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THE DIELECTRIC PROPERTIES OF ICE, SNOW AND WATER AT MICROWAVE FREQUENCIES AND THE MEASUREMENT OF THE THICKNESSES OF ICE AND SNOW LAYERS WITH RADAR

by

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ABSTRACT

Investigations are made of (a) the dielectric properties, at microwave frequencies, of fresh water ice, snow and water and (b) the possibility of using radar to measure the thicknesses of naturally occurring fresh water ice and snow layers. As concerns (b) above, it is shown that it should be possible to design an FM radar to measure the depths of several meters of ice and snow provided (a) the snow is not wet and (b) there is no water on top of the ice.

1. INTRODUCTION

Section 2 contains derivations for some of the equations which will be required. The equations pertain to (a) scattering from plane dielectric boundaries and (b) propagation in lossy dielectrics.

In Section 3 the dielectric properties (especially those at microwave frequencies) of ice, snow and water are examined.

The remainder of the document is concerned with an investigation of the possibility of using radar to measure the thicknesses of ice and snow layers. The situations investigated *(see Figure 1)* are for (a) the measurement of ice thickness where there is snow on top of, and water beneath, the ice and, (b) the measurement of snow thickness where the layer of snow is on top of either ice or soil.

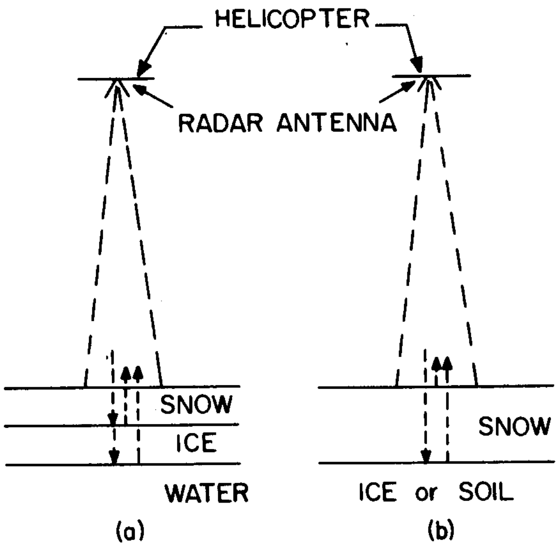


Fig. 1. Geometry for the measurement of the thicknesses of ice and snow layers.

The difficulty associated with measuring the thicknesses of ice and snow layers with radar is dependent on the relative amplitudes of the signals scattered from the boundaries. Calculations *(see Section 4)* of the above amplitudes show that it should be possible to design a suitable radar. The following observations can be made:

(a) The resolution requirements are such that a pulse radar would not be practical. However a linear FM system appears to be feasible.

(b) The radar antenna must be fairly close to the ice and snow surfaces. Therefore an airborne system would probably be mounted in a helicopter.

(c) Water has a very high rate of attenuation at microwave frequencies. The thickness of snow or ice which can be measured will, at least, be reduced therefore if the snow is wet or if there is water on top of the ice.

The effect of water in snow was indicated in a measurement program carried out in March 1972 by G.O. Venier and F.R. Cross[17] using a laboratory model X-band FM radar. Thickness measurements were made of both wet and dry snow. It was found that the amplitude of the signal scattered from the bottom surface of the snow was reduced when the snow was wet. Ice thickness measurements, where the ice was over water, were also made. The amplitude of the reflection from the ice-water boundary were found to be less than that indicated by calculations in this document. This may have been caused by water on top of the ice. Alternatively there may have been water pockets in the ice, particularly near the ice-water boundary.

2. SCATTERING OF SIGNALS NORMALLY INCIDENT ON A PLANE DIELECTRIC BOUNDARY AND PROPAGATION IN A LOSSY DIELECTRIC

The material in this section serves to (a) define, and (b) provide equations for computing certain scattering and propagation parameters which are required in subsequent sections. The equations are, of course, well known but are included here for ease of reference.

2.1 THE REFLECTION AND TRANSMISSION COEFFICIENTS FOR SIGNALS NORMALLY INCIDENT ON A PLANE DIELECTRIC BOUNDARY

Consider the case shown in Figure 2 where a plane wave signal in region 1 is normally incident on a plane dielectric boundary between regions 1 and 2.

Let

E_x^i, E_x^t and E_x^r = respectively, the incident, transmitted and reflected electric field intensities, and

H_y^i, H_y^t and H_y^r = respectively, the incident transmitted and reflected magnetic field intensities.

The subscripts for the E and H fields denote the direction of polarization.

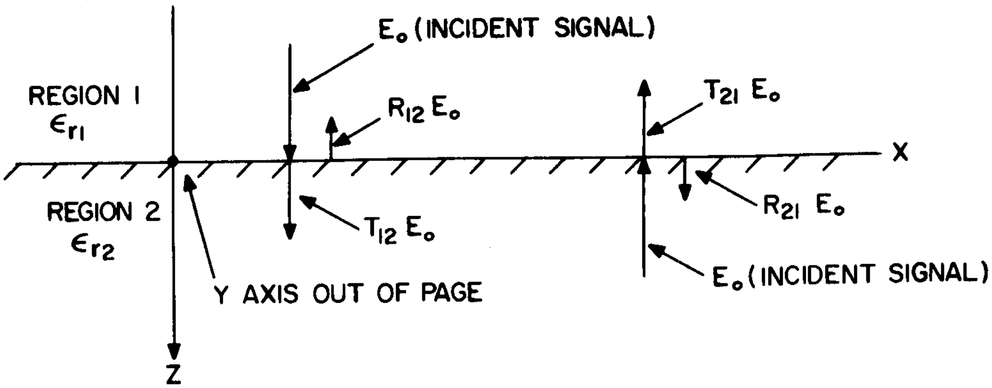


Fig. 2. Plane waves normally incident on a plane dielectric boundary.

If the incident electric field intensity is expressed as

$$E_x^i = E_o e^{-jk_1 z} , \qquad \text{.....(1)}$$

then it can be shown that

$$H_y^i = (E_o/\eta_1) e^{-jk_1 z} , \qquad \text{.....(2)}$$

$$E_x^t = T_{12}\ E_o\ e^{-jk_2 z} , \qquad \text{.....(3)}$$

$$H_y^t = (T_{12}\ E_o/\eta_2) e^{-jk_2 z} , \qquad \text{.....(4)}$$

$$E_x^r = R_{12}\ E_o\ e^{+jk_1 z} , \qquad \text{.....(5)}$$

$$H_y^r = -(R_{12}\ E_o/\eta_1) e^{+jk_1 z} , \qquad \text{.....(6)}$$

where

E_o is a constant

k_1 and k_2 are the propagation constants for regions 1 and 2, respectively. They are given by

$$k_1 = k_1' - jk_1'' = (2\pi f/c)\sqrt{\varepsilon_{r1}} \ , \qquad .....(7)$$

$$k_2 = k_2' - jk_2'' = (2\pi f/c)\sqrt{\varepsilon_{r2}} \ , \qquad .....(8)$$

where

f = frequency of signal in Hz

c = velocity of propagation in a vacuum

ε_{r1} and ε_{r2} are respectively the relative permittivities of regions 1 and 2. They are in general complex numbers, i.e.,

$$\varepsilon_{r1} = \varepsilon_{r1}' - j\varepsilon_{r1}'' \ , \qquad .....(9)$$

$$\varepsilon_{r2} = \varepsilon_{r2}' - j\varepsilon_{r2}'' \ . \qquad .....(10)$$

η_1 and η_2 are the wave impedances of regions 1 and 2, respectively, where

$$\eta_1 = \frac{120\pi}{\sqrt{\varepsilon_{r1}}} \ , \qquad .....(11)$$

and

$$\eta_2 = \frac{120\pi}{\sqrt{\varepsilon_{r2}}} \ . \qquad .....(12)$$

T_{12} and R_{12} are the transmission and reflection coefficients when the incident signal is on the region 1 side of the dielectric boundary.

Electromagnetic theory tells us that the components of E and H which are tangential to a dielectric boundary must be continuous across the boundary. Therefore when the boundary is at z = 0 *(see Figure 2)*

$$E_x^i(z=0) + E_x^r(z=0) = E_x^t(z=0) \ , \qquad .....(13)$$

and

$$H_y^i(z=0) + H_y^r(z=0) = H_y^t(z=0) \ . \qquad .....(14)$$

Equations (1) to (6), and (11) to (14) can be used to show that

$$T_{12} = 1 + R_{12} \ , \qquad .....(15)$$

$$R_{12} = \frac{1 - \eta_1/\eta_2}{1 + \eta_1/\eta_2} = \frac{1 - \sqrt{\varepsilon_{r2}/\varepsilon_{r1}}}{1 + \sqrt{\varepsilon_{r2}/\varepsilon_{r1}}} \ , \qquad .....(16)$$

and

$$T_{12} = \frac{2}{1 + \eta_1/\eta_2} = \frac{2}{1 + \sqrt{\varepsilon_{r2}/\varepsilon_{r1}}} \ . \qquad .....(17)$$

When the incident signal is on the region 2 side of the dielectric boundary *(see Figure 2)* let the reflection and transmission coefficients be R_{21} and T_{21}. R_{21} and T_{21} are related to R_{12} as follows:

$$R_{21} = -R_{12} \ , \qquad .....(18)$$

and

$$T_{21} = 1 - R_{12} \ . \qquad .....(19)$$

2.2 PROPAGATION IN A LOSSY DIELECTRIC

The equation for the field intensity of a signal which is propagating in the +z direction in region 'n' contains the function

$$e^{-jk_n z} = e^{-k''_n z} \, e^{-jk'_n z} \ , \qquad .....(20)$$

where from Equations (7) or (8)

$$k_n = k'_n - jk''_n = (2\pi f/c)\sqrt{\varepsilon'_{rn} - j\varepsilon''_{rn}} \ . \qquad .....(21)$$

It can be seen from equation (20) that

k'_n = the rate with which the phase of the signal decreases in the +z direction, and

k''_n = the rate of attenuation in the +z direction in nepers/unit length.

