Communications Research Centre

FREQUENCY DOMAIN OPTICAL PROCESSOR FOR SAR IMAGERY

by

E.B. Felstead

DEPARTMENT OF COMMUNICATIONS
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(Radio and Radar Branch)

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ABSTRACT

An interferometric method of generating a two-dimensional spatial filter is described. This filter is the key element in a frequency domain optical processor for producing imagery for synthetic aperture radars. The new method is not only simpler to implement but produces a better quality filter. In addition, a new form of the frequency domain processor is presented. In this new form, the processor can handle inputs of variable aspect ratio.

1. INTRODUCTION

Optical techniques have been used very successfully in producing imagery from data obtained by synthetic-aperture radar systems (SAR). From about 1957 to 1964 a compact method using the "axicon" lens was generally employed [1]. After 1964, the tilted-plane processor [2] replaced the axicon system providing more versatility and considerably improved imagery. A third method, originally noted in [1], involves a matched filtering operation in the spatial frequency domain. It is pointed out in [2] that Martin and Von Bieren synthesized the required filter by putting two cylindrical lenses in tandem with one tilted appropriately. Lee and Greer [3] suggest the use of a computer-generated matched filter and successfully implement such a system. Their method has all the advantages of the tilted-plane processor plus the advantage that readily available and inexpensive optical elements may be employed. Furthermore, frequency plane processing will likely be the key method for correcting aberrations due to range curvature. This curvature is a recording error found in radars with very large range such as in satellite-borne radars.

The purpose here is to describe an interferometric and a tilted lens method of generating the required matched filter. It is believed that the methods are not only simpler to implement but have potential for much higher quality than the computer-generated counterpart. In addition, an alternate configuration to that given by Lee and Greer [3] is presented for consideration. This new configuration is capable of handling inputs of variable scale.

2. FORM OF THE INPUT INTERFEROGRAM

The formulation of the problem and the notation loosely follows [2]. In flight, the SAR data is recorded on photographic film as an interferogram. For a single point source located on the ground at (x_o, r_o), where x_o is the azimuth location and r_o is the range measured from the aircraft, the amplitude transmittance of the interferogram is

$$t_i(x_f, r_f) = t_b + \sigma_o \,\mathrm{rect}\left\{\frac{r_f - \frac{r_o}{q}}{\frac{\Delta r}{q}}\right\} \times \cos\left[2\pi f_c x_f + \frac{2\pi p^2}{\lambda_r r_o}\left(x_f - \frac{x_o}{p}\right)^2 + \theta_o\right] \tag{1}$$

where

p = azimuth demagnification,

q = range demagnification,

(x_f, r_f) = coordinates on film,

f_c = spatial carrier frequency, cycles/cm,

λ_r = wavelength of radar signal

Δr = width of transmitted radar pulse in terms of distance

and σ_o = reflectivity of point reflector at (x_o, r_o).

The object is to process the interferogram so as to generate an image of the ground and preferably to have the range and azimuth scales equal. Methods employing operations in the spatial frequency plane are discussed below. First, the technique due to Lee and Greer [3] is described. Then, an alternate form, devised here, is presented.

3. PROCESSING METHOD OF LEE AND GREER [3]

In Figure 1 is shown the basic Lee and Greer [3] configuration in the azimuth and range dimension. The interferogram of transmittance (1) is placed in plane P_1. Note that the cosine term of (1) represents a zone plate along the azimuth direction x_f with an equivalent focal length of

$$F_x(r_o) = \pm \frac{\lambda_r}{2p^2\lambda} r_o \qquad (2)$$

where λ is the wavelength of light. Note that F_x is dependent on the range, r_o. The rect function describes the shape of the short rectangular pulse of width Δr. In some cases where increased range resolution is required, the rectangular pulse is replaced by a longer linear FM pulse. Then, the range pattern on the interferogram also has the form of a zone plate with equivalent focal length

$$F_r = \pm \frac{\pi c^2}{(q^2\lambda\ S)} \qquad (3)$$

where c = velocity of the radar signal and S is the frequency modulation rate of the linear FM. Plane parallel light impinging upon the transparency is focussed in the azimuth dimension to plane P_A, a distance $F_x(r_o)$ in front of the transparency, and in the range dimension to plane P_R, a distance F_r in front of the transparency.

