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TRACKING-FILTER STRUCTURES FOR AUTOMATIC TRACK-WHILE-SCAN SURVEILLANCE SYSTEMS

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A.W. Bridgewater

This work was sponsored by the Department of National Defence, Research and Development Branch under Project No. 33C78.

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by

A.W. Bridgewater

(Radar and Communications Technology Branch)

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TRACKING-FILTER STRUCTURES FOR AUTOMATIC TRACK-WHILE-SCAN SURVEILLANCE SYSTEMS

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A.W. Bridgewater

ABSTRACT

In an automatic track-while-scan air-surveillance radar system, the role of the target-tracking filter is to support the plot-to-track association process by providing reliable estimates of the current track state, on which to base predictions of subsequent track states. The Kalman filter is the most general solution of the recursive linear mean-square estimation problem, its drawback being its computational cost. An analysis of various one-dimensional forms of filter, derived from the Kalman, results in the complete specification of the α-β(-γ) and reduced-Kalman classes of tracking filter, which often form the basis for the design of the practical two- and three-dimensional trackers required in air-surveillance systems and which combine good track-following ability, ease of adaptation to changes in tracking conditions and low computational cost. Both recursive and steady-state adaptive versions of these one-dimensional structures are described. A functional relationship between the set of gain coefficients and a parameter which quantifies the current tracking conditions in terms of target manoeuvre uncertainty, radar measurement error and track update interval makes possible rapid and effective filter adaptation.

1. INTRODUCTION

In an automatic track-while-scan (TWS) air-surveillance system, the radar sensor reports measurements of target positions at regular intervals of time to a computer, which then assembles these reports, or "plots", from successive scans into tracks. The computer program must correctly associate new plots with existing tracks and initiate new tracks from reports received on air targets within the range of the radar. The association task is aided by tracking filters which combine noisy measurements with track predictions to obtain smoothed updated track estimates. The predicted position of the target for the next radar scan, based on the smoothed estimate of the current position of the target, is used together with the estimated standard deviation of the prediction to determine the location and size of the region of acceptability of new observations on that target. The tracking filter thus plays an essential role in the function of plot-to-track association, in addition to its role of providing accurate estimates of the position and motion of the target.

The literature on the techniques of track filtering (smoothing and prediction) is very large and diverse. This report attempts to construct a framework in which the majority of techniques could be placed and thereby more easily analyzed and compared.

An operational air-surveillance system must be capable of tracking many targets simultaneously, in an environment that may provide large numbers of false target indications due to fixed and moving clutter, man-made interference and system noise. The primary task of the tracking computer in such a system is that of track initiation and association, and these functions should be allotted most of the computational time available. In a system involving a network of sensors, the related task of track registration between different sensors should also be included in this preferred allotment. The measurement accuracy obtained and the tracking precision required by such systems do not demand the most computationally complex filtering operations. It is essential that the filtering operations give sufficient support to the association procedures; any complexity beyond that necessary for this task is of diminishing value. It is important however that the filter be sufficiently flexible to adjust quickly to changes in the tracking environment.

The general Kalman filter provides the starting point for the analyses which follow. The discrete form of the filter is presented, without derivation, and from it are derived the general forms of recursive and fixed-parameter (steady-state) filters which are various sub-optimal solutions to the estimation problem. The use of fixed-parameter filters eliminates the necessity of iteratively calculating new coefficients at every scan and thus greatly reduces the computational load of the filter portion of the automatic tracking system. The adaptation of the filter to changes in the tracking conditions (viz., manoeuvres, measurement error, update interval) is discussed, and the means by which the steady-state filter may adjust to such changes are described.

2. TRACK ESTIMATION

The role of an automatic tracking system is to provide a sequence of best estimates of the target's position and velocity, based on the available measurements and an assumed model of the target's behavior, without operator intervention. Powerful mathematical procedures exist with which to carry out the track-estimation operations. The Kalman filter[1] is the most general solution of the recursive, linear, mean-square estimation problem. It is conveniently expressed in matrix notation, as described in the following section.

2.1 KALMAN FILTER

The discrete form of the filter is described by the following equations:

Target Model

$$X_k = \Phi_{k-1}X_{k-1} + \Gamma_{k-1}U_{k-1} \tag{1}$$

Measurement Model

$$Y_k = M_kX_k + V_k \tag{2}$$

Forecast

$$\tilde{X}_k = \Phi_{k-1}\hat{X}_{k-1} \tag{3}$$

$$\tilde{P}_k = \Phi_{k-1}\hat{P}_{k-1}\Phi^t_{k-1} + \Gamma_{k-1}Q_{k-1}\Gamma^t_{k-1} \tag{4}$$

Estimation

$$K_{k(opt)} = \tilde{P}_kM^t_k(M_k\tilde{P}_kM^t_k + R_k)^{-1} \tag{5}$$

$$\hat{X}_k = \tilde{X}_k - K_{k(opt)}(M_k\tilde{X}_k - Y_k) \tag{6}$$

$$\hat{P}_k = \tilde{P}_k - K_{k(opt)}M_k\tilde{P}_k \tag{7}$$

The sampling instants at which measurements are taken and to which computed quantities apply are indicated by the k-subscripts. Forecasts and estimates are denoted by (˜) and (^) respectively. The superscript (t) denotes matrix transposition and the superscript (-1) denotes matrix inversion.

X_k is the state vector of the target in track, at the k^{th} instant; P_k is its covariance matrix.

U_k is a noise vector representing zero-mean random activity (model uncertainty); covariance Q_k.

V_k is a noise vector representing zero-mean random activity (measurement uncertainty); covariance R_k.

Γ_k is the excitation or manoeuvre matrix which specifies the effect of U_k on X_k.

Φ_k is the state transition matrix derived from the assumed model of the dynamical behavior of the target.

Y_k is the observation vector of the target in track.

M_k is the measurement or selection matrix which relates Y_k to X_k.

$K_{k(opt)}$ is the optimum gain matrix for the determination of the estimates $\hat{X}_k$ and $\hat{P}_k$.

The Kalman-filter algorithm combines a track forecast, which is derived from the previous best estimate in accordance with the equations of motion, with the most recent physical measurement to produce a weighted mean, the weighting factor K being chosen to minimize the variance. For strick optimality, the noise statistics must be Gaussian, and the noise terms must be uncorrelated from one sampling instant to the next. Because it provides for the inclusion of all possible couplings of covariance terms in its matrix formulation, the Kalman filter is independent of the coordinate system in which the state variables and measurement variables are expressed.

The drawback of the Kalman filter is its computational cost. The recursive procedures require, at each sampling instant and for each target, the multiplication of matrices or order nxn and the inversion of a matrix of order mxm (see eqns. (4) and (5); n is the length of the state vector X and m is the length of the observation vector Y). For a track-while-scan surveillance radar system, which may be required to track many targets simultaneously, this computational load could become prohibitive, particularly when one must take into account the additional load of the automatic track-association procedures. By dispensing with various components of the full apparatus of the Kalman filter, one can produce simpler approximations to the solution of the estimation problem, with corresponding reductions in computer loading. This must be accomplished, of course, without degrading the overall tracking performance of the system to an unacceptable degree.

2.2 SIMPLIFICATION

There are two approaches to the simplification of the filtering procedures. The first is to reduce the general matrix formulation to such an extent that it may be replaced by a small number of algebraic recursion relations. The necessary simplifying assumptions include the elimination of coordinate interaction terms in the covariance expressions, the reduction in the size of the state and measurement vectors with a corresponding reduction in the dimensions of the associated matrices, and the adoption of simple linear equations of motion derived from the transition matrix Φ (assumed invariant with k) and based on a constant sampling interval T. This results in a sub-optimal form of solution of the recursive estimation problem. Because of the coordinate decoupling, the choice of coordinate system in which to express the state and measurement variables can affect the filter performance. The filter does not propagate all possible covariance terms, as does the Kalman filter.

The second approach is to adopt a constant gain K_∞ in place of the recursively computed $K_{k(opt)}$ in the Kalman filter, using invariant forms for Φ, Γ, and M. This eliminates the need for iteratively computing the covariance matrix $\hat{P}_k$, during track updating. K_∞ is the steady-state gain or limiting value of $K_{k(opt)}$, with known or assumed values for the noise covariances Q and R. It is precomputed by iteration of eqns. (4), (5) and (7) starting with an initial value P_0 and is subsequently applied to the sequence of target observations using only eqns. (3) and (6). This form of solution to the estimation problem is sometimes called the Wiener filter.

These two approaches may be combined to form a constant-gain, reduced filter.

2.3 ADAPTIVE FILTERING

Whatever method is adopted for track filtering, it is usually necessary to combine it with some form of adaptation. An adaptive system is one which continually adjusts its own parameters in the course of time to meet a certain performance criterion. By this definition neither the recursive nor the steady-state filters outlined above can be termed adaptive. For the former, the sequence of values for the gain K_k could be computed off-line and stored prior to being applied to a sequence of target observations, given a set of initial values for the covariance terms.

On-line adaptation is required when significant changes occur in the target motion (manoeuvres), measurement accuracy or frequency of detection. The excitation noise covariance Q is a statistical quantity intended to cover uncertainties in the model of target motion described by Φ and Γ. Measurement accuracy is described statistically by the noise covariance R, and the interval between filter updates is given by the time T (which appears in Φ and Γ). Changes in the track environment must be reflected in the appropriate adjustment of these three filter parameters during the track-estimation process. Adaptive tracking requires the on-line computation of a figure-of-merit term, or track-performance indicator, which typically involves a weighted combination of the terms in the residual $MX'-Y$ in eqn. (6). It also requires a practical procedure for determining what quantitative adjustment should be made in the filter parameters.

3. ONE-DIMENSIONAL FILTER STRUCTURES

In this section, we analyze various forms of one-dimensional tracking filters, as derived from the general Kalman filter, demonstrating their recursive and adaptive features by means of algebraic equations, and where possible we derive closed-form functional relationships which describe completely their steady-state characteristics. These forms are important for the construction of the two- and three-dimensional tracking filters required for a practical air-surveillance system.

First, we analyze the class of so-called α-β($-\gamma$) filters, in which measurements are reported on target positions only. Various filters in this class have long been used in target-tracking applications. We show how all the types of filters in this class are related through a common derivation from the general Kalman form. We demonstrate their recursive behavior, verify certain optimal steady-state relationships between parameters in various filters reported in the literature and present new results for others. Second, we analyze the class of reduced Kalman filters, in which doppler measurements of velocity are available in addition to target position measurements. We develop a single structural framework, similar to that developed for the α-β($-\gamma$) filters, in which these filters may be conveniently described and demonstrate their recursive and steady-state behavior.

In the one-dimensional form of tracking filter, each coordinate x in the target's state vector is decoupled from the others and is treated separately.

3.1 α-β FILTER

The standard equations for the α-β filter are obtained by substituting

$$X_k = \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} ; \; \Phi_k = \Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}; \; K_k(\text{opt}) = \begin{bmatrix} \alpha_k \\ \beta_k/T \end{bmatrix}; \; Y_k = [y_k]; \; M_k = M = [1 \; 0]$$

in eqns. (3) and (6). This is a second-order, constant-velocity model for target motion, with only position measurements available and with a constant update interval T. For each coordinate, the tracking filter then reduces to the following algebraic equations:

Forecast

$$\tilde{x}_k = \hat{x}_{k-1} + T\hat{\dot{x}}_{k-1}; \; \tilde{\dot{x}}_k = \hat{\dot{x}}_{k-1} \tag{8}$$

Estimation

$$\hat{x}_k = \tilde{x}_k + \alpha_k(y_k-\tilde{x}_k); \; \hat{\dot{x}}_k = \tilde{\dot{x}}_k + \frac{\beta_k}{T}(y_k-\tilde{x}_k) \tag{9}$$

where x_k is the target position at the k^{th} instant, $\dot{x}_k$ is the target velocity, y_k is the measured target position, T is the sampling interval and α_k, β_k are the gain coefficients.

It is not necessary to propagate explicity the state covariance terms in order to calculate the filter gain at each iteration. The decoupling of the state coordinates and the use of only position measurements simplify the Kalman recursion relations to the extent that the gain coefficients at the k^{th} instant may be calculated directly. This is shown by expressing the elements of the system gain K_k(opt) and the state covariance P_k at the k^{th} instant in terms of the elements of the state covariance P_{k-1} at the $(k-1)^{th}$ instant, from eqns. (4), (5) and (7).

Set

$$\hat{P}_{k-1} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}_{(k-1)} = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

where a is the position covariance, d is the velocity covariance and b is the cross-covariance.

Then

$$K_{k(opt)} = \frac{1}{(R+a+2bT+dT^2)} \begin{bmatrix} a+2bT+dT^2 \\ b+dT \end{bmatrix} \tag{10a}$$

$$\hat{P}_k = \frac{1}{(R+a+2bT+dT^2)} \begin{bmatrix} R(a+2bT+dT^2) & R(b+dT) \\ R(b+dT) & (a+R)d-b^2 \end{bmatrix} \tag{10b}$$

The measurement covariance R is a scalar quantity. The excitation noise Q is assumed to be zero in this initial development. (The inclusion of non-zero Q is considered in subsequent sections).

From the above definition of $K_{k(opt)}$ in terms of α_k, β_k and from the added definition $p_{22(k)} = R\delta_k/T^2$, the state covariance (eqn. (10b)) may be rewritten

$$\hat{P}_k = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}_k = \begin{bmatrix} R\alpha_k & R\beta_k/T \\ R\beta_k/T & R\delta_k/T^2 \end{bmatrix} \tag{10b'}$$

where $p_{11(k)} = E\{w_k^2\}$, ($w_k = \hat{x}_k - E\{\hat{x}_k\}$, and $E\{\cdot\}$ denotes statistical expectation)

$$p_{12(k)} = p_{21(k)} = E\{w_k\dot{w}_k\} = E\left\{w_k \cdot \frac{(w_k-w_{k-1})}{T}\right\} \simeq \frac{1}{T} E\{w_k^2\},$$

$$p_{22(k)} = E\{\dot{w}_k^2\} = \frac{1}{T^2} E\left\{(w_k-w_{k-1})^2\right\} \simeq \frac{1}{T^2} (E\{w_k^2\} + E\{w_{k-1}^2\})$$

$$\simeq \frac{2}{T^2} E\{w_k^2\},$$ assuming that w_k, w_{k-1} are statistically uncorrelated.

