty}^{\infty} \left[R_{n\ell}(t - kT,\, s - kT) + \psi_{n\ell}(t - kT,\, s - kT)\right] k_{m\ell}(s - kT)\, ds. \qquad \ldots\ldots(B.9)$$

Substituting (B.1) and (B.2) in (B.9) gives the desired result, (B.7).

APPENDIX C

RESULTS FOR MULTIPATH CHANNELS

As an example to illustrate the performance of an optimum receiver, single-sequence signalling (M = 1), in a multipath situation, is considered. The channel is considered to consist of two discrete, uncorrelated, non-selective paths. The combination of the two paths, of course, renders the channel selective in a randomly time varying manner. The impulse response of such a channel is represented by

$$a(t,\alpha) = c_1(t)\ \delta(\alpha) + c_2(t)\ \delta(\alpha - \Delta) \qquad \text{.....(C.1)}$$

where $c_1(t)$ and $c_2(t)$ are the time-variant attenuations of each path and Δ is the delay difference. Each path has a non-zero mean, or estimated, value so that, with the assumption of wide sense stationary statistics,

$$c_1(t) = c_1 + \tilde{c}_1(t) \qquad \text{.....(C.2)}$$

where $\tilde{c}_1(t)$ is a random variable and c_1 is the mean value of $c_1(t)$; the properties of the second path are similarly defined. $\tilde{c}_1(t)$ and $\tilde{c}_2(t)$ are assumed to have identical Gaussian-shaped autocorrelation functions

$$\chi(\tau) = \exp - \frac{1}{2}\left(\xi\,\frac{\tau}{T}\right)^2. \qquad \text{.....(C.3)}$$

ξ is a measure of the decorrelation time of the channel with respect to the signalling interval. The random components $\tilde{c}_1(t)$ and $\tilde{c}_2(t)$ of the two paths may have different variances, $\sigma_1^2\ \gamma^2$ and $\sigma_2^2\ \gamma^2$ where

$$\sigma_1^2 + \sigma_2^2 = 1 \qquad \text{.....(C.4)}$$

and, since the paths are uncorrelated, the total variance is γ^2.

We shall consider the case of a single white transmitted sequence, so that the transmitted signal is

$$s(t) = \textstyle\sum_{k}.\theta_k p(t - kT) \qquad \text{.....(C.5)}$$

and

$$E\left\{\theta_j\theta_k\right\} = \delta_{k-j}. \qquad \text{.....(C.6)}$$

The pulse shape chosen is a raised cosine pulse

$$p(t) = \sqrt{\frac{2E_t}{3T}} \left(1 + \cos\frac{2\pi t}{T}\right), \quad -\frac{T}{2} \le \tau \le \frac{T}{2}$$

$$= 0 \text{ elsewhere} \qquad \text{.....(C.7)}$$

where E_t is the energy in a single pulse. With these definitions the estimate of the received waveform is

$$z(t) = c_1 p(t) + c_2 p(t - \Delta) \qquad \text{.....(C.8)}$$

and the energy in this waveform is

$$E_r = \int \bar{z}^2(t)\, dt. \qquad \text{.....(C.9)}$$

In a time-variant, exactly known channel with an impulse response identical to the estimated response of the present channel the signal-to-noise ratio at the receiver would be the ratio E_r/N_0. This ratio therefore provides a basis for assessing the effect of the randomness of the channel. Some algebra, starting with (B.4) shows that the autocovariance function of the channel output, with additive noise is

$$R(t,s) = \frac{\gamma^2}{2} \chi(t - r) \sum_k \Big[\sigma_1^2 p(t - kT)\, p(s - kT) + \sigma_2^2 p(t - \Delta - kT)\, p(s - \Delta - kT)\Big]. \qquad \text{.....(C.10)}$$

In (C.10) it is clear that γ^2 is a measure of the ratio of random energy received to transmitted energy. A similar measure for the ratio of deterministic energy received to transmitted energy is

$$g^2 = \frac{E_r}{E_t}. \qquad \text{.....(C.11)}$$

The two quantities γ^2 and g^2 can be combined to give a quality factor for the channel estimate:

$$\eta = \frac{g^2}{\gamma^2}. \qquad \text{.....(C.12)}$$

Thus a large value of η implies a good estimate of the channel (and only minor random time variance) and a small value of η indicates a poor estimate. The additive noise is assumed to be white, with spectral density N_0. Results will be plotted as a function of the ratio of the energy E_r in the expected or estimated received pulse to the noise spectral density N_0.

