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LINEAR-PHASE BANDPASS FILTERS

by

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ABSTRACT

The filters discussed in this Technical Note were developed from the analysis of a class of bandpass filters by Robert Lerner which simultaneously offer a good approximation to both constant magnitude and linear phase over the pass-band. Measurements on a typical broad-band IF filter, having 20 MHz bandwidth centered at 20 MHz, show a performance which can be closely predicted by computer analysis.

1. INTRODUCTION

The filters discussed in this Technical Note were developed from the analysis of linear phase bandpass filters by Robert Lerner[1]. The technique used has resulted in very flat pass-band magnitude and linear phase characteristics over most of the nominal bandwidth. Unlike most of the previous filters designed, which tend to concentrate on magnitude response only, closer approximation to the ideal magnitude response by using additional poles does not increase the phase distortion of the Lerner filter. It maintains its linear phase characteristic.

Besides offering a good approximation to both constant magnitude and linear phase, the Lerner filter design has several other advantages. The design procedure used in practice is simple and does not involve time-consuming calculations. Shunt inductance and stray capacitance can be easily taken into account in the filter design. Actually, they can be utilized to improve the response. The presence of resistance in series with the coils causes mainly flat attenuation in the transmission. Lerner describes his filter by means of pole-residue locations only. The necessity for knowing where the zeros are located is removed by using the partial fraction expansion of the transfer function to give a summation of pole-residue expressions rather than a product of pole and zero factors.

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1 Lerner, Robert. *Band-pass filters with linear phase.* Proceedings of the IEEE, March, 1964, p 249.

Before presenting the actual performance of the circuits used, an outline of the practical design approach of Lerner filters will be discussed.

2. THE LERNER FILTER

Simultaneous approximation to both constant amplitude and linear phase over most of the bandpass was obtained by placing poles at equal intervals along a line parallel to s = jω in the pole-zero plot. The residues were made equal except for alternating sign. A distinctive feature of this design is the use of two 'corrector' poles near the band edges to improve the control over the network behaviour at these points. The s-plane pole pattern is shown in Figure 1.

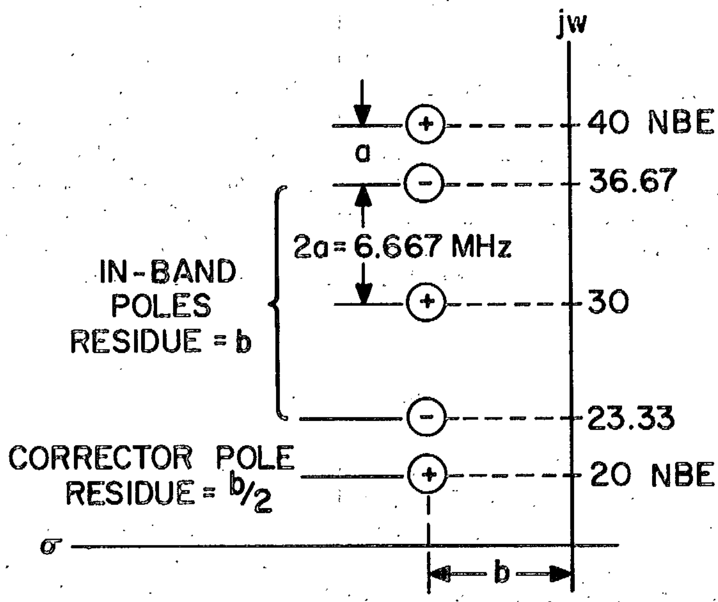

Fig. 1. S-plane pole pattern.

The filter design used to display these characteristics with realizable components is shown in Figure 2. The nominal bandwidth is 20 MHz to 40 MHz. This is the theoretical bandpass range. The nominal band edge is the theoretical band edge (20 MHz and 40 MHz in this case). The filter utilizes as basic components a transformer with centre-tapped secondary with two groups of series resonant branches attached to the secondary so that one group is driven in opposite phase to the other. The number of series resonant circuits corresponds to the number of poles. All branches except the two correctors are in-band resonators with equal inductances of 1.875 μH. The corrector resonators have inductances of 2 x 1.875 μH. The series capacitors are chosen to provide one branch resonance at each pole frequency at intervals of 2a across the pass-band, alternating their secondary branch positions as frequency is increased. This alternation, which will give the changing sign specified in the above paragraph, also applies to the corrector resonators which are tuned one interval 'a' above and below the in-band resonance range. The inductances were determined for a 50 Ω source (from the secondary) and load by the relation given by Lerner

$$R = \frac{4}{\pi} X_L$$

where X_L is calculated at 2a Hz, and R is defined in Figure 2.

