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THE BAYESIAN RECEIVER FOR INTERFERING DIGITAL SIGNALS

by

R.R. Bowen

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ABSTRACT

The Bayesian decision procedure to determine the digits of an infinite m-ary digit sequence that have been transmitted synchronously at a high rate over a known noisy dispersive linear communications channel is derived. This decision procedure is optimum whenever a digit-by-digit modulation procedure is used and the significant intersymbol interference at the receiver input is between a finite number of digits.

This result is then applied to the case in which the additive noise is white and Gaussian. The receiver that carries out the Bayesian decision procedure is described. This receiver can be simplified if pulse amplitude modulation is used. The receiver for binary pulse amplitude modulated digits is described in more detail, and its performance is compared with that of the linear equalizer and the decision feedback equalizer, two suboptimum receivers.

1. INTRODUCTION

Increasing use is being made of noisy dispersive channels for the transmission of digital information at high rates. A widely used modulation technique is to represent the message as a sequence of m-ary digits, to choose a distinctive pulse to represent each of the m possible digits, and to transmit a sequence of these pulses at a regular rate over the channel. (Examples of this type of modulation are m-ary pulse amplitude modulation and m-ary frequency-shift keying.) To use the channel efficiently, the data is sent so rapidly that intersymbol interference between several successive pulses occurs at the receiver input. The receiver must recover the original digit sequence with a very small probability of error in the presence of both this intersymbol interference and additive noise.

The receiver that does this in such a way that the average risk associated with making a decision about a digit is minimized, that is, the Bayesian receiver, is described in this paper, and its performance is given. This performance is compared with that of several suboptimum receivers and with the performance that would be possible if there were no intersymbol interference.

Several suboptimum receivers for the task outlined above have been described. Among these are the optimum linear receivers[1-4] and the decision feedback equalizer[5]. A distinction must be made at this point between two related problems: the detection of digits of an infinite sequence, and the detection of digits of a finite sequence of specified length. If the digit sequence is treated as a finite one the signal representing the complete message is received before any part of that message is determined. The optimum linear receiver for detecting such a sequence has been described by Tufts[1], and by Aaron and Tufts[2]. However, the statistically optimum receiver for detecting a finite sequence is nonlinear. It has been described by Bowen[6,7], and by Abend, Harley, Fritchman and Gumacos[8].

A different but related problem is the detection of the digits of an infinite sequence, or a sequence that is too long for the above receivers to be feasible. The optimum linear, time-invariant receiver for the detection of digits of an infinite sequence was described by George[3] and by Berger and Tufts[4]. As in the finite sequence case, however, the statistically optimum receiver is nonlinear. Austin[5] has shown that the receiver that uses previous decisions to coherently cancel the intersymbol interference performs better than the optimum linear receiver. Hancock and Quincy[9] have described the receiver that uses a restricted amount of signal to minimize the average risk when the digit alphabet is binary. However, it will be shown here that, in general, the correct use of a larger amount of receiver signal reduces the average risk. Gonsalves[10] has described the maximum likelihood receiver that uses the received signal over an arbitrarily large time interval, but this derivation is restricted to those cases in which the intersymbol interference is between adjacent digits only, and in which the digits are binary. A method of extending these results to the detection of m-ary digits and to combat more complex intersymbol interference was suggested, but the proposed method is suboptimum.

In this report the optimum method of detecting digits of an infinite sequence is extended to include digits from an m-ary alphabet, and to combat intersymbol interference between any finite number of consecutive digits. The results apply whenever a digit-by-digit modulation technique is used. The receiver is optimum for any digit loss matrix specified by the user, and for any known a priori probability distribution.

A mathematical description of the problem is given in the next section. The Bayesian decision procedure is then derived and discussed in Section 3. Synthesis of the Bayesian receiver when the noise is white and Gaussian is described in Section 4 for any digit-by-digit modulation procedure, and then in a simpler form for the detection of pulse amplitude modulated digits. (This synthesis method can also be used to mechanize the Bayesian receiver for a finite digit sequence.[6-8]) The performance of the receiver for binary pulse amplitude modulated digits is then described, and compared with that of several suboptimum receivers.

A more detailed description of the Bayesian receiver and of its performance can be found in the author's Ph.D. thesis[7]. The present report was completed for possible publication in March, 1970. Since that time, a derivation of the Bayesian decision procedure, very similar to that in section 3 of this report, has been reported by Abend and Fritchman[16]. However there are significant differences between the implementation discussed in section 4 and that described in reference 16. It is shown that several filters, matched to portions of the received isolated pulse, should be used rather than the single sharp cutoff filter that is suggested in reference 16.

2. MATHEMATICAL DESCRIPTION OF PROBLEM

The message is assumed to be an infinite sequence of independent m-ary digits from an ergodic random process. A digit is transmitted every T seconds. B(j) represents the digit that is transmitted at time $t = jT$. It is one of m digits b_i, $i = 1,,2,..., m$. The receiver makes the decision $\hat{B}(j)$ about B(j) $\hat{B}(j)$ is also one of the m digits $\{b_i\}$.

If $B(j) = b_k$, the pulse $s_k(\tau)$, one of m pulses of the set $\{s_i(\tau);\ i = 1,2,...,m;\ \tau > 0\}$ is transmitted in the interval $jT \leq t < (j+K+1)T$. The one-to-one mapping between $\{b_i;\ i = 1,2,...,m\}$ and $\{s_i(\tau);\ i = 1,2,...,m\}$, and the members of $\{s_i(\tau)\}$, are assumed to be known at the receiver, but no other assumptions are made about the modulator. Most nonlinear modulators, as well as the linear pulse amplitude modulator, can be described by this model. The channel is represented by a linear time-invariant filter, with known impulse response $c(\tau)$, followed by a source of additive noise n(t). It is assumed that n(t) is an element of an ergodic random process, that its statistical properties are known, and that it is statistically independent in different baud intervals $kT \leq t \leq (k+1)T$.

The received waveform x(t) in the interval $jT \leq t < (j+1)T$ is

$$x(t) = \sum_{k=0}^{\infty} q_{i(j-k)}\,(t - (j-k)T) + n(t) \qquad .....(1)$$

where

$$q_i(\tau) = s_i(\tau) * c(\tau), \quad i = 1,2,...,m \qquad .....(2)$$

In equation (1) i is a random function of the time index j-k, determined by the message digit sequence. It is now assumed, to make the problem tractable, that each of the m pulses $q_i(\tau)$ is zero outside the interval $0 \leq \tau < (K+1)T$

for some finite positive integer K. (This assumption, or approximation, can be made as accurate as desired by choosing a K sufficiently large. It will be seen that K is a parameter of the Bayesian receiver.) Equation (1) then becomes

$$x(t) = \sum_{k=0}^{K} q_{i(j-k)}(t - (j-k)T) + n(t) \qquad .....(3)$$

In the interval $jT \leq t < (j+1)T$, $x(t)$ is dependent on the digits $B(j),\ldots,$ $B(j-K)$, and on $n(t)$. In the next baud interval it is dependent on $B(j+1),$ $\ldots, B(j-K+1)$, and on $n(t)$. It has been shown[11] that the receiver "noiseless" signal

$$r(t) \triangleq x(t) - n(t) \qquad .....(4)$$

is the output of a Markov source with m^{K+1} states. Properties of a Markov random process are used to derive the Bayesian decision rule to determine $B(j)$. The received signal $x(t)$ over a baud interval $jT \leq t \leq (j+1)T$ can be represented by the vector $\vec{X}(j)$, where the components of $\vec{X}(j)$ are the coefficients of some expansion of $x(t)$. Equation (4) can be replaced by the vector equation

$$\vec{X}(j) = \vec{R}(j) + \vec{N}(j). \qquad .....(5)$$

$x(t)$ over a larger interval $r_1T \leq t \leq r_2T$ can be represented by the vector

$$\vec{V}(r_1,r_2) \triangleq \{\vec{X}(r_1),\ldots,\vec{X}(r_2-1)\}. \qquad .....(6)$$

It is required that the receiver use $x(t)$ over some specified interval $r_1T \leq t \leq r_2T$ to make a final decision about $B(j)$. (Sequential decision rules, in which r_1 and/or r_2 are functions of the received signal, are not considered.) The receiver can be described by a decision rule or function D that maps all possible received signal vectors $\vec{V}(r_1,r_2)$ onto a set of m possible decisions $\{b_i\}$. The average risk taken when this decision rule is used is

$$\rho(r_1,r_2,D) = \sum_{i=1}^{m} P_{B(j)}[b_i] \int_{\vec{V}} L\{b_i,D(\vec{V})\}p[\vec{V}|B(j) = b_i]\, d\vec{V} \qquad .....(7)$$

where $P_{B(j)}[b_i] = P[b_i]$ is the a priori probability that $B(j) = b_i$, $L\{b_i,D(\vec{V})\} = L\{i,k\}$ is the loss suffered when $B(j) = b_i$ and $D(\vec{V}) \triangleq \hat{B}(j) = b_k$, and $p[\vec{V}|B(j) = b_i]$ is the probability density function of $\vec{V}(r_1,r_2)$, given that $B(j) = b_i$. The decision rule $D(\vec{V})$ that minimizes (7) is the Bayesian decision rule, and the device that carries out that decision rule, or its equivalent, is the Bayesian receiver. The decision rule is derived in the next section. The receiver is allowed to observe $x(t)$ over the interval

$-\infty < t \leq (j+K+M+1)T$ before making the decision $\hat{B}(j)$. M is any non-negative integer, and becomes a parameter of the Bayesian receiver.