Even for such an apparently simple channel as the one just described it is clear that a considerable number of parameters is required to specify it and several examples are discussed below to illustrate the effects of the various channel parameters on system performance.

The first example is a case in which one path has such a low variance that it is almost deterministic or specular, while the other path is entirely random. Thus $c_1 = 1$, $c_2 = 0$, $\sigma_1^2 = 0.05$ and $\sigma_2^2 = 0.95$. The total variance of the channel is fairly high, giving a quality factor $\eta = 2$, and the channel is varying at rates comparable to the signalling rate, $\xi = 1$. There is therefore no deterministic intersymbol interference but the random signal due to one pulse in the transmitted sequence will overlap with the deterministic signal due to another. For this reason the tapped delay line portion of the receiver degenerates to a single tap or multiplier. The receiver therefore consists only of the filter $k(-t)$. Figure C.1 shows the performance of the optimum receiver and a simple matched filter. The results are plotted for path delay differences of 2T/3 and 5T/6. It is clear that in each case the optimum receiver achieves significant improvement in performance over the matched filter. The reason for this is that the covariance function of the random channel output, $R(t,r)$ is time dependent and the optimum receiver concentrates on the time intervals when it is small.

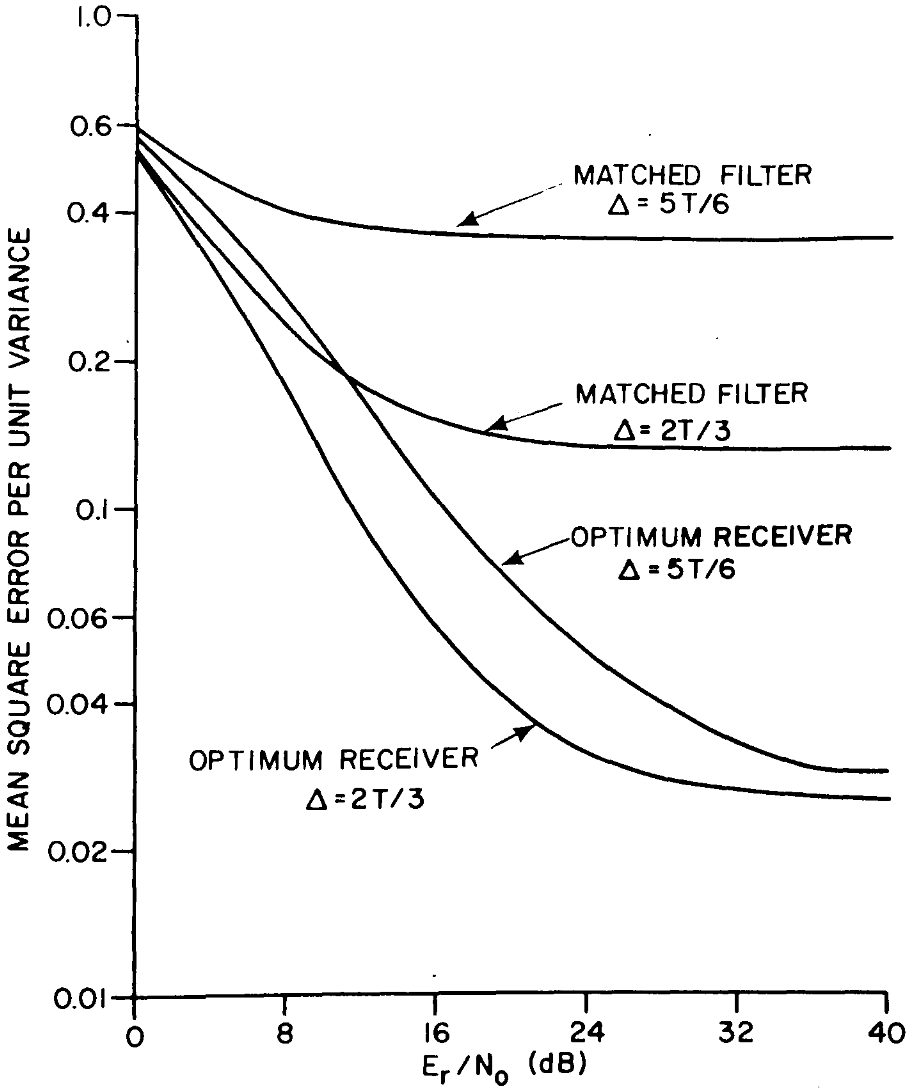

Fig. C.1. Performance for a multipath channel with one almost specular path ($C_1 = 1$, $\sigma_1^2 = 0.05$) and one random path ($C_2 = 0$, $\sigma_2^2 = 0.95$) with $\eta = 2$.

This is well illustrated by Figure C.2(b) which shows the impulse response of the optimum receiver for $\Delta = 2T/3$, compared with that of the matched filter, which is just the inverse of the averaged received pulse shape, shown in Figure C.2(a). The negative excursions of the impulse response indicate that the receiver uses the correlation properties of the multiplicative noise to minimize the effect by averaging over the signalling interval. The reason for the higher error when the delay difference is 5T/6 is that, in this case, the random reception from one pulse is very nearly in synchronism with the deterministic reception from the next. The effect is akin to the well known fact that intersymbol interference in a multipath situation is most severe when the delay difference is almost equal to a signalling interval.

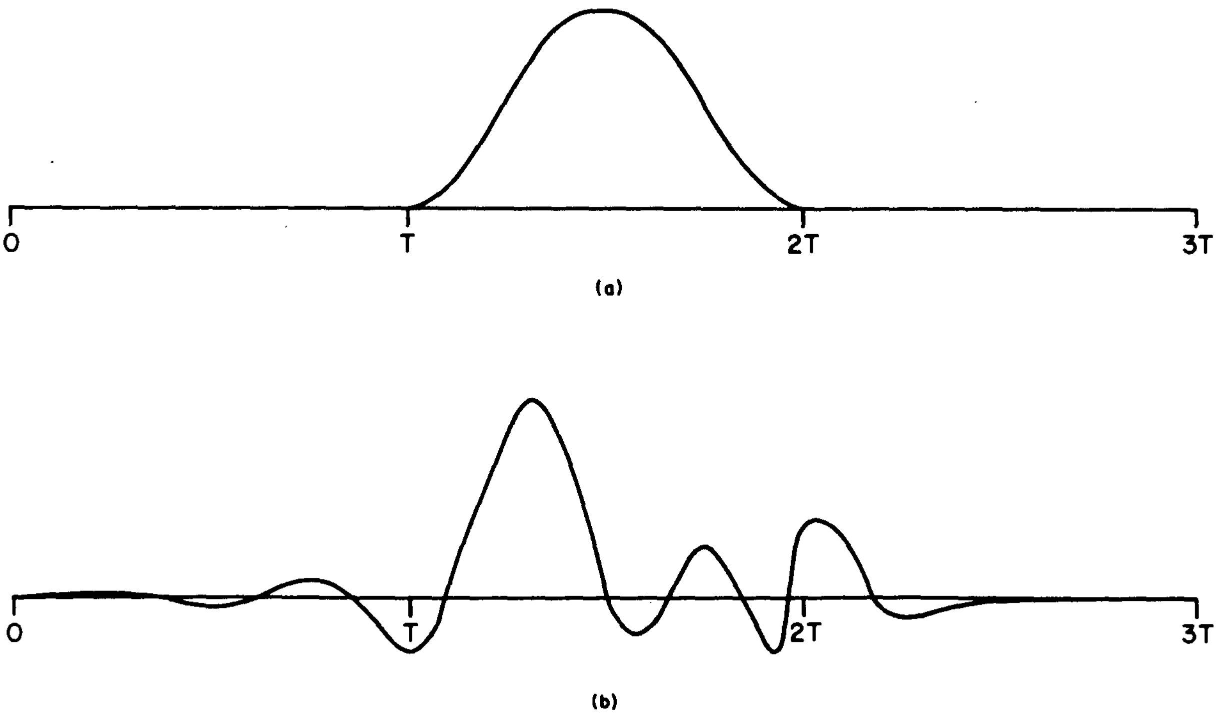

Fig. C.2. Impulse response of a matched filter (a) and the optimum receiver filter (b) for the conditions of Fig. C.1 at high signal-to-noise ratio and a path delay difference $\Delta = 2T/3$.