Finally, we obtain by inspection of eqns. (10b) and (10b') a set of three algebraic recursion relations, for α_k, β_k and δ_k, defining an additional parameter D_k for ease of notation:

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \delta_{k-1} \tag{11a}$$

$$\alpha_k = \frac{D_k - 1}{D_k} \tag{11b}$$

$$\beta_k = \frac{\beta_{k-1} + \delta_{k-1}}{D_k} \tag{11c}$$

$$\delta_k = \delta_{k-1} - \beta_k^2 D_k \tag{11d}$$

From eqns. (11) the coefficients α_k, β_k both approach zero asymptotically with increasing k, as the filter relies more and more on its own forecast and subsequent measurements receive progressively less weight in the estimation. An explicit value for R for each target coordinate is not required. Since R is the variance in a measurement of target position ($R = E\{V_k^2\} = r$, a scalar quantity) we may also define it to be the a priori uncertainty of the target position at the initiation of the tracking filter (i.e., $p_{11(o)} = r$). This assignment leads to a set of initial values for the filter gain coefficients, $\alpha_o=1$, $\beta_o=1$, and $\delta_o=2$. With α_o, β_o and δ_o taking on these numerical values it is possible to obtain the following explicit expressions for α_k, β_k, δ_k as functions of the iteration number k.

$$\alpha_k = \frac{2(2k+3)}{(k+2)(k+3)}; \quad \beta_k = \frac{6}{(k+2)(k+3)};$$

and

$$\delta_k = \frac{12}{(k+1)(k+2)(k+3)} \tag{12}$$

Figure 1 shows how α_k, β_k, δ_k vary as a function of k, in accordance with eqns. (11) or (12), using this initial assignment.

3.1.1 Random-velocity Models

In radar target tracking one would not permit α and β to decrease to zero, recognizing the need to provide for some uncertainty in the target model. This can be done by truncating the $(\alpha, \beta, \delta)_k$ values at some designed non-zero minimum. An alternative form of α-β filter includes this model uncertainty directly by providing for a non-zero Q. In eqn. (1) let U_k be a zero-mean random variable in velocity and Q_k, its covariance, be a scalar quantity q_v for each coordinate. The effect of this random velocity is included in the track-estimation process by means of the excitation matrix $\Gamma = (0\ 1)^t$, and serves to allow for target manoeuvres or deviations from the modelled dynamics of the target. The only change in the recursion relations (11) is the following:

$$\delta_k = \delta_{k-1} + \phi_1 - \beta_k^2 D_k \tag{11d'}$$

where $\phi_1 = q_v T^2/r$ is a system parameter.

The filter achieves a steady state with non-zero values for α, β and δ which are independent of the initial state of the filter $(\alpha_0, \beta_0, \delta_0)$ and depend only on ϕ_1. The transient behavior of the filter does depend on the initial state. Figure 2 gives an example of the dependence of the recursive filter coefficients α_k and β_k, as functions of k, on the parameter ϕ_1.

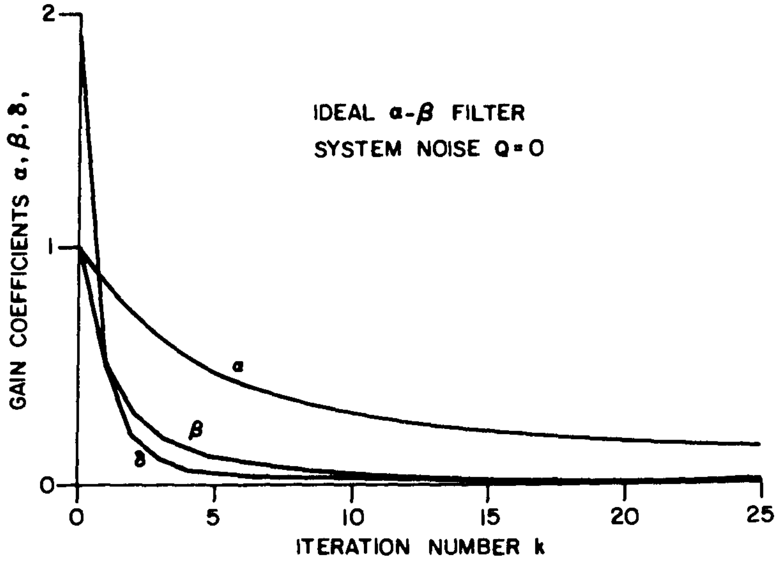

Figure 1. Asymptotic dependence of ideal α-β filter on iteration number

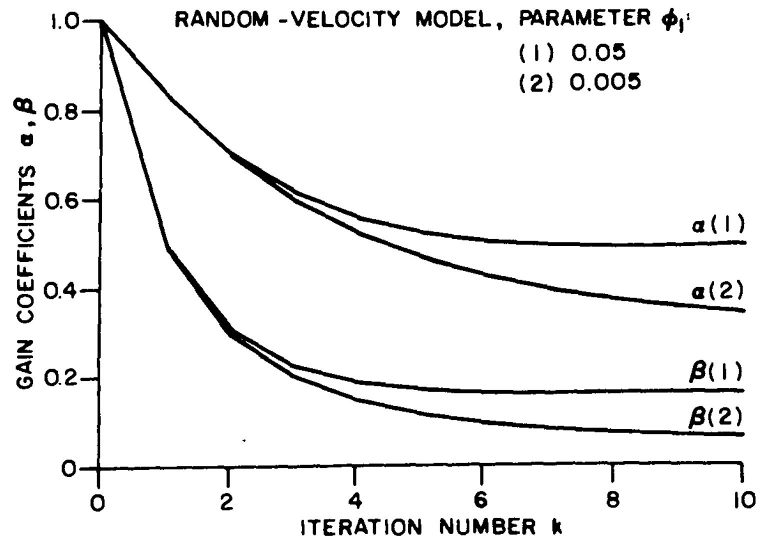

Figure 2. Transient behavior of α-β filter (random-velocity model)

As a modification to this form of filter, a different excitation matrix $\Gamma = (T\ 1)^t$ allows the random velocity variable to influence target position as well. The recursion relations must be rewritten as follows:

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \delta_{k-1} + \phi_1 \tag{13a}$$

$$\alpha_k = \frac{D_k - 1}{D_k} \tag{13b}$$

$$\beta_k = \frac{\beta_{k-1} + \delta_{k-1} + \phi_1}{D_k} \tag{13c}$$

$$\delta_k = \delta_{k-1} + \phi_1 - \beta_k^2 D_k \tag{13d}$$

The transient behavior differs very slightly from that of the preceding filter, but the final values for the coefficients again depend only on ϕ_1.

3.1.2 Adaptation

The sets of equations (11') and (13), in conjunction with eqns. (8) and (9), retain all the features of the original matrix formulation of the Kalman filter with the restrictions that (i) only target position is measured, (ii) only position and velocity are estimated and (iii) target coordinates can be decoupled and treated separately. When changes in Q, R or T occur, the Kalman filter incorporates them directly, from one iteration to the next, in its recursion equations. Normal operation of an α-β filter assumes constant values for the corresponding parameters q_v, r and T, but on-line adaptation to changes in these terms can be accommodated nevertheless. If filter adaptation is required between iterations (k-1) and (k), the coefficients α_{k-1}, β_{k-1}, δ_{k-1} in eqns. (11'), or (13), are first replaced by the coefficients

$$\alpha'_{k-1} = \left(\frac{r}{r'}\right) \alpha_{k-1};\ \beta'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)\beta_{k-1};\ \delta'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)^2 \delta_{k-1}, \tag{14}$$

and the system parameter is recomputed: $\phi'_1 = q'_v(T')^2/r'$. The coefficients α_k, β_k and δ_k are then obtained from the equations as usual. The superscript (') denotes the new or adapted values of the terms in question.

3.1.3 Steady-state Analysis

To simplify this α-β filter still further, one can eliminate the iterative computation of the filter coefficients by using the steady-state (S.S.) values appropriate to the current value of the system parameter ϕ_1. Only when ϕ_1 changes during the course of the tracking operation are the filter coefficients recalculated, and then once only, for all subsequent sampling intervals, or until ϕ_1 changes again. The analysis leading to the closed-form solution for $(\alpha,\beta)_{s.s.}$ as functions of ϕ_1 is outlined below.

For the recursive filter, it has been shown[2] that the number n(n+1)/2 of simultaneous equations that must generally be solved to obtain the steady-state gain matrix (where n is the size of the state vector) can be reduced to (np) simultaneous equations (where p is the size of the measurement vector), when $p < (n+1)/2$. For the α-β filter described by the set of equations (13), with a state vector of length 2 (position and velocity), we expect initially to have to solve three simultaneous, quadratically nonlinear equations. These equations are obtained by imposing the steady-state conditions $\alpha_k = \alpha_{k-1} = \alpha$, etc. and substituting in eqns. (13):

$$\alpha(\alpha + 2\beta + \delta + \phi_1) = 2\beta + \delta + \phi_1 \tag{15a}$$

$$\beta(\alpha + 2\beta + \delta + \phi_1) = \delta + \phi_1 \tag{15b}$$

$$\phi_1(1 + \alpha + 2\beta + \delta + \phi_1) = (\beta + \delta + \phi_1)^2 \tag{15c}$$

These reduce to two simultaneous equations, since only position measurements are available in this case (p=1). Recalling eqn. (13b) and noting that $D_k = D = 1+\alpha+2\beta+\delta+\phi_1$, divide (15c) by (15b), and add (15a) to (15b) to obtain the following results:

$$\beta^2 = \phi_1(1-\alpha) \tag{16}$$

$$\alpha(\alpha+\beta) = 2\beta \tag{17}$$

Equations (16) and (17) form the reduced set of simultaneous quadratically nonlinear equations. Equation (17), first obtained by Benedict and Bordner[3] and usually appearing in its alternative form as $\beta = \alpha^2/(2-\alpha)$, has often been quoted as an optimal design criterion for steady-state α-β filters. It specifies how the "optimal damping factor" β depends on the "system bandwidth" α, for all values of α. The original analysis left open the specification of α, a free parameter to be selected depending on the application. Equation (16) completes the analysis to provide the optimal specification of α as well, in terms of the global parameter ϕ_1. The same two equations would result from an analysis commencing with eqns. (11').

Combining eqns. (16) and (17) we obtain a linear quartic equation

$$\beta^4 - \phi_1\beta^3 - 2\phi_1\beta^2 - \phi_1^2\beta + \phi_1^2 = 0 \tag{18}$$

which may be solved by standard algebraic techniques. With the constraints that α and β must always be ≥ 0, and that $\alpha \leq 1$, the following unique solution is obtained:

$$\alpha = \frac{\phi_1}{8}\left(1 + \sqrt{1 + \frac{16}{\phi_1}}\right)\left\{\sqrt{2\left(1 + \sqrt{1 + \frac{16}{\phi_1}}\right)} - 2\right\} ;$$

$$\beta = \frac{\phi_1}{8}\sqrt{2\left(1 + \sqrt{1 + \frac{16}{\phi_1}}\right)}\left\{\sqrt{2\left(1 + \sqrt{1 + \frac{16}{\phi_1}}\right)} - 2\right\} \tag{19}$$

The limiting values are as follows: (i) as $\phi_1 \to 0$, α and β → 0 jointly; (ii) as

$\phi_1 \to \infty$, α and $\beta \to 1$. Figure 3 shows how $(\alpha,\beta)_{s.s.}$ vary with ϕ_1.

These expressions determine the optimal filtering coefficients of a steady-state α-β tracking filter for a system specified by the dimensionless parameter $\phi_1 = q_v T^2/r$, where target manoeuvres are described statistically by an additive zero-mean random velocity with covariance q_v. These expressions enable the steady-state filter to adapt immediately to any change in q_v, the measurement parameter r, or the update interval T. Values of α and β are computed using eqn. (19) only at track initiation, or when ϕ_1 changes during track life. Otherwise, only eqns. (8) and (9) are needed for track updating at each observation interval.

