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STABILITY OF A DUAL SPIN SATELLITE WITH TWO DAMPERS IN CIRCULAR ORBIT

by
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CANADA

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ABSTRACT

The motion of a dual spin satellite in circular orbit is studied to determine the effect of gravitational torques and damping when the spin vector is approximately normal to the orbit plane. Stability results are assessed by the Floquet method for selected cases to show that gravitational torques are important when spin rates are low. Solutions are obtained analytically for high spin cases using the method of averaging to demonstrate the effect of gravitational torque and damping on both rotor and despun platform. For the high spin cases, stability criteria are identical with criteria previously derived for a freely spinning untorqued dual spin satellite.

1. INTRODUCTION

The mechanics of dual spin satellites has received a great deal of attention in the recent literature because of the immediate application to spin stabilized communications satellites in synchronous orbit. A satellite composed of a rotor, a platform and one or more dampers is the configuration usually studied because it has the important features found in live applications. Extensive study of the motion of this configuration in 'free space' conditions (i.e., when all external torques are absent) has resulted in valuable insight regarding the role of the rotor and dampers on platform stability and pointing (much of the progress is documented[1-7]).

For the study of satellite stability in orbit, it would seem important to consider gravity torques (in addition to flexibility and damping), as it is not obvious at first sight (at least to this writer), that they may be ignored. For example, one notes that in studies of stability of spinning symmetric completely rigid bodies, the effect of gravity profoundly alters the 'maximum moment of inertial rule' obtained for free space conditions[8-12]. Also 'resonance lines' of parametric excitation arise as a result of gravity torques[13,14].

Equilibrium and stability studies of dual spin satellites in synchronous orbit accounting for gravity torques have been published[11-21]. In most cases it was assumed that damping was absent or was not included explicitly. The equivalence of the dual and single-body problems has been established[15,20,21], and this enables one to draw on many of the results established for single rigid bodies. In recent work resonance bands of parametric excitation are also explored for completely rigid dual spin satellites[20,21].

In this report a dual spin satellite composed of a platform, a rotor, a platform damper, and a rotor damper will be studied from a slightly different viewpoint with intent to determine how gravity torques and damping influence the motion when the spin vector is approximately normal to the orbit plane. The motion equations will be linearized at the outset and hence conclusions drawn concerning stability will refer to 'infinitesimal stability'. Stability will be assessed by application of Floquet theory for some cases. The equations will be solved by the Method of Averaging of references 21 - 24 for the 'high spin' case, and analytical stability criteria will be obtained.

2. EQUATIONS OF MOTION

Consider a dual-spin satellite composed of a platform which contains a pendulum type damper and a rotor which contains an internal damper, as shown in Figure 1. The axes (O'x'y'z') are assigned to the body so that when the damper springs are in their unstretched state, the axis O'z' is a common principal axis of the two bodies (the nominal axis of rotation). The point O' coincides with the mass center of the composite body and the axes O'x' and O'y' are principal axes fixed on the platform. In this study, it will be assumed that both rotor and platform are symmetric about the O'z' axis. The rotor rotates with respect to the platform about the O'z' axis with angle γ, and the rotation rate is maintained by supplying a torque with an internal motor. The platform damper, which is in static equilibrium with its mass on the O'z' axis, is located a distance 'a' from the mass center O', and is constrained to oscillate in the O'x' direction. The rotor damper consists of a sphere in a cavity located at O', and is constrained to oscillate about an axis transverse to the rotor, as shown in Figure 1. An additional set of axes (Oxyz) are assigned to be parallel to the (O'x'y'z') body fixed axes, so that O coincides with the instantaneous mass center of the configuration as the platform damper oscillates.

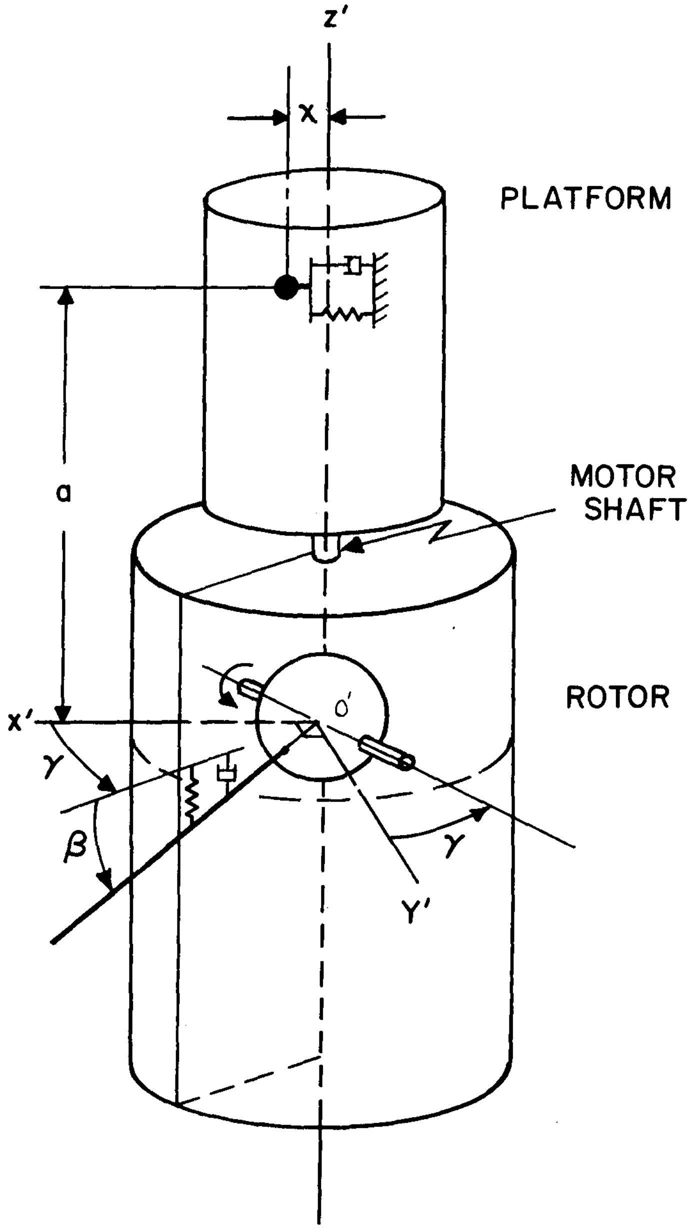

Figure 1

The satellite is in a circular earth orbit of radius R and orbital rate Ω. The satellite axes (Oxyz) are referenced to orbital axes (O $\underline{A}_1$ $\underline{A}_2$ $\underline{A}_3$) by a set of Euler angles (ψ, θ, ϕ) generated by the right hand rotation scheme,

i) ψ about $\underline{A}_1$, leading to axes (O $\underline{B}_1$, $\underline{B}_2$, $\underline{B}_3$)

ii) θ about $\underline{B}_2$, leading to axes (O $\underline{C}_1$, $\underline{C}_2$, $\underline{C}_3$)

iii) ϕ about $\underline{C}_3$, leading to axes (Oxyz).

The rotation is shown schematically in Figure 2.