Figure C.3 shows some results for a channel that causes deterministic as well as random intersymbol interference. The channel is slowly time varying ($\xi = 0.1$), with large variance ($\eta = 2$) equally divided between the paths, and a path separation of 5T/6. Curves 1 and 2, respectively, show the performance of the optimum receiver and the receiver designed for intersymbol interference and additive noise alone* on the basis of the estimated channel response. The deterministic energy per pulse from the second path is one quarter of that from the first ($c_2/c_1 = 0.5$). The difference in performance is not so great as in the previous case because there is no time interval in which the instanteous ratio of deterministic energy to random energy is large. However, the optimum receiver is able to cope with unequal division of the deterministic energy

---

* This is the optimum receiver for the exactly known channel as described by George[1] and Tufts[2].

between the two path responses while the suboptimum receiver is not. This fact is illustrated by observing that, when the deterministic energy is equally divided, the performance of the optimum receiver is almost unchanged while the suboptimum receiver is able to do better than before as illustrated by Curve 3.

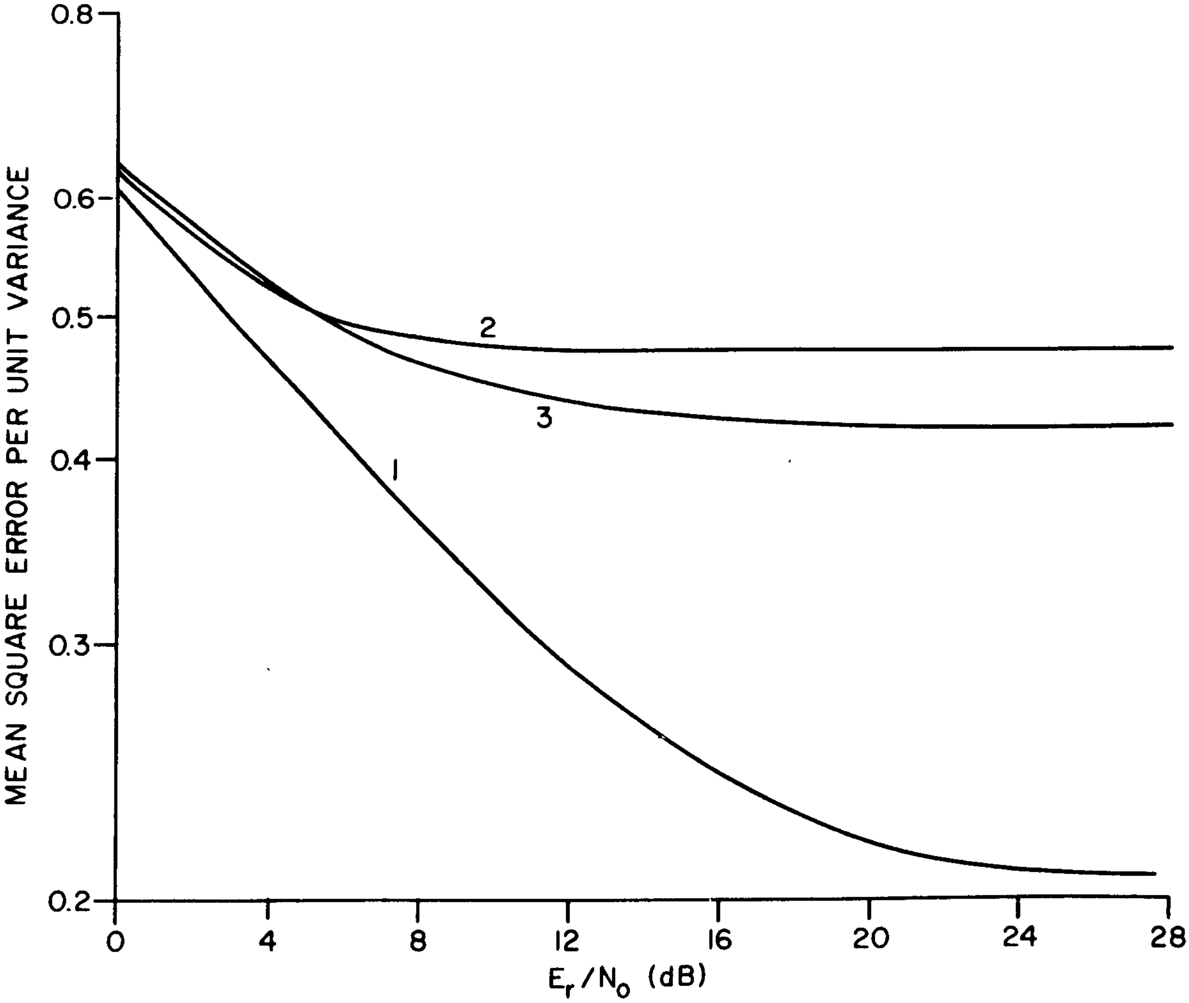

Fig. C.3. Performance for a multipath channel with two paths of equal variance ($\sigma_1^2 = \sigma_2^2 = 0.5$, $\eta = 2$) and separation $\Delta = 5T/6$.

Curve 1: The optimum receiver when $C_2/C_1 = 0.5$ and when $C_2/C_1 = 1$

Curve 2: The receiver of George and Tufts when $C_2/C_1 = 0.5$

Curve 3: The receiver of George and Tufts when $C_2/C_1 = 1$.

Figure C.4 shows the effect of different path delay differences. The other parameters are the same as those just specified for Figure C.3 but with E_r/N_0 = 20 dB. Curve 1 shows the performance of the optimum receiver and Curve 2 that of the suboptimum receiver previously mentioned. The deterioration in performance as the delay difference approaches the signalling interval is clearly shown.