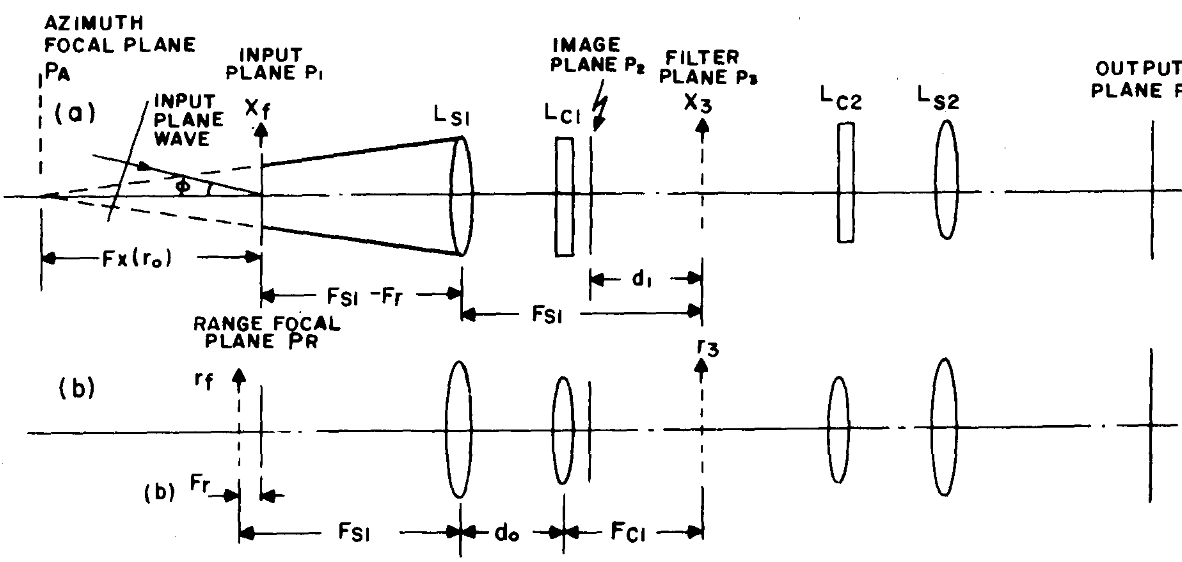

Figure 1. Schematic of the Lee and Greer optical processor a) azimuth dimension and b) range dimension.

Lee and Greer have described the system entirely in terms of imaging where the system is designed to bring both the range focal plane P_R and the azimuth focal plane P_A into focus at the same output plane, P_4. They describe the filter in terms of a lens whose focal length varies as a function of range r_3. From an alternate point of view, the x dimension operation can be described in terms of matched filtering. We take this approach here as it provides a good analytic basis for describing the requirements for the interferometrically generated filter.

The input transmittance (1) is a linearly frequency-modulated carrier wave with an offset x_o/p in the x_f direction. Because it is a linear FM, an obvious method of resolving the position x_o is to cross-correlate this function in the x_f dimension with a suitable linear FM reference function. This may be done by matched filtering in the frequency domain by means of a Vander Lugt filter [4]. Since the rate of frequency sweep is a function of range r_o, a different matched filter is required for every different range value. Thus, each range element must be processed separately in the frequency domain. This is accomplished, as shown in Figure 1, by imaging plane P_R onto plane P_3 in the range dimension while Fourier transforming in the azimuth dimension between P_1 and P_3. Thus, the Fourier transform in the x-dimension is present for every value of range.

The input plane P_1 is illuminated with plane-parallel monochromatic light of amplitude

$$A_o \, e^{-j2\pi\frac{\sin\phi}{\lambda} x_f}$$

where ϕ is the angle of incidence and is adjusted so that $\sin\phi/\lambda = f_c$. If range compression is used, it is to be understood that the rect function in (1) is an approximation to the compressed signal and that r_f is located in plane P_R rather than P_1. The width $\Delta r/q$ is now the effective compressed width. The amplitude distribution, at plane P_3 is then

$$a_3(x_3,r_3) = t_b\delta(x_3 + \lambda F_{s1}f_c) + \mathrm{rect}\left[\frac{\frac{r_3}{M_{r1}} - \frac{r_o}{q}}{\frac{\Delta r}{q}}\right] e^{j\frac{k}{2F_{c1}}\left(1 - \frac{d_o - F_{s1}}{F_{c1}}\right)r_3^2}$$

$$\times \; c_1 \, e^{j\frac{k}{2F_{s1}}\left(1 - \frac{F_{s1} - F_r}{F_{s1}}\right)x_3^2} \left\{ e^{j\theta_c} \int_{-\infty}^{\infty} e^{j\frac{2\pi p^2}{\lambda_r r_o}\left(x_f - \frac{x_o}{p}\right)^2} e^{-j2\pi f_x x_f} dx_f + c.c(-2f_c) \right\}$$

(4)

where $M_{r1} = F_{c1}/F_{s1}$, $k = 2\pi/\lambda$, $c.c(-2f_c)$ means complex conjugate of the previous term centred on $f_x = -2f_c$, $f_x = x_3/(\lambda F_{s1})$, and c_1 is a complex constant. Note that $d_o - F_{s1} = -F_{c1}$ and use the fact that

$$\int_{-\infty}^{\infty} e^{jaq^2}\, e^{j2\pi bq}\, dq = \sqrt{\frac{\pi}{a}}\, e^{j\frac{\pi}{4}}\, e^{-j\pi^2 b^2/a} \tag{5}$$

to obtain

$$a_3(x_3,r_3) = t_b\delta(x_3 + \lambda F_{s1}f_c) + c_2 \operatorname{rect}\left[\frac{\frac{qr_3}{M_{r1}} - r_o}{\Delta r}\right] e^{j\frac{k}{2F_{s1}}\left(1 - \frac{F_{s1}-F_r}{F_{s1}}\right)x_3^2}$$
$$\times \left\{ e^{-j\pi^2\left(\frac{x_3}{\lambda F_{s1}}\right)^2 \frac{\lambda_r r_o}{2\pi p^2}}\, e^{-j2\pi\frac{x_3}{\lambda F_{s1}}\frac{x_o}{p}} + c.c(-2f_c)\right\} \tag{6}$$