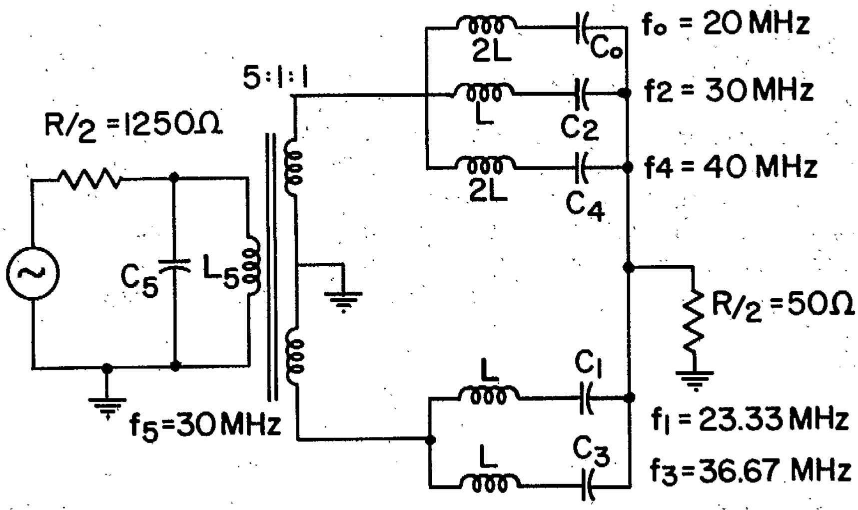

Fig. 2. Five-pole filter.

The response of an infinite series of poles similar to the in-band poles of Figure 1 can be expressed by the transfer function

$$H(\omega) = \sum_{-\infty}^{\infty} \frac{b(-1)^n}{j(\omega - 2na) + b} .$$

This formula assumes, after Lerner, that the in-band pole residues and their distances from the jω axis (their σ values) are equal. Thus the residue in the numerator and the σ value in the denominator are both expressed as 'b'. This transfer function is a periodic function of ω and Fourier analysis shows the ripple to have a relative amplitude of η^2 in which $\eta = \exp\ (-\ \pi b/2a)$. It also approximates a time delay of $\pi/2a$ seconds to within a maximum periodic error of η^2 radians. Also, in the theoretical zero-ripple case, the in-band residue at each pole can be determined as

$$b = \frac{P}{\sqrt{R_1 R_2}} = \frac{4a}{\pi R_2} \text{ or } \frac{4a}{\pi R_1}$$

where p is a scale factor, R_1 is the source resistance, and R_2 is the load resistance.

The expression for the magnitude and phase ripple provides a means of control. By varying the ratio b/a, the magnitude of this in-band ripple can be varied. However, this feature is more useful in controlling the band edge response.

The corrector poles are placed at the band edge to try to maintain the linear-phase approximation across the band. They are placed the same distance from the $j\omega$ axis as the in-band poles but have one-half the residue of the in-band poles. This design suppresses the phase hump just inside the band and causes it to occur just outside the band edge. Also, there is still some magnitude ripple. The b/a ratio of the correctors can be changed easily by varying the poles from the normal position a Hz above and below the in-band pole range. The hump in the phase characteristics near the band edges gets larger as b/a becomes smaller.

The magnitude response of the filter is improved when the load or source is shunted by a parallel tuned LC branch, resonant at the band centre. The magnitude ripple is decreased. Also, it is apparent from Lerner's work that this shunt compensation not only reduces the in-band phase error but also reduces the phase error outside the band edge at the hump in the phase characteristic. Thus the problem of using transformers with their shunt inductance and capacity can be eliminated; they become part of the parallel LC circuit. A capacitor placed across the transformer primary can tune this parallel LC branch to resonance at the band centre. The loaded Q of the shunt compensation can be expressed as

$$Q_p = k \frac{f_o}{\Delta f}$$

where Q_p is the Q of the parallel LC branch, f_o is the centre frequency, Δf is the nominal bandwidth, and k is the slope at f_o of the graph of frequency versus the admittance of the out-of-band resonances seen at the input terminals. This theoretical admittance can be determined through an equivalence transformation of the lattice network. The slope varies from 5/3 for small N to $\pi/2$ for large N. 'N' is the number of in-band poles. Lerner also suggests that

$$Q_B = NQ_p$$

$$L_p = \frac{L_B}{Q_B}$$

where L_B and Q_B are the inductance and Q of the individual in-band filter branch resonances.