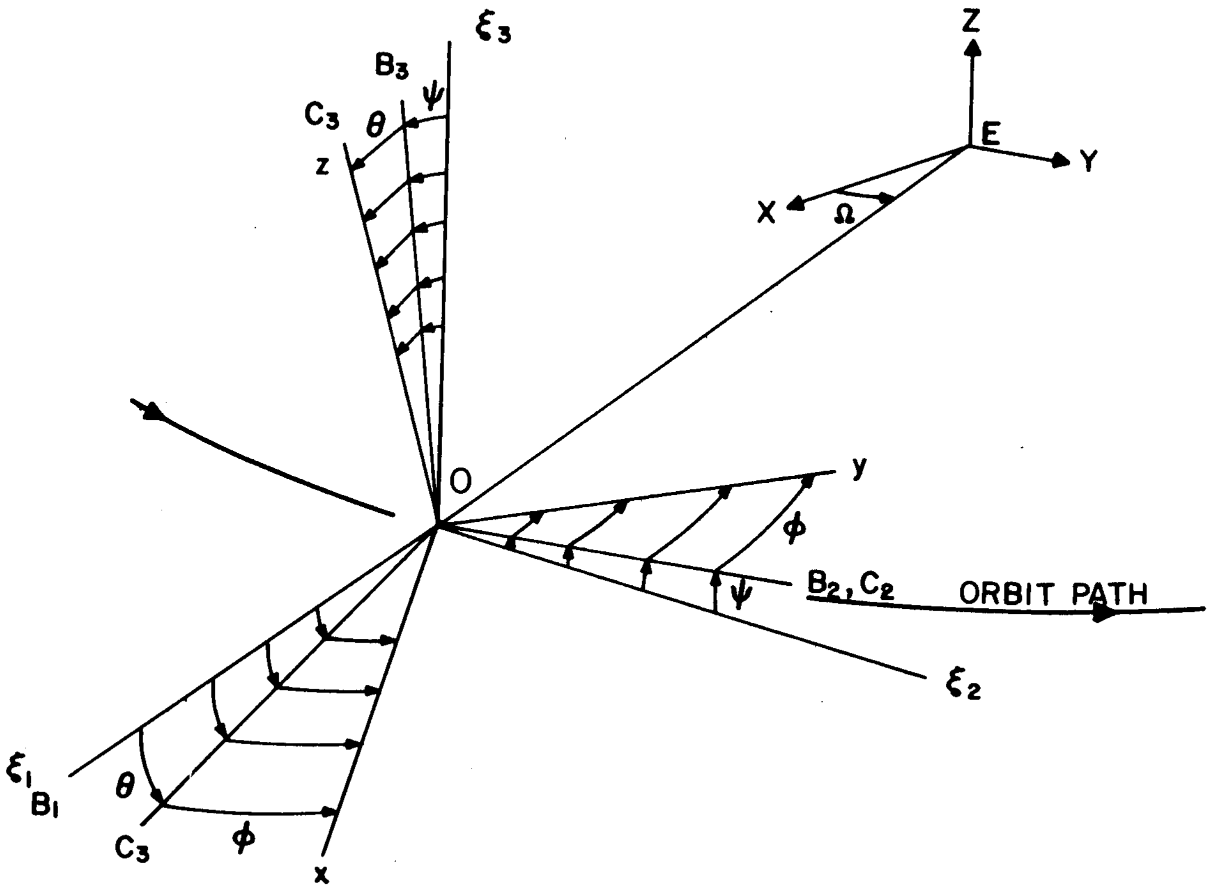

Figure 2

Equations of motion for this satellite may be derived by application of momentum laws, or by variational principles. Since the development of motion equations for similar configurations has been reported elsewhere,[1,2,15] much of the derivation will be bypassed, and equations of interest will be quoted directly. (A detailed development of these and other motion equations for satellites of interest in the Canadian Telesat program is reported in reference 25.)

The equilibrium solution of interest is the one that corresponds to pure rotation of the satellite about axis $O\underline{A}_3$ (and hence about (O'z')), with the platform pointing at the earth, i.e.,

$$\psi = \theta = \phi = \omega_x = \omega_y = \chi = \beta = 0 \qquad \text{.....(1)}$$

$$\Omega = \text{constant}$$

where $(\omega_x, \omega_y, \omega_z)$ are component absolute angular velocities about the (Oxyz) axes, and χ and β are the linear and angular deflections of the platform and rotor dampers, respectively. Accordingly, the motion equations may be linearized in these variables.

Three equations arising from application of the momentum laws for the total system about 0 are,

$$A_0\dot{\omega}_x - ma\dot{\chi}\omega_z - ma\chi\dot{\omega}_z - I_s\ddot{\beta}\sin\gamma - I_s\dot{\beta}\dot{\gamma}\cos\gamma + (C_0 - A_0)\omega_y\omega_z$$

$$+ C_R\dot{\gamma}\omega_y - (I_s\dot{\beta}\cos\gamma + ma\dot{\chi})\ {}_z = 0 \qquad .....(2)$$

$$A_0\dot{\omega}_y + I_s\ddot{\beta}\cos\gamma - I_s\dot{\beta}\dot{\gamma}\sin\gamma + ma\ddot{\chi} - (C_0 - A_0)\omega_x\omega_z - ma\omega_z^2\chi$$

$$- I_s\dot{\beta}\omega_z\sin\gamma - C_R\dot{\gamma}\omega_x = 3\Omega^2\,[(A_0 - C_0)\theta + ma\chi] \qquad .....(3)$$

$$C_0\dot{\omega}_z + C_R\ddot{\gamma} = 0. \qquad .....(4)$$

The equation of motion for the rotor is

$$C_R\,(\dot{\omega}_z + \ddot{\gamma}) = -\,c\dot{\gamma} + T_m(t) \qquad .....(5)$$

where $c\dot{\gamma}$ is a friction torque in the motor bearing assembly, and $T_m(t)$ is a torque supplied by the motor, and will be designed to maintain $\dot{\gamma}$ constant in in this instance.

Kinematical relations relating ω_x, ω_y, and ω_z to the Euler angles and their rates of change are, in linearized form,

$$\dot{\theta} = \omega_y - \Omega\psi \qquad .....(6)$$

$$\dot{\psi} = \omega_x + \Omega\theta \qquad .....(7)$$

$$\dot{\phi} = \omega_z - \Omega. \qquad .....(8)$$

The equations of motion for the platform and rotor dampers are,

$$m(1 - \mu)\ddot{\chi} + ma\dot{\omega}_y + ma\omega_z\omega_x + \bar{c}_1\dot{\chi} + [\bar{k}_1 - m(\omega_z^2 + 2\Omega^2)(1 - \mu)]\chi = 0 \qquad .....(9)$$

$$I_s\ddot{\beta} - I_s\dot{\gamma}\omega_x\cos\gamma - I_s\dot{\omega}_x\sin\gamma - I_s\dot{\gamma}\omega_y\sin\gamma + I_s\dot{\omega}_y\cos\gamma$$

$$+ \bar{c}_2\dot{\beta} + \bar{k}_2\beta = 0 \qquad .....(10)$$

where m is the mass of the platform dampers, μ is the ratio of m to the total satellite mass, I_s is the inertia of the spherical damper, $\bar{c}_1$ and $\bar{c}_2$ are the damping constants of the dampers, and $\bar{k}_1$ and $\bar{k}_2$ are the spring constants of the dampers.

Equations (4), (5) and (8) possess a 'steady state' solution

$$\omega_z = \Omega;\ \gamma = \dot{\gamma}_0 t\ \ ;\ \phi = 0 \qquad .....(11)$$

(where $\dot{\gamma}_0$ is a constant), when the torque $T_m(t)$ is designed to overcome friction, to damp out relative oscillations between platform and rotor, and to make the platform point towards the earth (i.e., to make $\phi = 0$) (for example, $T_m(t) = c\dot{\gamma}_0 + T_1 \sin\phi + T_2\dot{\phi}$, where T_1 and T_2 are constants, will achieve the required result).