where c_2 = a complex constant. If the spatial carrier f_c is sufficiently large compared to the bandwidth of the azimuth linear FM signal then the bias term $t_b\delta(x_3 - \lambda F_{s1}f_c)$ and the $c.c(-2f_c)$ term can be blocked off at plane P_3. Only the component

$$a_3(x_3,r_3) = c_2 \operatorname{rect}\left[\frac{\frac{qr_3}{M_{r1}} - r_o}{\Delta r}\right] e^{j\frac{k}{2F_{s1}}\left(1 - \frac{F_{s1}-F_r}{F_{s1}}\right)x_3^2}\, e^{-j\frac{\pi\lambda_r r_o x_3^2}{2p^2(\lambda F_{s1})^2}}\, e^{-\frac{j2\pi x_3 x_o}{p\lambda F_{s1}}} \tag{7}$$

is transmitted. Due to the proper selection of the angle of incidence ϕ, this component is centred in the x_3 dimension, on the optical axis. The first quadratic phase term in (7) is due to the fact that lens L_{s1} is not exactly a focal length from P_R and it will be seen how it may be cancelled by the filter. The second quadratic phase term is the spectrum of the input azimuth linear FM. For matched filtering we need to multiply this term by its complex conjugate. The remaining complex exponential term describes a plane wave tilted at an angle proportional to x_o. This wave corresponds to a point image at infinity. In normal Vander Lugt filtering this point image is brought in from infinity to the focal plane of an additional lens. Here we wish the point to be imaged at plane P_2 located a distance d_1 in front of plane P_3. This is accomplished by multiplying $a_3(x_3,r_3)$ by a term $\exp(jkx_3^2/2d_1)$ which corresponds to a quadratic approximation of a spherical wave due to a point source at plane P_2. In summary, we wish to construct a filter whose transmittance is proportional to

$$e^{-j\frac{k}{2}\left(\frac{1}{F_{s1}}-\frac{F_{s1}-F_r}{F_{s1}^2}-\frac{1}{d_1}\right)x_3^2} \times e^{+j\frac{\pi\lambda_r ro}{2p\,(\lambda F_{s1})^2}x_3^2} \quad . \qquad (8)$$

Once the azimuth image is cast on to plane P_2, the explanation of the remainder of the processing is identical to that given in Lee and Greer.

In Sections 5 and 6 will be given two different methods of constructing the required filter. The form of both is that of a Vander Lugt filter wherein a spatial carrier is utilized so that complex valued functions may be implemented as real amplitude variations. The carrier, in both cases, is introduced during manufacture by use of an offset reference beam.

Other methods of implementing filter function (8) are possible. First, note that transmittance (8) is equivalent to two thin lenses together; one with a fixed focal length and the other with a focal length that varies with range. Direct fabrication of such a lens combination is considered impractical [2]. According to [2], Martin and von Bieren in unpublished work showed that two cylindrical lenses in tandem, one suitably tilted, are equivalent to the desired filter. However, this "filter" is distributed along the optical axis and cannot be said to be at the frequency plane. Although nothing seems to have come of this method, it did lead to the tilted-plane processor. Furthermore, this technique is shown in Section 6 to be useful for the construction of our required filter.

An indirect method of implementing the equivalent lens is by use of a kinoform lens [5]. In this technique the phase shifts of the lens are calculated modulo 2π, computer plotted, photoreduced and bleached. These lenses might be useful for our filter. In [5], an example of a simple cylindrical lens is given and there seems no reason why our more complicated lens combination cannot be implemented.

Lenses implement the required phase shifts by a thickness variation. Whereas normal lenses use continuous thickness variation, kinoform lenses use a thickness that returns to zero each time the phase has increased by 2π radians. In comparison, a holographic lens and a Vander Lugt filter use a modulation of a spatial carrier to implement the required phase variation.

Like a normal lens, the kinoform directs most of the incident light into the image. Although ordinary holographic lenses diffract only a small part of the incidental light into the image, methods of greatly increasing the efficiency are known and are discussed in Section 7. It is the author's opinion that an extensive development program would be required to construct high quality kinoform lenses. Thus, it is recommended that initial work be directed towards implementing the Vander Lugt type of filter.

4. A FREQUENCY PLANE PROCESSOR FOR VARIABLE K INPUTS

If, in the method of Lee and Greer, the input demagnification p or q are changed, it is necessary to change the frequency plane filter at plane P_3. However, this change may be avoided if an alternate method of processing, shown in Figure 2, is utilized.