3. THE BAYESIAN DECISION PROCEDURE

The decision procedure $D(\vec{V})$ that minimizes ρ is derived here in a form that can be mechanized with a realizable fixed-sized machine. At time $t = (j+K+M+1)T$ the receiver must use $\vec{V}(-\infty, j+K+M+1)$ to make the decision $\hat{B}(j)$. If the receiver were to make the observation $\vec{V}(-\infty, j+K+M+1)$ and use a decision rule such that $D(\vec{V}) = b_k$, the risk taken in making this decision would be

$$\tau_j(k) = \frac{\sum_{i=1}^{m} L\{i,k\} p[\vec{V}(-\infty, j+K+M+1) | B(j) = b_i] P[b_i]}{\sum_{i=1}^{m} p[\vec{V}(-\infty, j+K+M+1) | B(j) = b_i] P[b_i]} \qquad .....(8)$$

(There is such a risk $\tau_j(\ell)$ associated with making each of the decisions $\hat{B}(j) = b_\ell$, $\ell = 1,2,...,m$.) Blackwell and Girshick (12, pp. 175-176) have shown that (7) is minimized when $\hat{B}(j) = b_k$ if $\tau_j(k)$ is the smallest risk of the set $\{\tau_j(\ell);\ \ell = 1,2,...,m\}$. Thus the Bayesian receiver must determine these m a posteriori risks, or m terms equivalent to them, each time it makes a decision.

The denominator of (8) is independent of k, and so $\tau_j(k)$ can be replaced by

$$\gamma(k) \triangleq \sum_{i=1}^{m} L\{i,k\} p[\vec{V}(-\infty, j+K+M+1) | B(j) = b_i] P[b_i] \qquad .....(9)$$

The a priori probabilities $\{P[b_i]\}$ are known, and the losses $\{L\{i,k\}\}$ are specified by the communication link user, and so the decision problem becomes that of calculating the m probability density values $p[\vec{V}(-\infty, j+K+M+1) | B(j) = b_i]$, $i = 1,2,...,m$. Calculation of these quantities can be simplified by using the fact that the "noiseless" received signal in any one baud interval $\ell T \leq t < (\ell+1)T$ is determined by the (K+1) digits $B(\ell),...,B(\ell-K)$, and in the next baud interval by the digits $B(\ell+1),...,B(\ell-K+1)$. Thus r(t) may be thought of as coming from one of m^{K+1} Markov states $A(\ell)$, where

$$A(\ell) \triangleq \{B(\ell),\ B(\ell-1),...,B(\ell-K)\} \qquad .....(10)$$

The occurrences of these states are, of course, mutually exclusive events, and one must occur, so

$$p[\vec{V}(-\infty,\ j+K+M+1)|B(j) = b_i]$$

$$= \sum_{\ell=1}^{m^{(K+1)}} p[\vec{V}(-\infty,\ j+K+M+1)|A(j+K) = a_\ell]\cdot P[A(j+K) = a_\ell|B(j) = b_i] \qquad .....(11)$$

Thus the problem of determining the probability density values $p[\vec{V}(-\infty,\ j+K+M+1)|B(j) = b_i]$ can be made one of calculating the terms $p[\vec{V}(-\infty,\ j+K+M+1)|A(j+K) = a_\ell]$, $\ell = 1,2,\ldots,m^{K+1}$. Because $A(j+K)$ is a Markov state, this calculation can be made easier by factoring $p[\vec{V}(-\infty,\ j+K+M+1)|A(j+K) = a_\ell]$ into the two terms

$$p[\vec{V}(-\infty,\ j+K+1)|A(j+K) = a_\ell]\cdot p[\vec{V}(j+K+1,\ j+K+M+1)|A(j+K) = a_\ell] \qquad .....(12)$$

Neither of these factors can be calculated directly. However, each of them can be further expanded to a form that can be calculated, as shown in Appendix A.

The first term can be expanded in the following way:

$$p[\vec{V}(-\infty,\ j+K+1)|A(j+K) = a_\ell]$$

$$= p[\vec{X}(j+K)|A(j+K) = a_\ell]\cdot\{\sum_{n=1}^{m^{(K+1)}} P[A(j+K) = a_\ell|A(j+K-1) = a_n]\cdot$$

$$p[\vec{V}(-\infty,j+K)|A(j+K-1) = a_n]\cdot P[A(j+K-1) = a_n]\} \qquad .....(13)$$

This expansion is over all the possible states that could occur in the interval $(j+K-1)T \leq t < (j+K)T$. It is done so that the term

$$p[\vec{X}(j+K)|A(j+K) = a_\ell]$$

$$= p_{\vec{N}}[\vec{X}(j+K) - \vec{R}_\ell] \qquad .....(14)$$

can be isolated. ($\vec{R}_\ell$ is the vector representation of the noiseless received signal when $A(j+K) = a_\ell$.) This term, or a term proportional to it, can be determined directly from the input waveform $x(t)$ and knowledge of the statistical properties of the additive noise.

The terms $P[A(j+K) = a_\ell | A(j+K-1) = a_n]$ and $P[A(j+K-1) = a_n]$ depend only on the digit a priori probabilities $\{P[b_i]\}$ and the method of indexing the Markov states. The terms $p[\vec{V}(-\infty, j+K) | A(j+K-1) = a_n]$ are the same as the term $p[\vec{V}(-\infty, j+K+1) | A(j+K) = a_\ell]$, except for a shift in the timing index. Moreover, these terms are used to determine $B(j-1)$, and so a recursive detection algorithm can be used, enabling one to use the received signal over the infinite time interval $-\infty < t \leq (j+K+1)T$ in an optimum way with a finite receiver of fixed size.

The other term of equation (12) can be expanded in a similar way. To do this, let us define a digit sequence

$$C(j+K+1,\ j+K+M+1) \triangleq \{B(j+K+M), \ldots, B(j+K+1)\} \qquad \ldots\ldots(15)$$

There are m^M such sequences. $A(j+K)$ and $C(j+K+1,\ j+K+M+1)$ specify the noiseless received signal $r(t)$ in the interval $(j+K+1)T \leq t < (j+K+M+1)T$. Thus

$$p[\vec{V}(j+K+1,\ j+K+M+1) | A(j+K) = a_\ell]$$

$$= \sum_{i=1}^{m^M} \{ \prod_{k=1}^{m} p[\vec{X}(j+K+k) | A(j+K+k) = a_n] \} P[C(j+K+1,\ j+K+M+1) = c_i] \qquad \ldots\ldots(16)$$

where the condition that $A(j+K+k) = a_n$ is consistent with the conditions that $A(j+K) = a_\ell$ and that $C(j+K+1,\ j+K+M+1) = c_i$.

Thus both terms in equation (12) can be evaluated from terms of the form $p[\vec{X}(j+K+k) | A(j+K+k) = a_n]$ and from the a priori probabilities of the digits and digit sequences. Equations (13) and (16), which show this, are derived in Appendix A.

The Bayesian decision procedure to use $x(t)$ over the interval $-\infty < t \leq (j+K+M+1)T$ to determine $B(j)$, then, is to:

1. Determine the m^{K+1} terms $p[\vec{X}(j+K+M) | A(j+K+M) = a_i]$, $i = 1, 2, \ldots, m^{K+1}$, from $x(t)$ in the last interval $(j+K+M)T \leq t \leq (j+K+M+1)T$. (Similar quantities that were determined in the previous M baud intervals are also used, but they are available from evaluation of $B(j-1), B(j-2), \ldots, B(j-M)$.)

2. Calculate the m^{K+1} terms $\{p[\vec{V}(-\infty,\ j+K+1) | A(j+K) = a_n]\}$ from the

m^{K+1} terms $\{p[\vec{V}(-\infty, j+K) | A(j+K-1) = a_k]\}$ and the m^{K+1} terms $\{p[\vec{X}(j+K) | A(j+K) = a_n]\}$, as shown in equation (13).

3. Calculate the m^{K+1} terms $\{p[\vec{V}(-\infty, j+K+M+1) | A(j+K) = a_n]\}$ from the results of Step 2 and the Mm^{K+1} terms $\{p[\vec{X}(j+K+k) | A(j+K+k) = a_i];\ i = 1,2,\ldots,m^{K+1};\ k = 1,2,\ldots,M\}$.
4. Calculate the m probabilities $\{p[\vec{V}(-\infty, j+K+M+1) | B(j) = b_i]\}$ from the results of Step 3 and equation (11).
5. Calculate the m a posteriori risks $\tau(k)$, or $\gamma(k)$, from the results of Step 4 and equation (9).
6. Set $\hat{B}(j)$ equal to the digit with the smallest a posteriori risk.

