Communications Research Centre

ON THE GENERATION AND REPRESENTATION OF LINE DRAWINGS

by

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EXECUTIVE SUMMARY

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A programme of research in image communications, currently underway in the Directorate of Data Systems and Networks R&D involves studying visual communications systems which use standard telephone lines for transmitting images. One of the problem areas being investigated relates to the efficient generation, representation and transmission of general line drawings.

The Image Communication program of CRC established a joint research program with the Royal Military College, Kingston, Ontario in 1975. This Technical Note is a report on one of the joint research topics being investigated.

The results of the work outlined in the Technical Note relate to a problem area being experienced by other government departments, specifically the Department of the Environment. Discussions are underway with DOE concerning the use of the findings of this research in the efficient coding and subsequent transmission of weather map information in the department weather network across Canada.

The outcome of this research will also directly benefit other research topics within the Directorate that relate to the provision of future new home/office communication systems and service.

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La Direction générale des systèmes et des réseaux de données, Recherche et développement, procède actuellement à l'exécution d'un programme de recherches sur la transmission des images. Ce programme vise l'étude des systèmes de communications visuelles qui emploient les lignes téléphoniques classiques pour transmettre les images. L'un des problèmes à l'étude porte sur la production, la représentation et la transmission efficaces de graphiques.

En 1975, le groupe du Programme du CRC sur la transmission des images a, en collaboration avec le Collège militaire royal de Kingston (Ontario), mis sur pied un programme mixte de recherches. La présente note technique vise à présenter un rapport sur l'un des sujets de recherche qui font actuellement l'objet du programme mixte.

La note technique décrit les résultats des travaux relatifs à un problème avec lequel d'autres ministères de l'État, notamment celui de l'Environnement, sont aux prises. Des pourparlers sont en cours avec le ministère de l'Environnement en vue d'utiliser les conclusions tirées de ces recherches pour le codage efficace et la transmission subséquente des données que contiennent les cartes météorologiques à l'ensemble du réseau météorologique de ce Ministère, d'un océan à l'autre.

Les résultats de ces recherches auront pour avantage de faire progresser la recherche sur d'autres sujets relatifs à la prestation future de nouveaux systèmes et d'un nouveau service de communication entre le foyer et le bureau, entreprise par la Direction générale.

COMMUNICATIONS RESEARCH CENTRE

DEPARTMENT OF COMMUNICATIONS
CANADA

ON THE GENERATION AND REPRESENTATION OF LINE DRAWINGS

by

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ON THE GENERATION AND REPRESENTATION OF LINE DRAWINGS

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ABSTRACT

The problem of efficiently generating or representing line drawings from the point of view of storage and transmission requirements is considered. A proposed solution is the use of mathematical techniques to characterize the various line segments of the line drawing. Towards that end, the B-spline technique is described and a new recursive relationship which simplifies the computations is given. The quadratic B-spline is then considered in detail and shown to be particularly useful as the basis of a line drawing generation algorithm.

1. INTRODUCTION

As Newman and Sproull remark[1], the graphical display is without a doubt one of the most fascinating devices that computer technology has produced. Further, its usefulness as a research and design tool is well illustrated by the numerous scientific journals devoted to the field of computer graphics. Its spectrum of applications is interestingly diverse. For example, at Regie Renault in France, computer graphics are commonly used in the styling and engineering design of their cars[2]. In architecture, computer graphics permit buildings to be designed and then displayed from various perspectives. In electronics, complex integrated circuits can be analysed through the use of computer aided design packages.

Many of the applications of computer graphics involve the generation and subsequent storage and/or transmission of line drawings. Hence, a problem that arises naturally is that of generating line drawings in a manner that simplifies the storage or transmission requirements. The possibility of generating or approximating existing line drawings through the use of mathematical curve generation techniques suggests itself. Such a mathematical representation of line drawings is attractive not only from the point of view of the data reduction possibilities it offers, but also because such a description can reveal otherwise hidden relations and structures of the various curves making up the line drawing.

In this report, we pursue this idea and we describe a powerful interactive curve generation technique which allows an operator to generate or approximate a line-drawing simply and economically.

The report is divided as follows. In Section 2, we describe the so-called B-spline curve generation technique from a linear system's theory viewpoint. This leads to a new derivation of some known results as well as to some new results which simplifies the computational aspects. In Section 3, we discuss in detail the quadratic B-spline in the context of line drawing generation and amplify the theory of Section 2 with a number of concrete examples. Finally in Section 4, we conclude with some suggestions for possible future work.

