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AN AIRBORNE LINEAR-SWEEP FM RADAR SYSTEM FOR MEASURING ICE THICKNESS

by

G.O. VENIER AND F.R. CROSS

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ABSTRACT

This report describes the design of a high resolution radar for use in measuring fresh-water ice thickness from a helicopter. The feasibility of ice-thickness measurement with an FM radar of this type has been proved in previous work. Of particular concern in this report is the data acquisition system required to record ice-thickness information at two metre intervals when the aircraft is moving at a speed of about 80 miles/h. The radar difference-frequency signal is sampled, converted to digital form and recorded on computer-compatible magnetic tape along with a time reference. The spectral analysis necessary to extract the depth information is accomplished on the ground by means of a Fast Fourier Transform routine in a digital computer. The equipment was designed for quick installation on a helicopter. Ice measurement results are not yet available.

1. INTRODUCTION

Over the past three years, experiments have been underway at the Communications Research Centre, Department of Communications, to determine the feasibility of measuring the thicknesses of ice and snow layers with a high range resolution X-band FM radar.

The first system developed was used to make fixed position tests near Shirley Bay, during the winter of 1971/72. These tests showed that it was feasible to measure the thicknesses of ice and snow layers with radar[1].

This radar system was modified and further tests were made, (in co-operation with the Department of the Environment, Floating Ice Section), during the winter of 1972/73. These tests were carried out on the St. Lawrence River between Wolfe Island and Kingston and on the Ottawa River at Shirley Bay. The radar was mounted on a sled and towed by a snowmobile in the first series of tests, and then was installed in an air cushion vehicle (ACV) and flown over the ice in the second series of tests. In addition to the field tests, the radar was evaluated in the CRC laboratory prior to the field tests and in the DOE laboratory during the summer of 1973[2]. The results from all field and laboratory tests have shown that snow depth and fresh-water ice thickness can be measured by an X-band linear-sweep FM radar.

As a result of the success achieved in the tests, the Department of the Environment requested the Communications Research Centre to develop an improved but lighter version of the radar for airborne use by the Floating Ice Section of DOE. The aim of the project was to produce a downward-looking X-band FM radar, with a range solution capability of 10 cms in ice, that could be installed and removed from a Jet Ranger helicopter in a minimum amount of time. The radar should have the capability of measuring ice thickness at two metre intervals along a flight line when flown over the ice surface at an altitude of 100 feet and a velocity of 80 mi/h. The radar was tested in the laboratory and has been installed in a Ministry of Transport helicopter. This system is intended to be used for measurements of the ice thickness along the St. Lawrence Seaway. Problems in realizing adequate sweep linearity in the transmitter had not been resolved in time to carry out measurements during the winter of 1973/74, and it is planned to make measurements during the following winter.

This report discusses the basic design parameters of the radar, the implementation of the data acquisition portion, and the processing of the recorded data.

2. RADAR DESIGN PARAMETERS

In a linear-sweep FM radar the range is determined from the frequency difference between the transmitted signal, whose frequency is a linearly increasing function of time, and the received waveform which is a delayed version of it. This is illustrated in Figure 1 for a single point target. The difference frequency, f_d, is proportional to the time delay between the transmitted and received signals. Therefore, if the spectrum of the difference frequency signal formed by heterodyning the transmitted and received signals is plotted, the resulting curve directly represents reflection amplitude as a function of range.

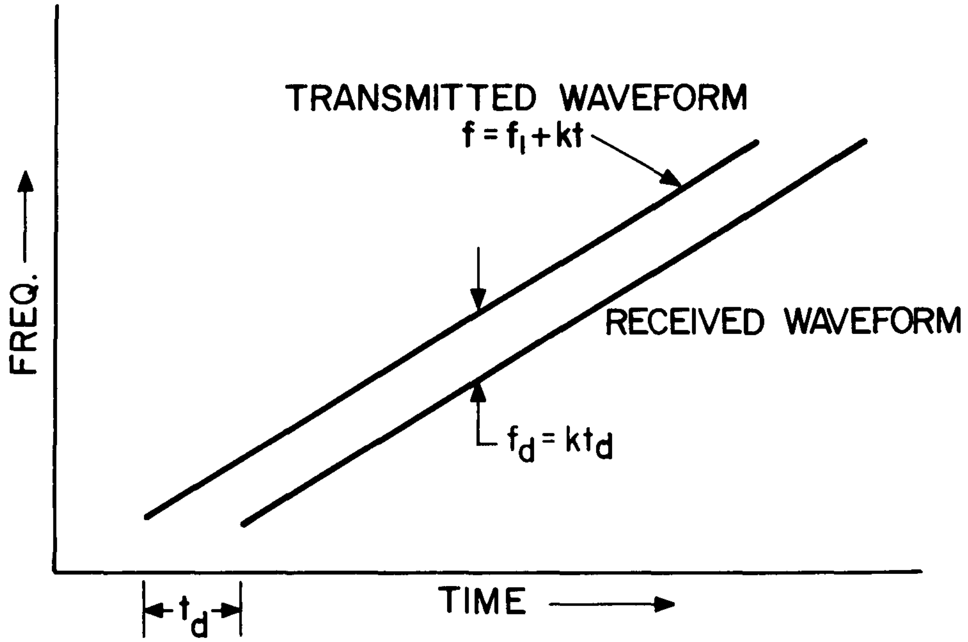


Figure 1. Frequency Difference As A Function of Time For A Single Point Target.

2.1 FREQUENCY

The major problems in the design of the radar arise from the very high range resolution requirements. A range resolution of about 10 cm is desired, and since the two-way velocity of propagation in ice is about 9 cm per nanosecond, a time delay resolution of about one nanosecond is required. Thus the bandwidth of the waveform must be about 1 GHz in order to satisfy the requirement (see Appendix D) that the bandwidth be equal to the inverse of the desired time-delay resolution. The necessary 1 GHz frequency sweep appears to rule out the use of frequencies below X-band, where percentage bandwidths become rather large, since there are stringent requirements on sweep linearity and amplitude levelling. Since attenuation through the ice, and any thin layers of water which may occur, increases with frequency[3], X-band seems the best choice for the measurement of fresh-water ice thickness.

2.2 ANTENNA BEAMWIDTH

The high range resolution requirement also influences the antenna size. For an antenna illuminating a flat surface at normal incidence, the distance from the antenna to the surface increases with the off-boresight angle to the surface point. This will cause a "smearing" of the returned signal and hence a degradation in resolution. The antenna beam must be narrow enough so that the range difference between the boresight path and a path at the beam edge is small, relative to the radar resolution. Reference 1 gives the maximum angle from the vertical for which the range error does not exceed some value d_ε (assuming a flat surface at a range R), as follows:

$$\theta_m = \sqrt{\frac{2d_\varepsilon}{R}} \, . \tag{1}$$

From this equation it follows that in order to keep the error less than 3 cm from an altitude of 100 feet, the beam should be confined to within 2.5° of the vertical. This implies that a 5° pencil beam is required. Since the antenna boresight may not always be perfectly vertical, and since there will be significant energy radiated outside the nominal (6dB two-way) beamwidth, a 3 dB beamwidth specification of 3° seems reasonable. This will produce an illuminated spot about 1.6 metres in diameter on the ice, an area which should be small enough to avoid inclusion of significant variations in ice thickness. A 3° pencil beam at X-band requires a parabolic antenna about 3 feet in diameter.