In the next baud interval, $(j+K+M+1)T \leq t \leq (j+K+M+2)T$, the above six steps are repeated with incremented time indices to determine $B(j+1)$. Thus the sequence $\{B(j)\}$ is detected sequentially with an algorithm that uses $x(t)$ from $t \to -\infty$ to $(K+M+1)T$ seconds after it is sent and MT seconds after the waveform representing that digit is received. No other algorithm could use the same signal to determine $B(j)$ with a lower average risk.

4. THE BAYESIAN RECEIVER

The Bayesian receiver, the device that carries out the above algorithm, is realizable and has a fixed finite size. The first part of this receiver is used to determine the m^{K+1} terms $p[\vec{X}(j+K+M) | A(j+K+M) = a_i]$, $i = 1,2,\ldots,m^{K+1}$. This part of the receiver is very dependent on the detailed characteristics of the channel, i.e., on the shape of the pulses $\{q_i(\tau)\}$ and on the statistical properties of the additive noise. The second part of the receiver uses these quantities to estimate $B(j)$ as specified in Steps 2 to 6 of the above algorithm. It is specified by the parameters m, K, and M, the losses $L\{i,k\}$, and the digit a priori probabilities $P[b_i]$, $i = 1,2,\ldots,m$, but is independent of the detailed characteristics of the channel. Note that the first part of this receiver is the same as the first part of the Bayesian receiver for a finite digit sequence[6,7,8,11]. That receiver determines the Nm^{K+1} terms $\{p[\vec{X}(i) | A(i) = a_j]$, $i = 1,2,\ldots,N;\ j = 1,2,\ldots,m^{K+1}\}$, where N is the length of the sequence, and then calculates the sequence $\{B(j)\}$ with a different but similar algorithm.

Special purpose digital computer techniques can be used to synthesize the second part of the receiver. A memory with at least $(M+3)m^{K+1}$ data locations is required, and the amount of calculation required to determine a digit is proportional to $m^{(K+M+1)}$. (This contrasts with an amount proportional to m^{2K+1} when $x(t)$ in the restricted interval $jT \leq t \leq (j+K+1)T$ is used to determine $B(j)$, as explained in $B(j)$.[8]) This calculation must either be done in one baud interval or be divided into sequential steps that require a baud interval or less each, because a new estimate $\hat{B}(j)$ must be made every T seconds. If the algorithm is divided into N such portions, the final decision about a digit is made $(K+M+N+1)T$ seconds after transmission of the signal representing that digit is started.

The first part of the receiver cannot be described in detail until the channel and the modulator are specified explicitly. It determines the quantities $\{p_{\vec{N}}[\vec{X}(j) - \vec{R}_i],\ i = 1,\ldots,m^{K+1}\}$, and so its form depends on the statistical characteristics of the noise. The example examined here is that in which the noise is a sample of a white Gaussian process with an autocorrelation function $N_o\delta(\tau)$. The first part of the receiver is described for any modulation procedure, and then is simplified for the case in which pulse amplitude modulation is used. Finally, the complete receiver for binary pulse amplitude modulation is specified for the case in which minimum probability of digit error is the performance criterion.

4.1 Bayesian Receiver when the Additive Noise is Gaussian

In this case $p_{\vec{N}}[\vec{X}(k) - \vec{R}_i]$ can be written in the form

$$p_{\vec{N}}[\vec{X}(k) - \vec{R}_i]$$

$$= \frac{1}{(2\pi)^{\ell/2}|\phi_n|^{\frac{1}{2}}} \cdot \exp\{-\frac{1}{2}(\vec{X}(k) - \vec{R}_i)' \phi_n^{-1} (\vec{X}(k) - \vec{R}_i)\} \quad \ldots\ldots(17)$$

$$\propto \exp\{\vec{X}'(k)\,\phi_n^{-1}\,\vec{R}_i - \frac{1}{2}\vec{R}_i'\,\phi_n^{-1}\,R_i\}$$

$$= \exp\{\frac{1}{N_o}\int_0^T x(t+kT)r_i(t)dt - \frac{1}{2N_o}\int_0^T r_i^2(t)dt\}$$

$$= \exp\frac{1}{N_o}\{\int^T x(t+kT)r_i(t)dt - \frac{E_i}{2}\} \triangleq z_i\ (k) \quad \ldots\ldots(18)$$

where ϕ_n is the correlation matrix of the components of $\vec{N}$, ℓ is the number of terms in $\vec{N}$, and $\vec{X}'$ is the transpose of $\vec{X}$. The factors in (17) that are independent of the state index i can be ignored, because all m^{K+1} such terms have the same factors, and so their omission multiplies each a posteriori risk $\tau_i(j)$ by the same factor, and so cannot change $\hat{B}(j)$. E_i is the known energy in the pulse $r_i(\tau)$ over the interval $0 \leq t \leq T$. $z_i(k)$ can be determined by processing the input $x(t)$ with the nonlinear circuit shown in Figure 1.

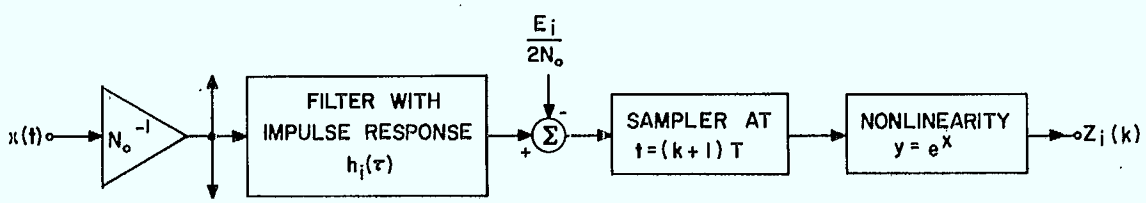

Fig. 1. Nonlinear Circuit for i^{th} Markov State

The filter impulse response $h_i(\tau)$ is

$$h_i(\tau) = r_i(T-\tau) \quad , \quad 0 \leq \tau \leq T$$

$$= 0 \quad , \quad \text{otherwise} \qquad \text{.....(19)}$$

that is, the filter is "matched" to $r_i(\tau)$. The Bayesian decision procedure specifies that the weighted sum of m^K terms such as $z_i(k)$ be evaluated. (The weights are dependent on $x(t)$ in other baud intervals.) This sum of exponentials, with the input signal part of the exponents, eliminates the possibility that there may be a linear realization of the Bayesian receiver.

The above realization, requiring m^{K+1} matched filters and nonlinear circuits such as that shown in Figure 1, can be simplified by utilizing the fact that although there are m^{K+1} possible waveforms $r_i(\tau)$, and each is composed of K+1 baud length portions of the pulses $q_i(\tau)$, there are altogether only m(K+1) baud length pulses. Thus the filter matched to $r_i(\tau)$ can be made by combining the outputs of filters matched to these baud length pulses, in much the same way that this combining is done in the channel and modulator. Let

$$f_{\ell,k}(\tau) = q_\ell(kT-\tau) \quad , \quad 0 \leq \tau \leq T$$

$$= 0 \quad , \quad \text{otherwise} \qquad \text{.....(20)}$$

for $\ell = 1,2,\ldots,m$ and $k = 1,2,\ldots,(K+1)$. Then

$$h_i(\tau) = \sum_{\ell=1}^{m} \sum_{k=1}^{K+1} c(\ell,k,i) f_{\ell,k}(\tau) \qquad \text{.....(21)}$$

where $c(\ell,k,i)$ is unity when $A(j) = a_i$ is such that $B(j+1-k) = b_\ell$, and is zero otherwise. Thus only m(K+1) filters are necessary, rather than m^{K+1} filters.

The nonlinear circuit shown in Figure 2 utilizes the above relationship to determine the m^{K+1} terms $\{z_n(j)\}$ from $x(t)$ over the interval $jT \leq t \leq (j+1)T$. The output of the $m(K+1)$ filters is sampled at time $(j+1)T$. These samples are then multiplied by the appropriate term $c(\ell,k,n)$, the products are summed, the bias $-E_n/2N_o$ is added, and the sum is passed through the nonlinear memoryless device with an input-output relationship $y = e^x$. The output of this circuit is $z_n(j)$.