2. B-SPLINE - THEORY AND TECHNIQUES

2.1 AN ENGINEERING VIEWPOINT

The classical theory of curve generation is both extensive and well-established[3]. It may be divided into two broad categories of techniques. These are, interpolation and curve fitting techniques. In the first category, we are concerned with the problem of generating a curve which must pass through a set of pre-determined points. Representative techniques in this category are: interpolating polynomials, piece-wise polynomial interpolation, trigonometric interpolation, etc. In the second categories of methods, the generated curve may or may not pass through any of the data points since we are basically concerned with obtaining a "best" fit. The method of least squares belongs to this category. To these two category of methods, a third may be added which represents a new approach to the curve generation problem and accounts for most of the current research in this field. Techniques belonging to this category evolved as a result of advancements in computer graphics technology and/or in response to specific problems in the computer aided design field. Here the work of Bezier[2] may be cited as an example with other references to be found in the excellent works of Newman and Sproull[1] and that of Rogers and Adams[4]. To this category belongs a powerful curve generation technique commonly referred to as B-spline[4]. From the point of view of the application here considered, this technique has a number of significant properties. For instance, it has a local flexibility in that it allows for segments of a curve to be modified without affecting the entire curve. Also, and perhaps most importantly it provides the user with a "feel" for the curve generation mechanism in the sense that the curve bears a close relationship to a defining polygon. However, to the non-mathematician, a survey of the published literature[5] on B-spline, reveals that the basic idea underlying the theory are not apparent in the rigorous and formal mathematical treatment[6]. In this report, the main results of B-spline theory are re-derived from an engineering viewpoint using simple concepts of linear systems theory. This has the advantage of not only providing a useful conceptual framework but also leads to some new results which prove significant in implementing the theory.

2.2 CURVE GENERATION AND LINEAR SYSTEMS

A parametric curve will be generated in the following manner. A set of points $\{P_0, P_1, \ldots, P_n\}$ is defined by the computer graphics system's operator. If we denote by $P_{x,i}$ and $P_{y,i}$, the x and y coordinates of the point P_i, the parametric equations X(t) and Y(t) will be the output of the linear time-invariant system when the impulse sequences $\{P_{x,i}\}$ and $\{P_{y,i}\}$ are applied as inputs. This is illustrated in Figure 1.

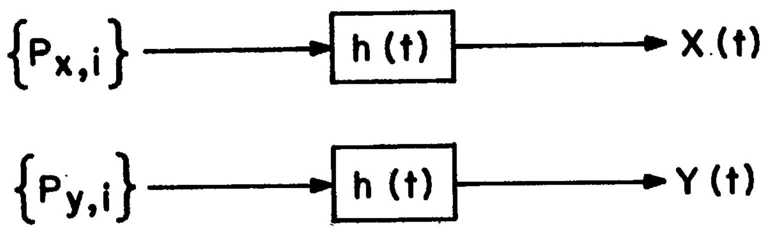

Figure 1. System Representation of Parametric Curve

The function h(t) is referred to as the unit impulse response of the system. The parametric form of the input sequences of Figure 1 may be written as follows

$$P(t) = \sum_{j=0}^{n} P_j \delta(t-j) \tag{1}$$

where $P_j\delta(t-j)$ denotes an impulse[11] of strength P_j located at t=j.

The parametric response is obtained as a convolution of P(t) with h(t)[7]. Denoting the response by R(t), we have for the convolution integral

$$R(t) = \int_{-\infty}^{\infty} h(\tau)P(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau) \sum_{j=o}^{n} P_j \delta(t-j-\tau)\, d\tau \tag{2}$$

Interchanging integration and summation, (2) may be re-written as

$$R(t) = \sum_{j=o}^{n} P_j h(t-j) \tag{3}$$

Hence, if h(t) is chosen to be continuous and everywhere differentiable, then R(t) will also posess these properties. We next discuss a special form of h(t) which leads to the B-spline technique.

2.3 THE CANONICAL B-SPLINE BASIS

Let us assume that the unit impulse response of the system is that shown in Figure 2, which we refer to as a 1st order system.

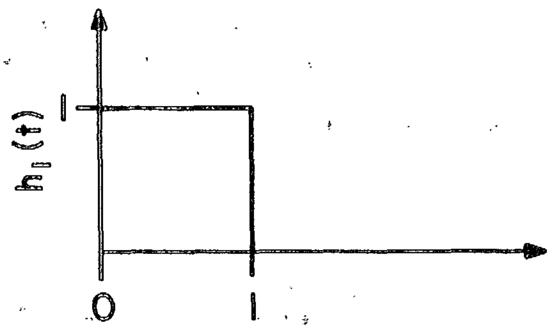

Figure 2. First Order Impulse Response

Other impulse response may be obtained by convolving $h_1(t)$ with itself. Let us then denote by $h_m(t)$ the impulse response obtained as a result of convolving $h_1(t)$ with itself m times. Using Laplace transform techniques,[8] the equation describing $h_m(t)$ may be obtained as follows. Since the Laplace transform of $h_1(t)$ is

$$L\left[H_1(t)\right] = \frac{1}{s}\left(1-e^{-s}\right) \tag{4}$$

where L denotes the one-sided Laplace transform operator, then the Laplace transform of $h_m(t)$ is

$$L\left[h_m(t)\right] = \left[\frac{1}{s}\left(1-e^{-s}\right)\right]^m \tag{5}$$

Hence,

$$h_m(t) = L^{-1}\left\{\left[\frac{1}{s}\left(1-e^{-s}\right)\right]^m\right\} \tag{6}$$