The following parameters are functions of k'_n and k''_n

$\lambda_n = 2\pi/k'_n$ is the wavelength in region n .....(22)

$v_n = 2\pi f/k'_n$ is the velocity of propagation in region n .....(23)

$A_n = (20 \log_{10} e)k''_n = 8.686\ k''_n$ is the rate of attenuation in region n in dB/unit length. .....(24)

When the right hand side of equation (21) is separated into real and imaginary parts it is found that k_n' and k_n'' are

$$k_n' = (2\pi f/c)\sqrt{\varepsilon_{rn}'}\,[1 + (\varepsilon_{rn}''/\varepsilon_{rn}')^2]^{1/4} \times \cos[(1/2)\tan^{-1}(\varepsilon_{rn}''/\varepsilon_{rn}')] , \qquad .....(25)$$

$$k_n'' = (2\pi f/c)\sqrt{\varepsilon_{rn}'}\,[1 + (\varepsilon_{rn}''/\varepsilon_{rn}')^2]^{1/4} \times \sin[(1/2)\tan^{-1}(\varepsilon_{rn}''/\varepsilon_{rn}')] . \qquad .....(26)$$

Provided $\varepsilon_{rn}''/\varepsilon_{rn}' \ll 1$, the following equations can be used to compute k_n' and k_n''.

$$k_n' \simeq (2\pi f/c)\sqrt{\varepsilon_{rn}'} , \qquad .....(27)$$

$$k_n'' \simeq (\pi f/c)\sqrt{\varepsilon_{rn}'}\,[\varepsilon_{rn}''/\varepsilon_{rn}']. \qquad .....(28)$$

Equations (27) and (28) are in error by less than 1 per cent if $\varepsilon_{rn}''/\varepsilon_{rn}' < 0.28$. Note that $\varepsilon_{rn}''/\varepsilon_{rn}'$ is often referred to as the loss tangent, i.e.,

$$\tan\delta_n = \varepsilon_{rn}''/\varepsilon_{rn}' . \qquad .....(29)$$

3. THE ELECTRICAL PROPERTIES OF ICE, WATER AND SNOW

In this section the dielectric properties of ice, water and snow are investigated.

Over limited frequency ranges it has been found that the relative permittivities of ice and water approximately obey the Debye equation. This equation is investigated below in Section 3.1. For a more complete treatment see Chapter X of Reference 14.

3.1 THE DEBYE EQUATION

Water is an example of a compound whose molecules have a permanent electric-charge dipole moment. That is, the molecules have a dipole moment even in the absence of an applied electric field. The variation with frequency of the relative permittivity of these substances can sometimes be described by equation (30), the Debye equation.

$$\varepsilon_r = \varepsilon_{r\infty} + \frac{\varepsilon_{ro} - \varepsilon_{r\infty}}{1 + jf/f_m} \quad . \qquad \text{.....(30)}$$

The real and imaginary parts of ε_r, as given by the above equation, can be shown to be

$$\varepsilon_r' = \varepsilon_{r\infty} + \frac{\varepsilon_{ro} - \varepsilon_{r\infty}}{1 + (f/f_m)^2} \; , \qquad \text{.....(31)}$$

and

$$\varepsilon_r'' = \frac{(\varepsilon_{ro} - \varepsilon_{r\infty})\,(f/f_m)}{1 + (f/f_m)^2} \; . \qquad \text{.....(32)}$$

See Figures 3, 8 and 9 for typical curves of ε_r' and ε_r'' versus frequency for substances whose dielectric properties are described by the Debye equation. The parameters ε_{ro}, $\varepsilon_{r\infty}$ and f_m are constants with respect to frequency and

ε_{ro} = the limiting low frequency value of ε_r',

$\varepsilon_{r\infty}$ = the limiting high frequency value of ε_r',

f_m = the relaxation frequency.

Note that some authors use relaxation time τ instead of f_m. τ and f_m are related by

$$f_m = 1/(2\pi\tau).$$

The peak value of ε_r'' occurs when $f = f_m$. The following relationships can be derived from equations (31) and (32).

$$\varepsilon_r'(f \ll f_m) \simeq \varepsilon_{ro} \; , \qquad \text{.....(33)}$$

$$\varepsilon_r'(f = f_m) = (\varepsilon_{ro} + \varepsilon_{r\infty})/2 \; , \qquad \text{.....(34)}$$

$$\varepsilon_r'(f \gg f_m) \simeq \varepsilon_{r\infty} \; , \qquad \text{.....(35)}$$

$$\varepsilon_r''(f \ll f_m) \simeq (\varepsilon_{ro} - \varepsilon_{r\infty})(f/f_m) \qquad \text{.....(36)}$$

$$\varepsilon_r''(f = f_m) = (\varepsilon_{ro} - \varepsilon_{r\infty})/2, \qquad \text{.....(37)}$$

$$\varepsilon_r''(f \gg f_m) \simeq (\varepsilon_{ro} - \varepsilon_{r\infty})/(f/f_m) \; . \qquad \text{.....(38)}$$

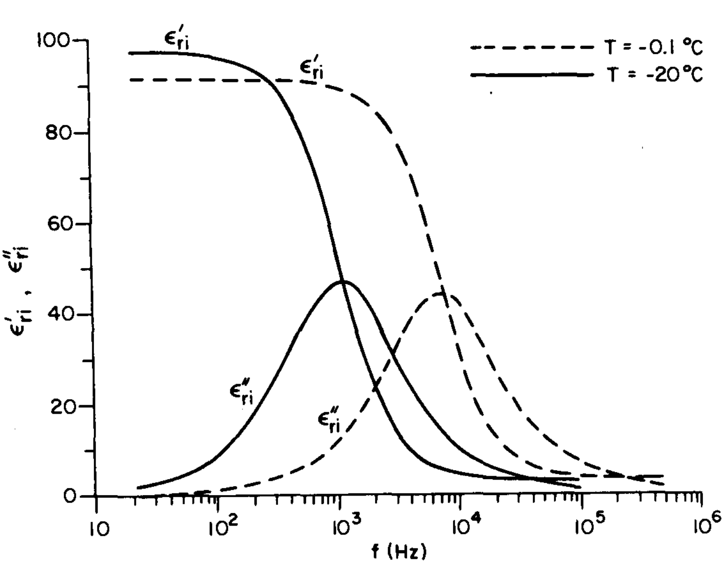


Fig. 3. Relative permittivity or ice at low frequencies.

The rate of attenuation A in a dielectric whose permittivity is described by equation (30) is typified by that for ice *(see Figure 4)*. It can be seen that A approaches a limiting value as f is increased. Equations (24), (28), (35) and (38) can be used to show this value of A to be

$$A(f \gg f_m) \simeq \frac{8.686\pi\, f_m\, (\varepsilon_{ro} - \varepsilon_{r\infty})}{c\sqrt{\varepsilon_{r\infty}}} \qquad \ldots..(39)$$

Equation (30) is appropriate for dipolar compounds which have a single relaxation frequency. For substances which have a distribution of relaxation frequencies, Cole and Cole[15], proposed the following empirical equation

$$\varepsilon_r = \varepsilon_{r\infty} + \frac{\varepsilon_{ro} - \varepsilon_{r\infty}}{1 + (jf/f_m)^{1-\alpha}} \qquad \ldots..(40)$$

where α is called the distribution parameter and $0 < \alpha < 1$. When $\alpha = 0$, equations (30) and (40) are identical.

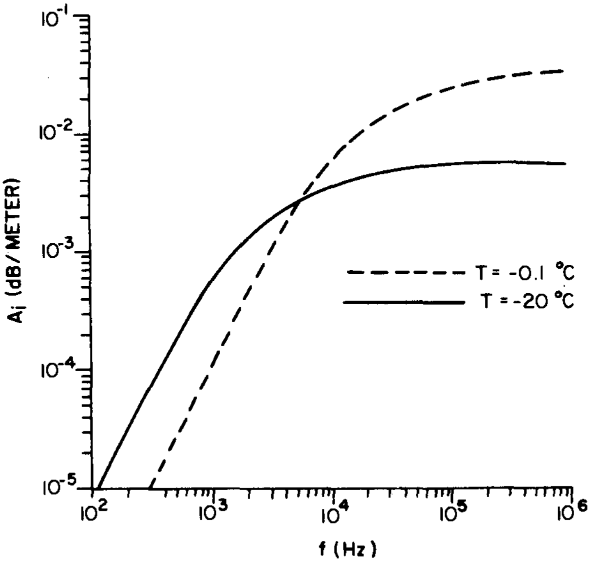


Fig. 4. Low frequency attenuation in ice.

3.2 THE ELECTRICAL PROPERTIES OF ICE

The relaxation frequency f_m for ice is below 8 kHz. Auty and Cole[1], measured ε_r' and ε_r'' at frequencies comparable to f_m and were thus able to determine the Debye equation (equation (30)) parameters ε_{ro}, $\varepsilon_{r\infty}$ and f_m. These are reproduced in Table 1 below. It can be seen that $\varepsilon_{r\infty}$ remains essentially constant when T is changed. However ε_{ro} and especially f_m are functions of temperature.

Equation (30) and the information in Table 1 were used to compute the plots of ε_{ri}' and ε_{ri}'' versus frequency given in Figure 3. Figure 4 shows the corresponding rates of attenuation. In Figure 7 the high frequency limiting rates of attenuation in ice, as predicted by the Debye equation, with the Auty and Cole parameters, are compared with measured values. It can be seen that the Debye equation rates of attenuation are not valid at microwave frequencies and therefore at these frequencies, measured values must be relied upon.

TABLE 1

Auty and Cole Debye Equation Parameters

T(°C)	ε_{ro}	$\varepsilon_{r\infty}$	f_m(Hz)
-0.1	91.5	3.10	7230
-10.8	95.0	3.08	2650
-20.9	97.4	3.10	970
-32.0	100	3.00	279
-44.7	104	3.10	63.2
-56.8	114	3.10	13.2
-65.8	133	3.10	3.54

Table 2 lists investigators who have measured the high frequency dielectric properties of ice.

TABLE 2

Investigators of the Dielectric Properties of Ice (150 MHz to 24 GHz)

Investigators	f GHz	Temp. Range °C	ε'_{ri}
Westphal, (glacier ice),[2]	.15, .30, .50	-1 to -60	2.90 to 2.95
Westphal, (glacier ice),[3]	2.5 (approx.)	-1 to -60	-
Cumming,[4]	9.375	0 to -18	3.15
Lamb,[5]	10	-1 to -49	3.17
Lamb and Turney,[6]	24	0 to -185	3.18

Westphal's measurements were made on glacier ice. In general, glacier ice contains small pockets of air and is therefore less dense than pure ice. The density of the glacier ice used by Westphal[2] was 0.868 gms/cm^3. This compares with a density of 0.916 gms/cm^3 for pure ice. The reduced density of Westphal's glacier ice accounts, at least in part, for the fact that his values of ε'_{ri} were smaller than the others listed in Table 2. The relative permittivity of the ice component of Westphal's glacier ice can be computed using a dielectric mixture equation. When this is done, Westphal's measurements indicate that ε'_{ri} is between 3.08 and 3.13. The spread in values for ε'_{ri} as measured by the investigators shown in Table 2 (i.e., 3.08 to 3.18) is not large and could be the result of experimental error. In the calculations of Section 4, Cumming's value of 3.15 for ε'_{ri} will be used.

