A SIMULATION MODEL FOR A TELEPHONE COMPANY

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CHAPTER I

Introduction

This progress report deals with the work done since October 1st on the Simulation Model of Bell Canada. We report here the final results obtained for the estimation of demand equations (Chapter II) and production function (Chapter III). We also introduce some material on the preliminary testing of the appropriate model to describe the behavior of Bell Canada. Thus, in Chapter IV, we use a simultaneous equation model which is then used to estimate simultaneously the production function and the side conditions for labor hiring. The results from this joint estimation yield parameters of the production function almost identical to the ones obtained from direct estimations of the production function.

Also during this period, we have developed and implemented a non-linear simulation package to be used for the simulation of the model.

We have also experimented quite extensively with the estimation of a multiple output production function to be used in the multiple output complete model of Bell Canada. We still do not have final results on the appropriate multiple output function parallel to the estimation of the simultaneous model of Bell Canada.

We have been studying the analytics of regulation in the context of a multiple output model. In this model, local services are provided on demand, toll services and other services are decision variables for the company.

CHAPTER II

DEMAND FOR TELEPHONE SERVICES: LOCAL, TOLL AND OTHER SERVICES

2.1 Introduction:

In this chapter we re-estimate the demand equations of the previous year project. There are four main differences between the present estimates and the one from last year's project.

First, the sample period has been extended to 1976 and actual instead of extrapolated values have been used for the years 1973-1974-1975. This was made possible by the new information made available through the CRTC Exhibit of January 1977 as well as by direct information provided by Bell to DOC.

Second, we consider an alternative model which was not used last time, that is the "habit formation model", which has been used successfully for services which have a habit formation element in the past.

Third, we allow for contemporaneous correlation in the disturbances across equations and estimate all equations simultaneously using Zellner's seemingly unrelated procedure.

Fourth, we re-define the quantity and price variables by using Divisia quantity and price indexes.

In the estimation of demand equations, two alternative specifications are considered: 1) a double log equation and 2) a "habit formation" equation.

2.2 The Models

In the double log formulation the demand equation is given by:

$$(1)\quad \ln SO_{it} = \alpha_0 + \alpha_1 \ln \frac{P_{1t}}{PD_t} + \alpha_2 \ln \frac{P_{2t}}{PD_t} + \alpha_3 \ln \frac{P_{3t}}{PD_t} + \alpha_4 \ln YD_t$$

where SO_{it} is the quantity demanded of the service i (1 local, 2 toll, 3 other services) in period t, P_{it} is the price of service i in period t, PD_t is a price deflator for period t and YD_t is real income.

The second model used is of the habit formation type, and is based on the assumption that the demand for a type of telephone service is a function of income, prices and a state variable S_t proportional to last period's demand, and representing the stock of accumulated telephone habits.

Thus, the "habit formation" model is given by the following pair of equations:

$$\ln SO_{it} = \beta_o + \beta_1 \ln \frac{P_{1t}}{PD_t} + \beta_2 \ln \frac{P_{2t}}{PD_t} + \beta_3 \ln \frac{P_{3t}}{PD_t} + \beta_4 \ln YD_t + \beta_5 \ln S_t$$

$$\ln S_t = \Theta \ln SO_{it-1}$$

Replacing the second equation in the first we obtain:

$$(2) \quad \ln SO_{it} = \beta_o + \beta_1 \ln \frac{P_{1t}}{PD_t} + \beta_2 \ln \frac{P_{2t}}{PD_t} + \beta_3 \ln \frac{P_{3t}}{PD_t} + \beta_4 \ln YD_t + \beta_5 \Theta \ln SO_{it-1}$$

A priori, we expect $\beta_1 < 0$, $\beta_4 > 0$ and $\beta_5 \Theta > 0$. Due to the presence of a lagged endogenous variable on the right-hand side of this equation, ordinary least squares would yield inconsistent estimates if the disturbances of this equation are auto-correlated. We therefore, in our estimation, begin by assuming a first-order auto-regressive process for the disturbances, and use the maximum likelihood estimation procedure to estimate simultaneously the coefficient of the auto-regressive process and the coefficients of the equation.

2.3 The Data

Before proceeding to analyse the estimated results we will describe the data used.

a) Quantity Demanded

We work with three outputs: Local, Toll and Other Services. For local services the quantity demanded is measured as the revenue from these types of services at 1967 prices. In the case of toll services, the quantity demanded is measured as a divisia quantity index of the three types of toll services. That is, Intra-Bell Telephone Message Toll Service, Trans-Canada Telephone Message Toll Service and the Canada-US and Overseas Telephone Message Toll Service. Each of these services is measured as the revenue from each type of services (including uncollectables) at 1967 prices.

