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**OPTIMUM CRYSTAL ORIENTATION FOR
A BARIUM SODIUM NIOBATE
TRANSVERSE LIGHT MODULATOR**

by
D.C. Johnson

DEPARTMENT OF COMMUNICATIONS
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CRC REPORT NO. 1229

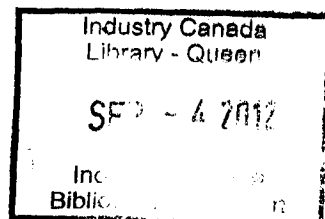
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D.C. Johnson

(Informatique Directorate)

CRC REPORT NO. 1229

Published February 1972

OTTAWA

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OPTIMUM CRYSTAL ORIENTATION FOR A BARIUM SODIUM NIOBATE TRANSVERSE LIGHT MODULATOR

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D.C. Johnson

ABSTRACT

A theoretical expression has been derived for the half-wave voltage required for a transverse modulator constructed from a crystal of barium sodium niobate in an arbitrary orientation. From this expression the optimum crystal orientation which produces the lowest half-wave voltage was found. Although this work was complicated by the fact that not all the signs of the electro-optic coefficients are known, it is shown that modulators can be constructed with half-wave voltages significantly lower than was previously available. For example, in the case of amplitude modulation it should be possible to construct a modulator with a half-wave voltage of 690 or 836 volts depending on whether the sign of the r_{51} coefficient is positive or negative. Whereas, in the normally used crystal orientation, the half-wave voltage is 1,750 volts.

1. INTRODUCTION

The electro-optical coefficients of barium sodium niobate are large^(1,2) making its use for light modulation very promising. However, the light modulators constructed from this crystal usually orient the crystal such that the directions for the applied field and the light propagation are along the c and a crystallographic axes respectively. Since this crystal orientation does not necessarily produce the lowest half-wave voltage for modulation, it may not be the most favourable one for constructing a modulator. The purpose of this report is to derive an expression for the phase retardation of a light wave propagating in an arbitrary direction through a crystal of barium sodium niobate on which a transverse electric field is applied. From this

expression, the optimum crystal orientation which produces the lowest half-wave voltage for modulation will be determined. This optimization will be carried out for both the high and low frequency values of the electro-optic coefficients and for both amplitude and phase modulation.

2. ELECTRO-OPTIC COEFFICIENTS OF BARIUM SODIUM NIOBATE

Barium sodium niobate which belongs to the point group 2mm has five independent electro-optic coefficients, r_{13} , r_{23} , r_{33} , r_{42} and r_{51} . The equation of the index ellipsoid is

$$\left[\frac{1}{n_x^2} + r_{13} E_z \right] x^2 + \left[\frac{1}{n_y^2} + r_{23} E_z \right] y^2 + \left[\frac{1}{n_z^2} + r_{33} E_z \right] z^2 + 2r_{42} E_y yz + 2r_{51} E_x xz = 1. \quad \dots(1)$$

where n_x , n_y and n_z are the principal refractive indices and E_x , E_y and E_z are the components of the applied field. Both the high and low frequency coefficients in barium sodium niobate have been measured^(1,2) and listed in Table I. Unfortunately, the signs of the r_{42} and r_{51} coefficients have not been determined and this ambiguity must be taken into account when determining the optimum crystal orientation. As shown in Table II there are four possible sign combinations. However, it is known that above 300°C barium sodium niobate becomes tetragonal (4mm) and in this state, crystal symmetry requires that $r_{42} = r_{51}$ and $r_{13} = r_{23}$. Hence the sign combinations C and D listed in Table II in which r_{42} and r_{51} have different signs, are not very probable and will be neglected in this work.

TABLE I

Electro-optic Coefficients	Low Frequency Value	High Frequency Value	Sign
	($\times 10^{12} \frac{\text{Meter}}{\text{Volt}}$)	($\times 10^{12} \frac{\text{Meter}}{\text{Volt}}$)	
r_{13}	15 ± 1	7	+
r_{23}	13 ± 1	8	+
r_{33}	48 ± 2	29	+
r_{42}	92 ± 4	79	?
r_{51}	90 ± 4	95	?