3.1.4 Random-acceleration Model

An alternative to the velocity model is one which assumes U_k in eqn. (1) to be a zero-mean random acceleration with covariance $Q = q_a$ (a scalar quantity), which is coupled into the filter equations by means of the excitation matrix $\Gamma = (T^2/2,\ T)^t$. The following recursion relations for this α-β filter are obtained, for each coordinate of the target:

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \delta_{k-1} + \phi_2 \tag{20a}$$

$$\alpha_k = \frac{D_k - 1}{D_k} \tag{20b}$$

$$\beta_k = \frac{\beta_{k-1} + \delta_{k-1} + 2\phi_2}{D_k} \tag{20c}$$

$$\delta_k = \delta_{k-1} + 4\phi_2 - \beta_k^2 D_k \tag{20d}$$

where $\phi_2 = q_a T^4/4r$ is the system parameter, and T and r are as defined previously. Figure 4 gives an example of the dependence on ϕ_2 of the recursive filter coefficients α_k and β_k, as functions of k. The closed-form solution to the corresponding steady-state filter is obtained in similar fashion as before. First we write the three simultaneous nonlinear equations:

$$\alpha(\alpha + 2\beta + \delta + \phi_2) = 2\beta + \delta + \phi_2 \tag{21a}$$

$$\beta(\alpha + 2\beta + \delta + \phi_2) = \delta + 2\phi_2 \tag{21b}$$

$$4\phi_2(1 + \alpha + 2\beta + \delta + \phi_2) = (\beta + \delta + 2\phi_2)^2 \tag{21c}$$

Then we reduce these to a set of two simultaneous nonlinear equations in the same way as was outlined for the random-velocity model (Section 3.1.3):

$$\beta^2 = 4\phi_2(1-\alpha) \tag{22}$$

$$(\alpha + \beta/2)^2 = 2\beta \tag{23}$$

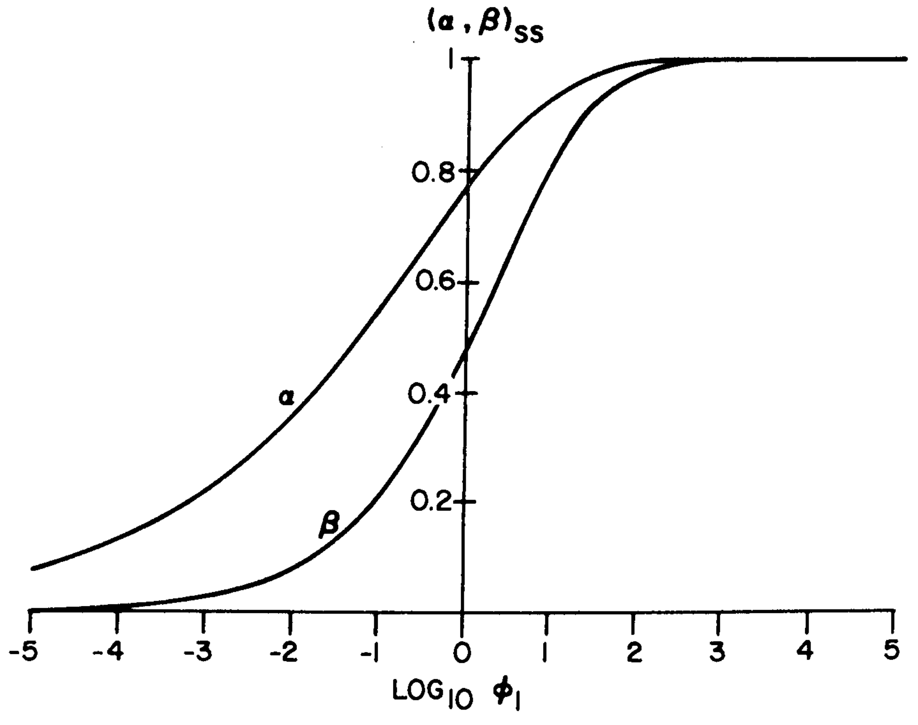

Figure 3. Dependence of steady-state α-β filter coefficients on random-velocity parameter ϕ_1

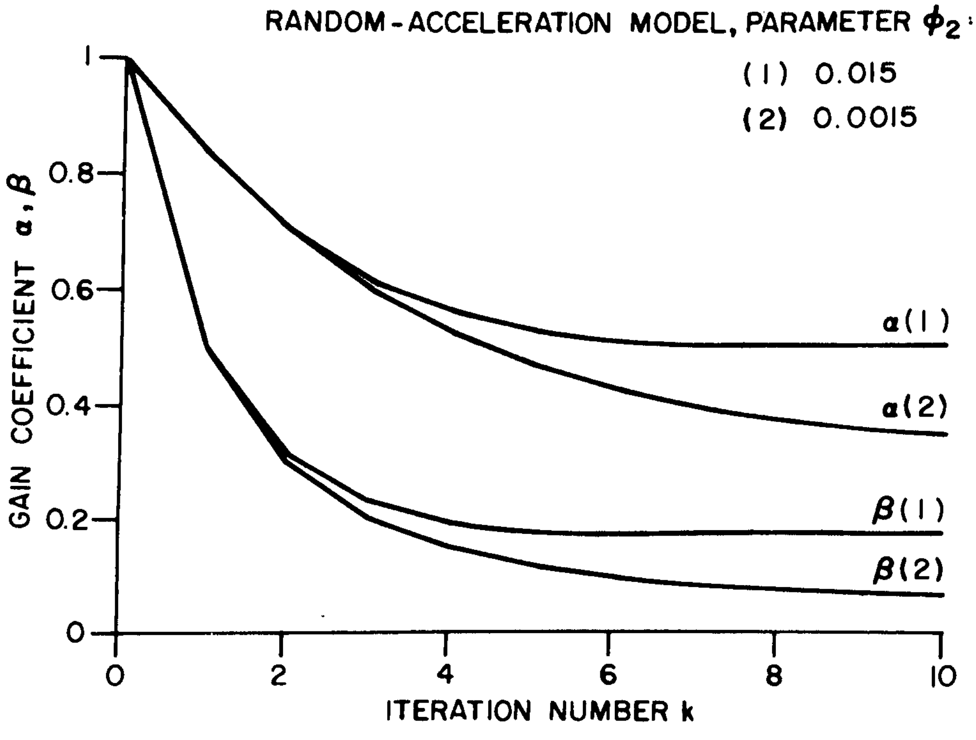

Figure 4. Transient behavior of α-β filter (random-acceleration model)

Equation (23) has appeared in the literature[4] in the form $\alpha = \sqrt{2\beta} - \beta/2$, defining the optimal α-β relationship for this filter model. Again the specification of one of the coefficients was left open. With the addition of eqn. (22) the optimal specification of both coefficients is obtainable in terms of the system parameter ϕ_2. Solving eqns. (22) and (23) we obtain the quartic equation:

$$\beta^4 - 4\phi_2\beta^3 + (4\phi_2^2 - 8\phi_2)\beta^2 - 16\phi_2^2\beta + 16\phi_2^2 = 0 \tag{24}$$

which may be solved by standard procedures. With the same system constraints as before ($\alpha,\beta > 0$, $\alpha \leq 1$) the following unique solution is obtained:

$$\alpha = \frac{1}{2}\sqrt{\phi_2 + 4\sqrt{\phi_2}}\left\{\sqrt{\phi_2} + 2 - \sqrt{\phi_2 + 4\sqrt{\phi_2}}\right\} \; ;$$
$$\beta = \sqrt{\phi_2}\left\{\sqrt{\phi_2} + 2 - \sqrt{\phi_2 + 4\sqrt{\phi_2}}\right\} \tag{25}$$

The limiting values are:

(i) as $\phi_2 \to 0$, α and $\beta \to 0$ jointly:

(ii) as $\phi_2 \to \infty$, $\alpha \to 1$ and $\beta \to 2$.

Figure 5 shows how $(\alpha,\beta)_{s.s.}$ vary with ϕ_2.

This result (eqn. (25)) is equivalent to one obtained previously by Friedland[5], which was cast in a different form.

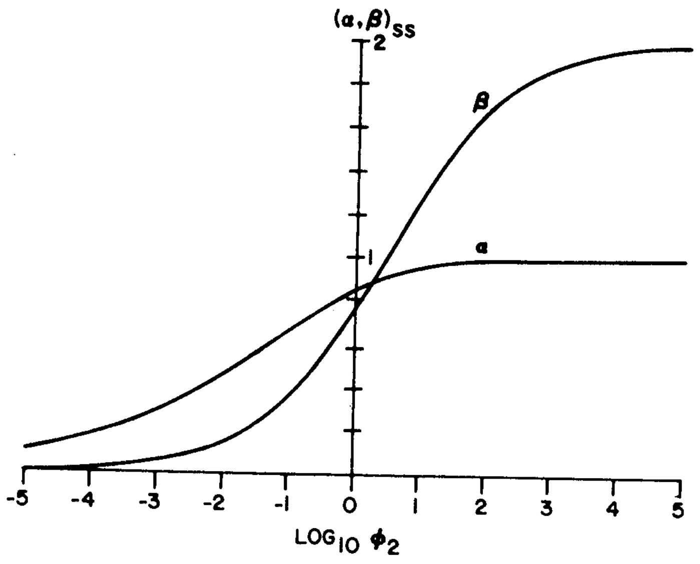

Figure 5. Dependence of steady-state α-β coefficients on random-acceleration parameter ϕ_2

3.1.5 Random-jerk Model

It is possible to extend these analyses to a filter model in which target manoeuvres are covered statistically by an additive zero-mean random "jerk" (rate of change of acceleration), with a covariance $Q=q_j$ coupled into the filter equations by means of the excitation matrix $\Gamma = (T^3/6,\ T^2/2)^t$. The following recursion relations are obtained:

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \delta_{k-1} + \phi_3 \tag{26a}$$

$$\alpha_k = \frac{D_k - 1}{D_k} \tag{26b}$$

$$\beta_k = \frac{\beta_{k-1} + \delta_{k-1} + 3\phi_3}{D_k} \tag{26c}$$

$$\delta_k = \delta_{k-1} + 9\phi_3 - \beta_k^2 D_k \tag{26d}$$

where $\phi_3 = q_j T^6/36r$ is the system parameter. Figure 6 gives an example of the transient behavior under this model.

For the steady-state version of the filter, we proceed analogously as before. The set of three simultaneous nonlinear equations for the steady-state condition is:

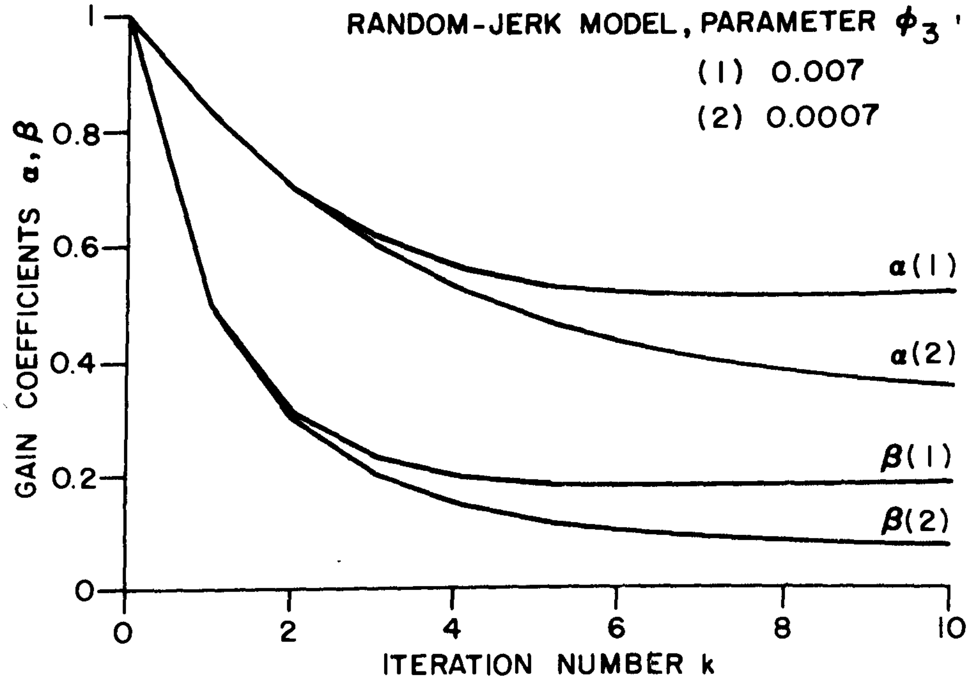

Figure 6. Transient behavior of α-β filter (random-jerk model)

$$\alpha(\alpha + 2\beta + \delta + \phi_3) = 2\beta + \delta + \phi_3 \tag{27a}$$

$$\beta(\alpha + 2\beta + \delta + \phi_3) = \delta + 3\phi_3 \tag{27b}$$

$$9\phi_3(1 + \alpha + 2\beta + \delta + \phi_3) = (\beta + \delta + 3\phi_3)^2 \tag{27c}$$

These may be reduced to the following two simultaneous nonlinear equations:

$$\beta^2 = 9\phi_3(1-\alpha) \tag{28}$$

$$(\alpha + 2\beta/3)(\alpha + \beta/3) = 2\beta \tag{29}$$

These equations specify completely the steady-state characteristics of the filter as a function of the parameter ϕ_3. Combining eqns. (28) and (29) gives the quartic equation

$$\beta^4 - 9\phi_3\beta^3 + 18\phi_3(\phi_3-1)\beta^2 - 81\phi_3^2\beta + 81\phi_3^2 = 0 \tag{30}$$

Which may be solved, under the same system constraints as before, to give the following unique solution:

$$\begin{aligned} \alpha &= \frac{\phi_3}{8}\sqrt{10 + 6\sqrt{1 + \frac{16}{\phi_3}}}\left\{3 + \sqrt{1 + \frac{16}{\phi_3}} - \sqrt{10 + 6\sqrt{1 + \frac{16}{\phi_3}}}\right\}\ ; \\ \beta &= \frac{3\phi_3}{4}\left\{3 + \sqrt{1 + \frac{16}{\phi_3}} - \sqrt{10 + 6\sqrt{1 + \frac{16}{\phi_3}}}\right\} \end{aligned} \tag{31}$$

The limiting values are:

(i) as $\phi_3 \to 0$, α and $\beta \to 0$ jointly;

(ii) as $\phi_3 \to \infty$, $\alpha \to 1$ and $\beta \to 1.5$.

Figure 7 shows how $(\alpha,\beta)_{s.s.}$ vary with ϕ_3.

This form of α-β filter has not previously appeared in the literature.