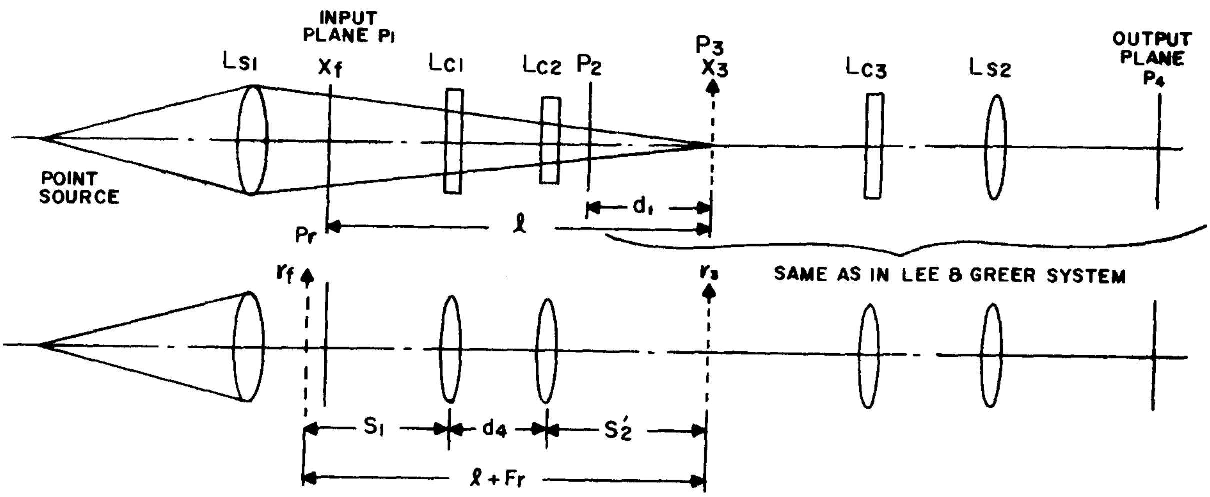

Figure 2. Schematic of an alternate frequency plane optical processor, a) azimuth direction and b) range dimension.

Consider the azimuth dimension first. Initially the azimuth input is $u_1(p_o x_f - x_o)$. The configuration shown in Figure 2(a) gives a variable scale Fourier transform [6] between plane P_1 and P_3 so that u_1 is transformed to $U(f_x)\exp(-j2\pi f_{x3} x_o)$ where ℓ_o is the initial separation between P_1 and P_3 and $f_{x3} = x_3/(p_o\lambda\ell_o)$. The matched filter $U^*(f_{x3})$ is placed at plane P_3. Suppose that the azimuth demagnification is now changed to p_1. To compensate we merely change the spacing ℓ_o to ℓ_1 where $\ell_1 = p_o/p_1 \ell_o$ so that the input $u_1(p_1 x_f - x_o)$ is transformed to $U[x_3/(p_1\lambda\ell_1)] = U[x_3/(p_o\lambda_o\ell_o)]$. Our original matched filter may still be employed. In changing the separation ℓ, the imaging in the range dimension is unfortunately upset. To compensate for this, two cylindrical lenses are used to image between plane P_R and P_3. It is required that the magnification M_{r1}, between P_R and P_3 remain constant while their separation $\ell + F_r$ varies from $\ell_o + F_r$ to $\ell_1 + F_r$. Equations for the selection of object distance S_1 of lens L_{c1} and the image distance S_2' of lens L_{c2} can be derived from geometrical optics that give any desired M_{r1} for any given $\ell + F_r$.

If the range demagnification q_o is changed to q_1 then the range magnification M_{r1} is changed to $(q_1/q_o)M_{r1}$ so that at plane P_3 the azimuth scale remains unaltered. Thus the original filter may be retained. The change of M_{r1} with constant ℓ is obtained, as mentioned above, by suitable selection of the spacings S_1 and S_2'.

If both p and q are altered then both ℓ and M_{r1} must be varied according to the methods described above.

5. INTERFEROMETRIC CONSTRUCTION OF THE FILTER

5.1 FOR THE LEE AND GREER SYSTEM

A filter of transmittance given by (8) is to be constructed following the general techniques originally given by Vander Lugt [4]. The basic configuration is shown in Figure 3. In plane P_1 is placed an opaque screen with a curved slit where the distance of the slit from the x_1 axis is proportional to x_1^2. Its transmittance is

$$t_1(x_1,r_1) = \mathrm{rect}\left(\frac{x_1}{w}\right)\mathrm{rect}\left[\frac{r_1 - \gamma x_1^2}{\varepsilon}\right] \qquad (9)$$

where γ is a constant, ε is the width of the slit in the r_1 direction, and w is total width of slit in the x_1 direction. Such a transparency is very simple to construct. It is simply drawn by hand or computer at a large scale and then photoreduced to the appropriate size.

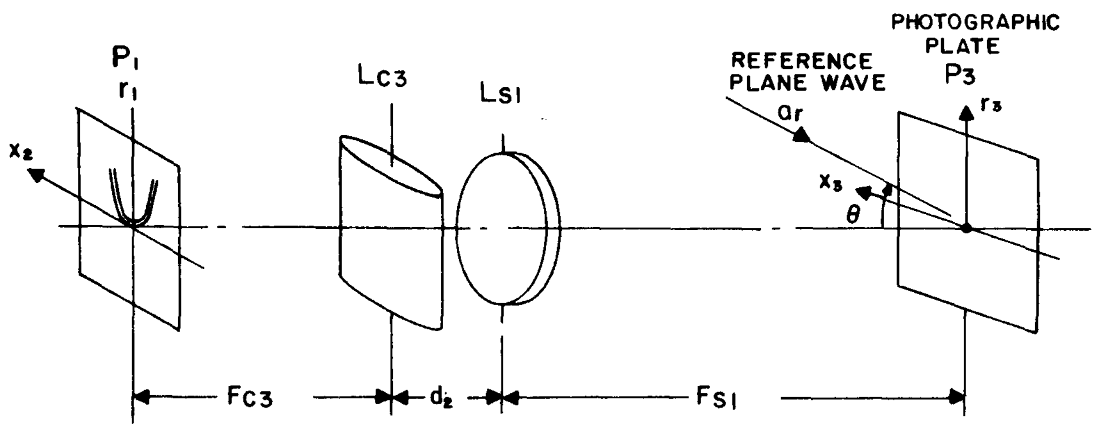

Figure 3. Schematic of system for generating the required filter.