Having outlined the basic design procedures for the filter and its interpretation on the pole-zero plot, a description of the actual filter response under various conditions follows.

3. FILTER RESPONSE CHARACTERISTICS

The curves in Figure 3 indicate the calculated magnitude and phase response of the source matched filter as the corrector poles are moved by various quantities of 'a' outside the nominal band edge (NBE). Also shown is the phase error from a constant delay. With the corrector poles in their normal positions of 20 MHz and 40 MHz at the nominal band edges, the magnitude variation from a constant value of −20.4 dB is a maximum of 1.5 dB over 90 per cent of the nominal pass-band. (Note: the gain calculated is a voltage gain; 14 dB of the apparent loss is due to the 5:1 input voltage transformation, the remaining

6 dB is due to power division between the source and load resistances.) This error is approached only at the band edges where the magnitude develops small peaks. The average rate of attenuation expressed as a ratio of the 60 dB bandwidth to the 6 dB bandwidth is 0.428. The phase response is linear to within ± 5 degrees over 47 per cent of the pass-band. Again, this error is approached only at the NBE where the phase loses its linearity. Here the error rises to form a large hump reaching a maximum error of + 97 degrees at one end and - 88 degrees at the other. Most of the hump occurs just outside the NBE. This is not of concern in such a filter because it is only within the bandpass that linear phase is desired.

As the correctors are moved further outside the band edge, the ratio b/a for the correctors becomes larger. This will decrease the η^2 phase ripple. Figure 3 shows that as the corrector poles are moved out, the filter phase more closely approximates a delay line. Less than a 5 degree error occurs over a greater percentage of the pass-band, as may be seen in Table 1. The ± 5 degree limit is used rather than the maximum error because the former tends to give a truer representation of the phase response over the bandwidth. It can also be seen that the phase error humps become smaller and are shifted further from the band edges. However, the phase curves show that with a sufficient movement of the correctors, the phase line is over-corrected and follows a path on the other side of the constant delay before following the slope of the original phase hump. In effect, the phase ripple frequency is increased. This shows up in the phase error graph in the appearance of a negative of the original phase error. This accounts for the decrease of the ± 5 degree error range in Table 1. The situation suggests an optimum corrector location (with all other parameters constant) beyond which the error begins to increase again. This optimum location, however, is not consistent with the best magnitude response. As the correctors are moved further from the NBE, the dip in the magnitude response increases, along with a more prominent peak. The in-band magnitude response is shown in Table 1 as the maximum error over 90 per cent of the pass-band; this gives a better indication of the magnitude response than does the use of the full 100 per cent bandwidth. As the distance to the corrector poles from the NBE is increased, the error decreases from 1.5 dB to 0.9 dB and then increases to 2.0 dB. Also included in Table 1 is an indication of the rate of attenuation outside the bandwidth. This is expressed as a ratio of the 60 dB bandwidth to the 6 dB bandwidth. The variation is not significant because the greatest change is 0.024. The ideal response would be unity.

TABLE 1

Response Characteristics

Corrector Position Outside NBE (a)	Maximum Magnitude Error over 90% of Pass-band (dB)	Rate of Attenuation $\frac{\text{6dB bandwidth}}{\text{60dB bandwidth}}$ (dB)	Bandwidth within ± 5° Phase Error (%)
0.0	1.5	0.428	47
0.1	1.4	0.412	58
0.3	0.9	0.416	72
0.6	1.7	0.426	95
1.0	2.0	0.436	75

