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Babinet's principle applied to bridged knife edges

by
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ABSTRACT

In knife-edge diffraction problems, Babinet's principle allows the original arrangement of diffracting screens to be replaced by two new arrangements in which any chosen diffracting screen is absent in one of the new arrangements, and is replaced by its complement in the other. The two new problems may be easier to solve than the original one. The principle is here extended to bridged knife edges, in which the spaces between successive diffracting screens are bridged over by perfectly reflecting plane surfaces that connect the knife edges. Any such structure with a 'valley' can be replaced by other structures that do not have that valley. Again, the new problems may be easier to solve than the original one. This formulation may also be applied to the multiple (unbridged) knife edge problem, which is a special case of the more general problem.

RÉSUMÉ

Dans le contexte des problèmes sur la diffraction, le principe de Babinet permet le remplacement de l'arrangement original des écrans de diffraction par deux autres arrangements pour lesquels un écran au choix est absent dans l'un des arrangements et est remplacé par son complément dans l'autre. Les solutions des deux nouveaux problèmes peuvent être plus faciles à déterminer que la solution du problème original. On généralise la solution aux cas des arêtes vives reliées par des surfaces planes parfaitement réfléchissantes. Une telle structure qui comporte une 'vallée' peut être remplacée par d'autres structures qui ne comportent pas cette vallée. Encore une fois, les nouveaux problèmes peuvent parfois être plus faciles à solutionnier que le problème original. Cette approche s'applique également à la question des arêtes vives multiples (sans surface réfléchissante reliant les arêtes), cas particulier du problème plus général.

EXECUTIVE SUMMARY

The context of this report is continuing research on the prediction of radio signal strengths on terrestrial paths at VHF and UHF. On short obstructed paths, the most important propagation mechanism is diffraction. For at least two methods of calculating diffraction attenuation, the calculation fails if there are sufficiently deep valleys in the terrain profile, and succeeds if there are no valleys. It would be of value, therefore, to have a method of transforming terrain diffraction problems with valleys into equivalent problems without valleys. This report describes such a method.

Babinet's principle, which comes from physical optics, rests on the idea of a thin opaque screen (a diffracting screen) that partly blocks the space between a source of waves and a point where the wave field strength is to be calculated. One can think of another diffracting screen, complementary to the first, that blocks the area left open by the first screen and leaves open the area blocked by the first screen. Babinet's principle states that the wave field received behind the first screen is equal to the field received with no screen minus the field received behind the complementary screen. These last two quantitites may be easier to calculate than the first one. If there is more than one diffracting screen, the principle may be applied to any one of them. If the edge of a diffracting screen is straight, it is commonly referred to as a knife edge.

If the terrain along a radiowave propagation path is a series of narrow ridges transverse to the path, it can be accurately modelled as a series of knife edges. However, if the terrain is gently rolling, a better model is a series of flat reflecting surfaces. Even though such a model is physically quite different from knife edges, it can be treated mathematically in a similar way. This is because the discontinuities between the flat surfaces block reflected waves in much the same way that knife edges block unreflected waves. Knife edges may be assumed to be present at these discontinuities without affecting the behaviour of the model since the associated diffracting screens are underground. They allow the transformation of the problem into knife-edge problems, to which Babinet's principle can be applied.

For both knife-edge models and bridged-knife-edge models, valleys in the terrain profile create computational difficulties. This report shows that by suitable transformations, terrain profiles with valleys can be replaced by profiles without valleys. The transformations make use of both Babinet's principle and mirror images.

1. INTRODUCTION

1.1 BACKGROUND

Babinet's principle as applied to waves diffracted by thin screens is well known (Born and Wolf [1, p.381], Kong [2, p.372]). It allows a diffraction problem to be replaced by two others which may be easier to solve. In the multiple-knife-edge problem, each diffracting screen occupies a half plane, and its boundary is a straight line (the knife edge). If the diffracting screen occupies the lower half plane, then the complementary screen occupies the upper half plane.

This report describes an extension of the use of Babinet's principle to a new type of problem in which the spaces between diffracting screens have been bridged over by perfectly reflecting plane surfaces connecting the knife edges. In this new problem, intended primarily for representing propagation at VHF and UHF over terrain, the knife edges are hidden. That is, the reflecting surfaces represent the terrain, and the knife edges are retained only as a mathematical convenience. This extended use of Babinet's principle contains its application to multiple knife edges as a special case.

A series solution of the multiple-knife-edge diffraction problem has been developed by Vogler [3], and used by Sharples and Mehler [4], and Whitteker [5]. The motivation for the present report is the fact that Vogler's series does not converge if valleys in the modelled terrain are too deep. By the use of Babinet's principle, terrain diffraction problems with valleys can be replaced by equivalent problems with the valleys removed. Although the motivation for the present work comes from [5], the use of Babinet's principle for diffraction obstacles of this type does not depend on using the particular formulation described in that paper. Vogler [6] describes a procedure, equivalent to Babinet's principle, of partitioning the integrals of the multiple-knife-edge problem. While the method of [6] is purely algebraic, the method described in the present report is based more on geometry, and it applies to bridged knife edges as well as to (unbridged) multiple knife edges.

1.2 BABINET'S PRINCIPLE FOR KNIFE EDGES

To begin, consider the multiple-knife-edge problem (without bridging surfaces). The field due to a source in the presence of the knife edges may be expressed as the field obtained in the absence of a particular diffracting screen minus the field obtained with the knife edge inverted. With reference to Figure 1, the desired field $E_{\uparrow 0 \uparrow 0 \uparrow}$ is given by

$$E_{\uparrow 0 \uparrow 0 \uparrow} = E_{\uparrow 0 \uparrow} - E_{\uparrow 0 \downarrow 0 \uparrow} \tag{1}$$

The subscripted arrows indicate upright ($\uparrow$) and inverted ($\downarrow$) knife edges. The zeros between the arrows are included for consistency with notation used later. (They indicate that there are no reflections on surfaces bridging the knife edges.) As the diagram indicates, the configurations of knife edges associated with $E_{\uparrow 0 \uparrow}$ and $E_{\uparrow 0 \downarrow 0 \uparrow}$ no longer contain the 'valley' associated with the middle knife edge in the original

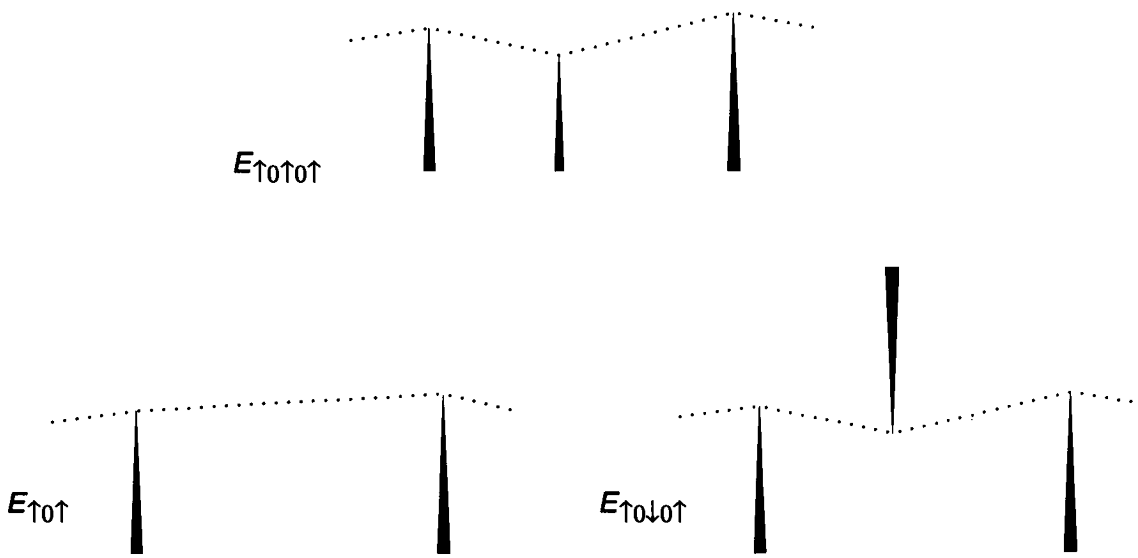

Figure 1. Babinet's principle applied to knife edges. The original problem (top) is decomposed into two other problems (bottom) which may be easier to solve. Each configuration of knife edges is labelled with a symbol, e.g. $E_{\uparrow 0\uparrow}$, representing the field due to that configuration. See (1) in text.

configuration. This is Babinet's principle for knife edges, and it is also a geometric representation of the partitioning of integrals described in [6]. It is one of the two building blocks out of which the transformations described in this report are built. The other building block, to be described in Sections 2.4 and 2.5, is the replacement of the source and other objects by their images, with the removal of bridging surfaces.