TABLE II
Possible Sign Combinations

	r_{13}	r_{23}	r_{33}	r_{42}	r_{51}
A	+	+	+	+	+
B	+	+	+	-	-
C	+	+	+	-	+
D	+	+	+	+	-

3. THEORETICAL TECHNIQUE

The technique used to optimize the crystal orientation of barium sodium niobate is similar to that used for potassium dihydrogen phosphate and lithium niobate⁽³⁻⁷⁾. The procedure is to choose a coordinate system X, Y and Z in which Z is along \bar{r} and arbitrary direction of light propagation through the crystal (see Figure 1). The index ellipsoid in Equation (1) is then transformed to this new coordinate system. As shown in Figure 1, the orthogonal axes x, y and z coincide with the crystallographic axes a, b and c respectively. The orientation of \bar{r} with respect to the crystallographic axes is determined by the angle ρ between z and \bar{r} and the angle θ between the planes rz and xz. Since the applied field is normal to \bar{r} , it can be specified by the angle ξ its direction makes with the rz plane. The appropriate transformation for rotating the axes to the new coordinate system is

$$\begin{aligned}x &= -X \sin \theta - Y \cos \theta \cos \rho + Z \cos \theta \sin \rho \\y &= X \cos \theta - Y \sin \theta \cos \rho + Z \sin \theta \sin \rho \\z &= Y \sin \rho + Z \cos \rho\end{aligned}$$

When Equation (1) is transformed, a new expression for the ellipsoid will be formed. However, to determine the retardation of a light wave, it is only necessary to find the equation for the ellipse in the plane perpendicular to the direction of propagation, (i.e., $Z = 0$). It will have the form

$$A_{11} X^2 + A_{22} Y^2 + 2A_{12} XY = 1, \quad \text{.....(2)}$$

where

$$A_{11} = \frac{1}{n_1^2} + Q_1 E, \quad \text{.....(3)}$$

$$A_{22} = \frac{1}{n_2^2} + Q_1 E, \quad \text{.....(4)}$$

$$A_{21} = \left[\frac{1}{n_x^2} - \frac{1}{n_y^2} \right] \sin 2\theta \cos \rho + Q_{21} E, \quad \text{.....(5)}$$

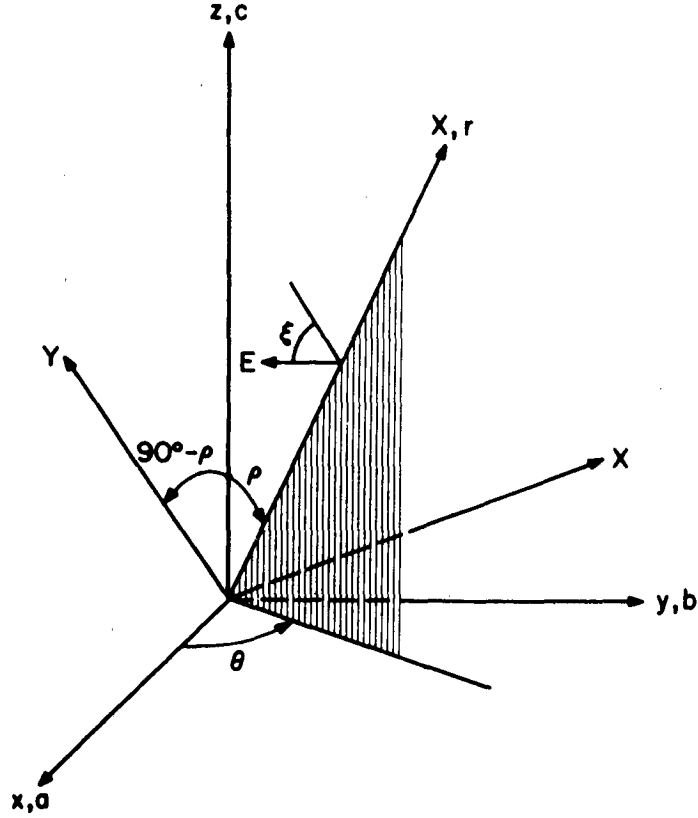


Fig. 1. A schematic diagram showing the orientation of the light propagation direction r with respect to the crystallographic axes a , b and c . The direction for light propagation is specified by the angle ρ between r and c and the angle θ between the planes rc and ac . E is the applied electric field which is perpendicular to r but at angle ξ to the rc plane.