Changes in temperature and frequency have little effect on ε'_{ri} (providing $f >> f_m$). However (refer to Table 3 and Figure 5) the same cannot be said for ε''_{ri}.

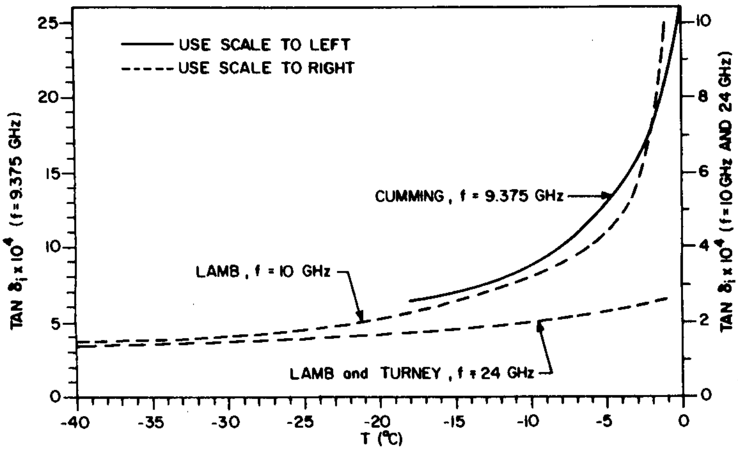


Fig. 5. Loss tangent in ice as measured by Lamb and Turney, Lamb and Cumming.

Rates of attenuation in ice, at selected temperatures, and as computed using the above measured dielectric properties, are plotted in Figure 7.

It should be noted that (a) at most of the frequencies above 150 MHz we have only one investigator's measurements of tan δ_i and some of these were made on glacier ice and (b) Cumming's measurements at 9.375 GHz are in significant disagreement with those made by Lamb at 10 GHz.

Table 3 lists Westphal's loss tangent measurements in glacier ice at 150 MHz, 300 MHz and 500 MHz. In addition to the above, Westphal privately communicated to Evans[3] measurements of tan δ_i for glacier ice at about 2.5 GHz. Although these values are not included in Table 3 (because Evan's Figure 7 could not be read accurately), they were used to compute the rates of attenuation in ice (A_i) at 2.5 GHz included in Figure 7.

For temperatures down to -40°C the measurements of tan δ_i obtained by Lamb, Lamb and Turney, and Cumming are reproduced in Figure 5. The corresponding rates of attenuation in ice were computed and are shown in Figure 6 as functions of temperature.

TABLE 3

Loss Tangent of Glacier Ice as Measured Westphal[2] (ρ_i = .868 gms/cm³)

T(°C)	150 MHz	tan δ_i x 10^4 300 MHz	500 MHz
-1	20.0	10.8	5.2
-5	14.4	8.0	4.0
-10	11.0	5.6	2.8
-20	6.8	3.4	1.9
-40	2.6	1.3	0.8
-60	0.5	0.3	0.3

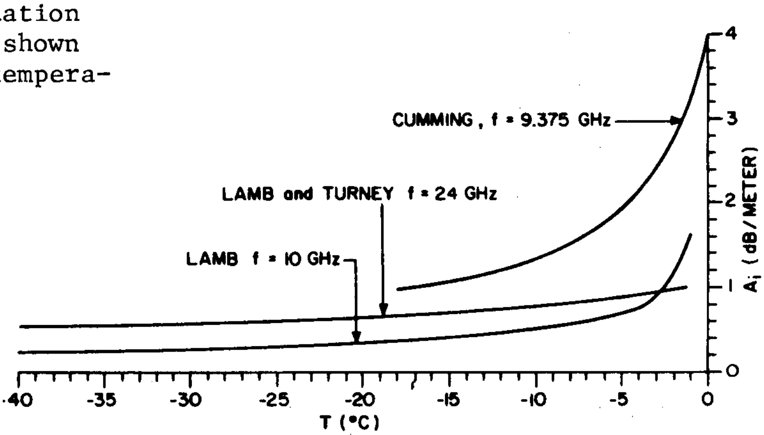


Fig. 6. Rate of attenuation in ice as computed using loss tangents shown in Figure 5.

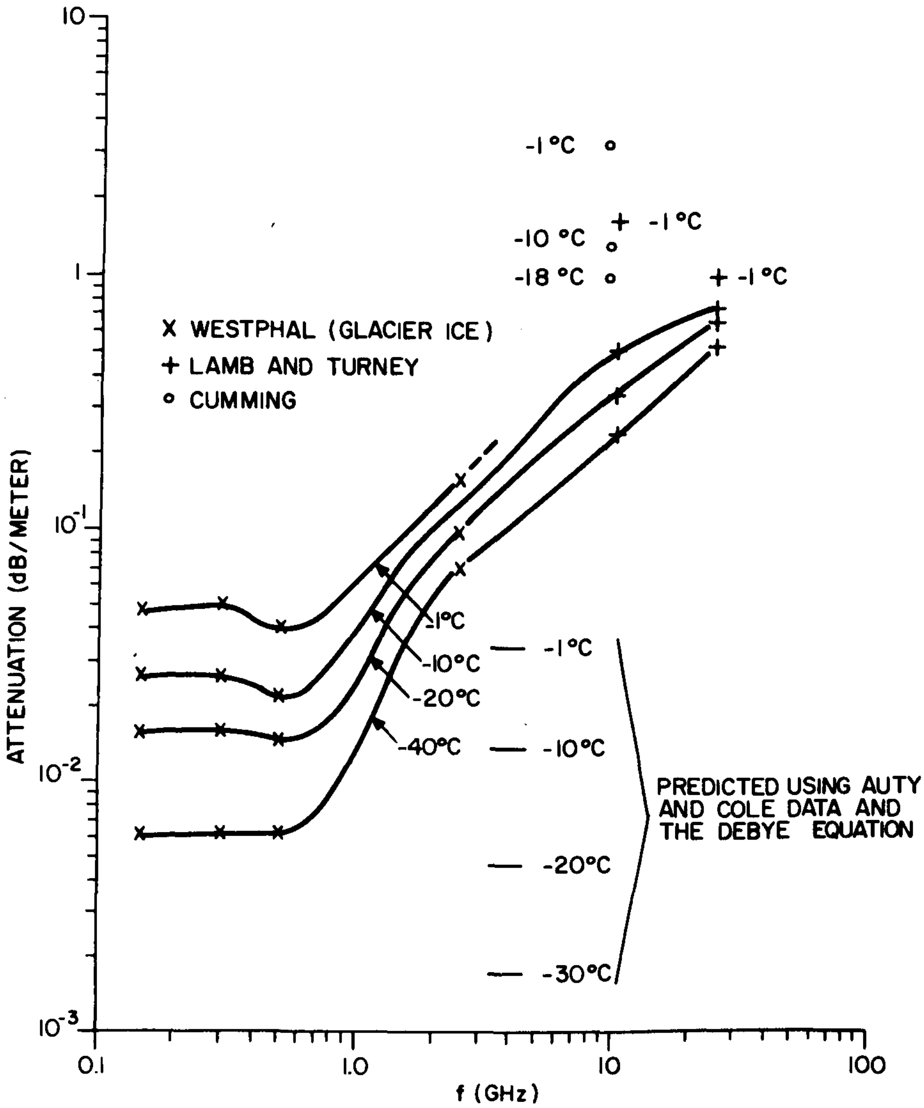


Fig. 7. Attenuation in ice.

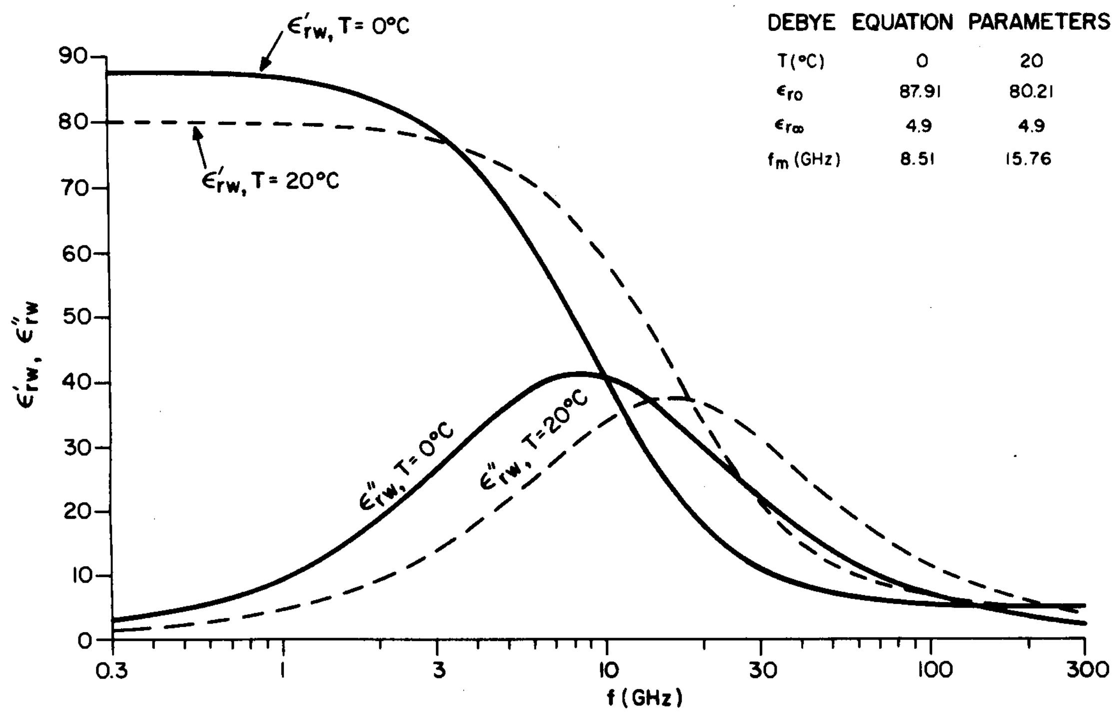


Fig. 8. Relative permittivity of water at T = 0°C and T = 20°C using the Debye equation.