2. APPLICATION TO BRIDGED KNIFE EDGES

2.1 SYMBOLS AND CONVENTIONS

The time variation of the field is assumed to be $e^{i\omega t}$. Knife edges are numbered from 1 to N, with the variable n. The x axis is directed horizontally along the path of propagation, the y axis is horizontal and perpendicular to the path of propagation, and the z axis is vertical. Knife edges are always assumed to be parallel to the y axis. To 'invert a knife edge' is to replace the associated diffracting screen with its complement. By the symbol E will be meant the scalar value of the field vector at point B due to a source at A. Subscripts of E, when they occur, specify whether particular knife-edges are inverted, and may specify particular modes of propagation. Superscripts indicate that the space on the left or right of some point has been replaced by its image.

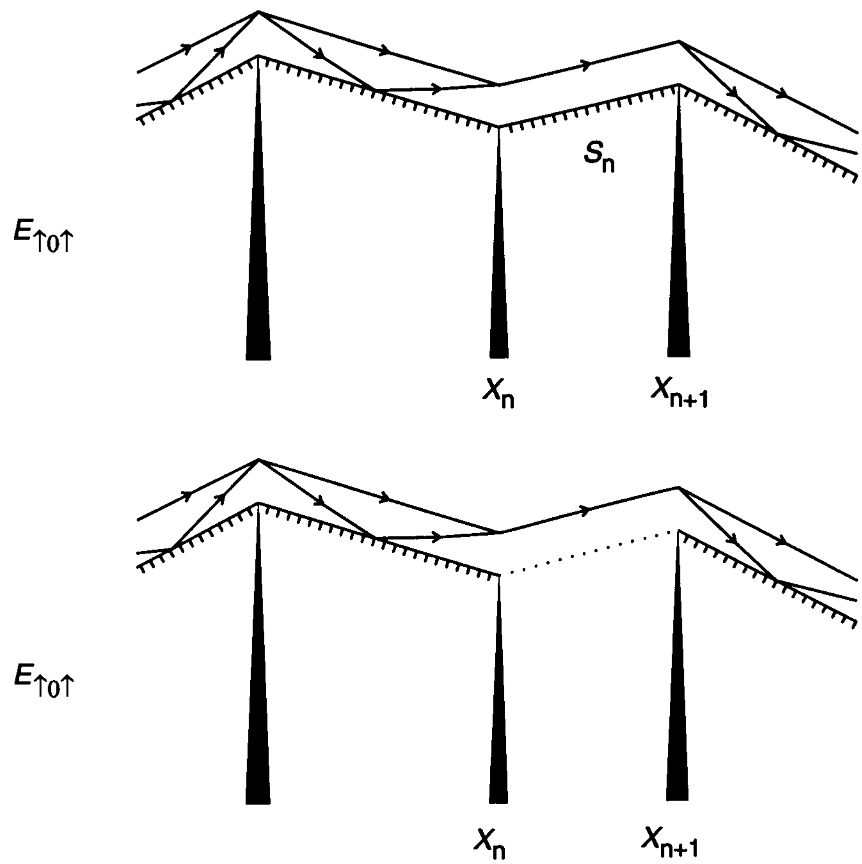

Figure 2. Non-reflected mode. The directed line segments indicate possible wave paths, which may pass over any path segment either directly or by reflection. The subscripts of E refer to the path segment x_n to x_{n+1}. For any mode not reflected in surface S_n (top drawing), surface S_n may be removed (bottom drawing) without changing the field due to that mode.

2.2 CONSTRUCTION OF BRIDGED KNIFE EDGES

The diffracting obstacle of interest here is not a series of knife edges, but rather a perfectly reflecting surface composed of connected planar strips. It can be constructed by bridging the spaces between knife edges with perfectly reflecting surfaces. See the upper drawing of Figure 2.

There may be any number (N) of knife edges, but the diagram shows only three of them. Since the motivation of this work is to calculate terrain attenuation of radio propagation, the diffracting obstacle will be referred to as terrain.

2.3 MODES

Suppose that there are N knife edges with $N-1$ bridging surfaces between them. (Reflecting surfaces can also be placed on either side of all the knife edges, but such surfaces can be eliminated immediately by using source and field-point images [5], and these surfaces will not be considered here.) A wave propagating over this structure can be decomposed into modes, each defined by the integers $m_n, n = 1, \ldots, N-1$. The wave corresponding to each mode passes from the n'th knife edge at x_n to the $n+1$'th at x_{n+1} either directly ($m_n = 0$) or by reflection in bridging surface S_n ($m_n = 1$). There are evidently 2^{N-1} modes of this type.

The words 'directly' and 'by reflection' require definition. Consider some point with coordinate x_{n+1} located above the $n+1$'th knife edge. The field at this point may be found by a Huygens'-principle integration of the field over a surface surrounding the point, *e.g.* [2, p.376]. In an extension of the Kirchhoff approximation [7], the integration may be limited to two plane surfaces: the vertical surface $x = x_n$ above the n'th knife edge, and the reflecting surface S_n. Given this approximation, by a wave passing 'directly' from x_n to x_{n+1} is meant that a Huygens'-principle integration to find the field at x_{n+1} is performed only over the vertical surface above the n'th knife edge. Similarly, for a wave passing by 'reflection', the integration is performed only over the reflecting surface S_n. The calculated field must therefore be the sum of the fields due to each mode.

The reason for decomposing the wave into modes is that, for each propagation mode, there is an equivalent propagation problem in which any chosen bridging surface is removed. If the bridging surfaces on both sides of a knife edge are removed, the knife edge reverts to being simply an (unbridged) knife edge, and Babinet's principle can then be used in the ordinary way. Removing and restoring bridging surfaces is the basis of the application of Babinet's principle to bridged knife edges.

2.4 REMOVAL OF BRIDGING SURFACES

To see how a bridging surface may be removed, first consider any propagation mode in which the wave is not reflected in S_n. Since there is no reflection in S_n, S_n may simply be removed (for this mode) with no other change required. This situation is illustrated in Figure 2.

Next consider any propagaton mode in which the wave is reflected in the n'th bridging surface, designated S_n, as illustrated in Figure 3.

From the point of view of geometric optics, S_n is a mirror, and an observer looking into S_n from the right sees the half space to the left of S_n reflected in the plane of S_n. All objects in this half space are seen to be inverted, as illustrated in the lower drawing of Figure 3. Therefore let us replace the original diffraction problem with one in which all the objects (the terrain and the source of radiation) in the half space to the left of S_n are replaced with their reflected counterparts. After this is done, the calculated field due to this mode will remain unchanged, except for a change in sign due to a reflection coefficient of -1. That is,

$$E_{\uparrow 1 \uparrow} = -E^{(l)}_{\downarrow 0 \uparrow} \tag{2}$$

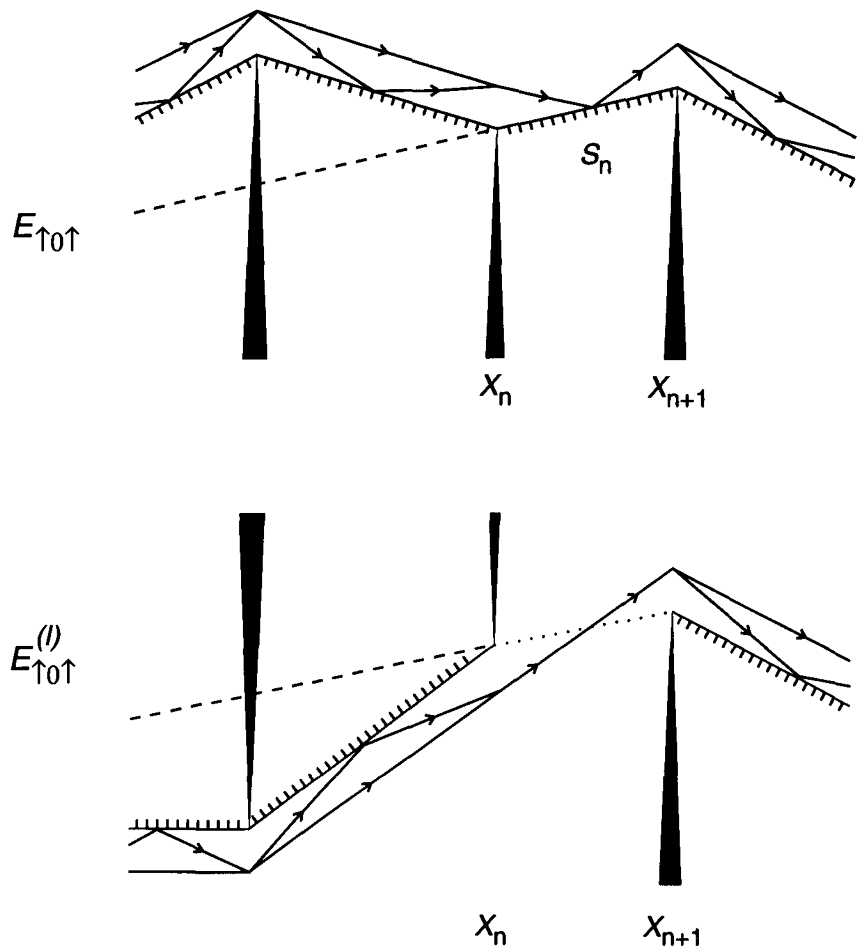

Figure 3. Reflected mode. Any mode reflected in surface S_n (top drawing) is equivalent to a mode with S_n omitted (bottom drawing), provided everything to the left of surface S_n is reflected in the projection of S_n, indicated by a broken line. See (2) in text. (Knife edges would not be vertical after reflection in a tilted surface. They are nevertheless shown as vertical to avoid irrelevant complication.)

where the subscripted arrows indicate whether the knife edges on either side of S_n are upright or inverted, the numerals between them indicate the mode parameter m_n, and the superscript (l) indicates that the world on the left of S_n has been reflected through S_n.

