Library Circo UNLIMITED

REFERENCE COPY

Communications Research Centre

ANALYSIS OF IMPEDANCE PROFILES AND STRUCTURAL RETURN LOSS OF HIGH QUALITY COAXIAL CABLES

by

D.W. Rice

DEPARTMENT OF COMMUNICATIONS
MINISTÈRE DES COMMUNICATIONS

CRC REPORT NO. 1224

This document was prepared for, and is the property of Department of National Defence, Defence Research Board.



COMMUNICATIONS RESEARCH CENTRE

DEPARTMENT OF COMMUNICATIONS
CANADA

Industry Canada Library - Queen

SEP - 4 777

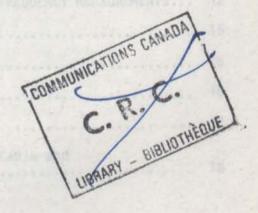
Industrie Canada Bibliothèque - Queen

ANALYSIS OF IMPEDANCE PROFILES AND STRUCTURAL RETURN LOSS OF HIGH QUALITY COAXIAL CABLES

by

D.W. Rice

(Radio Research Directorate)



CRC REPORT NO. 1224

DRB PROJECT NO. 7611

DRB TELS REPORT NO. 11

Published March 1972 OTTAWA

This document was prepared for, and is the property of, Department of National Defence, Defence Research Board.

CAUTION

This information is furnished with the express understanding that proprietary and patent rights will be protected.

TABLE OF CONTENTS

ABSTRACT	1
INTRODUCTION	1
LIST OF SYMBOLS	2
THE REFLECTION COEFFICIENT	4
IMPEDANCE AND VOLTAGE TRANSFORMATIONS	5
THE RETURN LOSS	6
CALCULATION OF RETURN LOSS FROM THE IMPEDANCE PROFILE	7
CALCULATION OF THE IMPEDANCE PROFILE FROM THE REFLECTION COEFFICIENT	10
CALCULATION OF THE IMPEDANCE PROFILE FROM SWEPT FREQUENCY MEASUREMENTS	12
SUMMARY	15
ACKNOWLE DGEMENTS	16
REFERENCES	16
APPENDIX A - The Return Loss Computer Program	17
APPENDIX B - Typical Phase Errors for Various Cable and	18

ANALYSIS OF IMPEDANCE PROFILES AND STRUCTURAL RETURN LOSS OF HIGH QUALITY COAXIAL CABLES

bу

D.W. Rice

ABSTRACT

Transmission-line theory is applied to the calculation of the transmission and reflection errors which occur in a transmission line that has small impedance discontinuities along its length. The discontinuities are typical of those arising in coaxial cables as a result of the manufacturing process. Errors arising from mismatches in the source and terminating impedances are also calculated. The results indicate that errors in phase up to several degrees are possible in the CRC High Frequency Direction Finding array, due to such imperfections.

The solution to the inverse problem of computing the transmission line impedance profile from measurements of the reflection characteristics of the line is also outlined, and some restrictions which apply when the reflection properties are measured with swept-frequency equipment are derived.

INTRODUCTION

High quality coaxial cable systems may be tested in either the frequency or the time domain. The older method involves measuring, in the frequency domain, the impedance or the voltage standing wave ratio (VSWR) at the cable input, and interpreting departures from the ideal in terms of signals reflected back to the source from the various inhomogeneities along the cable. However, this method gives no information about the distance along the cable of the various imperfections causing the reflections. More

recently, time domain testing has become possible with the introduction of time domain reflectometers (TDR's)^{1,2}. In TDR testing a voltage step function is introduced at the cable input, and an oscilloscope is used to observe the nature of the signals reflected back to the input, much like a radar.

This report reviews some well-known transmission-line theory, and applies it to the case in which the transmission line may be considered to consist of segments of uniform characteristic impedance, with discontinuous steps in impedance at the segment boundaries. This cable model is shown to represent adequately many of the important characteristics of the coaxial cable used in the CRC High Frequency Direction Finding antenna array. Small series inductive or shunt capacitive discontinuities are neglected, because their effects are negligible in the frequency range of interest, 2-30 MHz. This calculation of frequency domain properties from a known impedance profile is made directly via transmission line equations, and a computer program has been written to perform the calculation for a wide variety of situations. The results are in good agreement with experimental measurements.

The report also reviews the Fourier transform relation between time and frequency domain coaxial cable properties, and shows how the impedance profile of a cable may be computed via a Fourier transform from frequency domain measurements.

The motivation for this report was the observation that many of the co-axial cables supplied for the High Frequency Direction Finding antenna array had occasional impedance discontinuities or steps of the order of 0.5 ohms, and it was necessary to assess the magnitude and nature of errors contributed by these anomalies. A second application is the assessment of phase errors due to the fact that neither the antenna elements nor the receivers used in the system have internal impedances perfectly matched to the cable characteristic impedance. The analysis may also be of interest to those in the cable television industry, where many of the problems are similar.

LIST OF SYMBOLS

The following symbols are used in this report:

- c velocity of light
- f(t) step function response
- f'(t) impulse response
- f_s(t) step function response calculated from swept frequency measurements
- $f'_{s}(t)$ impulse response calculated from swept frequency measurements
- h(t) total response 1 + f(t)
- i $\sqrt{-1}$
- I current
- L cable electrical length

```
SRL
         structural return loss
         frequency sweep rate
s
         time
t
         rise time
Tr
v
         voltage
         voltage travelling in -z direction
V<sub>1</sub>
V_2
         voltage travelling in +z direction
V_{20}
         amplitude of voltage travelling in +z direction
         generator voltage
VSWR
         voltage standing wave ratio
         cable shunt admittance per unit length
٧
Z
         cable series impedance per unit length
Z
         impedance
Z_0
         transmission line characteristic impedance
Zg
         generator impedance
Z_{\tau}
         load or terminating impedance
z
         distance along cable from origin
         attenuation constant
α
В
         phase constant 2\pi/\text{wavelength}
         complex propagation constant \alpha + i\beta
γ
\delta(t)
         Dirac delta function
φ
         phase angle
         phase of voltage after travelling down the cable and
Φı
         back again
ф2
         phase of the downgoing signal at the origin z = 0
         complex voltage reflection coefficient
ρ
         complex voltage reflection coefficient of Z_{\tau}
\rho_{T}
         swept frequency complex voltage reflection coefficient
\rho_{\mathbf{s}}
         angular frequency in radians/sec
ω
         upper limit of a swept frequency measurement
\omega_{\mathbf{a}}
         initial angular frequency of swept frequency measurement.
\omega_0
```

THE REFLECTION COEFFICIENT

Following Weeks³, the reflection coefficient as measured at the input of a uniform cable which is terminated in an arbitrary impedance may be calculated with reference to Figure 1. At any point on a uniform transmission line, the voltage may be considered as the sum of two voltages travelling in opposite directions,

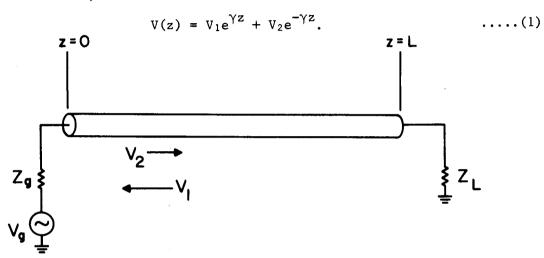


Fig. 1. Uniform transmission line with arbitrary source and load impedances.

