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RADIO-WAVE REFLECTIONS FROM A SPHERICAL EARTH – PREDICTIONS AT VHF AND UHF

by

J.H. WHITTEKER

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ABSTRACT

The field strength of radio waves at VHF and UHF can be affected by reflections from the ground or water. Except for the shortest paths, the reflecting surface must be considered to be spherical rather than flat. This report describes procedures that may be programmed into a computer for automatically finding the reflection point, for calculating the effective coefficient of reflection, and for estimating the effect of any obstructions in the path of the reflected ray. The description is intended to be complete enough to be used for writing a computer program to do the calculations. New formulas are derived, and references are given for formulas obtained elsewhere. A comparison of the results with CCIR curves for propagation around a spherical earth shows agreement within about 1 dB in the region for which reflection calculations are used.

1. INTRODUCTION

1.1 MOTIVATION

For the prediction of field strength at VHF and UHF, one of the effects that may be taken into consideration is the reflection of radio waves from the ocean, lakes, or flat ground. At the point of reception, the reflected wave and the direct wave combine, and the resulting field strength depends on the amplitude of the reflected wave and on its phase relative to that of the direct wave. The object of the procedures described here is to find this amplitude and phase. The computer must find the point of reflection by exploring the path profile numerically. It also must examine the terrain near the reflection point for roughness and ground cover. Finally, it must check for obstructions in the reflected path. These procedures have been incorporated into the CRC VHF/UHF propagation prediction program[1], which calculates field strength automatically, starting from a numerical representation of a path profile.

1.2 RELATIONSHIP TO PREVIOUS WORK

Many formulas relevant to this work are given in NBS Technical Note #101[2], hereinafter referred to as TN101. The relevant material is on pages 5-1 to 5-9, and in the Annex, pages III-1 to III-16. Some formulas are also found in Hall[3], section 4.2. Various equations in this work are quoted from these references. Occasional errors are corrected. In addition, new equations are derived that are required for completely automatic calculation by computer.

1.3 LIMITATIONS

Reflections are taken into account only for line-of-sight paths. On a path obstructed by hills, a reflected wave may indeed reach the receiving antenna, but its contribution is unlikely to be important. This is because the reflected wave usually suffers much more diffraction attenuation than the more direct one, since it goes deeper into the shadow of the obstruction. In any case, calculating the phases of diffracted waves is difficult to do reliably.

Only horizontal surfaces are assumed to reflect radio waves. This is partly because smooth horizontal surfaces (oceans and lakes and sedimentary basins) are much more common than smooth sloping surfaces. It is also partly because reflections from sloping surfaces are difficult to deal with in any case. This is because the calculation is done from a path profile, i.e. elevations along a line. The slope of the terrain perpendicular to the line is not known. The reflected ray may be deflected to the side, and miss the receiving antenna, or rays may reach the antenna from points on either side of the line. (The procedure does not demand an exactly horizontal surface, but any variation in height near the reflection point counts towards its calculated roughness.)

In the derivations, it is assumed that the angle between the incident ray and the reflecting surface is small. That is, the height of the antennas above the ground is small compared to the distance between them. It is also assumed that all distances are small compared to the radius of the earth.

The amplitude of the reflected wave is subject to some uncertainty due to our lack of detailed knowledge of the reflecting surface, in particular its roughness. So while the position and general appearance of maxima and minima may be expected to be correct, the amplitude may be only approximate. Even on paths over water, the surface roughness is not perfectly known, since it depends on the weather.

1.4 EFFECTIVE EARTH RADIUS

In this report, no explicit attention is paid to the refractive properties of the atmosphere. It is assumed that the earth is always represented with its effective radius a_e, defined as

$$\frac{1}{a_e} = \frac{1}{a} + \frac{n'}{n}$$

where a is the radius of the earth

n is the refractive index of the air

n' is the vertical gradient of n.

(In the rest of this report the subscript is dropped from a_e.)

It is well known that ray trajectories become straight lines when the earth's radius is changed to its effective value, but it is also true that the lengths of these lines represent the phase paths. This assertion may be justified by examining the properties of the wave equation in spherical coordinates, as is done, for example, by Bremmer[4] (p. 147) and by Wait[5] (p.117). The wave equation with a linear vertical gradient of refractive index is equivalent to the same equation without a gradient, but with the earth's radius replaced by its effective value. The only restriction is that heights above the surface must be small compared with the earth's radius. Since the rays of geometric optics are specialized solutions of the wave equation, it follows that they are correctly transformed by using an effective earth radius.

1.5 NOTE ON THE DIAGRAMS

Diagrams of propagation paths, if drawn to scale, would be very long (horizontally) and thin (vertically). In order to make them easier to work with, they can be modified in two ways: (1) The diagram can be drawn as if the length of the path were comparable to the radius of the earth. The advantage of this is that vertical and horizontal scales are equal, and right angles look like right angles. The disadvantage is that horizontal distances appear to depend on height above sea level, which for the shorter paths actually used, they do not. (2) The diagram can be drawn with a realistic path length, but with the vertical scale stretched. The advantage of this is that horizontal distances do not appear to depend on height. The disadvantage is that since the vertical and horizontal scales are different, angles are distorted. Both types of modification are used in this report.

2. FINDING THE POINT OF REFLECTION

2.1 GENERAL PROCEDURE

If the terrain is entirely horizontal (the ocean, for example), finding the point of reflection is not difficult (Section 2.4). However, we might be faced with the sort of terrain illustrated in Figure 1, in which there are

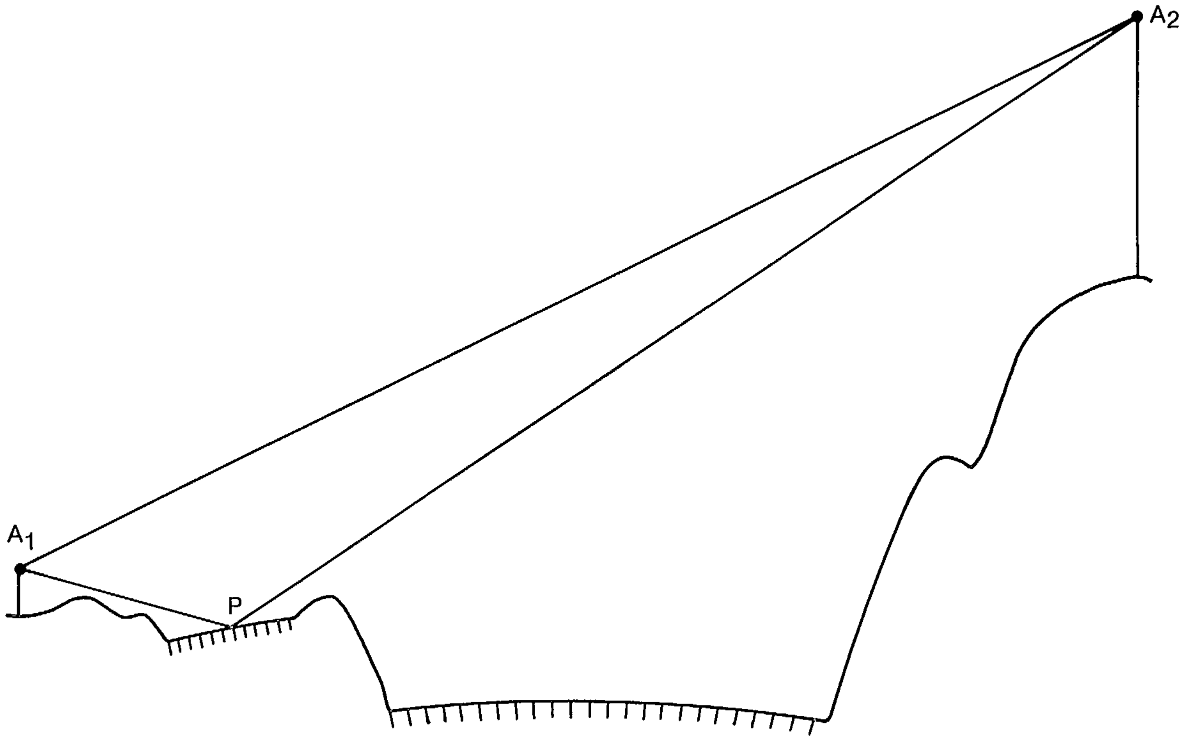

Figure 1. The profile of a hypothetical propagation path which traverses two lakes and irregular ground in between. The antennas are at A1 and A2, and the reflection point is at P.

lakes at different levels, and irregular terrain in between. Which lake is in the right place to reflect a ray from one antenna to the other?

We can begin by finding points along the path between which the geometrical point of reflection must occur (Section 2.2). Then we find the average elevation between these limits, and obtain a reflection point for this elevation (Section 2.4). The position of this point is only approximate, because the average elevation on which it is based may be different from the elevation at the reflection point. Now the elevation "at" the reflection point is really the average elevation within all or part of the first Fresnel zone around it (Section 2.3). Therefore we next find the average elevation "at" the estimated reflection point, and use this new elevation to find a new point. After a few iterations, we have the correct reflection point. If the terrain profile is smooth enough in the appropriate region to support reflections, this procedure converges rapidly. The following sub-sections describe the procedure in more detail.