and

$$\frac{1}{n_1^2} = \frac{\sin^2 \theta}{n_x^2} + \frac{\cos^2 \theta}{n_y^2} , \quad \dots\dots(6)$$

$$\frac{1}{n_2^2} = \frac{\cos^2 \theta \cos^2 \rho}{n_x^2} + \frac{\sin^2 \theta \cos^2 \rho}{n_y^2} + \frac{\sin^2 \rho}{n_z^2} , \quad \dots\dots(7)$$

$$Q_1 = r_{23} + \{(r_{13} - r_{23}) \sin^2 \theta\} \cos \xi \sin \rho , \quad \dots\dots(8)$$

$$\begin{aligned}
Q_2 = & \frac{1}{2} (r_{42} - r_{51}) \sin \xi \sin 2\rho \sin 2\theta \\
& + \left\{ (2r_{42} + r_{23}) - (2r_{51} + r_{13}) \right\} \cos \xi \sin \rho \sin^2 \theta \\
& + (2r_{51} + r_{13}) \sin \rho \cos^2 \rho \cos \xi \\
& + r_{33} \cos \xi \sin^3 \rho , \quad \dots(9)
\end{aligned}$$

$$\begin{aligned}
Q_{21} = & \frac{1}{2} (r_{13} - r_{23}) \cos \xi \sin 2\rho \sin 2\theta \\
& - (r_{42} - r_{51}) (2 \sin \xi \sin \rho \cos^2 \theta + \frac{1}{2} \cos \xi \sin 2\rho \sin 2\theta) \\
& - 2r_{51} \sin \xi \sin \rho . \quad \dots(10)
\end{aligned}$$

In the derivation of the above expressions, the applied field was expressed in terms of the angles ρ , θ and ξ as shown below;

$$\begin{aligned}
E_x &= E (\sin \xi \sin \theta - \cos \xi \cos \rho \cos \theta) \\
E_y &= -E (\sin \xi \cos \theta + \cos \xi \cos \rho \sin \theta) \\
E_z &= E \cos \xi \sin \rho .
\end{aligned}$$

In general, the effect of an applied field is not only to change the shape of the index ellipsoid, but also to rotate the principal axes. This can be seen in Equation (2) where the presence of the cross term $2A_{12}XY$ indicates that X and Y are not the principal axes of the ellipse. By transforming the ellipse in Equation (2) to its principal axes, it can be shown that they are rotated about the Z axis by an amount α where α is given by

$$\tan 2\alpha = \frac{2A_{12}}{A_{11} - A_{22}} .$$

However, from a consideration of the values of A_{11} , A_{22} and A_{12} in Equations (3), (4) and (5), it can be seen that α will be very small except when ρ is close to zero. Since, as will be seen later, the minimum half-wave voltage occurs for a non-zero ρ , the rotation of the principal axes can be neglected. Hence the axes, X and Y can be taken as the principal axes of the ellipse. The change in refractive index due to the applied field for a light wave propagating along \bar{r} and polarized along X is then given by

$$\Delta n_1 = - \frac{1}{2} n_1^3 Q_1 E ,$$

where n_1 and Q_1 are given in Equations (6) and (8) respectively. The retardation of this wave after passing through a crystal of length ℓ will be

$$\Gamma_1 = \frac{2\pi\ell}{\lambda} n_1 - \frac{\pi\ell}{\lambda} n_1^3 Q_1 E , \quad \dots(11)$$

where λ is the wavelength of the light. The analogous expression for the retardation of a light wave polarized along the Y axis is

$$\Gamma_2 = \frac{2\pi\ell}{\lambda} n_2 - \frac{\pi\ell}{\lambda} n_2^3 Q_2 E, \quad \dots\dots(12)$$

where n_2 and Q_2 are given in Equations (7) and (9) respectively. In Equations (11) and (12) we have the desired expressions for the phase retardation produced by a transverse electric field on a light wave when it passes through a barium sodium niobate crystal in an arbitrary orientation.