The time dependent factor $e^{i\omega t}$ is considered to be absorbed into the constants V_1 and V_2 . Thus ${V_1}e^{\gamma z}$ represents a wave travelling in the -z direction, and ${V_2}e^{-\gamma z}$ represents a wave travelling in the +z direction. γ is the complex propagation constant,

$$\gamma = (yz)^{\frac{1}{2}} = \alpha + i\beta, \qquad \dots (2)$$

where Y is the shunt admittance per unit length, Z is the series impedance per length, α is the attenuation constant, and β = $2\pi/w$ avelength is the phase constant.

The current is given by

$$I(z) = -\frac{1}{Z} \frac{dV}{dz}. \qquad \dots (3)$$

From the boundary condition that the ratio V/I at z=L must equal the terminating impedance Z_L , it is readily shown that

$$\frac{Z_{L}}{Z_{0}} = \frac{1 + \frac{V_{1}}{V_{2}} e^{2\gamma L}}{1 - \frac{V_{1}}{V_{2}} e^{2\gamma L}} \dots (4)$$

where $Z_0 = \frac{Z}{\gamma} = \left(\frac{Z}{y}\right)^{\frac{1}{2}}$, the characteristic impedance of the cable.

The voltage reflection coefficient of the load $\boldsymbol{\rho}_L$ may then be defined as

$$\rho_{L} = \frac{Y_{1}}{V_{2}} e^{2\gamma L} = \rho (0)e^{2\gamma L}, \qquad \dots (5)$$

where $\rho(0)$ is the reflection coefficient at the input end of the cable. Equation (5) gives the ratio of reflected-to-incident voltage at the termination, in terms of the same ratio $\rho(0)$ measured at the input end of the line, and the length of the line L.

From equations (4) and (5), the usual expression for the reflection coefficient in terms of the impedances is given by

$$\rho_{L} = \frac{Z_{L} - Z_{0}}{Z_{1} + Z_{0}} . \qquad(6)$$

This equation for the reflection coefficient is quite general and may be applied anywhere along the cable, i.e., if Z_L is reinterpreted as the actual impedance at a point (the impedance one would measure with a bridge at that point) and Z_0 is the characteristic impedance of the cable, then the reflection coefficient at that point is given by equation (6).

Another quantity of interest is the voltage standing wave ratio or VSWR, Which is given by

$$VSWR = \frac{1 + |\rho(0)|}{1 - |\rho(0)|}(7)$$

IMPEDANCE AND VOLTAGE TRANSFORMATIONS

Taking the ratio V/I at z = 0 (using equations (1) and (3), and using the fact that, from equations (5) and (6),

$$\frac{V_1}{V_2} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma L} , \qquad \dots (8)$$

it is readily shown that

$$Z(0) = Z_0 \frac{Z_L \cosh \gamma L + Z_0 \sinh \gamma L}{Z_0 \cosh \gamma L + Z_L \sinh \gamma L} \qquad(9)$$

This relates the impedance Z(0) at the input of a length L of cable to the terminating impedance $Z_{\rm L}$ at its other end.

In a similar manner, the ratio of the voltage at the load (z = L) to that at the input (z = 0) is given by

$$\frac{V(L)}{V(0)} = \frac{(1 + \rho_L)e^{-\gamma L}}{1 + \rho_L e^{-2\gamma L}}$$
(10)

THE RETURN LOSS

The return loss, or structural return loss as it is sometimes called (because it is related to the structure in the impedance profile of a cable) is defined by

$$SRL = -20 \log |\rho(0)| dB.$$
(11)

The frequency domain quantities that are most frequently measured are the VSWR and the return loss. However, in both of these the phase information is lost or discarded, and only relatively crude estimates of the nature and location of cable faults can be inferred from the variation of these quantities with frequency. However, they do serve as good indicators of overall cable quality.

A very useful bridge circuit for the measurement of reflection coefficient (and therefore the return loss) is shown in Figure 2. Normally, the generator impedance and all of the bridge impedances Z are made equal to the characteristic impedance Z_0 of the cable. Then it can be shown that the voltage measured by the bridge voltmeter is given by

$$\frac{V}{V_g} = \frac{1}{8} \frac{Z(0) - Z_0}{Z(0) + Z_0}$$
,(12)

or by equation (6)

$$\rho(0) = 8 \frac{V}{V_g}$$
,(13)

i.e., the umbalance voltage V measured by the bridge is proportional to the reflection coefficient $\rho(0)$ of the impedance Z(0), which is the impedance seen looking into the coaxial cable attached to the bridge terminals. It should be noted that the bridge presents a source of impedance Z_0 to the unknown load Z(0); and, when $Z(0) = Z_0$, the impedance seen looking into the bridge input terminals is also Z_0 . If the bridge voltage V is measured in both amplitude and phase, then the complex reflection coefficient can be measured as a function of frequency. Since the generator, with internal impedance Z_0 , would develop a voltage $V_0/2$ across a matched load, the insertion loss of the bridge is a factor of 4, or 12 dB.

By using matched precision resistors, return loss bridges can be made with a balance of about 40 dB. However, by using broadband transformer

techniques, bridges with a balance of 55 dB over the frequency range 4 to 300 MHz can be manufactured.*

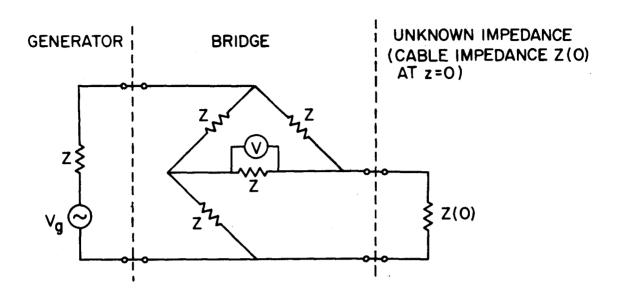


Fig. 2. Return loss bridge equivalent circuit. Normally $Z = Z_0$, the characteristic impedance of the cable to be measured.

