Communications Research Centre

SOLUTION OF THE CONTINUITY EQUATION FOR A DIODE WITH SPATIALLY VARYING MINORITY CARRIER LIFETIME

by

R.H. HUM, S.S. KAMAL, AND A.L. BARRY

Department of Communications
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EXECUTIVE SUMMARY

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TITLE: Solution of the Continuity Equation for a Diode with Spatially Varying Minority Carrier Lifetime

AUTHOR(S): R.H. Hum, S.S. Kamal and A.L. Barry

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This note describes the mathematical solution to a problem related to the effects of space radiation on GaAs light-emitting diodes.

The diodes under investigation were intended for use in the Communications Technology Satellite. Complete information on the effects of space radiation on these diodes was not previously available. It was found that behaviour of the devices in a radiation environment did not follow the simple degradation law predicted by theory, and which is obeyed by most light-emitting diodes. The work reported here is a first step toward explaining this and other anomalous behaviours in terms of special properties of the recently developed technology of amphoteric-silicon-doping of GaAs.

This solution also has potential relevance to other devices of interest for space applications, such as optical isolators.

SOMMAIRE À L'INTENTION DE LA DIRECTION

N° DU DOCUMENT: Note technique du CRC n° 679

TITRE: Solution de l'équation de continuité pour une diode ont la durée de vie des porteurs minoritaires est variable dans l'espace

AUTEUR(S): R.H. Hum, S.S. Kamal et A.L. Barry

DATE: Février 1976

La présente note décrit la solution mathématique du problème des effets de la radiation spatiale sur les diodes à émission de lumière à arséniure de gallium.

Les diodes étudiées devaient être utilisées dans le satellite technologique de télécommunications. Des renseignements complets sur l'effet de la radiation spatiale sur ces diodes n'étaient pas disponibles auparavant. On a démontré que le comportement de ces diodes, sous l'effet de la radiation, ne suivait pas la loi simple de dégénération prévue théoriquement, loi suivie par la plupart des diodes à émission de lumière. Le travail décrit dans le présent document est une première étape vers l'explication de ce phénomène ainsi que d'autres comportements anormaux en fonction des caractéristiques spéciales d'une nouvelle technique de dopage de l'arséniure de gallium au silicium amphotère.

Cette solution pourrait aussi être applicable à d'autres dispositifs utilisés dans les applications spatiales, tels les isolateurs optiques.

COMMUNICATIONS RESEARCH CENTRE

DEPARTMENT OF COMMUNICATIONS
CANADA

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LIST OF SYMBOLS

A, A', A"	constants of integration
Ai (z)	Airy function
b	x_a/L_{NR} (equation 16)
B, B', B"	constants of integration
D_n	diffusion constant for electrons in p-type semiconductor
ε	electric field
I_n (x)	modified Bessel function of first kind, of order n
J_n (x)	Bessel function of first kind, of order n
k	Boltzmann's constant
K	radiation damage constant
K_n (x)	modified Bessel function of second kind of order n
L_n	diffusion length of electrons in p-type semiconductor with constant lifetime
L_{NR}	"non-radiative diffusion length" of electrons in p-type semiconductor with spatially varying lifetime
n (x)	excess minority carrier (electron) density in p-type semiconductor
n_o	excess minority carrier (electron) density at edge of depletion region
n_{po}	equilibrium minority carrier (electron) density in p-type semiconductor
p_p	majority carrier (hole) density in p-type semiconductor
q	electronic charge
t	time
T	absolute temperature
u	$= (1 + x/x_a)$. Change of variable in equation (15)