Using, the binomial expansion for the term $(1-e^{-s})^m$, we may write

$$h_m(t) = \sum_{\ell=o}^{m}\left(-1\right)^{\ell}\binom{m}{\ell} L^{-1}\left\{\frac{e^{-s\ell}}{s^m}\right\} \tag{7}$$

From Laplace Transform tables[9], we find

$$h_m(t)=\frac{1}{(m-1)!}\sum_{\ell=o}^{m}\left(-1\right)^{\ell}\binom{m}{\ell}(t-\ell)^{m-1} \text{ with } \ell \leq t \tag{8}$$

The condition $\ell \leq t$ implies that the upper limit of the summation may or may not be reached depending on the value of the variable t. The above is another derivation of the so-called B-spline basis[5]. To further clarify 8, $h_3(t)$ is next computed in detail. From equation 8, we have,

$$\begin{array}{ll} 0 \leq t < 1 & h_3(t)=\frac{1}{2}\,t^2 \\ 1 \leq t < 2 & h_3(t)=\frac{1}{2}\left[t^2-3(t-1)^2\right] \\ 2 \leq t < 3 & h_3(t)=\frac{1}{2}\left[t^2-3(t-1)^2+3(t-2)^2\right] \\ 3 \leq t < 00 & h_3(t)=\frac{1}{2}\left[t^2-3(t-1)^2+3(t-2)^2-(t-3)^2\right] = 0 \end{array}$$

In Figure 3, $h_3(t)$ is plotted along with $h_1(t)$ and $h_2(t)$.

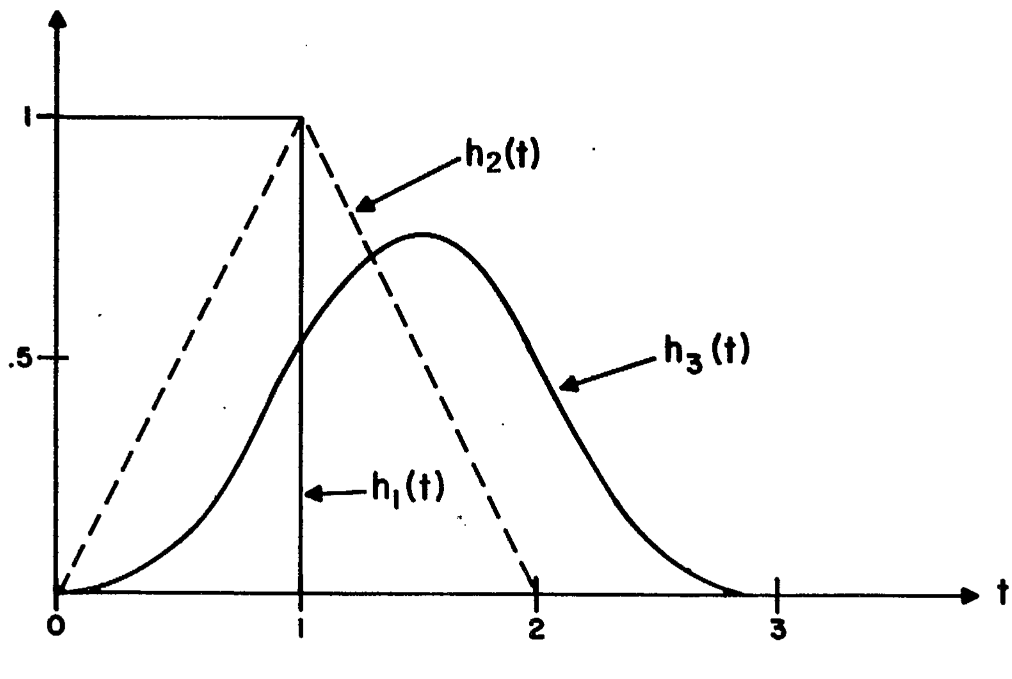

Figure 3. Plot of $h_1(t)$, $h_2(t)$ and $h_3(t)$

Substitution of (8) into (3) gives

$$R(t)=\sum_{j=o}^{n} P_j h(t-j)=\frac{1}{(m-1)!}\sum_{j=o}^{n} P_j \sum_{\ell=o}^{m} (-1)^{\ell}\binom{m}{\ell}(t-j-\ell)^{m-1} \qquad \text{with } t \geq j+\ell \tag{9}$$

It is interesting to note that equation (8) can be considered to be the impulse response of a low pass filter whose frequency response is given by the Fourier transform of $h_m(t)$ or

$$F\left[h_m(t)\right] = \frac{(1-e^{-jw})}{(jw)^m}^{m} = e^{-j\frac{wm}{2}}\left[\frac{\sin\frac{w}{2}}{\frac{w}{2}}\right]^m \tag{9a}$$

Equation (9) can then be considered a smoothing operation, where the high frequencies in the impulses $P_j\delta(t-j)$ are removed by the low pass filter.

From (9), we obtain the parametric equations for x(t) and y(t) given the data point set $\{P_o,P_1,\ldots,P_n\}$. In that $h_m(t)$ extends over m units of the variable t, every point will have a limited influence on the overall curve. Also, if we wish to have the curve start at P_o and end at P_n, it will be necessary to repeat a number of times the first and last point. This will be brought out more clearly in the following section.