CALCULATION OF RETURN LOSS FROM THE IMPEDANCE PROFILE

Equations (9) and (10) may be used as the basis for the calculation of the return loss and input-to-output phase error of a cable if its impedance profile is known. The cable may be regarded as a series of segments, each with a constant impedance, and with a discontinuous step in impedance at the segment boundaries. The load impedance is first transformed to the other end of the last segment by equation (9), that impedance is then used as a load impedance for the next segment, etc., until the impedance has been transformed all the way back to the input. The reflection coefficient and return loss can then be calculated by equations (6) and (11). The voltage developed across the load impedance can then be calculated by repeated application of equation (10), using as a starting point the voltage V(0) at the cable input, where

$$V(0) = \frac{V_g Z(0)}{Z_g + Z(0)} , \qquad \dots (14)$$

where V_g is the generator voltage, Z_g is the generator impedance, and Z(0) is the impedance looking into the cable, computed earlier by the repeated application of equation (9).

The above procedures are the basis of a computer program which has been

^{*} The Jerrold Corporation

written in FORTRAN IV, and which is listed in Appendix A. The program computes the following quantities:

- a) the magnitude of the return loss for an arbitrary terminating impedance,
- b) the transmission loss,
- c) the transmission amplitude and phase errors, relative to a matched, perfect cable of the same length, and
- the return signal phase error, for an open-circuited termination.

The return signal phase error for a short-circuit termination is the same as for an open-circuit termination except for a change in sign. (Measurements of the return signal phase for open and short-circuited terminations can be used to obtain the precise lengths of cables provided the approximate length is known well enough that the 2π ambiguity can be removed. More details are given in Appendix B.)

Figure 3 shows an X-Y chart recording of the TDR trace for a 354 meter length of one-half inch diameter foamed dielectric cable, type AL1250P manufactured by Canada Wire and Cable Ltd. The steps in impedance are believed to be caused by pauses in the cable manufacturing process. Provided the impedance steps are small enough that multiple reflections may be neglected, the steps in the TDR trace may be interpreted directly as changes in the characteristic impedance of the cable. However, the general slope of the TDR trace between impedance steps should not be similarly interpreted; this feature is attributable to distortion of the test pulse by frequency-dependent losses of

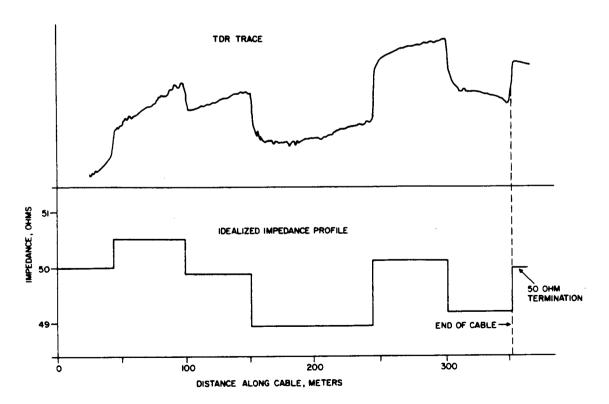
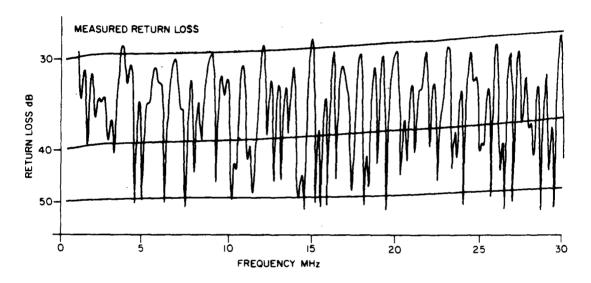


Fig. 3. Time domain reflectometer trace of a 354 meter cable, and the idealized impedance profile derived from the TDR trace.

various kinds in the $cable^2$. With this proviso, the idealized impedance profile of the cable is shown in the lower part of Figure 3. This interpretation was confirmed by comparisons of the cable characteristic impedance at each end with known impedance standards.

Figure 4 shows the measured return loss for the cable of Figure 3, along with the return loss computed on the basis of the idealized profile for the cable. The good general agreement between the predominant features of the return loss curves obtained by the two methods provides confidence that the phase errors which are computed from the model (but are not readily measurable) will also be representative of the real phase errors, at least in a worst-case or statistical sense. The return loss agreement is better in detail at the lower end of the frequency range, where small errors in the positions of the various impedance steps are less important to the computations.



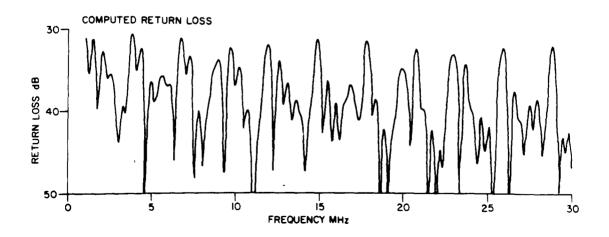


Fig.4. Comparison of measured and computed return loss, for the cable of Figure 3.

The slightly higher overall measured return loss can be attributed to a number of possible causes:

- (a) in deriving the impedance profile of Figure 3 from the TDR measurements, no correction was made for cable attenuation,
- (b) small but numerous impedance variations, too small to be resolved with a TDR, contribute to the total reflected signal,
- (c) errors in the return loss bridge itself may cause a slightly pessimistic return loss measurement.

For the magnitudes of cable imperfections of this example, the input-to-output phase error is less than 0.1°. However, the return phase error for open and short-circuited terminations amounts to several degrees, although their average is always negligibly small. Further details are given in Appendix B.

CALCULATION OF THE IMPEDANCE PROFILE FROM THE REFLECTION COEFFICIENT

Given measurements of the reflection coefficient over a range of frequencies at a cable input, it is possible to calculate the step-function response of the cable, that is, the response as seen by a Time Domain Reflectometer¹. As mentioned earlier, the step function response is very nearly equal to the impedance profile of the cable, provided the impedance discontinuities are small enough that multiple reflections may be neglected.

From linear system theory, the impulse response of a system is the Fourier transform of the reflection coefficient, or

$$f'(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho(\omega) e^{i\omega t} d\omega, \qquad \dots (15)$$

where f'(t) is the impulse response, and $\rho(\omega)$ is the reflection coefficient at the cable input. As well, the step function response is the integral of the time impulse response, or

$$f(t) = \int_{0}^{t} f'(t) dt,$$
(16)

where f(t) is the step function response. Finally, the total response h(t) observed by a TDR is the sum of the incident step and the response, or

$$h(t) = 1 + f(t)$$
.(17)

For example, for the uniform cable with mismatched load impedance \mathbf{Z}_{L} , as shown in Figure 1, the reflection coefficient is:

$$\rho(\omega) = \rho_{L} e^{-2\gamma L} , \qquad \dots (18)$$

where $\boldsymbol{\rho}_L$ is given by equation (6).