3.1.6 Summary

This completes the theoretical treatment of the α-β filter. The relationship to the general Kalman filter has been shown and the recursive and adaptive behaviors have been described. Three different models for incorporating the uncertainty in the target dynamics were treated (the question of the actual choice of a particular model for a given application was beyond the scope of this report). The recursion relationships led to

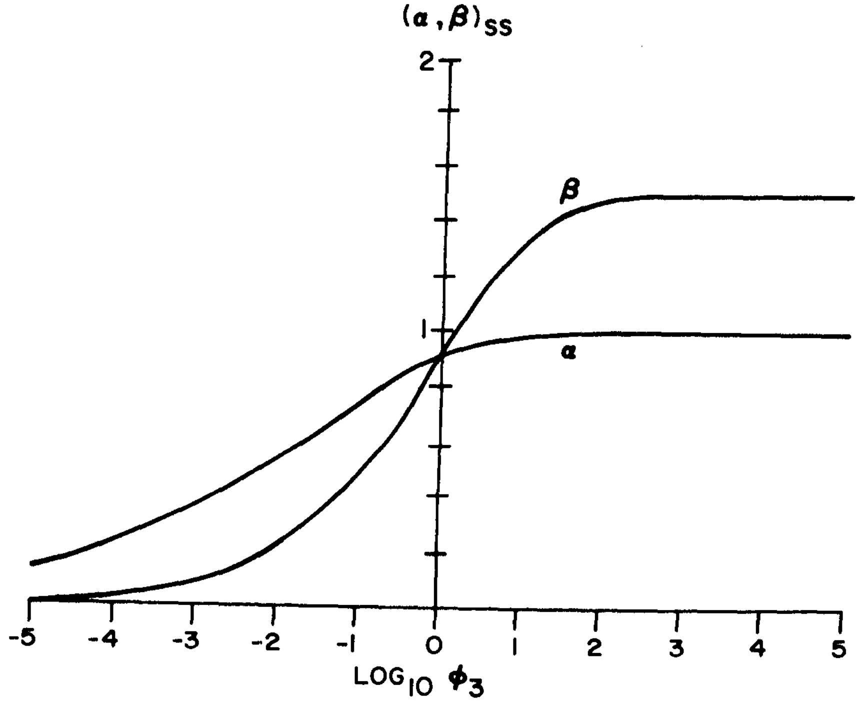

Figure 7. Dependence of steady-state α-β filter coefficients on random-jerk parameter ϕ_3

analyses of the steady-state filter and closed-form solutions for the optimal filtering coefficients were obtained for each model. The well-known Benedict-Bordner[3] relationship for steady-state α-β values was shown to arise out of a random-velocity model, and a complete specification of $(\alpha,\beta)_{s.s.}$ in terms of a single system parameter was given. An alternative relationship for $(\alpha,\beta)_{s.s.}$ mentioned elsewhere[4] in the literature was shown to arise out of a random-acceleration model; the complete specification of the filter corresponded to one described earlier by Friedland[5]. A new specification was also given in terms of a random-jerk model. These studies in steady-state tracking filters have therefore brought together earlier results hitherto unconnected and have indicated new forms which may be applied to practical problems. They have also suggested the possibility of using a steady-state adaptive filter for which optimal α-β filter coefficients can be computed as functions of the system parameters, thereby avoiding the necessity of recursively computing updated coefficients at every observation interval.

The question of the choice of a particular model to satisfy a given application will be examined in a subsequent report.

3.2 α-β-γ FILTER

The natural extension of the α-β filter is one which includes target acceleration as an explicit term in the state vector in addition to position and velocity. The resulting α-β-γ filter models a constant-acceleration target, with measurements made only on target position. Again it is assumed that each coordinate can be treated separately. We substitute in eqns. (3) and (6) the following expressions:

$$x_k = \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix} ; \; \Phi_k = \Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} ; \; K_{k(opt)} = \begin{bmatrix} \alpha_k \\ \beta_k/T \\ \gamma_k/T^2 \end{bmatrix} ;$$

$$y_k = [y_k]; \; M_k = M = [1 \; 0 \; 0]$$

to reduce the general Kalman formulation to the third-order algebraic equations:

Forecast

$$\tilde{x}_k = \hat{x}_{k-1} + T\hat{\dot{x}}_{k-1} + \frac{T^2}{2}\hat{\ddot{x}}_{k-1}; \; \tilde{\dot{x}}_k = \hat{\dot{x}}_{k-1} + T\hat{\ddot{x}}_{k-1}; \; \tilde{\ddot{x}}_k = \hat{\ddot{x}}_{1-1} \tag{32}$$

Estimation

$$\hat{x}_k = \tilde{x}_k + \alpha_k(y_k - \tilde{x}_k); \; \hat{\dot{x}}_k = \tilde{\dot{x}}_k + \frac{\beta_k}{T}(y_k - \tilde{x}_k); \; \hat{\ddot{x}}_k = \tilde{\ddot{x}}_k + \frac{\gamma_k}{T^2}(y_k - \tilde{x}_k) \tag{33}$$

where $\ddot{x}_k$ is the target acceleration, γ_k is the acceleration coefficient of the tracking filter, and all other terms are as defined in Section 3.1.

Under the assumption of an accurate model for target motion (Q=0), the filter coefficients are obtained at each observation instant, for constant T, by means of the set of recursion relations (34) given in Table 1. These are developed by a procedure exactly analogous to that outlined in Section 3.1 for the α-β filter. The coefficients α_k, β_k, etc. approach zero asymptotically with increasing k.

The terms γ_k, ε_k and η_k are derived from the additional independent elements required in the state covariance matrix P_k for a third-order filter:

$$\hat{P}_k = r \begin{bmatrix} \alpha_k & \beta_k/T & \gamma_k/T^2 \\ \beta_k/T & \delta_k/T^2 & \varepsilon_k/T^3 \\ \gamma_k/T^2 & \varepsilon_k/T^3 & \eta_k/T^4 \end{bmatrix}$$

Refer to Eqn. (10b') for comparison.

3.2.1 Random-acceleration Model

In the case of the second-order α-β filter, uncertainties in the modelled behavior of the target were covered by the excitation noise covariance Q. This was represented by a scalar term q_v, q_a or q_j, being the variance of a postulated zero-mean random activity in velocity, acceleration or jerk respectively, and was intended to accommodate estimation errors in the modelled position and velocity of the target. For the third-order α-β-γ filter, we use a similar scalar variance term q_a or q_j for zero-mean

TABLE 1

Recursive Equations for α-β-γ Filter With No Excitation Noise

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/4 \qquad (34a)$$

$$\alpha_k = \frac{D_{k-1}}{D_k} \qquad (34b)$$

$$\beta_k = \frac{\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + (3\varepsilon_{k-1} + \eta_{n-1})/2}{D_k} \qquad (34c)$$

$$\gamma_k = \frac{\gamma_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/2}{D_k} \qquad (34d)$$

$$\delta_k = \delta_{k-1} + 2\varepsilon_{k-1} + \eta_{k-1} - \beta_k^2 D_k \qquad (34e)$$

$$\varepsilon_k = \varepsilon_{k-1} + \eta_{k-1} - \beta_k \delta_k D_k \qquad (34f)$$

$$\eta_k = \eta_{k-1} - \gamma^2{}_k D_k \qquad (34g)$$

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random activity in acceleration or jerk to cover errors in the modelled position, velocity *and* acceleration of the target. Note that the term q_a in the third-order filter represents the variance about an estimated (possible non-zero) acceleration of the target, whereas in the second-order filter it represents the variance about an unestimated (assumed zero) target acceleration.

Analogously to the development of the α-β filter, the simplest way of introducing a target manoeuvre compensation would be by means of a zero-mean random acceleration term U_k, with a covariance Q_k coupled into the general filter eqn. (4) via the excitation matrix $\Gamma = (0\ 0\ 1)^t$. This would result in a set of equations (34') identical to the set (34) in Table 1 except for

$$\eta_k = \eta_{k-1} + \psi_1 - \gamma_k^2 D_k \qquad (34g')$$

where the system parameter is $\psi_1 = q_a T^4/r$, and r is the (scalar) position measurement variance and T is the update interval of the filter. Alternatively, the excitation matrix $\Gamma = (T^2/2, T, 1)^t$ may be used, which allows the random acceleration variable to influence the position and velocity terms directly. This yields a similar set of equations (35) differing slightly in detail (see Table 2). These two versions of the random-acceleration model produce very similar α-β-γ filters which display slightly different transient behavior but which reach identical steady-state values. Figure 8 gives an example of the transient behavior of the principal coefficients of this filter model, for two different values of ψ_1.

3.2.2 Adaptation

The inclusion of adaptive features in this recursive α-β-γ filter, when q_a, r and T are subject to change during the life of the track, is exactly analogous to the case for the α-β filter (Section 3.1.2). If adaptation is required between iterations (k-1) and (k), the six coefficients in (34') or (35), are first replaced by the coefficients:

$$\alpha'_{k-1} = \left(\frac{r}{r'}\right) \alpha_{k-1}; \quad \beta'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)\beta_{k-1}; \quad \gamma'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)^2 \gamma_{k-1};$$

$$\delta'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)^2 \delta_{k-1}; \quad \varepsilon'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)^3 \varepsilon_{k-1}; \quad \eta'_{k-1} = \left(\frac{r}{r'}\right)\left(\frac{T'}{T}\right)^4 \eta_{k-1}; \tag{36}$$

and the system parameter is recomputed: $\psi'_1 = q'_a(T')^4/r'$. The coefficients α_k, β_k, etc. are then obtained from the recursion relations as usual.

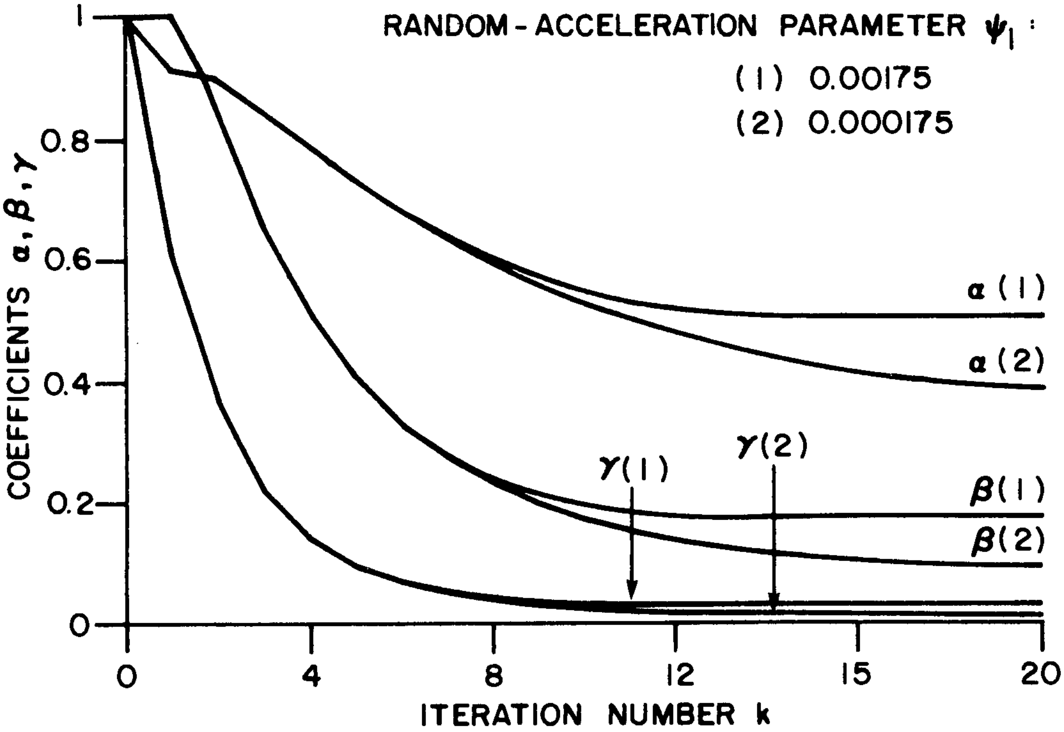

Figure 8. Transient behavior of α-β-γ filter (random-acceleration model)

TABLE 2

Recursive Equations for α-β-γ Filter With System Parameter ψ_1

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/4 + \psi_1/4 \qquad (35a)$$

$$\alpha_k = \frac{D_{k-1}}{D_k} \qquad (35b)$$

$$\beta_k = \frac{\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + (3\varepsilon_{k-1} + \eta_{k-1})/2 + \psi_1/2}{D_k} \qquad (35c)$$

$$\gamma_k = \frac{\gamma_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/2 + \psi_1/2}{D_k} \qquad (35d)$$

$$\delta_k = \delta_{k-1} + 2\varepsilon_{k-1} + \eta_{k-1} + \psi_1 - \beta_k^2 D_k \qquad (35e)$$

$$\varepsilon_k = \varepsilon_{k-1} + \eta_{k-1} + \psi_1 - \beta_k \gamma_k D_k \qquad (35f)$$

$$\eta_k = \eta_{k-1} + \psi_1 - \gamma_k^2 D_k \qquad (35g)$$

3.2.3 Steady-state Analysis

The steady-state values of the filter coefficients α, β, and γ can be calculated in closed form as a function of the system parameter ψ_1. Since the length of the state vector is n=3, we expect n(n+1)/2 = 6 simultaneous equations to solve for the steady-state coefficients. These correspond to the six unknowns represented by the coefficients α, β, γ, δ, ε, η. However since the length of the observation vector is p=1, it is sufficient to solve the reduced set of np=3 equations, which correspond to the three essential coefficients of the α-β-γ tracking filter. Setting $\alpha_{k+1} = \alpha_k = \alpha$, etc. in eqns. (35), the six simultaneous, quadratically nonlinear equations may be written as shown in Table 3 (eqn. (37)). After some algebraic manipulation these reduce to the following three equations:

$$\gamma^2 = \psi_1(1-\alpha) \qquad (38)$$

$$\beta^2 = 2\alpha\gamma \qquad (39)$$

$$\alpha(\alpha + \beta + \gamma/2) = 2\beta \qquad (40)$$

Equations (38), (39), and (40) form the reduced set of simultaneous quadrati-

TABLE 3

Simultaneous Equations for the Steady-state α-β-γ Filter Using ψ_1

$$\alpha(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4) = 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4 \qquad (37a)$$

$$\beta(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4) = \gamma + \delta + (3\varepsilon+\eta)/2 + \psi_1/2 \qquad (37b)$$

$$\gamma(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4) + \psi_1/4) = \varepsilon + \eta/2 + \psi_1/2 \qquad (37c)$$

$$(2\varepsilon + \eta + \psi_1)(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4)$$
$$= [\beta + \gamma + \delta + (3\varepsilon+\eta)/2 + \psi_1/2]^2 \qquad (37d)$$

$$(\eta + \psi_1)(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4)$$
$$= [\beta + \gamma + \delta + (3\varepsilon+\eta)/2 + \psi_1/2][\gamma + \varepsilon + \eta/2 + \psi_1/2] \qquad (37e)$$

$$\psi_1(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_1/4) = [\gamma + \varepsilon + \eta/2 + \psi_1/2]^2 \qquad (37f)$$

---

cally nonlinear equations which fully characterize this steady-state α-β-γ filter. Equation (40) was first obtained by Simpson[6], a result of numerical analyses extending the original work of Benedict and Bordner[3] on α-β filters. Equation (39) was later presented along with eqn. (40) by Neal[7] as a result of an analysis of a Kalman filter model with a Gauss-Markov random-acceleration model. Neal's result leaves one free parameter to be specified by the application. The analysis presented here derives eqn. (38) as well, and completes the specification of the filter as a function of a single system parameter ψ_1.