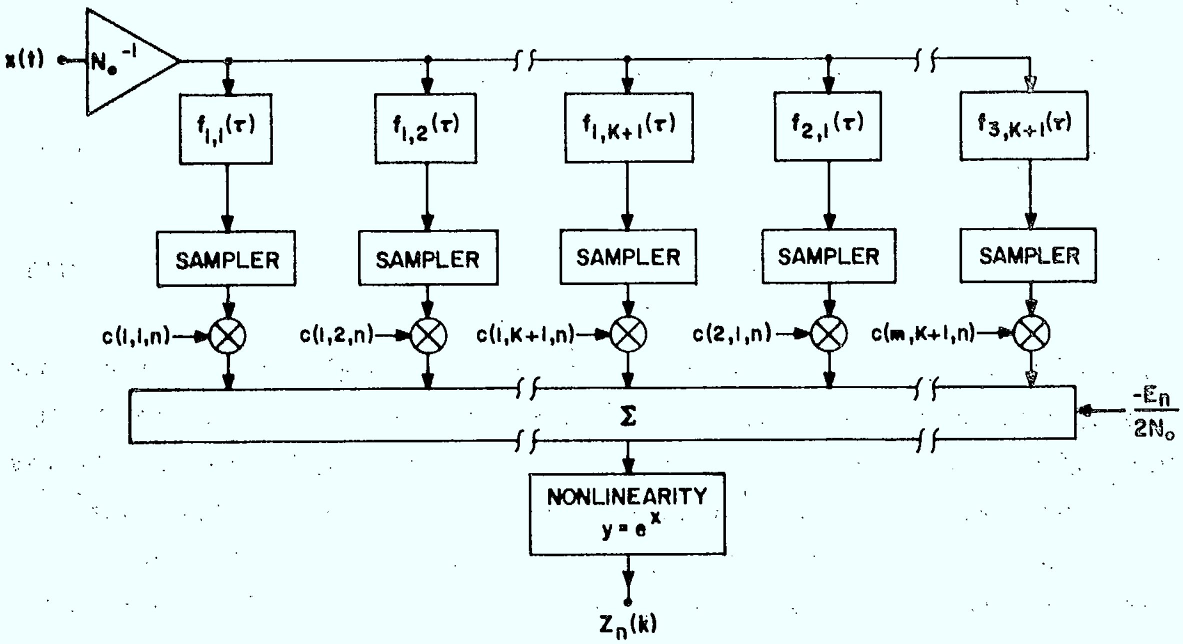

Fig. 2. Alternate form of Bayesian Receiver.

The outputs $\{z_n(j)\}$ could be obtained with m^{K+1} parallel circuits, or sequentially with the circuit shown in Figure 2, or in some series-parallel combination. Or, if the baud interval T is sufficiently long, the $m(K+1)$ filter outputs could be sampled at $t = (j+1)T$ and these samples processed in a general purpose digital computer.

4.2 Bayesian Receiver for Pulse Amplitude Modulated Digits when the Additive Noise is Gaussian

When pulse amplitude modulation is used the transmitted pulses are

$$s_i(t-jT) = b_i\, s(t-jT), \quad i = 1,2,\ldots,m, \qquad \ldots\ldots(22)$$

and so the received isolated pulses are

$$q_i(t-jT) = b_i\, s(t-jT)*c(\tau) \triangleq b_i\, q(t-jT) \qquad \ldots\ldots(23)$$

The receiver filter impulse responses $\{f_{\ell,k}(\tau)\}$ become

$$f_{\ell,k}(\tau) = b_\ell\, q(kT-\tau) \quad , \quad 0 \leq \tau \leq T \qquad .....(24)$$
$$= 0 \qquad , \quad \text{otherwise},$$

There are only (K+1) separate impulse responses, rather than m(K+1) in the more general case. Let us define a set of impulse responses $\{u_k(\tau);\ k = 1,2,\ldots,K+1\}$ by the relation

$$u_k(\tau) = q(kT-\tau) \quad , \quad 0 \leq \tau \leq T$$
$$= 0 \qquad , \quad \text{otherwise}. \qquad .....(25)$$

Then

$$h_n(\tau) = \sum_{k=1}^{K+1} d(k,n)u_k(\tau) \qquad .....(26)$$

where d(k,n) is equal to the value of B(j+1-k) when $A(j) = a_n$. A circuit that can be used to determine $z_n(j)$ in the P.A.M. case is shown in Figure 3. The outputs of the K+1 filters at the sampling time t = (j+1)T can either be used in m^{K+1} nonlinear memoryless circuits to determine $z_n(j)$, $n = 1,2,\ldots,m^{K+1}$, or they can be used to determine the $z_n(j)$ sequentially by supplying the appropriate values of the weights d(k,n) and the biases $-E_n/2N_o$, as in the more general case.

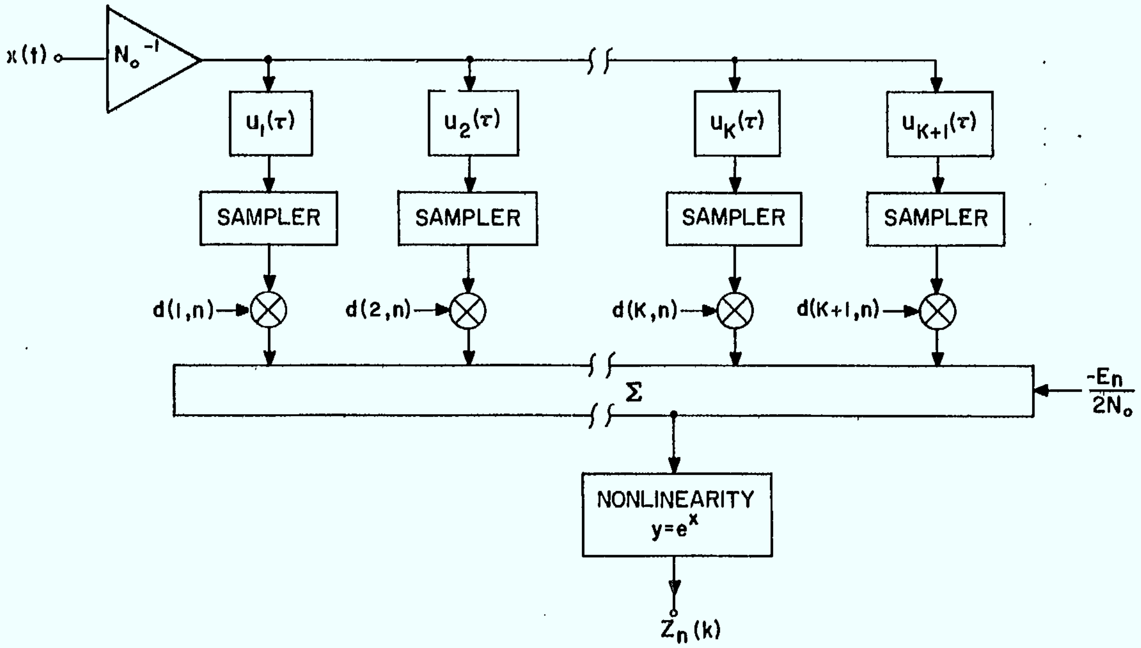

Fig. 3. Alternate form of Bayesian Receiver for Amplitude Modulated Digits

4.3 Receiver that Minimizes the Digit Error Probability of Binary Pulse Amplitude Modulated Digits when the Additive Noise is Gaussian

In the above two examples, circuits that could be used to evaluate terms proportional to $p[\vec{X}(j)|A(j) = a_n]$ were described. It was assumed that these values would be used in a special purpose computer to carry out the decision algorithm that was described in Section 3. A more specific example is considered here, in which the message is a sequence of independent binary digits that are either +1 or −1 with equal probability, and the transmitted pulses are either either $+s(\tau)$ or $-s(\tau)$. As in the previous examples, the additive noise is assumed to be white and Gaussian. In this example the receiver is to minimize the digit error probability. This implies that

$$L\{i,k\} = 1 \quad , \quad i \neq k$$

$$= 0 \quad , \quad i = k \qquad \text{.....(27)}$$

If M is set equal to zero, i.e., $\vec{V}(-\infty, j+K+1)$ is used to determine B(j), then it is sufficient for the receiver to calculate a quantity proportional to

$$\lambda(j) = \sum_{n=1}^{2^{(K+1)}} b_i \cdot z_n(j+K) \cdot P[A(j+K) = a_n|\vec{V}(-\infty, j+K)] \qquad \text{.....(28)}$$

where $A(j+K) = a_n$ is such that $B(j) = b_i$, and then to set $\hat{B}(j) = +1$ if $\lambda(j) \geq 0$ and −1 if $\lambda(j) < 0$. The receiver is shown in Figure 4. (Control