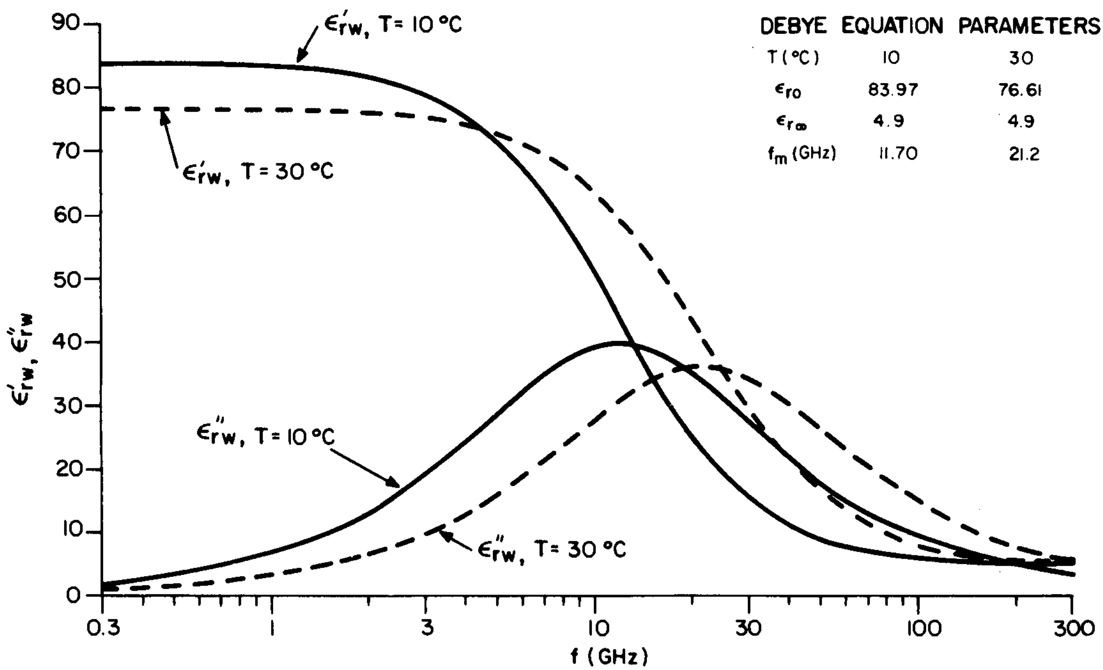


Fig. 9. Relative permittivity of water at T = 10°C and T = 30°C using the Debye equation.

3.3 THE MICROWAVE PROPERTIES OF WATER

Many measurements have been made of the dielectric properties of water at microwave frequencies. Some of the investigators found that the measured values of permittivity, as a function of frequency, were satisfactorily accounted for by equation (30), the Debye equation. Others concluded that equation (40) was more appropriate. In the latter case, the parameter α was found to be small ($\alpha \simeq 0.02$). The author of this document has found that equation (30), (i.e., $\alpha = 0$), where ε_{ro}, $\varepsilon_{r\infty}$ and f_m are as given in (a), (b) and (c) below, gave values for ε'_{rw} and ε''_{rw} which were in good agreement with those measured. The above comparisons between calculated and measured values were made over the frequency and temperature ranges,

$$577 \text{ MHz} \leq f \leq 24.2 \text{ GHz}$$

$$0°C \leq T \leq 30°C \ .$$

Refer to Table 6 for the particular case when T = 0°C.

(a) Where $0°C \leq T \leq 40°C$ Vidulich, Evans and Kay[7] found that the following empirical equation accurately predicted their measured values of ε_{ro} for water.

$$\varepsilon_{ro} = 10^{(1.94404 - 1.991 \times 10^{-3}\, T)} \qquad .....(41)$$

where T is in degrees Centigrade.

Table 4 lists some values for ε_{ro} given by equation (41)

TABLE 4

ε_{ro} for Water

T(°C)	ε_{ro}
0	87.91
10	83.97
20	80.21
30	76.61
40	73.18

(b) The magnitude of $\varepsilon_{r\infty}$, is not known accurately, but it appears to be between 4.5 and 5.5. For the purpose of computing the microwave properties of water in this document, Saxton's[8] value of 4.9 will be used.

(c) Saxton[8] found f_m to be as shown in Table 5.

TABLE 5

Relaxation Frequency for Water

T(°C)	f_m (GHz)
0	8.51
10	11.70
20	15.76
30	21.2
40	27.0
50	33.9

Equation (30) with the above values for ε_{ro}, $\varepsilon_{r\infty}$ and f_m were used to compute the relative permittivity of water curves plotted in Figure 8 and 9. The corresponding rates of attenuation appear in Figure 10. Note that water is very lossy at microwave frequencies. For example at f = 10 GHz and T = 0°C water attenuates at the rate of about 5000 dB/m.

TABLE 6

Measured and Calculated Values of the Permittivity of Water at T = 0°C

f GHz	Ref.	Measured ε_r'	Measured ε_r''	Calculated ε_r'	Calculated ε_r''
0.577	(9)	88.1	5.85	87.5	5.60
1.744	(9)	85.3	16.5	84.6	16.3
3.00	(10)	79.7	24.7	78.7	26.0
3.25	(11)	79.9	26.5	77.3	27.7
9.13	(11)	46.0	41.1	43.5	41.4
9.35	(10)	44.8	41.6	42.5	41.3
19.0	(12)	19.0	30.4	18.8	31.0
23.6	(10)	16.2	28.3	14.4	26.5
23.7	(11)	14.5	27.5	14.4	26.4
24.2	(13)	14.9	26.3	14.0	26.0

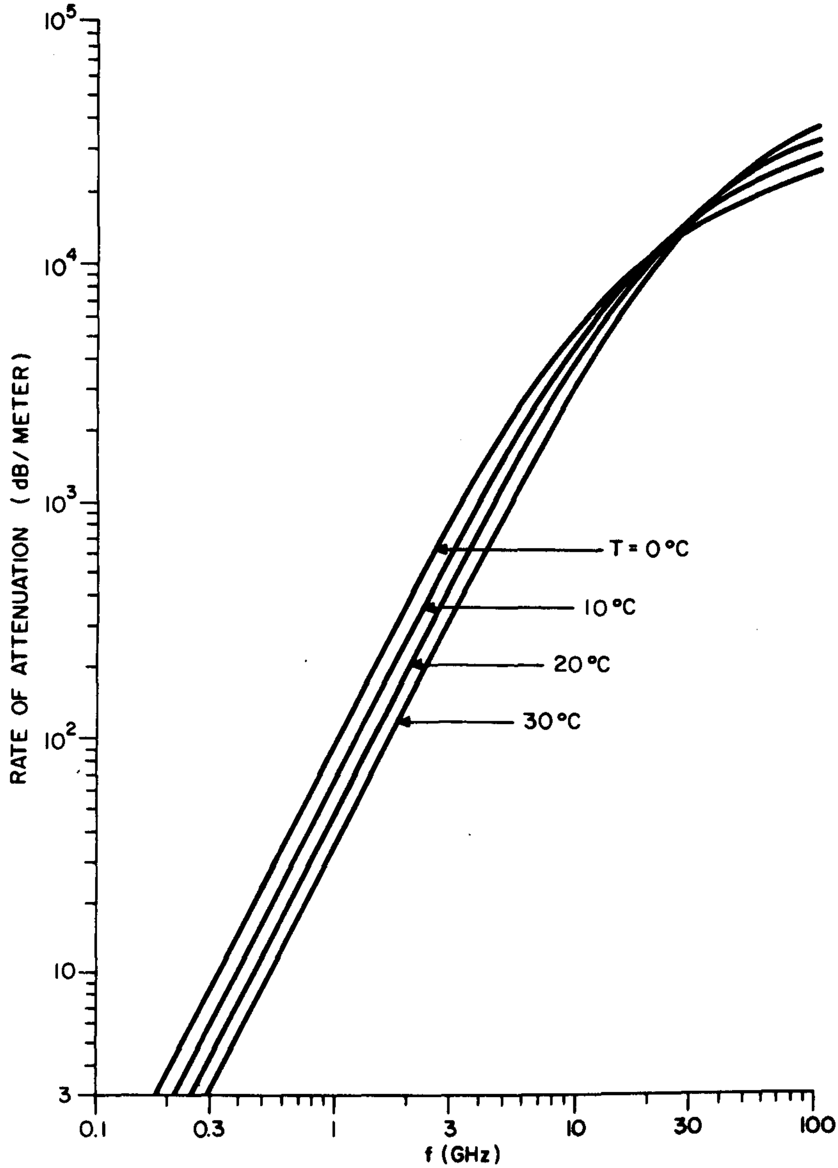


Fig. 10. Rate of attenuation in water.

3.4 THE MICROWAVE PROPERTIES OF SNOW

Snow is a mixture of two dielectrics, ice and air. Weiner (see Evans[3]) derived the following equation which expresses the relative permittivity of a mixture of two dielectrics in terms of the permittivities of its constituents.

$$\frac{\varepsilon_{rm} - 1}{\varepsilon_{rm} + u} = p\,\frac{\varepsilon_{r1} - 1}{\varepsilon_{r1} + u} + (1-p)\,\frac{\varepsilon_{r2} - 1}{\varepsilon_{r2} + u} \quad .....(42)$$

where

ε_{rm} = relative permittivity of mixture,

ε_{r_1} = relative permittivity of medium 1,

ε_{r_2} = relative permittivity of medium 2,

p = the proportion of a volume of the mixture which is occupied by medium 1,

u - is called the Formzahl, $0 \leq u \leq \infty$.

Let medium 1 be distributed through medium 2 in the form of particles. The value of the Formzahl is dependent on (a) the shape of the medium 1 particles and (b) the orientation of the medium 1 particles with respect to the polarization of the signal. For the general behaviour of u refer to the sketches above Figures 13 and 14.

In snow, medium 2 is air and $\varepsilon_{r_2} = 1-j0$. The second term on the right hand side of equation (42) is therefore zero and, as a result the equation becomes

$$\frac{\varepsilon_{rs} - 1}{\varepsilon_{rs} + u} = p \frac{\varepsilon_{ri} - 1}{\varepsilon_{ri} + u} , \qquad \text{.....(43)}$$

where

$\varepsilon_{rs} = \varepsilon'_{rs} - j\varepsilon''_{rs}$ is the relative permittivity of snow,

$\varepsilon_{ri} = \varepsilon'_{ri} - j\varepsilon''_{ri}$ is the relative permittivity of ice,

p = the proportion of a volume of the snow which is occupied by ice.