2.3 SWEEP DURATION

Since the illuminated spot is moving, the measurement time must be short enough that the spot displacement during one measurement (one RF sweep) is small relative to the spot diameter. However, the shorter the sweep time the higher will be the sweep rate, and the greater the bandwidth of difference-frequency signals which must be accommodated. A sweep duration of 10 milliseconds is a good compromise, permitting a displacement of only 0.36 metres at 80 mi/h and resulting in difference frequencies of 200 Hz per foot (656 Hz/metre) of range in air.

Errors due to vertical motion of the aircraft also constrain the usable range of sweep time. This constraint is analyzed in Appendix C, and it is shown in eqns. (C1) and (C11) that the sweep time T must satisfy the following requirements:

$$T \ll \frac{C}{2vF} \tag{2}$$

$$T < \sqrt{\frac{4\pi^2 CS}{a_{max} F}} \tag{3}$$

where

v is the maximum vertical velocity component of antenna motion

C is the velocity of light

F is the RF frequency sweep width

a_{max} is the peak vertical acceleration and

S is the ratio of maximum acceptable sideband amplitude in the difference frequency signal to the amplitude of the desired spectral component from the target. Since there is a one-to-one correspondence between the difference frequency and range, these sidebands will appear as range sidelobes associated with the true range response, and will henceforth be referred to as "range sidelobes".

It is expected that the vertical velocity of the aircraft can be held to 3 metres per second or less and that acceleration should not exceed 1g. Substituting these values and the required 1 GHz sweep width into (2) and (3) yields an upper bound on sweep duration for any sidelobe to mainlobe ratio. For sidelobes 30 dB below the main peak, $T \ll 50$ ms and $T < 0.2$ seconds must be satisfied. Thus the 10 millisecond sweep duration should be short enough.

2.4 SWEEP LINEARITY

The operation of this type of radar assumes linearity of the frequency sweep. The effect of deviations from this linearity are analyzed in Appendix A where it is shown (eqn. A15) that for a sinusoidal error in slope, the allowable peak percentage error r_{max} is given by

$$r_{max} < \frac{200\ S}{F\ t_d} \qquad (4)$$

where t_d is the propagation delay corresponding to the range of interest. For the ice-thickness measuring radar flown at 100 feet, t_d is about 200 ns. Thus, for maximum sidelobes of -30 dB, the maximum allowable error in the frequency-sweep slope is 0.03%.

2.5 AMPLITUDE FLUCTUATIONS

Amplitude variations in the received signal, caused by gain differences over the swept frequency band, result in spectral sidebands or sidelobes in range. The effect of these variations is analyzed in Appendix B. It is shown in eqn. (B4) that for a sinusoidal amplitude variation the maximum peak-to-trough amplitude ratio should be less than

$$\gamma = \frac{1 + 2S}{1 - 2S}. \qquad (5)$$

For maximum sidelobes of -30 dB, it follows that the peak-to-trough ratio should be kept less than 1.1 dB.

2.6 REPETITION RATE

Where one measurement results from each radar transmission, the repetition rate is determined by the density of sample points required along the flight path, and one point every two metres is a density acceptable to the Department of Environment. For the expected helicopter speed of 80 mi/h, the required repetition rate is about 20 Hz. For convenience in the monitoring system, a rate of twice this is used, but the radar return is recorded only on alternate transmissions.

2.7 RF REFLECTIONS

Reflections in the RF system (for example from the antenna feed) can cause replicas of the local oscillator reference signal, separated from the true reference by small delays. These, in turn, mix with the received signal to cause images of the received target responses, and as a result confuse the output response.

It is difficult to specify, directly, the allowable level of reflections since the mixing process is nonlinear; it will therefore be specified indirectly by a statement that reflection levels must be low enough that any difference frequency signals which result from them are at least 30 dB below the true difference frequency signal.

2.8 TRANSMITTED POWER

The required transmission energy is given by eqn. (9) of Ref. 1 as follows:

$$E_T = \frac{2.165 \times 10^{-18} R_s^2 N \ (S/N)}{G^2 |\rho|^2 \lambda^2} \text{ joules,} \tag{5}$$

where

R_s is the range

N is the noise factor

G is the antenna gain

(S/N) is the signal-to-noise ratio

ρ is the reflection coefficient; and

λ is the wavelength.

For the radar being developed at CRC,

R_s = 30.5 metres

$G \approx 4000$

$\lambda \approx 0.03$ metres.

For a conservative value of N = 100 and a signal-to-noise ratio of 1000 (30 dB) on a reflection coefficient of 0.001, eqn. (5) gives

$$E_T = 1.4 \times 10^{-8} \text{ joules.}$$

For a signal duration of 10 ms, a peak power of only 1.4 microwatts is required. However, past experience with an earlier experimental radar has shown interference from other sources to be a far more severe restriction than thermal noise, and we consider it advisable to have an output power of at least 10 milliwatts.

2.9 SUMMARY OF RADAR SPECIFICATIONS

Frequency	X-band
Sweep width	1 GHz
Sweep duration	10 ms
Repetition rate	40 Hz (monitoring) 20 Hz (recording)
Sweep linearity	slope accurate within 0.03%
Amplitude variations	less than 1.1 dB
Internal reflections	difference-frequency signal images must be 30 dB below true return
Transmitted power	10 milliwatts

3. THE RADAR SYSTEM

The radar system consists of two main parts, the RF subsystem and the Data Recording System. The main emphasis of this section will be on the latter. The final processing of the recorded video signal is performed on the ground with the aid of a digital computer. This processing will be dealt with separately in Section 4.

3.1 RF SUBSYSTEM

The RF Subsystem consists of a 3 foot parabolic antenna and RF components mounted in a box fastened to the antenna. The function of this subsystem is to generate a linearly swept FM waveform at X-band, transmit it downward vertically to the ice surface, and heterodyne the return signal with the transmitted one. The resulting output difference-frequency signal, which has frequencies in the kilohertz range, is fed to the Data Recording System inside the aircraft by means of a coaxial cable. A second coaxial cable brings timing pulses from the Recording system to the RF subsystem.

3.2 DATA RECORDING SYSTEM

The Data Recording System samples and digitizes the difference-frequency signal output from the RF receiver and records the resulting data along with time information on digital magnetic tape in a computer-compatible format. It also provides timing pulses to the RF transmitter, and displays some of the incoming data in a format useful to the operator in monitoring the operation. A simplified block diagram of the Data Recording System is shown in Figure 2.