2.4 A RECURSIVE FORMULA FOR COMPUTING WITH B-SPLINES

Equation (9) describes the parametric curve in a manner which does not suggest readily an efficient computational algorithm. A simplification of the computational problem is obtained through the use of a recursive relationship discovered by deBoor[6] and also by Cox[8]. In this section we also give a recursive relationship for computation with B-splines which is simple and easily implementable.

In the previous section, we have seen that the parametric curve equation R(t) could be viewed as the result of convolving the input $\sum_{i=o}^{n} P_i\delta(r-i)$ with the impulse response $h_m(t)$. In the Laplace transform domain, this operation may be written as

$$L\left[R(t)\right] = L\left[\sum_{i=o}^{n} P_i\delta(t-i)\right] \times L\left[h_m(t)\right] \tag{10}$$

Since

$$L\left[P_i\delta(t-i)\right] = P_i\ e^{-is},\ \text{then we have}$$

$$L\left[R(t)\right] = \left[\sum_{i=o}^{n} P_i e^{-is}\right]\left[\sum_{\ell=o}^{m} (-1)^{\ell}\binom{m}{\ell}\frac{e^{-s\ell}}{s^m}\right] \tag{11}$$

$$= \sum_{j=o}^{n+m}\left[\sum_{i=o}^{i} (-1)^{i}\binom{m}{i} P_{j-i}\right]\frac{e^{-js}}{s^m} \tag{12}$$

Transform inverting (12), we have

$$R(t) = \frac{1}{(m-1)!}\sum_{j=o}^{n+m}\left[\sum_{i=o}^{j} (-1)^{i}\binom{m}{i} P_{j-i}\right](t-j)^{m-1} \quad \text{with } t \geq j \tag{13}$$

Since $t \geq j$, for $0 \leq k \leq t < k+1$, k a positive integer we may re-write (13) as

$$R_k(t) = \frac{1}{(m-1)!}\sum_{j=o}^{k}\left[\sum_{i=o}^{j} (-1)^{i}\binom{m}{i} P_{j-i}\right](t-j)^{m-1} \tag{14}$$

Hence, for $k+1 \leq t < k+2$, we have

$$R_{k+1}(t) = \frac{1}{(m-1)!}\sum_{j=o}^{k+1}\left[\sum_{i=o}^{j} (-1)^{i}\binom{m}{i} P_{j-i}\right](t-j)^{m-1} \tag{15}$$

$$= R_k(t) + \frac{1}{(m-1)!}\left[\sum_{i=o}^{k+1} (-1)^{i}\binom{m}{i} P_{k+1-i}\right](t-k-1)^{m-1}$$

From equation (15), we see that on every interval $k+1 \leq t < k+2$, the function $R(t)$ is a polynomial of degree $m-1$. Also, $R(t)$ and its derivatives of order $1,2,\ldots,m-2$ are all continuous, over the entire curve. The usefulness of equation (15), lies in the fact that it is a simple recursive relation, so that the polynomial $R_{k+1}(t)$ may be obtained from the polynomial $R_k(t)$ by applying a correction term. We note that the correction polynomial $\frac{1}{(m-1)!}\left[\sum_{i=o}^{k+1} (-1) \binom{m}{i} P_{k+1-i}\right](t-k-1)^{m-1}$ depends at most only on $(m+1)$ points i.e. on the points $P_{k+1}, P_k, \ldots P_{k+1-m}$.

Let us next consider the "steady state" form of (15) (i.e. where $k+1 \geq m$). For $k+1 \leq t < k+2$, we may then write

$$R_{k+1}(t)=R_k(t) + \left[\frac{1}{(m-1)!} \sum_{i=o}^{m} (-1)^i \binom{m}{i} P_{k+1-i}\right] (t-k-1)^{m-1} \tag{16a}$$

or

$$R_{k+1}(t)=R_k(t) + \left[\frac{1}{(m-1)!} \sum_{i=o}^{m} (-1)^i \binom{m}{i} P_{k+1-i}\right] \sum_{\ell=o}^{m-1} (-1)^{\ell} \binom{m-1}{\ell} (k+1)^{\ell} t^{m-1-\ell} \tag{16b}$$

To insure that the curve described by the above equation begins at P_o, we set $R_o(t)=P_o=P_{-1}=P_{-2}= \ldots P_{-m+1}$ i.e., we repeat the first point m times, so that it has "multiplicity" m. We now illustrate the application of (16) by considering an example. Figure 4 shows three B-spline curves of different degrees each defined by the set of points (0,0),(3,9),(6,3),(9,6).