The other service category was measured as a divisia quantity index of Other Toll Revenues, Directory Adversiting and Other Miscellaneous Revenues. Again, each of these individual variables was measured as the revenue from each service at 1967 prices. The Toll and Other Services, divisia quantity indexes, were normalized to the 1967 dollar revenues from these services.* The source of information for the revenue figures was provided by Bell to DOC.

b) The Price of Each Telephone Service

For local services, the price index is taken directly from Bell data. For Toll services, the price index is defined as the ratio of the current dollar revenues from these types of services and the

* That is, the scale of the computed quantity indexes was defined in such a way that the value of these indexes for 1967 was equal to the dollar revenue from these services in 1967.

normalized divisia quantity index of this service. For other services, a procedure similar to the one for Toll services was used.

c) The Price Deflator

Bell Canada operates in the Quebec-Ontario regions, thus the price deflator used in our computation is an harmonic price index of the two provinces with weight given by the current dollar Gross Domestic Products of each of the two Provinces.

d) The Real Income Variable

The demand equations that we estimate are aggregated for Business and Household. This is caused by the inexistence, up to now, of disaggregated data on the public domain. Thus, the income variable that we require is a variable related to the overall level of economic activity in these two provinces. Indeed, for the income variable we used the sum of the Gross Provincial Products at 1967 prices of Quebec and Ontario. Where the deflators used were the consumer price indexes of Toronto and Montreal.

e) Other Exogenous Variables

We also study if there is a shift in the demand for toll services caused by advertising and/or Post Office strikes. For this purpose the following variables are defined:

(i) Advertising expenditure by Bell Canada divided the price deflator defined above.

(ii) Sum of the Advertising, Commercial, Marketing and Directory expenditures deflated by prices.

(iii) An index of strikes in the Post Office. This index is defined as the ratio of the man-hour striked in each year to the employment in man-hours for that year. Since the post office annual report provides employment information only about full-time and part-time employees, they are considered respectively as 250 and 125 days work per year, then multiplying the number of persons by days worked, we obtain the man days worked in a year.

2.4 The Empirical Results

We start by analysing the results of the double log model with and without correction for auto-correlation. Furthermore, we use two estimation procedures, estimation equation by equation and estimation by Zellner's seemingly unrelated regression procedure.

In Table 2.1 we present the results for the estimation without correction for auto-correlation and using only the own price variable in each equation. All the results from this table indicate strong auto-correlation in the disturbances. Therefore, the computed t-values are meaningless and no statistical inference can be based on computed value of R^2. Thus, we proceed to the results corrected for auto-correlation that appear in Table 2.2. From this table we observe that after correction for auto-correlation only the disturbances of the equation for local services are still auto-correlated. What is disturbing from these results is the very high value for the auto-regressive coefficient RHO which is close to one in all cases. This is an indication that something very systematic has been left out of our equations. Furthermore, the own price elasticity of Toll services is below one in absolute value, a result difficult to accept

In Tables 2.3 and 2.4 we allow for cross price effects by introducing the prices of the other two services in each equation. Now each equation has the same set of regressors, therefore, the estimation equation by equation and the estimation by Zellner's procedure yield the same results. Thus, only the results of the estimation equation by equation are presented here.

Again, as before, there is positive auto-correlation in the disturbances of the equations estimated by OLSQ (Table 2.3). Also, when a correction for auto-correlation is performed, the value of RHO is very close to one in all equations. Thus, again we conclude that some systematic variable has been left out.

In Table 2.5, we present the results for the Habit Formation model with correction for autocorrelation.

From the results of the Zellner's estimation procedure, we observe that RHO is significant only in the local service equation. Furthermore, the estimated value of RHO is only around .5. The long run price elasticities of local, toll and other services are -.809, -1.208 and -1.413 respectively. On the other hand, the long run income elasticities are .271, 1.017 and 8.772 respectively.