Although the equivalence just introduced is most easily visualized with geometric optics, it does not rely on the geometric-optics approximation. If the surface S_n is a perfect conductor, it imposes the boundary condition that all tangential electric fields vanish on the surface. The mirror image of all the sources and scatterers to the left of S_n must produce on S_n a field with tangential components identical to those produced by the original sources and scatterers. If this field is negated and superimposed on the original field, the boundary condition is satisfied, *cf.* [2, p.358].

2.5 DECOMPOSITION OF THE FIELD INTO MODES

First, a generalization of the original problem is required. The generalization is that not all modes need be present in the original problem. Let a parameter μ_n specify whether modes reflected in S_n (modes with $m_n = 1$) are present. Let $\mu_n = 1$ if they are, and $\mu_n = 0$ if they are not. A physical model for setting $\mu_n = 0$ would be bridged-knife-edge terrain in which the the reflecting surface S_n is absent. If $\mu_n = 0$ for all n, $n = 1, \ldots, N-1$, the problem reduces to multiple knife edges. Similarly, let $\lambda_n = 1$ if direct modes (modes with $m_n = 0$) are present and $\lambda_n = 0$ if they are not. There is no physical model for the possibility $\lambda_n = 0$, but it is an artifice that is required in repeated applications of Babinet's principle.

Now the total field can be expressed as

$$E = \lambda_n E_{\uparrow 0 \uparrow} + \mu_n E_{\uparrow 1 \uparrow} \tag{3}$$

Equation (3) divides the modes into two sets, those for which $m_n = 0$ and those for which $m_n = 1$. Using (2), (3) becomes

$$E = \lambda_n E_{\uparrow 0 \uparrow} - \mu_n E^{(l)}_{\downarrow 0 \uparrow} \tag{4}$$

Instead of reflecting the world in the half space on the left of S_n, we could have reflected the world on the right. The result of the decomposition would then be

$$E = \lambda_n E_{\uparrow 0 \uparrow} - \mu_n E^{(r)}_{\uparrow 0 \downarrow} \tag{5}$$

where the superscript (r) indicates a reflection of the world on the right of S_n.

The transformation represented by (4) or (5) is not particularly useful by itself, but it and Babinet's principle for knife edges are the building blocks for the transformations that follow.

3. APPLICATION TO FIRST OR LAST KNIFE EDGE

3.1 SPECIALIZATION OF TRANSFORMATION

Babinet's principle can be applied to the first or last knife edge of a series more easily than to an interior knife edge. This is because transformations (4) and (5), which are needed in preparation for applying it, can be done in such a way that the only object in the reflected half space is the source point or field point.

Consider a path profile in which knife edge 1 lies below the line between source A and knife edge 2, as illustrated in Figures 4 and 5, i.e. knife edge 1 is in a 'valley' that we wish to remove. The top drawings in these diagrams represent the same source and terrain. In Figure 4, modes not reflected in surface S_1 ($m_1 = 0$) are considered, and in Figure 5, modes reflected in S_1 ($m_1 = 1$) are considered. For ($m_1 = 0$) modes, S_1 can be removed with no effect (second drawing in Figure 4). For ($m_1 = 1$) modes, S_1 can be removed along with a world reflection (second drawing in Figure 5). Equation (4) becomes

$$E = \lambda_1 E_{\uparrow 0 \uparrow} - \mu_1 E^{(l)}_{\downarrow 0 \uparrow} \tag{6}$$

3.2 BABINET'S PRINCIPLE

Knife edge 1 is now free of bridging surfaces, and Babinet's principle can be applied to it, giving

$$E_{\uparrow 0 \uparrow} = E_{\uparrow} - E_{\downarrow 0 \uparrow} \tag{7}$$

(side-by-side drawings in Figure 4) and

$$E^{(l)}_{\downarrow 0 \uparrow} = E^{(l)}_{\uparrow} - E^{(l)}_{\uparrow 0 \uparrow} \tag{8}$$

(side-by-side drawings in Figure 5). Equations (7) and (8) may be obtained from (1) by removing the subscripted symbol referring to the left-hand knife edge (and the accompanying zero), since here only two knife edges are under consideration.

3.3 REVERTING TO UPRIGHT TERRAIN

It is most convenient to have all the terrain upright in the final problems to be solved. Of the four terms on the right hand side of (7) and (8), only $E_{\downarrow 0 \uparrow}$ in (7) contains the symbol for an inverted knife edge. Applying (2) in reverse results in upright terrain and a mode that is reflected in S_1 (bottom of Figure 4). That is,

$$E_{\downarrow 0 \uparrow} = -E^{(l)}_{\uparrow 1 \uparrow} \tag{9}$$

Also, the trivial addition of surface S_1 leads to the configuration at the bottom of Figure 5.

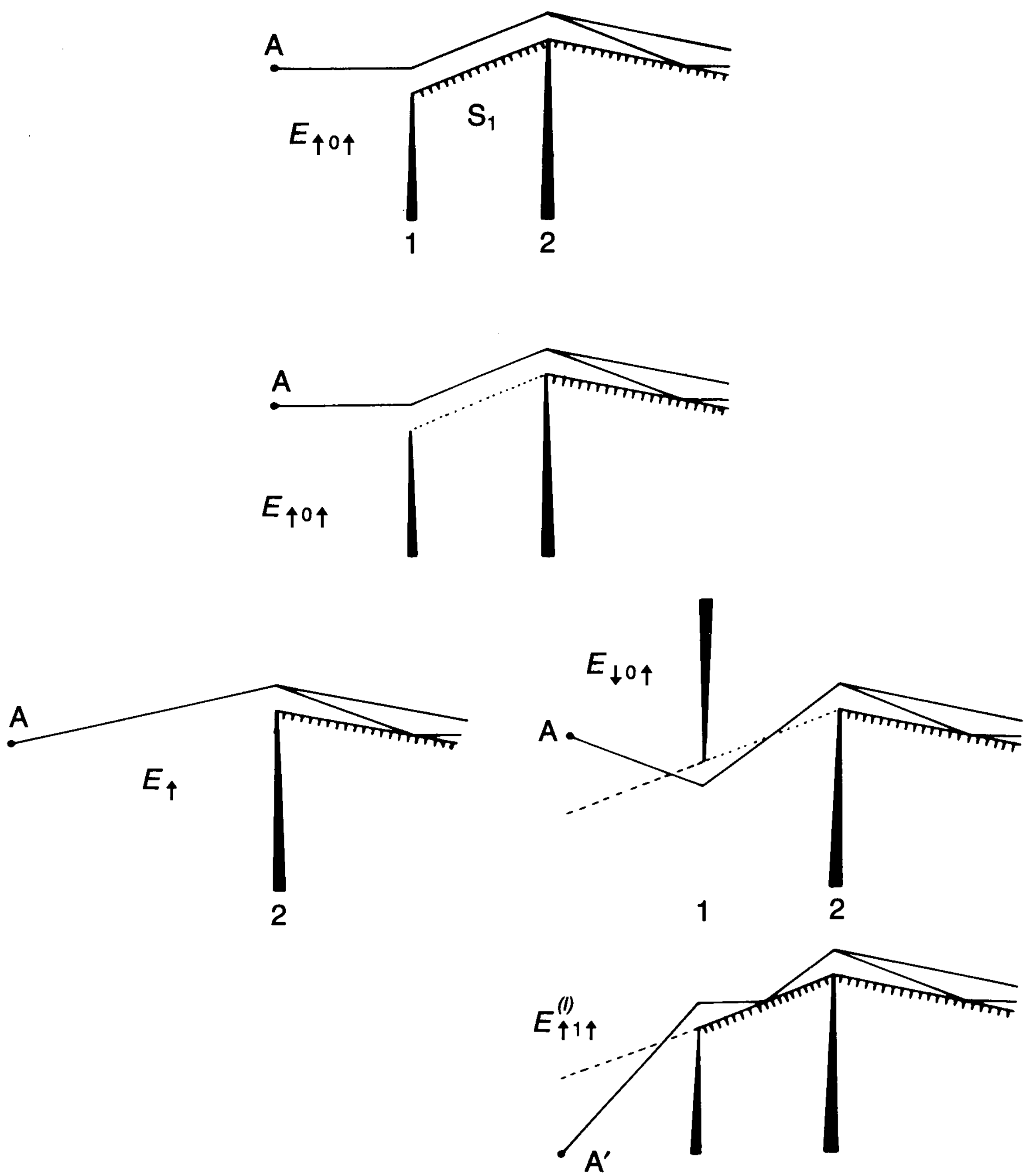

Figure 4. Elimination of a valley at the location of the first bridged knife edge of series, for modes in which $m_1 = 0$ (top of diagram). The transformation proceeds from the top of the diagram to the bottom. The final result is that the original problem (with field $E_{\uparrow 0\uparrow}$) is transformed into the two labelled with the symbols $E_{\uparrow}$ and $E^{(l)}_{\uparrow 1\uparrow}$. See Section 3 in text.