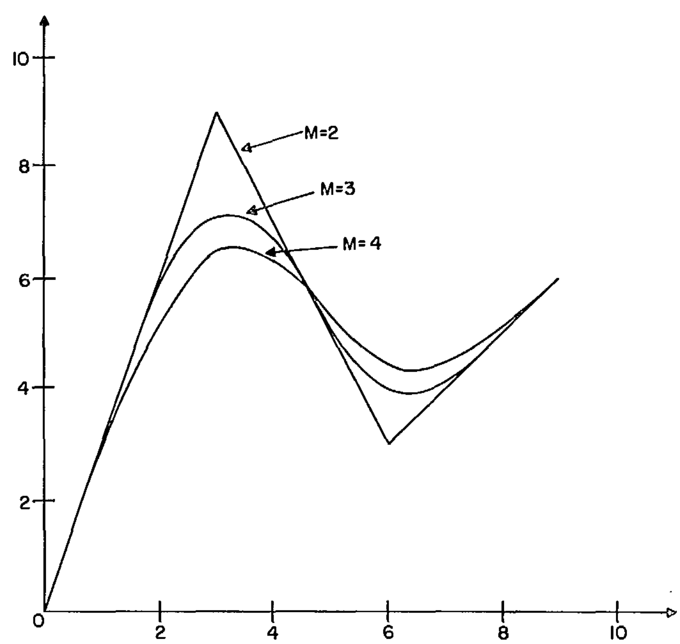

Figure 4. B-Spline Curves

For the case M=3, the following equations are obtained from (16b).

$k=o$, $1\leq t<2$

$R_{o,x}(t) = 1.5t^2-31+1.5$

$R_{o,y}(t) = 4.5t^2-9t+4.5$

t	1.0	1.2	1.4	1.6	1.8	2.0
x	0	.02	.24	.54	.96	1.5
y	0	.18	.72	1.6	2.9	4.5

$k=1$, $2\leq t<3$

$R_{1,x}(t) = 3\ t-4.5$

$R_{1,y}(t) = 7.5^2+39t-43.5$

t	2.0	2.2	2.4	2.6	2.8	3.0
x	1.5	2.1	2.7	3.3	3.9	4.5
y	4.5	6	6.9	7.2	6.9	6

$k=2$, $3\leq t<4$

$R_{2,x}(t) = 3t-4.5$

$R_{2,y}(t) = 4.5t^2-33t+64.5$

t	3.0	3.2	3.4	3.6	3.8	4.0
x	4.5	5.1	5.7	6.3	6.9	7.5
y	6.0	5.0	4.3	4.0	4.1	4.5

$k=3$, $4\leq t<5$

$R_{3,x}(t) = -1.5t^2+15t-28.5$

$R_{3,y}(t) = 1.5t^2+15t-31.5$

t	4.0	4.2	4.4	4.6	4.8	5.0
x	7.5	8.0	8.5	8.8	8.9	9.0
y	4.5	5.0	5.5	5.8	5.9	6.0

In computing the curves of Figure 4, the first and last points have a multiplicity greater than one i.e. are repeated, m times for the first point and (m-1) for the last point. It is of course possible to repeat any intermediary vertex. This is illustrated in Figure 5 where both of the curves have m=4 and the defining polygon is again $\{(0,0),(3,9),(6,3),(9,6)\}$ but now, for one of the curves, the second vertex (3,9) is repeated twice. From the Figure, it is seen that this has the effect of pulling the curve closer to that particular vertex.

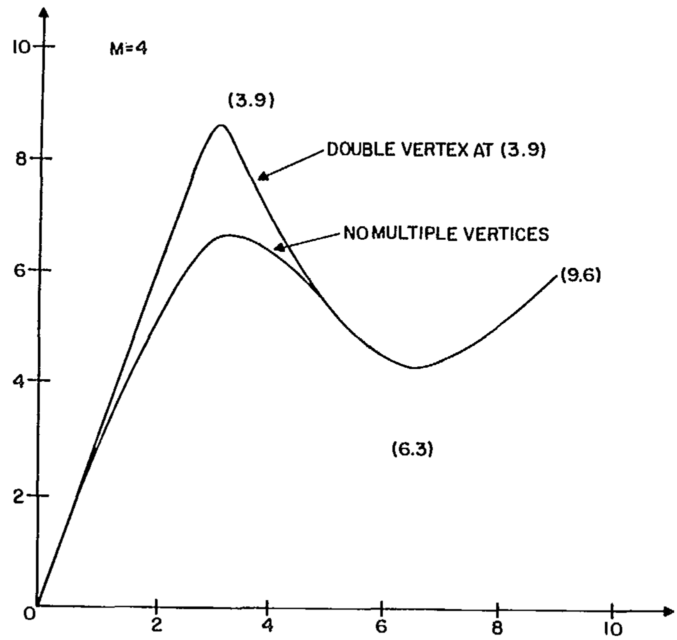

Figure 5. Multiple Vertex B-Spline Curve

Finally Figure 6 demonstrates how local changes can be made without affecting the entire shape of a curve. Each curve is obtained from polynomials of degree 3 (m=4) as described by equation (16). The only difference between each curve is that the fifth vertex is moved to a new position. It can be seen that the first part of each curve remains unchanged. This behaviour is the result of the local nature of the B-spline basis as illustrated in Figure 3.

