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THE TELEPHONE DEMAND MODEL FOR BELL CANADA: PRELIMINARY RESULTS

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1. Introduction

The purpose of this paper is to explicate the demand for the telephone services provided by Bell Canada. We formulate econometric models which examine the determinants of disaggregated (or component) demand and aggregated (or total) demand. These components are accordingly defined as local, toll, local plus toll, and local plus toll plus directory.

The study of demand behavior for telephone services is an important undertaking, not only for its own sake, but also for the analysis of the complete telephone industry. Indeed, demand systems already exist depicting the Canadian telephone industry, in general, (see R. Dobell et.al. [2] and L. Waverman [7]). Moreover, other important works have focused on particular demand aspects, as in, V. Corbo [1] and I.I.Q.E. [4]. Our immediate interest is in the general structural form concerning the telephone demand relations.

The demand formulations, that one usually encounters in connection with Bell Canada or with the total industry (as in Dobell [2] and Waverman [7]), is some variant of the H.S. Houthakker and L.D. Taylor [3] model. However, not only is this model rather ad-hoc in nature, it generally does not perform very well according to both economic and statistical criteria. Because of these considerations we have elected to postulate alternative hypotheses which have a firmer economic foundation and will produce meaningful results.

Three different theoretical specifications were selected for each category of telephone services. The first (and simplest) is referred to as the "linear demand model". In this framework the quantity demanded (telephone services) is expressed as a linear function of real income (which is defined as the gross products of Ontario and Quebec deflated by the price index of this gross product), and relative prices (the price index of a particular service deflated by the price index of the provinces' gross product). The 'a priori' results, based on economic theory, delimit that with an increase in real income (relative prices fixed) there will be an increase in the quantity demanded. On the other hand, we expect that with an increase in the relative prices (real income fixed) that quantity demanded will decrease. Thus we feel that telephone services are superior commodities (there are positive income effects and negative price effects).

The second structural form we consider is the "double-log model". In this case the identical variables are used as in the linear form, except now the natural logarithm of the variables are the regressors and regressand. The difference between the linear and double-log models originates from the implicit assumption concerning the nature of the demanders' objective function with respect to Bell Canada's telephone services.

Finally, the third enumeration is the "Rotterdam model" (see H. Theil [6]). This model incorporates the differences (in logarithmic values) of income, prices and the quantity demanded. In the following sections we will observe that the Rotterdam model, which has never before been adapted to the Canadian telephone industry, clearly yields the best results, both, in economic and statistical terms.

The remainder of this paper will develop the three theoretical relations, in section 2, and the empirical implementation, in section 3.

2. The Theoretical Models

2.1 The Linear Demand Model

Individual demand behavior, according to economic theory, suggests that given the objectives of demanders (preferences for individuals and generally profits for firms), the quantity demanded in period t (x_t) is a function of real income (y_t) and relative prices (p_t).

Hence,

$$x_t = h(p_t, y_t) \quad , \qquad (1)$$

where h is the demand function. Economic theory does not restrict the form of the demand function (h), although it does impose restrictions on the pattern of price and income effects in systems of demand behavior. For example because we use the price of telephone services and gross product, divided by the price index of the gross product, then we are incorporating the restriction that the demand functions must be homogeneous of degree zero in prices and income. This means that proportional increases in the price of the services, gross product and its price index will leave the quantity demanded unaltered. For the empirical applications of equation (1), it is necessary to specialize the general form of the demand equation and to account for stochastic phenomena.

In this section, we assume h is linear so that (1) becomes,

$$x_t = \beta_0 + \beta_1 p_t + \beta_2 y_t + e_t \qquad (2)$$

where $\beta_1<0$, $\beta_2>0$ and e_t represents the disturbance that can occur because h may not be strictly linear, measurement errors, etc.

2.2 The Double-Log Demand Model

In the double log model we begin with the general demand equation (1) but instead of assuming h is linear we assume that it is multiplicative,

$$x_t = \alpha_0 p_t^{\beta_1} y_t^{\beta_2} u_t \;. \qquad (3)$$

Equation (3), upon taking logarithms becomes,

$$\log x_t = \beta_0 + \beta_1 \log p_t + \beta_2 \log y_t + e_t \;, \qquad (4)$$

where $\beta_0 = \log \alpha_0$ and $e_t = \log u_t$. Manifestly, the values of β_0, β_1, β_2 in equation (4) will be different from those found in (2), nevertheless the signs of the coefficients should be identical, i.e. $\beta_1<0$, $\beta_2>0$.