Noting that the axis of the cylindrical lens L_{c3} is vertical, we see that between plane P_1 and P_3 there is imaging with magnification $M_1 = F_{s1}/F_{c3}$ in the azimuth dimension and Fourier transformation in the range dimension. The amplitude distribution at plane P_3 is thus,

$$g(x_3,r_3) = c_3 \operatorname{rect}\left(\frac{x_3}{M_1 w}\right) e^{j\frac{k}{2F_{s1}}\left(1-\frac{d_2-F_{c3}}{F_{s1}}\right)x_3^2}\ \varepsilon\ \frac{\sin(\pi\varepsilon r_3/\lambda F_{s1})}{\pi\varepsilon r_3/\lambda F_{s1}}$$

$$e^{-j2\pi\gamma\left(\frac{x_3}{M_1}\right)^2\left(\frac{r_3}{\lambda F_{s1}}\right)} e^{j\frac{k}{2F_{s1}}\left(1-\frac{d_2+F_{c3}}{F_{s1}}\right)r_3^2} \qquad (10)$$

where c_3 is a complex constant. The line width ε is made sufficiently small that $\operatorname{sinc}(\varepsilon r_3/\lambda F_{s1}) \simeq 1$ over the entire range dimension used. Thus,

$$g_3(x_3,r_3) \simeq c_4 \operatorname{rect}\left(\frac{x_3}{M_1 w}\right) e^{-j2\pi\gamma' x_3^2 r_3} e^{j\frac{k}{2}\left(\frac{1}{F_{s1}}-\frac{d_2-F_{c3}}{F_{s1}^2}\right)x_3^2}$$

$$e^{j\frac{k}{2F_{s1}}\left(1-\frac{d_2+F_{c3}}{F_{s1}}\right)r_3^2} \qquad (11)$$

where

$$\gamma' = \frac{\gamma}{M_1^2\ \lambda F_{s1}} \qquad (12)$$

To form the Vander Lugt filter a plane reference wave

$$a_r(r_3) = a_{ro}\ e^{-j2\pi\frac{\sin\theta}{\lambda} r_3} \qquad (13)$$

is added to g_3 where τ is the angle of incidence as shown in Figure 3 and a_{ro} is a constant. The intensity $|g_3 + a_r|^2$ illuminates a photographic plate placed at plane P_3 and the plate is exposed and developed to give an amplitude transmittance proportional to the intensity distribution (by exposing in the linear region of the plate's t_a-E curve [6]). The amplitude transmittance of the plate is thus

$$t_3(x_3,r_3) = c_5\Bigg\{a_{ro} + |g_3(x_3,r_3)|^2 + a_{ro}g_3(x_3,r_3)\ e^{j\frac{2\pi}{\lambda}\sin\theta\ r_3}$$

$$+\ a_{ro}g_3^*(x_3,r_3)\ e^{-j\frac{2\pi}{\lambda}\sin\theta\ r_3}\Bigg\} \qquad (14)$$

where c_5 is a constant.

This transparency is used as the filter in plane P_3 of Figure 1. The amplitude of the light emerging from plane P_3 is now a_3t_3 where a_3 is given by (7) and t_3 by (14). If θ is sufficiently large, then the 4 components of (14) result in spatially separable spectra in the Fourier transform plane that exists to the right of plane P_3. All the spectra except the one due to the term of (14) containing g_3* are blocked. It is this term that corresponds to the desired filter (8). Thus, only the last term of (14) contributes to the output so that the effective amplitude emerging from plane P_3 is

$$a_3'(x_3,r_3) = c_6 \operatorname{rect}\left(\frac{x_3}{M_1 w}\right) \operatorname{rect}\left[\frac{\frac{qr_3}{M_{r1}} - r_o}{\Delta r}\right] e^{-j\frac{k}{2F_{s1}}\left(1-\frac{d_2+F_{c3}}{F_{s1}}\right)r_3^2} e^{-j\frac{2\pi}{\lambda}\sin\theta\, r_3}$$

$$\times e^{j\frac{k}{2}\left(\frac{1}{F_{s1}} - \frac{F_{s1}-F_r}{F_{s1}^2}\right)x_3^2} e^{-j\frac{k}{2}\left(\frac{1}{F_{s1}} - \frac{d_2-F_{c3}}{F_{s1}^2}\right)x_3^2}$$

$$\times e^{-j2\pi\frac{\lambda_r r_o}{4p^2\lambda^2F_{s1}^2}x_3^2} e^{+j2\pi\gamma' r_3 x_3^2} \times e^{-j\frac{2\pi}{\lambda F_{s1}}\frac{x_3 x_o}{p}} \qquad (15)$$

The first three terms that are functions of r_3, describe the range image multiplied by a quadratic phase factor and by a linear phase factor. Since all subsequent operations in the range dimension involve imaging only, the phase factors are merely imaged to the output plane along with the desired image. Upon detection the phase factor becomes unity. Thus, the phase factors are now omitted.