2.2 LIMITS ON POSSIBLE REFLECTION POINTS

We are given a terrain profile and the elevation above sea level of the antenna at each end. To begin, assume we know nothing about the terrain profile. Some possible reflecting surfaces (curved lines) are illustrated in Figures 2 and 3. These surfaces are concentric about the centre of the earth,

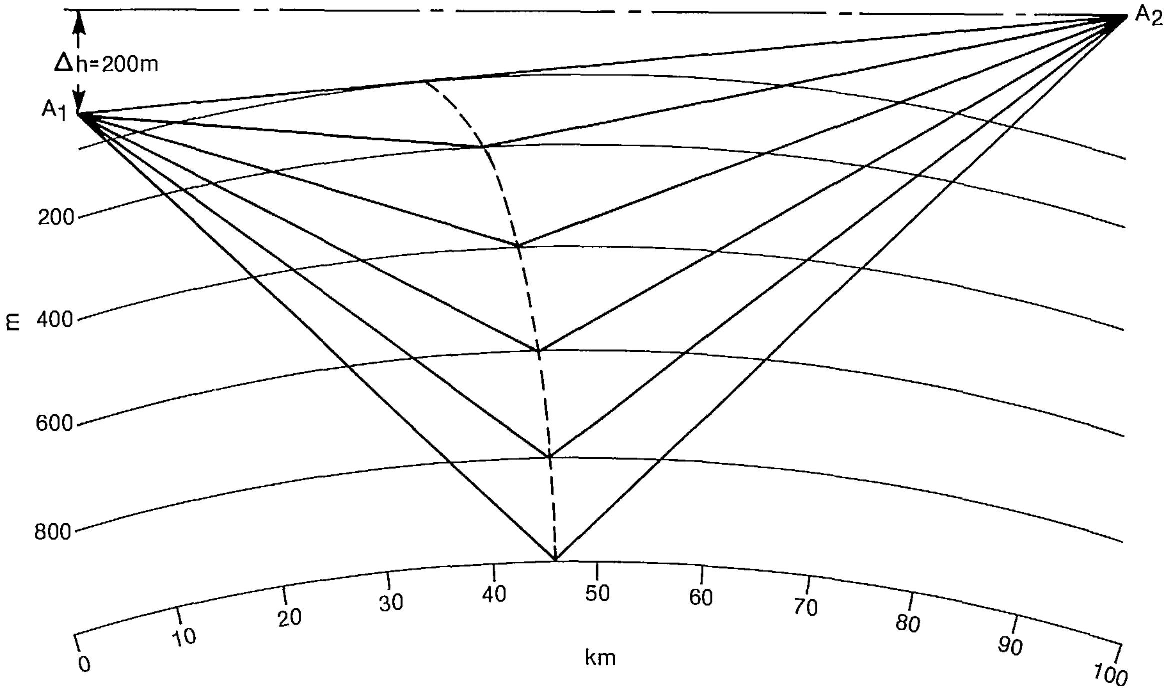

Figure 2. Antennas separated in height by 200 m, and a number of possible spherical reflecting surfaces. For each surface, the reflected ray is shown. The locus of all reflection points is shown as a broken line.

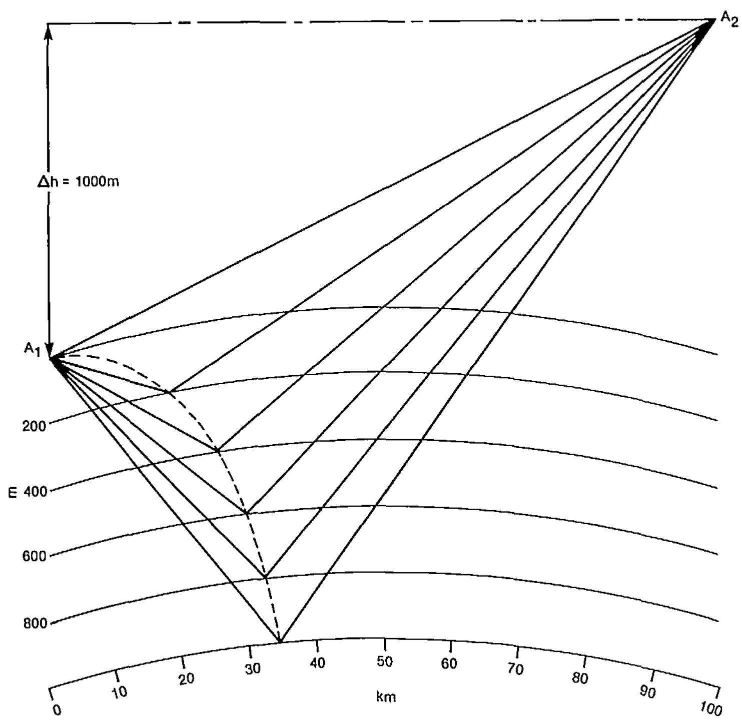

Figure 3. The same diagram as Figure 2, except that the antennas are separated in height by 1000 m.

and, to a sufficient approximation, they all have the same radius of curvature, the effective earth's radius. For each of these, we can find the horizontal position of the point of reflection (Section 2.4). The diagrams show the locus of such points. What we must search for is the point at which the path profile intersects this locus.

We can put limits on the region in which we must search. One limit is the mid-point of the path, since the reflection must occur in the half of the path occupied by the lower antenna. The other limit is illustrated in Figure 2; it is the point at which the direct ray is tangent to one of the hypothetical reflecting surfaces. This particular surface is the highest one for which the propagation path can be considered line-of-sight. The horizontal coordinate of this limit, measured from A_1, is

$$x = \frac{d}{2} - \frac{a}{d}(h_2 - h_1) \qquad (1)$$

where d is the distance from one antenna to the other,

a is the effective radius of the earth

h_1, h_2 are the heights of the antennas above sea level. (Under the assumption that $d << a$, it makes no difference whether x and d are measured along a curved or straight line between the antennas.) This equation may be derived by noting that, apart from an additive constant, the equation of any of the horizontal curved surfaces is

$$y = -\frac{\left(x - \frac{d}{2}\right)^2}{2a} \qquad \left|x - \frac{d}{2}\right| << a \qquad (2)$$

and the slope of the surface is therefore $-(x - d/2)/a$. The result (1) follows by equating this slope with the slope of the line joining the antennas.

If the difference in the antenna heights is greater than $d^2/2a$, then x, as given by equation 1, falls outside of the propagation path, and the end of the path must be taken as the limit. This situation is illustrated in Figure 3. We now know that the geometrical point of reflection lies between the centre of the propagation path and the point given by equation 1. This concludes the first procedure mentioned in Section 2.1.

2.3 FRESNEL ZONES - RELFECTION ZONE

We must now take note of the fact that the specular reflection of waves does not take place at a point. Rather it takes place, and requires a smooth surface, over a finite region for which ray path lengths do not vary by more than some fraction of a wavelength. If this fraction is $\lambda/2$, the region is known as the first Fresnel zone. For low ray angles, the first Fresnel zone is much longer along the propagation path than perpendicular to it (e.g. Hall, Section 4.3.2, p. 91). Therefore, it is reasonable to suppose that a smooth surface extends for a sufficient distance perpendicular to the path, and to be concerned only about its extent along the path. Imagine that the reflecting surface has boundaries that are straight lines perpendicular to the propagation path. Then the curves of Figure 4 represent the amplitude of the reflected wave as a function of the distance of one or both boundaries from the geometrical point of reflection.

From either curve, it may be concluded that the first Fresnel zone ($\delta r/\lambda < 0.5$) is unnecessarily large for representing the reflecting region. A surface bounded at $\delta r/\lambda = 0.2$ or 0.3 would reflect just as well. The choice is quite arbitrary, but for the prediction program, a 'reflection zone' has been defined to be one bounded at $\delta r/\lambda = 0.3$. In any case, the chosen boundary should lie to the right of $x = 0.6$ ($\delta r/\lambda = 0.18$), since otherwise there is a danger that in a later procedure (Section 3.3) the part of the reflecting surface just beyond the reflection zone will be incorrectly counted as an obstruction.