4. OPTIMIZATION OF CRYSTAL ORIENTATION FOR MINIMUM HALF-WAVE VOLTAGE

4.1 Amplitude Modulation

In the case of amplitude modulation, the crystal is placed between crossed polarizers with the orientation of the first polarizer such that light incident on the crystal is polarized at 45 degrees to the X axis. On entering the crystal, the light is resolved into two waves polarized along the X and Y axes respectively. Each of these waves as they pass through the crystal will experience a different phase retardation which results in an elliptical light wave emerging from the crystal. It is the passage of this elliptical wave through the second polarizer that produces the light intensity modulation. Using Equations (11) and (12), it can be shown that the phase difference produced between the two waves in the crystal is given by

$$\Delta\Gamma = \frac{2\pi\ell}{\lambda} (n_1 - n_2) + \frac{\pi\ell}{\lambda} (n_2^3 Q_2 - n_1^3 Q_1) E.$$

The first term which does not depend on the electric field is due to the crystal birefringence. In principle its effect on modulator performance can be eliminated. The second term in the phase difference represents an electrically induced birefringence. The voltage required to produce a phase difference of π radians is called the half-wave voltage and is given by

$$V_{1/2} = \frac{\lambda}{n_2^3 Q_2 - n_1^3 Q_1}. \quad \dots\dots(13)$$

For the best modulator performance it is desirable to optimize the crystal orientation such that the half-wave voltage in Equation (13) is a minimum. As can be seen in Equations (6), (7), (8) and (9) all the quantities n_1 , n_2 , Q_1 and Q_2 depend on crystal orientation. However, since the refractive indices n_1 , n_2 vary slowly with θ and ρ , their functional dependence will be ignored in the following analysis. Using the standard methods of calculus for minimization, analytical expressions for the optimum values for ξ and ρ can be found. From a consideration of the magnitudes of the electro-optic coefficients, it can be shown that these optimum values are very close to zero. Furthermore, this is true for either the A or the B sign combinations of the electro-optic coefficients listed in Table II. With the optimum values for

both ξ and θ as zero, A_{21} in Equation (5) is zero, verifying that there is no rotation of the principal axes. Also, the half-wave voltage in Equation (13) now depends only on the coefficients r_{13} , r_{23} , r_{33} , r_{51} and the angle ρ . Continuing to ignore the dependence of the refractive indices on ρ , an expression for the optimum value for ρ can be found analytically. It is given by

$$\sin \rho = \left[\frac{n_z^3 (2r_{51} + r_{13}) - n_1^3 r_{23}}{3n_2^3 (2r_{51} + r_{13} - r_{33})} \right]^{\frac{1}{2}} . \quad \text{.....(14)}$$

Since n_z is really a function of ρ (as shown in Equation (7)), the above equation is transcendental. Using the values $n_x = 2.3222$, $n_y = 2.3205$, and $n_z = 2.2177$ for the principal refractive indices at $\lambda = 0.6328 \mu\text{m}$ and the positive low frequency values for the electro-optic coefficients listed in Table I, Equation (14) was solved to give a value of 39.9 degrees for the optimum value of ρ .

The half-wave voltage given by Equation (13) was also evaluated numerically as a function of ρ . In this way, the minimum value for the half-wave voltage could be found without excluding the dependence of the refractive indices on ρ . The calculations were done using the refractive indices for barium sodium niobate at $\lambda = 0.6328 \mu\text{m}$. Results were obtained for both the low and high frequency values of the electro-optical coefficients listed in Table I and for both a positive and a negative sign for the r_{51} coefficient. In each of these cases, a graph of the half-wave voltage as a function of ρ was plotted (see Figures 2 to 5). A summary of the minimum half-wave voltages obtained along with the corresponding values for ρ is given in Table III. Thus the minimum half-wave voltages determined in this work can be compared with the half-wave voltages for light modulators constructed from a crystal in the normally used orientation. As can be seen in Table III, depending on the frequency of modulation and the sign of the r_{51} coefficient, a 52 percent to 76 percent reduction in the half-wave voltage is possible by suitably optimizing the crystal orientation. It should also be noted, that the optimum value of ρ is in the region of 30 to 40 degrees. This is to be expected, since r_{51} is the largest electro-optic coefficient and the crystal orientation should be such as to make maximum use of this coefficient. An inspection of Equation (9) indicates that this would occur when $\sin \rho \cos^3 \rho$ has its largest value, i.e., at $\rho = 35.3$ degrees. The optimum values of ρ higher than 35 degrees result when r_{51} and r_{33} have the same sign (i.e., sign combination A) and their effect on the refractive index is additive. Whereas, those values of ρ less than 35 degrees result when r_{51} and r_{33} have opposite signs and produce opposing changes in the refractive index. This is clearly illustrated in Figures 3 and 5 where for values of ρ around 70 degrees, the effects of r_{51} and r_{33} on the refractive index are equal and opposite resulting in a theoretically infinitely large value for the half-wave voltage.