The closed-form solution of the reduced set is less straightforward than in the cases described in Section 3.1. The reduced set of three simultaneous equations produces one sextic polynomial equation which is not generally solvable. Without a specific factorization to render it solvable a more circuitous method must be used. By combining eqns. (39) and (40) the relationship $(\alpha+\beta/2)^2 = 2\beta$ is obtained (Neal also noted this relationship in the equivalent form $(2\alpha+\beta)^2 = 8\beta$), identical to that for the random-acceleration model for the α-β filter (eqn. (23)). This means that, for both second-order and third-order filters, each pair of optimal α-β values is related by the same equation (23), and is obtained from a unique value of ϕ_2 for the

α-β filter or from a corresponding unique value of ψ_1 for the α-β-γ filter. From eqns. (25), (38) and (39), this correspondence may be expressed by

$$\psi_1 = \frac{4\phi_2^2}{\phi_2 + 4\sqrt{\phi_2}} \tag{41}$$

Rewriting this equation as a cubic polynomial and solving for ϕ_2 explicitly yields

$$\phi_2 = \frac{\psi_1}{6}\left[1 - \frac{1}{2}\left\{\sqrt[3]{1 - \frac{864}{\psi_1}\left(1 + \sqrt{1 - \frac{\psi_1}{432}}\right)} + \sqrt[3]{1 - \frac{864}{\psi_1}\left(1 - \sqrt{1 - \frac{\psi_1}{432}}\right)}\right\}\right]$$

$$\text{for } \psi_1 \leq 432;$$

$$= \frac{\psi_1}{3}\text{, for } \psi_1 = 432;$$

$$= \frac{\psi_1}{6}\left[1 + \cos\left\{\frac{1}{3}\cos^{-1}\left(\frac{864}{\psi_1} - 1\right)\right\}\right], \quad \text{for } \psi_1 \geq 432. \tag{42}$$

For a given value of system parameter ψ_1, a corresponding value of ϕ_2 is computed by means of eqn. (42). Explicit values of α, β, and γ are then computed from eqns. (25) and (39). Figure 9 shows how $(\alpha, \beta, \gamma)_{s.s}$ vary with ψ_1.

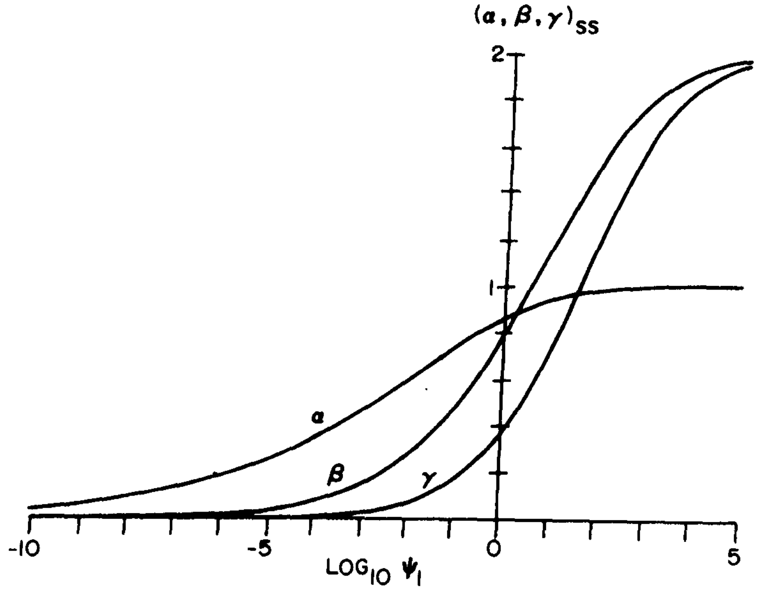

Figure 9. Dependence of steady-state α-β-γ filter coefficients on random-acceleration parameter ψ_1

The solution for the steady-state filter was obtained through the happy occurrence of an identical functional relationship between the coefficients α and β for both the second-order (α-β) filter and third-order (α-β-γ) filter when the same model for a random-acceleration perturbation on the target is included. This does not mean that the two filters are identical in other respects, in spite of the fact that their respective global parameters ϕ_2 and ψ_1 have similar structures. For matching values of q_a, r and T the resulting value for $\psi_1 = q_a T^4/r$ in the case of the third-order filter will in turn produce a corresponding value of ϕ_2 which is not the same as that which could be obtained for the second-order filter from $\phi_2 = q_a T^4/4r$. Therefore, the respective coefficients α and β in the two cases would not correspond, and the filters would differ in behavior. This is because, as stated at the beginning of Section 3.2.1, the term q_a refers to two quite different acceleration variances in the two types of filter. Using the same quantitative value for q_a in both instances would indicate an attempt to cover two different types of unmodelled accelerative behavior by the same noise statistics.

The use of the third-order steady-state filter in an adaptive fashion follows analogously from the description given at the end of Section 3.1.3, and the new closed-form solution for the optimal steady-state condition can contribute to its implementation. However, their close interrelationship suggests the possibility that in a practical system the track-estimation operation could be switched back and forth between a second-order and a third-order filter depending on the current behavior of the target. The second-order filter is better suited to producing smoothed estimates of a constant-velocity (straight-line) target track while the third-order filter can better handle a constant-acceleration (turning) target.

3.2.4 Random-jerk Model

As with the second-order filters (Section 3.1.5), it is possible to extend the analysis to a target model which covers manoeuvres by a zero-mean random jerk, covariance q_j, now coupled into the third-order filter equations by means of the excitation matrix $\Gamma = (T^3/6, T^2/2, T)^t$. The resulting recursion relations (43) are given in Table 4. Figure 10 gives an example of the transient behavior of the principal coefficients of this filter model, for two different values of the system parameter ψ_2 as defined in Table 4.

For the steady-state version of the filter, we can obtain a corresponding set of six simultaneous, quadratically nonlinear equations (Table 5, Eqn. (44)) which reduces to the following three simultaneous nonlinear equations:

$$\gamma^2 = 36\psi_2(1-\alpha) \tag{45}$$

$$\beta^2 + \gamma^2/12 = 2\alpha\gamma \tag{46}$$

$$(\alpha + \beta/2 + \gamma/12)(\alpha + \beta/2 - \gamma/12) = 2\beta \tag{47}$$

Equations (45), (46) and (47) fully characterize this steady-state third-order filter.

A closed-form solution to this set of equations has not been found. The eighth-order polynomial resulting from these equations is, of course, not generally solvable, and this writer has been unable to discover a

TABLE 4

Recursive Equations for α-β-γ Filter With System Parameter ψ_2

$$D_k = 1 + \alpha_{k-1} + 2\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/4 + \psi_2 \tag{43a}$$

$$\alpha_k = \frac{D_k - 1}{D_k} \tag{43b}$$

$$\beta_k = \frac{\beta_{k-1} + \gamma_{k-1} + \delta_{k-1} + (3\varepsilon_{k-1} + \eta_{k-1})/2 + 3\psi_2}{D_k} \tag{43c}$$

$$\gamma_k = \frac{\gamma_{k-1} + \varepsilon_{k-1} + \eta_{k-1}/2 + 6\psi_2}{D_k} \tag{43d}$$

$$\delta_k = \delta_{k-1} + 2\varepsilon_{k-1} + \eta_{k-1} + 9\psi_2 - \beta_k^2 D_k \tag{43e}$$

$$\varepsilon_k = \varepsilon_{k-1} + \eta_{k-1} + 18\psi_2 - \beta_k \gamma_k D_k \tag{43f}$$

$$\eta_k = \eta_{k-1} + 36\psi_2 - \delta_k^2 D_k \tag{43g}$$

where

$$\psi_2 = q_j T^6/36r.$$

---

specific factorization which could render it so. Further, there is apparently no corresponding functional relationship with the second-order random-jerk filter model (Section 3.1.5) as occurred between the second- and third-order random-acceleration models (Sections 3.1.4, 3.2.3). The steady-state values of the filter coefficients α, β and γ can be obtained by iterative computation from the recursion relationships (43), and their variation as a function of the system parameter ψ_2 is shown in Figure 11. The limiting values are:

(i) as $\psi_2 \to 0$, $\alpha, \beta, \gamma \to 0$ jointly;

(ii) as $\psi_2 \to \infty$, $\alpha \to 1$, $\beta \to \sqrt{3}$ and $\gamma \to 6(2-\sqrt{3})$.

TABLE 5

Simultaneous Equations for the Steady-state α-β-γ Filter Using ψ_2

$$\alpha(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2) = 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2 \tag{44a}$$

$$\beta(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2) = \gamma + \delta + (3\varepsilon+\eta)/2 + 3\psi_2 \tag{44b}$$

$$\gamma(\alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2) = \varepsilon + \eta/2 + 6\psi_2 \tag{44c}$$

$$(2\varepsilon + \eta + 9\psi_2)(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2)$$

$$= [\beta + \gamma + \delta + (3\varepsilon+\eta)/2 + 3\psi_2]^2 \tag{44d}$$

$$(\eta + 18\psi_2)(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2)$$

$$= [\beta + \gamma + \delta + (3\varepsilon+\eta)/2 + 3\psi_2][\gamma + \varepsilon + \eta/2 + 6\psi_2] \tag{44e}$$

$$36\psi_2(1 + \alpha + 2\beta + \gamma + \delta + \varepsilon + \eta/4 + \psi_2) = [\gamma + \varepsilon + \eta/2 + 6\psi_2]^2 \tag{44f}$$

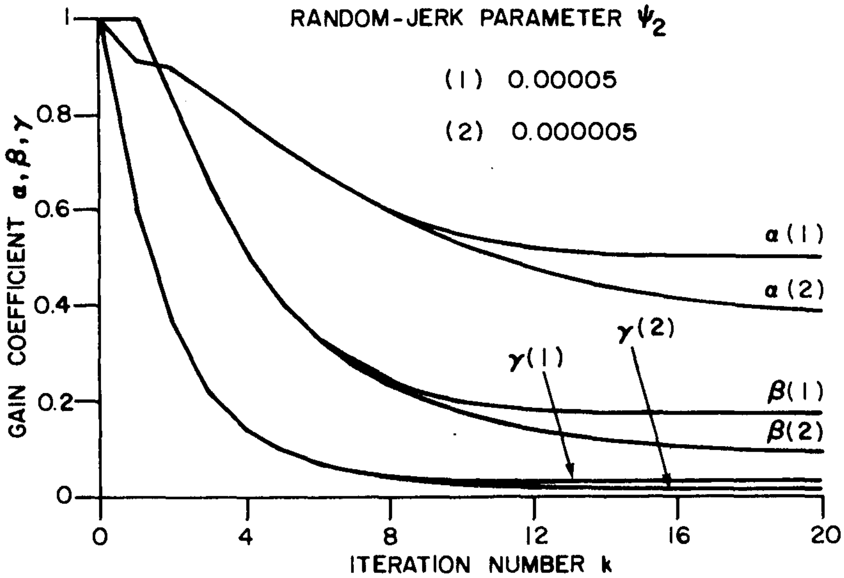

Figure 10. Transient behavior of α-β-γ filter (random-jerk model)

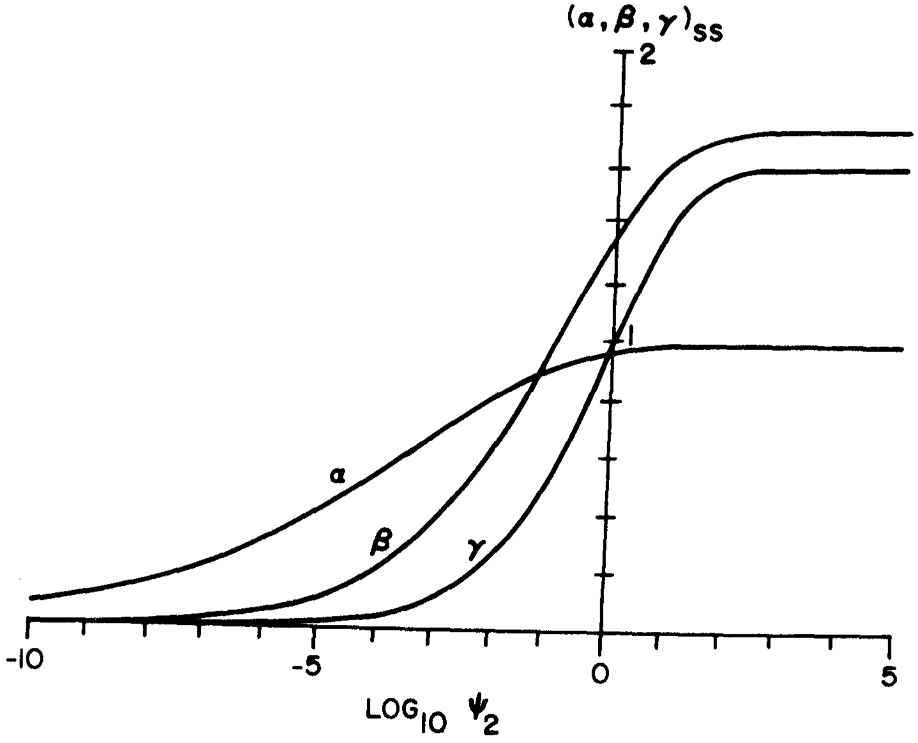

Figure 11. Dependence of steady-state α-β-γ filter coefficients on random-jerk parameter ψ_2

3.2.5 Summary

This completes the theoretical treatment of the α-β-γ tracking filter. Two different models for target manoeuvre were considered; the recursive and steady-state behaviors have been analyzed and the resulting filter structures have been described. Like the second-order α-β filter, these third-order filters are applied to radar measurements of target position only, but in addition they yield estimates of target acceleration (as well as position and velocity) and are therefore better suited to handling manoeuvring targets. This analysis has extended the earlier work of Simpson[6] and Neal[7] on the random-acceleration model for a steady-state third-order filter and has completed the specification of the filter coefficients by means of a closed-form functional relationship with a single system parameter. The simultaneous equations characterizing an alternative, random-jerk model for the third-order filter were also derived, but without a closed-form solution.