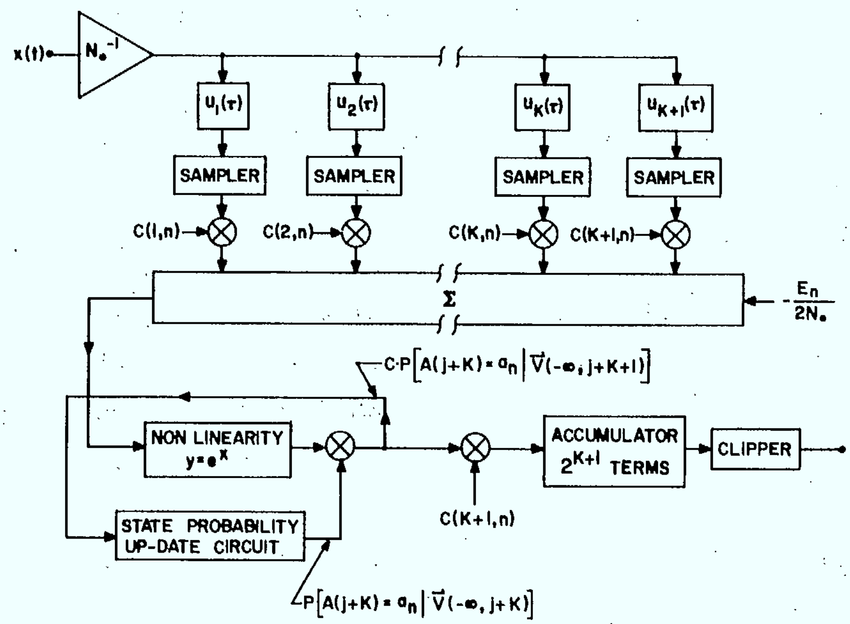

Fig. 4. Bayesian Receiver for Binary Pulse Amplitude Modulated Digits, M = 0.

circuitry that must feed the correct terms to the multipliers so that each of the 2^{K+1} Markov states is considered is not shown.) The "state probability up-date" block converts the 2^{K+1} terms

$$z_n(j+K)\cdot P[A(j+K) = a_n|\vec{V}(-\infty,\ j+K)]$$

$$= C\cdot P[A(j+K) = a_n|\vec{V}(-\infty,\ j+K+1)]$$

that were used to determine B(j) to the a priori probabilities

$$P[A(j+K+1) = a_i|\vec{V}(-\infty,\ j+K+1)]$$

$$= \sum_{n=1}^{2^{K+1}} m(i,n)\cdot z_n(j+K)\cdot P[A(j+K) = a_n|\vec{V}(-\infty,\ j+K)] \qquad .....(29)$$

to be used to determine B(j+1). The weighting factors m(i,n) are

$$m(i,n) = C^{-1}\cdot P[A(j+K+1) = a_i|A(j+K) = a_n]$$

$$= \frac{1}{2}\,C^{-1} \text{ or } 0 \qquad .....(30)$$

where

$$C = \sum_{n=1}^{2^{K+1}} z_n(j+K)\cdot P[A(j+K) = a_n|\vec{V}(-\infty,\ j+K)] \qquad .....(31)$$

(This factor C arises because the factors that were independent of the state index were eliminated in equation (18). The performance of the receiver is independent of the value of C that is used, except that if no normalization were done the receiver would saturate or overflow after a few digits were processed.)

If $M > 0$, i.e., if it is decided to use x(t) also in the interval $(j+K+1)T \leq t < (j+K+M+1)T$ to determine B(j), then the receiver must be modified after the exponential circuit. The terms $\{z_n(j+K+i);\ n = 1,2,\ldots,2^{K+1};\ i = 0,1,\ldots,M\}$ must be stored in a buffer memory that is up-dated in each baud interval. The a priori probability calculation is done in the same way as when $M = 0$. In addition, the terms $\{z_n(j+K+i);\ n = 1,2,\ldots,2^{K+1};\ i = 1, 2,\ldots,M\}$ are used to calculate 2^{K+1} terms proportional to $p[\vec{V}(j+K+1,j+K+M+1)|A(j+K) = a_n]$, as shown in equation (16). These terms are multiplied by the corresponding value of $b_i\cdot z_n(j+K)\cdot P[A(j+K) = a_n|\vec{V}(-\infty,\ j+K)]$ and the resulting 2^{K+1} terms are summed to determine $\lambda(j)$.

It is evident that the Bayesian receiver is quite complex. The reason for this is that every possible combination of digits that can influence the received signal at any one time is considered each time a digit estimate is made. The receiver complexity is proportional to m^{K+1} for this reason, although no more than $m\cdot(K+1)$ filters are required. As well, if $M > 0$ a calculation with a complexity proportional to m^M is necessary for each state, and so the overall complexity of the calculation is proportional to m^{M+K+1}. Also, a buffer memory with at least $(M+3)$ locations is necessary for each state, and so the memory must have $(M+3)m^{K+1}$ locations.

This receiver is much more complex than suboptimum receivers such as the optimum linear receiver[3], or the decision feedback equalizer[5], that have been proposed to do the same task. The performance of the Bayesian receiver will be examined in the next section, and compared with the performance of these suboptimum receivers.

5. PERFORMANCE OF THE BAYESIAN RECEIVER

In principle, the performance of the Bayesian receiver, or any other receiver, is given by equation (7) for any given set of digit a priori probabilities, losses $L\{i,j\}$, and probability density functions $p[\vec{V}|B(j) = b_i]$, $i = 1,2,\ldots,m$. Unfortunately, this average risk is very difficult to evaluate because the receiver is complex and nonlinear, and also because the input signal over the interval $-\infty < t < (j+K+M+1)T$ is used to determine $B(j)$. Hancock and Quincy[9] have evaluated the error probability when independent binary digits are transmitted, when the additive noise is white and Gaussian, and when the signal that is used is restricted to that in the interval $jT \leq t < (j+K+1)T$. This method has been generalized to determine the digit error probability when $x(t)$ over the larger interval $(j-L)T \leq t < (j+K+M+1)T$ is used to determine $B(j)$[7]. However, the number of integrals that must be evaluated in this computation is proportional to $m^{(K+L+M+1)}$, and the dimensionality of each integral is also $m^{(K+L+M+1)}$. No solution is available in closed form when $L \to \infty$, the case of interest here.

Because of these computational difficulties, the receiver performance was measured by simulating the transmitter, channel, and receiver on a general purpose digital computer. The system simulated was that in which the message digits were binary, pulse amplitude modulation was used, the noise was white and Gaussian, and the receiver delay parameter M was 0, 1, 2, or 3. (The Bayesian receiver for this situation is shown in Figure 4.) Several representative pulse shapes $q(\tau)$ were used. Among these were the time-constrained pulse

$$q(\tau) = A \quad , \; 0 \leq \tau \leq P$$
$$= 0 \quad , \text{ otherwise} \qquad \ldots\ldots(32)$$

and the unconstrained pulse

$$q(\tau) = A\tau e^{-\alpha\tau} \quad , \quad \tau > 0 \qquad \ldots\ldots(33)$$

The simulation was done in such a way that the normal difficulties associated with representing continuous signals, continuous filters, and white Gaussian noise on a digital computer were avoided. This method is described in Appendix B. The linear equalizer and the decision feedback equalizer were also simulated in a similar way, as described in Appendix C, for comparison purposes.

The performance of the Bayesian receiver when $q(\tau)$ is a rectangular pulse two baud intervals long is shown in Figure 5. The measured error probability $\hat{P}[\varepsilon]$ is shown as a function of the pulse energy to noise power spectral density ratio

$$\frac{E}{N_o} = \frac{1}{N_o}\int_0^\infty q^2(\tau)\ d\tau. \qquad .....(34)$$

The error function lower bound,

$$P[\varepsilon] = 0.5\ \{1 - \mathrm{erf}(E/2N_o)^{\frac{1}{2}}\}\ , \qquad .....(35)$$

the error probability that could be achieved if there were no intersymbol interference, is also shown. The performance of the Bayesian receiver improved each time M was increased, that is, each time more signal outside the interval $jT \leq t \leq (j+2)T$ was used. At high values of E/N_o there is a 0.5 dB improvement when M is increased from 2 to 3. Note that no digit $B(k)$ influences the signal in both the interval $jT \leq t < (j+2)T$ and also $(j+4)T \leq t < (j+5)T$, and yet a significant improvement can be obtained by using this signal correctly. This result verifies the conclusion reached in Section 3, that in general any increase in the amount of signal $x(t)$ that is used correctly will improve the receiver performance, even though $q(\tau)$ is strictly time constrained.

The performance of other receivers in the same situation is shown in Figure 6. The performance of the decision feedback equalizer with 13 forward taps is very similar to that of the Bayesian receiver with M = 0, but 1.8 dB poorer than that of the Bayesian receiver with M = 3. The performance of the linear equalizer is much poorer in this situation. It "bottoms" at an error probability that is determined by the number of taps in the delay line. The matched filter, the optimum linear receiver with only one tap, bottoms at an error probability of 0.22. The 5 tap linear equalizer bottomed at an error probability of about 8×10^{-3}.

The performance of the receivers when $q(\tau)$ is $A\tau e^{-\tau}$, with T = 1, is shown in Figure 7. Note that when $q(\tau)$ is not time constrained the value of K is not specified. At E/N_o equal to 12.0 dB it was found that no improvement could be achieved by increasing K beyond 6. (At higher signal-to-noise ratios it is expected that a larger value of K should be used to take advantage of the energy in $q(\tau)$ at $\tau > 7T$.) It was also found that in this case the Bayesian receiver with M = 0 performed as well as the ones with M > 0.