3.4 TOTAL FIELD

Combining (6), (7), (8), and (9), gives the field due to the original terrain and source:

$$E = \lambda_1(E_\uparrow + E^{(l)}_{\uparrow 1 \uparrow}) - \mu_1(E^{(l)}_\uparrow - E^{(l)}_{\uparrow 0 \uparrow}) \tag{10}$$

The original problem seems to be replaced by four new ones, but not all of them must be solved. If the original problem is the ordinary one in which $\lambda_1 = \mu_1 = 1$, (10) becomes

$$E = E_\uparrow - E^{(l)}_\uparrow + E^{(l)}_{\uparrow\uparrow} \tag{11}$$

where $E^{(l)}_{\uparrow\uparrow} = E^{(l)}_{\uparrow 0 \uparrow} + E^{(l)}_{\uparrow 1 \uparrow}$ is the field due to the original bridged-knife-edge configuration with an image source. The absence of numerals in the subscripts in (11) indicates that all modes are to be included, i.e. the new problems are ordinary bridged-knife-edge problems. There are three new problems in this case.

If, however, we begin with a problem in which $\lambda_1 = 0$ or $\mu_1 = 0$, then we must use (10), but only two of the four new problems have to be solved. Note that if we begin with a problem in which $\mu_1 = 0$ (reflecting surface not present), we get in (10) a problem ($E^{(l)}_{\uparrow 1 \uparrow}$) in which a direct mode is not present. This is the non-physical part of the generalization mentioned in Section 2.5.

In summary, (10) and (11) give the result of applying Babinet's principle to the first knife edge. The field E due to a terrain profile in which the first knife edge lies below the line joining the source to the second knife edge, forming a valley, is given in terms of fields due to terrain models with the valley removed.

Similar considerations apply to the N'th, or final, knife edge, and a field point beyond it. The result is

$$E = \lambda_{N-1}(E_\uparrow + E^{(r)}_{\uparrow 1 \uparrow}) - \mu_{N-1}(E^{(r)}_\uparrow - E^{(r)}_{\uparrow 0 \uparrow}) \tag{12}$$

which, if $\lambda_{N-1} = \mu_{N-1} = 1$, becomes

$$E = E_\uparrow - E^{(r)}_\uparrow + E^{(r)}_{\uparrow\uparrow} \tag{13}$$

Here, a single arrow refers to the $N-1$'th knife edge, and two arrows refer to the $N-1$'th and N'th knife edges.

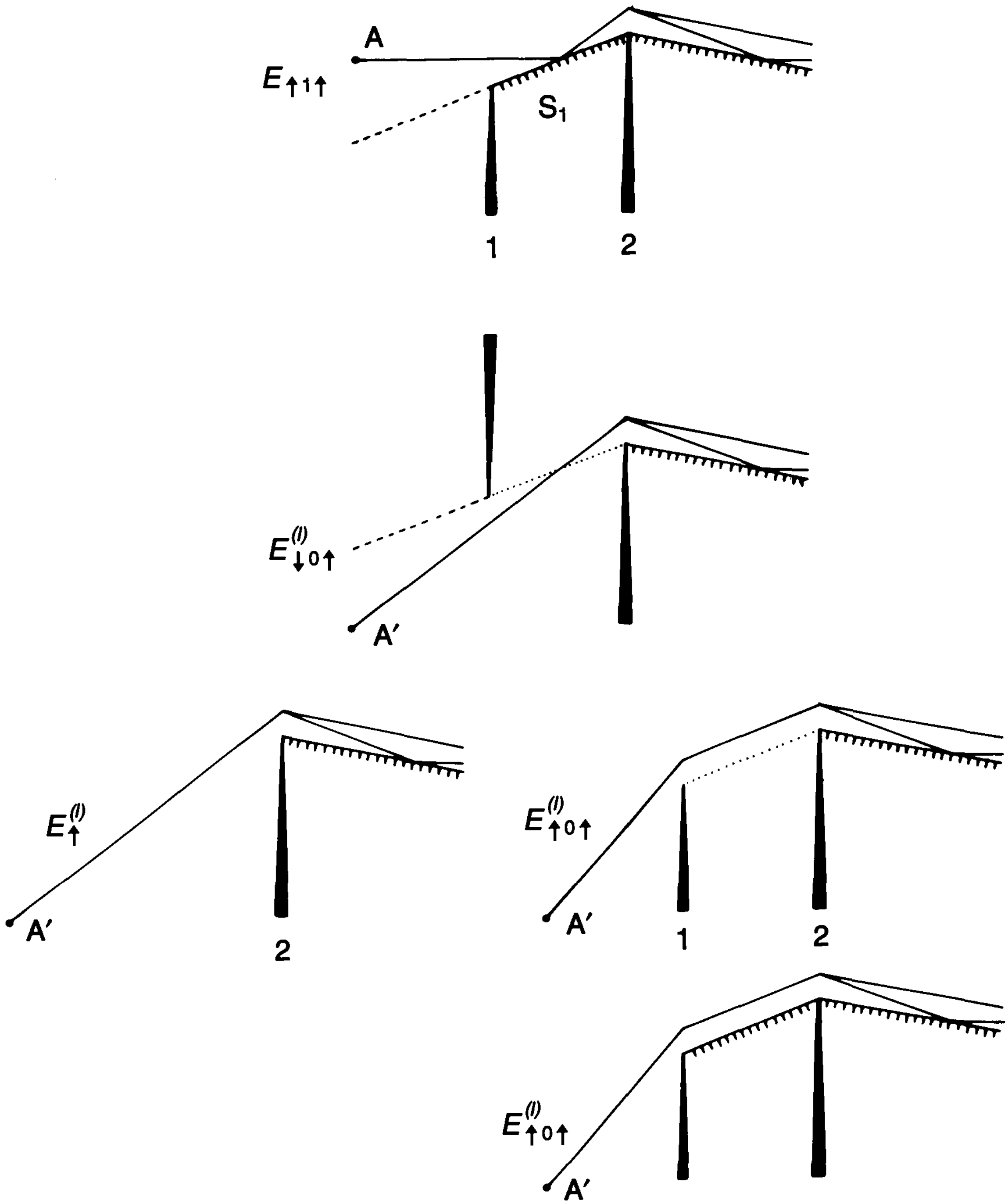

Figure 5. Elimination of a valley at the location of the first bridged knife edge of series, for modes in which $m_1 = 1$ (top of diagram). The final result is that the original problem (with field $E_{\uparrow 1 \uparrow}$) is transformed into the two labelled with the symbols $E^{(l)}_{\uparrow}$ and $E^{(l)}_{\uparrow 0 \uparrow}$. See Section 3 in text.

4. APPLICATION TO AN INTERIOR KNIFE EDGE

4.1 DECOMPOSITION OF FIELD

This section describes the removal of an interior valley in the terrain profile. Consider a path profile in which the n'th knife edge lies below the line between the $n-1$'th and $n+1$'th knife edge, as illustrated in Figures 6–9. Here, there are two bridging surfaces to eliminate, S_{n-1} and S_n. There are four sets of modes to consider, corresponding to the four possible pairs of values of m_{n-1} and m_n. That is, the field to be found may be expressed as

$$E = \lambda_{n-1}\lambda_n E_{\uparrow 0\uparrow 0\uparrow} + \lambda_{n-1}\mu_n E_{\uparrow 0\uparrow 1\uparrow} + \mu_{n-1}\lambda_n E_{\uparrow 1\uparrow 0\uparrow} + \mu_{n-1}\mu_n E_{\uparrow 1\uparrow 1\uparrow} \tag{14}$$

Equation (14) is the same as (3), except that the wave is divided into four sets of modes rather than two. Figures 6–9 illustrate configurations for the four terms on the right-hand side of (14), for which m_{n-1} and m_n are successively 0 and 1.