A performance analysis comparing these third-order filters with the second-order α-β filters is planned for a subsequent report.

3.3 REDUCED KALMAN FILTER

A further extension of the α-β form of filter is one which includes measurements of target velocity as well as position. In practice, this type of filter would be used with radar sensors which provide measurements of the target's radial velocity, or range-rate, by means of doppler processing. This form is often called the reduced Kalman filter, where it is again assumed that each coordinate can be treated independently. One-dimensional recursive filter structures can be developed for this form, but owing to their greater complexity in comparison with the α-β(-γ) filters, it has not been possible to develop closed-form solutions for the steady-state conditions. Steady-state values of the filter coefficients must be computed by iteration of the recursion relations.

3.3.1 Second-order Structures

For filters which model only position and velocity in their state vector representation, the following forms are appropriate. In the general equations (3) and (6) substitute

$$x_k = \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} ; \; \Phi_k = \Phi = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} ; \; K_{k(opt)} = \begin{bmatrix} \alpha_k & b_k T \\ \beta_k/T & d_k \end{bmatrix} ; \; Y_k = \begin{bmatrix} y_k \\ \dot{y}_k \end{bmatrix} ; \; M_k = M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then the tracking filter is described by the equations:

Forecast (Eqn. (8), Section 3.1)

$$\tilde{x}_k = \hat{x}_{k-1} + T\hat{\dot{x}}_{k-1}; \; \tilde{\dot{x}}_k = \hat{\dot{x}}_{k-1}$$

Estimation

$$\begin{aligned} \hat{x}_k &= \tilde{x}_k + \alpha_k(y_k - \tilde{x}_k) + b_k T(\dot{y}_k - \tilde{\dot{x}}_k) \\ \hat{\dot{x}}_k &= \tilde{\dot{x}}_k + \frac{B_k}{T}(y_k - \tilde{x}_k) + d_k(\dot{y}_k - \tilde{\dot{x}}_k) \end{aligned} \tag{48}$$

where b_k and d_k are the additional gain coefficients necessary to absorb the velocity measurement $\dot{y}$, and all other terms are as defined in Section 3.1.

The covariance of the measurement noise vector V_k in eqn. (2) is

$$R_k = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}_k$$

where the scalars r_1 and r_2 are, respectively, the variances associated with the position measurement y_k and the velocity measurement $\dot{y}_k$. These measurements are uncorrelated. In order to cover the uncertainty in the chosen model for the target motion, the covariance Q_k of the excitation noise vector U_k is used, as before. The three types of model uncertainty considered previously are invoked once again: random velocity, random acceleration or random jerk, with their corresponding scalar covariance terms q_v, q_a, or q_j. To develop the appropriate recursive filter equations, the same system parameters are required as previously: $\phi_1 = q_v T^2/r_1$, $\phi_2 = q_a T^4/4r_1$, or $\phi_3 = q_j T^6/36r_1$, depending on the choice of model (refer to Section 3.1 on the α-β filter). In addition, a second system parameter is required: $\phi_r = r_2 T^2/r_1$, the position-velocity measurement term, which is common to all three models. The following three recursion relations (49b,c,d) are then obtained, with the term $\overline{D}_k$ being introduced (49a) as a notational convenience:

$$\overline{D}_k = AD-B^2 \tag{49a}$$

$$\alpha_k = \frac{\overline{D}_k - D}{\overline{D}_k} \tag{49b}$$

$$\beta_k = \frac{\phi_r B}{\overline{D}_k} = \phi_r b_k \tag{49c}$$

$$d_k = \frac{\overline{D}_k - \phi_r A}{\overline{D}_k} \tag{49d}$$

The terms A, B and D depend on the choice of model. The corresponding expressions for them are given in Table 6, together with the appropriate forms for the excitation matrix Γ used in deriving them. Figures 12, 13 and 14 illustrate the transient behavior of these three second-order reduced Kalman filters, for selected values of the system parameters.

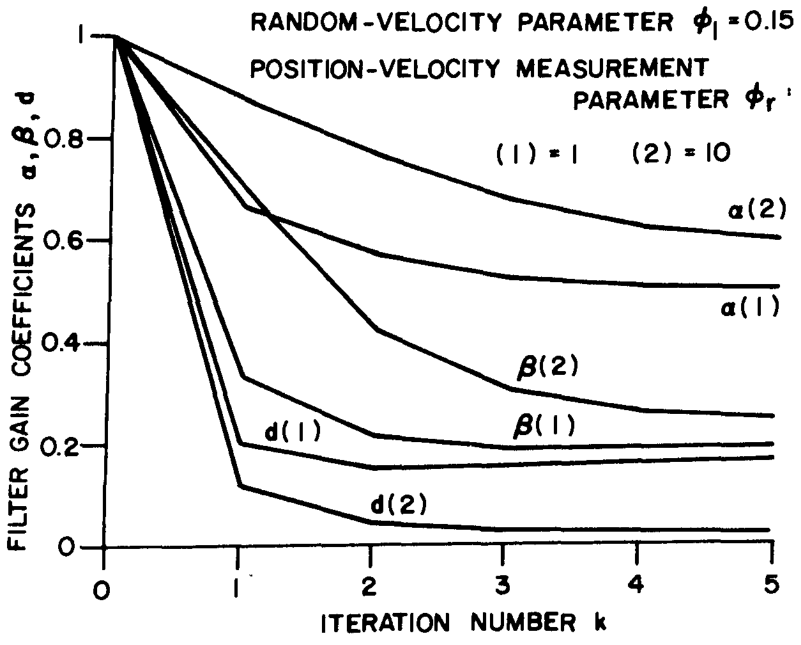

Figure 12. Transient behavior of gain coefficients: second-order reduced-Kalman filter (random-velocity model)

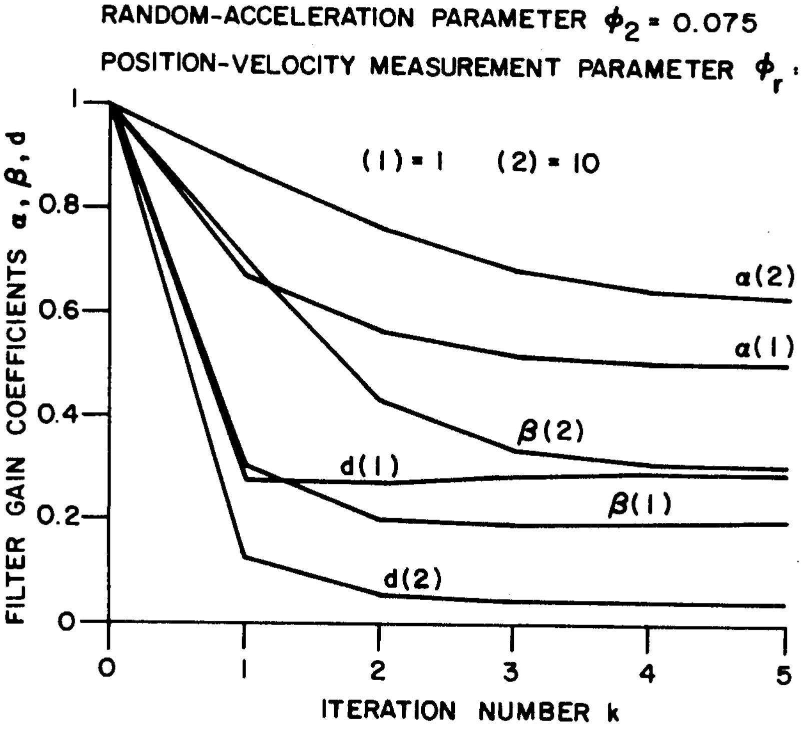

Figure 13. Transient behavior of gain coefficients: second-order reduced-Kalman filter (random-acceleration model)

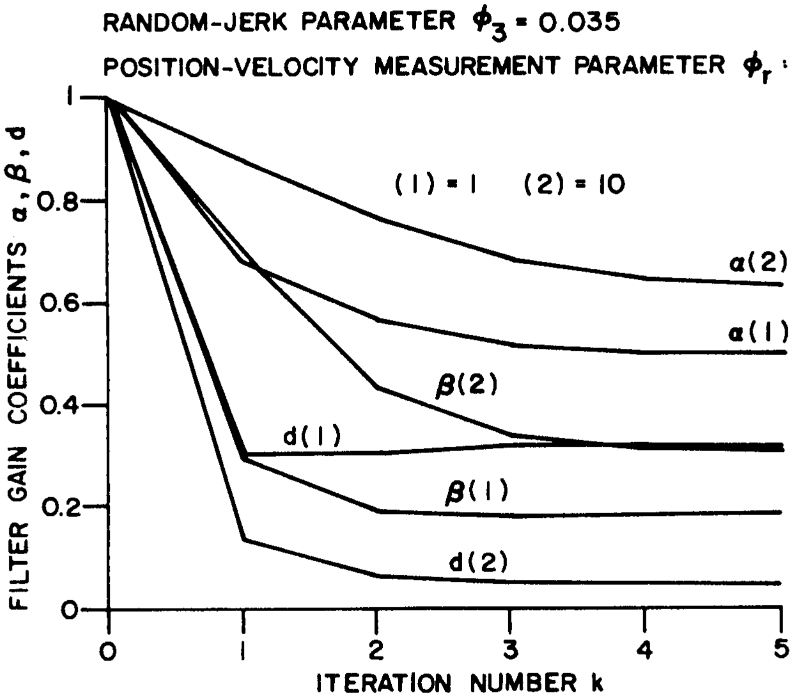

Figure 14. Transient behavior of gain coefficients: second-order reduced-Kalman filter (random-jerk model)

TABLE 6

Elements of the 2nd-order Reduced Kalman Filter

	RANDOM-VELOCITY MODEL	RANDOM-ACCELERATION MODEL	RANDOM-JERK MODEL
A	$1+\alpha_{k-1}+2\beta_{k-1}+\phi_1+\phi_r d_{k-1}$	$1+\alpha_{k-1}+2\beta_{k-1}+\phi_2+\phi_r d_{k-1}$	$1+\alpha_{k-1}+2\beta_{k-1}+\phi_3+\phi_r d_{k-1}$
B	$\beta_{k-1}+\phi_1+\phi_r d_{k-1}$	$\beta_{k-1}+2\phi_2+\phi_r d_{k-1}$	$\beta_{k-1}+3\phi_3+\phi_r d_{k-1}$
D	$\phi_1+\phi_r(1+d_{k-1})$	$4\phi_2+\phi_r(1+d_{k-1})$	$9\phi_3+\phi_r(1+d_{k-1})$
Γ	$(T,1)^t$	$(T^2/2,T)^t$	$(T^3/6,T^2/2)^t$

3.3.2 Third-order Structures

For filters which model position, velocity and acceleration, the following forms apply, substituting in eqns. (3) and (6):

$$x_k = \begin{bmatrix} x_k \\ \dot{x}_k \\ \ddot{x}_k \end{bmatrix}; \Phi_k = \Phi = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}; K_{k(opt)} = \begin{bmatrix} \alpha_k & b_k T \\ \beta_k/T & d_k \\ \gamma_k/T^2 & e_k/T \end{bmatrix}$$

$$Y_k = \begin{bmatrix} y_k \\ \dot{y}_k \end{bmatrix}; M_k = M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The tracking filter is described by the equations:

Forecast (Eqn. (32), Section 3.2)

$$\tilde{x}_k = \hat{x}_{k-1} + T\hat{\dot{x}}_{k-1} + \frac{T^2}{2}\hat{\ddot{x}}_{k-1}$$

$$\tilde{\dot{x}}_k = \hat{\dot{x}}_{k-1} + T\hat{\ddot{x}}_{k-1}; \quad \tilde{\ddot{x}}_k = \hat{\ddot{x}}_{k-1}$$

Estimation

$$\hat{x}_k = \tilde{x}_k + \alpha_k(y_k-\tilde{x}_k) + b_k T(\dot{y}_k-\tilde{\dot{x}}_k)$$

$$\hat{\dot{x}}_k = \tilde{\dot{x}}_k + \frac{\beta_k}{T}(y_k-\tilde{x}_k) + d_k(\dot{y}_k-\tilde{\dot{x}}_k)$$

$$\hat{\ddot{x}}_k = \tilde{\ddot{x}}_k + \frac{\gamma_k}{T^2}(y_k-\tilde{x}_k) + \frac{e_k}{T}(\dot{y}_k-\tilde{\dot{x}}_k) \qquad (50)$$

where e_k is a further gain coefficient necessary to absorb the velocity measurement into the acceleration estimate, and all other terms are as defined previously (Sections 3.1-3.3).