Equation (11) may now be substituted into equations (2), (3), (6), (7), (9) and (10), and the result expressed in the dimensionless form,

$$W'_x + (\Delta + J\alpha)\, W_y - 2R\xi' - I\beta'' \sin\gamma - I\beta'(\alpha + 1)\cos\gamma = 0 \qquad \ldots\ldots(12)$$

$$W'_y - (\Delta + J\alpha)\, W_x + R(\xi'' - 4\xi) + I\beta'' \cos\gamma - I\beta'\,(\alpha + 1)\sin\gamma + 3\Delta\theta = 0 \qquad \ldots\ldots(13)$$

$$(1 - \mu)\xi'' + c_1\xi' + k_1\xi + W'_y + W_x - 3\theta = 0 \qquad \ldots\ldots(14)$$

$$\beta'' + c_2\beta' + k_2\beta - \alpha W_x \cos\gamma - W'_x \sin\gamma - \alpha W_y \sin\gamma + W'_y \cos\gamma = 0 \qquad \ldots\ldots(15)$$

$$\theta' = W_y - \psi \qquad \ldots\ldots(16)$$

$$\psi' = W_x + \theta \qquad \ldots\ldots(17)$$

where $\tau = \Omega t$, the primes denote differentiation with respect to τ, and

$$\alpha = \dot{\gamma}_0/\Omega \ , \qquad \gamma = \alpha\tau \ ,$$

$$W_x = \omega_x/\Omega \ , \qquad W_y = \omega_y/\Omega$$

$$\xi = \chi/a \ ,$$

$$\Delta = (C_0 - A_0)/A_0 \ , \qquad J = C_R/A_0 \ ,$$

$$I = I_s/A_0 \ , \qquad R = ma^2/A_0 \ ,$$

$$c_1 = \bar{c}_1/m\Omega \ , \qquad c_2 = \bar{c}_2/I_s\Omega \ ,$$

$$k_1 = \{\bar{k}_1 - 3(1 - \mu)\}/m\Omega^2 \ , \qquad k_2 = \bar{k}_2/I_s\Omega^2 .$$

Note that $\alpha > 0$ by definition, i.e., if $\alpha < 0$ one must invert the definition of 'platform' and 'rotor'.

It is found helpful to reduce the above set further by substitution of (16) and (17) into (12) to (15), to obtain,

$$\psi'' + (\Delta + J\alpha)\psi + (\Delta + J\alpha - 1)\theta' = \varepsilon\Lambda_1 \qquad \ldots\ldots(18a)$$

$$-(\Delta + J\alpha - 1)\psi' + \theta'' + (4\Delta + J\alpha)\,\theta = \varepsilon\Lambda_2 \qquad \text{.....(18b)}$$

$$(1 - \mu)\xi'' + c_1\xi' + k_1\xi = -\theta'' - 2\psi' + 4\theta \qquad \text{.....(19a)}$$

$$\beta'' + c_2\beta' + k_2\beta = \alpha(\psi' - \theta)\cos\gamma + (\psi'' - \theta')\sin\gamma$$
$$+ \alpha(\theta' + \psi)\sin\gamma - (\theta'' + \psi')\cos\gamma, \qquad \text{.....(19b)}$$

where

$$\Lambda_1 = \varepsilon^{-1}\,\{2R\xi' + I\beta''\sin\gamma + I(\alpha + 1)\beta'\cos\gamma\}$$

$$\Lambda_2 = \varepsilon^{-1}\,\{R(-\xi'' + 4\xi) - I\beta''\cos\gamma + I(\alpha + 1)\beta'\sin\gamma\}.$$

In equations (18) a 'small parameter', ε, has been introduced (in rather an artificial way) as an aid to the analyses in later sections.

3. SOLUTION OF EQUATIONS (18) WHEN THE DAMPERS ARE ABSENT

When the dampers are absent, $R = I = 0$, which implies $\Lambda_1 = \Lambda_2 = 0$. Under these conditions, equations (18) become linear equations with constant coefficients, and may be solved in closed form. These same equations have been investigated[15,21]; however, it proves worthwhile to study them again from a slightly different viewpoint.

The solution of equations (18) (when $\Lambda_1 = \Lambda_2 = 0$) is

$$\psi = A\cos p_1\tau + B\sin p_1\tau + C\cos p_2\tau + D\sin p_2\tau \qquad \text{.....(20a)}$$

$$\theta = K_1 A\sin p_1\tau - K_1 B\cos p_1\tau + K_2 C\sin p_2\tau - K_2 D\cos p_2\tau \qquad \text{.....(20b)}$$

where A, B, C, and D are arbitrary constants, K_1 and K_2 are given by either one of two expressions,

$$K_i = -(-p_i^2 + \Delta + J\alpha)/p_i(\Delta + J\alpha - 1) = -\,p_i(\Delta + J\alpha - 1)/(-p_i^2 + 4\Delta + J\alpha)$$
$$i = 1,\ 2 \qquad \text{.....(21)}$$

and p_1^2 and p_2^2 are the two roots of

$$p^4 - \{(\Delta + J\alpha)^2 + 3\Delta + 1\}\,p^2 + (\Delta + J\alpha)(4\Delta + J\alpha) = 0. \qquad \text{.....(22)}$$

Infinitesimal stability is determined by the sign of p_i^2 $(i = 1,2)$, i.e., $p_i^2 \geq 0$ indicates stability, and $p_i^2 < 0$ instability. The complete properties of equation (22) in this regard may be summarized in chart form by plotting $J\alpha$ vs Δ as is done in Figure 3. The chart is equivalent to Figure 3, [15], but is more suitable in this analysis because it arises in a natural way from the equations, and the regions of instability fall within a finite region of the chart, namely $|J\alpha|<4$.