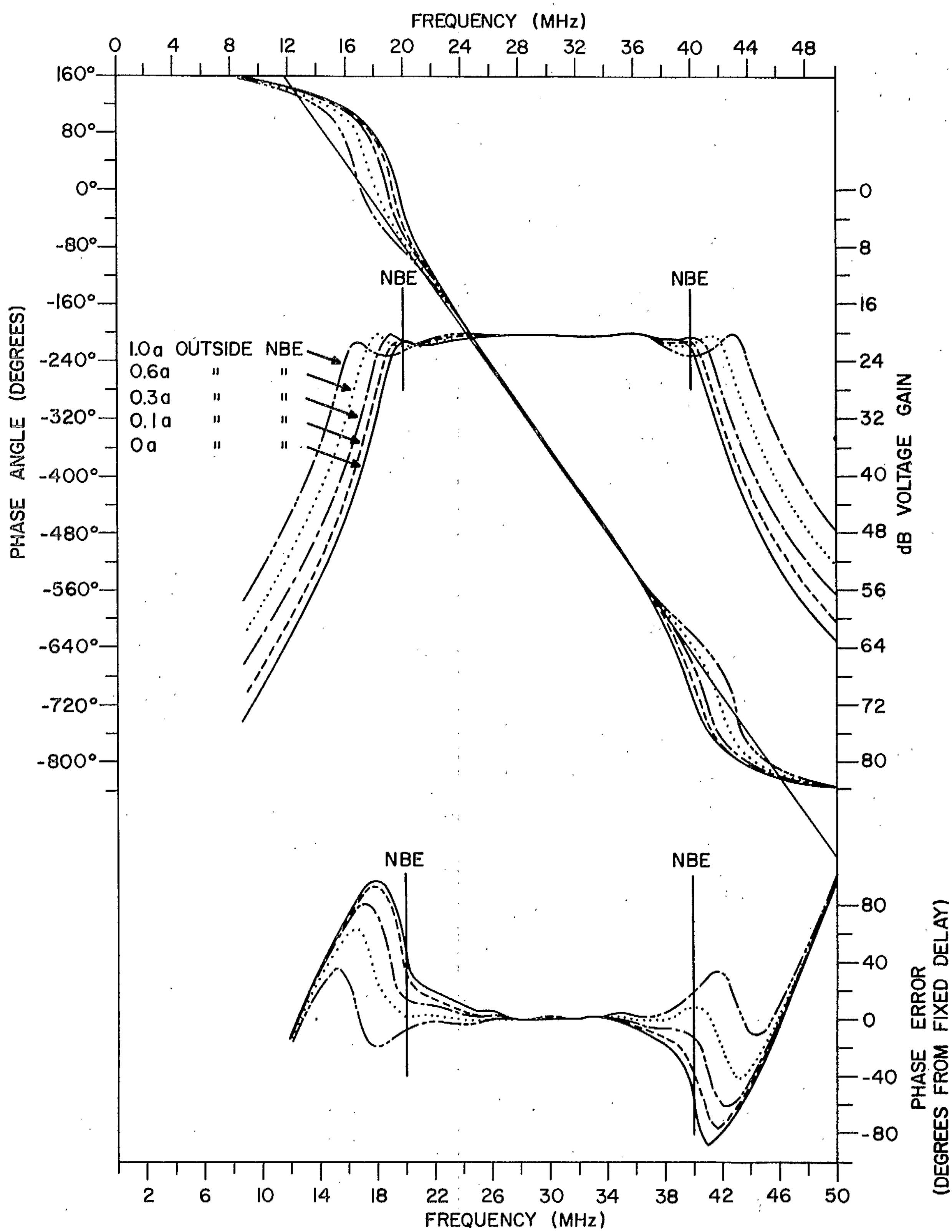

Fig. 3. Variation of corrector pole position outside nominal band edge.

Of the variations in corrector position tried, the 0.6a curve gave the best phase response ± 5 degrees over 95 per cent of the pass-band. This same curve, however, had an increased maximum peak-to-peak ripple of 1.9 dB compared to a low of 0.7 dB. Thus, without the addition of further corrector poles or other circuit refinements, a compromise must be made by the designer, depending on whether he is more concerned with linear phase or constant magnitude. Another drawback in the magnitude characteristic with the best phase condition is an increase in the bandwidth. This appears to defeat the original purpose of moving the correctors--that of flattening the phase error near the NBE. Now the effective band edge has been shifted away from the centre frequency so that the phase error we have succeeded in moving out of the pass-band is still in the transmission frequency range. Again, compromise is required, depending on the specifications of the application. The effective total bandwidth, however, can be decreased by the insertion of out-of-band transmission zeros. This is done by bridging series tuned circuits with the same inductance as the in-band resonators, across the source and/or load. An alternative is to increase the number of in-band poles. As the order is increased, the rate of attenuation at the band edges is increased while the phase error remains relatively constant. The magnitude ripple frequency is increased, but the ripple magnitude is affected very little.