The use of a random-acceleration form for the zero-mean excitation noise U_k is the simplest way to cover uncertainties in the model of target motion and to compensate for target manoeuvre (see Section 3.2.1). As before, the acceleration covariance is $Q=q_a$ (a scalar quantity) and the system parameter is $\psi_1 = q_a T^4/r_1$. The excitation matrix may take two different forms: $\Gamma = (0\ 0\ 1)^t$, or $\Gamma = (T^2/2,T,1)^t$. These will result in similar sets of recursion relations, exhibiting slightly different transient behavior but reaching identical steady-state values. Alternatively, a zero-mean random jerk, covariance q_j, may be used to compensate for target model uncertainties. The system parameter becomes $\psi_2 = q_j T^6/36r_1$, where the excitation matrix is $\Gamma = (T^3/6,T^2/2,T)^t$.

We can write a general set of recursion relations for these three versions of the third-order filter:

$$\bar{D}_k = AD-B^2 \tag{51a}$$

$$\alpha_k = \frac{\bar{D}_k-D}{\bar{D}_k} \tag{51b}$$

$$\beta_k = \frac{\phi_r B}{\bar{D}_k} = \phi_r b_k \tag{51c}$$

$$d_k = \frac{\bar{D}_k-\phi_r A}{\bar{D}_k} \tag{51d}$$

$$\gamma_k = \frac{CD-BE}{\bar{D}_k} \tag{51e}$$

$$e_k = \frac{AE-BC}{\bar{D}_k} \tag{51f}$$

$$f_k = f_{k-1} + \frac{1}{\phi_r}(z - \gamma_k C - e_k E) \tag{51g}$$

where $\phi_r = r_2T^2/r_1$ as before, f_k is an additional term required for the recursion, and z is replaced in eqn. (51g) by ψ_1 in the case of a random-acceleration model or by $36\psi_2$ in the case of the random-jerk model. The expressions represented by terms A,B,C,D and E are found in Table 7. They depend on the choice of filter model. Illustrations of the transient behavior of these filters are given in Figures 15 and 16.

TABLE 7

Elements of the 3rd-order Reduced Kalman Filter

	RANDOM-ACCELERATION MODEL I	RANDOM-ACCELERATION MODEL II	RANDOM-JERK MODEL
A	$1+\alpha_{k-1}+2\beta_{k-1}+\gamma_{k-1}+\phi_r(d_{k-1}+e_{k-1}+f_{k-1}/4)$	$1+\alpha_{k-1}+2\beta_{k-1}+\gamma_{k-1}+\psi_1/4+$ $\phi_r(d_{k-1}+e_{k-1}+f_{k-1}/4)$	$1+\alpha_{k-1}+2\beta_{k-1}+\gamma_{k-1}+\psi_2+$ $\phi_r(d_{k-1}+e_{k-1}+f_{k-1}/4)$
B	$\beta_{k-1}+\gamma_{k-1}+\phi_r(d_{k-1}+(3e_{k-1}+f_{k-1})/2)$	$\beta_{k-1}+\gamma_{k-1}+\psi_1/2+$ $\phi_r(d_{k-1}+(3e_{k-1}+f_{k-1})/2)$	$\beta_{k-1}+\gamma_{k-1}+3\psi_2+$ $\phi_r(d_{k-1}+(3e_{k-1}+f_{1-1})/2)$
C	$\gamma_{k-1}+\phi_r(e_{k-1}+f_{k-1}/2)$	$\gamma_{k-1}+\psi_1/2+\phi_r(e_{k-1}+f_{k-1}/2)$	$\gamma_{k-1}+6\psi_2+\phi_r(e_{k-1}+f_{k-1}/2)$
D	$\phi_r(1+d_{k-1}+2e_{k-1}+f_{k-1})$	$\psi_1+\phi_r(1+d_{k-1}+2e_{k-1}+f_{k-1})$	$9\psi_2+\phi_r(1+d_{k-1}+2e_{k-1}+f_{k-1})$
E	$\phi_r(e_{k-1}+f_{k-1})$	$\psi_1+\phi_r(e_{k-1}+f_{k-1})$	$18\psi_2+\phi_r(e_{k-1}+f_{k-1})$
Γ	$(0\ 0\ 1)^t$	$(T^2/2,\ T,\ 1)^t$	$(T^3/6,\ T^2/2,\ T)^t$

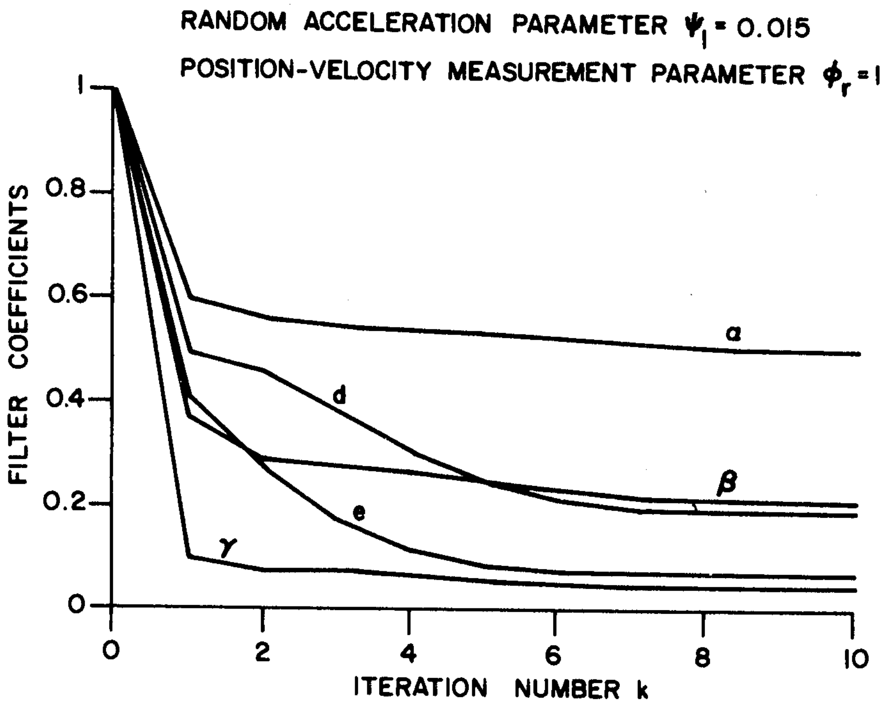

Figure 15. Transient behavior of gain coefficients: third-order reduced-Kalman filter (random-acceleration model, I and II)

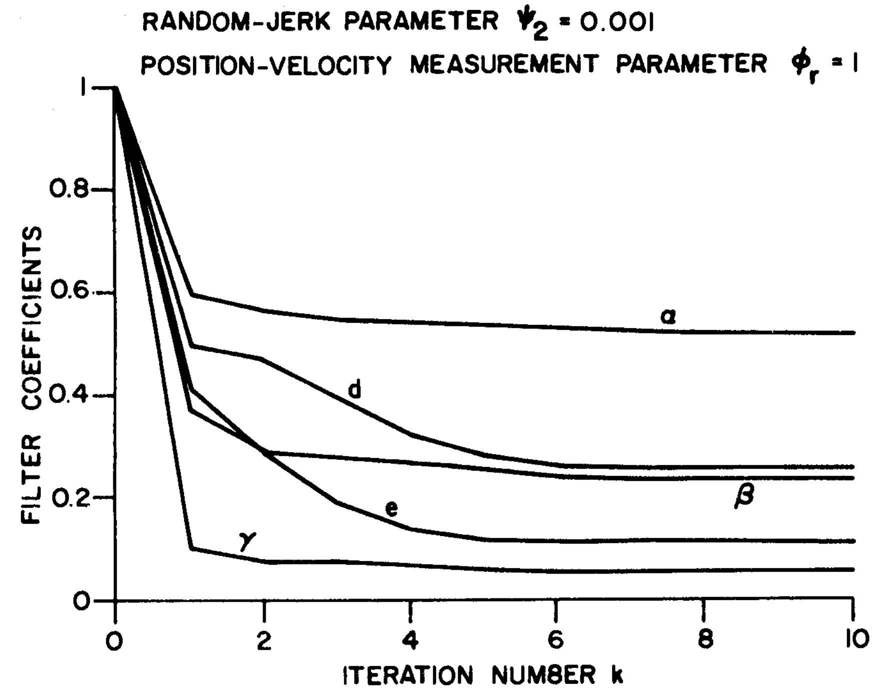

Figure 16. Transient behavior of gain coefficients: third-order reduced-Kalman filter (random-jerk model)

3.3.3 Adaptation

The inclusion of adaptive features in these recursive reduced Kalman filters is similar to the cases described under the α-β(-γ) class of filters. If adaptation is required between iterations (k-1) and (k), owing to a change in the value of one or more of the factors making up the system parameters ϕ_r, $\phi_{(.)}$ or $\psi_{(.)}$, then the appropriate gain coefficients (from eqn. (49), or (51), and Table 6 or 7) are first replaced by the coefficients

$$\alpha'_{k-1} = \left(\frac{r_1}{r'_1}\right)\alpha_{k-1};\quad \beta'_{k-1} = \left(\frac{r_1}{r'_1}\right)\left(\frac{T'}{T}\right)\beta_{k-1};\quad \gamma_{k-1} = \left(\frac{r_1}{r'_1}\right)\left(\frac{T'}{T}\right)^2\gamma_{k-1};$$

$$b'_{k-1} = \left(\frac{r_2}{r'_2}\right)\left(\frac{T}{T'}\right)b_{k-1};\quad d'_{k-1} = \left(\frac{r_2}{r'_2}\right)d_{k-1};\quad e'_{k-1} = \left(\frac{r_2}{r'_2}\right)\left(\frac{T'}{T}\right)e_{k-1};$$

$$f'_{k-1} = \left(\frac{r_2}{r'_2}\right)\left(\frac{T'}{T}\right)^2 f_{k-1} \tag{52}$$

The necessary system parameters are then recomputed: $\phi'_1 = q'_v(T')^2/r'_1$, $\phi'_2 = q'_a(T')^4/4r'_1$, $\phi'_3 = q'_j(T')^6/36r'_1$, $\psi'_1 = q'_a(T')^4/4r'_1$, or $\psi'_2 = q'_j(T')^6/36r'_1$; and $\phi'_r = r'_2(T')^2/r'_1$. Finally the coefficients α_k, β_k, etc., are computed from the appropriate recursion relations.

3.3.4 Steady-state Behavior

The steady-state values of the gain coefficients of these reduced Kalman filters depend on two system parameters, which together describe the effects of the measurement error, the choice of update interval and the uncertainty in the target dynamics. Figures 17, 18 and 19 illustrate, for several values of the parameter ϕ_r, the steady-state dependence of the filter gain coefficients on the parameter ϕ_1, ϕ_2 and ϕ_3 (that is, for the second-order random-velocity, -acceleration, and -jerk models). Similar examples for the third-order models, as functions of the parameter ψ_1 or ψ_2 (random-acceleration and -jerk models), for several values of ϕ_r, are given in Figures 20 and 21. These graphs were all computed by iteration, using eqns. (49) and (51).

For the one-dimensional second-order filters, the size of the state vector is n=2 and the size of the measurement vector is p=2. It is required that n(n+1)/2 = 3 simultaneous nonlinear equations be solved to obtain the steady-state gain matrix[2], since p is not less than (n+1)/2. The structure of such a set of equations has not yet been investigated for the possibility of a closed-form solution to the steady-state condition. For the third-order filters, we see that n=3 and p=2, and p is again not less than (n+1)/2. Therefore, n(n+1)/2 = 6 simultaneous nonlinear equations would have to be solved. In this case, the use of iterative numerical methods for obtaining steady-state values of the gain coefficients seems to be the only possible approach.

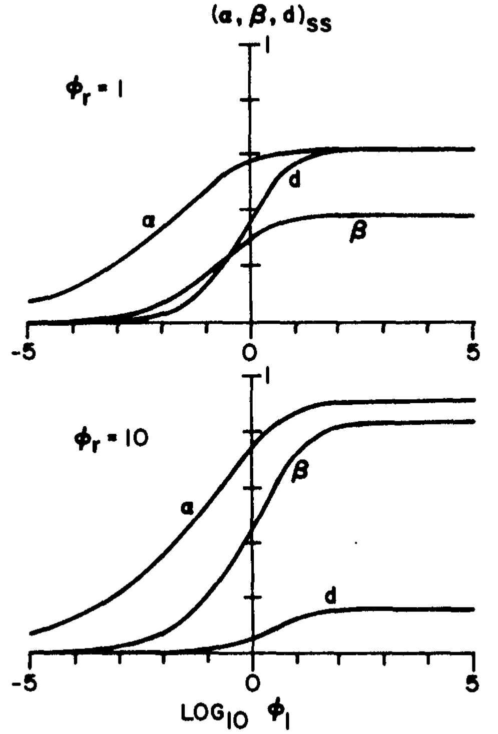

Figure 17. Dependence of steady-state second-order reduced-Kalman filter coefficients on random-velocity parameter ϕ_1, for two values of measurement parameter ϕ_r

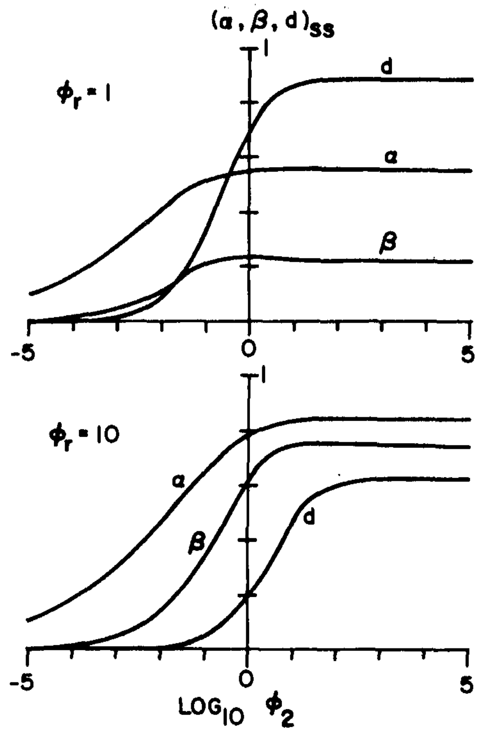

Figure 18. Dependence of steady-state second-order reduced-Kalman filter coefficients on random-acceleration parameter ϕ_2, for two values of measurement parameter ϕ_r