V_j	voltage appearing at junction of ideal diode (excluding voltage drop across ohmic series resistance)
x	distance into neutral p region from edge of depletion region
x_a	distance from edge of depletion region at which radiative and non-radiative recombination rates are equal
$Y_n\ (x)$	Bessel function of second kind of order n
α	constant proportional to the ionized acceptor concentration gradient on the p side of the junction
ϕ	radiation fluence, or dose
τ_{NR}	non-radiative lifetime of electrons in p-type semiconductor. Proportional to the reciprocal of the non-radiative recombination rate.
τ_R	radiative lifetime of electrons in p-type semiconductor. Proportional to the reciprocal of the radiative recombination rate.
τ_{Ro}	radiative lifetime at edge of depletion region (x=o). Proportional to the reciprocal of radiative recombination rate at this point.
τ_T	total minority carrier lifetime in a semiconductor, taking into account all radiative processes.
τ_o	initial value of τ_T before degradation by radiation

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ABSTRACT

The classical steady state solution of the continuity equation for minority carriers injected into a neutral region on one side of a p-n junction assumes a minority carrier lifetime which is constant throughout the neutral region. This produces an exponential decrease of excess carriers with distance from the edge of the depletion region. In some high-efficiency light-emitting diodes, minority carrier recombination is dominated by a radiative recombination process, which is dependent on the density of ionized acceptors in the p-region. In a graded-junction diode, the density of such acceptors increases with distance from the metallurgical junction, resulting in a recombination rate, and hence a lifetime, which is also a function of this distance. In this report, the continuity equation is solved for this situation, assuming a linear increase in ionized acceptors with distance. It is shown that the derived expression reduces to the exponential form for large values of a characteristic distance, x_a, which represents the point at which radiative and non-radiative recombination rates are equal. Possible implications of this result are discussed.

1. INTRODUCTION

In a "one-sided" p-n junction, the classical solution of the continuity equation in one dimension, assuming uniform minority carrier lifetime and negligible electric field in the neutral region, yields an exponential decrease of excess minority carriers with distance from the edge of the depletion region. This is reviewed in Section 2. While the assumption of constant minority carrier lifetime may be valid for many diodes, it is not generally valid for high-efficiency light-emitting diodes (LED's)[1]. These can have quantum efficiencies in excess of 50%; that is, the radiative recombination rate exceeds the non-radiative rate. If the density of radiative recombination centres is not spatially uniform, but varies over the light-emitting region, then the minority carrier lifetime in these high-efficiency diodes must also vary over this region. This produces a departure from the exponential decrease of excess minority carriers with distance into the neutral region, as exists for the case of constant lifetime. No previous solution to the case of non-uniform lifetime appears to have been published.

2. THE CONTINUITY EQUATION UNDER STEADY-STATE CONDITIONS

For minority carrier electrons injected into a field-free p-region on one side of a p-n junction diode*, the continuity equation in one dimension is[2]

$$\frac{dn_p}{dt} = D_n \frac{\partial^2 n_p}{\partial x^2} - \frac{(n_p - n_{po})}{\tau_T} \tag{1}$$

where n_p is the minority carrier (electron) density in the p-region as a function of distance x, n_{po} is the equilibrium minority carrier density, and D_n is the diffusion constant for electrons in the p-type semiconductor. The minority carrier lifetime, τ_T, is the "total lifetime", taking account of all recombination processes whose corresponding lifetimes add reciprocally, i.e.

$$\frac{1}{\tau_T} = \frac{1}{\tau_1} + \frac{1}{\tau_2} - - - + \frac{1}{\tau_k} \tag{2}$$

where $1/\tau_1$, $1/\tau_2$, etc., are the recombination rates of the various recombination processes which are assumed to be mutually independent.

Under steady-state conditions, $\frac{dn_p}{dt} = 0$, and equation (1) becomes

$$\frac{d^2 n_p}{dx^2} - \frac{n_p - n_{po}}{D_n \tau_T} = 0 \quad . \tag{3}$$

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* *For simplicity, we shall consider only electrons injected into the p-region, and neglect the hole current in the n region. This situation usually exists in practical LED's.*

Boundary conditions are:

at $x = \infty$ $\quad n_p = n_{po}$ (4)

at $x = 0$, $\quad n_p - n_{po} = n_{po} \exp (qV_j/kT)$. (5)