If light modulators are to be constructed from barium sodium niobate in the optimum orientation, it is important to resolve the ambiguity of the sign of the r_{51} coefficient. This can be done directly by cutting a crystal so that light propagates through it in the ac plane but at an angle of 38.6 degrees to the c axis. A measurement of the half-wave voltage for transverse modulation would then yield a value near 690 volts if r_{51} is positive or 883 volts if r_{51} is negative.

One final point is to note the small effect the dependence of the refractive indices on ρ has on the optimum value for ρ . The value of 39.9 degrees for the optimum value of ρ determined from Equation (14), an analytically derived expression, is only slightly different from the value of 37.9 degrees (see Figure 2) found numerically.

TABLE III
Amplitude Modulation

Electro-optic Coefficients	Sign Combination	ρ opt. Degrees	$V_{1/2}$ Volts	$V_{1/2}$ $\rho = 90^\circ$	% Decrease
Low frequency	A	37.9	689.7	1752.4	60.6
	B	31.1	835.5	1752.4	52.3
High frequency	A	36.1	691.6	2925.0	76.4
	B	32.2	796.7	2925.0	72.8

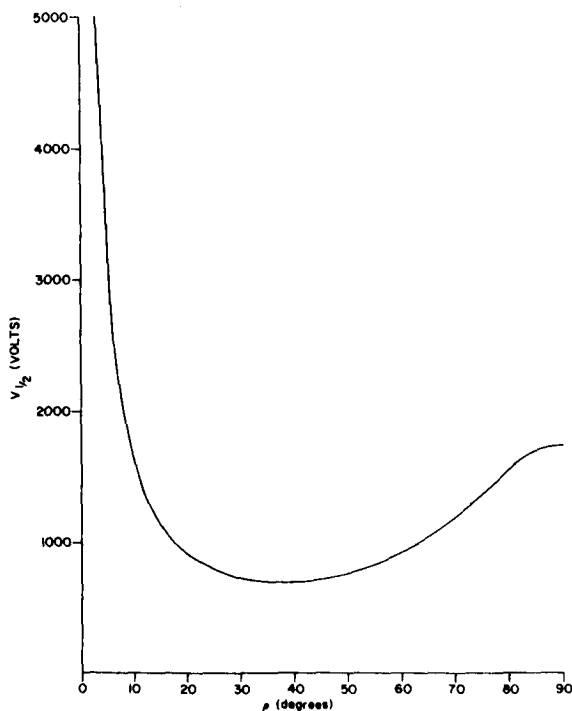


Fig. 2. A graph of the half-wave voltage for low frequency amplitude modulation as a function of ρ . In this case r_{51} is positive.

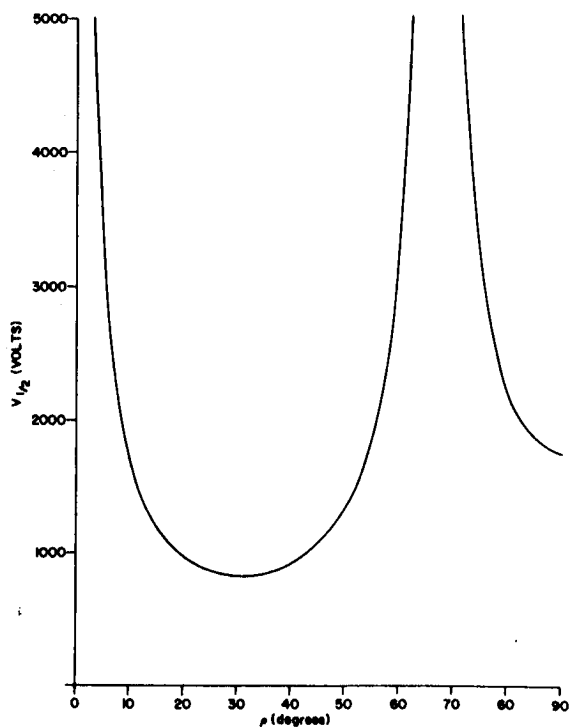


Fig. 3. A graph of the half-wave voltage for low frequency amplitude modulation as a function of ρ . In this case r_{51} is negative.