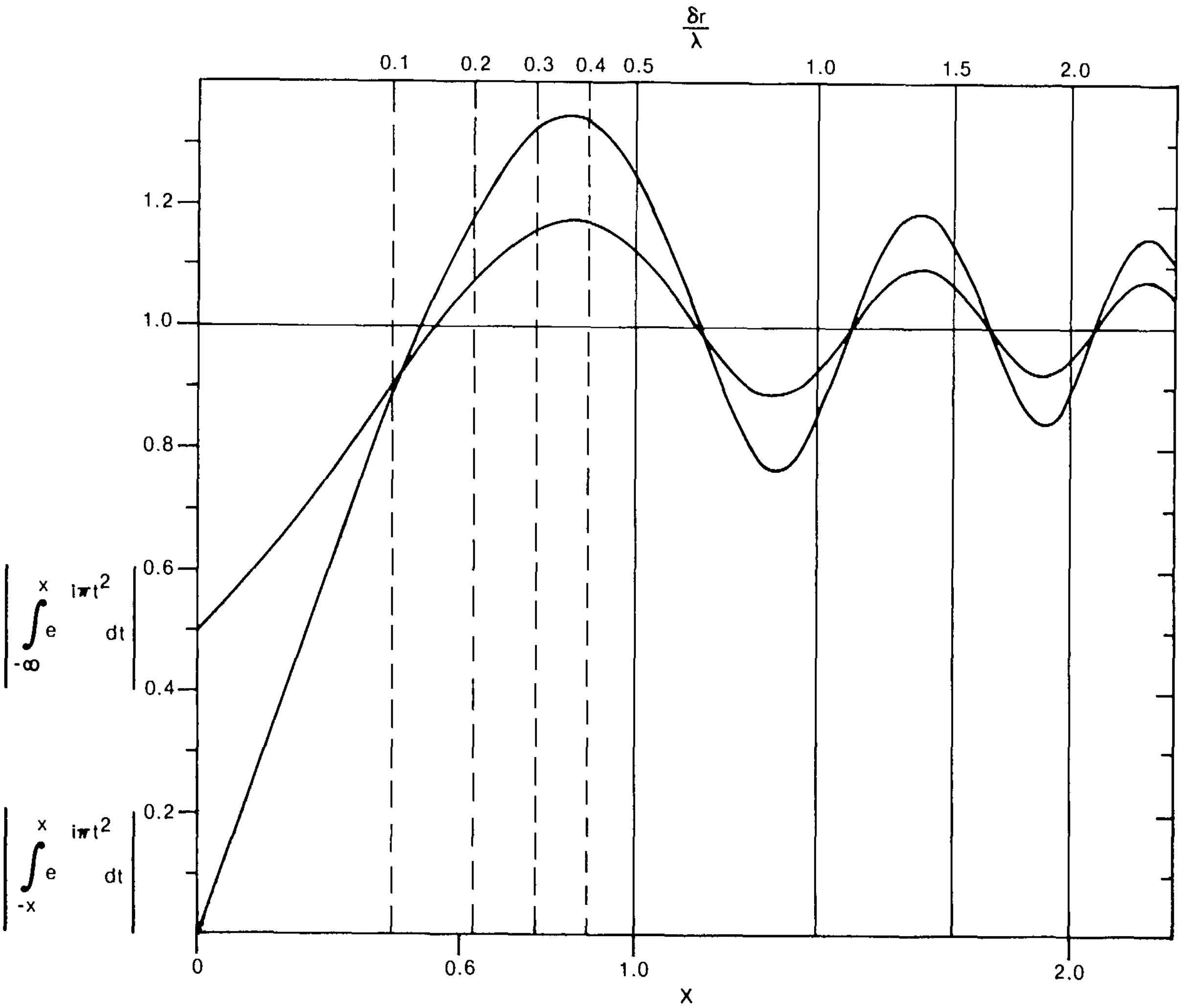

Figure 4. Fresnel integrals. The factor multiplying t^2 in the integrand has been chosen as π in order to normalize the integrals in a way suitable for this diagram. The Fresnel zones are separated by vertical lines. When $x = 1$, the first Fresnel zone is clear, and when $x = 2$, the first four Fresnel zones are clear. The top scale gives the path length relative to that of the geometrically reflected ray (δr) in units of wavelength (λ).

A method for finding Fresnel-zone or 'reflection-zone' limits is given in Appendix A3. Once a reflection zone has been defined, the average elevation within it may be found in order to continue with the search for the correct reflection point. Later, after the correct reflection point has been found, the zone may be examined for surface cover (Section 3.1) and for roughness (Section 3.2.3).

2.4 POINT OF REFLECTION, GIVEN A HORIZONTAL REFLECTING SURFACE

We are given the elevation of a reflecting surface. This elevation is an average over some part of the propagation path, either the initial region of Section 2.2, or a reflection zone of Section 2.3. We wish to find the geometrical reflection point. Let h_1 and h_2 be the heights of the antennas above the surface. The relevant geometry is illustrated in Figure 5. At point P, a plane touches the curved reflecting surface. The separation between this tangent plane and reflecting surface is $(x-x_p)^2/2a$ (from equation 2), where x_p is at P. Then the height of antenna A_1 above the tangent plane is

$$h_1' = h_1 - \frac{x_1^2}{2a} \qquad (3)$$

and the angle between the incident ray and the tangent plane is

$$\psi_1 = \frac{h_1'}{x_1} = \frac{h_1}{x_1} - \frac{x_1}{2a} \qquad (4a)$$

Similarly,

$$\psi_2 = \frac{h_2'}{x_2} = \frac{h_2}{x_2} - \frac{x_2}{2a} \qquad (4b)$$

The geometrical point of reflection is found where

$$\Delta\psi = \psi_2 - \psi_1 = \frac{h_2}{x_2} - \frac{h_1}{x_1} - \frac{x_2 - x_1}{2a} = 0 \qquad (5)$$

with the constraint that $x_1 + x_2 = d$, the (constant) total path length. This may be solved for x_1 (See Appendix A2). Once we know the reflection point, we can calculate the difference in path length between the direct and reflected ray (Appendix A2). This path difference, together with the reflection coefficient (Section 3.2.1), is required to find the phase of the reflected wave relative to that of the direct wave.

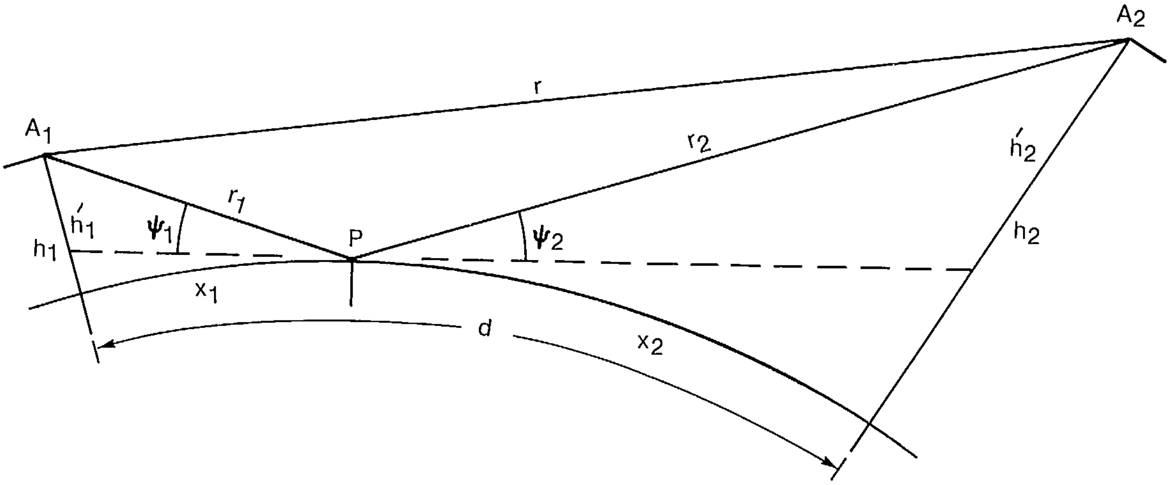

Figure 5. Reflection from a spherical earth. The antennas are at A_1 and A_2. P is any point chosen on the reflecting surface. The broken line represents the plane tangent at P. h_1 and h_2 are the heights of the antennas above the spherical earth, while h_1' and h_2' are their heights above the tangent plane. In the important special case when angles ψ_1 and ψ_2 are equal, P is the geometrical point of reflection, and the distances x_1 and x_2 are designated d_1 and d_2.

3. AMPLITUDE OF THE REFLECTED WAVE

3.1 TYPE OF GROUND COVER

Trees and buildings are expected to be too rough to support reflections. Therefore the amplitude of the reflected wave is multiplied by the fraction of the reflection zone not covered by trees or buildings.

3.2 REFLECTION COEFFICIENT

3.2.1 Reflection Coefficient for a Plane Surface

The reflection coefficients for a plane surface having a given conductivity and dielectric constant are given by both Hall (his equations 4.10 and 4.11) and TN101 (pages III-3, III-4). The complex reflection coefficient, as given by Hall, is

$$\rho = \frac{p \sin\psi - \left(n^2 - \cos^2\psi\right)^{\frac{1}{2}}}{p \sin\psi + \left(n^2 - \cos^2\psi\right)^{\frac{1}{2}}} \tag{6}$$

where n is the refractive index of the ground.

ψ is the angle between the ray and the surface.

$p = n^2$ for vertical polarization, and

$p = 1$ for horizontal polarization. The square of the refractive index is the complex number

$$n^2 = \varepsilon_r + i\ 60\ \sigma\ \lambda \tag{7}$$

where ε_r is the ratio of the permittivity of the ground to that of air

σ is the ground conductivity in Siemens/metre

λ is the wavelength in metres.

60 represents $1/2\pi\varepsilon_0 c$, where ε_0 is the permittivity of free space, and c is the speed of light.

(Although Hall gives a negative sign in this expression, appropriate for a wave of form exp $i(\omega t - kx)$, it must be positive for a wave of the form exp $i(kx - \omega t)$. See, for example, Panofsky and Phillips[6], equation 11-15.) TN101 gives the same coefficients expressed in terms of amplitude and phase. This avoids the use of complex numbers, but at the price of much more complicated expressions. Both TN101 and Hall display graphs of reflection coefficient for different types of ground. Those in TN101 are much more detailed.

Conductivities for different types of surface, as recommended by CCIR[7], are displayed in Figure 6. Permittivities (relative to air) are almost independent of frequency at VHF and UHF, and are given in the table below:

Sea water	75
Fresh water	80
Wet Ground	30
Medium dry ground	15
Very dry ground	3

These values are from CCIR[7], except for sea water, which is from CCIR[8].

In most cases, the values used for the ground constants are not critical, since for small values of ψ, the absolute value of the reflection coefficient is close to unity, and the phase close to π, with only a weak dependence on the constants. Vertical polarization over seawater is the only exception to this. In the prediction program, the ground constants are averaged over the reflection zone.