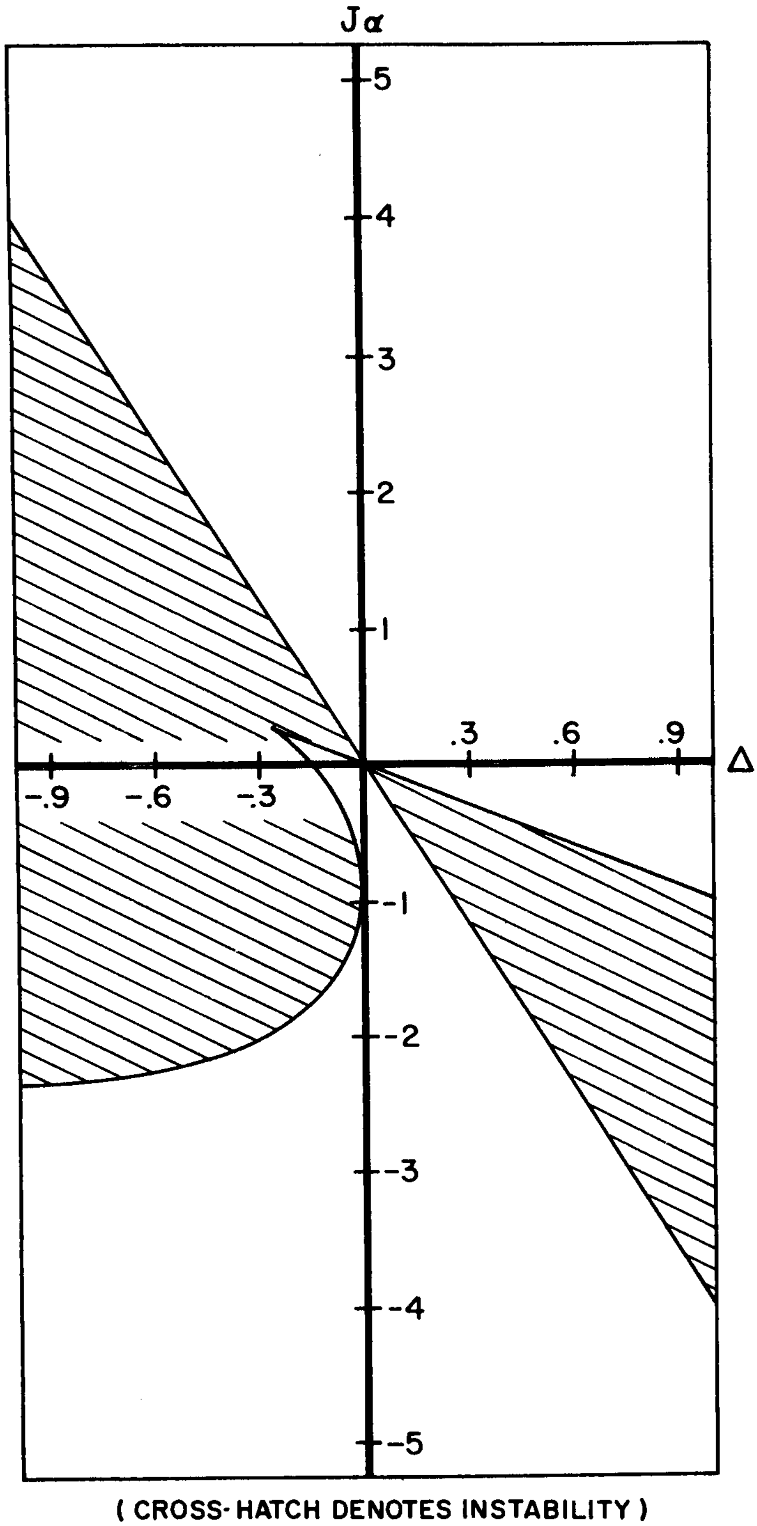

Figure 3

4. STUDY OF STABILITY BY FLOQUET ANALYSIS

Equations (12) to (17) may be easily rearranged into the form,

$$\underset{\sim}{A}\underline{z}' = \underset{\sim}{B}\underline{z} \quad , \qquad .....(23)$$

where $\underline{z}$ is an 8-dimensional column matrix

$$\underline{z} = \{\psi, \theta, W_x, W_y, \xi, \beta, \xi', \beta'\}^T \quad ,$$

and $\underset{\sim}{A}$ and $\underset{\sim}{B}$ are eight-by-eight matrix functions, periodic with period $(2\pi/\alpha)$. Stability of the solutions of the equations may be determined using Floquet theory programmed appropriately for digital computation (an account of this procedure is given in several recent papers, e.g., references 6, 15).

The method as applied to equation (23) gives some meaningful and interesting results. It is found convenient to retain the background grid of Figure 3 for displaying them, denoting a stable point by a '0' and an unstable point by an 'X'.

Stability was assessed first for check cases with R and I set equal to zero. Agreement with the chart of Figure 3 was found.

The sequence of Figures 3, 4, and 5, show that the effect of increasing the flexibility of the springs is to destroy stability. A notable exception is the point (0.3, -1.), where the increase of flexibility has rendered an unstable point stable.

Figures 6 and 7 show the effect of introducing platform damping and rotor damping to the configuration of Figure 5. In both cases, the effect is seen to be small but destabilizing (there is not sufficient evidence to conclude that damping is always destabilizing, however).

4.1 DISCUSSION

The above results demonstrate that consistent, unambiguous results are obtainable by the Floquet method in this problem, at least when $|J\alpha|<10$. However, in current applications (such as Telesat or Intelsat) $J\alpha$ is very large--of the order of 2×10^5. As a consequence, both very large and very small numbers are generated in the numerical computation process, and one becomes justifiably suspicious of the validity of the computed results under these conditions.

As well, it is evident above that the method is not well suited for obtaining a concise statement regarding stability in this problem, because of the large number of parameters involved.

The above considerations motivate one to turn to other analytical methods. A promising choice which will be pursued is the Method of Averaging.