If the density of the snow is known, then p is given by

$$p = \rho_s/0.916 , \qquad \text{.....(44)}$$

where ρ_s = density of the snow in gms/cm^3.

When equation (43) is solved for the real and imaginary components of ε_{rs}, it is found that

$$\varepsilon'_{rs} = \left[\left(\varepsilon'_{ri}(1 + up) + u(1 - p)\right)\left(\varepsilon'_{ri}(1 - p) + u + p\right) + (\varepsilon''_{ri})^2 (1 + up)(1 - p)\right] \Big/ \left[\left(\varepsilon'_{ri}(1 - p) + u + p\right)^2 + \left(\varepsilon''_{ri}(1 - p)\right)^2\right] , \qquad \text{.....(45)}$$

and

$$\varepsilon''_{rs} = \frac{\varepsilon''_{ri}\, p(1 + u)^2}{\left(\varepsilon'_{ri}(1 - p) + u + p\right)^2 + \left(\varepsilon''_{ri}(1 - p)\right)^2} . \qquad \text{.....(46)}$$

In Section 3.2, it was shown that at microwave frequencies, $\varepsilon''_{ri} << \varepsilon'_{ri}$. Some parts of equation (45) and (46) can therefore be omitted. The functions which result are

$$\varepsilon'_{rs} = \frac{\varepsilon'_{ri}(1 + up) + u(1 - p)}{\varepsilon'_{ri}(1 - p) + u + p} , \qquad .....(47)$$

$$\varepsilon''_{rs} = \frac{\varepsilon''_{ri} p \ (1 + u)^2}{[\varepsilon'_{ri}(1 - p) + u + p]^2} . \qquad .....(48)$$

Therefore, provided that the above equations are valid, the dielectric properties of snow can be computed if the Formzahl and the dielectric properties of ice are known.

Cumming[4] has measured tan δ and ε'_r for both ice and snow at f = 9.375 GHz. His results are shown in Figures 11 and 12. In addition to Cumming's values for ε'_{rs}, Figure 12 includes curves which were computed using equation (47). Note that for the purpose of these calculations the value 3.15 was used for ε'_{ri}. It can be seen that the curve obtained when u = 3.5 agrees quite well with the experimental points. Therefore in the remainder of this document, where ε'_{rs} is used, it will be that given by equation (47) with u = 3.5 and ε'_{ri} = 3.15. Note that Cumming measured ε'_{rs} at T = −18°C. His values should however be valid at other temperatures because ε'_{ri} is independent of temperature.

Weiner's equation does not predict ε''_{rs} as accurately as ε'_{rs}. This is shown in Figure 13, where (a) the curves were computed using equation (48) and (b) the experimental points were found using the following equation

$$\varepsilon''_{rs}/\varepsilon'_{ri} = (\varepsilon'_{rs} \tan \delta_s)/(3.15 \tan \delta_i),$$

where

ε'_{rs} is that given by equation (47) with u = 3.5 and ε_{ri} = 3.15.

tan S_s and tan S_i are as measured by Cumming, see Figures 11 and 5, respectively.

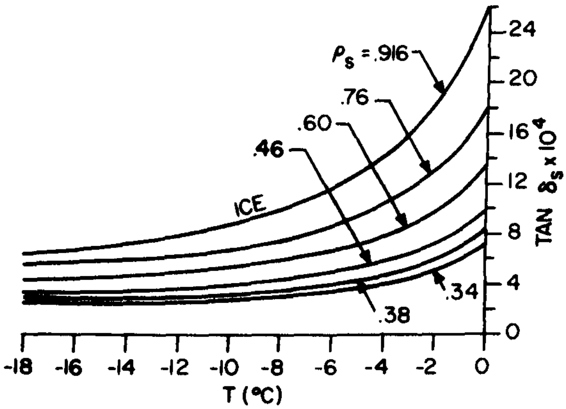


Fig. 11. Loss tangent of snow as measured by Cumming at f = 9.375 GHz ρ_s *= snow density (gms/cm³).*

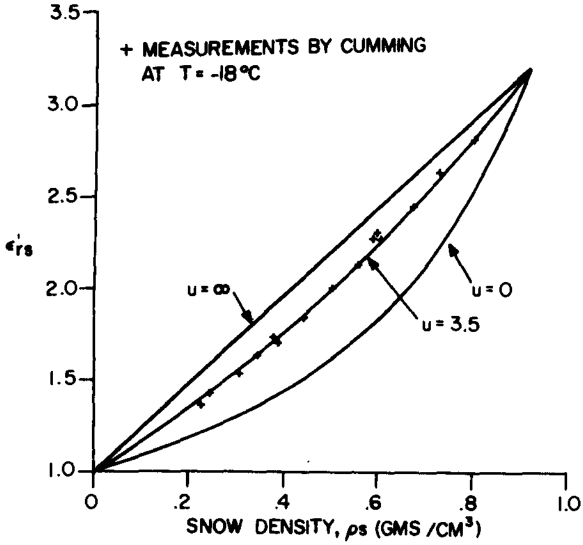


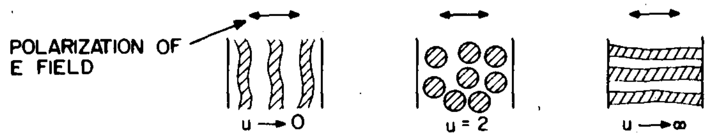
Fig. 12. ε'_{rs} *as measured by Cumming and predicted by Weiner's dielectric mixture equation.*

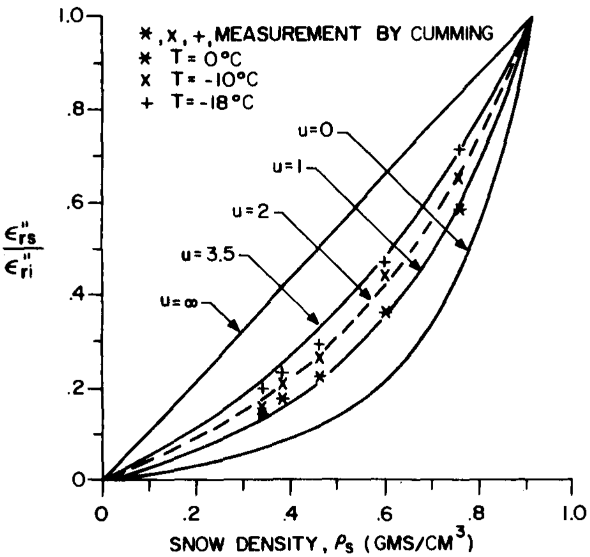
The experimental points do not lie close to any 'constant u' curve but are spread between the curves u = 1 and u = 3.5. The curve where u = 2, falls approximately half-way between those where u = 1 and u = 3.5.

The ratio of the rate of attenuation in snow to that in ice can be shown, (using equations (24) and (28)), to be given by

$$\frac{A_s}{A_i} = \frac{\varepsilon''_{rs}/\varepsilon''_{ri}}{\sqrt{\varepsilon'_{rs}/\varepsilon'_{ri}}} \qquad \ldots\ldots(49)$$

The A_s/A_i curves in Figure 14 were computed using equation (49) where, (a) $\varepsilon'_{rs}/\varepsilon'_{ri}$ was calculated using equation (47) with u and ε'_{ri} being 3.5 and 3.15, respectively, and (b) $\varepsilon''_{rs}/\varepsilon''_{ri}$ was computed using equation (48) with u = 1.0, 2.0 and 3.5. As was shown above, Cumming's data indicates that $\varepsilon''_{rs}/\varepsilon''_{ri}$ falls between the curves u = 1 and u = 3.5 and that the median curve was that where u ≃ 2. A_s/A_i will have the same behaviour as $\varepsilon''_{rs}/\varepsilon''_{ri}$. Therefore the values A_s used in Section 4 were computed using, (a) A_s/A_i as given by the u = 2 curve in Figure 14, and (b) values of A_i taken from Figure 6.





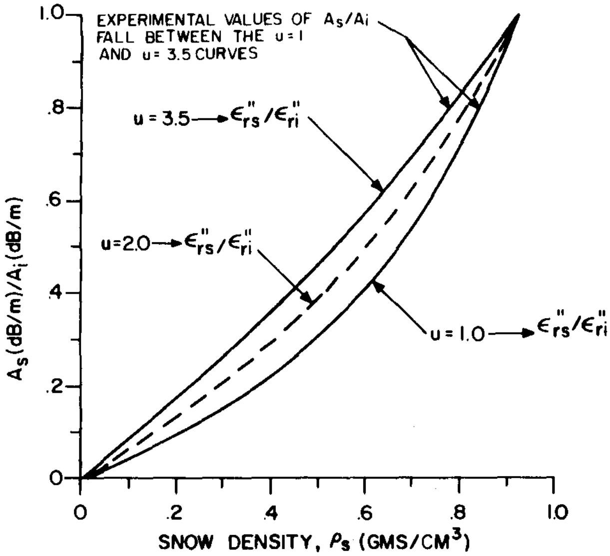


Fig. 13. $\varepsilon''_{rs}/\varepsilon''_{ri}$ *as measured by Cumming and predicted by Weiner's dielectric mixture equation.*

Fig. 14. A comparison between the rates of attenuation in snow and ice. u = 3.5 for computing $\varepsilon'_{rs}/\varepsilon'_{ri}$.

4. SCATTERING FROM ICE OVER WATER, SNOW OVER ICE AND SNOW OVER SOIL

In this section, comparisons are made between the magnitudes of the signals scattered from the top and bottom surfaces of ice and snow layers. This parameter is important because a radar system, which is used to measure ice and snow thickness, must be able to resolve the reflections from these surfaces. The difficulty associated with doing this depends on the relative magnitudes of the two signals (i.e., the greater the difference in magnitude the greater the difficulty).

Consider the situations shown in Figures 15 and 16, where (a) the incident signal (labelled E_o) is normally incident on the top boundary, and (b) the boundaries are plane and parallel to each other. Let

E_o = the rms magnitude of the electric field associated with the incident signal

$|R_s|E_o$, $|R_i|E_o$, $|R_w|E_o$, $|R_g|E_o$ = The magnitudes of the signals reflected from the top surfaces of the snow, ice, water and soil (ground), respectively.