4. RESULTS

The results plotted in Figure 3 were calculated using the DRTE computer. Figure 4 compares these calculated and measured results for a source matched filter whose corrector poles are moved 0.3a outside the NBE. Also shown is the response with an unmatched source of 50 kΩ. The magnitude responses for the matched source were in close agreement; the peaks were in approximately the same positions and of the same size. The measured rate of dB attenuation at the lower band edge was less than for the calculated response. The difference can be attributed to leakage reactance, stray capacitance, and component approximations, although these errors have been kept as small as possible. The measured phase error has more in-band ripple, but the hump in the characteristics does not occur as close to the NBE. The lower phase error hump is smaller in magnitude, while the higher frequency one is larger. The calculated magnitude response with the unmatched source shows a greater ripple--a maximum peak-to-peak ripple of 4.7 dB, as compared with 1.5 dB with the matched source. The same is seen in the phase error curve in which the ripple has become more pronounced. Both phase error humps have increased to 82 degrees--an increase of 20 degrees over the matched source for the higher frequency characteristic hump.

Figure 5 indicates the response of the 5-pole filter driven from the single-stage common base amplifier shown in Figure 6. The corrector poles are 0.3a outside the NBE. The output of the amplifier has a very high impedance--approximately 1 MΩ, so that the source is not matched. The response should be similar to the curve in Figure 4 with the 50 kΩ source impedance. The two curves are similar, with the one in Figure 5 more irregular due to the higher source impedance. The ripple magnitude is the same but the slope of the ripple has changed so that it no longer varies about the zero-ripple line. To a lesser extent this state can also be seen in Figure 4. The phase error ripple is about the same, except that it has become more pronounced in those areas where it had previously been flattened out. It can be seen that the humps in the