At low error probabilities the decision feedback equalizer requires about 1.6 dB more signal strength than the Bayesian receiver to achieve the same error probability, and the linear equalizer requires an additional 4.0 dB of signal.

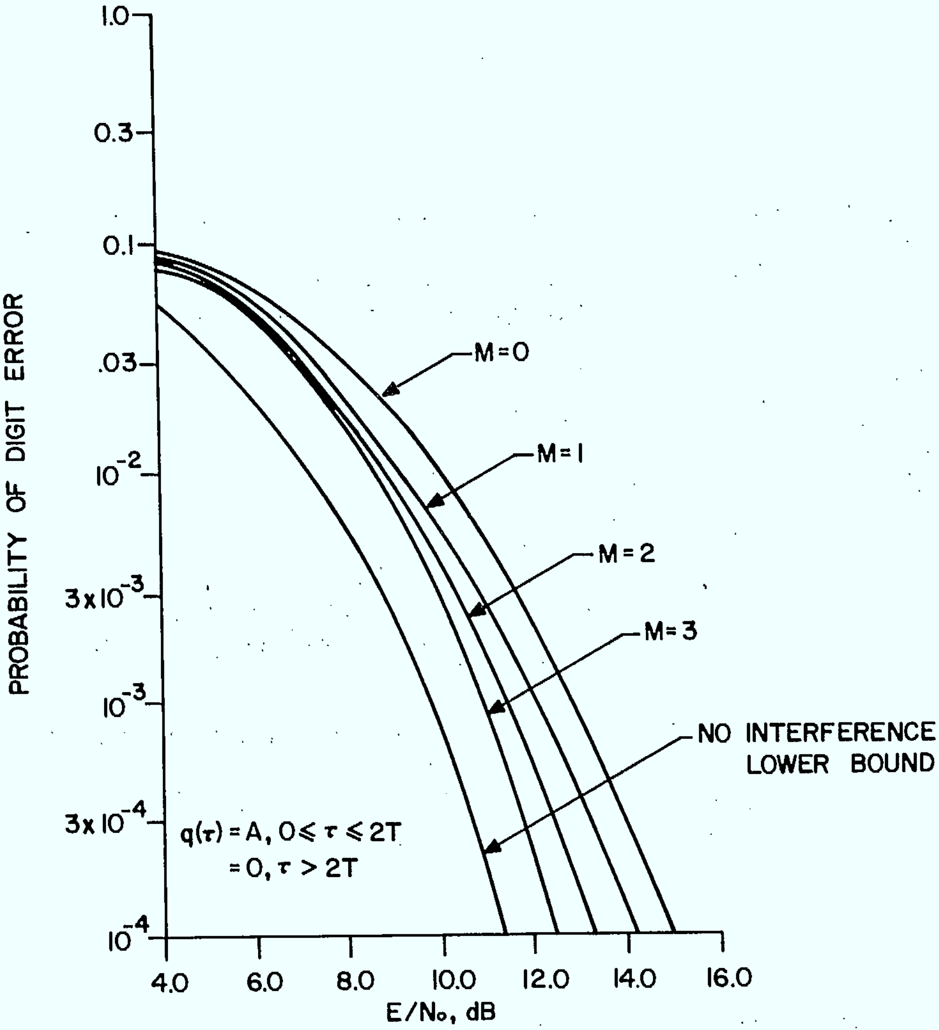

Fig. 5. Performance of Bayesian Receivers with Different Delays when $q(\tau)$ is a 2-baud length rectangular pulse

The amount of "excess" signal strength required by a specific receiver to achieve a specified error probability, that is, the amount of signal in excess of that required when there is no intersymbol interference, is, of course, a function of the data transmission rate. The amount of excess signal strength to achieve a 10^{-3} digit error probability when $q(\tau)$ is $A\tau e^{-\alpha\tau}$ is shown in Figure 8 as a function of the data transmission rate R. (R is proportional to T^{-1} for any channel. For the channel with impulse response $A\tau e^{-\alpha\tau}$ it was defined to be $1/\alpha T$.) As shown, the excess signal strength required by the Bayesian receiver increases by approximately 4 dB for each octave increase in R at high data rates. At high data rates the decision feedback equalizer requires about 2 dB more signal strength than the Bayesian receiver, independent of the data rate. In contrast, the linear equalizer requires a 9 dB increase to achieve the same error probability when the rate is doubled. Thus a substantial improvement can be obtained by using the Bayesian or decision feedback receiver, rather than the linear equalizer, at very high data rates.

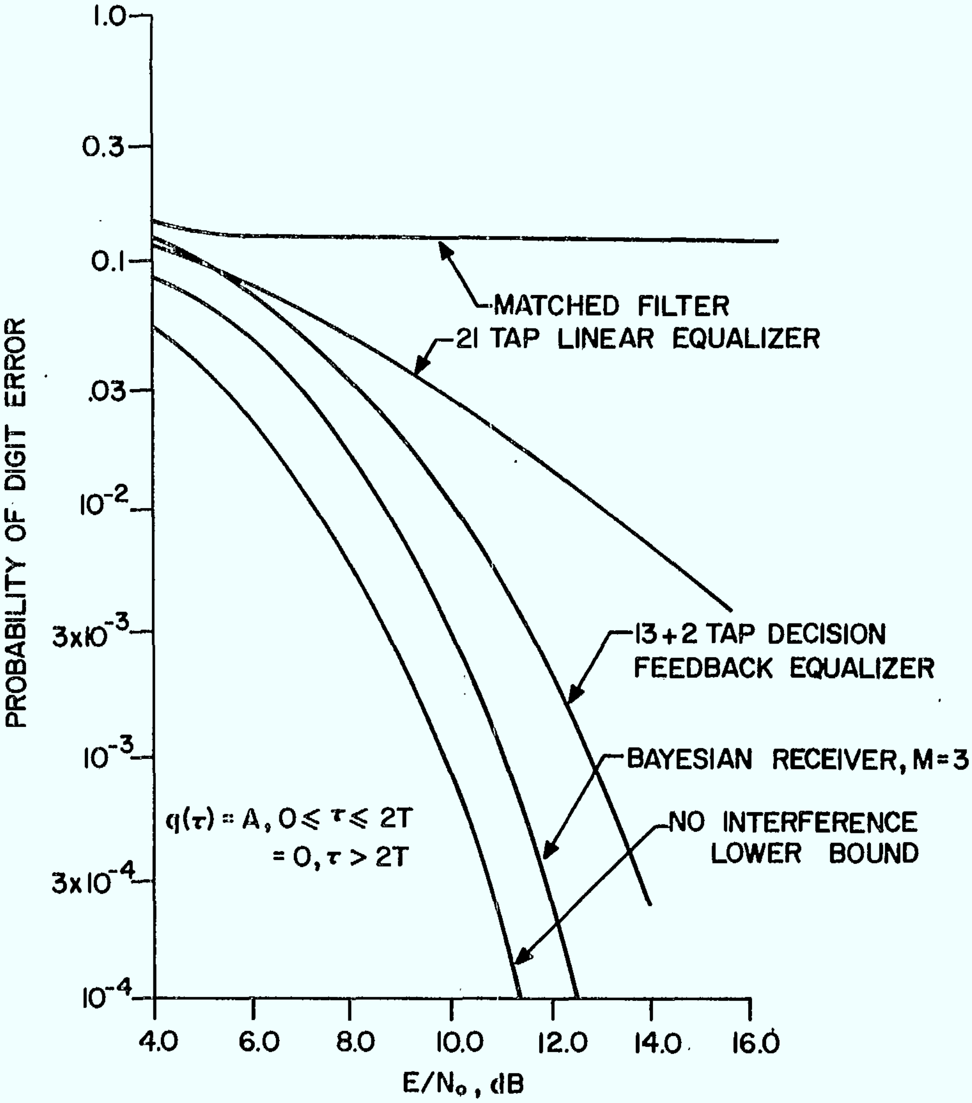

Fig. 6. Comparison of Receiver Performances

The dotted curves in Figure 8 show the performance of the Bayesian receiver, and the decision feedback equalizer, when the data rate is increased without increasing the size of the receiver. (Note that both receivers are designed on the assumption that $q(\tau)$ was strictly time constrained.) When the energy in the unconsidered "tail" of the pulse becomes significant, these receivers no longer operate effectively. However, this can be corrected in the Bayesian receiver case by increasing K, and in the decision feedback equalizer case by increasing the number of feedback taps.

The performance curves shown in Figures 5, 6, and 7 are all translations of the error function curve, described by equation (35), at low error rates. (An exception is the performance curve when the receiver bottoms.) These performance curves can be described at low error rates by the empirical expression

$$P[\varepsilon] = 0.5 \{1 - \mathrm{erf}\{\eta(R)E/2N_O\}^{\frac{1}{2}}\} \qquad \text{.....(36)}$$

where $\eta(R)$, the efficiency of the modem, is always between zero and one. $10 \log_{10}(\eta(R))^{-1}$ is shown in Figure 8 for each of the three simulated receivers when $q(\tau)$ is $A\tau e^{-\alpha\tau}$ as a function of $10 \log_{10} R$.