The impulse response then is

$$f'(t) = \frac{1}{2\pi} \int_{-\omega_a}^{\omega_a} \rho_L e^{-2\gamma L} e^{i\omega t} d\omega \qquad \dots (19)$$

or

$$f'(t) = \frac{1}{\pi} \rho_L e^{-2\alpha L} \frac{\sin \omega_a (t - \frac{2L}{c})}{t - \frac{2L}{c}} . \qquad (20)$$

In the limit as $\omega_a \rightarrow \infty$,

$$f'(t) = \rho_L e^{-2\alpha L} \delta(t - \frac{2L}{c}), \qquad \dots (21)$$

where $\delta(t - \frac{2L}{c})$ is the Dirac delta function.

This means that the system impulse response is a delta function, delayed by the two-way propagation time 2L/c, and reduced in amplitude from the incident impulse by the reflection coefficient of the load ρ_L , and the two-way attenuation of the cable $e^{-2\alpha L}$.

Then, by equation (16), the step response is

$$f(t) = \frac{1}{\pi} \rho_L e^{-2\alpha L} \int_0^t \frac{\sin \omega_a (t - \frac{2L}{c})}{t - \frac{2L}{c}} dt. \qquad \dots (22)$$

This can be expressed in terms of the sine integral $Si(z) = \int_0^z \frac{\sin p}{p} \, dp$ which is tabulated in reference 4. The step response obtained in this way is shown in Figure 5. The response has undershoot and overshoot, as a result of the finite bandwidth $2\omega_a$ over which the measurement is made. The 10-90 per cent rise time of the step is

$$T_r = \frac{2.8}{\omega_a} . \qquad \dots (23)$$

This may be taken as the time resolution of the method. For example, if $\frac{\omega_a}{2\pi}$ = 30 MHz, T_r = .015 microseconds.

This theoretical development used two simplifying assumptions. First, the attenuation constant α has been assumed to be independent of frequency, which of course it is not, and second, the reflection coefficient has been assumed to be independent of frequency. However, digital implementation of the Fourier transform of (measured) $\rho(\omega)$ would remove both of these limitations. In addition, the data could be 'windowed' thereby reducing the undershoot and overshoot in the computed step response, with some sacrifice in the rise time.

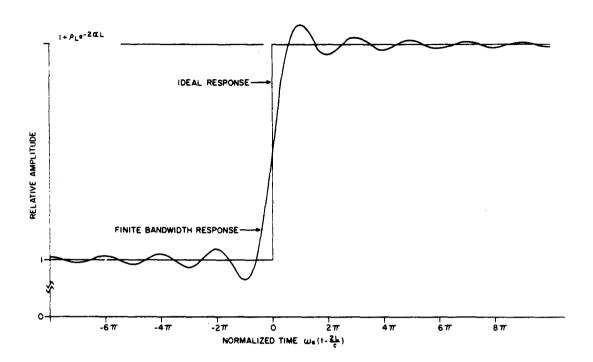


Fig. 5. The step-function response to an impedance discontinuity, obtained by Fourier transformation and integration of the reflection coefficient.

Normally, real data are taken for positive frequencies only, whereas the limits on the integral (19) are from $-\omega$ to $+\omega$. However, for real time functions, the spectrum is such that $\rho(-\omega) = \rho^*(\omega)$, i.e., the negative part of the spectrum is the complex conjugate of the positive one. The digital Fourier transform thus can be implemented using measurements of the positive spectrum only.

CALCULATION OF THE IMPEDANCE PROFILE FROM SWEPT FREQUENCY MEASUREMENTS

The foregoing discrete Fourier transform analysis requires measurements of the reflection coefficient $\rho(\omega)$ at a series of fixed frequencies across the bandwidth ω_a . However, it is more convenient to make measurements on a swept frequency basis, and this method is planned for the periodic testing of the cables in the High Frequency Direction Finding Array. In this case, the result is somewhat different. Consider in Figure 1 that the down-going voltage V_2 is a constant amplitude, linear frequency ramp. Then at z=0,

$$V_2 = V_{20} e^{i(\omega_0 + 2\pi st)t}$$
,(24)

where $V_{2\,0}$ is the amplitude, ω_0 the initial radian frequency, and s the frequency sweep rate. After time t, the accumulated phase of the signal V_2 is

$$\phi_2(t) = \int_0^t (\omega_0 + 2\pi st) dt = \omega_0 t + \pi s t^2.$$
 (25)

The signal after propagation down the cable and back has a phase

$$\phi_1(t) = \int_0^t -\frac{2L}{c} (\omega_0 + 2\pi st) dt = \omega_0 (t - \frac{2L}{c}) + \pi s (t - \frac{2L}{c})^2.$$
....(26)

The phase difference ϕ_1 - ϕ_2 is then

$$\phi_1 - \phi_2 = -(\omega - 2\pi s \frac{L}{c}) \frac{2L}{c}, \qquad \dots (27)$$

where $\omega=\omega_0+2\pi st$ is the instantaneous radian frequency of the input signal V_2 , and $\omega-2\pi s\,\frac{L}{c}$ is the average frequency of the signal propagating down the cable and back to the source. The ratio V_1/V_2 of the returned to the downgoing signal is

$$\rho_{s}(\omega) = \frac{V_{1}}{V_{2}} = \rho_{L}e^{-2\alpha L}e^{-i(\omega - 2\pi s \frac{L}{c})\frac{2L}{c}}, \qquad \dots (28)$$

where the subscript s refers to a swept frequency measurement. Using equations (18) and (2)

$$\rho_{s}(\omega) = \rho(\omega)e^{i\pi s(2L/c)^{2}}. \qquad(29)$$

Taking the Fourier transform of (28) and then integrating, one obtains

$$f_s(t) = e^{i\pi s (2L/c)^2} f(t),$$
(30)

where f (t) is the 'step-function response' obtained from the swept frequency measurement of the reflection coefficient, $\rho_{\rm S}(\omega)$.

The quantity $f_s(t)$ is not the true time domain step-function response because it is complex. However for reasonable sweep rates and cable lengths, $\pi_S(\frac{2L}{c})^2$ is small enough that the real part of $f_s(t)$ dominates. In any case, since the exponetial $e^{i\pi_S(2L/c)^2}$ has unity magnitude,

$$|f_s(t)| = |f(t)|$$
(31)

i.e., the magnitude of $f_s(t)$ is the same as the magnitude of the step response f(t).