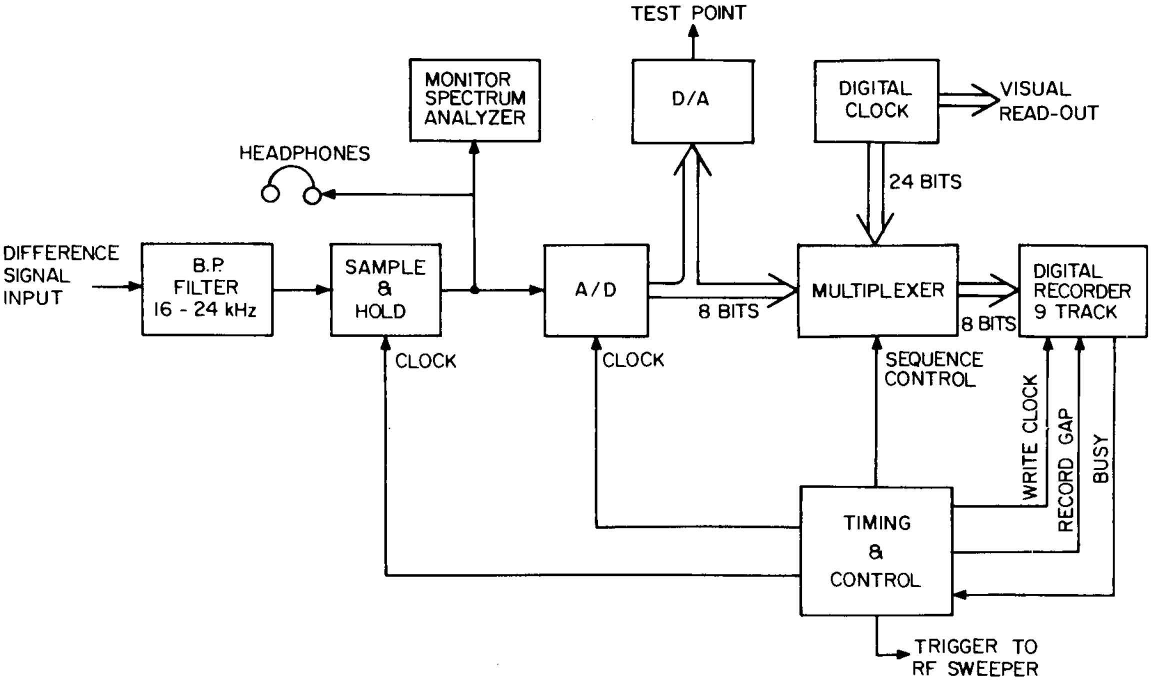


Figure 2. Block Diagram — Data Acquisition System.

3.2.1 Sampling

The radar makes about 20 measurements per second, each one providing information on the reflectivity at all ranges of interest. This information is recorded, in unprocessed form, on digital magnetic tape. The processing required to produce amplitude-versus-range plots (and A-type display) is carried out later on the ground in a digital computer. The raw signal, which is to be recorded, has a duration of 10 milliseconds and contains frequencies of about 200 Hz per foot of range. Since the intended aircraft height is 100 ft. above the ice, frequencies around 20 kHz will be produced, and to allow for aircraft height fluctuations as well as maximum ice thickness of up to 3 or 4 feet, the system must be able to record frequencies as high as about 24 kHz. This means that ordinarily, a sampling rate in excess of 48 kHz would be required for digital recording to prevent spectrum foldover. However, since only the range of frequencies between about 16 kHz and 24 kHz (80 to 120 feet) is of interest, this requirement can be reduced by the method illustrated in Figure 3. A sample rate of 16 kHz is used, and all frequencies outside the range of 16 to 24 kHz are filtered out before sampling. This results in a replica of the spectrum, after sampling, in the 0 to 8 kHz range, a reversed spectrum in the 8 to 16 kHz range and repetitions of these every 16 kHz. However, as long as no frequencies outside the 16 to 24 kHz range are allowed to be sampled, no intrusions of the unwanted spectra into the 0 to 8 kHz range will occur, and hence the range information in the desired 80 to 120 foot range can be recorded without degradation at the 16 kHz sample rate.

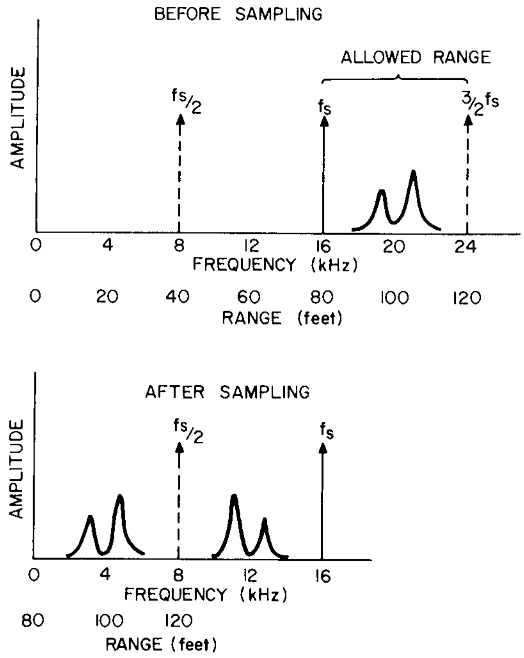


Figure 3. Signal Sampling Method.

3.2.2 Recording

The 16 kHz sample rate allows direct recording of the digitized samples on a 9 track digital recorder running in the continuous mode at 20 inches per second with 800 bytes per inch recording density. Samples are converted to eight bit words, each being recorded as one byte on the tape. The tape continues to run during the approximately 30 milliseconds of dead time between transmissions to provide inter-record gaps of the correct length. Thus each transmission will result in one record on the tape. The tape format is shown in Figure 4. Although the 10 millisecond transmission will yield only 160 samples at the 16 kHz clock rate, the use of a 256 word record simplifies the design. A 1200 foot tape will run for 12 minutes and accumulate about 15,000 records, from which 15,000 A-type plots could be produced.