If we construct the filter function t_3 so that

$$\gamma' r_3 = \frac{\lambda_r}{4p^2\lambda^2F_{s1}^2} r_o,$$

i.e.,

$$\gamma = \frac{r_o}{r_3}\frac{\lambda_r F_{s1}}{4p^2\lambda F_{c3}^2}, \qquad (16)$$

then the quadratic phase factors that are functions of r_o cancel to give unity. Furthermore, if d_2 is selected so that

$$-\frac{F_{s1} - F_r}{F_{s1}^2} + \frac{d_2 - F_{c3}}{F_{s1}^2} = +\frac{1}{d_1}$$

i.e.,

$$d_2 = \frac{F_{s1}^2}{d_1} + F_{s1} + F_{c3} - F_r, \tag{17}$$

then

$$a_3'(x_3, r_3) = c_6 \operatorname{rect}\left[\frac{\frac{qr_3}{M_{r1}} - r_o}{\Delta r}\right] e^{j \frac{k}{2d_1} x_3^2} e^{-j2\pi \left(\frac{x_3}{\lambda F_{s1}}\right) \frac{x_o}{p}}, \tag{18}$$

where the quadratic phase term describes a spherical wave diverging from a point at plane P_2 and the linear phase term in x_3 results in an offset of the point by an amount proportional to x_o/p. Thus, we have the desired result. Subsequent processing is identical to that described in Lee and Greer.

The ratio r_o/r_3 in (16) is found by noting that a point on the ground at r_o is located at $r_1 = r_o/q$ in plane P_1 and is imaged to $r_3 = (r_o/q)M_{r1} = -(F_{c1}/F_{s1})(r_o/q)$. Thus, (16) becomes

$$\gamma = - \frac{\lambda_r F_{s1}^2 q}{4p^2 F_{c1} F_{c3}^2 \lambda}. \tag{19}$$

The system magnification ratio K' is given by [3]

$$K' = \frac{M_r}{M_a} = \frac{1}{4}\left(\frac{F_{c1}}{F_{c2}}\right)\left[1 + \left(1 + 4\frac{F_{c2}}{d_1}\right)^{\frac{1}{2}}\right]^2 \tag{20}$$

where M_r is total range magnification and M_a is total azimuth magnification. If K' is made equal to the input aspect ratio $K = q/p$ then the output image has an aspect ratio of unity.

The positions of lenses L_{c3} and L_{s1} in Figure 2 may be interchanged, in certain cases, and the technique will work satisfactorily. Some of the equations must be altered appropriately. Similarly, lenses L_{s1} and L_{c1} in Figure 1 can presumably be interchanged. The analysis has not yet been made to determine which combination of positions gives the best quality results.

5.2 FOR THE VARIABLE SCALE SYSTEM

The details for constructing the filter for the variable scale system described in Section 4 have not yet been worked out. However, construction follows the general principles described in Section 5.1 above. Note that during construction, the reference transparency is illuminated by a converging beam of light. It is recommended that the same lenses be used for constructing the filter as will be used during actual processing. A discussion of Vander Lugt filtering using variable scale Fourier transformation is found in Reference [7].

6. TILTED LENS METHOD OF CONSTRUCTING FILTER

As mentioned above, Martin and von Bieren, in unpublished work showed that two cylindrical lenses in tandem, one suitably tilted, is equivalent to the desired filter. Such a combination is considered here for the construction of our required Vander Lugt filter.

Cylindrical lens L_{c4} focusses collimated light to a line image at plane P_L. Cylindrical lens L_{c5} is tilted at an angle θ_5 to the optical axis. It is assumed, if the beam is sufficiently wide in the r direction and sufficiently well collimated, that diffraction effects in the r direction are negligible. Thus, at any plane, the amplitude distribution is constant in the vertical direction. Both the lens L_{c5} and the photographic plate plane are tilted by an angle $\theta_3 = \theta_5$ to optic axis. The r_3 and r_5 axes are taken in the tilted plane so that after development the photographic plate may be placed directly in plane P_3 of the processor of Figure 1. The distances r_3 and r_5 in the plane are the distances $r_1 \sin\theta_3$ and $r_5 \sin\theta_5$ in the vertical direction. (See page 14)

The amplitude distribution of light impinging upon lens L_{c5} is $a_5 \exp[j(k/z)x_5^2]$ where $z = z_1 + r_5 \sin\theta_5$, a_5 is a constant and z_1 is the distance along the optical axis between P_4 and lens L_{c5}. The light emerging from lens L_{c5} is

$$a_5 \, e^{j \frac{k}{2} x_5^2 \left(\frac{1}{z} - \frac{1}{F_{c5}}\right)} \tag{21}$$

which is the amplitude which would be produced by a line source located a distance

$$d_1 = \frac{1}{\left(\frac{1}{z} - \frac{1}{F_{c5}}\right)} \tag{22}$$

to the left of the particular point on the lens. If plane P_3 is tilted at an angle $\theta_3 = \theta_5$ then the point source is a distance $d_2 = d_1 + z_2$ to the left of plane P_3 where z_2 is the distance between lens L_{c5} and plane P_3. The amplitude at plane P_3 is

$$h_3(x_3, r_3) = h_o \, e^{j \frac{k}{2d_2} x_3^2} \tag{23}$$

where h_o is a constant and

$$\frac{1}{d_2} = \frac{1}{\dfrac{(z_1 + r_3 \sin\theta_3)\, F_{c5}}{F_{c5} - z_1 - r_3 \sin\theta_3} + z_2} .$$