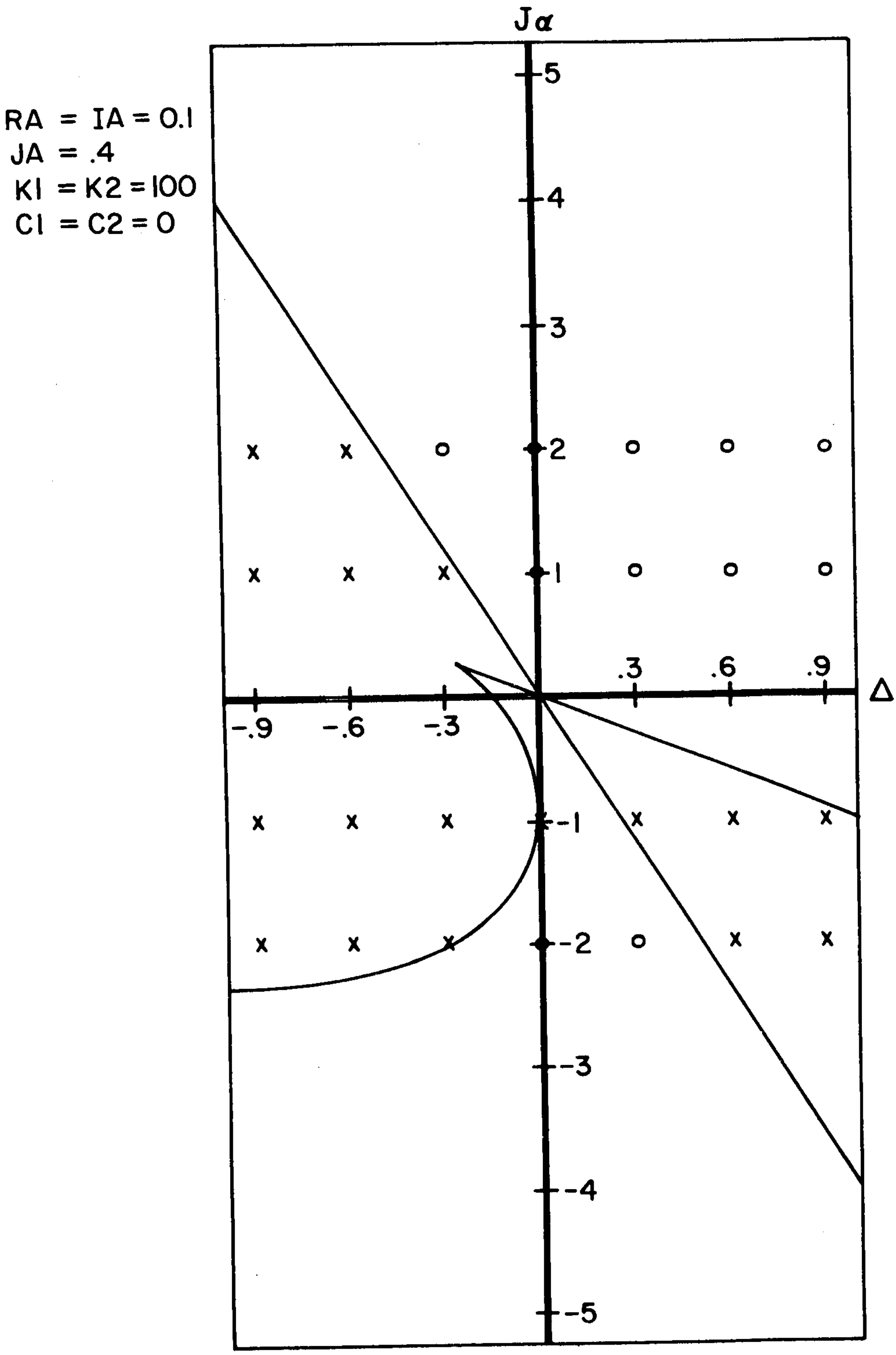

Figure 4

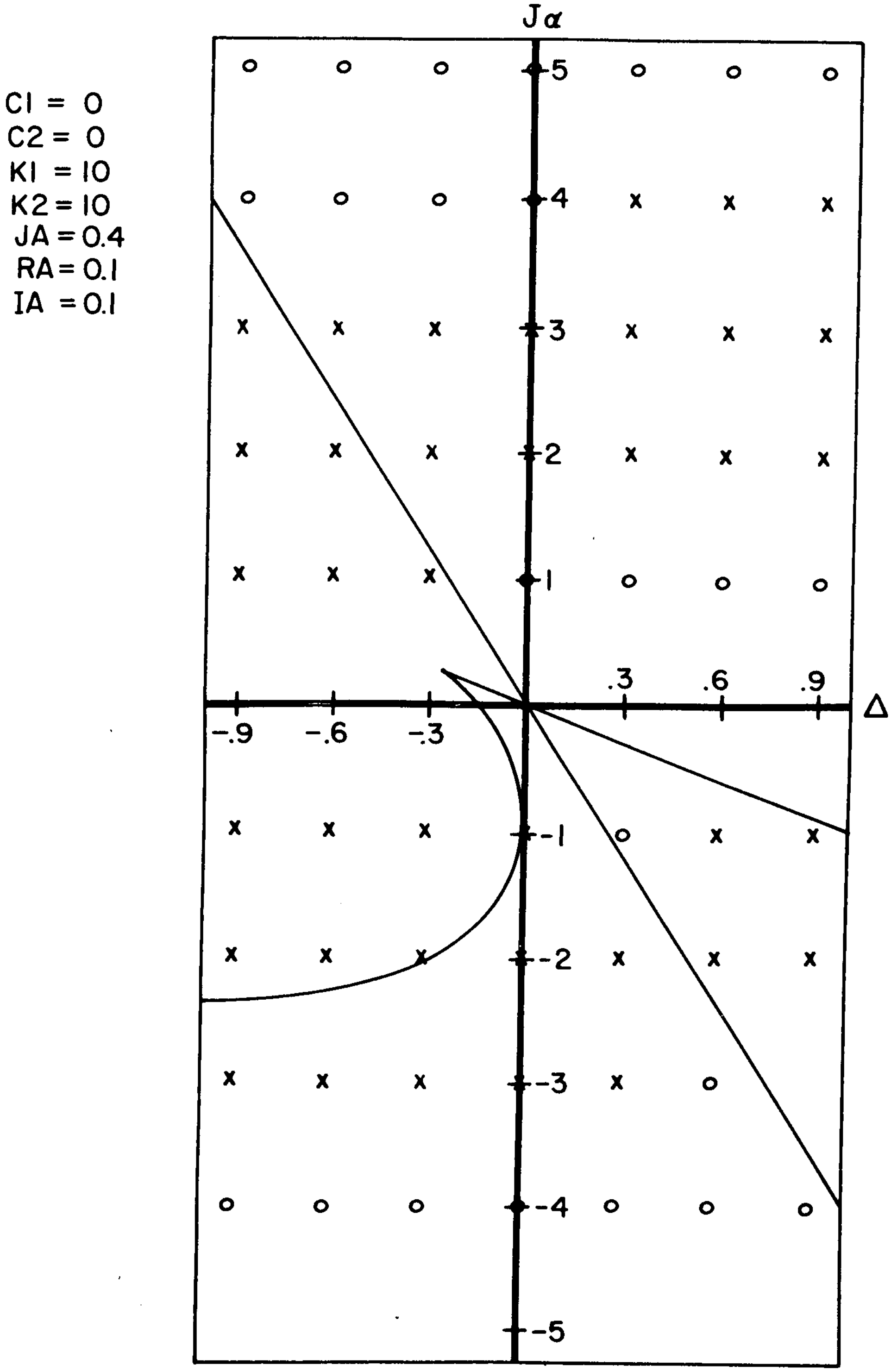

Figure 5

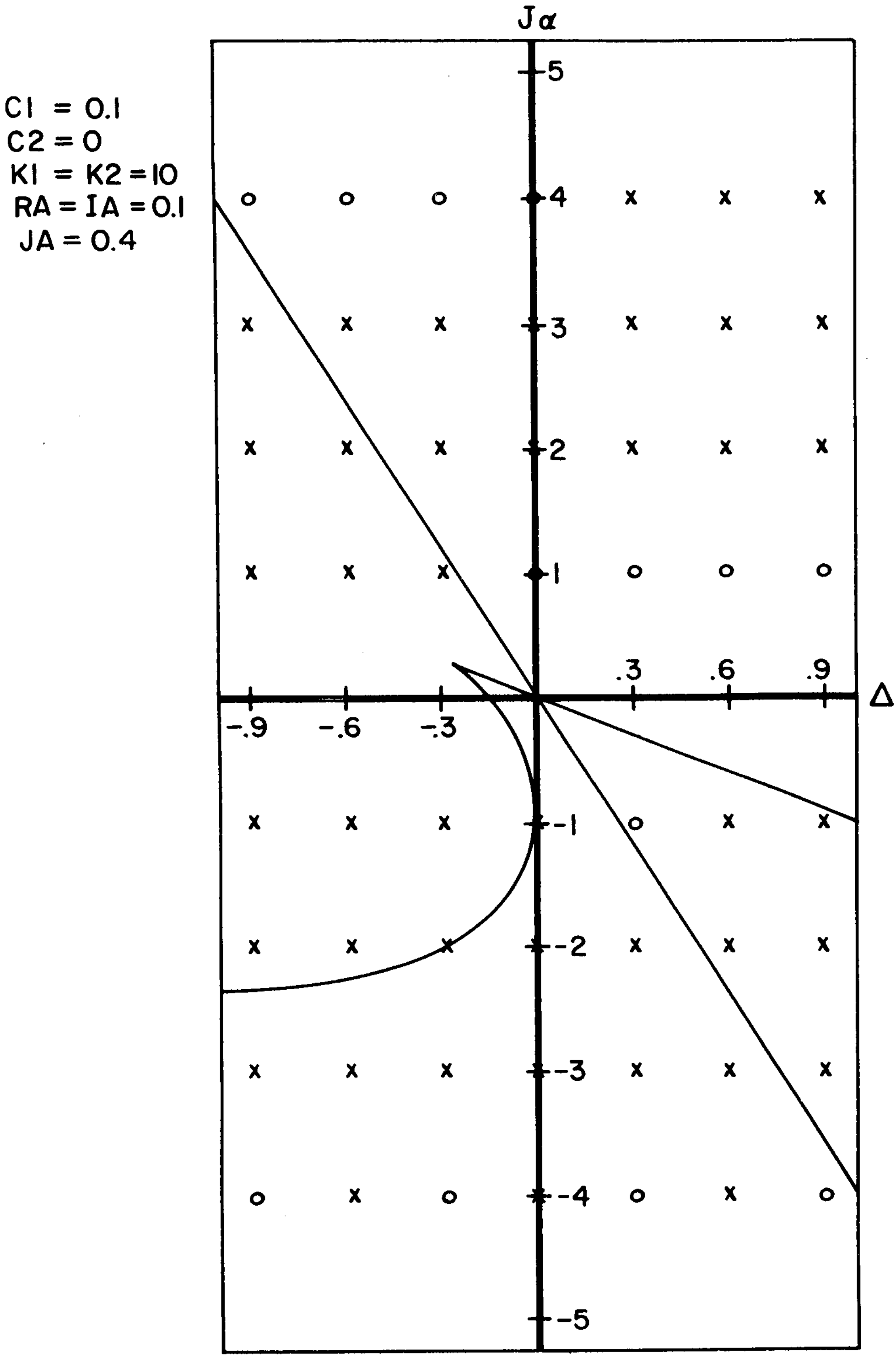

Figure 6

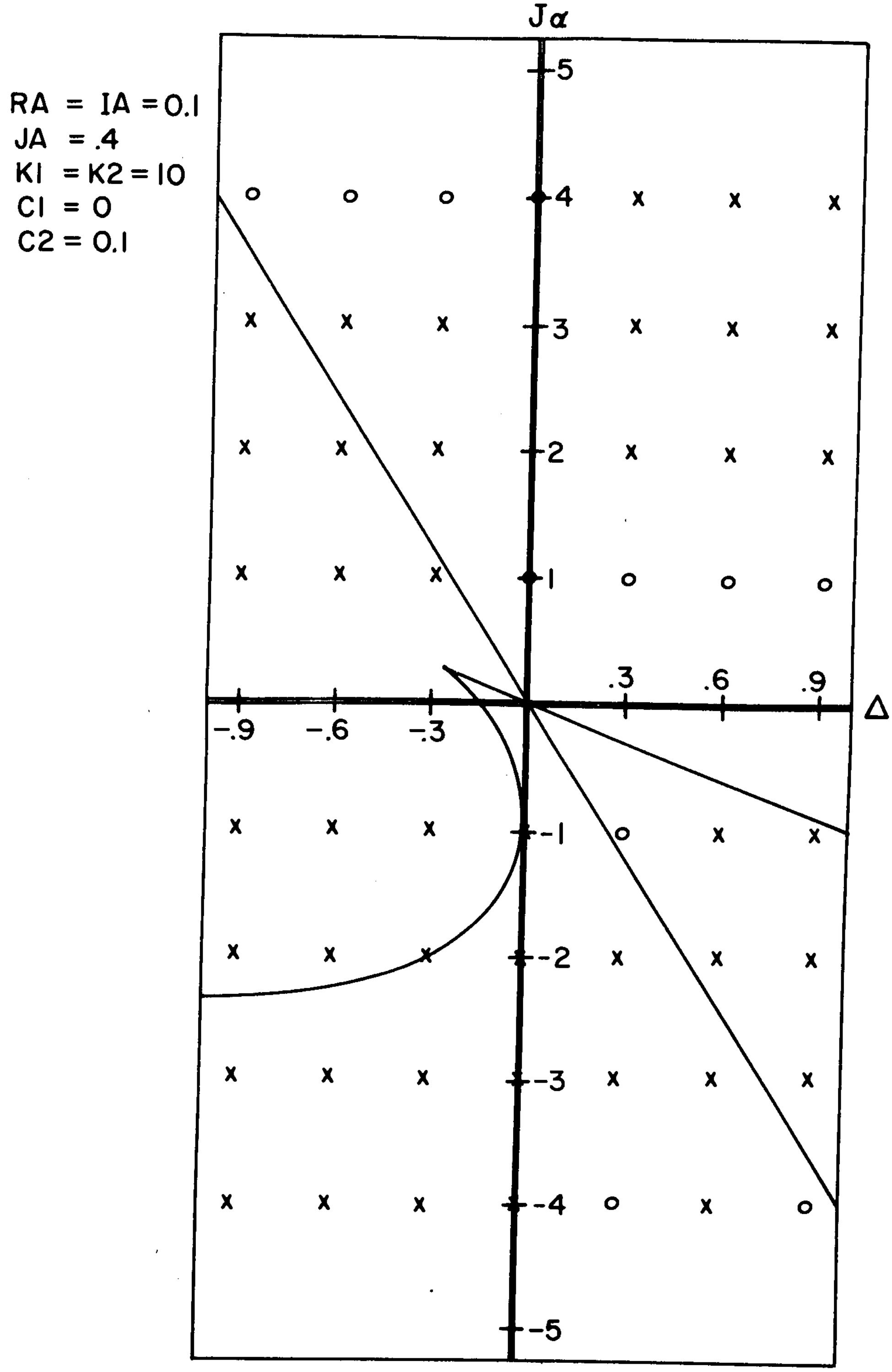

Figure 7

5. SOLUTION BY THE METHOD OF AVERAGING

5.1 TRANSFORMATION OF EQUATIONS (18) and (19)

The parameters R and I are very small in practice. Upon recognizing this, equations (18) and (19) may be transformed to a form suitable application of the formal Method of Averaging of references 22 - 24. The 'small parameter', ε, artificially introduced in equation (18) as an aid to applying this method, is indeed small when R and I are small with respect to unity.

The new variables $A(\tau)$, $B(\tau)$, $C(\tau)$ and $D(\tau)$ will be introduced to replace θ, ψ, and their first derivatives by a transformation motivated by equation (20);

$$\psi = A \cos p_1\tau + B \sin p_1\tau + C \cos p_2\tau + D \sin p_2\tau \quad \text{.....(24a)}$$

$$\psi' = -Ap_1 \sin p_1\tau + Bp_1 \cos p_1\tau - Cp_2 \sin p_2\tau + Dp_2 \cos p_2\tau \quad \text{.....(24b)}$$

$$\theta = K_1A \sin p_1\tau - K_1B \cos p_1\tau + K_2C \sin p_2\tau - K_2D \cos p_2\tau \quad \text{.....(24c)}$$

$$\theta' = K_1Ap_1 \cos p_1\tau + K_1Bp_1 \sin p_1\tau + k_2p_2C \cos p_2\tau + K_2p_2D \sin p_2\tau. \quad \text{.....(24d)}$$

Differentiation of equation (24a) and use of (24b) results in

$$A' \cos p_1\tau + B' \sin p_1\tau + C' \cos p_2\tau + D' \sin p_2\tau = 0. \quad \text{.....(25a)}$$

Similarly, equation (14c) and (14d) yield,

$$K_1A' \sin p_1\tau - K_1B' \cos p_1\tau + K_2C' \sin p_2\tau - K_2D' \cos p_2\tau = 0. \quad \text{.....(25b)}$$

Substitution of equations (24) into (18) yields,

$$- p_1A' \sin p_1\tau + p_1B' \cos p_1\tau - p_2C' \sin p_2\tau + p_2D' \cos p_2\tau = \varepsilon\Lambda_1 \quad \text{.....(25c)}$$