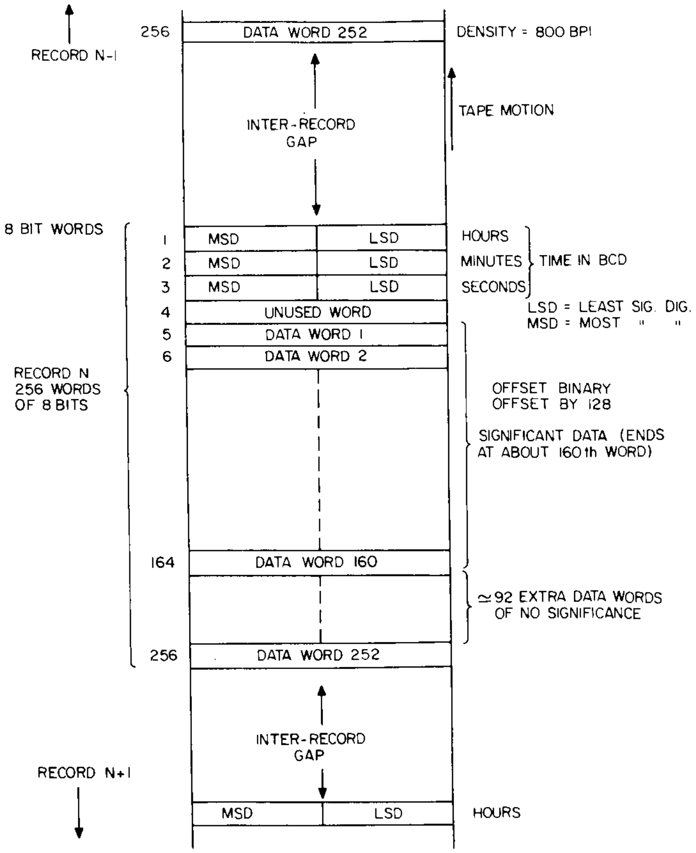


Figure 4. Magnetic Tape Format.

3.2.3 Timing and Control

The Timing and Control block shown in Figure 2 provides all the clock waveforms to sample, convert, and write on magnetic tape. It also controls switching of the multiplexer. The function of the multiplexer is to feed a binary-coded-decimal representation of hours, minutes and seconds from the digital clock into the first three words of each record, and to initiate end-of-record gaps. The timing and control block also provides a trigger to the RF sweeper to synchronize the recording with the transmission. Recording of each physical record is initiated by the end of the "Busy" signal from the recorder, indicating that the inter-record gap has been completed.

3.2.4 Performance Monitor

The A/D converter converts each sample of the bipolar input signal to an eight-bit offset binary number for recording. The three most significant bits from the 8-bit A/D converter are ANDed together to drive an indicating lamp, which shows the operator when signal peaks exceed 75 percent of the allowable value and allows him to set the gain accordingly. Normally the operator will set the gain slightly below that which will cause some flashing of the light, in order to prevent any significant amount of overflow.

A D/A converter is used to provide an analog monitoring point which allows the operation of the system to be tested. Headphones may be connected to the Sample-and-Hold output to monitor the operation, and a crude spectrum analyzer connected at this point provides a coarse A-type display over a limited range of about five feet. The centre of this monitored range may be varied over the 80 to 120 foot recording range by means of a manual control. The spectrum monitor should be useful in determining the height of the aircraft for possible corrective action.

A block diagram of the spectrum analyzer is shown in Figure 5. The output of the first mixer is a one kHz sweep whose centre can be positioned anywhere in the range 10.5 to 16.5 kHz, depending on the setting of the 18-24 kHz variable oscillator. This mixer output is heterodyned with the filtered input signal in the second mixer and fed to the narrow band filter centred at 9.5 kHz.

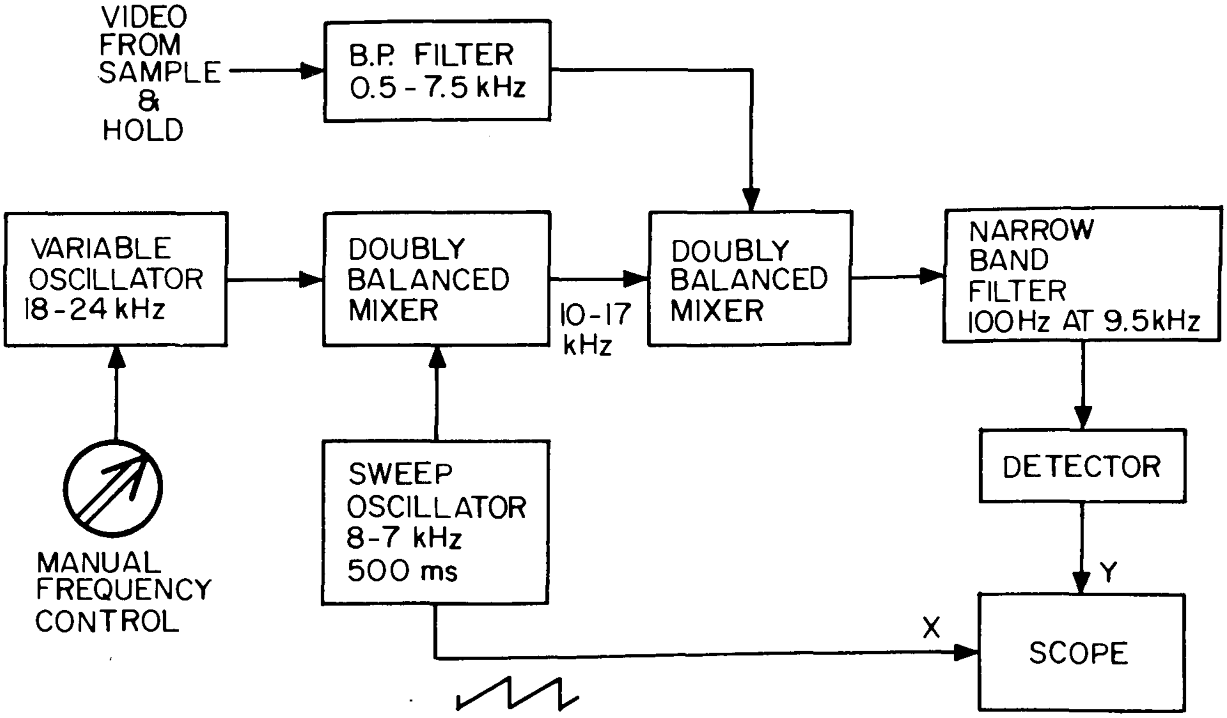


Figure 5. Block Diagram -- Spectrum Analyzer.

Figure 6 illustrates the mode of operation of the analyzer. The top graph shows the possible input frequency range (the changing amplitude is used only to show the inversion of the spectrum when heterodyned). The other three graphs show typical situations in which the 1 kHz segments 1, 2 and 3 are analyzed. The frequency ranges shown are those occurring at the input to the 9.5 kHz narrowband filter. The sweep oscillator range at the output of the first mixer is also shown. This is controlled by the manual setting of the variable oscillator; the oscillator setting appropriate to each figure is given to the right of that figure.

The technique of analyzing only a portion of the frequency range at any one time allows the analyzer to complete a scan in about one-half second instead of the much longer time required to scan the full range. Once the analyzed range has been positioned manually, its extent is sufficient (about 1.5 metres in air and about 0.9 metres in ice) to allow viewing of fairly thick ice on the display, although manual corrections may be necessary from time to time to compensate for aircraft vertical motion.