١

If the Fourier transform of $\rho_{\mathbf{S}}(\omega)$ is implemented using the positive part of the spectrum only, and assuming that the negative spectrum is the complex conjugate of the positive one, then

$$f'_{s}(t) = \frac{1}{2\pi} \int_{-\omega_{a}}^{0} \rho_{s}(\omega) e^{i\omega t} d\omega + \frac{1}{2\pi} \int_{0}^{\omega_{a}} \rho_{s}(\omega) e^{i\omega t} d\omega , \qquad \dots (32)$$

or

$$f_{s}^{\bullet}(t) = \frac{1}{2\pi} \int_{0}^{\omega_{a}} \rho_{s}^{\star}(\omega) e^{-i\omega t} d\omega + \frac{1}{2\pi} \int_{0}^{\omega_{a}} \rho_{s}(\omega) e^{i\omega t} d\omega. \qquad \dots (33)$$

For example, for a uniform cable of length L and load reflection coefficient ρ_L , the reflection coefficient $\rho_s(\omega)$ at the cable input, as measured on a swept frequency basis, is given by equation (28). Putting this in equation (33), one obtains;

$$f'_{s}(t) = \frac{\rho_{L}}{\pi} e^{-2\alpha L} \left\{ \left[\cos \pi s \left(\frac{2L}{c} \right)^{2} \right] \left[\frac{\sin \omega_{a} \left(t - \frac{2L}{c} \right)}{t - \frac{2L}{c}} \right] - \left[\sin \pi s \left(\frac{2L}{c} \right)^{2} \right] \left[\frac{1 - \cos \omega_{a} \left(t - \frac{2L}{c} \right)}{t - \frac{2L}{c}} \right] \right\} \dots (34)$$

and the step function is the integral of equation (34),

$$f_{s}(t) = \frac{\rho_{L}}{\pi} e^{-2\alpha L} \left\{ \left[\cos \pi s \left(\frac{2L}{c} \right)^{2} \right] \left[\int_{0}^{t} \frac{\sin \omega_{a} \left(t - \frac{2L}{c} \right)}{t - \frac{2L}{c}} dt \right] - \left[\sin \pi s \left(\frac{2L}{c} \right)^{2} \right] \left[\int_{0}^{t} \frac{1 - \cos \omega_{a} t - \left(\frac{2L}{c} \right)}{t - \frac{2L}{c}} dt \right] \right\} \dots (35)$$

The first integral in equation (35) can be expressed in terms of the sine ingegral $Si(z) = \int_0^z \frac{\sin p}{p} \, dp$, while the second integral can be expressed in terms of the modified cosine integral $Cin(z) = \int_0^z \frac{1-\cos p}{p} \, dp$. Both of these are tabulated in Reference 4. However, for reasonable sweep rates s and cable lengths L, equation (35) can be simplified as follows: for $\omega_a \, \frac{2L}{c} \, < \, 13,000$, the cosine integral has a value less than 10. Thus for

 $\pi s(\frac{2L}{c})^2 < .001$, the second term may be neglected with less than one per cent error. Equation (35) then reduces to

$$f_{s}(t) = \frac{\rho_{L}}{\pi} e^{-2\alpha L} \int_{0}^{t} \frac{\sin \omega_{a} \left(t - \frac{2L}{c}\right)}{\left(t - \frac{2L}{c}\right)} dt, \qquad \dots (36)$$

which is the same as equation (22).

Thus, the step function response may be obtained to an accuracy of one percent or better, by Fourier transformation and integration of the reflection coefficient measured on a swept frequency basis, provided

$$\pi s \left(\frac{2L}{c}\right)^2 < .001, \qquad(37)$$

and

For example, for a cable of L = 1000 meters, the inequalities are satisfied for sweep rates s less than 10 MHz/sec, and swept bandwidths $\frac{\omega}{2\pi}$ less than 300 MHz.

SUMMARY

Some well-known transmission-line theory has been reviewed and applied to the calculation of the frequency domain properties (reflection coefficient, return loss, and phase error) of a coaxial cable from the cable impedance profile. This analysis is useful in assessing the phase errors which small impedance discontinuities and mismatched terminating impedances will cause in the CRC High Frequency Direction Finding system. A computer program to perform this calculation, together with some examples of calculated results, are included in Appendices A and B.

The solution to the inverse problem of calculating the impedance profile from measurements of the complex reflection coefficient at the cable input terminals has also been outlined. The method involves a Fourier Transform of the frequency-domain reflection coefficient measurements to obtain the time domain impulse response of the cable, followed by an integration to obtain the step-function response. The restrictions which apply when the reflection coefficient is measured using a swept frequency signal source were derived. This analysis will have application in the swept-frequency cable testing program which is being implemented for the CRC High Frequency Direction Finding array, to monitor long term variations in cable length due to seasonal temperature changes or other effects.

ACKNOWLEDGEMENTS

I am indebted to E. Lloyd Winacott and J.P. Raymond, whose program of cable measurements suggested the foregoing analysis. They also provided the experimental data for Figures 3 and 4.

This work was supported by the Defence Research Board, Department of National Defence, Canada.

REFERENCES

- 1. Hewlett-Packard Journal, Vol. 15, No. 6, (February 1964).
- 2. Tektronix Service Scope, No. 45, (August 1967).
- 3. Weeks, W.L., Electromagnetic theory for engineering applications, John Wiley and Sons, New York, 1964, Chapter 2.
- 4. Ambramowitz, M., and I.A. Stegun, eds. Handbook of mathematical functions with formulas, graphs, and mathematical tables, NBS Applied Math. Series 55, (June 1964).

APPENDIX A

The Return Loss Computer Program

The computer program listed below is based on equations (9) and (10) of this report. The program as written can handle calculations for cables composed of up to 20 segments; this could be increased if desired by enlarging the size of the necessary arrays in the program. In single precision on the XDS Sigma 7 computer, round-off error gives a residual return loss of about 120 dB for a uniform, matched cable of 20 segments, however, the program is written such that calculated return losses greater than 100 dB are arbitrarily set to 100 dB.

At the end of the listing is a sample of the line printer output.

```
C
C
C
C
C
C
C
C
C
С
C
C
С
C
C
C
C
C
C
 C
 C
 C
 C
 C
 C
 C
 C
 C
 C
 C
 CCCC
 Ċ
 C
 С
 C
```

C

40

30

PROGRAM 'CABLE' TO COMPUTE THE RETURN LOSS, TRANSMISSION LOSS, AND PHASE SHIFT ERROR OF A TRANSMISSION LINE COMPOSED OF ARBITRARY LENGTH SEGMENTS OF ARBITRARY CHARACTERISTIC IMPEDANCE.