In Table 2.6, we estimate the habit formation model allowing for cross price elasticities. One major problem with these results is the high collinearity among the price variables. Thus, the estimated values of the own price elasticities become very unreliable. In particular, the own price elasticity of other services becomes positive and significant. Also the income elasticity of the demand for other services is negative and significant. Thus, we go back to the

"Habit Formation Model" without cross price elasticities. To complete our estimations we re-estimate the model of Table 2.5 introducing total advertising and a variable for Post-Office strikes in the demand equation for toll services. The variable for Post-Office strikes had a positive coefficient as expected but it was not significant. Of course, this could be due to a problem of time aggregation. The effect of Post-Office strikes in the demand for toll services could affect the monthly and/or quarterly demand equations but they do not show up in the annual demand equation. On the other hand, total advertising expenditures had a positive and significant effect on the demand for toll services.

The results that we obtain when total advertising expenditures are introduced in the demand for toll services of Table 2.5 appear in Table 2.7.

TABLE 2.1

DOUBLE LOG-DEMAND MODEL*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$		D.W.	R^2
a) OLSQ EQUATION BY EQUATION						
Local	-9.531 (-10.17)	.177 (0.74)	1.451 (16.44)		.5353	.9887
Toll	-10.193 (-11.74)	-.721 (-4.11)	1.453 (17.97)		.7563	.9964
Others	-16.898 (-19.26)	1.005 (2.57)	1.985 (23.88)		.9825	.9791
Total	-9.7289 (10.04)	-.0895 (-.37)	1.519 (16.72)		.5584	.9914
b) ZELLNER'S PROCEDURE						
Local	-10.405 (-29.47)	.4167 (5.05)	1.533 (46.08)		.6418	.9882
Toll	-11.055 (-34.41)	-.5396 (-8.78)	1.533 (51.14)		.7880	.9962
Others	-16.739 (-39.14)	.9076 (5.36)	1.970 (48.63)		.9503	.9790

* D.W. is the Durbin-Watson Statistic, R^2 is the multiple determination coefficient and the terms in parenthesis are t-values computed under the null hypothesis that the true value of the respective coefficient is zero.

TABLE 2.2

DOUBLE LOG MODEL: CORRECTED FOR AUTO-CORRELATION*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	RHO	D.W.	R^2
	a) MAXIMUM LIKELIHOOD EQUATION BY EQUATION					
Local	7.835 (4.32)	-.224 (-2.29)	0.131 (1.69)	0.981 (156.01)	1.194	.9997
Toll	43.778 (0.08)	-.537 (-2.89)	.509 (2.68)	.999 (62.33)	2.354	.9987
Others	9.271 (1.954)	-1.608 (-3.13)	-.321 (-0.87)	.944 (43.83)	2.516	.9960
Total	8.191 (3.21)	-.428 (-3.40)	.190 (1.95)	.983 (126.81)	2.166	.9996
	b) ZELLNER'S PROCEDURE					
Local	8.186 (8.34)	-.194 (-4.14)	.123 (3.03)	.982 (314.37)	1.161	.9997
Toll	7.814 (.69)	-.616 (-6.77)	.520 (5.16)	.993 (112.73)	2.383	.9986
Other	8.546 (3.50)	-1.697 (-6.54)	-.270 (-1.39)	.940 (80.31)	2.472	.9959

* See Note to Table 2.1

TABLE 2.3

DOUBLE LOG MODEL WITH CROSS PRICE ELASTICITIES*

	OLSQ EQUATION BY EQUATION							
	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$		D.W.	R^2
Local	-5.099 (-4.15)	-2.297 (-3.38)	.214 (0.58)	2.201 (4.67)	1.035 (9.03)		1.062	.9947
Toll	-7.658 (-7.92)	-1.954 (-3.65)	-.251 (-.86)	1.452 (3.91)	1.214 (13.44)		1.399	.9980
Others	-9.483 (-3.47)	-2.424 (-1.60)	-.523 (-.63)	3.415 (3.26)	1.293 (5.06)		1.258	.9851

* See Note to Table 2.1

TABLE 2.4

DOUBLE LOG MODEL WITH CROSS PRICE ELASTICITIES: CORRECTED FOR AUTO-CORRELATION*

	MAXIMUM LIKELIHOOD EQUATION BY EQUATION							
	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	RHO	D.W.	R^2
Local	7.838 (3.85)	-.307 (-1.84)	.078 (.75)	.021 (.12)	.139 (1.69)	.982 (140.87)	1.241	.9997
Toll	10.438 (.202)	-.328 (-.768)	.421 (-.591)	.139 (.307)	.545 (2.59)	.995 (46.28)	2.484	.9987
Others	9.179 (1.81)	.112 (.146)	.277 (.58)	-1.966 (-2.39)	-.297 (-.761)	.947 (43.71)	2.479	.9961