3. LINE DRAWINGS - GENERATION AND REPRESENTATION

3.1 PRELIMINARIES

In this section, we consider the problem of generating and/or approximating a line drawing in a manner that is efficient from the point of view of storage and transmission. Our approach is to employ a mathematical curve generation technique based on the theory described in Section 2, to efficiently represent the various curves which define the line drawing. Further, the process is to be an interactive one i.e. a human operator will use the curve generation scheme either to originate the line drawing or create an approximation which represents a "good fit" to an existing one. We are then dealing with a man-machine interface imposing certain basic requirements on the overall system. From the hardware side, computer speed and memory size are limiting factors. From the operator side, real-time operation as well as simplicity and flexibility in the algorithms and software will be other constraining factors.

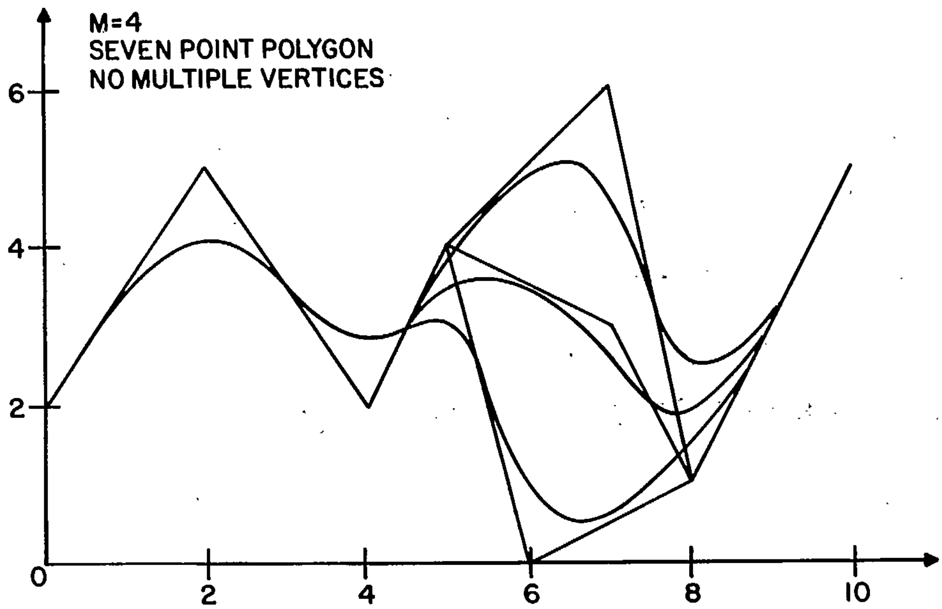

Figure 6. Local Control of B-Spline Curves

With the above in mind, we next describe a special case of the general curve generation algorithm of Section 2 which we used as the basis of a versatile line-drawing generation package[10].

3.2 QUADRATIC B-SPLINE

A quadratic B-spline is obtained if we set m=3 in equations (9) or (16). The usefulness of the quadratic B-spline arises from the fact that it represents the lowest value of m which can be used to represent line segments other than straight lines. Given that the operator has chosen the set of points $\{P_0, P_1, P_2, \ldots P_n\}$, the resulting curve could be calculated from equation (16). We choose, however, to derive the computational algorithm on the basis of equation (9) for reasons which will become apparent when we study the properties of the quadratic B-spline.

Equation (9) is graphically depicted in Figure 7. In the interval $i \leq t < i+1$, with $i \geq 1$ to avoid transient effects at the beginning, we have from Figure 7,

$$R_i(t) = P_i h_3(t-i) + P_{i-1}\, h_3(t-i+1) + P_{i-2}\, h_3(t-i+2) \tag{17}$$

Using the results of section 3 for $h_3(t)$, we then have

$$R_i(t) = 1/2 \left\{ P_i (t-i)^2 + P_{i-1} \left[(t-i+1)^2 - 3(t-i)^2 \right] + P_{i-2} \left[(t-i+2)^2 - 3(t-i+1)^2 + 3(t-i)^2 \right] \right\} \tag{18}$$

Figures 8 and 9 provide examples of curves obtained on the basis of equation (18).