Plane P_3 is positioned so that $z_2 = F_{c5}$. Then

$$\frac{1}{d_2} = \frac{(F_{c5} - z_1) - r_3 \sin\theta_3}{F_{c5}^2} . \tag{24}$$

A plane wave reference beam of amplitude a_r impinges on the recording plate at an angle θ_r as shown in Figure 4 so that the intensity is $|h_3(x_3,r_3) + a_r(r_3)|^2$. The plate is exposed and developed to give an amplitude transmittance $t_3(x_3,r_3)$ proportional to the incident intensity. Thus,

$$t_3(x_3,r_3) = c_6 \left\{ a_r^2 + |h_3(x_3,r_3)|^2 + a_r h_3(x_3,r_3)\, e^{j \frac{2\pi}{\lambda} \sin\theta_r r_3} + a_r h_3^*(x_3,r_3)\, e^{-j \frac{2\pi}{\lambda} \sin\theta_r r_3} \right\}. \tag{25}$$

Following the same arguments as in Section 5, we use only the last term of (25) so that

$$t_3'(x_3,r_3) = c_7\, e^{j \frac{k}{2} \left(\frac{F_{c5}-z_1}{F_{c5}^2}\right) x_3^2} \times e^{-j \frac{2\pi}{\lambda} \sin\theta_r r_3} \times e^{j \frac{k}{2} \left(\frac{\sin\theta_3}{F_{c5}^2}\right) r_3 x_3^2}. \tag{26}$$

If, in the construction of the filter, θ_3 and z_1 were set according to

$$\frac{1}{F_{s1}} - \frac{F_{s1} - F_r}{F_{s1}^2} - \frac{F_{c5} - z_1}{F_{c5}^2} = \frac{1}{d_1} \tag{27}$$

and

$$\frac{\sin\theta_3}{2\lambda F_{c5}^2}\, r_3 = \frac{\lambda_r}{4p^2\lambda^2 F_{s1}^2}\, r_o, \tag{28}$$

then the desired filter is obtained. Rearranging, we obtain,

$$z_1 = F_{c5}^2 \left(\frac{1}{d_1} - \frac{F_r}{F_{s1}^2}\right) + F_{c5} \tag{29}$$

and

$$\theta_3 = \sin^{-1} \left\{ \frac{r_o}{r_3} \frac{\lambda_r F_{c5}}{2p^2 \lambda F_{s1}^2} \right\}. \tag{30}$$

Noting from Section 5 that $r_o/r_3 = -q\, F_{s1}/F_{c1}$, we obtain

$$\theta_3 = \sin^{-1}\left\{-\frac{q\lambda_r F_{c5}^2}{2p^2\lambda F_{s1} F_{c1}}\right\}. \tag{31}$$

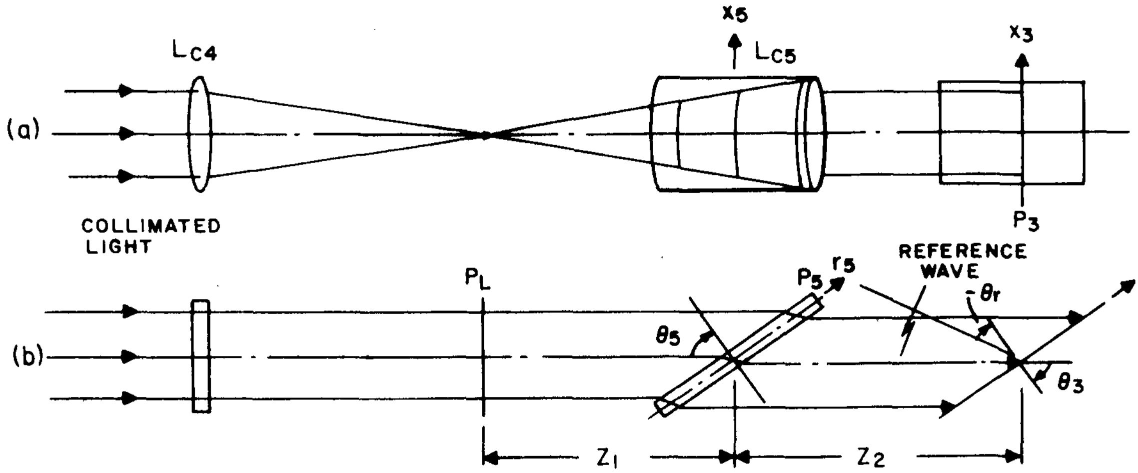

Figure 4. Schematic of system for generating desired filter for the tilted plane method, a) azimuth dimension and b) range dimension

The question arises, can the filter be constructed using only a single cylindrical lens to form a line focus and a tilted recording plate? This configuration results in a quadratic phase variation that is inversely proportional to range rather than directly proportional as is required. Thus, this simpler technique cannot be used to construct the required filter. Interestingly, it could be used to construct the interferometric form of the axicon lens.