2.3 The Rotterdam Demand Model

The Rotterdam model, as applied to the demand for telephone services, imposes a more complicated restriction upon equation (1). The Rotterdam formulation is given by,

$$x_t = \left[\alpha_0 \left(\frac{p_t}{p_{t-1}}\right)^{\beta_1} \left(\frac{y_t}{y_{t-1}}\right)^{\beta_2} u_t\right]^{\frac{1}{\alpha_t}} x_{t-1} \;. \qquad (5)$$

Taking logs of (5) and rearranging we find,

$$\alpha_t(\log x_t - \log x_{t-1}) = \beta_0 + \beta_1(\log p_t - \log p_{t-1}) + \beta_2 (\log y_t - \log y_{t-1}) + e_t \;, \qquad (6)$$

where $\alpha_t = \frac{a_t + a_{t+1}}{2}$ and a_t is the proportion of the value of demand for a particular service to the total demand. Clearly $\alpha_t = 1$ for aggregate demand. Equation (6) incorporates, not only logs, but the proportional change in the variables from period to period along with a weighted dependent variable.

We are now in a position to estimate equations (2), (4) and (6).

3. The Empirical Results

3.1 The Linear Demand Model

In order to estimate equation (2) one must find the appropriate empirical definitions of quantity demanded, relative prices and real income. For the demand variable, we selected the appropriate revenue and deflated it by the price index for that revenue category. In all cases we excluded other and miscellaneous revenues on the grounds that they were essentially random elements which do not reflect output. However, we did included uncollectibles in the revenue categories because these uncollectibles do reflect output, even though Bell was not, up to a particular point in time, fully paid for their services.

The relative price variable is comprised of the price index of the particular revenue category divided by the price index of the gross product of Ontario and Quebec. Finally the real income variable was defined by the gross product of Ontario and Quebec deflated by its price index.

Table 1 presents the results for the linear demand model using the ordinary least squares estimation method.

From table 1 we can observe that the linear model does not appear to be an adequate representation of the structure.

Table 1

Linear Demand Models: O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	-178.135 (-1.226)	26.126 (.278)	.0126 (10.777)	.630	.993
Toll	-251.932 (-1.820)	72.404 (.945)	.009 (6.610)	.562	.984
Local + Toll	-232.204 (-.795)	-16.732 (-.093)	.020 (7.891)	.570	.992
Local + Toll + Directory	-197.141 (-.690)	-40.736 (-.226)	.020 (8.519)	.571	.992
Total	-92.393 (-.268)	-143.141 (-.656)	.021 (7.549)	.5184	.991

Table 2

Linear Demand Models: C-O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	16.154 (.114)	-96.951 (-1.065)	.011 (8.516)	1.275	.996
Toll	-89.085 (-.833)	-34.211 (-.577)	.008 (7.071)	1.90	.994
Local + Toll	-59.948 (-.226)	-138.143 (-.853)	.019 (7.767)	1.515	.996
Local + Toll + Directory	-18.354 (-.066)	-165.279 (-.958)	.019 (7.644)	1.445	.996
Total	-19.802 (-.060)	-211.814 (-1.034)	.021 (7.243)	1.436	.996

We find that for all categories relative prices are insignificant and indeed for the local and toll components the price coefficient (β_1) has the wrong sign. The real income variable is significant and indeed the coefficient β_2 is positive. Moreover, this model suffers from positive autocorrelation reflected by the D.W. statistic being significantly lower than 2. To remedy the presence of autocorrelation we transformed the equation and the new results appear in table 2. We can see that although the D.W. statistic increased we still have serial correlation. Moreover, although the relative price variable has a negative impact on the quantity demanded, it is still not significant. We conclude then that the linear model is just too simple a structure to capture the essence of the demand for telephone services.

3.2 The Double-Log Model

In this case we utilized the same definition of the variables as in the linear model but now before estimating the equation we take the logs of the variables.