The second boundary condition implies the usual assumptions of "low-level injection" i.e. $n_p << p_p$, and negligible electric field in the neutral p-region. For forward bias levels of practical interest, (i.e. $V_j >> kT/q$), we may write n as the excess electron density $(n_p - n_{po})$, so that equation (3) becomes

$$\frac{d^2n}{dx^2} - \left(\frac{1}{D_n \tau_T}\right) n = 0 \tag{6}$$

with the boundary conditions

at $x = \infty$, $\quad n = 0$ (7)

at $x = 0$, $\quad n = n_{po} \exp (qV_j/kT)$ (8)

$\quad = n_o$

For the case of constant τ_T, equations (6), (7), and (8) yield the well-known result

$$n = n_o \exp (-x/L_n) \tag{9}$$

where $L_n = \sqrt{D_n \tau_T}$ is defined as the diffusion length of the minority carrier electrons in the p-type semiconductor.

3. CASE OF NON-UNIFORM ELECTRON LIFETIME IN P-REGION

We shall now assume that minority carrier recombination processes can be divided into non-radiative and radiative components, and that these are mutually independent. Non-radiative processes are assumed to be uniformly probable throughout the p-region; i.e., the non-radiative lifetime, τ_{NR}, is independent of x. The radiative recombination process is assumed to take place via silicon acceptor centres[3], the concentration of which is assumed to increase linearly with distance from the junction in a graded-junction diode[4]. This situation is believed to hold in many graded-junction LED's, and is illustrated in Figure 1. We may thus express the radiative recombination rate, $1/\tau_R$, as

$$\frac{1}{\tau_R} = \frac{1}{\tau_{Ro}} + \alpha x \tag{10}$$

where $1/\tau_{Ro}$ is the radiative recombination rate at the edge of the depletion region (proportional to the acceptor density at this point) and α is proportional to the ionized acceptor concentration gradient on the p side of the junction. τ_R is defined as the radiative lifetime. If the net acceptor concentration at the edge of the depletion region is small, so that $1/\tau_R \ll 1/\tau_{NR}$ at this point, then $1/\tau_{Ro} \simeq 0$ and

$$\frac{1}{\tau_R} = \alpha x \qquad (11)$$

with x = 0 representing the edge of the depletion region. (In a practical diode, of course, acceptor density does not continue to increase indefinitely with x. However, provided this linear increase holds for two or three diffusion lengths, negligible error will result from the assumption of (11)).