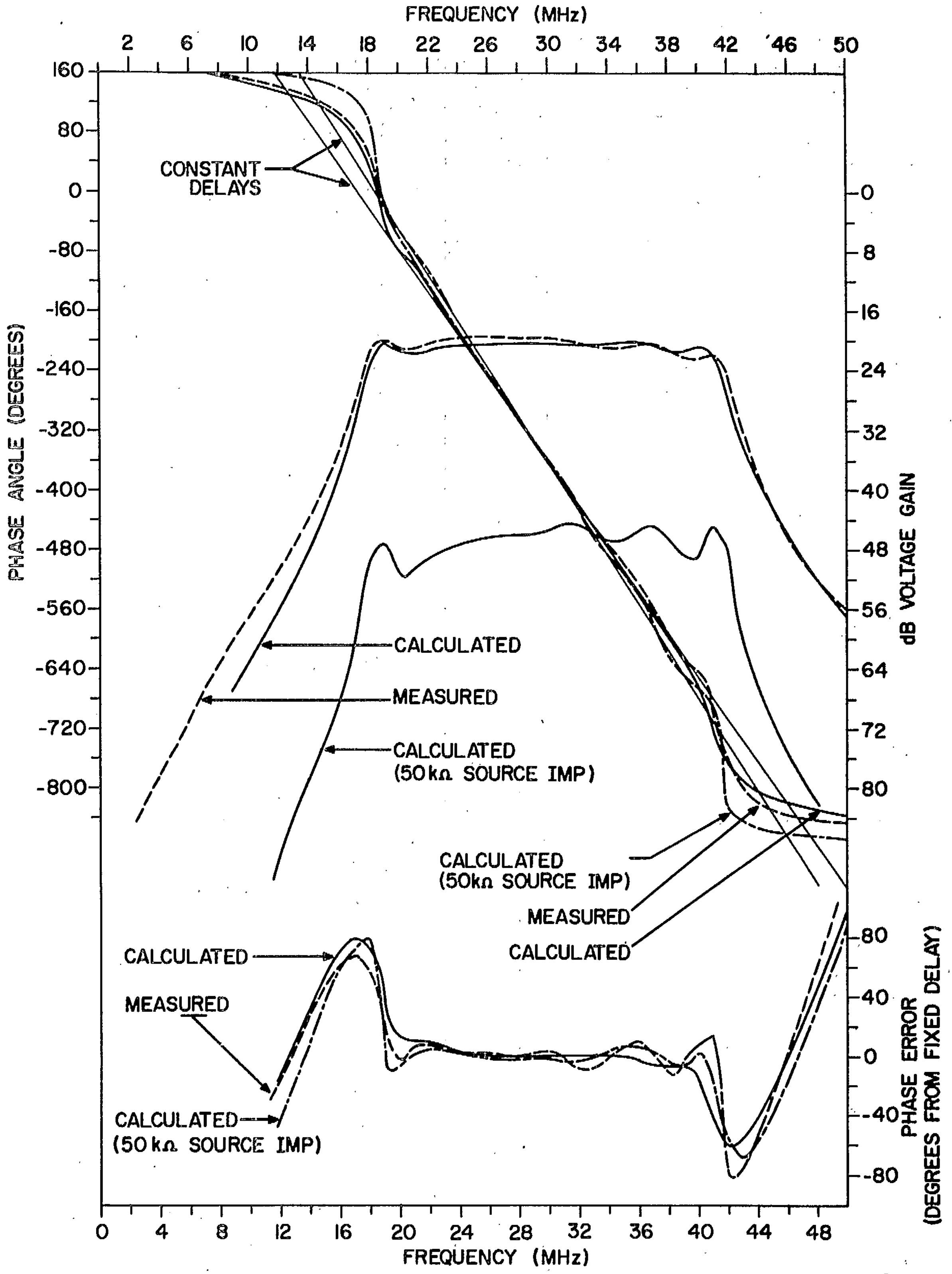

Fig. 4. Five pole Lerner bandpass filter calculated and measured comparison.