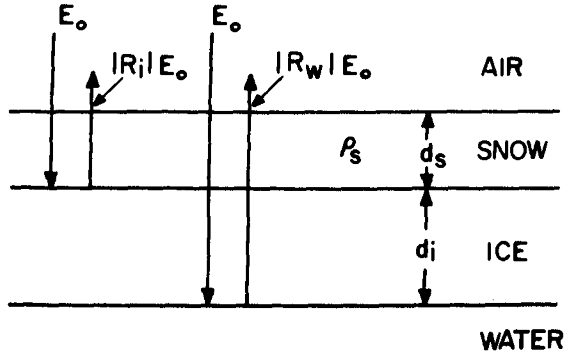


Fig. 15. Scattering from the top and bottom surfaces of ice where there is snow over ice over water (ρ_s *= snow density).*

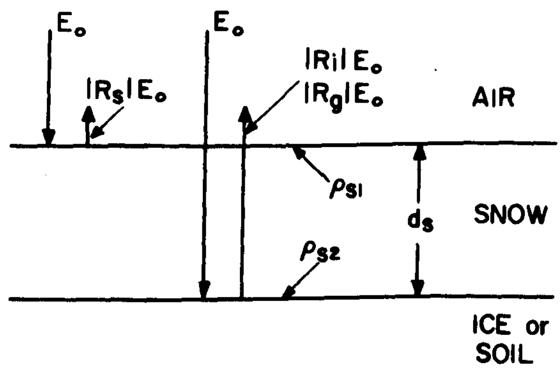


Fig. 16. Scattering from the top and bottom surfaces of ice where there is snow over ice or snow over soil (ρ_{s1}, ρ_{s2} *= snow density at respectively the top and bottom surfaces of snow).*

These magnitudes are those that the signals assume in the air, after leaving the snow. $|R_s|$, $|R_i|$, $|R_w|$ and $|R_g|$ are functions of, (a) the rates of attenuation in, and depths of, the snow and ice layers, and (b) the boundary reflection and transmission coefficients, i.e.,

$$|R_s| = |R_{as}| \qquad \text{.....(50)}$$

$$|R_i| = |T_{as}|\ |R_{si}|\ |T_{sa}| 10^{-(A_s\ 2d_s)/20} \qquad \text{.....(51)}$$

$$|R_w| = |T_{as}|\ |T_{si}|\ |R_{iw}|\ |T_{is}|\ |T_{sa}| 10^{-(A_s 2d_s + A_i 2d_i)/20} \qquad \text{.....(52)}$$

$$|R_g| = |T_{as}|\ |R_{sg}|\ |T_{sa}| 10^{-(A_s\ 2d_s)/20} \qquad \text{.....(53)}$$

where

d_s and d_i = the depths of snow and ice (meters), respectively,

A_s and A_i = the rates of attenuation in snow and ice (dB/meter), respectively, and can be found in Section 3.

$|R_{as}|$, $|T_{as}|$, $|R_{si}|$, etc., are the magnitudes of the boundary reflection and transmission coefficients. They are functions of the dielectric properties of the materials on either side of the boundary (see Section 2).

The magnitudes of the above boundary reflection and transmission coefficients were computed and are shown in Figures 17 to 20. At microwave frequencies and except for $|R_{iw}|$, the coefficients are independent of frequency. Below f = 10 GHz $|R_{iw}|$ changes only slightly with frequency. For the purpose of computing $|R_{sg}|$, (the magnitude of the reflection coefficient at the snow-ground boundary), the relative permittivity of the ground ε_{rg} was assumed to be 2.5. This is near the values given for dry sand and foam, as listed in Reference 16 for a frequency of 3 GHz. The other dielectric properties required, (i.e., those for snow, ice and water), can be found in Section 3.

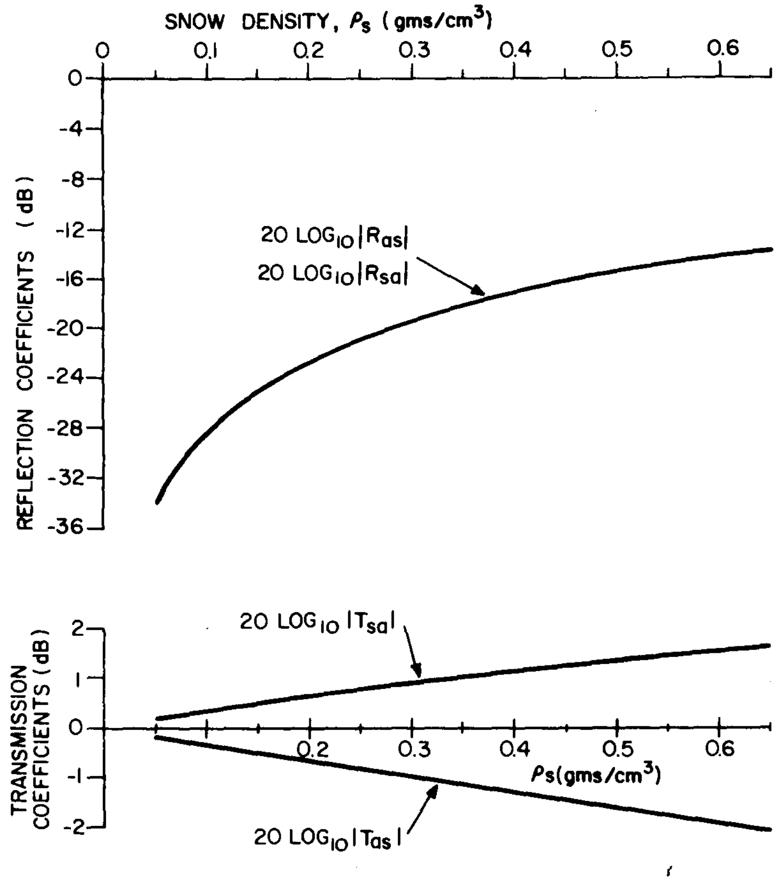


Fig. 17. Normal incidence reflection and transmission coefficients at an air-snow boundary.

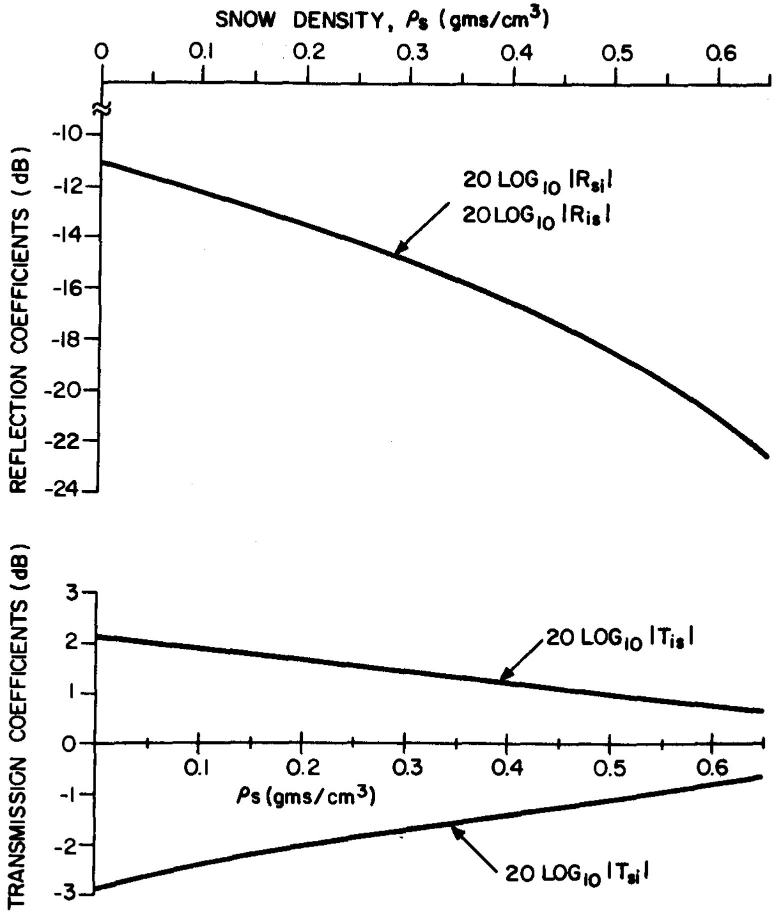


Fig. 18. Normal incidence reflection and transmission coefficients at a snow-ice boundary. Note that when $\rho_s = 0$ the reflection and transmission coefficients are for an air-ice boundary.

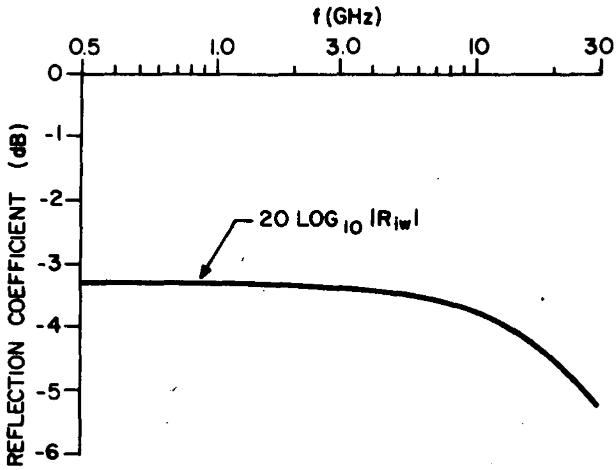


Fig. 19. Normal incidence reflection coefficient at a ice-water boundary, T = 0°C.

Note, it can be shown that

$$S_s/S_o = E_o^2|R_s|^2/|E_o|^2 = |R_s|^2 \quad .....(54)$$

$$S_i/S_o = |R_i|^2 \quad .....(55)$$

$$S_w/S_o = |R_w|^2 \quad .....(56)$$

$$S_g/S_o = |R_g|^2 \quad .....(57)$$

$$S_w/S_i = |R_w|^2/|R_i|^2 \quad .....(58)$$

$$S_i/S_s = |R_i|^2/|R_s|^2 \quad .....(59)$$

$$S_g/S_s = |R_g|^2/|R_s|^2 \quad .....(60)$$

where

S_o = power density (watts/unit area) of incident signal, and

S_s, S_i, S_w and S_g = the power densities of the signals scattered from the top surfaces of the snow, ice water and ground, respectively.