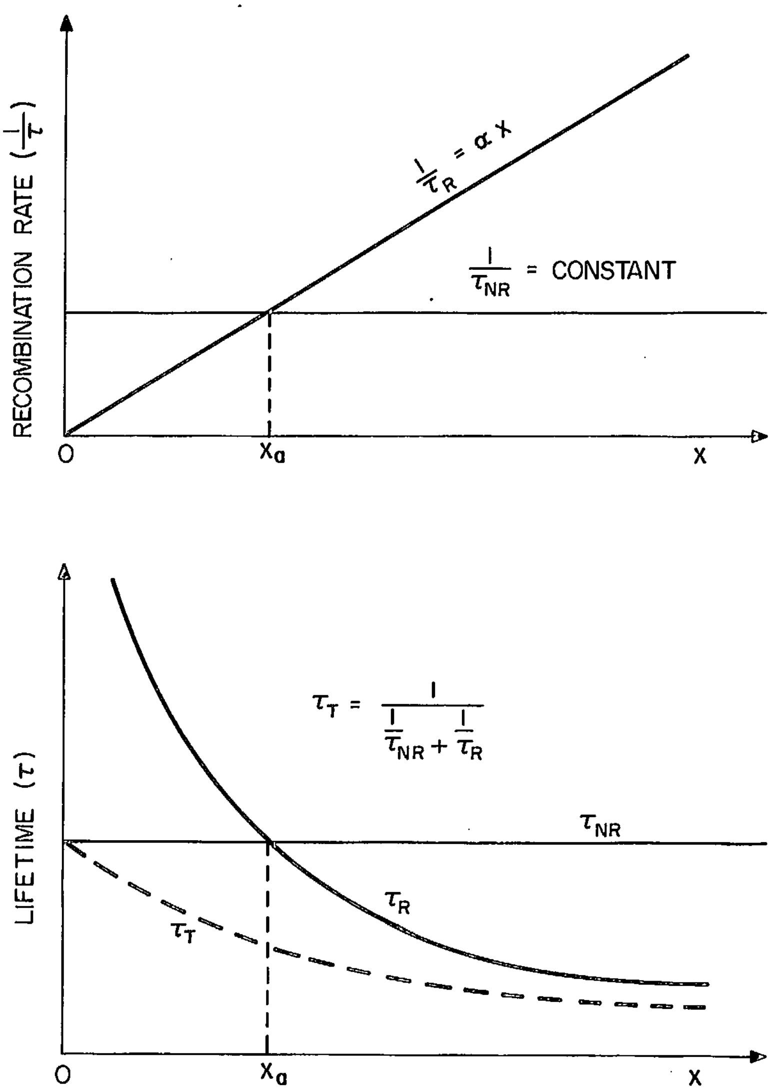

Figure 1. Variation of Recombination Rate and Lifetime With Distance From Edge of Depletion Region (x = 0), Indicating Definition of x_a

It is convenient at this point to define a characteristic distance x_a, at which the radiative and non-radiative lifetimes are equal. (See Figure 1).

$$\text{i.e.} \qquad \frac{1}{\tau_{NR}} = \alpha x_a \tag{12}$$

The total lifetime is now

$$\frac{1}{\tau_T} = \frac{1}{\tau_{NR}}\left(1 + \frac{x}{x_a}\right) \tag{13}$$

Substituting this x-dependent lifetime into (6), we obtain

$$\frac{d^2n}{dx^2} - \frac{1}{D_n\tau_{NR}}\left(1 + \frac{x}{x_a}\right)n = 0 \tag{14}$$

With the change of variable $u = (1 + x/x_a)$, this has the form of Stokes' equation[5], but with a negative coefficient in the second term, i.e.,

$$\frac{d^2n}{du^2} + (jb)^2un = 0 \tag{15}$$

where $b = x_a/L_{NR}$ and $j = \sqrt{-1}$. The quantity $L_{NR} = (D_n\tau_{NR})^{1/2}$ may be interpreted as a diffusion length corresponding to the non-radiative lifetime, τ_{NR}. Equation (15) has the following general solution in terms of Bessel functions of the first and second kind[5]

$$n = u^{1/2}\left[AJ_{1/3}\,(2/3\;jbu^{3/2}) + BY_{1/3}\,(2/3\;jbu^{3/2})\right] \tag{16}$$

To avoid the imaginary arguments of the J and Y functions, we may write an alternative solution in terms of modified Bessel functions of the first and second kind,

$$n = u^{1/2}\left\{A'I_{1/3}\left(2/3\;b\;u^{3/2}\right) + B'K_{1/3}\left(2/3\;b\;u^{3/2}\right)\right\} \tag{17}$$

From the boundary conditions of equation (7) we can immediately set $A' = 0$, since $I_{1/3}\,(\infty) \neq 0$ and $K_{1/3}\,(\infty) = 0$. For $u \geq 0$, we now have

$$n = B'u^{1/2}\,K_{1/3}\left[2/3\;b\;u^{3/2}\right] \tag{18}$$

This may be expressed in an alternative and perhaps more useful form, with the aid of a relationship between the third-order modified Bessel function of the second kind, $K_{1/3}(x)$, and the Airy function, $Ai(x)$, viz:[6]–

$$K_{1/3}(z) = \sqrt{3}\pi\;(3/2\;z)^{-1/3}\;Ai\;\left[(3/2\;z)^{2/3}\right] \tag{19}$$