* See Note to Table 2.1

TABLE 2.5

HABIT FORMATION MODEL: CORRECTED FOR AUTO-CORRELATION*

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	D.W.	R^2
a) OLSQ EQUATION BY EQUATION							
Local	-.454 (-2.97)	-.119 (-3.58)	.017 (.192)	.866 (25.11)	.310 (1.59)	2.125	.9998
Toll	3.592 (-4.66)	-.579 (-4.66)	.518 (1.94)	.520 (4.94)	.160 (.747)	2.128	.9989
Others	-.879 (-.98)	-.166 (-1.45)	.822 (1.49)	.884 (12.02)	-.122 (-.488)	2.036	.9934
Total	-.652 (2.35)	-.156 (-2.94)	.153 (1.11)	.858 (16.82)	-.042 (-.304)	1.917	.9995
b) ZELLNER'S PROCEDURE							
Local	-.618 (-7.26)	-.158 (-8.18)	.048 (1.05)	.823 (39.13)	.478 (6.28)	2.177	.9998
Toll	-3.555 (-10.76)	-.569 (-10.87)	.479 (3.98)	.529 (11.88)	.024 (.243)	1.900	.9989
Others	-.585 (-1.23)	-.130 (-2.14)	.807 (2.79)	.908 (23.13)	-.031 (-.273)	2.234	.9933

* See Note to Table 2.1

TABLE 2.6

HABIT FORMATION MODEL WITH CROSS PRICE ELASTICITIES: CORRECTED FOR AUTO-CORRELATION*

a) OLSQ EQUATION BY EQUATION

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	DW	R^2
Local	-1.046 (-3.11)	-.239 (-1.61)	.070 (.97)	.124 (.98)	.207 (4.15)	.814 (19.53)	.277 (1.22)	2.057	.999[illegible]
Toll	-4.296 (-3.70)	-.263 (-.48)	-.442 (-2.04)	2.46 (.62)	.666 (4.14)	.482 (3.82)	.046 (.17)	2.053	.999[illegible]
Others	3.429 (1.924)	-.964 (-1.499)	-.528 (-1.646)	.982 (1.970)	-.271 (-1.359)	.900 (9.085)	-.498 (-2.106)	2.314	.9956

b) ZELLNER'S PROCEDURE

	Constant	$\ln \frac{P_{1t}}{PD_t}$	$\ln \frac{P_{2t}}{PD_t}$	$\ln \frac{P_{3t}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	RHO	DW	R^2
Local	-1.091 (-6.818)	-.229 (-3.408)	.082 (2.338)	.091 (1.559)	.231 (8.901)	.811 (39.90)	.347 (3.758)	2.145	.9998
Toll	-3.891 (-7.755)	-.289 (-1.235)	-.410 (-4.393)	.260 (1.587)	.609 (8.717)	.519 (9.367)	-.090 (-.843)	1.968	.9990
Others	3.359 (3.877)	-.959 (-3.068)	-.521 (-3.337)	.981 (4.045)	-.263 (-2.722)	.899 (18.633)	-.473 (-4.185)	2.341	.9958

* See Note to Table 2.1

TABLE 2.7

HABIT FORMATION MODEL: Corrected for Auto-correlation with Advertising Expenditures in Demand for Toll*

a) OLSQ EQUATION BY EQUATION

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	$\ln ADV_t$	RHO	DW	R^2
Local			SAME AS TABLE 2.5					
Toll	-3.587 (-4.59)	-5.778 (-4.60)	.461 (1.67)	.522 (4.93)	.056 (.618)	.108 (.462)	2.113	.9989
Others			SAME AS TABLE 2.5					
Total			SAME AS TABLE 2.5					

b) ZELLNER'S PROCEDURE

	Constant	$\ln \frac{P_{it}}{PD_t}$	$\ln YD_t$	$\ln SO_{it-1}$	$\ln ADV_t$	RHO	DW	R^2
Local	-.624 (-7.48)	-.160 (-8.48)	.076 (1.66)	.822 (40.14)		.450 (6.13)	2.122	.9998
Toll	-3.574 (-11.61)	-.577 (-11.80)	.465 (4.29)	.527 (12.78)	.098 (3.67)	-.139 (-1.37)	1.748	.9988
Others	-.885 (-1.71)	-1.65 (-2.52)	.783 (2.67)	.884 (20.62)		.019 (.164)	2.266	.9933

* See Note to Table 2.1

We analyse now the results of Table 2.7, for the estimation using Zellner's Procedure. First, we observe that only the demand for local services shows a significant auto-regressive coefficient (RHO) with respect to the long run own price elasticities. These are -.899 for Local Services; -1.219 for Toll Services and -1.422 for Other Services, hence, local services are price inelastic, a result that has definitive implications for the formulation of a simulation model of the carrier.[1] On the other hand, the long run income elasticities are .427 for Local Services; .983 for Toll Services and 6.75 for Other Services.