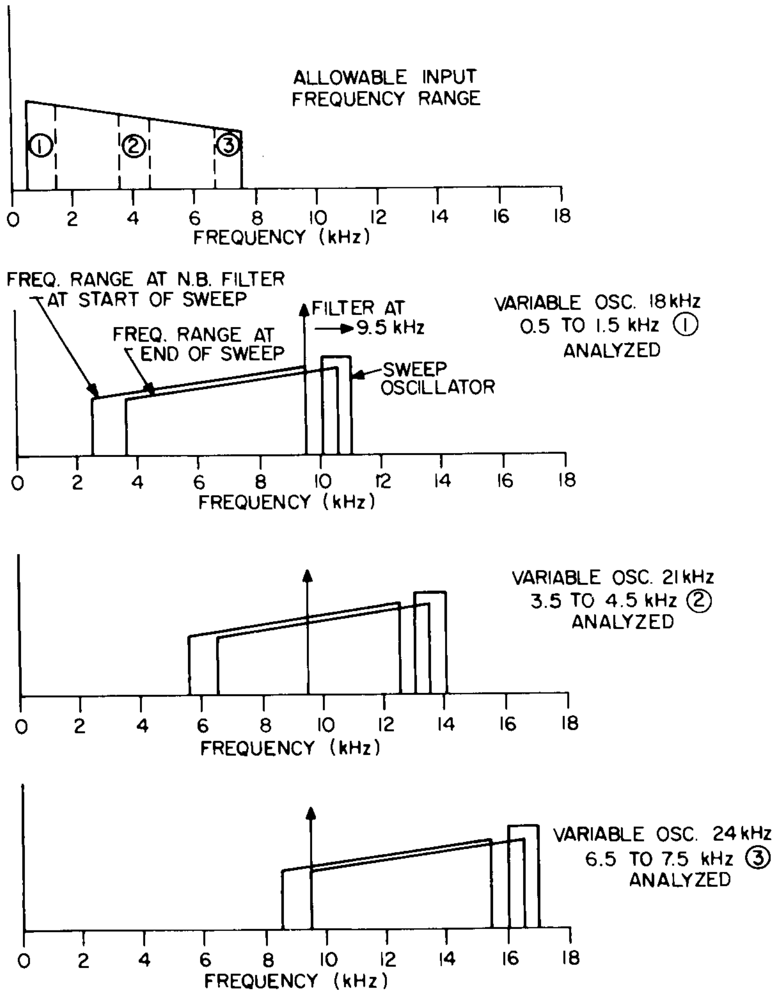


Figure 6. Spectrum Analyzer Operation.

Figure 7 is a photograph of the analyzer display when the input to the system is a single-frequency difference signal (simulating a return from a single target). The full width of the screen represents 1 kHz in frequency (the trace appears shortened due to the exposure time of the camera). Thus each division on the screen represents 100 Hz or 9 centimetres in ice. The "bumps" in the displayed signal are due to the fact that the transmitted signal is active for only 10 milliseconds out of approximately 25 (the simulated signal return was made to correspond in time to the normal transmitted signal); the "bumps" should therefore be ignored and only the envelope considered. The digital clock which can be seen just below the analyzer display, allows the time of passing of landmarks to be noted by the operator, on the same time scale as that which is automatically inserted in the recordings.

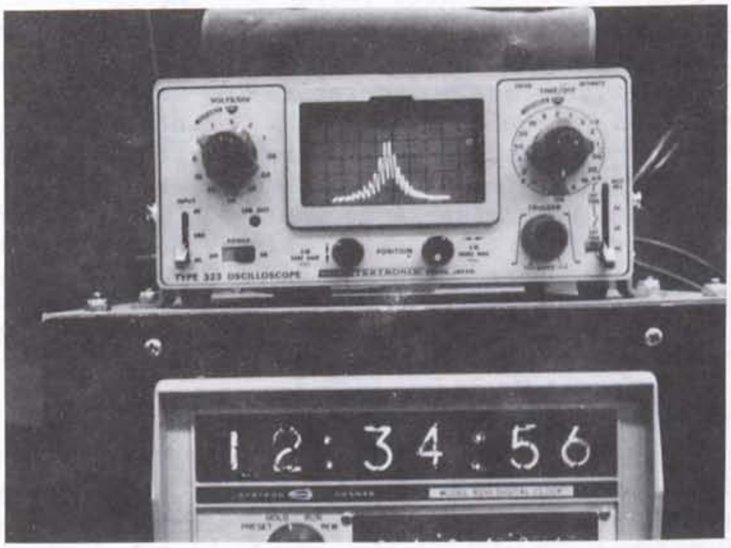


Figure 7. Spectrum Analyzer Display.

4. SIGNAL PROCESSING

The recorded digital representation of the radar video signal is processed in a digital computer to provide amplitude-versus-range information. Since the frequency of the signal is proportional to the range of the reflection causing it, the required processing is a Fourier Transformation, which is accomplished with a Fast Fourier Transform (FFT) routine.

The FFT routine computes the spectrum from zero frequency to 8 kHz (half the sampling rate). However, in the original video signal this corresponds to the frequency range of 16 to 24 kHz. The relationship between range and frequency is 200 Hz per foot (656 Hz per metre) for free space propagation. Thus the FFT output corresponds to the 40 foot range between 80 and 120 feet.

4.1 INTERPOLATION

As shown in the previous section, a record consists of 256 words, of which only about 160 are significant as radar data, covering a time period of 10 milliseconds. For a single point target, the recorded waveform will have a single frequency and the transform will therefore be of Sin x/x shape with a main lobe width of 200 Hz between nulls. This sets the basic resolution at about 100 Hz (equivalent to about 15 centimetres in air or 9 centimetres in ice). The FFT will provide discrete outputs only, separated in frequency by the inverse of the input record length. Thus if only the 160 input points were used, just two output points would occur within the main lobe envelope. While the transform is completely specified by the set of output points, the data is not easy to interpret by eye without some appropriate interpolation procedure. Interpolation can be accomplished by the addition of zeros to the end of the input data to extend its length and thereby reduce the spacing of the output points. Padding the input record of 512 samples provides output points every 31 Hz (about 3 centimetres in ice), which, when plotted, give quite a smooth curve. The maximum error in ice-depth measurement from a simple interpretation of the plot should be 3 centimetres (assuming a reasonably good signal-to-noise ratio) with average error much less than this. Since the penalty paid for the interpolation by means of padding with zeros is computing time, it may be advisable to use a 256 point transform when the accuracy requirements are not so great. Since the padding will be done at the time of computation, this decision can be made for the particular requirements at the time of processing.