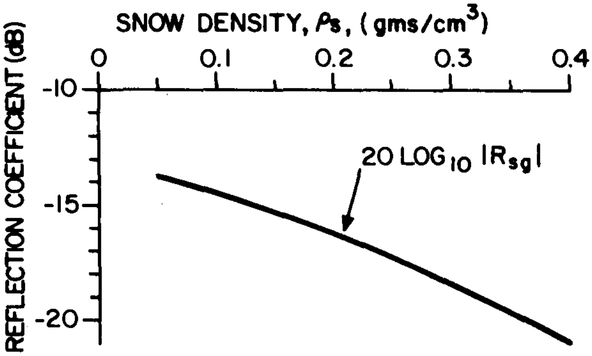


Fig. 20. Normal incidence reflection coefficient at a snow-ground boundary, where the relative permittivity of the ground (ε_{rg}) is 2.5.

In this section, values for $|R_s|$, $|R_i|$, $|R_w|/|R_i|$ are expressed in dB where, for example

$$|R_i|(\text{dB}) = 20 \log_{10}|R_i| \qquad \text{.....(61)}$$

Hence from equation (55)

$$|R_i|(\text{dB}) = 10 \log_{10} S_i/S_o.$$

These values can therefore be thought of as the power density ratios given in equations (54) to (60), expressed in dBs.

4.1 SCATTERING FROM ICE OVER WATER

Consider first the case where there is no snow on top of the ice. For the reason given at the beginning of this section it would be useful if upper and lower bounds could be obtained for $|R_w|/|R_i|$. Note that, the greater the attenuation in the ice layer, the smaller the valve of $|R_w|/|R_i|$. At a given frequency, the upper bound therefore occurs when $A_i = 0$. This corresponds to $d_i \to 0$ and/or $T \ll 0°C$. For a given depth, d_i, and frequency, the lower bound occurs when A_i has its maximum value. This happens when $T = 0°C$. In this document $A_i(T = -1°C)$ was used to compute the lower bound as Lamb did not give a value for $A_i(T = 0°C)$. Therefore from the information in Table 7 it can be seen that

$$-6.24 \text{ dB} \leq |R_w|/|R_i| \leq 6.64 \qquad \text{.....(62)}$$

when

$d_s = 0$,

$d_i \leq 2$ meters,

$f = 9.375$ GHz,

A_i as given by Cumming,

0_r, (also from Table 7)

$$0.16 \text{ dB} \leq |R_w|/|R_i| \leq 6.60 \text{ dB} \qquad .....(63)$$

when

$d_s = 0$,

$d_i \leq 2$ meters,

$f = 10$ GHz,

A_i as given by Lamb.

TABLE 7

Ice over Water

$|R_w|$ as Compared to $|R_i|, T_w = 0°C$

d_i meters	T_i °C	f GHz	$\|R_w\|$ dB	$\|R_i\|$ dB	$\|R_w\|/\|R_i\|$ dB	Comments
-		9.375	-4.44	-11.08	+6.64	A_i = 0 dB/m
1	-1	9.375	-10.88	-11.08	0.20	A_i (Cumming)
2	-1	9.375	-17.32	-11.08	-6.24	A_i (Cumming)
-	-	10	- 4.48	-11.08	+6.60	A_i = 0 dB/m
1	-1	10	- 7.70	-11.08	+3.38	A_i (Lamb)
2	-1	10	-10.92	-11.08	+0.16	A_i (Lamb)

Snow on top of the ice has the effect of decreasing the magnitude of the signals scattered from the top as compared to that scattered from the bottom ice surface. The more dense the snow the greater the above effort. Note that dry snow densities range from about 0.1 gms/cm^3 for newly fallen snow to about 0.4 gms/cm^3 for hard-packed snow. By using the information in Table 8 and reasoning similar to that used to obtain (62) and (63), it can be shown that

$$-0.18 \text{ dB} \leq |R_w|/|R_i| \leq +12.70 \text{ dB}, \qquad .....(64)$$

when

$\rho_s = 0.4$ gms/cm^3,

$d_i \leq 2$ meters,

$f = 9.375$ GHz,

A_i as given by Cumming.

Or using the lower bound from (62) and the upper bound from (64)

$$-6.24 \text{ dB} \leq |R_w|/|R_i| \leq +12.70 \text{ dB}, \qquad .....(65)$$

when

$\rho_s \leq 0.4$ gms/cm^3,

$d_i \leq 2$ meters

$f = 9.375$ GHz,

A_i as given by Cumming.

Similarly when Lamb's values for $A_i(T = 1°C)$ are used, then

$$0.16 \text{ dB} \leq |R_w|/|R_i| < 12.66 \text{ dB}, \qquad \text{.....(66)}$$

when

$\rho_s \leq 0.4$ gms/cm^3,

$d_i \leq 2$ meters,

$f = 10$ GHz,

A_i as given by Lamb.

Note that 3 meters of ice (A_i - Cumming, $T = -1°C$) will decrease the lower bound of equation (65) to -12.68 dB and, therefore

$$-12.7 \text{ dB} < |R_w|/|R_i| < 12.7, \qquad \text{.....(67)}$$

when

$\rho_s \leq 0.4$ gms/cm^3,

$d_i \leq 3$ meters,

$f = 9.375$ GHz,

A_i as given by Cumming.

It can be shown, provided $f >> f_m = 7.2$ kHz when $T = 0°C$ (see Table 1), or $f > 1$ MHz that (67) is valid with small error (< 0.5 dB) in the bounds shown, when the condition $f = 9.375$ GHz is replaced by $f < 9.375$ GHz. Therefore, the greatest amplitude difference for signals reflected from the top and bottom surfaces of ice over water (conditional that $d_i < 3$ meters and $f < 9.375$ GHz), is about 13 dB.

TABLE 8

Snow Over Ice Over Water, $|R_w|$ as compared to $|R_i|$, d_s = 1 meter T_w = 0°C

d_i meters	ρ_s gms/cm³	T_i °C	f GHz	$\|R_w\|$ dB	$\|R_i\|$ dB	$\|R_w\|/\|R_i\|$ dB	Comments
-	.2	-	9.375	-4.17	-13.59	+9.42	$A_s = A_i = 0$
1	.2	-1	9.375	-11.49	-14.47	+2.98	A_s, A_i (Cumming)
2	.2	-1	9.375	-17.93	-14.47	-3.46	A_s, A_i (Cumming)
-	.4	-	9.375	-4.10	-16.80	+12.70	$A_s = A_i = 0$
1	.4	-1	9.375	-12.46	-18.72	+6.26	A_s, A_i (Cumming)
2	.4	-1	9.375	-18.90	-18.72	-0.18	A_s, A_i (Cumming)
-	.2	-	10	-4.21	-13.59	+9.38	$A_s = A_i = 0$
1	.2	-1	10	-7.87	-14.03	+6.16	A_s, A_i (Lamb)
2	.2	-1	10	-11.09	-14.03	+2.94	A_s, A_i (Lamb)
-	.4	-	10	-4.14	-16.80	±12.66	$A_s = A_i = 0$
1	.4	-1	10	-8.32	-17.76	+9.44	A_s, A_i (Lamb)
2	.4	-1	10	-11.54	-17.76	+6.22	A_s, A_i (Lamb)

4.2 SCATTERING FROM SNOW OVER ICE AND SNOW OVER SOIL

Consider the situation shown in Figure 16 where there is a layer of snow over either ice or soil. Let

ρ_{s1}, ρ_{s2} = the density of the snow at the top and bottom snow surfaces, respectively.

Table 9 lists values for $|R_i|/|R_s|$ and $|R_g|/|R_s|$ when A_s = 0 dB/meter. These values are therefore upper limits for the values of ρ_{s1} and ρ_{s2} shown. It can be seen that

$$0.43 \text{ dB} \leq |R_i|/|R_s| \leq 16.11 \text{ dB}, \qquad \text{.....(68)}$$

$$-3.82 \text{ dB} \leq |R_g|/|R_s| \leq 13.89 \text{ dB}, \qquad \text{.....(69)}$$

when

$0.1 \text{ gms/cm}^3 < \rho_{s1} < 0.4 \text{ gms/cm}^3$,

$\rho_{s1} < \rho_{s2} < 0.4 \text{ gms/cm}^3$

A_s = 0 dB/meter.

Attenuation in the snow will decrease $|R_i|/|R_s|$ and $|R_g|/|R_s|$. When Cumming's value for A_s at $T = -1°C$, $f = 9.375$ GHz and $\rho_s = 0.4$ gms/cm^3 is used, $|R_i|/|R_s|$ and $|R_g|/|R_s|$ in dB, become equal to the negative of the upper bounds shown in (68) and (69) when $d_s = 8.6$ and 5.2 meters, respectively. Therefore conditional that

$$0.1 \text{ gms/cm}^3 < \rho_{s1} < 0.4 \text{ gms/cm},$$

$$\rho_{s1} < \rho_{s2} < 0.4 \text{ gms/cm},$$

$$A_s \text{ as given by Cummings},$$

$$f < 9.375 \text{ GHz},$$

then

$$-16.1 \text{ dB} < |R_i|/|R_s| < 16.1 \text{ dB}, \qquad .....(70)$$

when

$$d_s < 8.6 \text{ meters},$$

and

$$-13.9 \text{ dB} < |R_g|/|R_s| < +13.9 \text{ dB}, \qquad .....(71)$$

when

$$d_s < 5.2 \text{ meters}.$$

TABLE 9

Snow over Ice and Snow over Soil (ground), $|R_i|$ and $|R_g|$ as compared to $|R_s|$, A_s = 0 dB/m

ρ_{s1} gms/cm^3	ρ_{s2} gms/cm^3	$\lvert R_s\rvert$ dB	$\lvert R_i\rvert$ dB	$\lvert R_g\rvert$ dB	$\lvert R_i\rvert/\lvert R_s\rvert$ dB	$\lvert R_g\rvert/\lvert R_s\rvert$ dB
.1	.1	-28.38	-12.27	-14.49	16.11	13.89
.1	.4	-28.38	-16.65	-20.89	11.73	7.49
.2	.2	-22.69	-13.60	-16.28	9.09	6.41
.2	.4	-22.69	-16.68	-20.93	6.01	1.76
.3	.3	-19.46	-15.08	-18.40	4.38	1.06
.3	.4	-19.46	-16.73	-20.98	2.73	-1.52
.4	.4	-17.23	-16.80	-21.05	0.43	-3.82

5. BANDWIDTH AND RESOLUTION OF A SNOW AND ICE THICKNESS MEASURING RADAR

For a pulse radar, the required video and RF bandwidths are given nominally by

$$\Delta f_{vid} \simeq \frac{c}{\sqrt{\varepsilon_r'}} \frac{1}{2\Delta r} \quad ,$$

$$\Delta f_{rf} \simeq \frac{c}{\sqrt{\varepsilon_r'}} \frac{1}{\Delta r} \quad ,$$

where

$\Delta r \simeq$ difference in range between two targets, below which the returns from the targets cannot be resolved by the radar,

ε_r' = real part of the relative permittivity of the medium between the above two targets. It is assumed that $\varepsilon_r''/\varepsilon_e' \ll 1$.