Combining all constants, and with the substitution $u = (1 + 1/b \cdot x/L_{NR})$ we obtain

$$n = B'' \, Ai\left[b^{2/3}\left(1 + b^{-1} \, x/L_{NR}\right)\right] \tag{20}$$

The constant B" may now be evaluated by the boundary condition of equation (8), i.e. $n = n_o$ at $x = 0$. This gives

$$B'' = n_o / Ai\left(b^{2/3}\right) \tag{21}$$

The complete solution for the distribution of excess minority carrier density with distance from the edge of the depletion region is thus

$$n = n_o \frac{Ai\left[b^{2/3}\left(1 + b^{-1} \cdot x/L_{NR}\right)\right]}{Ai\left(b^{2/3}\right)} \tag{22}$$

4. ASYMPTOTIC SOLUTION FOR LARGE ARGUMENTS

For large arguments of the Airy function, the following asymptotic expansion holds[7]

$$Ai\ (z) = \frac{1}{2}\pi^{-1/2} z^{-1/4} \exp\left(-\frac{2}{3} z^{3/2}\right) \tag{23}$$

Then for large values of b, equation (22) becomes

$$n = n_o \left(1 + b^{-1} \cdot x/L_{NR}\right)^{-1/4} \exp\left\{\frac{2}{3} b \left[1 - \left(b^{-1} \cdot x/L_{NR}\right)^{3/2}\right]\right\} \tag{24}$$

This is valid for all values of x, if $x_a >> L_{NR}$. In practice, a negligible error results for all values of x provided only that $x_a/L_{NR} > 1$. This is demonstrated in Figure 2 which shows n/n_o as a function of normalized x for various values of x_a/L_n as a parameter.

For $x_a \to \infty$, (i.e. for radiative recombination rate which is negligible compared with the non-radiative rate throughout the entire p-region), equation (24) reduces to the form of equation (9),