Finally, advertising expenditures have a positive and significant effect on the demand for toll services.

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1 See Chapter IV, below

CHAPTER III

A ONE OUTPUT PRODUCTION FUNCTION FOR BELL CANADA

In the study of the technology of Bell Canada, we choose to start with a general form of production function which can be considered as a production function by itself or as a second order approximation to any production function. Where the approximation is made about a point in which the logarithms of each of the inputs are made equal to zero. This form of production function is the Transcendental Logarithmic Function (translog)1/, it has the advantage to reduce to a Cobb-Douglas form as a special case.

1/ See Christensen, L.R., D.W. Jorgenson and L.J. Lau, Conjugate duality and the transcendental logarithmic production function (abstract), Econometrica, 39,4, 255-256, 1971.

Other references on translog production functions include:

a- E. Berndt and L. Christensen, The Translog function and the substitution of equipment, structures, and labour in U.S. manufacturing 1929-1968, Journal of Econometrics 1,1, 81-113.

b- Vittorio Corbo and Patricio Meller, The Translog production function: some evidence from establishment data. Mimeo, July 1977, see also:

c- Vittorio Corbo *et al*, Rate Adjustment Guidelines for Regulated Industries: A Model for Bell Canada, IAER, May 1976.

The unconstrained transcendental logarithmic translog production function for one output and three inputs with symmetry imposed ($\gamma_{sk} = \gamma_{ks}$), allowing for technical change, can be written as:

$$(1) \qquad \ln Q_t = \alpha_0 + \alpha_1 \ln L_t + \alpha_2 \ln M_t + \alpha_3 \ln K_t$$
$$+ 1/2\ \gamma_{11} (\ln L_t)^2 + \gamma_{12} (\ln L_t)(\ln M_t)$$
$$+ \gamma_{13} (\ln L_t)(\ln K_t) + 1/2\ \gamma_{22} (\ln M_t)^2$$
$$+ \gamma_{23} (\ln M_t)(\ln K_t) + 1/2\ \gamma_{33} (\ln K_t)^2$$
$$+ \beta_\circ D_t$$

where Q_t: is the total revenue minus indirect taxes in millions of 1967 dollars, constructed as a divisia quantity index of: Local Services, Intra-Bell. The variable was normalized to make the average equal to one.

L_t: Weighted man-hours where the weights are the relative hourly wage rate of the different labour categories in 1967. The variable is normalized as above.

M_t: Intermediate inputs ("raw materials" for short), measured as a divisia quantity index of Cost of materials, services, rent and supplies, uncollectables, plus indirect taxes, all of them in constant 1967 dollars. The variable is also normalized.

K_t: Net capital stock in millions of 1967 dollars, normalized as above.

D_t: Percentage of calls Direct Distance Dialed.

When we estimate the translog production function (1), correcting for auto-correlation, we obtain the following:

TABLE 1: UNCONSTRAINED ONE OUTPUT TRANSLOG PRODUCTION FUNCTION

	Estimated Coefficient	t- Statistic
α_0	-.1080	-.719
α_1	.3646	2.453
α_2	.1217	1.003
α_3	1.2071	4.056
γ_{11}	5.2367	1.086
γ_{22}	-.7157	-.237
γ_{33}	.7384	.402
γ_{12}	-2.6178	-.789
γ_{13}	1.5492	.632
γ_{23}	.4561	.198
β	-.0854	-.215
ρ	.5565	2.229
R^2	.9997	
DW	2.2920	
SSR	.00230830	

Next we test the hypothesis of constant returns to scale. Constant returns to scale (CRTS) imply the following restrictions on the parameters:

(i) $\sum_{k=1}^{3} \alpha_k = 1$

(ii) $\sum_{k=1}^{3} \gamma_{sk} = 0$, $s=1,2,3$

(iii) $\sum_{s=1}^{3} \gamma_{sk} = 0$, $k=1,2,3$

(iv) $\sum_{s=1}^{3} \sum_{k=1}^{3} \gamma_{sk} = 0$

With symmetry imposed *a priori*, restrictions (iii) and (iv) are not independent of (i) and (ii). Therefore, we test for constant returns to scale in model (1) by imposing constraints (i) and (ii) on the parameters.