3.2.2 Divergence Factor

The amplitude of the reflected wave is reduced by divergence due to reflection from a convex mirror. TN101 and Hall give the divergence factor as

$$D = \left(1 + \frac{2\ d_1\ d_2}{a\ d\ \tan\ \psi}\right)^{-\frac{1}{2}} \tag{8}$$

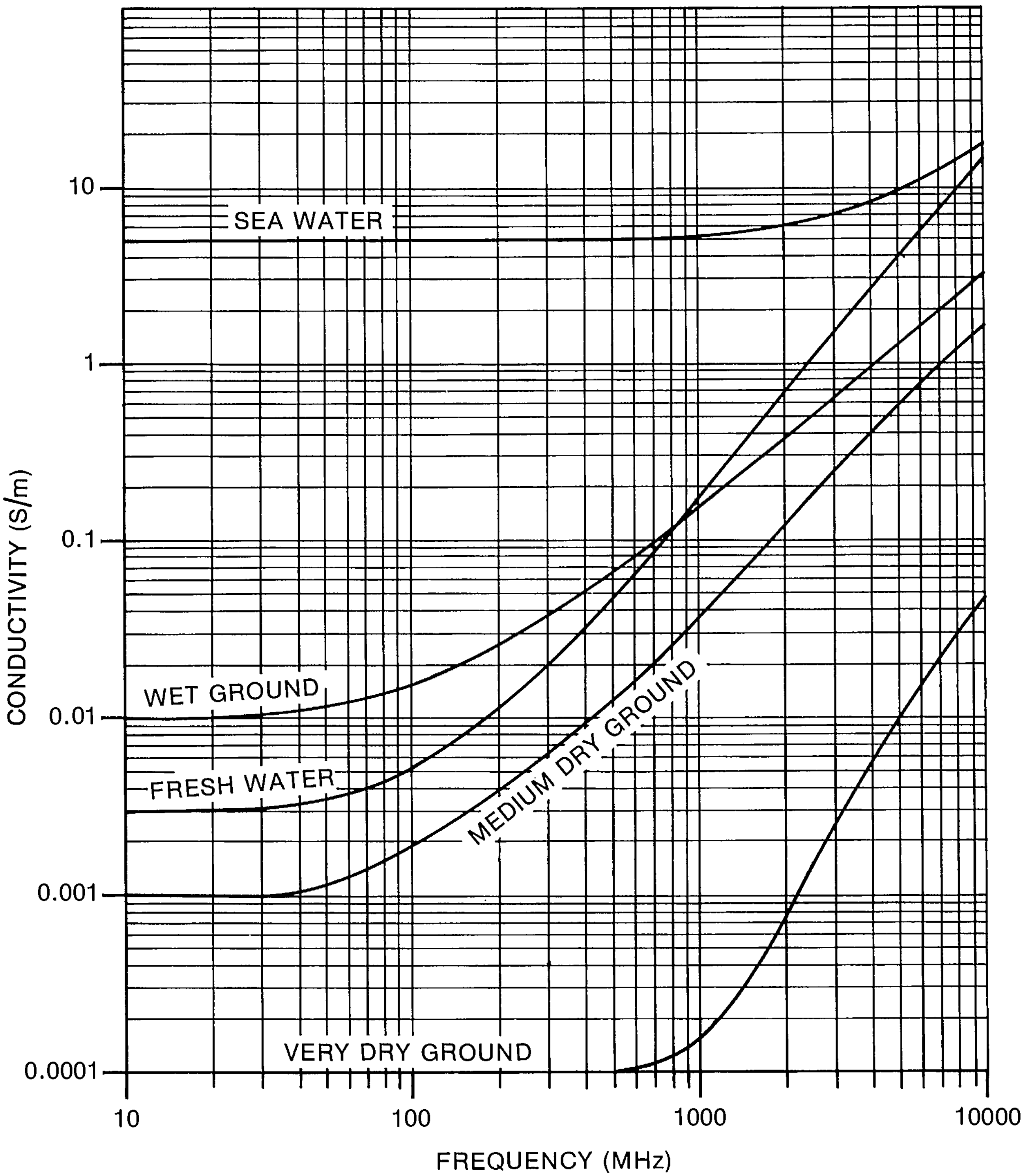

Figure 6. The conductivity of various types of ground at VHF and UHF, adapted from CCIR[7].

where d_1 and d_2 are the distances from the antennas to the point of reflection. (In Hall, a misprint has deprived the exponent of its minus sign.)

3.2.3 Terrain Roughness

A rough surface reflects less well than a smooth one. Hall (his equation 4.8) gives the factor for the reduction in amplitude as

$$\exp\left\{-\frac{1}{2}\left[4\,\pi\,\sigma_h\,\sin\psi\,/\lambda\right]^2\right\} \qquad (9)$$

where σ_h is the rms height variation, and λ is the wavelength. (TN101 and Hall do not agree on this. The expression quoted above is evidently the correct one, since it is obtained from Beckmann and Spizzichino[9], who derive it from first principles.) In the prediction program, the height variation is obtained from the path profile within the reflection zone. For any part of the reflection zone not occupied by water, an rms height variation of 3.3 m is added to account for small-scale variations that may be concealed within the 7.6 m (25 ft) contour intervals of topographic maps. Water surfaces are assumed to have an RMS height variation of 0.3 m.

3.3 ATTENUATION DUE TO OBSTRUCTIONS

Even if the direct ray clears all obstructions, this may not be so for the reflected ray, since it follows a lower trajectory. To obtain an estimate of the attenuation of the reflected wave, it is necessary to find the Fresnel-zone penetration of any obstruction under consideration. Consider the reflected wave from the point of reflection P to antenna A_2 (Figure 7): The wave here is approximately the same as if it came from the image A_1'. (It is exactly the same only very close to the ray, since a spherical mirror is not aberration-free.) Therefore the diffraction attenuation due to obstructions between P and A_2 may be calculated as if the wave actually came from A_1'. The attenuation between A_1 and P may be similarly estimated, using image A_2'. Of course, the part of the terrain surface that does the reflecting should not be counted as an obstruction. Therefore the search for obstructions begins beyond the edge of the reflection zone. The location of the image is derived in Appendix B.

A simple algorithm is used to calculate the diffraction attenuation of the reflected wave, since great accuracy is not needed. (Since the nature of the reflecting surface, in particular its roughness, is usually not known precisely, an elaborate calculation of diffraction loss is not warranted here.) On each side of the reflection point, the obstruction with the greatest Fresnel-zone penetration is found, and the following formula is applied to it:

$$\begin{aligned} A(\mathrm{dB}) &= 16.66\left(\frac{h}{R} + 0.6\right) & \frac{h}{R} &> -0.6 \\ A &= 0 & \frac{h}{R} &< -0.6 \end{aligned} \qquad (10)$$

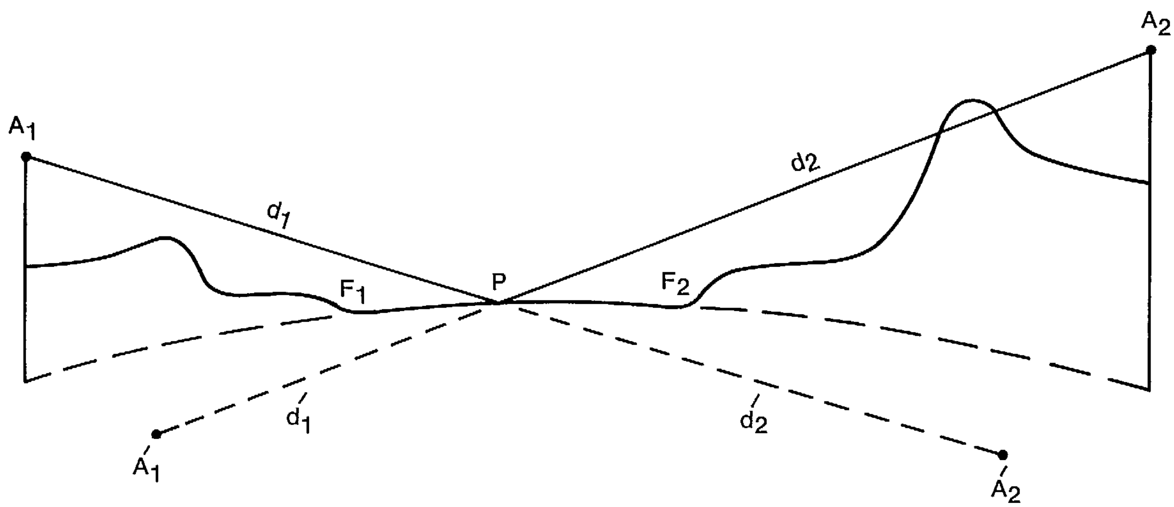

Figure 7. The construction used for estimating the diffraction loss of the reflected wave. A_1' and A_2' are the images of the antennas at A_1 and A_2. F_1 and F_2 are the reflection-zone boundaries, which, in this diagram, happen to coincide with the edges of a lake. For the path A_1' to A_2, the terrain is examined for obstructions between F_2 and A_2.

Here, A is the attenuation, h is the height of the obstruction relative to the line of sight, and R is the radius of the first Fresnel zone for diffraction. This formula is an approximation to one of the curves given by de Assis[10]. To choose one of his curves, his parameter α has been arbitrarily assigned the value 0.5. The attenuation factor is given here in decibels, and must be changed to an amplitude factor before use.

Note that this formula cuts off at $h/R = -0.6$, i.e. when the path difference becomes greater than 0.18 λ. (Refer back to Section 2.3 and Figure 4.) Therefore any part of the reflecting surface beyond the reflection zone defined earlier will easily escape being considered an obstruction.

The application of equation 10 to the reflected wave ignores any phase change that the obstruction might cause. An examination of Cornu's spiral, found in many textbooks on physical optics, shows that the phase of a wave diffracted by a knife edge differs from that of an unobstructed wave by no more than 16°, provided $h/R < 0.16$. (For rounded obstructions the phase difference may be greater.) However, as h/R increases beyond 0.16, the phase difference becomes large. Now at $h/R = 0.16$, equation 10 gives 0.23 as the attenuation factor. Therefore, the phase change due to diffraction is large only when the attenuation is severe. Nevertheless, phase considerations imply that the effect of an obstructed reflected wave can be calculated with only moderate accuracy.