4.2 WINDOWING OF THE DATA

For a single point target the transform will have a (sin x)/x shape. The high sidelobes of this function can mask the returns from other smaller targets, or may themselves be interpreted as separate returns. These sidelobes can be greatly reduced, at the expense of slight broadening of the main lobe, by the use of a window function on the input waveform. A useful window function for this application is the cosine squared function; the sampled time waveform is multiplied pointwise by samples of the function $\cos^2(\phi)$ over the range $-\pi/2$ to $+\pi/2$. The end points of this half-cycle are made to coincide with the first and last data samples. Detailed information on window functions is available in Ref. 4.

Figure 8 shows a computer plot of the FFT from a recording made on the data acquisition system with a single difference frequency input (simulating the output of the RF system for a single point target). The difference frequency was 20 kHz, which was translated by the sampling process to 4 kHz. Cosine squared weighting was used to reduce sidelobes, and the plotter pen was left down between points to produce a continuous curve. The input was padded with zeros to make 512 samples, giving a 256 point plot in the 0-8 kHz range of interest.

Figure 9 is a similar plot when two frequencies were used to simulate two targets (such as upper and lower ice surfaces). The frequency separation was about 330 Hz, corresponding to about 30 cm ice thickness.

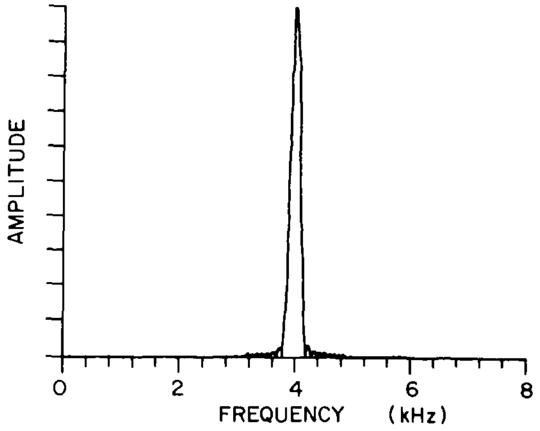


Figure 8. Fourier Transform for Single Target.

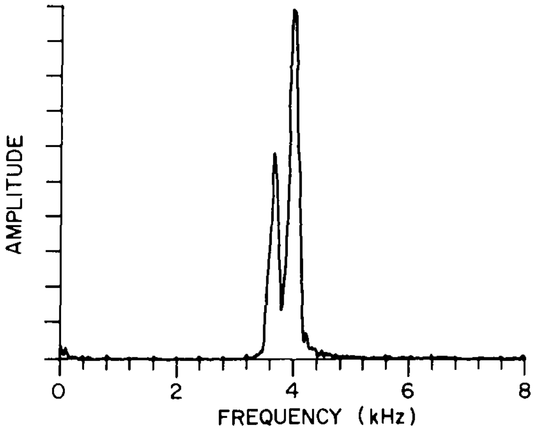


Figure 9. Fourier Transform for Two Targets.

4.3 DATA PRESENTATION TECHNIQUES

Until some experience is gained with the output data, the best form of output appears to be computer-generated plots of amplitude versus range, from which the ice depth can be scaled after a selection of the returns corresponding to the upper and lower surface reflections. However, since there will be about 20 returns per second of flight time, it becomes apparent that there is too much information to process all of it in this way. It may be sufficient to process only every 20th or 100th record in this manner, and to plot every record over only limited parts of the path, where more detailed information is required.

Amplitude-versus-range curves from successive radar sweeps may be plotted on the same page if the excursion in the y-direction is kept small and the x-axis is moved incrementally down the page for each curve; possibly up to five curves per inch could be accommodated. An even higher density of plotted data could be achieved by translating the amplitude of the return to a binary-valued function of the range variable, having the value one if some suitably fixed threshold is exceeded, and zero otherwise. Each translated curve would then be plotted along a straight line and the value of the function would determine the condition "pen down" or "pen up". Using this technique, up to 100 measurements per inch could be displayed, with the loss of some accuracy of depth determination.

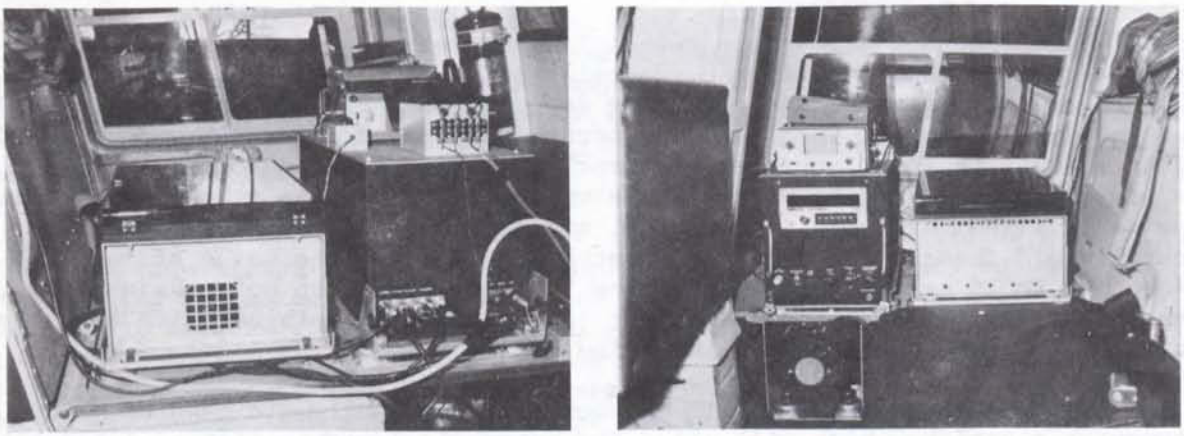
It is hoped that it will be possible, eventually, to develop a computer algorithm which will allow the automatic determination of ice depth, with the computer output being a thickness profile along the flight path. Development of such an algorithm will depend on experience gained from A-type plots, since there will be difficulties in identifying the correct returns as a result of undesired effects. For example, reflections from layers within the ice, and multiple internal reflections between the upper and lower surfaces of the ice may result in more than two peaks. Also, a snow layer on the ice surface will complicate the situation. The expected variations in the relative amplitudes of the reflections from two ice surfaces can be a problem as well.

5. HELICOPTER INSTALLATION

Photographs of the radar installation in a Ministry of Transport (MOT) Jet Ranger Helicopter are shown in Figures 10, A to D inclusive. Photographs A and B show the front and rear views of the equipment that was carried in the rear seat area of the cabin, while Photograph C shows the antenna mounted between the landing struts under the helicopter. The helicopter, with the radar installed, is shown in flight in Photograph D.

The equipment inside the cabin was mounted on and under a metal tray that was bolted to the seat belt anchors at the rear, and to the bulkhead behind the pilot at the front. The total weight of the complete radar system was 206.5 pounds. Photograph A shows the rotary generator/power converter (28V DC to 115V AC) mounted under the tray between the seat edge and the bulkhead, the standard Air Transport Racking (ATR) box and oscilloscope on top of the tray to the left, and the tape recorder to the right. Photograph B shows the rear view of this equipment.