The results of estimating equation (1) subject to restrictions (i) and (ii), follows:

TABLE 2: TRANSLOG PRODUCTION FUNCTION SUBJECT TO CRTS

	Estimated Coefficient	t-Statistic
α_0	-.4176	-9.513
α_1	.3288	3.029
α_2	.0156	.143
α_3	.6556	5.284
γ_{11}	.4466	1.463
γ_{22}	-2.6545	-1.194
γ_{33}	-1.2527	-.909
γ_{12}	.4775	.854
γ_{13}	-.9245	-2.024
γ_{23}	2.1767	1.260
β	.7385	6.979
ρ	.6775	3.800
R^2	.9995	
DW	2.0143	
SSR	.00365904	

In order to test for the existence of constant returns to scale, we perform an F-test.

We compute F-statistics using the sum of squares of the residuals (SSR) from Tables 2 and 3.

The F-statistic used can be written as:

$$F = \frac{\dfrac{SSR^{TLCRTS} - SSR^{TL}}{4}}{\dfrac{SSR^{TL}}{24-12}}$$

where SSR^{TLCRTS} = the sum of squares of the residuals for the translog subject to CRTS (from Table 2)

SSR^{TL} = the sum of squares of the residuals for the translog (unconstrained, from Table 1)

Our null hypothesis is the presence of CRTS. If the null hypothesis is true, the above statistic is distributed as F with 4 and 12 degrees of freedom.

The computed F value is 1.7631, the 5% $F\ (4,12)$ is 3.26, therefore, we cannot reject the hypothesis of constant returns to scale.

Having accepted CRTS, next we test for complete global separability, i.e. whether or not the function is of the Cobb-Doublas type, (that is, with $\gamma_{11} = \gamma_{22} = \gamma_{33} = \gamma_{12} = \gamma_{13} = \gamma_{23} = 0$).

The results of estimating a one-output Cobb-Douglas production function follows:

TABLE 3: COBB-DOUGLAS PRODUCTION FUNCTION WITH CRTS

	Estimated Coefficient	t-statistic
α_0	-.5029	-16.037
α_1	.5771	9.051
α_2	.0403	.401
α_3	.3826	5.261
β	.9889	15.285
ρ	.5121	2.845
R^2	.9993	
DW	1.7934	
SSR	.00563706	

Using the results of Table 3, now we run a test to see whether or not we accept the Cobb-Douglas technology. The test is

$$F = \frac{\dfrac{SSR^{CDCRTS} - SSR^{TLCRTS}}{6}}{\dfrac{SSR^{TLCRTS}}{24-8}}$$

The computed F is 1.4416 , the 5% F (6,16) is 2.74, therefore, we cannot reject the Cobb-Douglas specification of technology.

We observe a very low t-value for the estimated coefficient of "raw materials", α_2. The t-test for α_2 leads to the acceptance of the hypothesis that $\alpha_2=0$. Thus, "raw materials" do not belong in the equation.

Next, we estimate a two-input Cobb-Doublas production function with CRTS. The two inputs are labour (weighted man-hours) and net capital. The result is the following:

TABLE 4: TWO-INPUT COBB-DOUGLAS PRODUCTION FUNCTION WITH CRTS

	Estimated Coefficient	t-statistic
α_0	-.5058	-16.870
α_1	.5929	12.373
α_3	.4070	8.507
β	.9965	16.572
ρ	.5448	3.277
R^2	.9993	
DW	1.7549	
SSR	.00567737	

Thus, the final production function is of the form:

$$Q=\text{Min}\left[\frac{F(K,L)}{a}, \frac{M}{b}\right]$$

If both constraints are binding, then we have:

$$Q_t = \frac{F(L_t, K_t)}{a} = \frac{M_t}{b}$$

where:

$$Q_t = .794 \frac{L_t^{.5929} K_t^{.4070} e^{.9965 D_t} Q_{t-1}^{.5448}}{L_{t-1}^{.323} K_{t-1}^{.222} e^{.5429 D_{t-1}}}$$

and

$$Q_t = \frac{M_t}{b}$$

Finally, if we go back to the original units of the variables we have:

$$Q_t = 1.234 \frac{L_t^{.5929} K_t^{.4070} e^{.9965 D_t} Q_{t-1}^{.5448}}{L_{t-1}^{.323} K_{t-1}^{.222} e^{.5429 D_{t-1}}}$$

$$\underset{(29.282)}{M_t = .183\ Q_t}$$

CHAPTER IV

A ONE OUTPUT BELL CANADA MODEL: Simultaneous Estimation of Production Function and Side Conditions.