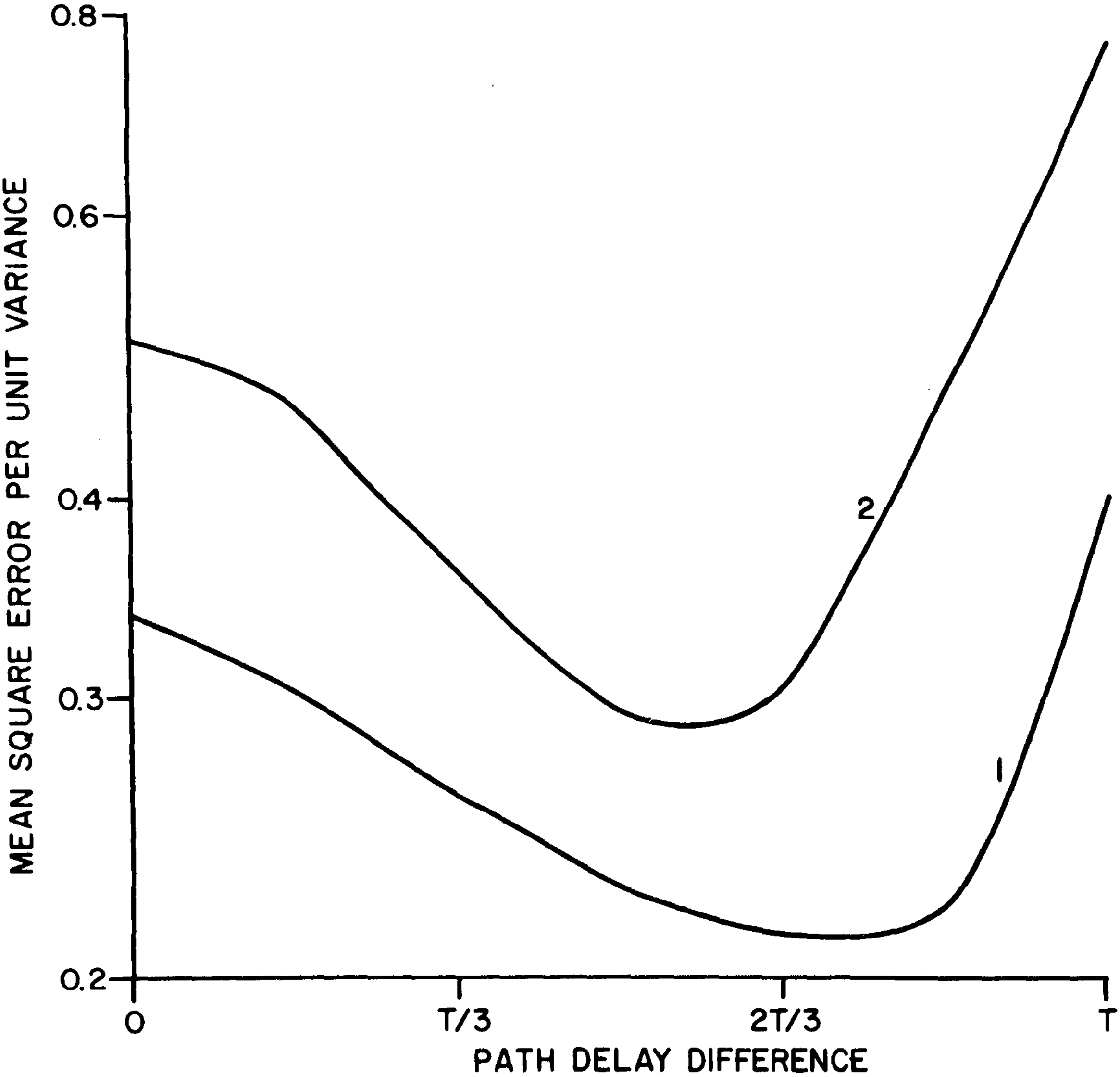

Fig. C.4. Performance for the channel of Figure C.3 with $C_2/C_1 = 0.5$ as a function of delay difference at $E_r/N_0 = 20$ dB.

Curve 1: The optimum receiver
Curve 2: The receiver of George and Tufts.

The curves shown have been for fairly bad channels, η small. Naturally as η gets larger and the channel becomes almost time invariant and exactly known the optimum receiver converges to the receiver for intersymbol interference and additive noise alone and they have converging performance.

The effect of more rapidly varying random components of the channel (large ξ) is to spread the random energy over a wider bandwidth so that, for a fixed variance, the mean square error decreases. For $\xi < 0.1$ the channel can be regarded as time invariant and the mean square error is almost independent of ξ.

APPENDIX D

AUTOMATIC ADJUSTMENT OF TAPS

In this Appendix it is assumed that the continuous filters in the receiver are predetermined but that the tap gains are to be adjusted automatically. It will be shown that the partial derivative of the error of the estimate of each transmitted message with respect to each tap gain can be obtained from measurements made at the receiver output.

Let $u_{km}(jT)$ be the signal at the k^{th} tap of the m^{th} delay line at $t = jT$:

$$u_{km}(jT) = \sum_{n=1}^{N} \int_{-\infty}^{\infty} k_{mn} \left[t - (k + j)T\right] w_n(t)\, dt. \qquad \text{.....(D.1)}$$

Then, from (10), we have:

$$\theta_{ji} = \sum_{k} \sum_{m=1}^{M} a_{km}^{i}\, u_{km}(jT). \qquad \text{.....(D.2)}$$

Now let

$$\varepsilon_i(jT) = \theta_{ji} - \hat{\theta}_{ji}. \qquad \text{.....(D.3)}$$

Assuming that the receiver makes no errors, $\varepsilon_i(jT)$ can be obtained from the receiver output as shown in Figure 6. Experience has shown that an adaptive algorithm based on this assumption will not be seriously affected provided the error rate is better than one in ten. Now the mean square error

$$e_i = E\left\{\varepsilon_i^2\,(jT)\right\} \qquad \text{.....(D.4)}$$

is independent of j because of the stationariness of the message sequence and the channel. Substituting in (D.4) from (D.2) and (D.3) and differentiating with respect to a_{km}^{i} gives

$$\frac{\partial e_i}{\partial a_{km}^{i}} = 2E\left\{\varepsilon_i(jT)\, u_{km}(jT)\right\} \qquad \text{.....(D.5)}$$

which is again independent of j for the same reasons as for (D.4). An approximation to $\partial e_i/\partial a_{km}^{i}$ can be obtained by measuring a time average of the form

$$\tilde{e}_i = \frac{2}{J} \sum_{j=1}^{J} \varepsilon_i(jT)\, u_{km}(jT) \qquad \text{.....(D.6)}$$

and measurements of this type can be used to increment or decrement the taps.