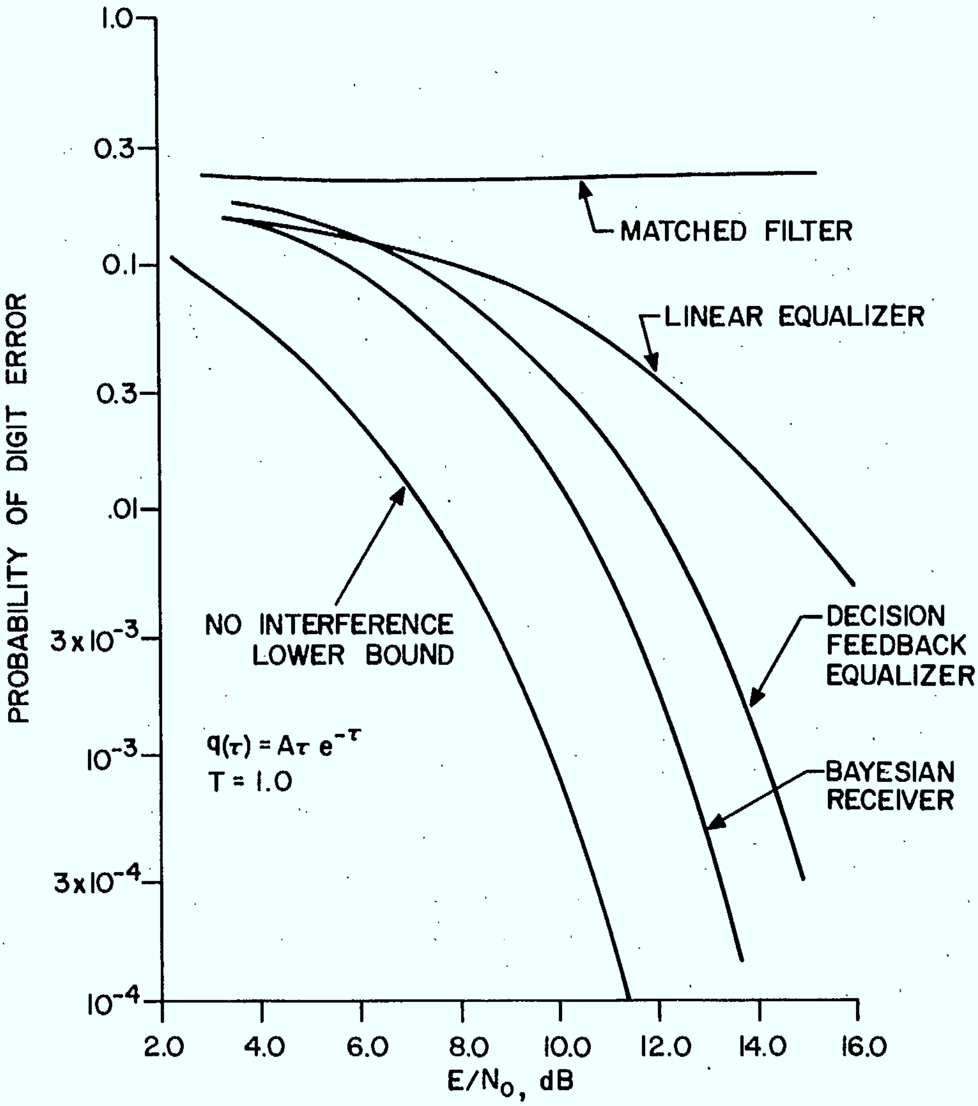

Fig. 7. Comparison of Receiver Performances

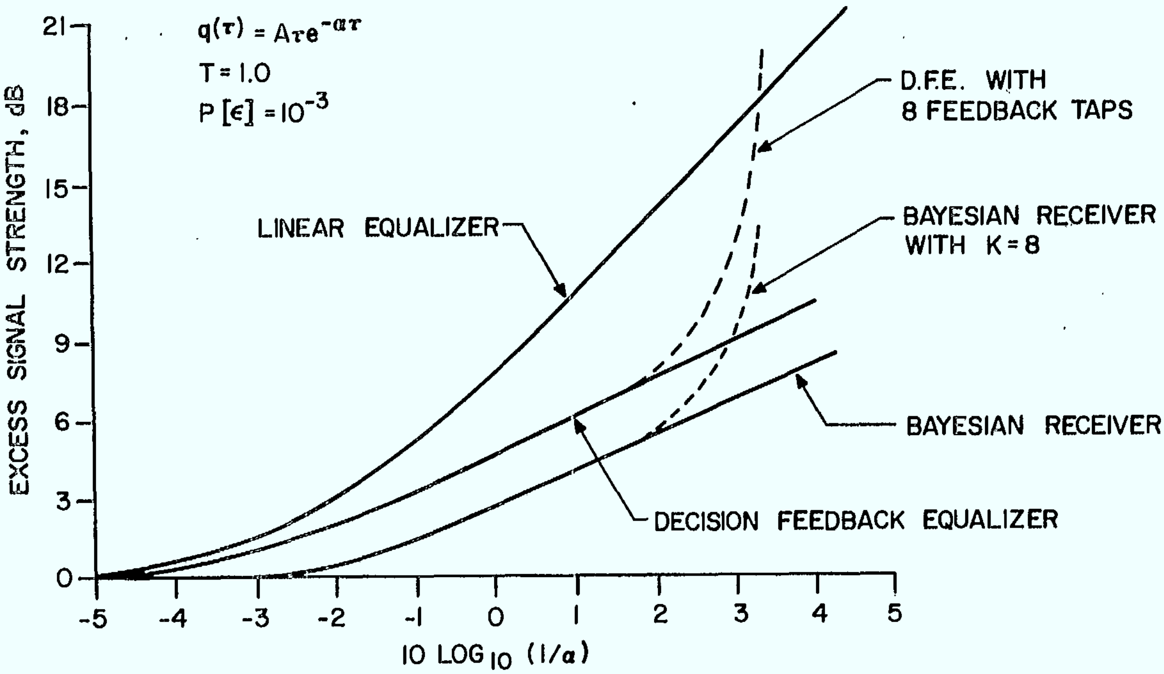

Fig. 8. Comparison of Receiver Performances at Different Data Transmission Rates

6. CONCLUSIONS

The receiver that detects a member of the sequence of m-ary digits with minimum average risk in the presence of both additive noise and intersymbol interference from K adjacent digits has been described. This receiver is quite complex, because it considers m^{K+1} possible digit sequences in each baud interval. Computer simulation studies show that the performance of the receiver is considerably better than that of the transversal equalizer, the optimum linear receiver. A significant practical result is that the decision feedback equalizer, a receiver that is no more complex than the transversal equalizer, performs much better than the linear equalizer and almost as well as the Bayesian receiver at very high data rates.

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APPENDIX A

To carry out the Bayesian decision procedure the probabilities $p[\vec{V}(-\infty,\ j+K+M+1)|A(j+K) = a_\ell]$; $\ell = 1,2,\ldots,m^{K+1}$, must be evaluated. This can be made easier by using the fact that

$$p[\vec{V}(-\infty,\ j+K+M+1)|A(j+K)]\cdot P[A(j+K)]$$

$$= p[\vec{V}(-\infty,\ j+K+1),\ \vec{V}(j+K+1,\ j+K+M+1),\ A(j+K)]$$

$$= p[\vec{V}(-\infty,\ j+K+1)|\vec{V}(j+K+1,\ j+K+M+1),\ A(j+K)]\cdot$$

$$\cdot\ p[\vec{V}(j+K+1,\ j+K+M+1)|A(j+K)]\cdot P[A(j+K)] \qquad .....(37)$$

However, given $A(j+K)$, $\vec{V}(j+K+1,\ j+K+M+1)$ provides no further information about $\vec{V}(-\infty,\ j+K+1)$, and so

$$p[\vec{V}(-\infty,\ j+K+1)|\vec{V}(j+K+1,\ j+K+M+1),\ A(j+K)]$$

$$= p[\vec{V}(-\infty,\ j+K+1)|A(j+K)] \qquad .....(38)$$

Equation (12) follows directly from this result.