4. SUMMARY OF METHOD

The objective of all the procedures described here is to find the phase and amplitude of the reflected wave, in each case relative to that of the direct wave. A prerequisite for finding these quantities is to find the geometrical point of reflection, by solving equation 5 for the appropriate reflecting height. Another is to find the extent of the reflection zone around this point, by solving equations A8 and A6.

The phase is the sum of the phase lag due to reflection and the phase lag due to the excess path. The phase lag due to reflection is the argument of the complex reflection coefficient ρ, given in equation 6. The phase lag due to the excess path is $2\pi\Delta r_o/\lambda$, where Δr_o is given in equation A7.

The amplitude is the absolute value of the reflection coefficient ρ, multiplied by a number of other factors, listed below:

The fraction of the 'reflection zone' that is reflective (Section 3.1).

The divergence factor, given in equation 8.

The roughness factor, given in equation 9.

The attenuation due to obstructions, given in equation 10.

5. VERIFICATION

5.1 COMPARISON WITH CCIR CALCULATIONS

Figure 8 shows a comparison between CCIR ground-wave propagation curves[11] and the results of the procedures described here in the case of vertical polarization over sea water. The results are similar for horizontal polarization and for other ground constants. In the CRC prediction program, the methods of this Note are used up to the point where the curve crosses the free-space curve for the last time.

The agreement between the two sets of curves is very close, except near the limit of use just mentioned, where they differ at most by about 1 dB. The discrepancy here is probably due to the interaction of the second Fresnel zone of the direct wave with the earth's surface, which is not taken into account in the reflection method. (If the reflection curves were extended farther to the right than shown here, they would approach the free-space curve. This is because of the increasing defocusing of the reflected ray, and the neglect of the attenuation of the direct wave as the earth penetrates its first Fresnel zone.)

5.2 EXAMPLES OVER SMOOTH AND IRREGULAR TERRAIN

Figure 9 shows the signal strength over the ocean as a function of distance, as calculated by the CRC prediction program. The part of the curve near 100 km corresponds to the curve for h_2 = 200 m in Figure 8. Note the discontinuity as the program changes from the reflection procedure to the CCIR curve. At shorter distances, the interference between the direct and reflected waves produce a series of maxima and nulls.

Figure 10 shows the signal strength on a irregular path obtained from a topographic data base. The path extends from CRC property, northward across the Ottawa River, onto farmland beyond, and finally into the Gatineau Hills. The receiving antenna is assumed to move at a constant height above the terrain. Very close to the transmitter, reflections are unimportant because the reflected wave is attenuated by ground roughness at the large values of ψ. One null appears as the receiving antenna crosses the river, and two more appear close together as it climbs the hill beyond. Beyond the crest of the hill, diffraction loss dominates, until the receiving antenna goes high enough on the final hill, where the signal regains its free-space value. Here, reflections from the river have no effect both because of roughness within the reflection zone and because the central hill is an obstruction to the reflected wave.

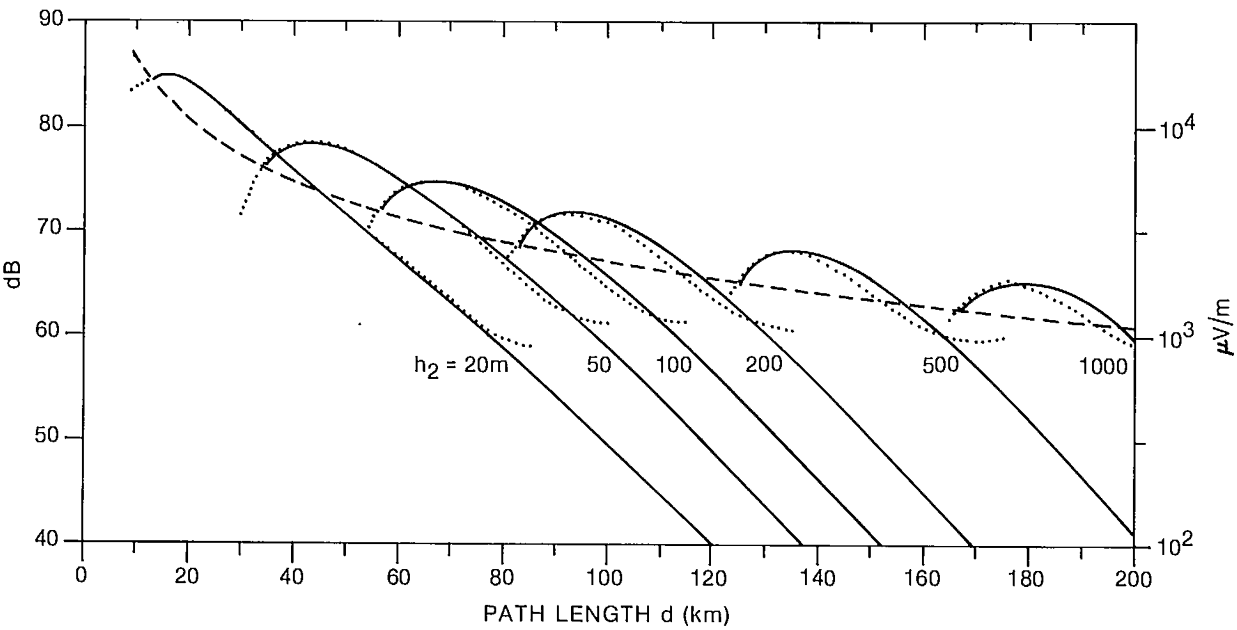

Figure 8. Comparison of CCIR field-strength curves (solid lines) with the results of reflection calculations (dotted lines). The CCIR curves are from p. 118 of CCIR Atlas[11]. They correspond to one kilowatt at 200 MHz, transmitted from a vertical dipole 500 m above the ocean. The field strength is calculated at height h_2. The broken line represents the free-space field strength.

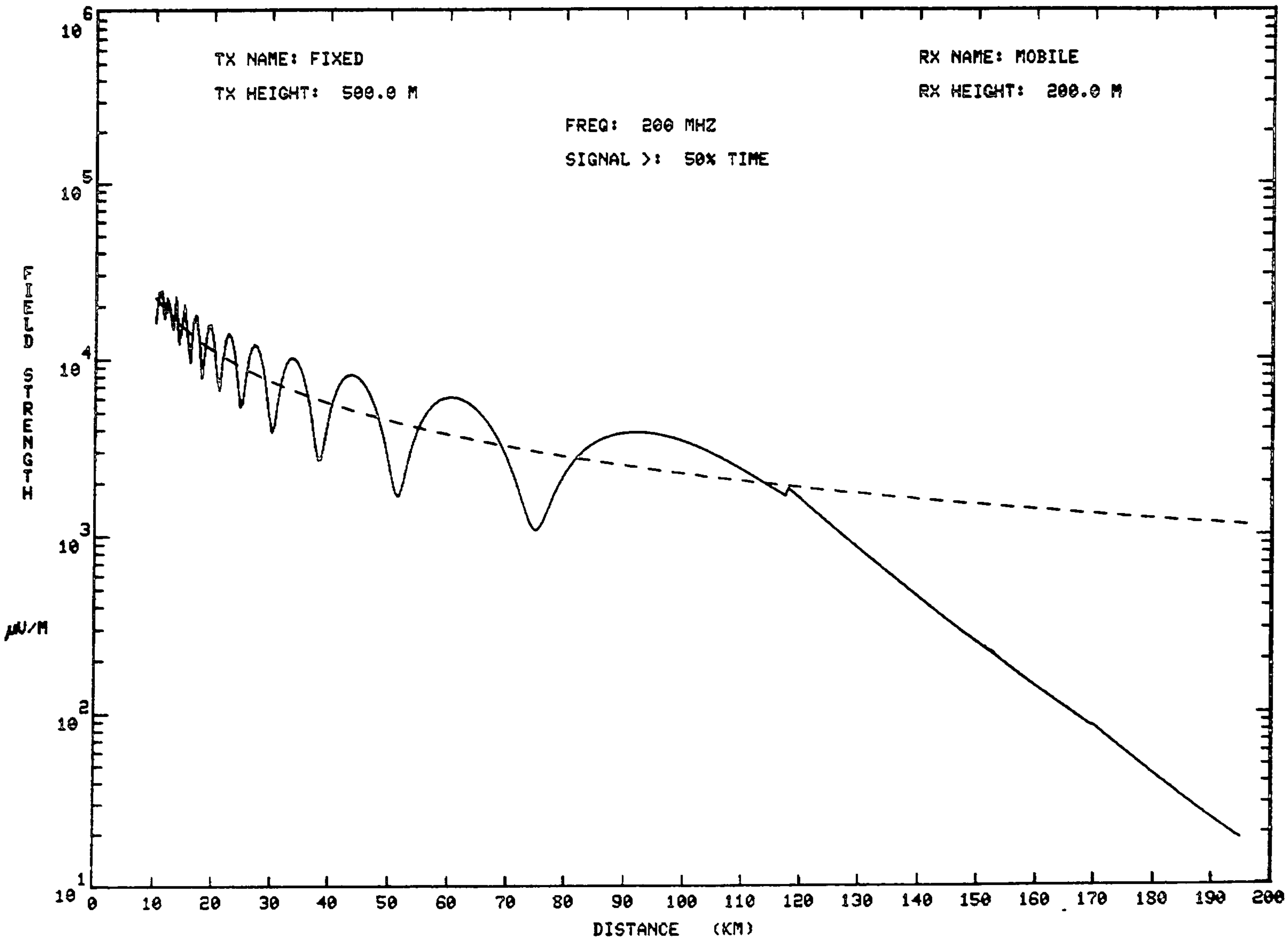

Figure 9. Results from the CRC prediction program, as plotted by the program, for the same conditions as in Figure 8, with $h_2 = 200$ m. In this case, the reflection calculation is used for distances up to 118 km, at which point the diffraction calculation takes over.