$$n = n_o \exp\left(-x/L_{NR}\right) \tag{25}$$

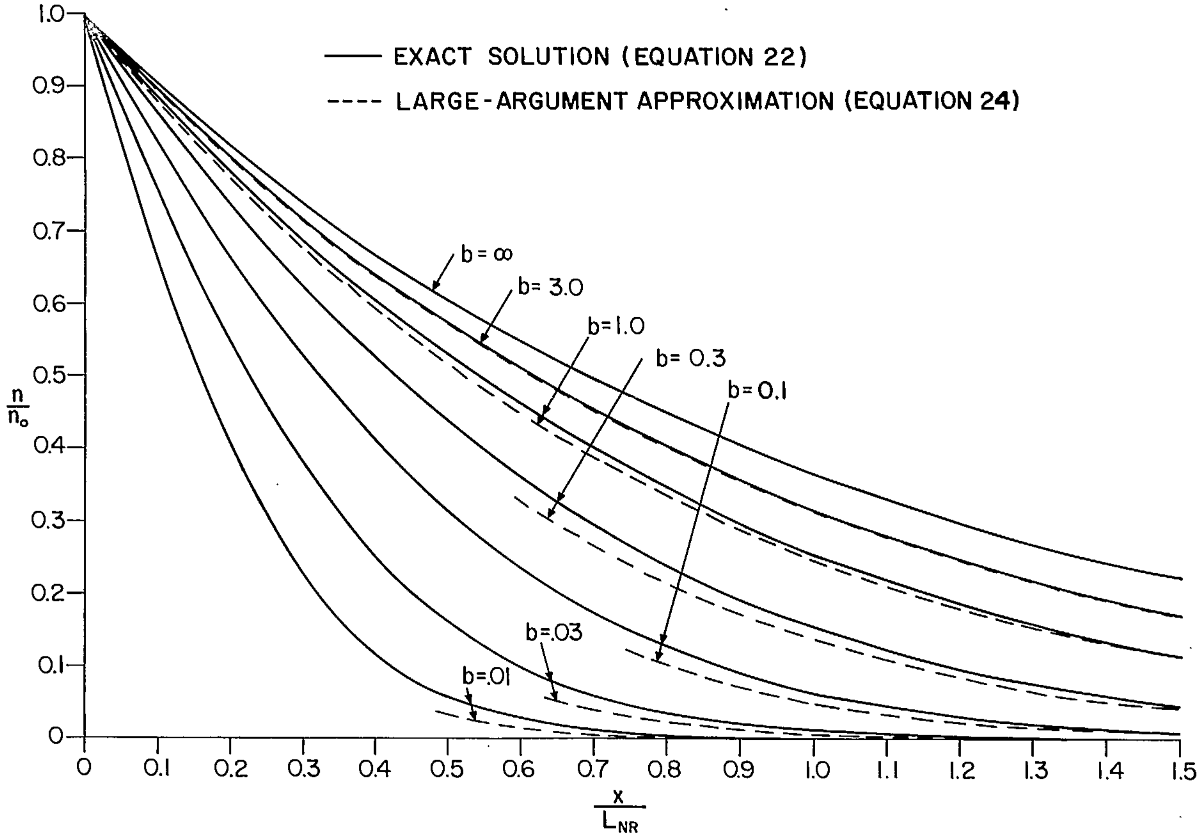

Figure 2. Distribution of Excess Minority Carriers Injected Into Neutral Region, Normalized to Value at x = 0.

5. CONCLUSIONS

A solution has been derived for the spatial distribution of minority carriers diffusing into a neutral semiconductor region having non-uniform lifetime. This may have applications in the following areas:

i) Explanation of Radiation Damage to LED's - Normally, degradation with radiation is assumed to take place via the decrease in total minority carrier lifetime[8], τ_T, i.e.

$$\frac{1}{\tau_T} = \frac{1}{\tau_o} + K\,\phi \tag{26}$$

where τ_o is the initial (pre-irradiation) lifetime, ϕ is the radiation flux (dose), and K is the conventional radiation damage constant. In the diodes considered here, lifetime degradation may be more complex than indicated by equation (26), since τ_{NR} and τ_R will probably degrade in a different manner.

ii) Direct Measurements of Lifetime - Two commonly used methods for the direct measurement of minority carrier lifetime in LED's are the methods of reverse-charge-recovery analysis[9], and the measurement of luminescence decay[1]. The analysis and interpretation of the results of each of these methods may require modification to take into account the effect evaluated here.

iii) Explanation of Anomalous LED Behaviour - Any observed anomalous behaviour of high-efficiency LED's should be considered in the light of the non-exponential decrease of minority carriers with distance in the neutral region. One possible application might be to the recently observed anomalous current component introduced by radiation. Work on this problem is continuing.

Finally, it should be pointed out that care must be taken in applying this theory to some GaAs LED's employing amphoteric-silicon doping. Anomalous behaviour of some of these devices at moderate and high current densities has been attributed by Byer[10] to the presence of a drift field in the highly-compensated neutral region. The existence of a significant electric field, ε, would invalidate the analysis given here, as the ε term in the continuity equation has been neglected in this analysis.

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8. ABSTRACT: A classical steady state solution of the continuity equation for minority carriers injected into a neutral region on one side of a p-n junction assumes a minority carrier lifetime which is constant throughout the neutral region. This produces an exponential decrease of excess carriers with distance from the edge of the depletion region. In some high-efficiency light-emitting diodes, minority carrier recombination is dominated by a radiative recombination process, which is dependent on the density of ionized acceptors in the p-region. In a graded-junction diode, the density of such acceptors increases with distance from the metallurgical junction, resulting in a recombination rate, and hence a lifetime, which is also a function of this distance. In this report, the continuity equation is solved for this situation, assuming a linear increase in ionized acceptors with distance. It is shown that the derived expression reduces to the exponential form for large values of a characteristic distance, x_a, which represents the point at which radiative and non-radiative recombination rates are equal. Possible implications of this result are discussed.

9. CITATION:

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