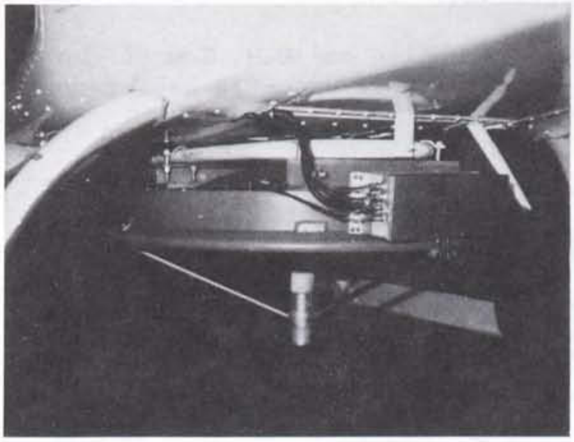
The three-foot parabolic antenna, with an attached chassis that contained the RF electronics, was fitted with brackets that clamped onto the external cargo frame as shown in Photograph C. (This cargo frame normally supports the cargo hook.) The power and signal cables were fed into the cabin through existing ports; therefore, no fuselage cutting or drilling was necessary. The equipment configuration was such as to allow installation or removal in approximately 30 minutes.



~~(a)~~ (b)

~~(b)~~ (a)

Figure 10. Radar Installation – MOT Helicopter



(c)

(d)

6. SUMMARY

This report has described the design of a helicopter-borne radar system for the measurement of fresh-water ice thickness. The radar was designed to operate from a helicopter flying at about 80 mi/h at a height of about 100 feet above the ice, and to record ice thickness data at two metre intervals along the flight path.

The radar transmits a long linear FM waveform, and range is determined from the frequency difference between the transmitted waveform and the received version of it delayed by the 2-way transmission time to the surface. The difference-frequency signal formed by heterodyning the transmitted and received waveforms is sampled, digitized, and recorded on digital magnetic tape. The spectral analysis necessary to extract the range information is accomplished on the ground by means of a Fast Fourier Transform (FFT) routine in a digital computer. Some of the problems involved in the presentation of the resulting data have been discussed.

The effects of deviations of the transmitted waveform from the ideal case and of the undesired motions of the helicopter on the radar performance, have been investigated. On the basis of this analysis, the radar parameters were chosen to satisfy the measurement requirements. A crude spectrum analyzer built into the radar system allows the performance to be monitored in the air while the data are being recorded.

The radar can be installed on a Jet Ranger helicopter in about 30 minutes. Flight tests for air-worthiness have been performed, but problems in achieving adequate sweep linearity in the transmitter were not resolved in time to carry out measurements during the winter of 1973/74. It is now planned to carry out these measurements during the winter of 1974/75.

7. ACKNOWLEDGEMENT

The authors wish to thank Messrs. D.R. Faire and D.H. Gow of the Scientific Design Technology Section of CRC for their valuable assistance during the course of this project.

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APPENDIX A

Effect of FM Slope Error

Let the desired functional relationship of frequency on time in the transmitted waveform be

$$f = f_o + kt. \tag{A1}$$

For a point target with delay t_d we will have a difference frequency

$$f_d = kt_d. \tag{A2}$$

Now suppose the FM slope k is not constant, but is a function of time. It may be written in the form

$$k = k_o \left(1 + \frac{r(t)}{100} \right). \tag{A3}$$

If the percentage error r(t) is a slowly changing function of time relative to the propagation time delays of interest, then eqn. (A3) may be substituted in (A2) giving

$$f_d(t) = \left[k_o \left(1 + \frac{r(t)}{100} \right) \right] t_d. \tag{A4}$$

The resulting error in f_d relative to the desired value will be

$$f_\varepsilon(t) = k_o\, t_d\, \frac{r(t)}{100}\ . \tag{A5}$$

This frequency variation will cause the spectrum of the difference signal to be degraded, broadening the main response or giving rise to sidelobes which may be confused with other returns.

As a first requirement, the maximum value of frequency error must be much less than the frequency resolution 1/T where T is the sweep duration. That is,

$$\frac{k_o t_d r_{max}}{100} \ll \frac{1}{T}$$

or

$$r_{max} \ll \frac{100}{k_o t_d T} , \qquad (A6)$$

where r_{max} is the peak percentage error in the slope of the sweep waveform. From (A1), the frequency sweep width is

$$\begin{aligned} F &= kT \\ &\approx k_o T. \end{aligned} \qquad (A7)$$

Substituting this in (A6) we get

$$r_{max} \ll \frac{100}{Ft_d} . \qquad (A8)$$

Consider the case of sinusoidal slope variation, i.e.,

$$r(t) = r_{max} \cos 2\pi f_m t. \qquad (A9)$$

where f_m is the frequency of the sinusoidal variation and r_{max} is the peak value of $r(t)$. Thus we have a frequency modulation

$$f_\varepsilon(t) = \frac{k_o t_d}{100} r_{max} \cos 2\pi f_m t, \qquad (A10)$$

or a phase modulation

$$\begin{aligned} \theta_\varepsilon(t) &= 2\pi \int f_\varepsilon(t)dt \\ &= \frac{k_o t_o}{100 f_m} r_{max} \sin 2\pi f_m t \end{aligned} \qquad (A11)$$

If the sweep time is T, then the number of cycles of frequency variation in one sweep is

$$n = f_m T. \qquad (A12)$$

Substituting (A12) and (A7) in (A11) we have

$$\theta_{\varepsilon}(t) = \frac{Ft_d}{100\ n}\ r_{max} \sin \frac{2\pi nt}{T}\ . \tag{A13}$$

For peak phase variations which are small relative to one radian, this modulation will give rise in the spectrum to sidebands above and below the desired response by n/T Hz, and of amplitude ($Ft_d/200\ n\ r_{max}$) times the desired response[5].

The amplitude of these undesired sidelobes is inversely proportional to n. Since the resolution of the spectrum for a total sample length is about 1/T, when n is less than unity the sidelobes will merge into the main response. We may take the worst case therefore as that with $n = 1$, and for this case the ratio of sidelobe to desired response amplitude will be

$$S = \frac{Ft_d}{200}\ r_{max}. \tag{A14}$$

If we wish to specify the maximum sidelobe level S, then the maximum allowable peak slope error is given by

$$r_{max} < \frac{200\ S}{Ft_d} \tag{A15}$$

This will be a more stringent requirement than (A8) for values of S significantly less than 0.5.

APPENDIX B

Effect of Amplitude Modulation

Let the difference-frequency signal from a point target in the ideal case be

$$V_1 = A \cos \omega_d t. \tag{B1}$$

Consider the effect of an undesired sinusoidal amplitude modulation which modifies eqn. (B1) to give

$$V_2 = A[1 + a \cos \omega_m t] \cos \omega_d t \tag{B2}$$

where a is the ratio of the modulation amplitude to the signal amplitude and ω_m is the modulation frequency. It is well known that eqn. (B2) gives rise to spectral sidebands of amplitude $S = a/2$ relative to the main response.