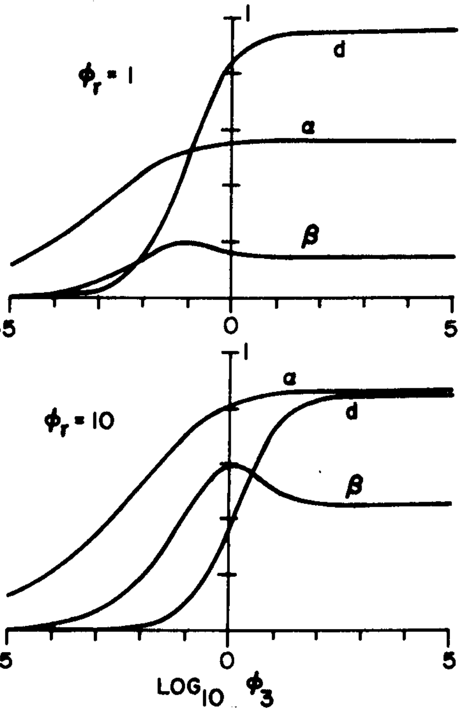

Figure 19. Dependence of steady-state second-order reduced-Kalman filter coefficients on random-jerk parameter ϕ_3, for two values of measurement parameter ϕ_r

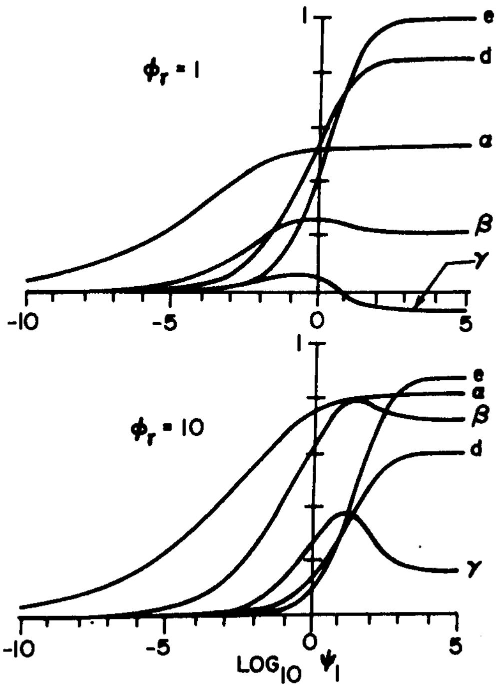

Figure 20. Dependence of steady-state third-order reduced-Kalman filter coefficients on random-acceleration parameter ψ_1, for two values of measurement parameter ϕ_r

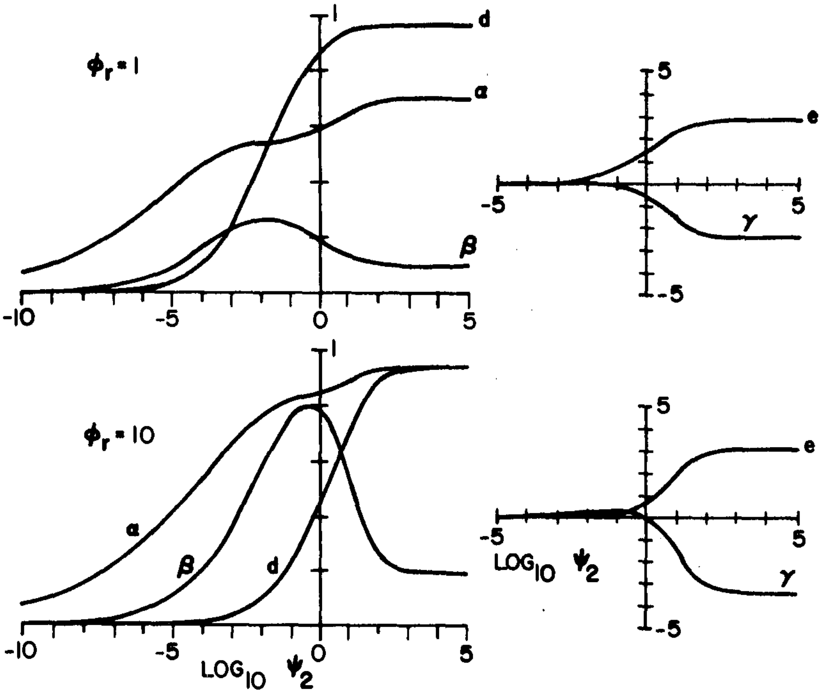

Figure 21. Dependence of steady-state third-order reduced-Kalman filter coefficients on random-jerk parameter ψ_2, for two values of measurement parameter ϕ_r

3.3.5 Summary

This completes the treatment of the reduced Kalman tracking filter. A new way of specifying the one-dimensional form of tracking filter has been described, for the case where doppler measurements of the target's velocity are available in addition to the position measurements. The same models for target manoeuvre as in the α-β(-γ) filters were used; the recursive and adaptive properties have been described and the steady-state behavior treated briefly. The algebraic equations which describe these one-dimensional filters are more complex than those arising in the α-β(-γ) filters. However, they are less expensive in terms of computation (number of operations per iteration) and memory (storage of covariance terms and gain coefficients) than the general recursive form. As in the α-β(-γ) filters they permit all the adaptive features to be employed. With the aid of a suitable look-up table, pre-computed by means of these recursive equations, adaptive steady-state versions of the reduced Kalman filters can be designed.

3.4 ONE-DIMENSIONAL FORMULATIONS

The filter structures described in the preceding sections are the most efficient ways of describing the α-β (position measurements only) class and reduced-Kalman (position and doppler measurements) class of tracking filters. Wherever decoupled filter computations may be used in practical two- and three-dimensional surveillance systems, these forms may be applied directly for a reduced computational and memory load. This is done by using algebraic formulations rather than matrix structures and avoiding the computation of all unnecessary covariance terms.

4. FILTER DESIGN

We now look at various approaches to the implementation of tracking filters. No attempt is made at complete designs but some of the implications of the foregoing analyses are examined from the point of view of a designer of an air-surveillance radar-tracking system.

4.1 FILTER CATEGORIES

Tracking filters may be classified under a number of headings, the most common being:

(a) Recursive or steady-state (fixed parameters);

(b) Adaptive or non-adaptive;

(c) Coupled or decoupled coordinates (a third category: "combined" coordinates should be included);

(d) Third or second order (the two most often adopted, although first order or higher-than-third order filters are possible);

(e) Three dimensional or two dimensional (depending on the radar sensors).

The category mentioned first in each listed item is the more computationally complex. In heading (c), the category "combined" coordinates refers to the case where the same (one-dimensional) set of filter coefficients is applied to all coordinates.

4.2 FILTER INITIALIZATION

When a new target track is acquired, a smoothing and prediction filter is initialized for that track. An estimate of the state of the target is made: position and velocity (for a second-order filter), and acceleration (added, for a third-order filter). From a knowledge of the characteristics of the radar sensor and the signal-processing and plot-detection operations, a suitable estimate of the measurement covariance R is determined. With the assumption of a particular model for target dynamics to be employed by the filter and an estimation of the manoeuvring abilities of the class of targets to be tracked, an a priori specification of the model uncertainty covariance Q is made. Finally, with the inclusion of the update interval T, the tracking filter is initialized to carry out smoothing and prediction on subsequent measurements of the target.

In the recursive, coupled (i.e., full Kalman) filter we use the full matrix specification for R and Q. In the decoupled (i.e., reduced Kalman, and α-β(-γ)) filter, both recursive and steady-state, we select specific elements of R and Q to compute the appropriate system parameter(s) for each coordinate. In the combined-coordinate filter, we extract scalar values from R and Q (i.e., a combination of the separate elements of each) to compute the system parameter(s) applicable to all coordinates of the target position.

4.3 COORDINATE TRANSFORMATIONS

The radar sensor provides target measurements in a line-of-sight coordinate frame (i.e., the polar coordinates of range, azimuth, and possibly elevation angle). However, the filtering of track data in these coordinates can lead to large dynamic errors when a linear model for target motion is used, as in the formulations described in the preceding sections. Simple constant-velocity target tracks appear nonlinear in these coordinates, and artificial acceleration components are generated. This problem does not arise if track filtering is done in a fixed Cartesian reference frame. It is generally desirable to use fixed (earth-referenced) Cartesian coordinates for TWS filtering, particularly when one must consider distributed multi-sensor surveillance systems and the problem of track registration. The use of decoupled coordinates is strictly valid only in a line-of-sight coordinate system, for which the component measurement-errors are independent. The fully-coupled Kalman filter, operating in fixed Cartesian coordinates, absorbs the resulting cross-terms in the measurement covariance matrix R directly. A decoupled filter ignores them from the outset.

The cost of the fully coupled filter is measured in computation time and memory space per track. One method by which this cost can be reduced is to carry out full Kalman filtering (which includes all cross-terms) in the following way: first, compute the reduced set of gain coefficients, assuming a decoupled line-of-sight coordinate system, using the appropriate one-dimensional formulations as described in Section 3; second, compute the fully coupled set of gain coefficients by means of a coordinate rotation into a fixed Cartesian system; and third, carry out all filter operations (smoothing and prediction) in the coupled coordinates. This technique is an alternative to the computation of the gain matrix directly in fixed coordinates using the fully-coupled matrix formulation of the Kalman filter. It allows the decoupled form of filter to be used without loss of generality. It may be applied to both recursive and steady-state versions of the tracking filter. In the case of the combined-coordinate filter, of course, the technique does not apply, since the computation of gain coefficients is intentionally collapsed into one dimension.

An alternative method involves computing the decoupled gain coefficients from the one-dimensional algebraic equations (as in the first method) but implementing the track smoothing or estimation operations in decoupled line-of-sight coordinates as well. Track-prediction operations would be done in the fixed Cartesian frame using a coordinate transformation of the state vector of the track.

Both these methods will be examined in detail in a subsequent report, which will deal more fully with tracking-filter implementations.

4.4 FILTER ADAPTATION

The adaptation of recursive tracking filters to changes in the environmental parameters Q, R and T has been discussed in Section 3.1.2. For the class of optimal steady-state filters developed in this report, it is only required to recalculate the relevant global system parameter, which in turn determines the new gain coefficients to be applied. Changes in the measurement covariance R (brought about by a significant change in the position of the target-in-track with respect to the origin of the tracking coordinates) and the update interval T (brought about by missed plots on individual scans, or by asynchronous combining of reports on one target from several sensors) will occur as a matter of course during the life of the track and are easily identified. To determine whether a change in the excitation noise Q is necessary, some form of manoeuvre detector is required.

Referring to eqns. (5) and (6), the track residual $(MX'-Y)$ has a covariance $Z = (MP'M^t+R)$. The normalized squared residual, a scalar quantity $NSR = (MX'-Y)^t(Z^{-1})(MX'-Y)$, is often used as an on-line measure of track quality. When this figure is found to be consistently (that is, for two or three consecutive updates) greater than some upper threshold or less than some lower threshold, it indicates that an alteration in the size of the elements of Q should be made. Otherwise, the filter is well adapted to the manoeuvre characteristics of the target. The choice of the threshold values is a matter of designer's judgement. Note that if the residual may be assumed multivariate-normal-distributed, then NSR will be χ^2-distributed.

4.5 COMPUTATION OF COEFFICIENTS

In the case of constant-gain filters, when the computation of the optimal steady-state coefficients need be done only once or a small number of times over the life of the track, it would be satisfactory to carry out explicitly, as required, the rather involved calculations implied by eqns. (19), (25), or (31) in the case of second-order filters, with the addition of eqn. (42) when a third-order filter is used. The elimination of the need for recursive gain calculations at each update interval over all tracks results in a considerable saving of computing time.

However, with the inclusion of the adaptive feature, where filter gains may be recomputed a number of times during the life of the track, a more practical and efficient approach would be to use a table look-up procedure. A list of about 100 entries of filter coefficients against the corresponding global system parameter(s) would be more than sufficient, considering the precision of the estimates of Q and R. If the entries of ϕ or ψ are uniformly distributed in the table according to the logarithm of their values, the largest deviation from the correct value of gain coefficient resulting from the selection of nearest match in the list to the calculated value of ϕ or ψ would be about 2%. The table could be contained in less than 1K word of computer memory. An even greater speed advantage could be realized with the use of a small content-addressable memory unit. The table look-up method would be necessary, of course, for those filters which do not have a closed-form solution to the steady-state condition.

4.6 SUMMARY

This section has comprised only a brief look at the problem of tracking-filter implementation as it relates to the one-dimensional filter structures developed in the preceding sections. The principal elements of the problem were:

i) the transformations required to couple the one-dimensional filter structures into the appropriate coordinates of the target's trajectory;

ii) the adaptive techniques required to respond to changes in tracking conditions; and

iii) the computational considerations involved in choosing recursive or steady-state forms for the filters.

All these elements must be examined with regard to necessary tracking performance. A more detailed treatment will be attempted in a subsequent report.

5. FUTURE WORK

The analyses presented in this report represent only the starting point for considerable further work in automatic tracking techniques for radar surveillance systems.

First, the cost and performance of the various filter structures must be compared in order to establish their respective domains of application. Second, the integration of the one-dimensional forms into the practical two- and three-dimension tracking and surveillance systems must be accomplished. Third, automatic track initiation and plot-to-track association must be designed to exploit the features of these estimation and prediction filters to the full. Fourth, the performance of an automatic tracking system employing different categories of filters must be measured, in order to assess the relative effectiveness and cost of such filters. This would be done using simulated or recorded real data as input to the system. Finally, the application of these filter structures in tracking and surveillance systems using electronically agile sensors might be explored.

6. ACKNOWLEDGEMENT

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