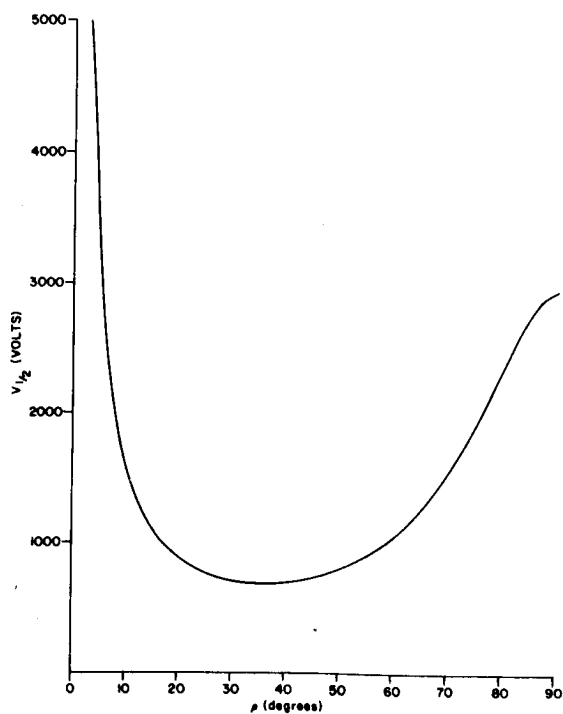


Fig. 4. A graph of the half-wave voltage for high frequency amplitude modulation as a function of ρ . In this case r_{51} is positive.

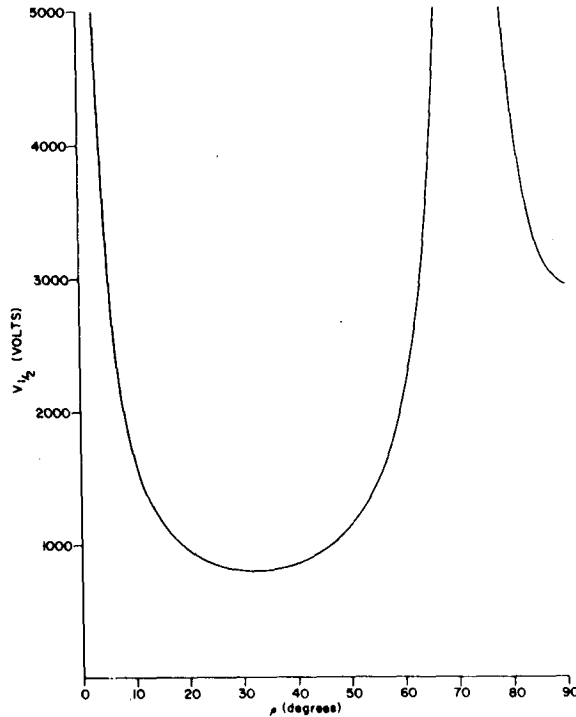


Fig. 5. A graph of the half-wave voltage for high frequency amplitude modulation as a function of ρ . In this case r_{51} is negative.

4.2 Phase Modulation

In phase modulation, light is passed through a crystal polarized parallel to either the X or Y axes. The phase of the light emerging from the crystal can be varied through the electro-optic effect by an applied field. Due to the relative magnitudes of the various electro-optic coefficients of barium sodium niobate, the applied field produces the greatest retardation for light polarized along the Y axis. Hence only phase modulation with light polarized along the Y axis is of interest in this work.

From Equation (12) it can be shown that the voltage required to produce a phase change of π radians for light polarized along the Y axis is

$$V_{1/2} (Y) = \frac{\lambda}{n_2^3 Q_2} \quad \text{.....(15)}$$

Using the same technique as outlined for amplitude modulation, it can be shown that the minimum value of the half-wave voltage given by Equation (15) occurs for $\xi = 0$, $\theta = 0$, and ρ given by

$$\sin \rho = \left[\frac{2r_{51} + r_{13}}{3(2r_{51} + r_{13} - r_{33})} \right]^{1/2}.$$

As before, the dependence of the refractive indices on crystal orientation was ignored in minimizing Equation (15).

Numerical calculations of the half-wave voltage given by Equation (15) were also calculated as a function of ρ . The results obtained are plotted in Figures 6 to 9, and summarized in Table IV. For comparison purposes, the half-wave voltages for phase modulation with light propagating along the a axis are also listed in Table IV. The results indicate that a large decrease in the half-voltage for phase modulation can be obtained by suitably orienting the crystal.