The analyses are given algebraically in the following sections, but the easiest way to arrive at the final equations is to follow the diagrams, in which each configuration is labelled with the corresponding field symbol. Where Babinet's principle is applied, the diagram splits left and right, and the field becomes the sum of two fields. The only detail not given in the diagrams is the sign of the terms. The rule for the sign is that it changes on each addition or deletion of a reflecting surface, and it changes for the right-hand drawing where the diagram splits.

4.1.1 Analysis of $E_{\uparrow 0\uparrow 0\uparrow}$

For the set of modes illustrated in Figure 6, both S_{n-1} and S_n can be removed with no effect. As illustrated in the second drawing from the top, knife edge n has been left free, and Babinet's principle can be applied to it, giving

$$E_{\uparrow 0\uparrow 0\uparrow} = E_{\uparrow 0\uparrow} - E_{\uparrow 0\downarrow 0\uparrow} \tag{15}$$

where the terms on the right hand side of (15) are illustrated by side-by-side drawings in Figure 6.

On the left-hand side of Figure 6, a new bridging surface may be introduced between knife edges $n-1$ and $n+1$ with no effect, and the symbol $E_{\uparrow 0\uparrow}$ is retained.

Starting from the configuration at the middle right of Figure 6, the world to the left of knife edge n is reflected in S_{n-1}, and the world to the right is reflected in S_n, by applying equation (2) for half-space reflection in reverse. Because (2) is used twice, the sign does not change. The result is denoted by $(E^{(l,r)}_{\downarrow 1\downarrow 1\downarrow})$, in which the superscript (l,r) indicates that a single reflection has occurred on the left and on the right. This results in the whole space being upside down. The diagram may be inverted without affecting the field. If the inversion is done by reflecting everything in the plane bridging the

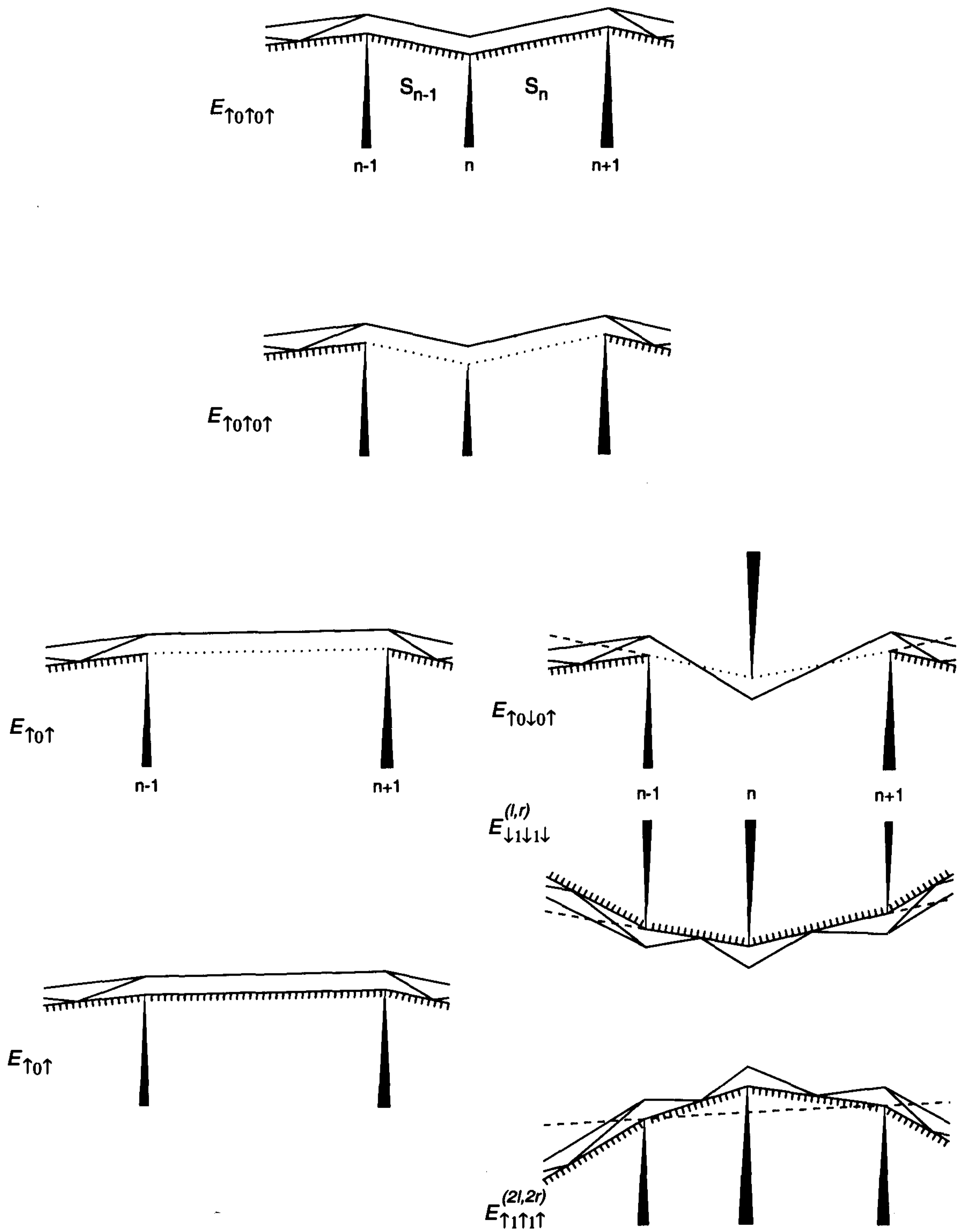

Figure 6. Elimination of a valley at the location of the n'th bridged knife edge, for modes in which $m_{n-1} = 0$ and $m_n = 0$ (top of diagram). The problem is transformed into the two illustrated at the bottom left and right. See Section 4.1.1 in text.

knife edges $n-1$ and $n+1$, the resulting diagram is similar in some respects to the diagram at the lower left, and to the diagrams corresponding to the other three terms. Summarizing the last two transformations of Figure 6, we have

$$E_{\uparrow 0\downarrow 0\uparrow} = E^{(l,r)}_{\downarrow 1\downarrow 1\downarrow} = E^{(2l,2r)}_{\uparrow 1\uparrow 1\uparrow} \tag{16}$$

where the superscript $(2l, 2r)$ indicates two reflections of both the left-hand and right-hand worlds. Then combining (15) and (16) gives

$$E_{\uparrow 0\uparrow 0\uparrow} = E_{\uparrow 0\uparrow} - E^{(2l,2r)}_{\uparrow 1\uparrow 1\uparrow} \tag{17}$$

4.1.2 Analysis of $E_{\uparrow 0\uparrow 1\uparrow}$

For the set of modes illustrated in Figure 7, S_{n-1} can be removed with no effect, and S_n can be removed along with a half-space reflection. The result is

$$E_{\uparrow 0\uparrow 1\uparrow} = -E^{(r)}_{\uparrow 0\uparrow 0\downarrow} \tag{18}$$

where the reflection indicated by the superscript (r) is in surface S_n. As illustrated in the second drawing of Figure 7, knife edge n has been left free, and Babinet's principle can be applied to it, giving

$$E^{(r)}_{\uparrow 0\uparrow 0\downarrow} = E^{(r)}_{\uparrow 0\downarrow} - E^{(r)}_{\uparrow 0\downarrow 0\downarrow} \tag{19}$$

where the terms on the right hand side of (19) are illustrated in the side-by-side drawings in Figure 7.

Equation (2) for half-space reflection may be applied to $E^{(r)}_{\uparrow 0\downarrow}$ in reverse, resulting in upright terrain and a mode that is reflected in the surface that bridges knife edges $n-1$ and $n+1$, as illustrated in the lower left of Figure 7. The half space to the right of S_n has now undergone two reflections, one in S_n and another in the new bridge that spans the valley that knife edge n occupied. That is

$$E^{(r)}_{\uparrow 0\downarrow} = -E^{(2r)}_{\uparrow 1\uparrow} \tag{20}$$

where the superscript $(2r)$ indicates that the world on the right of S_n has been twice reflected.