$$K_1p_1A' \cos p_i\tau + K_1p_1B' \sin p_1\tau + K_2p_2C' \cos p_2\tau + K_2p_2D' \sin p_2\tau = \varepsilon\Lambda_2. \quad \text{.....(25d)}$$

The above equations (25) may be written in matrix form, and then solved algebraically by Cramer's Rule for A', B', C', and D', to obtain (after a lengthy but straightforward calculation),

$$A' = \frac{\varepsilon}{\Xi} \{(p_2K_2^2 - p_1K_1K_2)\, \Lambda_1 \sin p_1\tau + (p_1K_2 - p_2K_1)\, \Lambda_2 \cos p_1\tau\} \quad \text{.....(26a)}$$

$$B' = \frac{\varepsilon}{\Xi} \{-(p_2K_2^2 - p_1K_1K_2)\, \Lambda_1 \cos p_1\tau + (p_1K_2 - p_2K_1)\, \Lambda_2 \sin p_1\tau\} \quad \text{.....(26b)}$$

$$C' = \frac{\varepsilon}{\Xi} \{(p_1K_1^2 - p_2K_1K_2)\, \Lambda_1 \sin p_2\tau - (p_1K_2 - p_2K_1)\, \Lambda_2 \cos p_2\tau\} \quad \text{.....(26c)}$$

$$D' = \frac{\varepsilon}{\Xi} \{-(p_1K_1^2 - p_2K_1K_2)\, \Lambda_1 \cos p_1\tau - (p_1K_2 - p_2K_1)\, \Lambda_2 \sin p_2\tau\} \quad \text{.....(26d)}$$

where $\Xi = K_1K_2(p_1^2 + p_2^2) - p_1p_2(K_1^2 + K_2^2) = (p_1K_1 - p_2K_2)(p_1K_2 - p_2K_1)$.

Combining equations (24) with equations (19) yields, after some reduction,

$$\xi'' + c_1\xi' + k_1\xi = (p_1^2 - 2p_1 + 4)(-A \sin p_1\tau + B \cos p_1\tau) + (p_2^2 + 2p_2 + 4)(C \sin p_2\tau - D \cos p_2\tau) - \varepsilon\Lambda_1 \quad \text{.....(27a)}$$

$$\beta'' + c_2\beta' + k_2\beta = (p_1 - 1)(p_1 + \alpha)\{-A \sin (p_1 + \alpha)\tau + B \cos (p_1 + \alpha)\tau\} + (p_2 + 1)(p_2 - \alpha)\{C \sin (p_2 - \alpha)\tau - D \cos (p_2 - \alpha)\tau\} + \varepsilon(\Lambda_1 \sin \gamma - \Lambda_2 \cos \gamma). \quad \text{.....(27b)}$$

Equations (26) and (27) are exact, in the sense that they are derived from (18) and (19) with no approximations.