D W RICE 17 FEB 1971

EXAMPLE OF INPUT DATA REQUIRED: N=3, ZO(0)=50.0

EL(1)=10.0, ZO(1)=51.0

EL(2)=20.0, ZO(2)=52.0

EL(3)=10.0, ZO(3)=53.0

Z0(4) = 50.0

F=1.0,30.0,1.0 *

UP TO 20 SEGMENTS MAY BE SPECIFIED.

IF DATA IS INPUT FROM A DISC FILE, THE FILE SHOULD BE CREATED WITH CR OFF.

THE INPUT VARIABLES HAVE THE FOLLOWING MEANINGS:

VARIABLE	TYPE	MEANING
N	INTEGER	NUMBER OF SEGMENTS IN CABLE
ZO(0)	REAL	SOURCE IMPEDANCE, OHMS
EL(J)	REAL	THE ELECTRICAL LENGTH OF THE
		J-TH SEGMENT, METERS
ZO(J)	REAL	THE CHARACTERISTIC IMPEDANCE OF
		THE J-TH SEGMENT, OHMS
ZO(N+1)	REAL	THE TERMINATING IMPEDANCE, OHMS
F	REAL ARRA	Y REPEAT TRIPLE SPECIFYING INITIAL.
		FINAL, AND STEP INTERVAL FOR THE
		FREQUENCIES AT WHICH COMPUTATIONS
		ARE REQUIRED, MHZ

THE VALUE 0.816 IS ASSUMED FOR THE PROPAGATION VELOCITY FACTOR. THE ATTENUATION FACTOR IS 0.26 DB/100 FT. AT 10 MHZ, AND VARIES AS (FREQUENCY)**0.53 (WHICH FITS THE PUBLISHED ATTENUATION CHARACTERISTIC OF 1/2 INCH ALUCEL AL1250P).

COMPLEX Z(0:21), V(20), VSTD, ZP, RHO(3), RHOP, ZTCPLX DIMENSION ZO(0:21), EL(20), F(3), ZT(3), ROANG(2) NAME LIST N, ZO, EL, F DATA VP, PI/0.816, 3.141592653/ CALL EOFSET(101S)

INPUT(105) ELTOT=0.0 DO 40 J=1.N

ELTOT=ELTOT+EL(J)

WRITE(108,30) N, ELTOT, ZO(0), ZO(N+1), (J, EL(J), ZO(J), J=1, N) FORMAT(1H1,3X,12,1X, 'SEGMENT CABLE OF ',F7.2,1X,

C'METERS TOTAL ELECTRICAL LENGTH',
C//4X, 'SOURCE IMPEDANCE', 1X, G. 3, 'OHMS, LOAD IMPEDANCE',

C1X, G. 3, 'OHMS',

```
C//4X, 'SEGMENT', 4X, 'ELECTRICAL', 3X, 'IMPEDANCE',
     C/15X, 'LENGTH, M', 6X, 'OHMS',
     C/20(/6X, 12, 7X, F8, 2, 5X, F7, 2))
      WRITE(108,31)
31
      FORMAT(//3X, 'FREQ', 4X, 'RETURN', 3X, 'TRANSMISSION',
     C5X, 'TRANSMISSION ERROR', 4X, 'RETURN PHASE'
     C/4X, 'MHZ', 2(3X, 'LOSS, DB'), 5X, 'MAG, DB', 3X,
     C'PHA, DEGREES', 3X, 'ERROR, DEG'/59X, '(OUTPUT OC)')
C
      REPEAT 100, FOR EF=(F(1),F(2),F(3))
      FREQ=EF*1.0E6
      XLOSS=0.26*(EF/10.0)**0.53
C
C
      COMPUTE RETURN SIGNAL FOR OPEN AND TERMINATED CABLES
C
      ZT(1)=1.0E20
      ZT(2) = ZO(N+1)
      DO 52 K=1.2
      Z(N+1)=CMPLX(ZT(K),0.0)
      DO 50 J=N.1.-1
      CALL ZTSFRM(Z(J+1), ZO(J), FREQ, EL(J), VP, XLOSS, Z(J))
50
      CONTINUE
      RHO(K) = (Z(1) - ZO(0))/(Z(1) + ZO(0))
52
      CONTINUE
      ROMAG=CABS(RHO(2))
      IF(ROMAG-1.0E-5) 20,20,21
20
      RETL=100.0
      GO TO 22
21
      RETL=-20.0*LOGIO(ROMAG)
55
      CONTINUE
С
С
      COMPUTE REFLECTION FROM PERFECT OPEN-CIRCUITED CABLE
С
      ZTCPLX=CMPLX(ZT(1),0.0)
      CALL ZTSFRM(ZTCPLX,50.0,FREQ,ELTOT, VP,XLOSS,ZP)
      RHOP=(ZP-ZO(O))/(ZP+ZO(O))
      ROPRE=REAL(RHOP)
      ROPIM=AIMAG(RHOP)
      ROPANG=180.0/PI*ATAN(ROPIM,ROPRE)
      RHORE=REAL(RHO(1))
      RHOIM=AIMAG(RHO(1))
C
C
      COMPUTE RETURN PHASE ERROR
C
      ROANG(1)=180.0/PI*A1AN(RHOIM, RHORE)-ROPANG
53
C
      COMPUTE VOLTAGE AT LOAD END OF CABLE, COMPARE WITH RESULT
C
      FOR PERFECT CABLE
C
C
      V(1)=2.0*Z(1)/(Z(1)+Z0(0))
C
      V(1)=VOLTAGE AT CABLE INPUT
C
C
      TRANSFORM V(1) TO OUTPUT END OF CABLE
C
```

```
DO 51 J=1.N
      CALL VTSFRM(V(J),Z(J+1),ZO(J),FREQ,EL(J),VP,XLOSS,V(J+1))
51
      CONTINUE
С
С
      COMPUTE PHASE SHIFT AND ATTENUATION OF A 'PERFECT' CABLE
C
      CALL VTSFRM((1.0,0.0),(50.0,0.0),50.0,FREU,ELTOT,VP,XLOSS,VSTD)
      VSTDM=CABS(VSTD)
      VSTDRE=REAL(VSTD)
      VSTDIM=AIMAG(VSTD)
      ANGSTD=180.0/PI*ATAN(VSTDIM, VSTDRE)
      VMAG=CABS(V(N+1))
      VRE=REAL (V(N+1))
      VIM=AIMAG(V(N+1))
      ANGV=180.0/PI*ATAN(VIM, VRE)
C
      ANGERR=ANGV-ANGSTD
      VERR=VMAG/VSTDM
      VERR=20.0*LOG10(VERR)
C
      TLOSS=1.0/VMAG
      TLOSS=20.0*LOG10(TLOSS)
C
      WRITE(108,10) EF, RETL, TLOSS, VERR, ANGERR, KOANG(1)
10
      FORMAT(1X, F6.3, 2X, F8.2, 2X, F9.2, 5X, F8.2, 5X, F7.2, 3X, F9.2)
100
      CONTINUE
101
      STOP
      END
      SUBROUTINE ZISFRM(ZA, ZO, F, EL, VP, XLOSS, ZB)
C
      "ZTSFRM" COMPUTES THE IMPEDANCE ZB SEEN AT ONE END OF A
C
      TRANSMISSION LINE WHEN THE LINE IS TERMINATED AT THE OTHER
C
      END BY AN IMPEDANCE ZA.
C
С
      THE INPUT VARIABLES ARE:
C
С
      VARIABLE
                  TYPE
                             MEANING
C
C
      ZA
                  CMPLK
                             TERMINATING IMPEDANCE, OHMS
С
      20
                  REAL
                             LINE CHARACTERISTIC IMPEDANCE, OHMS
C
      F
                  REAL
                             FREQUENCY, HERTZ
C
                             LINE ELECTRICAL LENGTH, METERS
      EL
                  REAL
                             PROPAGATION VELOCITY FACTOR
С
      VΡ
                  REAL
                             LINE ATTENUATION CONSTANT, DB/100 FT
С
                  REAL
      XL0SS
Č
C
      THE OUTPUT VARIABLE IS:
C
                             THE COMPUTED LINE INPUT IMPEDANCE, OHMS
C
       ZB
                  CMPLX
C
      COMPLEX ZA, ZB, GAMMAL, ZAN
      DATA PI, CLT/3.141592653, 2.997925E8/
       CONST=20.0*30.4799*0.4342945
```