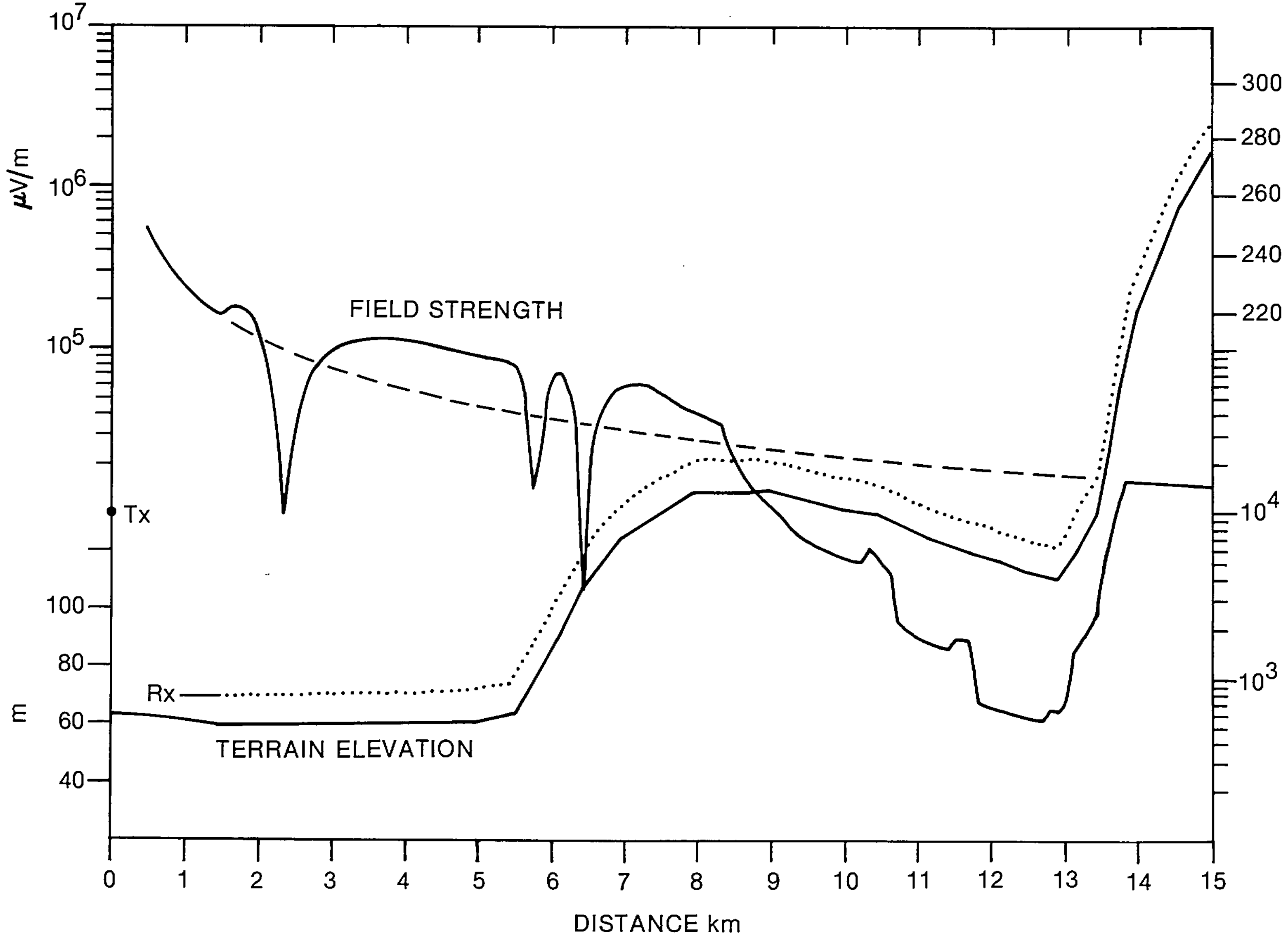

Figure 10. A terain profile and the field strength along it, as calculated by the CRC prediction program (solid lines). An antenna transmitting at 500 MHz is on a 64-metre tower, and the receiving antenna is 10 m above the terrain at each point (dotted line). The free-space field strength is represented by a broken line.

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APPENDIX A

The Reflection Point and Reflection Zone

A1. SOLVING FOR THE REFLECTION POINT

A rapid way of solving equation 5 is Newton's method. In general, to find the solution of $y(x) = 0$ by Newton's method, the n+1'th approximation is found from the n'th approximation by

$$x_{(n+1)} = x_{(n)} - \frac{y\left(x_{(n)}\right)}{\frac{dy}{dx}\left(x_{(n)}\right)} \tag{A1}$$

To solve equation 5 this way, we require the derivative

$$\frac{d(\Delta\psi)}{dx_1} = \frac{h_2}{x_2^{\,2}} + \frac{h_1}{x_1^{\,2}} + \frac{1}{a} \tag{A2}$$

Before relying on Newton's method, it is worthwhile to verify that it will converge to a unique solution. This will be so if the function to be solved has the form illustrated in Figure A1(a) on the domain of interest ($0 < x_1 < d$). That the function given in equation 5 does indeed have this form may be seen by noting (1) that the first derivative is positive for all x_1 between 0 and d, provided that h_1 and h_2 are positive, i.e. provided that both antennas are above the reflecting surface, and (2) that the second derivative,

$$\frac{d^2(\Delta\psi)}{dx_1^{\,2}} = \frac{2h_2}{x_2^{\,3}} - \frac{2h_1}{x_1^{\,3}} \tag{A3}$$

is negative for small x_1, positive for small x_2 (x_1 close to d), and changes sign exactly once in the range 0 to d.

A convenient first trial value for x_1 is

$$x_{1(1)} = \frac{h_1\, d}{h_1 + h_2} \tag{A4}$$

which is the reflection point for a flat reflector.

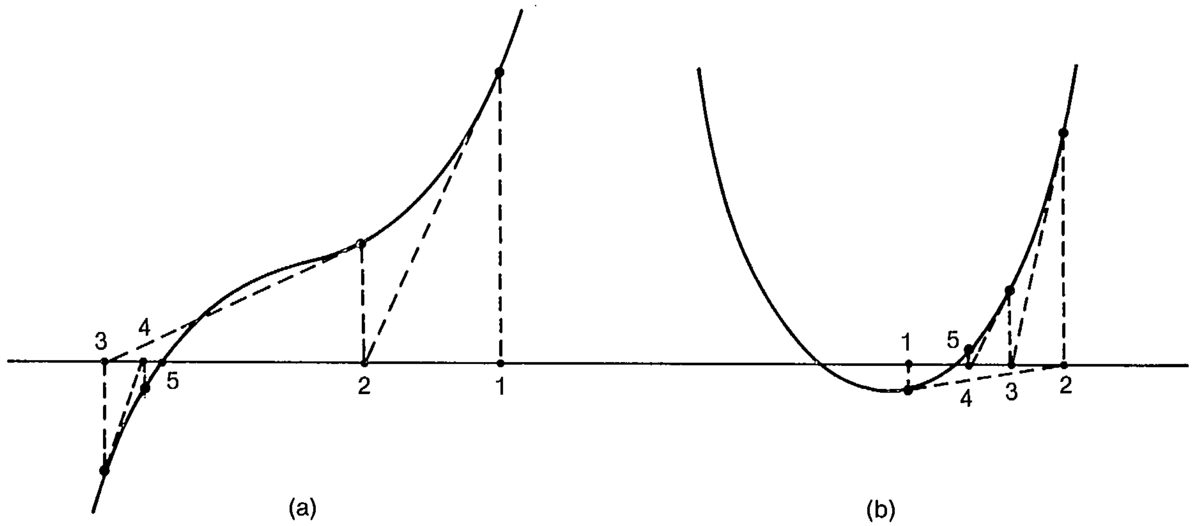

Figure A1. The form of two types of function whose roots can be found by the use of Newton's method. The horizontal line represents zero. Trial values used in Newton's method are numbered consecutively.

A2. PATH DIFFERENCE, DIRECT AND REFLECTED RAYS

We require the path difference between the direct ray and a ray reflected from a horizontal surface. We want this difference not only for the vector addition of the two waves at the end of the calculation, but also in order to find Fresnel-zone or reflection-zone boundaries. With this second purpose in mind, we allow the angles of incidence and reflection to be different.

Refer again to Figure 5. Consider triangle $PA_1\ A_2$. The angle at P is $\pi - \psi_1 - \psi_2$. We can find r in terms of r_1 and r_2 and the angle at P:

$$\begin{aligned} r^2 &= r_1{}^2 + r_2{}^2 - 2r_1r_2 \cos\left(\pi - \psi_2 - \psi_2\right) \\ &= r_1{}^2 + r_2{}^2 + 2r_1r_2 \cos\left(\psi_2 + \psi_2\right) \end{aligned} \qquad \text{(A5)}$$

But assuming small angles, we can represent $\cos(\psi_1 + \psi_2)$ as $1-(\psi_1 + \psi_2)^2/2$, which, with a little re-arrangement, leads to

$$r^2 = \left(r_1 + r_2\right)^2 \left[1 - \frac{r_1r_2\left(\psi_1 + \psi_2\right)^2}{\left(r_1 + r_2\right)^2}\right]$$

Again assuming $\psi_1 + \psi_2$ to be small, we can extract the square root:

$$r = r_1 + r_2 - \frac{1}{2}\,\frac{r_1r_2\left(\psi_1 + \psi_2\right)^2}{r_1 + r_2}$$

In the last term, we may replace r_1 by x_1 and r_2 by x_2. Therefore the difference between the indirect and direct paths is

$$\Delta r = r_1 + r_2 - r = \frac{x_1 \; x_2}{2d} \left(\psi_2 + \psi_2\right)^2 \tag{A6}$$

Suppose, as a special case, that P is the geometrical point of reflection. Then $\psi_1 = \psi_2$, and $(\psi_1 + \psi_2)^2 = 4\psi_1 \; \psi_2$. Then recalling also (equation 4) that $x_1 \; \psi_1 = h_1'$ and $x_2 \; \psi_2 = h_2'$, the path difference between the reflected and direct rays is

$$\Delta r_o = \frac{2h_1' \; h_2'}{d} \tag{A7}$$

where h_1' and h_2' are given in equation 3. This is equation 5.9 of TN101 and equation 4.6 in Hall. Δr_o is the path difference between the direct ray and the specularly reflected ray.