Starting from the configuration at the middle right of Figure 7, the next step is to reflect the half space to the left of S_{n-1} in S_{n-1}. This results in the whole space being upside down ($E^{(l,r)}_{\downarrow 1\downarrow 0\downarrow}$). The diagram may be inverted without affecting the field. As before, the inversion is done by reflecting everything in the plane bridging the knife edges $n-1$ and $n+1$. Summarizing the last two transformations of Figure 7, we have

$$E^{(r)}_{\uparrow 0\downarrow 0\downarrow} = -E^{(l,r)}_{\downarrow 1\downarrow 0\downarrow} = -E^{(2l,2r)}_{\uparrow 1\uparrow 0\uparrow} \tag{21}$$

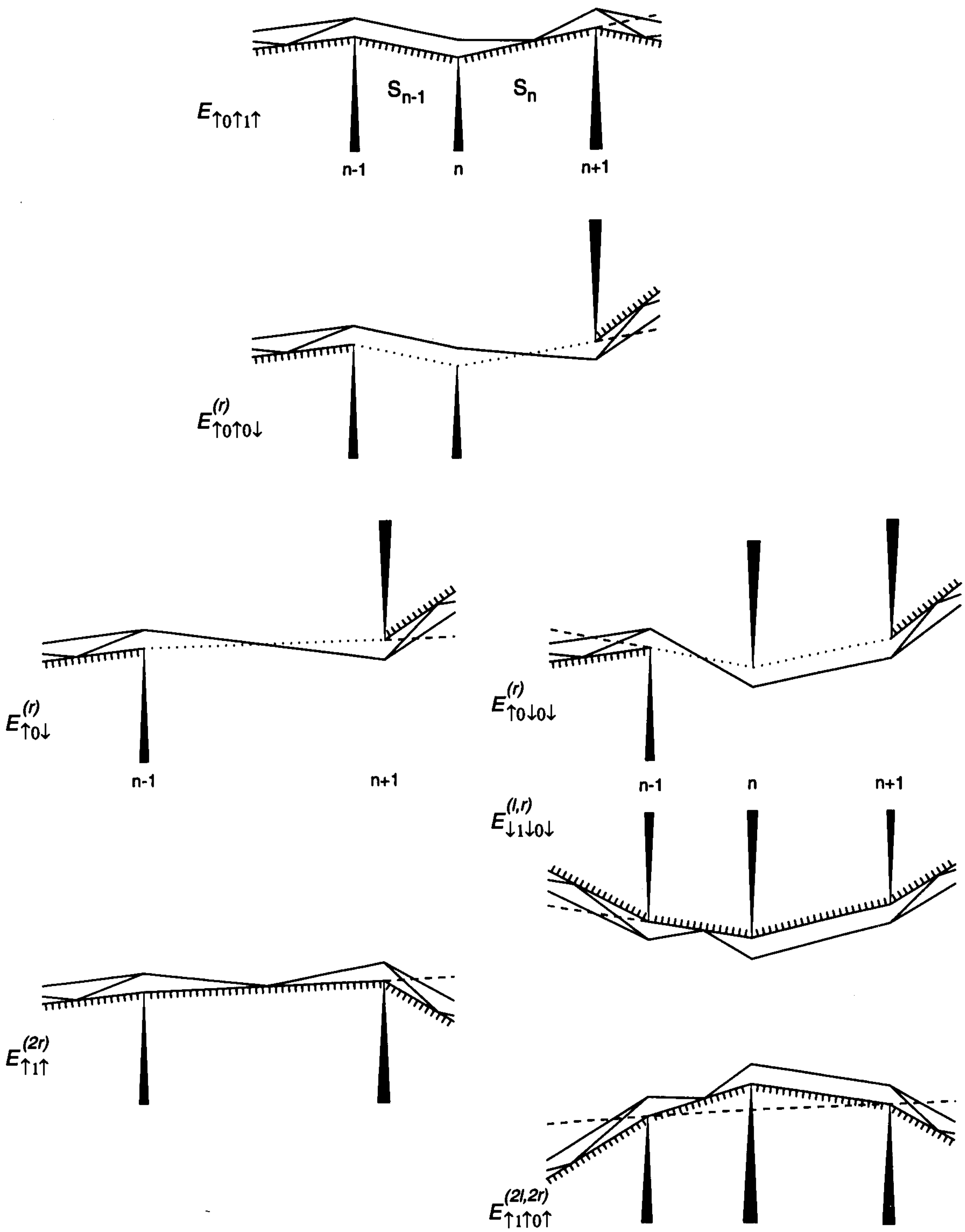

Figure 7. Elimination of a valley at the location of the n'th bridged knife edge, for modes in which $m_{n-1} = 0$ and $m_n = 1$ (top of diagram). The problem is transformed into the two illustrated at the bottom left and right. See Section 4.1.2 in text.

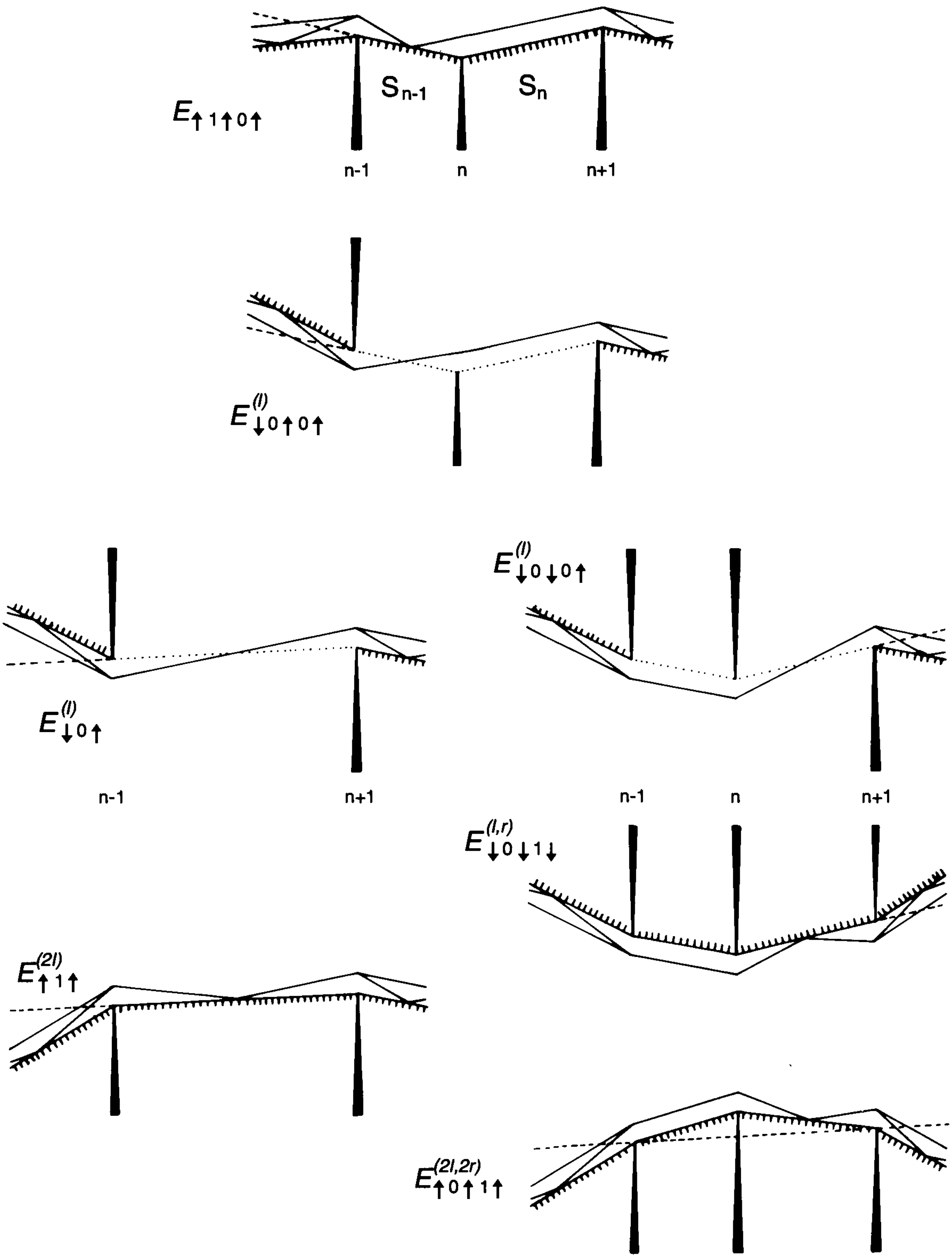

Figure 8. Elimination of a valley at the location of the n'th bridged knife edge, for modes in which $m_{n-1} = 1$ and $m_n = 0$ (top of diagram). The problem is transformed into the two illustrated at the bottom left and right. See Section 4.1.3 in text.