BETA=2.0*PI*F/CLT ALPHA=XLOSS*VP/CONST

```
GAMMAL=CMPLX(ALPHA, BETA)*EL
ZAN=ZA/ZO
ZB=ZO*(ZAN*CCOSH(GAMMAL)+CSINH(GAMMAL))
C/(CCOSH(GAMMAL)+ZAN*CSINH(GAMMAL))
RETURN
END
```

SUBROUTINE VTSFRM(VA,ZT,ZO,F,EL,VP,XLOSS,VB)
'VTSFRM' COMPUTES THE VOLTAGE VB AT ONE END OF A TRANSMISSION LINE, GIVEN THE VOLTAGE VA AT THE OTHER END.

THE INPUT VARIABLES ARE:

VARIABLE	REAL	MEANING
V.A	CMPLX	INPUT VOLTAGE, VOLTS
ZT	CMPLX	TERMINATING IMPEDANCE
20	REAL	LINE CHARACTERISTIC IMPEDANCE, OHMS
F	REAL	FREWUENCY, HERTZ
EL	REAL	LINE ELECTRICAL LENGTH, METERS
٧P	REAL	PROPAGATION VELOCITY FACTOR
KLOSS -	REAL	LINE ATTENUATION CONSTANT, DB/100 FT

THE OUTPUT VARIABLE IS:

VB CMPLK VOLTAGE AT OTHER END OF LINE, VOLTS

DATA PI,CLT/3.141592653,2.997925E8/
COMPLEX VA,VB,ZT,RHO,GAMMAL,GAMMAL2
CONST=20.0*30.4799*0.4342945
RHO=(ZT-CMPLX(ZO,0.0))/(ZT+CMPLX(ZO,0.0))
BETA=2.0*PI*F/CLT
ALPHA=XLOSS*VP/CONST
GAMMAL=-CMPLX(ALPHA,BETA)*EL
GAMMAL2=2.0*GAMMAL
VB=VA*(CEXP(GAMMAL)*(1.0+RHO))
C/(1.0+RHO*CEXP(GAMMAL2))
RETURN
END

EXAMPLE OF PROGRAM OUTPUT:

3 SEGMENT CABLE OF 40.00 METERS TOTAL ELECTRICAL LENGTH

SOURCE IMPEDANCE 50.0 OHMS. LOAD IMPEDANCE 50.0 OHMS

SEGMENT	ELECTRICAL LENGTH. M	IMPEDANCE OHMS
1	10.00	51.00
2	20.00	52.00
3	10.00	53.00

					_
FREG	RETURN	TRANSMISSION	TRANS	MISSION ERROR	RETURN PHASE
MHZ	LOSS, DB	LOSS, DB	MAG. DB	PHA, DEGREES	ERROR, DEG
					(OUTPUT OC)
2.000	27.75	•13	01	01	1.26
4.000	35.16	• 17	00	01	• 49
6.000	27.72	•55	01	01	1.05
8.000	28.50	•25	01	00	04
10.000	29.98	•28	00	•03	-1.93
12.000	30.50	• 31	00	01	•69
14.000	31.07	• 34	00	•03	-2.62
16.000	30.96	•36	00	03	2.64
18.000	30.73	• 38	00	•01	7 0
50.000	30.10	• 41	00	 03	1.93
\$5.000	28.86	• 43	01	• 00	• 0 4
24.000	28.09	• 45	01	• 0 1	-1.08
26.000	36.16	• 46	00	•01	- • 44
28.000	28.22	• 49	01	• 01	-1.32
30.000	53.17	• 50	00	00	• 08

APPENDIX B

Typical Phase Errors for Various Cable and Termination Imperfections

The HFDF array utilizes cables in two matched sets; one set is approximately 170 meters long (electrically), and the other is approximately 750 meters long. To make valid phase comparisons between signals transmitted over the two different cable lengths, it is necessary to know these precise lengths. One way to make the required length measurement is to open- or short-circuit one end of the cable and to measure, at the other end of the cable, the phase of the reflected signal with respect to a signal source at the cable input, using the return loss bridge of Figure 2. This measures the two-way phase length of the cable. Since the cables are many wavelengths long, the length must be approximately known from a separate measurement (e.g., by a pulse reflection technique) to remove the 2π ambiguity.

If there are imperfections in the cable which cause reflections back to the source in addition to the desired one from the open- or short-circuited end, then the resultant phase of the total reflected signal may be in error. Some examples of the phase error in the return signal, computed for an open circuited cable of 170 meters length with 1 ohm impedance discontinuities at various locations, are shown in Figure B1.

It can be shown that the return signal phase error in a short-circuited cable is identical to that in an open-circuited cable except for a reversal of sign. Thus the effect of errors caused by such cable imperfections can be overcome by taking the average of an open-circuit and a short-circuit measurement.

Figure B2 shows the computed return signal phase error for the same circumstances as in Figure B1, except that the cable length is 750 meters rather than 170 meters. The results are similar except that the longer cable produces a more rapid variation of phase error with frequency, and the effects of cable attenuation are more pronounced.