A3. FRESNEL-ZONE AND REFLECTION-ZONE BOUNDARIES

TN101 (page III-5) gives closed-form expressions for the Fresnel-zone boundaries. However, they are not quite correct, since their derivation assumes that reflection is from a plane tangent to the earth's surface, rather from the spherical earth itself.

Reflection-zone boundaries for a spherical reflector may be found in the following way: At the edge of the reflection zone, as defined in this Note, the excess path length is

$$\Delta r - \Delta r_o = 0.3 \; \lambda$$

(or 0.5 λ at the edge of the first Fresnel zone). Therefore we want to find the roots of the equation

$$\Delta r - \Delta r_o - 0.3 \; \lambda = 0 \tag{A8}$$

As shown below, we may solve equation A8 for x_1 (and for x_2) by using Newton's method. The two roots thus found are the desired boundaries, illustrated in Figure A2.

A3.1 Convergence of Newton's Method for Reflection-Zone Boundaries

If the function on the left side of equation A8 has the form shown in Figure A1(b), there are two roots, and, as the diagram illustrates, Newton's method will always converge to the root on the same side of the minimum as the initial estimate.

It remains to show that the function $\Delta r - \Delta r_o - 0.3 \; \lambda$ does in fact have the form shown in Figure A1(b). We are going to need equations 4(a,b),

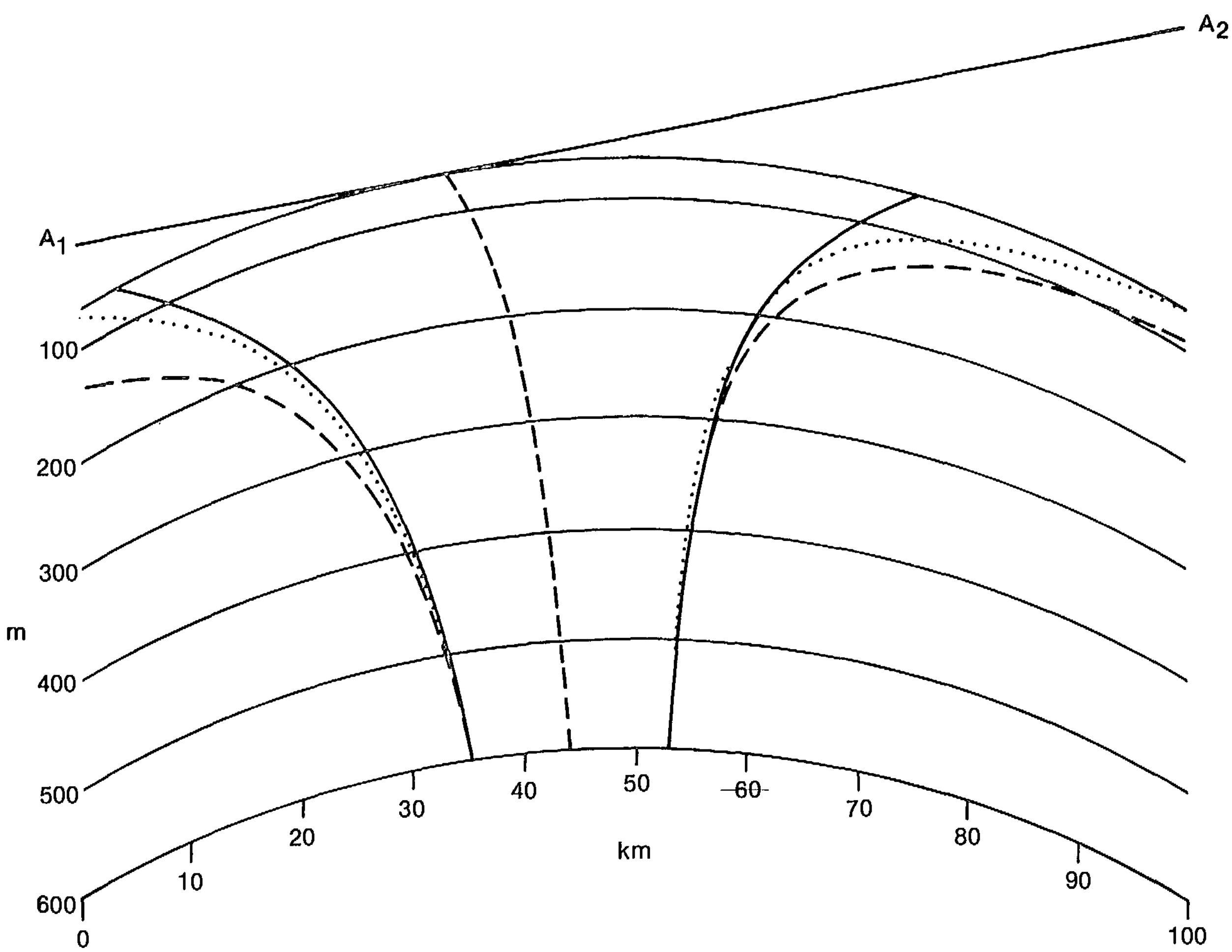

Figure A2. The ($\delta r/\lambda = 0.3$) reflection zone for $\lambda = 1$ m. Also the first Fresnel zone for $\lambda = 0.6$ m. Horizontal and vertical separations of the antennas are the same as in Figure 2. The broken line in the centre is the locus of reflection points, as in Figure 2. The solid lines are the reflection-zone boundaries, solutions of equation A8. The broken lines outside of these are from equation A14. The dotted lines are from equation A15.

and two equations that follow from them:

$$\psi_1 + \psi_2 = \frac{h_1}{x_1} + \frac{h_2}{x_2} - \frac{d}{2a} \tag{A9}$$

$$\frac{d}{dx_1}\left(\psi_2 + \psi_2\right) = \frac{\psi_2}{x_2} - \frac{\psi_2}{x_1} \tag{A10}$$

Now we can differentiate Δr (equation A6) to obtain

$$\frac{d\Delta r}{dx_1} = \frac{x_2 - x_1}{2d}\left(\psi_1 + \psi_2\right)^2 + \frac{x_1\, x_2}{d}\left(\psi_1 + \psi_2\right)\left(\frac{\psi_2}{x_2} - \frac{\psi_1}{x_1}\right)$$

Factoring out $(\psi_1 + \psi_2)$, collecting terms in the other factor, and remembering that $x_1 + x_2 = d$, we arrive at

$$\frac{d\Delta r}{dx_1} = \frac{1}{2}\left(\psi_2^{\,2} - \psi_1^{\,2}\right) \tag{A11}$$

This first derivative is required in the application of Newton's method, as well as in the proof of its suitability. Note that it is negative on the left side of the reflection point, and positive on the right side. This is the behavior illustrated in Figure A1(b).

In order to investigate the behavior of the curvature, we differentiate again, to obtain

$$\frac{d^2\Delta r}{dx_1^{\,2}} = \psi_1\left(\frac{h_1}{x_1^{\,2}} + \frac{1}{2a}\right) + \psi_2\left(\frac{h_2}{x_2^{\,2}} + \frac{1}{2a}\right) \tag{A12}$$

Since the angles ψ_1 and ψ_2 can be negative for some values of x_1, depending on the path length, it is not yet obvious whether the second derivative is always positive. Equations 4(a,b) can be used to express it in terms of x_1 and x_2.

$$\frac{d^2\Delta r}{dx_1^{\,2}} = \left(\frac{h_1^{\,2}}{x_1^{\,3}} + \frac{h_2^{\,2}}{x_2^{\,3}}\right) - \frac{d}{4a^2}$$

By further differentiation, it may be discovered that this derivative is a minimum when $x_2 = \sqrt{h_2/h_1}\; x_1$. Remembering that $x_1 + x_2 = d$, x_1 and x_2 may be found in terms of h_1, h_2 and d, leading to

$$\frac{d^2\Delta r}{dx_1^{\,2}} \geq \frac{1}{d^3}\left(\sqrt{h_1} + \sqrt{h_2}\right)^4 - \frac{d}{4a^2}$$

Now we must put some limit on the distance between the antennas. If we put $h_1' = 0$ in equation 3, we see that the horizon distance as seen by antenna A_1 is $\sqrt{2ah_1}$, and similarly for antenna A_2. On any line-of-sight path, the sum of the horizon distances must exceed the total path length d, and we must have

$$\sqrt{h_1} + \sqrt{h_2} > \frac{d}{\sqrt{2a}}$$

With this inequality and the one above, it finally becomes clear that the second derivative is everywhere positive. This completes the proof that Newton's method will converge to find the reflection-zone boundaries.