4.1 Introduction

The purpose of this chapter is to develop and estimate a one output model of Bell Canada. The model is then used for forecasting and policy simulations.

As we saw in chapter two, the demand for local services is price inelastic. Thus, the marginal revenue from local services is negative. This important feature of the operation of Bell Canada has to be incorporated into a model of the carrier. In the development of the one output model, we have a composite output which is a quantity index of local and non-local services. Furthermore, the quantity of local services provided by Bell is considered as exogenous. That is, firm's decisions about changes in total output are carried out only through changes in non-local services (i.e. toll and other services).

The one output characteristic of the model is given by the specification of technology where labor, raw materials and capital inputs are combined through a translog production function to produce a composite commodity.

The second main characteristic of the model is that the production function and the side conditions - for labor hiring and raw materials use - derived from profit maximization, are jointly estimated. Thus, the first order conditions for labor and raw materials are now estimated relations.

4.2 THE MODEL

The model that we use is of the Averch-Johnson type. The firm is supposed to maximize profits subject to a rate of return constraint. The firm produces a composite output (Q) which is the sum of local services in constant dollars (Q_L) and a divisia index of non-local services (Q_{NL}). Output is produced with: labor (x_1), raw materials (x_2) and capital (x_3). Initially, we assume that the firm hire factors at fixed prices. Thus, our model can be formulated in the following way:

(1) Max Profits: $P_L\, Q_L + P_{NL}\, Q_{NL} - r_1\, x_1 - r_2\, x_2 - r_3\, x_3$

subject to a technology constraint:

(2) $F[(Q_L + Q_{NL}),\ x_1,\ x_2,\ x_3] = 0$

and to a regulatory constraint:

(3) $P_L Q_L + P_{NL}\, Q_{NL} - r_1\, x_1 - r_2\, x_2 = s x_3$

where

P_L = Price Index of local services.

P_{NL} = Divisia Price Index of non-local services.

Q_L = Quantity of local services

Q_{NL} = Divisia Quantity index of non-local services

r_1 = Price Index of labor services

r_2 = Divisia Price index of raw materials

r_3 = Price Index of capital services

x_1 = Quantity of labor

x_2 = Divisia Quantity Index of raw materials

x_3 = Quantity of capital

s = "allowed price of capital services"

We assume that the firm chooses Q_{NL}, x_1, x_2, and x_3 to maximize the level of profit. On the other hand Q_L is exogenous. The lagrangian for this problem can be written as:

$$\Omega = P_L Q_L + P_{NL} Q_{NL} - r_1 x_1 - r_2 x_2 - r_3 x_3$$

$$-\lambda \, [P_L Q_L + P_{NL} Q_{NL} - r_1 x_1 - r_2 x_2 - s x_3]$$

$$- \mu \, [F \, [Q_L + Q_{NL}] \, x_1, x_2, x_3 \,]$$

The first order conditions for this problem are given by:

$$(1) \quad \frac{\partial \Omega}{\partial Q_{NL}} = P_{NL} \left[1 + \frac{1}{\eta_{NL}} \right] (1-\lambda) - \mu \frac{\partial F}{\partial (Q_L + Q_{NL})} = 0$$

$$(2) \quad \frac{\partial \Omega}{\partial x_1} = -(1-\lambda) \, r_1 - \mu \frac{\partial F}{\partial x_1} = 0$$

$$(3) \quad \frac{\partial \Omega}{\partial x_2} = -(1-\lambda) \, r_2 - \mu \frac{\partial F}{\partial x_2} = 0$$

$$(4) \quad \frac{\partial \Omega}{\partial x_3} = - r_3 + \lambda s - \mu \frac{\partial F}{\partial x_3} = 0$$

$$(5) \quad P_L Q_L + P_{NL} Q_{NL} = r_1 x_1 + r_2 x_2 + s x_3$$

$$(6) \quad F \, [(Q_L + Q_{NL}, x_1, x_2, x_3] = 0$$

Where λ and μ are lagrangian multipliers and η_{NL} is the price elasticity of demand for non-local services.