It can be verified that cable imperfections such as those of Figures Bl and B2 do not produce a significant error in the phase of a signal transmitted from one end of the cable to the other, provided the cable is properly terminated at both ends. However, the impedances of the antenna elements in the HFDF array are such that VSWR's up to 5:1 may be produced, while the receivers may result in VSWR's up to 1.4:1. These worst-case mismatches may cause significant phase errors, even in perfect cables, due to multiple reflections of the signal from the mismatches at the cable ends. Figure B3 shows the computed phase errors, as a function of frequency, for cables of 170 and 750 meters length, and with source and load mismatches corresponding to VSWR's of 5:1 and 1.4:1, respectively. These are errors in a perfect cable, with respect to the correct value when source and load are properly matched to the cable characteristic impedance.

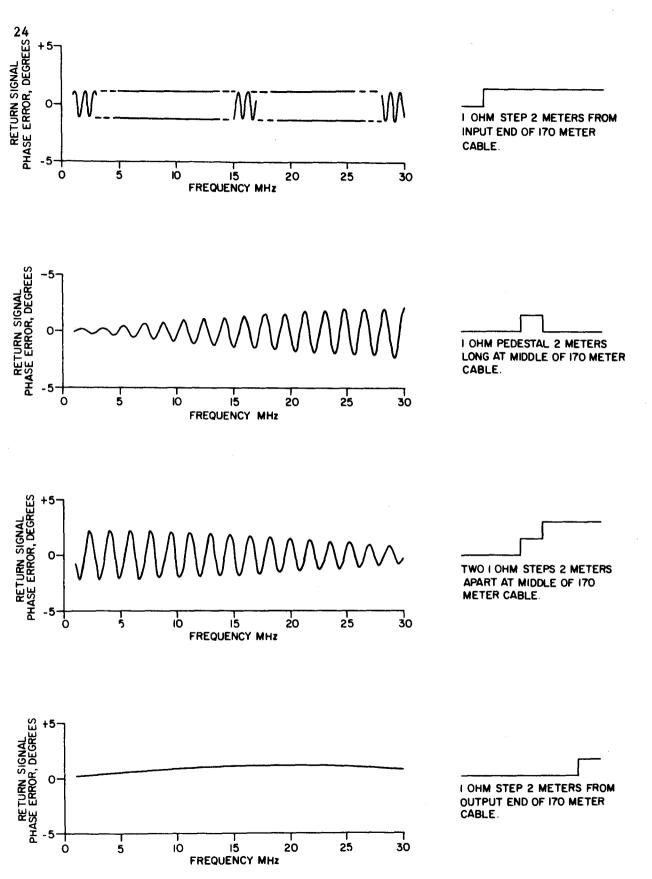


Fig. B1. Computed return phase error for various cable impedance discontinuities; cable terminal end open circuited; cable length 170 meters.

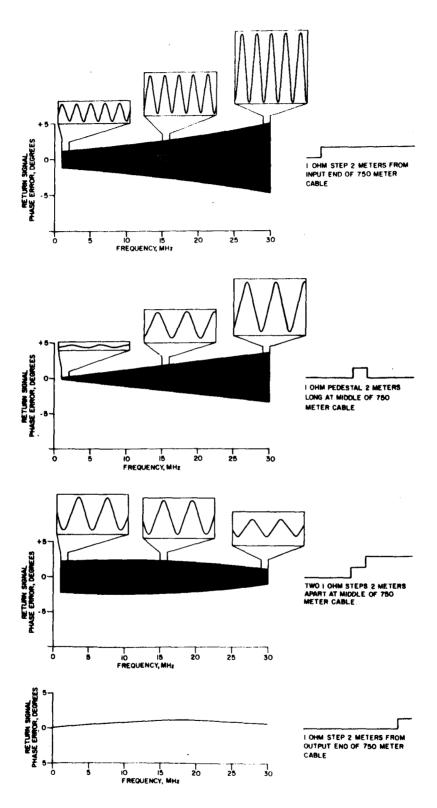


Fig.B2. Computed return phase error for various cable impedance discontinuities; cable terminal end open circuited; cable length 750 meters.

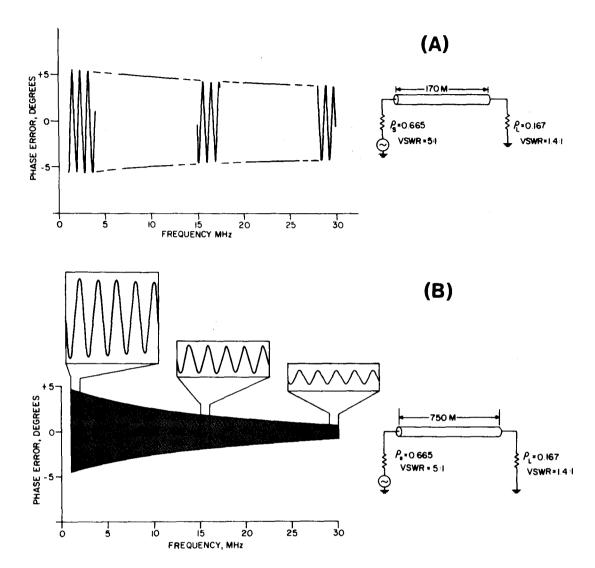


Fig. B3. Computed input-to-output phase errors, for cables with mismatched source and load. Source VSWR 5:1; load VSWR 1.4:1. (A) Cable of 170 meters length, (B) Cable of 750 meters length.

For cables matched in length and with identical (but mismatched) terminating impedances, the phase errors would be the same in all cables and hence of no consequence in the comparison of the phase of the signal from one antenna to another. However, in comparing the signals from two antennas via two different lengths, phase errors of 10° peak at the low frequency end due to mismatched source and terminating impedances, are possible. The phase error is reduced at higher frequencies, particularly in the longer cable because of cable attenuation.

All of the computations in this report assume cable characteristics corresponding to Canada Wire and Cable Alucell AL1250P. These are:

- nominal characteristic impedance 50 ohms
- velocity factor of propagation 0.816

• attenuation 0.26 dB/100 feet of physical length at 10 MHz, and varying with frequency raised to the power 0.53.

Cable lengths quoted are electrical length i.e., physical length divided by 0.816.

RICE, D. W.
--Analysis of impedance profiles
and structural return loss of

TK 5102.5 0673e #1224

DATE DUE

1011-		
1616		-
1 1 1 1	-	
		-
	-	
-		
1		
-		

LOWE-MARTIN No. 1137

CRC LIBRARY/BIBLIOTHEQUE CRC