When Δr = 0.1 meter then, (a) when ε_r' = 3.15 (ice), Δf_{vid} and Δf_{rf} are 0.84 GHz and 1.69 GHz, respectively; (b) when ε_r' = 1.16 (snow, ρ_s = 0.1 gms/cm) Δf_{vid} are 1.4 GHz and 2.79 GHz, respectively.

It would be very difficult to design a video system with the bandwidths shown in (a) and (b), above, and therefore a pulse radar would not appear to be practical. Consider next the linearly swept FM radar. This type of radar system requires about the same RF bandwidth as the pulse radar. The signal, after detection, is however, relatively narrow-band.

Let the signal, transmitted by the FM radar, be swept through Δf_{rf} in time Δt, as shown in Figure 21. The detected signal is obtained by selecting the beat note which results when the signals scattered from the targets are mixed with a portion of the transmitted signal. The beat frequency corresponding to a target at a given range r is

$$f_d = T\, \Delta f_{rf}/\Delta t, \qquad \text{.....(72)}$$

where

T = time for the signal to propagate from the radar to the target and then return to the radar.

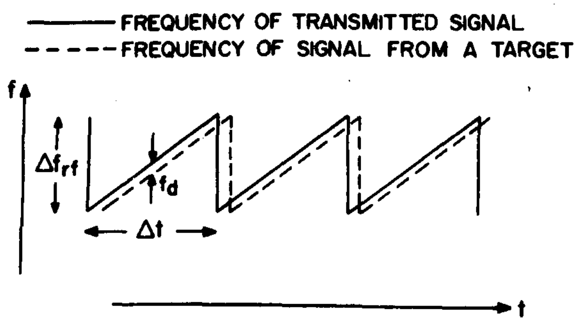


Fig. 21. Frequency-time relationships for a linearly-swept FM radar. The scatterer is a point target.

Since T = 2r/c, equation (72) can be written

$$f_d = (2r/c)(\Delta f_{rf}/\Delta t). \qquad \text{.....(73)}$$

It can be seen that f_d is a measure of range. The spectrum of the detected signal is therefore of interest. Let this spectrum be obtained during the time required for one sweep of the transmitted signal. In particular, consider the following signal

$$s(t) = \cos(2\pi f_d t), \quad t_1 < t < t_1 + \Delta t$$

$$= 0, \quad t < t_1 \text{ and } t > t_1 + \Delta t.$$

The energy density spectrum of this signal is a $(\sin(x)/x)^2$ curve centred at f_d *(see Figure 22)*. The zeroes in the spectrum occur at

$$f_d \pm m/\Delta t, \quad m = 1,2,3,\rightarrow . \qquad .....(74)$$

Therefore let 1/ t be the nominal frequency resolution. It can be seen, using equation (73), that

$$f_d + \frac{1}{\Delta t} = \frac{2\Delta f_{rf}}{c\Delta t}(r + \Delta r), \qquad .....(75)$$

or

$$\Delta r = c/(2\Delta f_{rf}), \qquad .....(76)$$

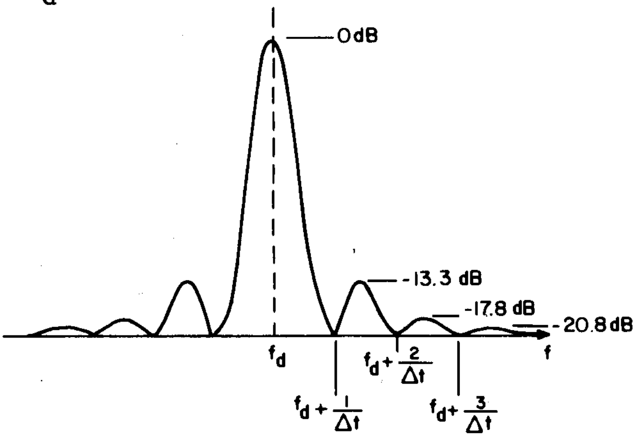


Fig. 22. Energy density spectrum of the detected signal in an FM radar. The scatterer is a point target and the spectrum is evaluated through time Δt.

where

Δr = the nominal range resolution, corresponding to the nominal frequency resolution $1/\Delta t$.

Note that if the relative permittivity (ε_r') of the dielectric between the targets to be resolved is not 1, then equation (76) becomes

$$\Delta r = c/(2\sqrt{\varepsilon_r'}\,\Delta f_{rf}). \qquad .....(77)$$

The required RF bandwidth can therefore be computed if Δr and ε_r' are known. For example if $\Delta r = 0.1$ meters then, (a) when $\varepsilon_r' = 3.15$ (ice), $\Delta f_{rf} = 0.84$ GHz and, (b) when $\varepsilon_r' = 1.16$ (snow, $\rho_s = 0.1$ gms/cm^3), $\Delta f_{rf} = 1.4$ GHz. These bandwidths indicate that the transmitted frequencies will have to be in the microwave band, (e.g., 9.0 GHz to 10.4 GHz).

There may be situations, during ice and snow depth measurements, when the range resolution as given by equation (77) will not be realized. For example, it was shown, *(see equation (70))*, that, (when $d_s < 8.6$ meters), $|R_i|/|R_s|$ can under certain circumstances reach +16 dB. It can be seen by referring to the second sidelobe of the $(\sin x/x)^2$ spectrum is 17.8 dB below that at f_d. Therefore nominally let the frequency resolution for signals which differ by 16 dB be

$$[\text{the frequency at the first zero in the } (\sin x/x)^2 \text{ spectrum above the second sidelobe}] - f_d$$

or

$$\text{frequency resolution} = [f_d + 3/\Delta t] - f_d = 3/\Delta t.$$

This frequency resolution is three times that used to derive equation (77), therefore Δr is three times that given by equation (77). It should be noted that the $(\sin(x)/x)^2$ sidelobes can be reduced somewhat by proper time or frequency weighting of the detected signal.

6. PROBLEMS ASSOCIATED WITH MEASURING THE THICKNESSES OF ICE AND SNOW LAYERS WITH RADAR

At f = 9.375 GHz Cumming[4] found that the presence of only 0.3 per cent by weight of water in the snow was sufficient to increase the loss tangent and hence A_s by a factor of 10. This is not unexpected because as is shown in Figure 10, A_w (f = 9.375 GHz, T = 0°C) ≃ 5000 dB/meter. The rate of attenuation in wet snow may therefore make it impossible to detect radar echoes from boundaries beneath the snow.

The range from the snow or ice to the radar antenna must be fairly small because;

(a) The greater the range the larger is the surface area illuminated. If the thickness being measured varies significantly within this area the snow and ice boundaries will appear smeared on the radar display.

(b) The range from the antenna to a plane surface is a function of the angle of incidence. This effect can also result in smearing on the display.

For example, let

r = minimum range,

r_1 = range through the 3 dB point on the antenna pattern,

θ_{BW} = 3 dB antenna beamwidth.

It can be shown that if $r_1 - r$ is to be less than 0.1 meters, when θ_{BW} is 5 degrees, then r must be less than 105 meters. It would appear that the radar, if airborne, should be mounted on a helicopter.

The signal amplitude ratios shown in Section 4 were computed assuming, (a) that the surfaces were parallel to each other, and (b) that the surfaces were smooth. If these conditions are not met then it would be possible to get amplitude ratios which are larger than those shown.

Two additional problems associated with the measurement of snow depth are:

(a) Snow is not, in general, homogenous and therefore confusing reflections could originate within the snow.

(b) The density of the snow and hence the velocity of propagation in it is, in general, unknown. The depth of the snow layer cannot therefore be precisely calculated. For example,

$$\varepsilon'_{rs} \; (\rho_s = 0.1 \text{ gms/cm}^3) = 1.16$$

$$\varepsilon'_{rs} \; (\rho_s = 0.4 \text{ gms/cm }) = 1.74,$$

and therefore

$$d_s(\rho_s = -.1 \text{ gms/cm}^3) = \frac{1}{\sqrt{\varepsilon'_{rs}}} \left(\frac{c\Delta t}{2}\right) = .928 \left(\frac{c\Delta t}{2}\right),$$

$$d_s(\rho_s = 0.4 \text{ gms/cm}^3) = .758 \left(\frac{c\Delta t}{2}\right),$$

where

Δt = time between arrival of signal from top and bottom snow surfaces.

7. SUMMARY

The dielectric properties of water are well known at microwave frequencies and they can be accurately computed using the Debye equation. In Section 3, it was shown that the rate of attenuation in water is very high in the microwave band. For example, at $T = 0°C$

$$A_w(f = 1 \text{ GHz}) \simeq 90 \text{ dB/m},$$

$$A_w(f = 10 \text{ GHz}) \simeq 5000 \text{ dB/m}.$$

Only a few measurements of the microwave properties of ice are available. The real part of the permittivity is 3.15 ± 0.05 and appears to be independent of frequency ($f > 1$ MHz) and temperature. The imaginary part of the permittivity, and hence the rate of attenuation, for ice are however both frequency and temperature dependent. Cumming's loss tangents for ice at 9.375 GHz give

$$A_i(T = -1°C) \simeq 3.2 \text{ dB/m},$$

$$A_i(T = -18°C) \simeq 1 \text{ dB/m}.$$

The corresponding rates of attenuation, computed using Lamb's data, for $f = 10$ GHz, are

$$A_i(T = -1°C) \approx 1.6 \text{ dB/m},$$

$$A_i(T = -18°C) \approx 0.37 \text{ dB/m}.$$

The dielectric properties of snow can be computed from those for ice and are of course dependent on the density of the snow. Both ε_r' and A (at a given temperature) are less for snow than they are for ice.

It was shown that for those conditions normally encountered it should be possible to use radar operating at X-band frequencies to measure the depths of ice and dry snow layers. Note that the measurements will probably have to be made when the snow is dry because the presence of water in the snow greatly increases its rate of attenuation.

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