TABLE IV
Phase Modulation

Electro-optic Coefficients	Sign Combination	ρ opt. Degrees	$V_{1/2}$ Volts	$V_{1/2}$ $\rho = 90^\circ$	% Decrease
Low frequency	A	39.7	620.8	1208.7	48.6
	B	29.7	937.2	1208.7	22.5
High frequency	A	37.1	649.3	2000.6	67.5
	B	31.3	853.2	2000.6	57.4

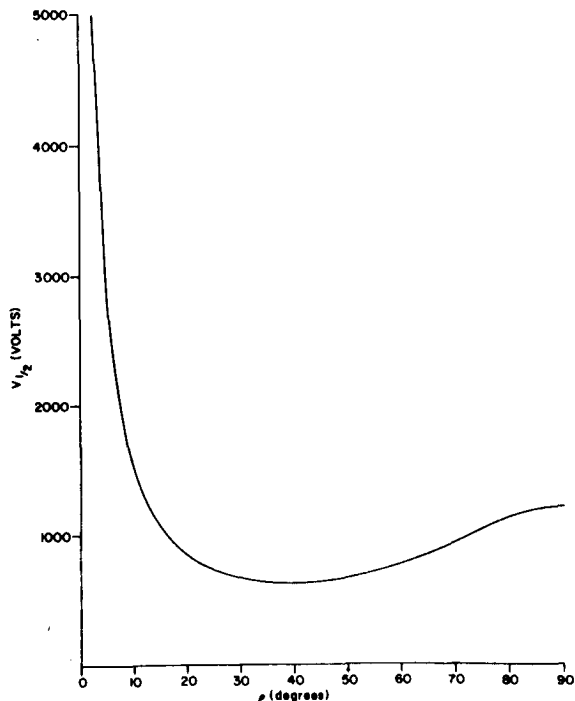


Fig. 6. A graph of the half-wave voltage for low frequency phase modulation as a function of ρ . In this case r_{51} is positive.

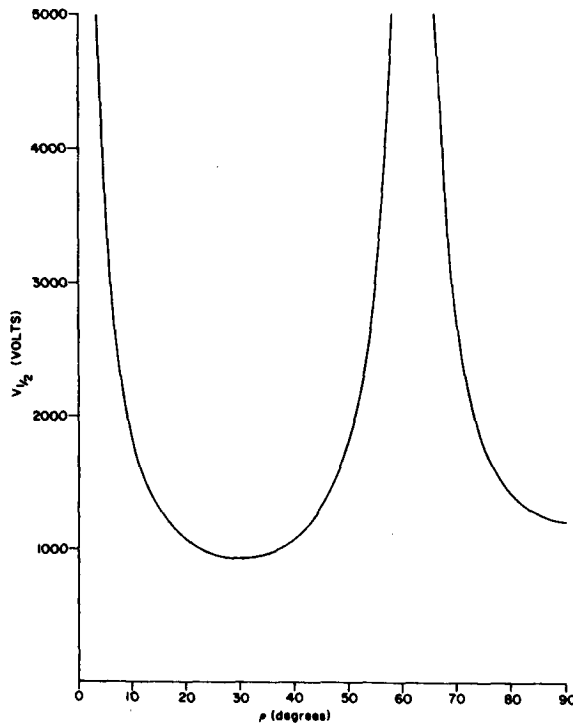


Fig. 7. A graph of the half-wave voltage for low frequency phase modulation as a function of ρ . In this case r_{51} is negative.

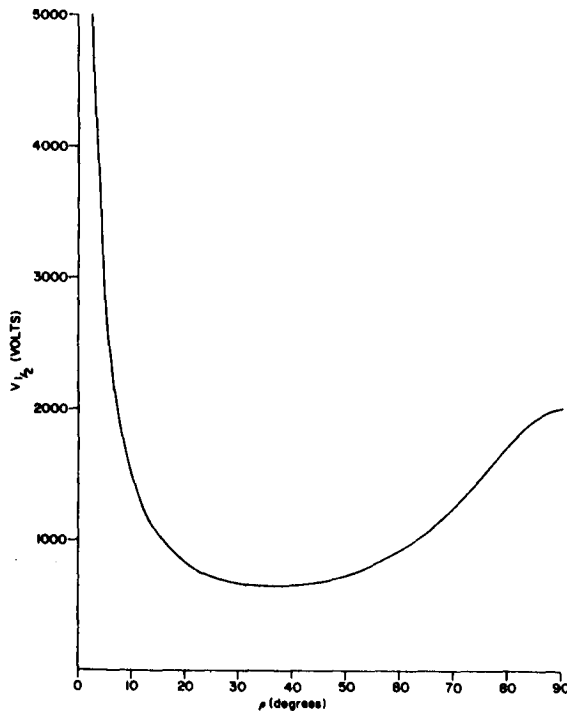


Fig. 8. A graph of the half-wave voltage for high frequency phase modulation as a function of ρ . In this case r_{51} is positive.