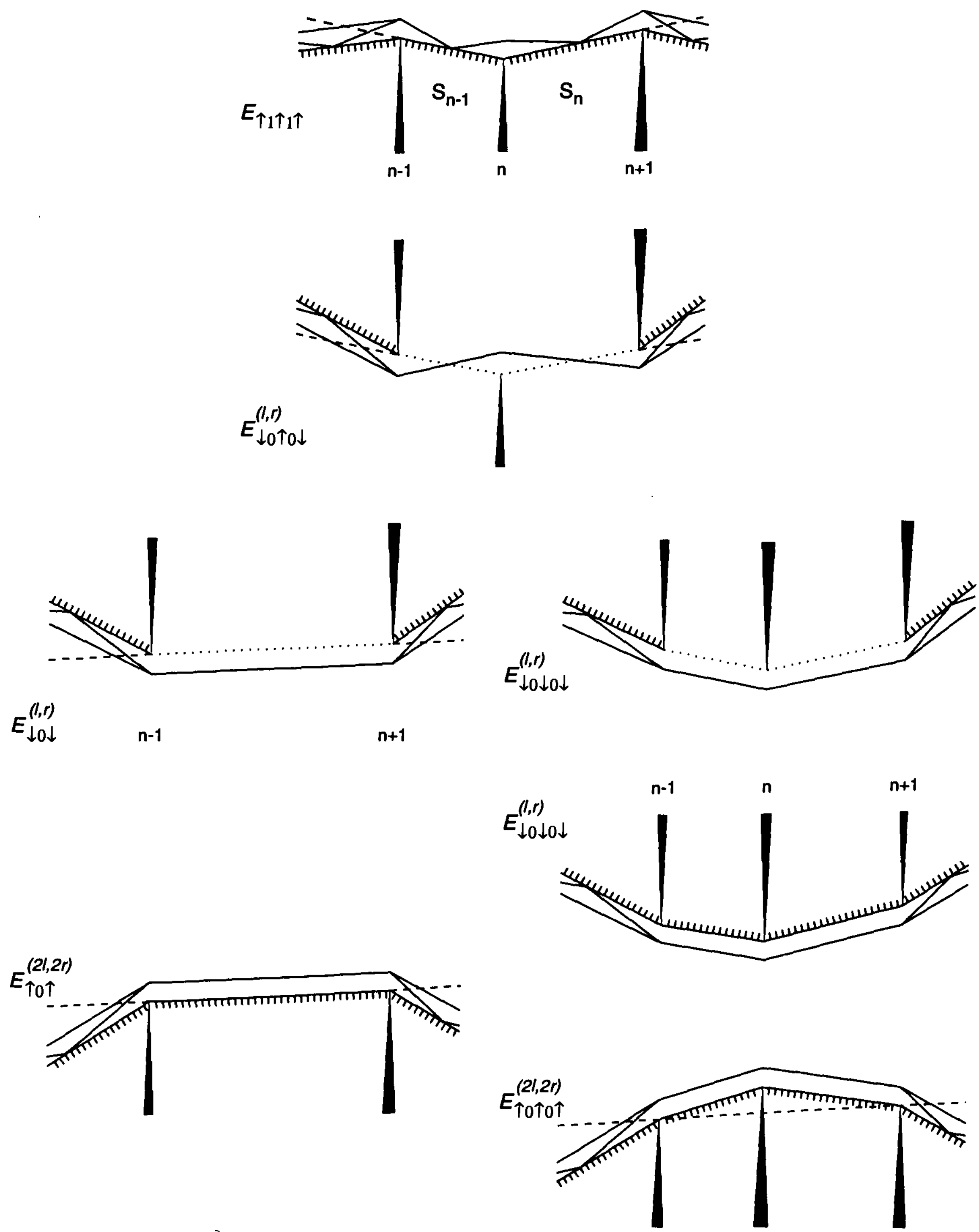

Figure 9. Elimination of a valley at the location of the n'th bridged knife edge, for modes in which $m_{n-1} = 1$ and $m_n = 1$ (top of diagram). The problem is transformed into the two illustrated at the bottom left and right. See Section 4.1.4 in text.

Then combining (18) to (21) gives

$$E_{\uparrow 0\uparrow 1\uparrow} = E^{(2r)}_{\uparrow 1\uparrow} - E^{(2l,2r)}_{\uparrow 1\uparrow 0\uparrow} \tag{22}$$

4.1.3 Analysis of $E_{\uparrow 1\uparrow 0\uparrow}$

For the set of modes illustrated in Figure 8, S_n can be removed with no effect, and S_{n-1} can be removed along with a half-space reflection. The analysis here is the same as in the last section, except that the roles of S_n and S_{n-1} are reversed. The result is

$$E_{\uparrow 1\uparrow 0\uparrow} = E^{(2l)}_{\uparrow 1\uparrow} - E^{(2l,2r)}_{\uparrow 0\uparrow 1\uparrow} \tag{27}$$

4.1.4 Analysis of $E_{\uparrow 1\uparrow 1\uparrow}$

For the set of modes illustrated in Figure 9, S_{n-1} and S_n can be removed along with a half-space reflection for each. The result is

$$E_{\uparrow 1\uparrow 1\uparrow} = E^{(l,r)}_{\downarrow 0\uparrow 0\downarrow} \tag{28}$$

Knife edge n has been left free, and Babinet's principle can be applied to it, giving

$$E^{(l,r)}_{\downarrow 0\uparrow 0\downarrow} = E^{(l,r)}_{\downarrow 0\downarrow} - E^{(l,r)}_{\downarrow 0\downarrow 0\downarrow} \tag{29}$$

where the terms on the right hand side of (29) are illustrated in the side-by-side drawings in Figure 9. On the left of Figure 9, it is now only necessary to insert a spanning reflecting surface, and to reflect everything in it, to arrive at

$$E^{(l,r)}_{\downarrow 0\downarrow} = E^{(2l,2r)}_{\uparrow 0\uparrow} \tag{30}$$

Similarly, on the right, it is only necessary to insert reflecting surfaces and to reflect everything in the spanning surface. This gives

$$E^{(l,r)}_{\downarrow 0\downarrow 0\downarrow} = E^{(2l,2r)}_{\uparrow 0\uparrow 0\uparrow} \tag{31}$$

Then combining (28) to (31) gives

$$E_{\uparrow 1\uparrow 1\uparrow} = E^{(2l,2r)}_{\uparrow 0\uparrow} - E^{(2l,2r)}_{\uparrow 0\uparrow 0\uparrow} \tag{32}$$

4.2 TOTAL FIELD

The final result for an interior knife edge is

$$\begin{aligned} E = {} & \lambda_{n-1}\lambda_n(E_{\uparrow 0\uparrow} - E^{(2l,2r)}_{\uparrow 1\uparrow 1\uparrow}) + \lambda_{n-1}\mu_n(E^{(2r)}_{\uparrow 1\uparrow} - E^{(2l,2r)}_{\uparrow 1\uparrow 0\uparrow}) \\ & + \mu_{n-1}\lambda_n(E^{(2l)}_{\uparrow 1\uparrow} - E^{(2l,2r)}_{\uparrow 0\uparrow 1\uparrow}) + \mu_{n-1}\mu_n(E^{(2l,2r)}_{\uparrow 0\uparrow} - E^{(2l,2r)}_{\uparrow 0\uparrow 0\uparrow}) \end{aligned} \tag{33}$$

In each of the eight new problems in (33), the half spaces are either reflected twice, or not at all. Where there are two reflections, one is in either S_{n-1} (l reflections) or S_n (r reflections), and the other is in the spanning bridge. Where the superscript $(2l, 2r)$ occurs, the middle, or n'th knife edge also undergoes two reflections, which may equally be considered part of the $2l$ or $2r$ reflections.

If $\lambda_{n-1} = \lambda_n = \mu_{n-1} = \mu_n = 1$, then (33) reduces to

$$E = E_{\uparrow 0 \uparrow} + E^{(2r)}_{\uparrow 1 \uparrow} + E^{(2l)}_{\uparrow 1 \uparrow} + E^{(2l,2r)}_{\uparrow 0 \uparrow} - E^{(2l,2r)}_{\uparrow\uparrow\uparrow} \tag{34}$$

There are five new problems in this case. In four of these, knife edge n has been omitted, and in the fifth, the valley at x_n has been replaced by a hill. If any of the λ's or μ's are zero, then (33) must be used, but at most four of the eight new problems in (33) have to be solved.

5. REPETITIVE APPLICATION

By use of (10), (12), or (33), the original problem is transformed into two or more new problems in which a valley has been removed from the model terrain profile. However, some of the new terrain profiles may contain other valleys. Therefore it may be necessary to do successive transformations. This is a straightforward matter, since the new problems have the same form as the original one, with the generalization mentioned in Section 2.5. To find the solution of each diffraction problem, it is first necessary to assign values to the λ_n and the μ_n.

In the original problem, $\lambda_n = 1$ for all values of n. Depending on whether a bridging surface is or is not present between knife edges n and $n+1$, $\mu_n = 1$ or 0.

In problems arising out of previous applications of Babinet's principle, values for the λ_n and μ_n are assigned as follows: If the numeral in the subscript of the symbol E in the position corresponding to surface S_n is a zero, then $\lambda_n = 1$ and $\mu_n = 0$. Conversely, if the numeral is a 1, then $\lambda_n = 0$ and $\mu_n = 1$. If there is no numeral, as in (11), then $\lambda_n = 1$ and $\mu_n = 1$. For any n not represented in the subscripts of E (attention is always limited to one or two values of n), the values of λ_n and μ_n are retained from the previous problem. After all necessary transformations have been done, the final values of the λ_n and the μ_n may be used in equation (35) of [5].