The ratio of peak-to-trough amplitude in the envelope of V_2 is

$$\gamma = \frac{(1+a)}{(1-a)}. \tag{B3}$$

Therefore if a maximum sidelobe level relative to the main response is specified as S, then the maximum allowable value of γ will be

$$\gamma = \frac{1 + 2S}{1 - 2S} . \tag{B4}$$

APPENDIX C

Effect of Aircraft Vertical Motion

Vertical motion of the aircraft can cause degradation of the received signal as a result of the change in range during the sweep time. This may be analyzed in the following ways:

a) Range Variation

For a sweep duration T and constant vertical velocity v, the range variation across the sweep will be vT. One requirement, which must be satisfied for the received signal to be not seriously degraded, is that this be small relative to the free space range resolution $R_r = C_m/2F$ (see Appendix D). That is

$$vT << \frac{C}{2F}$$

or

$$T << \frac{C}{2vF} . \qquad (C1)$$

Now suppose the vertical velocity varies in a sinusoidal fashion, i.e.,

$$v(t) = v_{max} \sin 2\pi f_m t, \qquad (C2)$$

where f_m is the frequency of the sinusoidal variation and v_{max} is the peak value of v(t). This will cause a range variation

$$R_\varepsilon(t) = - \frac{v_{max}}{2\pi f_m} \cos 2\pi f_m t , \qquad (C3)$$

and a frequency variation

$$f_\varepsilon(t) = \frac{2kR_\varepsilon(t)}{C}$$

$$= - \frac{v_{max} k}{\pi f_m C} \cos 2\pi f_m t,$$

where C is the velocity of light and k is the FM sweep rate as defined in Appendix A.

The phase variation will be

$$\theta_\varepsilon(t) = 2\pi \int f_\varepsilon(t)\ dt$$

$$= \frac{v_{max}k}{\pi f_m^2 C} \cdot \cos 2\pi f_m t. \qquad \text{(C4)}$$

This will give rise to sidebands separated from the true difference-frequency by multiples of f_m. However, if the peak deviation $v_{max}k/\pi f_m^2 C$ is much less than unity (as it must be for reasonable performance), only the two nearest sidebands at the difference frequency $\pm f_m$ will be significant and these will have an amplitude, relative to that of the true difference-frequency, given by[5]

$$S = \frac{v_{max}k}{2\pi f_m^2 C} . \qquad \text{(C5)}$$

The acceleration for this case is given by

$$a = \frac{d}{dt}\left(v_{max} \sin 2\pi f_m t\right)$$

$$= 2\pi f_m v_{max} \cos 2\pi f_m t, \qquad \text{(C6)}$$

and the peak acceleration is

$$a_{max} = 2\pi f_m v_{max}. \qquad \text{(C7)}$$

Thus, in terms of peak acceleration, (C5) becomes

$$S = \frac{a_{max}k}{4\pi^2 f_m^3 C} \qquad \text{(C8)}$$

For a particular value of a_{max}, the worst case will be for minimum f_m, that is $f_m = 1/T$. Then

$$S = \frac{a_{max} k T^3}{4\pi^2 C}$$

$$= \frac{a_{max} F T^2}{4\pi^2 C} \qquad (C9)$$

where F = kT is the frequency sweep width.

If we specify a maximum sidelobe level S, the maximum allowable peak acceleration is given by

$$a_{max} < \frac{4\pi^2 CS}{FT^2} \qquad (C10)$$

or

$$T < \sqrt{\frac{4\pi^2 CS}{a_{max} F}} \; . \qquad (C11)$$

b) Doppler Shift

Constant vertical velocity will cause a doppler shift which will add to the difference frequency. Since only the difference in range between the two surfaces is important, the shift itself is not too serious. However, since the transmitted signal has a relatively large percentage bandwidth, the doppler shift will be a function of the transmitted frequency and hence will cause the difference frequency for a point target to vary across the sweep. It will be shown that this gives exactly the same result as was obtained by consideration of the range variation, and that therefore these are really only two ways of looking at the same effect.

For a frequency sweep width F this doppler change will be $2v/c$ F. This change should be kept small relative to the difference frequency resolution, i.e.,

$$\frac{2v}{c} F \ll \frac{1}{T} \qquad (C12)$$

or

$$T \ll \frac{c}{2vF} \; . \qquad (C13)$$

This is identical to (C1), a not surprising result.

APPENDIX D

Range Resolution

It is well known that the frequency resolution possible in a spectrum measurement is a function of the time duration of the measured signal. For a signal duration T, the frequency resolution will be roughly 1/T.

We can see from Figure 1 that the relationship between propagation time delay t_d and difference frequency f_d is

$$t_d = \frac{f_d}{k} . \qquad \text{(D1)}$$

Let the time delay resolution corresponding to the difference frequency resolution f_{dr} be t_{dr}. Then from (D1)

$$t_{dr} = \frac{f_{dr}}{k} . \qquad \text{(D2)}$$

If the duration of the transmitted waveform is T, and T is much larger than propagation delays, then the duration of the difference frequency signal is also T, and

$$f_{dr} = \frac{1}{T}.$$

Substituting this in (D2) we have

$$t_{dr} = \frac{1}{kT} . \qquad \text{(D3)}$$

Now if the extent of the frequency sweep of the transmitted signal is F, then it is evident from Figure 1 that F = kT and (D3) becomes

$$t_{dr} = \frac{1}{F} . \qquad \text{(D4)}$$

That is, the time delay resolution is equal to the inverse of the frequency sweep width, or bandwidth, of the transmitted waveform. This is the value predicted by radar theory as the resolution capability of a waveform in terms of its bandwidth, and this indicates that the method of difference frequency measurement, for processing the return of a long duration linear FM waveform, makes full use of the waveform bandwidth.

The range resolution capability of the radar is given by

$$R_r = \frac{C_m t_{dr}}{2}$$

$$= \frac{C_m}{2F} , \qquad \text{(D5)}$$

where C_m is the velocity of electromagnetic wave propagation in the medium being measured. The factor 2 occurs because of the two-way path of the radar wave.

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8. ABSTRACT: This report describes the design of a high resolution radar for use in measuring fresh-water ice thickness from a helicopter. The feasibility of ice-thickness measurement with an FM radar of this type has been proved in previous work. Of particular concern in this report is the data acquisition system required to record ice-thickness information at two metre intervals when the aircraft is moving at a speed of about 80 miles/h. The radar difference-frequency signal is sampled, converted to digital form and recorded on computer-compatible magnetic tape along with a time reference. The spectral analysis necessary to extract the depth information is accomplished on the ground by means of a Fast Fourier Transform routine in a digital computer. The equipment was designed for quick installation on a helicopter. Ice measurement results are published elsewhere.

9. CITATION:

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