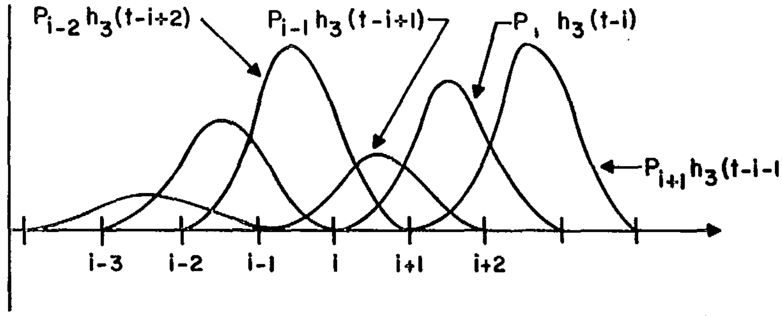

Figure 7. Graphical Representation of $R(t) = \sum_{i=0}^{n} P_i h_3(t-i)$

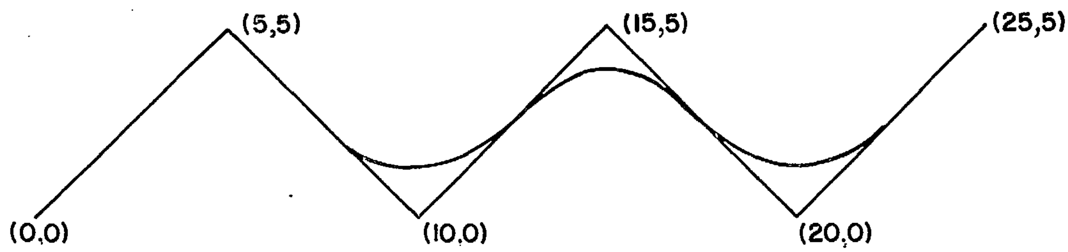

Figure 8. Quadratic B-Spline — Open Curve

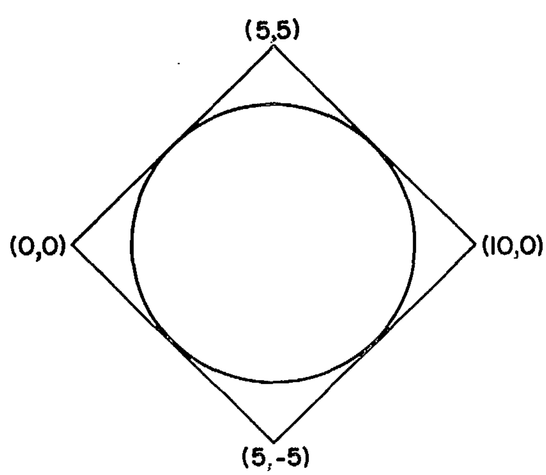

Figure 9. Quadratic B-Spline — Closed Curve

3.3 PROPERTIES OF THE QUADRATIC B-SPLINE

We next discuss some of the properties of the quadratic B-spline which make it useful for line drawing generation.

Continuity

That the curve generated from the quadratic B-spline is continuous is clearly seen from equation (16), which for m=3 and, for $k+1<t<k+2$ becomes

$$R_{k+1}(t) = R_k(t) + \left[\frac{1}{2}\sum_{i=0}^{2} (-1)^i \binom{2}{i} P_{k+1-i}\right](t-k-1)^2 \tag{19}$$

Differentiability

From equation (18), we have for $i \le t < i+1$,

$$R_i(t) = \frac{1}{2}\left\{P_i(t-i)^2 + P_{i-1}\left[(t-i+1)^2 - 3(t-i)^2\right] + P_{i-2}\left[(t-i+2)^2 - 3(t-i+1)^2 + 3(t-i)^2\right]\right\} \tag{20}$$

and for $(i-1) \le t < i$

$$R_{i-1}(t) = \frac{1}{2}\left\{P_{i-1}(t-i+1)^2 + P_{i-2}\left[(t-i+2)^2 - 3(t-i+1)^2\right] + P_{i-3}\left[(t-i+3)^2 - 3(t-i+2)^2 + 3(t-i+1)^2\right]\right\} \tag{21}$$

Differentiating (20) and (21) with respect to t gives

$$R_i'(t) = P_i(t-i) + P_{i-1}\left[(t-i+1) - 3(t-i)\right] + P_{i-2}\left[(t-i+2) - 3(t-i+1) + 3(t-i)\right] \tag{22}$$

$$R_{i-1}'(t) = P_{i-1}(t-i+1) + P_{i-2}\left[(t-i+2) - 3(t-i+1)\right] + P_{i-3}\left[(t-i+3) - 3(t-i+2) + 3(t-i+1)\right] \tag{23}$$

At the junction point t=i

$$R_i'(i) = R_{i-1}'(i) = P_{i-1} - P_{i-2} \tag{24}$$

so that the first derivation is everywhere continuous.

Relation to defining polygon

Let $P_o, P_1, \ldots, P_n$ represent the set of points chosen by the operator. If the point P_{i-1} is joined to P_i ,i=1,2,...n, we generate a polygon. The curve obtained from equation (18) bears a special relationship to that polygon.

For example, from (18) we have that

$$R_i(i) = \frac{P_{i-1}+P_{i-2}}{2} \tag{25}$$

from which we conclude that the curve will intersect at the midway point the line joining P_{i-2} to P_{i-1}. Also, from (24) we see that at the intersection point, the slope of the curve will be the same as the line segment from P_{i-2} to P_{i-1}.

The above properties are well illustrated in the curve of Figure 10 obtained from equation (18).

Straight Line Representation

Another useful property of the quadratic B-spline is that it allows for straight lines to be precisely generated. In equation (18), let the points P_i, P_{i-1}, and P_{i-2} be co-linear. We then have the relation

$$Pi-1 = \alpha(P_i-P_{i-2}) + P_{i-2} \tag{26}$$

where $0\leq\alpha\leq 1$. Substitution of (26) into (18) yields the following equation for $i\leq t<i+1$,

$$R_i(t) = (P_i-P_{i-2}) \left[(t-i)^2(1-3\alpha)+\alpha(t-i+1)^2\right] +\alpha P_{i-2} \tag{27}$$

From (27), we see that the slope of $R_i(t)$ is

$$\left[P_{i,y}-P_{(i-2),y}\right] \Big/ \left[P_{i,x}-P_{(i-2),x}\right] \tag{28}$$

a constant, so that in the interval $i\leq t<i+1$, $R_i(t)$ is a straight line.