6. IMAGE POSITIONS

6.1 DEFINITIONS AND ASSUMPTIONS

This section provides formulas for locating the reflected images of the source A, field point B, and terrain points. Let the slope of surface S_n be γ_n. Then if the height of the n'th knife edge is h_n, the height z_S of the reflecting surface as a function of x is

$$z_S = h_n + \gamma_n(x - x_n) \tag{35}$$

In the following it is assumed that all distances in the direction of propagation are much greater than any perpendicular distances. In particular, $|\gamma_n| << 1$.

6.2 SINGLE REFLECTIONS

Suppose that the world to the left of S_n is reflected through S_n. The image of any point (x, y, z), $x < x_n$ appears at (x', y, z'), where

$$\begin{aligned} z' &= z - 2(z - z_S) \\ x' &= x + \gamma_n(z - z') \end{aligned} \tag{36}$$

The horizontal displacements $x' - x$ have small magnitudes compared to the distances between knife edges, and must be taken into account only because the change in path length affects the phase of the field. Furthermore, only the shifts in the end points of the paths have any effect, since for intermediate points, the path is lengthened on one side and shortened by an equal amount on the other. Therefore the only horizontal displacements of importance are

$$\Delta x_{A,n} = \gamma_n(h_A - h_{A'}) \tag{37}$$

and

$$\Delta x_{B,n} = -\gamma_n(h_B - h_{B'}) \tag{38}$$

where $h_{A'}$ and $h_{B'}$ are the heights of the images A$'$ and B$'$ of the source at A and of the field point at B, found from (36) (see Figure 10).

The quantities $\Delta x_{A,n}$ and $\Delta x_{B,n}$ are the distances by which the image paths are shorter than the original paths. Therefore, in finding the fields due to reflected worlds on the left and right sides of S_n, horizontal coordinates can be left unchanged, provided the fields are multiplied by $\exp(ik\Delta x_{A,n})$ and $\exp(ik\Delta x_{B,n})$, respectively.

6.3 DOUBLE REFLECTIONS

Suppose now that the world on the left of S_{n-1} is reflected twice, once through S_{n-1} and once through the surface spanning S_{n-1} and S_n. (In Figure 11, the spanning surface coincides with the broken line between knife edges $n - 1$ and $n + 1$.)

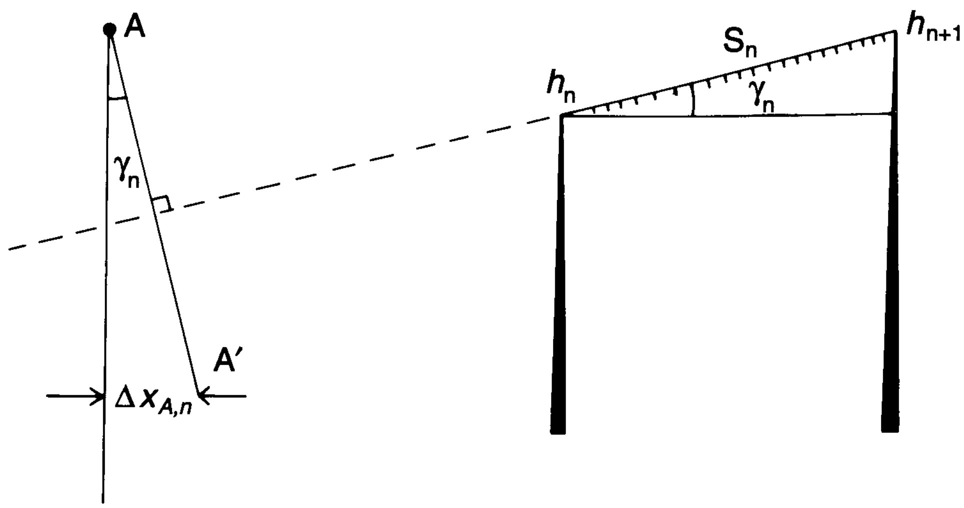

Figure 10. For a single reflection, the displacement of the image of the source with respect to the source. See (37) and (38) in text.

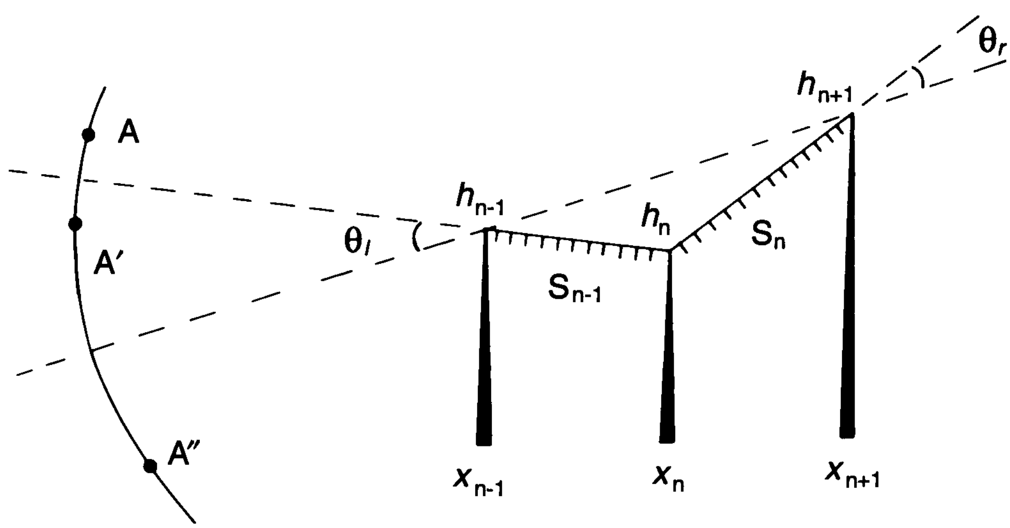

Figure 11. For a double reflection, the displacement of the image of the source with respect to the source. See (40) and (41) in text.

Consider the angle θ_l between S_{n-1} and the bridge spanning S_{n-1} and S_n (Figure 11). As a result of the two reflections, the world for $x \leq x_n$ pivots about knife edge $n-1$ by angle $2\theta_l$. Similarly, when the world on the right of S_n is reflected twice, once through S_n and once through the spanning surface, the world for $x \geq x_n$ pivots about knife edge $n+1$ by angle $2\theta_r$, where θ_r is the angle between S_n and the spanning bridge. (When knife edge n is present its reflection in the spanning bridge is consistent with both rotations.)

For double reflections of the left half-space, the image of any point (x, y, z), $x < x_n$ appears at (x'', y, z''), where

$$\begin{aligned} z'' &= z - 2\theta_l(x_{n-1} - x) \\ x'' &= x - \theta_l(z + z'' - 2h_{n-1}) \end{aligned} \tag{39}$$

As with single reflections, the only horizontal displacements of importance are

$$\Delta x_{A,n} = -\theta_l(h_A + h_{A''} - 2h_{n-1}) \tag{40}$$

and

$$\Delta x_{B,n} = \theta_r(h_B + h_{B''} - 2h_{n+1}) \tag{41}$$

where $h_{A''}$ and $h_{B''}$ are the heights of the images of A and B, found from (39). As with single reflections, $\Delta x_{A,n}$ and $\Delta x_{B,n}$ are the distances by which the image paths are shorter that the original paths. Therefore, in finding the fields due to reflected worlds on the left and right sides of x_n, horizontal coordinates can be left unchanged, provided the fields are multiplied by $\exp(ik\Delta x_{A,n})$ and $\exp(ik\Delta x_{B,n})$ respectively.

7. CONCLUSION

A terrain diffraction problem composed of bridged knife edges of arbitrary height can be replaced by other bridged-knife-edge problems in which there are no valleys. This replacement can be accomplished by repeated use of (10), (12), and (33). Equations (10) and (12) are used to remove valleys at the ends of the terrain profile, and (33) is used to remove interior valleys. These equations can be used to transform the original problem into new problems, which themselves can be transformed, and so on, until all the valleys are removed. In the final problems, a segmented line passing through the source, terrain vertices and field point is everywhere convex upward.

As a special case, all of the coefficients μ_n may be set to zero in the original problem. That is, all of the bridging surfaces may be omitted. The transformations given here then provide a way of finding the field due to multiple knife edges in cases where the series in [3] does not converge.

For both bridged and unbridged knife edges, Vogler's series [3] does converge in the presence of shallow valleys, and only fails to converge when the valleys are deep enough. The conditions for convergence in general have not been worked out. However, the series seems to always converge for terrain in which all valleys have been removed.

The transformations described here were used in the sample calculations of [5], and except for the three-knife-edge example, the calculations could not have been done over the whole range of parameters that are shown without these transformations.

8. REFERENCES

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