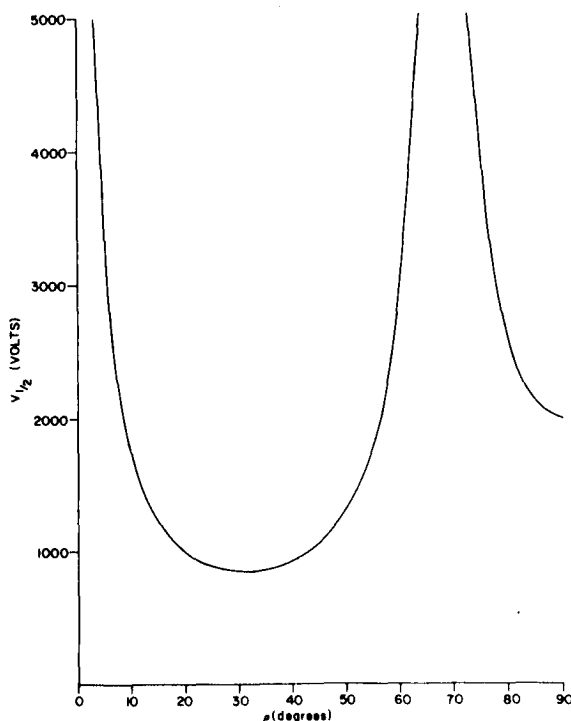


Fig. 9. A graph of the half-wave voltage for high frequency phase modulation as a function of ρ . In this case r_{51} is negative.

5. CONCLUSIONS

A theoretical analysis of the half-wave voltage required for transverse modulation using barium sodium niobate electro-optic crystals has been presented. The analysis was complicated somewhat by the fact that not all the signs of the electro-optic coefficients are known. However, it was shown that, by suitably orienting the crystal, transverse modulators can be constructed with half-wave voltages considerably lower than those made with the usual crystal orientation. In the case of low frequency amplitude modulation it should be possible to construct a modulator with a half-wave voltage of 690 or 836 volts, depending on whether the sign of the r_{51} coefficient is positive or negative.

6. REFERENCES

1. Geusic, J.E., H.J. Levinstein, J.J. Rubin, S. Singh and L.G. Van Uitert. *The Nonlinear Optical Properties of $Ba_2 Na Nb_5 O_{15}$* . Appl. Phys. Letters, 11, pp. 269-270, 1 November, 1967. Errata, Appl. Phys. Letters, 12, pp. 224, 15 March, 1968.

2. Singh, S., D.A. Draegert and J.E. Geusic. *Optical and Ferroelectric Properties of Barium Sodium Niobate*. Phys. Rev. B, Vol. 2, pp. 2700-2724, 1 October, 1970.
3. Kalymnios, D., J.M. Ley and K. Rashidi. *Optimisation of 45° y' Cut Modulators*. Electronics Letters. Vol. 6, pp. 771-773, November, 1970.
4. Francois, G.E., F.M. Librecht and J.J. Engelen. *Optimum Cut in XDP Crystals for Transverse Light Modulation*. Electronics Letters. Vol. 6, pp. 778-779, November, 1970.
5. Kalymnios, D. *Optimisation of Electro-optic Crystal Modulators*. Electronics Letters. Vol. 6, pp. 804-805, December, 1970.
6. Librecht, F.M., and G.E. Francois. *Optimum Cut for a LiNbO₃ Transverse Light Modulator*. IEEE Journal of Quantum Electronics. Vol. QE-7, pp. 374-376, July, 1971.
7. Hookabe, K., and Y. Matsuo. *Optimum Cut of KDP-Type Crystals for Light Modulation*. Trans. of the Institute of Electronics and Communications Engineers of Japan. Vol. 54-B, pp. 237-277, May, 1971.

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