5.2 THE METHOD OF AVERAGING

At this point it becomes evident that equations (26) and (27) may be cast into the form

$$x' = \varepsilon X(x,y) \quad \text{.....(28a)}$$
$$y' = Y_0(x,y) + \varepsilon Y_1(x,y) \quad \text{.....(28b)}$$

where x and y are 4-dimensional column vectors

$$x = \{A, B, C, D\}^T \quad , \quad y = \{\xi, \beta, \xi', \beta'\}^T.$$

Solutions of equations (28) have been found and established by the formal Method of Averaging in references 22, 23, and 24. Approximate solutions valid to any desired degree of accuracy may be obtained. Briefly, one seeks a solution of the form,

$$x = \overline{x} + \sum_{n=1}^{\infty} \varepsilon^k u^k \quad \text{.....(29a)}$$

$$y = \overline{y} + \sum_{n=1}^{\infty} \varepsilon^k v^k \quad \text{.....(29b)}$$

where $\overline{x}$ and $\overline{y}$ are the 'averaged' solutions, and u^k and v^k are time-varying functions. The functions $\overline{x}$, $\overline{y}$, u^k and v^k are obtained by solving differential equations constructed by formal procedures outlined in reference 22, and are usually easier to solve than the original equations (28). To obtain a solution valid to a 'first approximation', (i.e., $x = \overline{x}$, $y = \overline{y}$), one first solves equations (28) with $\varepsilon = 0$, to obtain a solution

$$x = \text{constant} = \overline{x} \quad \text{.....(30a)}$$

$$y = \zeta(x,\tau) \quad \text{.....(30b)}$$

and then constructs the equations for $\overline{x}$ and $\overline{y}$,

$$\frac{d\overline{x}}{d\tau} = \lim_{T\to\infty} \frac{1}{T} \int_{\tau}^{\tau+T} \varepsilon X[\overline{x}, \zeta(\overline{x},t)]\, dt \qquad \text{.....(31a)}$$

$$\overline{y} = \zeta(\overline{x},\tau). \qquad \text{.....(31b)}$$

5.3 THE FIRST APPROXIMATION SOLUTION FOR THE HIGH SPIN CASE

Although the solution of equations (26) and (27) by the method outlined above is straightforward in principle, it requires a great deal of effort. To make the solution tractable, the 'high spin' approximation, which implies that $J\alpha$ is large, will be introduced.

One may verify that the roots of equation (22) are obtained from the expansion,

$$p^2 = \frac{(J\alpha)^2}{2} \left[\left\{1 + \frac{2\Delta}{J\alpha} + \frac{\Delta^2 + 3\Delta + 1}{(J\alpha)^2}\right\} - \left\{1 + \frac{2\Delta}{J\alpha} + \frac{\Delta^2 + 3\Delta - 1}{(J\alpha)^2}\right\} + 0\left\{\frac{1}{(J\alpha)^3}\right\}\right].$$

When $J\alpha$ is large, p_1 and p_2 are then given approximately by

$$p_1^2 = 1, \quad p_2^2 = (J\alpha)^2.$$

Hence, for high spin, all roots are real (indicating rigid body stability). For the transformation equations (24), p_1 and p_2 will be taken to be the positive roots

$$p_1 = 1\ , \quad p_2 = J\alpha. \qquad \text{.....(32)}$$

Substitution of these values into k_i and Ξ of expressions (21) and (26) yields

$$K_1 = -1, \quad K_2 = 1, \quad \Xi = -J^2\alpha^2,$$

and equations (26) become,

$$A' = \frac{\varepsilon}{J\alpha}\{-\Lambda_1 \sin p_1\tau - \Lambda_2 \cos p_1\tau\} \qquad \text{.....(33a)}$$

$$B' = \frac{\varepsilon}{J\alpha}\{\Lambda_1 \cos p_1\tau - \Lambda_2 \sin p_1\tau\} \qquad \text{.....(33b)}$$

$$C' = \frac{\varepsilon}{J\alpha}\{-\Lambda_1 \sin p_2\tau + \Lambda_2 \cos p_2\tau\} \qquad \text{.....(33c)}$$

$$D' = \frac{\varepsilon}{J\alpha}\{\Lambda_1 \cos p_2\tau + \Lambda_2 \sin p_2\tau\}. \qquad \text{.....(33d)}$$

The Method of Averaging as outlined in the preceding section will now be applied directly to equations (33) and (27) (without transforming to the form of (28)) to obtain a 'first approximation' solution. The equations for the solution corresponding to equations (30) are

$$A' = 0\ , \quad B' = 0\ , \quad C' = 0\ , \quad D' = 0\ , \qquad \text{.....(34)}$$

together with equations (27). Substitution of (32) and the solution to the above equations (namely A = const., etc.) into equation (27), leads to the solution $\langle\xi\rangle$ and $\langle\beta\rangle$, (i.e., the $\langle\rangle$ denote average values).

$$\langle\xi\rangle = \Phi_1 \, [- A \sin (\tau - \phi_1) + B \cos (\tau - \phi_1)]$$

$$+ Z_1 \, [C \sin (J\alpha\tau - \zeta_1) - D \cos (J\alpha\tau - \zeta_1)] \qquad \text{.....(35a)}$$

$$\langle\beta\rangle = Z_2 \, [C \sin \{(J\alpha - \alpha)\tau - \zeta_2\} - D \cos \{(J\alpha - \alpha)\tau - \zeta_2\}], \qquad \text{.....(35b)}$$

where

$$\Phi_1 = 3/\{(k_1 - 1)^2 + c_1^2\}^{1/2}$$

$$Z_1 = \{(J\alpha)^2 + 2J\alpha + 4\}/\{(k_1 - J^2\alpha^2)^2 + (c_1 J\alpha)^2\}^{1/2}$$

$$Z_2 = (J\alpha + 1)(J\alpha - \alpha)/[\{k_2 - (J\alpha - \alpha)^2\}^2 + \{c_2 \, (J\alpha - \alpha)\}^2]^{1/2}$$

$$\tan \phi_1 = c_1/(k_1 - 1)$$

$$\tan \zeta_1 = c_1 J\alpha/(k_1 - J^2\alpha^2)$$

$$\tan \phi_2 = c_2(1 + \alpha)/\{k_2 - (1 + \alpha)^2\}$$

$$\tan \zeta_2 = c_2(J\alpha - \alpha)/\{k_2 - (J\alpha - \alpha)^2\}$$

and $1 - \mu \doteq 1$. Substitution of equations (35) into (33) and "averaging" as is indicated in equation (31a) results in (after lengthy but straightforward calculation) the following differential equations.

$$\langle A' \rangle = a_{11} \langle A \rangle - a_{21} \langle B \rangle \qquad \text{..... (36a)}$$

$$\langle B' \rangle = a_{21} \langle A \rangle + a_{11} \langle B \rangle \qquad \text{..... (36b)}$$

$$\langle C' \rangle = d_{11} \langle C \rangle - d_{21} \langle D \rangle \qquad \text{..... (36c)}$$

$$\langle D' \rangle = d_{21} \langle C \rangle + d_{11} \langle D \rangle \qquad \text{..... (36d)}$$

where

$$a_{11} = -9c_1 R/[2|J\alpha|\{(k_1 - 1)^2 + c_1^2\}]$$

$$a_{21} = 3R(k_1 - 1)/[2|J\alpha|\{(k_1 - 1)^2 + c_1\}^2]$$

$$d_{11} = -\frac{c_1 R\{(J\alpha)^2 + 2J\alpha + 4\}^2}{2[(k_1 - J^2\alpha^2)^2 + (c_1 J\alpha)^2]} - \frac{c_2 I\alpha^2 (J-1)^3 (J\alpha + 1)^2}{2J[\{k_2 - \alpha^2(J-1)^2\}^2 + \{c_2\alpha(J-1)\}^2]}$$

$$d_{21} = \frac{R\{(J\alpha)^2 + 2J\alpha + 4\}(k_1 - J^2\alpha^2)}{2J\alpha[(k_1 - J^2\alpha^2)^2 + (c_1 J\alpha)^2]} + \frac{I\alpha(J-1)^2(J\alpha+1)^2 \{k_2 - \alpha^2(J-1)^2\}}{2J\,[\{k_2 - \alpha^2(J-1)^2\}^2 + \{c_2\alpha(J-1)\}^2]}$$

and the < > denotes averaged value.