3.4 GENERATING LINE-DRAWINGS

In the preceding section, we suggested that the quadratic B-spline, provided a simple and easily implementable curve generation technique that could be used to represent line drawings efficiently, in the sense that the various curves making up the line drawing could be described using a few parameters. The properties that make the quadratic B-spline useful in generating curves are in summary:

i) the generated curve intersects at the midway point, the straight line joining two successive vertices,

ii) at the intersection point, the slope is the same as that of the straight line it intersects,

iii) straight lines can be generated precisely,
iv) only three vertices are needed to generate a curve segment,
v) computational requirements are minimal,
vi) the generated curve is continuous as well as its first derivative.

An interactive program has been written[10] which demonstrated experimentally the usefulness of the quadratic B-spline in generating or approximating line drawings. As an example of the capabilities of a curve generation technique based on the quadratic B-spline, we offer Figure 11 which shows possible isobars distribution over the Canadian western provinces. Fifty-five polygon vertices were used to draw the isobars and 28 for the background map. It is conjectured that the above technique in applications such as weather map transmission, could offer significant savings in transmission and storage costs.

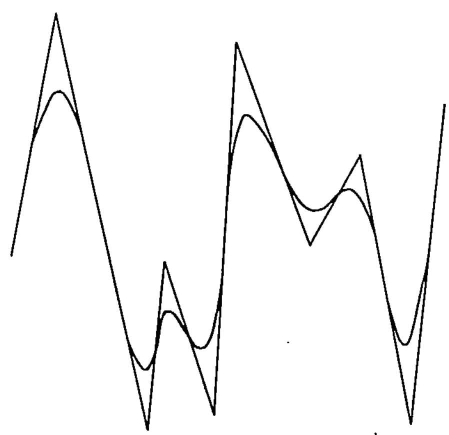

Figure 10. A Quadratic B-Spline Curve

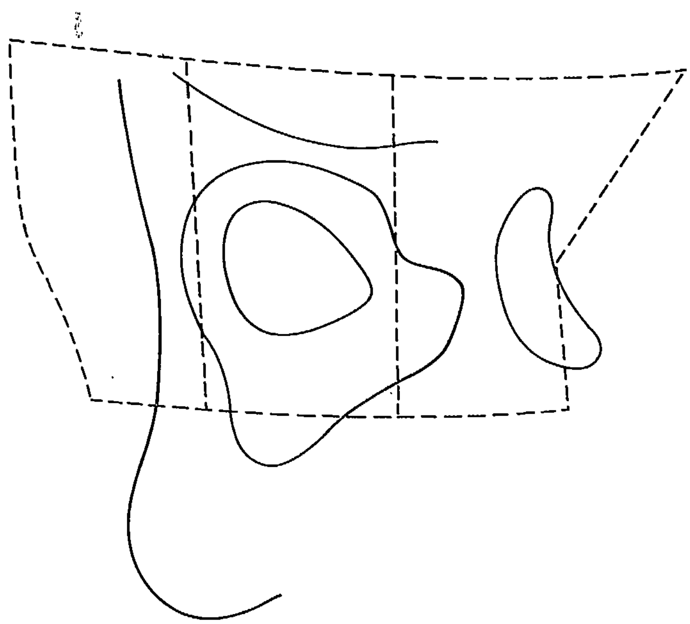

Figure 11. Quadratic B-Spline Line Drawings

4. CONCLUDING REMARKS

This report has dealt with a problem basic to computer graphics, namely the efficient generation and representation of line drawings. The approach taken has been to use mathematical techniques in describing the various curves making up the line drawing. This leads us to the consideration of the B-spline technique, a powerful curve generation technique that we described in the framework of linear system theory. In Section 3, we discussed at length a special case of the B-spline. An interactive computer program[10] for curve generation, based on the quadratic B-spline was written, for the purpose of experimentation. It was found that even a novice user could quickly get the "feel" of curve generation and required only a short familiarization time to efficiently use the system for line drawing generation or approximation. In Figure 11, we gave a possible application of such a scheme, that of efficiently encoding weather information.

Possible avenues for further work include the extension of the theory of section 2 to surface generation and determining the optimum set of vertices for line drawing approximation which would meet a certain performance criteria.

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8. **ABSTRACT:** The problem of efficiently generating or representing line drawings from the point of view of storage and transmission requirements is considered. A proposed solution is the use of mathematical techniques to characterize the various line segments of the line drawing. Towards that end, the B-spline technique is described and a new recursive relationship which simplifies the computation is given. The quadratic B-spline is then considered in detail and shown to be particularly useful as the basis of a line drawing generation algorithm.

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