Adding to these first order conditions the demand equation for non-local services we obtain a system of seven equations in seven unknowns: Q_{NL}, P_{NL}, x_1, x_2, x_3, λ and μ.

We can get rid of μ by working with equation (1) to (4): Thus,

$$(7) \quad \frac{\partial \, (Q_1 + Q_{n1})}{\partial \, x_1} = - \frac{\dfrac{\partial F}{\partial x_1}}{\dfrac{\partial F}{\partial (Q_L + Q_{NL})}} = \frac{r_1}{P_{NL} \left(1 + \dfrac{1}{\eta_{NL}}\right)}$$

$$(8)\quad \frac{\partial\,(Q_L+Q_{NL})}{\partial x_1} = -\frac{\dfrac{\partial F}{\partial x_2}}{\dfrac{\partial F}{\partial\,(Q_L+Q_{NL})}} = \frac{r_2}{P_{NL}\,[1+\dfrac{1}{\eta_{NL}}]}$$

$$(9)\quad \frac{\partial\,(Q_L+Q_{NL})}{\partial x_3} = -\frac{\dfrac{\partial F}{\partial x_3}}{\dfrac{\partial F}{\partial\,(Q_L+Q_{NL})}} = \frac{r_3-\lambda s}{(1-\lambda)\,P_{NL}\,[1+\dfrac{1}{\eta_{NL}}]}$$

The system of equations (5), (6), (7), (8), (9) and the demand equation for non-local services conform now a system of six equations in six unknowns: Q_{NL}, P_{NL}, x_1, x_2, x_3 and λ.

We saw in Chapter III that the translog production function for Bell Canada reduces to a Constant Returns to Scale two input Cobb-Douglas for labor and capital and to a Leontief for raw materials.

In this chapter, as a validation of our model, we will estimate simultaneously equations (6) and (7). With equation (6) specified as a CRTS Cobb-Douglas function on x_1 and x_3. If our model is an appropriate description of the decision making process of Bell, then the direct estimation of the production function and the simultaneous estimation of the production function and side condition for labor hiring should give similar results.

The two equations that we estimate are:

$$(10)\quad \ln Q_t = \alpha_0 + \alpha_1 \ln L_t + (1-\alpha_1)\ \ln K_t + \beta\ D_t$$

and

$$(11)\quad \frac{r_{1t} L_t}{P_{T,t}\ Q_t} = \alpha_1\ [1 + \frac{1}{\eta_{NL}}]\quad \frac{P_{NL,t}}{P_{T,t}}.$$

where

$Q = Q_L + Q_{NL}$

$L = x_1$

$K = x_3$

D_t = Percentage of Calls Direct Distance Dialed.

$P_{T,t}$ = Divisia Price Index of total output.

In order to estimate equations (10) and (11), we need a value for the price elasticity of the aggregation of non-local services (η_{NL}). For this purpose, we use the results obtained in the demand chapter for the price elasticity of toll services (η_{toll}= -1.220) and the price elasticity of other services (η_{other}= -1.418), and we weighted them by the means of the revenues from each service, to obtain the composite elasticity for non-local services (η_{NL}). This gives a result of η_{NL} = -1.2728

Using the value for η_{NL} just computed, we obtained the following results when we estimated simultaneously equations (10) and (11):

TABLE 5: One Output Cobb-Douglas Production Function Estimated jointly with Side Condition for Labour Hiring

	Estimated Coefficient	t-statistic
α_0	.4647	4.768
α_1	.5938	19.081
α_3	.4062	13.055
β	.9967	25.579
ρ	.5489	5.112
ρ_{sc}	.9717	144.609

where

ρ: Autocorrelation coefficient of the production function (equation 10)

ρ_{sc}: Autocorrelation coefficient of the side condition (equation 11)

Thus, from the joint estimation of the production function and the side condition for labor hiring, we obtain estimates of the coefficients of the production function very similar to the ones obtained from the estimation of the production function alone. These results provide a strong validation of the appropriatness of a model of profit maximization to describe the behavior of Bell Canada. We should also mention that equations (10) and (11) also apply to a model of profit maximization without rate of return regulation.

A SIMULATION MODEL FOR A TELEPHONE COMPANY.