Equations (36) are readily solved to give

$$\langle A\rangle = e^{a_{11}\tau}\{A_o \cos a_{21}\tau - B_o \sin a_{21}\tau\} \quad \text{.....(37a)}$$

$$\langle B\rangle = e^{a_{11}\tau}\{A_o \sin a_{21}\tau + B_o \cos a_{21}\tau\} \quad \text{.....(37b)}$$

$$\langle C\rangle = a^{d_{11}\tau}\{C_o \cos d_{21}\tau - D_o \sin d_{21}\tau\} \quad \text{.....(37c)}$$

$$\langle D\rangle = a^{d_{11}\tau}\{C_o \sin d_{21}\tau + D_o \cos d_{21}\tau\} \quad \text{.....(37d)}$$

In accordance with previous discussion, a first approximation solution is,

$$A = \langle A\rangle\ ,\ B = \langle B\rangle\ ,\ C = \langle C\rangle\ ,\ D = \langle D\rangle\ , \quad \text{.....(38a)}$$

$$\xi = \langle\xi\rangle\ ,\ \beta = \langle\beta\rangle\ . \quad \text{.....(38b)}$$

The above solution may be expected to be valid whenever the right hand sides of equations (33) are 'small', which is true when R and I are sufficiently small. Inaccuracy may arise when the dampers are excited at 'near resonance' conditions, in which case χ and β (and consequently the right hand sides of (33)) are large.

The solution accuracy may be improved at the expense of laborious but straightforward calculation by invoking the theory for the higher order approximations, as outlined in references 22 - 24.

5.4 COMPARISON WITH RESULTS OF OTHER WORK

The free spin of a dual spin satellite (i.e., the problem posed herein, but with gravity torque absent) has been previously studied by the author[7] by the Method of Averaging. The results of that analysis for high spin are found to be embodied exactly in equations (36c) and (36d). The appearance of equations (36a) and (36b) thus stems from gravity torques.

The validity of the Method of Averaging solution for this class of problems is demonstrated in reference 7, where analytical and numerically-obtained solutions are compared.

5.5 STABILITY CRITERIA

The analytical stability criteria deduced from equations (37) are $a_{11} < 0$, $d_{11} < 0$.

Since J, α, c_1, and c_2 are always positive, it follows that a_{11}, and the first term of d_{11} are less than zero. If rotor damping is present, and if $J < 1$, d_{11} can be made greater than zero. The stability criterion then becomes

$d_{11} < 0$ implies stability; $d_{11} > 0$ implies instability. .....(39)

From the above discussion, it is noted that A and B are always bounded, the platform damping is always stabilizing, and the rotor damping is stabilizing if and only if $J > 1$ (i.e., $C_R/A_0 > 1$), in the high spin case.

6. DISCUSSION

In this work the linearized equations have been dealt with, and hence conclusions regarding stability must be interpreted in the sense of 'infinitesimal stability', i.e., stability is indicated, but not guaranteed. Results obtained by the Method of Averaging may be interpreted in the same light as those given by the 'exact' Floquet analysis in this regard since the method is well established and its validity has been demonstrated for this class of problems in reference 7. It is noted in Section 4 than when R, I, and $(1/J\alpha)$ are small (as they are for dual spin communications satellites), inaccuracies arise in computation of Floquet exponents which jeopardize the validity of stability results obtained by that method. In contrast, the method of averaging solution may be expected to approach the true solution with increasing accuracy as R, I, and $(1/J\alpha)$ tend to smaller values.

The analytical results obtained herein as the first approximation solution of the linearized equations are exactly what one could obtain by a similar first approximation treatment of the corresponding non-linear equations. Resonance lines of parametric excitation (which are not displayed in this work, but are important in some instances) have invariably been constructed from solutions of the linearized equations [13, 14, 20, 21] and could be generated in a 'second approximation' treatment along the lines set out in this report (it is possible that they may also be generated from a somewhat revised first approximation treatment). This extension of the analysis remains for future investigation.

7. CONCLUDING REMARKS

The following general observations and conclusions may be made.

The results of Sections 3 and 4 show that gravity torques are an important consideration when spin rates are low (i.e., when $|J\alpha| <$ about 10).

For high spin cases (i.e., when $J\alpha$ is large), the stability criterion obtained are identical with those obtained by the analysis of a freely spinning untorqued satellite (namely $J > 1$ for stability). The gravity effects are present in the final solution, but do not contribute to stability criteria.

The Method of Averaging as developed herein appears to be well suited for the class of spin problems where the damper masses are small and the spin is large.

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NOMENCLATURE

$O'x'y'z'$ - axes fixed on principal axes of undeformed satellite.

$Oxyz$ - axes which move so that O is on the mass center at all times.

M - total mass.

$\psi, \theta, \phi,$ - Euler orientation angles.

$\omega_x, \omega_y, \omega_z,$ - angular velocity components resolved in (Oxyz).

χ - deflection of platform damper.

β - angular deflection of rotor damper.

A_0 - transverse moment of inertia of whole satellite when undeformed.

C_0 - spin axis moment of inertia of whole satellite when undeformed.

C_R - spin axis moment of inertia of the rotor (including damper).

C_P - spin axis moment of inertia of the platform (including damper).

I_S - moment of inertia of the spherical damper.

m - mass of platform damper.

a - distance from mass center of satellite to m.

t, superscript dot - time.

γ - angle of turn of the rotor with respect to the platform.

c - damping constant of the motor - bearing assembly.

$T_m(t)$ - control torque supplied by internal motor.

T_1, T_2 - torque parameters of $T_m(t)$.

$\bar{c}_1$ - damping constant of platform damper.

$\bar{c}_2$ - damping constant of rotor damper.

$\bar{k}_1$ - spring constant of platform damper.

$\bar{k}_2$ - spring constant of rotor damper.

Ω - angular frequency of circular orbit.

$\dot{\gamma}_0$ - angular speed of rotor with respect to the platform.

τ, superscript primes - Ωt, dimensionless time

W_x - $\omega_x\Omega$, dimensionless angular rate.

W_y - ω_y/Ω, dimensionless angular rate.

ξ - χ/a, dimensionless displacement of platform damper.

Δ - $(C_0 - A_0)/A_0$

J - C_R/A_0

α - $\dot{\gamma}_0/\Omega$

$J\alpha$ - dimensionless angular momentum of rotor

R - ma^2/A_0

I - I_s/A_θ

c_1 - $\bar{c}_1/m\Omega$

c_2 - $\bar{c}_2/I_s\Omega$

k_1 - $\{\bar{k}_1 - 3(1 - u)\}/m\Omega^2$

k_2 - $\bar{k}_2/I_s\Omega^2$

Λ_1, Λ_2 - perturbation function due to dampers.

ε - small parameter

A, B, C, D, - dimensionless variables, functions of τ

p, p_1, p_2 - resonant frequencies

K_1, K_2 - constants

Ξ - value of determinant

$\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underline{z}$ - matrix functions

x, y, X, Y, $\zeta(x,t)$, u^k, v^k - variables used in describing Method of Averaging

ϕ_1, ϕ_2, ζ_1, ζ_2 - phase lag angles of damper responses

Φ_1, Z_1, Z_2 - magnification factors of damper deflections

a_{11}, d_{11}, - stability parameters

a_{21}, d_{21}, - coning frequencies