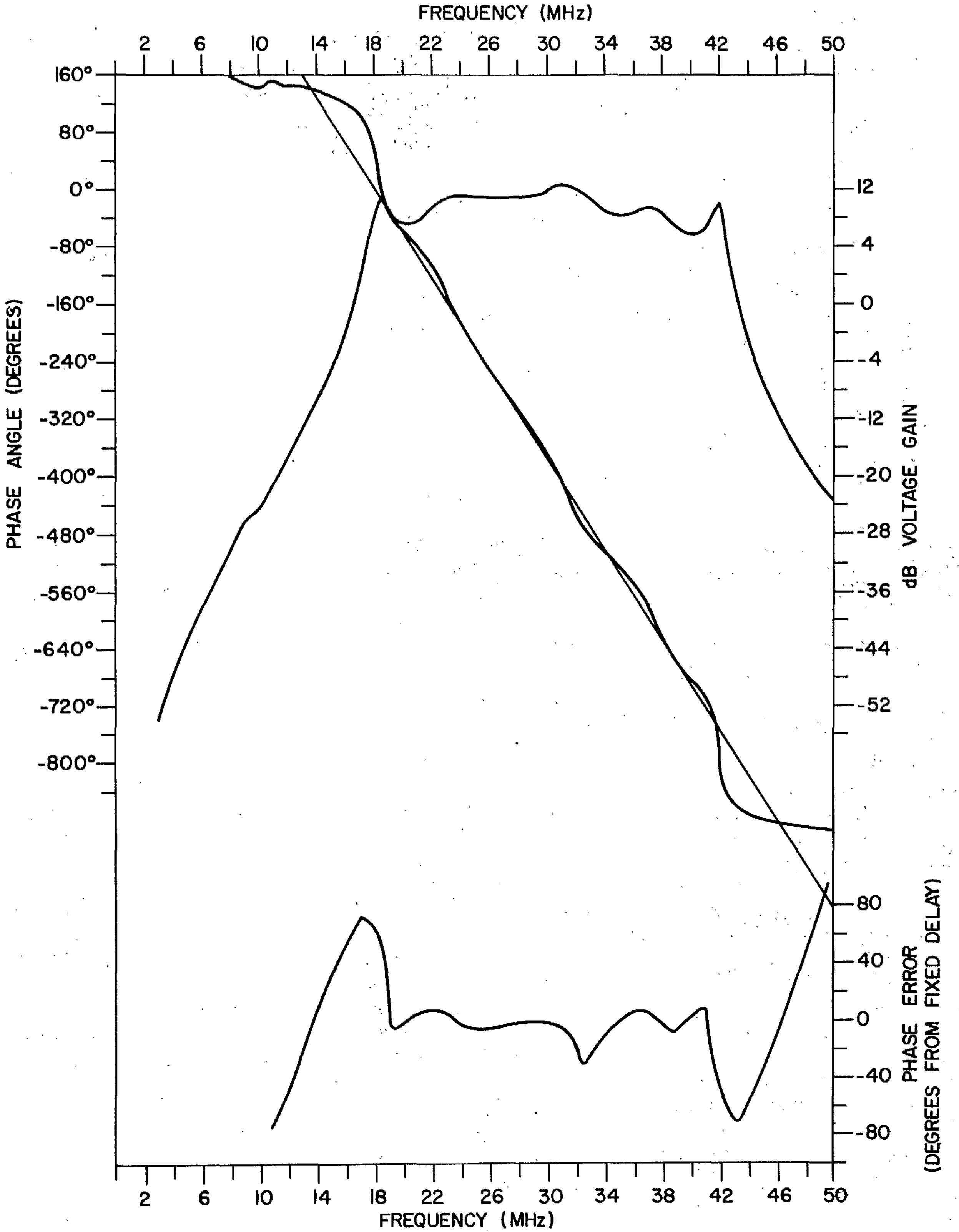

Fig. 5. Five pole Lerner bandpass filter driven by common base amplifier.

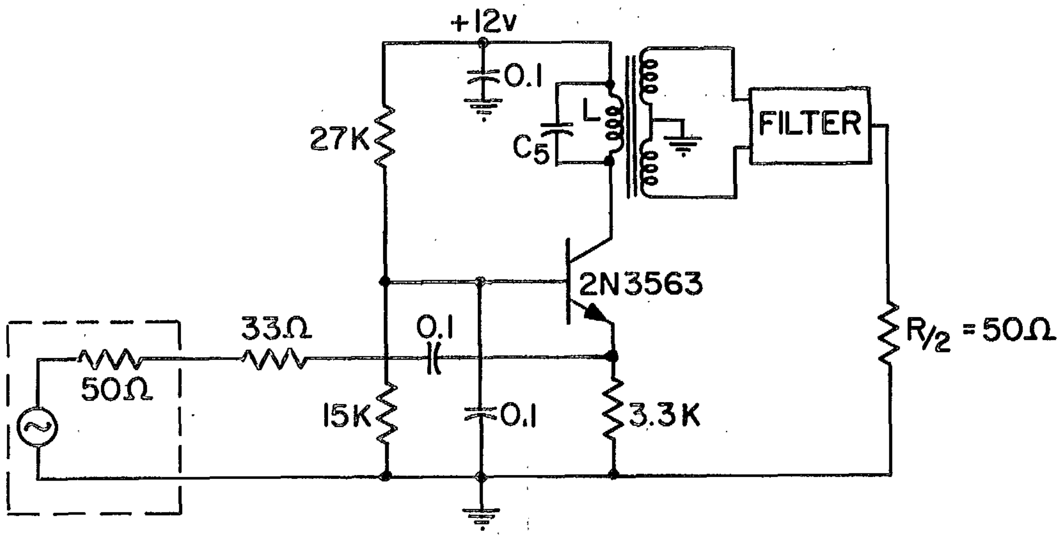

Fig. 6. Transistor driven filter.

phase error characteristics outside the nominal band edges are ± 72 degrees. The response could be improved by using a better matching transformer, although the leakage inductance will become more of a problem. Another alternative would be to bridge the current source of the transistor with the required matching source resistance of 1250 Ω. However, this would also result in a loss of one-half of the output power.

5. CONCLUSION

When a rectangular pass-band and linear phase-shift are both necessary, it is customary to use a Butterworth-Thomson combination which allows the designer to achieve results in an indirect way. The Lerner filter discussed in this Technical Note provides a single filter approximation to both constant bandpass magnitude and constant delay. These networks are easy to calculate and respond well to variations in parameters and terminal conditions. It is easy to experiment by computation with variations in the residue and position of the corrector pole and other poles near the band edge. Phase errors of ± 3 degrees over 94 per cent of the pass-band and magnitude ripple of 0.9 dB over 90 per cent of the pass-band were not difficult to achieve. The 6:60 dB attenuation ratio reached 0.436, which compares favourably with corresponding 5-pole Chebyshev and Butterworth functions. This ratio improves even more as the number of poles increases, without increasing the phase error. Matching the source impedance to the load gives a much smaller phase and magnitude ripple. In one case, the in-band magnitude variation was decreased by 5.7 dB with a peak phase error decrease of 7 degrees.