A3.2 Initial Estimate for Reflection-Zone Boundary

There is another use for the second derivative, namely to construct an initial estimate for a reflection-zone boundary. If Δr is expanded in a Taylor series about the reflection point at d_1, and if we ignore terms of order higher than 2, we have

$$\Delta r = \Delta r_o + a_2 \left(x_1 - d_1\right)^2$$

where

$$a_2 = \frac{1}{2} \frac{d^2 \Delta r}{dx_1^{\,2}} \Bigg|_{x_1 = d_1}$$

At the reflection point, where $x_1 = d_1$, the angles ψ_1 and ψ_2 are equal, and from equation A12 we obtain

$$\frac{d^2 \Delta r}{dx_1^{\,2}} \Bigg|_{x_1 = d_1} = \psi_1 \left(\frac{h_1}{d_1^{\,2}} + \frac{h_2}{d_2^{\,2}} + \frac{1}{a}\right) \tag{A13}$$

Then if we specify $\Delta r - \Delta r_o = \delta r$, where δr might be 0.3 λ for example, the initial estimates of the reflection-zone boundaries may be taken to be

$$x_{1(1)} = d_1 \pm \sqrt{\frac{\delta r}{a_2}} \tag{A14}$$

This approximation to the reflection-zone boundary is shown in Figure A2. It is very close to the exact value when the line-of-sight between the antennas is well clear of the earth, but diverges from it for grazing incidence. This need not cause any trouble, since the starting value for Newton's method can simply be constrained to lie between the end points of the path, and the method will find the correct solution anyway.

Nevertheless, the starting value of x_1 may be improved by an arbitrary modification to equation A14, in which the two values of $x_{1(1)}$ become:

$$x_{1(1)} = d_1 - \sqrt{\frac{\delta r}{a_2 + \frac{\delta r}{d_1^{\,2}}}}$$

$$x_{1(1)} = d_1 + \sqrt{\frac{\delta r}{a_2 + \frac{\delta r}{d_2^{\,2}}}} \tag{A15}$$

These modified values are also shown in Figure A2.

APPENDIX B

Focal Properties of a Convex Mirror

B1. FOCAL LENGTH

The focal length depends on the radius of curvature 'a' of the mirror and on the angle of incidence. Consider Figure B1. Rays incident normally (downward in the diagram) diverge after reflection because the direction of the tangent to the surface is different for the two rays. From elementary optics, the focal length of the mirror in this case is a/2. Rays incident obliquely at the same points diverge by the same relative angle, because the surface tangents are the same as for the normal rays. However, for oblique incidence, the perpendicular distance between the rays is smaller than for normal incidence, by a factor of sin ψ , where ψ is defined in the diagram. Therefore, if the reflected rays are projected backward, they meet at a point which is closer by the same factor. Therefore the focal length of the mirror for oblique incidence is

$$f = -\frac{a}{2}\sin\psi \tag{B1}$$

The sign is negative because the image is a virtual one. From the lens formula, the distance d_1' of the (virtual) image of antenna A_1 from the point of reflection is given by

$$\frac{1}{d_1'} = \frac{1}{d_1} + \frac{2}{a\psi} \tag{B2}$$

where sin ψ has been approximated by ψ.

To see how the image distance is used, refer back to Figure 7. The virtual path for finding the diffraction attenuation between P and A_2 is the line A_1' to A_2. Its length is $d_1' + d_2$. Similarly, the other virtual path is the line A_1 to A_2'.

B2. IMAGE HEIGHTS

There remains one more detail to take care of, namely to find the height of the images in a rectilinear coordinate system in which height is zero on the reflection surface at the ends of the path. (The coordinate system used in the prediction program is one easy step removed from this one; there, the height is zero at sea level at the ends of the path.) For this, we require the angle α between the tangent plane at the point of reflection and the base line of the coordinate system. Refer to Figure B2. This angle may be obtained from the slope of the reflecting surface at distance x from

A_1, which is $-(x - d/2) / a$ (See Section 2.2). Since, at the point of reflection, $x = d_1$,

$$\alpha = \frac{d_2 - d_1}{2a} \tag{B3}$$

The height of A_1' may then be found by projecting backwards along the reflected ray:

$$h_1'' = h_2 - (\psi + \alpha)\left(d_2 + d_1'\right) \tag{B4}$$

Similarly, the height of A_2' is

$$h_2'' = h_1 - (\psi - \alpha)\left(d_1 + d_2'\right) \tag{B5}$$

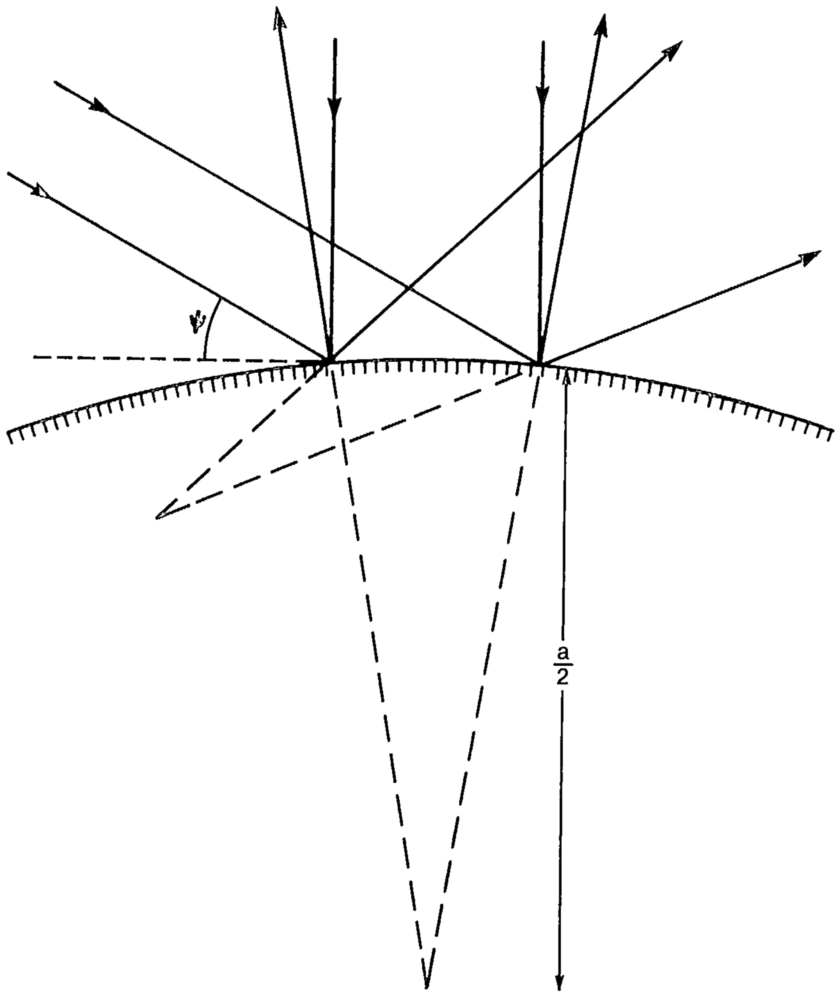

Figure B1. The focal length of a convex mirror having radius of curvature 'a'. Rays are shown for vertical and oblique incidence, and in both cases, the reflected rays are projected backward to meet at a virtual focal point.

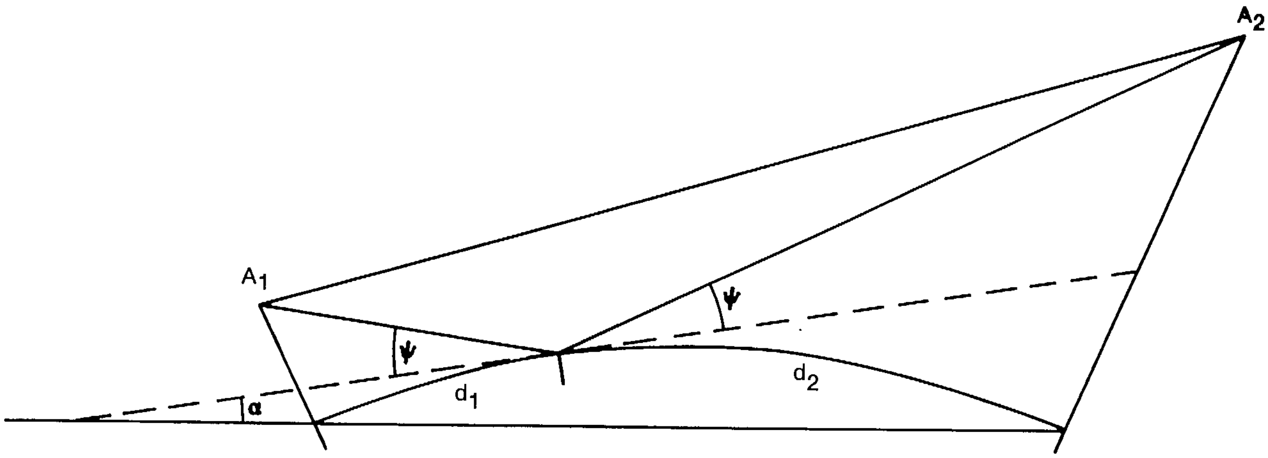

Figure B2. The relationship between the coordinates used for finding the reflection point, and those used in describing the terrain profile. The required parameter is the angle α between the plane tangent at the reflection point and the line joining two points at the same elevation at the ends of the path.

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The field strength of radio waves at VHF and UHF can be affected by reflections from the ground or water. Except for the shortest paths, the reflecting surface must be considered to be spherical rather than flat. This report describes procedures that may be programmed into a computer for automatically finding the reflection point, for calculating the effective coefficient of reflection, and for estimating the effect of any obstructions in the path of the reflected ray. The description is intended to be complete enough to be used for writing a computer program to do the calculations. New formulas are derived, and references are given for formulas obtained elsewhere. A comparison of the results with CCIR curves for propagation around a spherical earth shows agreement within about 1 dB in the region for which reflection calculations are used.

9. **CITATION:**

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