The results for the estimation of equation (4) using ordinary least squares are presented in table 3. As we can observe from table 3 there is a high degree of positive auto-correlation because of the low D.W. In addition whenever the price variable has the right sign (negative) it is an insignificant variable and whenever it is significant relative prices enter the equation with the wrong sign (positive). Again, as in the linear model, the income term has a positive effect and is

Table 3

Double-Log Demand Model: O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	-13.085 (-11.318)	.6687 (2.556)	1.784 (16.443)	.901	.991
Toll	-14.220 (-11.234)	-.067 (-.305)	1.827 (15.500)	1.479	.997
Local + Toll	-13.365 (-11.242)	.499 (2.017)	1.848 (16.620)	1.111	.994
Local + Toll + Directory	-13.142 (-11.031)	.543 (2.090)	1.832 (16.432)	1.011	.992
Total	-13.520 (-11.966)	.469 (1.887)	1.875 (17.729)	1.125	.994

significant. The obvious question to ask concerns the nature of the changes in our results when we adjust for autocorrelation. These results are presented in table 4. From table 4 we find an enormous improvement in the estimated structure. The income variable is significant and has a positive influence, while the price variable has a negative effect and is significant. Although, we must observe that, except for the toll category, there still is the presence of autocorrelation. In any event table 4 shows us that we are on the right track toward capturing the correct structural form.

3.3 The Rotterdam Demand Model

Finally we come to the most complex case; that is, we estimate demand behavior which is governed by equation (6). In this, the dependent variable is a composite of the present quantity demanded and the one-period lagged quantity demanded. As we stated earlier, the fact that equations differ with respect to the nature of the manner in which the variables enter the equation does not alter our 'a priori' expectations regarding the signs of the coefficients.

Table 5 presents the results for the Rotterdam model using ordinary least squares. We can observe that the signs of the income and price coefficients in all of the demand categories have the expected sign. The problem, once again, is that the equations suffer from serious autocorrelation, indeed the toll

Table 4

Double-Log Demand Model: C-O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	1.421 (.986)	-.556 (-3.320)	.460 (3.500)	.817	.999
Toll	-9.600 (-4.686)	-.740 (-2.322)	1.397 (7.313)	2.057	.997
Local + Toll	.962 (.570)	-.718 (-3.563)	.540 (3.508)	1.381	.999
Local + Toll + Directory	1.211 (.718)	-.749 (-3.615)	.518 (3.343)	1.264	.999
Total	1.484 (.869)	-.821 (-3.957)	.505 (3.245)	1.263	.999

Table 5

Rotterdam Demand Model: O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	.043 (9.061)	-.014 (-.154)	.069 (.813)	.696	.041
Toll	.018 (4.217)	-.163 (-2.300)	.122 (1.519)	2.462	.374
Local + Toll	.060 (8.136)	-.188 (-1.226)	.203 (1.503)	1.549	.214
Local + Toll + Directory	.066 (8.090)	-.177 (-1.052)	.154 (1.039)	1.383	.133
Total	.073 (9.802)	-.257 (-1.655)	.151 (1.133)	1.800	.216

component exhibits a high degree of negative autocorrelation. Therefore, although in some of the equations, given by table 5, the variables are marginally significant, we must first adjust for the autocorrelation and this will, among other things, tend to enable us to exact a clearer picture of the importance of the variables in explaining demand behavior.

The final table (table 6) shows the empirical magnitudes for the Rotterdam model after we have corrected for the auto-correlation. Now not only are the signs of the coefficients consistent with economic theory but the t-values show that indeed the price and income variables play the important role in determining demand.

We feel that the Rotterdam model as represented by equation (6) and the empirical results found in table 6 provide a very good explanation of the determinants of demand for Bell Canada's services.

Table 6

Rotterdam Demand Model: C-O.L.S.
(t-values in parentheses)

Output Category	β_0	β_1	β_2	D.W.	R^2
Local	.036 (5.218)	-.163 (-2.566)	.105 (2.292)	2.276	.575
Toll	.018 (4.468)	-.144 (-2.063)	.137 (1.592)	2.057	.420
Local + Toll	.059 (7.207)	-.246 (-1.432)	.207 (1.610)	1.965	.257
Local + Toll + Directory	.063 (6.785)	-.290 (-1.582)	.169 (1.281)	2.041	.300
Total	.072 (9.180)	-.293 (1.701)	.146 (1.059)	1.951	.226

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