Equation (38) can be expanded by considering all the m^{K+1} possible states $A(j+K-1)$ in the previous interval.

$$p[\vec{V}(-\infty,\ j+K+1)|A(j+K)]\cdot P[A(j+K) = a_\ell]$$

$$= \sum_{n=1}^{m^{K+1}} p[\vec{V}(-\infty,\ j+K+1),\ A(j+K) = a_\ell,\ A(j+K-1) = a_n]$$

$$= \sum_{n=1}^{m^{K+1}} p[\vec{V}(-\infty,\ j+K),\ \vec{X}(j+K),\ A(j+K) = a_\ell,\ A(j+K-1) = a_n]$$

$$= \sum_{n=1}^{m^{K+1}} p[\vec{X}(j+K)|\vec{V}(-\infty,\ j+K),\ A(j+K) = a_\ell,\ A(j+K-1) = a_n]\cdot$$

$$\cdot p[\vec{V}(-\infty,\ j+K)|A(j+K) = a_\ell,\ A(j+K-1) = a_n]\cdot P[A(j+K-1) = a_n|A(j+K) = a_\ell]\cdot$$

$$\cdot P[A(j+K) = a_\ell] \qquad .....(39)$$

The properties of a Markov source can be used to simplify this expression.

$$p[\vec{X}(j+K)\,|\,\vec{V}(-\infty,\ j+K),\ A(j+K) = a_\ell,\ A(j+K-1) = a_n]$$

$$= p[\vec{X}(j+K)\,|\,A(j+K) = a_\ell] \qquad \ldots\ldots(40)$$

independent of n, and

$$p[\vec{V}(-\infty,\ j+K)\,|\,A(j+K) = a_\ell,\ A(j+K-1) = a_n]$$

$$= p[\vec{V}(-\infty,\ j+K)\,|\,A(j+K-1) = a_n] \qquad \ldots\ldots(41)$$

because $B(j+K)$ does not contribute to $x(t)$ before $t = (j+K)T$. Equation (13) follows immediately when (40) and (41) are used in (39).

The term $p[\vec{V}(j+K+1,\ j+K+M+1)\,|\,A(j+K) = a_\ell]$ can also be expanded, in this case by considering all possible sequences $C(j+K+1,\ j+K+M+1)$ that could be transmitted in the interval $(j+K+1)T \leq t < (j+K+M+1)T$.

$$p[\vec{V}(j+K+1,\ j+K+M+1)\,|\,A(j+K) = a_\ell]\cdot P[A(j+K) = a_\ell]$$

$$= \sum_{i=1}^{m^M} p[\vec{V}(j+K+1,\ j+K+M+1),\ A(j+K) = a_\ell,\ C(j+K+1,\ j+K+M+1) = c_i]$$

$$= \sum_{i=1}^{m^M} p[\vec{X}(j+K+1),\ldots,\vec{X}(j+K+M)\,|\,A(j+K) = a_\ell,\ C(j+K+1,\ j+K+M+1) = c_i]\cdot$$

$$\cdot P[C(j+K+1,\ j+K+M+1) = c_i\,|\,A(j+K) = a_\ell]\cdot P[A(j+K) = a_\ell] \qquad \ldots\ldots(42)$$

Since the digits $\{B(j)\}$ are statistically independent,

$$P[C(j+K+1,\ j+K+M+1) = c_i\,|\,A(j+K) = a_\ell]$$

$$= P[C(j+K+1,\ j+K+M+1) = c_i] \qquad \ldots\ldots(43)$$

Also, given $A(j+K)$ and $C(j+K+1,\ j+K+M+1)$, or $\{B(\ell);\ \ell = j,\ldots,j+K+M+1\}$, the knowledge about any one member of the sequence $\{\vec{X}(j+K+k);\ k = 1,2,\ldots,M\}$ is not improved by knowledge of the other members. Thus

$$p[\vec{X}(j+K+1),\ldots,\vec{X}(j+K+M)\,|\,A(j+K)= a_\ell,\ C(j+K+1,\ j+K+M+1) = c_i]$$

$$= \prod_{k=1}^{M} p[\vec{X}(j+K+k)\,|\,A(j+K+k) = a_n] \qquad \ldots..(44)$$

where $A(j+K+k) = a_n$ is the digits $\{B(j+k),\ldots,B(j+k+K)\}$ when $A(j+K) = a_\ell$ and $C(j+K+1,\ j+K+M+1) = c_i$. Equation (16) is obtained by substituting (44) and (43) into (42).

APPENDIX B

The Bayesian receiver may be thought of as consisting of two parts. The first part filters the input signal $x(t)$ in $(K+1)$ parallel filters $\{u_i(\tau) = q(iT-\tau);\ i = 0,1,\ldots,K\}$, and produces $K+1$ samples

$$y(i,k) = N_0^{-1} \int_0^T x(\tau+kT)q(iT+\tau)d\tau, \quad i = 0,1,\ldots,K \qquad \ldots..(45)$$

at the end of each baud interval. The second part of the receiver is a digital computer that uses these samples $\{y(i,k)\}$ to estimate the digits $\{B(j)\}$. This part of the receiver can be simulated easily on a general purpose digital computer, since there is no need to do real-time simulation.

Generation of the samples $\{y(i,k)\}$ on a digital computer is more difficult, because they are outputs of a continuous system. One of the problems that must be investigated by simulation is the choice of the parameter K when $q(\tau)$ is not strictly time limited. This is done by setting $q(\tau)$ equal to zero after $\tau = (L+1)T$, where $L \geq K$. The input $x(t)$ in the interval $kT \leq t < (k+1)T$ is then

$$x(kT+\tau) = \sum_{\ell=0}^{L} B(k-\ell)q(\ell T+\tau) + n(kT+\tau) \ , \quad 0 \leq \tau \leq T \qquad \ldots..(46)$$

On substituting (46) into (45),

$$y(i,k) = \int_0^T q(iT+\tau)\{\sum_{\ell=0}^{L} B(k-\ell)q(\ell T+\tau)\}\ d\tau$$

$$+ \int_0^T q(iT+\tau)n(kT+\tau)d\tau$$

$$= \sum_{\ell=0}^{L} B(k-\ell)Q(i,\ell) + n_i(k) \qquad \ldots..(47)$$

where

$$Q(i,k) \triangleq \int_0^T q(iT+\tau)q(\ell T+\tau)d\tau \qquad \ldots..(48)$$

The output noise samples $\{n_i(k)\}$ are Gaussian random variables that are statistically independent in different baud intervals, and have correlation coefficients

$$E[n_i(k)n_j(k)] = N_0 Q(i,k) \qquad \ldots..(49)$$

These samples $\{n_i(k);\ i = 0,\ldots,K\}$ are generated by first generating K+1 independent Gaussian noise samples by the "direct method" that is described by Muller[13], and then converting these samples to K+1 properly correlated samples with a linear transformation that was described by Marsaglia[14]. With these samples generated, the outputs $\{y(i,k)\}$ can be calculated from equation (47), and used as specified by the Bayesian algorithm. The only approximation that is necessary in this simulation is that $q(\tau)$ is zero for $\tau \geq (L+1)T$. In the tests described here, $L \leq 23$ and $K \leq 8$.

APPENDIX C

The transversal equalizer and the decision feedback equalizer were simulated to compare their performances with that of the Bayesian receiver. These equalizers filter the signal $x(t)$ with a filter matched to $q(\tau)$. Let $y(j)$ equal the output of this filter at $t = jT$. The linear equalizer makes the estimate $\hat{B}(j-N-L) = +1$ if

$$z(j) = \sum_{i=0}^{2N} y(j-i)a(i) \geq 0 \qquad \ldots\ldots(50)$$

or -1 if $z(j) < 0$. Similarly, the decision feedback equalizer makes the estimate $\hat{B}(j-L-M) = +1$ if

$$v(j) = \sum_{i=0}^{M} y(j-i)c(i) - \sum_{\ell=1}^{L} \hat{B}(j-L-M-\ell)f(\ell)$$

$$\geq 0 \qquad \ldots\ldots(51)$$

or -1 if $v(j) < 0$.

The matched filter output $y(j)$ is

$$y(j) = \int_0^{(L+1)T} x((j-L-1)T+\tau)q(\tau)d\tau$$

$$= \sum_{k=0}^{2L+1} B(j-k)f(k) + n_o(j) \qquad \ldots\ldots(52)$$

where

$$f(L+1+i) = f(L+1-i)$$

$$= \int_0^{(L+1)T} q(\tau)q(\tau+iT)d\tau$$

$$\triangleq \phi_q(i) \; ; \quad i = 0,1,\ldots,L \qquad \ldots\ldots(53)$$

The filter output noise sequence $\{\ldots n_o(j-1), n_o(j), n_o(j+1), \ldots\}$ is a sequence of Gaussian random variables with zero mean and a correlation function

$$\phi_n(iT) = N_o \; \phi_q(i) \qquad \ldots\ldots(54)$$

This sequence can be generated from a sequence of independent Gaussian random samples $\{n_i(k)\}$. If the fourier transform of $\phi_q(i)$ is a rational function, then $n_o(j)$ can be generated with a baud-rate digital filter[15] that is described by the equation

$$n_o(j) = \sum_{i=0}^{I} d(i) \cdot n_i(j+i) + \sum_{n=1}^{N} e(n) n_o(j-n) \qquad \ldots\ldots(55)$$

If $\Phi_n(\omega)$ is not a rational function, then a more closely spaced digital filter can be used to produce an output sequence with an arbitrarily close correlation function. Equations (52) and either (51) or (50) are then used to calculate the receiver output sequence.

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