7. EFFICIENCY

The major disadvantage of using a Vander Lugt filter is its very low diffraction efficiency defined as the ratio of the total light intensity diffracted into the desired order to the total incident light. For a planar amplitude-only filter, the maximum diffraction efficiency is 6.25% and even that cannot be achieved if reasonably linear output is required [8].

By comparison, coated lenses have nearly 100% transmission efficiency. Since the tilted-plane processor uses lenses only, there is a substantial efficiency advantage over the frequency-plane processor. Fortunately bleaching may be used to convert the amplitude filter to a phase filter and in doing so, the diffraction efficiency can be increased to as high as 65% [8]. Better still, if a volume filter is constructed using dichromated gelatin, the diffraction efficiency can approach 100% along with low noise [8]. Thus, with relatively little extra effort, the major disadvantage of holographic filtering can be overcome.

8. COMPARISON OF THE TWO METHODS OF CONSTRUCTION

Due to lack of experimental experience with the two methods of constructing the filter, the remarks in this section are quite subjective and are certainly open to debate. The methods are compared on two aspects: i) ease of construction and ii) quality of filter.

To obtain the correct scaling of the required filter using the interferometric method of construction, the distance d_2 and the reference mask scale factor γ must be accurately set. A distance is relatively simple to set accurately. Since the reference mask will likely be drawn by computer the value of γ is easily controlled with the accuracy of the photoreduction being the limiting factor. For the tilted-lens method, the distance z_1 and the angles θ_3 and θ_5 must be set accurately. Since z_1 is measured to a tilted plane some difficulty may be experienced in finding the desired location on the tilted plane to which one measures. Furthermore, setting angles (and there are two of them) is not as simple as setting distances or scale factors. Therefore, the interferometric method is probably easier to implement.

In the tilted-lens method there are three factors that may degrade the quality of the filter. First, a cylindrical lens is used that is tilted with respect to the impinging wave. It is not clear what aberrations will result from light passing obliquely through this lens. If θ_5 and θ_3 are small then the resulting aberrations may be small. Second, there is no focussing control in the vertical direction with the assumption that the collimation can adequately contain the beam. This assumption is an approximation that may lead to degradation of the resulting filter. Third, two cylindrical lenses are used, compared to only one in the interferometric method. Because of the difficulty in manufacturing good quality cylindrical lenses, it is desirable to keep the number of these elements to a minimum. Therefore, it appears that the interferometric method may lead to better quality filters.

9. K VALUE

The value of K is the aspect ratio of the interferogram recorded inflight and is given by

$$K = \frac{q}{p}. \tag{32}$$

Since K is generally greater than unity, the azimuth recording is generally stretched out relative to the range recording. The value of p and q depend on many factors such as aircraft velocity, recording film velocity, electron beam velocity at recording CRT, optical aperture, etc. Restricting all these factors to maintain always the same K value certainly limits the operation of the radar system. It is preferable to have a processor that can handle inputs of various K values. Additionally, it is desired to have a processor magnification ratio $K' = M_r/M_a$ chosen so that $K'=K$. Then, the output azimuthal and range scales are equal. In this section we discuss the effects due to changing K.

In the axicon system a change in either p *or* q would require an entirely new axicon lens. Thus, a change of K is very undesirable because axicon lenses of high quality are very difficult to construct. Furthermore, to obtain an output of unity aspect ratio both the input signal film and the output recording film must both be moving at precisely controlled velocities with the signal-to-recording film velocities in the exact ratio p/q.

For most forms of the tilted plane processor [2], the focal length of the lenses are chosen to give a system magnification $K'=K$. Therefore, once the lenses are selected, the value of K' is fixed. If the input K were changed then new lenses would be required to keep $K'/K=1$. However, if one were willing to tolerate a non-unity output scale ratio then K and K' need not be equal. An image would still be formed but the range-to-azimuth dimension ratio would be K'/K. One form of the tilted-plane processor can, in fact, accommodate signal films of various input aspect ratios. Extra imaging capability is added in one dimension to permit variable scaling in that dimension relative to the other.

In all tilted plane processors the tilt angle of the input plane is reduced $1/K'^2$ times in the imaging process. Therefore K' must be greater than 1 and preferably much greater. It is not clear if this is the reason why typical inputs have K values of 10 or more.

For the frequency-plane processor of Lee and Greer a change in either p *or* q would require a new frequency-plane filter. Fortunately, by the methods presented in Section 5, it is quite simple to construct a new filter. In fact, it is not unreasonable to construct say, 50 filters for 50 possible combinations of p and q. The value of K' can, of course, be kept equal to K for unity aspect ratio. The alternate method proposed in Section 4 not only can handle any reasonable value of p or q without change of filter, but it appears that the output aspect ratio remains unity. Further analysis is required to prove this statement. Note that frequency plane processors can accommodate input signals with $K < 1$. This feature may be useful.

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8. **ABSTRACT:**

An interferometric method of generating a two-dimensional spatial filter is described. This filter is the key element in a frequency domain optical processor for producing imagery for synthetic aperture radars. The new method is not only simpler to implement but produces a better quality filter. In addition, a new form of the frequency domain processor is presented. In this new form, the processor can handle inputs of variable aspect ratio.

9. **CITATION:**